TOPOLOGICAL GEOMETRODYNAMICS: AN OVERVIEW: PART II

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0.1 PREFACE

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. The approximate localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family and Kalevi and Ritva Tikkanen and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 45 lonely years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During the last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss my work. Pertti Kärkkäinen is my old physicist friend and has provided continued economic support for a long time. I have also had stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Tommi Ullgren has provided both economic support and encouragement during years. Pekka Rapinoja has offered his help in this respect and I am especially grateful to him for my Python skills.

During the last five years I have had inspiring discussions with many people in Finland interested in TGD. We have had video discussions with Sini Kunnas and had podcast discussions with Marko Manninen related to the TGD based view of physics and consciousness. Marko has also helped in the practical issues related to computers and quite recently he has done a lot of testing of chatGPT helping me to get an overall view of what it is. The discussions in a Zoom group involving Marko Manninen, Tuomas Sorakivi and Rode Majakka have given me the valuable opportunity to clarify my thoughts.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation in CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. I am grateful to Mark McWilliams, Paul Kirsch, Gary Ehlenberg, and Ulla Matfolk and many others for providing links to possibly interesting websites and articles. We have collaborated with Peter Gariaev and Reza Rastmanesh. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the public through the iron wall of academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as an individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without direct support from power holders- even in archives like arXiv.org.

Situation changed as Andrew Adamatsky proposed the writing of a book about TGD when I had already gotten used to the thought that my work would not be published during my lifetime. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loophole. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy.

And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his 73th birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During the period when the right wing held political power this trend was steadily strengthening and the situation is the same as I am writing this. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

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Matti Pitkänen

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Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K5].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A57]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of

electromagnetic fields are nonvanishing. The correlations functions for weak fields are non-vanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A29] [B25, B17, B18]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. “Topological” in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B16]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A48, A56, A39, A53].

The identification of the space-time as a sub-manifold [A49, A70] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

[1] There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.1.5 Construction of scattering amplitudes

Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A62, A77, A87]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K67]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their M^8-H duals in M_c^8 are defined for both reals and various p-adic number fields. This is true if they are defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p=3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M_c^8$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K63].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \to f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M_c^8

$Y^4 \subset M_c^8$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M_c^4 point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M_c^4 \subset M_c^8$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} . \qquad (1.1.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \to Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \to 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M_c^8$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M_c^4 \to SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to M^8-H duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.
2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.
3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as M^8-H duality requires and deduce an explicit formula for the point. This means a realization of the dream.
4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the M^8-H image of Y^4 satisfies the generalized holomorphy.
5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same.

The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of M^8-H duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4\times CP_2$?

1. The selection of $SU(3)\subset G_2$ for ordinary M^8-H duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3\subset M^8_c$ and its image $H^3\subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3)\subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of M^8-H duality as something more fundamental than the ordinary M^8-H duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3\times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1)\times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1\oplus 3\oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3\oplus\overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8\oplus 3\oplus\overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3)\times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x)\times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that

Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E18] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K89].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K80, K81, K79]) support the view that dark matter might be a key player in living matter.

Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K100]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A57]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   $M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L63].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L52]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t-m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

1.2 Bird's Eye of View about the Topics of the Book

This book tries to give an overall view about quantum TGD as it stands now. The topics of this book are following.

1. In the first part of the book I will try to give an overall view about the evolution of TGD and about quantum TGD in its recent form. I cannot avoid the use of various concepts without detailed definitions and my hope is that reader only gets a bird's eye of view about TGD. Two visions about physics are discussed. According to the first vision physical states of the Universe correspond to classical spinor fields in the world of the classical worlds identified as 3-surfaces or equivalently as corresponding 4-surfaces analogous to Bohr orbits and identified as special extrema of Kähler action. TGD as a generalized number theory vision leading naturally also to the emergence of p-adic physics as physics of cognitive representations is the second vision.

2. The second part of the book is devoted to the vision about physics as infinite-dimensional configuration space geometry. The basic idea is that classical spinor fields in infinite-dimensional "world of classical worlds", space of 3-surfaces in $M^4 \times CP_2$ describe the quantum states of the Universe. Quantum jump remains the only purely quantal aspect of quantum theory in this approach since there is no quantization at the level of the configuration space. Space-time surfaces correspond to special extremals of the Kähler action analogous to Bohr orbits and define what might be called classical TGD discussed in the first chapter. The construction of the configuration space geometry and spinor structure are discussed in remaining chapters.

3. The third part of the book describes physics as generalized number theory vision. Number theoretical vision involves three loosely related approaches: fusion of real and various p-adic physics to a larger whole as algebraic continuations of what might be called rational physics; space-time as a hyper-quaternionic surface of hyper-octonion space, and space-time surfaces as a representations of infinite primes.

4. The first chapter in the third part of the book summarizes the basic ideas related to Neumann algebras known as hyper-finite factors of type II_1 about which configuration space Clifford algebra represents canonical example.

Second chapter is devoted to the basic ideas related to the hierarchy of Planck constants and related generalization of the notion of embedding space to a book like structure.

$M^8 - H$ duality:

5. The physical applications of TGD are the topic of the second part of the book. The cosmological and astrophysical applications of the many-sheeted space-time are summarized and the applications to elementary particle physics are discussed at the general level. TGD explains particle families in terms of generation genus correspondences (particle families correspond to 2-dimensional topologies labelled by genus). The notion of elementary particle vacuum functional is developed leading to an argument that the number of light particle families is three is discussed. The general theory for particle massivation based on p-adic thermodynamics is discussed at the general level. The detailed calculations of elementary particle masses are not however carried out in this book.

1.2.1 Organization of "TGD: an Overview: Part II"

The book consists of 2 parts.

1. The first chapter in the 1st part of the book summarizes the basic ideas related to von Neumann algebras known as hyper-finite factors of type II_1 about which configuration space Clifford algebra represents a canonical example.

   Second chapter is devoted to the basic ideas related to the hierarchy of Planck constants and related generalization of the notion of imbedding space to a book like structure. Third chapter is about $M^8 - H$ duality.

2. The physical applications of TGD are the topic of the 2nd part of the book. The cosmological and astrophysical applications of the many-sheeted space-time are summarized and the applications to elementary particle physics are discussed at the general level. TGD explains particle families in terms of generation genus correspondences (particle families correspond to 2-dimensional topologies labelled by genus). The notion of elementary particle vacuum functional is developed leading to an argument that the number of light particle families is three is discussed. The general theory for particle massivation based on p-adic thermodynamics is discussed at the general level. The detailed calculations of elementary particle masses are not however carried out in this book.

1.3 Sources

The eight online books about TGD [K109, K101, K84, K73, K26, K68, K49, K92] and nine online books about TGD inspired theory of consciousness and quantum biology [K98, K20, K78, K19, K46, K59, K61, K91, K97] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 PART I: HYPERFINITE FACTORS OF TYPE II_1, HIERARCHY OF PLANCK CONSTANTS, AND $M^8 - H$ DUALITY

Evolution of Ideas about Hyper-finite Factors in TGD

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors, could provide the mathematics needed to develop a more explicit view about the construction of M-matrix generalizing the notion of S-matrix in zero energy ontology (ZEO). In this chapter I will discuss various aspects of hyper-finite factors and their possible physical interpretation in TGD framework.

1. Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. Therefore also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.
2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.
3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.
4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. One can start from a local octonionic Clifford algebra in M^8. Associativity (co-associativity) condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the induced gamma matrices associated with the Kähler action span a complex quaternionic (complex co-quaternionic) sub-space at each point of the sub-manifold. This associative (co-associative) sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative (co-associative(algebra and thus to HFF of type II_1.

2. Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be

used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology (ZEO): the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions meaning the analog of state function collapse in zero modes fixing the classical conserved charges equal to the quantal counterparts. Classical charges would be parameters characterizing zero modes.

A concrete construction of M-matrix motivated the recent rather precise view about basic variational principles is proposed. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.

3. Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In ZEO $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$

interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

4. Analogs of quantum matrix groups from finite measurement resolution?

The notion of quantum group replaces ordinary matrices with matrices with non-commutative elements. In TGD framework I have proposed that the notion should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution.

In this article I will consider the notion of quantum matrix inspired by recent view about quantum TGD and it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution. The basic idea is to replace complex matrix elements with operators expressible as products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers.

The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. The weak definition of determinant meaning its development with respect to a fixed row or column does not pose additional conditions. Strong definition of determinant requires its invariance under permutations of rows and columns. The permutation of rows/columns turns out to have interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

The resulting quantum matrices define a more general structure than quantum group but provide a concrete representation and interpretation for quantum group in terms of finite measurement resolution if q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by sign factor invariant under the permutations of both rows and columns. One could also understand the fractal structure of inclusion sequences of hyper-finite factors resulting by recursively replacing operators appearing as matrix elements with quantum matrices.

5. Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

Does TGD predict spectrum of Planck constants?

The quantization of Planck constant has been the basic theme of TGD since 2005. The basic idea was stimulated by the suggestion of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, where the velocity parameter v_0 has the approximate value $v_0 \simeq 2^{-11}$ for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The recent version of the chapter represents the evolution of ideas about quantization of Planck constants from a perspective given by seven years's work with the idea. A very concise summary about the situation is as follows.

1. Basic physical ideas

The basic phenomenological rules are simple.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value

of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Effective embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies.

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. The interpretation of $\hbar_{gr}$ introduced by Nottale in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in astronomical scales. The gravitational Compton length $GM/v_0 = r_S/2v_0$ does not depend on m so that all particles around say Sun say same gravitational Compton length.

   By the independence of gravitational acceleration and gravitational Compton length on particle mass, it is enough to assume that only microscopic particles couple to the dark gravitons propagating along flux tubes mediating gravitational interaction. Therefore $h_{gr} = h_{eff}$ could be true in microscopic scales and would predict that cyclotron energies have no dependence on the mass of the charged particle meaning that the spectrum ordinary photons resulting in the transformation of dark photons to ordinary photons is universal. An attractive identification of these photons would be as bio-photons with energies in visible and UV range and thus inducing molecular transitions making control of biochemistry by dark photons. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation. The energy of the graviton is gigantic unless the emission is assume to take place from a microscopic systems with large but not gigantic h_{gr}.

3. Why Nature would like to have large - maybe even gigantic - value of effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

2. Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of embedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology (ZEO), it became clear that the notion of singular covering space of the embedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. In ZEO 3-surfaces are unions of space-like 3-surface at opposite boundaries of CD. The non-determinism of Kähler action due to the huge vacuum degeneracy would naturally explain the existence of several space-time sheets connecting the two 3-surfaces at the opposite boundaries of CD. Quantum criticality suggests strongly conformal invariance and the identification of n as the number of conformal equivalence classes of these space-time sheets. Also a connection with the notion of negentropic entanglement emerges.

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part I

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces.

In the sequel I shall consider the following topics.

1. I will discuss basic notions of algebraic geometry such as algebraic variety, surface, and curve, all rational point of variety central for TGD view about cognitive representation, elliptic curves and surfaces, and rational and potentially rational varieties. Also the notion of Zariski topology and Kodaira dimension are discussed briefly. I am not a mathematician and what hopefully saves me from horrible blunders is physical intuition developed during 4 decades of TGD.

2. It will be shown how $M^8 - H$ duality could reduce TGD at fundamental level to octonionic algebraic geometry. Space-time surfaces in M^8 would be algebraic surfaces identified as zero loci for imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial of complexified octonionic variable o_c decomposing as $o_c = q_c^1 + q_c^2 I^4$ and projected to a Minkowskian subspace M^8 of complexified O. Single real valued polynomial of real variable with algebraic coefficients would determine space-time surface! As proposed already earlier, spacetime surfaces would form commutative and associative algebra with addition, product and functional composition.

   One can interpret the products of polynomials as correlates for free many-particle states with interactions described by added interaction polynomial, which can vanish at boundaries of CDs thanks to the vanishing in Minkowski signature of the complexified norm $q_c\overline{q_c}$ appearing in $RE(P)$ or $IM(P)$ caused by the quaternionic non-commutativity. This leads to the same picture as the view about preferred extremals reducing to minimal surfaces near boundaries of CD. Also zero zero energy ontology (ZEO) could emerge naturally from the failure of number field property for for quaternions at light-cone boundaries.

The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients can give rise to associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

   The hierarchy of notions involved is well-ordering for 1-D structures, commutativity for complex numbers, and associativity for quaternions. This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy Riemann conditions are linear and constant value manifolds are 1-D and thus well-ordered.

Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

In fact, all algebras obtained by Cayley-Dickson construction adding imaginary units to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and $M^8 - H$ correspondence could generalize.

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k, $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i. In the generic case this gives 3-D surface.

   In this generic case $M^8 - H$ duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H, and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

   One can also have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i. Space-time surface would be analogous to a polynomial with a multiple root. The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

   One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces except at 3-D boundary surfaces. These regions correspond to interaction regions in which Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes to the level of complex/co-complex surfaces assigned with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces?

   I have proposed commutativity or co-commutatitivity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commutative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part II

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to octonionic algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces.

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4. The super variant of the octonionic geometry relying on octonionic triality makes sense and the geometry of the space-time variety correlates with fermion and antifermion numbers assigned with it. This new view about super-geometry involving also automatic SUSY breaking at the level of space-time geometry.

Also a sketchy proposal for the description of interactions is discussed.

1. The surprise that $RE(P) = 0$ and $IM(P) = 0$ conditions have as singular solutions light-cone interior and its complement and 6-spheres $S^6(t_n)$ with radii t_n given by the roots of the real $P(t)$, whose octonionic extension defines the space-time variety X^4. The intersections $X^2 = X^4 \cap S^6(t_n)$ are tentatively identified as partonic 2-varieties defining topological interaction vertices.

   The idea about the reduction of zero energy states to discrete cognitive representations suggests that interaction vertices at partonic varieties X^2 are associated with the discrete set of intersection points of the sparticle lines at light-like orbits of partonic 2-surfaces belonging to extension of rationals.

2. CDs and therefore also ZEO emerge naturally. For CDs with different origins the products of polynomials fail to commute and associate unless the CDs have tips along real (time) axis. The first option is that all CDs under observation satisfy this condition. Second option allows general CDs.

   The proposal is that the product $\prod P_i$ of polynomials associated with CDs with tips along real axis the condition $IM(\prod P_i) = 0$ reduces to $IM(P_i) = 0$ and criticality conditions guaranteeing associativity and provides a description of the external particles. Inside these CDs $RE(\prod P_i) = 0$ does not reduce to $RE(\prod P_i) = 0$, which automatically gives rise to geometric interactions. For general CDs the situation is more complex.

3. The possibility of super octonionic geometry raises the hope that the twistorial construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY generalizes to TGD in rather straightforward manner to a purely geometric construction. Functional integral over WCW would reduce to summations over polynomials with coefficients in extension of rationals and criticality conditions on the coefficients could make the summation well-defined by bringing in finite measurement resolution.

Scattering diagrams would be determined by points of space-time variety, which are in extension of rationals. In adelic physics the interpretation is as cognitive representations.

1. Cognitive representations are identified as sets of rational points for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8- and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces. For the earlier work related to adelic TGD and cognitive representations see [?]

2. Some aspects related to homology charge (Kähler magnetic charge) and genus-generation correspondence are discussed. Both topological quantum numbers are central in the proposed model of elementary particles and it is interesting to see whether the picture is internally consistent and how algebraic variety property affects the situation. Also possible problems related to $h_{eff}/h = n$ hierarchy []adelicphysics realized in terms of n-fold coverings of space-time surfaces are discussed from this perspective.

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III

Cognitive representations are the basic topic of the third chapter related to $M^8 - H$ duality. Cognitive representations are identified as sets of points in extension of rationals for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8- and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces.

The notion is applied in various cases and the connection with $M^8 - H$ duality is rather loose.

1. Extensions of rationals are essentially coders of information. There the possible analogy of extensions of rationals with genes deserves discussion. Extensions, which are not extensions of extensions would be analogous to genes. The notion of conserved gene as number theoretical analogy for Galois extensions as the Galois group of extension which is normal subgroup of Galois extension.
2. The possible physical meaning of the notion of perfectoid introduced by Peter Scholze is discussed in the framework of p-adic physics. Extensions of p-adic numbers involving roots of the prime defining the extension are involved and are not considered previously in TGD framework. There there possible physical meaning deserves discussion.
3. The construction of cognitive representation reduces to a well-known mathematical problem of finding the points of space-time surface with embedding space coordinates in given extension of rationals. The work of Kim and Coates represents new ideas in this respect and there is a natural connection with TGD.
4. One expects that large cognitive representations are winners in the number theoretical fight for survival. Strong form of holography suggests that it is enough to consider cognitive representations restricted to string world sheets and partonic 2-surfaces. If the 2-surface possesses large group of symmetries acting in extension of rationals, one can have large cognitive representations as orbit of point in extension. Examples of highly symmetric 2-D surfaces are geodesic spheres assignable to partonic 2-surfaces and cosmic strings and elliptic curves assignable with string world sheets and cosmic strings.
5. Rationals and their extensions give rise to a unique discretizations of space-time surface (for instance) - cognitive representation - having interpretation in terms of finite measurement resolution. There are however many open questions. Should one allow only octonionic polynomials defined as algebraic continuations of real polynomials or should one allow also analytic functions and regard polynomials as approximations. Zeta functions are especially interesting analytic functions and Defekind zetas characterize extensions of rationals and one can pose physically motivated questions about them.

Could quantum randomness have something to do with classical chaos?

Tim Palmer has proposed that classical chaos and quantum randomness might be related. It came as a surprise to me that these to notions could a have deep relationship in TGD framework.

1. Strong form of Palmer's idea stating that quantum randomness reduces to classical chaos certainly fails but one can consider weaker forms of the idea. Even these variants fail in Copenhagen interpretation since strictly speaking there is no classical reality, only wave function coding for the knowledge about the system. Bohr orbits should be more than approximation and in TGD framework space-time surface as preferred extremal of action is analogous to Bohr orbit and classical physics defined by Bohr orbits is an exact part of quantum theory.
2. In the zero energy ontology (ZEO) of TGD the idea works in weaker form and has very strong implications for the more detailed understanding of ZEO and $M^8 - M^4 \times CP_2$ duality. Ordinary ("big") state functions (BSFRs) meaning the death of the system in a universal sense and re-incarnation with opposite arrow of time would involve quantum criticality accompanied by classical chaos assignable to the correspondence between geometric time and subjective time identified as sequence of "small" state function reductions (SSFRs) as analogs

of weak measurements. The findings of Minev et al give strong support for this view and Libet's findings about active aspects of consciousness can be understood if the act of free will corresponds to BSFR.

M^8 picture identifies 4-D space-time surfaces X^4 as roots for "imaginary" or "real" part of octonionic polynomial P_2P_1 obtained as a continuation of real polynomial $P_2(L-r)P_1(r)$, whose arguments have origin at the the tips of B and A and roots a the light-cone boundaries associated with tips. Causal diamond (CD) is identified intersection of future and past directed light-cones light-cones A and B. In the sequences of SSFRs $P_2(L-r)$ assigned to B varies and $P_1(r)$ assigned to A is unaffected. L defines the size of CD as distance $\tau = 2L$ between its tips.

Besides 4-D space-time surfaces there are also brane-like 6-surfaces corresponding to roots $r_{i,k}$ of $P_i(r)$ and defining "special moments in the life of self" having $t_i = r_{i,k}$ ball as M^4_+ projection. The number of roots and their density increases rapidly in the sequence of SSFRs. The condition that the largest root belongs to CD gives a lower bound to it size L as largest root. Note that L increases.

Concerning the approach to chaos, one can consider three options.

Option I: The sequence of steps consisting of unitary evolutions followed by SSFR corresponds to a functional factorization at the level of polynomials as sequence $P_2 = Q_1 \circ Q_2 \circ ...Q_n$. If the size of CD is assumed to increase, also the tip of active boundary of CD must shift so that the argument of P_2 $r - L$ is replaced in each iteration step to with updated argument with larger value of L.

Option II: A completely unexpected connection with the iteration of analytic functions and Julia sets, which are fractals assigned also with chaos interpreted as complexity emerges. In a reasonable approximation quantum time evolution by SSFRs could be induced by an iteration of a polynomial or even an analytic function: $P_2 = P_2 \to P_2^{\circ 2} \to ....$ For $P_2(0) = 0$ the roots of the iterate consists of inverse images of roots of P_2 by $P_2^{\circ -k}$ for $k = 0, ..., N-1$.

Suppose that M^8 and X^4 are complexified and thus also $t = r$ and "real" X^4 is the projection of X^4_c to real M^8. Complexify also the coefficients of polynomials P. If so, the Mandelbrot and Julia sets (`http://tinyurl.com/cplj9pe` and `http://tinyurl.com/cvmr83g`) characterizing fractals would have a physical interpretation in ZEO.

One approaches chaos in the sense that the $N-1$:th inverse images of the roots of P_2 belonging to filled Julia set approach to points of Julia set of P_2 as the number N of iterations increases. Minimal L would increase with N if CD is assumed to contain all roots. The density of the roots in Julia set increases near L since the size of CD is bounded by the size Julia set. One could perhaps say that near the $t = L$ in the middle of CD the life of self when the size of CD has become almost stationary, is the most intensive.

Option III: A conservative option is to consider also real polynomials $P_2(r)$ with real argument r. Only non-negative real roots r_n are of interest whereas in the general case one considers all values of r. For a large N the new roots with possibly one exception would approach to the real Julia set obtained as a real projection of Julia set for complex iteration.

How the size L of CD is determined and when can BSFR occur?

Option I: If L is minimal and thus given by the largest (non-exceptional) root of iterate of P_2 in Julia set, it is bound to increase in the iteration (this option is perhaps too deterministic). L should smaller than the sizes of Julia sets of both$A$ and B since the iteration gives no roots outside Julia sets.

Could BSFR become probable when L as the largest allowed root for iterate P_2 is larger than the size of Julia set of A? There would be no more new "special moments in the life of self" and this would make death (in universal sense) and re-incarnation with opposite arrow of time probable. The size of CD could decrease dramatically in the first iteration for P_1 if it is determined as the largest allowed root of P_1: the re-incarnated self would have childhood.

Option II: The size of CD could be determined in SSFR statistically as an allowed root of P_2. Since the density of roots increases, one would have a lot of choices and quantum criticality and fluctuations of the order of clock time $\tau = 2L$: the order of subjective time would not anymore correspond to that for clock time. BSFR would occur for the same reason as for the first option.

TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors

In this chapter 4 topics are discussed. McKay correspondence, SUSY, and twistors are discussed from TGD point of view, and new aspects of $M^8 - H$ duality are considered.

1. McKay correspondence in TGD framework

There are two mysterious looking correspondences involving ADE groups. McKay correspondence between McKay graphs characterizing tensor products for finite subgroups of $SU(2)$ and Dynkin diagrams of affine ADE groups is the first one. The correspondence between principal diagrams characterizing inclusions of hyper-finite factors of type II_1 (HFFs) with Dynkin diagrams for a subset of ADE groups and Dynkin diagrams for affine ADE groups is the second one.

These correspondences are discussed from number theoretic point of view suggested by TGD and based on the interpretation of discrete subgroups of $SU(2)$ as subgroups of the covering group of quaternionic automorphisms $SO(3)$ (analog of Galois group) and generalization of these groups to semi-direct products $Gal(K) \triangleleft SU(2)_K$ of Galois group for extension K of rationals with the discrete subgroup $SU(2)_K$ of $SU(2)$ with representation matrix elements in K. The identification of the inclusion hierarchy of HFFs with the hierarchy of extensions of rationals and their Galois groups is proposed.

A further mystery whether $Gal(K) \triangleleft SU(2)_K$ could give rise to quantum groups or affine algebras. In TGD framework the infinite-D group of isometries of "world of classical worlds" (WCW) is identified as an infinite-D symplectic group for which the discrete subgroups characterized by K have infinite-D representations so that hyper-finite factors are natural for their representations. Symplectic algebra SSA allows hierarchy of isomorphic sub-algebras SSA_n. The gauge conditions for SSA_n and $[SSA_n, SSA]$ would define measurement resolution giving rise to hierarchies of inclusions and ADE type Kac-Moody type algebras or quantum algebras representing symmetries modulo measurement resolution.

A concrete realization of ADE type Kac-Moody algebras is proposed. It relies on the group algebra of $Gal(K) \triangleleft SU(2)_K$ and free field representation of ADE type Kac-Moody algebra identifying the free scalar fields in Kac-Moody Cartan algebra as group algebra elements defined by the traces of representation matrices (characters).

2. New aspects of $M^8 - H$ duality

$M^8 - H$ duality is now a central part of TGD and leads to new findings. $M^8 - H$ duality can be formulated both at the level of space-time surfaces and light-like 8-momenta. Since the choice of M^4 in the decomposition of momentum space $M^8 = M^4 \times E^4$ is rather free, it is always possible to find a choice for which light-like 8-momentum reduces to light-like 4-momentum in M^4 - the notion of 4-D mass is relative. This leads to what might be called $SO(4) - SU(3)$ duality corresponding to the hadronic and partonic views about hadron physics. Particles, which are eigenstates of mass squared are massless in $M^4 \times CP_2$ picture and massive in M^8 picture. The massivation in this picture is a universal mechanism having nothing to do with dynamics and results in zero energy ontology automatically if the zero energy states are superpositions of states with different masses. p-Adic thermodynamics describes this massivation. Also a proposal for the realization of ADE hierarchy emerges.

4-D space-time surfaces correspond to roots of octonionic polynomials $P(o)$ with real coefficients corresponding to the vanishing of the real or imaginary part of $P(o)$. These polynomials however allow universal roots, which are not 4-D but analogs of 6-D branes and having topology of S^6. Their M^4 projections are time =constant snapshots $t = r_n, r_M \leq r_n$ 3-balls of M^4 light-cone (r_n is root of $P(x)$). At each point the ball there is a sphere S^3 shrinking to a point about boundaries of the 3-ball. These special values of M^4 time lead to a deeper understanding of ZEO based quantum measurement theory and consciousness theory.

3. Is the identification of twistor space of M^4 really correct?

The critical questions concerning the identification of twistor space of M^4 as $M^4 \times S^2$ led to consider a more conservative identification as CP_3 with hyperbolic signature (3,-3) and replacement of H with $H = cd_{conf} \times CP_2$, where cd_{conf} is CP_2 with hyperbolic signature (1,-3). This approach reproduces the nice results of the earlier picture but means that the hierarchy of

CDs in M^8 is mapped to a hierarchy of spaces cd_{conf} with sizes of CDs. This conforms with Poincare symmetry from which everything started since Poincare group acts in the moduli space of octonionic structures of M^8. Note that also the original form of $M^8 - H$ duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$.

The outcome of octo-twistor approach applied at level of M^8 together with modified M^8-H duality leads to a nice picture view about twistorial description of massive states based on quaternionic generalization of twistor (super-)Grassmannian approach. A radically new view is that descriptions in terms of massive and massless states are alternative options, and correspond to two different alternative twistorial descriptions and leads to the interpretation of p-adic thermodynamics as completely universal massivation mechanism having nothing to do with dynamics. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory relying on the universal roots of octonionic polynomials of M^8, which are not 4-D but analogs of 6-D branes. This part of article is not a mere side track since by $M^8 - H$ duality the finite sub-groups of $SU(2)$ of McKay correspondence appear quite concretely in the description of the measurement resolution of 8-momentum.

The Recent View about SUSY in TGD Universe

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge would appear as space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with the precise understanding of quantum criticality and understand the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials. The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is indeed bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$,

$D_{\alpha,s}\Gamma_s^{\alpha} = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

Super-Dirac action vanishes on-mass-shell. The proposed construction relying on ZEO allows however to get scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation is however needed and makes possible to express the derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution conforms with the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory

The understanding of the unitarity of the S-matrix has remained a major challenge of Topological Geometrodynamics (TGD) for 4 decades. It has become clear that some basic principle is still lacking. Assigning S-matrix to a unitary evolution works in non-relativistic theory but fails already in the generic quantum field theory (QFT). The solution of the problem turned out to be extremely simple. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space would replace unitary S-matrix.

An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of Teichmueller matrix. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong from of holography (SH), the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmueller condition guaranteeing non-negative probabilities.

Equivalence Principle generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

Breakthrough in understanding of $M^8 - H$ duality

A critical re-examination of $M^8 - H$ duality is discussed. $M^8 - H$ duality is one of the cornerstones of Topological Geometrodynamics (TGD). The original version of $M^8 - H$ duality assumed that space-time surfaces in M^8 can be identified as associative or co-associative surfaces. If the surface has associative tangent or normal space and contains a complex or co-complex surface, it can be mapped to a 4-surface in $H = M^4 \times CP_2$.

Later emerged the idea that octonionic analyticity realized in terms of real polynomials P algebraically continued to polynomials of complexified octonion could fulfill the dream. The vanishing of the real part $Re_Q(P)$ (imaginary part $Im_Q(P)$) in the quaternionic sense would give rise to an associative (co-associative) space-time surface.

The realization of the general coordinate invariance motivated the notion of strong form of holography (SH) in H allowing realization of a weaker form of $M^8 - H$ duality by assuming that associativity/co-associativity conditions are needed only at 2-D string world sheet and partonic 2-surfaces and possibly also at their light-like 3-orbits.

The outcome of the re-examination yielded both positive and negative surprises.

1. Although no interesting associative space-time surfaces are possible, every distribution of normal associative planes (co-associativity) is integrable.
2. Another positive surprise was that Minkowski signature is the only possible option. Equivalently, the image of M^4 as real co-associative subspace of O_c (complex valued octonion norm squared is real valued for them) by an element of local G_2 or rather, its subgroup $SU(3)$, gives a real co-associative space-time surface.
3. The conjecture based on naive dimensional counting, which was not correct, was that the polynomials P determine these 4-D surfaces as roots of $Re_Q(P)$. The normal spaces of these surfaces possess a fixed 2-D commuting sub-manifold or possibly their distribution allowing the mapping to H by $M^8 - H$ duality as a whole.

   If this conjecture were correct, strong form of holography (SH) would not be needed and would be replaced with extremely powerful number theoretic holography determining space-time surface from its roots and selection of real subspace of O_c characterizing the state of motion of a particle. erate
4. The concrete calculation of the octonion polynomial was the most recent step - carried already earlier [L47, L48, L49] but without realizing the implications of the extremely simple outcome. The imaginary part of the polynomial is proportional to the imaginary part of octonion itself. It turned out that the roots $P = 0$ of the octonion polynomial P are 12-D complex surfaces in O_c rather than being discrete set of points defined as zeros $X = 0, Y = 0$ of two complex functions of 2 complex arguments. The analogs of branes are in question. Already earlier 6-D real branes assignable to the roots of the real polynomial P at the light-like boundary of 8-D light-cone were discovered: also their complex continuations are 12-D [L73, L80].
5. P has quaternionic de-composition $P = Re_Q(P) + I_4 Im_Q(P)$ to real and imaginary parts in a quaternionic sense. The naive expectation was that the condition $X = 0$ implies that the resulting surface is a 4-D complex surface X^4_c with a 4-D real projection X^4_r, which could be co-associative.

   The expectation was wrong! The equations $X = 0$ and $Y = 0$ involve the same(!) complex argument o_c^2 as a complex analog for the Lorentz invariant distance squared from the tip of the light-cone. This implies a cold shower. Without any additional conditions, $X = 0$ conditions have as solutions 7-D complex mass shells H^7_c determined by the roots of P. The explanation comes from the symmetries of the octonionic polynomial.

   There are solutions $X = 0$ and $Y = 0$ only if the two polynomials considered have a common a_c^2 as a root! Also now the solutions are complex mass shells H^7_c.

How could one obtain 4-D surfaces X^4_c as sub-manifolds of H^7_c? One should pose a condition eliminating 4 complex coordinates: after that a projection to M^4 would produce a real 4-surface X^4 .

1. The key observation is that G_2 acts as the automorphism group of octonions respects the co-associativity of the 4-D real sub-basis of octonions. Therefore a local G_2 gauge transformation

applied to a 4-D co-associative sub-space M^4 gives a co-associative four-surface as a real projection. Octonion analyticity would correspond to G_2 gauge transformation: this would realize the original idea about octonion analyticity.

2. A co-associative X^4_c satisfying also the conditions posed by the existence of $M^8 - H$ duality is obtained by acting with a local SU_3 transformation g to a co-associative plane $M^4 \subset M^8_c$. If the image point $g(p)$ is invariant under $U(2)$, the transformation corresponds to a local CP_2 element and the map defines $M^8 - H$ duality even if the co-associativity in geometric sense were not satisfied.

   The co-associativity of the plane M^4 is preserved in the map because G_2 acts as an automorphism group of the octonions. If this map also preserves the value of 4-D complex mass squared, one can require that the intersections of X^4_c with H^7_c correspond to 3-D complex mass shells. One obtains holography with mass shells defined by the roots of P giving boundary data. The condition H images are analogous to Bohr orbits, corresponds to number theoretic holography.

   The group $SU(3)$ has interpretation as a Kac-Moody type analog of color group and the map defining space-time surface. This picture conforms with the H-picture in which gluon gauge potentials are identified as color gauge potentials. Note that at QFT limit the gauge potentials are replaced by their sums over parallel space-time sheets to give gauge fields as the space-time sheets are approximated with a single region of Minkowski space.

3. Octonionic Dirac equation as analog of momentum space variant of ordinary Dirac equation forces the interpretation of M^8 as an analog of momentum space and Uncertainty Principle forces to modify the map $M^4 \subset M^8 \rightarrow M^4 \subset H$ from an identification to an almost inversion. The octonionic Dirac equation reduces to the mass shell condition $m^2 = r_n$, where r_n is a root of the polynomial P defining the 4-surface but only in the co-associative case.

This picture combined with zero energy ontology leads also to a view about quantum TGD at the level of M^8. A local $SU(3)$ element defining 4-surface in M^8, which suggests a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by P. The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

1.4.2 PART II: SOME APPLICATIONS

Cosmology and Astrophysics in Many-Sheeted Space-Time

This chapter is devoted to the applications of TGD to astrophysics and cosmology.

1. Many-sheeted cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the vapor phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

It should be made clear that many-sheeted cosmology involves a vulnerable assumption. It is assumed that single-sheeted space-time surface is enough to model the cosmology. This need not to be the case. GRT limit of TGD is obtained by lumping together the sheets of many-sheeted space-time to a piece of Minkowski space and endowing it with an effective metric, which is sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Hence the proposed models make sense only if GRT limits allowing imbedding as a vacuum extremal of Kähler action have special physical role.

The most important differences are following.

1. Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.

2. TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.
3. The new view about energy means provided by zero energy ontology (ZEO) means that the notion of energy and also other quantum numbers is length scale dependent. This allows to understand the apparent non-conservation of energy in cosmological scales although Poincare invariance is exact symmetry. In ZEO any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.
4. Dark matter hierarchy with dynamical quantized Planck constant implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4. Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.
5. Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations of the microwave background reflect the quantum criticality of the critical period rather than amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

2. Cosmic strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically. Concerning the understanding of cosmic strings a decisive breakthrough came through the identification of gravitational four-momentum as the difference of inertial momenta associated with matter and antimatter and the realization that the net inertial energy of the Universe vanishes. This forced to conclude cosmological constant in TGD Universe is non-vanishing. p-Adic length fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet. The recent value of the cosmological constant comes out correctly. The gravitational energy density described by the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and of magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

3. Dark matter and quantization of gravitational Planck constant

The notion of gravitational Planck constant having possibly gigantic values is perhaps the most radical idea related to the astrophysical applications of TGD. D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis comes from empirical data.

By Equivalence Principle and independence of the gravitational Compton length on particle mass m it is enough to assume g_{gr} only for flux tubes mediating interactions of microscopic objects with central mass M. In TGD framework h_{gr} relates to the hierarchy of Planck constants $h_{eff} = n \times h$ assumed to relate directly to the non-determinism and to the quantum criticality of Kähler action.

Dark matter can be identified as large h_{eff} phases at Kähler magnetic flux tubes and dark energy as the Kähler magnetic energy of these flux tubes carrying monopole magnetic fluxes. No currents are needed to create these magnetic fields, which explains the presence of magnetic fields in cosmological scales.

Overall View About TGD from Particle Physics Perspective

Topological Geometrodynamics is able to make rather precise and often testable predictions. In this and two other articles I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

In the first chapter I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.
- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the embedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces.

  What GRT limit of TGD and Equivalence Principle mean in TGD framework have are problems which found a solution only quite recently (2014). GRT space-time is obtained by lumping together the sheets of many-sheeted space-time to single piece of M^4 provided by an effective metric defined by the sum of Minkowski metric and the deviations of the induced metrics of space-time sheets from Minkowski metric. Same description applies to gauge potentials of gauge theory limit. Equivalence Principle as expressed by Einstein's equations reflects Poincare invariance of TGD.

  Super-conformal symmetries imply generalization of the space-time supersymmetry in TGD framework consistent with the supersymmetries of minimal supersymmetric variant of the standard model. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization of the Yangian symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.
- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electro-weak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realized quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.
- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical constructing involving the concept of Dirac operator. As a matter fact, the construction of TGD involves four Dirac operators.

  1. The Kähler Dirac equation holds true in the interior of space-time surface: the well-definedness of em charge as quantum number of zero modes implies localization of the modes of the induced spinor field to 2-surfaces. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective

metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet.

2. Assuming measurement interaction term for four-momentum, the boundary condition for Kähler-Dirac operator gives essentially massless M^4 Dirac equation in algebraic form coupled to what looks like Higgs term but gives a space-time correlate for the stringy mass formula at stringy curves at the space-like ends of space-time surface.
3. The ground states of the Super-Virasoro representations are constructed in terms of the modes of embedding space spinor fields which are massless in 8-D sense.
4. The fourth Dirac operator is associated with super Virasoro generators and super Virasoro conditions defining Dirac equation in WCW. These conditions characterize zero energy states as modes of WCW spinor fields and code for the generalization of S-matrix to a collection of what I call M-matrices defining the rows of unitary U-matrix defining unitary process.

- Twistor approach has inspired several ideas in quantum TGD during the last years. The basic finding is that M^4 resp. CP_2 is in a well-defined sense the only 4-D manifold with Minkowskian resp. Euclidian signature of metric allowing twistor space with Kähler structure. It seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness: contrary to original expectations the stringy character of the amplitudes seems necessary to guarantee finiteness.

Particle Massivation in TGD Universe

This chapter represents the most recent (2014) view about particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters. In this chapter my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

1. Physical states as representations of super-symplectic and Super Kac-Moody algebras

The basic constraint is that the super-conformal algebra involved must have five tensor factors. The precise identification of the Kac-Moody type algebra has however turned out to be a difficult task. The recent view is as follows. Electroweak algebra $U(2)_{ew} = SU(2)_L \times U(1)$ and symplectic isometries of light-cone boundary ($SU(2)_{rot} \times SU(3)_c$) give 2+2 factors and full supersymplectic algebra involving only covariantly constant right-handed neutrino mode would give 1 factor. This algebra could be associated with the 2-D surfaces X^2 defined by the intersections of light-like 3-surfaces with $\delta M^4_\pm \times CP_2$. These 2-surfaces have interpretation as partons.

For conformal algebra there are several candidates. For symplectic algebra radial light-like coordinate of light-cone boundary replaces complex coordinate. Light-cone boundary $S^2 \times R_+$ allows extended conformal symmetries which can be interpreted as conformal transformations of S^2 depending parametrically on the light-like coordinate of R_+. There is infinite-D subgroup of conformal isometries with S^2 dependent radial scaling compensating for the conformal scaling in S^2. Kähler-Dirac equation allows ordinary conformal symmetry very probably liftable to embedding space. The light-like orbits of partonic 2-surface are expected to allow super-conformal symmetries presumably assignable to quantum criticality and hierarchy of Planck constants. How these conformal symmetries integrate to what is expected to be 4-D analog of 2-D conformal symmetries remains to be understood.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams [**?**] The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces. The preliminary discussion of what is involved can be found in [**?**]

2. Particle massivation

Particle massivation can be regarded as a generation of thermal mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The obvious objection is that Poincare invariance is lost. One could argue that one calculates just the vacuum expectation of conformal weight so that this is not case. If this is not assumed, one would have in positive energy ontology superposition of ordinary quantum states with different four-momenta and breaking of Poincare invariance since eigenstates of four-momentum are not in question. In Zero Energy Ontology this is not the case since all states have vanishing net quantum numbers and one has superposition of time evolutions with well-defined four-momenta. Lorentz invariance with respect to the either boundary of CD is achieved but there is small breaking of Poincare invariance characterized by the inverse of p-adic prime p characterizing the particle. For electron one has $1/p = 1/M_{127} \sim 10^{-38}$.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

1. Instead of energy, the Super Kac-Moody Virasoro (or equivalently super-symplectic) generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics (and also in its real version assuming it exists). The fact that mass squared is thermal expectation of conformal weight guarantees Lorentz invariance. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)). What basically matters is the number of tensor factors involved and five is the favored number.
2. There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.
3. A natural identification of the non-integer contribution to the mass squared is as stringy contribution to the vacuum conformal weight (strings are now "weak strings"). TGD predicts Higgs particle and Higgs is necessary to give longitudinal polarizations for gauge bosons. The notion of Higgs vacuum expectation is replaced by a formal analog of Higgs vacuum expectation giving a space-time correlate for the stringy mass formula in case of fundamental fermions. Also gauge bosons usually regarded as exactly massless particles would naturally receive a small mass from p-adic thermodynamics. The theoretetical motivation for a small mass would be exact Yangian symmetry which broken at the QFT limit of the theory using GRT limit of many-sheeted space-time.
4. Hadron massivation requires the understanding of the CKM mixing of quarks reducing to different topological mixing of U and D type quarks. Number theoretic vision suggests that the mixing matrices are rational or algebraic and this together with other constraints gives strong constraints on both mixing and masses of the mixed quarks.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.
2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than

the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

New Physics Predicted by TGD

TGD predicts a lot of new physics and it is quite possible that this new physics becomes visible at LHC. Although the calculational formalism is still lacking, p-adic length scale hypothesis allows to make precise quantitative predictions for particle masses by using simple scaling arguments.

The basic elements of quantum TGD responsible for new physics are following.

1. The new view about particles relies on their identification as partonic 2-surfaces (plus 4-D tangent space data to be precise). This effective metric 2-dimensionality implies generalizaton of the notion of Feynman diagram and holography in strong sense. One implication is the notion of field identity or field body making sense also for elementary particles and the Lamb shift anomaly of muonic hydrogen could be explained in terms of field bodies of quarks.

   4-D tangent space data must relate to the presence of strings connecting partonic 2-surfaces and defining the ends of string world sheets at which the modes of induced spinor fields are localized in the generic case in order to achieve conservation of em charge. The integer characterizing the spinor mode should charactize the tangent space data. Quantum criticality suggests strongly and super-conformal invariance acting as a gauge symmetry at the light-like partonic orbits and leaving the partonic 2-surfaces at their ends invariant. Without the fermionic strings effective 2-dmensionality would degenerate to genuine 2-dimensionality.

2. The topological explanation for family replication phenomenon implies genus generation correspondence and predicts in principle infinite number of fermion families. One can however develop a rather general argument based on the notion of conformal symmetry known as hyper-ellipticity stating that only the genera $g = 0, 1, 2$ are light. What "light" means is however an open question. If light means something below CP_2 mass there is no hope of observing new fermion families at LHC. If it means weak mass scale situation changes.

   For bosons the implications of family replication phenomenon can be understood from the fact that they can be regarded as pairs of fermion and antifermion assignable to the opposite wormhole throats of wormhole throat. This means that bosons formally belong to octet and singlet representations of dynamical SU(3) for which 3 fermion families define 3-D representation. Singlet would correspond to ordinary gauge bosons. Also interacting fermions suffer topological condensation and correspond to wormhole contact. One can either assume that the resulting wormhole throat has the topology of sphere or that the genus is same for both throats.

3. The view about space-time supersymmetry differs from the standard view in many respects. First of all, the super symmetries are not associated with Majorana spinors. Super generators correspond to the fermionic oscillator operators assignable to leptonic and quark-like induced spinors and there is in principle infinite number of them so that formally one would have $\mathcal{N} = \infty$ SUSY. I have discussed the required modification of the formalism of SUSY theories and it turns out that effectively one obtains just $\mathcal{N} = 1$ SUSY required by experimental constraints. The reason is that the fermion states with higher fermion number define only short range interactions analogous to van der Waals forces. Right handed neutrino generates this super-symmetry broken by the mixing of the M^4 chiralities implied by the mixing of M^4 and CP_2 gamma matrices for induced gamma matrices. The simplest assumption is that particles and their superpartners obey the same mass formula but that the p-adic length scale can be different for them.

4. The new view about particle massivation involves besides p-adic thermodynamics also Higgs particle but there is no need to assume that Higgs vacuum expectation plays any role. All particles could be seen as pairs of wormhole contacts whose throats at the two space-time sheets are connected by flux tubes carrying monopole flux: closed monopole flux tube involving two space-time sheets would be ion question. The contribution of the flux tube to particle mass would dominate for weak bosons whereas for fermions second wormhole contact would dominate.

5. One of the basic distinctions between TGD and standard model is the new view about color.
   (a) The first implication is separate conservation of quark and lepton quantum numbers implying the stability of proton against the decay via the channels predicted by GUTs. This does not mean that proton would be absolutely stable. p-Adic and dark length scale hierarchies indeed predict the existence of scale variants of quarks and leptons and proton could decay to hadons of some zoomed up copy of hadrons physics. These decays should be slow and presumably they would involve phase transition changing the value of Planck constant characterizing proton. It might be that the simultaneous increase of Planck constant for all quarks occurs with very low rate.
   (b) Also color excitations of leptons and quarks are in principle possible. Detailed calculations would be required to see whether their mass scale is given by CP_2 mass scale. The so called leptohadron physics proposed to explain certain anomalies associated with both electron, muon, and τ lepton could be understood in terms of color octet excitations of leptons.
6. Fractal hierarchies of weak and hadronic physics labelled by p-adic primes and by the levels of dark matter hierarchy are highly suggestive. Ordinary hadron physics corresponds to $M_{107} = 2^{107}-1$ One especially interesting candidate would be scaled up hadronic physics which would correspond to $M_{89} = 2^{89}-1$ defining the p-adic prime of weak bosons. The corresponding string tension is about 512 GeV and it might be possible to see the first signatures of this physics at LHC. Nuclear string model in turn predicts that nuclei correspond to nuclear strings of nucleons connected by colored flux tubes having light quarks at their ends. The interpretation might be in terms of M_{127} hadron physics. In biologically most interesting length scale range 10 nm-2.5 μm there are four Gaussian Mersennes and the conjecture is that these and other Gaussian Mersennes are associated with zoomed up variants of hadron physics relevant for living matter. Cosmic rays might also reveal copies of hadron physics corresponding to M_{61} and M_{31}
7. Weak form of electric magnetic duality implies that the fermions and antifermions associated with both leptons and bosons are Kähler magnetic monopoles accompanied by monopoles of opposite magnetic charge and with opposite weak isospin. For quarks Kähler magnetic charge need not cancel and cancellation might occur only in hadronic length scale. The magnetic flux tubes behave like string like objects and if the string tension is determined by weak length scale, these string aspects should become visible at LHC. If the string tension is 512 GeV the situation becomes less promising.

In this chapter the predicted new physics and possible indications for it are discussed.

Part I

HYPER-FINITE FACTORS OF TYPE II_1, HIERARCHY OF PLANCK CONSTANTS, AND M^8-H duality

Chapter 2

Evolution of Ideas about Hyper-finite Factors in TGD

2.1 Introduction

This chapter has emerged from a splitting of a chapter devote to the possible role of von Neumann algebras known as hyper-finite factors in quantum TGD. Second chapter emerging from the splitting is a representation of basic notions to chapter "Was von Neumann right after all?" [K112] representing only very briefly ideas about application to quantum TGD only briefly.

In the sequel the ideas about TGD applications are reviewed more or less chronologically. A summary about evolution of ideas is in question, not a coherent final structure, and as always the first speculations - in this case roughly for a decade ago - might look rather weird. The vision has however gradually become more realistic looking as deeper physical understanding of factors has evolved slowly.

The mathematics involved is extremely difficult for a physicist like me, and to really learn it at the level of proofs one should reincarnate as a mathematician. Therefore the only practical approach relies on the use of physical intuition to see whether HFFs might the correct structure and what HFFs do mean. What is needed is a concretization of the extremely abstract mathematics involved: mathematics represents only the bones to which physics should add flesh.

2.1.1 Hyper-Finite Factors In Quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.
2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.
3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space

$L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K90] . Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in M^8. Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the Kähler-Dirac gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality [K113, K30] this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II_1.

2.1.2 Hyper-Finite Factors And M-Matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

2.1.3 Connes Tensor Product As A Realization Of Finite Measurement Resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.
2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.
3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$ "averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

2.1.4 Concrete Realization Of The Inclusion Hierarchies

A concrete construction of M-matrix motivated by the recent rather precise view about basic variational principles of TGD allows to identify rather concretely the inclusions of HFFs in TGD framework and relate them to the hierarchies of broken conformal symmetries accompanying quantum criticalities.

1. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.
2. The formulation of scattering amplitudes in terms of Yangian of the super-symplectic algebra leads to a rather detailed view about scattering amplitudes [K100]. In this formulation scattering amplitudes are representations for sequences of algebraic operations connecting collections of elements of Yangian and sequences produce the same result. A huge generalization of the duality symmetry of the hadronic string models is in question.
3. The reduction of the hierarchy of Planck constants $h_{eff}/h = n$ to a hierarchy of quantum criticalities accompanied by a hierarchy of sub-algebras of super-symplectic algebra acting as conformal gauge symmetries leads to the identification of inclusions of HFFs as inclusions of WCW Clifford algebras characterizing by $n(i)$ and $n(i+1) = m(i) \times n(i)$ so that hierarchies of von Neuman algebras, of Planck constants, and of quantum criticalities would be very closely related. In the transition $n(i) \rightarrow n(i+1) = m(i) \times n(i)$ the measurement accuracy indeed increases since some conformal gauge degrees of freedom are transformed to physical ones. An open question is whether one could interpret $m(i)$ as the integer characterizing inclusion: the problem is that also $m(i) = 2$ with $\mathcal{M} : \mathcal{N} = 4$ seems to be allowed whereas Jones inclusions allow only $m \geq 3$.

Even more, number theoretic universality and strong form of holography leads to a detailed vision about the construction of scattering amplitudes suggesting that the hierarchy of algebraic extensions of rationals relates to the above mentioned hierarchies.

2.1.5 Analogs of quantum matrix groups from finite measurement resolution?

The notion of quantum group replaces ordinary matrices with matrices with non-commutative elements. In TGD framework I have proposed that the notion should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution.

In this article I will consider the notion of quantum matrix inspired by recent view about quantum TGD and it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution. The basic idea is to replace complex matrix elements with operators expressible as products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers.

The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. The weak definition of determinant meaning its development with respect to a fixed row or column does not pose additional conditions. Strong definition of determinant requires its invariance under permutations of rows and columns. The permutation of rows/columns turns out to have interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

The resulting quantum matrices define a more general structure than quantum group but provide a concrete representation and interpretation for quantum group in terms of finite measurement resolution if q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by sign factor invariant under the permutations of both rows and columns. One could also understand the fractal structure of inclusion sequences of hyper-finite factors resulting by recursively replacing operators appearing as matrix elements with quantum matrices.

2.1.6 Quantum Spinors And Fuzzy Quantum Mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

This chapter represents a summary about the development of the ideas with last sections representing the recent latest about the realization and role of HFFs in TGD. I have saved the reader from those speculations that have turned out to reflect my own ignorance or are inconsistent with what I regarded established parts of quantum TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L22].

2.2 A Vision About The Role Of HFFs In TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFs of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by ZEO and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

2.2.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A|| ||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A28] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.
2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0,1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.
3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.
4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X,μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.
2. A positive linear functional is weight with $\omega(1)$ finite.
3. A state is a weight with $\omega(1) = 1$.
4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.
5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$ For factors of type II_1 the values span the range $[0,1]$ and for factors of type II_∞ n the range $[0,\infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

2. Let

$$L^{\infty}(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \qquad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{2.2.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*{}^* = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.

4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A40, A63] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \to \sigma_t^{\omega} = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_{∞} factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines

the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a way to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1h_1(g_2), h_1h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1g_1(a_2), g_1g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K112] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A1] and those of factors of type III by Alain Connes [A30] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the embedding.

The basic facts proved by Jones are following [A1] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$
\begin{aligned}
& a) \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ , \quad h \geq 3 \ , \\
& b) \ \mathcal{M} : \mathcal{N} \geq 4 \ .
\end{aligned} \tag{2.2.2}
$$

   the numbers at right hand side are known as Beraha numbers [A54] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B33] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed. The Dynkin graphs of Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. $E_6, E_7, andE_8$ correspond to symmetry groups of tetrahedron, octahedron/cube, and icosahedron/dodecahedron. The group for octahedron/cube is missing: what could this mean?

   For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of $SU(2)$ and the interpretation proposed in [A85] is following-

   The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: A_∞ corresponding to $SU(2)$ itself, $A_{-\infty,\infty}$ corresponding to circle group $U(1)$, and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection.

   One can construct also inclusions for which the diagrams corresponding to finite subgroups $G \subset SU(2)$ are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with $n = 6, 7, 8$ for tetrahedron, cube, dodecahedron. These extensions correspond to ADE type Kac-Moody algebras.

   The extension is constructed by constructing first factor R as infinite tensor power of $M_2(C)$ (complexified quaternions). Sub-factor R_0 consists elements of of R of form $Id \otimes x$. $SU(2)$ preserves R_0 and for any subgroup G of $SU(2)$ one can identify the inclusion $N \subset M$ in terms of $N = R_0^G$ and $M = R^G$, where $N = R_0^G$ and $M = R^G$ consists of fixed points of R_0 and R under the action of G. The principal graph for $N \subset M$ is the extended Coxeter-Dynk graph for the subgroup G.

   Physicist might try to interpret this by saying that one considers only sub-algebras R_0^G and R^G of observables invariant under G and obtains extended Dynkin diagram of G defining an ADE type Kac-Moody algebra. Could the condition that Kac-Moody algebra elements with non-vanishing conformal weight annihilate the physical states state that the state is invariant under R_0 defining measurement resolution. Besides this the states are also invariant under finite group G? Could R_0^G and R^G correspond just to states which are also invariant under finite group G.

Connes tensor product

The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$
\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N} \ . \tag{2.2.3}
$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M}\otimes\mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n\times n$ matrices acts on V from right, V can be regarded as a space formed by $m\times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n\times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m\times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.
2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn}\otimes e^{nr}$$

in the tensor product $V\otimes W$.
3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger\otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m=r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.
4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A75, A40, A63] . There are good arguments showing that in HFFs of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A63] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E\in\mathcal{M}(\mathcal{O})$ can be written as WW^* with $W\in\mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W=1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.
2. Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.
3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of

type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ , \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ .$$

An infinite hierarchy of inclusions of HFFs of type III_1s is induced by set theoretic inclusions.

2.2.2 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?
2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and ZEO, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.
3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?
4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at embedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?
2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?
3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the Kähler-Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the Kähler-Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?
2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

ZEO and factors

The first question concerns the identification of the Hilbert space associated with the factors in ZEO. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.
2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in ZEO.

1. In ZEO M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFs is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.
2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \to 0$ limit. In quantum TGD the exponent of Kähler-Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Kähler-Dirac action can therefore be regarded as a "square root" of Kähler action.
3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in ZEO based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the Kähler-Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFs involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of Kähler-Dirac action [K113] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.
2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.
3. Quantum criticality means that Kähler-Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two super-conformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.
4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.
5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.
2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.
3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L_{-n}^*$, $n \neq 0$ and $L_0 = L_0^*$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.
2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem

to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.
4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K42] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.
2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.
3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?
4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

2.2.3 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

A proposal for M-matrix

The proposed general picture reduces the core of U-matrix to the construction of S-matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized eigenvalue $p^k\gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-light geodesics of embedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.

4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K100].

   The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface

the vertices would be represented by partonic 2-surfaces. In [K100] the interpretation of scattering ampiltudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are "free". At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K15] is a remnant of an "idea that came too early". The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond topartial wave in the space S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??** (`http://tgdtheory.fi/appfigures/elparticletgd.jpg` `http://tgdtheory.fi/appfigures/tgdgrpahs.jpg`) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K100] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

Quantum TGD as square root of thermodynamics

ZEO (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A75] [K112]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A75], [K112] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the Kähler-Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes.

Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the embedding space [K42]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. This implies that classical symplectic Noether charges vanish for this sub-algebra. The quantal conformal charges associated with induced spinor fields annihilate the physical states. Therefore it seems that the measured quantities are the symplectic charges and there is not need to introduce any measurement interaction term and the formalism simplifies dramatically.

The resolution increases with $h_{eff}/h = n$. Also the number of of strings connecting partonic 2-surfaces (in practice elementary particles and their dark counterparts plus bound states generated by connecting dark strings) characterizes physically the finite measurement resolution. Their presence is also visible in the geometry of the space-time surfaces through the conditions that induced W fields vanish at them (well-definedness of em charge), and by the condition that the canonical momentum currents for Kähler action define an integrable distribution of planes parallel to the string world sheet. In spirit with holography, preferred extremal is constructed by fixing string world sheets and partonic 2-surfaces and possibly also their light-like orbits (should one fix wormhole contacts is not quite clear). If the analog of AdS/CFT correspondence holds true, the value of Kähler function is expressible as the energy of string defined by area in the effective metric defined by the anti-commutators of K-D gamma matrices.

Super-symplectic algebra, whose charges are represented by Noether charges associated with strings connecting partonic 2-surfaces extends to a Yangian algebra with multi-stringy generators [K100]. The better the measurement resolution, the larger the maximal number of strings associated with the multilocal generator.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

The inclusions of super-symplectic Yangians form a hierarchy and would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra. As h_{eff} increases, infinite number of these gauge degrees of freedom become dynamical and measurement resolution is increased. This picture is definitely in conflict with the original view but the reduction of criticality in the increase of h_{eff} forces it.

Summary

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

2.2.4 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in ZEO

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.
2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.
3. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.
4. ZEO gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish ZEO allows a symmetry breaking respecting a chosen Cartan algebra.
5. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced

Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFs as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.
2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.
3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \tag{2.2.4}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{2.2.5}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{2.2.6}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times Tr_N(SS^{\dagger}) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \ . \tag{2.2.7}$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of embedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of embedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

1. In ZEO the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.
2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.
3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.
4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A13] .
5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A13] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.
6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{2.2.8}$$

   for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.
2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.
3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \tag{2.2.9}$$

for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the Kähler-Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain Kähler-Dirac gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A58] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be

between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.
2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.
3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

2.2.5 Questions about quantum measurement theory in Zero Energy Ontology

The following summary about quantum measurement theory in ZEO is somewhat out-of-date and somewhat sketchy. For more detailed view see [K63, K108, K9].

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

Quantum measurements in ZEO

ZEO based quantum measurement theory leads directly to a theory of conscious entities. The basic idea is that state function reduction localizes the second boundary of CD so that it becomes a piece of light-cone boundary (more precisely $\delta M^4_\pm \times CP_2$).

Repeated reductions are possible as in standard quantum measurement theory and leave the passive boundary of CD. Repeated reduction begins with U process generating a superposition of CDs with the active boundary of CD being de-localized in the moduli space of CDs, and is followed by a localization in this moduli space so that single CD is the outcome. This process tends to increase the distance between the ends of the CD and has interpretation as a space-time correlate for the flow of subjective time.

Self as a conscious entity corresponds to this sequence of repeated reductions on passive boundary of CD. The first reduction at opposite boundary means death of self and its re-incarnation at the opposite boundary of CD. Also the increase of Planck constant and generation of negentropic entanglement is expected to be associated with this state function reduction.

Weak form of NMP is the most plausible variational principle to characterize the state function reduction. It does not require maximal negentropy gain for state function reductions but allows it. In other words, the outcome of reduction is n-dimensional eigen space of density matrix space but this space need not have maximum possible dimension and even 1-D ray is possible in which case the entanglement negentropy vanishes for the final state and system becomes isolated from the rest of the world. Weak form of NMP brings in free will and can allow also larger negentropy gain than the strong form if n is a product of primes. The beauty of this option is that one can understand how the generalization of p-adic length scale hypothesis emerges.

Hyper-finite factors of type II_1 and quantum measurement theory with a finite measurement resolution

The realization that the von Neumann algebra known as hyper-finite factor of type II_1 is tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

Hyper-finite factor of type II_1 has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the embedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the embedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in ZEO states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.
2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.
3. This leads also to the notion of quantum group. For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.
4. For HFFs the dimension of infinite-dimensional state space is finite and 1 by convention. For included HFF $\mathcal{N} \subset \mathcal{M}$ the dimension of the tensor factor space containing only the degrees of freedom which are above measurement resolution is given by the index of inclusion $d = \mathcal{M} : \mathcal{N}$. One can say that the dimension associated with degrees of freedom below measurement resolution is $D = 1/d$. This number is never large than 1 for the inclusions and contains a set of discrete values $d = 4cos^2(2\pi/n)$, $n \geq 3$, plus the continuum above it. The fractal generalization of the formula for entanglement entropy gives $S = -log(1/D) = -log(d) \leq 0$ so that one can say that the entanglement negentropy assignable to the projection operators to the sub-factor is positive except for $n = 3$ for which it vanishes. The non-measured degrees of freedom carry information rather than entropy.
5. Clearly both HFFs of type I and II allow entanglement negentropy and allow to assign it with finite measurement resolution. In the case of factors its is not clear whether the weak form of NMP allows makes sense.

As already explained, the topology of the many-sheeted space-time encourages the generalization of the notion of quantum entanglement in such a way that unentangled systems can possess entangled sub-systems. One can say that the entanglement between sub-selves is not visible in the resolution characterizing selves. This makes possible sharing and fusion of mental images central for TGD inspired theory of consciousness. These concepts find a deeper justification from the quantum measurement theory for hyper-finite factors of type II_1 for which the finite measurement resolution is basic notion.

Hierarchies of conformal symmetry breakings, Planck constants, and inclusions of HFFs

The basic almost prediction of TGD is a fractal hierarchy of breakings of symplectic symmetry as a gauge symmetry.

It is good to briefly summarize the basic facts about the symplectic algebra assigned with $\delta M^4_\pm \times CP_2$ first.

1. Symplectic algebra has the structure of Virasoro algebra with respect to the light-like radial coordinate r_M of the light-cone boundary taking the role of complex coordinate for ordinary conformal symmetry. The Hamiltonians generating symplectic symmetries can be chosen to be proportional to functions $f_n(r_M)$. What is the natural choice for $f_n(r_M)$ is not quite clear. Ordinary conformal invariance would suggests $f_n(r_M) = r_M^n$. A more adventurous possibility is that the algebra is generated by Hamiltonians with $f_n(r_M) = r^{-s}$, where s is a root of Riemann Zeta so that one has either $s = 1/2 + iy$ (roots at critical line) or $s = -2n$, $n > 0$ (roots at negative real axis).

2. The set of conformal weights would be linear space spanned by combinations of all roots with integer coefficients $s = n - iy$, $s = \sum n_i y_i$, $n > -n_0$, where $-n_0 \geq 0$ is negative conformal weight. Mass squared is proportional to the total conformal weight and must be real demanding $y = \sum y_i = 0$ for physical states: I call this conformal confinement analogous to color confinement. One could even consider introducing the analog of binding energy as "binding conformal weight".

   Mass squared must be also non-negative (no tachyons) giving $n_0 \geq 0$. The generating conformal weights however have negative real part -1/2 and are thus tachyonic. Rather remarkably, p-adic mass calculations force to assume negative half-integer valued ground state conformal weight. This plus the fact that the zeros of Riemann Zeta has been indeed assigned with critical systems forces to take the Riemannian variant of conformal weight spectrum with seriousness. The algebra allows also now infinite hierarchy of conformal sub-algebras with weights coming as n-ples of the conformal weights of the entire algebra.

3. The outcome would be an infinite number of hierarchies of symplectic conformal symmetry breakings. Only the generators of the sub-algebra of the symplectic algebra with radial conformal weight proportional to n would act as gauge symmetries at given level of the hierarchy. In the hierarchy n_i divides n_{i+1}. In the symmetry breaking $n_i \rightarrow n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Gauge degrees of freedom would transform to physical degrees of freedom.

4. What about the conformal Kac-Moody algebras associated with spinor modes. It seems that in this case one can assume that the conformal gauge symmetry is exact just as in string models.

The natural interpretation of the conformal hierarchies $n_i \rightarrow n_{i+1}$ would be in terms of increasing measurement resolution.

1. Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and correspond to generators with conformal weight proportional to n_i. Conformal hierarchies and associated hierarchies of Planck constants and n-fold coverings of space-time surface connecting the 3-surfaces at the ends of causal diamond would give a concrete realization of the inclusion hierarchies for hyper-finite factors of type II_1 [K112].

   n_i could correspond to the integer labelling Jones inclusions and associating with them the quantum group phase factor $U_n = exp(i2\pi/n)$, $n \geq 3$ and the index of inclusion given by $|M : N| = 4cos^2(2\pi/n)$ defining the fractal dimension assignable to the degrees of freedom above the measurement resolution. The sub-algebra with weights coming as n-multiples of the basic conformal weights would act as gauge symmetries realizing the idea that these degrees of freedom are below measurement resolution.

2. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

   This interpretation might make sense for the symplectic algebra of $\delta M^4_\pm \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

3. If Kähler action has vanishing total variation under deformations defined by the broken conformal symmetries, the corresponding conformal charges are conserved. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N} = 4$ symmetric gauge theories. The deformations defined by symplectic transformations acting gauge symmetries the second variation vanishes and there is not contribution to WCW Kähler metric.
4. One can interpret the situation also in terms of consciousness theory. The larger the value of h_{eff}, the lower the criticality, the more sensitive the measurement instrument since new degrees of freedom become physical, the better the resolution. In p-adic context large n means better resolution in angle degrees of freedom by introducing the phase $exp(i2\pi/n)$ to the algebraic extension and better cognitive resolution. Also the emergence of negentropic entanglement characterized by $n \times n$ unitary matrix with density matrix proportional to unit matrix means higher level conceptualization with more abstract concepts.

The extension of the super-conformal algebra to a larger Yangian algebra is highly suggestive and gives and additional aspect to the notion of measurement resolution.

1. Yangian would be generated from the algebra of super-conformal charges assigned with the points pairs belonging to two partonic 2-surfaces as stringy Noether charges assignable to strings connecting them. For super-conformal algebra associated with pair of partonic surface only single string associated with the partonic 2-surface. This measurement resolution is the almost the poorest possible (no strings at all would be no measurement resolution at all!).
2. Situation improves if one has a collection of strings connecting set of points of partonic 2-surface to other partonic 2-surface(s). This requires generalization of the super-conformal algebra in order to get the appropriate mathematics. Tensor powers of single string super-conformal charges spaces are obviously involved and the extended super-conformal generators must be multi-local and carry multi-stringy information about physics.
3. The generalization at the first step is simple and based on the idea that co-product is the "time inverse" of product assigning to single generator sum of tensor products of generators giving via commutator rise to the generator. The outcome would be expressible using the structure constants of the super-conformal algebra schematically a $Q^1_A = f_A^{BC} Q_B \otimes Q_C$. Here Q_B and Q_C are super-conformal charges associated with separate strings so that 2-local generators are obtained. One can iterate this construction and get a hierarchy of n-local generators involving products of n stringy super-conformal charges. The larger the value of n, the better the resolution, the more information is coded to the fermionic state about the partonic 2-surface and 3-surface. This affects the space-time surface and hence WCW metric but not the 3-surface so that the interpretation in terms of improved measurement resolution makes sense. This super-symplectic Yangian would be behind the quantum groups and Jones inclusions in TGD Universe.
4. n gives also the number of space-time sheets in the singular covering. One possible interpretation is in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

It is not an accident that quantum phases are assignable to Yangian algebras, to quantum groups, and to inclusions of HFFs. The new deep notion added to this existing complex of high level mathematical concepts are hierarchy of Planck constants, dark matter hierarchy, hierarchy of criticalities, and negentropic entanglement representing physical notions. All these aspects represent new physics.

2.2.6 Planar Algebras And Generalized Feynman Diagrams

Planar algebras [A18] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [A41] . In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [K24] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

1. One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.
2. One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.
3. One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.
4. It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

1. Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.
2. Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.
3. The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.
4. The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

5. One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

1. The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.
2. The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

   [Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].
3. Planar diagrams contain lines, which begin and return to the same disk boundary. Also "vacuum bubbles" are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.
4. Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).
5. There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of WCW ("world of classical worlds") whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

2.2.7 Miscellaneous

The following considerations are somewhat out-of-date: hence the title "Miscellaneous".

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A85] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A32] .

Fusion rules are indeed something more intricate that the naïve product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A38] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naïve tensor product with something more intricate. The naïvest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naïve an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K29] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A46] .

1. The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K7] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K38] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A46] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say 3-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

2.3 Fresh View About Hyper-Finite Factors In TGD Framework

In the following I will discuss the basic ideas about the role of hyper-finite factors in TGD with the background given by a work of more than half decade. First I summarize the input ideas which I combine with the TGD inspired intuitive wisdom about HFFs of type II_1 and their inclusions allowing to represent finite measurement resolution and leading to notion of quantum spaces with algebraic number valued dimension defined by the index of the inclusion.

Also an argument suggesting that the inclusions define "skewed" inclusions of lattices to larger lattices giving rise to quasicrystals is proposed. The core of the argument is that the included HFF of type II_1 algebra is a projection of the including algebra to a subspace with dimension $D \leq 1$. The projection operator defines the analog of a projection of a bigger lattice to the included lattice. Also the fact that the dimension of the tensor product is product of dimensions of factors just like the number of elements in finite group is product of numbers of elements of coset space and subgroup, supports this interpretation.

One also ends up with a detailed identification of the hyper-finite factors in orbital degrees of freedom in terms of symplectic group associated with $\delta M^4_\pm \times CP_2$ and the group algebras of their discrete subgroups define what could be called "orbital degrees of freedom" for WCW spinor fields. By very general argument this group algebra is HFF of type II, maybe even II_1.

2.3.1 Crystals, Quasicrystals, Non-Commutativity And Inclusions Of Hyperfinite Factors Of Type II_1

I list first the basic ideas about non-commutative geometries and give simple argument suggesting that inclusions of HFFs correspond to "skewed" inclusions of lattices as quasicrystals.

1. Quasicrystals (see `http://tinyurl.com/67kz3qo`) (say Penrose tilings) [A20] can be regarded as subsets of real crystals and one can speak about "skewed" inclusion of real lattice to larger lattice as quasicrystal. What this means that included lattice is obtained by projecting the larger lattice to some lower-dimensional subspace of lattice.
2. The argument of Connes concerning definition of non-commutative geometry can be found in the book of Michel Lapidus at page 200. Quantum space is identified as a space of equivalence classes. One assigns to pairs of elements inside equivalence class matrix elements having the element pair as indices (one assumes numerable equivalence class). One considers irreducible representations of the algebra defined by the matrices and identifies the equivalent irreducible representations. If I have understood correctly, the equivalence classes of irreps define a discrete point set representing the equivalence class and it can often happen that there is just

single point as one might expect. This I do not quite understand since it requires that all irreps are equivalent.

3. It seems that in the case of linear spaces - von Neumann algebras and accompanying Hilbert spaces - one obtains a connection with the inclusions of HFFs and corresponding quantum factor spaces that should exist as analogs of quantum plane. One replaces matrices with elements labelled by element pairs with linear operators in HFF of type II_1. Index pairs correspond to pairs in linear basis for the HFF or corresponding Hilbert space.

4. Discrete infinite enumerable basis for these operators as a linear space generates a lattice in summation. Inclusion $N \subset M$ defines inclusion of the lattice/crystal for N to the corresponding lattice of M. Physical intuition suggests that if this inclusion is "skewed" one obtains quasicrystal. The fact the index of the inclusion is algebraic number suggests that the coset space M/N is indeed analogous to quasicrystal.

   More precisely, the index of inclusion is defined for hyper-finite factors of type II_1 using the fact that quantum trace of unit matrix equals to unity $Tr(Id(M)) = 1$, and from the tensor product composition $M = (M/N) \times N$ given $Tr(Id(M)) = 1 = Ind(M/N)Tr(P(M \to N))$, where $P(M \to N$ is projection operator from M to N. Clearly, $Ind(M/N) = 1/Tr(P(M \to N))$ defines index as a dimension of quantum space M/N.

   For Jones inclusions characterized by quantum phases $q = exp(i2\pi/n)$, $n = 3, 4, ...$ the values of index are given by $Ind(M/N) = 4cos^2(\pi/n)$, $n = 3, 4, ...$ There is also another range inclusions $Ind(M/N) \geq 4$: note that $Tr(P(M \to N))$ defining the dimension of N as included sub-space is never larger than one for HFFs of type II_1. The projection operator $P(M \to N)$ is obviously the counterpart of the projector projecting lattice to some lower-dimensional sub-space of the lattice.

5. Jones inclusions are between linear spaces but there is a strong analogy with non-linear coset spaces since for the tensor product the dimension is product of dimensions and for discrete coset spaces G/H one has also the product formula $n(G) = n(H) \times n(G/H)$ for the numbers of elements. Noticing that space of quantum amplitudes in discrete space has dimension equal to the number of elements of the space, one could say that Jones inclusion represents quantized variant for classical inclusion raised from the level of discrete space to the level of space of quantum states with the number of elements of set replaced by dimension. In fact, group algebras of infinite and enumerable groups defined HFFs of type II under rather general conditions (see below).

   Could one generalize Jones inclusions so that they would apply to non-linear coset spaces analogs of the linear spaces involved ? For instance, could one think of infinite-dimensional groups G and H for which Lie-algebras defining their tangent spaces can be regarded as HFFs of type II_1? The dimension of the tangent space is dimension of the non-linear manifold: could this mean that the non-linear infinite-dimensional inclusions reduce to tangent space level and thus to the inclusions for Lie-algebras regarded hyper-finite factors of type II_1 or more generally, type II? This would would rise to quantum spaces which have finite but algebraic valued quantum dimension and in TGD framework take into account the finite measurement resolution.

6. To concretize this analogy one can check what is the number of points map from 5-D space containing aperiodic lattice as a projection to a 2-D irrational plane containing only origin as common point with the 5-D lattice. It is easy to get convinced that the projection is 1-to-1 so that the number of points projected to a given point is 1. By the analogy with Jones inclusions this would mean that the included space has same von Neumann dimension 1 - just like the including one. In this case quantum phase equals $q = exp(i2\pi/n)$, $n = 3$ - the lowest possible value of n. Could one imagine the analogs of $n > 3$ inclusions for which the number of points projected to a given point would be larger than 1? In 1-D case the rational lines $y = (k/l)x$ define 1-D rational analogs of quasi crystals. The points $(x, y) = (m, n)$, $m \ mod \ l = 0$ are projected to the same point. The number of points is now infinite and the ratio of points of 2-D lattice and 1-D crystal like structure equals to l and serves as the analog for the quantum dimension $d_q = 4cos^2(\pi/n)$.

To sum up, this this is just physicist's intuition: it could be wrong or something totally trivial from the point of view of mathematician. The main message is that the inclusions of HFFs might define also inclusions of lattices as quasicrystals.

2.3.2 HFFs And Their Inclusions In TGD Framework

In TGD framework the inclusions of HFFs have interpretation in terms of finite measurement resolution. If the inclusions define quasicrystals then finite measurement resolution would lead to quasicrystals.

1. The automorphic action of N in $M \supset N$ and in associated Hilbert space H_M where N acts generates physical operators and accompanying stas (operator rays and rays) not distinguishable from the original one. States in finite measurement resolution correspond to N-rays rather than complex rays. It might be natural to restrict to unitary elements of N.

   This leads to the need to construct the counterpart of coset space M/N and corresponding linear space H_M/H_N. Physical intuition tells that the indices of inclusions defining the "dimension" of M/N are algebraic numbers given by Jones index formula.
2. Here the above argument would assign to the inclusions also inclusions of lattices as quasicrystals.

Degrees of freedom for WCW spinor field

Consider first the identification of various kinds of degrees of freedom in TGD Universe.

1. Very roughly, WCW ("world of classical worlds") spinor is a state generated by fermionic creation operators from vacuum at given 3-surface. WCW spinor field assigns this kind of spinor to each 3-surface. WCW spinor fields decompose to tensor product of spin part (Fock state) and orbital part ("wave" in WCW) just as ordinary spinor fields.
2. The conjecture motivated by super-symmetry has been that both WCW spinors and their orbital parts (analogs of scalar field) define HFFs of type II_1 in quantum fluctuating degrees of freedom.
3. Besides these there are zero modes, which by definition do not contribute to WCW Kähler metric.
   (a) If the zero zero modes are symplectic invariants, they appear only in conformal factor of WCW metric. Symplectically invariant zero modes represent purely classical degrees of freedom - direction of a pointer of measurement apparatus in quantum measurement - and in given experimental arrangement they entangle with quantum fluctuating degrees of freedom in one-one manner so that state function reduction assigns to the outcome of state function reduction position of pointer. I forget symplectically invariant zero modes and other analogous variables in the following and concentrate to the degrees of freedom contributing WCW line-element.
   (b) There are also zero modes which are not symplectic invariants and are analogous to degrees of freedom generated by the generators of Kac-Moody algebra having vanishing conformal weight. They represent "center of mass degrees of freedom" and this part of symmetric algebra creates the representations representing the ground states of Kac-Moody representations. Restriction to these degrees of freedom gives QFT limit in string theory. In the following I will speak about "cm degrees of freedom".

The general vision about symplectic degrees of freedom (the analog of "orbital degrees of freedom" for ordinary spinor field) is following.

1. WCW (assignable to given CD) is a union over the sub-WCWs labeled by zero modes and each sub-WCW representing quantum fluctuating degrees of freedom and "cm degrees of freedom" is infinite-D symmetric space. If symplectic group assignable to $\delta M^4_+ \times CP_2$ acts as as isometries of WCW then "orbital degrees of freedom" are parametrized by the symplectic group or its coset space (note that light-cone boundary is 3-D but radial dimension is light-like so that symplectic - or rather contact structure - exists).

   Let S^2 be $r_M = constant$ sphere at light-cone boundary (r_M is the radial light-like coordinate fixed apart from Lorentz transformation). The full symplectic group would act as isometries of WCW but does not - nor cannot do so - act as symmetries of Kähler action except in the huge vacuum sector of the theory correspond to vacuum extremals.

2. WCW Hamiltonians can be deduced as "fluxes" of the Hamiltonians of $\delta M^4_+ \times CP_2$ taken over partonic 2-surfaces. These Hamiltonians expressed as products of Hamiltonians of S^2 and CP_2 multiplied by powers r_M^n. Note that r_M plays the role of the complex coordinate z for Kac-Moody algebras and the group G defining KM is replaced with symplectic group of $S^2 \times CP_2$. Hamiltonians can be assumed to have well-defined spin (SO(3)) and color (SU(3)) quantum numbers.
3. The generators with vanishing radial conformal weight ($n = 0$) correspond to the symplectic group of $S^2 \times CP_2$. They are not symplectic invariants but are zero modes. They would correspond to "cm degrees of freedom" characterizing the ground states of representations of the full symplectic group.

Discretization at the level of WCW

The general vision about finite measurement resolution implies discretization at the level of WCW.

1. Finite measurement resolution at the level of WCW means discretization. Therefore the symplectic groups of $\delta M^4_+ \times CP_2$ *resp.* $S^2 \times CP_2$ are replaced by an enumerable discrete subgroup. WCW is discretized in both quantum fluctuating degrees of freedom and "center of mass" degrees of freedom.
2. The elements of the group algebras of these discrete groups define the "orbitals parts" of WCW spinor fields in discretization. I will later develop an argument stating that they are HFFs of type II - maybe even II_1. Note that also function spaces associated with the coset spaces of these discrete subgroups could be considered.
3. Discretization applies also in the spin degrees of freedom. Since fermionic Fock basis generates quantum counterpart of Boolean algebra the interpretation in terms of the physical correlates of Boolean cognition is motivated (fermion number 1/0 and various spins in decomposition to a tensor product of lower-dimensional spinors represent bits). Note that in ZEO fermion number conservation does not pose problems and zero states actually define what might be regarded as quantum counterparts of Boolean rules $A \rightarrow B$.
4. Note that 3-surfaces correspond by the strong form of GCI/holography to collections of partonic 2-surfaces and string world sheets of space-time surface intersecting at discrete set of points carrying fermionic quantum numbers. WCW spinors are constructed from second quantized induced spinor fields and fermionic Fock algebra generates HFF of type II_1.

Does WCW spinor field decompose to a tensor product of two HFFs of type II_1?

The group algebras associated with infinite discrete subgroups of the symplectic group define the discretized analogs of waves in WCW having quantum fluctuating part and cm part. The proposal is that these group algebras are HFFs of type II_1. The spinorial degrees of freedom correspond to fermionic Fock space and this is known to be HFF. Therefore WCW spinor fields would defined tensor product of HFFs of type II_1. The interpretation would be in terms of supersymmetry at the level of WCW. Super-conformal symmetry is indeed the basic symmetry of TGD so that this result is a physical "must". The argument goes as follows.

1. In non-zero modes WCW is symplectic group of $\delta M^4_+ \times CP_2$ (call this group just *Sympl*) reduces to the analog of Kac-Moody group associated with $S^2 \times CP_2$, where S^2 is $r_M =$ constant sphere of light-cone boundary and z is replaced with radial coordinate. The Hamiltonians, which do not depend on r_M would correspond to zero modes and one could not assign metric to them although symplectic structure is possible. In "cm degrees of freedom" one has symplectic group associated with $S^2 \times CP_2$.
2. Finite measurement resolution, which seems to be coded already in the structure of the preferred extremals and of the solutions of the Kähler-Dirac equation, suggests strongly that this symplectic group is replaced by its discrete subgroup or symmetric coset space. What this group is, depends on measurement resolution defined by the cutoffs inherent to the solutions. These subgroups and coset spaces would define the analogs of Platonic solids in WCW!

3. Why the discrete infinite subgroups of *Sympl* would lead naturally to HFFs of type II? There is a very general result stating that group algebra of an enumerable discrete group, which has infinite conjugacy classes, and is amenable so that its regular representation in group algebra decomposes to all unitary irreducibles is HFF of type II. See for examples about HFFs of type II listed in Wikipedia article (see `http://tinyurl.com/y8445w8q`) [A10].

4. Suppose that the group algebras associated the discrete subgroups *Sympl* are indeed HFFs of type II or even type II_1. Their inclusions would define finite measurement resolution the orbital degrees of freedom for WCW spinor fields. Included algebra would create rays of state space not distinguishable experimentally. The inclusion would be characterized by the inclusion of the lattice defined by the generators of included algebra by linearity. One would have inclusion of this lattice to a lattice associated with a larger discrete group. Inclusions of lattices are however known to give rise to quasicrystals (Penrose tilings are basic example), which define basic non-commutative structures. This is indeed what one expects since the dimension of the coset space defined by inclusion is algebraic number rather than integer.

5. Also in fermionic degrees of freedom finite measurement resolution would be realized in terms of inclusions of HFFs- now certainly of type II_1. Therefore one could obtain hierarchies of lattices included as quasicrystals.

What about zero modes which are symplectic invariants and define classical variables? They are certainly discretized too. One might hope that one-one correlation between zero modes (classical variables) and quantum fluctuating degrees of freedom suggested by quantum measurement theory allows to effectively eliminate them. Besides zero modes there are also modular degrees of freedom associated with partonic 2-surfaces defining together with their 4-D tangent space data basis objects by strong form of holography. Also these degrees of freedom are automatically discretized. But could one consider finite measurement resolution also in these degrees of freedom. If the symplectic group of $S^2 \times CP_2$ defines zero modes then one could apply similar argument also in these degrees of freedom to discrete subgroups of $S^2 \times CP_2$.

2.3.3 Little Appendix: Comparison Of WCW Spinor Fields With Ordinary Second Quantized Spinor Fields

In TGD one identifies states of Hilbert space as WCW spinor fields. The analogy with ordinary spinor field helps to understand what they are. I try to explain by comparison with QFT.

Ordinary second quantized spinor fields

Consider first ordinary fermionic QFT in fixed space-time. Ordinary spinor is attached to an space-time point and there is $2^{D/2}$ dimensional space of spin degrees of freedom. Spinor field attaches spinor to every point of space-time in a continuous/smooth manner. Spinor fields satisfying Dirac equation define in Euclidian metric a Hilbert space with a unitary inner product. In Minkowskian case this does not work and one must introduce second quantization and Fock space to get a unitary inner product. This brings in what is essentially a basic realization of HFF of type II_1 as allowed operators acting in this Fock space. It is operator algebra rather than state space which is HFF of type II_1 but they are of course closely related.

Classical WCW spinor fields as quantum states

What happens TGD where one has quantum superpositions of 4-surface/3-surfaces by GCI/partonic 2-surfaces with 4-D tangent space data by strong form of GCI.

1. First guess: space-time point is replaced with 3-surface. Point like particle becomes 3-surface representing particle. WCW spinors are fermionic Fock states at this surface. WCW spinor fields are Fock state as a functional of 3-surface. Inner product decomposes to Fock space inner product plus functional integral over 3-surfaces (no path integral!). One could speak of quantum multiverse. Not single space-time but quantum superposition of them. This quantum multiverse character is something new as compared to QFT.

2. Second guess: forced by ZEO, by geometrization of Feynman diagrams, etc.

(a) 3-surfaces are actually not connected 3-surfaces. They are collections of components at both ends of CD and connected to single connected structure by 4-surface. Components of 3-surface are like incoming and outgoing particles in connected Feynman diagrams. Lines are identified as regions of Euclidian signature or equivalently as the 3-D light-like boundaries between Minkowskian and Euclidian signature of the induced metric.

(b) Spinors(!!) are defined now by the fermionic Fock space of second quantized induced spinor fields at these 3-surfaced and by holography at 4-surface. This fermionic Fock space is assigned to all multicomponent 3-surfaces defined in this manner and WCW spinor fields are defined as in the first guess. This brings integration over WCW to the inner product.

3. Third, even more improved guess: motivated by the solution ansatz for preferred extremals and for Kähler-Dirac equation [K113] giving a connection with string models.

   The general solution ansatz restricts all spinor components but right-handed neutrino to string world sheets and partonic 2-surfaces: this means effective 2-dimensionality. String world sheets and partonic 2-surfaces intersect at the common ends of light-like and space-like braids at ends of CD and at along wormhole throat orbits so that effectively discretization occurs. This fermionic Fock space replaces the Fock space of ordinary second quantization.

2.4 The idea of Connes about inherent time evolution of certain algebraic structures from TGD point of view

Jonathan Disckau asked me about what I think about the proposal of Connes represented in the summary of progress of noncommutative geometry in "Noncommutative Geometry Year 2000" [A31] (see `https://arxiv.org/abs/math/0011193`) that certain mathematical structures have inherent time evolution coded into their structure.

I have written years ago about Connes's proposal. At that time I was trying to figure out how to understand the construction of scattering amplitudes in the TGD framework and the proposal of Connes looked attractive. Later I had to give up this idea. However, the basic idea is beautiful. One should only replace the notion of time evolution from a one-parameter group of automorphisms to something more interesting. Also time evolution as increasing algebraic complexity is a more attractive interpretation.

The inclusion hierarchies of hyperfinite factors (HFFs) - closely related to the work of Connes - are a key element of TGD and crucial for understanding evolutionary hierarchies in TGD. Is it possible that mathematical structure evolves in time in some sense? The TGD based answer is that quantum jump as a fundamental evolutionary step - moment of subjective time evolution - is a necessary new element. The sequence of moments of consciousness as quantum jumps would have an interpretation as hopping around in the space of mathematical structures leading to increasingly complex structures.

The generalization of the idea of Connes is discussed in this framework. In particular, the inclusion hierarchies of hyper-finite factors, the extension hierarchies of rationals, and fractal inclusion hierarchies of subalgebras of supersymplectic algebra isomorphic with the entire algebra are proposed to be more or less one and the same thing in TGD framework.

The time evolution operator of Connes could corresponds to super-symplectic algebra (SSA) to the time evolution generated by $exp(iL_0\tau)$ so that the operator Δ of Connes would be identified as $\Delta = exp(L_0)$. This identification allows number theoretical universality if τ is quantized. Furthermore, one ends up with a model for the subjective time evolution by small state function reductions (SSFRs) for SSA with SSA_n gauge conditions: the unitary time evolution for given SSFR would be generated by a linear combination of Virasoro generators not annihilating the states. This model would generalize the model for harmonic oscillator in external force allowing exact S-matrix.

2.4.1 Connes proposal and TGD

In this section I develop in more detail the analog of Connes proposal in TGD framework.

What does Connes suggest?

One must first make clear what the automorphism of HFFs discovered by Connes is.

1. Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. I have described the theory earlier [K67, K43].

First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.
2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \qquad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{2.4.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*{}^* = \mathcal{M}$.
3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.
4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.
5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A40, A63] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.
3. $\omega \to \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

The definition of Δ^{it} reduces in eigenstate basis of Δ to the definition of complex function d^{it}. Note that is positive so that the logarithm of d is real.

In TGD framework number theoretic universality poses additional conditions. In diagonal basis $e^{log(d)it}$ must exist. A simply manner to solve the conditions is $e = exp(m/r)$ existing p-adically for an extension of rational allowing r:th root of e. This requires also quantization of as a root of unity so that the exponent reduces to a root of unity.

2. Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.
2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists of real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.
3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

3. Objections against the idea of Connes

One can represent objections against this idea.

1. Ordinary time evolution in wave mechanics is a unitary automorphism, so that in this framework they would not have physical meaning but act as gauge transformations. If outer automorphisms define time evolutions, they must act as gauge transformations. One would have an analog of gauge field theory in HFF. This would be of course highly interesting: when I gave up the idea of Connes, I did not consider this possibility. Super-symplectic algebras having fractal structure are however extremely natural candidate for defining HFF and there is infinite number of gauge conditions.
2. An automorphism is indeed in question so that the algebraic system would not be actually affected. Therefore one cannot say that HFF has inherent time evolution and time. However, one can represent in HFF dynamical systems obeying this inherent time evolution. This possibility is highly interesting as a kind of universal gauge theory.

   On the other hand, outer automorphisms affect the trace of the projector defining the identity matrix for a given factor. Does the scaling factor Λ represent some kind of renormalization operation? Could it relate to the action of scalings in the TGD framework where scalings replace time translations at the fundamental level? What the number theoretic vision of TGD could mean? Could this quantize the continuous spectrum of the scalings Λ for HFFs so that they belong to the extension? Could one have a spectrum of Λ for each extension of rationals? Are different extensions related by inclusions of HFFs?
3. The notion of time evolution itself is an essentially Newtonian concept: selecting a preferred time coordinate breaks Lorentz invariance. In TGD however time coordinate is replace by scaling parameter and the situation changes.
4. The proposal of Connes is not general enough if evolution is interpreted as an increase of complexity.

For these reasons I gave up the automorphism proposed by Connes as a candidate for defining time evolution giving rise to scattering amplitudes in TGD framework.

Two views about TGD

The two dual views about what TGD is described briefly in [L107].

1. Physics as geometry of the world of "world of classical worlds" (WCW) identified as the space of space-time surfaces in $M^4 \times CP_2$ [K85]. Twistor lift of TGD [K87] implies that the space-time surfaces are minimal surfaces which can be also regarded as extermals of the Kähler action. This implies holography required by the general coordinate invariance in TGD framework.
2. TGD as generalized number theory forcing to generalize physics to adelic physics [L52] fusing real physics as correlate of sensory experience and various p-adic physics as correlates of cognition. Now space-times are naturally co-associative surfaces in complexified M^8 (complexified octonions) defined as "roots" of octonionic polynomials determined by polynomials with rational coefficients [L99, L100, L117]. Now holography extends dramatically: finite number of rational numbers/roots of rational polynomial/points of space-time region dictate it.

$M^8 - H$ duality relates these two views and is actually a generalization of Fourier transform and realizes generalization of momentum-position duality.

The notion of time evolution in TGD

Concerning various time evolutions in TGD, the general situation is now rather well understood.

There are two quantal time evolutions: geometric one assignable to single CD and and subjective time evolution which reflects the generalization of point-like particle to a 3-surface and the introduction of CD as 4-D perceptive field of particle in ZEO [L80].

1. Geometric time evolution corresponds to the standard scattering amplitudes for which I have a general formula now in terms of zero energy ontology (ZEO) [L111, L99, L100, L117]. The analog of S-matrix corresponds to entanglement coefficients between members of zero energy state at opposite boundaries of causal diamond (CD).
2. Subjective time evolution of conscious entity corresponds to a sequence of "small" state function reductions (SSFRs) as moments of consciousness: each SSFR is preceded by an analog of unitary time evolution, call it U. SSFRs are the TGD counterparts of "weak" measurements.

   $U(t)$ is generated by the scaling generator L_0 scaling light-like radial coordinate of light-cone boundary and is a generalization of corresponding operator in superconformal and string theories and defined for super-symplectic algebras acting as isometries of the world of classical worlds (WCW) [L117]. $U(t)$ is not the exponential of energy as a generator of time translation as in QFTs but an exponential of the mass squared operator and corresponds to the scaling of radial light-like coordinate r of the light-like boundary of CD: r is analogous to the complex coordinate z in conformal field theories.

   Also "big" SFRs (BSFRs) are possible and correspond to "ordinary" SFRs and in TGD framework mean death of self in the universal sense and followed by reincarnation as time reversed subjective time evolution [L69].
3. There is also classical time evolution at the level of space-time surfaces. Here the assumption that X^4 belongs to $H = M^4 \times CP_2$ defines Minkowski coordinates of M^4 as almost unique space-time coordinates of X^4 is the M^4 projection of X^4 is 4-D. This generalizes also to the case of M^8. Symmetries make it possible to identify an essentially a unique time coordinate.

   This means enormous simplification. General coordinate invariance is a marvellous symmetry but it leads to the problem of specifying space-time coordinates that is finding preferred coordinates. This seems impossible since 3-metric is dynamical. M^4 provides a fixed reference system and the problem disappears. M^4 is dynamical by its Minkowskian signature and one can speak about classical signals.
4. There is also classical time evolution for the induced spinor fields. At the level of H the spinor field is a superposition of modes of the massless Dirac operator (massless in 8-D sense). This spinor field is free and second quantized. Second quantization of induced spinor trivializes and this is absolutely crucial for obtaining scattering amplitudes for fermions and avoiding the usual problems for quantization of fermions in curved background.

The induced spinor field is a restriction of this spinor field to the space-time surface and satisfies modified Dirac equation automatically. There is no need for second quantization at the level of space-time surface and propagators etc.... are directly calculable. This is an enormous simplification.

There are therefore as many as 4 time evolutions and subjective time evolution by BSFRs and possibly also by SSFRs is a natural candidate for time evolution as genuine evolution as emergence of more complex algebraic structures.

Could the inherent time evolution of HFF have a physical meaning in TGD after all?

The idea about inherent time evolution defined by HFF itself as one parameter group of outer automorphisms is very attractive by its universality: physics would become part of mathematics.

1. Thermodynamic interpretation, with inverse temperature identified as an analog of time coordinate, comes first in mind but need not be the correct interpretation.
2. Outer automorphisms should act at a very fundamental level analogous to the state space of topological field theories. Fundamental group is after all in question! The assignment of the S-matrix of particle physics to the outer automorphism does not look reasonable since the time evolution would be with respect to the linear Minkowski coordinate, which is not Lorentz invariant.

For these reasons I gave up the idea of Connes when considering it for the first time. However, TGD inspired theory of consciousness as a generalization of quantum measurement theory has evolved since then and the situation is different now.

The sequence of SSFRs defines subjective time evolution having no counterpart in QFTs. Each SSFR is preceded by a unitary time evolution, which however corresponds to the scaling of the light-like radial coordinate of the light-cone boundary [L117] rather than time translation. Hamiltonian is replaced with the scaling generator L_0 acting as Lorentz invariant mass squared operator so that Lorentz invariance is not lost.

Could the time evolution assignable to L_0 correspond to the outer automorphism of Connes when one poses an infinite number of gauge conditions making inner automorphisms gauge transformations? The connection of Connes proposal with conformal field theories and with TGD is indeed suggestive.

1. Conformally invariant systems obey infinite number of gauge conditions stating that the conformal generators L_n, $n > 0$, annihilate physical states and carry vanishing Noether charges.

   These gauge conditions bring in mind the condition that infinitesimal inner automorphisms do not change the system physically. Does this mean that Connes outer automorphism generates the time evolution and inner automorphisms act as gauge symmetries? One would have an analog of gauge field theory in HFF.
2. In TGD framework one has an infinite hierarchy of systems satisfying conditions analogous to the conformal gauge conditions. The generators of the super-symplectic algebra (SCA) acting as isometries of the "world of classical worlds" (WCW) are labelled by non-negative conformal weight n and it has infinite hierarchy of algebras SCA$_k$ isomorphic to it with conformal weights given by k-multiple of those of the entire algebra, $k = 1, 2, ...$.

   Gauge conditions state for SCA_k that the generators of SCA_k and its commutator with SCA annihilate physical states. The interpretation is in terms of a hierarchy of improving measurement resolutions with degrees of freedom below measurement resolution acting like gauge transformations.

   The Connes automorphism would "see" only the time evolution in the degrees of freedom above measurement resolution and as k increases, their number would increase.

In the case of hyperfinite factors of type II_1 (HFFs) the fundamental group of corresponding factor II_∞ consists of all reals: I hope I am right here.

1. The hyperfinite factors of type II_1 and corresponding factors II_∞ are natural in the TGD context. Therefore the spectrum would consist of reals unless one poses additional conditions.

2. Could the automorphisms correspond to the scalings of the lightcone proper time, which replace time translations as fundamental dynamics. Also in string models scalings take the role of time translations.
3. In zero energy ontology (ZEO) the scalings would act in the moduli space of causal diamonds which is finite-dimensional. This moduli space defines the backbone of the "world of classical worlds". WCW itself consists of a union of sub-WCs as bundle structures over CDs [L136]. The fiber consists of space-time surfaces inside a given CD analogous to Bohr orbits and satisfying holography reducing to generalized holomorphy. The scalings as automorphisms scale the causal diamonds. The space of CDs is a finite-dimensional coset space and has also other symmetry transformations.
4. The number theoretic vision suggests a quantization of the spectrum of Λ so that for a given extension of rationals the spectrum would belong to the extension. HFFs would be labelled at least partially by the extensions of rationals. The recent view of M^8-H duality [L138] is dramatically simpler than the earlier view [L99, L100, ?] and predicts that the space-time regions are determined by a pair of analytic functions with rational coefficients forced by number theoretical universality meaning that the space-time surfaces have interpretation also as p-adic surfaces.

   The simplest analytic functions are polynomials with integer coefficients and if one requires that the coefficients are smaller than the degree of the polynomial, the number of polynomials is finite for a given degree. This would give very precise meaning for the concept of number theoretic evolution.

   There would be an evolutionary hierarchy of pairs of polynomials characterized by increasing complexity and one can assign to these polynomials extension of rationals characterized by ramified primes depending on the polynomials. The ramified primes would have interpretation as p-adic primes characterizing the space-time region considered. Extensions of rationals and ramified primes could also characterize HFFs. This is a rather dramatic conjecture at the level of pure mathematics.
5. Scalings define renormalization group in standard physics. Now they scale the size of the CD. Could the scalings as automorphisms of HFFs correspond to discrete renormalization operations?

Three views about finite measurement resolution

Evolution could be seen physically as improving finite measurement resolution: this applies to both sensory experience and cognition. There are 3 views about finite measurement resolution (FMR) in TGD.

1. Hyper finite factors (HFFs) and FMR

HFFs are an essential part of Connes's work and I encountered them about 15 years ago or so [K112, K43].

The inclusions of hyper-finite factors HFFs provide one of the three - as it seems equivalent - ways to describe finite measurement resolution (FMR) in TGD framework: the included factor defines an analog for gauge degrees of freedom which correspond to those below measurement resolution.

2. Cognitive representations and FMR

Another description for FMR in the framework of adelic physics would be in terms of cognitive representations [L73]. First some background about M^8-H duality.

1. There are number theoretic and geometric views about dynamics. In algebraic dynamics at the level of M^8, the space-time surfaces are roots of polynomials. There are no partial differential equations like in the geometric dynamics at the level of H.
2. The algebraic "dynamics" of space-time surfaces in M^8 is dictated by co-associativity, which means that the normal space of the space-time surface is associative and thus quaternionic. That normal space rather than tangent space must be associative became clear last year [L99, L100].

3. $M^8 - H$ duality maps these algebraic surfaces in M^8 to $H = M^4 \times CP_2$ and the one obtains the usual dynamics based on variational principle giving minimal surfaces which are non-linear analogs for the solutions of massless field equations. Instead of polynomials the natural functions at the level of H are periodic functions used in Fourier analysis [L117].

At level of complexified M^8 cognitive representation would consist of points of co-associative space-time surface X^4 in complexified M^8 (complexified octonions), whose coordinates belong to extension of rationals and therefore make sense also p-adically for extension of p-adic numbers induced by extension of rationals. $M^8 - H$ duality maps the cognitive representations to H.

Cognitive representations form a hierarchy: the larger the extension of rationals, the larger the number of points in the extension and in the unique discretization of space-time surface. Therefore also the measurement resolution improves.

The surprise was that the cognitive representations which are typically finite, are for the "roots" of octonionic polynomials infinite [L99, L100]. Also in this case the density of points of cognitive representation increases as the dimension of extensions increases.

The understanding of the physical interpretation of $M^8 - H$ duality increased dramatically during the last half year.

1. X^4 in M^8 is highly analogous to momentum space (4-D analog of Fermi ball one might say) and H to position space. Physical states correspond to discrete sets of points - 4-momenta - in X^4. This is just the description used in particle physics for physical states. Time and space in this description are replaced by energy and 4-momentum. At the level of H one space-time and classical fields and one talks about frequencies and wavelengths instead of momenta.
2. $M^8 - H$ duality is a generalization of Fourier transform. Hitherto I have assumed that the space-time surface in M^8 is mapped to H. The momentum space interpretation at the level of M^8 however requires that the image must be a superposition of translates of the image in plane wave with some momentum: only the translates inside some bigger CD are allowed - this means infrared cutoff.

   The total momentum as sum of momenta for two half-cones of CD in M^8 is indeed well-defined. One has a generalization of a plane wave over translational degrees of freedom of CD and restricted to a bigger CD.

   At the limit of infinitely large size for bigger CD, the result is non-vanishing only when the sum of the momenta for two half-cones of CD vanishes: this corresponds to conservation of 4-momentum as a consequence of Poincare invariance rather than assumption as in the earlier approach [L117].

   This generalizes the position-momentum duality of wave mechanics lost in quantum field theory. Point-like particle becomes a quantum superposition of space-time surfaces inside the causal diamond (CD). Plane wave is a plane wave for the superposition of space-time surfaces inside CD having the cm coordinates of CD as argument.

3. Inclusion hierarchy of supersymplectic algebras and FMR

The third inclusion hierarchy allowing to describe finite measurement resolution is defined by supersymplectic algebras acting as the isometries of the "world of classical worlds" (WCW) consisting of space-time surfaces are preferred extremals ("roots" of polynomials in M^8 and minimal surfaces satisfying infinite-D set of additional "gauge conditions" in H).

At a given level of hierarchy generators with conformal weight larger than n act like gauge generators as also their commutators with generators with conformal weight smaller than n correspond to vanishing Noether charges. This defines "gauge conditions".

To sum up, there are therefore 3 hierarchies allowing to describe finite measurement resolution and they must be essentially equivalent in TGD framework.

Three evolutionary hierarchies

There are three evolutionary hierarchies: hierarchies of extensions of extensions of... ofrationals...; inclusions of inclusions of .... of HFFs, and inclusions of isomorphic super symplectic algebras.

1. Extensions of rationals

The extensions of rationals become algebraically increasingly complex as their dimension increases. The co-associative space-time surfaces in M^8 are "roots" of real polynomials with rational coefficients to guarantee number theoretical universality and this means space-time surfaces are characterized by extension of rationals.

Each extension of rationals defines extensions for p-adic number fields and entire adele. The interpretation is as a cognitive leap: the system's intelligence/algebraic complexity increases when the extension is extended further.

The extensions of extensions of .... define hierarchies with Galois groups in certain sense products of extensions involved. Exceptional extensions are those which do not allow this decomposition. In this case Galois group is a simple group. Simple groups are primes of finite groups and correspond to elementary particles of cognition. Kind of fundamental, non-decomposable ideas. Mystic might speak of pure states of consciousnesswith no thoughts.

In the evolution by quantum jumps the dimension of extension increases in statistical sense and evolution is unavoidable. This evolution is due to subjective time evolution by quantum jumps, something which is in spirit with Connes proposal but replaces time evolution by a sequence of evolutionary leaps.

2. Inclusions of HFFs as a hierarchy

HFFs are fractals. They have infinite inclusion hierarchies in which sub-HFF isomorphicto entire HFFs is included to HFF.

Also the hierarchies of inclusions define evolutionary hierarchies: HFF which is isomorphic with original becomes larger and in some sense more complex than the included factor. Also now one has sequences of inclusions of inclusions of.... These sequences would correspond to sequences for extensions of extensions... of rationals. Note that the inclusion hierarchy would be the basic object: not only single HFF in the hierarchy.

3. Inclusions of supersymplectic algebras as an evolutionary hierarchy

The third hierarchy is defined by the fractal hierarchy of sub-algebras of supersymplectic algebra isomorphic to the algebra itself. At a given level of hierarchy generators with conformal weight larger than n correspond to gauge degrees of freedom. As n increases the number of physical degrees of freedom above measurement resolution increases which means evolution. This hierarchy should correspond rather concretely to that for the extensions of rationals. These hierarchies would be essentially one and the same thing in the TGD Universe.

TGD based model for subjective time development

The understanding of subjective time development as sequences of SSFRs preceded by unitary "time" evolution has improved quite considerably recently [L117]. The idea is that the subjective time development as a sequence of scalings at the light-cone boundary generated by the vibrational part $\hat{L}_0$ of the scaling generator $L_0 = p^2 - \hat{L}_0$ (L_0 annihilates the physical states). Also p-adic mass calculations use $\hat{L}_0$.

For more than 10 years ago [K67, K43], I considered the possibility that Connes time evolution operator that he assigned with thermo-dynamical time could have a significant role in the definition of S-matrix in standard sense but had to give up the idea.

It however seems that for super-symplectic algebra $\hat{L}_0$ generates an outer automorphism since the algebra has only generators with conformal with $n > 0$ and its extension to included also generators with $n \leq 0$ is required to introduce L_0: since L_0 contains annihilation operators, it indeed generates outer automorphism in SCA. The two views could be equivalent! Whereas Connes considered thermo-dynamical time evolution, in TGD framework the time evolution would be subjective time evolution by SSFRs.

1. The guess would be that the exponential of the scaling operator L_0 gives the time evolution. The problem is that L_0 annihilates the physical states. The solution of the problem would be the same as in p-adic thermodynamics. L_0 decomposes as $L_0 = p^2 - \hat{L}_0$ and the vibrational part $\hat{L}_0$ this gives mass spectrum as eigenvalues of p^2. The thermo-dynamical state in p-adic thermodynamics is $p^{\hat{L}_0\beta}$. This operator exists p-adically in the p-adic number field defined by prime p.

2. Could unitary subjective time development involve the operator $exp(i2\pi L_0\tau)$ $\tau = log(T/T_0)$? This requires $T/T_0 = exp(n/m)$ guaranteeing that exponential is a root of unity for an eigenstate of L_0. The scalings are discretized and scalings come as powers of $e^{1/m}$. This is possible using extensions of rationals generated by a root of e. The unique feature of p-adics is that e^p is ordinary p-adic number. This alone would give periodic time evolution for eigenstates of L_0 with integer eigenvalues n.

SSA and SSA_n

Supersymplectic algebra SSA has fractal hierarchies of subalgebras SSA_n. The integers in a given hierarchy are of forn $n_1, n_1n_2, n_1n_2n_3, ...$ and correspond naturally to hierarchies of inclusions of HFFs. Conformal weights are positive: $n > 0$. For ordinary conformal algebras also negative weights are allowed. Yangians have only non-negative weights. This is of utmost importance.

SSA_n with generators have radial light-like conformal weights coming as multiples of n. SSA_n annihilates physical states and $[SSA_n, SSA]$ does the same. Hence the generators with conformal weight larger than n annihilate the physical states.

What about generators with conformal weights smaller than n? At least a subset of them need not annihilate the physical states. Since L_n are superpositions of creation operators, the idea that analogs of coherent states could be in question.

It would be nice to have a situation in which L_n, $n < m$ commute. $[L_k, L_l] = 0$ effectively for $k + l \geq m$.

The simplest way to obtain a set of effectively commuting operators is to take the generators L_k, $[m/2] < k < m$, where $[m/2]$ is nearest integer larger than $m/2$.

This raises interesting questions.

1. Could the Virasoro generators $O(\{c_k\}) = \sum_{k\in[m/2],m]} c_kL_k$ as linear combinations of creation operators generate a set of coherent states as eigenstates of their Hermitian conjugates.
2. Some facts about coherent states are in order.
   (a) When one adds to quantum harmonic oscillator Hamiltonian oscillator a time dependent perturbation which lasts for a finite the vacuum state evolves to an oscillator vacuum whose position is displacemented. The displacement is complex and is a Fourier component of the external force $f(t)$ corresponding to the harmonic oscillator frequency ω. Time evolution picks up only this component.
   (b) Coherent state property means that the state is eigenstate of the annihilation creation operator with eivengeu $\alpha = -ig(\omega)$ where $g(omega) = \int f(u)exp(-i\omega u)du$ is Fourier transform of $f(t)$.
   (c) Coherent states are not orthogonal and form an overcomplete set. The overlaps of coherent states are proportional to a Gaussian depending on the complex parameters characterizing them. One can however develop any state in terms of coherent states as a unique expansion since one can represent unitary in terms of coherent states.
   (d) Coherent state obtained from the vacuum state by time evolution in presence of $f(t)$ by a unitary displacement operator $D(\alpha) = exp(\alpha a^\dagger - \overline{\alpha}a)$. (`https://en.wikipedia.org/wiki/Displacement_operator`).
   The displacement operator is a unitary operator and in the general case the displacement is complex. The product of two displacement operators would be apart from a phase factor a displacement operator associated with the sum of displacements.
   (e) Harmonic oscillator coherent states are indeed maximally classical since wave packets have minimal width in both q and p space. Furthermore, the classical expectation values for q and p obey classical equations of motion.

These observations raise interesting questions about how the evolution by SSFRs could be modelled.

1. Instead of harmonic oscillator in q-space, one would have time evolution in the space of scalings of causal diamond parameterized by the scaling parameter $\tau = log(T/T_0)$, where T can be identified as the radial light-like coordinate of light-cone boundary.
   The analogs of harmonic oscillator states would be defined in this space and would be essentially wave packets with ground state minimizing the width of the wave packet.

2. The role of harmonic oscillator Hamiltonian in absence of external force would be taken by the generator $\hat{L}_0$ ($L_0 = p^2 - \hat{L}_0$ acts trivially) and gives rise to mass squared quantization. The situation would be highly analogous to that in p-adic thermodynamics. The role of ω would be taken by the minimal conformal weight h_{min} such that the eigenvalues of L_0 are its multiples. It seems that this weight must be equal to $h_{min} = 1$.

   The commutations of $\hbar L_0$ with L_k, $k > 0$ would be as for L_0 so what the replacement should not affect the situation.

3. The scaling parameter τ is analogous to the spatial coordinate q for the harmonic oscillator. Can one identify the analog of the external force $f(t)$ acting during unitary evolution between two SSFRs? Or is it enough to use only the analog of $g(\omega \rightarrow h_{min} = 1)$ - that is the coefficients C_k.

   To identify $f(t)$, one needs a time coordinate t. This was already identified as τ. This one would have $q = t$, which looks strange. The space in which time evolution is the space of scalings and the time evolutions are scalings and thus time evolution means translation in this space. The analog for this would be Hamiltonian $H = i\hbar d/dq$.

   Number theoretical universality allows only the values of $\tau = r/s$ whose exponents give roots of unity. Also $exp(n\tau)$ makes sense p-adically for these values. This would mean that the Fourier transform defining g would become discrete and be sum over the values $f(\tau = r/s)$.

4. What happens if one replaces $\hat{L}_0$ with L_0. In this case one would have the replacement of ω with $h_{vac} = 0$. Also the analog of Fourier transform with zero frequency makes sense. $\hat{L_0} = p^2 - L_0$ is the most natural choice for the Hamiltonian defining the time evolution operator but is trivial. Could $\Delta^{i\tau}$ describe the inherent time evolution. It would be outer automorphism since it is not defined solely in terms of SCA. So: could one have $\Delta = exp(\hat{L_0})$ so that $\Delta^{i\tau}$ coincide with $exp(i\hat{L_0}\tau)$? This would mean the identification

$$\Delta = exp(\hat{L_0}) \ ,$$

   which is a positive definite operator. The exponents coming from $exp(iL_0\tau)$ can be number theoretically universal if $\tau = log(T/T_0)$ is a rational number implying $T/T_0 = exp(r/s)$, which is possible number theoretically) and the extension of rationals contains some roots of e.

5. Could one have $\Delta = L_0$? Also now that positivity condition would be satisfied if SSA conformal weights satisfy $n > 0$.

   The problem with this operation is that it is not number theoretically universal since the exponents $exp(ilog(n)\tau)$ do not exist p-adically without introducing infinite-D extension of p-adic number making $log(n)$ well-defined.

   What is however intriguing is that the "time" evolution operator $\Delta^{i\tau}$ in the eigenstate basis would have trace equal to $Tr(\Delta^{i\tau}) \sum d(n) n^{i\tau}$, where $d(n)$ is the degeneracy of the state. This is a typical zeta function: for Riemann Zeta one has $d(n) = 1$.

   For $\Delta = exp(L_0)$ option $Tr(\Delta^{i\tau}) = \sum d(n) exp(in\tau)$ exists for $\tau = r/s$ if r:th root of e belongs to the extension of p-adics.

To sum up, one would have Gaussian wave packet as harmonic oscillator vacuum in the space of scaled variants of CD. The unitary time evolution associated with SSFR would displace the peak of the wave packet to a larger scalings. The Gaussian wave function in the space of scaled CDs has been proposed earlier.

Could this time evolution make sense and be even realistic?

1. The analogs of harmonic oscillator states are defined in the space of scalings as Gaussians and states obtained from them using oscillator operators. There would be a wave function in the moduli space of CDs analogous to a state of harmonic oscillator.

2. SSFR following the time evolutions would project to an eigenstate of harmonic oscillator having in general displaced argument. The unitary displacement operator D should commute with the operators having the members of zero energy states at the passive boundary of CD as eigenstates. This poses strong conditions. At least number theoretic measurements could satisfy these conditions.

3. SSFRs are identified as weak measurements as near as possible to classical measurements. Time evolution by the displacement would be indeed highly analogous to classical time evolution for theeharmonic oscillator.

4. The unitary displacement operator corresponds to the arbitrary external force on the harmonic oscillator and it seems that it would be selected in SSFR for the unitary evolution after SSFR. This means fixing the coefficients C_k in the operator $\sum C_k L_k$.

   What is the subjective "time" evolution operator when in the case of SSA_n?

1. The scaling analog of the unitary displacement operator D as $D = \sum exp(\sum C_k L_k - \overline{C}_k L_{-k})$ is highly suggestive and would take the oscillator vacuum to a coherent state. Coefficients C_k would be proportional to τ. There would be a large number of choices for the unitary displacement operator. One can also consider complex values of τ since one has complexified M^8.

2. There should be a normalization for the coefficients: without this it is not possible to talk about a special value of τ does not make sense. For instance, the sum of their moduli squared could be equal to 1. This would give interpretation as a quantum state in the degrees of freedom considered. The width of the Gaussian would increase slowly during the unitary time evolution and be proportional to $log(T/T_0)$.

   The width of the Gaussian would increase slowly as a function of T during the unitary time evolution and be proportional to $log(T/T_0)$. The condition that c_k are proportional the same complex number times τ is too strong.

3. The arbitrariness in the choice of C_k would bring in a kind of non-determinism as a selection of this superposition. The ability to engineer physical systems is in conflict with the determinism of classical physics and also difficult to understand in standard quantum physics. Could one interpret this choice as an analog for engineering a Hamiltonian as in say quantum computation or build-up of an electric circuit for some purpose? Could goal directed action correspond to this choice?

   If so engineerable degrees of freedom would correspond to intermediate degrees of freedom associated with L_k, $[m/2] \leq k \leq m$. They would be totally absent for $k = 1$ and this would correspond to a situation analogous to the standard physics without any intentional action.

2.5 MIP*= RE: What could this mean physically?

I received a very interesting link to a popular article (`https://cutt.ly/sfd5UQF`) explaining a recently discovered deep result in mathematics having implications also in physics. The article [A65] (`https://cutt.ly/rffiYdc`) by Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen has a rather concise title "**MIP*=RE**". In the following I try to express the impressions of a (non-mainstream) physicist about the result.

The following is the result expressed using the concepts of computer science about which I know very little at the hard technical level. The results are however told to state something highly non-trivial about physics.

1. RE (recursively enumerable languages) denotes all problems solvable by computer. P denotes the problems solvable in a polynomial time. NP does not refer to a non-polynomial time but to "non-deterministic polynomial acceptable problems" - I hope this helps the reader- I am a little bit confused! It is not known whether P = NP is true.

2. IP problems (P is now for "prover" that can be solved by a collaboration of an interrogator and prover who tries to convince the interrogator that her proof is convincing with high enough probability. MIP involves multiple l provers treated as criminals trying to prove that they are innocent and being not allowed to communicate. MIP* is the class of solvable problems in which the provers are allowed to entangle.

The finding, which is characterized as shocking, is that *all* problems solvable by a Turing computer belong to this class: MIP*=RE. All problems solvable by computer would reduce to problems in the class MIP*! Quantum computation would indeed add something genuinely new to the classical computation.

Quantum entanglement would play an essential role in quantum computation. Also the implications for physics are highly non-trivial.

1. Connes embedding problem asking whether all infinite-D matrices can always be approximated by finite-D matrices has a negative solution.Therefore MIP*= RE does not hold true for hyperfinite factors of type II_1 (HFFs) central in quantum TGD. Also the Tirelson problem finds a solution. The measurements of commuting observers performed by two observers are equivalent to the measurements of tensor products of observables only in finite-D case and for HFFs. That quantum entanglement would not have any role in HFFs is in conflict with intuition.

2. In the TGD framework finite measurement resolution is realized in terms of HFFs at Hilbert space level and in terms of cognitive representations at space-time level defined purely number-theoretically. This leads to a hierarchy of adeles defined by extensions of rationals and the Hilbert spaces must have algebraic extensions of rationals as a coefficient field. Therefore one cannot in general case find a unitary transformation mapping the entangled situation to an unentangled one, and quantum entanglement plays a key role. It seems that computationalism formulated in terms of recursive functions of natural numbers must be formulated for the hierarchy of extensions of rationals in terms of algebraic integers.
3. In TGD inspired theory of consciousness entanglement between observers could be seen as a kind of telepathy helping to perform conscious quantum computations. Zero energy ontology also suggests a modification of the views about quantum computation. TGD can be formulated also for real and p-adic continua identified as correlates of sensory experience and cognition, and it seems that computational paradigm need not apply in the continuum cases.

2.5.1 Two physically interesting applications

There are two physically interesting applications of the theorem interesting also from the TGD point of view and force to make explicit the assumptions involved.

About the quantum physical interpretation of MP*

To proceed one must clarify the quantum physical interpretation of MIP*.

Quantum measurement requires entanglement of the observer O with the measured system M. What is basically measured is the density matrix of M (or equivalently that of O). State function reduction gives as an outcome a state, which corresponds to an eigenvalue of the density matrix. Note that this state can be an entangled state if the density matrix has degenerate eigenvalues. Quantum measurement can be regarded as a question to the measured system: "What are the values of given commuting observables?". The final state of quantum measurement provides an eigenstate of the observables as the answer to this question. M would be in the role of the prover and O_i would serve as interrogators.

In the first case multiple interrogators measurements would entangle M with unentangled states of the tensor product $H_1 \otimes H_2$ for O followed by a state function reduction splitting the state of M to un-entangled state in the tensor product $M_1 \otimes M_2$.

In the second case the entire M would be interrogated using entanglement of M with entangled states of $H_1 \otimes H_2$ using measurements of several commuting observables. The theorem would state that interrogation in this manner is more efficient in infinite-D case unless HFFs are involved.

3. This interpretation differs from the interpretation in terms of computational problem solving in which one would have several provers and one interrogator. Could these interpretations be dual as the complete symmetry of the quantum measurement with respect to O and M suggests? In the case of multiple provers (analogous to accused criminals) it is advantageous to isolate them. In the case of multiple interrogators the best result is obtained if the interrogator does not effectively split itself into several ones.

Connes embedding problem and the notion of finite measurement/cognitive resolution

Alain Connes formulated what has become known as Connes embedding problem. The question is whether infinite matrices forming factor of type II_1 can be *always* approximated by finite-D matrices that is imbedded in a *hyperfinite* factor of type II_1 (HFF). Factors of type II and their HFFs are special classes of von Neumann algebras possibly relevant for quantum theory.

This result means that if one has measured of a complete set of for a product of commuting observables acting in the full space, one can find in the finite-dimensional case a unitary transformation transforming the observables to tensor products of observables associated with the factors of a tensor product. In the infinite-D case this is not true.

What seems to put alarms ringing is that in TGD there are excellent arguments suggesting that the state space has HFFs as building bricks. Does the result mean that entanglement cannot help in quantum computation in TGD Universe? I do not want to live in this kind of Universe!

Tsirelson problem

Tsirelson problem (see this) is another problem mentioned in the popular article as a physically interesting application. The problem relates to the mathematical description of quantum measurement.

Three systems are considered. There are two systems O_1 and O_2 representing observers and the third representing the measured system M. The measurement reducing the entanglement between M and O_1 or O_2 can regarded as producing correspondence between state of M and O_1 or O_2, and one can think that O_1 or O_2 measures only obserservables in its own state space as a kind of image of M. There are two ways to see the situation. The provers correspond now to the observers and the two situations correspond to provers without and with entanglement.

Consider first a situation in which one has single Hilbert space H and single observer O. This situation is analogous to IP.

1. The state of the system is described statistically by a density matrix - not necessarily pure state -, whose diagonal elements have interpretation as reduction probabilities of states in this bases. The measurement situation fixes the state basis used. Assume an ensemble of identical copies of the system in this state. Assume that one has a complete set of commuting observables.

2. By measuring all observables for the members of the ensemble one obtains the probabilities as diagonal elements of the density matrix. If the observable is the density matrix having no- degenerate eigenvalues, the situation is simplified dramatically. It is enough to use the density matrix as an observable. TGD based quantum measurement theory assumes that the density matrix describing the entanglement between two subsystems is in a universal observable measure in state function reductions reducing their entanglement.

3. Can one deduce also the state of M as a superposition of states in the basic chosen by the observer? This basis need not be the same as the basis defined by - say density matrix if the system has interacted with some system and this ineracton has led to an eigenstate of the density matrix. Assume that one can prepare the latter basis by a physical process such as this kind of interaction.

   The coefficients of the state form a set of N^2 complex numbers defining a unitary $N \times N$ matrix. Unitarity conditions give N conditions telling that the complex rows and unit vectors: these numbers are given by the measurement of all observables. There are also $N(N-1)$ conditions telling that the rows are orthogonal. Together these $N + N(N-1) = N^2$ numbers fix the elements of the unitary matrix and therefore the complex coefficients of the state basis of the system can be deduced from a complete set of measurements for all elements of the basis.

Consider now the analog of the MIS* involving more than one observer. For simplicity consider two observers.

1. Assume that the state space H of M decomposes to a tensor product $H = H_1 \otimes H_2$ of state spaces H_1 and H_2 such that O_1 measures observables X_1 in H_1 and O_2 measuresobservables X_2 in H_2. The observables X_1 and X_2 commute since they act in different tensor factors.

The absence of interaction between the factors corresponds to the inability of the provers to communicate. As in the previous case, one can deduce the probabilities for the various outcomes of the joint measurements interpreted as measurements of a complete set of observables $X_1 \otimes X_2$.

2. One can also think that the two systems form a single system O so that O_1 and O_2 can entangle. This corresponds to a situation in which entanglement between the provers is allowed. Now X_1 and X_2 are not in general independent but also now they must commute. One can deduce the probabilities for various outcomes as eigenstates of observables X_1X_2 and deduce the diagonal elements of the density matrix as probabilities.

Are these ways to see the situation equivalent? Tsirelson demonstrated that this is the case for finite-dimensional Hilbert spaces, which can indeed be decomposed to a tensor product of factors associated with O_1 and O_2. This means that one finds a unitary transformation transforming the entangled situation to an unentangled one and to tensor product observables.

For the infinite-dimensional case the situation remained open. According to the article, the new result implies that this is not the case. For hyperfinite factors the situation can be approximated with a finite-D Hilbert space so that the situations are equivalent in arbitrary precise approximation.

2.5.2 The connection with TGD

The result looks at first a bad news from the TGD point of view, where HFFs are highly suggestive. One must be however very careful with the basic definitions.

Measurement resolution

Measurement resolution is the basic notion.

1. There are intuitive physicist's arguments demonstrating that in TGD the operator algebras involved with TGD are HFFs provides a description of finite measurement resolution. The inclusion of HFFs defines the notion of resolution: included factor represents the degrees of freedom not seen in the resolution used [K112, K43] (`http://tgdtheoryd.fi/pfpool/vNeumann.pdf`) and `http://tgdtheoryd.fi/pfpool/vNeumannnew.pdf`).

   Hyperfinite factors involve new structures like quantum groups and quantum algebras reflecting the presence of additional symmetries: actually the "world of classical worlds" (WCW) as the space of space-time surfaces as maximal group of isometries and this group has a fractal hierarchy of isomorphic groups imply inclusion hierarchies of HFFs. By the analogs of gauge conditions this infinite-D group reduces to a hierarchy of effectively finite-D groups. For quantum groups the infinite number of irreps of the corresponding compact group effectively reduces to a finite number of them, which conforms with the notion of hyper-finiteness.

   It looks that the reduction of the most general quantum theory to TGD-like theory relying on HFFs is not possible. This would not be surprising taking into account gigantic symmetries responsible for the cancellation of infinities in TGD framework and the very existence of WCW geometry.

2. Second TGD based approach to finite resolution is purely number theoretic [L53] and involves adelic physics as a fusion of the real physics with various p-adic physics as correlates of cognition. Cognitive representations are purely number theoretic and unique discretizations of space-time surfaces defined by a given extension of rationals forming an evolutionary hierarchy: the coordinates for the points of space-time as a 4-surface of the embedding space $H = M^4 \times CP_2$ or of its dual M^8 are in the extension of rationals defining the adele. In the case of M^8 the preferred coordinates are unique apart from time translation. These two views would define descriptions of the finite resolution at the level of space-time and Hilbert space. In particular, the hierarchies of extensions of rationals should define hierarchies of inclusions of HFFs.

For hyperfinite factors the analog of MIP*=RE cannot hold true. Doesn't the TGD Universe allow a solution of all the problems solvable by Turing Computer? There is a loophole in this argument.

1. The point is that for the hierarchy of extensions of rationals also Hilbert spaces have as a coefficient field the extension of rationals! Unitary transformations are restricted to matrices with elements in the extension. In general it is not possible to realize the unitary transformation mapping the entangled situation to an un-entangled one! The weakening of the theorem would hold true for the hierarchy of adeles and entanglement would give something genuinely new for quantum computation!
2. A second deep implication is that the density matrix characterizing the entanglement between two systems cannot in general be diagonalized such that all diagonal elements identifiable as probabilities would be in the extension considered. One would have stable or partially stable entanglement (could the projection make sense for the states or subspaces with entanglement probability in the extension). For these bound states the binding mechanism is purely number theoretical. For a given extension of p-adic numbers one can assign to algebraic entanglement also information measure as a generalization of Shannon entropy as a p-adic entanglement entropy (real valued). This entropy can be negative and the possible interpretation is that the entanglement carries conscious information.

What about transcendental extensions?

During the writing of this article an interesting question popped up.

1. Also transcendental extensions of rationals are possible, and one can consider the generalization of the computationalism by also allowing functions in transcendental extensions. Could the hierarchy of algebraic extensions could continue with transcendental extensions? Could one even play with the idea that the discovery of transcendentals meant a quantum leap leading to an extension involving for instance e and π as basic transcendentals? Could one generalize the notion of polynomial root to a root of a function allowing Taylor expansion $f(x) = \sum q_n x^n$ with rational coefficients so that the roots of $f(x) = 0$ could be used define transcendental extensions of rationals?

2. Powers of e or its root define and infinite-D extensions having the special property that they are finite-D for p-adic number fields because e^p is ordinary p-adic number. In the p-adic context e can be defined as a root of the equation $x^p - \sum p^n/n! = 0$ making sense also for rationals. The numbers $log(p_i)$ such that p_i appears a factor for integers smaller than p define infinite-D extension of both rationals and p-adic numbers. They are obtained as roots of $e^x - p_i = 0$.
3. The numbers $(2n+1)\pi$ $(2n\pi)$ can be defined as roots of $sin(x) = 0$ $(cos(x) = 0$. The extension by π is infinite-dimensional and the conditions defining it would serve as consistency conditions when the extension contains roots of unity and effectively replaces them.
4. What about other transcendentals appearing in mathematical physics? The values $\zeta(n)$ of Riemann Zeta appearing in scattering amplitudes are for even values of n given by $\zeta(2n) = (-1)^{n+1}B_{2n}(2\pi)^{2n}/2(2n+1)!$. This follows from the functional identity for Riemann zeta and from the expression $\zeta(-n) = (-1)^n B_{n+1}/(n+1)$ ($(B(-1/2) = -1/2)$ (`https://cutt.ly/dfgtgmw`). The Bernoulli numbers B_n are rational and vanish for odd values of n. An open question is whether also the odd values are proportional to π^n with a rational coefficient or whether they represent "new" transcendentals.

What about the situation for the continuum version of TGD?

At least the cognitively finitely representable physics would have the HFF property with coefficient field of Hilbert spaces replaced by an extension of rationals. Number theoretical universality would suggest that HFF property characterizes also the physics of continuum TGD. This leads to a series of questions.

1. Does the new theorem imply that in the continuum version of TGD all quantum computations allowed by the Turing paradigm for real coefficients field for quantum states are not possible: $MIP* \subset RE$? The hierarchy of extensions of rationals allows utilization of entanglement, and one can even wonder whether one could have $MIP* = RE$ at the limit of algebraic numbers.

2. Could the number theoretic vision force change also the view about quantum computation? What does RE actually mean in this framework? Can one really assume complex entanglement coefficients in computation. Does the computational paradigm makes sense at all in the continuum picture?

   Are both real and p-adic continuum theories unreachable by computation giving rise to cognitive representations in the algebraic intersubsection of the sensory and cognitive worlds? I have indeed identified real continuum physics as a correlate for sensory experience and various p-adic physics as correlates of cognition in TGD: They would represent the computionally unreachable parts of existence.

   Continuum physics involves transcendentals and in mathematics this brings in analytic formulas and partial differential equations. At least at the level of mathematical consciousness the emergence of the notion of continuum means a gigantic step. Also this suggests that transcendentality is something very real and that computation cannot catch all of it.

3. Adelic theorem allows to express the norm of a rational number as a product of inverses of its p-adic norms. Very probably this representation holds true also for the analogs of rationals formed from algebraic integeres. Reals can be approximated by rationals. Could extensions of all p-adic numbers fields restricted to the extension of rationals say about real physics only what can be expressed using language?

Also fermions are highly interesting in the recent context. In TGD spinor structure can be seen as a square root of Kähler geometry, in particular for the "world of classical worlds" (WCW). Fermions are identified as correlates of Boolean cognition. The continuum case for fermions does not follow as a naïve limit of algebraic picture.

1. The quantization of the induced spinors in TGD looks different in discrete and continuum cases. Discrete case is very simple since equal-time anticommutators give discrete Kronecker deltas. In the continuum case one has delta functions possibly causing infinite vacuum energy like divergences in conserved Noether charges (Dirac sea).

2. In [L104] (`https://cutt.ly/zfftoK6`) I have proposed how these problems could be avoided by avoiding anticommutators giving delta-function. The proposed solution is based on zero energy ontology and TGD based view about space-time. One also obtains a long-sought-for concrete realization for the idea that second quantized induce spinor fields are obtained as restrictions of second quantized free spinor fields in $H = M^4 \times CP_2$ to space-time surface. The fermionic variant of $M^8 - H$-duality [L105] provides further insights and gives a very concrete picture about the dynamics of fermions in TGD.

These considerations relate in an interesting manner to consciousness. Quantum entanglement makes in the TGD framework possible telepathic sharing of mental images represented by sub-selves of self. For the series of discretizations of physics by HFFs and cognitive representations associated with extensions of rationals, the result indeed means something new.

What does one mean with quantum computation in TGD Universe?

The TGD approach raises some questions about computation.

1. The ordinary computational paradigm is formulated for Turing machines manipulating natural numbers by recursive algorithms. Programs would essentially represent a recursive function $n \to f(n)$. What happens to this paradigm when extensions of rationals define cognitive representations as unique space-time discretizations with algebraic numbers as the limit giving rise to a dense in the set of reals.

   The usual picture would be that since reals can be approximated by rationals, the situation is not changed. TGD however suggests that one should replace at least the quantum version of the Turing paradigm by considering functions mapping algebraic integers (algebraic rational) to algebraic integers.

   Quite concretely, one can manipulate algebraic numbers without approximation as a rational and only at the end perform this approximation and computations would construct recursive functions in this manner. This would raise entanglement to an active role even if one has

HFFs and even if classical computations could still look very much like ordinary computation using integers.

This suggests that computationalism usually formulated in terms of recursive functions of natural or rational numbers could be replaced with a hierarchy of computationalisms for the hierarchy of extensions of rationals. One would have recursively definable functions defined and having values in the extensions of rationals. These functions would be analogs of analytic functions (or polynomials) with the complex variable replaced with an integer or a rational of the extension. This poses very powerful constraints and there are good reasons to expect an increase of computational effectiveness. One can hope that at the limit of algebraic numbers of these functions allow arbitrarily precise approximations to real functions. If the real world phenomena can be indeed approximated by cognitive representations in the TGD sense, one can imagine a highly interesting approach to AI.

2. ZEO brings in also time reversal occurring in "big" (ordinary) quantum jumps and this modifies the views about quantum computation. In ZEO based conscious quantum computation halting means "death" and "reincarnation" of conscious entity, self? How the processes involving series of haltings in this sense differs from ordinary quantum computation: could one shorten the computation time by going forth and back in time.

There are many interesting questions to be considered. M^8-H duality gives justifications for the vision about algebraic physics. TGD leads also to the notion of infinite prime and I have considered the possibility that infinite primes could give a precise meaning for the dimension of infinite-D Hilbert space. Could the number-theoretic view about infinite be considerably richer than the idea about infinity as limit would suggest [K94].

The construction of infinite primes is analogous to a repeated second quantization of arithmetic supersymmetric quantum field theory allowing also bound states at each level and a concrete correspondence with the hierarchy of space-time sheets is suggestive. For the infinite primes at the lowest level of the hierarchy single particle states correspond to rationals and bound states to polynomials and therefore to the sets of their roots. This strongly suggests a connection with M^8 picture.

Could the number field of computable reals (p-adics) be enough for physics?

For some reason I have managed to not encounter the notion of computable number (see `https://cutt.ly/pTeSSfR`) as opposed to that of non-computable number (see `https://cutt.ly/gTeD9vF`). The reason is perhaps that I have been too lazy to take computationalism seriously enough.

Computable real number is a number, which can be produced to an arbitrary accuracy by a Turing computer, which by definition has a finite number of internal states, has input which is natural number and produces output which is natural numbers. Turing computer computes values of a function from natural numbers to itself by applying a recursive algorithm.

The following three formal definitions of the notion are equivalent.

1. The real number a is computable, if it can be expressed in terms of a computable function $n \rightarrow f(n)$ from natural numbers to natural numbers characterized by the property

$$\frac{f(n)-1}{n} \leq a \leq \ (\frac{f(n)+1}{n} .$$

For rational $a = q$, $f(n) = nq$ satisfies the conditions. Note that this definition does not work for p-adic numbers since they are not well-ordered.

2. The number a is computable if for an arbitrarily small rational number ϵ there exists a computable function producing a rational number r satisfying $|r - x \leq \epsilon$. This definition works also for p-adic numbers since it involves only the p-adic norm which has values which are powers of p and is therefore real valued.

3. a is computable if there exists a computable sequence of rational numbers r_i converging to a such that $|a - r_i| \leq 2^{-i}$ holds true. This definition works also for 2-adic numbers and its variant obtained by replacing 2 with the p-adic prime p makes sense for p-adic numbers.

The set R_c of computable real numbers and the p-adic counterparts $Q_{p,c}$ of R_c, have highly interesting properties.

1. R_c is enumerable and therefore can be mapped to a subset of rationals: even the ordering can be preserved. Also $Q_{p,c}$ is enumerable but now one cannot speak of ordering. As a consequence, most real (p-adic) numbers are non-computable. Note that the pinary expansion of a rational is periodic after some pinary digit. For a p-adic transcendental this is not the case.

2. Algebraic numbers are computable so that one can regard R_c as a kind of completion of algebraic numbers obtained by adding computable reals. For instance, π and e are computable. 2π can be computed by replacing the unit circle with a regular polygon with n sides and estimating the length as nL_n. L_n the length of the side. e can be computed from the standard formula. Interestingly, e^p is an ordinary p-adic number. An interesting question is whether there are other similar numbers. Certainly many algebraic numbers correspond to ordinary p-adic numbers.

3. R_c ($Q_{p,c}$) is a number field since the arithmetic binary operations $+, -\times, /$ are computable. Also differential and integral calculus can be constructed. The calculation of a derivative as a limit can be carried out by restricting the consideration to computable reals and there is always a computable real between two computable reals. Also Riemann sum can be evaluated as a limit involving only computable reals.

4. An interesting distinction between real and p-adic numbers is that in the sum of real numbers the sum of arbitrarily high digits can affect even all lower digits so that it requires computational work to predict the outcome. For p-adic numbers memory digits affect only the higher digits. This is why p-adic numbers are tailor made for computational purposes.Canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ used in p-adic mass calculations to map p-adic mass squared to its real counterpart [K60] maps p-adics to reals in a continuous manner. For integers this corresponds is 2-to-1 due to the fact that the p-adic numbers $-1 = (p-1)/(1-p)$ and $1/p$ are mapped to p.

5. For computable numbers, one cannot define the relation $=$. One can only define equality in some resolution ϵ. The category theoretical view about equality is also effective and conforms with the physical view.

   Also the relations $\leq$ and $\geq$ fail to have computable counterparts since only the absolute value $|x-y|$ can appear in the definition and one loses the information about the well-ordered nature of reals. For p-adic numbers there is no well-ordering so that nothing is lost. A restriction to non-equal pairs however makes order relation computable. For p-adic numbers the same is true.

6. Computable number is obviously definable but there are also definanable numbers, which are not computable. Examples are Gödel numbers in a given coding scheme for statements, which are true but not provable. More generally, the Gödel numbers coding for undecidable problems such as the halting problem are uncomputable natural numbers in a given coding scheme. Chaitin's constant, which gives the probability that random Turing computation halts, is a non-computable but definable real number.

7. Computable numbers are arithmetic numbers, which are numbers definable in terms of first order logic using Peano's axioms. First order logic does not allow statements about statements and one has an entire hierarchy of statements about... about statements. The hierarchy of infinite primes defines an analogous hierarchy in the TGD framework and is formally similar to a hierarchy of second quantizations [K94].

2.6 Analogs Of Quantum Matrix Groups From Finite Measurement Resolution?

The notion of quantum group [?]eplaces ordinary matrices with matrices with non-commutative elements. This notion is physically very interesting, and in TGD framework I have proposed that it should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution [?] These ideas have developed slowly through various side tracks.

In the sequel I will consider the notion of quantum matrix inspired by the recent view about quantum TGD relying on the notion of finite measurement resolution and show that under some additional conditions it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution.

1. The basic idea is to replace complex matrix elements with operators, which are products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers. Modulus and phase would be non-commuting and have commutation relation analogous to that between momentum and plane-wave in accordance with the idea about quantization of complex numbers.
2. The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. Strong/weak permutation symmetry of determinant requires its invariance apart from sign change under permutations of rows and/or columns. Weak permutation symmetry means development of determinant with respect to a fixed row or column and does not pose additional conditions. For weak permutation symmetry the permutation of rows/columns would however have a natural interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements and here quantum group structure emerges.
3. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

Quantum matrices define a more general structure than quantum group but provide a concrete representation for them in terms of finite measurement resolution, in particular when q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by a sign factor invariant under the permutations of both rows and columns. One can also understand the recursive fractal structure of inclusion sequences of hyper-finite factors resulting by replacing operators appearing as matrix elements with quantum matrices and a concrete connection with quantum groups emerges.

In Zero Energy Ontology (ZEO) M-matrix serving as the basic building brick of unitary U-matrix and identified as a hermitian square root of density matrix provides a possible application for this vision. Especially fascinating is the possibility of hierarchies of measurement resolutions represented as inclusion sequences realized as recursive construction of M-matrices. Quantization would emerge already at the level of complex numbers appearing as M-matrix elements.

This approach might allow to unify various ideas behind TGD. For instance, Yangian algebras emerging naturally in twistor approach are examples of quantum algebras. The hierarchy of Planck constants should have close relationship with inclusions and fractal hierarchy of sub-algebras of super-symplectic and other conformal algebras.

2.6.1 Well-definedness Of The Eigenvalue Problem As A Constraint To Quantum Matrices

Intuition suggests that the presence of degrees of freedom below measurement resolution implies that one must use density matrix description obtained by taking trace over the unobserved degrees of freedom. One could argue that in state function reduction with finite measurement resolution the outcome is not a pure state, or not even negentropically entangled state (possible in TGD framework) but a state described by a density matrix. The challenge is to describe the situation mathematically in an elegant manner.

1. There is present an infinite number of degrees of freedom below measurement resolution with which measured degrees of freedom entangle so that their presence affects the situation. One has a system with finite number degrees of freedom such as two-state system described by a quantum spinor. In this case observables as hermitian operators described by 2×2 matrices would be replaced by quantum matrices with elements, which in general do not commute.

   An attractive generalization of complex numbers appearing as elements of matrices is obtained by replacing them with products $H_{ij} = h_{ij}u_{ij}$ of hermitian operators h_{ij} with non-negative spectrum (modulus of complex number) and unitary operators u_{ij} (phase of complex number) suggests itself. The commutativity of h_{ij} and u_{ij} would look nice but is not necessary and is in conflict with the idea that modulus and phase of an amplitudes do not commute in quantum mechanics.

Very probably this generalization is trivial for mathematician. One could indeed interpret the generalization in terms of a tensor product of finite-dimensional matrices with possibly infinite-dimensional space of operators of Hilbert space. For the physicist the situation might be different as the following proposal for what hermitian quantum matrices could be suggests.

2. The modulus of complex number is replaced with a hermitian operator having non-negative eigenvalues. The representation as $h = AA^\dagger + A^\dagger A$ is would guarantee this. The phase of complex number would be replaced by a unitary operator U possibly allowing the representation $U = exp(iT)$, T hermitian. The commutativity condition

$$[h_{ij}, u_{ij}] = 0 \tag{2.6.1}$$

for a given matrix element is also suggestive but as already noticed, Uncertainty Principle suggests that modulus and phase do not commute as operators. The commutator of modulus and phase would naturally be equal to that between momentum operator and plane wave:

$$[h_{ij}, u_{ij}] = i\hbar \times u_{ij} \ , \tag{2.6.2}$$

Here $\hbar = h/2\pi$ can be chosen to be unity in standard quantum theory. In TGD it can be generalized to a hermitian operator H_{eff}/h with an integer valued spectrum of eigenvalues given by $h_{eff}/h = n$ so that ordinary and dark matter sectors would be unified to single structure mathematically.

3. The notions of eigenvalues and eigenvectors for a hermitian operator should generalize. Now hermitian operator H would be a matrix with formally the same structure as $N \times N$ hermitian matrix in commutative number field - say complex numbers - possibly satisfying additional conditions.

Hermitian matrix can be written as

$$H_{ij} = h_{ij}u_{ij} \quad \text{for i>j} \quad H_{ij} = u_{ij}h_{ij} \quad \text{for i<j} \ , \quad H_{ii} = h_i \ . \tag{2.6.3}$$

Hermiticity conditions $H_{ij} = H^\dagger_{ji}$ give

$$h_{ij} = h_{ji} \ , \quad u_{ij} = u^\dagger_{ji} \ . \tag{2.6.4}$$

Here it has been assumed that one has quantum SU(2). For quantum U(2) one would have $U_{11} = U^\dagger_{22} = h_a u_a$ with u_a commuting with other operators. The form of the conditions is same as for ordinary hermitian matrices and it is not necessary to assume commutativity $[h_{ij}, u_{ij}] = 0$. Generalization of Pauli spin matrices provides a simple illustration.

4. The well-definedness of eigenvalue problem gives a strong constraint on the notion of hermitian quantum matrix. Eigenvalues of hermitian operator are determined by the vanishing of determinant $det(H - \lambda I)$. Its expression involves sub-determinants and one must decide whether to demand that the definition of determinant is independent of which column or row one chooses to develop the determinant.

For ordinary matrix the determinant is expressible as sum of symmetric functions:

$$det(H - \lambda I) = \sum \lambda^n S_n(H) \ . \tag{2.6.5}$$

Elementary symmetric functions S_n - n-functions in following - have the property that they are sums of contributions from to n-element paths along the matrix with the property that path contains no vertical or horizontal steps. One has a discrete analog of path integral in which time increases in each step by unit. The analogy with fermionic path integral is also obvious. In the non-commutative case non-commutativity poses problems since different orderings of rows (or columns) along the same n-path give different results.

(a) For the first option one gives up the condition that determinant can be developed with respect to any row or column and defines determinant by developing it with respect to say first row or first column. If one developing with respect to the column (row) the permutations of rows (columns) do not affect the value of determinant or sub-determinants but permutations of columns (rows) do so unless one poses additional conditions stating that the permutations do not affect given contribution to the determinant or sub-determinant. It turns out that this option must be applied in the case of ordinary quantum group. For quantum phase $q = \pm 1$ the determinant is invariant under permutations of both rows and columns.

(b) Second manner to get rid of difficulty would be that n-path does not depend on the ordering of the rows (columns) differ only by the usual sign factor. For 2×2 case this would give

$$ad - bc = da - cb \ , \quad \text{(Option 2)} \tag{2.6.6}$$

These conditions state the invariance of the n-path under permutation group S_n permuting rows or columns.

(c) For the third option the elements along n-paths commute: paths could be said to be "classical". The invariance of N-path in this sense guarantees the invariance of all n-paths. In 2-D case this gives

$$[a, d] = 0 \ , \quad [b, c] = 0 \ . \quad \text{(Option 3)} \tag{2.6.7}$$

5. One should have a well-defined eigenvalue problem. If the n-functions commute, one can diagonalize the corresponding operators simultaneously and the eigenvalues problem reduces to possibly infinite number of ordinary eigenvalue problems corresponding to restrictions to given set of eigenvalues associated with $N - 1$ symmetric functions. This gives an additional constraint on quantum matrices.

   In 2-dimensional case one would have the condition

$$[ad - bc, a + d] = 0 \ . \tag{2.6.8}$$

   Depending on how strong S_2 invariance one requires, one obtains 0, 1, 2 nontrivial conditions for 2×2 quantum matrices and 1 condition from the commutativity of n-functions besides hermiticity conditions.

   For $N \times N$-matrices one would have $N! - 1$ non-trivial conditions from the strong form of permutation invariance guaranteeing the permutation symmetry of n-functions and $N(N - 1)/2$ conditions from the commutativity of n-functions.

6. The eigenvectors of the density matrix are obtained in the usual manner for each eigenvalue contributing to quantum eigenvalue. Also the diagonalization can be carried out by a unitary transformation for each eigenvalue separately. Hence the standard approach seems to generalize almost trivially.

   What makes the proposal non-trivial and possibly physically interesting is that the hermitian operators are not assumed to be just tensor products of $N \times N$ hermitian matrices with hermitian operators in Hilbert space.

The notion of unitary quantum matrix should also make sense. The naïve guess is that the exponentiation of a linear combination of ordinary hermitian matrices with coefficients, which are hermitian matrices gives quantum unitary matrices. In the case of U(1) the replacement of exponentiation parameter t in $exp(itX)$ with a hermitian operator gives standard expression for the exponent and it is trivial to see that unitary conditions are satisfied also in this case. Also in the case of SU(2) it is easy to verify that the guess is correct. One must also check that one indeed obtains a group: it could also happen that only semi-group is obtained.

In any case, one could speak of quantum matrix groups with coordinates replaced by hermitian matrices. These quantum matrix group need not be identical with quantum groups in the standard sense of the word. Maybe this could provide one possible meaning for quantization in the case of groups and perhaps also in the case of coset spaces G/H.

2.6.2 The Relationship To Quantum Groups And Quantum Lie Algebras

It is interesting to find out whether quantum matrices give rise to quantum groups under suitable additional conditions. The child's guess for these conditions is that the permutation of rows and columns correspond to braiding for the hermitian moduli h_{ij} defined by unitary operators U_{ij}.

Quantum groups and quantum matrices

The conditions for hermiticity and unitary do not involve quantum parameter q, which suggests that the naïve generalization of the notion of unitary matrix gives unitary group obtained by replacing complex number field with operator algebra gives group with coordinates defined by hermitian operators rather than standard quantum group. This turns out to be the case and it seems that quantum matrices provide a concrete representation for quantum group. The notion of braiding as that for operators h_{ij} can be said to emerge from the notion of quantum matrix.

1. Exponential of quantum hermitian matrix is excellent candidate for quantum unitary matrix. One should check the exponentiation indeed gives rise to a quantum unitary matrix. For $q = \pm 1$ this seems obvious but one should check this separately for other roots of unity. Instead of considering the general case, we consider explicit ansatz for unitary U(2) quantum matrix as $U = [a, b; -b^\dagger, a^\dagger]$. The conditions for unitary quantum group in the proposed sense would state the orthonormality and unit norm property of rows/columns.

   The explicit form of the conditions reads as

$$\begin{aligned} &ab - ba = 0 \ , \quad ab^\dagger = b^\dagger a \ , \\ &aa^\dagger + bb^\dagger = 1 \ , \quad a^\dagger a + b^\dagger b = 1 \ . \end{aligned} \tag{2.6.9}$$

   The orthogonality conditions are unique and reduce to the vanishing of commutators.

   Normalization conditions involve a choice of ordering. One possible manner to avoid the problem is to assume that both orderings give same unit length for row or column (as done above). If only the other option is assumed then only third or fourth equations is needed. The invariance of determinant under permutation of rows would imply $[a, a^\dagger] = [b, b^\dagger] = 0$ and the ordering problem would disappear.

2. One can look what conditions the explicit representation $U_{ij} = h_{ij}u_{ij}$ or equivalently $[h_a u_a, h_b u_b; -u_b^\dagger h_b, u_a^\dagger h_a]$ gives. The intuitive expectation is that U(2) matrix decomposes to a product of commutating SU(2) matrix and U(1) matrices. This implies that u_a commutes with the other matrices involved. One obtains the conditions

$$h_a h_b = h_b (u_b h_a u_b^\dagger) \ , \quad h_b h_a = (u_b h_a u_b^\dagger) h_b \ . \tag{2.6.10}$$

   These conditions state that the permutation of h_a and h_b analogous to braiding operation is a unitary operation.

For the purposes of comparison consider now the corresponding conditions for $SU(2)_q$ matrix.

1. The $\mathrm{SU}(2)_q$ matrix $[a, b; b^\dagger, a^\dagger]$ with *real* value of q (see `http://tinyurl.com/yb8tycag`) satisfies the conditions

$$\begin{aligned} &ba = qab \ , \quad b^\dagger a = qab^\dagger, \quad bb^\dagger = b^\dagger b \ , \\ &a^\dagger a + q^2 b^\dagger b = 1 \ , \quad aa^\dagger + bb^\dagger = 1 \ . \end{aligned} \tag{2.6.11}$$

This gives $[a^\dagger, a] = (1-q^2)b^\dagger b$. The above conditions would correspond to $q = \pm 1$ but with complex numbers replaced with operator algebra. q-commutativity obviously replaces ordinary commutativity in the conditions and one can speak of q-orthonormality.

For complex values of q - in particular roots of unity - the condition $a^\dagger a + q^2 b^\dagger b = 1$ is in general not self-consistent since hermitian conjugation transforms q^2 to its complex conjugate. Hence this condition must be dropped for complex roots of unity.

2. Only for $q = \pm 1$ corresponding to Bose-Einstein and Fermi-Dirac statistics the conditions are consistent with the invariance of n-functions (determinant) under permutations of both rows and columns. Indeed, if 2×2 q-determinant is developed with respect to column, the permutation of rows does not affect its value. This is trivially true also in $N \times N$ dimensional case since the permutation of rows does not affect the n-paths at all.

If the symmetry under permutations is weakened, nothing prevents from posing quantum orthogonality conditions also now and the decomposition to a product of positive and hermitian matrices give a concrete meaning to the notion of quantum group.

Do various $n-$functions commute with each other for $\mathrm{SU}(2)_q$? The only commutator of this kind is that for the trace and determinant and should vanish:

$$\left[b + b^\dagger, aa^\dagger + bb^\dagger\right] = 0 \ . \tag{2.6.12}$$

Since $a^\dagger a$ and $aa^\dagger$ are linear combinations of $b^\dagger b = b^\dagger b$, they vanish. Hence it seems that TGD based view about quantum groups is consistent with the standard view.

3. One can look these conditions in TGD framework by restricting the consideration to the case of SU(2) ($u_a = 1$) and using the ansatz $U = [h_a, h_b u_b; -u_b^\dagger h_b, h_a]$. Orthogonality conditions read as

$$h_a h_b = q h_b (u_b h_a u_b^\dagger) \ , \quad h_b h_a = q(u_b h_a u_b^\dagger) h_b \ .$$

If q is root of unity, these conditions state that the permutation of h_a and h_b analogous to a unitary braiding operation apart from a multiplication with quantum phase q. For $q = \pm 1$ the sign-factor is that in standard statistics. Braiding picture could help guess the commutators of h_{ij} in the case of $N \times N$ quantum matrices. The permutations of rows and columns would have interpretation as braidings and one could say that braided commutators of matrix elements vanish.

The conditions from the normalization give

$$h_a^2 + h_b^2 = 1 \ , \quad h_a^2 + q^2(u_b^\dagger h_b^2 u_b) = 1 \ . \tag{2.6.13}$$

For complex q the latter condition does not make sense since $h_a^2 - 1$ and $u_b^\dagger h_b^2 u_b$ are hermitian matrices with real eigenvalues. Also for real values of $q \neq \pm 1$ one obtains contradicion since the spectra of unitarily related hermitian operators would differ by scaling factor q^2. Hence one must give up the condition involving q^2 unless one has $q = \pm 1$. Note that the term proportional to q^2 does not allow interpretation in terms of braiding.

4. Roots of unity are natural number theoretically as values of q but number theoretical universality allows the generic value of q would be a complex number existing simultaneously in all p-adic number properly extended. This would suggest the spectrum of q to come as

$$q(m,n) = e^{1/m} exp(\frac{i2\pi}{n}) \ . \tag{2.6.14}$$

The motivation comes from the fact that e^p is ordinary p-adic number for all p-adic number fields so e and also any root of e defines a finite-dimensional extension of p-adic numbers [K111] [L25]. The roots of unity would be associated to the discretization of the ordinary angles in case of compact matrix groups. Roots of e would be associated with the discretization of hyperbolic angles needed in the case of non-compact matrix groups such as SL(2,C).

Also now unification of various values of q to single single operator Q, which is product of *commuting* hermitian and unitary operators and commuting with the hermitian operator H representing the spectrum of Planck constant would code the spectrum. Skeptic can of course wonder, whether the modulus and phase of Q can be assumed to commute. The relationship between integers associated with H and Q is interesting.

Quantum Lie algebras and quantum matrices

What about quantum Lie algebras? There are many notions of quantum Lie algebra and quantum group. General formulas for the commutation relations are well-known for Drinfeld-Jimbo type quantum groups (see `http://tinyurl.com/yb8tycag`). The simplest guess is that one just poses the defining conditions for quantum group, replaces complex numbers as coefficient module with operator algebra, and poses the above described conditions making possible to speak about eigenvalues and eigen vectors. One might however hope that this representation allows to realize the non-commutativity of matrix elements of quantum Lie algebra in a concrete manner.

1. For SU(2) the commutation relations for the elements X_+, X_-, h read as

$$[h, X_\pm] = \pm X_\pm \quad , \quad [X_+, X_-] = h \quad . \tag{2.6.15}$$

   Here one can use the 2×2 matrix representations for the ladder operators $X^\pm$ and diagonal angular momentum generator h.

2. For $SU(2)_q$ one has

$$[h, X_\pm] = \pm X_\pm \quad , \quad [X_+, X_-] = \frac{q^h - q^{-h}}{q - q^{-1}} \quad . \tag{2.6.16}$$

3. Using the ansatz for the generators but allowing hermitian operator coefficients in non-diagonal generators $X_\pm$, one obtains the condition
   For $SU(2)_q$ one would have

$$[X_+, X_-] = h_+^2 = h_-^2 = \frac{q^h - q^{-h}}{q - q^{-1}} \quad . \tag{2.6.17}$$

   Clearly, the proposal might make possible to have concrete representations for the quantum Lie algebras making the decomposition to measurable and directly non-measurable degrees of freedom explicit.

The conclusion is that finite measurement resolution does not lead automatically to standard quantum groups although the proposed realization is consistent with them. Also the quantum phases $q = \pm 1$ $n = 1, 2$ are realized and correspond to strong permutation symmetry and Bose-Einstein and Fermi statistics.

2.6.3 About Possible Applications

The realization for the notion of finite measurement resolution is certainly the basic application but one can imagine also other applications where hermitian and unitary matrices appear.

Density matrix description of degrees of freedom below measurement resolution

Density matrix ρ obtained by tracing over non-observable degrees of freedom is a fundamental example about a hermitian matrix satisfying the additional condition $Tr(\rho) = 1$.

1. A state function reduction with a finite measurement resolution would lead to a non-pure state. This state would be describable using $N \times N$-dimensional quantum hermitian quantum density matrix satisfying the condition $Tr(\rho) = 1$ (or more generally $Tr_q(\rho) = 1$), and satisfying the additional conditions allowing to reduce its diagonalization to that for a collection of ordinary density matrices so that the eigenvalues of ordinary density matrix would be replaced by N quantum eigenvalues defined by infinite-dimensional diagonalized density matrices.

2. One would have N quantum eigenvalues - quantum probabilities - each decomposing to possibly infinite set of ordinary probabilities assignable to the degrees of freedom below measurement resolution and defining density matrix for non-pure states resulting in state function reduction.

Some questions

Some further questions pop up naturally.

1. One might hope that the quantum counterparts of hermitian operators are in some sense universal, at least in TGD framework (by quantum criticality). Could the condition that the commutator of hermitian generators is proportional to $i\hbar$ times hermitian generator pose additional constraints? In 2-D case this condition is satisfied for quantum SU(2) generators and very probably the same is true also in the general case. The possible problems result from the non-commutativity but $(XY)^\dagger = Y^\dagger X^\dagger$ identity takes care that there are no problems.
2. One can also raise physics related questions. What one can say about most general quantum Hamiltonians and their energy spectra, say quantum hydrogen atom? What about quantum angular momentum? If the proposed construction is only a concretization of abstract quantum group construction, then nothing new is expected at the level of representations of quantum groups.
3. Could the spectrum of h_{eff} define a quantum h as a hermitian positive definite operator? Could this allow a description for the presence of dark matter, which is not directly observable? Same question applies to the quantum parameter q.
4. M-matrices are basic building bricks of scattering amplitudes in ZEO. M-matrix is produce of hermitian "complex" square root H of density matrix satisfying $H^2 = \rho$ and unitary S-matrix S. It has been proposed that these matrices commute. The previous consideration relying on basic quantum thinking suggests that they relate like translation generator in radial direction and phase defined by angle and thus satisfy $[H, S] = i(H_{eff}/h) \times S$. This would give enormously powerful additional condition to S-matrix. One can also ask whether M-matrices in presence of degrees of freedom below measurement resolution is quantum version of M-matrix in the proposed sense.
5. Fractality is of of the key notions of TGD and characterizes also hyperfinite factors. I have proposed some realizations of fractality such as infinite primes and finite-dimensional Hilbert spaces taking the role of natural numbers and ordinary sum and product replaced with direct sum and tensor product. One could also imagine a fractal hierarchy of quantum matrices obtained by replacing the operators appearing as matrix elements of quantum matrix element by quantum matrices. This hierarchy could relate to the sequence of inclusions of HFFs.

2.7 Jones Inclusions And Cognitive Consciousness

WCW spinors have a natural interpretation in terms of a quantum version of Boolean algebra. Beliefs of various kinds are the basic element of cognition and obviously involve a representation of the external world or part of it as states of the system defining the believer. Jones inclusions mediating unitary mappings between the spaces of WCWs spinors of two systems are excellent candidates for these maps, and it is interesting to find what one kind of model for beliefs this picture leads to.

The resulting quantum model for beliefs provides a cognitive interpretation for quantum groups and predicts a universal spectrum for the probabilities that a given belief is true. This spectrum depends only on the integer n characterizing the quantum phase $q = exp(i2\pi/n)$ characterizing the Jones inclusion. For $n \neq \infty$ the logic is inherently fuzzy so that absolute knowledge is impossible. $q = 1$ gives ordinary quantum logic with qbits having precise truth values after state function reduction.

2.7.1 Does One Have A Hierarchy Of U- And M-Matrices?

U-matrix describes scattering of zero energy states and since zero energy states can be illustrated in terms of Feynman diagrams one can say that scattering of Feynman diagrams is in question.

The initial and final states of the scattering are superpositions of Feynman diagrams characterizing the corresponding M-matrices which contain also the positive square root of density matrix as a factor.

The hypothesis that U-matrix is the tensor product of S-matrix part of M-matrix and its Hermitian conjugate would make U-matrix an object deducible by physical measurements. One cannot of course exclude that something totally new emerges. For instance, the description of quantum jumps creating zero energy state from vacuum might require that U-matrix does not reduce in this manner. One can assign to the U-matrix a square like structure with S-matrix and its Hermitian conjugate assigned with the opposite sides of a square.

One can imagine of constructing higher level physical states as composites of zero energy states by replacing the S-matrix with M-matrix in the square like structure. These states would provide a physical representation of U-matrix. One could define U-matrix for these states in a similar manner. This kind of hierarchy could be continued indefinitely and the hierarchy of higher level U and M-matrices would be labeled by a hierarchy of n-cubes, $n = 1, 2$,... TGD inspired theory of consciousness suggests that this hierarchy can be interpreted as a hierarchy of abstractions represented in terms of physical states. This hierarchy brings strongly in mind also the hierarchies of n-algebras and n-groups and this forces to consider the possibility that something genuinely new emerges at each step of the hierarchy. A connection with the hierarchies of infinite primes [K94] and Jones inclusions are suggestive.

2.7.2 Feynman Diagrams As Higher Level Particles And Their Scattering As Dynamics Of Self Consciousness

The hierarchy of inclusions of hyper-finite factors of II_1 as counterpart for many-sheeted space-time lead inevitably to the idea that this hierarchy corresponds to a hierarchy of generalized Feynman diagrams for which Feynman diagrams at a given level become particles at the next level. Accepting this idea, one is led to ask what kind of quantum states these Feynman diagrams correspond, how one could describe interactions of these higher level particles, what is the interpretation for these higher level states, and whether they can be detected.

Jones inclusions as analogs of space-time surfaces

The idea about space-time as a 4-surface replicates itself at the level of operator algebra and state space in the sense that Jones inclusion can be seen as a representation of the operator algebra $\mathcal{N}$ as infinite-dimensional linear sub-space (surface) of the operator algebra $\mathcal{M}$. This encourages to think that generalized Feynman diagrams could correspond to image surfaces in II_1 factor having identification as kind of quantum space-time surfaces.

Suppose that the modular S-matrices are representable as the inner automorphisms $\Delta(\mathcal{M}_k^{it}$ assigned to the external lines of Feynman diagrams. This would mean that $\mathcal{N} \subset \mathcal{M}_k$ moves inside $calM_k$ along a geodesic line determined by the inner automorphism. At the vertex the factors $calM_k$ to fuse along $\mathcal{N}$ to form a Connes tensor product. Hence the copies of $\mathcal{N}$ move inside $\mathcal{M}_k$ like incoming 3-surfaces in H and fuse together at the vertex. Since all $\mathcal{M}_k$ are isomorphic to a universal factor $\mathcal{M}$, many-sheeted space-time would have a kind of quantum image inside II_1 factor consisting of pieces which are $d = \mathcal{M} : \mathcal{N}/2$-dimensional quantum spaces according to the identification of the quantum space as subspace of quantum group to be discussed later. In the case of partonic Clifford algebras the dimension would be indeed $d \leq 2$.

The hierarchy of Jones inclusions defines a hierarchy of S-matrices

It is possible to assign to a given Jones inclusion $\mathcal{N} \subset \mathcal{M}$ an entire hierarchy of Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2 ...$, $\mathcal{M}_0 = N$, $\mathcal{M}_1 = M$. A possible interpretation for these inclusions would be as a sequence of topological condensations.

This sequence also defines a hierarchy of Feynman diagrams inside Feynman diagrams. The factor $\mathcal{M}$ containing the Feynman diagram having as its lines the unitary orbits of $\mathcal{N}$ under $\Delta_{\mathcal{M}}$ becomes a parton in $\mathcal{M}_1$ and its unitary orbits under $\Delta_{\mathcal{M}_1}$ define lines of Feynman diagrams in M_1. The concrete representation for M-matrix or projection of it to some subspace as entanglement coefficients of partons at the ends of a braid assignable to the space-like 3-surface representing a vertex of a higher level Feynman diagram. In this manner quantum dynamics would be coded and simulated by quantum states.

The outcome can be said to be a hierarchy of Feynman diagrams within Feynman diagrams, a fractal structure for which many particle scattering events at a given level become particles at the next level. The particles at the next level represent dynamics at the lower level: they have the property of "being about" representing perhaps the most crucial element of conscious experience. Since net conserved quantum numbers can vanish for a system in TGD Universe, this kind of hierarchy indeed allows a realization as zero energy states. Crossing symmetry can be understood in terms of this picture and has been applied to construct a model for M-matrix at high energy limit [K29] .

One might perhaps say that quantum space-time corresponds to a double inclusion and that further inclusions bring in N-parameter families of space-time surfaces.

Higher level Feynman diagrams

The lines of Feynman diagram in $\mathcal{M}_{n+1}$ are geodesic lines representing orbits of $\mathcal{M}_n$ and this kind of lines meet at vertex and scatter. The evolution along lines is determined by $\Delta_{\mathcal{M}_{n+1}}$. These lines contain within themselves $\mathcal{M}_n$ Feynman diagrams with similar structure and the hierarchy continues down to the lowest level at which ordinary elementary particles are encountered.

For instance, the generalized Feynman diagrams at the second level are ribbon diagrams obtained by thickening the ordinary diagrams in the new time direction. The interpretation as ribbon diagrams crucial for topological quantum computation and suggested to be realizable in terms of zero energy states in [K7] is natural. At each level a new time parameter is introduced so that the dimension of the diagram can be arbitrarily high. The dynamics is not that of ordinary surfaces but the dynamics induced by the $\Delta_{\mathcal{M}_n}$.

Quantum states defined by higher level Feynman diagrams

The intuitive picture is that higher level quantum states corresponds to the self reflective aspect of existence and must provide representations for the quantum dynamics of lower levels in their own structure. This dynamics is characterized by M-matrix whose elements have representation in terms of Feynman diagrams.

1. These states correspond to zero energy states in which initial states have "positive energies" and final states have "negative energies". The net conserved quantum numbers of initial and final state partons compensate each other. Gravitational energies, and more generally gravitational quantum numbers defined as absolute values of the net quantum numbers of initial and final states do not vanish. One can say that thoughts have gravitational mass but no inertial mass.

2. States in sub-spaces of positive and negative energy states are entangled with entanglement coefficients given by M-matrix at the level below.

To make this more concrete, consider first the simplest non-trivial case. In this case the particles can be characterized as ordinary Feynman diagrams, or more precisely as scattering events so that the state is characterized by $\hat{S} = P_{in}SP_{out}$, where S is S-matrix and P_{in} *resp.* P_{out} is the projection to a subspace of initial *resp.* final states. An entangled state with the projection of S-matrix giving the entanglement coefficients is in question.

The larger the domains of projectors P_{in} and P_{out}, the higher the representative capacity of the state. The norm of the non-normalized state $\hat{S}$ is $Tr(\hat{S}\hat{S}^{\dagger}) \leq 1$ for II_1 factors, and at the limit $\hat{S} = S$ the norm equals to 1. Hence, by II_1 property, the state always entangles infinite number of states, and can in principle code the entire S-matrix to entanglement coefficients.

The states in which positive and negative energy states are entangled by a projection of S-matrix might define only a particular instance of states for which conserved quantum numbers vanish. The model for the interaction of Feynman diagrams discussed below applies also to these more general states.

The interaction of $\mathcal{M}_n$ Feynman diagrams at the second level of hierarchy

What constraints can one pose to the higher level reactions? How Feynman diagrams interact? Consider first the scattering at the second level of hierarchy ($\mathcal{M}_1$), the first level $\mathcal{M}_0$ being assigned to the interactions of the ordinary matter.

1. Conservation laws pose constraints on the scattering at level $\mathcal{M}_1$. The Feynman diagrams can transform to new Feynman diagrams only in such a way that the net quantum numbers are conserved separately for the initial positive energy states and final negative energy states of the diagram. The simplest assumption is that positive energy matter and negative energy matter know nothing about each other and effectively live in separate worlds. The scattering matrix form Feynman diagram like states would thus be simply the tensor product $S \otimes S^\dagger$, where S is the S-matrix characterizing the lowest level interactions and identifiable as unitary factor of M-matrix for zero energy states. Reductionism would be realized in the sense that, apart from the new elements brought in by $\Delta_{\mathcal{M}_n}$ defining single particle free dynamics, the lowest level would determine in principle everything occurring at the higher level providing representations about representations about... for what occurs at the basic level. The lowest level would represent the physical world and higher levels the theory about it.

2. The description of hadronic reactions in terms of partons serves as a guide line when one tries to understand higher level Feynman diagrams. The fusion of hadronic space-time sheets corresponds to the vertices $\mathcal{M}_1$. In the vertex the analog of parton plasma is formed by a process known as parton fragmentation. This means that the partonic Feynman diagrams belonging to disjoint copies of $\mathcal{M}_0$ find themselves inside the same copy of $\mathcal{M}_0$. The standard description would apply to the scattering of the initial *resp.* final state partons.

3. After the scattering of partons hadronization takes place. The analog of hadronization in the recent case is the organization of the initial and final state partons to groups I_i and F_i such that the net conserved quantum numbers are same for I_i and F_i. These conditions can be satisfied if the interactions in the plasma phase occur only between particles belonging to the clusters labeled by the index i. Otherwise only single particle states in $\mathcal{M}_1$ would be produced in the reactions in the generic case. The cluster decomposition of S-matrix to a direct sum of terms corresponding to partitions of the initial state particles to clusters which do not interact with each other obviously corresponds to the "hadronization". Therefore no new dynamics need to be introduced.

4. One cannot avoid the question whether the parton picture about hadrons indeed corresponds to a higher level physics of this kind. This would require that hadronic space-time sheets carry the net quantum numbers of hadrons. The net quantum numbers associated with the initial state partons would be naturally identical with the net quantum numbers of hadron. Partons and they negative energy conjugates would provide in this picture a representation of hadron about hadron. This kind of interpretation of partons would make understandable why they cannot be observed directly. A possible objection is that the net gravitational mass of hadron would be three times the gravitational mass deduced from the inertial mass of hadron if partons feed their gravitational fluxes to the space-time sheet carrying Earth's gravitational field.

5. This picture could also relate to the suggested duality between string and parton pictures [K96] . In parton picture hadron is formed from partons represented by space-like 2-surfaces X^2_i connected by join along boundaries bonds. In string picture partonic 2-surfaces are replaced with string orbits. If one puts positive and negative energy particles at the ends of string diagram one indeed obtains a higher level representation of hadron. If these pictures are dual then also in parton picture positive and negative energies should compensate each other. Interestingly, light-like 3-D causal determinants identified as orbits of partons could be interpreted as orbits of light like string word sheets with "time" coordinate varying in space-like direction.

Scattering of Feynman diagrams at the higher levels of hierarchy

This picture generalizes to the description of higher level Feynman diagrams.

1. Assume that higher level vertices have recursive structure allowing to reduce the Feynman diagrams to ordinary Feynman diagrams by a procedure consisting of finite steps.
2. The lines of diagrams are classified as incoming or outgoing lines according to whether the time orientation of the line is positive or negative. The time orientation is associated with the time parameter t_n characterizing the automorphism $\Delta_{\mathcal{M}_\backslash}^{it_n}$. The incoming and outgoing net quantum numbers compensate each other. These quantum numbers are basically the quantum numbers of the state at the lowest level of the hierarchy.
3. In the vertices the $\mathcal{M}_{n+1}$ particles fuse and $\mathcal{M}_n$ particles form the analog of quark gluon plasma. The initial and final state particles of $\mathcal{M}_n$ Feynman diagram scatter independently and the S-matrix S_{n+1} describing the process is tensor product $S_n \otimes S_n^\dagger$. By the clustering property of S-matrix, this scattering occurs only for groups formed by partons formed by the incoming and outgoing particles $\mathcal{M}_n$ particles and each outgoing $\mathcal{M}_{n+1}$ line contains and irreducible $\mathcal{M}_n$ diagram. By continuing the recursion one finally ends down with ordinary Feynman diagrams.

2.7.3 Logic, Beliefs, And Spinor Fields In The World Of Classical Worlds

Beliefs can be characterized as Boolean value maps $\beta_i(p)$ telling whether i believes in proposition p or not. Additional structure is brought in by introducing the map $\lambda_i(p)$ telling whether p is true or not in the environment of i. The task is to find quantum counterpart for this model.

The spectrum of probabilities for outcomes in state function reduction with finite measurement resolution is universal

Consider quantum two-spinor as a model of a system with finite measurement resolution implying that state function reduction do not anymore lead to a spin state with a precise value but that one can only predict the probability distribution for the outcome of measurement. These probabilities can be also interpreted as truth values of a belief in finite cognitive resolution.

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1 = (z^1\overline{z^1} + \overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2} + \overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.
2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as

$$X_1 = (1/2 + n_1 q^{n_2})r \ \ , \ \ X_2 = (1/2 + n_2 q^{n_1})r \ \ .$$

   The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \rightarrow \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \rightarrow \infty$.
3. The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2+N_1 n, 1/2+N_2 n)/[1+(N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. This also conforms with the frequency interpretation for probabilities. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \rightarrow \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.
4. A possible interpretation for the fuzziness is in terms of finite measurement resolution. The quantized probabilities could be assigned with diagonalized density matrix regarded as matrix with elements which are commuting hermitian operators. The generalized eigenvalues would be eigenvalues spectra. States would not be pure expect at the limits$N_1 \gg N_2$ and $N_2 \gg$

N_1. The non-purity of the state could be understood in terms of entanglement with the degrees of freedom below measurement resolution describable in terms of inclusion of von Neumann algebras. One could perhaps say that in finite measurement resolution the outcome of state function reduction is always non-pure state characterized by a universal density matrix obtained by tracing over non-visible degrees of freedom.

WCW spinors as logic statements

In TGD framework the infinite-dimensional WCW (CH) spinor fields defined in CH, the "world of classical worlds", describe quantum states of the Universe [K113] . WCW spinor field can be regarded as a state in infinite-dimensional Fock space and are labeled by a collection of various two valued indices like spin and weak isospin. The interpretation is as a collection of truth values of logic statements one for each fermionic oscillator operator in the state. For instance, spin up and down would correspond to two possible truth values of a proposition characterized by other quantum numbers of the mode.

The hierarchy of space-time sheet could define a physical correlate for the hierarchy of higher order logics (statements about statements about...). The space-time sheet containing N fermions topologically condensed at a larger space-time sheet behaves as a fermion or boson depending on whether N is odd or even. This hierarchy has also a number theoretic counterpart: the construction of infinite primes [K94] corresponds to a repeated second quantization of a super-symmetric quantum field theory.

Quantal description of beliefs

The question is whether TGD inspired theory of consciousness allows a fundamental description of beliefs.

1. Beliefs define a model about some subsystem of universe constructed by the believer. This model can be understood as some kind of representation of real word in the state space representing the beliefs.
2. One can wonder what is the difference between real and p-adic variants of WCW spinor fields and whether they could represent reality and beliefs about reality. WCW spinors (as opposed to spinor fields) are constructible in terms of fermionic oscillator operators and seem to be universal in the sense that one cannot speak about p-adic and real WCW spinors as different objects. Real/p-adic spinor fields however have real/p-adic space-time sheets as arguments. This would suggest that there is no fundamental difference between the logic statements represented by p-adic and real WCW spinors.
3. This vision is realized if the intersection of reality and various p-adicities corresponds to an algebraically universal set of consisting of partonic 2-surfaces and string world sheets for which defining parameters are WCW coordinates in an algebraic extension of rationals defining that for p-adic number fields. Induced spinor fields would be localized at string world sheets and their intersections with partonic 2-surfaces and would be number theoretically universal. If second quantized induced spinor fields are correlates of Boolean cognition, which is behind the entire mathematics, their number theoretical universality is indeed a highly natural condition. Also fermionic anticommutation relations are number theoretically universal. By conformal invariance the conformal moduli of string world sheets and partonic 2-surface would be the natural WCW coordinates for the 2-surfaces in question and I proposed their p-adicization already in p-adic mass calculations for two decades ago.

   This picture would provide an elegant realization for the p-adicization. There would be ne need to map real space-time surfaces directly to p-adic ones and vice versa and one would avoid problems related to general coordinate invariance (GCI) completely. Strong form of holography would assign to partonic surfaces the real and p-adic variants. Already p-adic mass calculations support the presence of cognition in all length scales.

These observations suggest a more concrete view about how beliefs emerge physically.

The idea that p-adic WCW spinor fields could serve as representations of beliefs and real WCW spinor fields as representations of reality looks very nice and conforms with the adelic vision that space-time is adele - a book-like structure contains space-time sheets in various number fields as pages glued together along back for which the parameters characterizing space-time surface are numbers in an algebraic extension of rationals. Real space-time surfaces would be correlates for sensory experience and p-adic space-time sheets for cognition.

2.7.4 Jones Inclusions For Hyperfinite Factors Of Type II_1 As A Model For Symbolic And Cognitive Representations

Consider next a more detailed model for how cognitive representations and beliefs are realized at quantum level. This model generalizes trivially to symbolic representations.

The Clifford algebra of gamma matrices associated with WCW spinor fields corresponds to a von Neumann algebra known as hyper-finite factor of type II_1. The mathematics of these algebras is extremely beautiful and reproduces basic mathematical structures of modern physics (conformal field theories, quantum groups, knot and braid groups,....) from the mere assumption that the world of classical worlds possesses infinite-dimensional Kähler geometry and allows spinor structure.

The almost defining feature is that the infinite-dimensional unit matrix of the Clifford algebra in question has by definition unit trace. Type II_1 factors allow also what are known as Jones inclusions of Clifford algebras $\mathcal{N} \subset \mathcal{M}$. What is special to II_1 factors is that the induced unitary mappings between spinor spaces are genuine inclusions rather than 1-1 maps.

The S-matrix associated with the real-to-p-adic quantum transition inducing belief from reality would naturally define Jones inclusion of CH Clifford algebra $\mathcal{N}$ associated with the real space-time sheet to the Clifford algebra $\mathcal{M}$ associated with the p-adic space-time sheet. The moduli squared of S-matrix elements would define probabilities for pairs or real and belief states.

In Jones inclusion $\mathcal{N} \subset \mathcal{M}$ the factor $\mathcal{N}$ is included in factor $\mathcal{M}$ such that $\mathcal{M}$ can be expressed as $\mathcal{N}$-module over quantum space $\mathcal{M}/\mathcal{N}$ which has fractal dimension given by Jones index $\mathcal{M} : \mathcal{N} = 4cos^2(\pi/n) \leq 4$, $n = 3, 4, ...$ varying in the range $[1, 4]$. The interpretation is as the fractal dimension corresponding to a dimension of Clifford algebra acting in $d = \sqrt{\mathcal{M} : \mathcal{N}}$-dimensional spinor space: d varies in the range $[1, 2]$. The interpretation in terms of a quantal variant of logic is natural.

Probabilistic beliefs

For $\mathcal{M} : \mathcal{N} = 4$ $(n = \infty)$ the dimension of spinor space is $d = 2$ and one can speak about ordinary 2-component spinors with $\mathcal{N}$-valued coefficients representing generalizations of qubits. Hence the inclusion of a given $\mathcal{N}$-spinor as M-spinor can be regarded as a belief on the proposition and for the decomposition to a spinor in N-module $\mathcal{M}/\mathcal{N}$ involves for each index a choice $\mathcal{M}/\mathcal{N}$ spinor component selecting super-position of up and down spins. Hence one has a superposition of truth values in general and one can speak only about probabilistic beliefs. It is not clear whether one can choose the basis in such a way that $\mathcal{M}/\mathcal{N}$ spinor corresponds always to truth value 1. Since WCW spinor field is in question and even if this choice might be possible for a single 3-surface, it need not be possible for deformations of it so that at quantum level one can only speak about probabilistic beliefs.

Fractal probabilistic beliefs

For $d < 2$ the spinor space associated with $\mathcal{M}/\mathcal{N}$ can be regarded as quantum plane having complex quantum dimension d with two non-commuting complex coordinates z^1 and z^2 satisfying $z^1z^2 = qz^2z^1$ and $\overline{z^1z^2} = \overline{q}\overline{z^2z^1}$. These relations are consistent with hermiticity of the real and imaginary parts of z^1 and z^2 which define ordinary quantum planes. Hermiticity also implies that one can identify the complex conjugates of z^i as Hermitian conjugates.

The further commutation relations $[z^1, \overline{z^2}] = [z^2, \overline{z^1}] = 0$ and $[z^1, \overline{z^1}] = [z^2, \overline{z^2}] = r$ give a closed algebra satisfying Jacobi identities. One could argue that $r \geq 0$ should be a function $r(n)$ of the quantum phase $q = exp(i2\pi/n)$ vanishing at the limit $n \to \infty$ to guarantee that the algebra becomes commutative at this limit and truth values can be chosen to be non-fuzzy. $r = sin(\pi/n)$ would be the simplest choice. As will be found, the choice of $r(n)$ does not however affect at all the spectrum for the probabilities of the truth values. $n = \infty$ case corresponding to non-fuzzy quantum logic is also possible and must be treated separately: it corresponds to Kac Moody algebra instead of quantum groups.

The non-commutativity of complex spinor components means that z^1 and z^2 are not independent coordinates: this explains the reduction of the number of the effective number of truth values to $d < 2$. The maximal reduction occurs to $d = 1$ for $n = 3$ so that there is effectively only single truth value and one could perhaps speak about taboo or dogma or complete disappearance of the notions of truth and false (this brings in mind reports about meditative states: in fact $n = 3$ corresponds to a phase in which Planck constant becomes infinite so that the system is maximally quantal).

As non-commuting operators the components of d-spinor are not simultaneously measurable for $d < 2$. It is however possible to measure simultaneously the operators describing the probabilities $z^1\overline{z^1}$ and $z^2\overline{z^2}$ for truth values since these operators commute. An inherently fuzzy Boolean logic would be in question with the additional feature that the spinorial counterparts of statement and its negation cannot be regarded as independent observables although the corresponding probabilities satisfy the defining conditions for commuting observables.

If one can speak of a measurement of probabilities for $d < 2$, it differs from the ordinary quantum measurement in the sense that it cannot involve a state function reduction to a pure qubit meaning irreducible quantal fuzziness. One could speak of fuzzy qbits or fqbits (or quantum qbits) instead of qbits. This picture would provide the long sought interpretation for quantum groups.

The previous picture applies to all representations $M_1 \subset M_2$, where M_1 and M_2 denote either real or p-adic Clifford algebras for some prime p. For instance, real-real Jones inclusion could be interpreted as symbolic representations assignable to a unitary mapping of the states of a subsystem M_1 of the external world to the state space M_2 of another real subsystem. $p_1 \to p_2$ unitary inclusions would in turn map cognitive representations to cognitive representations. There is a strong temptation to assume that these Jones inclusions define unitary maps realizing universe as a universal quantum computer mimicking itself at all levels utilizing cognitive and symbolic representations. Subsystem-system inclusion would naturally define one example of Jones inclusion.

The spectrum of probabilities of truth values is universal

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1 = (z^1\overline{z^1} + \overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2} + \overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.

2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as

$$X_1 = (1/2 + n_1 q^{n_2})r \ \ , \ \ X_2 = (1/2 + n_2 q^{n_1})r \ \ .$$

   The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \to \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \to \infty$.

3. The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2+N_1 n, 1/2+N_2 n)/[1+(N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. This also conforms with the frequency interpretation for probabilities. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \to \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.

4. A possible interpretation for the fuzziness is in terms of finite measurement resolution. The quantized probabilities could be assigned with diagonalized density matrix regarded as matrix

with elements which are commuting hermitian operators. The generalized eigenvalues would be eigenvalues spectra. States would not be pure expect at the limits$N_1 \gg N_2$ and $N_2 \gg N_1$. The non-purity of the state could be understood in terms of entanglement with the degrees of freedom below measurement resolution describable in terms of inclusion of von Neumann algebras. One could perhaps say that in finite measurement resolution the outcome of state function reduction is always non-pure state characterized by a universal density matrix obtained by tracing over non-visible degrees of freedom.

How to define variants of belief quantum mechanically?

Probabilities of true and false for Jones inclusion characterize the plausibility of the belief and one can ask whether this description is enough to characterize states such as knowledge, misbelief, doubt, delusion, and ignorance. The truth value of $\beta_i(p)$ is determined by the measurement of probability assignable to Jones inclusion on the p-adic side. The truth value of $\lambda_i(p)$ is determined by a similar measurement on the real side. β and λ appear completely symmetrically and one can consider all kinds of triplets $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ assuming that there exist unitary S-matrix like maps mediating a sequence $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ of Jones inclusions. Interestingly, the hierarchies of Jones inclusions are a key concept in the theory of hyper-finite factors of type II_1 and pair of inclusions plays a fundamental role.

Let us restrict the consideration to the situation when $\mathcal{M}_1$ corresponds to a real subsystem of the external world, $\mathcal{M}_2$ its real representation by a real subsystem, and $\mathcal{M}_3$ to p-adic cognitive representation of M_3. Assume that both real and p-adic sides involve a preferred state basis for qubits representing truth and false.

Assume first that both $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\mathcal{M}_2 \subset \mathcal{M}_3$ correspond to $d = 2$ case for which ordinary quantum measurement or truth value is possible giving outcome true or false. Assume further that the truth values have been measured in both M_2 and M_3.

1. Knowledge corresponds to the proposition $\beta_i(p) \wedge \lambda_i(p)$.
2. Misbelief to the proposition $\beta_i(p) \wedge \neq \lambda_i(p)$.
   Knowledge and misbelief would involve both the measurement of real and p-adic probabilities .
3. Assume next that one has $d < 2$ form $\mathcal{M}_2 \subset \mathcal{M}_3$. Doubt can be regarded neither belief or disbelief: $\beta_i(p) \wedge \neq \beta_i(\neq p)$: belief is inherently fuzzy although proposition can be non-fuzzy.
   Assume next that truth values in $\mathcal{M}_1 \subset \mathcal{M}_2$ inclusion corresponds to $d < 2$ so that the basic propositions are inherently fuzzy.
4. Delusion is a belief which cannot be justified: $\beta_i(p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. This case is possible if $d = 2$ holds true for $\mathcal{M}_2 \subset \mathcal{M}_3$. Note that also misbelief that cannot be shown wrong is possible.
   In this case truth values cannot be quantum measured for $\mathcal{M}_1 \subset \mathcal{M}_2$ but can be measured for $\mathcal{M}_2 \subset \mathcal{M}_3$. Hence the states are products of pure $\mathcal{M}_3$ states with fuzzy $\mathcal{M}_2$ states.
5. Ignorance corresponds to the proposition $\beta_i(p) \wedge \neq \beta_i(\neq p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. Both real representational states and belief states are inherently fuzzy.

Quite generally, only for $d_1 = d_2 = 2$ ideal knowledge and ideal misbelief are possible. Fuzzy beliefs and logics approach to ordinary one at the limit $n \to \infty$, which according to the proposal of [K89] corresponds to the ordinary value of Planck constant. For other cases these notions are only approximate and quantal approach allows to characterize the goodness of the approximation. A new kind of inherent quantum uncertainty of knowledge is in question and one could speak about a Uncertainty Principle for cognition and symbolic representations. Also the unification of symbolic and various kinds of cognitive representations deserves to be mentioned.

2.7.5 Intentional Comparison Of Beliefs By Topological Quantum Computation?

Intentional comparison would mean that for a given initial state also the final state of the quantum jump is fixed. This requires the ability to engineer S-matrix so that it leads from a given state to single state only. Any S-matrix representing permutation of the initial states fulfills these conditions. This condition is perhaps unnecessarily strong.

Quantum computation is basically the engineering of S-matrix so that it represents a superposition of parallel computations. In TGD framework topological quantum computation based on the braiding of magnetic flux tubes would be represented as an evolution characterized by braid [K7] . The dynamical evolution would be associated with light-like boundaries of braids. This evolution has dual interpretations either as a limit of time evolution of quantum state (program running) or a quantum state satisfying conformal invariance constraints (program code).

The dual interpretation would mean that conformally invariant states are equivalent with engineered time evolutions and topological computation realized as braiding connecting the quantum states to be compared (beliefs represented as many-fermion states at the boundaries of magnetic flux tubes) could give rise to conscious computational comparison of beliefs. The complexity of braiding would give a measure for how much the states to be compared differ.

Note that quantum computation is defined by a unitary map which could also be interpreted as symbolic representation of states of system M_1 as states of system M_2 mediated by the braid of join along boundaries bonds connecting the two space-time sheets in question and having light-like boundaries. These considerations suggest that the idea about S-matrix of the Universe should be generalized so that the dynamics of the Universe is dynamics of mimicry described by an infinite collection of fermionic S-matrices representable in terms of Jones inclusions.

2.7.6 The Stability Of Fuzzy Qbits And Quantum Computation

The stability of fqbits against state function reduction might have deep implications for quantum computation since quantum spinors would be stable against state function reduction induced by the perturbations inducing de-coherence in the normal situation. If this is really true, and if the only dangerous perturbations are those inducing the phase transition to qbits, the implications for quantum computation could be dramatic. Of course, the rigidity of qbits could be just another way to say that topological quantum computations are stable against thermal perturbations not destroying anyons [K7] .

The stability of fqbits could also be another manner to state the stability of rational, or more generally algebraic, bound state entanglement against state function reduction, which is one of the basic hypothesis of TGD inspired theory of consciousness [K62] . For sequences of Jones inclusions or equivalently, for multiple Connes tensor products, one would obtain tensor products of quantum spinors making possible arbitrary complex configurations of fqbits. Anyonic braids in topological quantum computation would have interpretation as representations for this kind of tensor products.

2.7.7 Fuzzy Quantum Logic And Possible Anomalies In The Experimental Data For The EPR-Bohm Experiment

The experimental data for EPR-Bohm experiment [J7] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [J1] . The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The anomaly

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i,j), where i,j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha,\beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha-\beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha-\beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [J7] are not consistent with this prediction [J2] and this is identified as the anomaly.
2. The QM prediction $E(\alpha,\beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha - \beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [J2] .

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly b) but not anomaly a). A "mundane" explanation for anomaly a) is proposed.

Predictions of fuzzy quantum logic for the probabilities and correlations

1. The description of fuzzy quantum logic in terms statistical ensemble

The fuzzy quantum logic implies that the predictions $P_{i,j}$ for the probabilities should be replaced with ensemble averages over the ensembles defined by fuzzy quantum logic. In practice this means that following replacements should be carried out:

$$\begin{aligned} P_{i,j} &\rightarrow P^2 P_{i,j} + (1-P)^2 P_{i+1,j+1} \\ &+ P(1-P)\left[P_{i,j+1} + P_{i+1,j}\right] \ . \end{aligned} \tag{2.7.1}$$

Here P is one of the state dependent universal probabilities/fuzzy truth values for some value of n characterizing the measurement situation. The concrete predictions would be following

$$\begin{aligned} P_{0,0} = P_{1,1} &\rightarrow A\frac{cos^2(\alpha-\beta)}{2} + B\frac{sin^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{cos^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ P_{0,1} = P_{1,0} &\rightarrow A\frac{sin^2(\alpha-\beta)}{2} + B\frac{cos^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{sin^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ A &= P^2 + (1-P)^2 \ , \ B = 2P(1-P) \ . \end{aligned} \tag{2.7.2}$$

The prediction is that the graphs of probabilities as a function as function of the angle $\alpha - \beta$ are scaled by a factor $1 - 4P(1-P)$ and shifted upwards by $P(1-P)$. The value of P, and one might hope even the value of n labeling Jones inclusion and the integer m labeling the quantum state might be deducible from the experimental data as the upward shift. The basic prediction is that the maxima of curves measuring probabilities $P_(i,j)$ have minimum at $B/2 = P(1-P)$ and maximum is scaled down to $(A-B)/2 = 1/2 - 2P(1-P)$.

If the P is same for all pairs i,j, the correlation $E = \sum_i (P_{ii} - P_{i,i+1})$ transforms as

$$E(\alpha,\beta) \rightarrow [1 - 4P(1-P)]\, E(\alpha,\beta) \ . \tag{2.7.3}$$

Only the normalization of $E(\alpha,\beta)$ as a function of $\alpha - \beta$ reducing the magnitude of E occurs. In particular the maximum/minimum of E are scaled down from $E = \pm 1$ to $E = \pm(1 - 4P(1-P))$.

From the figure 1b) of [J2] the scaling down indeed occurs for magnitudes of E with same amount for minimum and maximum. Writing $P = 1 - \epsilon$ one has $A - B \simeq 1 - 4\epsilon$ and $B \simeq 2\epsilon$ so that the maximum is in the first approximation predicted to be at $1 - 4\epsilon$. The graph would give $1 - P \simeq \epsilon \simeq .025$. Thus the model explains the reduction of the magnitude for the maximum and minimum of E which was not however considered to be an anomaly in [J1, J2] .

A further prediction is that the identities $P(i,i) + P(i+1,i) = 1/2$ should still hold true since one has $P_{i,i} + P_{i,i+1} = (A-B)/2 + B = 1$. This is implied also by probability conservation.

The four curves corresponding to these identities do not however co-incide as the figure 6 of [J2] demonstrates. This is regarded as the basic anomaly in [J1, J2] . From the same figure it is also clear that below $\alpha-\beta<10$ degrees $P_{++}=P_{--}$ $\Delta P_{+-}=-\Delta P_{-+}$ holds true in a reasonable approximation. After that one has also non-vanishing ΔP_{ii} satisfying $\Delta P_{++}=-\Delta P_{--}$. This kind of splittings guarantee the identity $\sum_{ij}P_{ij}=1$. These splittings are not visible in E.

Since probability conservation requires $P_{ii}+P_{ii+1}=1$, a mundane explanation for the discrepancy could be that the failure of the conditions $P_{i,i}+P_{ii+1}=1$ means that the measurement efficiency is too low for P_{+-} and yields too low values of $P_{+-}+P_{--}$ and $P_{+-}+P_{++}$. The constraint $\sum_{ij}P_{ij}=1$ would then yield too high value for P_{-+}. Similar reduction of measurement efficiency for P_{++} could explain the splitting for $\alpha-\beta>10$ degrees.

Clearly asymmetry with respect to exchange of photons or of detectors is in question.

1. The asymmetry of two photon state with respect to the exchange of photons could be considered as a source of asymmetry. This would mean that the photons are not maximally entangled. This could be seen as an alternative “mundane” explanation.
2. The assumption that the parameter P is different for the detectors does not change the situation as is easy to check.
3. One manner to achieve splittings which resemble observed splittings is to assume that the value of the probability parameter P depends on the *polarization pair*: $P=P(i,j)$ so that one has $(P(-,+),P(+,-))=(P+\Delta,P-\Delta)$ and $(P(-,-),P(+,+))=(P+\Delta_1,P-\Delta_1)$. $\Delta\simeq .025$ and $\Delta_1\simeq\Delta/2$ could produce the observed splittings qualitatively. One would however always have $P(i,i)+P(i,i+1)\geq 1/2$. Only if the procedure extracting the correlations uses the constraint $\sum_{i,j}P_{ij}=1$ effectively inducing a constant shift of P_{ij} downwards an asymmetry of observed kind can result. A further objection is that there are no special reason for the values of $P(i,j)$ to satisfy the constraints.

2. Is it possible to say anything about the value of n in the case of EPR-Bohm experiment?

To explain the reduction of the maximum magnitudes of the correlation E from 1 to $\sim .9$ in the experiment discussed above one should have $p_1\simeq .9$. It is interesting to look whether this allows to deduce any information about the valued of n. At the limit of large values of N_in one would have $(N_1-N_2)/(N_1+N_2)\simeq .4$ so that one cannot say anything about n in this case. $(N_1,N_2)=(3,1)$ satisfies the condition exactly. For $n=3$, the smallest possible value of n, this would give $p_1\simeq .88$ and for $n=4$ $p_1=.41$. With high enough precision it might be possible to select between $n=3$ and $n=4$ options if small values of N_i are accepted.

2.7.8 Category Theoretic Formulation For Quantum Measurement Theory With Finite Measurement Resolution?

I have been trying to understand whether category theory might provide some deeper understanding about quantum TGD, not just as a powerful organizer of fuzzy thoughts but also as a tool providing genuine physical insights. Marni Dee Sheppeard (or Kea in her blog Arcadian Functor at `http://tinyurl.com/yb3lsbjq`) is also interested in categories but in much more technical sense. Her dream is to find a category theoretical formulation of M-theory as something, which is not the 11-D something making me rather unhappy as a physicist with second foot still deep in the muds of low energy phenomenology.

Locales, frames, Sierpinski topologies and Sierpinski space

The ideas below popped up when Kea mentioned in M-theory lesson 51 the notions of locale and frame [A6] . In Wikipedia I learned that complete Heyting algebras, which are fundamental to category theory, are objects of three categories with differing arrows. CHey, Loc and its opposite category Frm (arrows reversed). Complete Heyting algebras are partially ordered sets which are complete lattices. Besides the basic logical operations there is also algebra multiplication (I have considered the possible role of categories and Heyting algebras in TGD in [K25]). From Wikipedia I also learned that locales and the dual notion of frames form the foundation of pointless topology [A19] . These topologies are important in topos theory which does not assume axiom of choice.

The so called particular point topology [A17] assumes a selection of single point but I have the physicist's feeling that it is otherwise rather near to pointless topology. Sierpinski topology [A22] is this kind of topology. Sierpinski topology is defined in a simple manner: the set is open only if it contains a given preferred point p. The dual of this topology defined in the obvious sense exists also. Sierpinski space consisting of just two points 0 and 1 is the universal building block of these topologies in the sense that a map of an arbitrary space to Sierpinski space provides it with Sierpinski topology as the induced topology. In category theoretical terms Sierpinski space is the initial object in the category of frames and terminal object in the dual category of locales. This category theoretic reductionism looks highly attractive.

Particular point topologies, their generalization, and number theoretical braids

Pointless, or rather particular point topologies might be very interesting from physicist's point of view. After all, every classical physical measurement has a finite space-time resolution. In TGD framework discretization by number theoretic braids replaces partonic 2-surface with a discrete set consisting of algebraic points in some extension of rationals: this brings in mind something which might be called a topology with a set of particular algebraic points. Could this preferred set belongs to any open set in the particular point topology appropriate in this situation?

Perhaps the physical variant for the axiom of choice could be restricted so that only sets of algebraic points in some extension of rationals can be chosen freely and the choices is defined by the intersection of p-adic and real partonic 2-surfaces and in the framework of TGD inspired theory of consciousness would thus involve the interaction of cognition with the material world. The extension would depend on the position of the physical system in the algebraic evolutionary hierarchy defining also a cognitive hierarchy. Certainly this would fit very nicely to the formulation of quantum TGD unifying real and p-adic physics by gluing real and p-adic number fields to single super-structure via common algebraic points.

Analogs of particular point topologies at the level of state space: finite measurement resolution

There is also a finite measurement resolution in Hilbert space sense not taken into account in the standard quantum measurement theory based on factors of type I. In TGD framework one indeed introduces quantum measurement theory with a finite measurement resolution so that complex rays become included hyper-finite factors of type II_1 (HFFs).

1. Could topology with particular algebraic points have a generalization allowing a category theoretic formulation of the quantum measurement theory without states identified as complex rays?
2. How to achieve this? In the transition of ordinary Boolean logic to quantum logic in the old fashioned sense (von Neuman again!) the set of subsets is replaced with the set of subspaces of Hilbert space. Perhaps this transition has a counterpart as a transition from Sierpinski topology to a structure in which sub-spaces of Hilbert space are quantum sub-spaces with complex rays replaced with the orbits of subalgebra defining the measurement resolution. Sierpinski space {0,1} would in this generalization be replaced with the quantum counterpart of the space of 2-spinors. Perhaps one should also introduce q-category theory with Heyting algebra being replaced with q-quantum logic.

Fuzzy quantum logic as counterpart for Sierpinksi space

The program formulated above might indeed make sense. The lucky association induced by Kea's blog was to the ideas about fuzzy quantum logic realized in terms of quantum 2-spinor that I had developed a couple of years ago. Fuzzy quantum logic would reflect the finite measurement resolution. I just list the pieces of the argument.

Spinors and qbits: Spinors define a quantal variant of Boolean statements, qbits. One can however go further and define the notion of quantum qbit, qqbit. I indeed did this for couple of years ago (the last section of this chapter).

Q-spinors and qqbits: For q-spinors the two components a and b are not commuting numbers but non-Hermitian operators: $ab = qba$, q a root of unity. This means that one cannot measure both a and b simultaneously, only either of them. $aa^\dagger$ and $bb^\dagger$ however commute so that probabilities for bits 1 and 0 can be measured simultaneously. State function reduction is not possible to a state in which a or b gives zero. The interpretation is that one has q-logic is inherently fuzzy: there are no absolute truths or falsehoods. One can actually predict the spectrum of eigenvalues of probabilities for say 1. Obviously quantum spinors would be state space counterparts of Sierpinski space and for $q \neq 1$ the choice of preferred spinor component is very natural. Perhaps this fuzzy quantum logic replaces the logic defined by the Heyting algebra.

Q-locale: Could one think of generalizing the notion of locale to quantum locale by using the idea that sets are replaced by sub-spaces of Hilbert space in the conventional quantum logic. Q-openness would be defined by identifying quantum spinors as the initial object, q-Sierpinski space. a (resp. b for the dual category) would define q-open set in this space. Q-open sets for other quantum spaces would be defined as inverse images of a (resp. b) for morphisms to this space. Only for q=1 one could have the q-counterpart of rather uninteresting topology in which all sets are open and every map is continuous.

Q-locale and HFFs: The q-Sierpinski character of q-spinors would conform with the very special role of Clifford algebra in the theory of HFFs, in particular, the special role of Jones inclusions to which one can assign spinor representations of $SU(2)$. The Clifford algebra and spinors of the world of classical worlds identifiable as Fock space of quark and lepton spinors is the fundamental example in which 2-spinors and corresponding Clifford algebra serves as basic building brick although tensor powers of any matrix algebra provides a representation of HFF.

Q-measurement theory: Finite measurement resolution (q-quantum measurement theory) means that complex rays are replaced by sub-algebra rays. This would force the Jones inclusions associated with $SU(2)$ spinor representation and would be characterized by quantum phase q and bring in the q-topology and q-spinors. Fuzzyness of qqbits of course correlates with the finite measurement resolution.

Q-n-logos: For other q-representations of SU(2) and for representations of compact groups (Appendix) one would obtain something which might have something to do with quantum n-logos, quantum generalization of n-valued logic. All of these would be however less fundamental and induced by q-morphisms to the fundamental representation in terms of spinors of the world of classical worlds. What would be however very nice that if these q-morphisms are constructible explicitly it would become possible to build up q-representations of various groups using the fundamental physical realization - and as I have conjectured [K84] - McKay correspondence and huge variety of its generalizations would emerge in this manner.

The analogs of Sierpinski spaces: The discrete subgroups of $SU(2)$, and quite generally, the groups Z_n associated with Jones inclusions and leaving the choice of quantization axes invariant, bring in mind the n-point analogs of Sierpinski space with unit element defining the particular point. Note however that $n \geq 3$ holds true always so that one does not obtain Sierpinski space itself. If all these n preferred points belong to any open set it would not be possible to decompose this preferred set to two subsets belonging to disjoint open sets. Recall that the generalized embedding space related to the quantization of Planck constant is obtained by gluing together coverings $M^4 \times CP_2 \to M^4 \times CP_2/G_a \times G_b$ along their common points of base spaces. The topology in question would mean that if some point in the covering belongs to an open set, all of them do so. The interpretation would be that the points of fiber form a single inseparable quantal unit.

Number theoretical braids identified as as subsets of the intersection of real and p-adic variants of algebraic partonic 2-surface define a second candidate for the generalized Sierpinski space with a set of preferred points.

Chapter 3

Does TGD Predict a Spectrum of Planck Constants?

3.1 Introduction

The quantization of Planck constant has been the basic them of TGD since 2005 and the perspective in the earlier version of this chapter reflected the situation for about year and one half after the basic idea stimulated by the finding of Nottale [E18] that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$ for the inner planets. The general form of $\hbar_{gr}$ is dictated by Equivalence Principle. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales.

The second crucial empirical input were the anomalies associated with living matter. Mention only the effects of ELF radiation at EEG frequencies on vertebrate brain and anomalous behavior of the ionic currents through cell membrane. If the value of Planck constant is large, the energy of EEG photons is above thermal energy and one can understand the effects on both physiology and behavior. If ionic currents through cell membrane have large Planck constant the scale of quantum coherence is large and one can understand the observed low dissipation in terms of quantum coherence. This approach led to the formula $h_{eff} = n \times h$. Rather recently (2014) it became clear that for microscopic systems the identification $h_{eff} = h_{gr}$ makes sense and predicts universal energy spectrum for cyclotron energies of dark photons identifiable as energy spectrum of bio-photons in TGD inspired quantum biology.

3.1.1 Evolution Of Mathematical Ideas

The original formulation for the hierarchy of Planck constants was in terms of $h_{eff}/h = n$-fold singular coverings of the embedding space $H = M^4 \times CP_2$. Later it turned out that there is no need to postulate these covering spaces although they are a nice auxiliary tool allowing to understand why the phase of matter with different values of n behave like dark matter relative to each other: they are simply at different pages of the book-like structure formed by the covering spaces.

Few years ago it became clear that the hierarchy of Planck constants could be only effective but have the same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $h_{eff} = n \times h$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. This reduces the understanding of the effective hierarchy of Planck constants to quantum variant of multi-furcations for the dynamics of preferred extremals of Kähler action. The number of branches of multi-furcation defines the integer n in $\hbar_{eff} = n\hbar$.

One of the latest steps in the progress was the realization that the hierarchy of Planck constants can be understood in terms of quantum criticality of TGD Universe postulated from the beginning as a way to obtain a unique theory. In accordance with what is known about 2-D critical systems, quantum criticality should correspond to a generalization of conformal invariance. TGD indeed predicts several analogs of super-conformal algebras: so called super-symplectic algebra acting in $\delta M^4_\pm \times CP_2$ should act as isometries of WCW and its generators are labeled by conformal weights. Light-cone boundary $\delta M^4_\pm$ has an extension of conformal symmetries as conformal symmetries and an algebra isomorphic to the ordinary conformal algebra acts as its isometries. The light-like orbits of partonic 2-surfaces allow similar algebra of conformal symmetries and string world sheets and partonic 2-surfaces allow conformal symmetries.

The proposal is that super-symplectic algebra (at least it) defines a hierarchy of broken super-conformal gauge symmetries in the sense that the sub-algebra for which the conformal weights are n-ples of those for the entire algebra acts as gauge conformal symmetries. $n = h_{eff}/h$ giving a connection to the hierarchy of Planck constants would hold true. These sub-algebras are isomorphic to the full algebra and thus form a fractal hierarchy. One has infinite number of hierarchies of broken conformal symmetries defined by the sequences $n(i+1) = m_i \times n(i)$. In the phase transition increasing n conformal gauge symmetry is reduced and some gauge degrees of freedom transform to physical ones and criticality is reduced so that the transition takes place spontaneously. TGD Universe is like a ball at the top of hill at the top of hill at....

This view has far reaching implication for the understanding of living matter and leads to deep connections between different key ideas of TGD. The hierarchy has also a purely number theoretical interpretation in terms of hierarchy of algebraic extensions of rationals appearing naturally in the adelic formulation of quantum TGD. $n = h_{eff}/h$ would naturally correspond to an integer, which is product of so called ramified primes (rational primes for which the decomposition to primes of extension contains higher powers of these primes).

In this framework it becomes obvious that - instead of coverings of embedding space postulated in the original formulation - one has space-time surfaces representable as singular n-fold coverings. The non-determinism of Kähler action - key element of criticality - would be the basic reason for the appearance of singular coverings: two 3-surfaces at the opposite boundaries of CD are connected by n-sheeted space-time surfaces for which the sheets co-incide at the boundaries. Criticality must be accompanied by 4-D variant of conformal gauge invariance already described so that these space-time surfaces are replaced by conformal gauge equivalence classes.

These coverings are highly analogous to the covering space associated with the analytic function $w(z) = z^{1/n}$. If one uses w as a variable, the ordinary conformal symmetries generated by functions of z indeed correspond to the algebra generated by w^n and the sheets of covering correspond to conformal gauge equivalence classes not transformed to each other by conformal transformations.

3.1.2 The Evolution Of Physical Ideas

The evolution of physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The basic idea was that ordinary matter condenses around dark matter which is a phase of matter characterized by non-standard value of Planck constant.
2. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K77] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E18] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with minimum size of order Schwarstchild radius r_S of order scaled up Planck length: $r_S \sim \sqrt{\hbar G}$. Black hole entropy being inversely proportional to $\hbar$ is predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

3. Darkness is a relative concept and due to the fact that particles at different pages of book cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to different page of book are however possible and change Planck constant so that particle exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. This allows to conclude that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K103] , [I8] .

4. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially shocking outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L3, K103] , [L3] .

3.1.3 Basic Physical Picture As It Is Now

The basic phenomenological rules are simple and remained roughly the same during years.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K104].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K77] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. It was [E18] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

3. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

4. The interpretation of the hierarchy of Planck constants as labels for quantum critical systems is especially powerful in TGD inspired quantum biology and consciousness theory. The increase of Planck constant by integer factor occurs spontaneously and means an increase of complexity and sensory and cognitive resolution - in other words evolution. Living matter is however fighting to stay at the existing level of criticality. The reason is that the changes involves state function reduction at the opposite boundary of CD and means death of self followed by re-incarnation.

   Negentropy Maximization Principle [K63] saves the system from this fate if it is able to generate negentropic entanglement by some other means. Metabolic energy suggested already earlier to be a carrier of negentropic entanglement makes this possible. Also other metabolites can carry negentropy. Therefore living systems are eating each other to satisfy the demands of NMP! Why this non-sensical looking Karma's cycle? The sub-systems of self defining sub-selves (mental images) are dying and re-incarnating and generating negentropy: self is a gardener and sub-selves are the fruit trees and the longer self lives, the more fruits are produced. Hence this process, which Buddhist would call attachment to ego is the ways to generate what I have called "Akashic records". Everything has its purpose.

In this chapter I try to summarize the evolution of the ideas related to Planck constant. I have worked hardly to achieve internal consistency but the old theory layers are there and might cause confusion.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L22].

3.2 Experimental Input

In this section basic experimental inputs suggesting a hierarchy of Planck constants and the identification of dark matter as phases with non-standard value of Planck constant are discussed.

3.2.1 Hints For The Existence Of Large $\hbar$ Phases

Quantum classical correspondence suggests the identification of space-time sheets identifiable as quantum coherence regions. Since they can have arbitrarily large sizes, phases with arbitrarily large quantum coherence lengths and arbitrarily long de-coherence times seem to be possible in TGD Universe. In standard physics context this seems highly implausible. If Planck constant can have arbitrarily large values, the situation changes since Compton lengths and other quantum scales are proportional to $\hbar$. Dark matter is excellent candidate for large $\hbar$ phases.

The expression for $\hbar_{gr}$ in the model explaining the Bohr orbits for planets is of form $\hbar_{gr} = GM_1M_2/v_0$ [K89]. This suggests that the interaction is associated with some kind of interface between the systems, perhaps join along boundaries bonds/flux tubes connecting the space-time sheets associated with systems possessing gravitational masses M_1 and M_2. Also a large space-time sheet carrying the mutual classical gravitational field could be in question. This argument generalizes to the case $\hbar/\hbar_0 = Q_1Q_2\alpha/v_0$ in case of generic phase transition to a strongly interacting phase with α describing gauge coupling strength.

There exist indeed some experimental indications for the existence of phases with a large $\hbar$.

1. With inspiration coming from the finding of Nottale [E18] I have proposed an explanation of dark matter as a macroscopic quantum phase with a large value of $\hbar$ [K89]. Any interaction, if sufficiently strong, can lead to this kind of phase. The increase of $\hbar$ would make the fine structure constant α in question small and guarantee the convergence of perturbation series.
2. Living matter could represent a basic example of large $\hbar$ phase [K37, K11]. Even ordinary condensed matter could be "partially dark" in many-sheeted space-time [K39]. In fact, the realization of hierarchy of Planck constants leads to a considerably weaker notion of darkness stating that only the interaction vertices involving particles with different values of Planck constant are impossible and that the notion of darkness is relative notion. For instance, classical interactions and photon exchanges involving a phase transition changing the value of $\hbar$ of photon are possible in this framework.
3. There is claim about a detection in RHIC (Relativistic Heavy Ion Collider in Brookhaven) of states behaving in some respects like mini black holes [C56]. These states could have explanation as color flux tubes at Hagedorn temperature forming a highly tangled state and identifiable as stringy black holes of strong gravitation. The strings would carry a quantum coherent color glass condensate, and would be characterized by a large value of $\hbar$ naturally resulting in confinement phase with a large value of α_s [K90] . The progress in hadronic mass calculations led to a concrete model of color glass condensate of single hadron as many-particle state of super-symplectic gluons [K70, K64] - something completely new from the point of QCD - responsible for non-perturbative aspects of hadron physics. In RHIC events these color glass condensate would fuse to single large condensate. This condensate would be present also in ordinary black-holes and the blackness of black-hole would be darkness.
4. I have also discussed a model for cold fusion based on the assumption that nucleons can be in large $\hbar$ phase. In this case the relevant strong interaction strength is $Q_1Q_2\alpha_{em}$ for two nucleon clusters inside nucleus which can increase $\hbar$ so large that the Compton length of protons becomes of order atomic size and nuclear protons form a macroscopic quantum phase [K39, K37].

3.2.2 Quantum Coherent Dark Matter And $\hbar$

The argument based on gigantic value of $\hbar_{gr}$ explaining darkness of dark mater is attractive but one should be very cautious.

Consider first ordinary QED$e = \sqrt{\alpha 4\pi\hbar}$ appears in vertices so that perturbation expansion in powers of $\sqrt{\hbar}$ basically. This would suggest that large $\hbar$ leads to large effects. All predictions are however in powers of alpha and large $\hbar$ means small higher order corrections. What happens can be understood on basis of dimensional analysis. For instance, cross sections are proportional to $(\hbar/m)^2$, where m is the relevant mass and the remaining factor depends on $\alpha = e^2/(4\pi\hbar)$ only. In the more general case tree amplitudes with n vertices are proportional to e^n and thus to $\hbar^{n/2}$ and loop corrections give only powers of α which get smaller when $\hbar$ increases. This must relate to the powers of $1/\hbar$ from the integration measure associated with the momentum loop integrals affected by the change of α.

Consider now the effects of the scaling of $\hbar$. The scaling of Compton lengths and other quantum kinematical parameters is the most obvious effect. An obvious effect is due to the change of $\hbar$ in the commutation relations and in the change of unit of various quantum numbers. In particular, the right hand side of oscillator operator commutation and anti-commutation relations is scaled. A further effect is due to the scaling of the eigenvalues of the Kähler-Dirac operator $\hbar\Gamma^\alpha D_\alpha$.

The exponent $exp(K)$ of Kähler function K defining perturbation series in WCW degrees of freedom is proportional to $1/g_K^2$ and does not depend on $\hbar$ at all if there is only single Planck constant. The propagator is proportional to g_K^2. This can be achieved also in QED by absorbing e from vertices to e^2 in photon propagator. Hence it would seem that the dependence on α_K (and $\hbar$) must come from vertices which indeed involve Jones inclusions of the II_1 factors of the incoming and outgoing lines.

This however suggests that the dependence of the scattering amplitudes on $\hbar$ is purely kinematical so that all higher radiative corrections would be absent. This seems to leave only one option: the scale factors of covariant CD and CP_2 metrics can vary and might have discrete spectrum of values.

1. The invariance of Kähler action with respect to overall scaling of metric however allows to keep CP_2 metric fixed and consider only a spectrum for the scale factors of M^4 metric.
2. The first guess motivated by Schrödinger equation is that the scaling factor of covariant CD metric corresponds the ratio $r^2 = (\hbar/\hbar_0)^2$. This would mean that the value of Kähler action depends on r^2. The scaling of M^4 coordinate by r the metric reduces to the standard form but if causal diamonds with quantized temporal distance between their tips are the basic building blocks of WCW geometry as zero energy ontology requires, this scaling of $\hbar$ scales the size of CD by r so that genuine effect results since M^4 scalings are not symmetries of Kähler action.
3. In this picture r would code for radiative corrections to Kähler function and thus space-time physics. Even in the case that the radiative corrections to WCW functional integral vanish, as suggested by quantum criticality, they would be actually taken into account.

This kind of dynamics is not consistent with the original view about embedding space and forces to generalize the notion of embedding spaces since it is clear that particles with different Planck constants cannot appear in the same vertex of Feynman diagram. Somehow different values of Planck constant must be analogous to different pages of book having almost copies of embedding space as pages. A possible resolution of the problem cames from the realization that the fundamental structure might be the inclusion hierarchy of number theoretical Clifford algebras from which entire TGD could emerge including generalization of the embedding space concept.

3.2.3 The Phase Transition Changing The Value Of Planck Constant As A Transition To Non-Perturbative Phase

A phase transition increasing $\hbar$ as a transition guaranteeing the convergence of perturbation theory

The general vision is that a phase transition increasing $\hbar$ occurs when perturbation theory ceases to converge. Very roughly, this would occur when the parameter $x = Q_1Q_2\alpha$ becomes larger than one. The net quantum numbers for "spontaneously magnetized" regions provide new natural units for quantum numbers. The assumption that standard quantization rules prevail poses very strong restrictions on allowed physical states and selects a subspace of the original configuration space. One can of course, consider the possibility of giving up these rules at least partially in which case a spectrum of fractionally charged anyon like states would result with confinement guaranteed by the fractionization of charges.

The necessity of large $\hbar$ phases has been actually highly suggestive since the first days of quantum mechanics. The classical looking behavior of macroscopic quantum systems remains still a poorly understood problem and large $\hbar$ phases provide a natural solution of the problem.

In TGD framework quantum coherence regions correspond to space-time sheets. Since their sizes are arbitrarily large the conclusion is that macroscopic and macro-temporal quantum coherence are possible in all scales. Standard quantum theory definitely fails to predict this and the conclusion is that large $\hbar$ phases for which quantum length and time scales are proportional to $\hbar$ and long are needed.

Somewhat paradoxically, large $\hbar$ phases explain the effective classical behavior in long length and time scales. Quantum perturbation theory is an expansion in terms of gauge coupling strengths inversely proportional to $\hbar$ and thus at the limit of large $\hbar$ classical approximation becomes exact. Also the Coulomb contribution to the binding energies of atoms vanishes at this limit. The fact that we experience world as a classical only tells that large $\hbar$ phase is essential for our sensory perception. Of course, this is not the whole story and the full explanation requires a detailed anatomy of quantum jump.

The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar_{gr} = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength $\alpha Q_i Q_j$, where α is the appropriate coupling strength and $Q_i Q_j$ represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician. A primitive formulation for this criterion is the condition $\alpha Q_i Q_j \geq 1$.

The first working hypothesis was the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule turned out to be quite too specific. The mathematically plausible formulation predicts that in principle any rational value for $r = \hbar(M^4)/\hbar(CP_2)$ is possible but there are certain number theoretically preferred values of r such as those coming powers of 2.

3.3 A Generalization of the Notion of Embedding Space as a Realization of the Hierarchy of Planck Constants

In the following the basic ideas concerning the realization of the hierarchy of Planck constants are summarized and after that a summary about generalization of the embedding space is given. In [K77] the important delicacies associated with the Kähler structure of generalized embedding space are discussed. The background for the recent vision is quite different from that for half decade ago. Zero energy ontology and the notion of causal diamond, number theoretic compactification leading to the precise identification of number theoretic braids, the realization of number theoretic universality, and the understanding of the quantum dynamics at the level of Kähler-Dirac action fix to a high degree the vision about generalized embedding space.

3.3.1 Basic Ideas

The first key idea in the geometric realization of the hierarchy of Planck constants emerges from the study of Schrödinger equation and states that Planck constant appears a scaling factor of M^4 metric. Second key idea is the connection with Jones inclusions inspiring an explicit formula for Planck constants. For a long time this idea remained heuristic must-be-true feeling but the recent view about quantum TGD provide a justification for it.

Scaling of Planck constant and scalings of CD and CP_2 metrics

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term has no dependence on it. This makes the scaling of $\hbar$ a non-trivial transformation. If the contravariant metric scales as $r = \hbar/\hbar_0$ the effect of scaling of Planck constant is realized at the level of embedding space geometry provided it is such that it is possible to compare the regions of generalized embedding space having different value of Planck constant.

In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

1. The minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined in such a way that spinor connection corresponds to the quantity $\hbar eQA$, where denotes A gauge potential, and there is no natural manner to separate $\hbar e$ from it.
2. The contravariant CD metric scales like $\hbar^2$. In the case of Dirac operator in $M^4 \times CP_2$ one can assign separate Planck constants to Poincare and color algebras and the scalings of CD and CP_2 metrics induce scalings of corresponding values of $\hbar^2$. As far as Kähler action is considered, CP_2 metric could be always thought of being scaled to its standard form.
3. Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^i k_i$ rather than four-momentum squared $p^2 = p^i p_i$ in CD degrees of freedom and its analog in CP_2 degrees of

freedom. The values of k^2 are proportional to $1/r^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$: analogous conclusion applies in CP_2 degrees of freedom. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

This consideration generalizes to the case of the induced gamma matrices and induced metric in X^4, Kähler-Dirac operator, and Kähler action which carry dynamical information about the ratio $r = \hbar_{eff}/\hbar_0$.

Kähler function codes for a perturbative expansion in powers of $\hbar(CD)/\hbar(CP_2)$

Suppose that one accepts that the spectrum of CD *resp.* CP_2 Planck constants is accompanied by a hierarchy of overall scalings of covariant CD (causal diamond) metric by $(\hbar(M^4)/\hbar_0)^2$ and CP_2 metric by $(\hbar(CP_2)/\hbar_0)^2$ followed by overall scaling by $r^2 = (\hbar_0/\hbar(CP_2))^2$ so that CP_2 metric suffers no scaling and difficulties with isometric gluing procedure of sectors are avoided.

The first implication of this picture is that the Kähler-Dirac operator determined by the induced metric and spinor structure depends on r in a highly nonlinear manner but there is no dependence on the overall scaling of the H metric. This in turn implies that the fermionic oscillator algebra used to define WCW spinor structure and metric depends on the value of r. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite series in powers of r.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over WCW defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant WCW metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of r. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semi-classical" approximation, which might be actually exact, automatically would give a full expansion in powers of r. This is in a sharp contrast to the usual quantization approach.

Jones inclusions and hierarchy of Planck constants

From the beginning it was clear that Jones inclusions of hyper-finite factors of type II_1 are somehow related to the hierarchy of Planck constants. The basic motivation for this belief has been that WCW Clifford algebra provides a canonical example of hyper-finite factor of type II_1 and that Jones inclusion of these Clifford algebras is excellent candidate for a first principle description of finite measurement resolution.

Consider the inclusion $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 [K112]. A deep result is that one can express $\mathcal{M}$ as $\mathcal{N} : \mathcal{M}$-dimensional module over N with fractal dimension $\mathcal{N} : \mathcal{M} = B_n$. $\sqrt{b_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{b_n} = 2cos(\pi/n)$ varying thus in the range $[1, 2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra. The interpretation is that finite measurement resolution introduces correlations between components of quantum spinor implying effective reduction of the dimension of quantum spinors providing a description of the factor space $\mathcal{N}/\mathcal{M}$.

This would suggest that somehow the hierarchy of Planck constants must represent finite measurement resolution and since phase factors coming as roots of unity are naturally associated with Jones inclusions the natural guess was that angular resolution and coupling constant evolution associated with it is in question. This picture would suggest that the realization of the hierarchy of Planck constant in terms of a book like structure of generalized embedding space provides also a geometric realization for a hierarchy of Jones inclusions.

The notion of number theoretic braid and realization that the modified Dirac operator has only finite number of generalized eigenmodes -thanks to the vacuum degeneracy of Kähler action-finally led to the understanding how the notion of finite measurement resolution is coded to the Kähler action and the realized in practice by second quantization of induced spinor fields and how these spinor fields endowed with q-anti-commutation relations give rise to a representations of finite-quantum dimensional factor spaces $\mathcal{N}/\mathcal{M}$ associated with the hierarchy of Jones inclusions having generalized embedding space as space-time correlate. This means enormous simplification since infinite-dimensional spinor fields in infinite-dimensional world of classical worlds are replaced with finite-quantum-dimensional spinor fields in discrete points sets provided by number theoretic braids.

The study of a concrete model for Jones inclusions in terms of finite subgroups G of $SU(2)$ defining sub-algebras of infinite-dimensional Clifford algebra as fixed point sub-algebras leads to what looks like a correct track concerning the understanding of quantization of Planck constants.

The ADE diagrams of A_n and D_{2n} characterize cyclic and dihedral groups whereas those of E_6 and E_8 characterize tetrahedral and icosahedral groups. This approach leads to the hypothesis that the scaling factor of Planck constant assignable to Poincare (color) algebra corresponds to the order of the maximal cyclic subgroup of $G_b \subset SU(2)$ ($G_a \subset SL(2,C)$) acting as symmetry of space-time sheet in CP_2 (CD) degrees of freedom. It predicts arbitrarily large CD and CP_2 Planck constants in the case of A_n and D_{2n} under rather general assumptions.

There are two ways for how G_a and G_b can act as symmetries corresponding to G_i coverings and factors spaces. These coverings and factor spaces are singular and associated with spaces $\hat{CD}\backslash M^2$ and $CP_2\backslash S^2_I$, where S^2_I is homologically trivial geodesic sphere of CP_2. The physical interpretation is that M^2 and S^2_I fix preferred quantization axes for energy and angular moment and color quantum numbers so that also a connection with quantum measurement theory emerges.

3.3.2 The Vision

A brief summary of the basic vision behind the generalization of the embedding space concept needed to realize the hierarchy of Planck constants is in order before going to the detailed representation.

1. The hierarchy of Planck constants cannot be realized without generalizing the notions of embedding space and space-time because particles with different values of Planck constant cannot appear in the same interaction vertex. Some kind of book like structure for the generalized embedding space forced also by p-adicization but in different sense is suggestive. Both M^4 and CP_2 factors would have the book like structure so that a Cartesian product of books would be in question.
2. The study of Schrödinger equation suggests that Planck constant corresponds to a scaling factor of CD metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in CD factor is possible so that the interpretation for scaled up value of $\hbar$ is as scaling of the size of causal diamond CD.
3. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states, and the infinite-D spaces of light-like 3-surfaces inside scaled variants of CD define the fundamental building brick of WCW (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces the effect of scaling on classical and quantum dynamics is non-trivial and a coupling constant evolution results and the coding of radiative corrections to the geometry of space-time sheets becomes possible. The basic geometry of CD suggests that the allowed sizes of CD come in the basic sector $\hbar = \hbar_0$ as powers of two. This predicts p-adic length scale hypothesis and lead to number theoretically universal discretized p-adic coupling constant evolution. Since the scaling is accompanied by a formation of singular coverings and factor spaces, different scales are distinguished at the level of topology. p-Adic length scale hierarchy affords similar characterization of length scales in terms of effective topology.
4. The idea that TGD Universe is quantum critical in some sense is one of the key postulates of quantum TGD. The basic ensuing prediction is that Kähler coupling strength is analogous to critical temperature. Quantum criticality in principle fixes the p-adic evolution of various coupling constants also the value of gravitational constant. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized embedding space. Quantum criticality of TGD Universe means that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the very definition of number theoretic braids to these critical sub-manifolds.

At the boundaries of CD associated with positive and negative energy parts of zero energy state in a given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes to regions corresponding to different values of Planck constant much like matter decomposes to several phases at criticality.

The connection with Jones inclusions was originally a purely heuristic guess, and it took half decade to really understand why and how they are involved. The notion of measurement resolution is the key concept.

1. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and the this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized embedding space could indeed serve as correlates for Jones inclusions.
2. The dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field automatically implying cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups associated with Jones inclusions. The interpretation of the Clifford algebra spanned by the fermionic oscillator operators is as a realization for the concept of the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution for angle measurement in the sense that the action of this algebra on zero energy state has no detectable physical effects. $\mathcal{M}$ takes the role of complex numbers in quantum theory and makes physics non-commutative. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.
3. G invariance for the elements of the included algebra can be interpreted in terms of finite measurement resolution in the sense that action by G invariant Clifford algebra element has no detectable effects. Quantum groups realize this view about measurement resolution for angle measurement. The G-invariance of the physical states created by fermionic oscillator operators which by definition are not G invariant guarantees that quantum states as a whole have non-fractional quantum numbers so that the leakage between different pages is possible in principle. This hypothesis is consistent with the TGD inspired model of quantum Hall effect [K77].
4. Concerning the formula for Planck constant in terms of the integers n_a and n_b characterizing orders of the maximal cyclic subgroups of groups G_a and G_b defining coverings and factor spaces associated with CD and CP_2 the basic constraint is that the overall scaling of H metric has no effect on physics. What matters is the ratio of Planck constants $r = \hbar(M^4)/\hbar(CP_2)$ appearing as a scaling factor of M^4 metric. This leaves two options if one requires that the Planck constant defines a homomorphism. The model for dark gravitons suggests a unique choice between these two options but one must keep still mind open for the alternative.
5. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 and CP_2 in some sense. The alternative interpretation assigning the inclusions to the two different geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom and is therefore not convincing.
6. The natural question is why the hierarchy of Planck constants is needed. Is it really necessary? Number theoretic Universality suggests that this is the case. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in the p-adic context. All that one can achieve naturally is the notion of phase defined as a root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases purely geometrically and quantum phases q assignable to

Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on the hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

3.3.3 Hierarchy Of Planck Constants And The Generalization Of The Notion Of Embedding Space

In the following the recent view about structure of embedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E18] that the orbits of the 4 inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K89] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.
2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K90]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or overcritical mass density and the "pressure" associated with these cosmologies is negative.
3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the embedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.
4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K103].
5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K77]. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E18] can be understood. Dark matter would resemble

to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L3, K103], [L3].

The most general option for the generalized embedding space

Simple physical arguments pose constraints on the choice of the most general form of the embedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

    (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the Kähler-Dirac gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

    (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds

to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the embedding space to another one.

1. How the gluing of copies of embedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to $n_a n_b$ sheets and

multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} was proposed to define favored as values of n_a in living matter [K38].

The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1 + i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG [K38]. This hypothesis - to be referred to as Mersenne hypothesis - replaces the rather ad hoc proposal $r = \hbar/\hbar_0 = 2^{11k}$ for the preferred values of Planck constant.

How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the embedding space sector with given values of Planck constants are proportional to the corresponding Planck. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

3.4 Updated View About The Hierarchy Of Planck Constants

During last years the work with TGD proper has transformed from the discovery of brave visions to the work of clock smith. The challenge is to fill in the details, to define various notions more precisely, and to eliminate the numerous inconsistencies.

Few years has passed from the latest formulation for the hierarchy of Planck constant. The original hypothesis was that the hierarchy is real. In this formulation the embedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. It was no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

It seems that the time is ripe for checking whether some polishing of this formulation might be needed. In particular, the work with TGD inspired quantum biology suggests a close connection between the hierarchy of Planck constants and negentropic entanglement. Also the connection with anyons and charge fractionalization (see `http://tinyurl.com/y89xp4bu`) has remained somewhat fuzzy [K77]. In particular, it seems that the formulation based on multi-furcations of space-time surfaces to N branches is not general enough: the N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural: as a matter fact, this interpretation was the original one and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

3.4.1 Basic Physical Ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K104].
2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a

   new interpretation for FQHE (see `http://tinyurl.com/y89xp4bu`) (fractional quantum Hall effect) [K77] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.
3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale (see `http://tinyurl.com/ya6f3s4l`) [E18] who first introduced

the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the Kähler-Dirac equation.

4. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

3.4.2 Space-Time Correlates For The Hierarchy Of Planck Constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of embedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the embedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the Kähler-Dirac gamma matrices [K113] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of embedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book). What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches coincide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations (see `http://tinyurl.com/2swb2p`) represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

3.4.3 The Relationship To The Original View About The Hierarchy Of Planck Constants

Originally the hierarchy of Planck constant was assumed to correspond to a book like structure having as pages the n-fold coverings of the embedding space for various values of n serving therefore as a page number. The pages are glued together along a 4-D "back" at which the branches of n-furcations are degenerate. This leads to a very elegant picture about how the particles belonging to the different pages of the book interact. All vertices are local and involve only particles with the same value of Planck constant: this is enough for darkness in the sense of particle physics. The interactions between particles belonging to different pages involve exchange of a particle travelling from page to another through the back of the book and thus experiencing a phase transition changing the value of Planck constant.

Is this picture consistent with the picture based on n-furcations? This seems to be the case. The conservation of energy in n-furcation in which several sheets are realized simultaneously is consistent with the conservation of classical conserved quantities only if the space-time sheet before n-furcation involves n identical copies of the original space-time sheet or if the Planck constant is $h_{eff} = nh$. This kind of degenerate many-sheetedness is encountered also in the case of branes. The first option means an n-fold covering of embedding space and h_{eff} is indeed effective Planck constant. Second option means a genuine quantization of Planck constant due to the fact the value of Kähler coupling strength $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is scaled down by $1/n$ factor. The scaling of Planck constant consistent with classical field equations since they involve α_K as an overall multiplicative factor only.

3.4.4 Basic Phenomenological Rules Of Thumb In The New Framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elementary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar/m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n + 1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.
2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) automatically.
2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.
3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

3.4.5 Charge Fractionalization And Anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization (see http://tinyurl.com/26tmhoe) the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

   Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionization and fractionization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N+1$: th branch corresponds to the original one. This suggests that angular momentum fractionization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

   The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

   The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in embedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionization however leads to problems with fractionization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

3.4.6 Negentropic Entanglement Between Branches Of Multi-Furcations

The application of negentropic entanglement and effective hierarchy of Planck constants to photosynthesis and metabolism (see http://tinyurl.com/yd7j9f5j) [K57] suggests that these two notions might be closely related. Negentropic entanglement is possible for rational (and even algebraic) entanglement probabilities. If one allows number theoretic variant of Shannon entropy (see http://tinyurl.com/y6v73ryc) based on the p-adic norm for the probability appearing as argument of logarithm [K63], it is quite possible to have negative entanglement entropy and the interpretation is as genuine information carried by entanglement. The superposition of state pairs $a_i \otimes b_i$ in entangled state would represent instances of a rule. In the case of Schrödinger cat the rule states that it is better to not open the bottle: understanding the rule consciously however requires that cat is somewhat dead! Entanglement provides information about the relationship between two systems. Shannon entropy represents lack of information about single particle state.

Negentropic entanglement would replace metabolic energy as the basic quantity making life possible. Metabolic energy could generate negentropic entanglement by exciting biomolecules to negentropically entangled states. ATP providing the energy for generating the metabolic entanglement could also itself carry negentropic entanglement, and transfer it to the target by the emission of large $\hbar$ photons.

How the large $\hbar$ photons could carry negentropic entanglement? There are several options to consider and at this stage it is not possible to pinpoint anyone of them as the only possible one. Several of them could also be realized.

1. In zero energy ontology large $\hbar$ photons could carry the negentropic entanglement as entanglement between positive and negative energy parts of the photon state.
2. The negentropic entanglement of large $\hbar$ photon could be also associated with its positive or energy part or both. Large $\hbar_{eff} = n\hbar$ photon with n-fold energy $E = n \times hf$ is n-sheeted structure consisting of n-photons with energy $E = hf$ de-localized in the discrete space formed by the N space-time sheets. The n single photon states can entangle and since the branches effectively form a discrete space, rational and algebraic entanglement is very natural. There are many options for how this could happen. For instance, for N-fold branching the superposition of all $N!/(N-n)!n!$ states obtained by selecting n branches are possible and the resulting state is entangled state. If this interpretation is correct, the vacuum degeneracy and multi-furcations implied by it would the quintessence of life.
3. A further very attractive possibility discovered quite recently is that large $h_{eff} = nh$ is closely related to the negentropic entanglement between the states of *two* n-furcated - that is dark - space-time sheets. In the most recent formulation negentropic entanglement corresponds to a state characterized by $n \times n$ identity matrix resulting from the measurement of density matrix. The number theoretic entanglement negentropy is positive for primes dividing p and there is unique prime for which it is maximal.

The identification of negentropic entanglement as entanglement between branches of a multi-furcation is not the only possible option.

1. One proposal is that non-localized single particle excitations of cyclotron condensate at magnetic flux tubes give rise to negentropic entanglement relevant to living matter. Dark photons could transfer the negentropic entanglement possibly assignable to electron pairs of ATP molecule.

   The negentropic entanglement associated with cyclotron condensate could be associated with the branches of the large $\hbar$ variant of the condensate. In this case single particle excitation would not be sum of single particle excitations at various positions of 3-space but at various sheet of covering representing points of WCW . If each of the n branches carries $1/n$: th part of electron one would have an anyonic state in WCW .
2. One can also make a really crazy question. Could it be that ATP and various bio-molecules form n-particle states at the n-sheet of n-furcation and that the bio-chemistry involves simultaneous reactions of large numbers of biomolecules at these sheets? If so, the chemical reactions would take place as large number of copies.

Note that in this picture the breaking of time reversal symmetry [K9] in the presence of metabolic energy feed would be accompanied by evolution involving repeated multi-furcations leading to increased complexity. TGD based view about the arrow of time implies that for a given CD this evolution has definite direction of time. At the level of ensemble it implies second law but at the level of individual system means increasing complexity.

3.4.7 Dark Variants Of Nuclear And Atomic Physics

During years I have in rather speculative spirit considered the possibility of dark variants of nuclear and atomic - and perhaps even molecular physics. Also the notion of dark cyclotron state is central in the quantum model of living matter. One such notion is the idea that dark nucleons could realize vertebrate genetic code [K107].

Before the real understanding what charge fractionization means it was possible to imagine several variants of say dark atoms depending on whether both nuclei and electrons are dark or whether only electrons are dark and genuinely fractionally charged. The recent picture however fixes these notions completely. Basic building bricks are just ordinary nuclei and atoms and they form n-particle states associated with n-branches of N-furcation with $n = 1, ..., N$. The fractionization for a single particle state de-localized completely to the discrete space of N branches as the analog of plane wave means that single branch carriers charge $1/N$.

The new element is the possibility of n-particle states populating n branches of the N-furcation: note that there is superposition over the states corresponding to different selections of these n branches. $N-k$ and k-nuclei/atoms are in sense conjugates of each other and they can fuse to form N-nuclei/N-atoms which in fermionic case are analogous to Fermi sea with all states filled.

Bio-molecules seem to obey symbolic dynamics which does not depend much on the chemical properties: this has motivated various linguistic metaphors applied in bio-chemistry to describe the interactions between DNA and related molecules. This motivated the wild speculation was that N-atoms and even N-molecules could make possible the emergence of symbolic representations with $n \leq N$ serving as a name of atom/molecule and that k- and $N-k$ atom/molecule would be analogous to opposite sexes in that there would be strong tendency for them to fuse together to form N-atom/-molecule. For instance, in bio-catalysis k- and $N-k$-atoms/molecules would be paired. The recent picture about n and $N-k$ atoms seems to be consistent with these speculations which I had already given up as too crazy. It is difficult to avoid even the speculation that bio-chemistry could replace chemical reactions with their n-multiples. Synchronized quantum jumps would allow to avoid the disastrous effects of state function reductions on quantum coherence. The second manner to say the same thing is that the effective value of Planck constant is large.

3.4.8 What About The Relationship Of Gravitational Planck Constant To Ordinary Planck Constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g_{eff}^{\alpha\beta}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the Kähler-Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.
2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g_{eff}^{kl} = K^2 m^{kl}$.

   One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g_{eff}^{\alpha\beta} p_\alpha p_\beta = g_{eff}^{\alpha\beta} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g_{eff}^{kl} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{3.4.1}$$

   Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g_{eff}^{kl} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} \quad .$$

   $\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.
3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0}\frac{1}{L} \quad . \tag{3.4.2}$$

For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of the 4 inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta}L_\alpha L_\beta = g_{eff}^{kl}L_k L_l = l(l+1)\hbar^2 \quad . \tag{3.4.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{3.4.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

One might counter argue that if gravitational 4- momentum square is proportional to inertial 4-momentum squared, then Equivalence Principle implies that h_{gr} can have only single value. In ZEO however all wormhole throats - also virtual - are massless and the argument fails. The varying h_{gr} can be assigned only with longitudinal and transversal momentum squared separately but not to the ratio of gravitational and inertial 4-momenta squared which both vanish.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m_{eff}^{kl} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m_{eff}^{kl}.

3.4.9 Hierarchy Of Planck Constants And Non-Determinism Of Kähler Action

Originally the hierarchy of Planck constant was inspired by empirical inputs from neuroscience, biology, and from Nottale's observations. That it is possible to understand the hierarchy in terms of non-determinism of Kähler action - the fundamental difference between TGD and quantum field theories and string models - is a victory for TGD approach (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`, or **Fig. ??** in the appendix of this book).

Recall that non-determinism means that all space-time surfaces with CP_2 projection, which is Lagrangian sub-manifold (at most 2-D) of CP_2, carries a vanishing induced Kähler form and is vacuum extremal. The first guess would be that there is a finite number n of space-time sheets connecting given pair of 3-surfaces at the ends of space-time surface at the light-like boundaries of causal diamond (CD). Planck constant would be given as $h_{eff} = n \times h$ in accordance with the earlier interpretation. The degenerate extremals would have same Kähler action and conserved quantities as assumed also in the earlier approach. That the degenerate extremals co-incide at the ends of space-time surface was motivated by mathemtical aesthetics in the earlier approach but finds an interpretation in terms of non-uniqueness of the preferred extremals.

It is essential that these n degrees of freedom are regarded as genuine physical degrees of freedom, which are however discrete. Negentropic entanglement and dark matter would be associated with them naturally. The effective description would be in terms of n-fold singular covering of embedding space becoming singular at the ends of the space-time surface.

I have also assigned hierarchy of Planck constants with the quantum criticality. Quantum criticality means the existence of an entire continuous family of deformations of space-time sheet with same Kähler action and conserved quantities. The deformations would by definition vanish at the ends of space-time surface. The critical deformations would act as gauge transformations identifiable as conformal symmetries indeed expected to be presents since WCW isometries form a conformal algebra and there is also Kac-Moody type algebra present. The proposal has been that the sub-algebras of conformal algebra for which conformal weights are integer multiples of integer $n = 1, 2, ..$ defined a hierarchy of gauge algebras so that the dynamical algebra reduces to n-dimensional one.

These two identifications seem to be mutually inconsistent. The resolution of the conflict comes from the gauge invariance. For a given Kähler action and conserved quantities there would be n conformal equivalence classes of these 4-surfaces rather than n surfaces, and one would have n-fold degeneracy but for conformal equivalence classes of 4-surfaces rather than 4-surfaces. In Minkowskian regions the degenerate preferred extremals are sheets (graphs of a map from M^4 to CP_2).

3.5 Vision About Dark Matter As Phases With Non-Standard Value Of Planck Constant

3.5.1 Dark Rules

It is useful to summarize the basic phenomenological view about dark matter.

The notion of relative darkness

The essential difference between TGD and more conventional models of dark matter is that darkness is only relative concept.

1. Generalized embedding space forms a book like structure and particles at different pages of the book are dark relative to each other since they cannot appear in the same vertex identified as the partonic 2-surface along which light-like 3-surfaces representing the lines of generalized Feynman diagram meet.
2. Particles at different space-time sheets act via classical gauge field and gravitational field and can also exchange gauge bosons and gravitons (as also fermions) provided these particles can leak from page to another. This means that dark matter can be even photographed [I8]. This interpretation is crucial for the model of living matter based on the assumption that dark matter at magnetic body controls matter visible to us. Dark matter can also suffer a phase transition to visible matter by leaking between the pages of the Big Book.
3. The notion of standard value $\hbar_0$ of $\hbar$ is not a relative concept in the sense that it corresponds to rational $r = 1$. In particular, the situation in which both CD and CP_2 correspond to trivial coverings and factor spaces would naturally correspond to standard physics.

Is dark matter anyonic?

In [K77] a detailed model for the Kähler structure of the generalized embedding space is constructed. What makes this model non-trivial is the possibility that CP_2 Kähler form can have gauge parts which would be excluded in full embedding space but are allowed because of singular covering/factor-space property. The model leads to the conclusion that dark matter is anyonic if the partonic 2-surface, which can have macroscopic or even astrophysical size, encloses the tip of CD within it. Therefore the partonic 2-surface is homologically non-trivial when the tip is regarded as a puncture. Fractional charges for anyonic elementary particles imply confinement to the partonic 2-surface and the particles can escape the two surface only via reactions transforming them to ordinary particles. This would mean that the leakage between different pages of the big book is a rare phenomenon. This could partially explain why dark matter is so difficult to observe.

Field body as carrier of dark matter

The notion of "field body" implied by topological field quantization is essential. There would be em, Z^0, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

p-Adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

3.5.2 Phase Transitions Changing Planck Constant

The general picture is that p-adic length scale hierarchy corresponds to p-adic coupling constant evolution and hierarchy of Planck constants to the coupling constant evolution related to phase resolution. Both evolutions imply a book like structure of the generalized embedding space.

Transition to large $\hbar$ phase and failure of perturbation theory

One of the first ideas was that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces the value of gauge coupling strength $\alpha \propto 1/\hbar$ so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

A justification for this picture would be that in non-perturbative phase large quantum fluctuations are present (as functional integral formalism suggests). At space-time level this could mean that space-time sheet is near to a non-deterministic vacuum extremal -at least if homologically trivial geodesic sphere defines the number theoretic braids. At certain critical value of coupling constant strength one expects that the transition amplitude for phase transition becomes very large. The resulting phase would be of course different from the original since typically charge fractionization would occur.

One should understand why the failure of the perturbation theory (expected to occur for $\alpha Q_1Q_2 > 1$) induces the reduction of Clifford algebra, scaling down of CP_2 metric, and whether the

G-symmetry is exact or only approximate. A partial understanding already exists. The discrete G symmetry and the reduction of the dimension of Clifford algebra would have interpretation in terms of a loss of degrees of freedom as a strongly bound state is formed. The multiple covering of $M^4_\pm$ accompanying strong binding can be understood as an automatic consequence of G-invariance. A concrete realization for the binding could be charge fractionization which would not allow the particles bound on large light-like 3-surface to escape without transformation to ordinary particles.

Two examples perhaps provide more concrete view about this idea.

1. The proposed scenario can reproduce the huge value of the gravitational Planck constant. One should however develop a convincing argument why non-perturbative phase for the gravitating dark matter leads to a formation of $G_a\times$ covering of $CD\backslash M^2 \times CP_2\backslash S^2_I$ with the huge value of $\hbar_{eff} = n_a/n_b \simeq GM_1M_2/v_0$. The basic argument is that the dimensionless parameter $\alpha_{gr} = GM_1M_2/4\pi\hbar$ should be so small that perturbation theory works. This gives $\hbar_{gr} \geq GM_1M_2/4\pi$ so that order of magnitude is predicted correctly.
2. Color confinement represents the simplest example of a transition to a non-perturbative phase. In this case A_2 and $n = 3$ would be the natural option. The value of Planck constant would be 3 times higher than its value in perturbative QCD. Hadronic space-time sheets would be 3-fold coverings of $M^4_\pm$ and baryonic quarks of different color would reside on 3 separate sheets of the covering. This would resolve the color statistics paradox suggested by the fact that induced spinor fields do not possess color as spin like quantum number and by the facts that for orbifolds different quarks cannot move in independent CP_2 partial waves assignable to CP_2 cm degrees of freedom as in perturbative phase.

The mechanism of phase transition and selection rules

The mechanism of phase transition is at classical level similar to that for ordinary phase transitions. The partonic 2-surface decomposes to regions corresponding to difference values of $\hbar$ at quantum criticality in such a way that regions in which induced Kähler form is non-vanishing are contained within single page of embedding space. It might be necessary to assume that only a region corresponding to single value of $\hbar$ is possible for partonic 2-surfaces and $\delta CD \times CP_2$ so that quantum criticality would be associated with the intermediate state described by the light-like 3-surface. One could also see the phase transition as a leakage of X^2 from given page to another: this is like going through a closed door through a narrow slit between door and floor. By quantum criticality the points of number theoretic braid are already in the slit.

As in the case of ordinary phase transitions the allowed phase transitions must be consistent with the symmetries involved. This means that if the state is invariant under the maximal cyclic subgroups G_a and G_b then also the final state must satisfy this condition. This gives constraints to the orders of maximal cyclic subgroups Z_a and Z_b for initial and final state: $n(Z_{a_i})$ *resp.* $n(Z_{b_i}))$ must divide $n(Z_{a_f})$ *resp.* $n(Z_{b_f}$ or vice versa in the case that factors of Z_i do not leave invariant the states. If this is the case similar condition must hold true for appropriate subgroups. In particular, powers of prime Z_{p^n}, $n = 1, 2, ...$ define hierarchies of allowed phase transitions.

3.5.3 Coupling Constant Evolution And Hierarchy Of Planck Constants

If the overall vision is correct, quantum TGD would be characterized by two kinds of couplings constant evolutions. p-Adic coupling constant evolution would correspond to length scale resolution and the evolution with respect to Planck constant to phase resolution. Both evolution would have number theoretic interpretation.

Evolution with respect to phase resolution

The coupling constant evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i2\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only iterated square root operation are number theoretically very special since they correspond to algebraic extensions of p-adic numbers obtained by an iterated square root operation, which should emerge first. Therefore systems involving these values of q should be especially abundant in Nature. That arbitrarily high square roots are involved as becomes clear by studying the case $n = 2^k$: $cos(\pi/2^k) = \sqrt{[1 + cos(\pi/2^{k-1})]/2}$.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K71].

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constant assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

The Dynkin diagrams of exceptional Lie groups E_6 and E_8 are exceptional as subgroups of rotation group in the sense that they cannot be reduced to symmetry transformations of plane. They correspond to the symmetry group $S_4 \times Z_2$ of tetrahedron and $A_5 \times Z_2$ of dodecahedron or its dual polytope icosahedron (A_5 is 60-element subgroup of S_5 consisting of even permutations). Maximal cyclic subgroups are Z_4 and Z_5andthus their orders correspond to Fermat polygons. Interestingly, $n = 5$ corresponds to minimum value of n making possible topological quantum computation using braids and also to Golden Mean.

Is there a correlation between the values of p-adic prime and Planck constant?

The obvious question is whether there is a correlation between p-adic length scale and the value of Planck constant. One-to-one correspondence is certainly excluded but loose correlation seems to exist.

1. In [L63] the information about the number theoretic anatomy of Kähler coupling strength is combined with input from p-adic mass calculations predicting α_K to be the value of fine structure constant at the p-adic length scale associated with electron. One can also develop an explicit expression for gravitational constant assuming its renormalization group invariance on basis of dimensional considerations and this model leads to a model for the fraction of volume of the wormhole contact (piece of CP_2 type extremal) from the volume of CP_2 characterizing gauge boson and for similar volume fraction for the piece of the CP_2 type vacuum extremal associated with fermion.
2. The requirement that gravitational constant is renormalization group invariant implies that the volume fraction depends logarithmically on p-adic length scale and Planck constant (characterizing quantum scale). The requirement that this fraction in the range $(0, 1)$ poses a correlation between the rational characterizing Planck constant and p-adic length scale. In particular, for space-time sheets mediating gravitational interaction Planck constant must be larger than $\hbar_0$ above length scale which is about .1 Angstrom. Also an upper bound for $\hbar$ for given p-adic length scale results but is very large. This means that quantum gravitational effects should become important above atomic length scale [L63].

3.6 Some Applications

Below some applications of the hierarchy of Planck constants as a model of dark matter are briefly discussed. The range of applications varying from elementary particle physics to cosmology and I hope that this will convince the reader that the idea has strong physical motivations.

3.6.1 A Simple Model Of Fractional Quantum Hall Effect

The generalization of the embedding space suggests that it could possible to understand fractional quantum Hall effect [D2] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{3.6.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9..., 1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [D2].

The model of Laughlin [D20] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [D15]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of embedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

1. The easiest manner to understand the observed fractions is by assuming that both CD and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [D2, D8]. The fractionized charge is $e/4$ in the latter case [D8, D4]. Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [D15]. $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene

the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6\times10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather ad hoc. Therefore the model can be taken as a warm-up exercise only. In [K77], where the delicacies of Kähler structure of generalized embedding space are discussed, also a more detailed of QHE is discussed.

3.6.2 Gravitational Bohr Orbitology

The basic question concerns justification for gravitational Bohr orbitology [K89]. The basic vision is that visible matter identified as matter with $\hbar = \hbar_0$ ($n_a = n_b = 1$) concentrates around dark matter at Bohr orbits for dark matter particles. The question is what these Bohr orbits really mean. Should one in improved approximation relate Bohr orbits to 3-D wave functions for dark matter as ordinary Bohr rules would suggest or do the Bohr orbits have some deeper meaning different from that in wave mechanics. Anyonic variants of partonic 2-surfaces with astrophysical size are a natural guess for the generalization of Bohr orbits.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7\pm.7$ km/s giving $v_0/c = 4.6\times10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K89] .

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation -or at least Bohr rules with appropriate interpretation - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

Prediction for the parameter v_0

One of the key questions relate to the value of the parameter v_0. Before the introduction of the hierarchy of Planck constants I proposed that the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n-fold much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. The planetary mass ratios can be produced with an accuracy better than 10 per cent assuming ruler and compass phases.

Further predictions

The study of inclinations (tilt angles with respect to the Earth's orbital plane) leads to a concrete model for the quantum evolution of the planetary system. Only a stepwise breaking of the rotational symmetry and angular momentum Bohr rules plus Newton's equation (or geodesic equation) are needed, and gravitational Schrödinger equation holds true only inside flux quanta for the dark matter.

1. During pre-planetary period dark matter formed a quantum coherent state on the (Z^0) magnetic flux quanta (spherical cells or flux tubes). This made the flux quantum effectively a single rigid body with rotational degrees of freedom corresponding to a sphere or circle (full SO(3) or SO(2) symmetry).
2. In the case of spherical shells associated with inner planets the $SO(3) \rightarrow SO(2)$ symmetry breaking led to the generation of a flux tube with the inclination determined by m and j and a further symmetry breaking, kind of an astral traffic jam inside the flux tube, generated a planet moving inside flux tube. The semiclassical interpretation of the angular momentum algebra predicts the inclinations of the inner planets. The predicted (real) inclinations are 6 (7) resp. 2.6 (3.4) degrees for Mercury resp. Venus). The predicted (real) inclination of the Earth's spin axis is 24 (23.5) degrees.
3. The $v_0 \rightarrow v_0/5$ transition allowing to understand the radii of the outer planets in the model of Da Rocha and Nottale can be understood as resulting from the splitting of (Z^0) magnetic flux tube to five flux tubes representing Earth and outer planets except Pluto, whose orbital parameters indeed differ dramatically from those of other planets. The flux tube has a shape of a disk with a hole glued to the Earth's spherical flux shell.

   It is important to notice that effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question. This allows to consider also alternative explanations. Perhaps external gravitational perturbations have kicked dark matter from the orbit or Earth to $n = 5k$, $k = 2, 3, ..., 7$ orbits: the fact that the tilt angles for Earth and all outer planets except Pluto are nearly the same, supports this explanation. Or perhaps there exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter and these orbits satisfy $n \; mod \; 5 = 0$ for some reason.
4. A remnant of the dark matter is still in a macroscopic quantum state at the flux quanta. It couples to photons as a quantum coherent state but the coupling is extremely small due to the gigantic value of $\hbar_{gr}$ scaling alpha by $\hbar/\hbar_{gr}$: hence the darkness.

   The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with $n = 1$ orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Comparison with Bohr quantization of planetary orbits

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}} n_{F_{b,2}}/n_{F_{b,1}} n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.
2. Nottale [E18] has suggested that also the harmonics and sub-harmonics of $\hbar_{gr}$ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary [K89]). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in solar system [K89] $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}] * X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet) n_{F,b}(Earth)/n_{F,a}(Earth) n_{F,b}(planet)$. The deviations are at most 2 per cent.

$planet$	Me	V	E	M	J
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9\times 5\times 17$	$2^8\times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
$planet$	S	U	N	P	
y	$2^{14}\times 3\times 5\times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^4\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 3.1: Table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k\prod_k F_{n_k}$, where $F_i = 2^{2^i}+1$, $i = 0,1,2,3,4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254}\times 5\times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [K38]. For $v_0 = 4.6\times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1+\epsilon)$ of the total mass for Earth gives

$$1+\epsilon = \frac{17}{16} \tag{3.6.2}$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1-\epsilon) \simeq 4.6\times 10^{-4}$ equals with $v_0(eff) = 1/(2^7\times F_2) = 4.5956\times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v\cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This would suggest that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.
2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of CD seems to resolve these problems.

1. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

2. One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of CD, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

Quantum Hall effect and dark anyonic systems in astrophysical scales

Bohr orbitology could be understood if dark matter concentrates on 2-dimensional partonic surfaces usually assigned with elementary particles and having size of order CP_2 radius. The interpretation is in terms of wormhole throats assignable to topologically condensed CP_2 type extremals (fermions) and pairs of them assignable to wormhole contacts (gauge bosons). Wormhole throat defines the light-like 3-surface at which the signature of metric of space-time surface changes from Minkowskian to Euclidian.

Large value of Planck constant would allow partons with astrophysical size. Since anyonic systems are 2-dimensional, the natural idea is that dark matter corresponds to systems carrying large fermion number residing at partonic 2-surfaces of astrophysical size and that visible matter condenses around these. Not only black holes but also ordinary stars, planetary systems, and planets could correspond at the level of dark matter to atom like structures consisting of anyonic 2-surfaces which can have complex topology (flux tubes associated with planetary orbits connected by radial flux tubes to the central spherical anyonic surface). Charge and spin fractionization are key features of anyonic systems and Jones inclusions inspiring the generalization of embedding space indeed involve quantum groups central in the modelling of anyonic systems. Hence one has could hopes that a coherent theoretical picture could emerge along these lines.

This seems to be the case. Anyons and charge and spin fractionization are discussed in detail [K77] and leads to a precise identification of the delicacies involved with the Kähler gauge potential of CP_2 Kähler form in the sectors of the generalized embedding space corresponding to various pages of boook like structures assignable to CD and CP_2. The basic outcome is that anyons correspond geometrically to partonic 2-surfaces at the light-like boundaries of CD containing the tip of CD inside them. This is what gives rise to charge fractionization and also to confinement like effects since elementary particles in anyonic states cannot as such leak to the other pages of the generalized embedding space. G_a and G_b invariance of the states imply that fractionization occurs only at single particle level and total charge is integer valued.

This picture is much more flexible that based on G_a symmetries of CD orbifold since partonic 2-surfaces do not possess any orbifold symmetries in CD sector anymore. In this framework various astrophysical structures such as spokes and circles would be parts of anyonic 2-surfaces with complex topology representing quantum geometrically quantum coherence in the scale of say solar system. Planets would have formed by the condensation of ordinary matter in the vicinity of the anyonic matter. This would predict stars, planetary system, and even planets to have onion-like structure consisting of shells at the level of dark matter. Similar conclusion is suggested also by purely classical model for the final state of star predicting that matter is strongly concentrated at the surface of the star [K106].

Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} =$

$2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic.

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

3.6.3 Accelerating Periods Of Cosmic Expansion As PhaseTransitions Increasing The Value Of Planck Constant

There are several pieces of evidence for accelerated expansion, which need not mean cosmological constant, although this is the interpretation adopted in [E11, E5]. Quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the value of the gravitational Planck constant. This assumption provides explanation for the apparent cosmological constant. Also planets are predicted to expand in this manner. This provides a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering the entire surface of Earth but with radius which was one half of the recent one.

The four pieces of evidence for accelerated expansion

1. Supernovas of type Ia

Supernovas of type Ia define standard candles since their luminosity varies in an oscillatory manner and the period is proportional to the luminosity. The period gives luminosity and from this the distance can be deduced by using Hubble's law: $d = cz/H_0$, H_0 Hubble's constant. The observation was that the farther the supernova was the more dimmer it was as it should have been. In other words, Hubble's constant increased with distance and the cosmic expansion was accelerating rather than decelerating as predicted by the standard matter dominated and radiation dominated cosmologies.

2. Mass density is critical and 3-space is flat

It is known that the contribution of ordinary and dark matter explaining the constant velocity of distance stars rotating around galaxy is about 25 per cent from the critical density. Could it be that total mass density is critical?

From the anisotropy of cosmic microwave background one can deduce that this is the case. What criticality means geometrically is that 3-space defined as surface with constant value of cosmic time is flat. This reflects in the spectrum of microwave radiation. The spots representing small anisotropies in the microwave background temperature is 1 degree and this correspond to flat 3-space. If one had dark matter instead of dark energy the size of spot would be.5 degrees!

Thus in a cosmology based on general relativity cosmological constant remains the only viable option. The situation is different in TGD based quantum cosmology based on sub-manifold gravity and hierarchy of gravitational Planck constants.

3. The energy density of vacuum is constant in the size scale of big voids

It was observed that the density of dark energy would be constant in the scale of 10^8 light years. This length scale corresponds to the size of big voids containing galaxies at their boundaries.

4. Integrated Sachs-Wolf effect

Also so called integrated Integrated Sachs-Wolf effect supports accelerated expansion. Very slow variations of mass density are considered. These correspond to gravitational potentials. Cosmic expansion tends to flatten them but mass accretion to form structures compensates this effect so that gravitational potentials are unaffected and there is no effect of CMB. Situation changes if dark matter is replaced with dark energy the accelerated expansion flattening the gravitational potentials wins the tendency of mass accretion to make them deeper. Hence if photon passes by an over-dense region, it receives a little energy. Similarly, photon loses energy when passign by an under-dense region. This effect has been observed.

Accelerated expansion in classical TGD

The minimum TGD based explanation for accelerated expansion involves only the fact that the embeddings of critical cosmologies correspond to accelerated expansion. A more detailed model allows to understand why the critical cosmology appears during some periods.

The first observation is that critical cosmologies (flat 3-space) imbeddable to 8-D embedding space H correspond to negative pressure cosmologies and thus to accelerating expansion. The negativity of the counterpart of pressure in Einstein tensor is due to the fact that space-time sheet is forced to be a 4-D surface in 8-D embedding space. This condition is analogous to a force forcing a particle at the surface of 2-sphere and gives rise to what could be called constraint force. Gravitation in TGD is sub-manifold gravitation whereas in GRT it is manifold gravitation. This would be minimum interpretation involving no assumptions about what mechanism gives rise to the critical periods.

Accelerated expansion and hierarchy of Planck constants

One can go one step further and introduce the hierarchy of Planck constants. The basic difference between TGD and GRT based cosmologies is that TGD cosmology is quantum cosmology. Smooth cosmic expansion is replaced by an expansion occurring in discrete jerks corresponding to the increase of gravitational Planck constant. At space-time level this means the replacement of 8-D embedding space H with a book like structure containing almost-copies of H with various values of Planck constant as pages glued together along critical manifold through which space-time sheet can leak between sectors with different values of $\hbar$. This process is the geometric correlate for the phase transition changing the value of Planck constant.

During these phase transition periods critical cosmology applies and predicts automatically accelerated expansion. Neither genuine negative pressure due to “quintessence” nor cosmological constant is needed. Note that quantum criticality replaces inflationary cosmology and predicts a unique cosmology apart from single parameter. Criticality also explains the fluctuations in microwave temperature as long range fluctuations characterizing criticality.

Accelerated expansion and flatness of 3-cosmology

Observations 1) and 2) about super-novae and critical cosmology (flat 3-space) are consistent with this cosmology. In TGD dark energy must be replaced with dark matter because the mass density is critical during the phase transition. This does not lead to wrong sized spots since it is the increase of Planck constant which induces the accelerated expansion understandable also as a constraint force due to embedding to H.

The size of large voids is the characteristic scale

The TGD based model in its simplest form model assigns the critical periods of expansion to large voids of size 10^8 ly. Also larger and smaller regions can express similar periods and dark space-time sheets are expected to obey same universal "cosmology" apart from a parameter characterizing the duration of the phase transition. Observation 3) that just this length scale defines the scale below which dark energy density is constant is consistent with TGD based model.

The basic prediction is jerk-wise cosmic expansion with jerks analogous to quantum transitions between states of atom increasing the size of atom. The discovery of large voids with size of order 10^8 ly but age much longer than the age of galactic large voids conforms with this prediction. One the other hand, it is known that the size of galactic clusters has not remained constant in very long time scale so that jerk-wise expansion indeed seems to occur.

Do cosmic strings with negative gravitational mass cause the phase transition inducing accelerated expansion

Quantum classical correspondence is the basic principle of quantum TGD and suggest that the effective antigravity manifested by accelerated expansion might have some kind of concrete space-time correlate. A possible correlate is super heavy cosmic string like objects at the center of large voids which have negative gravitational mass under very general assumptions. The repulsive gravitational force created by these objects would drive galaxies to the boundaries of large voids. At some state the pressure of galaxies would become too strong and induce a quantum phase transition forcing the increase of gravitational Planck constant and expansion of the void taking place much faster than the outward drift of the galaxies. This process would repeat itself. In the average sense the cosmic expansion would not be accelerating.

3.6.4 Phase Transition Changing Planck Constant And Expanding Earth Theory

TGD predicts that cosmic expansion at the level of individual astrophysical systems does not take place continuously as in classical gravitation but through discrete quantum phase transitions increasing gravitational Planck constant and thus various quantum length and time scales. The reason would be that stationary quantum states for dark matter in astrophysical length scales cannot expand. One would have the analog of atomic physics in cosmic scales. Increases of $\hbar$ by a power of two are favored in these transitions but also other scalings are possible.

This has quite far reaching implications.

1. These periods have a highly unique description in terms of a critical cosmology for the expanding space-time sheet. The expansion is accelerating. The accelerating cosmic expansion can be assigned to this kind of phase transition in some length scale (TGD Universe is fractal). There is no need to introduce cosmological constant and dark energy would be actually dark matter.
2. The recently observed void which has same size of about 10^8 light years as large voids having galaxies near their boundaries but having an age which is much higher than that of the large voids, would represent one example of jerk-wise expansion.
3. This picture applies also to solar system and planets might be perhaps seen as having once been parts of a more or less connected system, the primordial Sun. The Bohr orbits for inner and outer planets correspond to gravitational Planck constant which is 5 times larger for outer planets. This suggests that the space-time sheet of outer planets has suffered a phase transition increasing the size scale by a factor of 5. Earth can be regarded either as n=1 orbit for Planck constant associated with outer planets or n= 5 orbit for inner planetary system. This might have something to do with the very special position of Earth in planetary system. One could even consider the possibility that both orbits are present as dark matter structures. The phase transition would also explain why n=1 and n=2 Bohr orbits are absent and one only n=3, 4, and 5 are present.
4. Also planets should have experienced this kind of phase transitions increasing the radius: the increase by a factor two would be the simplest situation.

The obvious question - that I did not ask - is whether this kind of phase transition might have occurred for Earth and led from a completely granite covered Earth - Pangeia without seas - to the recent Earth. Neither it did not occur to me to check whether there is any support for a rapid expansion of Earth during some period of its history.

Situation changed when my son visited me last Saturday and told me about a Youtube video [F8] by Neal Adams, an American comic book and commercial artist who has also produced animations for geologists. We looked the amazing video a couple of times and I looked it again yesterday. The video is very impressive artwork but in the lack of references skeptic probably cannot avoid the feeling that Neal Adams might use his highly developed animation skills to cheat you. I found also a polemic article [F1] of Adams but again the references were lacking. Perhaps the reason of polemic tone was that the concrete animation models make the expanding Earth hypothesis very convincing but geologists refuse to consider seriously arguments by a layman without a formal academic background.

The claims of Adams

The basic claims of Adams were following.

1. The radius of Earth has increased during last 185 million years (dinosaurs [I1] appeared for about 230 million years ago) by about factor 2. If this is assumed all continents have formed at that time a single super-continent, Pangeia, filling the entire Earth surface rather than only 1/4 of it since the total area would have grown by a factor of 4. The basic argument was that it is very difficult to imagine Earth with 1/4 of surface containing granite and 3/4 covered by basalt. If the initial situation was covering by mere granite -as would look natural- it is very difficult for a believer in thermodynamics to imagine how the granite would have gathered to a single connected continent.
2. Adams claims that Earth has grown by keeping its density constant, rather than expanded, so that the mass of Earth has grown linearly with radius. Gravitational acceleration would have thus doubled and could provide a partial explanation for the disappearance of dinosaurs: it is difficult to cope in evolving environment when you get slower all the time.
3. Most of the sea floor is very young and the areas covered by the youngest basalt are the largest ones. This Adams interprets this by saying that the expansion of Earth is accelerating. The alternative interpretation is that the flow rate of the magma slows down as it recedes from the ridge where it erupts. The upper bound of 185 million years for the age of sea floor requires that the expansion period - if it is already over - lasted about 185 million years after which the flow increasing the area of the sea floor transformed to a convective flow with subduction so that the area is not increasing anymore.
4. The fact that the continents fit together - not only at the Atlantic side - but also at the Pacific side gives strong support for the idea that the entire planet was once covered by the super-continent. After the emergence of subduction theory this evidence as been dismissed.
5. I am not sure whether Adams mentions the following objections [F2]. Subduction only occurs on the other side of the subduction zone so that the other side should show evidence of being much older in the case that oceanic subduction zones are in question. This is definitely not the case. This is explained in plate tectonics as a change of the subduction direction. My explanation would be that by the symmetry of the situation both oceanic plates bend down so that this would represent new type of boundary not assumed in the tectonic plate theory.
6. As a master visualizer Adams notices that Africa and South-America do not actually fit together in absence of expansion unless one assumes that these continents have suffered a deformation. Continents are not easily deformable stuff. The assumption of expansion implies a perfect fit of *all* continents without deformation.

Knowing that the devil is in the details, I must admit that these arguments look rather convincing to me and what I learned from Wikipedia articles supports this picture.

The critic of Adams of the subduction mechanism

The prevailing tectonic plate theory [F5] has been compared to the Copernican revolution in geology. The theory explains the young age of the seafloor in terms of the decomposition of the litosphere to tectonic plates and the convective flow of magma to which oceanic tectonic plates participate. The magma emerges from the crests of the mid ocean ridges representing a boundary of two plates and leads to the expansion of sea floor. The variations of the polarity of Earth's magnetic field coded in sea floor provide a strong support for the hypothesis that magma emerges from the crests.

The flow back to would take place at so called oceanic trenches [F3] near continents which represent the deepest parts of ocean. This process is known as subduction. In subduction oceanic tectonic plate bends and penetrates below the continental tectonic plate, the material in the oceanic plate gets denser and sinks into the magma. In this manner the oceanic tectonic plate suffers a metamorphosis returning back to the magma: everything which comes from Earth's interior returns back. Subduction mechanism explains elegantly formation of mountains [F4] (orogeny), earth quake zones, and associated zones of volcanic activity [F6] .

Adams is very polemic about the notion of subduction, in particular about the assumption that it generates steady convective cycle. The basic objections of Adams against subduction are following.

1. There are not enough subduction zones to allow a steady situation. According to Adams, the situation resembles that for a flow in a tube which becomes narrower. In a steady situation the flow should accelerate as it approaches subduction zones rather than slow down. Subduction zones should be surrounded by large areas of sea floor with constant age. Just the opposite is suggested by the fact that the youngest portion of sea-floor near the ridges is largest. The presence of zones at which both ocean plates bend down could improve the situation. Also jamming of the flow could occur so that the thickness of oceanic plate increases with the distance from the eruption ridge. Jamming could increase also the density of the oceanic plate and thus the effectiveness of subduction.
2. There is no clear evidence that subduction has occurred at other planets. The usual defense is that the presence of sea is essential for the subduction mechanism.
3. One can also wonder what is the mechanism that led to the formation of single super continent Pangeia covering 1/4 of Earth's surface. How probable the gathering of all separate continents to form single cluster is? The later events would suggest that just the opposite should have occurred from the beginning.

Expanding Earth theories are not new

After I had decided to check the claims of Adams, the first thing that I learned is that Expanding Earth theory [F2], whose existence Adams actually mentions, is by no means new. There are actually many of them.

The general reason why these theories were rejected by the main stream community was the absence of a convincing physical mechanism of expansion or of growth in which the density of Earth remains constant.

1. 1888 Yarkovski postulated some sort of aether absorbed by Earth and transforming to chemical elements (TGD version of aether could be dark matter). 1909 Mantovani postulated thermal expansion but no growth of the Earth's mass.
2. Paul Dirac's idea about changing Planck constant led Pascual Jordan in 1964 to a modification of general relativity predicting slow expansion of planets. The recent measurement of the gravitational constant imply that the upper bound for the relative change of gravitational constant is 10 time too small to produce large enough rate of expansion. Also many other theories have been proposed but they are in general conflict with modern physics.
3. The most modern version of Expanding Earth theory is by Australian geologist Samuel W. Carey. He calculated that in Cambrian period (about 500 million years ago) all continents were stuck together and covered the entire Earth. Deep seas began to evolve then.

Summary of TGD based theory of Expanding Earth

TGD based model differs from the tectonic plate model but allows subduction which cannot imply considerable back-flow of magma. Let us sum up the basic assumptions and implications.

1. The expansion is or was due to a quantum phase transition increasing the value of gravitational Planck constant and forced by the cosmic expansion in the average sense.
2. Tectonic plates do not participate to the expansion and therefore new plate must be formed and the flow of magma from the crests of mid ocean ridges is needed. The decomposition of a single plate covering the entire planet to plates to create the mid ocean ridges is necessary for the generation of new tectonic plate. The decomposition into tectonic plates is thus prediction rather than assumption.
3. The expansion forced the decomposition of Pangeia super-continent covering entire Earth for about 530 million years ago to split into tectonic plates which began to recede as new non-expanding tectonic plate was generated at the ridges creating expanding sea floor. The initiation of the phase transition generated formation of deep seas.
4. The eruption of plasma from the crests of ocean ridges generated oceanic tectonic plates which did not participate to the expansion by density reduction but by growing in size. This led to a reduction of density in the interior of the Earth roughly by a factor 1/8. From the upper bound for the age of the seafloor one can conclude that the period lasted for about 185 million years after which it transformed to convective flow in which the material returned back to the Earth interior. Subduction at continent-ocean floor boundaries and downwards double bending of tectonic plates at the boundaries between two ocean floors were the mechanisms. Thus tectonic plate theory would be more or less the correct description for the recent situation.
5. One can consider the possibility that the subducted tectonic plate does not transform to magma but is fused to the tectonic layer below continent so that it grows to an iceberg like structure. This need not lead to a loss of the successful predictions of plate tectonics explaining the generation of mountains, earthquake zones, zones of volcanic activity, etc...
6. From the video of Adams it becomes clear that the tectonic flow is East-West asymmetric in the sense that the western side is more irregular at large distances from the ocean ridge at the western side. If the magma rotates with slightly lower velocity than the surface of Earth (like liquid in a rotating vessel), the erupting magma would rotate slightly slower than the tectonic plate and asymmetry would be generated.
7. If the planet has not experienced a phase transition increasing the value of Planck constant, there is no need for the decomposition to tectonic plates and one can understand why there is no clear evidence for tectonic plates and subduction in other planets. The conductive flow of magma could occur below this plate and remain invisible.

The biological implications might provide a possibility to test the hypothesis.

1. Great steps of progress in biological evolution are associated with catastrophic geological events generating new evolutionary pressures forcing new solutions to cope in the new situation. Cambrian explosion indeed occurred about 530 years ago (the book "Wonderful Life" of Stephen Gould [I14] explains this revolution in detail) and led to the emergence of multicellular creatures, and generated huge number of new life forms living in seas. Later most of them suffered extinction: large number of phylae and groups emerged which are not present nowadays.

   Thus Cambrian explosion is completely exceptional as compared to all other dramatic events in the evolution in the sense that it created something totally new rather than only making more complex something which already existed. Gould also emphasizes the failure to identify any great change in the environment as a fundamental puzzle of Cambrian explosion. Cambrian explosion is also regarded in many quantum theories of consciousness (including TGD) as a revolution in the evolution of consciousness: for instance, micro-tubuli emerged at this time. The periods of expansion might be necessary for the emergence of multicellular life forms on planets and the fact that they unavoidably occur sooner or later suggests that also life develops unavoidably.
2. TGD predicts a decrease of the surface gravity by a factor 1/4 during this period. The reduction of the surface gravity would have naturally led to the emergence of dinosaurs 230 million years ago as a response coming 45 million years after the accelerated expansion ceased. Other reasons led then to the decline and eventual catastrophic disappearance of the dinosaurs.

The reduction of gravity might have had some gradually increasing effects on the shape of organisms also at microscopic level and manifest itself in the evolution of genome during expansion period.

3. A possibly testable prediction following from angular momentum conservation ($\omega R^2 = constant$) is that the duration of day has increased gradually and was four times shorter during the Cambrian era. For instance, genetically coded bio-clocks of simple organisms during the expansion period could have followed the increase of the length of day with certain lag or failed to follow it completely. The simplest known circadian clock is that of the prokaryotic cyanobacteria. Recent research has demonstrated that the circadian clock of Synechococcus elongatus can be reconstituted in vitro with just the three proteins of their central oscillator. This clock has been shown to sustain a 22 hour rhythm over several days upon the addition of ATP: the rhythm is indeed faster than the circadian rhythm. For humans the average innate circadian rhythm is however 24 hours 11 minutes and thus conforms with the fact that human genome has evolved much later than the expansion ceased.

4. Scientists have found a fossil of a sea scorpion with size of 2.5 meters [I3], which has lived for about 10 million years for 400 million years ago in Germany. The gigantic size would conform nicely with the much smaller value of surface gravity at that time. The finding would conform nicely with the much smaller value of surface gravity at that time. Also the emergence of trees could be understood in terms of a gradual growth of the maximum plant size as the surface gravity was reduced. The fact that the oldest known tree fossil is 385 million years old [I9] conforms with this picture.

Did intra-terrestrial life burst to the surface of Earth during Cambrian expansion?

The possibility of intra-terrestrial life [**?**] is one of the craziest TGD inspired ideas about the evolution of life and it is quite possible that in its strongest form the hypothesis is unrealistic. One can however try to find what one obtains from the combination of the IT hypothesis with the idea of pre-Cambrian granite Earth. Could the harsh pre-Cambrian conditions have allowed only intra-terrestrial multicellular life? Could the Cambrian explosion correspond to the moment of birth for this life in the very concrete sense that the magma flow brought it into the day-light?

1. Gould emphasizes the mysterious fact that very many life forms of Cambrian explosion looked like final products of a long evolutionary process. Could the eruption of magma from the Earth interior have induced a burst of intra-terrestrial life forms to the Earth's surface? This might make sense: the life forms living at the bottom of sea do not need direct solar light so that they could have had intra-terrestrial origin. It is quite possible that Earth's mantle contained low temperature water pockets, where the complex life forms might have evolved in an environment shielded from meteoric bombardment and UV radiation.

2. Sea water is salty. It is often claimed that the average salt concentration inside cell is that of the primordial sea: I do not know whether this claim can be really justified. If the claim is true, the cellular salt concentration should reflect the salt concentration of the water inside the pockets. The water inside water pockets could have been salty due to the diffusion of the salt from ground but need not have been same as that for the ocean water (higher than for cell interior and for obvious reasons). Indeed, the water in the underground reservoirs in arid regions such as Sahara is salty, which is the reason for why agriculture is absent in these regions. Note also that the cells of marine invertebrates are osmoconformers able to cope with the changing salinity of the environment so that the Cambrian revolutionaries could have survived the change in the salt concentration of environment.

3. What applies to Earth should apply also to other similar planets and Mars [E6] is very similar to Earth. The radius is .533 times that for Earth so that after quantum leap doubling the radius and thus Schumann frequency scale (7.8 Hz would be the lowest Schumann frequency) would be essentially same as for Earth now. Mass is.131 times that for Earth so that surface gravity would be.532 of that for Earth now and would be reduced to.131 meaning quite big dinosaurs! have learned that Mars probably contains large water reservoirs in it's interior and that there is an un-identified source of methane gas usually assigned with the presence of life. Could it be that Mother Mars is pregnant and just waiting for the great quantum leap when

it starts to expand and gives rise to a birth of multicellular life forms. Or expressing freely how Bible describes the moment of birth: in the beginning there was only darkness and water and then God saidLet the light come!

To sum up, TGD would provide only the long sought mechanism of expansion and a possible connection with the biological evolution. It would be indeed fascinating if Planck constant changing quantum phase transitions in planetary scale would have profoundly affected the biosphere.

3.6.5 Allais Effect As Evidence For Large Values Of Gravitational Planck Constant?

Allais effect [E1, E28] is a fascinating gravitational anomaly associated with solar eclipses. It was discovered originally by M. Allais, a Nobelist in the field of economy, and has been reproduced in several experiments but not as a rule. The experimental arrangement uses so called paraconical pendulum, which differs from the Foucault pendulum in that the oscillation plane of the pendulum can rotate in certain limits so that the motion occurs effectively at the surface of sphere.

Experimental findings

Consider first a brief summary of the findings of Allais and others [E28].

a) In the ideal situation (that is in the absence of any other forces than gravitation of Earth) paraconical pendulum should behave like a Foucault pendulum. The oscillation plane of the paraconical pendulum however begins to rotate.

b) Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

c) Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5\times 10^{-4}$ [E1, E26] which happens to correspond to the constant $v_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

d) There is also quite recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [E31].

TGD based models for Allais effect

I have already earlier proposed an explanation of the effect in terms of classical Z^0 force [K14]. If the Z^0 charge to mass ratio of pendulum varies and if Earth and Moon are Z^0 conductors, the resulting model is quite flexible and one might hope it could explain the high variation of the experimental results.

The rapid variation of the effect during the eclipse is however a problem for this approach and suggests that gravitational screening or some more general interference effect might be present. Gravitational screening alone cannot however explain Allais effect.

A model based on the idea that gravitational interaction is mediated by topological light rays (MEs) and that gravitons correspond to a gigantic value of the gravitational Planck constant however explains the Allais effect as an interference effect made possible by macroscopic quantum coherence in astrophysical length scales. Equivalence Principle fixes the model to a high degree and one ends up with an explicit formula for the anomalous gravitational acceleration and the general order of magnitude and the large variation of the frequency change as being due to the variation of the distance ratio $r_{S,P}/r_{M,P}$ (S, M, and P refer to Sun, Moon, and pendulum respectively). One can say that the pendulum acts as an interferometer.

3.6.6 Applications To Elementary Particle Physics, Nuclear Physics, And Condensed Matter Physics

The hierarchy of Planck constants could have profound implications for even elementary particle physics since the strong constraints on the existence of new light particles coming from the decay widths of intermediate gauge bosons can be circumvented because direct decays to dark matter are not possible. On the other hand, if light scaled versions of elementary particles exist they must be dark since otherwise their existence would be visible in these decay widths. The constraints on the existence of dark nuclei and dark condensed matter are much milder. Cold fusion and some other anomalies of nuclear and condensed matter physics - in particular the anomalies of water- might have elegant explanation in terms of dark nuclei.

Leptohadron hypothesis

TGD suggests strongly the existence of lepto-hadron [K104]. Lepto-hadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of orto-positronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that lepto-pions (or electro-pions) decay to lepto-nucleon pair $e^+_{ex}e^-_{ex}$ first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged [C49]. Towards the end of 2008 CDF anomaly [C15] gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly [K104] led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

Cold fusion, plasma electrolysis, and burning salt water

The article of Kanarev and Mizuno [D16] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

In nuclear string model nucleon are connected by color bonds representing the color magnetic body of nucleus and having length considerably longer than nuclear size. One can consider also dark nuclei for which the scale of nucleus is of atomic size [L3], [L3]. In this framework can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the catode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

1. The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [E16]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.
2. The so called $H_{1.5}O$ anomaly of water [D17, D14, D19, D10] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.
3. The mysterious behavior burning salt water [D1] can be also understood in the same framework.
4. The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. This kind of transmutations have been reported also in living matter long time ago [C13, C66]. Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

3.6.7 Applications To Biology And Neuroscience

The notion of field or magnetic body regarded as carrier of dark matter with large Planck constant and quantum controller of ordinary matter is the basic idea in the TGD inspired model of living matter.

Do molecular symmetries in living matter relate to non-standard values of Planck constant?

Water is exceptional element and the possibility that G_a as symmetry of singular factor space of CD in water and living matter is intriguing.

1. There is evidence for an icosahedral clustering in [D21] [D18]. Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by CD points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of CD points by CP_2 could be involved.
2. It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.
3. What makes realization of G_a as a symmetry of singular factor space of CD is that the biomolecules most relevant for the functioning of brain (DNA nucleotides, amino-acids acting as neurotransmitters, molecules having hallucinogenic effects) contain aromatic 5- and 6-cycles.

These observations led to an identification of the formula for Planck constant (two alternatives were allowed by the condition that Planck constant is algebraic homomorphism) which was not consistent with the model for dark gravitons. If one accepts the proposed formula of Planck constant, the dark space-time sheets with large Planck constant correspond to factor spaces of both $\hat{CD}\backslash M^2$ and of $CP_2\backslash S^2_I$. This identification is of course possible and it remains to be seen whether it leads to any problems. For gravitational space-time sheets only coverings of both CD and CP_2 make sense and the covering group G_a has very large order and does not correspond to geometric symmetries analogous to those of molecules.

High T_c super-conductivity in living matter

The model for high T_c super-conductivity realized as quantum critical phenomenon predicts the basic scales of cell membrane [K21] from energy minimization and p-adic length scale hypothesis. This leads to the vision that cell membrane and possibly also its scaled up dark fractal variants define Josephson junctions generating Josephson radiation communicating information about the nearby environment to the magnetic body.

Any model of high T_c superconductivity should explain various strange features of high T_c superconductors. One should understand the high value of T_c, the ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap temperature $T_{c_1} > T_c$ and scaling law for resistance for $T_c \leq T < T_{c_1}$, the role of fluctuating charged stripes which are anti-ferromagnetic defects of a Mott insulator, the existence of a critical doping, etc... [D13, D5].

There are reasons to believe that high T_c super-conductors correspond to quantum criticality in which at least two (cusp catastrophe as in van der Waals model), or possibly three or even more phases, are competing. A possible analogy is provided by the triple critical point for water vapor, liquid phase and ice coexist. Instead of long range thermal fluctuations long range quantum fluctuations manifesting themselves as fluctuating stripes are present [D13].

The TGD based model for high T_c super-conductivity [K21] relies on the notions of quantum criticality, general ideas of catastrophe theory, dynamical Planck constant, and many-sheeted space-time. The 4-dimensional spin glass character of space-time dynamics deriving from the vacuum degeneracy of the Kähler action defining the basic variational principle would realize space-time correlates for quantum fluctuations.

1. Two kinds of super-conductivities and ordinary non-super-conducting phase would be competing at quantum criticality at T_c and above it only one super-conducting phase and ordinary conducting phase located at stripes representing ferromagnetic defects making possible formation of $S = 1$ Cooper pairs.
2. The first super-conductivity would be based on exotic Cooper pairs of large $\hbar$ dark electrons with $\hbar = 2^{11}\hbar_0$ and able to have spin $S = 1$, angular momentum $L = 2$, and total angular momentum $J = 2$. Second type of super-conductivity would be based on BCS type Cooper pairs having vanishing spin and bound by phonon interaction. Also they have large $\hbar$ so that gap energy and critical temperature are scaled up in the same proportion. The exotic Cooper pairs are possible below the pseudo gap temperature $T_{c_1} > T_c$ but are unstable against decay to BCS type Cooper pairs which above T_c are unstable against a further decay to conduction electrons flowing along stripes. This would reduce the exotic super-conductivity to finite conductivity obeying the observed scaling law for resistance.
3. The mere assumption that electrons of exotic Cooper pairs feed their electric flux to larger space-time sheet via *two* elementary particle sized wormhole contacts rather than only *one* wormhole contacts implies that the throats of wormhole contacts defining analogs of Higgs field must carry quantum numbers of quark and anti-quark. This inspires the idea that cylindrical space-time sheets, the radius of which turns out to be about about 5 nm, representing zoomed up dark electrons of Cooper pair with Planck constant $\hbar = 2^{11}\hbar_0$ are colored and bound by a scaled up variant of color force to form a color confined state. Formation of Cooper pairs would have nothing to do with direct interactions between electrons. Thus high T_c super-conductivity could be seen as a first indication for the presence of scaled up variant of QCD in mesoscopic length scales.

This picture leads to a concrete model for high T_c superconductors as quantum critical superconductors [K21]. p-Adic length scale hypothesis stating that preferred p-adic primes $p \simeq 2^k$, k integer, with primes (in particular Mersenne primes) preferred, makes the model quantitative.

1. An unexpected prediction is that coherence length ξ is actually $\hbar_{eff}/\hbar_0 = 2^{11}$ times longer than the coherence length 5-10 Angstroms deduced theoretically from gap energy using conventional theory and varies in the range $1-5$ μm, the cell nucleus length scale. Hence type I super-conductor would be in question with stripes as defects of anti-ferromagnetic Mott insulator serving as duals for the magnetic defects of type I super-conductor in nearly critical magnetic field.

2. At quantitative level the model reproduces correctly the four poorly understood photon absorption lines and allows to understand the critical doping ratio from basic principles.

3. The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same. One of the characteristic absorption lines has energy of.05 eV which corresponds to the Josephson energy for neuronal membrane for activation potential $V = 50$ mV. Hence the idea that axons are high T_c superconductors is highly suggestive. Dark matter hierarchy coming in powers $\hbar/\hbar_0 = 2^{k11}$ suggests hierarchy of Josephson junctions needed in TGD based model of EEG [K38].

Magnetic body as a sensory perceiver and intentional agent

The hypothesis that dark magnetic body serves as an intentional agent using biological body as a motor instrument and sensory receptor is consistent with Libet's findings about strange time delays of consciousness. Magnetic body would carry cyclotron Bose-Einstein condensates of various ions. Magnetic body must be able to perform motor control and receive sensory input from biological body.

Cell membrane would be a natural sensor providing information about cell interior and exterior to the magnetic body and dark photons at appropriate frequency range would naturally communicate this information. The strange quantitative co-incidences with the physics of cell membrane and high T_c super-conductivity support the idea that Josephson radiation generated by Josephson currents of dark electrons through cell membrane is responsible for this communication [K38].

Also fractally scaled up versions of cell membrane at higher levels of dark matter hierarchy (in particular those corresponding to powers $n = 2^{k11}$) are possible and the model for EEG indeed relies on this hypothesis. The thickness for the fractal counterpart of cell membrane thickness would be 2^{44} fold and of order of depth of ionosphere! Although this looks weird it is completely consistent with the notion of magnetic body as an intentional agent.

Motor control would be most naturally performed via genome: this is achieved if flux sheets traverse through DNA strands. Flux quantization for large values of Planck constant requires rather large widths for the flux sheets. If flux sheet contains sequences of genomes like the page of book contains lines of text, a coherent gene expression becomes possible at level of organs and even populations and one can speak about super- and hyper-genomes. Introns might relate to the collective gene expression possibly realized electromagnetically rather than only chemically [K21, K22].

Dark cyclotron radiation with photon energy above thermal energy could be used for co-ordination purposes at least. The predicted hierarchy of copies of standard model physics leads to ask whether also dark copies of electro-weak gauge bosons and gluons could be important in living matter. As already mentioned, dark W bosons could make possible charge entanglement and non-local quantum bio-control by inducing voltage differences and thus ionic currents in living matter.

The identification of plasmoids as rotating magnetic flux structures carrying dark ions and electrons as primitive life forms is natural in this framework. There exists experimental support for this identification [I12] but the main objection is the high temperature involved: this objection could be circumvented if large $\hbar$ phase is involved. A model for the pre-biotic evolution relying also on this idea is discussed in [**?**].

At the level of biology there are now several concrete applications leading to a rich spectrum of predictions. Magnetic flux quanta would carry charged particles with large Planck constant.

1. The shortening of the flux tubes connecting biomolecules in a phase transition reducing Planck constant could be a basic mechanism of bio-catalysis and explain the mysterious ability of biomolecules to find each other. Similar process in time direction could explain basic aspects of symbolic memories as scaled down representations of actual events.

2. The strange behavior of cell membrane suggests that a dominating portion of important biological ions are actually dark ions at magnetic flux tubes so that ionic pumps and channels are needed only for visible ions. This leads to a model of nerve pulse explaining its unexpected thermodynamical properties with basic properties of Josephson currents making it

un-necessary to use pumps to bring ions back after the pulse. The model predicts automatically EEG as Josephson radiation and explains the synchrony of both kHz radiation and of EEG.

3. The DC currents of Becker could be accompanied by Josephson currents running along flux tubes making possible dissipation free energy transfer and quantum control over long distances and meridians of chinese medicine could correspond to these flux tubes.
4. The model of DNA as topological quantum computer assumes that nucleotides and lipids are connected by ordinary or "wormhole" magnetic flux tubes acting as strands of braid and carrying dark matter with large Planck constant. The model leads to a new vision about TGD in which the assignment of nucleotides to quarks allows to understood basic regularities of DNA not understood from biochemistry.
5. Each physical system corresponds to an onion-like hierarchy of field bodies characterized by p-adic primes and value of Planck constant. The highest value of Planck constant in this hierarchy provides kind of intelligence quotient characterizing the evolutionary level of the system since the time scale of planned action and memory correspond to the temporal distance between tips of corresponding causal diamond (CD). Also the spatial size of the system correlates with the Planck constant. This suggests that great evolutionary leaps correspond to the increase of Planck constant for the highest level of hierarchy of personal magnetic bodies. For instance, neurons would have much more evolved magnetic bodies than ordinary cells.
6. At the level of DNA this vision leads to an idea about hierarchy of genomes. Magnetic flux sheets traversing DNA strands provide a natural mechanism for magnetic body to control the behavior of biological body by controlling gene expression. The quantization of magnetic flux states that magnetic flux is proportional to $\hbar$ and thus means that the larger the value of $\hbar$ is the larger the width of the flux sheet is. For larger values of $\hbar$ single genome is not enough to satisfy this condition. This leads to the idea that the genomes of organs, organism, and even population, can organize like lines of text at the magnetic flux sheets and form in this manner a hierarchy of genomes responsible for a coherent gene expression at level of cell, organ, organism and population and perhaps even entire biosphere. This would also provide a mechanism by which collective consciousness would use its biological body - biosphere.

DNA as topological quantum computer

I ended up with the recent model of TQC in bottom-up manner and this representation is followed also in the text. The model which looks the most plausible one relies on two specific ideas.

1. Sharing of labor means conjugate DNA would do TQC and DNA would "print" the outcome of TQC in terms of mRNA yielding amino-acids in the case of exons. RNA could result also in the case of introns but not always. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of TQC also electromagnetically in terms of standardized field patterns as Gariaev's findings suggest [I5]. Also speech would be a form of gene expression. The quantum states braid (in zero energy ontology) would entangle with characteristic gene expressions. This argument turned out to be based on a slightly wrong belief about DNA: later I learned that both strand and its conjugate are transcribed but in different directions. The symmetry breaking in the case of transcription is only local which is also visible in DNA replication as symmetry breaking between leading and lagging strand. Thus the idea about *entire* leading strand devoted to printing and second strand to TQC must be weakened appropriately.
2. The manipulation of braid strands transversal to DNA must take place at 2-D surface. Here dancing metaphor for topological quantum computation [C26] generalizes. The ends of the space-like braid are like dancers whose feet are connected by thin threads to a wall so that the dancing pattern entangles the threads. Dancing pattern defines both the time-like braid, the running of classical TQC program and its representation as a dynamical pattern. The space-like braid defined by the entangled threads represents memory storage so that TQC program is automatically written to memory as the braiding of the threads during the TQC. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic

reticulum included are good candidates for dancing halls. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

One ends up to the model also in top-down manner.

1. Darwinian selection for which standard theory of self-organization [B5] provides a model, should apply also to TQC programs. TQC programs should correspond to asymptotic self-organization patterns selected by dissipation in the presence of metabolic energy feed. The spatial and temporal pattern of the metabolic energy feed characterizes the TQC program - or equivalently - sub-program call.
2. Since braiding characterizes the TQC program, the self-organization pattern should correspond to a hydrodynamical flow or a pattern of magnetic field inducing the braiding. Braid strands must correspond to magnetic flux tubes of the magnetic body of DNA. If each nucleotide is transversal magnetic dipole it gives rise to transversal flux tubes, which can also connect to the genome of another cell.
3. The output of TQC sub-program is probability distribution for the outcomes of state function reduction so that the sub-program must be repeated very many times. It is represented as four-dimensional patterns for various rates (chemical rates, nerve pulse patterns, EEG power distributions, ...) having also identification as temporal densities of zero energy states in various scales. By the fractality of TGD Universe there is a hierarchy of TQC's corresponding to p-adic and dark matter hierarchies. Programs (space-time sheets defining coherence regions) call programs in shorter scale. If the self-organizing system has a periodic behavior each TQC module defines a large number of almost copies of itself asymptotically. Generalized EEG could naturally define this periodic pattern and each period of EEG would correspond to an initiation and halting of TQC. This brings in mind the periodically occurring sol-gel phase transition inside cell near the cell membrane.
4. Fluid flow must induce the braiding which requires that the ends of braid strands must be anchored to the fluid flow. Recalling that lipid mono-layers of the cell membrane are liquid crystals and lipids of interior mono-layer have hydrophilic ends pointing towards cell interior, it is easy to guess that DNA nucleotides are connected to lipids by magnetic flux tubes and hydrophilic lipid ends are stuck to the flow.
5. The topology of the braid traversing cell membrane cannot affected by the hydrodynamical flow. Hence braid strands must be split during TQC. This also induces the desired magnetic isolation from the environment. Halting of TQC reconnects them and make possible the communication of the outcome of TQC.
6. There are several problems related to the details of the realization. How nucleotides A, T, C, G are coded to strand color and what this color corresponds to? The prediction that wormhole contacts carrying quark and anti-quark at their ends appear in all length scales in TGD Universe resolves the problem. How to split the braid strands in a controlled manner? High T_c super conductivity provides a partial understanding of the situation: braid strand can be split only if the supra current flowing through it vanishes. From the proportionality of Josephson current to the quantity $sin(\int 2eVdt)$ it follows that a suitable voltage pulse V induces DC supra-current and its negative cancels it. The conformation of the lipid controls whether it it can follow the flow or not. How magnetic flux tubes can be cut without breaking the conservation of the magnetic flux? The notion of wormhole magnetic field saves the situation now: after the splitting the flux returns back along the second space-time sheet of wormhole magnetic field.

To sum up, it seems that essentially all new physics involved with TGD based view about quantum biology enter to the model in crucial manner.

Quantum model of nerve pulse and EEG

In this article a unified model of nerve pulse and EEG is discussed.

1. In TGD Universe the function of EEG and its variants is to make possible communications from the cell membrane to the magnetic body and the control of the biological body by the

magnetic body via magnetic flux sheets traversing DNA by inducing gene expression. This leads to the notions of super- and hyper-genome predicting coherent gene expression at level of organs and population.

2. The assignment the predicted ranged classical weak and color gauge fields to dark matter hierarchy was a crucial step in the evolution of the model, and led among other things to a model of high T_c superconductivity predicting the basic scales of cell, and also to a generalization of EXG to a hierarchy of ZXGs, WXGs, and GXGs corresponding to Z^0, W bosons and gluons.

3. Dark matter hierarchy and the associated hierarchy of Planck constants plays a key role in the model. For instance, in the case of EEG Planck constant must be so large that the energies of dark EEG photons are above thermal energy at physiological temperatures. The assumption that a considerable fraction of the ionic currents through the cell membrane are dark currents flowing along the magnetic flux tubes explains the strange findings about ionic currents through cell membrane. Concerning the model of nerve pulse generation, the newest input comes from the model of DNA as a topological quantum computer and experimental findings challenging Hodgkin-Huxley model as even approximate description of the situation.

4. The identification of the cell interior as gel phase containing most of water as structured water around cytoskeleton - rather than water containing bio-molecules as solutes as assumed in Hodkin-Huxley model - allows to understand many of the anomalous behaviors associated with the cell membrane and also the different densities of ions in the interior and exterior of cell at qualitative level. The proposal of Pollack that basic biological functions involve phase transitions of gel phase generalizes in TGD framework to a proposal that these phase transitions are induced by quantum phase transitions changing the value of Planck constant. In particular, gel-sol phase transition for the peripheral cytoskeleton induced by the primary wave would accompany nerve pulse propagation. This view about nerve pulse is not consistent with Hodkin-Huxley model.

The model leads to the following picture about nerve pulse and EEG.

1. The system would consist of two superconductors- microtubule space-time sheet and the space-time sheet in cell exterior- connected by Josephson junctions represented by magnetic flux tubes defining also braiding in the model of TQC. The phase difference between two super-conductors would obey Sine-Gordon equation allowing both standing and propagating solitonic solutions. A sequence of rotating gravitational penduli coupled to each other would be the mechanical analog for the system. Soliton sequences having as a mechanical analog penduli rotating with constant velocity but with a constant phase difference between them would generate moving kHz synchronous oscillation. Periodic boundary conditions at the ends of the axon rather than chemistry determine the propagation velocities of kHz waves and kHz synchrony is an automatic consequence since the times taken by the pulses to travel along the axon are multiples of same time unit. Also moving oscillations in EEG range can be considered and would require larger value of Planck constant in accordance with vision about evolution as gradual increase of Planck constant.

2. During nerve pulse one pendulum would be kicked so that it would start to oscillate instead of rotating and this oscillation pattern would move with the velocity of kHz soliton sequence. The velocity of kHz wave and nerve pulse is fixed by periodic boundary conditions at the ends of the axon implying that the time spent by the nerve pulse in traveling along axon is always a multiple of the same unit: this implies kHz synchrony. The model predicts the value of Planck constant for the magnetic flux tubes associated with Josephson junctions and the predicted force caused by the ionic Josephson currents is of correct order of magnitude for reasonable values of the densities of ions. The model predicts kHz em radiation as Josephson radiation generated by moving soliton sequences. EEG would also correspond to Josephson radiation: it could be generated either by moving or standing soliton sequences (latter are naturally assignable to neuronal cell bodies for which $\hbar$ should be correspondingly larger): synchrony is predicted also now.

3.7 Appendix

3.7.1 About Inclusions Of Hyper-Finite Factors Of Type Ii$_1$

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A67]. It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [A67] for inclusions with $\mathcal{M}:\mathcal{N}\leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.
2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A67]. For any amenable group G the inclusion is also unique apart from outer automorphism [A50].

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II$_1$ factor [A67]. The construction of Jones leads to a standard inclusion sequence $\mathcal{N}\subset\mathcal{M}\subset\mathcal{M}^1\subset ....$ This sequence means addition of projectors e_i, $i<0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty=\cup_i\mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M}\otimes_\mathcal{N}\mathcal{M}....\otimes_\mathcal{N}\mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A67].

1. Jones inclusions

For hyper-finite factors of type II$_1$ Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M}:\mathcal{N}=r$ with $r\in\{4cos^2(\pi/n)|n\geq 3\}\cap[4,\infty)$ [A67]. They are defined for an algebra defined by projectors e_i, $i\geq 1$. All but nearest neighbor projectors commute. $\lambda=1/r$ appears in the relations for the generators of the algebra given by $e_ie_je_i=\lambda e_i$, $|i-j|=1$. $\mathcal{N}\subset\mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i\geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A67]. It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r\leq 4$ inclusions.

Irreducibility holds true for $r<4$ in the sense that the intersection of $Q'\cap P=P'\cap P=C$. For $r\geq 4$ one has $dim(Q'\cap P)=2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P)=Q$ is $\sigma(e_i)=e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r<4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis that only the groups $G_a\times G_b\subset SU(2)\times SU(2)\subset SL(2,C)\times SU(3)$ define orbifold coverings of $H_\pm=CD\times CP_2\rightarrow H_\pm/G_a\times G_b$.

2. Wasserman's inclusion

Wasserman's construction of $r=4$ factors clarifies the role of the subgroup of $G\subset SU(2)$ for these inclusions. Also now $r=4$ inclusion is characterized by a discrete subgroup $G\subset SU(2)$ and is given by $(1\otimes\mathcal{M})^G\subset(M_2(C)\times\mathcal{M})^G$. According to [A67] Jones inclusions are irreducible also for $r=4$. The definition of Wasserman inclusion for $r=4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2,C)\otimes\mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r=4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d=4$ for finite G.

For $r<4$ inclusion is defined as $M^G\subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $...\subset\mathcal{N}\subset\mathcal{M}\subset...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r<4$ inclusion $\mathcal{N}=\mathcal{M}^G\subset\mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r=4$ inclusion to $\mathcal{N}_1\subset\mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm\rightarrow H_\pm/G_a\times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n\geq 4$) are allowed for $r=4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r<4$, remains open.

3.7.2 Generalization From $Su(2)$ To Arbitrary Compact Group

The inclusions with index $\mathcal{M}:\mathcal{N}<4$ have one-dimensional relative commutant $\mathcal{N}'\cup\mathcal{M}$. The most obvious conjecture that $\mathcal{M}:\mathcal{N}\geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A55] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2=(q-1)e_i+q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q=1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q=exp(i2\pi/l)$, $l=4,5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k\geq 2$. For $SU(2)$ also the value $l=3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M}:\mathcal{N} = \prod_{1\leq r<s\leq k}\frac{sin^2\left((\lambda_r-\lambda_s+s-r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} . \tag{3.7.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1\leq k\leq l-1$ holds true. Only Young diagrams satisfying the condition $l-k=\lambda_1-\lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the embedding space in such a way that only cyclic group Z_n appears in the covering of $M^4\rightarrow M^4/G_a$ or $CP_2\rightarrow CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the embedding space. In the case of $SU(2)$ the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the embedding space level so that Z_3 statistics would be in question.

Chapter 4

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part I

4.1 Introduction

There are good reasons to hope that TGD is integrable theory in some sense. Classical physics is an exact part of quantum physics in TGD and during years I have ended up with several proposals for the general solution of classical field equations (classical TGD is an exact part of quantum TGD).

4.1.1 Various approaches to classical TGD

World of classical worlds

The first approach is based on the geometry of the "world of classical worlds" (WCW) [K52, K31, K85].

1. The study of classical field equations led rather early to the realization that preferred extremals for the twistor lift of Kähler action with Minkowskian signature of induced metric define a slicing of space-time surfaces defined by 2-D string world sheets and partonic two-surfaces locally orthogonal to them. The interpretation is in terms of position dependent light-like momentum vector and polarization vector defining the local decompositions $M^2(x) \times E^2(x)$ of tangent space integrating to a foliation by partonic 2-surfaces and string world sheets. I christened this structure Hamilton-Jacobi structure. Its Euclidian counterpart is complex structure in Euclidian regions of space-time surface.
2. The formulation of quantum TGD in terms of spinor fields in WCW [K113] leads to the conclusion that WCW must have Kähler geometry [K52, K31] and has it only if it has maximal group of isometries identified as symplectic transformations of $\delta M^4_\pm \times CP_2$, where $\delta M^4_\pm$ denotes light cone boundary two which upper/lower boundary of causal diamond (CD) belongs. Symplectic Lie algebra extends naturally to supersymplectic algebra (SSA).
3. Space-time surfaces would be preferred extremals of twistor lift of Kähler action [K87] and the conditions realizing strong form of holography (SH) would state that sub-algebra of SSA isomorphic with it and its commutator with SSA give rise to vanishing Noether charges and these charges annihilate physical states or create zero norm states from them. One should solve these conditions.
4. The dynamics involves also fermions. Induced spinor fields are located inside space-time surface but for some yet not completely understood reason only the information about spinor at 2-D string world sheets is needed in the construction of scattering amplitudes. This dynamics would be 2-dimensional. The construction of twistor amplitudes even suggests that

it is 1-dimensional in the sense that 1-D light-like curves at light-like partonic orbits defining boundaries of Minkowskian and Euclidian regions determines the scattering amplitudes. String world sheets are however needed only as correlates for entanglement between fermions at different partonic orbits.

The 2-D character of fermionic dynamics conforms with the strong form of holography (SH) but how the string world sheets and partonic 2-surfaces are selected from Hamilton-Jacobi slicing? Electromagnetic neutrality could select string worlds sheets but one can actually always find a gauge in which the induced classical electroweak field at these surfaces is purely electromagnetic.

Twistor lift of TGD

Second approach to preferred extremals is based on TGD version [K100, L30, K13, K87] of twistor Grassmann approach [B21, B43, B26].

1. The twistor lift of TGD leads to a proposal that space-time surfaces can be represented as sections in their 6-D twistor spaces identified as twistor bundles in the product $T(H) = T(M^4) \times T(CP_2)$ of 6-D twistor spaces of M^4 and CP_2. Twistor structure would be induced from $T(H)$. Kähler action can be lifted to the level of twistor spaces only for $M^4 \times CP_2$ since only for these spaces twistor space allows Kähler structure [A57]. Twistors were originally introduced by Penrose with the motivation that one could apply algebraic geometry in Minkowskian signature. The bundle property is extremely powerful and should be consistent with the algebraic geometrization at the level of M^8_c. The challenge is to formulate the twistor lift at the level of M^8.
2. The twistor lift of Kähler action contains also volume term. Field equations have two kinds of solutions. For the solutions of first kind the dynamics of volume term and Käction are coupled and the interpretation is in terms of interaction regions. Solutions of second kind are minimal surfaces and extremals of both Kähler action and volume term, whose dynamics decouple completely and all coupling constants disappear from the dynamics. These extremals are natural candidates for external particles. For these solutions at least the field equations reduce to the existence of Hamilton-Jacobi structure. The completely universal dynamics of these regions suggests interpretation in terms of maximal quantum criticality characterized by the extension of the usual conformal invariance to its quaternionic analog.
3. A connection with zero energy ontology (ZEO) emerges. Causal diamond (CD, intersection of future and past directed light-cones of M^4 with points replaced by CP_2) would naturally determine the interaction region to which external particles enter through its 2 future and past boundaries. But where does ZEO emerge?

$M^8 - H$ duality

The third approach is based on number theoretic vision [K95, K96, K94, K111].

1. M^8-H duality [K96, K111, K10] means that one can see space-times as 4-surfaces in either M^8 or $H = M^4 \times CP_2$. One could speak "number theoretical compactification" having however nothing to do with stringy version of compactification, which is dynamical. $M^8 - H$ duality suggests that space-time surfaces in $H = M^4 \times CP_2$ are images of space-time surfaces in M^8 or actually of M^8 projections of complexified space-time surfaces in M^8_c identified as space of complexified octonions. These space-time surfaces could contain the integrated distributions of string world sheets and partonic 2-surfaces mentioned in the previous item. Space-time surfaces must have associative tangent or normal space for $M^8 - H$ correspondence to exist.
2. The fascinating possibility mentioned already earlier is that in M^8 these surfaces could correspond to zero loci for real or imaginary parts of real analytic octonionic polynomials $P(o) = RE(P) + IM(P)I_4$, I_4 an octonionic imaginary unit orthogonal to quaternionic ones. The condition $IM(P) = 0$ ($RE(P) = 0$) would give associative (co-associative) space-time surface. In the simplest case these functions would be polynomials so that one would have algebraic geometry for algebraically 4-D complex surfaces in 8-D complex space.

Remark: The naive guess that space-time surfaces reduce to quaternionic curves in quaternionic plane fails due to the non-commutativity of quaternions meaning that one has $P(o) = P(q_1, q_2, \overline{q}_1, \overline{q}_2)$ rather than $P(o) = P(q_1, q_2)$.

Remark: Why not rational functions expressible as ratios $R = P_1/P_2$ of octonionic polynomials? It has become clear that one can develop physical arguments in favor of this option. The zero loci for $IM(P_i)$ would represent space-time varieties. Zero loci for $RE(P_1/P_2) = 0$ and $RE(P_1/P_2) = \infty$ would represent their interaction presumably realized as wormhole contacts connecting these varieties. In the sequel most considerations are for polynomials: the replacement of polynomials with rational functions does not introduce big differences and its discussed in the section "Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view" of [L48].

3. The objection against this proposal is obvious. $M^8 - H$ correspondence cannot hold true since the dynamics defined by octonionic polynomials in M^8 contains no coupling constants whereas the dynamics of twistor lift of Kähler action depends on coupling constants in the generic space-time region. However, for space-time surfaces representing external particles entering inside CD at its boundaries this is however not the case! They could satisfy $M^8 - H$ correspondence!

   This suggests that inside CDs the space-time surfaces are not associative/co-associative in M^8 so that $M^8 - H$ correspondence cannot map them to H and the twistor lifted Kähler action and SH take care of the dynamics. External particles are associative and quantum critical and $M^8 - H$ correspondence makes sense. The quantum criticality and associativity at the boundaries of CD poses extremely powerful conditions and allows to satisfy infinite number of vanishing conditions for SSA charges.

   It has later turned out [L64] that it might be possible to take the associativity conditions to extreme in the sense that they would hold everywhere apart from a set of discrete points and space-time surface would be minimal surfaces at all points except this finite set of points. There would be transfer of conserved quantities assignable to the volume term and the 4-D Kähler action (coming as dimensionally reduced 6-D Kähler action for the twistor lift of TGD) only at these points and elementary fermions would be naturally assignable to these points.

4. This picture is consistent with the the explicit formulation of the associativity conditions $Re(P) = 0$ and $IM(P) = 0$ for varieties. The calculations shows that associativity can be realized either by posing a condition making them 3-dimensional except, when the situation is critical in the sense that the 4-D variety is analogous to a double root of polynomial: now however the polynomial corresponds to prime polynomial decomposing to product of polynomials in the extension of rationals such that the product contains higher powers of the factors. One has ramification at the level of polynomial primes so that the criticality condition does not bring anything new but need not make the situation associative. At most 3 conditions need to be applied to guarantee associativity and they might leave the space-time surface 4-D.

5. The coordinates of M^4 as octonionic roots $x + iy$ of the 4 real polynomials need not be real. Space-time surface must have M_c^4 projection, which reduces to M^4. There are two options.

   (a) The original proposal was that the *projection* from M_c^8 to real M^4 (for which M^1 coordinate is real and E^3 coordinates are imaginary with respect to i!) defines the real space-time surface mappable by $M^8 - H$ duality to CP_2. One can howeerver critize the allowance of a nonvanishing imaginary part of space-time surface in M_c^4.

   (b) A more stringent condition is that the roots of the 4 vanishing polynomials as coordinates of M_c^4 belong automatically to M^4 so that m^0 would be real root and m^k, $k = 1, ..., 3$ imaginary with respect to $i \to -i$. M_c^8 coordinates would be invariant ("real") under combined conjugation $i \to -i, I_k \to -I_k$. In the following I will speak about this property as *Minkowskian reality.*

   This could allow to identify CDs in very elegant way: outside CD these 4 conditions would not hold true. This option looks more attractive than the first one. Why these conditions can be true just inside CD, should be understood.

6. This octonionic view as also lower-dimensional quaternionic counterpart. In this case one considers 2-D commutative/co-commutative surfaces tentatively identifiable as string world

sheets and partonic 2-surfaces. Why not all 2-surfaces appearing in the Hamilton-Jacobi slicing are not selected? The above mechanism would work also now. The commutativity conditions reduce in the generic case give 1-D curve as a solution. The interpretation would be as orbit of point like particle at 3-D partonic orbit appearing in the construction of twistorial amplitudes. In critical situation one would obtains string world sheet serving as a correlate for entanglement between point like particles at its ends: one would have quantum critical bound state.

I have considered also other attempts to define what quaternion structure could mean.

1. One could also consider the possibility that the tangent spaces of space-time surfaces in H are associative or co-associative [K111]. This is not necessary although it seems that this might be the case for the known extremals. If this holds true, one can construct further preferred extremals by functional composition by generalization of M^8-H correspondence to $H-H$ correspondence.
2. I have considered also the possibility of quaternion analyticity in the sense of generalization of Cauchy-Riemann equations, which tell that left- or right quaternionic differentiation makes sense [L39]. It however seems that this approach is not promising. The conditions are quite too restrictive and bring nothing essentially new. Octonion/quaternion analyticity in the above mentioned sense does not require the analogs of Cauchy-Riemann conditions.

4.1.2 Could one identify space-time surfaces as zero loci for octonionic polynomials with real coefficients?

The identification of space-time surfaces as zero loci of real or imaginary part of octonionic polynomial has several extremely nice features.

1. Octonionic polynomial is an algebraic continuation of a real valued polynomial on real line so that the situation is effectively 1-dimensional! Once the degree of polynomial is known, the value of polynomial at finite number of points are needed to determine it and cognitive representation could give this information! This would strengthen the view strong form of holography (SH) - this conforms with the fact that states in conformal field theory are determined by 1-D data.
2. One can add, sum, multiply, and functionally compose these polynomials provided they correspond to the same quaternionic moduli labelled by CP_2 points and share same time-line containing the origin of quaternionic and octonionic coordinates and real octonions (or actually their complexification by commuting imaginary unit). Classical space-time surfaces - classical worlds - would form an associative and commutative algebra. This algebra induces an analog of group algebra since these operations can be lifted to the level of functions defined in this algebra. These functions form a basic building brick of WCW spinor fields defining quantum states.
3. One can interpret the products of polynomials as correlates for free many-particle states with interactions described by added interaction polynomial, which can vanish at boundaries of CDs. This leads to the same picture as the view about preferred extremals reducing to minimal surfaces near boundaries of CD [L28]. Also zero zero energy ontology (ZEO) could be forced by the failure of number field property for quaternions at light-cone boundaries. It indeed turns out that light-cone boundary emerges quite generally as singular zero locus of polynomials $P(o)$ containing no linear part: this is essentially due to the non-commutativity of the octonionic units. Also the emergence of CDs can be understood. At this surface the region with $RE(P)=0$ can transform to $IM(P)=0$ region. In Euclidian signature this singularity corresponds to single point. A natural conjecture is that also the light-like orbits of partonic 2-surfaces correspond to this kind of singularities for non-trivial Hamilton-Jacobi structures.
4. The reduction to algebraic geometry would mean enormous boost to the vision about cognition with cognitive representations identified as generalized rational points common to reals rationals and various p-adic number fields defining the adele for given extension of rationals. Hamilton-Jacobi structure would result automatically from the decomposition of quaternions to real and imaginary parts which would be now complex numbers.

5. Also a connection with infinite primes is suggestive [K96]. The light-like partonic orbits, partonic 2-surfaces at their ends, and points at the corners of string world sheets might be interpreted in terms of singularities of varying rank and the analog of catastrophe theory emerges.

The great challenge is to prove rigorously that these approaches - or at least some of them - are indeed equivalent. Also it remains to be proven that the zero loci of real/imaginary parts of octonionic polynomials with real coefficients are associative or co-associative. I shall restrict the considerations of this article mostly to $M^8 - H$ duality. The strategy is simple: try to remember all previous objections against $M^8 - H$ duality and invent new ones since this is the best way to make real progress.

4.1.3 Topics to be discussed

Key notions and ideas of algebraic geometry

Before going of octonionic algebraic geometry, I will discuss basic notions of algebraic geometry such as algebraic variety (see `http://tinyurl.com/hl6sjmz`), - surface (see `http://tinyurl.com/y8d5wsmj`), and - curve (see `http://tinyurl.com/nt6tkey`), rational point of variety central for TGD view about cognitive representation, elliptic curves (see `http://tinyurl.com/lovksny`) and - surfaces (see `http://tinyurl.com/yc33a6dg`), and rational points (see `http://tinyurl.com/ybbnnysu`) and potentially rational varieties (see `http://tinyurl.com/yablk4xt`). Also the notion of Zariski topology (see `http://tinyurl.com/h5pv4vk`) and Kodaira dimension (see `http://tinyurl.com/yadoj2ut`) are discussed briefly. I am not a mathematician. What hopefully saves me from horrible blunders is physical intuition developed during 4 decades of TGD.

Much of algebraic geometry is counting numbers of say rational points or of varieties satisfying some conditions. One can also count dimensions of moduli spaces. Hence the basic notions and methods of enumerative geometry are discussed. There is also a discussion of Gromow-Witten invariants and Riemann-Roch theorem having Atyiah-Singer index theorem as a generalization. These notions will be applied in the second part of the article [L48].

$M^8 - H$ duality

M^8-H duality [K10, K96, K111] would reduce classical TGD to the algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces. Space-time surfaces in M^8 would be algebraic varieties identified as zero loci for imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial of complexified octonionic variable o decomposing as $o = q_c^1 + q_c^2 I_4$ and projected to a Minkowskian sub-space M^8 of o. Single real valued polynomial of real variable with algebraic coefficients would determine space-time surface! As proposed already earlier, spacetime surfaces in M^8 would form commutative and associative algebra with addition, product and functional composition.

As already noticed, the associativity conditions do not allow 4-D solutions except for criticality so that $M^8 - H$ correspondence can hold true only in these space-time regions and one has these nice features at the level of M^8. In critical regions $M^8 - H$ correspondence is true and these features have H counterparts

The basic problem is to understand the map mediating $M^8 - H$ duality mapping the point (m, e) of $M^8 = M_0^4 \times E^4$ to a point (m, s) of $M_0^4 \times CP_2$, where M_0^4 point is obtained as a projection to a suitably chosen $M_0^4 \subset M^8$ and CP_2 point parameterizes the tangent space as quaternionic sub-space containing preferred $M_0^2(x) \subset M^4(x)$. This map involves slightly non-local information and could allow to understand why the preferred extremals at the level of H are determined by partial differential equations rather than algebraic equations. Also the generalization to the level of twistor lift is briefly touched.

Challenges of the octonionic algebraic geometry

The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients determine associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

   This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy Riemann conditions are linear. Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

   In fact, all algebras obtained by Cayley-Dickson construction (see `http://tinyurl.com/ybuyla2k`) by adding imaginary unit repeatedly to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and a $M^8 - H$ correspondence could generalize (maybe even TGD!).

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k, $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i. In the generic case this gives 3-D surface.

   In this generic case $M^8 - H$ duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H, and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

   One can have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i. Space-time surface would be analogous to a polynomial with a multiple root. The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory [A47] emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

   One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces except at 3-D boundary surfaces. These regions correspond to interaction regions in which Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes also to the level of complex/co-complex surfaces associated with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces? I have proposed commutativity or co-commutatitivity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commu-

tative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

The easiest way to kill $M^8 - H$ duality in the form it is represented here is to prove that 4-D zero loci for imaginary/real parts of octonionic polynomials with real coefficients can never be associative/co-associative being always 3-D. I hope that some professional mathematician would bother to check this.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); Strong Form of GCI (SGCI); Quantum Criticality (QC); Strong Form of Holography (SH); World of Classical Worlds (WCW); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Causal Diamond (CD); Number Theoretical Universality (NTU) are the most often occurring acronyms.

4.2 Some basic notions, ideas, results, and conjectures of algebraic geometry

In this section I will summarize very briefly the basic notions of algebraic geometry needed in the sequel.

4.2.1 Algebraic varieties, curves and surfaces

The basic notion of algebraic geometry is algebraic variety.

1. One considers affine space A^n with n coordinates $x^1, ..., x^n$ having values in a number field K usually assumed to be algebraically closed (note that affine space has no preferred origin like linear space). Algebraic variety is defined as a solution of one or more algebraic equations stating the vanishing of polynomials of n variables: $P^i(x^1, ..., x^n) = 0$, $i = 1, ..., r \leq n$. One can restrict the coefficients of polynomials to p-adic number field or or its extension to an extension of rationals. One talks about polynomials on $k \subset K$.

2. The basic condition is that the variety is not a union of disjoint varieties. This for instance happens, when the polynomial $P(x^1, .., x^n)$ defining co-dimension 1 manifold is product of polynomials $P = \prod_r P_r$. Algebraic variety need not be a manifold meaning that it can have singular points. For instance, for co-dimension 1 variety the Jacobian matrix $\partial P/\partial x^i$ of the polynomial can vanish at singularity.

3. One can define projective varieties (see `http://tinyurl.com/ybsqvy3r`) in projective space P^n having coordinatization in terms of $n+1$ homogenous coordinates $(x^1, ..., x^{n+1})$ in K with points differing by an overall scaling identified. Projective variety is defined as zero locus of homogenous polynomials of $n + 1$ coordinates so that solutions remain solutions under the overall scaling of all coordinates. By identifying the points related by scaling one obtains a surface in P^n. Grassmannian of linear space V^n (not affine space!) is a projective spaces defined as space of k-planes of V^n. These spaces are encountered in twistor Grassmannian approach to scattering amplitudes.

For polynomials of single variable one obtains just the roots of $P_n(x) = 0$ in an algebraic extension assignable to the polynomial. For several variables one can in principle proceed step by step by solving variable x^1 as algebraic function of others from $P_1(x^1, ..., x^n) = 0$, proceed to solve x^2 from $P_2(x^1(x^2, ...), x^2, ...) = 0$ as as algebraic function of the remaining variables, and so one. The algebraic functions involved get increasingly complex but in some exceptional situations the solution has parametric representation in terms of *rational* rather than algebraic functions of parameters t^k. For co-dimension $d_c > 1$ case the intersection of surfaces $P^i = 0$ need not be complete and the tangent spaces of the hyper-surfaces $P^i = 0$ need not intersect transversally in the generic case. Therefore $d_c > 1$ case is not gained so much attention as $d_c = 1$ case.

A more advanced treatment relies on ring theory by assigning to polynomials a ring as the ring of polynomials in the space involved divided by the ring of polynomials vanishing at zero loci of polynomials P^i.

1. The notion of ideal is central and determined as a subring invariant under the multiplication by elements of ring. Prime ideal generalizes the notion of prime and one can say that the notion of integer generalizes to that of ideal. One can also define the notion of fractional ideal.
2. Zariski topology (see http://tinyurl.com/h5pv4vk) replacing the topology based on real norm is second highly advanced notion. The closed sets in this topology are algebraic varieties of various dimensions. Since the complement of any algebraic variety is open set this topology and open also in the ordinary real topology, this topology is considerable rougher than the ordinary than the ordinary topology.

Some remarks from the point of view of TGD are in order.

1. In the scenario inspired by M^8-H duality one has co-dimension 4 surfaces in 8-D complex space. Octonionicity of polynomials however implies huge symmetries since the polynomial is determined by single real polynomial of real variable, whose values at finite number of points determined the polynomial.
2. In TGD the extension of rationals can be assumed to contain also powers for some root of e since in p-adic context this gives rise to a finite-dimensional extensions due to the fact that e^p is ordinary p-adic number. Also a restriction to a finite field are possible and restriction of rational coefficients to their modulo p counterparts reduces the polynomial to polynomial in finite field. This reduction is used as a technical tool. In the case of Diophantine equations (see http://tinyurl.com/nt6tkey and http://tinyurl.com/y8hm4zce) the coefficients are restricted to be integers.
3. In adelic TGD [L52] [L51] the number fields involved are reals and extensions of p-adic numbers. The coefficient field for the coefficients of polynomials would be naturally extension of rationals or extension of p-adics induced by it. The coefficients of polynomials serve as coordinates of adelic WCW. p-Adic numbers are not algebraically closed and one must assume an extension of p-adic numbers from that for the coefficients one to allow maximal number of roots.

   This suggests an evolutionary process [L54] extending the number field for the coefficients of polynomials. Arbitrary root of polynomial for given extension can be realized only if the original extension is extended further. But this allows polynomial coefficients in this new extension: WCW is now larger. Now one has however roots in even larger extension so that the unavoidable outcome is number theoretic evolution as increase of complexity.
4. What is so remarkable is that octonionic polynomials with rational coefficients could be determined by their values at finite set of points for a polynomial of real argument once the order of polynomial is fixed. Real coordinate corresponds to preferred time axis naturally. A cognitive representation consisting of finite number of rational points could fix the entire space-time surface! This would extend ordinary holography to its discrete variant!
5. Algebraic variety is rather simple object as compared to the solutions of partial differential equations encountered in physics - say those for minimal surfaces. Now one must fix boundary values or initial values at $n-1$-dimensional surface to fix the solution. For integrable theories the situation can change. In TGD SH suggests that the classical solutions are determined by data at 2-surfaces, which together with conformal invariance could reduce the data to one-dimensional data specified by a polynomial. M^8-H correspondence allows to consider this option seriously.
6. M^8-H duality suggests that space-time surfaces are co-dimension $d_c=4$ algebraic curves in M^8. Could space-time surfaces define closed sets for the analog of Zariski topology? Could string world sheets and partonic 2-surfaces do the same inside space-time surfaces? An interesting question is whether this generalizes also to the level of embedding space H and could perhaps define a topology rougher than real topology in better accord with the notion of finite measurement resolution.

4.2.2 About algebraic curves and surfaces

The realization M^8-H correspondence to be considered allows to understand space-time surfaces as 4-D complex algebraic surfaces X_c^4 in the space o of complexified octonions projected to real sub-space of O^c with Minkowskian signature. Due to the non-commutativity of quaternions, the reduction of space-time surfaces to curves in quaternionic plane is not possible. Despite this it is instructive to start from the algebraic geometry of curves and surfaces.

Degree and genus of the algebraic curve

Algebraic curve is defined as zero locus of a polynomial $P(x^1, x^2, ..., x^n)$ with x^n in some - preferably algebraically closed - number field K and coefficients in some number field $k \subset K$. In adelic physics K corresponds to real or complex numbers and k to the extension of rationals defining adeles. In p-adic sectors k corresponds to tje extension of p-adic numbers induced by k. In general roots belong to an extension of k.

Degree, genus, and Euler characteristic are the basic characterizers of algebraic curve.

1. The degree d of algebraic curve corresponds to the highest power for the variables appearing in the polynomial. One can also define multi-degree in an obvious manner. A useful geometric interpretation for the degree is that line intersects curve (also complex) of degree d in at most d points as is clear from the fact that the equation of curve reduces the equation for curve to an equation for the roots of d:th order polynomial of single variable.
2. Also the genus g of the curve (see `http://tinyurl.com/ybm3wfue`) is important characteristic. One can distinguish between topological genus, geometric genus and arithmetic genus. For curves these notions are equivalent. The connection between genus and degree d of non-singular algebraic curve is very useful:

$$g = \frac{(d-1)(d-2)}{2} \ . \tag{4.2.1}$$

   Spherical topology for complex curves corresponds to $n = 1$ and $n = 2$.

   A more general formula reads as:

$$g = \frac{(d-1)(d-2)}{2} + \frac{n_s}{2} \ . \tag{4.2.2}$$

   Here n_s is the number of holes of the curve behaving like holes and increasing the genus.
3. Euler characteristic (for Euler characteristic see `http://tinyurl.com/pp52zd4`) is a homological invariant making sense in arbitrary dimension and also for manifolds. Homological definition based on simplicial homology relies on counting of simplexes of various dimension. The definition in terms of dimensions of homology groups H_n is given by

$$\chi = b_0 - b_1 + b_2 ... + (-1)^n b_n \ , \tag{4.2.3}$$

   where b_k is the dimension of k:th homology group (see `http://tinyurl.com/j48ojys`).

The following gives the engineering rules for obtaining Euler characteristic of the surface obtained from simpler building blocks. Note that algebraic variety property is not essential here.

1. Euler characteristic is homotopy invariant so that it does not change one adds homologically trivial space such as E^n as a Cartesian factor.
2. χ is additive under disjoint union. Inclusion-exclusion principle states that if M and N intersect, one has $\chi(M \cup N) = \chi(M) + \chi(N) - \chi(M \cap N)$.
3. Euler characteristic for the connected sum $A\#B$ of n-dimensional manifolds obtained by drilling balls B^n from summands, giving opposite orientation to the boundaries of the hole, and connecting with cylinder $D \times S^{n-1}$ is given by $\chi(A) + \chi(B) - \chi(S^{n-1})$. One has $\chi(S^2) = 2$ and $\chi(D^2) = 1$.
4. The Euler characteristic for product $M \times N$ is $\chi(M) \times \chi(N)$.
5. The Euler characteristic for N-fold covering space M_n is $N \times \chi(M)$ with a correction term coming from the singularities of the covering (ramified covering space).

6. For a fibration $M \to B$ with fiber S, which differs from fiber bundle in that the fibers are only homeomorphic, one has $\chi(M) = \chi(B) \times \chi(S)$.

Euler characteristic and the genus of 2-surface (or complex) curve are related by the equation

$$\chi = 2(1-g) \ . \tag{4.2.4}$$

having values $2, 0, -2, ...$ If the 2-surface has n_s holes (punctures), one has

$$\chi = 2(1-g) - n_s \ . \tag{4.2.5}$$

Punctures must be distinguished from singularities at which some sheets of covering meet at single point.

A formal generalization of the definition of genus for varieties in terms of Euler characteristic makes sense.

$$g = -\frac{\chi}{2} + 1 - \frac{n_s}{2} \ . \tag{4.2.6}$$

Disk has genus 1/2 and drilling of n holes increases genus by $n/2$. Pair of holes gives same contribution to g and the cylinder connecting the holes. Note that for complex curves the definition of puncture is obvious. For real curves the puncture would mean missing point of the curve.

The latter definitions of genus can be identified in terms of Euler characteristic also for higher-dimensional varieties. For curves these notions are equivalent if there are no singularities. For algebraic curves g is same for the real and complex variants of the curve in RP_1 and CP_1 respectively.

Elliptic curves and elliptic surfaces

Elliptic curves (see `http://tinyurl.com/lovksny`) are cubic curves with no singularities (cusps or self-intersections) having representation of form $y^2 - x^3 - ax - b = 0$. These singularities can occur only at special values of parameters $((a = 0, b = 0)$. Since the degree equals to $d = 3$, elliptic curve has genus $g = 1$.

Elliptic curves allow a group of Abelian symmetries generated by a finite number of generators. The emergence of abelian group structure can be intuitively understood as follows.

1. Given line intersects the curve of degree 3 in at most 3 points. Let P and Q be two of these points. Then there can be also a third intersection point R and by the Z^2 symmetry changing the sign of y also the reflection of R - identify it as $-R$ - belongs to the curve. Define the sum of $P + Q$ to be $-R$.

   The actual proof is slightly more complicated since the number of intersection points for the line with curve can be also 2 or 1. By writing explicit expressions for the coordinates x_R and y_R, one can also find that they are indeed rational if the points P and Q are rational. If the elliptic curve as single rational point it has infinite number of them.

2. The generators with finite order give rise to torsion. The rank of generators of infinite order is called rank and conjectured to be arbitrarily large (see `http://tinyurl.com/lovksny`) . Therefore elliptic curve is an Abelian group and one talks about Abelian variety. If elliptic curve contains a rational point it contains entire lattice of rational points obtained as shifts of this point.

Remark: Complex elliptic curves are 2-surfaces in complex projective plane CP_2 and therefore highly interesting from TGD point of view. $g = 1$ partonic 2-surfaces would in TGD framework correspond to second generation fermions [K28]. Abelian varieties define a generalization of elliptic curves to higher dimensions and simplest space-time surfaces allowing also large cognitive representations could correspond to such.

Elliptic surfaces (see `http://tinyurl.com/yc33a6dg`) are fibrations with an algebraic curve as base space and elliptic curve as fiber (fibration is more general notion than fiber space since the fibers are only homeomorphic). The singular fibers failing to be elliptic curves have been classified by Kodaira.

4.2.3 The notion of rational point and its generalization

The notion of algebraic integer (see http://tinyurl.com/y8z389a7) makes sense for any number field as a root of a monic polynomial (polynomial with integer coefficients with coefficient of highest power equal to unity). The field of fractions for given number field consists of ratios of algebraic integers. The same is true for the notion of prime. The more precise definition forces to replace integers and primes with ideals.

Rational varieties are expressible as maps defined by rational functions with rational coefficients in some extension of Q and contain infinite number of rational points. If the variety is not rational, one can ask whether it could allow a dense set of rational points with rational number replaced with the ratio of algebraic integers for some extension of Q. This leads to the idea of potentially rational point, and one can classify algebraic varieties according to whether they are potentially rational or not. The variety is potentially rational if it allows a parameteric representation using rational functions. Otherwise the parametric representation involves algebraic functions such as roots of rational functions.

The interpretation in terms of cognition would be that large enough extension makes the situation "cognitively easy" since cognitive representations involving fermions at the rational points and defining discretizations of the algebraic variety could be arbitrary large. The simpler the surface is cognitively, the large the number of rational points or potentially rational points is.

Complexity of algebraic varieties is measured by Kodaira dimension d_K (see http://tinyurl.com/yadoj2ut). The spectrum for this dimension varies in the range $(-\infty, 0, 1, 2, ...d)$, where d is the algebraic dimension of the variety. Maximal value equals to the ordinary topological dimension d and corresponds to maximal complexity: in this case the set of rational points is finite. Minimal Kodaira dimension is $d_K = -\infty$: in this case the set of rational points is infinite. Rational surfaces are maximally simple and this corresponds to the existence of parametric representations using only rational functions.

Rational points for algebraic curves

The sets of rational points for algebraic curves are rather well understood. Mordelli conjecture proved by Falting as a theorem (see http://tinyurl.com/y9oq37ce) states that a curve over Q with genus $g = (d-1)(d-2)/2 > 1$ (degree $d > 3$) has only finitely many rational points.

1. Sphere CP_1 in CP_2 has rational points as a dense set. Quite generally rational surfaces, which by definition allow parametric representation using polynomials with rational coefficients (encountered in context of Du Val singularities characterized by the extended Dynkin diagrams for finite subgroups of $SU(2)$) allow dense set of rational points [A61, A69]).

   $g = 0$ does not yet guarantee that there is dense set of rational points. It is possible to have complex conics (quadratic surface) in CP_2 with no rational points. Note however that this depends on the choice of the coordinates: if origin belongs to the surface, there is at least one rational point

2. Elliptic curve $y^2 - x^3 - ax - b$ in CP_2 (see http://tinyurl.com/lovksny) has genus $g = 1$ and has a union of lattices of rational points and of finite cyclic groups of them since it has origin as a rational point. This lattice of points are generated by translations. Note that elliptic curve has no singularities that is self intersections or cusps (for $a = 0, b = 0$ origin is a singularity).

   $g = 1$ does not guarantee that there is infinite number of rational points. Fermat's last theorem and CP_2 as example. $x^d + y^d = z^d$ is projectively invariant statement and therefore defines a curve with genus $g = (d-1)(d-2)/2$ in CP_2 (one has $g = 0, 0, 2, 3, 6, 10, ...$). For $d > 2$, in particular $d = 3$, there are no rational points.

3. $g \geq 2$ curves do not allow a dense set of rational points nor even potentially dense set of rational points.

Remark: In TGD framework algebraic varieties could be zero loci of octonionic polynomials and have algebraic dimension 4 so that the classification for algebraic curves does not help. Octonion analyticity must bring in symmetries which simplify the situation.

Enriques-Kodaira classification

The tables of (see http://tinyurl.com/ydelr4np) give an overall view about the Enriques-Kodaira classification of algebraic curves, surfaces, and varieties in terms of Kodaira dimension (see http://tinyurl.com/yadoj2ut).

1. For instance, general curves ($g \geq 2$) have $d_K = 1$, elliptic curves ($g = 1$) have $d_K = 0$ and projective line ($g = 0$) has $d_K = -\infty$. $CP_1 \subset CP_2$ is a rational curve so that rational points are dense. Elliptic curves allow infinite number or rational points forming an Abelian group if they containing single rational point and are therefore cognitively easy.
2. Algebraic varieties (with real dimension $d_R = 4$ in complex case) with $d_K = 2$ are surfaces of general type, elliptic surfaces (see http://tinyurl.com/yc33a6dg) have $d_K = 1$, surfaces with attribute abelian, hyper-elliptic, K3, and Enriques, have $d_K = 0$.

   Remark: All real 2-surfaces are hyper-elliptic for $g \leq 2$, in other words allow Z_2 as global conformal symmetry. Genus-generation correspondence [K28] for fermions allows to assign to the 3 lowest generations of fermions hyper-elliptic partonic 2-surfaces with genus $g = 0, 1, 2$. These surfaces would have $d_K = 0$ and be rather simple as real surfaces in Kodaira classification. Could one regard them as M^4 projection of complex hyper-elliptic surfaces of real dimension $d_R = 4$? $d_K = -\infty$ holds true for rational surfaces and ruled surfaces, which allow straight line through any point are maximally simple. In complex case the line would be CP_1.
3. The Wikipedia article gives also a table about the classification of algebraic 3-folds. Real algebraic 3-surfaces might well occur in TGD framework. The twistor space for space-time surface might allow realization as complex 3-fold and since it has S^2 has fiber, it would naturally correspond to an uni-ruled surface with $d_K = -\infty$. The table shows that one can build higher dimensional algebraic varieties with $d_K < d$ from lower-dimensional ones as fiber-space like structures, which based or fiber having $d_K < d$. 3-D Abelian varieties and Calabi-Yau 3-folds are complex manifolds with $d_K = 0$, which cannot be engineered in this manner.
4. Space-time surfaces would be surfaces of algebraic dimension 4. Wikipedia tables do not give direct information about this case but one can make guesses on basis of the three tables. Octonionic polynomials are analytic continuations of real polynomials of real variable, which must mean a huge simplification, which also favor cognitive representability. The best that one might have infinite sets of rational points. The examples about extremals of Kähler action does not however favor this wish.

Bombieri-Lang conjecture (see http://tinyurl.com/y887yn5b) states that, for any variety X of general type over a number field k, the set of k-rational points of X fails to be Zariski dense (see http://tinyurl.com/jm9fh74) in X. This means that , the k-rational points are contained in a *finite* union of lower-dimensional sub-varieties of X. In dimension 1, this is exactly Faltings theorem, since a curve is of general type if and only if it has $g \geq 2$. The conjecture of Vojta (see http://tinyurl.com/y9sttuu4) states that varieties of general type cannot be potentially dense. As will be found, these conjectures might be highly relevant for TGD.

4.3 About enumerative algebraic geometry

Algebraic geometry is something very different from Riemann geometry, Kähler geometry, or sub-manifold geometry based on local notions. Sub-manifolds are replaced with sub-varieties defined as zero loci for polynomials with coefficients in the field of rationals or extension of rationals. Partial differential equations are replaced with algebraic ones. One can generalize algebraic geometry to any number field.

String theorists have worked with algebraic geometry with motivation coming from various moduli spaces emerging in string theory. The moduli spaces for closed and open strings possibly in presence of branes are involved. Also Calabi-Yau compacticication leads to algebraic geometry, and topological string theories of type A and B involve also moduli spaces and enumerative algebraic geometry.

In TGD the motivation for enumerative algebraic geometry comes from several sources.

1. Twistor lift of TGD lifts space-time surfaces to their 6-D twistor spaces representable as surfaces in the product of 6-D twistor spaces of M^4 and CP_2 and possessing Kähler structure - this makes these spaces completely unique and strongly suggests the role of algebraic geometry, in particular in the generalization of twistor Grassmannian approach [L48].
2. There are three threads in number theoretic vision: p-adic numbers and adelics, classical number fields, and infinite primes. Adelic physics [L52] as physics of sensory experience and cognition unifies real physics and various p-adic physics in the adele characterized by an extension of rationals inducing those of p-adic number fields. This leads to algebraic geometry and counting of points with embedding space coordinates in the extension of rationals and defining a discrete cognitive representation. The core of the scattering amplitude would be defined by this cognitive representation identifiable in terms of points appearing as arguments of n-point function in QFT picture [L46].
3. $M^8 - M^4 \times CP_2$ duality is the analog of the rather adhoc spontaneous compactification in string models but would be non-dynamical and thus allow to avoid landscape catastrophe. Classical physics would reduce to octonionic algebraic geometry at the level of complexified octonions with several special features due to non-commutativity and non-associativity: space-time could be seen as 4-surface in the complexification of of octonions. The commuting imaginary unit would make possible the realization of algebraic extensions of rationals.

   The moduli space for the varieties is discrete if the coefficients of the polynomials are in the extension of rationals. If one poses additional conditions such as associativity of 4-surfaces, the moduli space is further reduced by the resulting criticality conditions realizing quantum criticality at the fundamental level raising hopes about extremely simple formulation of scattering amplitudes at the level of M^8 [L48].

   Also complex and co-complex sub-manifolds of associative space-time surface are important and would realize strong form of holography (SH). For non-associative regions of space-time surface it might not be possible to define complex and co-complex surfaces in unique manner since the basic $M^2 \subset M^4$ local flag structure is missing. String world sheets and partonic 2-surfaces and their moduli spaces are indeed in key role and the topology of partonic surfaces plays a key role in understanding of family replication phenomenon in TGD [L46].

In this framework one cannot avoid enumerative algebraic geometry.

1. One might want to know the number of points of sub-variety belonging to the number field defining the coefficients of the polynomials. This problem is very relevant in M^8 formulation of TGD, where these points are carriers of sparticles. In TGD based vision about cognition [L52] they define cognitive representations as points of space-time surface, whose M^8 coordinates can be thought of as belonging to both real number field and to extensions of various p-adic number fields induced by the extension of rationals. If these cognitive representations define the vertices of analogs of twistor Grassmann diagrams in which sparticle lines meet, one would have number theoretically universal adelic formulation of scattering amplitudes and a deep connection between fundamental physics and cognition.
2. Second kind of problem involves a set algebraic surfaces represented as zero loci for polynomials - lines and circles in the simplest situations. One must find the number of algebraic surfaces intersecting or touching the surfaces in this set. Here the notion of incidence is central. Point can be incident on line or two lines (being their intersection), line on plane, etc.. This leads to the notion of Grassmannians and flag-manifolds.

Moduli spaces parameterizing sub-varieties of given kind - lines, circles, algebraic curves of given degree, are central for the more advanced formulation of algebraic geometry. These moduli spaces emerge also in the formulation of TGD. The moduli space of conformal equivalence classes of partonic 2-surfaces is one example involved with the explanation of family replication phenomenon [K28]. One can assign moduli spaces also to octonion and quaternion structures in M^8 (or equivalently with the complexification of E^8). One can identify CP_2 as a moduli space of quaternionic sub-spaces of octonions containing preferred complex sub-space.

One cannot avoid these moduli spaces in the formulation of the scattering amplitudes and this leads to $M^8 - H$ duality. The hard core of the calculation should however reduce to the understanding of the algebraic geometry of 4-surfaces in octonionic space. Clearly, M^8 picture seems to provide the simplest formulation of the number theoretic vision.

4.3.1 Some examples about enumerative algebraic geometry

Some examples give an idea about what enumerative algebraic geometry (see `http://tinyurl.com/y7yzt67b`) is.

1. Consider 4 lines in 3-D space. What is the number of lines intersecting these 4 lines [A82] (see `http://tinyurl.com/ycrbr5aj`). One could deduce the number of lines and lines by writing the explicit equations for the lines with each line characterized by 2+3=5 parameters specifying direction t vector and arbitarily chosen point x_0 on the line. 2+3=5 parameters characterize each sought-for line.

   For intersection points x_i of sought for line with i:th one has $x_i = x_0 + k_i t_0$, $i = 1, ..., 4$ for the sought for line with direction t_0. At the intersection points at the 4 lines one has $x_i = x_{0i} + K_i t_i$ with fixed directions t_i. Combining the two equations for each line one has $4 \times 3 = 12$ equations and 3+4+2 parameters for the sought for line plus 4 parameters K_i for the four lines. This gives 13 unknown parameters corresponding to x_0, k_i, K_i. One would have one parameter set of solutions: something goes wrong.

   One has however projective invariance: one can shift x_0 along the line by $x_0 \rightarrow x_0 - at$, $k_i \rightarrow k_i + a$ and using this freedom assume for instance $k_1 = 0$. This reduces the number of parameters to 12 and one has finite number of solutions in the generic case. Actually the number is 2 in the generic case but can be infinite in some special cases. The challenge is to deduce the number of the solutions by geometric arguments.Below Schubert's argument proving that the number of solutions is 2 will be discussed.

   The idea of enumerative geometry is to do this using general geometric arguments allowing to deform the problem topologically to a simpler one in which case the number of solutions is obvious which in the most abstract formulation become topological.

2. Apollonius can be seen as founder of enumerative algebraic geometry. Apollonian circles (see `http://tinyurl.com/ycvxe688`) represent second example. One has 3 circles in plane. What is the number of circles tangential to all these 3 circles. Wikipedia link represents the geometric solution of the problem. The number of circles is 8 in the generic case but there are exceptional cases.

3. In Steiner's conic problem (see `http://tinyurl.com/yahshsjo`) one have 5 conical sections (circles, cones, ellipsoids, hyperbole) in plane. How many different conics tangential to the conics there exist? This problem is rather difficult and the thumb rules of enumerative geometry (dimension counting, Bezout's rule, Schubert calculus) fail. This is a problem in projective geometry where one is forced to introduce moduli space for conics tangential to given conic. This space is algebraic sub-variety of all conics in plane which is 5-D projective space. One must be able to deduce the number of points in the intersection of these sub-varieties so that the original problem in 2-D plane is replaced with a problem in moduli space.

4.3.2 About methods of algebraic enumerative geometry

A brief summary about methods of algebraic geometry is in order to give some idea about what is involved (see `http://tinyurl.com/y7yzt67b`).

1. Dimension counting is the simplest method. If two geometric objects of n-D space have dimensions k and l, there intersection is $n - k - l$-dimensional for $n - k - l \geq 0$ or empty in the generic case. For $k + l = n$ one obtains discrete set of intersection points.

2. Bezout's theorem is a more advanced method. Consider for instance, curves in plane defined by the curves polynomials $x = P^m(y)$ and $x = P^n(y)$ of degrees $k = m$ and $k = n$. The number N of intersection points in the generic case is bounded above by $N = m \times n$ (in this case all roots are real). One can understand this by noticing that one has m roots y_k or given x giving rise to a m-branched graph of function $y = f(x)$. The number of intersections for the graphs of the two polynomials is at most $m \times n$. If one has curve in plane represented by polynomial equation $P^{m,n}(x, y) = 0$, one can also estimate immediately the minimal multi-degree (m, n) for this polynomials.

3. Schubert calculus http://tinyurl.com/y766ddw2) is a more advanced but not completely rigorous method of enumerative geometry [A82] (see http://tinyurl.com/ycrbr5aj).

   Schubert's vision was that the number of intersection points is stable against deformations in the generic case. This is not quite true always but in exceptional cases one can say that two separate solutions degenerate to single one, just like roots of polynomial can do for suitable values of coefficients.

   For instance, Schubert's solution to the already mentioned problem of finding a line intersecting 4 lines in generic position relies on this assumption. The idea is to deform the situation so that one has two intersecting pairs of lines. One solution to the problem is a line going through the intersection points for line pairs. Second solution is obtained as intersection of the planes. It can happen that planes are parallel in which case this does not work.

   Schubert calculus it applies to linear sub-varieties but can be generalized also to non-linear varieties. The notion of incidence allowing a general formulation for intersection and tangentiality (touching) is central. This leads to the notions of flag, flag manifold, and Schubert variety as sub-variety of Grassmannian.

   Flag is a hierarchy of incident subspaces $A_0 \subset A_1 \subset A_2 ... \subset A_n$ with the property that the dimension $d_i \leq n$ of A_i satisfies $d_i \geq i$. As a special case this notion leads to the notion of Grassmannian $G(k,n)$ consisting of k-planes in n-dimensional space: in this case A_0 corresponds to k-planes and A_2 to space A_n. More general flag manifolds are moduli spaces and sub-varieties of Grassmannian providing a solution to some conditions. Flag varieties as sub-varieties of Grassmannians are Schubert varieties (see http://tinyurl.com/y7ehcrzg). They are also examples of singular varieties. More general Grassmannians are obtained as coset spaces of G/P, where G is algebraic group and P is parabolic sub-group of G.

   Remark: CP_2 corresponds to the space of complex lines in C^3. CP_2 can be also understood as the space of quaternionic planes in octonionic 8-space containing fixed 2-plane so that also now one has flag. String world sheets inside space-time surfaces define curved flags with 2-D and 4-D tangent spaces defining an integrable distribution of local flags.

4. Cohomology combined with Poincare duality allows a rigorous formulation of Schubert calculus. Schubert's idea about possibility to deform the generic position corresponds to homotopy invariance, when the degeneracies of the solutions are taken into account. Homology and cohomology become basic tools and the so called cup product for cohomology together with Poincare duality and Künneth formula for the cohomology of Cartesian product in terms of cohomologies of factors allows to deduce intersection numbers algebraically. Schubert cells define a basis for the homology of Grassmannian containing only even-dimensional generators.

   Grassmannians play a key role in twistor Grassmannian approach as auxiliary manifolds. In particular, the singularities of the integrand of the scattering amplitude defined as a multiple residue integral over $G(k,n)$ define a hierarchy of Schubert cells. The so called positive Grassmannian [B22] defines a subset of singularities appearing in the scattering amplitudes of $\mathcal{N} = 4$ SUSY. This hierarchy and its CP_2 counterpart are expected also in TGD framework.

   Remark: Schubert's vision might be relevant for the notion of conscious intelligence. Could problem solving involve the transformation of a problem to a simple critical problem, which is easy but for which some solutions can become degenerate? The transformation of general position for 4 lines to a pair of intersecting lines would be example of this. One can wonder whether quantum criticality could help problem solving by finding critical cases.

5. Moduli spaces of curves and varieties provide the most refined methods. Flag manifolds define basic examples of moduli spaces. Quantum cohomology represents even more refined conceptualization: the varieties (branes in M-theory terminology) are said to be connected or intersect if each of them has a common point with the same pseudo-holomorphic variety ("string world sheet"). Pseudo-holomorphy - which could have minimal surface property as counterpart - implies that the connecting 2-surface is not arbitrary.

   Quantum intersection for the "string world sheet" and "brane" is possible also when it is not stable classically (the co-dimension of brane is smaller than 2). Even in the case that it possible classically quantum intersection makes possible kind of "telepathic" quantum contact mediated by the "string world sheet" naturally involved with the description of quantum entanglement in TGD framework.

4.3.3 Gromow-Witten invariants

Gromow-Witten invariants repreent example of so called quantum invariants natural in string models and M-theory. They provide new invariants in algebraic and symplectic geometry.

Formal definition

Consider first the definition of Gromow-Witten (G-W) invariants (see `http://tinyurl.com/y9b5vbcw`). G-W invariants are rational number valued topological invariants useful in algebraic and symplectic geometry. These quantum invariants give information about these geometries not provided by classical invariants. Despite being rational numbers in the general case G-W invariants in some sense give the number of string world sheets connecting given branes.

1. One considers collection of n surfaces ("branes") with even dimensions in some symplectic manifold X of dimension $D = 2k$ (say Kähler manifold) and pseudo-holomorphic curves ("string world sheets") X^2, which have the property that they connect these n surfaces in the sense that they intersect the "branes" in the marked points x_i, $i = 1, .., n$.

   "Connect" does not reduce to intersection in topologically stable sense since connecting is possible also for branes with dimension smaller than $D - 2$. One allows all surfaces that X^2 that intersects the n surfaces at marked points if they are pseudo-holomorphic even if the basic dimension rule is not satisfied. In 4-dimensional case this does not seem to have implications since partonic 2-surfaces satisfy automatically the dimension rule. The n branes intersect or touch in quantum sense: there is no concrete intersection but intersection with the mediation of "string world sheet".

2. Pseudo-holomorphy means that the Jacobian df of the embedding map $f : X^2 \to X$ commutes with the symplectic structures j *resp.* J of X^2 *resp.* X: i.e. one has $df(jT) = Jdf(T)$ for any tangent vector T at given point of X^2. For $X^2 = X = C$ this gives Cauchy-Riemann conditions.

   In the symplectic case X^2 is characterized topologically by its genus g and homology class A as surface of X. In algebraic geometry context the degree d of the polynomial defining X^2 replaces A. In TGD X^2 corresponds to string world sheet having also boundary. X^2 has also n marked points $x_1, ..., x_n$ corresponding to intersections with the n surfaces.

3. G-W invariant $GW^{X,A}_{g,n}$ gives the number of pseudo-holomorphic 2-surfaces X^2 connecting n given surfaces in X - each at single marked point. In TGD these surfaces would be partonic 2-surfaces and marked points would be carriers of sparticles.

The explicit definition of G-W invariant is rather hard to understand by a layman like me. I however try to express the basic idea on basis of Wikipedia definition (see `http://tinyurl.com/y9b5vbcw`). I apologize for my primitive understanding of higher algebraic geometry. The article of Vakil [L35] (see `http://tinyurl.com/ybobccub`) discusses the notion of G-W invariant in detail.

1. The situation is conformally invariant meaning that one considers only the conformal equivalence classes for the marked pseudo-holomorphic curves X^2 parameterized by the points of so called Deligne-Mumford moduli space $\overline{M}_{g,n}$ of curves of genus g with n marked points (see `http://tinyurl.com/yaq8n6dp`): note that these curves are just abstract objects without no embedding as surface to X assumed. $\overline{M}_{g,n}$ has *complex* dimension

$$d_0 = 3(g-1) + n \ .$$

   n corresponds complex dimensions assignable to the marked points and $3(g - 1)$ correspond to the complex moduli in absence of marked points. This space appears in TGD framework in the construction of elementary particle vacuum functionals [K28].

2. Since these curves must be represented as surfaces in X one must introduces the moduli space $\overline{M}_{g,n}(X, A)$ of their maps f to X with given homology equivalence class. The elements in this space are of form $(C, x_1, .., x_n, f)$ where C is one particular representative of A.

3. The complex dimension d of $\overline{M}_{g,n}(X, A)$ can be calculated. One has

$$d = d_0 + c_1^X(A) + (g-1)k \ .$$

Here $c_1^X(A)$ is the first Chern class defining element of second cohomology of X evaluated for A. For Calabi-Yau manifolds one has $c_1 = 0$. The contribution $(g-1)k$ to the dimension vanishing for torus topology should have some simple explanation.

4. One defines so called evaluation map ev from $\overline{M}_{g,n}(X,A) \rightarrow Y$, $Y = \overline{M}_{g,n} \times X^n$ in terms of stabilization $st(C, x_1, ..., x_n) \in \overline{M}_{g,n}(X,A)$ of C (I understand that stabilization means that the automophism group of the stabilized surface defined by f is finite [A80] (see `http://tinyurl.com/y8r44uhl`). I am not quite sure what the finiteness of the automorphism group means. One might however think that conformal transformations must be in question. One has

$$ev(C, x_1, .., x_n, f) = (st(C, x_1, .., x_n), f(x_1), ...f(x_n)) \ .$$

   Evaluation map assigns to the concrete realization of string world sheet with marked points the abstract curve $st(C, x_1, .., x_n)$ and points $(f(x_i), ..., f(x_n)) \in X^n$ possibly interpretable as positions $f(x_i)$ of n particles. One could say that one has many particle system with particles represented by surfaces of X_i of X connected by X^2 - string world sheet - mediating interaction between X_i via the intersection points.

5. Evaluation map takes the fundamental class of $\overline{M}_{g,n}(X,A)$ in $H_d(\overline{M}_{g,n}(X,A))$ to an element of homology group $H_d(Y)$. This homology equivalence class defines G-W invariant, which is rational valued in the general case.

6. One can make this more concrete by considering homology equivalence class β in $\overline{M}_{g,n}$ and homology equivalence classes α_i, $i = 1, ..., n$ represented by the surfaces X_i. The co-dimensions of these $n+1$ homology equivalence classes must sum up to d. The homologies of $\overline{M}_{g,n}$ and $Y = \overline{M}_{g,n} \times X^n$ induce homology of Y by Künneth formula (see `http://tinyurl.com/yd9ttlfr`) implying that Y has class of $H_d(Y)$ given by the product $\beta \cdot \alpha_1 ... \cdot \alpha_n$.

   One can identify the value of $GW_{g,n}^{X,A}$ for a given class $\beta \cdot \alpha_1 ... \cdot \alpha_n$ as the coefficients in its expansion as sum of all elements in $H_d(Y)$. This coefficient is the value of its intersection product of $GW_{g,n}^{X,A}$ with the product $\beta \cdot \alpha 1 ... \cdot \alpha_n$ and gives element of $H_0(Q)$, which is rational number.

7. There are two non-classical features. Classically intersection must be topologically stable. This would require α_i to have codimension 2 but all even co-dimensions are allowed. That the value for the number of connecting string world sheets is rational number does not conform with the classical geometric intuition. The Wikipedia explanation is that the orbifold singularities for the space $\overline{M}_{g,n}(X,A)$ of stable maps are responsible for rational number.

Application to string theory

Topological string theories give a physical realization of this picture. Here the review article *Instantons, Topological Strings, and Enumerative Geometry* of Szabo [A80] (see `http://tinyurl.com/y8r44uhl`) is very helpful.

1. In M-theory framework and for topological string models of type A and B the physical interpretation for the varieties associated with α_i would be as branes of various dimensions needed to satisfy Dirichlet boundary conditions for strings.

2. In topological string theories one considers sigma model with target space X, which can be rather general. The symplectic or complex structure of X is however essential. X is forced to be 3-D (in complex sense) Calabi-Yau manifold by consistency of quantum theory. Interestingly, the super twistor space $CP(3|4)$ is super Calabi-Yau manifold although CP_3 is not and must therefore have trivial first Chern class c_1 appearing in the formula for the dimension d above. I must admit that I do not understand why this is the case.

   Closed topological strings have no marked points and one has $n = 0$. Open topological strings world sheets meet n branes at points x_i, where they satisfy Dirichlet boundary conditions. Branes an be identified as even-dimensional Lagrangian sub-manifolds with vanishing induced symplectic form.

3. For topological closed string theories of type A one considers holomorphically imbedded curves in X characterized by genus g and homology class A: one speaks of world sheet instantons. $A = \sum n_i S_i$ is sum over the generating classes S_i with integer coefficients. For given g and A one has analog of product of non-interacting systems at temperatures $1/t_i$ assignable to the homology classes S_i with energies identifiable as n_i. One can assign Boltzmann weight labelled by (g, A) as $Q^\beta = \prod_i Q_i^{n_i}$, $Q_i = exp(-t_i)$.

   One can construct partition function for the entire system as sum over Boltzmann weights with degeneracy factors telling the number of world sheet instantons with given (g, A). One can calculate free energy as sum $\sum N_{g,\beta} Q^\beta$ over contributions labelled by (g, A). The coefficients $N_{g,\beta}$ count the rational valued degeneracies of the world sheet instantons of given type and reduce to G-W invariants $GW_{g,0}^{X,A}$.

   Remark: If one allows powers of a root $e^{-1/n}$, $t = n$, in the extension of rationals or replace e^{-t} with p^n, partition functions make sense also in the p-adic context.

4. For topological open string theories of type A one has also branes. Homology equivalence classes are relative to the brane configuration. The coefficients $N_{g,\beta}$ are given by $GW_{g,n}^{X,A}$ for a given configuration of branes: the above described general formulas correspond to these.

5. For topological string theories of type B, string world sheets reduce to single point and thus correspond to constant solutions to the field equations of sigma model. Quantum intersection reduces to ordinary intersection and one has $x_1 = x_2 ... = x_n$. G-W invariants involve only classical cohomology and give for $n = 2$ the number of common points for two surfaces in X with dimension d_1 and $d_2 = n - d$. The duality between topological string theories of type A and B related to the mirror symmetry supports the idea that one could generalize the calculation of these invariants in theories B to theories A. It is not clear whether this option as any analog in TGD.

The so called Witten conjecture (see `http://tinyurl.com/yccahv3q`) proved by Kontsevich states that the partition function in one formulation of stringy quantum gravity and having as coefficients of free energy G-W invariants of the target space is same as the partition function in second formulation and expressible in terms of so called tau function associated with KdV hierarchy. This leads to non-trivial identities. Witten conjecture actually follows from the invariance of partition function with respect to half Virasoro algebra and Virasoro conjecture (see `http://tinyurl.com/y7xcc9hm`) stating just this generalizes Witten's conjecture.

4.3.4 Riemann-Roch theorem

Riemann-Roch theorem (RR) is also part of enumerative geometry albeit more abstract. Instead of counting of numbers of points, one counts dimensions of various function spaces associated with Riemann surfaces. RR provides information about the dimensions for the spaces of meromorphic functions and 1-forms with prescribed zeros and poles.

Basic notions

Riemann surface is the basic notion. Riemann surface is orientable is characterized by its genus g and number of holes/punctures in it. Riemann surface can also possess marked points, which seem to be equivalent with punctures. The moduli space of these complex curves is parameterized by a moduli space $\overline{M}_{g,n}$ of curves of genus g with n marked points (see `http://tinyurl.com/yaq8n6dp`) (see `http://tinyurl.com/yaq8n6dp`).

Dolbeault cohomology (see `http://tinyurl.com/y7cvs5sx`) generalizes the notion of differential form so that it has has well-defined degrees with respect to complex coordinates and their conjugates: one can write in general complex manifold this kind of form as

$$\omega = \omega_{i_1 i_2 .. i_n, j_1 j_2 ... j_n} dz^{i_1} \wedge dz^{i_2} ... dz^{i_n} d\overline{z}^{j_1} \wedge d\overline{z}^{j_2} ... dz^{j_n} \quad .$$

The ordinary exterior derivative d is replaced with its holomorphic counterpart ∂ and its conjugate. One can construct the counterparts of cohomology groups (Hodge theory) $H^{p,q} = H^{q,p}$. Betti numbers as numbers $h_{i,j}$ defining the dimensions of the cohomology groups forms of degrees i and

j with respect to dz^i and $d\bar{z}^j$. One can define the holomorphic Euler's characteristic as $\chi_C = h_{0,0} - h_{01} = 1 - g$ whereas orinary Euler characteristic is $\chi_R = h_{0,0} - (h_{01} + h_{10}) + h_{1,1} = 2(1-g)$.

One considers meromorphic functions having poles and zeros as the only singularities (points at which the map does not preserve angles): rational functions provide the basic example. Riemann zeta provides example of meromorphic function not reducing to rational function. Holomorphic functions have only zeros and entire functions have neither zeros nor poles. If analytic functions on Riemann surfaces can be interpreted as maps of compact Riemann surface to itself rather than to complex plane, meromorphy reduces to holomorphy since the point ∞ belongs to the Riemann surface.

The elements of free group of divisors are defined as formal sums of integers associated with the points P of Riemann surface. Divisors $D = \sum_P n(P)$, where (P) is integer, are analogous to integer valued "wave functions" on Riemann surface. The number of points with $n(P) \neq 0$ is countable. The degree of divisor is obtained as the ordinary sum $deg(D)$ of the integers defining the divisor.

Although divisors can be seen as purely formal objects, they are in practice associated to both meromorphic functions and 1-forms. The divisor of a meromorphic function is known as principal divisor. Meromorphic functions and 1-forms differing by a multiplication with meromorphic function are regarded as linearly equivalent - in other words, one can add to a given divisor a divisor of a meromorphic function without changing its equivalence class. It can be shown that all divisors associated with meromorphic 1-forms linearly equivalent and one can talk about canonical divisor. Note that $deg(D)$ is linear invariant since the degree of globally meromorphic function is zero.

The motivation for the divisors is following. Consider the space of meromorphic functions h with the property that the degrees of poles associated with the poles of these functions are not higher than given integers $n(P)$. In other words, one has $\langle h(P)\rangle + D(P) \geq 0$ for all points P ($\langle h\rangle$ is the divisor of h). For $D(P) > 0$ the pole has degree not higher than $D(P)$. For non-positive $D(P)$ the function has zero of order $D(P)$ at least.

Formulation of RR theorem

With these prerequisites it is possibly to formulate RR (for Wikipedia article see `http://tinyurl.com/mdmbcx6`). The Wikipedia article is somewhat confusing and a more precise description of RR can be found in the article "Riemann-Roch theorem" by Vera Talovikova [A86] (see `http://tinyurl.com/ktww7ks`).

Let $l(D)$ be the dimension of the space of meromorphic functions with principal divisor D or 1-forms linearly equivalent with canonical divisor K. Then the equality

$$l(D) - l(K - D) = deg(D) - g + 1 \tag{4.3.1}$$

is true for both meromorphic functions and canonical divisors. For $D = K$ one obtains using $l(0) = 1$

$$l(K) = deg(K) - g + 2 \tag{4.3.2}$$

giving the dimension of the space of canonical divisors. $l(K) > 0$ in general is not in conflict with the fact that canonical divisors are linearly equivalent. $deg(K) = 2g - 2$ in the above formula gives $l(K) = g$.

$l(K) = 0$ for $g = 0$ case looks strange: one should actually make notational distinction between dimensions of spaces of meromorphic functions and one-forms (this is done in the article of Talivakova). The explanation is that $l(K)$ here is not the dimension of the space of canonical 1-forms but that of the holomorphic functions with the divisor of K. The canonical form K for the sphere has second order pole at ∞ so that one cannot have meromorphic forms holomorphic outside P.

Riemann's inequality

$$l(D) \geq deg(D) - g + 1 \tag{4.3.3}$$

follows from $l(K-D) \geq 0$, which can be seen as a correction term to the formula

$$l(D) = deg(D) - g + 1 \ . \tag{4.3.4}$$

In what sense this is true, becomes clear from what follows.

The dimension of the space meromorphic functions corresponding to given divisor

The simplest divisor associated with meromorphic function involves only one point. Multiplying a function, which is non-vanishing and finite at P by $(z - z(P))^{-n}$ gives a pole of order n (zero has negative order in this sense). One is interested on the dimension $l(nP)$ of the space nP of meromorphic functions and RR allows to deduce information about $l(nP)$. One is interested on the behavior of $l(nP)$ as function of genus g of Riemann surface (more general situation would allow also punctures). For $n = 0$ one has entire function without poles and zeros. Only constant function is possible: $l(0) = 1$.

First some general observations. K has degree $deg(K) = 2g - 2$, which gives $l(K) = g$. For $n = deg(D) > deg(K) = 2g - 2$ the correction term vanishes since $deg(K - D)$ becomes negative, and one has $l(D) = deg(D) - g + 1$. This gives $l(n = 2g - 1) = g$. Therefore $n \in \{2g - 1, 2g, ...\}$ corresponds to $l(nP) \in \{g, g + 1, ...\}$. $n < 2g - 2$ corresponds to $l(nP) = 1$. What about the range $n \in \{2, ..., 2g - 2\}$? Note that $2g - 2$ is the negative of the Euler character of Riemann surface.

1. $g = 0$ case. K on sphere. dz canonical 1-form on Riemann sphere covered by two complex coordinate patches. $z \rightarrow w = 1/z$ relates the coordinates. There is second order pole at infinity ($dw = -dz/z^2$). One has therefore $deg(K) = -2$ for sphere in accordance with the general formula $deg(K) = 2g - 2$. The formula $l(nP) = deg(D) + 1$ holds for all n and there is no correction term now. One as $l(nP) = n + 1$.
2. $g = 1$ case.

   One has $deg(K) = 2g - 2 = 0$ for torus reflecting the fact that the canonical form $\omega = dz$ has no poles or zeros (torus is obtained by identifying the cells of a periodic lattice in complex plane). Correction term vanishes since it would have negative degree for all n and one has $l(nP) \in \{1, 1, 2, 3, ...\}$.
3. $g = 2$ case.

   For $n = deg(D) \geq 2 \times 2 - 1 = 3$ gives $l(D) = n - 1$ giving for $n \geq 3$ $l(nP) \in \{2, 3, ...\}$. What about $n = g = 2$? For generic points one has $l(2) = 1$. There are 6 points at which one has $l(D) = 2$ so that there is additional meromorphic function having pole of order 2 at this kind of point. These points are fixed points under Z_2 defining hyper-ellipticity. Note that $g \leq 2$ Riemann surfaces are always hyper-elliptic in the sense that it allows Z_2 as conformal symmetry (see `http://tinyurl.com/y9sdu4o3`).

   One has therefore $l(nP) \in \{1, 1, 1, 2, ..\}$ for a generic point and $l(nP) \in \{1, 1, 2, 2......\}$ for 6 points fixed under Z_2. An interesting question is whether this phenomenon could have physical interpretation in TGD framework.
4. $g > 2$ case.

   For $g > 2$. $l(nP)$ in the range $\{2, 2g - 2\}$ can depend on point and even on the conformal moduli. There are more special points in which correction term differs from that in the generic case. $g = 3$ illustrates the situation. $n \in \{1, 1, 1, 1, 1, 2, ...\}$ is obtained for a generic point. At special points and for $n < 3$ there are also other options for $l(nP)$. Also the dependence of $l(nP)$ on moduli emerges for $g \geq 3$. The natural guess layman is that these points are fixed points of conformal symmetries. Also now hyper-elliptic surfaces allowing projective Z_2 covering are special. In the general case hyper-ellipticity is not possible.

In TGD framework Weierstrass points(see `http://tinyurl.com/y9wehsml`) are of special interest physically.

1. My layman guess is that special points known as Weierstrass points (see `http://tinyurl.com/y9wehsml`) correspond to singularities for projective coverings for which conformal symmetries permute the sheets of the covering. Several points coincide for the covering since a sub-group of conformal symmetries would act trivially on the Weierstrass point.

Note that for $g > 2$ Z_2 covering is not possible except for hyper-elliptic surfaces, and one can wonder whether this relates to the experimental absence fo $g > 2$ fermion families [K28]. Second interesting point is that elementary particles indeed correspond to double sheeted structures from the condition that monopole fluxes flow along closed flux tubes (there are no free magnetic monopoles).

2. There is an obvious analogy with the coverings associated with the cognitive representation defined by the points of space-time surface with coordinates in an extension of rationals [L52, L46] [L51]. Fixed points for a sub-group of Galois group generate singularities at which sheets touch each other. These singular points are physically the most interesting and could carry sparticles. The action of discrete conformal groups restricted to cognitive representation could be represented as the action of Galois group on points of cognitive representation. Cognitive representation would indeed represent!

   Remarkably, if the tangent spaces are not parallel for the touching sheets, these points are mapped to several points in H in M^8-H correspondence. If this picture is correct, the hyper-elliptic symmetry Z_2 of genera $g \leq 2$ could give rise to this kind of exceptional singularities for $g \geq 2$.

   What is worrying that there are two views about twistorial amplitudes. One view relying on the notion of octonionic super-space M^8 [L46] is analogous to that of SUSYs: sparticles can be seen as completely local composites of fermions. Second view relies on embedding space $M^4 \times CP_2$ [K87] and on the identification sparticles as non-local many-fermion states at partonic 2-surfaces. These two views could be actually equivalent by $M^8 - H$ duality.

3. When these singular points are present at partonic 2-surfaces at boundaries of CD and at vertices, the topology of partonic 2-surface is in well-defined sense between g and $g+1$ external particles: one has criticality. The polynomials representing external particles indeed satisfy criticality conditions guaranteeing associativity or co-associativity (quantum criticality of TGD Universe is the basic postulate of quantum TGD). At partonic orbits the touching pieces of partonic 2-surface could separate (g) or fuse ($g+1$). Could this topological mixing give rise to CKM mixing of fermions [K28, K60, K70]?

RR for algebraic varieties and bundles

RR can be generalized to algebraic varieties (see `http://tinyurl.com/y9asz4qg`). In complex case the real dimension is four so that this generalization is interesting from TGD point of view and will be considered later. The generalization involves rather advanced mathematics such as the notion of sheaf (see `http://tinyurl.com/nudhxo6`). Zeros and poles appearing in the divisor are for complex surfaces replaced with 2-D varieties so that the generalization is far from trivial.

The following is brief summary based on Wikipedia article.

1. Genus g is replaced with algebraic genus and $deg(D)$ plus correction term is replaced with the intersection number (see `http://tinyurl.com/y7dcffb6`) for D and $D-K$, where K is the canonical divisor associated with 2-forms, which is also unique apart from linear equivalence Points of divisor are replaced with 2-varieties.

2. The generalization to complex surfaces (with real dimension equal to 4) reads as

$$\chi(D) = \chi(0) + \frac{1}{2} D \cdot (D-K) \ . \tag{4.3.5}$$

   $\chi(D)$ is holomorphic Euler characteristic associated with the divisor. $\chi(0)$ is defined as $\chi(0) = h_{0,0} - h_{0,1} + h_{0,2}$, where $h_{i,j}$ are Betti numbers for holomorphic forms. '$\cdot$' denotes intersection product in cohomology made possibly by Poincare duality. K is canonical two-form which is a section of determinant bundle having unique divisor (their is linear equivalence due to the possibility to multiply with meromorphic function.

   One has $\chi(0) = 1 + p_a$, where p_a is arithmetic genus. Noether's formula gives

$$\chi(0) = \frac{c_1^2 + c_2}{12} = \frac{K \cdot K + e}{12} \ . \tag{4.3.6}$$

c_1^2 is Chern number and $e = c_2$ is topological Euler characteristic.

Clearly the information given by $\chi(D)$ is about Dolbeault homology. For comparison note that RR for curves states $l(D) - l(K - D) = \chi(D) = \chi(0) + deg(D)$.

RR can be also generalized so that it applies to vector bundles. Ordinary RR can be interpreted as applying to a bundle for which the fiber is point. This requires the notion of the inverse bundle defined as a bundle with the property that its direct sum (Whitney sum) with the bundle itself is trivial bundle. One ends up with various characteristic classes, which represent homologically non-trivial forms in the base spaces characterizing the bundle. For instance, the generalizations of RR give information about the dimensions of the spaces of sections of the vector bundle.

Atyiah-Singer index theorem (see `http://tinyurl.com/k6daqco`) deals with so called elliptic operators in compact manifolds and represents a generalization important in recent theoretical physics, in particular gauge theories and string models. The theorem relates analytical index - typically characterizing the dimension for the spectrum of solutions of elliptic operator to a topological index. Elliptic operator is assigned with small perturbations for a given solution of field equations. Perturbations of a given solution of say Yang-Mills equations is a representative example.

4.4 Does $M^8 - H$ duality allow to use the machinery of algebraic geometry?

The machinery of algebraic geometry is extremely powerful. In particular, the number theoretical universality of algebraic geometry implies that same equations make sense for all number fields: this is just what adelic physics [L52] [L51] demands. Therefore it would be extremely nice if one could somehow use this machinery also in TGD framework as it is used in string models. How this could be achieved? There are several guide lines.

1. Twistor lift of TGD [K100, L30, K13, K87] is now a rather well-established idea although a lot of work remains to be done with the details. Twistors were originally introduced in order to be able to use this machinery and involves complexification of Minkowski space M^4 to M^4_c as an auxiliary tool. Complexification in sufficiently general sense seems to be a necessary auxiliary tool but it cannot be a trick (like Wick rotation) but something fundamental and here complexification at the level of M^8 is suggestive. In the sequel I will used M^4 for M^4_c and M^8 for M^8_c unless it is necessary to emphasize that M^8_c is in question. The essential point is that the Euclidian metric is complexified and it reduces to a real metric in various subspaces defining besides Euclidian space also Minkowski spaces with varying signature when the complex coordinates are real or imaginary.

2. If $M^8 - H$ duality holds true, one can solve field equations in $M^8 = M^4 \times E^8$ by assuming that either the tangent space or normal space of the space-time surface X^4 is associative (quaternionic) at each point and contains preferred M^2 in its tangent space. M^2 could depend on x but $M^2(x)$:s should integrate to a 2-surface. This allows to map space-time surface M^8 to a surface in $M^4 \times CP_2$ since tangent spaces are parameterized by points of CP_2 and CP_2 takes the role of moduli space. The image of tangent space as point of CP_2 is same irrespective of whether one has quaternions or complexified quaternions (H_c).

   It came a surprise that associativity/co-associativity is possible only if the space-time surface is critical in the sense that some gradients of 8 complex components of the octonionic polynomial P vanish without posing them as additional conditions reducing thus the dimension of the space-time surface. This occurs when the coefficients of P satisfy additional conditions. One obtains associative/co-associative space-time regions and regions without either property and they correspond nicely to two solution types for the twistor lift of Kähler action.

3. Contrary to the original expectations, $M^4 \subset M^8_c$ must be identified as co-associative (co-quaternionic) subspace so that E^4 is the associative/quaternionic sub-space. This allows to have light-cone boundary as the counterpart of point-like singularity in ordinary algebraic geometry and also allows to understand the emergence of CDs and ZEO.

Remark: A useful convention to be used in the sequel. $RE(o)$ and $IM(o)$ denote the real and imaginary parts of the octionion in the decomposition $o = RE(o) + IM(o)I_4$ and $Re(o)$ and $Im(o)$ its real number valued and purely imaginary parts in the usual decomposition.

The problems related to the signature of M^4 have been a longstanding head-ache of M^8 duality.

1. The intuitive vision has been that the problems can be solved by replacing M^8 with its complexification M^8_c identifiable as complexified octonions o. This requires introduction of imaginary unit i commuting with the octonionic units $E^k \leftrightarrow (1, I_1, ..., I_7)$. The real octonionic components are thus replaced with ordinary complex numbers $z_i = x_i + iy_i$.
2. Importantly, complex conjugation $o \rightarrow \overline{o}$ changes only the sign of I_i but *not!* that of i so that the octonionic inner product $(o_1, o_2) = o_1\overline{o}_2 = o_1^k o_2^l \delta_{k,l}$ becomes complex valued. Norm is equal to $O\overline{O} = \sum_i z_i^2$. Both norm and inner product are in general complex valued and real valued only in sub-spaces in which octonionic coordinates are real or imaginary. Sub-spaces have all possible signatures of metric. These sub-spaces are not closed under multiplication and this has been an obstacle in the earlier attempts based on the notion of octonion analyticity. This argument applies also to quaternions and one obtains signatures $(1,1,1,1)$, $(1,1,1,-1)$, $(1,1,-1,-1)$, and $(1,-1,-1,-1)$. Why just the usual Minkowskian signature $(1,-1,-1,-1)$ is physical, should be understood.

   The convention consistent with that used in TGD corresponds to a negative length squared for space-like vectors and positive for time-like vectors. This gives $m = (o^0, io^1, ..., io^7)$ with real o^k. The projection $M^8_c \rightarrow M^8$ defines the projection of $X^4_c \subset M^8_c$ to $X^4 \subset M^8$ serving as the pre-image of $X^4 \subset M^8$ in $M^8 - H$ correspondence.
3. o is not field anymore as is clear from the fact that $1/o = \overline{o}/o\overline{o}$ is formally infinite in Minkowskian sub-spaces, when octonion defines a light-like vector. o (and H_c) remains however a ring so that sum and products are well-defined but division can lead to problems unless one stays inside 7+7-dimensional light-cone with $Re(o\overline{o}) > 0$ ($Re(q\overline{q}) > 0$).

   Although the number field structure is lost, one can still define polynomials needed to define algebraic varieties by requiring their simultaneous vanishing and rational functions make sense inside the light-cone. Also rational functions can be defined but poles are replaced with light-cones in Minkowskian section. Algebraic geometry would thus be forced by the complexification of octonions. This looks to me highly non-trivial! The extension of zeros and poles to light-cones making propagation possible could be a good reason for why Minkowskian signature is physical. Interestingly, the allowed octonionic momenta are light-like quaternions [K87].
4. An interesting question is whether ZEO and the emergence of CDs relates to the failure of field property. It seems now clear that CDs must be assigned even with elementary particles. I have asked whether they could form an analog for the covering of manifold by coordinate patches (in TGD inspired theory of consciousness CDs would be correlates for perceptive fields for conscious entities assignable to CDs [L54]). These observations encourage to ask whether the tips of CD should correspond to a pair formed by two poles/two zeros or by pole and zero assignable to positive and negative energy states.

   It turns out that the space-time surfaces in the interior of CD would naturally correspond to non-associative surfaces and only their 3-D boundaries would have associative 4-D tangent spaces allowing mapping to H by M^8-duality, which is enough by holography.
5. The relationship between light-like 3-surface bounding Minkowskian and Euclidian space-time regions and light-like boundaries of CDs is interesting. Could also the partonic orbits be understood a singularities of octonionic polynomials with $IM(P) = RE(P) = 0$?

4.4.1 What does one really mean with $M^8 - H$ duality?

The original proposal was that M^8 duality should map the associative tangent/normal planes of associative/co-associative space-time surface containing preferred M^2, call it M^2_0, to CP_2: the map read as $(m, e) \in M^4 \times E^4 \rightarrow (m, s) \in M^4 \times CP_2$. Eventually it became clear that the choice of M^2 can depend on position with $M^2(x)$ forming an integrable distribution to CP_2: this would define what I have called Hamilton-Jacobi structures [K10]. String like objects have minimal surface as

M^4 projection for almost any general coordinate invariant action, and internal consistency requires that $M^2(x)$ integrate to a minimal surface. The details are however not understood well enough.

1. M^4 coordinate would correspond simply to projection to a fixed M_0^4 in the decomposition $M^8 = M_0^4 \times E_0^4$. One can however challenge this interpretation. How M_0^4 is chosen? Is it possible to choose it uniquely? Could also M^4 coordinates represent moduli analogous to CP_2 coordinates? What about ZEO?

   There is an elegant general manner to formulate the choice of M_0^4 at the level of M^8. The complexified quaternionic sub-spaces of M_c^8 (M^8) are parameterized by moduli space defining the quaternionic moduli. The moduli space in question is CP_2. The choice of M_0^4 corresponds to fixing of the quaternionic moduli by fixing a point of CP_2.

   Warning: Note that one should be very careful in distinguishing between quaternionic as sub-spaces of M^8 and as the tangent space M^8 of given point of M^8.

2. One can ask whether there could be a connection with ZEO, where CDs play a key role. Indeed, the complexified Minkowski inner product means that the complexified octonions (quaternions) inside M_c^8 (M_c^4) have inverse only inside 7-D (4-D) complexified light-cone and this would motivate the restriction of space-time surfaces inside future or past light-cone or both but not yet force CD.

   If one allows rational functions and even meromorphic functions of octonionic or quaternionic variable, one could consider the possibility of restricting the space-time surface defined as their zeros to a maximally sized region containing no poles.

3. Consider complexified quaternions H_c. Poles $(q\overline{q})^{-n}$, $n \geq 1$ would correspond M^4 light-cone boundaries since $q\overline{q} = 0$ at them. Also zeros $q\overline{q} = 0$, for $n \geq 1$ correspond to light-like boundaries. Could one have two poles with with time-like distance defining CD or a pair of pole and zero?

   There is also a possible connection with the notion of infinite primes [K94]. The notion of infinite prime leads to the proposal that rationals defined as ratios of infinite integers but having unit real norm (and also p-adic norms) could correspond pairs of positive and negative energy states with identical total quantum numbers and located at opposite boundaries of CD. Infinite rationals can be mapped to rational functions. Could positive energy states correspond to the numerators with zeros at second boundary of CD and negative energy states to denominators with zeros at opposite boundary of CD?

Is the choice of the pair (M_0^2, M_0^4) consistent with the properties of known extremals in H

It should be made clear that the notion of associativity/co-associativity (quaternionicity/co-quaternionicity) of the tangent/normal space need not make sense at the level of H. I shall however study this working hypothesis in the sequel.

The choice of the pair (M_0^2, M_0^4) means choosing preferred co-commutative (commutative) sub-space M_0^2 of M^8 defining a subspace of fixed co-quaternionic (quaternionic) sub-space $M_0^4 \subset M^8$.

Remark: M^4 should indeed be the co-associative/co-quaternionic subspace of M^8 if the argument about emergence of CDs is accepted and if $M^8 - H$ correspondence maps associative to associative and co-associative to co-associative.

M_0^4 in turn contains preferred M_0^2 defining co-commutative (hyper-complex) structure. Both M_0^2 and M_0^4 are needed in order to label the choice by CP_2 point (that is as a point of Grassmannian).

Is the projection to a fixed factor $M_0^4 \subset M_0^4 \times E^4$ as a choice of co-quaternionic moduli consistent with what we know about the extremals of twistor lift of Kähler action in H? How could one fix M_0^4 from the data about the extremal in H? One can make similar equations about the choice of M_0^2 as a fixed co-complex moduli characterized by a unit quaternion. Note that this choice is expected to relate closely to the twistor structure and Kähler structure.

It is best to check the proposal for the known extremals in H [K10]. Consider first CP_2 type extremals for which M^4 projection is a piece of light-like geodesic.

1. The CP_2 projection for the image of $X^4 \subset M^8$ differs from single point only if the tangent space isomorphic to M^4 and parameterized by CP_2 point varies. Consider CP_2 type extremals for the twistor lift of Kähler action [?]n H having light-like geodesic as M^4 projection as an example. The light-like geodesic defines a light-like vector in the tangent space of CP_2 type extremal. This light-like vector together with its dual spans fixed M^2, which however does not belong to the tangent space so that associative surface would not be in question.

   What about co-associativity or associativity (the latter is favored by above argument)? This property should hold true for the pre-image of CP_2 type extremal in M^8 but I am not able to say anything about this. It is questionable to require this property at the level H but one can of course look what it would give.

   What about associativity for CP_2 tangent space? The normal space of CP_2 type extremal is 3-D (!) since the only the light-like tangent vector of the geodesic and 2 vectors orthogonal to it are orthogonal to CP_2 tangent vectors. For Euclidian signature this would mean that tangent space is 5-D and cannot be associative but now the tangent space is 4-D. Can one still say that tangent space is associative. The co-associativity of the tangent space makes sense trivially. Can one conclude that CP_2 is co-associative.

   The associativity for CP_2 tangent space might make sense since the tangent space is 4-D. The light-like vector k defines M^2_0. The associativity conditions involving two tangent space vectors of CP_2 and the light-like vector k contracted with the corresponding octonion components. The contributions from the components of k to the associator should cancel each other. Since one can change the relative sign of the components of k, this mechanism does not seem to work for all components. Hence associativity cannot hold true. Neither does M^2_0 belong to the normal space since k and its dual are not orthogonal.

   Could one conclude that CP_2 type extremal is co-associative in accordance with the original belief thanks to the light-like projection to M^4? This does not conform with what the singularity considerations for the octonionic polynomials would suggest. Or is it simply not correct to try to apply associativity at the level of H. Or does $M^8 - H$ correspondence map associative tangent spaces to co-associative ones?

2. The normal space M^4 of CP_2 type extremal have all orientations characterized by its CP_2 projection. The normal space must contain the M^2_0 determined by the tangent of the light-like geodesic and this is indeed the case. Note that CP_2 type extremals cannot have entire CP_2 as CP_2 projection: they necessarily have hole at either end, which would be naturally be at the boundary of CD.

   CP_2 type extremals seem to be consistent with $M^8 - H$ correspondence. It however seems that one cannot fix the choice of M^4_0 uniquely in terms of the properties of the extremal. There is a moduli space for M^4_0:s defined by CP_2 and obviously codes for moduli for quaternion structures in octonionic space. The distributions of $M^2(x)$ (minimal surfaces) would code for quaternion structures (decomposition of octonionic coordinates to quaternionic coordinates in turn decomposing to pairs of complex coordinates).

Consider next the associativity condition for cosmic strings in $X^2 \times Y^2 \subset M^4 \times CP_2$. Now CP_2 projection is 2-D complex surfaces and M^4 projection is minimal surface. Situation is clearly associative. How unique the choice of M^4_0 is now?

1. Now $M^2(x)$ depends on position but $M^2(x)$:s define an integrable distribution defining string orbit X^2 as a minimal surface. M^4_0 must contain all surfaces $M^2(x)$, which would fix M^4_0 to a high degree for complex enough cosmic strings.

2. Each point of X^2 corresponds to the same partonic surface $Y^2 \subset CP_2$ labelling the tangent spaces for its pre-image in M^8. All the tangent surfaces $M^2(x) \times E^2(y)$ for $X^2 \times Y^2 \subset M^8$ share only $M^2(x) \subset M^4_0$. M^4_0 must contain all tangent spaces $M^2(x)$ and the inverse image of $Y^2 \subset CP_2$ must belong to the orthogonal complement E^4 of M^4_0. This is completely analogous with the condition $X^2 = X^2 \times Y^2 \subset M^4 \times CP_2$.

   Consider a decomposition $M^8 = M^4_0 \times E^4$, $M^4_0 = M^2_0 \times E^2_0$. If the inverse image of Y^2 at point x belongs to E^4, the M^4_0 projection belongs to M^4_0 also in M^8. If this does not pose any condition on the tangent spaces assignable to the points of Y^2 defining points of CP_2, there are no problems. What could happen that the tangent spaces assignable to Y^2 could force the projection of the inverse image of Y^2 to intersect M^4_0.

One should also understand massless extremals (MEs). How to choose M_0^4 in this case?

1. MEs are given as zeros of arbitrary functions of CP_2 coordinates and 2 M^4 coordinates u and v representing local light-like direction and polarization direction orthogonal to it. In the simplest situation these directions are constant and define $M_0^4 = M_0^2 \times E_0^2$ decomposition everywhere so that M_0^4 is uniquely defined. Same applies also when the directions are not constant. In the general case light-like direction would define the local tangent plane of string world sheet and local polarization plane. Since the dimension of M^4 projection is 4 there seems to be no problems involved.
2. Tangent plane of X^4 is parameterized by CP_2 coordinates depending on 2 coordinates u and v. The surface $X^4 \subset M^8$ must be graph for a map $M_0^4 \rightarrow E^4$ so that a 2-parameter deformation of M_0^4 as tangent plane is in question. The distribution of tangent planes of $X^4 \subset M^8$ is 2-D as is also the CP_2 projection in H.

To sum up, the original vision about the associativity properties of the known extremals at level of H survives. On the other hand, CDs emerge if M^4 corresponds to the co-associative part of O. Does this mean that $M^8 - H$ correspondence maps associative to co-associative by multiplying the quaternionic tangent space in M^8 by I_4 to get that in H and vice versa or that the notions of associative and co-associative do not make sense at the level of H? This does not affect the correspondence since the same CP_2 point parametrizes both associative tangent space and its complement.

Space-time surfaces as co-dimension 4 algebraic varieties defined by the vanishing of real or imaginary part of octonionic polynomial?

If the theory intended to be a theory of everything, the solution ansatz for the field equations defining space-time surfaces should be ambitious enough: nothing less than a general solution of field equations should be in question.

1. One cannot exclude the possibility that all analytic functions of complexified octonionic variable with real Taylor or even Laurent coefficients. These would would a commutative and associative algebra. Space-time surfaces would be identified as their zero loci. This option is however number theoretically attractive and can also leads to problems with adelic physics. Since Taylor series at rational point need not anymore give a rational value.
2. Polynomials of complexified octonion variable o with real coefficients define the simplest option but also rational functions formed as ratios of this kind of polynomials must be considered. Polynomials form a non-associative ring allowing sum, product, and functional decomposition as basic operations. If the coefficients o_n of polynomials are complex numbers $o_n = a_n + ib_n$, a_n, b_n real, where i refers to the commutative imaginary unit complexifying the octonions, the ring is associative. It is essential to allow only powers o^n (or $(o - o_0))^n$ with $o_0 = a_0 + ib_0$, a_0, b_0 real numbers). Physically this means that a preferred time axis is fixed. This time axis could connect the tips of CD in ZEO.

   One can write

$$P(o) = \sum_k p_k o^k \equiv RE(P)(q_1, q_2, \overline{q_1}, \overline{q}_2) + IM(P)(q_1, q_2, \overline{q_1}, \overline{q}_2) \times I_4 \quad , p_k \ \text{real} \quad , \tag{4.4.1}$$

   where the notations

$$o = q_1 + q_2 I_4 \quad , \quad q_i = z_i^1 + z_i^2 I_2 \quad , \quad \overline{q}_i = z_i^1 - z_i^2 I_2 \quad , z_i^j = x_i^j + i y_i^j \tag{4.4.2}$$

   Note that the conjugation does *not* change the sign of i. Due to the non-commutativity of octonions P^i as functions of quaternions are in general *not* analytic in the sense that they would be polynomials of q_i with real coefficients! They are however analytic functions of z_i. The real and imaginary parts of x_i^j correspond to Minkowskian and Euclidian signatures.

In adelic physics coefficients o_n of the octonionic polynomials define WCW coordinates and should be rational numbers or rationals in the extension of rationals defining the adele. The polynomials form an associative algebra since associativity holds for powers o^n multiplied by real number. Thus complex analyticity crucial in algebraic geometry would be a key element of adelic physics.

3. If the preferred extremals correspond to the associative algebra formed by these polynomials, one could construct a completely general solution of the field equations as zero loci of their real or imaginary parts and build up of new solutions using algebra operation sum, product, and functional decomposition. One could identify space-time regions as associative or co-associative algebraic varieties in terms of these polynomials and they would form an algebra.

The motivation for this dream comes from 2-D electrostatics, where conducting surfaces correspond to curves at which the real part u or imaginary part v of analytic function $w = f(z) = u + iv$ vanishes. In electrostatics curves form families with curves orthogonal to each other locally and the map $w = u + iv \rightarrow v - iu$ defines a duality in which curves of constant potential and the curves defining their normal vectors are mapped to each other.

1. The generalization to the recent situation would be vanishing of the imaginary part $IM(P)$ or real part $RE(P)$ of the octonionic polynomial, where real and imaginary parts are defined via $o = q_c^1 + q_c^2 I_4$. One can consider also the possibility that imaginary or real part has constant value c are restricted to be rational so that one can regard the constant value set also as zero set for a polynomial with constant shift. Note that the rationals could be also complexified by addition of i. One would have

$$RE(P)(z_i^k) \quad \text{or} \quad IM(P)(z_i^k) = c \ , \quad c = c_0 \ \text{rational} \ . \tag{4.4.3}$$

   c_0 must be real. These two options should correspond to the situations in which tangent space or normal space is associative (associativity/co-associativity). Complexified space-time surfaces X_c^4 corresponding to different constant values c of imaginary or real part (with respect to i) would define foliations of M_c^8 by locally orthogonal 4-dimensional surfaces in M_c^8 such that normal space for surface X_c^4 would be tangent space for its co-surface.

   CDs and ZEO emerges naturally if the $IM(o)$ corresponds to co-quaternionic part of octonion.

2. It must be noticed that one has moduli space for the quaternionic structures even when M_0^4 is fixed. The simplest choices of complexified quaternionic space $H_c = M_{c,0}^4$ containing preferred complex plane $M_{c,0}^2$ and its orthogonal complement are parameterized by CP_2. More complex choices are characterized by the choice of distribution of $M^2(x)$ integrable to (presumably minimal) 2-surface in M^4. Also the choice of the origin matters as found and one has preferred coordinates. Also the 8-D Lorentz boosts give rise to further quaternionic moduli. The physically interesting question concerns the interpretation of space-time surfaces with different moduli. For instance, under which conditions they can interact?

The proposal has several extremely nice features.

1. Single real valued polynomial of real coordinate extended to octonionic polynomial and fixed by real coefficients in extension of rationals would determine space-time surfaces.

2. The notion of analyticity needed in concrete equations is just the ordinary complex analyticity forced by the octonionic complexification: there is no need for the application to have left- or right quaternion analyticity since quaternionic derivatives are not needed. Algebraically one has the most obvious guess for the counterpart of real analyticity for polynomials generalized to octonionic framework and there is no need for the quaternionic generalization of Cauchy-Riemann equations [A88, A66] [A88, A66] (`http://tinyurl.com/yb8l34b5`) plagued by the problems with the definition of differentiation in non-commutative and non-associative context. There would be no problems with non-associativity and non-commutativity thanks to commutativity of complex coordinates with octonionic units.

3. The vanishing of the real or imaginary part gives rise to 4 conditions for 8 complex coordinates z_1^k and z_2^k allowing to solve z_2^k as algebraic functions $z_2^k = f^k(z_1^l)$ or vice versa. The conditions would reduce to algebraic geometry in complex co-dimension $d_c = 4$ and all methods and concepts of algebraic geometry can be used! Algebraic geometry would become part of TGD as it is part of M-theory too.

4.4.2 Is the associativity of tangent-/normal spaces really achieved?

The non-trivial challenge is to prove that the tangent/normal spaces are indeed associative for the two options. The surfaces X_c^4 are indeed associative/co-associative if one considers the *internal* geometry since points are in M_c^4 or its orthogonal complement.

One should however prove that X_c^4 are also associative *as sub-manifolds* of O and therefore have quaternionic tangent space or normal space at each point parameterized by a point of CP_2 in the case that tangent space containing position dependent M_c^2, which integrate to what might be called a 2-D complexified string world sheet inside M_c^4.

1. The first thing to notice that associativity and quaternionicity need not be identical concepts. Any surface with complex dimension $d < 4$ in O is associative and any surface with dimension $d > 4$ co-associative. Quaternionic and co-quaternionic surfaces are 4-D by definition. One can of course ask whether one should consider a generalization of brane hierarchy of M-theory also in TGD context and allow associativity in its most general sense. In fact, the study of singularity of o^2 shows that 6 and 5-dimensional surfaces are allowed for which the only interpretation would be as co-associative spaces. This exceptional situation is due to the additional symmetries increasing the dimension of the zero locus.
2. One has clearly quaternionicity at the level of o obtained by putting $Y = 0$ and at the level of the tangent space for the resulting surface. The tangent space should be quaternionic. The Jacobian of the map defined by P is such that it takes fixed quaternionic subspace $H_c \to M_{0,c}^4$ of O to a quaternionic tangent space of X^4. The Jacobian applied to the vectors of H_c gives the octonionic tangent vectors and they should span a quaternionic sub-space.
3. The notion of quaternionic surface is rigorous. $M^8 - H$ correspondence could be actually interpreted in terms of the construction of quaternionic surface in M^8. One has 4-D integrable distribution of quaternionic planes in O with given quaternion structure labelled by points of CP_2 and has representation at the level of H as space-time surface and should be preferred extremals. These quaternion planes should integrate to a slicing by 4-surfaces and their duals. One obtains this slicing by fixing the values 4 of the suitably defined octonionic coordinates P^i, $i = 1,..,8$, to a real constants depending on the surface of the slicing. This gives a space-time surfaces for which tangent space-spaces or normal spaces are quaternionic.

   The first guess for these coordinates P^i be as real or imaginary parts of real polynomials $P(o)$. But how to prove and understand this?

Could the following argument be more than wishful thinking?

1. In complex case an analytic function $w(z) = u + iv$ of $z = x + iy$ mediates a map between complex planes Z and W. One can interpret the imaginary unit appearing in w locally as a tangent vector along $u = constant$ coordinate line.
2. One can interpret also octonionic polynomials with real coefficients as mediating a map from octonionic plane O to second octonionic plane, call if W. The decomposition $P = P^{1)} + P^{2)} I_4$ would have interpretation in terms of coordinates of W with coordinate lines representing quaternions and co-quaternions.
3. This would suggests that the quaternionic coordinate lines in W can be identified as coordinate curves in O - that space-time surfaces - which are quaternionic/co-quaternionic surfaces for $P^1 = constant/P^2 = constant$ lines. One would have a representation of the same thing in two spaces, and if sameness includes also quaternionicity/co-quaternionicity as attributes, then also associativity and co-associativity should hold true.

The most reasonable approach is based on generality. Associativity/quaternionicity means a slicing of octonion space by orthogonal quaternionic and co-quaternionic 4-D surfaces defined by constant value surfaces of octonionic polynomial with real coefficients. This slicing should make sense also for quaternions: one should have a slicing by complex and co-complex (commutative/co-commutative) surfaces and in TGD string world sheets and partonic 2-surfaces assignable to Hamilton-Jacobi structure would define this kind of slicing. In the case of complex numbers one has a slicing in terms of constant value curves for real and imaginary parts of analytic function and Cauchy-Riemann equations should define the property and co-property. The first guess that the tangent space of the curve is real or imaginary is wrong.

Could associativity and commutativity conditions be seen as a generalization of Cauchy-Rieman conditions?

Quaternionicity in the octonionic case, complexity in quaternionic case, and what-ever-it-is in complex case should be seen as a 3-levelled hierarchy of geometric conditions satisfied by polynomial maps with real coefficients for polynomials in case of octonions and quaternions. Of course, also Taylor and even Laurent series might be considered. The "Whatever it is" cannot be nothing but Cauchy-Riemann conditions defining complex analyticity for complex maps.

The hierarchy looks obvious. In the case of Cauchy-Riemann conditions one has commutative and associative structure and Cauchy-Riemann conditions are linear in the partial derivatives. In the case of commutative sub-manifolds of quaternionic space the conditions are quadratic in the partial derivatives. In the case of associative sub-manifolds of octonionic space the conditions are trilinear in partial derivatives. One would have nothing but a generalization of Cauchy-Riemann equations to multilinear equations in dimensions $D = 2^k$, $k = 1, 2, 3$: k-linearity with $k = 1, 2, 3$!

One can continue the hierarchy of division algebras by assuming only algebra property by using Cayley-Dickson construction (see `http://tinyurl.com/ybuyla2k`) by adding repeatedly a non-commuting imaginary unit to the structure already obtained and thus doubling the dimension of the algebra each time. Polynomials with real coefficients should still define an associative and commutative algebra if the proposal is to make sense. All these algebras are indeed power associative: one has $x^m x^n = x^{m+n}$. For instance, sedenions define 16-D algebra. Could this hierarchy corresponds to a hierarchy of analyticities satisfying generalized Cauchy-Riemann conditions?

Complex curves in real plane cannot have real tangent space

Going from octonions to quaternions to complex numbers, could constant value curves of real and imaginary parts of ordinary analytic function in complex plane make sense? The curves $u = 0$ and $v = 0$ of functions $f(z) = u + iv$, $z = x + iy$ define a slicing of plane by orthogonal curves completely analogous to its octonionic and quaternionic variants. Can one say that the tangent vectors for these curves are real/imaginary? For $u = 0$ these curves have tangent $\partial_x u + i\partial_y u$, which is not real unless one has $f(z) = k(x + iy)$, k real.

Reality condition is clearly too strong. In fact, it is the well-ordering of the points of the 1-dimensional curve, which is the property in question and lost for complex numbers and regained at $u = 0$ and $v = 0$ curves. The reasonable interpretation is in terms of hierarchy of conditions multilinear in the gradients of coordinates proposed above and linear Cauchy-Riemann conditions is the only option in the case of complex plane. What is special in this curves that the tangent vectors define flows which by Cauchy-Riemann conditions are divergenceless and irrotational locally.

Pessimistic would conclude that since the conjecture fails except for linear polynomials in complex case, it fails also in the case of quaternions and octonions. For quaternionic polynomial q^2 the conditions are however satisfied and it turns out that the resulting conditions make sense also in the general case. Optimistic would argue that reality condition is not analogous to commutativity and associativity so that this example tells nothing. Less enthusiastic optimist might admit that the reality condition is a natural generalization to complex case but that the conjecture might be true only for a restricted set of polynomials - in complex case of for $f(z) = kz$, k real. In quaternionic and octonionic case but hopefully for a larger set of polynomials with real coefficients, maybe even all polynomials with real coefficients.

Associativity and commmutativity conditions as a generalization of Cauchy-Rieman conditions?

Quaternionicity in the octonionic case, complexity in quaternionic case, and what-ever-it-is in complex case should be seen as a 3-levelled hierarchy of geometric conditions satisfied by polynomial maps with real coefficients for polynomials in case of octonions and quaternions. Of course, also Taylor and even Laurent series might be considered. The "whatever-it-is" cannot be nothing but Cauchy-Riemann conditions defining complex analyticity for complex maps.

The hierarchy looks obvious. In the case of Cauchy-Riemann conditions one has commutative and associative structure and Cauchy-Riemann conditions are linear in the partial derivatives. In the case of commutative sub-manifolds of quaternionic space the conditions are quadratic in the partial derivatives. In the case of associative sub-manifolds of octonionic space the conditions are trilinear in partial derivatives. One would have nothing but a generalization of Cauchy-Riemann equations to multilinear equations in dimensions $D = 2^k$, $k = 1, 2, 3$: k-linearity with $k = 1, 2, 3$!

One can continue the hierarchy of number fields by assuming only algebra property by adding additional imaginary units as done in Cayley-Hamilton construction (see `http://tinyurl.com/ybuyla2k`) by adding repeatedly a non-commuting imaginary unit to the algebra already obtained and thus doubling the dimension of the algebra each time. Polynomials with real coefficients should still define an associative and commutative algebra if the proposal is to make sense. All these algebras are indeed power associative: one has $x^m x^n = x^{m+n}$. For instance, sedenions define 16-D algebra. Could this hierarchy corresponds to a hierarchy of analyticities satisfying generalized Cauchy-Riemann conditions? Could this hierarchy corresponds to a hierarchy of analyticities satisfying generalized Cauchy-Riemann conditions?

One would have also a nice physical interpretation: in the case of quaternions one would have "quaternionic conformal invariance" as conformal invariances inside string world sheets and partonic 2-surfaces in a nice agreement with basic vision about TGD. At the level of octonions would have "quaternionic conformal invariance" inside space-time surfaces and their duals. What selects the preferred commutative or co-commutative surfaces is of course an interesting problem. Is a gauge choice in question? Are these surfaces selected by some special property such as singular character? Or does one have wave function in the set of these surfaces for a given space-time surface?

Could quaternionic polynomials define complex and co-complex surfaces in H_c?

What about complex and co-complex (commutative/co-commutative) surfaces in the space of quaternions? One would have a slicing of the quaternionic space by pairs of complex and co-complex surfaces and would have natural identification as quaternion/Hamilton-Jacobi structure and relate to the decomposition of space-time to string world sheets and partonic 2-surfaces. Now the condition of associativity would be replaced with commutativity.

1. In the quaternionic case the tangent vectors of the 2-D complex sub-variety would be commuting. Can this be the case for the zero loci real polynomials $P(q)$ with $IM(P) = 0$ or $RE(P) = 0$? In this case the commutativity condition is that the tangent vectors have imaginary parts (as quaternions) proportional to each other but can have different real parts. The vanishing of cross product is the condition now and involves only two vectors whereas associativity condition involves 3 vectors and is more difficult.
2. The tangent vectors of a commutative 2-surface commute: $[t^1, t^2] = 0$. The commutator reduces to the vanishing of the cross product for the imaginary parts:

$$Im(t^1) \times Im(t^2) = 0 \ . \tag{4.4.4}$$

3. Expressing z_1^i as a function of z_2^k and using (z_1^i, z_2^k) as coordinates in quaternionic space, the tangent vectors in quaternionic spaces can be written in terms of partial derivatives $\partial z_1^{1)}/\partial z_2^{k)}$ as

$$t_k^i = (\frac{\partial z_1^{i)}}{\partial z_2^{k)}}, \delta_k^i) \ , \tag{4.4.5}$$

Here the first part corresponds to $RE(t^i)$ as quaternion and second part to $IM(t^i)$ as quaternion.

The condition that the vectors are parallel implies

$$\frac{\partial z_1^{1)}}{\partial z_2^{k)}} = 0 \ . \quad (4.4.6)$$

At the commutative 2-surface X^2 $z_1^{1)}$ is constant and $z_1^{2)}$ is a function of $z_2^{1)}$ and $z_2^{2)}$. One would have a graph of a function $z_1^{2)} = f_2(z_2^{k)})$ at X^2 but not elsewhere. One could regard $z_1^{1)}$ as an extremum of a function $z_1^{1)} = f_1(z_2^{k)})$.

How to interpret this result?

1. In the generic case this condition eliminates 1 dimension so that 2-D surface would reduce to a 1-D curve.
2. If one poses constraints on the coefficients of $P(q)$ analogous to the conditions forcing the potential function for say cusp catastrophe to have degenerate extrema at the boundaries of the catastrophe one can get 2-D solution. For these values of parameters the conditions would be equivalent with $RE(P) = 0$ or $IM(P) = 0$ conditions.

   The vanishing of the gradient of z_1^1 would indeed correspond in the case of cups catastrophe to the condition for the co-incidence of two roots of the behavior variable x as extremum of potential function $V(x, a, b)$ fixing the control variable a as function of b.

   This would pose constraints on the coefficients of P not all polynomials would be allowed. This kind of conditions would realize the idea of quantum criticality of TGD at the level of quaternion polynomials. This option is attractive if realizable also at the level of octonion polynomials. This turns out to be the case.
3. One would thus have two kinds of commutative/co-commutative surfaces. The generic 1-D surfaces and 2-D ones which are commutative/commutative and critical and assignable to string world sheets and partonic 2-surfaces. 1-D surfaces would correspond to fermion lines at the orbits of partonic 2-surfaces appearing in the twistor amplitudes in the interaction regions defined by CDS. 2-D surfaces would correspond to the orbits of fermionic strings connecting point-like fermions at their ends and serving as correlates for bound state entanglement for external fermions arriving into CD. This picture would allow also to understand why just some string world sheets and partonic 2-surfaces are selected.

The simplest manner to kill the proposal is to look for $P = q^2$ and $RE(P(q^2)) = 0$ surface. In this case this condition is indeed satisfied. One has

$$\begin{aligned}
&RE(P) = X^{1)} + X^{2)}I_1 \ , \\
&X^{1)} = (z_1^{1)})^2 - (z_1^{2)})^2 + (z_2^{1)})^2 - (z_2^{2)})^2 \ , \quad X^{2)} = 2z_1^{1)}z_1^{2)}I_1 \ , \\
&IM(P) = Y^{1)} + Y^{2)}I_1 \ , \\
&Y^{1)} = (z_2^{1)} + \overline{z_2^{1)}})z_1^{1)} \ , \qquad Y^{2)} = (z_2^{2)} + \overline{z_2^{2)}})z_1^{2)}
\end{aligned} \quad (4.4.7)$$

$X^{2)} = 0$ gives $z_1^{1)}z_1^{2)} = 0$ so that one has either $z_1^{1)} = 0$ or $z_1^{2)} = 0$. $X^{1)} = 0$ gives for $z_1^{1)} = 0$ $z_1^{2)} = \pm\sqrt{(z_2^{1)})^2 + (z_2^{2)})^2}$.

The partial derivative $\partial z_1^{1)}/\partial z_2^{k)}$ is from implicit function theorem - following from the vanishing of the differential $d(RE(P))$ along the surface - proportional $\partial X^{1)}/\partial z_2^{k)}$, but vanishes as required.

Clearly, the quaternionic variant of the proposal survives in the simplest case its simplest test. 2-D character of the surface would be due to the criticality of q^2 making it possible to satisfy the conditions without the reduction of dimension.

Explicit form of associativity/quaternionicity conditions

Consider now the explicit conditions for associativity in the octonionic case.

1. One should calculate the octonionic tangent (normal) vectors t^i for $X = 0$ in associative ($Y = 0$ in co-associative case) and show that there associators $Ass(t^i, t^j, t^k)$ vanish for all possible or all possible combinations i, j, k. In other words, one that one has

$$Ass(t^i,t^j,t^k) = 0 \ \ , \ \ i,j,k \in \{1,..,4\} \ \ , \ \ Ass(a,b,c) \equiv (ab)c - a(bc) \ \ . \tag{4.4.8}$$

   One can cast the condition to simpler from by expressing t^i as octonionic vectors $t^i_k E^k$:

$$\begin{array}{l} Ass(E^a,E^b,E^b) =\equiv f^{abcd}E_d \ \ , \ \ a,b,c,d \in \{1,..,7\} \ \ , \\ f^{abcd} = \epsilon^{abe}\epsilon_e^{\ cd} - \epsilon^{aed}\epsilon^{bc}_{\ \ e} = 2\epsilon^{abe}\epsilon_e^{\ cd} \ \ . \end{array} \tag{4.4.9}$$

   The permutation symbols for a given triplet i, j, k are structures constants for quaternionic inner product and completely antisymmetric (see `http://tinyurl.com/p42tqsq`).. $\epsilon_{ijk} = 1$ is true for the seven triplets $123, 145, 176, 246, 257, 347, 365$ defining quaternionic sub-spaces with 1-D intersections. The anti-associativity condition $(E_iE_j)E_k = -(E_iE_j)E_k$ holds true so that one has obtains the simpler expression for f^{ijks} having values ± 2.
   Using this representation $Ass(t^i, t^j, t^k)$ reduces to 7 conditions for each triplet:

$$t^i_r t^j_s t^k_t f^{rstu} = 0 \ \ , \ \ i,j,k \in \{1,..,4\} \ \ , \ \ r,s,t,u \in \{1,..,7\} \ \ . \tag{4.4.10}$$

2. If the vanishing condition $X = 0$ or $Y = 0$ is crucial for associativity then every polynomial is its own case to be studied separately and a general principle behind associativity should be identified: the proposal is as a non-linear generalization of Cauchy-Riemann conditions. As the following little calculation shows, the vanishing condition indeed appears as one calculates partial derivatives $\partial z_1^{k)}/\partial z_2^{l)}$ in the expression for the tangent vectors of the surface deduced from the vanishing gradient of X or Y.

3. I have proposed the octonionic polynomial ansatz already earlier but failed to prove that it gives associative tangent or normal spaces. Besides the intuitive geometric argument I failed to notice that the complex 8-D tangent vectors in coordinates $z_1^{k)}$ or $z_2^{k)}$ for complexified space-time surface and coordinates $(z_1^{k)}, z_2^{k)})$ for o have components

$$\begin{array}{l} \frac{\partial o^i}{\partial z^1_k} \leftrightarrow (\delta^i_k, \frac{\partial z_2^{i)}}{\partial z_1^{k)}}) \\ \\ \qquad \text{or} \\ \\ (\frac{\partial o^i}{\partial z^2_k}) \leftrightarrow (\frac{\partial z_1^{i)}}{\partial z_2^{k)}}, \delta^i_k) \ \ . \end{array} \tag{4.4.11}$$

   These vectors correspond to complexified octonions O_i given by

$$\delta^i_k E^k + \frac{\partial z_2^{i)}}{\partial z_1^{k)}} E^k E_4 \ \ , \tag{4.4.12}$$

   where the unit octonions are given by $(E_0, E_1, E_2, E_3) = (1, I_1, I_2, I_3)$ and $(E_5, E_5, E_7, E_8) = (1, I_1, I_2, I_3)E_4$. The vanishing of the associators stating that one has

4. One can calculate the partial derivatives $\frac{\partial z_i^k}{\partial z_j^k}$ explicitly without solving the equations or the complex valued quaternionic components of $RE(P)\equiv X=0$ or $IM(P)\equiv Y=0$ (note that X and Y have for complex components labelled by X^i and Y^i respectively.

$$Y^i(z_1^{k)},z_2^{l)}) = c\in R \ , \ \ i=1,...,4 \ , \quad \text{associativity} \ ,$$

or

$$X^i(z_1^{k)},z_2^{l)}) = c\in R \ , \ \ i=1,...,4 \ , \quad \text{co-associativity} \ . \tag{4.4.13}$$

explicitly and check whether associativity holds true. The derivatives can be deduced from the constancy of Y or X.

5. For instance, if one has $z_2^{k)}$ as function of $z_1^{k)}$, one obtains in the associative case

$$RE(Y)^i{}_k + IM(Y)^i{}_k\frac{\partial z_2^{r)}}{\partial z_1^{k)}} = 0$$
$$RE(Y)^i{}_k \equiv \frac{\partial Y^i}{\partial z_1^{k)}} \ , \qquad\qquad IM(Y)^i{}_k \equiv \frac{\partial Y^i}{\partial z_2^{k)}} \ . \tag{4.4.14}$$

In co-associative case one must consider normal vectors expressible in terms of Y^i so that X is replaced with Y in these equations.
This allows to solve the partial derivatives needed in associator conditions

$$\frac{\partial z_2^{i)}}{\partial z_1^{k)}} = \left[Im(Y)^{-1}\right]^i{}_r\, Re(Y)^r{}_k \ . \tag{4.4.15}$$

6. The vanishing conditions for the associators are however multilinear and one can multiply each factor by the matrix $IM(P)$ without affecting the condition so that $IM(P)^{-1}$ disappears and one obtains the conditions for vectors

$$T_r^iT_s^jT_t^kf^{rstu} = 0 \ , \qquad\qquad i,j,k\in\{1,..,4\} \ , \quad r,s,t,u\in\{1,..,7\} \ ,$$
$$T^i = IM(Y)^i{}_kE^k - RE(Y)^i{}_kE^kE_4 \ . \tag{4.4.16}$$

This form of conditions is computationally much more convenient.

How to solve these equations?

1. The antisymmetry of f^{rstu} with respect to the first two indices r,s leads one to ask whether one could have

$$T_r^iT_s^jT_t^k = 0 \tag{4.4.17}$$

for the 7 quaternionic triplets. This is guaranteed if one has either $RE(Y)^i{}_k = \partial Y^i/\partial z_1^k = 0$ (coquaternionic part of T^i) or $IM(Y)^i{}_k = \partial Y^i/\partial z_2^k = 0$ (co-quaternionic part of T^i) for *one* member in each triplet.
The study of the structure constants listed above shows that indices 1,2,3 are contained in all 7 triplets. Same holds for the indices appearing in any quaternionic triplet. Hence it is enough to require that three gradients $RE(Y)^ik = 0$ or $IM(Y)^i{}_k = 0$ $k\in\{1,2,3\}$ vanish. This condition is obviously too strong since already single vanishing condition reduces the dimension of space-time variety to 3 in the generic case and it becomes trivially associative.

Octonionic automorphism group G_2 gives additional basis with their own quaternion triplets and the general condition would be that 3 partial derivatives vanish for a triplet obtained from the basic triplet $\{1, 2, 3\}$ by G_2 transformation. It is not quite clear to me whether the G_2 transformation can depend on position on space-time surface.

2. As noticed, the vanishing of all triplets is an un-necessarily strong condition. Already the vanishing of single gradient $RE(Y)^i{}_k$ or $IM(Y)^i{}_k$ reduces the dimension of the surface from 4 to 3 in the generic case. If one accepts that the dimension of associative surface is lower than 4 then single criticality condition is enough to obtain 3-D surface.

   In the generic case associativity holds true only at the 4-D tangent spaces of 3-surfaces at the ends of CD (at light-like partonic orbits it holds true trivially in 4-D) and that the twistor lift of Kähler action determines the space-time surfaces in their interior.

   In this case one can map only the boundaries of space-time surface by $M^8 - H$ duality to H. The criticality at these 3-surfaces dictates the boundary conditions and provides a solution to infinite number of conditions stating the vanishing of SSA Noether charges at space-like boundaries. These space-time regions would correspond to the regions of space-time surfaces inside CDs identifiable as interaction regions, where Kähler action and volume term couple and dynamics depends on coupling constants.

   The mappability of M^8 dynamics to H dynamics in all space-time regions does not look feasible: the dynamics of octonionic polynomials involves no coupling constants whereas twistor lift of Kähler action involves couplings parameters. The dynamics would be non-associative in the geometric sense in the interior of CDs. Notice that also conformal field theories involve slight breaking of associativity and that octonions break associativity only slightly ($a(bc) = -(ab)c$ for octonionic imaginary units). I have discussed the breaking of associativity from TGD viewpoint in [K53] .

3. Twistor lift of Kähler action allows also space-time regions, which are minimal surfaces [L28] and for which the coupling between Kähler action and volume term vanishes. Preferred extremal property reduces to the existence of Hamilton-Jacobi structure as image of the quaternionic structure at the level of M^8. The dynamics is universal as also critical dynamics and independent of coupling constants so that $M^8 - H$ duality makes sense for it. External particles arriving into CD via its boundaries would correspond to critical 4-surfaces: I have discussed their interpretation from the perspective of physics and biology in [L29].

4. One should be able to produce associativity without the reduction of dimension. One can indeed hope of obtaining 4-D associative surfaces by posing conditions on the coefficients of the polynomial P by requiring that one $RE(Y)^i_k$ or $IM(Y)^i_k$, $i = i_1$ -call it just X_1 - should vanish so that Y^i would be critical as function of z_1^k or z_2^k.

   At $X_1 = 0$ would have degenerate zero at the 4-surface. The decomposition of X_1 to a product of monomial factors with root in extension of rationals would have one or more factors appearing at least twice. The associative 4-surfaces would be ramified. Also the physically interesting p-adic primes are conjectured to be ramified in the sense that their decomposition to primes of extension of rationals contains powers of primes of extension. The ramification of the monomial factors is nothing but ramification for polynomials primes in field of rationals in terms of polynomial primes in its extension.

   This could lead to vanishing of say one triplet while keeping $D = 4$. This need not however give rise to associativity in which case also second $RE(Y)^i_i$ or $IM(Y)^i_k$, $i = i_2$, call it X_2, should vanish. The maximal number of X_i would be $n_{max} = 3$. The natural condition consistent with quantum criticality of TGD Universe would be that the variety is associative but maximally quantum critical and has therefore dimension $D = 3$ or $D = 4$. Stronger condition allows only $D = 4$.

   These $n \leq 3$ additional conditions make the space-time surface analogous to a catastrophe with $n \leq 3$ behavior variables in Thom's classification of 7 elementary catastrophes with less than 11 control variables [A47]. Thom's theory does not apply now since it has only one potential function $V(x)$ (now $n \leq 3$ corresponding to the critical coordinates Y^i!) as a function of behaviour variables and control variables). Also the number of non-vanishing coefficients in the polynomial having values in an extension of rationals and acting as control variables is unlimited. In quaternionic case the number of potential functions is indeed 1 but the number of control variables unlimited.

5. One should be able to understand the $D = 3$ associative objects - say light-like 3-surfaces or 3-surfaces at the boundaries of CD - as 3-surfaces along which 4-D associative (co-associative) and non-associative (non-co-associative) surfaces are glued together.

   Consider a product P of polynomials allowing 3-D surfaces as necessarily associative zero loci to which a small interaction polynomial vanishing at the boundaries of CD (proportional to o^n, $n > 1$) is added. Could P allow 4-D surface as a zero locus of real or imaginary part and containing the light-like 3-surfaces thanks to the presence of additional parameters coming from the interaction polynomial. Can one say that this small interaction polynomial would generate 4-D space-time in some sense? 4-D associative space-time regions would naturally emerge from the increasing algebraic complexity both via the increase of the degree of the polynomial and the increase of the dimension of the extension of rationals making it easier to satisfy the criticality conditions!

   There are two regions to be considered: the interior and exterior of CD. Could associativity/co-associativity be possible outside CD but not inside CD so that one would indeed have free external particles entering to the non-associative interaction region. Why associativity conditions would be more difficult to satisfy inside CD? Certainly the space-likeness of M^4 points with respect to the preferred origin of M^8 in this region should be crucial since Minkowski norm appears in the expressions of $RE(P)$ and $IM(P)$.

Do the calculations for the associative case generalize to the co-associative case?

1. Suppose that one has possibly associative surface having $RE(P) = 0$. One would have $IM(P) = 0$ for dual space-time surface defining locally normal space of $RE(P) = 0$ surface. This would transform the co-associativity conditions to associativity conditions and the preceding arguments should go through essentially as such.

   Associative and co-associative surfaces would meet at singularity $RE(P) = IM(P) = 0$, which need not be point in Minkowskian signature (see $P = o^2$ example in the Appendix) and can be even 4-D! This raises the possibility that the associative and co-associative surfaces defined by $RE(P) = 0$ and $IM(P) = 0$ meet along 3-D light-like orbits partonic surfaces or 3-D ends of space-time surfaces at the ends of CD.

2. If $D = 3$ for associative surfaces are allowed besides $D = 4$ as boundaries of 4-surfaces, one can ask why not allow $D = 5$ for co-associative surfaces. It seems that they do not have a reasonable interpretation as a surface at which co-associative and non-co-associative 4-D space-time regions would meet. Or could they in some sense be geometric "co-boundaries" of 4-surfaces like branes in M-theory serve as co-boundaries of strings? Could this mean that 4-D space-time-surface is boundary of 5-D co-associative surface defining a TGD variant of brane with strings world sheets replaced with 4-D space-time surfaces?

   What came as a surprise that $P = o^2$ allows 5-D and 6-D surfaces as zero loci of $RE(P)$ or $IM(P)$ as shown in Appendix. The vanishing of the entire o^2 gives 4-D interior or exterior of CD forced also by associativity/co-associativity and thus maximal quantum criticality. This is very probably due to the special properties of o^2 as polynomial: in the generic case the zero loci should be 4-D.

This discussion can be repeated for complex/co-complex surfaces inside space-time surfaces associated with fermionic dynamics.

1. Associativity condition does not force string world sheets and partonic 2-surfaces but they could naturally correspond to commutative or co-commutative varieties inside associative/co-associative varieties.

   In the generic case commutativity/co-commutativity allows only 1-D curves - naturally light-like fermionic world lines at the boundaries of partonic orbits and representing interacting point-like fermions inside CDs and used in the construction of twistor amplitudes [L30, K87]. There is coupling between Kähler part and volume parts of modified Dirac action inside CDs so that coupling constants are visible in the spinor dynamics and in dynamics of string world sheet.

2. At criticality one obtains 2-D commutative/co-commutative surfaces necessarily associated with external particles quantum critical in 4-D sense and allowing quaternionic structure.

String world sheets would serve as correlates for bound state entanglement between fermions at their ends. Criticality condition would select string world sheets and partonic 2-surfacs from the slicing of space-time surface provided by quaternionic structure (having Hamilton-Jacobi structure as H-counterpart).

If associativity holds true and fixed M_c^2 is contained in the tangent space of space-time surface, one can map the M^4 projection of the space-time surface to a surface in $M^4 \times CP_2$ so that the quaternionic tangent space at given point is mapped to CP_2 point. One obtains 4-D surface in $H = M^4 \times CP_2$.

1. The condition that fixed M_c^2 belongs to the tangent space of X_c^4 is true in the sense that the coordinates $z_2^{k)}$ do not depend on $z_1^{1)}$ and $z_1^{2)}$ defining the coordinates of M_c^2. It is not clear whether this condition can be satisfied in the general case: octonionic polynomials are expected to imply this dependence un-avoidably.

   The more general condition allows M_c^2 to depend on position but assumes that M_c^2:s associated with different points integrate to a family 2-D surfaces defining a family of complexified string world sheets. In the similar manner the orthogonal complements E_c^2 of M_c^2 would integrate to a family of partonic 2-surfaces. At each point these two tangent spaces and their real projections would define a decomposition analogous to that define by light-like momentum vector and polarization vector orthogonal to it. This decomposition would define decomposition of quaternionic sub-spaces to complexified complex subspace and its co-complex normal space. The decomposition would correspond to Hamilton-Jacobi structure proposed to be central aspect of extremals [K10].

2. What is nice that this decomposition of M_c^4 (M^4) would (and of course should!) follow automatically from the octonionic decomposition. This decomposition is lower-dimensional analog to that of the complexified octonionic space induced by level sets of real octonionic polymials but at lower level and extremely natural due to the inclusion hierarchy of classical number fields. Also M_c^2 could have similar decomposition orthogonal complex curves by the value sets of polynomials. The hierarchy of grids means the realization of the coordinate grid consisting of quaternionic, complex, and real curves for complexified coordinates o^k and their quaternionic and complex variants and is accompanied by corresponding real grids obtained by projecting to M^4 and mapping to CP_2.

   Thus these decompositions would be obtained from the octonionic polynomial decomposing it to real quaternionic and imaginary quaternionic parts first to get a grid of space-time surfaces as constant value sets of either real or imaginary part, doing the same for the non-constant quaternionic part of the octonionic polynomial to get similar grid of complexified 2-surfaces, and repeating this for the complexified complex octonionic part.

Unfortunately, I do not have computer power to check the associativity directly by symbolic calculation. I hope that the reader could perform the numerical calculations in non-trivial cases to this!

General view about solutions to $RE(P) = 0$ and $IM(P) = 0$ conditions

The first challenge is to understand at general level the nature of $RE(P) = 0$ and $IM(P) = 0$ conditions. Appendix shows explicitly for $P(o) = o^2$ that Minkowski signature gives rise to unexpected phenomena. In the following these phenomena are shown to be completely general but not quite what one obtains for $P(o) = o^2$ having double root at origin.

1. Consider first the octonionic polynomials $P(o)$ satisfying $P(0) = 0$ restricted to the light-like boundary δM_+^8 assignable to 8-D CD, where the octonionic norm of o vanishes.

   (a) $P(o)$ reduces along each light-ray of δM_+^8 to the same real valued polynomial $P(t)$ of a real variable t apart from a multiplicative unit $E = (1 + in)/2$ satisfying $E^2 = E$. Here n is purely octonion-imaginary unit vector defining the direction of the light-ray.
   $IM(P) = 0$ corresponds to quaterniocity. If the E^4 ($M^8 = M^4 \times E^4$) projection is vanishing, there is no additional condition. 4-D light-cones $M_\pm^4$ are obtained as solutions of $IM(P) = 0$. Note that $M_\pm^4$ can correspond to any quaternionic subspace.

If the light-like ray has a non-vanishing projection to E^4, one must have $P(t) = 0$. The solutions form a collection of 6-spheres labelled by the roots t_n of $P(t) = 0$. 6-spheres are not associative.

(b) $RE(PE) = 0$ corresponding to co-quaternionicity leads to $P(t) = 0$ always and gives a collection of 6-spheres.

2. Suppose now that $P(t)$ is shifted to $P_1(t) = P(t) - c$, c a real number. Also now $M^4_\pm$ is obtained as solutions to $IM(P) = 0$. For $RE(P) = 0$ one obtains two conditions $P(t) = 0$ and $P(t - c) = 0$. The common roots define a subset of 6-spheres which for special values of c is not empty.

The above discussion was limited to δM^8_+ and light-likeness of its points played a central role. What about the interior of 8-D CD?

1. The natural expectation is that in the interior of CD one obtains a 4-D variety X^4. For $IM(P) = 0$ the outcome would be union of X^4 with M^4_+ and the set of 6-spheres for $IM(P) = 0$. 4-D variety would intersect M^4_+ in a discrete set of points and the 6-spheres along 2-D varieties X^2. The higher the degree of P, the larger the number of 6-spheres and these 2-varieties.
2. For $RE(P) = 0$ X^4 would intersect the union of 6-spheres along 2-D varieties. What comes in mind that these 2-varieties correspond in H to partonic 2-surfaces defining light-like 3-surfaces at which the induced metric is degenerate.
3. One can consider also the situation in the complement of 8-D CD which corresponds to the complement of 4-D CD. One expects that $RE(P) = 0$ condition is replaced with $IM(P) = 0$ condition in the complement and $RE(P) = IM(P) = 0$ holds true at the boundary of 4-D CD.

6-spheres and 4-D empty light-cones are special solutions of the conditions and clearly analogs of branes. Should one make the (reluctant-to-me) conclusion that they might be relevant for TGD at the level of M^8.

1. Could M^4_+ (or CDs as 4-D objects) and 6-spheres integrate the space-time varieties inside different 4-D CDs to single connected structure with space-time varieties glued to the 6-spheres along 2-surfaces X^2 perhaps identifiable as pre-images of partonic 2-surfaces and maybe string world sheets? Could the interactions between space-time varieties X^4_i assignable with different CDs be describable by regarding 6-spheres as bridges between X^4_i having only a discrete set of common points. Could one say that X^2_i interact via the 6-sphere somehow. Note however that 6-spheres are not dynamical.
2. One can also have Poincare transforms of 8-D CDs. Could the description of their interactions involve 4-D intersections of corresponding 6-spheres?
3. 6-spheres in $IM(P) = 0$ case do not have image under $M^8 - H$ correspondence. This does not seem to be possible for $RE(P) = 0$ either: it is not possible to map the 2-D normal space to a unique CP_2 point since there is 2-D continuum of quaternionic sub-spaces containing it.

4.4.3 $M^8 - H$ duality: objections and challenges

In the following I try to recall all objections against the reduction of classical physics to octonionic algebraic geometry and against the notion of $M^8 - H$ duality and also invent some new counter arguments and challenges.

Can on really assume distribution of $M^2(x)$?

Hamilton-Jacobi structure means that $M^2(x)$ depends on position and $M^2(x)$ should define an integrable distribution integrating to a 2-D surface. For cosmic string extremals this surface would be minimal surface so that the term "string world sheet" is appropriate. There are objections.

1. It seems that the coefficients of octonionic polynomials cannot contain information about string world sheet, and the only possible choice seems to be that string world sheets and partonic 2-surfaces parallel to it assigned with integrable distribution of orthogonal complements

$E^2(x)$ should be coded by quaternionic moduli. It should be possible to define quaternionic coordinates q_i decomposing to pairs of complex coordinates to each choice of $M^2(x) \times E^2(x)$ decomposition of given M_0^4. Octonionic coordinates would be given by $o = q_1 + q_2 I_4$ where q_i are associated with the same quaternionic moduli. The choice of Hamilton-Jacobi structure would not be ad hoc procedure anymore but part of the definition of solutions of field equations at the level of M^8.

2. It would be very nice if the quaternionic structure could be induced from a fixed structure defined for M_c^8 once the choice of curvilinear M^4 coordinates is made. Since Hamiltoni-Jacobi structure [K10] involves a choice of generalized Kähler form for M^4 and since quaternionic structure means that there is full S^2 of Kähler structures determined by quaternionic imaginary units (ordinary Kähler form for sub-space $E^8 \subset M_c^8$) the natural proposal is that Hamilton-Jacobi structures is determined by a particular local choice of the Kähler form for M^4 involving fixing of quaternionic imaginary unit fixing $M^2(x) \subset M_0^4$ identifiable as point of S^2. This might relate closely also to the fixing of twistor structure, which indeed involves also self-dual Kähler form and a similar choice.
3. One can argue that it is not completely clear whether massless extremals (MEs) [K10] allow a general Hamilton-Jacobi structure. It is certainly true that if the light-like direction and orthogonal polarization direction are constant, MEs exist. It is clear that if the form of field equations is preserved and thus reduces to contractions of various tensors with second fundamental form one obtains only contractions of light-like vector with itself or polarization vector and these contractions vanish. For spatially varying directions one could argue that light-like direction codes for a direction of light-like momentum and that problems with local conservation laws expressed by field equations might emerge.

Can one assign to the tangent plane of $X^4 \subset M^8$ a unique CP_2 point when M^2 depends on position

One should show that the choice $s(x) \in CP_2$ for a given distribution of $M^2(x) \subset M^4(x)$ is unique in order to realize the $M^8 - H$ correspondence as a map $M^8 \to H$. It would be even better if one had an analytic formula for $s(x)$ using tangent space-data for $X^4 \subset H$.

1. If $M^2(x) = M_0^2$ holds true but the tangent space $M^4(x)$ depends on position, the assignment of CP_2 point $s(x)$ to the tangent space of $X^4 \subset M^8$ is trivial. When $M^4(x)$ is not constant, the situation is not so easy.
2. The space $M^2(x) \subset M^4(x)$ satisfies also the constraint $M^2(x) \subset M_0^4$ since quaternionic moduli are fixed. To avoid confusion notice that $M^4(x)$ denotes tangent space of X^4 and is different from M_0^4 fixing the quaternionic moduli.
3. $M^2(x)$ determines the local complex subspace and its completion to quaternionic tangent space $M^4(x)$ determines a point $s(x)$ of CP_2. The idea is that M_0^2 defines a standard reference and that one should be able to map $M^2(x)$ to M_0^2 by G_2 automorphism mapping also the $s(x)$ to a unique point $s_0(x) \in CP_2$ defining the CP_2 point assignable to the point of $X^4 \subset M^8$.
4. One can assign to the point x quaternionic unit vector $n(x)$ determining $M^2(x)$ as the direction of the preferred imaginary unit. The G_2 transformation must rotate $n(x)$ to n_0 defining M_0^2 and acts on s. G_2 transformation is not unique since $u_1 g u_2$ has the same effect for $u_i \subset U(2)$ leaving invariant the point of CP_2 for initial and final situation. Hence the equivalence classes of transformations should correspond to a point of 6-dimensional double coset space $U(2)\backslash G_2/U(2)$. Intuitively it seems obvious that the $s_0(x)$ is unique but proof is required.

What about the inverse of $M^8 - H$ duality?

$M^8 - H$ duality should have inverse in the critical regions of $X^4 \subset M^8$, where associativity conditions are satisfied. How could one construct the inverse of $M^8 - H$ duality in these regions? One should map space-time points $(m, s) \in M^4 \times CP_2$ to points $(m, e) = (m, f(m, s)) \in M^8$. $M_0^4 \supset M_0^2$ parameterized by CP_2 point can be chosen arbitrarily and one can require that it corresponds to some space-time point $(m_0, s_0) \in H$. CP_2 point $s(x)$ characterizes the quaternionic tangent space containing $M^2(x)$ and can choose M_0^2 to be $M^2(x_0)$ for conveniently chosen x_0. Coordinates x can be used also for $X^4 \subset M^8$.

One obtains set of points $(m,e) = (m(x), f(m(x), s(x)) \in M^8$ and a distribution of 4-D spaces of labelled by $s(x)$. This requires that the 4-D tangent space spanned by the gradients of $m(x)$ and $f(m(x), s(x))$ and characterized by $s_1 \subset CP_2$ for given $M^2(x)$ by using the above procedure mapping the situation to that for M_0^2 is same as the tangent space determined by $s(x)$: $s(x) = s_1(x)$. Also the associativity conditions should hold true. One should have a formula for s_1 as function of tangent vectors of space-time surface in M^8. The ansatz based on algebraic geometry in M_c^8 should be equivalent with this ansatz. The problem is that the ansatz leads to algebraic functions which cannot be found explicitly. It might be that in practice the correspondence is easy only in the direction $M^8 \to H$.

What one can say about twistor lift of $M^8 - H$ duality?

One can argue that the twistor spaces CP_1 associated with M^4 and E^4 are in no way visible in the dynamics of octonion polynomials and in $M^8 - H$ duality. Hence one could argue that they are not needed for any reasonable purpose. I cannot decide whether this is indeed the case. There I will consider the existence of twistor lift of the M^8 and also the twistor lift $M^8 - H$ duality in the space-time regions, where the tangent spaces satisfy the conditions for the existence of the duality as a map $(m,e) \in M^8 \to (m,s) \in M^4 \times CP_2$ must be considered. This involves some non-trivial delicacies.

1. The twistor bundles of M_c^4 and E_c^4 would be simply $M_c^4 \times CP_1$ and $E_c^4 \times CP_1$. $CP_1 = S^2$ parameterizes direction vectors in 3-D Euclidian space having interpretation as unit quaternions so that this interpretation might make sense. The definition of twistor structure means a selection of a preferred quaternion unit and its representation as Kähler form so that these twistor bundles would have thus Kähler structure. Twistor lift replaces complex quaternionic surfaces with their twistor spaces with induced twistor structure.
2. In M^8 the radii of the spheres CP_1 associated with M^4 and E^4 would be most naturally identical whereas in $M^4 \times CP_2$ they can be different since CP_2 is moduli space. Is the value of the CP_2 radius visible at all in the classical dynamics in the critical associative/co-associative space-time regions, where one has minimal surfaces. Criticality would suggest that besides coupling constants also parameters with dimension of length should disappear from the field equations. At least for the known extremals such as massless extremals, CP_2 type extremals, and cosmic strings CP_2 radius plays no role in the equations. CP_2 radius comes however into play only in interaction regions defined by CDs since $M^8 - H$ duality works only at the 3-D ends of space-time surface and at the partonic orbits. Therefore the different radii for the CP_1 associated with CP_2 and E^4 cause no obvious problems.

Consider now the idea about twistor space as real part of octonionic twistor space regarded as quaternion-complex space.

1. One can regard $CP_1 = S^2$ as the space of unit quaternions and it is natural to replace it with the 6-sphere S^6 of octonionic imaginary units at the level of complexified octonions. The sphere of complexified (by i) unit octonions is non-compact space since the norm is complex valued and this generalization looks neither attractive nor necessary since the projection to real numbers would eliminate the complex part.

   The equations determining the twistor bundle of space-time surface can be indeed formulated as vanishing of the quaternionic imaginary part of S^6 coordinates, and one obtains a reduction to quaternionic sphere S^2 at space-time level.

   If S^2 is identified as sub-manifold $S^2 \subset S^6$, it can be chosen in very many ways (this is of course not necessary). The choices are parameterized by $SO(7)/SO(3) \times SO(4)$ having dimension $D = 12$. This choice has no physical content visible at the level of H. Note that the Kähler structure determining Hamilton-Jaboci structure is fixed by the choice of preferred direction ($M^2(x)$). If all these moduli are allowed, it seems that one has something resembling multiverse, the description at the level of M^8 is deeper one and one must ask whether the space-time surfaces with different twistorial, octonionic, and quaternionic moduli can interact.
2. The resulting octonionic analog of twistor space should be mapped by $M^8 - H$ corresponds to twistor space of space-time surface $T(M^4) \times T(CP_2)$. The radii of twistor spheres of $T(M^4)$ and $T(CP_2)$ are different and this should be also understood. It would seem that the radius

of $T(M^4)$ at $H = M^4 \times CP_2$ side should correspond to that of $T(M^4)$ at M^8 side and thus to that of S^6 as its geodesic sphere: Planck length is the natural proposal inspired by the physical interpretation of the twistor lift. The radius of $T(CP_2)$ twistor sphere should correspond to that of CP_2 and is about 2^{12} Planck lengths.

Therefore the scale of CP_2 would emerge as a scale of moduli space and does not seem to be present at the level of M^8 as a separate scale. M^8 level would correspond to what might be called Planckian realm analogous to that associated with strings before dynamical compactification which is now replaced with number theoretic compactification. The key question is what determines the ratio of the radii of CP_2 scale to Planck for which favored value is 2^{12} [K13]. Could quantum criticality determine this ratio?

4.5 Appendix: o^2 as a simple test case

Octonionic polynomial o^2 serves as a simple testing case. o^2 is not irreducible so that its properties might not be generic and it might be better to study polynomial of form $P(o) = o + po^2$ instead.

Before continuing, some conventions are needed.

1. The convention is that in $M^8 = M^1 \times E^7$ E^7 corresponds to purely imaginary complexified octonions in both octonionic sense and in the sense that they are proportional to i. M^1 corresponds to octonions real in both senses. This corresponds to the signature $(1, -1, -1, -1, ...)$ for M^8 metric obtained as restriction of complexified metric. For $M^4 = M^1 \times E^3$ analogous conventions hold true.
2. Conjugation $o = o_0 + o_k I_k \rightarrow \overline{o} \equiv o_0 - I_k o_k$ does not change the sign of i. Quaternions can be decomposed to real and imaginary parts and some notation is needed. The notation $q = Re(q) + Im(q)$ seems to be the least clumsy one: here $Im(q)$ is 3-vector.

The explicit expression in terms of quaternionic decomposition $o = q_1 + q_2 I_4$ reads as

$$P(o) = o^2 = q_1^2 - q_2\overline{q}_2 + (q_1 q_2 + q_2\overline{q}_1)I_4 \quad . \tag{4.5.1}$$

o corresponds to complexified octonion and there are two options concerning the interpretation of M^4 and E^4. M^4 could correspond to quaternionic or co-quaternionic sub-space. I have assumed the first interpretation hitherto but actually the identification is not obvious. This two cases are different and must be treated both.

With these notations quaternionic inner product reads as

$$\begin{array}{l} q_1 q_2 = Re(q_1 q_2) + Im(q_1 q_2) \ , \\ Re(q_1 q_2) = Re(q_1)Re(q_2) - Im(q_1) \cdot Im(q_2) \ , \\ Im(q_1 q_2) = Re(q_1)Im(q_2) + Re(q_2)Im(q_1) + Im(q_1) \times Im(q_2) \ . \end{array} \tag{4.5.2}$$

Here $a \cdot b$ denotes the inner product of 3-vectors and $a \times b$ their cross product.

Note that one has real and imaginary parts of octonions as two quaternions and real and imaginary parts of quaternions. To avoid confusion, I will use RE and IM to denote the decomposition of octonions to quaterions and Re and Im for the decomposition of quaternions to real and imaginary parts.

One can express the $RE(o^2)$ as

$$\begin{array}{l} RE(o^2) \equiv X \equiv q_1^2 - q_2\overline{q}_2 \ , \\ Re(X) = Re(q_1)^2 - Im(q_1) \cdot Im(q_2) - (Re(q_2)^2 + Im(q_2) \cdot Im(q_2)) \ , \\ Im(X) = Im(q_1^2) = 2Re(q_1)Im(q_1) \ . \end{array} \tag{4.5.3}$$

For $IM(o^2)$ one has

$$IM(o^2) \equiv Y = q_1 q_2 + q_2\overline{q}_1$$
$$Re(Y) = 2Re(q_1)Re(q_2) \ ,$$
$$Im(Y) = Re(q_1)Im(q_2) - Re(q_2)Im(q_1) + Im(q_1) \times Im(q_2) \ . \tag{4.5.4}$$

The essential point is that only $RE(o^2)$ contains the complexified Euclidian norm $q_2\overline{q_2}$ which becomes Minkowskian of Euclidian norm depending on whether one identifies M^4 as associative or co-associative surface in o_c^8.

4.5.1 Option I: M^4 is quaternionic

Consider first the condition $RE(o^2) = 0$. The condition decomposes to two conditions stating the vanishing of quaternionic real and imaginary parts:

$$Re(X) = Re(q_1)^2 - Im(q_1) \cdot Im(q_2) - (Re(q_2)^2 + Im(q_2) \cdot Im(q_2)) \equiv N_{M^4}(q_1) - N_{E^4}(q_2) = 0 \ ,$$
$$Im(X) = Im(q_1^2) = 2Re(q_1)Im(q_1) = 0 \ . \tag{4.5.5}$$

$Im(X) = 0$ is satisfied for $Re(q_1) = 0$ or $Im(q_1) = 0$ so that one has two options. This gives 1-D line in time direction of 3-D hyperplane as a solution for M^4 factor.

$Re(X) = 0$ states $N_{M^4}(q_1) = N_{E^4}(q_2)$. q_2 coordinate itself is free. $N_{E^4}(q_2)$ is negative so that q_1 must be space-like with respect to the N_{M^4} so that only the solution $Re(q_1) = 0$ is possible. Therefore one has $Re(q_1) = 0$ and $N_{M^4}(q_1) = N_{E^4}(q_2)$.

One can assign to each E^4 point a section of hyperboloid with $t = 0$ hyper-plane giving a sphere and the surface is 6-dimensional sphere bundle like variety! This is completely unexpected result and presumably is due to the additional accidental symmetries due to the octonionicity. Also the fact that o^2 is not irreducible polynomial is a probably reason since for o the surface is 4-D. The addition of linear term is expected to remove the degeneracy.

Consider next the case $IM(o^2) = 0$. The conditions read now as

$$Re(Y) = 2Re(q_1)Re(q_2) = 0 \ ,$$
$$Im(Y) = Re(q_1)Im(q_2) - Re(q_2)Im(q_1) + Im(q_1) \times Im(q_2) = 0 \ . \tag{4.5.6}$$

Since cross product is orthogonal to the factors $Im(Y) = 0$ condition requires that $Im(q_1)$ and $Im(q_2)$ are parallel vectors: $Im(q_1) = \lambda Im(q_2)$ and one has the condition $Re(q_1) = \lambda Re(q_2)$ implying $q_1 = \Lambda q_2$. Therefore to each point of E^4 is associated a line of M^4. The surface is 5-dimensional.

It is interesting to look what the situation is if both conditions are true so that one would have a singularity. In this case $Re(q_1) = 0$ and $Re(q_1) = \lambda Re(q_2)$ imply $\lambda = 0$ so that $q_1 = 0$ is obtained and the solution reduces to 4-D E^4, which would be co-associative.

4.5.2 Option II: M^4 is co-quaternionic

This case is obtained by the inspection of the previous calculation by looking what changes the identification of M^4 as co-quaternionic factor means. Now q_1 is Euclidian and q_2 Minkowskian coordinate and $q_2\overline{q}_2$ gives Minkowskian rather than Euclidian norm.

Consider first $RE(o^2) = 0$ case.

$$Re(X) = Re(q_1)^2 - Im(q_1) \cdot Im(q_2) - (Re(q_2)^2 + Im(q_2) \cdot Im(q_2)) \equiv N_{M^4}(q_1) - N_{M^4}(q_2) = 0 \ ,$$
$$Im(X) = Im(q_1^2) = 2Re(q_1)Im(q_1) = 0 \ . \tag{4.5.7}$$

$N_{M^4}(q_1) - N_{M^4}(q_2) = 0$ condition holds true now besides the condition $Re(q_1) = 0$ or $Im(q_1) = 0$ so that one has also now two options.

1. For $Re(q_1) = 0$ $N_{M^4}(q_1)$ is non-positive and this must be the case for $N_{M^4}(q_2)$) so that the *exterior* of the light-cone is selected. In this case the points of M^4 with fixed N_{M^4} give rise to a 2-D intersection with $Re(q_1) = 0$ hyper-plane that is sphere so that one has 6-D surface, kind of sphere bundle.
2. For $Im(q_1) = 0$ Minkowski norm is positive and so must be corresponding norm in E^4 so that in E^4 surface has future ligt-cone as projection. This surface is 4-D. The emergence of future light-cone might provide justification for the emergence of CDs and zero energy ontology.

For $IM(o^2)$ the discussion is same as in quaternionic case since norm does not appear in the equations.

At singularity both $RE(o^2)$ and $IM(o^2) = 0$ vanish. The condition $q_1 = \Lambda q_2$ reduces to $\Lambda = 0$ so that $q_1 = 0$ is only allowed. This leaves only light-cone boundary under consideration.

The appearance of surfaces with dimension higher than 4 raises the question whether something is wrong. One could of course argue that associativity allows also lower than 4-D surfaces as associative surfaces and higher than 4-D surfaces as co-associative surfaces. At H-level one can say that one has 4-D surfaces. A good guess is that this behavior disappears when the linear term is absent and origin ceases to be a singularity.

Chapter 5

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part II

5.1 Introduction

There are good reasons to hope that TGD is integrable theory in some sense. Classical physics is an exact part of quantum physics in TGD and during years I have ended up with several proposals for the general solution of classical field equations (classical TGD is an exact part of quantum TGD).

5.1.1 Could one identify space-time surfaces as zero loci for octonionic polynomials with real coefficients?

The identification of space-time surfaces as zero loci of real or imaginary part of octonionic polynomial has several extremely nice features.

1. Octonionic polynomial is an algebraic continuation of a real valued polynomial on real line so that the situation is effectively 1-dimensional! Once the degree of polynomial is known, the value of polynomial at finite number of points are needed to determine it and cognitive representation could give this information! This would strengthen the view strong form of holography (SH) - this conforms with the fact that states in conformal field theory are determined by 1-D data.

   Remark: Why not rational functions expressible as ratios $R = P_1/P_2$ of octonionic polynomials? It has become clear that one can develop physical arguments in favor of this option. The zero loci for $IM(P_i)$ would represent space-time varieties. Zero loci for $RE(P_1/P_2) = 0$ and $RE(P_1/P_2) = \infty$ would represent their interaction presumably realized as wormhole contacts connecting these varieties. In the sequel most considerations are for polynomials: the replacement of polynomials with rational functions does not introduce big differences and its discussed in the section "Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view".

2. One can add, sum, multiply, and functionally compose these polynomials provided they correspond to the same quaternionic moduli labelled by CP_2 points and share same time-line containing the origin of quaternionic and octonionic coordinates and real octonions (or actually their complexification by commuting imaginary unit). Classical space-time surfaces - classical worlds - would form an associative and commutative algebra. This algebra induces an analog of group algebra since these operations can be lifted to the level of functions defined in this algebra. These functions form a basic building brick of WCW spinor fields defining quantum states.

3. One can interpret the products of polynomials as correlates for free many-particle states with interactions described by added interaction polynomial, which can vanish at boundaries

of CDs. This leads to the same picture as the view about preferred extremals reducing to minimal surfaces near boundaries of CD [L28]. Also zero zero energy ontology (ZEO) could be forced by the failure of number field property for quaternions at light-cone boundaries. It indeed turns out that light-cone boundary emerges quite generally as singular zero locus of polynomials $P(o)$ containing no linear part: this is essentially due to the non-commutativity of the octonionic units. Also the emergence of CDs can be understood. At this surface the region with $RE(P) = 0$ can transform to $IM(P) = 0$ region. In Euclidian signature this singularity corresponds to single point. A natural conjecture is that also the light-like orbits of partonic 2-surfaces correspond to this kind of singularities for non-trivial Hamilton-Jacobi structures.

4. The reduction to algebraic geometry would mean enormous boost to the vision about cognition with cognitive representations identified as generalized rational points common to reals rationals and various p-adic number fields defining the adele for given extension of rationals. Hamilton-Jacobi structure would result automatically from the decomposition of quaternions to real and imaginary parts which would be now complex numbers.

5. Also a connection with infinite primes is suggestive [K96]. The light-like partonic orbits, partonic 2-surfaces at their ends, and points at the corners of string world sheets might be interpreted in terms of singularities of varying rank and the analog of catastrophe theory emerges.

The great challenge is to prove rigorously that these approaches - or at least some of them - are indeed equivalent. Also it remains to be proven that the zero loci of real/imaginary parts of octonionic polynomials with real coefficients are associative or co-associative. I shall restrict the considerations of this article mostly to $M^8 - H$ duality. The strategy is simple: try to remember all previous objections against $M^8 - H$ duality and invent new ones since this is the best way to make real progress.

5.1.2 Topics to be discussed

Challenges of the octonionic algebraic geometry

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to octonionic algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces. The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients determine associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

   This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy Riemann conditions are linear. Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

   In fact, all algebras obtained by Cayley-Dickson construction (see `http://tinyurl.com/ybuyla2k`) by adding imaginary unit repeatedly to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and a $M^8 - H$ correspondence could generalize (maybe even TGD!).

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k, $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i. In the generic case this gives 3-D surface.

   In this generic case $M^8 - H$ duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H, and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

   One can also have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i. Space-time surface would be analogous to a polynomial with a multiple root.

   Various components of octonion polynomial P of degree n are polynomials of same degree. Could criticality reduces to the degeneracy of roots for some component polynomials? Could P as a polynomial of real variable have degenerate roots?

   The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory [A47] emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

   One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces except at 3-D boundary surfaces. These regions correspond to interaction regions in which Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes also to the level of complex/co-complex surfaces associated with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces? I have proposed commutativity or co-commutatitivity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commutative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

4. The super variant of the octonionic geometry relying on octonionic triality makes sense and the geometry of the space-time variety correlates with fermion and antifermion numbers assigned with it. This new view about super-geometry involving also automatic SUSY breaking at the level of space-time geometry.

Description of interactions

Also a sketchy proposal for the description of interactions is discussed.

1. $IM(P_1P_2) = 0$ is satisfied for $IM(P_1) = 0$ and $IM(P_2) = 0$ since $IM(o_1o_2)$ is linear in $IM(o_i)$ and one obtains union of space-time varieties. $RE(P_1P_2) = 0$ cannot be satisfied in this way since $RE(o_1o_2)$ is not linear in $RE(o_i)$ so that the two varieties interact and this interaction could give rise to a wormhole contact connecting the two space-time varieties.

2. The surprise that $RE(P) = 0$ and $IM(P) = 0$ conditions have as singular solutions light-cone interior and its complement and 6-spheres $S^6(t_n)$ with radii t_n given by the roots of the real $P(t)$, whose octonionic extension defines the space-time variety X^4. The intersections $X^2 = X^4 \cap S^6(t_n)$ are tentatively identified as partonic 2-varieties defining topological interaction vertices. S^6 and therefore also X^2 are doubly critical, S^6 is also singular surface.

   The idea about the reduction of zero energy states to discrete cognitive representations suggests that interaction vertices at partonic varieties X^2 are associated with the discrete set of intersection points of the sparticle lines at light-like orbits of partonic 2-surfaces belonging to extension of rationals.

3. CDs and therefore also ZEO emerge naturally. For CDs with different origins the products of polynomials fail to commute and associate unless the CDs have tips along real (time) axis. The first option is that all CDs under observation satisfy this condition. Second option allows general CDs.

   The proposal is that the product $\prod P_i$ of polynomials associated with CDs with tips along real axis the condition $IM(\prod P_i) = 0$ reduces to $IM(P_i) = 0$ and criticality conditions guaranteeing associativity and provides a description of the external particles. Inside these CDs $RE(\prod P_i) = 0$ does not reduce to $RE(\prod P_i) = 0$, which automatically gives rise to geometric interactions. For general CDs the situation is more complex.

4. The possibility of super-octonionic geometry raises the hope that the twistorial construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY generalizes to TGD in rather straightforward way to a purely geometric construction. Functional integral over WCW would reduce to summations over polynomials with coefficients in extension of rationals and criticality conditions on the coefficients could make the summation well-defined by bringing in finite measurement resolution.

   If scattering diagrams are associated with discrete cognitive representations, one obtains a generalization of twistor formalism involving polygons. Super-octonions as counterparts of super gauge potentials are well-defined if octonionic 8-momenta are quaternionic. Indeed, Grassmannians have quaternionic counterparts but not octonionic ones. There are good hopes that the twistor Grassmann approach to $\mathcal{N} = 4$ SUSY generalizes. The core part in the calculation of the scattering diagram would reduce to the construction of octonionic 4-varieties and identifying the points belonging to the appropriate extension of rationals.

   Twistor Grassmannian construction of scattering amplitudes at the level of M^8 looks feasible. The amplitudes decompose to M^4 and CP_2 parts with similar structure with E^4 spin (electroweak isospin) replacing ordinary spin. The residue integrals over Grassmannians emerging from the conservation of M^4 and E^4 4-momenta would have same form and guarantee Yangian supersymmetry in both sectors. The counterpart for the product of delta functions associated with the "negative helicities" (weak isospins with negative sign) would be expressible as a delta function in the complement of $SU(3)$ Cartan algebra $U(1)\times U(1)$ by using exponential map.

About the analogs of Gromow-Witten invariants and branes in TGD

Gromov-Witten (G-W) invariants belong to the realm of quantum enumerative geometry briefly discussed in [L47]. They count numbers of points in the intersection of varieties ("branes") with quantum intersection identified as the existence of "string world sheet(s)" intersecting the branes. Also octonionic geometry gives rise to brane like objects. G-W invariants are rational numbers but it is proposed that they could be integers in TGD framework.

Riemann-Roch theorem (RR) and its generalization Atyiah-Singer index theorem (AS) relate dimensions of various kinds of moduli spaces to topological invariants. The possible generalizations of RR and AS to octonionic framework and the implications of $M^8 - H$ duality for the possible generalizations are discussed. The adelic hierarchy of extensions of rationals and criticality conditions make the moduli spaces discrete so that one expects kind of particle in box type quantization selecting discrete points of moduli spaces about the dimension.

The discussion of RR as also the notion of infinite primes and infinite rationals as counterparts of zero energy states suggests that rational functions $R = P_1/P_2$ could be more appropriate than mere polynomials. The construction of space-time varieties would not be modified in essential way: one would have zero loci of $IM(P_i)$ identifiable as space-time sheets and zero- and ∞-loci of $RE(P_1/P_2)$ naturally identifiable as wormhole contacts connecting the space-time sheets.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); Strong Form of GCI (SGCI); Quantum Criticality (QC); Strong Form of Holography (SH); World of Classical Worlds (WCW); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Causal Diamond (CD); Number Theoretical Universality (NTU) are the most often occurring acronyms.

5.2 Some challenges of octonionic algebraic geometry

Space-time surfaces in $H = M^4 \times CP_2$ identified as preferred extremals of twistor lift of Kähler action leads to rather detailed view about space-time surfaces as counterparts of particles. Does this picture follow from $X^4 \subset M^8$ picture and does this description bring in something genuinely new?

5.2.1 Could free many-particle states as zero loci for real or imaginary parts for products of octonionic polynomials

In algebraic geometry zeros for the products of polynomials give rise to disjoint varieties, which are disjoint unions of surfaces assignable to the individual surfaces and possibly having lower-dimensional intersections. For instance, for complex curves these intersections consist of points. For complex surfaces they are complex curves.

In the case of octonionic polynomial $P = RE(P) + IM(P)I_4$ (Re and Im are defined in quaternionic sense) one considers zeros of quaternionic polynomial $RE(P)$ or $IM(P)$.

1. Product polynomial $P = P_1P_2$ decomposes to

$$P = RE(P_1)RE(P_2) - IM(P_1)IM(P_2) + (RE(P_1)IM(P_1) + IM(P_1)RE(P_2)I_4 \ .$$

   One can require vanishing of $RE(P)$ or $IM(P)$.

   (a) $IM(P)$ vanishes for

$$(RE(P_1) = 0, RE(P_2) = 0)$$

   or

$$I(m(P_1) = 0, IM(P_2) = 0) \ .$$

   (b) $RE(P)$ vanishes for

$$(RE(P_1) = 0, IM(P_2) = 0)$$

   or

$$IM(P_1) = 0, RE(P_2) = 0) \ .$$

   One could reduce the condition $RE(P) = 0$ to $IM(P) = 0$ by replacing $P = P_1 + P_2I_4$ with $P_2 - P_1I_4$. If this condition is satisfied for the factors, it is satisfied also for the product. The set of surfaces is a commutative and associative algebra for the condition $IM(P) = 0$. Note that the quaternionic moduli must be same for the members of product. If one has quantum superposition of quaternionic moduli, the many-particle state involves a superposition of products with same moduli.

   As found, the condition $IM(P) = 0$ can transform to $RE(P) = 0$ at singularities having $RE(P) = 0, IM(P) = 0$.

2. The commutativity of the product means that the products are analogous to many-boson states. P^n would define an algebraic analog of Bose-Einstein condensate. Does this surface correspond to a state consisting of n identical particles or is this artefact of representation? As a limiting case of product of different polynomials it might have interpretation as genuine n-boson states.
3. The product of two polynomials defines a union of disjoint surfaces having discrete intersection in Euclidian signature. In Minkowskian signature the vanishing of $q\overline{q}$ (conjugation does not affect the sign of i and changes only the sign of I_k!) can give rise to 3-D light-cone. The non-commutativity of quaternions indeed can give rise to combinations of type $q\overline{q}$ in $RE(P)$ and $IM(P)$.

What about interactions?

1. Could one introduce interaction by simply adding a polynomial P_{int} to the product? This polynomial should be small outside interaction region. CD would would define naturally interaction regions and the interaction terms should vanish at the boundaries of CD. This might be possible in Minkowskian signature, where $f(q^2)$ multiplying the interaction term might vanish at the boundary of CD: in Euclidian sector $q\overline{q} = 0$ would imply $q = 0$ but in Minkowskian sector it would give light-cone as solution. One should arrange $IM(P_{int})$ to be proportional to $q\overline{q}$ vanishing at the boundary of CD. Minkowskian signature could be crucial for the possibility to "turning interactions on".
2. If the imaginary part of the interaction term is proportional $f_1(q^2)f_2((q-T)^2)$ (T is real and corresponds to the temporal distance between the tips of CD) with $f_i(0) = 0$, one could obtain asymptotic states reducing to disjoint unions of zero loci of P^i at the boundaries of CD. If the order of of the perturbation terms is higher than the total order of polynomials P^i, one would obtain new roots and particle emission. Non-perturbative situation would correspond to a dramatic modification of the space-time surface as a zero locus of $IM(P)$. This picture would be M^8 counterpart for the reduction of preferred extremals to minimal surfaces analogous to geodesic lines near the boundaries of CD: preferred extremals reduce to extremals of both Kähler action and volume term in these regions [L28].

The singularities of scattering amplitudes at algebraic varieties of Grassmann manifolds are central in the twistor Grassmann program [B21, B43, B26]. Since twistor lift of TGD seems to be the correct manner to formulate classical TGD in H, one can wonder about the connection between space-time surfaces in M^8_c and scattering amplitudes. Witten's formulation of twistor amplitudes in terms of algebraic curves in CP_3 suggests a formulation of scattering amplitudes in terms of the 4-D algebraic varieties in M^8_c as of course, also TGD itself [L30, K87]! Could the huge multi-local Yangian symmetries of twistor Grassmann amplitudes reduce to octonion analyticity.

5.2.2 Two alternative interpretations for the restriction to M^4 subspace of M^8_c

One must complexify M^8 so that one has complexified octonions M^8_c. This means the addition of imaginary unit i commuting with octonionic imaginary units. The vanishing of real or imaginary part of octonionic polynomial in quaternionic sense ($o = q_1 + Jq_2$) defines the space-time surface. Octonionic polynomial itself is obtained from a real polynomial by algebraic continuation so that in information theoretic sense space-time is 1-D. The roots of this real polynomial fix the polynomial and therefore also space-time surface uniquely. 1-D line degenerates to a discrete set of points of an extension in information theoretic sense. In p-adic case one can allow p-adic pseudo constants and this gives a model for imagination.

The octonionic roots $x + iy$ of the real polynomial need not however be real. There are two options.

1. The original proposal in [L46, L48] was that the *projection* from M^8_c to real M^4 (for which M^1 coordinate is real and E^3 coordinates are imaginary with respect to i!) defines the real space-time surface mappable by $M^8 - H$ duality to CP_2.
2. An alternative option is that only the roots of the 4 vanishing polynomials as coordinates of M^4_c belong to M^4 so that m^0 would be real root and m^k, $k = 1, ..., 3$ imaginary with

respect to $i \rightarrow -i$. M^8_c coordinates would be invariant ("real") under combined conjugation $i \rightarrow -i, I_k \rightarrow -I_k$. In the following I will speak about this property as *Minkowskian reality.* This could make sense.

What is remarkable that this could allow to identify CDs in very elegant manner: outside CD these 4 conditions would not hold true. This option looks more attractive than the first one. Why these conditions can be true just inside CD, should be understood.

Consider now this in detail.

1. One can think of starting from one of the 4 vanishing conditions for the components of octonionic polynomial guaranteeing associativity. Assuming real roots and continuing one by one through all 4 conditions to obtain 4-D Minkowskian real regions. The time coordinate of M^4 coordinates is real and others purely imaginary with respect to $i \rightarrow -i$. If this region does not connect 3-D surface at the boundaries of real CD, one must make a new trial.

   Cusp catastrophe determined as the zero locus of third order polynomial provides an example. There are regions with single real root, regions with two real roots (complex roots become real and identical) defining V-shaped boundary of cusp and regions with 3 real roots (the interior of the cusp).

2. The restriction of the octonionic polynomial to time axis m^0 identifiable as octonionic real axes is a real polynomial with algebraic coefficients. In this case the root and its conjugate with respect to i would define the same surface. One could say that the Galois group of the real polynomial characterizes the space-time surface although at points other than those at real axis (time axis) the Galois group can be different.

   One could consider the local Galois group of the fourth quaternionic valued polynomial, say the part of quaternionic polynomial corresponding to real unit 1 when other components are required to vanish and give rise to coordinates in $M^8 \subset M^8_c$ - Minkowskian reality. The extension and its Galois group would depend on the point of space-time surface.

   An interesting question is how strong conditions Minkowskian reality poses on the extension. Minkowskian reality seems to imply that E^3 roots are purely real so that for an octonionic polynomial obtained as a continuation of a *real* polynomial one expects that both root and complex conjugate should be allow and that Galois group should contain Z_2 reflection $i \rightarrow -i$. Space-time surface would be at least 2-sheeted. Also the model for elementary particles forces this conclusion on physical grounds. Real as opposite to imagined would mean Minkowskian reality in mathematical sense. In the case of polynomials this description would make sense in p-adic case by allowing the coefficients of the polynomial be pseudo constants.

3. What data one could use to fix the space-time surface? Can one start directly from the real polynomial and regard its coefficients as WCW coordinates? This would be easy and elegant. Space-time surface could be determined as Minkowskian real roots of the octonionic polynomial. The condition that the space-time surface has ends at boundaries of given CD and the roots are not Minkowskian real outside it would pose conditions on the polynomial. If the coefficients of the polynomial are p-adic pseudo constants, this condition might be easy to satisfy.

The situation depends also on the coordinates used. For linear coordinates such as Minkowski coordinates Minkowskian reality looks natural. One can however consider also angle like coordinates representable only in terms of complex phases p-adically and coming as roots of unity and requiring complex extension: at H-side they are very natural. For instance, for CP_2 all coordinates would be naturally represented in this manner. For future light-cone one would have hyperbolic angle and 2 ordinary angles plus light-cone proper time which would be real and positive coordinate.

This picture conforms with the proposed picture. The point is that the time coordinate m^k can be real in the sense that they are linear combinations of complex roots, say powers for the roots of unity. $E^4_c \subset M^8_c$ could be complex and contain also complex roots since $M^8 - H$ duality does not depend on whether tangent space is complex or not. Therefore would could have complex extensions.

5.2.3 Questions related to ZEO and CDs

Octonionic polynomials provide a promising approach to the understanding of ZEO and CDs. Light-like boundary of CD as also light-cone emerge naturally as zeros of octonionic polynomials. This does not yet give CDs and ZEO: one should have intersection of future and past directed light-cones. The intuitive picture is that one has a hierarchy of CDs and that also the space-time surfaces inside different CDs an interact.

Some general observations about CDs

It is good to list some basic features of CDS, which appear as both 4-D and 8-D variants.

1. There are both 4-D and 8-D CDs defined as intersections of future and past directed light-cones with tips at say origin 0 at real point T at quaternionic or octonionic time axis. CDs can be contained inside each other. CDs form a fractal hierarchy with CDs within CDs: one can add smaller CDs with given CD in all possible ways and repeat the process for the sub-CDs. One can also allow overlapping CDs and one can ask whether CDs define the analog of covering of O so that one would have something analogous to a manifold.
2. The boundaries of two CDs (both 4-D and 8-D) can intersect along light-like ray. For 4-D CD the image of this ray in H is light-like ray in M^4 at boundary of CD. For 8-D CD the image is in general curved line and the question is whether the light-like curves representing fermion orbits at the orbits of partonic 2-surfaces could be images of these lines.
3. The 3-surfaces at the boundaries of the two 4-D CDs are expected to have a discrete intersection since 4 + 4 conditions must be satisfied (say $RE(P_i^k)) = 0$ for $i = 1,2$, $k = 1,4$. Along line octonionic coordinate reduces effectively to real coordinate since one has $E^2 = E$ for $E = (1+in)/2$, n octonionic unit. The origins of CDs are shifted by a light-like vector kE so that the light-like coordinates differ by a shift: $t_2 = t_1 - k$. Therefore one has common zero for real polynomials $RE(P_1^k(t))$ and $RE(P_2^k(t-k))$.

   Are these intersection points somehow special physically? Could they correspond to the ends of fermionic lines? Could it happen that the intersection is 1-D in some special cases? The example of o^2 suggest that this might be the case. Does 1-D intersection of 3-surfaces at boundaries of 8-D CDs make possible interaction between space-time surfaces assignable to separate CDs as suggested by the proposed TGD based twistorial construction of scattering amplitudes?
4. Both tips of CD define naturally an origin of quaternionic coordinates for $D = 4$ and the origin of octonionic coordinates for $D = 8$. Real analyticity requires that the octonionic polynomials have real coefficients. This forces the origin of octonionic coordinates to be along the real line (time axis) connecting the tips of CD. Only the translations in this specified direction are symmetries preserving the commutativity and associativity of the polynomial algebra.
5. One expects that also Lorentz boosts of 4-D CDs are relevant. Lorentz boosts leave second boundary of CD invariant and Lorentz transforms the other one. Same applies to 8-D CDs. Lorentz boosts define non-equivalent octonionic and quaternionic structures and it seems that one assume moduli spaces of them.

One can of course ask whether the still somewhat ad hoc notion of CD general enough. Should one generalize it to the analog of the polygonal diagram with light-like geodesic lines as its edges appearing in the twistor Grassmannian approach to scattering diagrams? Octonionic approach gives naturally the light-like boundaries assignable to CDs but leaves open the question whether more complex structures with light-like boundaries are possible. How do the space-time surfaces associated with different quaternionic structures of M^8 and with different positions of tips of CD interact?

The emergence of causal diamonds (CDs)

CDs are a key notion of zero energy ontology (ZEO). They should emerge from the number-theoretic dynamics somehow. How? In the following this question is approached from two different directions.

1. One can ask whether the emergence of CDs could be understood in terms of singularities of octonion polynomials located at the light-like boundaries of CDs. In Minkowskian case the complex norm $q\overline{q}_i$ is present in P. Could this allow to blow up the singular point to a 3-D boundary of light-cone and allow to understand the emergence of causal diamonds (CDs) crucial in ZEO. This question will be considered below.
2. These arguments were developed before the realization that the Minkowskian reality condition discussed in the previous section is natural for the space-time surfaces as roots of the 4 polynomials defining real or imaginary part of octonionic polynomial in quaternionic sense and giving M^4 point as a solution. Minkowskian reality can hold only in some regions of M^4 and an attractive conjecture is that it fails outside CD. CD would be a prediction of number theoretical dynamics and have counterpart also at the level of H.

Consider now the second approach in more detail. The study of the special properties for zero loci of general polynomial $P(o)$ at light-rays of O indeed demonstrated that both 8-D land 4-D light-cones and their complements emerge naturally, and that the M^4 projections of these light-cones and even of their boundaries are 4-D future - or past directed light-cones. What one should understand is how CDs as their intersections, and therefore ZEO, emerge.

1. One manner to obtain CDs naturally is that the polynomials are sums $P(t) = \sum_k P_k(o)$ of products of form $P_k(o) = P_{1,k}(o)P_{2,k}(o - T)$, where T is real octonion defining the time coordinate. Single product of this kind gives two disjoint 4-varieties inside future and past directed light-cones $M^4_+(0)$ and $M^4_-(T)$ for either $RE(P) = 0$ (or $IM(P) = 0$) condition. The complements of these cones correspond to $IM(P) = 0$ (or $RE(P) = 0$) condition.
2. If one has nontrivial sum over the products, one obtains a connected 4-variety due the interaction terms. One has also as special solutions $M^4_\pm$ and the 6-spheres associated with the zeros $P(t)$ or equivalently $P_1(t_1) \equiv P(t)$, $t_1 = T - t$ vanishing at the upper tip of CD. The causal diamond $M^4_+(0) \cap M^4_-(T)$ belongs to the intersection.

   Remark: Also the union $M^4_-(0) \cup M^4_+(T)$ past and future directed light-cones belongs to the intersection but the latter is not considered in the proposed physical interpretation.
3. The time values defined by the roots t_n of $P(t)$ define a sequence of 6-spheres intersecting 4-D CD along 3-balls at times t_n. These time slices of CD must be physically somehow special. Space-time variety intersects 6-spheres along 2-varieties X^2_n at times t_n. The varieties X^2_n are perhaps identifiable as 2-D interaction vertices, pre-images of corresponding vertices in H at which the light-like orbits of partonic 2-surfaces arriving from the opposite boundaries of CD meet.

   The expectation is that in H one as generalized Feynman diagram with interaction vertices at times t_n. The higher the evolutionary level in algebraic sense is, the higher the degree of the polynomial $P(t)$, the number of t_n, and more complex the algebraic numbers t_n. $P(t)$ would be coded by the values of interaction times t_n. If their number is measurable, it would provide important information about the extension of rationals defining the evolutionary level. One can also hope of measuring t_n with some accuracy! Octonionic dynamics would solve the roots of a polynomial! This would give a direct connection with adelic physics [L52] [L53].

   Remark: Could corresponding construction for higher algebras obtained by Cayley-Dickson construction solve the "roots" of polynomials with larger number of variables? Or could Cartesian product of octonionic spaces perhaps needed to describe interactions of CDs with arbitrary positions of tips lead to this?
4. Above I have considered only the interiors of light-cones. Also their complements are possible. The natural possibility is that varieties with $RE(P) = 0$ and $IM(P) = 0$ are glued at the boundary of CD, where $RE(P) = IM(P) = 0$ is satisfied. The complement should contain the external (free) particles, and the natural expectation is that in this region the associativity/co-associativity conditions can be satisfied.
5. The 4-varieties representing external particles would be glued at boundaries of CD to the interacting non-associative solution in the complement of CD. The interaction terms should be non-vanishing only inside CD so that in the exterior one would have just product $P(o) = P_{1,k_0}(o)P_{2,k_0}(o-T)$ giving rise to a disjoint union of associative varieties representing external particles. In the interior one could have interaction terms proportional to say $t^2(T - t)^2$

vanishing at the boundaries of CD in accordance with the idea that the interactions are switched one slowly. These terms would spoil the associativity.

Remark: One can also consider sums of the products $\prod_k P_k(o - T_k)$ of n polynomials and this gives a sequence CDs intersecting at their tips. It seems that something else is required to make the picture physical.

5.2.4 About singularities of octonionic algebraic varieties

In Minkowskian signature the notion of singularity for octonionic polynomials involves new aspects as the study of o^2 singular at origin shows (see Appendix). The region in which $RE(o^2) = 0, IM(o^2) = 0$ holds true is 4-D rather than a discrete set of points as one would naïvely expect.

1. At singularity the local dimension of the algebraic variety is reduced. For instance, double cone of 3-space has origin as singular point where it becomes 0-dimensional. A more general example is local pinch in which cylinder becomes infinitely thin at some point. This kind of pinching could occur for fibrations as the fiber contracts to a lower-dimensional space along a sub-variety of the base space.

   A very simple analogy for this kind of singularity is the singularity of $P(x, y) = y^2 - x = 0$ at origin: now the sheets $y = \pm\sqrt{x}$ co-incide at origin. The algebraic functions $y \mp \sqrt{x}$ defining the factorization of $P(x, y)$ co-incide at origin. Quite generally, two or more factors in the factorization of polynomial using algebraic functions co-incide at the singularity. This is completely analogous to the degeneracy or roots of polynomials of single variable.

   The signature of the singularity of algebraic variety determined by the conditions $P^i(z^j) = 0$ is the reduction of the maximal rank r for the matrix formed by the partial derivatives $P^i_j \equiv \partial IM(P)^i/\partial z^j$ ("RE" could replace "IM"). Rank corresponds to the largest dimension of the minor of P^i_j with non-vanishing determinant. Determinant vanishes when two rows of the minor are proportional to each other meaning that two tangent vectors become linearly dependent. When the rank is reduced by Δr, one has $r = r_{max} - \Delta r$ and the local dimension is locally reduced by Δr. One has hierarchy of singularities within singularities.

   The conditions that all independent minors of the P^i_j have reduced rank gives additional constraints and define a sub-variety of the algebraic variety. Note that the dimension of the singularity corresponds to $d_s = \Delta r$ in the sense that the dimension of tangent space at singularity is effectively d_s.

2. In the recent case there are 4 polynomials and 4 complex variables so that $IM(P)^i_j$ is 4×4-matrix. Its rank r can have values in $r = 1, 2, 3, 2, 4$. One can use Thom's catastrophe theory as a guideline. Catastrophe decomposes to pieces of various dimensions characterized by the reduction of the rank of the matrix defined by the second derivatives $V_{ij} = \partial_i \partial_j V$ of the potential function defining the catastrophe. For instance, for cusp catastrophe with $V(x, a, b) = x^4 + ax^2 + bx$ one has V-shaped region in (a, b) plane with maximal reduction of rank to $r = 0$ ($\partial_x^2 V = 0$) at the tip $(a, b) = 0$ at reduction to $r = 1$ at the sides of V, where two roots of $\partial_x V = 4x^3 + 2ax + b = 0$ co-incide requiring that the discriminant of this equation vanishes.

3. In the recent case $IM(P)$ takes the role of complex quaternion valued potential function and the 4 coordinates $z_1^{k)}$ that of behavior variable x for cusp and $z_2^{k)}$ that of control parameters (a, b). The reduction of the rank of $n \times n$ matrix by Δr means that there are r linearly independent rows in the matrix. These give Δr additional conditions besides $IM(P) = 0$ so that the sub-variety along which the singularity takes places as dimension r. One can say that the r-dimensional tangent spaces integrate to the singular variety of dimension r.

   The analogy with branes would be realized as a hierarchical structure of singularities of the spacetime surfaces. This hierarchy of singularities would realize space-time correlates for quantum criticality, which is basic principle of quantum TGD. For instance, the reduction by 3-units would correspond to strings - say at the ends of CD and along the partonic orbits (fermion lines), and maximal reduction might correspond to discrete points - say the ends of fermion lines at partonic 2-surfaces. Also isolated intersection points can be regarded as singularities and are stably present but it does not make sense to add fermions to these points so that cognitive representations are not possible.

4. Note that also the associativity - and commutativity conditions already discuss involved the gradients of $IM(P)^i$ and $RE(P)^i$, which would suggests that these regions can be interpreted as singularities for which the dimension is not lowered by on unit since the vanishing conditions hold true identically by criticality.

There are two cases to be considered. The usual Euclidian case in which pinch reducing the dimension and the Minkowskian case in which metric dimension is reduced locally.

Consider first the Euclidian case.

1. In Euclidian case it is difficult to tell whether all values of Δr are possible since octonion analyticity poses strong conditions on the singularities. The pinch could correspond to the singularity of the covering associated with the space-time surface defined by Galois group for the covering associated with $h_{eff}/h = n$ identifiable as the dimension of the extension [L43]. Therefore there would be very close connection between the extensions of rationals defining the Galois group and the extension of polynomial ring of 8 complex variables z_i^k, $i = 1, 2$, $k = 1, .., 4$ by algebraic functions. At the pinch, which would be algebraic point, the Galois group would have subgroup leaving the coordinates of the point invariant and some sheets of the covering defining roots would co-incide.
2. A very simple analogy for this kind of singularity is the singularity of $P(x, y) = y^2 - x = 0$ at origin: now the sheets $y = \pm\sqrt{x}$ co-incide at origin. The algebraic functions $y \mp \sqrt{x}$ defining the factorization of $P(x, y)$ co-incide at origin. Quite generally, two or more factors in the factorization of polynomial using algebraic functions co-incide at the singularity. This is completely analogous to the degeneracy or roots of polynomials of single variable.
3. Quaternion structure predicts the slicing of M^4 by string world sheets inducing that of space-time surfaces. One must ask whether singular space-time sheets emerge already for the slicing of M^4 by string world sheets. String world sheets could be considered as candidates for $\Delta r = 2$ singularities of this kind. The physical intuition strongly suggests that there indeed physically preferred string world sheets and identification as $\Delta r = 2$ singularities of Euclidian type is attractive. Partonic 2-surfaces are also candidates in this respect. Could some sheets of the $h_{eff}/h = n$ covering co-incide at string world sheets?

Consider next the Minkowskian case. At the level of H the rank of the induced metric is reduced. This reduction need not be same as that for the matrix P_j^i and it is of course not obvious that the partonic orbit allows description as a singularity of algebraic variety.

1. Could the matrix P_j^i take a role analogous to the dual of induced metric and one might hope that the change of the sign for P_j^i for a fixed polynomial at singular surface could be analogous to the change of the sign of $\sqrt{g_4}$ so that the idea about algebraization of this singularity at level of M^8 might make sense. The information about metric could come from the fact that $IM(P)$ depends on complex valued quaternion norm reducing to Minkowskian metric in Minkowskian sub-space.
2. The condition for the reduction of rank from its maximal value of $r = 4$ to $r = 3$ occurs if one has $det(P) = 0$, which defines co-dimension 1 surface as a sub-variety of space-time surface. The interpretation as co-incidence of two roots should make sense if $IM(P) = 0$. Root pairs would now correspond now to the points at different sides of the singular 3-surface.

   Minkowskian singularity cannot be identified as the 3-D space-like boundary of many-sheeted space-time surface located at the boundary of CD (induced metric is space-like).

   Could this sub-variety be identified as partonic orbit, the common boundary of the Euclidian and Minkowskian regions? This would require that associative region transforms to co-associative one here. $IM(P) = 0$ condition can transform to $RE(P) = 0$ condition if one has $P = 0$ at this surface. Minkowskian variant of point singularity (P_j^i vanishes) would explode it to a light-like partonic orbit.

   What does this imply about the rank of singularity? The condition $IM(P) = RE(P) = 0$ does not reduce the rank if P is linear polynomial and one could consider a hierarchy of reductions of rank. Since $q\overline{q}$ vanishes in Minkowskian sub-space at light-cone boundary rather than at point $q = 0$ only, there are reasons to expect that it appears in P and reduces the rank by $\Delta r = 4$ (see Appendix for the discussion of o^2 case). The rank of the induced 4-metric is however reduced only by $\Delta r = 1$ at partonic orbit. If the complexified complex norm $z\overline{z}$, $z = z_1 + z_2 I_2$ can take the role of $q\overline{q}$, one has $\Delta r = 2$.

3. The reduction of rank to $r = 2$ would give rise to 2-surfaces, which are at the boundaries of 3-D singularities. If partonic orbits correspond to $\Delta r = 1$ singularities one could identify them as partonic 2-surfaces at the ends partonic orbits.

   Could the singularity at partonic 2-surface correspond to the reduction of the rank of the induced metric by 2 units? This is impossible in strict sense since there is only one light-like direction in signature $(1, -1, -1, -1)$. Partonic 2-surface singularity would however correspond to a corner for both Euclidian and Minkowskian regions at which the metrically 2-D but topologically 3-D partonic orbit meets the the space-like 3-surface along the light-like boundary of CD. Also the radial direction for space-like 3-surface could become light-like at partonic 2-surface if the CP_2 coordinates have vanishing gradient with respect to the light-like radial coordinate r_M at the partonic 2-surface. In this sense the rank could be reduced by 2 units. The situation is analogous to that for fold singularity $y^2 - x = 0$.

   String world sheets cannot be subsets of $r = 3$ singularities, which suggests different interpretation for partonic 2-surfaces and string world sheets.

What could this different interpretation be?

1. Perhaps the most convincing interpretation of string world sheets/partonic 2-surfaces has been already discussed (this interpretation would generalize to associative space-time surfaces). They could be commutative/co-commutative (here permutation might be allowed!) sub-manifolds of associative regions of the space-time surface allowing quaternionic tangent spaces so that the notions of commutative and co-commutative make sense. The criticality conditions are satisfied without the reduction of dimension from $d = 2$ to $d = 1$. In non-associative regions string world sheets would reduce to 1-D curves. This would happen at the boundaries of partonic orbits and 3-surfaces at the ends of space-time surface and only the ends of strings at partonic orbits carrying fermion number would be needed to determine twistorial scattering amplitudes [L30, K87].

2. I have also considered an interpretation in terms of singularities of space-time surfaces represented as a sections of their own twistor bundle. Self-intersections of the space-time surface would correspond to 2-D surfaces in this case [L43] and perhaps identifiable as string world sheets. The interpretation mentioned above would be in terms of Euclidian singularities. If this is true, the question is only about whether these two interpretations are consistent with each other.

If I were forced to draw conclusion on basis of these notices, it would be that only $r = 4$ Minkowskian singularities could be interesting and at them $RE(P) = 0$ regions could be transformed to $IM(P) = 0$ regions. Furthermore, the reduction of rank for the induced metric cannot be equal to the reduction of the rank for P^i_j.

5.2.5 The decomposition of space-time surface to Euclidian and Minkowskian regions in octonionic description

The unavoidable outcome of H picture is the decomposition of space-time surface to regions with Minkowskian or Euclidian signature of the induced metric. These regions are bounded by 3-D regions at which the signature of the induced metric is $(0, -1, -1, -1)$ due to the vanishing of the determinant of the induced metric. The boundary is naturally the light-like orbit of partonic 2-surface although one can consider also the possibility that these regions have boundaries intersecting along light-like curves defining boundaries of string world sheets. A more detailed view inspired by the study of extremals is following.

1. Let us assume that the above picture about decomposition of space-time surfaces in H to two kinds regions takes place. The regions where the dynamicis universal minimal surface dynamics have associative pre-image in M^8. The regions where Kähler action and volume term couple the associative pre-image in M^8 exists only at the 3-D boundary regions and M^8 dynamics determines the boundary conditions for H dynamics, which by holography is enough.

2. In the space-time regions having associative pre-image in M^8 one has a fibration of X^4 with with partonic surface as a local base and string world sheet as local fiber. In the interior of space-time region there are no singularities but at the boundary 2-D string world sheets becomes metrically 1-D as 1-D string boundary reduces metrically to 0-D structure analogous to a point. This reduction of dimension would be metric, but not topological.

   The singularity for plane curve $P(x,y) = y^2 - x^3 = 0$ at origin illustrates the difference between Minkowskian and Euclidian singularity. One has $(\partial_x P, \partial_y P) = (-3x^2, 2y)$ vanishing at origin so that $\Delta r = 1$ singularity is in question and the dimension of singular manifold is indeed $r = 0$. From $y = \pm x^{3/2}$, $x \geq 0$. The induced metric $g_{xx} = 1 + (dy/dx)^2 = 1 + (9/4)x$, $x \geq 0$ is however non-singular at origin.

3. If the Euclidian region with pre-image corresponds to a deformation of wormhole contact, the identification as image of a co-associative space-time region in M^8 is natural so that normal space is associative and contains also the preferred $M^2(x)$. In Minkowskian regions the identification as image of associative space-time region in M^8 is natural.

What can one say about the relationship of the M^8 counterparts of neighboring Minkowskian and Euclidian regions?

1. Do these regions intersect along light-like 3-surfaces, 1-D light-like curve (orbit of fermion) or is the intersection disrete set of points possibly assignable to the partonic 2-surface at the boundaries of CD? The M^4 projections of the inverse image of the light-like partonic orbit should co-incide but E^4 projections need not do so. They could be however mappable to the same partonic two surface in $M^8 - H$ correspondence or the images could have at least have light-like curve as common.

2. Is seems impossible for the space-time surfaces determined as zeros of octonionic polynomials to have boundaries. Rather, it seems that the boundary must be between Minkowskian and Euclidian regions of the space-time surface determined by the same octonionic polynomial. At the boundary also associate region would transform to co-associative region suggesting that $IM(P) = RE(P) = 0$ holds allowing to change the condition from $IM(P) = 0$ to $RE(P) = 0$.

   Consider now in more detail whether this view can be realized.

1. In $H = M^4 \times CP_2$ the boundary between the Minkowskian and Euclidian space-time regions - light-like partonic 3-surface - is a singularity possible only in Minkowskian signature. Space-time surface X^4 at the boundary is effectively 3-D since one has $\sqrt{g_4} = 0$ meaning that tangent space is effectively 3-D. The 3-D boundary itself is metrically 2-D and this gives rise to the extended conformal invariance defining crucial distinction between TGD and super string models.

2. The singularities of $P(o)$ for o identified as linear coordinate of M_c^8 were already considered. The singularities correspond to the boundaries of light-cone and the emergence of CDs can be understood. Could also the light-like orbits of partonic 2-surfaces be understood in the same manner? Does the pre-image of this singularity in M^8 emerge as a singularity of an algebraic variety determined by the vanishing of $IM(P)$ for the octonionic polynomial?

   What is common is that the rank of the induced metric by one unit also now. Now one has however also $det(g_4) = 0$. The singularities correspond to curved light-like 3-surfaces inside space-time surfaces rather than light-like surfaces in M^8: induced metric matters rather than M^4 metric.

3. Could also these regions correspond to singularities of octonionic polynomials at which $P(o) = 0$ is satisfied and associative region transforms to a co-associative region? For $M^2(x) = M_0^2$ this is impossible. Partonic 2-surfaces are planes E^2 now. One should have closed partonic 2-surfaces.

   Could the allowance of quaternionic structures with slicing of X^4 by string world sheets and partonic 2-surfaces help? If one has slicing of string world sheets by dual light-like curves corresponding to light-like coordinates u and v, this slicing gives also rise to a slicing of light-like 3-surfaces and dual light-like coordinate. The pair (u,v) in fact defines the analog of z and $\overline{z}$ in hypercomplex case. Could the singularity of $P(o)$ using the quaternionic coordinates defined by (u,v) and coordinates of partonic 2-surface allow to identify light-like partonic orbits with $det(g_4) = 0$ as a generalization of light-cone boundaries in M^4?

The decomposition $M_0^4 = M_x^2 \times E^2(x)$ associated with quaternionic structure is independent of E^4. In the other hand, tangent space of space-time surface at point decomposes $M^2(x) \times E_T^2(x)$, where $E_T^2(x)$ is in general different from $E^2(x)$. Is this enough to obtain partonic 2-surfaces as singularities with $RE(P) = IM(P) = 0$?

The question whether the boundaries between Minkowskian and Euclidian can correspond to singular regions at which $P(o)$ vanishes and the surface $RE(P) = 0$ transforms to $IM(P) = 0$ surface remains open. What remains poorly understood is the role of the induced metric. My hope is that with a further work the picture could be made more detailed.

5.2.6 About rational points of space-time surface

What one can say about rational points of space-time surface?

1. An important special case corresponds to a generalization of so called rational surfaces for which a parametric representation in terms of 4 complex coordinates t^k exists such that o_1^k are *rational* functions of t^k. The singularities for 2-complex dimensional surfaces in C^3 or equivalently CP_3 are classified by Du Val [A61, A69] (see `http://tinyurl.com/ydz93hle`).
2. In [L43] [L38] I considered possible singularities of the twistor bundle. These would correspond typically 2-D self-intersections of the embedding of space-time surfaces as 4-D base space of 6-D twistor bundle with sphere as a fiber. They could relate to string world sheets and partonic 2-surfaces and - as already found - are different from singularities at the level of M_c^8. The singularities of string world sheets and partonic 2-surfaces as hyper-complex and co-complex surfaces consist of points and could relate to the singularities at octonionic level.

As already mentioned, Bombieri-Lang conjecture (see `http://tinyurl.com/y887yn5b`) states that, for any variety X of general type over a number field k, the set of k-rational points of X is not Zariski dense (see `http://tinyurl.com/jm9fh74`) in X. Even more, the k-rational points are contained in a *finite* union of lower-dimensional sub-varieties of X.

This conjecture is highly interesting from TGD point of view if one believes in $M^8 - H$ duality. Space-time surfaces $X^4 \subset M_c^8$ can be seen as $M^8 = M^4 \times E^4$ projections of zero loci for real or imaginary parts of octonionic polynomials in o. In complex sense they reduce to $M^4 \times E^4$ projections of algebraic co-dimension 4 surfaces in C^8. If Bombieri-Lang conjectures makes sense in this context, it would state that for a space-time surface $X^4 \subset M^8$ of general type the rational points are contained in a *finite* union of lower-dimensional sub-varieties. Also the conjecture of Vojta (see `http://tinyurl.com/y9sttuu4`) stating that varieties of general type cannot be potentially dense is known to be true for curves and support this general vision.

Could the finite union of sub-varieties correspond to string world sheets, partonic 2-surfaces, and their light-like orbits define singularities? But why just singular sub-varieties would be cognitively simple and have small Kodaira dimension d_K allowing large number of rational points? In the case of partonic orbits one might understand this as a reduction of metric dimension. The orbit is effectively 2-dimensional partonic surface metrically and for the genera $g = 0, 1$ rational points are dense. For string world sheets with handle number smaller than 2 the situation is same.

The proposed realizations of associativity and commutativity provide additional support for this picture. Criticality guaranteeing associativity/commutativity would select preferred space-time surfaces as also string world sheets and partonic 2-surfaces.

Concluding, the general wisdom of algebraic geometry conforms with SH and with the vision about the localization of cognitive representations at 2-surfaces. There are of many possible options for detailed interpretation and certainly the above sketch cannot be correct at the level of details.

5.2.7 About $h_{eff}/h = n$ as the number of sheets of Galois covering

The following considerations were motivated by the observation of a very stupid mistake that I have made repeatedly in some articles about TGD. Planck constant $h_{eff}/h = n$ corresponds naturally to the number of sheets of the covering space defined by the space-time surface.

I have however claimed that one has $n = ord(G)$, where $ord(G)$ is the order of the Galois group G associated with the extension of rationals assignable to the sector of "world of classical worlds" (WCW) and the dynamics of the space-time surface (what this means will be considered below).

This claim of course cannot be true since the generic point of extension G has some subgroup H leaving it invariant and one has $n = ord(G)/ord(H)$ dividing $ord(G)$. Equality holds true only for Abelian extensions with cyclic G. For singular points isotropy group is $H_1 \sup H$ so that $ord(H_1)/ord(H)$ sheets of the covering touch each other. I do not know how I have ended up to a conclusion, which is so obviously wrong, and how I have managed for so long to not notice my blunder.

This observation forced me to consider more precisely what the idea about Galois group acting as a number theoretic symmetry group really means at space-time level and it turned out that $M^8 - H$ correspondence [L46] (see `http://tinyurl.com/yd43o2n2`) gives a precise meaning for this idea.

Consider first the action of Galois group (see `http://tinyurl.com/y8grabt2` and `http://tinyurl.com/ydze5psx`).

1. The action of Galois group leaves invariant the number theoretic norm characterizing the extension. The generic orbit of Galois group can be regarded as a discrete coset space G/H, $H \subset G$. The action of Galois group is transitive for irreducible polynomials so that any two points at the orbit are G-related. For the singular points the isotropy group is larger than for generic points and the orbit is G/H_1, $H_1 \sup H$ so that the number of points of the orbit divides n. Since rationals remain invariant under G, the orbit of any rational point contains only single point. The orbit of a point in the complement of rationals under G is analogous to an orbit of a point of sphere under discrete subgroup of $SO(3)$.

   $n = ord(G)/ord(H)$ divides the order $ord(G)$ of Galois group G. The largest possible Galois group for n-D algebraic extension is permutation group S_n. A theorem of Frobenius states that this can be achieved for $n = p$, p prime if there is only single pair of complex roots (see `http://tinyurl.com/y8grabt2`). Prime-dimensional extensions with $h_{eff}/h = p$ would have maximal number theoretical symmetries and could be very special physically: p-adic physics again!

2. The action of G on a point of space-time surface with embedding space coordinates in n-D extension of rationals gives rise to an orbit containing n points except when the isotropy group leaving the point is larger than for a generic point. One therefore obtains singular covering with the sheets of the covering touching each other at singular points. Rational points are maximally singular points at which all sheets of the covering touch each other.

3. At QFT limit of TGD the n dynamically identical sheets of covering are effectively replaced with single one and this effectively replaces h with $h_{eff} = n \times h$ in the exponent of action (Planck constant is still the familiar h at the fundamental level). n is naturally the dimension of the extension and thus satisfies $n \leq ord(G)$. $n = ord(G)$ is satisfied only if G is cyclic group.

The challenge is to define what space-time surface as Galois covering does really mean!

1. The surface considered can be partonic 2-surface, string world sheet, space-like 3-surface at the boundary of CD, light-like orbit of partonic 2-surface, or space-time surface. What one actually has is only the data given by these discrete points having embedding space coordinates in a given extension of rationals. One considers an extension of rationals determined by irreducible polynomial P but in p-adic context also roots of P determine finite-D extensions since e^p is ordinary p-adic number.

2. Somehow this data should give rise to possibly unique continuous surface. At the level of $H = M^4 \times CP_2$ this is impossible unless the dynamics satisfies besides the action principle also a huge number of additional conditions reducing the initial value data ans/or boundary data to a condition that the surface contains a discrete set of algebraic points.

   This condition is horribly strong, much more stringent than holography and even strong holography (SH) implied by the general coordinate invariance (GCI) in TGD framework. However, preferred extremal property at level of $M^4 \times CP_2$ following basically from GCI in TGD context might be equivalent with the reduction of boundary data to discrete data if $M^8 - H$ correspondence [L46] (see `http://tinyurl.com/yd43o2n2`) is accepted. These data would be analogous to discrete data characterizing computer program so that an analog of computationalism would emerge [L40] (see `http://tinyurl.com/y75246rk`).

One can argue that somehow the action of discrete Galois group must have a lift to a continuous flow.

1. The linear superposition of the extension in the field of rationals does not extend uniquely to a linear superposition in the field reals since the expression of real number as sum of units of extension with real coefficients is highly non-unique. Therefore the naïve extension of the extension of Galois group to all points of space-time surface fails.
2. The old idea already due to Riemann is that Galois group is represented as the first homotopy group of the space. The space with homotopy group π_1 has coverings for which points remain invariant under subgroup H of the homotopy group. For the universal covering the number of sheets equals to the order of π_1. For the other coverings there is subgroup $H \subset \pi_1$ leaving the points invariant. For instance, for homotopy group $\pi_1(S^1) = Z$ the subgroup is nZ and one has $Z/nZ = Z_p$ as the group of n-sheeted covering. For physical reasons its seems reasonable to restrict to finite-D Galois extensions and thus to finite homotopy groups.

   $\pi_1 - G$ correspondence would allow to lift the action of Galois group to a flow determined only up to homotopy so that this condition is far from being sufficient.
3. A stronger condition would be that π_1 and therefore also G can be realized as a discrete subgroup of the isometry group of $H = M^4 \times CP_2$ or of M^8 ($M^8 - H$ correspondence) and can be lifted to continuous flow. Also this condition looks too weak to realize the required miracle. This lift is however strongly suggested by Langlands correspondence [K55, K56] (see `http://tinyurl.com/y9x5vkeo`).

The physically natural condition is that the preferred extremal property fixes the surface or at least space-time surface from a very small amount of data. The discrete set of algebraic points in given extension should serve as an analog of boundary data or initial value data.

1. $M^8 - H$ correspondence [L46] (see `http://tinyurl.com/yd43o2n2`) could indeed realize this idea. At the level of M^8 space-time surfaces would be algebraic varieties whereas at the level of H they would be preferred extremals of an action principle which is sum of Kähler action and minimal surface term.

   They would thus satisfy partial differential equations implied by the variational principle and infinite number of gauge conditions stating that classical Noether charges vanish for a subgroup of symplectic group of $\delta M^4_\pm \times CP_2$. For twistor lift the condition that the induced twistor structure for the 6-D surface represented as a surface in the 12-D Cartesian product of twistor spaces of M^4 and CP_2 reduces to twistor space of the space-time surface and is thus S^2 bundle over 4-D space-time surface.

   The direct map $M^8 \to H$ is possible in the associative space-time regions of $X^4 \subset M^8$ with quaternionic tangent or normal space. These regions correspond to external particles arriving into causal diamond (CD). As surfaces in H they are minimal surfaces and also extremals of Kähler action and do not depend at all on coupling parameters (universality of quantum criticality realized as associativity). In non-associative regions identified as interaction regions inside CDs the dynamics depends on coupling parameters and the direct map $M^8 \to CP_2$ is not possible but preferred extremal property would fix the image in the interior of CD from the boundary data at the boundaries of CD.
2. At the level of M^8 the situation is very simple since space-time surfaces would correspond to zero loci for $RE(P)$ or $IM(P)$ (RE and IM are defined in quaternionic sense) of an octonionic polynomial P obtained from a real polynomial with coefficients having values in the field of rationals or in an extension of rationals. The extension of rationals would correspond to the extension defined by the roots of the polynomial P.

   If the coefficients are not rational but belong to an extension of rationals with Galois group G_0, the Galois group of the extension defined by the polynomial has G_0 as normal subgroup and one can argue that the relative Galois group $G_{rel} = G/G_0$ takes the role of Galois group.

   It seems that $M^8 - H$ correspondence could allow to realize the lift of discrete data to obtain continuous space-time surfaces. The data fixing the real polynomial P and therefore also its octonionic variant are indeed discrete and correspond essentially to the roots of P.

3. One of the elegant features of this picture is that the at the level of M^8 there are highly unique linear coordinates of M^8 consistent with the octonionic structure so that the notion of a M^8 point belonging to extension of rationals does not lead to conflict with GCI. Linear coordinate changes of M^8 coordinates not respecting the property of being a number in extension of rationals would define moduli space so that GCI would be achieved.

Does this option imply the lift of G to π_1 or to even a discrete subgroup of isometries is not clear. Galois group should have a representation as a discrete subgroup of isometry group in order to realize the latter condition and Langlands correspondence supports this as already noticed. Note that only a rather restricted set of Galois groups can be lifted to subgroups of $SU(2)$ appearing in McKay correspondence and hierarchy of inclusions of hyper-finite factors of type II_1 labelled by these subgroups forming so called ADE hierarchy in 1-1 correspondence with ADE type Lie groups [K112, K43] (see `http://tinyurl.com/ybavqvvr`). One must notice that there are additional complexities due to the possibility of quaternionic structure which bring in the Galois group SO(3) of quaternions.

Remark: After writing this article a considerable progress in understanding of $h_{eff}/h = n$ as number of sheets of Galois covering emerged. By M^8-duality space-time surface can be seen as zero locus for real or imaginary part (regarding octonions as sums of quaternionic real and imaginary parts) allows a nice understanding of space-time surface as an $h_{eff}/h = n$-fold Galois covering. M^8 is complexified by adding an imaginary unit i commuting with octonionic imaginary units. Also space-time surface is complexified to 8-D surface in complexified M^8. One can say that ordinary space-time surface is the “real part” of this complexified space-time surface just like x is the real part of a complex number $x + iy$. Space-time surface can be also seen as a root of n:th order polynomial with n complex branches and the projections of complex roots to “real part” of M^8 define space-time surface as an n-fold covering space in which Galois group acts.

5.2.8 Connection with infinite primes

The idea about space-time surfaces as zero loci of polynomials emerged for the first time as I tried to understand the physical interpretation of infinite primes [K94], which were motivated by TGD inspired theory of consciousness. Infinite primes form an infinite hierarchy. At the lowest level the basic entity is the product $X = \prod_p p$ of all finite primes. The physical interpretation could be as an analog of fermionic sea with fermion states labelled by finite primes p.

1. The simplest infinite primes are of form $P = X \pm 1$ as is easy to see. One can construct more complex infinite primes as infinite integers of form $nX/r + mr$. Here r is square free integer, n is integer having no common factors with r, and m can have only factors possessed also by r.

   The interpretation is that r defines fermionic state obtained by kicking from Dirac sea the fermions labelled by the prime factors of r. The integers n and m define bosonic excitations in which k:th power of p corresponds to k bosons in state labelled by p. One can also construct more complex infinite primes as polynomials of X and having no rational factors. In fact, X becomes coordinate variable in the correspondence with polynomials.

2. This process can be repeated at the next level. Now one introduces product $Y = \prod_P P$ of all primes at the previous level and repeats the same construction. These infinite correspond to polynomials of Y with coefficients given by rational functions of X. Primality means irreducibility in the field of rational functions so that solving Y in terms of X would give algebraic function.

3. At the lowest level are ordinary primes. At the next level the infinite primes are indeed infinite in real sense but have p-adic norms equal to unity. They can be mapped to polynomials $P(x_1)$ with rational coefficients and the simplest polynomials are monomials with rational root. Higher polynomials are irreducible polynomials with algebraic roots. At the third level of hierarchy one has polynomials $P(x_2|x_1)$ of two variables. They are polynomials of x_1 with coefficients with are rational functions of x_1. This hierarchy can be continued.

   One can define also infinite integers as products of infinite primes at various levels of hierarchy and even infinite rationals.

4. This hierarchy can be interpreted in terms of a repeated quantization of an arithmetic supersymmetric quantum field theory with elementary particles labelled by primes at given level of hierarchy. Physical picture suggests that the hierarchy of second quantizations is realized also in Nature and corresponds to the hierarchy of space-time sheets.

5. One could consider a mapping $P(x_n|x_{n-1}|..|x_1)$ by a diagonal projection $x_i = x$ to polynomials of single variable x. One could replace x with complexified octonic coordinate o_c. Could this correspondence give rise to octonionic polynomials and could the connection with second quantization give classical space-time correlates of real quantum states assignable to infinite primes and integers? Even quantum states defining counterparts of infinite rationals could be considered. One could require that the real norm of these infinite rationals equals to one. They would define infinite number of real units with arbitrarily complex number theoretical anatomy. The extension of real numbers by these units would mean huge extension of the notion of real number and one could say that each real point corresponds to platonic defined by these units closed under multiplication.

   In ZEO zero energy states formed by pairs of positive and negative energy could correspond to these states physically. The condition that the ratio is unit would have also a physical interpretation in terms of particle content.

6. As already noticed, the notions of analyticity, quaternionicity, and octonionicity could be seen as a manifestation of polynomials in algebras defined by adding repeatedly a new non-commuting imaginary unit to already existing algebra. The dimension of the algebra is doubled in each step so that dimension comes as a power of 2. The algebra of polynomials with real coefficients is commutative and associative. This encourages the crazy idea that the spaces are indeed realized and the generalization of $M^8 - H$ duality holds true at each level. At level k the counterpart for CP_2 (for $k = 3$) would be as moduli space for sub-spaces of dimension 2^{k-1} for which tangent space reduces to the algebra at level $k-1$. For $k = 2$ CP_1 is the moduli space and could correspond to twistor sphere. Essentially Grassmannian $Gl(2^k, 2^{k-1})$ would be in question. This brings in mind twistor Grassmann approach involving hierarchy of Grassmannians too, which however allows all dimensions. What is interesting that the spinor bundle for space of even dimension d has fiber with dimension $2^{d/2}$.

   The number of arguments for the hierarchy of polynomials assignable to the hierarchy of infinite primes increases by one at each step. Hence these two hierarchies are different.

The vanishing of the octonionic polynomials indeed allow a decomposition to products of prime polynomials with roots which in general are algebraic numbers and an exciting possibility is that the prime polynomials have interpretation as counterparts of elementary particles in very general sense.

Infinite primes can be mapped to polynomials and the most natural counterpart for the infinite rational would be as a complexified octonionic rational function $P_1(t)/P_2(t-T)$, where T is real octonion, with coefficients in extension of rationals. This would naturally give the geometry CD. The assignment of opposite boundaries of CD to $P_1(t)$ and $P_2(t-T)$ is suggestive and identification of zero loci of $IM(P_1)$ and $IM(P_2)$ as incoming and outgoing particles would be natural. The zero and ∞ loci for $RE(P_1/P_2)$ would define interaction between these space-time varieties and should give rise to wormhole contacts connecting them. Note that the linearity of $IM(o_1o_2)$ in $IM(o_i)$ and non-linearity of $RE(o_1o_2)$ in $RE(o_i)$ would be a key element behind this identification. This idea will be discussed in more detail in the section "Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view".

5.3 Super variant of octonionic algebraic geometry and space-time surfaces as correlates for fermionic states

Could the octonionic level provide an elegant description of fermions in terms of super variant of octonionic algebraic geometry? Could one even construct scattering amplitudes at the level of M^8 using the variant of the twistor approach discussed in [L30, K87]?

The idea about super-geometry is of course very different from the idea that fermionic statistics is realized in terms of the spinor structure of "world of classical worlds" (WCW) but

$M^8 - H$ duality could however map these ideas and also number theoretic and geometric vision to each other. The angel of geometry and the devil of algebra could be dual to each other.

In the following I start from the notion of emergence generalized to the vision that entire physics emerges from the notion of number. This naturally leads to an identification of super-variants of various number fields, in particular of complexified octonions. After that super variants of $RE(P) = 0$ and $IM(P) = 0$ conditions are discussed, and the surprising finding is that the conditions might allow only single fermion states localized at strings. This would allow only single particle in the super-multiplet and would mean breaking of SUSY. This picture would be consistent with the earlier H picture about construction of scattering amplitudes [L30, K87]. Finally the problems related to the detailed physical interpretation are discussed.

5.3.1 About emergence

The notion of emergence is fashionable in the recent day physics, in particular, he belief is that 3-space emerges in some manner. In the sequel I consider briefly the standard view about emergence idea from TGD point of view, then suggest that the emergence in the deepest sense requires emergence of physics from the notion of number and that complexified octonions [L46] [L47, L48, L35, L45] are the most plausible candidate in this respect. After that I will show that number theory generalizes to super-number theory: super-number fields make sense and one can define the notion of super-prime. Every new step of progress creates worry about consistency with the earlier work, now the work done during last months with physics as octonionic algebraic geometry and also this aspect is discussed.

1. The notion of holography is behind the emergence of 3-space and implies that the notion of 2-space is taken as input. This could be justified by conformal invariance.
2. The key idea is that 3-space emerges somehow from entanglement. There is something that must entangle and this something must be labelled by points of space: one must introduce a discretised space. Then one must do some handwaving to make it 3-D - perhaps by arguing that holography based on 2-D holograms is unique by conformal invariance. The next hand-wave would replace this as a 3-D continuous space at infrared limit.
3. How to get space-time and how to get general coordinate invariance? How to get the symmetries of standard model and special relativity? Somehow all this must be smuggled into the theory when the audience is cheated to direct its attention elsewhere. This Münchausen trick requires a professional magician!
4. One attempt could take as starting point what I call strong form of holography (SH) in which 2-D data determine 4-D physics. Just like 2-D real analytic function determines analytic function of two complex variables in spacetime of 2 complex dimensions by analytic continuation (this hints strongly to quaternions). This is possible if conformal invariance is generalized to that for light-like 3-surfaces such as light-cone boundary. But the emergence magician should do the same without these.

   In TGD one could make this even simpler. Octonionic polynomials and rational functions are obtained from real polynomials of real variable by octonion-analytic continuation. And since polynomials and rational functions P_1/P_2 are in question their values at finite number of discrete points determined them if the orders of P_1 and P_2 are known!

   If one accepts adelic hierarchy based on extensions of rationals the coefficients of polynomials are in extensions of rationals and the situation simplifies further. The criticality conditions guaranteeing associativity for external particles is one more simplification: everything b becomes discrete. The physics at fundamental level could be incredibly simple: discrete number of points determines space-time surfaces as zero loci for $RE(P)$ or $IM(P)$ (octonions are decomposed to two quaternions gives $RE(o)$ and $IM(o)$).

   How this is mapped to physics leading to standard model emerging from the formulation in $M \times CP_2$ This map exists - I call it $M^8 - H$ duality - and takes space-time varieties in Minkowskian sector of complexified octonions to a space-time surface in $M^4 \times CP_2$ coding for standard model quantum numbers and classical fields.

How to get all this without bringing in octonionic embedding space: this is the challenge for the emergence-magician! I am afraid this this trick is impossible. I will however propose a deeper for what emergence is. It would not be emergence of space-time and all physics from entanglement but from the notion of number, which is at the base of all mathematics. This view led to a discovery of the notion of super-number field, a completely new mathematical concept, which should show how deep the idea is.

5.3.2 Does physics emerge from the notion of number field?

Concerning emergence one can start from a totally different point of view. Even if one gets rid of space as something fundamental from Hilbert sapce and entanglement, one has not reached the most fundamental level. Structures like Hilbert space, manifold, etc. are not fundamental mathematical structures: they require the notion of number field. Number field is the fundamental notion.

Could entire physics emerge from the notion of number field alone: space-time, fermions, standard model interactions, gravitation? There are good hopes about this in TGD framework if one accepts $M^8 - H$ duality and physics as octonionic algebraic geometry! One could however argue that fermions do not follow from the notion of number field alone. The real surprise was that formalizing this more precisely led to a realization that the very notion of number field generalizes to what one could call super-number field!

Emergence of physics from complexified octonionic algebraic geometry

Consider first the situation for number fields postponing the addition of attribute “super” later.

1. Number field endowed with basic arithmetic operations $+, -, \cdot, /$ is the basic notion for anyone wanting to make theoretical physics. There is a rich repertoire of number fields. Finite fields, rationals and their extensions, real numbers, complex numbers, quaternions, and octonions. There also p-adic numbers and their extensions induced by extensions of rationals and fusing into adele forming basic structure of adelic physics. Even the complex, quaternionic, and octonionic rationals and their extensions make sense. p-Adic variants of say octonions must be however restricted to have coefficients belonging to an extension of rationals unless one is willing to give up field property (the p-adic analog of norm squared can vanish in higher p-adic dimensions so that inverse need not exist).

   There are also function fields consisting of functions with local arithmetic operations. Analytic functions of complex variable provides the basic example. If function vanishes at some point its inverse element diverges at the same point. Function fields are derived objects rather than fundamental.

2. Octonions are the largest classical number field and are therefore the natural choice if one wants to reduce physics to the notion of number. Since one wants also algebraic extensions of rationals, it is natural to introduce the notion of complexified octonion by introducing an additional imaginary unit - call it i, commuting with the 7 octonionic imaginary units I_k. One obtains complexified octonions.

   That this is not a global number field anymore turns out to be a blessing physically. Complexified octonion z_kE^k has $z_k = z_k + iy_k$. The complex valued norm of octonion is given by $z_0^2 + ...z_7^2$ (there is no conjugation involved. The norm vanishes at the complex surface $z_0^2 + ...z_7^2 = 0$ defining a 7-D surface in 7-D O_c (the dimension is defined in complex sense). At this surface - complexified light-cone boundary - number field theory property fails but is preserved elsewhere since one can construct the inverse of octonion.

   At the real section M^8 (8-D Minkowski space with one real (imaginary) coordinate and 7 imaginary (real) coordinates the vanishing takes place also. This surface corresponds to the 7-D light-cone boundary of 8-D Minkowskian light-cone. This suggests that light-like propagation is basically due to the complexification of octonions implying local failure of the number field property. Same happens also in other real sections with $0 < n < 8$ real coordinates and $0 < m = 8 - n < 8$ imaginary coordinates and one obtains variant of light-cone with different signatures. Euclidian signature corresponding to $m = 0$ or $m = 8$ is an exception: light-cone boundary reduces to single point in this case and one has genuine number field - no propagation is possible in Euclidian signature.

Similar argument applies in the case of complexified quaternions Q_c and complexified complex numbers $z_1 + z_2 I \in C_c$, where I is octonionic imaginary unit. For Q_c one obtains ordinary 3-D light-cone boundary in real section and 1-D light-cone boundary in the case of C_c. It seems that physics demands complexification! The restriction to real sector follows from the requirement that norm squared reduces to a real number. All real sectors are possible and I have already considered the question whether this should be taken as a prediction of TGD and whether it is testable.

Super-octonionic algebraic geometry

There is also a natural generalization of octonionic TGD to super-octonionic TGD based on octonionic triality. $SO(1,7)$ allows besides 8-D vector representations also spinor representations 8_c and $\overline{8}_c$. This suggests that super variant of number field of octonions might make sense. One would have $o = o_8 + o_{c,8} + \overline{0}_{c,8}$.

1. Should one combine o_8, $o_{c,8}$ and $\overline{o}_{c,8}$ to a coordinate triplet $(o_8, o_{c,8}, \overline{o}_{c,8})$ as done in supersymmetric theories to construct super-fields? The introduction of super-fields as primary dynamical variables is a good idea now since the very idea is to reduce physics to algebraic geometry at the level of M^8. Polynomials of super-octonions defining space-time varieties as zero loci for their real or imaginary part in quaternionic sense could however take the role of super fields. Space-time surface would correspond to zero loci for $RE(P)$ or $IM(P)$.
2. The idea about super-octonions should be consistent with the idea that we live in a complexified number field. How to define the notion of super-octonion? The tensor product $8 \otimes 8_c$ contains 8_c and $8 \otimes 8_{\overline{c}}$ contains $8_{\overline{c}}$ and one can use Glebsch-Gordan coefficients to contract o and θ_c and o and $\overline{\theta}_{c,n}$. The tensor product of 8_c and $8_{\overline{c}}$ defined using structure constants defining octonion product gives 8. Therefore one must have

$$o_s = o + \Psi_c \times \theta_{\overline{c}} + \Psi_{\overline{c}} \times \theta_c \ , \tag{5.3.1}$$

   where the products are octonion products. Super parts of super-coordinates would not be just Grassmann numbers but octonionic products of Grassmann numbers with octonionic spinors in 8_c and $\overline{8}_c$. This would bring in the octonionic analogs of spinor fields into the octonionic geometry.

   This seems to be consistent with super field theories since octonionic polynomials and even rational functions would give the analogs of super-fields. What TGD would provide would be an algebraic geometrization of super-fields.
3. What is the meaning of the conditions $RE(P) = 0$ and $IM(P) = 0$ for super-octonions? Does this condition hold true for all $d_G = 2^{16}$ super components of $P(o_s)$ or is it enough to pose the condition only for the octonionic part of $P(o)$? In the latter case Ψ_c and $\Psi_{\overline{c}}$ would be free and this does not seem sensical and does not conform with octonionic super-symmetry. Therefore the first option will be studied in the sequel.

If super-octonions for a super variant of number field so that also inverse of super-octonion is well-defined, then even rational functions of complexified super-octonions makes sense and poles have interpretation in terms of 8-D light-fronts (partonic orbits at level of H). The notion must make sense also for other classical number fields, finite fields, rationals and their extensions, and p-adic numbers and their extensions. Does this structure form a generalization of number field to a super counter part of number field? The easiest manner to kill the idea is to check what happens in the case of reals.

1. The super-real would be of form $s = x + y\theta$, $\theta^2 = 0$. Sum and product are obviously well-defined. The inverse is also well-defined and given by $1/s = (x - y\theta))/x^2$. Note that for complex number $x + iy$ the inverse would be $\overline{z}/z\overline{z} = (x - yi)/(x^2 + y^2)$. The formula for super-inverse follows from the same formula as the inverse of complex number by defining conjugate of super-real s as $\overline{s} = x - y\theta$ and the norm squared of s as $|s|^2 = s\overline{s} = x^2$.

   One can identify super-integers as $N = m + n\theta$. One can also identify super-real units as number of unit norm. Any number $1_n = 1 + n\theta$ has unit norm and the norms form an Abelian group under multiplication: $1_m 1_n = 1_{m+n}$. Similar non-uniqueness of units occurs also for algebraic extensions of rationals.

2. Could one have super variant of number theory? Can one identify super-primes? Super-norm satisfies the usual defining property $|xy| = |x||y|$. Super-prime is defined only apart from the multiplicative factor 1_m giving not contribution to the norm. This is not a problem but a more rigorous formulation leads to the replacement of primes with prime ideals labelled by primes already in the ordinary number theory.

   If the norm of super-prime is ordinary prime it cannot decompose to a product of super-primes. Not all super-primes having given ordinary prime as norm are however independent. If super-primes $p + n\theta$ and $p + m\theta$ differ by a multiplication with unit $1_r = 1 + r\theta$, one has $n - m = pr$. Hence there are only p super-primes with norm p and they can be taken $p_s = p + k\theta$, $k \in \{0, p-1\}$. A structure analogous to a cyclic group Z_p emerges.

   Note that also θ is somewhat analogous to prime although its norm is vanishing.

3. Just for fun, one an ask what is the super counterpart of Riemann Zeta. Riemann zeta can be regarded as an analog of thermodynamical partition function reducing to a product for partition functions for bosonic systems labelled by primes p. The contribution from prime p is factor $1/(1-p^{-s})$. p^{-s} is analogous to Boltzmann weight $N(E)exp(-E/T)$, where $N(E)$ is number of states with energy E. The degeneracy of states labelled by prime p is for ordinary primes $N(p) = 1$. For super-primes the degeneracy is $N(p) = p$ and the weight becomes $1/(1-N(p)p^{-s}) = 1/(1-p^{-s+1})$. Super Riemann zeta is therefore $zeta(s-1)$ having critical line at $s = 3/2$ rather than at $s = 1/2$ and trivial zeros at real points $s = -1, -3, -5$, rather than at $s = -2, -4, -6, ...$

There are good reasons to expect that the above arguments work also for algebraic extensions of super-rationals and in fact for all number fields, even for super-variants of complex numbers, quaternions and octonions. This because the conditions for invertibility reduce to that for real numbers. One would have a generalization of number theory to super-number theory! Net search gives no references to anything like this. Perhaps the generalization has not been noticed because the physical motivation has been lacking. M^8-H duality would imply that entire physics, including fermion statistics, standard model interactions and gravitation reduces to the notion of number in accordance with number theoretical view about emergence.

Is it possible to satisfy super-variants of $IM(P) = 0$ and $RE(P) = 0$ conditions?

Instead of super-fields one would have a super variant of octonionic algebraic geometry.

1. Super variants of the polynomials and even rational functions make sense and reduce to a sum of octonionic polynomials $P_{kl}\theta_1^k\theta_2^l$, where the integers k and l would be tentatively identified as fermion numbers and θ_k is a shorthand for a monomial of k different thetas. The coefficients in $P_{kl} = P_{kl,n}o^n$ would be given by $P_{kl,n} = P_{n+k+l}B(n+k+l, k+l)$, where $B(r,s) = r!/(r-s)!s!$ is binomial coefficient. The space-time surfaces associated with P_{kl} would be different and they need not be simultaneously critical, which could give rise to a breaking of supersymmetry.

   One would clearly have an upper bound for k and l for given CD. Therefore these many-fermion states must correspond to fundamental particles rather than many-fermion Fock states. One would obtain bosons with non-vanishing fermion numbers if the proposed identification is correct. Octonionic algebraic geometry for single CD would describe only fundamental particles or states with bounded fermion numbers. Fundamental particles would be indeed fundamental also geometrically.

2. One can also now define space-time varieties as zero loci via the conditions $RE(P_s)(o_s) = 0$ or $IM(P_s)(o_s) = 0$. One obtains a collection of 4-surfaces as zero loci of P_{kl}. One would have a correlation with between fermion content and algebraic geometry of the space-time surface unlike in the ordinary super-space approach, where the notion of the geometry remains rather formal and there is no natural coupling between fermionic content and classical geometry. At the level of H this comes from quantum classical correspondence (QCC) stating that the classical Noether charges are equal to eigenvalues of fermionic Noether charges.

In the definition of the first variant of super-octonions I followed the standard idea about what super-coordinates assuming that the super-part of super-octonion is just an anti-commuting Grassmann number without any structure: I just replaced o with $o + \theta_k E^k + \overline{\theta}_k E^k$ regarding θ_k as anticommiting coordinates. Now θ_k receives octonionic coefficient: $\theta_k \rightarrow o_k \theta_k$. θ_k is now analogous to unit vector.

For the super-number field inspired formulation the situation is different since one assigns independent octonionic coordinates to anticommuting degrees of freedom. One has linear space with partially anti-commutative basis. O_c is effectively replaced with O_c^3 so that one has 8+8+8=24-dimensional Cartesian product (it is amusing that the magic dimension 24 for physical polarizations of bosonic string models emerges).

What is the number of equations in the new picture? For N super-coordinates one has 2^N separate monomials analogous to many-fermion states. Now one has $N = 8 + 8 = 16$ and this gives 2^{16} monomials! In the general case $RE = 0$ or $IM = 0$ gives 4 equations for each of the $d_G = 2^{16}$ monomials: the number of equations $RE = 0$ or $IM = 0$ is 4×2^{16} and exceeds the number $d_O = 24$ of octonion valued coordinates. In the original interpretation these equations were regarded as independent and gave different space-time variety for each many-fermion state.

In the new framework these equations cannot be treated independently. One has 24 octonionic coordinates and 2^{16} equations. In the generic case there are no solutions. This is actually what one hopes since otherwise one would have a state involving superposition of many-fermion states with several fermion numbers.

The freedom to pose constraints on the coefficients of Grassmann parameters however allows to reduce degrees of freedom. All coefficients must be however expressible as products of $3 \times 8 = 24$ components of super-octonion.

1. One can have solutions for which both 8_c part and $\overline{8}_c$ parts vanish. This gives the familiar 4 equations for 8 variables and 4-surfaces.
2. Consider first options, which fail. If 8_c- or $\overline{8}_c$ part vanishes one has $d_G = 2^8$ and $4 \times d_G = 4 \times 64$ equations for $dO_= 8 + 8 = 16$ variables having no solutions in the generic case. The restriction of 8_c to its 4-D quaternionic sub-space would give $d_O = 4$ and $4d_G = 4 \times 2^4 = 64$ conditions and 16 variables. The reduction to complex sub-space $z_1 + z_2 I$ of super-octonions would give $d_O = 2^2$ and $4 \times 2^2 = 16$ conditions for $8 + 2 = 10$ variables.
3. The restriction to 1-D sub-space of super-octonions would give $4 \times 2^1 = 8$ conditions and $8 + 1 = 9$ variables. Could the solution be interpreted as 1-D fermionic string assignable to the space-like boundary of space-time surface at the boundary of CD? Skeptic inside me asks whether this could mean the analog of $\mathcal{N} = 1$ SUSY, which is not consistent with H picture.

   Second possibility is restriction to light-like subspace for which powers of light-like octonion reduce effectively to powers of real coordinate. Fermions would be along light-lines in M^8 and along light-like curves in H. The powers of super-octonion have super-part, which belongs to the 1-D super-space in question: only single fermion state is present besides scalar state.
4. There are probably other solutions to the conditions but the presence of fermions certainly forces a localization of fermionic states to lower-dimensional varieties. This is what happens also in H picture. During years the localization of fermion to string worlds sheets and their boundaries has popped up again and again from various arguments. Could one hope that super-number theory provides the eventual argument.

   But how could one understand string world sheets in this framework? If they do not carry fermions at H-level, do they appear naturally as 2-D structures in the ordinary sense?

To sum up, although many details must be checked and up-dated, super-number theory provides and extremely attractive approach promising ultimate emergence as a reduction of physics to the notion of number. When physical theory leads to a discovery of new mathematics, one must take it seriously.

5.3.3 About physical interpretation

Super-octonionic algebraic geometry should be consistent with the H picture in which baryon and lepton numbers as well as other standard model quantum numbers can be understood. There are still many details, which are not properly understood.

The interpretation of theta parameters

The interpretation of theta parameters is not completely straightforward.

1. The first interpretation is that θ_c and $\theta_{\overline{c}}$ correspond to objects with opposite fermion numbers. If this is not the case, one could perhaps define the conjugate of super-coordinate as octonionic conjugate $\overline{o}_s = \overline{o} + \overline{\theta}_1 + \overline{\theta}_2$. This looks ugly but cannot be excluded.

   There is also the question about spinor property. Octonionic spinors are 2-spinors with octonion valued components. Could one say that the coefficients of octonion units have been replaced with Grassmann numbers and the entire 2-component spinor is represented as a pair of θ_c and $\theta_{\overline{c}}$? The two components of spinor in massless theories indeed correspond to massless particle and its antiparticle.

2. One should obtain particles and antiparticles naturally as also separately conserved baryon and lepton numbers (I have also considered the identification of hadrons in terms of anyonic bound states of leptons with fractional charges).

   Quarks and leptons have different coupling to the induced Kähler form at the level of H. It seems impossible to understand this at the level of M^8, where the dynamics is purely algebraic and contains no gauge couplings.

   The difference between quarks and leptons is that they allow color partial waves with triality $t = \pm 1$ and triality $t = 0$. Color partial waves correspond to wave functions in the moduli space CP_2 for $M^4_0 \supset M^2_0$. Could the distinction between quarks and leptons emerge at the level of this moduli space rather than at the fundamental octonionic level? There would be no need for gauge couplings to distinguish between quarks and leptons at the level of M^8. All couplings would follow from the criticality conditions guaranteeing 4-D associativity for external particles (on mass shell states would be critical).

   If so, one would have only the super octonions and θ_c and $\theta_{\overline{c}}$ would correspond to fermions and antifermions with no differentiation to quarks or leptons. Fermion number conservation would be coded by the Grassmann algebra. Quantum classical correspondence (QCC) however suggests that it should be possible to distinguish between quarks and leptons already at M^8 level. Is it really enough that the distinction comes at the level of moduli space for CDs?

One can imagine also other options but they have their problems. Therefore this option will be considered in the sequel.

Questions about quantum numbers

The first questions relate to fermionic statistics.

1. Do super-octonions really realize fermionic statistics and how? The polynomials of super-octonions can have only finite degree in θ and θ_c. One an say that only finite number of fermions are possible at given space-time point. As found, the conditions $IM(P) = 0$ and $RE(P) = 0$ might allow only single fermion strings as solutions perhaps assignable to partonic 2-surfaces.

   Can one allow for given CD arbitrary number of this kind of points as the idea that identical fermions can reside at different points suggests? Or is the number of fermions finite for given CD or correspond to the highest degree monomial of θ and θ_c in P?

   Finite fermion number of CD looks somewhat disappointing at first. The states with high fermion numbers would be described in terms of Cartesian products just like in condensed matter physics. Note however that space-time varieties with different octonionic time axes must be in any case described in this manner. It seems possible to describe the interactions using super-space delta functions stating that the interaction occur only in the intersection points of the space-time surfaces. The delta function would have also super-part as in SUSYs.

2. As found, the theta degree effectively reduces to $d = 1$ for the pointlike solutions, which by above argument are the only possible solutions besides purely bosonic solutions. Only single fermion would be allowed at given point. I have already earlier considered the question whether the partonic 2-surfaces can carry also many-fermion states or not [L30, K87], and adopted the working hypothesis that fermion numbers are not larger than 1 for given wormhole throat, possibly for purely dynamical reasons. This picture however looks too limited. The

many fermion states might not however propagate as ordinary particles (the proposal has been that their propagator pole corresponds to higher power of p^2).

The M^8 description of particle quantum numbers should be consistent with H description.

1. Can octonionic super geometry code for the quantum numbers of the particle states? It seems that super-octonionic polynomials multiplied by octonionic multi-spinors inside single CD can code only for the electroweak quantum numbers of fundamental particles besides their fermion and anti-fermion numbers. What about color?

   As already suggested, color corresponds to partial waves in CP_2 serving as moduli space for $M^4_0 \supset M^2_0$. Also four-momentum and angular momentum are naturally assigned with the translational degrees for the tip of CD assignable with the fundamental particle.

2. Quarks and leptons have different trialities at H level. How can one understand this at M^8 level. Could the color triality of fermion be determined by the color representation assignable to the color decomposition of octonion as $8 = 1 + 1 + 3 + \overline{3}$. This decomposition occurs for all 3 terms in the super-octonion. Could the octet in question correspond to the term $D(8 \otimes 8_c; 8_c)^{mn}_k o_{c,m}\theta_{c,n}E^k$ and analogous $\theta_{\overline{c}}$ term in super octonion. Only this kind of term survives from the entire super-octonion polynomial at fermionic string for the solutions found.

3. There is however a problem: $8 = 1 + 1 + 3 + \overline{3}$ decomposition is not consistent with the idea that θ_c and $\theta_{\overline{c}}$ have definite fermion numbers. Quarks appear only as 3, not $\overline{3}$. Why $\overline{3}$ from θ term and 3 from $\theta_{\overline{c}}$ term should drop out as allowed single fermion state?

There are also other questions.

1. What about twistors in this framework? $M^4 \times CP_1$ as twistor space with CP_1 coding for the choice of $M^2_0 \subset M^4_0$ allows projection to the usual twistor space CP_3. Twistor wave functions describing spin elegantly would correspond to wave functions in the twistor space and one expects that the notion of super-twistor is well-defined also now. The 6-D twistor space $SU(3)/U(2) \times U(1)$ of CP_2 would code besides the choice of $M^4_0 \supset M^2_0$ also quantization axis for color hypercharge and isospin.

2. The intersection of space-time surfaces with S^6 giving analogs of partonic 2-surfaces might make possible for two sparticle lines to fuse to form a third one at these surfaces. This would define sparticle 3-vertex in very much the same manner as in twistor Grassmann approach to $\mathcal{N} = 4$ SUSY.

   H-picture however supports the alternative option that sparticles just scatter but there is no contact interaction defining analog of 3-vertex. If the lines can carry only single fermion, the H picture about twistor diagrams [L30, K87] would be realized also at the level of M^8! This means breaking of SUSY since only single fermion states from the octonionic SUSY multiplet are realized. This would provide and easy - perhaps too easy - explanation for the failure to find SUSY at LHC.

3. What about the sphere S^6 serving as the moduli space for the choices of M^8_+? Should one have wave functions in S^6 or can one restrict the consideration to single M^8_+? As found, one obtains S^6 also as the zero locus of $Im(P) = 0$ for some radii identifiable as values t_n of time coordinates given as roots of $P(t)$: as matter of fact, $S^6(t_n)$ is a solution of both $RE(P) = 0$ and $IM(P) = 0$. Can one identify the intersections $X^4 \cap S^6$ are 2-D as partonic 2-surfaces serving as topological vertices?

5.4 Could scattering amplitudes be computed in the octonionic framework?

Octonionic algebraic geometry might provide incredibly simple framework for constructing scattering amplitudes since now variational principle is involved and WCW reduces to a discrete set of points in extension of rationals.

5.4.1 Could scattering amplitudes be computed at the level of M^8?

It would be extremely nice if the scattering amplitudes could be computed at the octonionic level by using a generalization of twistor approach in ZEO finding a nice justification at the level of M^8. Something rather similar to $\mathcal{N} = 4$ twistor Grassmann approach suggests itself.

1. In ZEO picture one would consider the situation in which the passive boundary of CD and members of state pairs at it appearing in zero energy state remain fixed during the sequence of state function reductions inducing stepwise drift of the active boundary of CD and change of states at it by unitary U-matrix at each step following by a localization in the moduli space for the positions of the active boundary.

2. At the active boundary one would obtain quantum superposition of states corresponding to different octonionic geometries for the outgoing particles. Instead of functional integral one would have sum over discrete points of WCW. WCW coordinates would be the coefficients of polynomial P in the extension of rationals. This would give undefined result without additional constraints since rationals are a dense set of reals.

   Criticality however serves as a constraint on the coefficients of the polynomials and is expected to realize finite measurement resolution, and hopefully give a well defined finite result in the summation. Criticality for the outgoing states would realize purely number theoretically the cutoff due to finite measurement resolution and would be absolutely essential for the finiteness and well-definedness of the theory.

5.4.2 Interaction vertices for space-time surfaces with the same CD

Consider interaction vertices for space-time surfaces associated with given CD. At the level of H the fundamental interactions vertices are partonic 2-surfaces at which 3 light-like partonic orbits meet. The incoming light-like sparticle lines scatter at this surface and they are not assumed to meet at single vertex. This assumption is motivated because it allows to avoid infinities but one must be ready to challenge it. It is essential that wormhole throats appear in pairs assignable to wormhole contacts and also contacts form pairs by the conservation of Kähler magnetic flux.

What could be the counterpart of this picture at level of M^8?

1. The simplest interaction could be associated with the common stable intersection points of the space-time regions. By dimensional consideration these intersections are stable and form a discrete set. This would however allow only 2-vertices involved in processes like mixing of states. In the generic case the intersection would consist of discrete points.

2. A stronger condition would be that these points belong to the extension of rationals defining adeles or is extension defined by the polynomial P. This would conform with the idea that scattering amplitudes involve only data associated with the points in the extension. The interaction points could be ramified points at which the action of a subgroup H of Galois group G would leave sheets of the Galois covering invariant so that some number of sheets would touch each other. I have discussed this proposal in [L43]. These points could be seen as analogs of interaction points in QFT description in terms of n-point functions and the sum over polynomials would give rise to the analog over integral over different n-point configurations.

3. A possible interpretation is that if the subgroup $H \subset G$ has k-elements the vertex represents meeting of k sparticle lines and thus k-vertex would be in question. This picture is not what the H view about twistor diagrams [K87] suggests: in these diagrams sparticle lines at the light-like orbits of partonic 2-surfaces do not meet at single point but only scatter at partonic 2-surface, where three light-like orbits of partonic 2-surfaces meet.

4. An alternative interpretation is that k-vertex describes the decay of particle to k fractional particles at partonic 2-surfaces and has nothing do with the usual interaction vertex.

This proposal need not describe usual particle scattering. Could the intersection of space-time varieties defined as zero loci for $RE(P_i)$ and $IM(P_i)$ with the special solutions $S^6(t_n)$ and $CD = M^4_+ \cap M^4_-$ define the loci of interaction? It is difficult to believe that these special solutions could be only a beauty spot of the theory. $X^2 = X^4 \cap S^6(t_n)$ is 2-D and $X^0 = X^4 \cap CD$ consists of discrete points.

Consider now the possible role of the singular ($RE(P) = IM(P) = 0$) maximally critical surface $S^6(t_n)$ in the scattering.

1. As already found, the 6-D spheres S^6 with radii t_n given by the zeros of $P(t)$ are universal and have interpretation as $t = t_n$ snapshots of 7-D spherical light front projection to $t = t_n$ 3-balls as cross sections of 4-D CD. Could the 2-D intersection $X^2 = X^4 \cap S^6(t_n)$ play a fundamental role in the description of interaction vertices?

2. Suppose that 3-vertices realize the dynamical realization of octonionic SUSY predicting large number of sparticles. Could one understand in this framework the 3-vertex for the orbits X_i^3 of partonic 2-surfaces meeting each other along their 2-D end defining partonic 2-surface and undersand how 3 fermions lines meet at single point in this picture?

3. Assume that 3 partonic orbits X_i^3, $i = 1, 2, 3$ meet at $X^2 = X^4 \cap S^6(t_n)$. That this occurs could be part of boundary conditions, which should follow from interaction consistency. If fermions just *go through* the X_i^2 in time direction they cannot meet at single point in the generic case. If the sparticle lines however can *move along* X^2 - maybe due the fact that an intersection $X^2 = X^4 \cap S^6(t_n)$ is in question - they intersect in the generic case and fuse to a third fermion line. Note that this portion of fermion line would be space-like whereas outside X^2 the line would be light-like. This can be used as an objection against the idea.

The picture allowing 3-vertices would be different from H picture in which fermion lines only scatter and only 2+2 fermion vertex assignable to topological 3-vertex is fundamental.

1. One would have 2 wormhole contacts carrying fermion and third one carrying fermion anti-fermion pair at its opposite throats and analogous to boson. Of course, one can reproduce the earlier picture by giving up the condition about supersymmetric fermionic 3-vertex. On the other hand, the idea that interactions occur only at discrete points in extension of rationals is extremely attractive.

2. The surprising outcome from the construction of solutions of super-variants of $RE(P) = 0$ and $IM(P) = 0$ conditions was that if the superpart of super-octonion is non-vanishing, the variety can be only 1-D string like entity carrying one-fermion state. This does allow strings with higher fermion number so that the 3-vertex would not be possible! This suggests that fermionic lines appear as sub-varieties of space-time variety.

   If so the original picture [K87] applying at the level of H applies also at the level of M^8. SUSY is broken dynamically allowing only single fermion states localized at strings and scattering of these occurs by classical interactions at the partonic 2-surfaces defining the topological vertices.

3. The only manner to have a point/line containing sparticle with higher fermion number would be as a singularity along which several branches of super-variety degenerate to single point/line: each variety would carry one fermion line. Unbroken octonionic SUSY would characterize singularities of the space-time varieties, which would be unstable so that SUSY would break. Singularities are indeed critical and thus unstable and also tend to possess enhanced symmetries.

What could be the interpretation of $X^0 = X^4 \cap CD$? For instance, could it be that these points code for 4-momenta classically so that quantum classical correspondence (QCC) would be realized also at the level of M^8 although there are no Noether charges now. But what about angular momenta? Could twistorialization realized in terms of the quaternionic structure of M_0^4 help here. What is the role of the intersections of 6-D twistor bundle of X^4 with 6-D twistor bundle of M_0^4 consisting of discrete points?

The interaction vertex would involve delta function telling that the interacting space-time varieties or their regions touch at same point of M^8. Delta function in theta parameter degrees of freedom and Grassmann integral over them would be also involved and guarantee fermion number conservation. Vertex factor should be determined by arguments used in Grassmannian twistor approach. I have developed a proposal in [K87] but this proposal allows only fermion number ± 1 at fermion lines. Now all members of the multiplet would be allowed.

5.4.3 How could the space-time varieties associated with different CDs interact?

The interaction of space-time surfaces inside given CD is well-defined in the octonionic algebraic geometry. The situation is not so clear for different CDs for which the choice of the origin of octonionic coordinates is in general different and polynomial bases for different CDs do not commute nor associate.

The intuitive expectation is that 4-D/8-D CDs can be located everywhere in M^4/M^8. The polynomials with different origins neither commute nor are associative. Their sum is a polynomial whose coefficients are not real. How could one avoid losing the extremely beautiful associative and commutative algebra of polynomials?

1. Should one assume that the physics observable by single conscious observer corresponds to single CD defining the perceptive field of this observer [L54].
2. Or should one give up associativity and allow products (but not sums since one should give up the assumption that the coefficients of polynomials are real) of polynomials associated with different CDs as an analog for the formation of free many-particle states.

Consider first what happens for the single particle solutions defined as solutions of either $RE(P_i) = 0$ or $IM(P_i) = 0$.

1. The polynomials associated with different 8-D CDs do not commute nor associate. Should one allow their products so that one would still *effectively* have a Cartesian product of commutative and associative algebras? This would realize non-commutative and non-associative physics emerging in conformal field theories also at the level of space-time geometry.
2. If the CDs differ by a *real* (time) translation $o_2 = o_1 + T$ one still obtains $IM(P_1) = 0$ and $IM(P_2) = 0$ as solutions to $IM(P_1P_2) = 0$. This applies also to states with more particles. The identification would be in terms of external particles. For $RE(P_1P_2) = 0$ this is not the case. If the interior of CD corresponds to $RE(P_1P_2) = 0$, the dynamics in the interior is not only non-trivial but also non-commutative and non-associative. Non-trivial interaction would be obtained even without interaction terms in the polynomial vanishing at the boundaries of CD!

   Could one consider allowing only CDs with tips at the same real axis but having all sizes scales? This hierarchy of CD would characterize a particular hierarchy of conscious observers - selves having sub-selves (sub-CDs) [L54]. The allowance of only these CD would be analogous to a fixing of quantization axes.
3. What happens if one allows CDs differing by arbitrary octonion translation? Consider external particles. For P_1 and P_2 RE and IM are defined for different decompositions $o_i = RE(o_i) + n_iIm(o_i)$, where n_i, $i = 1, 2$ is a unit octonion.

   What decomposition should one use for P_1P_2? The decomposition for P_1 or P_2 or some other decomposition? One can express $P_2(o_2)$ using o_1 as coordinate but the coefficients multiplying powers of o_1 from *right* would not be real numbers anymore implying $IM(P_2)_1 \neq IM(P_2)_2$. $IM(P_2)_1 = 0$ makes sense but the presence of particle 1 would have affected particle 2 or vice versa.

   Could one argue that the coordinate systems satisfying the condition that some external particles described by P_i have real coefficients and perhaps serving in the role of observers are preferred? Or could one imagine that o_{12} is a kind of center of mass coordinate? In this case the 4-varieties associated with both particles would be affected. What is clear that the choice of the octonionic coordinate origin would affect the space-time varieties of external particles even if they could remain associative/critical.
4. Are there preferred coordinates in which criticality is preserved? For instance, can one achiever criticality for P_2 on coordinates of o_1 if P_1 is critical. Could one see this as a kind of number theoretic observer effect at the level of space-time geometry?

   Remark: $P_i(o)$ would reduce to a real polynomial at light-like rays with origin for o_i irrespective of the octonionic coordinate used so that the spheres S^6_i with origin at the origin of o_i as solutions of $P_i(o) = 0$ would not be lost.

If one does not give up associativity and commutativity for polynomials, how can one describe the interactions between space-time surfaces inside different CDs at the level of M^8? The following proposal is the simplest one that one can imagine by assuming that interactions take place at discrete points of space-time surfaces with coordinates belonging an extension of rationals.

1. The most straightforward manner would be to introduce Cartesian powers of O and CD:s inside these powers to describe the interaction between CDs with different origin. This would be analogous to what one does in condensed matter physics. What seems clear is that M^8-H correspondence should map all the factors of $(M^8)^n$ to the same $M^4 \times CP_2$ by a kind of diagonal projection.

   In topological 3-parton vertex X^2 three light-like partonic orbits along X^4 would meet. X^2 would be the contact of X^4 with S^6 associated with second 8-D CD. Together with SH this gives hopes about an elegant description of interactions in terms of connected space-time varieties.

2. The intersection $X_i^4 \cap X_j^4$ consists of discrete set of points. This would suggest that the interaction means transfer of fermion between X_1^4 and X_2^4. The intersection of $X = S_1^6(t_m) \cap S_2^6(t_n)$ is 4-D and space-like. The intersection $X_i^4 \cap X$ consists of discrete points could these discrete points allow to construct interaction vertices.

To make this more concrete, assume that the external particles outside the interaction CD (CD_{int}) defining the interaction region correspond to associative (or co-associative) space-time varieties with different CDs.

Remark: CDs are now 8-dimensional.

1. One can assign the external particles to the Cartesian factors of $(M^8)^n$ giving $(P_1, ..., P_n)$ just like one does in condensed matter physics for particles in 3-space E^3. Inside CD_{int} the Cartesian factors would fuse to single factor and instead of Cartesian product one would have the octonionic product $P = \prod P_i$ plus the condition $RE(P) = 0$ (or $IM(P) = 0$: one should avoid too strong assumptions at this stage) would give to the space-time surface defining the interaction region.

2. $RE(P) = 0$ and $IM(P) = 0$ conditions make sense even, when the polynomials do not have origin at common real axis and give rise to 4 conditions for 8 polynomials of 8 complexified octonion components P^i. It is not possible to reduce the situation at the light-like boundaries of 8-D light-cone to a vanishing of polynomial $P(t)$ of real coordinate t anymore, and one loses the the surfaces S_i^6 as special solutions and therefore also the partonic 2-surfaces $X_i^2 = X^4 \cap S_i^6$. Should one assign all X_i^2 with the intersections of external particles with the two boundaries $\delta_\pm$ CD of CD defining the interaction region. They would intersect $\delta_\pm$CD at highly unique discrete points defining the sparticle interaction vertices. By 7-dimensionality of $\delta_\pm$CD the intersection points would be at the boundaries of 4-D CD and presumably at light-like partonic orbits at which the induced metric is singular at H side at least just as required by H picture.

   The most general external single-sparticle state would be defined by a product P of mutually commuting and associating polynomials with tips of CD along common real axis and satisfying $IM(P_i) = 0$ or $RE(P_i) = 0$. This could give both free and bound states of constituents.

3. Different orders and associations for $P = \prod P_i$ give rise to different interaction regions. This requires a sum over the scattering amplitudes $\sum_p T(\prod_i P_{p(i)})$ associated with the permutations p: $(1, ..., n) \rightarrow (p(1), .., p(n))$ and $T = \sum_p U(p)T(P_{p(1)}...P_{p(n)})$ ($T(AB) + T(BA)$ in the simplest case) with suitable phase factors $U(p)$. Note that one does *not* have a sum over the polynomials $P_{p(1)}...P_{p(n)}$ but over the scattering amplitudes associated with them.

4. Depending on the monomial of theta parameters in super-octonion part of P_i, one has plus or minus signs under the exchange of P_i and P_j. One can also have braid group as a lift of the permutation group. In this case given contribution to the scattering amplitude has a phase factor depending on the permutation (say $T = T(AB) + exp(i\theta)T(BA)$.

   One must also form the sum $T = \sum_{Ass} U(Ass)T(Ass(P))$ over all associations for a given permutation with phase factors $U(Ass)$. Here $T = T((AB)C) + UT(A(BC))$, U phase factor, is the simplest case. One has "association statistics" as the analog of braid statistics. Permutations and associations have now a concrete geometric meaning at the level of space-time geometry - also at the level of H.

5. The geometric realization of permutations and associations could relate to the basic problem encountered in the twistorial construction of the scattering amplitudes. One has essentially sum over the cyclic permutations of the external particles but does not know how to construct the amplitudes for general permutations, which correspond to non-planar Feynman diagrams. The geometric realization of the permutations and associations would solve this problem in TGD framework.

5.4.4 Twistor Grassmannians and algebraic geometry

Twistor Grassmannians provide an application of algebraic geometry involving the above described notions [B22] (see `http://tinyurl.com/yd9tf2ya`). This approach allows extremely elegant expressions for planar amplitudes of $\mathcal{N} = 4$ SYM theory in terms of amplitudes formulated in Grassmannians $G(k,n)$.

It seems that this approach generalizes to TGD in such a way that CP_2 degrees of freedom give rise to additional factors in the amplitudes having form very similar to the M^4 part of amplitudes and involving also $G(k,n)$ with ordinary twistor space CP_3 being replaced with the flag manifold $SU(3)/U(1)\times U(1)$: k would now correspond to the number sparticles with negative weak isospin. Therefore the understanding of the algebraic geometry of twistor amplitudes could be helpful also in TGD framework.

Twistor Grassmannian approach very concisely

I try to compress my non-professional understanding of twistor Grassmann approach to some key points.

1. Twistor Grassmannian approach constructs the scattering amplitudes by fusing 3-vertices (+,-,-) (one positive helicity) and (-,+,+) (one negative helicity) to a more complex diagrams. All particles are on mass shell and massless but complex. If only real massless momenta are allowed the scattering amplitudes would allow only collinear gluons. Incoming particles have real momenta.

   Remark: Remarkably, $M^4\times CP_2$ twistor lift of TGD predicts also complex Noether charges, in particular momenta, already at classical level. Quantal Noether charges should be hermitian operators with real eigenvalues, which suggests that total Noether charges are real. For conformal weights this condition corresponds to conformal confinement. Also M^8-H duality requires a complexification of octonions by adding commuting imaginary unit and allows to circumvent problems related to the Minkowski signature since the metric tensor can be regarded as Euclidian metric tensor defining complex value norm as bilinear $m^k m_{kl} m^l$ in complexified M^8 so that real metric is obtained only in sub-spaces with real or purely imaginary coordinates. The additional imaginary unit allows also to define what complex algebraic numbers mean.

   The unique property of 3-vertex is that the twistorial formulation for the conservation of four-momentum implies that in the vertex one has either $\lambda_1 \propto \lambda_2 \propto \lambda_3$ or $\overline{\lambda}_1 \propto \overline{\lambda}_2 \propto \overline{\lambda}_3$. These cases correspond to the 2 3-vertices distinguished notationally by the color of the vertex taken to be white or black [B22].

   Remark: One must allow octonionic super-space in M^8 formulation so that octonionic SUSY broken by CP_2 geometry reducing to the quaternionicity of 8-momenta in given scattering diagram is obtained.

2. The conservation condition for the total four-momentum is quadratic in twistor variables for incoming particles. One can linearize this condition by introducing auxiliary Grassmannian $G(k,n)$ over which the tree amplitude can be expressed as a residue integral. The number theoretical beauty of the multiple residue integral is that it can make sense also p-adically unlike ordinary integral.

   The outcome of residue integral is a sum of residues at discrete set of points. One can construct general planar diagrams containing loops from tree diagrams with loops by BCFW recursion. I have considered the possibility that BCFW recursion is trivial in TGD since coupling constants should be invariant under the addition of loops: the proposed scattering diagrammatics however assumed that scattering vertices reduce to scattering vertices for 2 fermions. The justification for renormalization group invariance would be number theoretical: there is no guarantee that infinite sum of diagrams gives simple function defined in all number fields with parameters in extension of rationals (say rational function).

3. The general form of the Grassmannian integrand in $G(k,n)$ can be deduced and follows from Yangian invariance meaning that one has conformal symmetries and their duals which expand to full infinite-dimensional Yangian symmetry. The denominator of the integrand of planar tree diagram is the product of determinants of $k \times k$ minors for the $k \times n$ matrix providing representation of a point of $G(k,n)$ unique apart from $SL(k,k)$ transformations. Only minors consisting of k consecutive columns are assumed in the product. The residue integral is determined by the poles of the denominator. There are also dynamical singularities allowing the amplitude to be non-vanishing only for some special configurations of the external momenta.

4. On mass-shell diagrams obtained by fusing 3-vertices are highly redundant. One can describe the general diagram by using a disk such that its boundary contains the external particles with positive or negative helicity. The diagram has certain number n_F of faces. There are moves, which do not affect the amplitude and it is possible to reduce the number of faces to minimal one: this gives what is called reduced diagram. Reduced diagrams with n_F faces define a unique $n_F - 1$-dimensional sub-manifold of $G(k,n)$ over which the residue integral can be defined. Since the dimension of $G(k,n)$ is finite, also n_F is finite so that the number of diagrams is finite.

5. On mass shell diagrams can be labelled by the permutations of the external lines. This gives a connection with 1+1-dimensional QFTs and with braid group. In 1+1-D integral QFTs however scattering matrix induces only particle exchanges.

   The permutation has simple geometric description: one starts from the boundary point of the diagram and moves always from left or right depending on the color of the point from which one started. One arrives some other point at the boundary and the final points are different for different starting points so that the process assigns a unique perturbation for a given diagram. Diagrams which are obtained by moves from each other define the same permutation. BFCW bridge which is a way to obtain new Yangian invariant corresponds to a permutation of consecutive external particles in the diagram.

6. The poles of the denominator determine the value of the multiple residue integrals. If one allowed all minors, one would have extremely complex structure of singularities. The allowance only cyclically taken minors simplifies the situation dramatically. Singularities correspond to n subgroups of more than 2 collinear k-vectors implying vanishing of some of the minors.

7. Algebraic geometry comes in rescue in the understanding of singularities. Since residue integral is in question, the choice is rather free and only the homology equivalence class of the cell decomposition matters. The poles for a hierarchy with poles inside poles since given singularity contains sub-singularities. This hierarchy gives rise to a what is known as cell composition - stratification - of Grassmannian consisting of varieties with various dimensions. These sub-varieties define representatives for the homology group of Grassmannian. Schubert cells already mentioned define this kind of stratification.

   Remark: The stratification has very strong analogy of the decomposition of catastrophe in Thom's catastrophe theory to pieces of various dimensions. The smaller the dimension, the higher the criticality involved. A connection with quantum criticality of TGD is therefore highly suggestive.

   Cyclicity implies a reduction of the stratification to that for positive Grassmannians for which the points are representable as $k \times n$ matrices with non-negative $k \times k$ determinants. This simplifies the situation even further.

   Yangian symmetries have a geometric interpretation as symmetries of the stratification: level 1 Yangian symmetries are diffeomorphisms preserving the cell decomposition.

Problems of twistor approach

Twistor approach is extremely beautiful and elegant but has some problems.

1. The notion of twistor structure is problematic in curved space-times. In TGD framework the twistor structures of M^4 and CP_2 (E^4) induce twistor structure of space-time surface and the problem disappears just like the problems related to classical conservation laws are circumvented. Complexification of octonions allows to solve the problems related to the metric signature in twistorialization.
2. The description of massive particles is a problem. In TGD framework M^8 approach allows to replace massive particles with particles with octonionic momenta light-like in 8-D sense belonging to quaternionic subspace for a given diagram. The situation reduces to that for ordinary twistors in this quaternionic sub-space but since quaternionic sub-space can vary, additional degrees of freedom bringing in CP_2 emerge and manifest themselves as transversal 8-D mass giving real mass in 4-D sense.
3. Non-planar diagrams are also a problem. In TGD framework a natural guess is that they correspond to various permutations of free particle octonionic polynomials. Their product defines interaction region in the interior of CD to which free particles satisfying associativity conditions (quantum criticality) arrive. If the origins of polynomials are not along same time axis, the polynomials do not commute nor associate. One must sum over their permutations and for each permutation over its associations.

5.4.5 About the concrete construction of twistor amplitudes

At H-side the ground states of super-conformal representations are given by the anti-symmetrized products of the modes of H-spinor fields labelled by four-momentum, color quantum numbers, and electroweak (ew) quantum numbers. At partonic 2-surface one has finite number of many fermion states. Single fermion states are assigned with H-spinor basis and the fermion states form a representation of a finite-D Clifford algebra.

M^8 picture should reproduce the physical equivalent of H picture: in particular, one should understand four-momentum, color quantum numbers, ew quantum numbers, and B and L. M^8-H correspondence requires that the super-twistorial description of scattering amplitudes in M^8 is equivalent with that in H.

The M^8 picture is roughly following.

1. The ground states of super-conformal representations expressible in terms of spinor modes of H correspond at level of M^8 wave functions in super variant of the product $T(M^4)\times T(CP_2)$ of twistor spaces of M^4 and CP_2. This twistor space emerges naturally in M^8-H correspondence from the quaternionicity condition for 8-momenta.
2. Bosonic M^8 degrees of freedom translate to wave functions in the product $T(M^4)\times T(CP_2)$ labelled by four-momentum and color. Super parts of the M^4 and CP_2 twistors code for spin and ew degrees of freedom and fermion numbers. Only a finite number of spin-ew spin states is possible for a given fundamental particle since one has finite-D Grassmann algebra.
3. Contrary to the earlier expectations [K87], the view about scattering diagrams is very similar to that in $\mathcal{N}=4$ SUSY. The analog of 3-gluon vertex is fundamental and emerges naturally from number theoretic vision in which scattering diagrams defines a cognitive representation and vertices of the diagram correspond to fusion of sparticle lines.

Identification of H quantum numbers in terms of M^8 quantum numbers

The first challenge is to understand how M^8-H correspondence maps M^8 quantum numbers to H quantum numbers. At the level of M^8 one does not have action principle and conservation laws must follow from the properties of wave functions in various moduli spaces assignable to 4-D and 8-D CDs that is quaternion and octonion structures. The symmetries of the moduli spaces would dictate the properties of wave functions.

There are three types of symmetries and quantum numbers.

1. WCW quantum numbers

At level of H the quantum numbers in WCW "vibrational" degrees of freedom are associated with the representations of super-symplectic group acting as isometries of WCW. Super-symplectic generators correspond to Hamiltonians labelled by color and angular momentum quantum numbers for $SU(3)\times SO(3)$. In $M^4_\pm$ there are also super-symplectic conformal weights assignable to the radial light-coordinate in $\delta M^4_\pm$. These conformal weights could be complex and might relate closely to the zeros of Riemann zeta [K41]. Physical states should however have integer valued conformal weights (conformal confinement).

At the level of M^8 WCW "vibrational"degrees of freedom are discrete and correspond to the degree of the octonionic polynomial P and its coefficients in the extension of rationals considered. WCW integration reduces to a discrete sum, which should be well-defined by the criticality conditions on the coefficients of the polynomials. M^8-H correspondence guarantees that 4-varieties in M^8 are mappable to space-time surfaces in H. Therefore also quantum numbers should be mappable to each other.

There are also spinorial degrees of freedom associated with WCW spinors with spin-like quantum numbers assignable to fermionic oscillator operators labelled by spin, ew quantum numbers, fermion numbers, and by super-symplectic conformal weights.

2. Quantum numbers assignable to isometries of H.

These quantum numbers are special assignable to the ground states of the representations of Kac-Moody algebras associated with light-like partonic orbits.

1. The isometry group of H consists of Poincare group and color group for CP_2. M^8 isometries correspond to $8-D$ Poincare group. Only G_2 respects given octonion structure and 8-D Lorentz transformations transform to each other different octonion structures. Quantum numbers consist of 8-momentum and analogs of spin and ew spin. M^8-H correspondence is non-trivial since one must map light-like quaternionic 8-momenta to 4-momenta and color quantum numbers.

2. There are quantum numbers assignable to cm spinor degrees of freedom. They correspond for both M^8 and H to 8-D spinors and give rise to spin and ew quantum numbers. For these quantum numbers M^8-H correspondence is trivial. At the level of H baryon and lepton numbers are assignable to the conserved chiralities of H-spinors.

Quantum classical correspondence (QCC) is a key piece of TGD.

1. At the level of H QCC states that the eigenvalues of the fermionic Noether charges are equal to the classical bosonic Noether charges in Cartan algebra implies that fermionic quantum number as also ew quantum numbers and spin have correlates at the level of space-time geometry.

2. A the level of M^8 QCC is very concrete. Both bosonic and superpart of octonions have the decomposition $1+\overline{1}+3+\overline{3}$ under color rotations. Each monomial of theta parameters characterizes one particular many-fermion state containing leptons/antileptons and quarks/antiquarks. Leptons/antileptons are assignable to complexified octonionic units $(1\pm iI_1)/\sqrt{2}$ defining preferred octonion plane M_2 and quarks/antiquarks are assignable to triplet and antitriplet, which also involve complexified octonion units. One obtains breaking of SUSY in the sense that space-time varieties assignable to different theta monomials are different (one can argue that the sum $8_s+\overline{8}_s$ can be regarded as real).

   Purely leptonic and antileptonic varieties correspond to 1 and $\overline{1}$ and quark and antiquark varieties to 3 and $\overline{3}$ and the monomial transforms as a tensor product of thetas. The monomial has well defined quark and lepton numbers and the interpretation is that it characterizes fundamental sparticle. At the level of H this kind of correspondence follows form QCC.

3. Also super-momentum leads to a characterization of spin and fermion numbers of the state since delta function expressing conservation of super-momentum codes the supersymmetry for scattering amplitudes and gives rise to vertices conserving fermion numbers. Does this mean QCC in the sense that the super parts of super-momentum and super twistor should be associated with space-time varieties with same fermion and spin content?

How the light-like quaternionic 8-momenta are mapped to H quantum numbers?

The key challenge is to understand how the light-like quaternionic 8-momenta are mapped to massive M^4 momenta and color quantum numbers.

1. One has wave function in the space of CP_2 quaternionic four-momenta. M^4_0 momentum can be identified as M^2_0 projection and in general massive unless M^2_0 and M^4_0 are chosen so that the light-like M^8 momentum belongs to M^2_0. The situation is analogous to that in the partonic description of hadron scattering.

   The space of quaternionic sub-spaces $M^4_0 \supset M^2_0$ with this property is parameterized by CP_2, and one obtains color partial waves. The inclusion of the choice of quantization axis extends this space to $T(CP_2) = SU(3)/U(1) \times U(1)$. Without quaternionicity/associativity condition the space of momenta would correspond to M^8.

   The wave functions in the moduli space for the position of the tip of CD and for the choice $M^2_0 \supset M^4_0$ specifying M^4_0 twistor structure and choice of quantization axis of spin correspond to wave functions in the twistor space CP_3 of $M^4_\pm$ coding for momentum and spin.

   Remark: The inclusion of M^4 spin quantization axis characterized by the choice of M^2_0 extends M^4_0 to geometric twistor space $T(M^4) = M^4_0 \times S^2 \supset M^2_0$ having bundle projection to CP_3. Twistorialization means essentially the inclusion of the choice of various quantization axis as degrees of freedom. This space is for symmetry group G the space G/H, where H is the Cartan sub-group of G. This description might make sense also at the level of super-symplectic and super-Kac-Moody symmetries.

2. Ordinary octonionic degrees of freedom for super-octonions in M^8 must be mapped to $M^4 \times CP_2$ cm degrees of freedom. Super octonionic parts should correspond to fermionic and spin and electroweak degrees of freedom. The space of super-twistorial states should same as the space of the super-symplectic grounds states describable in terms H-spinor modes.

3. One has wave function in the moduli space of CDs. The states in M^8 are labelled by quaternionic super-momenta. Bosonic part must correspond to four-momentum and color and super-part to spin and ew quantum numbers of CP_2. This part of the moduli space wave function is characterized by the spin and ew spin quantum numbers of the fundamental particle. Wave functions in the super counterpart of $T(M^4) \times T(CP_2)$ allow to characterize these degrees of freedom without the introduction of spinors and should correspond to the ground states of super-conformal representations in H.

It seems that H-description is an abstract description at the level moduli spaces and M^8 description for single space-time variety represents reduction to the primary level, where number theory dictates the dynamics.

Octonionic twistors and super-twistors

How to define octonionic twistors? Or is it enough to identify quaternionic/associative twistors as sub-spaces of octonionic twistors?

1. Ordinary twistors and super-twistors

Consider first how ordinary twistors and their super counterparts could be defined, and how they could allow an elegant description of spin and ew quantum numbers as quantum numbers analogous to angular momenta.

1. Ordinary twistors are defined as pairs of 2-spinors giving rise to a representation of four-momentum. The spinors are complex spinors transforming as a doublet representation of SL(2,C) and its conjugate.

   The 2-spinors are related by incidence relation, a linear condition in which M^4 coordinates represented as 2×2 matrix appears linearly [K87]. The expression of four-momentum is bilinear in the spinors and invariant under complex scalings of the 2-spinors compensating each other so that instead of 8-D space one has actually 6-D space, which reduces to CP_3 to which the geometric twistor space $M^4 \times S^2$ has a projection.

2. For light-like four-momenta p the determinant of the matrix having the two 2-spinors as rows and representing p as a point of M^4 vanishes. Wave functions in CP_3 allow to describe spin in terms of bosonic wave function. What is so beautiful is that this puts particles with different spin in a democratic position.

Super-twistors allow to integrate the states constructible as many-fermion states of $\mathcal{N}$ elementary fermions in the same representations involving several spins. The many-fermion states - sparticles - are in 1-1-correspondence with Grassmann algebra basis.

3. The description of massless particles in terms of M^4 (super-)twistors is elegant but one encounters problems in the case of massive particles [K100, L30, K87].

2. Octonionic twistors at the level of M^8?

How to define octonionic twistors at the level of M^8?

1. At the level of M^8 one has light-like 8-momenta. The M^4 momentum identified as M_0^4 projection can there be massive. This solves the basic problem of the standard twistor approach.

2. The additional assumption is that the 8-momenta in given vertex of scattering diagram belong to the same quaternionic sub-space $M_0^4 \subset M^8$ satisfying $M_0^4 \supset M_0^2$. This effectively transforms momentum space $M^4 \times E^4$ to $M^4 \times CP_2$. A stronger condition is that all momenta in a given diagram belong to the same sub-space $M_0^4 \supset M_0^2$.

   Remark: Quaternionicity implies that the 8-momentum is time-like or light-like if one requires that quaternionicity for an arbitrary choice of the octonionic structure (the action of 8-D Poincare group gives rise transforms octonionic structures to each other).

3. Complex 2-spinors are replaced with complexified octonionic spinors which must be consistent quaternionicity condition for 8-momenta. A good guess is that the spinors belong to a quaternionic sub-space of octonions too. This is expected to transform them effectively to quaternionic spinors. Without effective quaternionicity the number of 2-spinor components would be 8 rather than 4 times larger than for ordinary 2-spinors.

   Remark: One has complexified octonions (i commutes with the octonionic imaginary units E_k).

4. Octonionic/quaternionic twistors should be pairs of octonionic/quaternionic 2-spinors determined only modulo octonionic/quaternionic scaling. If quaternionicity holds true, the number of 2-spinor components is 4 times larger than usually. Does this mean that one has basically quaternionic twistors plus moduli space CP_2 for $M_0^4 \supset M_0^2$. One should be able to express octonionic twistors as bi-linears formed from 2 octonionic/quaternionic 2-spinors. Octonionic option should give the octonionic counterpart OP_3 of Grassmannian CP_3, which does not however exist.

   Remark: Octonions allow only projective plane OP_2 as the octonionic counterpart of CP_2 (see `http://tinyurl.com/ybwaeu2s`) but do not allow higher-D projective spaces nor Grassmannians (see `http://tinyurl.com/ybm8ubef`, whereas reals, complex numbers, and quaternions do so. The non-existence of Grassmannians for rings obtained by Cayley-Dickson construction could mean that $M^8 - H$ correspondence and TGD do not generalize beyond octonions.

   Does the restriction to quaternionic 8-momenta the Grassmannians to be quaternionic (subspaces of octonions). This would give quaternionic counterpart HP_3 of CP_3. Quaternions indeed allow projective spaces and Grassmannians and (see `http://tinyurl.com/y9htjstc` and `http://tinyurl.com/y87gpq8l`).

   Remark: One can wonder whether non-commutativity forces to distinguish between left- and right Grassmannians (points as lines $\{c(q_1,..,q_n)|c \in H\}$ or as lines as lines $\{(q_1,..,q_n)c|c \in H\}$.

5. Concerning the generalization to octonionic case, it is crucial to realize that the 2×2-matrix representing four-momentum as a pair 2-spinor can be regarded as an element in the subspace of complexified quaternions. The representation of four-momentum would be as sum of $p_8 = p_1^k \sigma_k + I_4 p_2^k \sigma_k$, where I_4 octonionic imaginary unit orthogonal to σ_k representing quaternionic units.

   No! The twistorial representation of the 4-momentum is already quaternionic! Choosing the decomposition of M^8 to quaternionic sub-space and its complement suitably, one has $IM(p_8) = 0$ for quaternionic 8-momenta and one obtains standard representation of 4-momentum in this sub-space! The only new element is that one has now moduli specifying the quaternionic sub-space. If the sub-space contains a fixed M_0^2 one obtains just CP_2 and ordinary twistor codes for the choices of M_0^2. If the choice of color quantization axes matters as it indeed does, one has twistor space $SU(3)/U(1)\times U(1)$ instead of CP_2. This would suggest that ordinary representation of scattering amplitudes reduces apart from the presence of CP_2 twistor to the usual representation.

One can hope for a reduction to ordinary twistors and projective spaces, moduli space CP_2 for quaternion structures, and moduli space for the choices of real axis of octonion structures. One can even consider the possibility [K87] of using standard M_0^2 with the property that M^8 momentum reduces to M_0^2 momentum and coding the information about real M_0^2 to moduli. This could reduce the twistor space to $RP(3)$ associated with M_0^2 is considered and solve the problems related to the signature of M^4. Note however that the complexification of octonions in any case allow to regard the metric as Euclidian albeit complexified so that these problems should disappear.

3. Octonionic super-twistors at the level of M^8?

Should one generalize the notion of super-twistor to octonionic context or can one do by using only the moduli space and the fact that octonionic geometry codes for various components of octonion as analog of super-field? It seems that super-twistors are needed.

1. It seems that super-twistors are needed. Octonionic super-momentum would appear in the super variant of momentum conserving delta function resulting in the integration over translational moduli. In twistor Grassmann approach this delta function is super-twistorialized and this leads to the amazingly simple expressions for the scattering amplitudes.
2. At the level of M^8 one should generalize ordinary momentum to super-momentum and perform super-twistorialization. Different monomials of theta parameters emerging from super part of momentum conserving delta function (for $\mathcal{N}=1$ one has $\delta(\theta-\theta_0)=exp(i\theta-\theta_0)/i)$ correspond to different spin states of the super multiplet and anti-commutativity guarantees correct statistics. At the level of H the finite-D Clifford algebra of 8-spinors at fixed point of H gives states obtained as monomials or polynomials for the components of super-momentum in M^8.
3. Octonionic super-momentum satisfying quaternionicity condition can be defined as a combination of ordinary octonionic 8-momentum and super-parts transforming like 8_s and $\overline{8}_s$. One can express the octonionic super-momentum as a bilinear of the super-spinors defining quaternionic super-twistor. Quaternionicity is assumed at least for the octonionic super-momenta in the same vertex. Hence the M^4 part of the super-twistorialization reduces to that in SUSYs and one obtains standard formulas. The new elements is the super-twistorialization of $T(CP_2)$.

   Remark: Octonionic SUSY involving $8+8_s+\overline{8}_s$ would be an analog of $\mathcal{N}=8$ SUSY associated with maximal supergravity (see `http://tinyurl.com/nv3aajy`) and in M^4 degrees of freedom twistorialization should be straightforward.

   The octonionic super-momentum belongs to a quaternionic sub-space labelled by CP_2 point and corresponds to a particular sub-space M_0^2 in which it is light-like (has no other octonionic components). M_0^2 is characterized by point of S^2 point of twistor space $M^4\times S^2$ having bundle projection to CP_3.
4. That the twistor space $T(CP_2)=SU(3)/U(1)\times U(1)$ coding for the color quantization axes rather than only CP_2 emerges must relate to the presence of electroweak quantum numbers related to the super part of octonionic momentum. Why the rotations of $SU(2)\times U(1)\subset SU(3)$ have indeed interpretation also as tangent space-rotations interpreted as electroweak rotations. The transformations having an effect on the choice of quantization axies are parameterized by S^2 relating naturally to the choice of $SO(4)$ quantization axis in E^4 and coded by the geometric twistor space $T(E^4)=E^4\times S^2$.
5. Since the super-structure is very closely related to the construction of the exterior algebra in the tangent space, super-twistorialization of $T(CP_2)$ should be possible. Octonionic triality could be also in a key role and octonionic structure in the tangent space of $SU(3)$ is highly suggestive. $SU(3)$ triality could relate to the octonionic triality.

$SU(3)/U(1) \times U(1)$ is analogous with the ordinary twistor space CP_3 obtained from C^4 as a projective space. Now however $U(1) \times U(1)$ instead of group of complex scalings would define the equivalence classes. Generalization of projective space would be in question. The super-part of twistor would be obtained as $U(1) \times U(1)$ equivalence class and gauge choice should be possible to get manifestly 6-D representation. One can ask whether the CP_2 counterparts of higher- D Grassmannians appear at the level of generalized twistor diagrams: could the spaces $SU(n)/G$, H Cartan group correspond to these spaces?

4. How the wave functions in super-counterpart of $T(CP_2)$ correspond to quantum states in CP_2 degrees of freedom?

In CP_2 spinor partial waves have vanishing triality $t = 0$ for leptonic chirality and $t = \pm 1$ for quarks and antiquarks. One can say that the triality $t \neq 1$ states are possible thanks to the anomalous hypercharge equal to fractional electromagnetic charge $Y_A = Q_{em}$ of quarks: this gives also correlation between color quantum numbers and electroweak quantum numbers which is wrong for spinor partial waves. The super-symplectic and super Kac-Moody algebras however bring in vibrationals degrees of freedom and one obtains correct quantum number assignments [K60].

This mechanism should have a counterpart at the level of the super variant of the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$. The group algebra of $SU(3)$ gives the scalar wave functions for all irreps of $SU(3)$ as matrix elements. Allowing only matrix elements that are left- or right invariant under $U(1) \times U(1)$ one obtains all irreps realized in $T(CP_2)$ as scalar wave functions. These representations have $t = 0$. The situation would be analogous for scalar functions in CP_2. One must however obtain also electroweak quantum numbers and $t \neq 0$ colored states. Here the octonionic algebraic geometry and superpart of the $T(CP_2)$ should come in rescue. The electroweak degrees of freedom in CP_2 should correspond to the super-parts of twistors.

The $SU(3)$ triplets assignable to the triplets 3 and $\overline{3}$ of space-time surfaces would make possible also the $t = \pm 1$ states. Color would be associated with the octonionic geometry. The simplest possibility would be that one has just tensor products of the triplets with $SU(3)/U(1) \times U(1)$ partial waves. In the case of CP_2 there is however a correlation between color partial waves and electroweak quantum numbers and the same is expected also now between super-part of the twistor and geometric color wave function: minimum correlation is via $Y_A = Q_{em}$. The minimal option is that the number theoretic color for the octonionic variety modifies the transformation properties of $T(CP_2)$ wave function only by a phase factor due to $Y_A = Q_{em}$ as in the case of CP_2.

The most elegant outcome would be that super-twistorial state basis in $T(M^4) times T(CP_2)$ is equivalent with the state basis defined by super-symplectic and super Kac-Moody representations in H.

About the analogs of twistor diagrams

There seems to be a strong analogy with the construction of twistor amplitudes in $\mathcal{N} = 4$ SUSY [B21, B43, B26] and one can hope of obtaining a purely geometric analog of SUSY with dynamics of fields replaced by the dynamics of algebraic super-octonionic surfaces.

1. Number theoretical vision leads to the proposal that the scattering amplitudes involve only data at discrete points of the space-time variety belonging to extension of rationals defining cognitive representation. The identification of these points has been already considered in the case of partonic orbits entering to the partonic 2-vertex and for the regions of space-time surfaces intersecting at discrete set of points. Scattering diagrams should therefore correspond to polygons with vertices of polygons defining cognitive representation and lines assignable to the external fundamental particles with given quark and lepton numbers having correlates at the level of space-time geometry. This occurs also in twistor Grassmannian approach [B21, B43, B26].

   Since polynomials determine space-time surfaces, this data is enough to determine the space-time variety completely. Indeed, the zeros of $P(t)$ determining the space-time variety give also rise to a set of spheres $S^6(t_n)$ and partonic 2-surfaces $X^2(t_n) = X^4 \cap S^6(t_n)$, where t_n is root of $P(t)$. The discretization need not mean a loss of information. The scattering amplitudes would be expressible as an analog of n-point function with points having coordinates in the extension of rationals.

2. (Super) octonion as"field"in X^4 is dynamically analogous to (super) gauge potentials and super-octonion to its super variant. (Super) gauge potentials are replaced with M^8 (super-) octonion coordinate and gauge interactions are geometrized. Here I encounter a problem with terminology. Neither sparticle nor sboson sounds good. Hence I will talk about sparticles.

3. The amplitude for a given space-time variety contains no information M^8-momentum. M^8-momentum emerges as a label for a wave function in the moduli space of 4-D and 8-D CDs involving both translational and orientational degrees of freedom. For fixed time axis the orientational degrees of freedom reduce to rotational degrees of freedom identifiable in terms of the twistor sphere S^2. The delta functions expressing conservation of 8-D quaternionic super-momentum in M^8 coming from the integration over the moduli space of 8-D translations.

   As found, quaternionicity of 8-momenta implies that standard M^4 twistor description of momenta applies but one obtains CP_2 twistors as additional contribution. This is of course what one would intuitively expect.

   8-D momentum conservation in turn translates to the conservation of momentum and color quantum numbers in the manner described. The amplitudes in momentum and color degrees of freedom reduce to kinematics as in SUSYs. It is however not clear whether one should also perform number theoretical discretization of various moduli spaces.

   In any case, it seems that all the details of the scattering amplitudes related to moduli spaces reduce to symmetries and the core of calculations reduces to the construction of space-time varieties as zero loci of octonionic polynomials and identification of the points of the 4-varieties in extension of rationals. Classical theory would indeed be an exact part of the quantum theory.

4. Quaternionic 8-D light-likeness reduces the situation to the level of ordinary complex and thus even positive (real) Grassmannians. This is crucial from the p-adic point of view. CP_2 twistors characterizes the moduli related to the choice of quaternionic sub-space, where 8-momentum reduces to ordinary 4-momentum. M^4 parts of the scattering amplitudes in twistor Grassmann approach should be essentially the same as in $\mathcal{N} = 4$ SUSY apart from the replacement of super degrees of freedom with super-octonionic ones. The challenge is to generalize the formalism so that it applies also to CP_2 twistors. The challenge would be to generalize the formalism so that it applies also to CP_2 twistors. The M^4 and CP_2 degrees of freedom are expected to factorize in twistorial amplitudes. A good guess is that the scattering amplitudes are obtained as residue integrals in the analogs of Grassmannians associated with $T(CP_2)$. Could one have Grassmannians also now?

   Consider the formula of tree amplitude for n gluons with k negative helicities conjectured Arkani-Hamed *et al* in the twistor Grassmannian approach [B26]. The amplitude follows from the twistorial representation for momentum conservation and is equal to an $k \times n$-fold multiple residue integral over the complex variables $C_{\alpha a}$ defining coordinates for Grassmannian $Gl(n,k)$ and reduces to a sum over residues. The integrand is the inverse for the product of all $k \times k$ minors of the matrix $C_{\alpha a}$ in cyclic order and the resides corresponds to zeros for one or more minors. This part does not depend on twistor variables. The dependence on n twistor variables comes from the product $\prod_{\alpha=1}^{k} \delta(C_{\alpha a} W^a)$ of k delta functions related to momentum conservation. W^a denotes super-twistors in the 8-D representation, which is linear. One has projective invariance and therefore a reduction to $T(M^4) = CP_3 = SU(4)/SU(3) \times U(1)$.

   Could this formula generalize almost as such to $T(CP_2)$ and come from the conservation of E^4 momentum? One has n sparticles to which super-twistors in $T(CP_2)$ are assigned. The first guess is that the sign of helicity are replaced by the sign of electroweak isospin - essentially E^4 spin at the level of M^8. For electromagnetic charge identified as the analog of helicity one would have problems in the case of neutrinos. $T(M^4) = CP_3 = SU(4)/SU(3) \times U(1)$ is replaced with $T(CP_2) = SU(3)/U(1) \times U(1)$. $T(CP_2)$ does not have a representation as a projective space but there is a close analogy since the group of complex scalings is replaced with $U(1) \times U(1)$. The (apparent) linearity is lost but one represent the points of $T(CP_2)$ as exponentials of $su(3)$ Lie-algebra elements with vanishing $u(1) \times u(1)$ part. The resulting 3 complex coordinates are analogous to two complex CP_2 coordinates. The basic difference between M^4 and CP_2 degrees of freedom would come from the exponential representation of twistors.

5. By Yangian invariance one should obtain very similar formulas for the amplitudes except that one has instead of $\mathcal{N} = 4$ SUSY $\mathcal{N} = 8$ octonionic SUSY analogous to $\mathcal{N} = 8$ SUGRA.

Trying to understand the fundamental 3-vertex

Due to its unique twistorial properties as far as realization of four-momentum conservation is considered 3-vertex is fundamental in the construction of scattering diagrams in twistor Grassmannian approach to $\mathcal{N} = 4$ SYM [B22] (see `http://tinyurl.com/yd9tf2ya`). Twistor Grassmann approach suggests that 3-vertex with complexified light-like 8-momenta represents the basic building brick representing from which more complex diagrams can be constructed using the BCFW recursion formula [B22]. In TGD 3-vertex generalized to 8-D light-like quaternionic momenta should be highly analogous to the 4-D 3-vertex and in a well-defined sense reduce to it if all momenta of the diagram belong to the same quaternionic sub-space M_0^4. It is however not completely clear how 3-vertex emerges in TGD framework.

1. A possible identification of the 3-vertex at the level of M^8 would be as a vertex at which 3 sparticle lines with light-like complexified quaternionic 8-momenta meet. This vertex would be associated with the partonic vertex $X^2(t_n) = X^4 \cap S^6(t_n)$. Incoming sparticle lines at the light-like partonic orbits identified as boundaries of string world sheets (for entangled states at least) would be light-like.

   Does the fusion of two sparticle lines to third one require that either or both fusing lines become space-like - say pieces of geodesic line inside the Euclidian space-time region- bounded by the partonic orbit? The identification of the lines of twistor diagrams as carriers of light-like complexified quaternionic momenta in 8-D sense does not encourage this interpretation (also classical momenta are complex). Should one pose the fusion of the light-like lines as a boundary condition? Or should one give up the idea that sparticle lines make sense inside interaction region?

2. As found, one can challenge the assumption about the existence of string world sheets as commutative regions in the non-associative interaction region. Could one have just fermion lines as light-like curves at partonic orbits inside CD? Or cannot one have even them?

   Even if the polynomial $\prod_i P_i$ defining the interaction region is product of polynomials with origins of octonionic coordinates not along the same real line, the 7-D light-cones of M^8 associated with the particles still make sense in the sense that $P_i(o_i) = 0$ reduces at it to $P_i(t_i) = 0$, t_i real number, giving spheres $S^6(t_i(n))$ and partonic 2-surfaces and vertices $X_2(t_i(n))$. The light-like curves as geodesics the boundary of 7-D light-cones mapped to light-like curves along partonic orbits in H would not be lost inside interaction regions.

3. At the level of H this relates to a long standing interpretational problem related to the notion of induced spinor fields. SH suggests strongly the localization of the induced spinor fields at string world sheets and even at sparton lines in absence of entanglement. Super-conformal symmetry however requires that induced spinor fields are 4-D and thus seems to favor de-localization. The information theoretic interpretation is that the induced spinor fields at string world sheets or even at sparton lines contain all information needed to construct the scattering amplitudes. One can also say that string world sheets and sparton lines correspond to a description in terms of an effective action.

Could the M^8 view about twistorial scattering amplitudes be consistent with the earlier H picture?

The proposed M^8 picture involving super coordinates of M^8 and super-twistors does not conform with the earlier proposal for the construction of scattering amplitudes at the level of H [K87]. In H picture the introduction of super-space does not look natural, and one can say that fundamental fermions are the only fundamental particles [L30, K87]. The H view about super-symmetry is as broken supersymmetry in which many fermion states at partonic 2-surfaces give rise to supermultiplets such that fermions are at different points. Fermion 4-vertex would be the fundamental vertex and involve classical scattering without fusion of fermion lines. Only a redistribution of fermion and anti-fermion lines among the orbits of partonic 2-surfaces would take place in scattering and one would have kind of OZI rule.

Could this H view conform with the recent M^8 view much closer to the SUSY picture. The intuitive idea without a rigorous justification has been that the fermion lines at partonic 2-surfaces correspond to singularities of many-sheeted space-time surface at which some sheets co-incide. M^8 sparticle consists effectively of n fermions at the same point in M^8. Could it be mapped by M^8-H duality to n fermions at distinct locations of partonic 2-surface in H?

$M^8 - H$ correspondence maps the points of $M^4 \subset M^4 \times E^4$ to points of $M^4 \subset M^4 \times CP_2$. The tangent plane of space-time surface containing a preferred M^2 is mapped to a point of CP_2. If the effective n-fermion state M^8 is at point at which n sheets of space-time surface co-incide and if the tangent spaces of different sheets are not identical, which is quite possible and even plausible, the point is indeed mapped to n points of H with same M^4 coordinates but different CP_2 coordinates and sparticle would be mapped to a genuine many-fermion state. But what happens to scalar sparticle. Should one regard it as a pure gauge degree of freedom in accordance with the chiral symmetry at the level of M^8 and H?

5.5 From amplituhedron to associahedron

Lubos has a nice blog posting (see `http://tinyurl.com/y7ywhxew`) explaining the proposal represented in the newest article by Nima Arkani-Hamed, Yuntao Bai, Song He, Gongwang Yan [**?**]see `http://tinyurl.com/ya8zstll`). Amplituhedron is generalized to a purely combinatorial notion of associahedron and shown to make sense also in string theory context (particular bracketing). The hope is that the generalization of amplituhedron to associahedron allows to compute also the contributions of non-planar diagrams to the scattering amplitudes - at least in $\mathcal{N} = 4$ SYM. Also the proposal is made that color corresponds to something less trivial than Chan-Paton factors.

The remaining problem is that 4-D conformal invariance requires massless particles and TGD allows to overcome this problem by using a generalization of the notion of twistor: masslessness is realized in 8-D sense and particles massless in 8-D sense can be massive in 4-D sense.

In TGD non-associativity at the level of arguments of scattering amplitude corresponds to that for octonions: one can assign to space-time surfaces octonionic polynomials and induce arithmetic operations for space-time surface from those for polymials (or even rational or analytic functions). I have already earlier [L46] demonstrated that associahedron and construction of scattering amplitudes by summing over different permutations and associations of external particles (space-time surfaces). Therefore the notion of associahedron makes sense also in TGD framework and summation reduces to "integration" over the faces of associahedron. TGD thus provides a concrete interpretation for the associations and permutations at the level of space-time geometry.

In TGD framework the description of color and four-momentum is unified at the level and the notion of twistor generalizes: one has twistors in 8-D space-time instead of twistors in 4-D space-time so Chan-Paton factors are replaced with something non-trivial.

5.5.1 Associahedrons and scattering amplitudes

The following describes briefly the basic idea between associahedrons.

Permutations and associations

One starts from a non-commutative and non-associative algebra with product (in TGD framework this algebra is formed by octonionic polynomials with real coefficients defining space-time surfaces as the zero loci of their real or imaginary parts in quaternionic sense. One can indeed multiply space-time surface by multiplying corresponding polynomials! Also sum is possible. If one allows rational functions also division becomes possible.

All permutations of the product of n elements are in principle different. This is due to non-commutativity. All associations for a given ordering obtained by scattering bracket pairs in the product are also different in general. In the simplest case one has either $a(bc)$ or $(ab)c$ and these 2 give different outcomes. These primitive associations are building bricks of general associations: for instance, abc does not have well-defined meaning in non-associative case.

If the product contains n factors, one can proceed recursively to build all associations allowed by it. Decompose the n factors to groups of m and $n-m$ factors. Continue by decomposing these two groups to two groups and repeat until you have have groups consisting of 1 or two elements.

You get a large number of associations and you can write a computer code computing recursively the number $N(n)$ of associations for n letters.

Two examples help to understand. For $n = 3$ letters one obviously has $N(n = 3) = 2$. For $n = 4$ one has $N(4) = 5$: decompose first $abcd$ to $(abc)d$, $a(bcd)$ and $(ab)(cd)$ and then the two 3 letter groups to two groups: this gives $N(4) = 2 + 2 + 1 = 5$associations and associahedron in 3-D space has therefore 5 faces.

Geometric representation of association as face of associahedron

Associations of n letters can be represented geometrically as so called Stasheff polytope (see http://tinyurl.com/q9ga785). The idea is that each association of n letters corresponds to a face of polytope in $n - 2$-dimensional space with faces represented by the associations.

Associahedron is constructed by using the condition that adjacent faces (now 2-D polygons) intersecting along common face (now 1-D edges). The number of edges of the face codes for the structure particular association. Neighboring faces are obtained by doing minimal change which means replacement of some $(ab)c$ with $a(bc)$ appearing in the association as a building bricks or vice versa. This means that the changes are carried out at the root level.

How does this relate to particle physics?

In scattering amplitude letters correspond to external particles. Scattering amplitude must be invariant under permutations and associations of the external particles. In particular, this means that one sums over all associations by assigning an amplitude to each association. Geometrically this means that one "integrates" over the boundary of associahedron by assigning to each face an amplitude. This leads to the notion of associahedron generalizing that of amplituhedron.

Personally I find it difficult to believe that the mere combinatorial structure leading to associahedron would fix the theory completely. It is however clear that it poses very strong conditions on the structure of scattering amplitudes. Especially so if the scattering amplitudes are defined in terms of "volumes" of the polyhedrons involved so that the scattering amplitude has singularities at the faces of associahedron.

An important constraint on the scattering amplitudes is the realization of the Yangian generalization of conformal symmetries of Minkowski space. The representation of the scattering amplitudes utilizing moduli spaces (projective spaces of various dimensions) and associahedron indeed allows Yangian symmetries as diffeomorphisms of associahedron respecting the positivity constraint. The hope is that the generalization of amplituhedron to associahedron allows to generalize the construction of scattering amplitudes to include also the contribution of non-planar diagrams of at $\mathcal{N} = 4$ SYM in QFT framework.

5.5.2 Associations and permutations in TGD framework

Also in the number theoretical vision about quantum TGD one encounters associativity constraings leading to the notion of associahedron. This is closely related to the generalization of twistor approach to TGD forcing to introduce 8-D analogs of twistors [L46] (see http://tinyurl.com/yd43o2n2).

Non-associativity is induced by octonic non-associativity

As found in [L46], non-associativity at the level of space-time geometry and at the level of scattering amplitudes is induced from octonionic non-associativity in M^8.

1. By $M^8 - H$ duality ($H = M^4 \times CP_2$) the scattering are assignable to complexified 4-surfaces in complexified M^8. Complexified M^8 is obtained by adding imaginary unit i commutating with octonionic units I_k, $k = 1, , .., 7$. Real space-time surfaces are obtained as restrictions to a Minkowskian subspace complexified M^8 in which the complexified metric reduces to real valued 8-D Minkowski metric. This allows to define notions like Kähler structure in Minkowskian signature and the notion of Wick rotations ceases to be ad hoc concept. Without complexification one does not obtain algebraic geometry allowing to reduces the dynamics defined by partial differential equations for preferred extremals in H to purely algebraic

conditions in M^8. This means huge simplications but the simplicity is lost at the QFT-GRT limit when many-sheeted space-time is replaced with slightly curved piece of M^4.

2. The real 4-surface is determined by a vanishing condition for the real or imaginary part of octonionic polynomial with $RE(P)$ and $IM(P)$ defined by the composition of octonion to two quaternions: $o = RE(o) + I_4 IM(o)$, where I_4 is octonionic unit orthogonal to a quaternionic sub-space and $RE(o)$ and $IM(o)$ are quaternions. The coefficients of the polynomials are assumed to be real. The products of octonionic polynomials are also octonionic polynomials (this holds for also for general power series with real coefficients (no dependence on I_k. The product is not however neither commutative nor associative without additional conditions. Permutations and their associations define different space-time surfaces. The exchange of particles changes space-time surface. Even associations do it. Both non-commutativity and non-associativity have a geometric meaning at the level of space-time geometry!

3. For space-time surfaces representing external particles associativity is assumed to hold true: this in fact guarantees $M^8 - H$ correspondence for them! For interaction regions associativity does not hold true but the field equations and preferred extremal property allow to construct the counterpart of space-time surface in H from the boundary data at the boundaries of CD fixing the ends of space-time surface.

   Associativity poses quantization conditions on the coefficients of the polynomial determining it. The conditions are interpreted in terms of quantum criticality. In the interaction region identified naturally as causal diamond (CD), associativity does not hold true. For instance, if external particles as space-time surfaces correspond to vanishing of $RE(P_i)$ for polynomials representing particles labelled by i, the interaction region (CD) could correspond to the vanishing of $IM(P_i)$ and associativity would fail. At the level of H associativity and criticality corresponds to minimal surface property so that quantum criticality corresponds to universal free particle dynamics having no dependence on coupling constants.

4. Scattering amplitudes must be commutative and associative with respect to their arguments which are now external particles represented by polynomials P_i This requires that scattering amplitude is sum over amplitudes assignable to 4-surfaces obtained by allowing all permutations and all associations of a given permutation. Associations can be described combinatorially by the associahedron!

   Remark:. In quantum theory associative statistics allowing associations to be represented by phase factors can be considered (this would be associative analog of Fermi statistics). Even a generalization of braid statistics can be considered.

Yangian variants of various symmetries are a central piece also in TGD although supersymmetries are realized in different manner and generalized to super-conformal symmetries: these include generalization of super-conformal symmetries by replacing 2-D surfaces with light-like 3-surfaces, supersymplectic symmetries and dynamical Kac-Moody symmetries serving as remnants of these symmetries after supersymplectic gauge conditions characterizing preferred extremals are applied, and Kac-Moody symmetries associated with the isometries of H . The representation of Yangian symmetries as diffeomorphisms of the associahedron respecting positivity constraint encourages to think that associahedron is a useful auxiliary tool also in TGD.

Is color something more than Chan-Paton factors?

Nima *et al* talk also about color structure of the scattering amplitudes usually regarded as trivial. It is claimed that this is actually not the case and that there is non-trivial dynamics involved. This is indeed the case in TGD framework. Also color quantum numbers are twistorialized in terms of the twistor space of CP_2, and one performs a twistorialization at the level of M^8 and $M^4 \times CP_2$. At the level of M^8 momenta and color quantum numbers correspond to associative 8-momenta. Massless particles are now massless in 8-D sense but can be massive in 4-D sense. This solves one of the basic difficulty of the ordinary twistor approach. A further bonus is that the choice of the embedding space H becomes unique: only the twistor spaces of S^4 (and generalized twistor space of M^4 and CP_2 have Kähler structure playing a crucial role in the twistorialization of TGD. To sum up, all roads lead to Rome. Everyone is well-come to Rome!

5.5.3 Questions inspired by quantum associations

Associations have (or seem to have) different meaning depending on whether one is talking about cognition or mathematics. In mathematics the associations correspond to different bracketings of mathematical expressions involving symbols denoting mathematical objects and operations between them. The meaning of the expression - in the case that it has meaning - depends on the bracketing of the expression. For instance, one has $a(b+c) \neq (ab)+c$, that is $ab+ac \neq ab+c$). Note that one can change the order of bracket and operation but not that of bracket and object.

For ordinary product and sum of real numbers one has associativity: $a(bc) = (ab)c$ and $a+(b+c) = (a+b)+c$. Most algebraic operations such as group product are associative. Associativity of product holds true for reals, complex numbers, and quaternions but not for octonions and this would be fundamental in both classical and quantum TGD.

The building of different associations means different groupings of n objects. This can be done recursively. Divide first the objects to two groups, divide these tow groups to two groups each, and continue until you jave division of 3 objects to two groups - that is abc divided into $(ab)c$ or $a(bc)$. Numbers 3 and 2 are clearly the magic numbers.

This inspire several speculative quetions related to the twistorial construction of scattering amplitudes as associative singlets, the general structure of quantum entanglement, quantum measurement cascade as formation of association, the associative structure of many-sheeted space-time as a kind of linguistic structure, spin glass as a strongly associative system, and even the tendency of social structures to form associations leading from a fully democratic paradise to cliques of cliques of ... .

1. In standard twistor approach 3-gluon amplitude is the fundamental building brick of twistor amplitudes constructed from on-shell-amplitudes with complex momenta recursively. Also in TGD proposal this holds true. This would naturally follow from the fact that associations can be reduced recursively to those of 3 objects. 2- and 3-vertex would correspond to a fundamental associations. The association defined 2-particle pairing (both associated particles having either positive or negative helicities for twistor amplitudes) and 3-vertex would have universal structure although the states would be in general decompose to associations.

2. Consider first the space-time picture about scattering [L46]. CD defines interaction region for scattering amplitudes. External particles entering or leaving CD correspond to associative space-time surfaces in the sense that the tangent space or normal space for these space-time surfaces is associative. This gives rise to $M^8 - H$ correspondence.

   These surfaces correspond to zero loci for the imaginary parts (in quaternionic sense) for octonionic polynomial with coefficients, which are real in octonionic sense. The product of $\prod_i P_i)$ of polynomials with same octonion structure satisfying $IM(P_i) = 0$ has also vanishing imaginary part and space-time surface corresponds to a disjoint union of surfaces associated with factors so that these states can be said to be non-interacting.

   Neither the choice of quaternion structure nor the choice of the direction of time axis assignable to the octonionic real unit need be same for external particles: if it is the particles correspond to same external particle. This requires that one treats the space of external particles (4-surfaces) as a Cartesian product of of single particle 4-surfaces as in ordinary scattering theory.

   Space-time surfaces inside CD are non-associative in the sense that the neither normal nor tangent space is associative: $M^8 - M^4 \times CP_2$ correspondence fails and space-time surfaces inside CD must be constructed by applying boundary conditions defining preferred extremals. Now the real part of $RE(\prod_i P_i)$ in quaternionic sense vanishes: there is genuine interaction even when the incoming particles correspond to the same octonion structure since one does not have union of surfaces with vanishing $RE(P_i)$. This follows from s rather trivial observation holding true already for complex numbers: imaginary part of zw vanishes if it vanishes for z and w but this does not hold true for the real part. If octonionic structures are different, the interaction is present irrespective of whether one assumes $RE(\prod_i P_i) = 0$ or $IM(\prod_i P_i) = 0$. $RE(\prod_i P_i) = 0$ is favoured since for $IM(\prod_i P_i) = 0$ one would obtain solutions for which $IM(P_i) = 0$ would vanish for the i:th particle: the scattering dynamics would select i:th particle as non-interacting one.

3. The proposal is that the entire scattering amplitude defined by the zero energy state - is associative, perhaps in the projective sense meaning that the amplitudes related to different associations relate by a phase factor (recall that complexified octonions are considered), which could be even octonionic. This would be achieved by summing over all possible associations.
4. Quantum classical correspondence (QCC) suggests that in ZEO the zero energy states - that is scattering amplitudes determined by the classically non-associative dynamics inside CD - form a representation for the non-associative product of space-time surfaces defined by the condition $RE(\prod_i P_i) = 0$. Could the scattering amplitude be constructed from products of octonion valued single particle amplitudes. This kind of condition would pose strong constraints on the theory. Could the scattering amplitudes associated with different associations be octonionic - may be differing by octonion-valued phase factors - and could only their sum be real in octonionic sense (recall that complexified octonions involving imaginary unit i commuting with the octonionic imaginary units are considered)?

One can look the situation also from the point of view of positive and negative energy states defining zero energy states as they pairs.

1. The formation of association as subset is like formation of bound state of bound states of ... . Could each external line of zero energy state have the structure of association? Could also the internal entanglement associated with a given external line be characterized in terms of association.

   Could the so called monogamy theorem stating that only two-particle entanglement can be maximal correspond to the decomposing of $n = 3$ association to one- and two-particle associations? If quantum entanglement is behind associations in cognitive sense, the cognitive meaning of association could reduce to its mathematical meaning.

   An interesting question relates to the notion of identical particle: are the many-particle states of identical particles invariant under associations or do they transform by phase factor under association. Does a generalization of braid statistics make sense?
2. In ZEO based quantum measurement theory the cascade of quantum measurements proceeds from long to short scales and at each step decomposes a given system to two subsystems. The cascade stops when the reduction of entanglement is impossible: this is the case if the entanglement probabilities belong to an extension of extension of rationals characterizing the extension in question. This cascade is nothing but a formation of an association! Since only the state at the second boundary of CD changes, the natural interpretation is that state function reduction mean a selection of association in 3-D sense.
3. The division of n objects to groups has also social meaning: all social groups tend to divide into cliques spoiling the dream about full democracy. Only a group with 2 members - Romeo and Julia or Adam and Eve - can be a full democracy in practice. Already in a group of 3 members 2 members tend to form a clique leaving the third member outside. Jules and Catherine, Jim and Catherine, or maybe Jules and Jim! Only a paradise allows a full democracy in which non-associativity holds true. In ZEO it would be realized only at the quantum critical external lines of scattering diagram and quantum criticality means instability. Quantum superposition of all associations could realize this democracy in 4-D sense.

A further perspective is provided by many-sheeted space-time providing classical correlate for quantum dynamics.

1. Many-sheeted space-time means that physical states have a hierarchical structure - just like associations do. Could the formation of association (AB) correspond basically to a formation of flux tube bond between A and B to give AB and serve as space-time correlate for (negentropic) entanglement. Could ((AB)C) would correspond to (AB) and (C) "topologically condensed" to a larger surface. If so, the hierarchical structure of many-sheeted space-time would represent associations and also the basic structures of language.
2. Spin glass (see `http://tinyurl.com/y9yyq8ga`) is a system characterized by so called frustrations. Spin glass as a thermodynamical system has a very large number of minima of free energy and one has fractal energy landscape with valleys inside valleys. Typically there is a competition between different pairings (associations) of the basic building bricks of the system.

Could spin glass be describable in terms of associations? The modelling of spin glass leads to the introduction of ultrametric topology characterizing the natural distance function for the free energy landscape. Interestingly, p-adic topologies are ultrametric. In TGD framework I have considered the possibility that space-time is like 4-D spin glass: this idea was originally inspired by the huge vacuum degeneracy of Kähler action. The twistor lift of TGD breaks this degeneracy but 4-D spin glass idea could still be relevant.

5.6 Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view

Gromov-Witten (G-W) invariants, Riemann-Roch theorem (RR), and Atyiah-Singer index theorem (AS) are applied in advanced algebraic geometry, and it is interesting to see whether they could have counterparts in TGD framework. The basic difference between TGD and conventional algebraic geometry is due to the adelic hierarchy demanding that the coefficients of polynomials involved are in given extension of rationals. Continuous moduli spaces are replaced with discrete ones by number theoretical quantization due to the criticality guaranteeing associativity of tangent or normal space. $M^8 - H$ duality brings in powerful consistency conditions: counting of allowed combinations of coefficients of polynomials on M^8 side and counting of dimensions on H side using AS should give same results. $M^8 - H$ duality might be in fact analogous to the mirror symmetry of M-theory.

5.6.1 About the analogs of Gromow-Witten invariants and branes in TGD

Gromow-Witten invariants, whose definition was discussed in [L47], play a central role in superstring theories and M-theory and are closely related to branes. For instance, partition functions can be expressed in terms of these invariants giving additional invariants of symplectic and algebraic geometries. Hence it is interesting to look whether they could be important also in TGD framework.

1. As such the definition of G-W invariants discussed in [L47] do not make sense in TGD framework. For instance, space-time surface is not a closed symplectic manifold whereas M^8 and H are analogs of symplectic spaces. Minkowskian regions of space-time surface have Hamilton-Jacobi structure at the level of both M^8 and H and this might replace the symplectic structure. Space-time surfaces are not closed manifolds.

   Physical intuition however suggests that the generalization exists. The fact that Minkowskian metric and Euclidian metric for complexified octonions are obtained in various sectors for which complex valued length squared is real suggests that signature is not a problem. Kähler form for complexified z gives as special case analog of Kähler form for E^4 and M^4.

2. The quantum intersection defines a description of interactions in terms of string world sheets. If I have understood G-W invariant correctly, one could have for $D > 4$-dimensional symplectic spaces besides partonic $2k-2$-D surfaces also surfaces with smaller but even dimension identifiable as branes of various dimensions. Branes would correspond to a generalization of relative cohomology. In TGD framework one has $2k = 4$ and the partonic 2-surfaces have dimension 2 so that classical intersections consisting of discrete points are possible and stable for string world sheets and partonic 2-surfaces. This is a unique feature of 4-D space-time.

   One might think a generalization of G-W invariant allowing to see string world sheets as connecting the spaced-like 3-surfaces at the boundaries of CDs and light-like orbits of partonic 2-surfaces. The intersection is not discrete now and marked points would naturally correspond to the ends points of strings at partonic 2-surfaces associated with the boundaries of CD and with the vertices of topological scattering diagrams.

3. The idea about 2-D string world sheet as interaction region could generalize in TGD to space-time surface inside CD defining 4-D interaction region. In [L48] one indeed ends up with amazingly similar description of interactions for n external particles entering CD and represented as zero loci for quaternion valued “real” part $RE(P)$ or “imaginary” part $IM(P)$ for the complexified octonionic polynomial.

Associativity forces quantum criticality posing conditions on the coefficients of the polynomials. Polynomials with the origin of octonion coordinate along the same real axis commute and associate. Since the origins are different for external particles in the general case, the polynomials representing particles neither commute nor associate inside the interaction region defined by CD but one can also now define zero loci for both $RE(\prod P_i)$ and $IM(\prod P_i)$ giving $P_i = 0$ for some i. Now different permutations and different associations give rise to different interaction regions and amplitude must be sum over all these.

3-vertices would correspond to conditions $P_i = 0$ for 3 indices i simultaneously. The strongest condition is that 3 partonic 2-surfaces X_i^2 co-incide: this condition does not satisfy classical dimension rule and should be posed as essentially 4-D boundary condition. Two partonic 2-surfaces $X_i^2(t_i(n))$ intersect at discrete set of points: could one assume that the sparticle lines intersect and there fusion is forced by boundary condition? Or could one imagine that partonic 2-surfaces turns back in time and second partonic 2-surface intersects it at the turning point?

4. In 4-D context string world sheets are associated with magnetic flux tubes connecting partonic orbits and together with strings serve as correlates for negentropic entanglement assignable to the p-adic sectors of the adele considered, to attention in consciousness theory, and to remote mental interactions in general and occurring routinely between magnetic body and biological body also in ordinary biology. This raises the question whether "quantum touch" generalizes from 2-D string world sheets to 4-D space-time surface (magnetic flux tubes) connecting 3-surfaces at the orbits and partonic orbits.

5. The above formulation applies to closed symplectic manifolds X. One can however generalize the formulation to algebraic geometry. Now the algebraic curve X^2 is characterized by genus g and order of polynomial n defining it. This formulation looks very natural in M^8 picture.

An interesting question is whether the notion of brane makes sense in TGD framework.

1. In TGD branes inside space-time variety are replaced by partonic 2-surfaces and possibly by their light-like orbits at which the induced metric changes signature. These surfaces are metrically 2-D. String world sheets inside space-time surfaces have discrete intersection with the partonic 2-surfaces. The intersection of strings as space-like *resp.* light-like boundaries of string world sheet with partonic orbit sheet *resp.* space-like 3-D ends of space-time surface at boundaries of CD is also discrete classically.

2. An interesting question concerns the role of 6-spheres $S^6(t_n)$ appearing as special solutions to the octonionic zero locus conditions solving both $RE(P_n) = 0$ and $IM(P_n) = 0$ requiring $P_n(o) = 0$. This can be true at 7-D light cone $o = et$, e light-like vector and t a real parameter. The roots t_n of $P(t) = 0$ give 6-spheres $S^6(t_n)$ with radius t_n as solutions to the singularity condition. As found, one can assign to each factor P_i in the product of polynomials defining many-particle state in interaction region its own partonic 2-surfaces $X^2(t_n)$ related to the solution of $P_i(t) = 0$

Could one interpret 6-spheres as brane like objects, which can be connected by 2-D "free" string world sheets as 2-varieties in M^8 and having discrete intersection with them implied by the classical dimension condition for the intersection. Free string world sheets would be something new and could be seen as trivially associative surfaces whereas 6-spheres would represent trivially co-associative surfaces in M^8.

The 2-D intersections of $S^6(t_n)$ with space-time surfaces define partonic 2-surfaces X^2 appearing at then ends of space-time and as vertices of topological diagrams. Light-like sparticle lines along parton orbits would fuse at the partonic 2-surfaces and give rise to the analog of 3-vertex in $\mathcal{N} = 4$ SUSY.

Some further TGD inspired remarks are in order.

1. Virasoro conjecture generalizing Witten conjecture involves half Virasoro algebra. Super-Virasoro algebra algebra and its super-symplectic counterpart (SSA) play a key role in the formulation of TGD at level of H. Also these algebras are half algebras. The analogs of super-conformal conformal gauge conditions state that sub-algebra of SSA with conformal weights coming as n-ples of those for entire algebra and its commutator with entire SSA give rise to vanishing Noether charges and annihilate physical states.

These conditions are conjecture to fix the preferred extremals and serve as boundary conditions allowing the formulation of M^8-H correspondence inside space-time regions (interaction regions), where the associativity conditions fail to be true and direct $M^8 - H$ correspondence does not make sense. Non-trivial solutions to these conditions are possible only if one assumes half super-conformal and half super-symplectic algebras. Otherwise the generators of the entire SSA annihilate the physical states and all SSA Noether charges vanish. The invariance of partition function for string world sheets in this sense could be interpreted in terms of emergent dynamical symmetries.

2. Just for fun one can consider the conjecture that the reduction of quantum intersections to classical intersections mediated by string world sheets implies that the numbers of string world sheets as given by the analog of G-W invariants are integers.

5.6.2 Does Riemann-Roch theorem have applications to TGD?

Riemann-Roch theorem (RR) (see `http://tinyurl.com/mdmbcx6`) is a central piece of algebraic geometry. Atyiah-Singer index theorem is one of its generalizations relating the solution spectrum of partial differential equations and topological data. For instance, characteristic classes classifying bundles associated with Yang-Mills theories (see `http://tinyurl.com/y9xvkhyy`) have applications in gauge theories and string models.

The advent of octonionic approach to the dynamics of space-time surfaces inspired by M^8-H duality [L46] [L47, L48] gives hopes that dynamics at the level of complexified octonionic M^8 could reduce to algebraic equations plus criticality conditions guaranteeing associativity for space-time surfaces representing external particles, in interaction region commutativity and associativity would be broken. The complexification of octonionic M^8 replacing norm in flat space metric with its complexification would unify various signatures for flat space metric and allow to overcome the problems due to Minkowskian signature. Wick rotation would not be a mere calculational trick.

For these reasons time might be ripe for applications of possibly existing generalization of RR to TGD framework. In the following I summarize my admittedly unprofessional understanding of RR discussing the generalization of RR for complex algebraic surfaces having real dimension 4: this is obviously interesting from TGD point of view.

I will also consider the possible interpretation of RR in TGD framework. One interesting idea is possible identification of light-like 3-surfaces and curves (string boundaries) as generalized poles and zeros with topological (but not metric) dimension one unit higher than in Euclidian signature.

Could a generalization of Riemann-Roch theorem be useful in TGD framework?

The generalization of RR for algebraic varieties, in particular for complex surfaces (real dimension equal to 4) exists. In M^8 picture the complexified metric Minkowskian signature need not cause any problems since the situation can be reduced to Euclidian sector. Clearly, this picture would provide a realization of Wick rotation as more than a trick to calculate scattering amplitudes.

Consider first the motivations for the desire of having analog of Riemann-Roch theorem (RR) at the level of space-time surfaces in M^8.

1. It would be very nice if partonic 2-surfaces would have interpretation as analogs of zeros or poles of a meromorphic function. RR applies to the divisors characterizing meromorphic functions and 2-forms, and one could hope of obtaining information about the dimensions of these function spaces giving rise to octonionic space-time varieties. Note however that the reduction to real polynomials or even rational functions might be already enough to give the needed information. Rational functions are required by the simplest generalization whereas the earlier approach assumed only polynomials. This generalization does not however change the construction of space-time varieties as zero loci of polynomials in an essential manner as will be found.

2. One would like to count the degeneracies for the intersections of 2-surfaces of space-time surface and here RR might help since its generalization to complex surfaces involves intersection form as was found in the brief summary of RR for complex surfaces with real dimension 4 (see Eq. 4.3.5).

In particular, one would like to know about the intersections of partonic 2-surfaces and string world sheets defining the points at which fermions reside. The intersection form reduces the problem via Poincare duality to 2-cohomology of space-time surfaces. More generally, it is known that the intersection form for 2-surfaces tells a lot about the topology of 4-D manifolds (see `http://tinyurl.com/y8tmqtef`). This conforms with SH. Gromow-Witten invariants [L35] (see `http://tinyurl.com/ybobccub`) are more advanced rational valued invariants but might reduce to integer valued in variants in TGD framework [L48].

There are also other challenges to which RR might relate.

1. One would like to know whether the intersection points for string world sheets and partonic 2-surfaces can belong in an extension of rationals used for adele. If the points belong to cognitive representations and subgroup of Galois group acts trivially then the number of points is reduces as the points at its orbit fuse together. The sheets of the Galois covering would intersect at point. The images of the fused points in H could be disjoint points since tangent spaces need not be parallel.
2. One would also like to have idea about what makes partonic 2-surfaces and string world sheets so special. In 2-D space-time one would have points instead of 2-surfaces. The obvious idea is that at the level of M^8 these 2-surfaces are in some sense analogous to poles and zeros of meromorphic functions. At the level of H the non-local character of M^8-H would imply that preferred extremals are solutions of an action principle giving partial differential equations.

What could be the analogs of zeros and poles of meromorphic function?

The basic challenge is to define what notions like pole, zero, meromorphic function, and divisor could mean in TGD context. The most natural approach based on a simple observation that rational functions need not define map of space-time surface to itself. Even though rational function can have pole inside CD, the point ∞ need not belong to the space-time variety defined the rational functions. Hence one can try the modification of the original hypothesis by replacing the octonionic polynomials with rational functions. One cannot exclude the possibility that although the interior of CD contains only finite points, the external particles outside CD could extend to infinity.

1. For octonionic analytic polynomials the notion of zero is well-defined. The notion of pole is well-defined only if one allows rational functions $R = P_1(o)/P_2(o)$ so that poles would correspond to zeros for the denominator of rational function. 0 and ∞ are both unaffected by multiplication and ∞ also by addition so that they are algebraically special. There are several variants of this picture. The most general option is that for a given variety zeros of both P_i are allowed.
2. The zeros of $IM(P_1) = 0$ and $IM(P_2) = 0$ would give solutions as unions of surfaces associated with P_i. This is because $IM(o_1o_2) = IM(o_1)RE(o_2) + IM(o_2)RE(o_1)$. There is no need to emphasize how important this property of IM for product is. One might say that one has two surfaces which behave like free non-interacting particles.
3. These surfaces should however interact somehow. The intuitive expectation is that the two solutions are glued by wormhole contacts connecting partonic 2-surfaces corresponding to $IM(P_1) = 0$ and $IM(P_2) = 0 = \infty$. For $RE(P_i) = 0$ and $RE(P_i) = \infty$ the solutions do not reduce to separate solutions $RE(P_1) = 0$ and $RE(P_2) = 0$. The reason is that the real part of o_1o_2 satisfies $Re(o_1o_2) = Re(o_1)Re(o_2) - Im(o_1)Im(o_2)$. There is a genuine interaction, which should generate the wormhole contact. Only at points for which $P_1 = 0$ and $P_2 = 0$ holds true, $RE(P_1) = 0$ and $RE(P_2) = 0$ are satisfied simultaneously. This happens in the discrete intersection of partonic 2-surfaces.
4. Elementary particles correspond even for $h_{eff} = h$ to two-sheeted structures with partonic surfaces defining wormhole throats. The model for elementary particles requires that particles are minimally 2-sheeted structures since otherwise the conservation of monopole Kähler magnetic flux cannot be satisfied: the flux is transferred between space-time sheets through wormhole contacts with Euclidian signature of induced metric and one obtains closed flux loop. Euclidian wormhole contact would connect the two Minkowskian sheets. Could the Minkowskian sheets corresponds to zeros $IM(P_i)$ for P_1 and P_2 and could wormhole contacts emerge as zeros of $RE(P_1/P_2)$?

One can however wonder whether this picture could allow more detailed specification. The simplest possibility would be following. The basic condition is that CD emerges automatically from this picture.

1. The simplest possibility is that one has $P_1(o)$ and $P_2(T-o)$ with the origin of octions at the "lower" tip of CD. One would have $P_1(0) = 0$ and $P_2(0) = 0$. $P_1(o)$ would give rise to the "lower" boundary of CD and $P_2(T-o)$ to the "upper" boundary of CD.

   ZEO combined with the ideas inspired by infinite rationals as counterparts of space-time surfaces connecting 3-surfaces at opposite boundaries of CD [K94] would suggest that the opposite boundaries of CD could correspond zeros and poles respectively and the ratio $P_1(o)/P_2(T-o)$ and to zeros of P_1 resp. P_2 assignable to different boundaries of CD. Both light-like parton orbits and string world sheets would interpolate between the two boundaries of CD at which partonic 2-surface would correspond to zeros and poles.

   The notion divisor would be a straightforward generalization of this notion in the case of complex plane. What would matter would be the rational function $P_1(t)/P_2(T-t)$ extended from the real (time) axis of octonions to the entire space of complexified octonions. Positive degree of divisor would multiply $P_1(t)$ with $(t-t_1)^m$ inducing a new zero at or increasing the order of existing zero at t_1. Negative orders n would multiply the denominator by $(t-t_1)^n$.

2. One can also consider the possibility that both boundaries of CD emerge for both P_1 and P_2 and without assigning either boundary of CD with P_i. In this case P_i would be sum over terms $P_{ik} = P_{ia_k}(o)P_{ib_k}(T-o)$ of this kind of products satisfying $P_{ia_k}(0) = 0$ and $P_{ib_k}(0) = 0$.

One can imagine also an alternative approach in which 0 and ∞ correspond to opposite tips of CD and have geometric meaning. Now zeros and poles would correspond to 2-surfaces, which need not be partonic. Note that in the case of Riemann surfaces ∞ can represent any point. This approach does not however look attractive.

Could one generalize RR to octonionic algebraic varieties?

RR is associated with complex structure, which in TGD framework seems to make sense independent of signature thanks to complexification of octonions. Divisors are the key notion and characterize what might be called local winding numbers. De-Rham cohomology is replaced with much richer Dolbeault cohomology (see `http://tinyurl.com/y7cvs5sx`) since the notion of continuity is replaced with that of meromorphy. Symplectic approach about which G-W invariants for symplectic manifolds provide an example define a different approach and now one has ordinary cohomology.

An interesting question is whether $M^8 - H$-duality corresponds to the mirror symmetry of string models (see `http://tinyurl.com/yc2m2e5m`) relating complex structures and symplectic structures. If this were the case, M^8 would correspond to complex structure and H to symplectic structure.

RR for curves gives information about dimensions for the spaces of meromorphic functions having poles with order not higher than specified by divisor. This kind of interpretation would be very attractive now since the poles and zeros represented as partonic 2-surfaces would have direct physical interpretation in terms of external particles and interaction vertices. RR for curves involves poles with orders not higher than specified by the divisor and gives a formula for the dimension of the space of meromorphic functions fora given divisor. As a special case give the dimension $l(nD)$ for a given divisor.

Could something similar be true in TGD framework?

1. Arithmetic genus makes sense for polynomials $P(t)$ since t can be naturally complexified giving a complex curve with well-defined arithmetic genus. What could correspond to the intersection form for 2-surfaces representing D and $K - D$? The most straightforward possibility is that partonic 2-surfaces correspond to poles and zeros.

   Divisor $-D$ would correspond to the inverse of P_2/P_1 representing it. $D - K$ would also a well-defined meaning provided the canonical divisor associated with holomorphic 2-form has well-defined meaning in the Dolbeault cohomology of the space-time surface with complex structure. RR would give direct information about the space of space-time varieties defined by $RE(P) = 0$ or $IM(P) = 0$ condition.

One could hope of obtaining information about intersection form for string world sheets and partonic 2-surfaces. Whether the divisor $D-K$ has anything to do string world sheets, is of course far from clear.

2. Complexification means that field property fails in the sense that complexified Euclidian norm vanishes and the inverse of complexified octonion/quaternion/complex number is infinite formally. For Euclidian sector with real coordinates this does not happen but does take place when some coordinates are real and some imaginary so that signature is effectively Minkowskian signature.

   At 7-D light-cone of M^8 the condition $P(o)=0$ reduces to a condition for real polynomial $P(t)=0$ giving roots t_n. Partonic 2-varieties are intersections of 4-D space-time varieties with 6-spheres with radii t_n. There are good reasons to expect that the 3-D light-like orbits of partonic 3-surfaces are intersections of space-time variety with 7-D light-cone boundary and their H counterparts are obtained as images under M^8-H duality.

   For light-like complefixied octonionic points the inverse of octonion does not exist since the complexified norm vanishes. Could the light-like 3-surfaces as partonic orbits correspond to images under M^8-H duality for zeros and/or poles as 3-D light-like surfaces? Could also the light-like boundaries of strings correspond to this kind of generalized poles or zeros? This could give a dynamical realization for the notions of zero and pole and increase the topological dimension of pole and zero for both 2-varieties and 4-varieties by one unit. The metric dimension would be unaffected and this implies huge extension of conformal symmetries central in TGD since the light-like coordinate appears as additional parameter in the infinitesimal generators of symmetries.

Could one formulate the counterpart of RR at the level of H? The interpretation of M^8-H duality as analog of mirror symmetry (see http://tinyurl.com/yc2m2e5m) suggests this. In this case the first guess for the identification of the counterpart of canonical divisor could be as Kähler form of CP_2. This description would provide symplectic dual for the description based on divisors at the level of M^8. G-W invariants and their possible generalization are natural candidates in this respect.

5.6.3 Could the TGD variant of Atyiah-Singer index theorem be useful in TGD?

Atyiah-Singer index theorem (AS) is one of the generalizations of RR and has shown its power in gauge field theories and string models as a method to deduce the dimensions of various moduli spaces for the solutions of field equations. A natural question is whether AS could be useful in TGD and whether the predictions of AS at H side could be consistent with M^8-H duality suggesting very simple counting for the numbers of solutions at M^8 side as coefficient combinations of polynomials in given extension of rationals satisfying criticality conditions. One can also ask whether the hierarchy of degrees n for octonion polynomials could correspond to the fractal hierarchy of generalized conformal sub-algebras with conformal weights coming as n-multiples for those for the entire algebras.

Atyiah-Singer index theorem (AS) and other generalizations of RR involve extremely abstract concepts. The best manner to get some idea about AS is to learn the motivations for it. The article http://tinyurl.com/yc49lljp gives a very nice general view about the motivations of Atyiah-Singer index theorem and also avoids killing the reader with details.

Solving problems of algebraic geometry is very demanding. The spectrum of solutions can be discrete (say number of points of space-time surface having linear M^8 coordinates in an extension of rationals) or continuous such as the space of roots for n:th order polynomials with real coefficients.

An even more difficult challenge is solving of partial differential equations in some space, call it X, of say Yang-Mills gauge field coupled to matter fields. In this case the set of solutions is typically continuous moduli space.

One can however pose easier questions. What is the number of solutions in counting problem? What is the dimension of the moduli space of solutions? Atiyiah-Singer index theorem relates this number - analytic index - to topological index expressible in terms of topological invariants assignable to complexified tangent bundle of X and to the bundle structure - call it field bundle - accompanying the fields for which field equations are formulated.

AS very briefly

Consider first the assumptions of AS.

1. The idea is to study perturbations of a given solution and linearize the equations in some manifold X often assumed to be compact. This leads to a linear partial differential equations defined by linear operator P. One can deduce the dimension of the solution space of P. This number defines the dimension of the tangent space of solution space of full partial differential equations, call it moduli space.

2. The idea is to assign to the partial differential operator P its symbol $\sigma(P)$ obtained by replacing derivatives with what might be called momentum components. The reversal of this operaion is familiar from elementary wave mechanics: $p_i \rightarrow id/dx^i$. This operation can be formulated in terms of co-tangent bundle. The resulting object is purely algebraic. If this matrix is reversible for all momentum values and points of X, one says that the operator is elliptic.

   Note that for field equations in Minkowski space M^4 the invertibility constraint is not satisfied and this produces problems. For instance, for massive M^4 d'Alembertian for scalar field the symbol is four-momentum squared, which vanishes, when on-mass shell condition is satisfied. Wick rotation is somewhat questionable manner to escape this problem. One replaces Minkowski space with its Euclidian counterpart or by 4-sphere. If all goes well the dimension of the solution space does not depend on the signature of the metric.

3. In the general case one studies linear equation of form $DP = f$, where f is homogenuity term representing external perturbation. f can also vanish. Quite generally, one can write the dimension of the solution space as

   $$Ind_{anal}(P) = dim(ker(P)) - dim(coker(P)) \ . \tag{5.6.1}$$

   $ker(P)$ denotes the solution space for $DP = 0$ without taking into account the possible restrictions coming from the fact that f can involve part f_0 satisfying $Df_0 = 0$ (for instance, f_0 corresponds to resonance frequency of oscillator system) nor boundary conditions guaranteing hermiticity. Indeed, the hermitian conjugate $D^\dagger$ of D is not automatically identical with D. $D^\dagger$ is defined in terms of the inner product for small perturbations as

   $$\langle D^\dagger P_1^* | DP_2\rangle = \langle P_1 | DP_2\rangle \ . \tag{5.6.2}$$

   The inner product involves integration over X and partial integrations transfer the action of partial derivatives from P_2 to P_1^*. This however gives boundary terms given by surface integral and hermiticity requires that they vanish. This poses additional conditions on P and contributes to $dim(coker(P))$.

The challenge is to calculate $Ind_{anal}(P)$ and here AS is of enormous help. AS relates analytical index $Ind_{anal}(P)$ for P to topological index $Ind_{top}(\sigma(P))$ for its symbol $\sigma(P)$.

1. $Ind_{top}(\sigma(P))$ involves only data associated with the topology X and with the bundles associated with field variables. In the case of Yang-Mills fields coupled to matter the bundle is the bundle associated with the matter fields with a connection determined by Yang-Mills gauge potentials. So called Todd class $Td(X)$ brings in information about the topology of complexified tangent bundle.

2. $Ind_{top}(\sigma(P))$ is not at all easy to define but is rather easily calculable as integrals of various invariants assignable to the bundle structure involved. Say instanton density for YM fields and various topological invariants expressing the topological invariants associated with the metric of the space. What is so nice and so non-trivial is that the dimension of the moduli space for non-linear partial differential equations is determined by topological invariants. Much of the dynamics reduces to topology.

The expression for $Ind_{top}(\sigma(P))$ involves besides σ_P topological data related to the field bundle and to the complexified tangent bundle. The expression Ind_{top} as a function of the symbol $\sigma(P)$ is given by

$$Ind_{top}(\sigma(P)) = (-1)^n \langle ch(\sigma(P)) \cdot Td(T_C(X), [X]) \rangle \ . \quad (5.6.3)$$

The expression involves various topological data.

1. Dimension of X.
2. The quantity $\langle x.y \rangle$ involving cup product $x.y$ of cohomology classes, which contains a contribution in the highest homology group $H^n(X)$ of X corresponding to the dimension of X and is contracted with this fundamental class $[X]$. $\langle x.y \rangle$ denotes matrix trace for the operator $ch(\sigma(P))$ formed as polynomial of $\sigma(P)$. $[X]$ denotes so called fundamental class fr X belonging to H^n and defines the orientation of X.
3. Chern character $ch_E(t)$ (see `http://tinyurl.com/ybavu66h`). I must admit that I ended up to a garden of branching paths while trying to understand the definition of ch_E is. In any case, $ch_E(t)$ characterizes complex vector bundle E expressible in terms of Chern classes (see `http://tinyurl.com/y8jlaznc`) of E. E is the bundle assignable to field variables, say Yang Mills fields and various matter fields.

   Both direct sums and tensor products of fiber spaces of bundles are possible and the nice feature of Chern class is that it is additive under tensor product and multiplicative under direct sum. The fiber space of the entire bundle is now direct sum of the tangent space of X and field space, which suggests that $Ind(top)$ is actually the analog of Chern character for the entire bundle.

   $t = \sigma P$ has interpretation as an argument appearing in the definition of Chern class generalized to Chern character. $t = \sigma(P)$ would naturally correspond to a matrix valued argument of the polynomial defining Chern class as cohomology element. $ch(\sigma(P))$ is a polynomial of the linear operator defined by symbol $\sigma(P)$. ch_E for given complex vector bundle is a polynomial, whose coefficients are relatively easily calculable as topological invariants assignable to bundle E. E must be the field bundle now.
4. Todd class $Td(T_C(X))$ for the complexified tangent bundle (see `http://tinyurl.com/yckv4w84`) appears also in the expression. Note that also now the complexification occurs. The cup product gives element in $H^n(X)$, which is contracted with fundamental class $[X]$ and integrated over X.

AS and TGD

The dynamics of TGD involves two levels: the level of complexified M^8 (or equivalently E^8) and the level of H related to $M^8 - H$ correspondence.

1. At the level of M^8 one has algebraic equations rather than partial differential equations and the situation is extremely simple as compared to the situation for a general action principle. At the level of H one has action principle and partial differential equations plus infinite number of gauge conditions selecting preferred extremals and making dynamics for partial differential equations dual to the dynamics determined by purely number theoretic conditions.

   The space-time varieties representing external particles outside CDs in M^8 satisfy associativity conditions for tangent space or normal space and reducing to criticality conditions for the real coefficients of the polynomials defining the space-time variety. In the interior of CDs associativity conditions are not satisfied but the boundary conditions fix the values of the coefficients to be those determined by criticality conditions guaranteing associativity outside the CD.

   In the interiors space-time surfaces of CDs M^8-duality does not apply but associativity of tangent spaces or normal spaces at the boundary of CD fixes boundary values and minimal surface dynamics and strong form of holography (SH) fixes the space-time surfaces in the interior of CD.

2. For the H-images of space-time varieties in H under $M^8 - H$ duality the dynamics is universal coupling constant independent critical dynamics of minimal surfaces reducing to holomorphy in appropriate sense. For minimal surfaces the 4-D Kähler current density vanishes so that the solutions are 4-D analogs of geodesic lines outside CD. Inside CD interactions are coupled on and this current is non-vanishing. Infinite number of gauge conditions for various half conformal algebras in generalized sense code at H side for the number theoretical critical conditions at M^8 side. The sub-algebra with conformal weights coming as n-ples of the entire algebra and its commutator with entire algebra gives rise to vanishing classical Noether charges. An attractive assumption is that the value of n at H side corresponds to the order n of the polynomials at M^8 side.
3. The coefficients of polynomials $P(o)$ determining space-time varieties are real numbers (also complexified reals can be considered without losing associativity) restricted to be numbers in extension of rationals. This makes it possible to speak about p-adic variants of the space-time surfaces at the level of M^8 at least.

Could Atyiah-Singer theorem have relevance for TGD?

1. For real polynomials it is easy to calculate the dimension of the moduli space by counting the number of independent real (in octonionic sense) coefficients of the polynomials of real variable (one cannot exclude that the coefficients are in complex extension of rationals). Criticality conditions reduce this number and the condition that coefficients are in extension of rationals reduces it further. One has quite nice overall view about the number of solutions and one can see them as subset of continuous moduli space. If $M^8 - H$ duality really works then this gives also the number of preferred extremals at H side.
2. This picture is not quite complete. It assumes fixing of 8-D CD in M^8 as well as fixing of the decomposition $M^2 \subset M^4 \subset M^4 \times E^4$. This brings in moduli space for different choices of octonion structures (8-D Lorentz group is involved). Also moduli spaces for partonic 2-surfaces are involved. Number theoretical universality seems to require that also these moduli spaces have only points with coordinates in extension of rationals involved.
3. In principle one can try to formulate the counterpart of AS at H side for the linearization of minimal surface equations, which are nothing but the counterpart of massless field equations in a fixed background metric. Note that additional conditions come from the requirement that the term from Kähler action reduces to minimal surface term.

   Discrete sets of solutions for the extensions of rationals should correspond to each other at the two sides. One can also ask whether the dimensions for the effective continuous moduli spaces labelled by n characterizing the sub-algebras of various conformal algebras isomorphic to the entire algebra and those for the polynomials of order n satisfying criticality conditions. One would have a number theoretic analog for a particle in box leading to the quantization of momenta.

All this is of course very speculative and motivated only by the general physical vision. If the speculations were true, they would mean huge amount of new mathematics.

5.7 Intersection form for 4-manifolds, knots and 2-knots, smooth exotics, and TGD

Gary Ehlenberger sent a highly interesting commentary related to smooth structures in R^4 discussed in the article of Gompf [A79] (`https://cutt.ly/eMracmf`) and more generally to exotics smoothness discussed from the point of view of mathematical physics in the book of Asselman-Maluga and Brans [A83] (`https://cutt.ly/DMuOdYr`). I am grateful for these links for Gary.

5.7.1 Basic ideas

The role of intersection forms in TGD

The intersection form of 4-manifold (`https://cutt.ly/jMriNdI`) characterizing partially its 2-homology is a central notion in the study of the smooth structures. I am not a topologist but have two good reasons to get interested on intersection forms.

1. In the TGD framework [L121], the intersection form describes the intersections of string world sheets and partonic 2-surfaces and therefore is of direct physical interest [K54, L48].
2. Knots have an important role in TGD. The 1-homology of the knot complement characterizes the knot. Time evolution defines a knot cobordism as a 2-surface consisting of knotted string world sheets and partonic 2-surfaces. A natural guess is that the 2-homology for the 4-D complement of this cobordism characterizes the knot cobordism. Also 2-knots are possible in 4-D space-time and a natural guess is that knot cobordism defines a 2-knot.

   The intersection form for the complement for cobordism as a way to classify these two-knots is therefore highly interesting in the TGD framework. One can also ask what the counterpart for the opening of a 1-knot by repeatedly modifying the knot diagram could mean in the case of 2-knots and what its physical meaning could be in the TGD Universe. Could this opening or more general knot-cobordism of 2-knot take place in zero energy ontology (ZEO) [L80, L120, L127] as a sequence of discrete quantum jumps leading from the initial 2-knot to the final one.

Why exotic smooth structures are not possible in TGD?

The existence of exotic 4-manifolds [A79, A83, A52] could be an anomaly in the TGD framework. In the articles [A79, A52] the term anomaly is indeed used. Could these anomalies cancel in the TGD framework?

The first naive guess was that the exotic smooth structures are not possible in TGD but it turned out that this is not trivially true. The reason is that the smooth structure of the space-time surface is not induced from that of H unlike topology. One could induce smooth structure by assuming it given for the space-time surface so that exotics would be possible. This would however bring an ad hoc element to TGD. This raises the question of how it is induced.

1. This led to the idea of a holography of smoothness, which means that the smooth structure at the boundary of the manifold determines the smooth structure in the interior. Suppose that the holography of smoothness holds true. In ZEO, space-time surfaces indeed have 3-D ends with a unique smooth structure at the light-like boundaries of the causal diamond $CD = cd \times CP_2 \subset H = M^4 \times CP_2$, where cd is defined in terms of the intersection of future and past directed light-cones of M^4. One could say that the absence of exotics implies that $D = 4$ is the maximal dimension of space-time.
2. The differentiable structure for $X^4 \subset M^8$, obtained by the smooth holography, could be induced to $X^4 \subset H$ by $M^8 - H$-duality. Second possibility is based on the map of mass shell hyperboloids to light-cone proper time $a = constant$ hyperboloids of H belonging to the space-time surfaces and to a holography applied to these.
3. There is however an objection against holography of smoothness (`https://cutt.ly/3MewYOt`). In the last section of the article, I develop a counter argument against the objection. It states that the exotic smooth structures reduce to the ordinary one in a complement of a set consisting of arbitrarily small balls so that local defects are the condensed matter analogy for an exotic smooth structure.

5.7.2 Intersection form in the case of 4-surfaces

Intersection form (`https://cutt.ly/jMriNdI`) for homologically trivial 2-surfaces of the space-time surface and 2-homology for the complement of these surfaces can be physically important in tGD framework.

Intersection forms in 2-D case

It is good to explain the notion of intersection form by starting from 1-homology. The intersection form for 1-homology is encountered for a cylinder with ends fixed. In this case, one has relative homology and homologically trivial curves are curves connecting the ends of string and characterized by a winding number.

In the case of torus obtained by identifying the ends of cylinder, one obtains two winding numbers (m, n) corresponding to to homologically non-trivial circles at torus. The intersection number for curves (m, n) and (p, q) at torus is $N = mq - np$ and for curves at cylinder one as $(m, n) = (1, n)$ giving $N = n - q$.

The antisymmetric intersection form is defined as 2×2 matrix defining intersections for the basis of the homology with $(m, n) = (1, 0)$ and $(n, m) = (0, 1)$ and is given by $(0, 1; -1, 0)$.

Intersection for 4-surfaces in TGD context

In TGD, the intersection form for a 4-surface identified as space-time surface could have a rather concrete physical interpretation and the stringy part of TGD physics would actually realize it concretely.

1. $M^8 - H$ duality requires that the 4-surface in M^8 has quaternionic/associative normal space: this distribution of normal spaces is integrable and integrates to the 4-surface in M^8.

   The normal must also contain a commutative (complex) sub-space at each point. Only this allows us to parametrize normal spaces by points of CP_2 and map them to space-time surfaces in $H = M^4 \times CP_2$. The integral distribution of these commutative sub-spaces defines a 2-D surface. Physically, these surfaces would correspond to string world sheets and partonic 2-surfaces.

2. String world sheets and partonic 2-surfaces, regarded as objects in relative homology (modulo ends of the space-time surfaces at the boundaries of causal diamond (CD)), can intersect as 2-D objects inside the space-time surface and the intersection form characterizes them.

   There is an analogy with the cylinder: time-like direction corresponds to the cylinder axis and a homologically non-trivial 2-surface of CP_2 corresponds to the circle at the cylinder.

3. If the second homology of the space-time surface is trivial, the naive expectation is that the intersections of string world sheets are not stable under large enough deformations of the string world sheets. Same applies to intersecting plane curves. At the cylinder, the situation is different since the relative first homology is non-trivial and spanned by two generators: the circle and a line connecting the ends of the cylinder.

   The intersection form is however non-trivial as in the case of the cylinder for 2-surfaces having 2-D homologically non-trivial CP_2 projection. They would represent M^4 deformations of 2-D homologically trivial surfaces of CP_2 just like a helical orbit along a cylinder surface. A 2-D generalization of CP_2 type extremal would have a light-like curve or light-like geodesic as M^4 projection and could define light-partonic orbit.

4. The intersection of string world sheet and partonic 2-surface can be stable however. Partonic 2-surface is a boundary of a wormhole contact connecting two space-time sheets.

   Consider a string arriving along space-time sheet A, going through the wormhole contact, and continuing along sheet B. The string has an intersection point with both wormhole throats. This intersection is stable against deformations. The orbit of this string intersects the light-like orbit of the partonic 2-surface along the light-like curve.

   One has a non-trivial intersection form with the number of intersections with partonic 2-surfaces equal to 1. In analogy with cylinder, also the intersections of 2-surfaces with 2-D homologically trivial CP_2 projection are unavoidable and reflect the non-trivial intersection form of CP_2.

5.7.3 About ordinary knots

Ordinary knots and 3-topologies are related and the natural expectation is that also 2-knots and 4-topologies are related.

About knot invariants

Consider first knot invariants (`https://cutt.ly/DMrgs14`)at the general level.

1. One important knot invariant of ordinary knots is the 1-homology of the complement and the associated first homotopy group whose abelianization gives the homology group.

2. The complement of the knot can be given a metric of a hyperbolic 3-manifold, which corresponds to a unit cell for a tessellation of the mass shell. $M^8 - H$ duality suggests that the intersection X^3 of 4-surface of M^8 with mass shell $H^3_m \subset M^4 \subset M^8$ is a hyperbolic manifold and identical with the hyperbolic manifold associated with the complement of a knot of H^3_a realized as light-cone proper time $a = constant$ hyperboloid of $M^4 \subset H$ and closed knotted and linked strings as ends of string world sheets at H^3_a.

   The evolution of the strings defined by the string world sheets would define a 1-knot cobordism. The 2-homology of the knot complement should characterize the topological evolution of the 1-homology of the knot.

Opening of knots and links by knot cobordisms

The procedure leading to the trivialization of knot or link can be used to define knot invariants and the procedure itself characterizes knot.

1. Ordinary knot is described by a knot diagram obtained as a projection of the knot to the plane. It contains intersections of lines and the intersection contains information telling which line is above and which line is below.
2. The opening of the knot or link to give a trivial knot or link, which is used in the construction of knot invariants, is a sequence of violent operations. In the basic step strings portions go through each other and therefore suffer a reconnection. This operation can therefore change the 1-homology of the 3-D knot complement.

   Knot or link can be modified by forcing two intersecting strands of the plane projection to go through each other. Locally the basic operation for two links is the same as for the pieces of knot. The transformation of the knot or link to a trivial knot or link corresponds to some sequence of these operations and can be used to define a knot invariants. This operation is not unique since there are moves which do not affect the knot.

   The basic opening operation can be also seen as a time evolution, knot cobordism, in which the first portion, call it A, remains unchanged and the second portion, call it B, draws a 2-D surface in E^3. A intersects the 2-D orbit at a single point.
3. The 2-homology for the string world sheets and partonic 2-surfaces as 2-surfaces in space-time serves as an invariant of knot cobordism and represents the topological dynamics of ordinary 1-knots of 3-surface and links formed by strings or flux tubes in 3-surface as cobordism defining the time evolution of a knot to another knot.

   In particular, the intersection form for the 2-homology of the complement of the cobordism defines an invariant of cobordism. This intersection form must be distinguished from the intersection form for the second homology of the space-time surface rather than the 2-knot complement.
4. One can also consider more general sequences of basic operations transforming two knots or links to each other as knot-/link cobordisms, which involve self intersections of the knots. Does this mean that the intersection form characterizes the knot cobordism. Could a string diagram involving reconnections describe the cobordism process.

Stringy description of knot cobordisms

$M^8 - H$ duality [L99, L100, L130, L129] requires string word sheets and partonic 2-surfaces. This implies that TGD physics represents the 2-homology of both space-time surfaces and the homology of the complement of the knotted links defined by them.

Although the "non-homological" intersections of string world sheets can be eliminated by a suitable deformation of the string world sheet, they should have a physical meaning. This comes from the observation that they affect nontrivially the 1-homology of the knot complement as 3-D time=constant slice.

The first thing that I am able to imagine is that strings reconnect. This is nothing but the trouser vertex for strings so that intersection form would define topological string dynamics in some sense. These reconnections play a key role in TGD, also in TGD inspired quantum biology.

The dynamics of partonic 2-surfaces and string world sheets could relate to knot cobordisms, possibly leading to the opening of ordinary knot,

5.7.4 What about 2-knots and their cobordisms?

2-D closed surfaces in 4-D space give rise to 2-knots. What is the physical meaning of 2-knots of string world sheets? What could 2-knots for orbits of linear molecules or associated magnetic flux tubes mean physically and from the point of view of quantum information theory? One can try to understand 2-knots by generalizing the ideas related to the ordinary knots.

1. Intuitively it seems that the cobordism of a 1-knot defines a 2-knot. It is not clear to me whether all 2-knots for space-time surfaces connecting the boundaries of CD can be regarded as this kind of cobordisms of 1-knots.
2. The 2-homology of the complement of 2-knot should define a 2-knot invariant. In particular, the intersection form should define a 2-knot invariant.
3. The opening of 1-knot by repeating the above described basic operation is central in the construction of knot invariants and the sequence of the operations can be said to be knot invariant modulo moves leaving the knot unaffected.

   The opening or a more general cobordism of a 2-knot could be seen as a time evolution with respect to a time parameter t_5 parametrizing the isotopy of space-time surface. The local cobordism can keep the first portion of 2-knot, call it A, unchanged and deform another portion, call it B, so that a 3-D orbit at the space-time surface is obtained. For each value of t_5, the portions A and B of 2-knot have in the generic case only points as intersections.

   This would suggest that an intersection point of A and B is generated in the operation and moves during the t_5 time evolution along A along 1-D curve during the process. This process would be the basic operation used repeatedly to open 2-knot or to transform it to another 2-knot.
4. In quantum TGD, a sequence of quantum jumps, quantum cobordism, would have the same effect as t_5 time evolution. This brings in mind DNA transcription and replication as a process proceeding along a DNA strand parallel to the monopole flux tube as a sequence of SFRs involving direct contact between DNA strand and enzymes catalyzing the process and also of corresponding flux tubes. An interesting possibility is that these quantum cobordisms appear routinely in biochemistry of the fundamental linear bio-molecules such as DNA, RNA, tRNA, and amino-acids [K46, K6, K107, K1, K117, L24] [L67].

   The quantum cobordism of 2-knot is possible only in ZEO, where the quantum state as a time= constant snapshot is replaced with a superposition of space-time surfaces.

5.7.5 Could the existence of exotic smooth structures pose problems for TGD?

The article of Gabor Etesi [A52] (`https://cutt.ly/2Md7JWP`) gives a good idea about the physical significance of the existence of exotic smooth structures and how they destroy the cosmic censorship hypothesis (CCH of GRT stating that spacetimes of GRT are globally hyperbolic so that there are no time-like loops.

Smooth anomaly

No compact smoothable topological 4-manifold is known, which would allow only a single smooth structure. Even worse, the number of exotics is infinite in every known case! In the case of non-compact smoothable manifolds, which are physically of special interest, there is no obstruction against smoothness and they typically carry an uncountable family of exotic smooth structures.

One can argue that this is a catastrophe for classical general relativity since smoothness is an essential prerequisite for tensory analysis and partial differential equations. This also destroys hopes that the path integral formulation of quantum gravitation, involving path integral over all possible space-time geometries, could make sense. The term anomaly is certainly well-deserved.

Note however that for 3-geometries appearing as basic objects in Wheeler's superspace approach, the situation is different since for $D < 3$ there is only a single smooth structure. If one has holography, meaning that 3-geometry dictates 4-geometry, it might be possible to avoid the catastrophe.

The failure of the CCH is the basic message of Etesi's article. Any exotic R^4 fails to be globally hyperbolic and Etesi shows that it is possible to construct exact vacuum solutions representing curved space-times which violate the CCH. In other words, GRT is plagued by causal anomalies.

Etesi constructs a vacuum solution of Einstein's equations with a vanishing cosmological constant which is non-flat and could be interpreted as a pure gravitational radiation. This also represents one particular aspect of the energy problem of GRT: solutions with gravitational radiation should not be vacua.

1. Etesi takes any exotic R^4 which has the topology of $S^3 \times R$ and has an exotic smooth structure, which is not a Cartesian product. Etesi maps maps R^4 to CP_2, which is obtained from C^2 by gluing CP_1 to it as a maximal ball B^3_r for which the radial Eguchi-Hanson coordinate approaches infinity: $r \to \infty$. The exotic smooth structure is induced by this map. The image of the exotic atlas defines atlas. The metric is that of CP_2 but $SU(3)$ does not act as smooth isometries anymore.
2. After this Etesi performs Wick rotation to Minkowskian signature and obtains a vacuum solution of Einstein's equations for any exotic smooth structure of R^4.

In TGD, the question of exotic smoothness is encountered both at the level of embedding space and associated fixed spaces and at the level of space-time surfaces and their 6-D twistor space analogies. Could TGD solve the smooth anomaly?

Can embedding space and related spaces have exotic smooth structure?

One can first worry about the exotic smooth structures possibly associated with the M^4, CP_2, $H = M^4 \times CP_2$, causal diamond CD= $cd \times CP_2$, where cd is the intersection of the future and past directed light-cones of M^4, and with M^8. One can also worry about the twistor spaces CP_3 *resp.* $SU(3)/U(1) \times U(1)$ associated with M^4 *resp.* CP_2.

The key assumption of TGD is that all these structures have maximal isometry groups so that they relate very closely to Lie groups, whose unique smooth structures are expected to determine their smooth structures.

1. The first sigh of relief is that all Lie groups have the standard smooth structure. In particular, exotic R^4 does not allow translations and Lorentz transformations as isometries. I dare to conclude that also the symmetric spaces like CP_2 and hyperbolic spaces such as $H^n = SO(1,n)/SO(n)$ are non-exotic since they provide a representation of a Lie group as isometries and the smoothness of the Lie group is inherited. This would mean that the charts for the coset space G/H would be obtained from the charts for G by an identification of the points of charts related by action of subgroup H.

   Note that the mass shell H^3, as any 3-surface, has a unique smooth structure by its dimension.
2. Second sigh of relief is that twistor spaces CP_3 and $SU(3)/U(1) \times U(1)$ have by their isometries and their coset space structure a standard smooth structure.

   In accordance with the vision that the dynamics of fields is geometrized to that of surfaces, the space-time surface is replaced by the analog of twistor space represented by a 6-surface with a structure of S^2 bundle with space-time surface X^4 as a base-space in the 12-D product of twistor spaces of M^4 and CP_2 and by its dimension $D = 6$ can have only the standard smooth structure unless it somehow decomposes to $(S^3 \times R) \times R^2$. Holography of smoothness would prevent this since it has boundaries because X^4 as base space has boundaries at the boundaries of CD.

   If exotic smoothness is allowed at the space-time level in the proposed sense ordinary smooth structure could be possible at the level of twistor space in the complement of a Cartesian product of the fiber space S^2 with a discrete set of points associated with partonic 2-surfaces.
3. cd is an intersection of future and past directed light-cones of M^4. Future/past directed light-cone could be seen as a subset of M^4 and implies standard smooth structure is possible. Coordinate atlas of M^4 is restricted to cd and one can use Minkowski coordinates also inside the cd. cd could be also seen as a pile of light-cone boundaries $S^2 \times R_+$ and by its dimension $S^2 \times R$ allows only one smooth structure.

4. M^8 is a subspace of complexified octonions and has the structure of 8-D translation group, which implies standard smooth structure.

The conclusion is that continuous symmetries of the geometry dictate standard smoothness at the level of embedding space and related structures.

Could TGD eliminate the smoothness anomaly or provide a physical interpretation for it?

The question of exotic smoothness is encountered both at the level of embedding space and associated fixed spaces and at the level of space-time surfaces and their 6-D twistor space analogies.

What does the induction of a differentiable structure really mean? Here my naive expectations turn out to be wrong. If a sub-manifold $S \subset H$ can be regarded as an embedding of smooth manifold N to $S \subset H$, the embedding $N \rightarrow S \subset H$ induces a smooth structure in S (https://cutt.ly/tMtvG79). The problem is that the smooth structure would not be induced from H but from N and for a given 4-D manifold embedded to H one could also have exotic smooth structures. This induction of smooth structure is of course physically adhoc.

It is not possible to induce the smooth structure from H to sub-manifold. The atlas defining the smooth structure in H cannot define the charts for a sub-manifold (surface). For standard R^4 one has only one atlas.

1. Could holography of smoothness make sense in the general case?

The first trial to get rid of exotics [A83] was based on the holography of smoothness and did not involve TGD. Could a smooth structure at the boundary of a 4-manifold could dictate that of the manifold uniquely. Could one speak of holography for smoothness? Manifolds with boundaries would have the standard smooth structure.

1. The obvious objection is that the coordinate atlas for 3-D boundary cannot determine 4-D atlas in any way because the boundary cannot have information of the topology of the interior.
2. The holography for smoothness is also argued to fail (https://cutt.ly/3MewYOt). Assume a 4-manifold W with 2 different smooth structures. Remove a ball B^4 belonging to an open set U and construct a smooth structure at its boundary S^3. Assume that this smooth structure can be continued to W. If the continuation is unique, the restrictions of the 2 smooth structures in the complement of B^4 would be equivalent but it is argued that they are not.
3. The first layman objection is that the two smooth structures of W are equivalent in the complement $W - B^3$ of an arbitrary small ball $B^3 \subset W$ but not in the entire W. This would be analogous to coordinate singularity. For instance, a single coordinate chart is enough for a sphere in the complement of an arbitrarily small disk.

   An exotic smooth structure would be like a local defect in condensed matter physics. In fact it turned out that this intuitive idea is correct: it can be shown that the exotic smooth structures are equivalent with standard smooth structure in a complement of a set having co-dimension zero (https://cutt.ly/7MbGqx2). This does not save the holography of smoothness in the general case but gives valuable hints for how exotic smoothness might be realized in TGD framework.

2. Could holography of smoothness make sense in the TGD framework?

Could $M^8 - H$ duality and holography make holography of smoothness possible in the TGD framework?

1. In the TGD framework space-time is 4-surface rather than abstract 4-manifold. 4-D general coordinate invariance, assuming that 3-surfaces as generalization of point-like particles are the basic objects, suggests a fully deterministic holography. A small failure of determinism is however possible and expected, and means that space-time surfaces analogous to Bohr orbits become fundamental objects. Could one avoid the smooth anomaly in this framework?

   The 8-D embedding space topology induces 4-D topology. My first naive intuition was that the 4-D smooth structure, which I believed to be somehow inducible from that of $H = M^4 \times CP_2$, cannot be exotic so that in TGD physics the exotics could not be realized. But can one really exclude the possibility that the induced smooth structure could be exotic as a 4-D smooth structure?

2. In the TGD framework and at the level of $H = M^4 \times cP_2$, one can argue that the holography implied by the general coordinate invariance somehow determines the smooth structure in the interior of space-time surface from the coordinate atlas at the boundary. One would have a holography of smoothness. It is however not obvious why this unique structure should be the standard one.

3. One has also holography in M^8 and this induces holography in H by $M^8 - H$ duality. The 3-surfaces X^3 inducing the holography in M^8 are parts of mass shells, which are hyperbolic spaces $H^3 \subset M^4 \subset M^8$. 3-surfaces X^3 could be even hyperbolic 3-manifolds as unit cells of tessellations of H^3. These hyperbolic manifolds have unique smooth structures as manifolds with dimension $D < 4$.

   The hypothesis is that one can assign to these 3-surfaces a 4-surface by a number theoretic dynamics requiring that the normal space is associative, that is quaternionic [L99, L100]. The additional condition is that the normal space contains commutative subspace makes it possible to parametrize normal spaces by points of CP_2. $M^8 - H$ duality would map a given normal space to a point of CP_2. $M^8 - H$ duality makes sense also for the twistor lift.

4. A more general statement would be as follows. A set of 3-surfaces as sub-manifolds of mass shells H^3_m determined by the roots of polynomial P having interpretation as mass square values defining the 4-surface in M^8 take the role of the boundaries. Mass-shells H^3_m or partonic 2-surfaces associated with them having particle interpretation could correspond to discontinuities of derivatives and even correspond to failure of manifold property analogous to that occurring for Feybman diagrams so that the holography of smoothness would decompose to a piece-wise holography.

   The regions of $X^4 \subset M^8$ connecting two sub-sequent mass shells would have a unique smooth structure induced by the hyperbolic manifolds H^3 at the ends.

It is important to notice that the holography of smoothness does not force the smooth 4-D structure to be the standard one.

3. Could the exotic smooth structures have a physical interpretation in the TGD framework?

In the TGD framework, exotic smooth structures could also have a physical interpretation. As noticed, the failure of the standard smooth structure can be thought to occur at a point set of dimension zero and correspond to a set of point defects in condensed matter physics. This could have a deep physical meaning.

1. The space-time surfaces in $H = M^4 \times CP_2$ are images of 4-D surfaces of M^8 by $M^8 - H$-duality. The proposal is that they reduce to minimal surfaces analogous to soap films spanned by frames. Regions of both Minkowskian and Euclidean signature are predicted and the latter correspond to wormhole contacts represented by CP_2 type extremals. The boundary between the Minkowskian and Euclidean region is a light-like 3-surface representing the orbit of partonic 2-surface identified as wormhole throat carrying fermionic lines as boundaries of string world sheets connecting orbits of partonic 2-surfaces.

2. These fermionic lines are counterparts of the lines of ordinary Feynman graphs, and have ends at the partonic 2-surfaces located at the light-like boundaries of CD and in the interior of the space-time surface. The partonic surfaces, actually a pair of them as opposite throats of wormhole contact, in the interior define topological vertices, at which light-like partonic orbits meet along their ends.

3. These points should be somehow special. Number theoretically they should correspond points with coordinates in an extension of rationals for a polynomial P defining 4-surface in H and space-time surface in H by $M^8 - H$ duality. What comes first in mind is that the throats touch each other at these points so that the distance between Minkowskian space-time sheets vanishes. This is analogous to singularities of Fermi surface encountered in topological condensed matter physics: the energy bands touch each other. In TGD, the partonic 2-surfaces at the mass shells of M^4 defined by the roots of P are indeed analogs of Fermi surfaces at the level of $M^4 \subset M^8$, having interpretation as analog of momentum space.

Could these points correspond to the defects of the standard smooth structure in X^4? Note that the branching at the partonic 2-surface defining a topological vertex implies the local failure of the manifold property. Note that the vertices of an ordinary Feynman diagram imply that it is not a smooth 1-manifold.

4. Could the interpretation be that the 4-manifold obtained by removing the partonic 2-surface has exotic smooth structure with the defect of ordinary smooth structure assignable to the partonic 2-surface at its end. The situation would be rather similar to that for the representation of exotic R^4 as a surface in CP_2 with the sphere at infinity removed [A52].

5. The failure of the cosmic censorship would make possible a pair creation. As explained, the fermionic lines can indeed turn backwards in time by going through the wormhole throat and turn backwards in time. The above picture suggests that this turning occurs only at the singularities at which the partonic throats touch each other. The QFT analog would be as a local vertex for pair creation.

6. If all fermions at a given boundary of CD have the same sign of energy, fermions which have returned back to the boundary of CD, should correspond to antifermions without a change in the sign of energy. This would make pair creation without fermionic 4-vertices possible.

   If only the total energy has a fixed sign at a given boundary of CD, the returned fermion could have a negative energy and correspond to an annihilation operator. This view is nearer to the QFT picture and the idea that physical states are Galois confined states of virtual fundamental fermions with momentum components, which are algebraic integers. One can also ask whether the reversal of the arrow of time for the fermionic lines could give rise to gravitational quantum computation as proposed in [A83].

4. A more detailed model for the exotic smooth structure associated with a topological 3-vertex

One can ask what happens to the 4-surface near the topological 3-particle vertex and what is the geometric interpretation of the point defect. The first is whether the description of the situation is possible both in M^8 and H. Here one must consider momentum conservation.

1. By Uncertainty Principle and momentum conservation at the level of M^8, the incoming real momenta of the particle reaction are integers in the scale defined by CD. In the standard QFT picture, the momenta at the vertex of physical particles are at different mass shells.

   In M^8 picture, the mass squared values of virtual fermions are in general algebraic and also complex roots of a polynomial defining the 3-D mass shells H^3_m of $M^4 \subset M^8$, determining 4-surface by associative holography.

   In the standard wave mechanical picture assumed also in TGD, a given topological vertex, describable in terms of partonic 2-surfaces, would correspond to a multi-local vertex in M^8 in accordance with the representation of a local n-vertex in M^4 as convolution of n-local vertices in momentum space realizing momentum conservation.

2. $M^8 - H$ duality maps M^4 momenta by inversion to positions in $M^4 \subset H$. This encourages the question whether the topological vertex could be described also in M^8 as a partonic surface at single algebraic mass shell in M^8, mapped by $M^8 - H$ duality to a single $a = constant$ hyperboloid in $M^4 \subset H$.

   The virtual momenta at the level of M^8 are algebraic, in general complex, integers. The algebraic mass squared values at the mass shell of M^8 would be the same for all particles of the vertex. This kind of correspondence does not make sense if $M^8 - H$ duality applies to the full algebraic momenta. The assumption has been that it applies to the rational parts of the momenta.

3. The rational parts of the algebraic integer valued 4-momenta of virtual fermions are in general not at the same mass shell. Could this make possible a description in terms of partonic 2-surfaces at fixed mass *resp.* $a = consant$ shell at the level of M^8 *resp.* H?

   The classical space-time surface in H, partonic 2-surfaces and fermion lines at them are characterized by classical momenta by Noether's theorem. Quantum classical correspondence, realized in ZEO as Bohr orbitology, suggests that the classical 4-momenta assignable to these objects correspond to the rational parts of the momenta at M^8 mass shell. Could the rational projections of M^8 momenta at H^3_n correspond to different mass squared values at given H^3?

4. Note that this additional symmetry for complexified momentum space and position space descriptions would be analogous to the duality of twistor amplitudes position space and the space of area momenta.

How to describe the topological vertex in H? The goal is to understand how exotic smooth structure and its point defects could emerge from this picture. The physical picture applied hitherto is as follows.

1. 3 partonic orbits meet at a vertex described by a partonic 2-surface. Assume that they are located to single $a = constant$ $H^3 \subset M^4 \subset H$.

2. The partonic wormhole throats appear as pairs at the opposite Minkowskian space-time sheets. There are three pairs corresponding to 3 external particle lines and one line which must be a bosonic line describing fermion-antifermion bound state disappears: this corresponds to a boson absorption (or emission).

   The opposite throats carry opposite magnetic monopole charges. The only possibility, not noticed before, is that the opposite wormhole throats for the partoni orbit, which ends at the vertex, must coincide at the vertex. The minimal option is that the exotic smooth structure is associated with this partonic orbit turning back in time. The two partonic orbits, which bind 4-D Euclidean regions as wormhole throats, would fuse to a larger 4-D surface with an exotic smooth structure.

   Fermion-antifermion annihilation occurs at a point at which fermion and antifermion lines meet. The first guess is that this point corresponds to the defect of the smooth structure.

3. There is an analogy with the construction of Etesi [A52]in which a homologically non-trivial ball CP_1 glued to the C^2 at infinity to construct an exotic smooth structure. One dimension disappears for the glued 3-surface at infinity.

   In the partonic vertex, one has actually two homologically non-trivial 2-surfaces with opposite homology charges as boundaries between wormhole contact and Minkowskian regions and they fuse together in the partonic vertex. Also now, one dimension disappears as the partonic 2-surfaces become identical so that 3-D wormhole contact contracts to single 2-D partonic 2-surface.

4. The defect for the smooth structure associated with the fusion of the pair of wormhole orbits should correspond to a point at which fermion and antifermion lines meet.

   This suggests that the throats do not fuse instantaneously but gradually. The fusion would start from a single touching point identifiable asd the fermion-antifermion vertex, serving as a seed of a phase transition, and would proceed to the entire wormhole contact so that it reduces to a partonic 2-surface.

   One can argue that one has a problem if this surface is homologically non-trivial. Could the process make the closed partonic 2-surface homologically trivial. A simplified example is the fusion of two circles with opposite winding numbers ± 1 on a cylinder. The outcome is two homologically non-trivial circles of opposite orientations on top of each other. The phase transition starting from a point would correspond to a touching of the circles.

A couple of further comments are in order.

1. The connection of the pair of wormhole throats to the associative holography is an interesting question. The 4-D tangent planes of $X^4 \subset M^8$ mass shell correspond to points of CP_2. They would be different at the two parallel sheets.

   At the mass shell H^3_m the branches would coincide. The presence of two tangent planes could give rise to two different holographic orbits, which coincide at the initial mass shell and gradually diverge from each other just as in the above model for the fusion of partonic 2-surfaces. The failure of the strict determinism for the associative holography at the partonic 2-surface would make in TGD the analogy of fermion-antifermion annihilation vertex possible.

2. There is also an analogy with the cusp catastrophe in which the projection of the cusp catastrophe as a 2-surface in 3-D space with behavior variable x and two control parameters

(a, b) has a boundary at which two real roots of a polynomial of degree 3 coincide. The projection to the (a, b) plane gives a sharp shape, whose boundary is a V-shaped curve in which the sides of V become parallel at the vertex. The vertex corresponds to maximal criticality. The particle vertex would be a critical phenomenon in accordance with the interpretation as a phase transition.

5.7.6 Is a master formula for the scattering amplitudes possible?

Marko Manninen asked whether TGD can in some sense be reduced to a single equation or principle is very interesting. My basic answer is that one could reduce TGD to a handful of basic principles but formula analogous to $F = ma$ is not possible. However, at the level of classical physics, one could perhaps say that general coordinate invariance $\rightarrow$ holography $\leftarrow$ 4-D generalization of holomorphy [**?**]educe the representations of preferred extremals as analogs of Bohr orbits for particles as 3-surfaces to a representation analogous to that of a holomorphic function.

Can one hope something analogous to happen at the level of scattering amplitudes? Is some kind of a master formula possible? I have considered many options, even replacing the S-matrix with the Kähler metric in the fermionic degrees of freedom [L112]. The motivation was that the rows of the matrix defining Kähler metric define unit vectors allowing interpretation in terms of probability conservation. However, it seems that the concept of zero energy state alone makes the definition unambiguous and unitarity is possible without additional assumptions.

1. In standard quantum field theory, correlation functions for quantum fields give rise to scattering amplitudes. In TGD, the fields are replaced by the spinor fields of the "world of classical worlds" (WCW) which can regarded as superpositions of pairs of multi-fermion states restricted at the 3-D surfaces at the ends of the 4-D Bohr orbits at the boundaries of CD.

   These 3-surfaces are extremely strongly but not completely correlated by holography implied by 4-D general coordinate invariance. The modes of WCW spinor fields at the 3-D surfaces correspond to irreducible unitary representations of various symmetries, which include supersymplectic symmetries of WCW and Kac-Moody type symmetries [K31, K85] [L121, L130, L137]. Hence the inner product is unitary.

2. Whatever the detailed form of the 3-D parts of the modes of WCW spinor fields at the boundaries of CD is, they can be constructed from ordinary many fermion states. These many-fermion state correspond in the number theoretic vision of TGD to Galois singlets realizing Galois confinement [L137, L132, L135]. They are states constructed at the level of M^8 from fermion with momenta whose components are possibly complex algebraic integers in the algebraic extension of rationals defining the 4-D region of M^8 mapped to H by $M^8 - H$ duality. Complex momentum means that the corresponding state decomposes to plane waves with a continuum of momenta. The presence of Euclidian wormhole contact makes already the classical momenta complex.

   Galois confined states have momenta, whose components are integers in the momentum scale defined by the causal diamond (CD). Galois confinement defines a universal mechanism for the formation of bound states. The induced spinor fields are second quantized free spinor fields in H and their Dirac propagators are therefore fixed. This means an enormou calculational simplification.

3. The inner products of these WCW spinor fields restricted to 3-surfaces determine the scattering amplitudes. They are non-trivial since the modes of WCW spinor fields are located at opposite boundaries of CD. These inner products define the zero energy state identifiable as such as scattering amplitudes. This is the case also in wave mechanics and quantum TGD is indeed wave mechanics for particles identified as 3-surfaces.

4. There is also a functional integral of these amplitudes over the WCW, i.e. over the 4-D Bohr orbits. This defines a unitary inner product. The functional integral replaces the path integral of field theory and is mathematically well-defined since the Kähler function, appearing in the exponent defining vacuum functional, is a non-local function of the 3-surface so that standard local divergences due to the point-like nature of particles disappear. Also the standard problems due to the presence of a Hessian coming from a Gaussian determinant is canceled by the square foot of the determinant of the Kähler metric appearing in the integration measure [K52, K85].

5. The restriction of the second quantized spinor fields to 4-surfaces and zero-energy ontology are absolutely essential. Induction turns free fermion fields into interacting ones. The spinor fields of H are free and define a trivial field theory in H. The restriction to space-time surfaces changes the situation. Non-trivial scattering amplitudes are obtained since the fermionic propagators restricted to the space-time surface are not anymore free propagators in H. Therefore the restriction of WCW spinors to the boundaries of CD makes the fermions interact in exactly the same way as it makes the induced spinor connection and the metric dynamical.

There are a lot of details involved that I don't understand, but it would seem that a simple "master formula" is possible. Nothing essentially new seems to be needed. There is however one more important "but".

Are pair production and boson emission possible?

The question that I have pondered a lot is whether the pair production and emission of bosons are possible in the TGD Universe. In this process the fermion number is conserved, but fermion and antifermion numbers are not conserved separately. In free field theories they are, and in the interacting quantum field theories, the introduction of boson fermion interaction vertices is necessary. This brings infinities into the theory.

1. In TGD, the second quantized fermions in H are free and the boson fields are not included as primary fields but are bound states of fermions and antifermions. Is it possible to produce pairs at all and therefore also bosons? For example, is the emission of a photon from an electron possible? If a photon is a fermion-antifermion pair, then the fermion and antifermion numbers cannot be preserved separately. How to achieve this?

2. If fundamental fermions correspond to light-like curves at light-like orbit of partonic 2-surfaces, pair creation requires that that fermion trajectory turns in time direction. At this point velocity is infinite and this looks like a causal anomaly. There are two options: the fermion changes the sign of its energy or transforms to antiferion with the same sign of energy.

   Different signs of energy is not possible since the annihilation operator creating the fermion with opposite energy would annihilate either the final state or some fermion in the final state so that both fermion and antifermion numbers of the final state would be the same as those of the initial state.

   On the other hand, it can be said that positive energy antifermions propagate backwards in time because in the free fermion field since the terms proportional to fermion creation operators and antifermion annihilation operators appear in the expression of the field as sum of spinor modes.

   Therefore a fermion-antifermion pair with positive energies can be created and corresponds to a pair of creation operators. It could also correspond to a boson emission and to a field theory vertex, in which the fermion, antifermion and boson occur. In TGD, however, the boson fields are not included as primary fields. Is such a "vertex without a vertex" possible at all?

3. Can one find an interpretation for this creation of a pair that is in harmony with the standard view. Space-time surfaces are associated with induced classical gauge potentials. In standard field theory, they couple to fermion-antifermion pairs, and pairs can be created in classical fields. The modified Dirac equation [K113] and the Dirac equation in H also have such a coupling. Now the modified Dirac equation holds true at the fermion lines at the light-like orbits of the partonic 2-surface. Does the creation of pairs happen in this way? It might do so: also in the path integral formalism of field theories, bosons basically correspond to classical fields and the vertex is just this except that in TGD fermions are restricted to 1-D lines.

Fundamental fermion pair creation vertices as local defects of the standard smooth structure of the space-time surface?

Here comes the possible connection with a very general mathematical problem of general relativity that I have already discussed.

1. Causal anomalies as time loops that break causality are more the rule than an exception in general relativity the essence of the causal anomaly is the reversal of the arrow of time. Causal anomalies correspond to exotic diffeo-structures that are possible only in dimension $D = 4$! Their number is infinite.
2. Quite generally, the exotic smooth structures reduce to defects of the usual differentiable structure and have measure zero. Assume that they are point like defects. Exotic differentiable structures are also possible in TGD, and the proposal is that the associated defects correspond to a creation of fermion-fermion pairs for emission of fermion pairs of of gauge bosons and Higgs particle identified in TGD as bound states of fermion-antifermion pairs. This picture generalizes also to the case of gravitons, which would involve a pair of vertices of this kind. The presence of 2 vertices might relate to the weakness of the gravitational interaction.

   The reversal of the fermion line in time direction would correspond to a creation of a fermion-antifermion pair: fermion and antiferion would have the same sign of energy. This would be a causal anomaly in the sense that the time direction of the fermion line is reversed so that it becomes an antifermion.

   I have proposed that this causal anomaly is identifiable as an anomaly of differentiable structure so that emission of bosons and fermion pairs would only be possible in dimension 4: the space-time dimension would be unique!
3. But why would a point-like local defect of the differentiable structure correspond to a fermion pair creation vertex. In TGD, the point-like fermions correspond to 1-D light-like curves at the light-like orbit of the partonic 2-surface.

   In the pair creation vertex in presence of classical induced gauge potentials, one would have a V-shaped world line of fermion turning backwards in time meaning that antifermion is transformed to fermion. The antifermion and fermion numbers are not separately conserved although the total fermion number is. If one assumes that the modified Dirac equation holds true along the entire fermion worldline, there would be no pair creation.

   If it holds true only outside the V-shaped vertex the modified Dirac action for the V-shaped fermion libe can be transformed to a difference of antifermion number equal to the discontinuity of the antifermion part of the fermion current identified as an operator at the vertex. This would give rise to a non-trivial vertex and the modified gamma matrices would code information about classical bosonic action.
4. The 1-D curve formed by fermion and antifermion trajectories with opposite time direction turns backwards in time at the vertex. At the vertex, the curve is not differentiable and this is what the local defect of the standard smooth differentiable structure would mean physically!

Master formula for the scattering amplitudes: finally?

Most pieces that have been identified over the years in order to develop a master formula for the scattering amplitudes are as such more or less correct but always partially misunderstood. Maybe the time is finally ripe for the fusion of these pieces to a single coherent whole. I will try to list the pieces into a story in the following.

1. The vacuum functional, which is the exponential Kähler function defined by the classical bosonic action defining the preferred extremal a an analog of Bohr orbit, is the starting point. Physically, the Kähler function corresponds to the bosonic action (e.g. EYM) in field theories.

   Because holography is almost unique, it replaces the path integral by a sum over 4-D Bohr trajectories as a functional integral over 3-surfaces plus discrete sum.
2. However, the fermionic part of the action is missing. I have proposed a long time ago a super symmetrization of the WCW Kähler function by adding to it what I call modified Dirac action. It relies on modified gamma matrices modified gamma matrices Γ^α, which are contractions $\Gamma_k T^{\alpha k}$ of H gamma matrices Γ_k with the canonical momentum currents $T^{\alpha k} = \partial L/\partial_{\partial_\alpha h^k}$ defined by the Lagrangian L. Modified Dirac action is therefore determined by the bosonic action from the requirement of supersymmetry. This supersymmetry is however quite different from the SUSY associated with the standard model and it assigns to fermonic Noether currents their super counterparts.

Bosonic field equations for the space-time surface actually follow as hermiticity conditions for the modified Dirac equation. These equations also guarantee the conservation of fermion number(s). The overall super symmetrized action that defines super symmetrized Kähler function in WCW would be unambiguous. One would get exactly the same master formula as in quantum field theories, but without the path integral.

3. The overall super symmetrized action is sum of contributions assignable to the space-time surface itself, its 3-D light-like parton orbits as boundaries between Minkowskian regions and Euclidian wormhole contact, 2-D string world sheets and their 1-D boundaries as orbits of point-like fermions. These 1-D boundaries are the most important and analogous to the lines of ordinary Feynman diagrams. One obtains a dimensional hierarchy.
4. One can assign to these objects of varying dimension actions defined in terms of the induced geometry and spinor structure. The supersymmetric actions for the preferred extremals analogous to Bohr orbit in turn give contributions to the super symmetrized Kähler function as an analogue of the YM action so that, apart from the reduction of path integral to a sum over 4-D Bohr orbits, there is a very close analogy with the standard quantum field theory.

However, some problems are encountered.

1. It seems natural to assume that a modified Dirac equation holds true. I have presented an argument for how it indeed emerges from the induction for the second quantized spinor field in H restricted to the space-time surface assuming modified Dirac action.

   The problem is, however, that the fermionic action, which should define vertex for fermion pair creation, disappears completely if Dirac's equation holds everywhere! One would not obtain interaction vertices in which pairs of fermions arise from classical induced fields. Something goes wrong. In this vertex total fermion number is conserved but fermion and antifermion numbers are changed since antifermion transforms to fermion at the V-shaped vertex: this condition should be essential.
2. If one gives up the modified Dirac equation, the fermionic action does not disappear. In this case, one should construct a Dirac propagator for the modified Dirac operator. This is an impossible task in practice.

   Moreover, the construction of the propagator is not even necessary and in conflict with the fact that the induced spinor fields are second quantized spinors of H restricted to the space-time surface and the propagators are therefore well-defined and calculable and define the propagation at the space-time surface.
3. Should we conclude that the modified Dirac equation cannot hold everywhere? What these, presumably lower-dimensional regions of space-time surface, are and could they give the interaction vertices as topological vertices?

The key question is how to understand geometrically the emission of fermion pairs and bosons as their bound states?

1. I have previously derived a topological description for reaction vertices. The fundamental 1 → 2 vertex (for example e→e+ gamma) generalizes the basic vertex of Feynman diagrams, where a fermion emits a boson or a boson decays into a pair of fermions. Three lines meet at the ends.

   In TGD, this vertex can topologically correspond to the decomposition of a 3-surface into two 3-surfaces, to the decomposition of a partonic 2-surface into two, to the decomposition of a string into two, and finally, to the turning of the fermion line backwards from time. One can say that the n-surfaces are glued together along their $n-1$-dimensional ends, just like the 1-surfaces are glued at the vertex in the Feynman diagram.
2. In the previous section, I already discussed how to identify vertex for fermion-antifermion pair creation as a V-shaped turning point of a 1-D fermion line. The fermion line turns back in time and fermion becomes an antifermion. In TGD, the quantized boson field at the vertex is replaced by a classical boson field. This description is basically the same as in the ordinary path integral where the gauge potentials are classical.

   The problem was that if the modified Dirac equation holds everywhere, there are no pair creation vertices. The solution of the problem is that the modified Dirac equation at the V-shaped vertex cannot hold true.

What this means physically is that fermion and antifermion numbers are not separately conserved in the vertex. The modified Dirac action for the fermion line can be transformed to the change of antifermion number as operator (or fermion number at the vertex) expressible as the change of the antifermion part of the fermion number. This is expressible as the discontinuity of a corresponding part of the conserved current at the vertex. This picture conforms with the appearance of gauge currents in gauge theory vertices. Notice that modified gamma matrices determined by the bosonic action appear in the current.

3. This argument was limited to 1-D objects but can be generalized to higher-dimensional defects by assuming that the modified Dirac equation holds true everywhere except at defects represented as vertices, which become surfaces. The modified Dirac action reduces to an integral of the discontinuity of say antifermion current at the vertex, i.e. the change of the antifermion charge as an operator.

What remains more precisely understood and generalized, is the connection with the irreducible exotic smooth structures possible only in 4-D space-time.

1. TGD strongly suggests that 0-dimensional vertices generalize to topological vertices representable as surfaces of dimension $n = 0, 1, 2, 3$ assignable to objects carrying induced spinor field. In the $1 \rightarrow 2$ vertex, the orbit of an $n < 4$- dimensional surface would turn back in the direction of time and would define a V-shaped structure in time direction. These would be the various topological vertices that I have previously arrived at, but guided by a physical intuition. Also now the vertex would boild down to the discontinuity of say antifermion current instead of the current itself at the vertex.
2. It is known that exotic smooth structures reduce to standard ones except in a set of defects having measure zero. Also non-point-like defects might be possible in contrast to what I assumed at first. If the defects are surfaces, their dimension is less than 4. If not, then only the direction of fermion lines could change.

   If the generalization is possible, also 1-D, 2-D, and 3-D defects, defining an entire hierarchy of particles of different dimensions, is possible. As a matter of fact, a longstanding issue has been whether this prediction should be taken seriously. Note that in topological condensed matter physics, defects with various dimensions are commonplace. One talks about bulk states, boundary states, edge states and point-like singularities. In this would predict hierarchy of fermionic object of various dimensions.

To summarize, exotic smooth structures would give vertices without vertices assuming only free fermions fields and no primary boson fields! And this is possible only in space-time dimension 4!

5.8 A possible connection with family replication phenomenon?

In TGD framework the genus g of the partonic 2-surfaces is proposed to label fermion families [K28, K60, K64]. One can characterize by genus g the topology of light-like partonic orbits and identify the three fermion generators as 2-surfaces with genus $g = 0, 1, 2$ with the special property that they are always hyper-elliptic. Quantum mechanically also topological mixing giving rise to CKM mixing is possible. The view is that given connected 3-surface can contain several light-like 3-surface with different genera. For instance, hadrons would be such surfaces.

There are however questions to be answered.

1. The genera $g = 0, 1, 2$ assigned with the free fermion families correspond to Riemann surfaces, which are always hyper-elliptic allowing therefore Z_2 as a global conformal symmetry. These complex curves correspond to degrees $n = 2, 3, 4$ for the corresponding polynomials. For $n \leq 4$ can write explicit solutions for the roots of the polynomials. Could there be a deep connection between particle physics and mathematical cognition?
2. The homology and genus for 2-surfaces of CP_2 correlate with each other [A76]: is this consistent with the proposed topologicization of color hypercharge implying color confinement?
3. $h_{eff}/h = n$ hypothesis means that dark variant of particle particle characterized by genus g is n-fold covering of this surface. In the general case the genus of covering is different. Is this consistent with the genus-generation correspondence?

4. The degree of complex curve correlates with the genus of the curve. Is generation-genus correspondence consistent with the assumption that partonic 2-surfaces have algebraic curve as CP_2 projection (this need not be the case)?

5.8.1 How the homology charge and genus correlate?

Complex surfaces in CP_2 are highly interesting from TGD point of view.

1. The model for elementary particles assumes that the partonic 2-surfaces carrying fermion number are homologically non-trivial, in other words they carry Kähler magnetic monopole flux having values $q = \pm 1$ and $q = \pm 2$. The idea is that color hyper charge $Y = \{\pm 2/3, \pm 1/3\}$ is proportional to n for quarks and color confinement topologizes to the vanishing of total homology charge [K64].
2. The explanation of the family replication phenomenon [K28] in terms of genus-generation correspondence states that the three quarks and lepton generations correspond to the three lowest genera $g = 0, 1, 2$ for partonic 2-surfaces. Only these genera are always hyper-elliptic allowing thus a global Z_2 conformal symmetry. The physical vision is that for higher genera the handles behave like free particles. Is this proposal consistent with the proposal for the topologization of color confinement?

There is a result [A76] (page 124) stating that if the homology charge q is divisible by 2 then one must have $g \geq q^2/4 - 1$. If q is divisible by h, which is odd power of prime, one has $g \geq (q^2/4 - 1) - (q^2/4h^2)$. For $q = 2$ the theorem allows $g \geq 0$ so that all genera with color hyper charge $Y = \pm 2/3$ are realized.

The theorem says however nothing about $q = 0, 1$. These charges can be assigned to the two different geodesic spheres of CP_2 with $g = 0$ remaining invariant under SO(3) and U(2) subgroups of SU(3) respectively. Is $g > 0$ possible for $q = 1$ as the universality of topological color confinement would require? For $q = 3$ one would have $g \geq 1$. For $q = 4$ $h = 2$ divides q and one has $g \geq 2$. It would seem $g \geq 5$. The conditions become more restrictive for higher q, which suggests that for $q = 0, 1$ one has $g \geq 0$ so that the topologization of color hypercharge would make sense.

5.8.2 Euler characteristic and genus for the covering of partonic 2-surface

Hierarchy of Planck constants $h_{eff}/h = n$ means a hierarchy of space-time surfaces identifiable as n-fold coverings. The proposal is that the number of sheets in absence of singularities is maximal possible and equals to the dimension of the extension dividing the order of its Galois group.

The Euler characteristic of n-fold covering in absence of singular points is $\chi_n = n\chi$. If there are singular (ramified) points these give a correction term given by Riemann-Hurwitz formula (see `http://tinyurl.com/y7n2acub`.)

In absence of singularities one has from $\chi = -2(g - 1)$ and $\chi_n = n\chi$

$$g_n = n(g - 1) + 1 \ . \qquad (5.8.1)$$

For $n = 1$ this indeed gives $g_1 = g$ independent of g. One can also combine this with the formula $g = (d - 1)(d - 2)/2$ holding for non-singular algebraic curves of degree d.

Singularities are unavoidable at algebraic points of cognitive representations at which some subgroup of Galois group leaves the point invariant (say rational point in ordinary sense). One can consider the possibility that fermions are located at the singular points at which several sheets of covering touch each other. This would give a correction factor to the formula. If the projection map from the covering to based is of form $\Pi(z) = z^n$ at the singular point P, one says that singularity has ramimifaction index $e_P = n$ and the algebraic genus would increase to

$$g_n = n(g - 1) + 1 + \frac{1}{2}\sum_P (e_P - 1) \ . \qquad (5.8.2)$$

Indeed, singularities mean that sheets touch each other at singular points and this increases connectivity.

Under what conditions the genus of dark partonic surface with $n > 1$ can be same as that of the ordinary partonic surface representing visible matter? For the genera $g = 0$ and $g = 1$ this is possible so that these genera would be in an exceptional role also from the point of view of dark matter.

1. For $g = 1$ one has $g_n = g = 1$ independent of n in absence of singular point. Torus topology (assignable to muon and (c,s) quarks) is exceptional. In presence of singularities the genus would increase by the $\sum_P (e_P - 1)/2$ independent of the value of n. The lattice of points for elliptic surfaces would suggest existence of infinite number of singular points if the abelian group operations preserve the singular character of the points so that the genus would become infinite.

2. For $g = 0$ one would have $g_n = -n + 1$ in absence of singularities. Only $n = 1$ - ordinary matter - is possible without singularities. Dark matter is however possible if singularities are allowed. For sphere one would obtain $g_n = -n + 1 + \sum_P (e_P - 1)/2 \geq 0$. The condition $n \leq \sum_P (e_P - 1)/2 + 1$ must therefore hold true for $g \geq 0$.

   The condition $g_n = -n + 1 + \sum_P (e_P - 1)/2 = g = 0$ gives $\sum_P (e_P - 1) = 2(n - 1)$. For spherical topology it is possible to have dense set of rational points so that it is possible create cognitive representations with arbitrary number of points which can be also singular. One might argue that this kind of situation corresponds to a non-perturbative phase.

3. For $g = 2$ one would have $g_n = n + 1 + \sum_P (e_P - 1)/2$ and genus would grow with n even in absence of singularities and would be very large for large values of h_{eff}. $g_n = 2$ is obtained with $n = 1$ (ordinary matter) and no singular points not even allowed for $n = 1$. $g_n = g = 2$ is not possible for $n > 1$.

   Note that dark $g \geq 2$ fermions cannot correspond to lower generation fermions with singular points of covering. More generally, one could say that $g \geq 2$ fermions can exists only with standard value of Planck constant unless they are singular coverings of $g < 2$ fermions.

What is clear that the model of dark matter predicts breaking of universality. This breaking is not seen in the standard model couplings but makes it visible in amore delicate manner and might allow to understand why the masses of fermions increase with generation index.

5.8.3 All genera are not representable as non-singular algebraic curves

Suppose for a moment that partonic 2-surfaces correspond to rational maps of algebraic curves in CP_2 to M^4 that is deformations of these curves in M^4 direction. This assumption is of course questionable but deserves to be sttudied.

The formula (for algebraic curve see `http://tinyurl.com/nt6tkey`)

$$g = \frac{(d-1)(d-2)}{2} + \frac{\sum \delta_s}{2} \quad ,$$

where $\delta_s > 0$ characterizes the singularity, does not allow all genera for algebraic curves for $\sum \delta_s = 0$: one has $g = 0, 0, 1, 3, 6, 10, ..$ for $d = 1, 2, ...$

For instance, $g = 2$, which would correspond in TGD to third quark or lepton generation is not possible without singularities for $d = 3$ curve having $g = 1$ without singularities!

This raises questions. Could the third fermion generation actually correspond to $g = 3$? Or does it correspond to $g = 2$ 2-surface of CP_2, which is more general surface than algebraic curve meaning that it is not representable as complex surface? Or could third generation fermions correspond to $g = 0$ or $g = 1$ curves with singular point of covering by Galois group so that several sheets touch each other?

To sum up, if the results for algebraic varieties generalize to TGD framework, they suggest notable differences between different fermion families. Universality of standard model interactions says that the only differences between fermion families are due to the differ masses. It is not clear whether the different masses could be due to the differences at number theoretical level and dark matter sectors.

1. All genera can appear as as ordinary matter ($d = 1$). Dark variants of $g = 1$ states have $g_d = 1$ automatically in absence of singular points. Dark variants of $g = 0$ states must have singular point in order to give $g_n = 0$. Dark variants of $g = 2$ states with $g_d = 2$ are obtained from $g = 1$ states with singularities. The special role of the two lowest is analogous to their special role for algebraic curves.
2. If one assumes that partonic 2-surfacs correspond to algebraic curves, one obtains again that $g = 2$ surfaces must correspond to singular $g = 0$ and $g = 1$ which could be dark in TGD sense.

5.9 Summary and future prospects

In the following I give a brief summary about what has been done. I concentrate on $M^8 - H$ duality since the most significant results are achieved here.

It is fair to say that the new view answers the following a long list of open questions.

1. When $M^8 - H$ correspondence is true (to be honest, this question emerged during this work!)? What are the explicit formulas expressing associativity of the tangent space or normal space of the 4-surface?

   The key element is the formulation in terms of complexified M^8 - M^8_c - identified in terms of octonions and restriction $M^8_c \rightarrow M^8$. One loses the number field property but for polynomials ring property is enough. The level surfaces for real and imaginary parts of octonionic polynomials with real coefficients define 4-D surfaces in the generic case.

   Associativity condition is an additional condition reducing the dimension of the space-time surface unless some components of $RE(P)$ or $IM(P)$ are critical meaning that also their gradients vanish. This conforms with the quantum criticality of TGD and provides a concrete first principle realization for it.

   An important property of $IM(P_1P_2)$ is its linearity with respect to $IM(P_i)$ implying that this condition gives the surfaces $IM(P_i) = 0$ as solutions. This generalizes by induction to $IM(P_1P_2...P_n)$. For $RE(P_1P_2) = 0$ linearity does not hold true and there is a genuine interaction. A physically attractive idea idea is that $RE(P_1P_2) = 0$ holds true inside CDs and for wormhole contacts between space-time sheets with Minkoskian signature. One can generalizes this also to $IM(P_1/P_2)$ and $RE(P_1/P_2)$ if rational functions are allowed. Note however that the origins of octonionic coordinates in P_i must be on the octonionic real line.
2. How this picture corresponds to twistor lift? The twistor lift of Kähler action (dimensionally reduced Kähler action in twistor space of space-time surface) one obtains two kinds of space-time regions. The regions, which are minimal surfaces and obey dynamics having no dependence on coupling constants, correspond naturally to the critical regions in M^8 and H.

   There are also regions in which one does not have extremal property for both Kähler action and volume term and the dynamics depends on coupling constant at the level of H. These regions are associative only at their 3-D ends at boundaries of CD and at partonic orbits, and the associativity conditions at these 3-surfaces force the initial values to satisfy the conditions guaranteeing preferred extremal property. The non-associative space-time regions are assigned with the interiors of CDs. . The particle orbit like space-time surfaces entering to CD are critical and correspond to external particles.

   It has later turned out [L64] that it might be possible to take the associativity conditions to extreme in the sense that they would hold everywhere apart from a set of discrete points and space-time surface would be minimal surfaces at all points except this finite set of points. There would be transfer of conserved quantities assignable to the volume term and the 4-D Kähler action (coming as dimensionally reduced 6-D Kähler action for the twistor lift of TGD) only at these points and elementary fermions would be naturally assignable to these points.
3. The surprise was that $M^4 \subset M^8$ is naturally co-associative. If associativity holds true also at the level of H, $M^4 \subset H$ must be associative. This is possible if $M^8 - H$ duality maps tangent space in M^8 to normal space in H and vice versa.
4. The connection to the realization of the preferred extremal property in terms of gauge conditions of subalgebra of SSA is highly suggestive. Octonionic polynomials critical at the

boundaries of space-time surfaces would determine by $M^8 - H$ correspondence the solution to the gauge conditions and thus initial values and by holography the space-time surfaces in H.

5. A beautiful connection between algebraic geometry and particle physics emerges. Free many-particle states as disjoint critical 4-surfaces can be described by products of corresponding polynomials satisfying criticality conditions. These particles enter into CD , and the non-associative and non-critical portions of the space-time surface inside CD describe the interactions. One can define the notion of interaction polynomial as a term added to the product of polynomials. It can vanish at the boundary of CD and forces the 4-surface to be connected inside CD. It also spoils associativity: interactions are switched on. For bound states the coefficients of interaction polynomial are such that one obtains a bound state as associative space-time surface.

6. This picture generalizes to the level of quaternions. One can speak about 2-surfaces of space-time surface with commutative or co-commutative tangent space. Also these 2-surfaces would be critical. In the generic case commutativity/co-commutativity allows only 1-D curves.

   At partonic orbits defining boundaries between Minkowskian and Euclidian space-time regions inside CD the string world sheets degenerate to the 1-D orbits of point like particles at their boundaries. This conforms with the twistorial description of scattering amplitudes in terms of point like fermions.

   For critical space-time surfaces representing incoming states string world sheets are possible as commutative/co-commutative surfaces (as also partonic 2-surfaces) and serve as correlates for (long range) entaglement) assignable also to macroscopically quantum coherent system ($h_{eff}/h = n$ hierarchy implied by adelic physics).

7. The octonionic polynomials with real coefficients form a commutative and associative algebra allowing besides algebraic operations function composition. Space-time surfaces therefore form an algebra and WCW has algebra structure. This could be true for the entire hierarchy of Cayley-Dickson algebras, and one would have a highly non-trivial generalization of the conformal invariance and Cauchy-Riemann conditions to their n-linear counterparts at the n:th level of hierarchy with $n = 1, 2, 3, ..$ for complex numbers, quaternions, octonions,... One can even wonder whether TGD generalizes to this entire hierarchy!

8. In the original version of this article I did not realize that there are two options for realizing the idea that the M^4_c projection of space-time surface in M^8_c must belong to M^4.

   (a) I proposed that the *projection* from M^8_c to real M^4 (for which M^1 coordinate is real and E^3 coordinates are imaginary with respect to i!) defines the real space-time surface mappable by $M^8 - H$ duality to CP_2 [L46].

   (b) An alternative option, which I have not considered in the original versions of [L46, L48] is that only the roots of the 4 vanishing polynomials as coordinates of M^4_c belong to M^4 so that m^0 would be real root and m^k, $k = 1, ..., 3$ imaginary with respect to $i \to -i$. M^8_c coordinates would be invariant ("real") under combined conjugation $i \to -i, I_k \to -I_k$. In the following I will speak about this property as *Minkowskian reality.* This could make sense. Outside CD these conditions would not hold true. This option looks more attractive than the first one. Why these condition can be true just inside CD, should be understood.

9. The use of polynomials or rational functions could be also an approximation. Analytic functions of real variable extended to octonionic functions would define the most general space-time surfaces but the limitations of cognition would force to use polynomial approximation. The degree n of the polynomial determining also $h_{eff} = nh_0$ would determine the quality of the approximation and at the same time the "IQ" of the system.

All big pieces of quantum TGD are now tightly interlinked.

1. The notion of causal diamond (CD) and therefore also ZEO can be now regarded as a consequence of the number theoretic vision and $M^8 - H$ correspondence, which is also understood physically.

2. The hierarchy of algebraic extensions of rationals defining evolutionary hierarchy corresponds to the hierarchy of octonionic polynomials.

3. Associative varieties for which the dynamics is critical are mapped to minimal surfaces with universal dynamics without any dependence on coupling constants as predicted by twistor lift of TGD. The 3-D associative boundaries of non-associative 4-varieties are mapped to initial values of space-time surfaces inside CDs for which there is coupling between Kähler action and volume term.
4. Free many particle states as algebraic 4-varieties correspond to product polynomials in the complement of CD and are associative. Inside CD the addition of interaction terms vanishing at its boundaries spoils associativity and makes these varieties connected.
5. The super variant of the octonionic algebraic geometry makes sense, and one obtains a beautiful correlation between the fermion content of the state and corresponding space-time variety. This suggests that twistorial construction indeed generalizes. Criticality for the external particles giving rise to additional constraints on the coefficients of polynomials could make possible to have well-define summation over corresponding varieties.

What mathematical challenges one must meet?

1. One should prove more rigorously that criticality is possible without the reduction of dimension of the space-time surface.
2. One must demonstrate that SSA conditions can be true for the images of the associative regions (with 3-D or 4-D). This would obviously pose strong conditions on the values of coupling constants at the level of H.

Concerning the description of interactions there are several challenges.

1. Do associative space-time regions have minimal surface extremals as images in H and indeed obeying universal critical dynamics? As found, the study of the known extremals supports this view.
2. Could one construct the scattering amplitudes at the level of M^8? Here the possible problems are caused by the exponents of action (Kähler action and volume term) at H side. Twistorial construction [K87] however leads to a proposal that the exponents actually cancel. This happens if the scattering amplitude can be thought as an analog of Gaussian path integral around single extremum of action and conforms with the integrability of the theory. In fact, nothing prevents from defining zero energy states in this manner! If this holds true then it might be possible to construct scattering amplitudes at the level of M^8.
3. What about coupling constants? Coupling constants make themselves visible at H side both via the vanishing conditions for Noether charges in sub-algebra of SSA and via the values of the non-vanishing Noether charges. M^8-H correspondence determining the 3-D boundaries of interaction regions within CDs suggests that these couplings must emerge from the level M^8 via the criticality conditions posing conditions on the coefficients of the octonionic polynomials coding for interactions.

   Could all coupling constant emerge from the criticality conditions at the level of M^8? The ratio of R^2/l_P^2 of CP_2 scale and Planck length appears at H level. Also this parameter should emerge from M^8-H correspondence and thus from criticality at M^8 level. Physics would reduce to a generalization of the catastrophe theory of Rene Thom!
4. The description of interactions at the space-time surface associated with single CD should be M^8 counterpart of the H picture in which 3 light-like partonic orbits meet at common end topological vertex - defined by a partonic 2-surface and fermions scatter without touching. Now one has octonionic sparticle lines and interaction vertex becomes possible. This conforms with the idea that interactions take place at discrete points belonging to the extension of rationals. The partonic 2-surfaces defining topological vertices would naturally correspond to the intersections $X^2 = X^4 \cap S^6(t_n)$. If sparticle lines are allowed to move along this space-like 2-surface (the line becomes space-like) they can intersect and give rise to a fusion vertex producing the third fermionic line.

   The partonic 2-surfaces defining topological vertices would naturally correspond to the intersections $X^2 = X^4 \cap S^6(t_n)$, which satisfy $RE(P) = IM(P) = 0$ and are singular and doubly critical. If sparticle lines are allowed to move along this space-like 2-surface (the line becomes space-like) they can intersect and give rise to a fusion vertex producing the third fermionic line.

5. Real analyticity requires that the octonionic polynomials have real coefficients. This forces the origin of octonionic coordinates to be at real line (time axis) in the octonionic sense, and guarantees the associativity and commutativity of the polynomials. Arbitrary CDs cannot be located along this line. Can one assume that all CDs involved with *observable* processes satisfy this condition?

   If not, how do the 4-varieties associated with octonionic polynomials with different origins interact? How could one avoid losing the extremely beautiful associative and commutative algebra? It seems that one cannot form their products and sums and must form the Cartesian product of M^8:s with different tips for CDS and formulate the interaction in this framework. In the case of space-time surfaces associated with different CDs the discrete intersections of space-time surfaces would define the interaction vertices.

6. Super-octonionic geometry suggests that the twistorial construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY generalizes to TGD in rather straightforward manner to a purely geometric construction. Functional integral over WCW would reduce to summations over polynomials with coefficients in an appropriate extension of rationals and criticality conditions on the coefficients could make the summation well-defined by bringing in finite measurement resolution.

   If scattering diagrams are associated with discrete cognitive representations, one obtains a generalization of super-twistor formalism involving polygons. Super-octonions as counterparts of super gauge potentials are well-defined if octonionic 8-momenta are quaternionic: indeed, Grassmannians have quaternionic counterparts but not octonionic ones. There are good hopes that the twistor Grassmann approach to $\mathcal{N} = 4$ SUSY generalizes. The core part in the calculation of the scattering diagram would reduce to the construction of octonionic 4-varieties and identifying the points belonging to the extension of rationals considered. The rest would be dictated by symmetries and integrations over various moduli spaces, which should be number theoretically universal so that residue calculus strongly suggests itself.

7. What is the connection with super conformal variant of Yangian symmetry, whose generalization in TGD framework is highly suggestive? Twistorial construction of scattering amplitudes at the level of M^8 looks highly promising idea and could also realize Yangian supersymmetry. The conjecture is that the twistorial amplitudes decompose to M^4 and CP_2 parts with similar structure with E^4 spin (electroweak isospin) replacing ordinary spin and that the integrands in Grassmannians emerging from the conservation of M^4 and E^4 4-momenta are identical in the two cases and thus guarantee Yangian supersymmetry in both sectors. The only difference would be due to the product of delta functions associated with the "negative helicities" (weak isospins with negative sign) expressible as a delta function in the complement of $SU(3)$ Cartan algebra $U(1) \times U(1)$ by using exponential map.

It is appropriate to close with a question about fundamentals.

1. The basic structure at M^8 side consists of complexified octonions. The metric tensor for the complexified inner product for complexified octonions (no complex conjugation with respect to i for the vectors in the inner product) can be taken to have any signature $(\epsilon_1, ..., \epsilon_8)$, $\epsilon_i = \pm 1$. By allowing some coordinates to be real and some coordinates imaginary one obtains effectively any signature from say purely Euclidian signature. What matters is that the restriction of complexified metric to the allowed sub-space is real. These sub-spaces are linear Lagrangian manifolds for Kähler form representing the commuting imaginary unit i. There is analogy with wave mechanics. Why M^8 -actually M^4 - should be so special real section? Why not some other signature?

2. The first observation is that the CP_2 point labelling tangent space is independent of the signature so that the problem reduces to the question why M^4 rather than some other signature $(\epsilon_1, .., \epsilon_4)$. The intersection of real subspaces with different signatures and same origin $(t, r) = 0$ is the common sub-space with the same signature. For instance, for $(1, -1, -1, -1)$ and $(-1, -1, -1, -1)$ this subspace is 3-D $t = 0$ plane sharing with CD the lower tips of CD. For $(-1, 1, 1, 1)$ and $(1, 1, 1, 1)$ the situation is same. For $(1, -1, -1, -1)$ and $(1, 1, -1, -1)$ $z = 0$ holds in the intersection having as common with the lower boundary of CD the boundary of 3-D light-cone. One obtains in a similar manner boundaries of 2-D and 1-D light-cones as intersections.

3. What about CDs in various signatures? For a fully Euclidian signature the counterparts for the interiors of CDs reduce to 4-D intervals $t \in [0, T]$ and their exteriors and thus the space-time varieties representing incoming particles reduce to pairs of points $(t, r) = (0, 0)$ and $(t, r) = (T, 0)$: it does not make sense to speak about external particles. For other signatures the external particles correspond to 4-D surfaces and dynamics makes sense. The CDs associated with the real sectors intersect at boundaries of lower dimensional CDs: these lower-dimensional boundaries are analogous to subspaces of Big Bang (BB) and Big Crunch (BC).

4. I have not found any good argument for selecting $M^4 = M^{1,3}$ as a unique signature. Should one allow also other real sections? Could the quantum numbers be transferred between sectors of different signature at BB and BC? The counterpart of Lorentz group acting as a symmetry group depends on signature and would change in the transfer. Conservation laws should be satisfied in this kind of process if it is possible. For instance, in the leakage from $M^4 = M^{1,3}$ to Mi, j, say $M^{2,2}$, the intersection would be $M^{1,2}$. Momentum components for which signature changes, should vanish if this is true. Angular momentum quantization axis normal to the plane is defined by two axis with the same signature. If the signatures of these axes are preserved, angular momentum projection in this direction should be conserved. The amplitude for the transfer would involve integral over either boundary component of the lower-dimensional CD.

   Could the leakage between signatures be detected as disappearance of matter for CDs in elementary particle scales or lab scales?

5. One can also raise a question about the role of WCW geometry as a continuous infinite-D geometry: could the discretization by cognitive representations making WCW effectively discrete mean its loss? It seems that this cannot be the case. At least in the real sector continuum must be present and the discretization reflects only the discreteness of cognitive representations. In principle continuous WCW could make sense also in p-adic sectors of the adele.

   The identification of space-time surfaces as zero loci of polynomials generalizes to rational functions and even transcendental functions although the existence of the p-adic counterparts of these functions requires additional conditions. Could one interpret the representation in terms of polynomials and possibly rational functions as an approximation? Could the hierarchy of approximations obtained in this manner give rise to a hierarchy of hyper-finite factors of type II_1 defining a hierarchy of measurement resolutions [K112]?

Chapter 6

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III

6.1 Introduction

In the third chapter about $M^8 - H$ duality the question whether the space-time surfaces in M^8 allow a global slicing by string world sheets X^2 defined by an integrable distribution of local tangent spaces $M^2(x) \subset M^4$ and their orthogonal duals or whether there is only a discrete set of surfaces X^2 is discussed. Discrete set is obtained by requiring that space-time surface or its normal space contains string world sheet as a complex (commutative) sub-manifold. By the strong form of holography (SH) this is enough to deduce the image of $X^4 \subset M^8$ in H from the boundary data consisting of the H-images of X^2 and metrically 2-D light-like partonic orbits X^3_L of topological dimension $D = 3$.

Also the relation of $M^8 - H$ duality to p-adic length scale hypothesis and dark matter hierarchy are discussed and it is shown that the notion of p-adic length scale emerging from p-adic mass calculations emerges also geometrically.

The fermionic aspects of $M^8 - H$ duality are discussed: the basic purely number theoretic elements are the octonionic realization of M^8 spinors and the replacement of Dirac equation as a partial differential equation with an algebraic equation for octonionic spinors. Dirac equation for octonionic spinors is analogous to the algebraic momentum space variant of the ordinary Dirac equation. This provides also considerable understanding about the bosonic aspects of $M^8 - H$ duality. In particular, the pre-images of $X^3_L \subset X^4 \subset H$ in M^8 correspond to mass shells for massless octonionic spinor modes realized as light-like 3-surfaces in M^8. One can say that M^8 picture realizes the momentum space dual of the modified Dirac equation in $X^4 \subset H$. Twistor Grassmannian picture supports the view that spinor modes also in H are localized to $X^3_L \subset X^4$, and obey the modified Dirac equation associated with Chern-Simons term.

Cognitive representations is the third basic topic of the chapter. Cognitive representations are identified as sets of points in an extension of rationals for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8- and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces. For the earlier work related to adelic TGD and cognitive representations see [L52, L38, L43].

The notion is applied in various cases and the connection with $M^8 - H$ duality is rather loose.

1. Extensions of rationals are essentially coders of information. There the possible analogy of extensions of rationals with genes deserves discussion. Extensions, which are not extensions of extensions would be analogous to genes. The notion of conserved gene as number theoretical analogy for Galois extensions as the Galois group of extension which is normal subgroup of

Galois extension.

2. The work of Peter Scholze [A72] based on the notion of perfectoid has raised a lot of interest in the community of algebraic geometers. One application of the notion relates to the attempt to generalize algebraic geometry by replacing polynomials with analytic functions satisfying suitable restrictions. Also in TGD this kind of generalization might be needed at the level of $M^4 \times CP_2$ whereas at the level of M^8 algebraic geometry might be enough. The notion of perfectoid as an extension of p-adic numbers Q_p allowing all p:th roots of p-adic prime p is central and provides a powerful technical tool when combined with its dual, which is function field with characteristic p.

   Could perfectoids have a role in TGD? The infinite-dimensionality of perfectoid is in conflict with the vision about finiteness of cognition. For other p-adic number fields Q_q, $q \neq p$ the extension containing p:th roots of p would be however finite-dimensional even in the case of perfectoid. Furthermore, one has an entire hierarchy of almost-perfectoids allowing powers of p^m:th roots of p-adic numbers. The larger the value of m, the larger the number of points in the extension of rationals used, and the larger the number of points in cognitive representations consisting of points with coordinates in the extension of rationals. The emergence of almost-perfectoids could be seen in the adelic physics framework as an outcome of evolution forcing the emergence of increasingly complex extensions of rationals [L44].

3. The construction of cognitive representation represents a well-known mathematical problem of finding the points of space-time surface with embedding space coordinates in given extension of rationals. Number theorist Minhyong Kim [A59, A68] has speculated about very interesting general connection between number theory and physics. The reading of a popular article about Kim's work revealed that number theoretic vision about physics provided by TGD has led to a very similar ideas and suggests a concrete realization of Kim's ideas [L79]. In the following I briefly summarize what I call identification problem. The identification of points of algebraic surface with coordinates, which are rational or in extension of rationals, is in question. In TGD framework the embedding space coordinates for points of space-time surface belonging to the extension of rationals defining the adelic physics in question are common to reals and all extensions of p-adics induced by the extension. These points define what I call cognitive representation, whose construction means solving of the identification problem.

   Cognitive representation defines discretized coordinates for a point of "world of classical worlds" (WCW) taking the role of the space of spaces in Kim's approach. The symmetries of this space are proposed by Kim to help to solve the identification problem. The maximal isometries of WCW necessary for the existence of its Kähler geometry provide symmetries identifiable as symplectic symmetries. The discrete subgroup respecting extension of rationals acts as symmetries of cognitive representations of space-time surfaces in WCW, and one can identify symplectic invariants characterizing the space-time surfaces at the orbits of the symplectic group.

4. One expects that large cognitive representations are winners in the number theoretical fight for survival. Strong form of holography suggests that it is enough to consider cognitive representations restricted to string world sheets and partonic 2-surfaces. If the 2-surface possesses large group of symmetries acting in extension of rationals, one can have large cognitive representations as orbit of point in extension. Examples of highly symmetric 2-D surfaces are geodesic spheres assignable to partonic 2-surfaces and cosmic strings and elliptic curves assignable with string world sheets and cosmic strings [L90].

5. Rationals and their extensions give rise to a unique discretizations of space-time surface (for instance) - cognitive representation - having interpretation in terms of finite measurement resolution. There are howevever many open questions. Should one allow only octonionic polynomials defined as algebraic continuations of real polynomials or should one allow also analytic functions and regard polynomials as approximations. Zeta functions are especially interesting analytic functions and Dekekind zetas characterize extensions of rationals and one can pose physically motivated questions about them [L68].

6.2 About $M^8 - H$-duality, p-adic length scale hypothesis and dark matter hierarchy

$M^8 - H$ duality, p-adic length scale hypothesis and dark matter hierarchy as phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ are basic assumptions of TGD, which all reduce to number theoretic vision. In the sequel $M^8 - H$ duality, p-adic length scale hypothesis and dark matter hierarchy are discussed from number theoretic perspective.

Several new results emerge. Strong form of holography (SH) allows to weaken strong form of $M^8 - H$ duality mapping space-time surfaces $X^4 \subset M^8$ to $H = M^4 \times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained.

M^8 duality allows to relate p-adic length scales L_p to differences for the roots of the polynomial defining the extension defining "special moments in the life of self" assignable causal diamond (CD) central in zero energy ontology (ZEO). Hence p-adic length scale hypothesis emerges both from p-adic mass calculations and $M^8 - H$ duality. It is proposed that the size scale of CD correspond to the largest dark scale nL_p for the extension and that the sub-extensions of extensions could define hierarchy of sub-CDs. Skyrmions are an important notion if nuclear and hadron physics, $M^8 - H$ dyality suggests an interpretation of skyrmion number as winding number as that for a map defined by complex polynomial.

6.2.1 Some background

A summary of the basic notions and ideas involved is in order.

p-Adic length scale hypothesis

In p-adic mass calculations [K60] real mass squared is obtained by so called canonical identification from p-adic valued mass squared identified as analog of thermodynamical mass squared using p-adic generelization of thermodynamics assuming super-conformal invariance and Kac-Moody algebras assignable to isometries ad holonomies of $H = M^4 \times CP_2$. This implies that the mass squared is essentially the expectation value of sum of scaling generators associated with various tensor factors of the representations for the direct sum of super-conformal algebras and if the number of factors is 5 one obtains rather predictive scenario since the p-adic temperature T_p must be inverse integer in order that the analogs of Boltzmann factors identified essentially as p^{L_0/T_p}.

The p-adic mass squared is of form $Xp + O(p^2)$ and mapped to $X/p + O(1/p^2)$. For the p-adic primes assignable to elementary particles ($M_{127} = 2^{127} - 1$ for electron) the higher order corrections are in general extremely small unless the coefficient of second order contribution is larger integer of order p so that calculations are practically exact.

Elementary particles seem to correspond to p-adic primes near powers 2^k. Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of period doubling in which prime $p = 2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p = 3$ (period tripling as approach to chaos). Many elementary particles and also hadron physics and electroweak physics seem to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2.

For given prime p also higher powers of p define p-adic length scales: for instance, for electron the secondary p-adic time scale is .1 seconds characterizing fundamental bio-rhythm. Quite generally, elementary particles would be accompanied by macroscopic length and time scales perhaps assignable to their magnetic bodies or causal diamonds (CDs) accompanying them.

This inspired p-adic length scale hypothesis stating the size scales of space-time surface correspond to primes near half-octaves of 2. The predictions of p-adic are exponentially sensitive to the value of k and their success gives strong support for p-adic length scale hypothesis. This hypothesis applied not only to elementary particle physics but also to biology and even astrophysics and cosmology. TGD Universe could be p-adic fractal.

Dark matter as phases of ordinary matter with $h_{eff} = nh_0$

The identification of dark matter as phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ is second key hypothesis of TGD. To be precise, these phases behave like dark matter and galactic dark matter could correspond to dark energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes.

There are good arguments in favor of the identification $h = 6h_0$ [L31, L60]. "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining space-time surface as a covering. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

$M^8 - H$ duality

$M^8 - H$ duality ($H = M^4 \times CP_2$) [L76] has taken a central role in TGD framework. $M^8 - H$ duality allows to identify space-time regions as "roots" of octonionic polynomials P in complexified M^8 - M^8_c - or as minimal surfaces in $H = M^4 \times CP_2$ having 2-D singularities.

Remark:O_c,H_c,C_c,R_c will be used in the sequel for complexifications of octonions, quaternions, etc.. number fields using commuting imaginary unit i appearing naturally via the roots of real polynomials.

The precise form of $M^8 - H$ duality has however remained unclear. Two assumptions are involved.

1. Associativity stating that the tangent or normal space of at the point of the space-time space-time surface M^8 is associative - that is quaternionic. There are good reasons to believe that this is true for the polynomial ansatz everywhere but there is no rigorous proof.
2. The tangent space of the point of space-time surface at points mappable from M^8 to H must contain fixed $M^2 \subset M^4 \subset M^8$ or an integrable distribution of $M^2(x)$ so that the 2-surface of M^4 determined by it belongs to space-time surface.

The strongest, global form of $M^8 - H$ duality states that $M^2(x)$ is contained to tangent spaces of X^4 at all points x. Strong form of holography (SH) states allows also the option for which this holds true only for 2-D surfaces - string world sheets and partonic 2-surfaces - therefore mappable to H and that SH allows to determined $X^4 \subset H$ from this data. In the following a realization of this weaker form of $M^8 - H$ duality is found. Note however that one cannot exclude the possibility that also associativity is true only at these surfaces for the polynomial ansatz.

Number theoretic origin of p-adic primes and dark matter

There are several questions to be answered. How to fuse real number based physics with various p-adic physics? How p-adic length scale hypothesis and dark matter hypothesis emerge from TGD?

The properties of p-adic number fields and the strange failure of complete non-determinism for p-adic differential equations led to the proposal that p-adic physics might serve as a correlate for cognition, imagination, and intention. This led to a development of number theoretic vision which I call adelic physics. A given adele corresponds to a fusion of reals and extensions of various p-adic number fields induced by a given extension of rationals.

The notion of space-time generalizes to a book like structure having real space-time surfaces and their p-adic counterparts as pages. The common points of pages defining is back correspond to points with coordinates in the extension of rationals considered. This discretization of space-time surface is in general finite and unique and is identified as what I call cognitive representation. The Galois group of extension becomes symmetry group in cognitive degrees of freedom. The ramified primes of extension are exceptionally interesting and are identified as preferred p-adic primes for the extension considered.

The basic challenge is to identify dark scale. There are some reasons to expect correlation between p-adic and dark scales which would mean that the dark scale would depend on ramified primes, which characterize roots of the polynomial defining the extensions and are thus not defined completely by extension alone. Same extension can be defined by many polynomials. The naïve guess is that the scale is proportional to the dimension n of extension serving as a measure for algebraic complexity (there are also other measures). p-Adic length scales L_p would be proportional

nL_p, p ramified prime of extension? The motivation would be that quantum scales are typically proportional to Planck constant. It turns out that the identification of CD scale as dark scale is rather natural.

6.2.2 New results about $M^8 - H$ duality

In the sequel some new results about $M^8 - H$ duality are deduced. Strong form of holography (SH) allows to weaken the assumptions making possible $M^8 - H$ duality. It would be enough to map only certain complex 2-D sub-manifolds of quaternionic space-time surface in M^8 to H: SH would allow to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds would be determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and they form a discrete set.

Strong form of holography (SH)

Ordinary 3-D holography is forced by general coordinate invariance (GCI) and loosely states that the data at 3-D surfaces allows to determined space-time surface $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

This poses additional strong conditions on the space-time surface.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SC_n with radial conformal weights coming as n-multiples of those for the entire algebra SC and its commutator $[SC_n, SC]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations.

   This hierarchy of minimal surfaces would naturally corresponds to the hierarchy of extensions of rationals with n identifiable as dimension of the extension giving rise to effective Planck constant. At the level of Hilbert spaces the inclusion hierarchies for extensions could also correspond to the inclusion hierarchies of hyper-finite factors of type I_1 [K112] so that $M^8 - H$ duality would imply beautiful connections between key ideas of TGD.

2. Second conjecture is that the preferred extremals (PEs) are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor structure from the product of twistor spaces of M^4 and CP_2 to 6-D S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [A57] so that TGD is unique.

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated either with

- light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian or
- the space-like 3-surfaces at the ends of CD to determine space-time surface as PE (in case that it exists).

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken strong form of $M^8 - H$ duality mapping space-time surfaces $X^4 \subset M^8$ to $H = M^4 \times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained.

Space-time as algebraic surface in M_c^8 regarded complexified octonions

The octonionic polynomial giving rise to space-time surface as its "root" is obtained from ordinary real polynomial P with rational coefficients by algebraic continuation. The conjecture is that the identification in terms of roots of polynomials of even real analytic functions guarantees associativity and one can formulate this as rather convincing argument [?] Space-time surface X_c^4 is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. These options correspond to complexified-quaternionic tangent- or normal spaces. For $P(x)=x^n+..$ ordinary roots are algebraic integers. The real 4-D space-time surface is projection of this surface from M_c^8 to M^8. One could drop the subscripts "$_c$" but in the sequel they will be kept.

M_c^4 appears as a special solution for any polynomial P. M_c^4 seems to be like a universal reference solution with which to compare other solutions.

One obtains also brane-like 6-surfaces as 6-spheres as universal solutions. They have M^4 projection, which is a piece of hyper-surface for which Minkowski time as time coordinate of CD corresponds to a root $t=r_n$ of P. For monic polynomials these time values are algebraic integers and Galois group permutes them.

One cannot exclude rational functions or even real analytic functions in the sense that Taylor coefficients are octonionically real (proportional to octonionic real unit). Number theoretical vision - adelic physics [?, ?] suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a+ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

How do the solutions assignable to the opposite boundaries of CD relate to each other?

CD has two boundaries. The polynomials associated with them could be different in the general formulation discussed in [L103, L109] but they could be also same. How are the solutions associated with opposite boundaries of CD glued together in a continuous manner?

1. The polynomials assignable to the opposite boundaries of CD are allowed to be polynomials of o *resp.* $(o-T)$: here T is the distance between the tips of CD.

2. CD brings in mind the realization of conformal invariance at sphere: the two hemispheres correspond to powers of z and $1/z$: the condition $z=\overline{1/z}$ at unit circle is essential and there is no real conjugation. How the sphere is replaced with 8-D CD which is also complexified. The absence of conjugation looks natural also now: could CD contain a 3-surface analogous to the unit circle of sphere at which the analog of $z=\overline{1/z}$ holds true? If so, one has $P(o,z)=P(1/o,z)$ and the solutions representing roots fo $P(o,z)$ and $P(1/o,z)$ can be glued together.

   Note that $1/o$ can be expressed as $\overline{o}/o\overline{o}$ when the Minkowskian norm squared $\overline{o}o$ is non-vanishing and one has polynomial equation also now. This condition is true outside the boundary of 8-D light-cone, in particular near the upper boundary of CD.

   The counter part for the length squared of octonion in Minkowskian signature is light-one proper time coordinate $a^2=t^2-r^2$ for M_+^8. Replacing o which scaled dimensionless variable $o_1=o/(T/2)$ the gluing take place along $a=T/2$ hyperboloid.

One has algebraic holomorphy with respect to o but also anti-holomorphy is possible. What could these two options correspond to? Could the space-time surfaces assignable to self and its time-reversal relate by octonionic conjugation $o\rightarrow\overline{o}$ relating two Fock vacuums annihilated by fermionic annihilation *resp.* creation operators?

In [L103, L109] the possibility that the sequence of SSFRs or BSFRs could involve iteration of the polynomial defining space-time surface - actually different polynomials were allowed for two boundaries. There are 3 options: each SSFR would involve the replacement $Q=P\circ..\circ P\rightarrow P\circ Q$, the replacement occurs only when new "special moments in the life of self" defined by the roots of P as $t=r_n$ balls of cd, or the replacement can occur in BSFR when the metabolic resources do not allow to continue the iteration (the increase of h_{eff} during iteration increases the needed metabolic feed).

The iteration is compatible with the proposed picture. The assumption $P(0) = 0$ implies that iterates of P contain also the roots of P as roots - they are like conserved genes. Also the 8-D light-cone boundary remains invariant under iteration. Even more general function decompositions $P \to Q \to P$ are consistent with the proposed picture.

Brane-like solutions

One obtains also 6-D brane-like solutions to the equations.

1. In general the zero loci for imaginary or real part are 4-D but the 7-D light-cone δM^8_+ of M^8 with tip at the origin of coordinates is an exception [L47, L48, L49]. At δM^8_+ the octonionic coordinate o is light-like and one can write $o = re$, where 8-D time coordinate and radial coordinate are related by $t = r$ and one has $e = (1 + e_r)/\sqrt{2}$ such that one as $e^2 = e$.

   Polynomial $P(o)$ can be written at δM^8_+ as $P(o) = P(r)e$ and its roots correspond to 6-spheres S^6 represented as surfaces $t_M = t = r_N$, $r_M = \sqrt{r_N^2 - r_E^2} \leq r_N$, $r_E \leq r_N$, where the value of Minkowski time $t = r = r_N$ is a root of $P(r)$ and r_M denotes radial Minkowski coordinate. The points with distance r_M from origin of $t = r_N$ ball of M^4 has as fiber 3-sphere with radius $r = \sqrt{r_N^2 - r_E^2}$. At the boundary of S^3 contracts to a point.

2. These 6-spheres are analogous to 6-D branes in that the 4-D solutions would intersect them in the generic case along 2-D surfaces X^2. The boundaries $r_M = r_N$ of balls belong to the boundary of M^4 light-cone. In this case the intersection would be that of 4-D and 3-D surface, and empty in the generic case (it is however quite not clear whether topological notion of "genericity" applies to octonionic polynomials with very special symmetry properties).

3. The 6-spheres $t_M = r_N$ would be very special. At these 6-spheres the 4-D space-time surfaces X^4 as usual roots of $P(o)$ could meet. Brane picture suggests that the 4-D solutions connect the 6-D branes with different values of r_n.

   The basic assumption has been that particle vertices are 2-D partonic 2-surfaces and light-like 3-D surfaces - partonic orbits identified as boundaries between Minkowskian and Euclidian regions of space-time surface in the induced metric (at least at H level) - meet along their 2-D ends X^2 at these partonic 2-surfaces. This would generalize the vertices of ordinary Feynman diagrams. Obviously this would make the definition of the generalized vertices mathematically elegant and simple.

   Note that this does not require that space-time surfaces X^4 meet along 3-D surfaces at S^6. The interpretation of the times t_n as moments of phase transition like phenomena is suggestive. ZEO based theory of consciousness suggests interpretation as moments for state function reductions analogous to weak measurements ad giving rise to the flow of experienced time.

4. One could perhaps interpret the free selection of 2-D partonic surfaces at the 6-D roots as initial data fixing the 4-D roots of polynomials. This would give precise content to strong form of holography (SH), which is one of the central ideas of TGD and strengthens the 3-D holography coded by ZEO alone in the sense that pairs of 3-surfaces at boundaries of CD define unique preferred extremals. The reduction to 2-D holography would be due to preferred extremal property realizing the huge symplectic symmetries and making $M^8 - H$ duality possible as also classical twistor lift.

   I have also considered the possibility that 2-D string world sheets in M^8 could correspond to intersections $X^4 \cap S^6$? This is not possible since time coordinate t_M constant at the roots and varies at string world sheets.

   Note that the compexification of M^8 (or equivalently octonionic E^8) allows to consider also different variants for the signature of the 6-D roots and hyperbolic spaces would appear for $(\epsilon_1, \epsilon_i, .., \epsilon_8)$, $epsilon_i = \pm 1$ signatures. Their physical interpretation - if any - remains open at this moment.

5. The universal 6-D brane-like solutions S^6_c have also lower-D counterparts. The condition determining X^2 states that the C_c-valued "real" or "imaginary" for the non-vanishing Q_c-valued

"real" or "imaginary" for P vanishes. This condition allows universal brane-like solution as a restriction of O_c to M^4_c (that is CD_c) and corresponds to the complexified time=constant hyperplanes defined by the roots $t = r_n$ of P defining "special moments in the life of self" assignable to CD. The condition for reality in R_c sense in turn gives roots of $t = r_n$ a hyper-surfaces in M^2_c.

Explicit realization of $M^8 - H$ duality

$M^8 - H$ duality allows to map space-time surfaces in M^8 to H so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D singularities in H satisfying an infinite number of additional conditions stating vanishing of Noether charges for super-symplectic algebra actings as isometries for the "world of classical worlds" (WCW). Twistor lift allows variants of this duality. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of extensions of rationals defining an evolutionary hierarchy. This forms the basis for the number theoretical vision about TGD.

M^8-H duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions.

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of $M^8 - H$ duality and means that tangent - or normal space is quaternionic.
2. The tangent space of space-time surface and thus space-time surface itself must contain a preferred $M^2_c \subset M^4_c$ or more generally, an integrable distribution of tangent spaces $M^2_c(x)$ and similar distribution of their complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense. $E^2_c(x)$ would correspond to partonic 2-surface.

One can imagine two realizations for this condition.

Option I: Global option states that the distributions $M^2_c(x)$ and $E^2_c(x)$ define slicing of X^4_c.

Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce space-time surfaces in H. This would be the minimal option.

That the selection between these options is not trivial is suggested by following.

1. For massless extremals (MEs, topological light rays) parameterized by light-like vector vector k defining $M^2 \subset M^2 \times E^2 \subset M^4$ at each point and by space-like polarization vector ϵ depending on single transversal coordinate of E^2 [K10].
2. CP_2 coordinates have an arbitrary dependence on both $u = k \cdot m$ and $w = \epsilon \cdot m$ and can be also multivalued functions of u and w. Single light-like vector k is enough to identify M^2. CP_2 type extremals having metric and Kähler form of CP_2 have light-like geodesic as M^4 projection defining M^2 and its complement E^2 in the normal space.
3. String like objects $X^2 \times Y^2 \subset M^4 \times CP_2$ are minimal surfaces and X^2 defines the distribution of $M^2(x) \subset M^4$. Y^2 ddefines the complement of this distribution.

   Option I is realized in all 3 cases. It is not clear whether M^2 can depend on position in the first 2 cases and also CP_2 point in the third case. It could be that only a discrete set of these string world sheets assignable to wormhole contacts representing massless particles is possible (**Option II**).

How these conditions would be realized?

1. The basic observation is that X^2c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M^2_c would be the simplest solution but also more general complex sub-manifolds $X^2_c \subset M^4_c$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

   These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. As found, there are also classes special global solutions for which the choice of M_c^2 is global and does not depend on space-time point. The interpretation would be in terms of modes of classical massless fields characterized by polarization and momentum. If the identification of M_c^2 is correct, these surfaces are however unstable against perturbations generating discrete string world sheets and orbits of partonic 2-surfaces having interpretation space-time counterparts of quanta. That fields are detected via their quanta was the revolutionary observation that led to quantum theory. Could quantum measurement induce the instability decomposing the field to quanta at the level of space-time topology?
3. One can generalize this condition so that it selects 1-D surface in X_c^2. By assuming that R_c-valued "real" or "imaginary" part of quaternionic part of P at this 2-surface vanishes. one obtains preferred M_c^1 or E_c^1 containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

   This option could be made possible by SH. SH states that preferred extremals are determined by data at 2-D surfaces of X^4. Even if the conditions defining X_c^2 have only a discrete set of solutions, SH at the level of H could allow to deduce the preferred extremals from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. Associativity and existence of $M^2(x)$ would be required only at the 2-D surfaces.
4. I have proposed that physical string world sheets and partonic 2-surfaces appear as singularities and correspond to 2-D folds of space-time surfaces at which the dimension of the quaternionic tangent space degenerates from 4 to 2 [L75] [K10]. This interpretation is consistent with a book like structure with 2-pages. Also 1-D real and imaginary manifolds could be interpreted as folds or equivalently books with 2 pages.

   For the singular surfaces the dimension quaternionic tangent or normal space would reduce from 4 to 2 and it is not possible to assign CP_2 point to the tangent space. This does not of course preclude the singular surfaces and they could be analogous to poles of analytic function. Light-like orbits of partonic 2-surfaces would in turn correspond to cuts.
5. What could the normal space singularity mean at the level of H? Second fundamental form defining vector basis in normal space is expected to vanish. This would be the case for minimal surfaces.
   (a) String world sheets with Minkowskian signature (in M^4 actually) are expected to be minimal surfaces. In this case T matters and string world sheets could be mapped to H by $M^8 - H$ duality and SH would work for them.
   (b) The light-like orbits of partonic 2-surfaces with Euclidian signature in H would serve as analogs of cuts. N is expected to matter and partonic 2-surfaces should be minimal surfaces. Their branching of partonic 2-surfaces is thus possible and would make possible (note the analogy with the branching of soap films) for them to appear as 2-D vertices in H.
   The problem is to identify the pre-images of partonic 2-surfaces in M^8. The light-likeness of the orbits of partonic 2-surfaces (induced 4-metric changes its signature and degenerates to 3-D) should be important. Could light-likeness in this sense define the pre-images partonic orbits in M^8?

Remark: It must be emphasized that SH makes possible $M^8 - H$ correspondence assuming that also associativity conditions hold true only at partonic 2-surfaces and string world sheets. Thus one could give up the conjecture that the polynomial ansatz implies that tangent or normal spaces are associative. Proving that this is the case for the tangent/normal spaces of these 2-surfaces should be easier.

Does $M^8 - H$ duality relate hadron physics at high and low energies?

During the writing of this article I realized that $M^8 - H$ duality has very nice interpretation in terms of symmetries. For $H = M^4 \times CP_2$ the isometries correspond to Poincare symmetries and color $SU(3)$ plus electroweak symmetries as holonomies of CP_2. For octonionic M^8 the subgroup $SU(3) \subset G_2$ is the sub-group of octonionic automorphisms leaving fixed octonionic imaginary unit invariant - this is essential for $M^8 - H$ duality. $SU(3)$ is also subgroup of $SO(6) \equiv SU(4)$ acting as rotation on $M^8 = M^2 \times E^6$. The subgroup of the holonomy group of $SO(4)$ for E^4 factor of $M^8 = M^4 \times E^4$ is $SU(2) \times U(1)$ and corresponds to electroweak symmetries. One can say that at the level of M^8 one has symmetry breaking from $SO(6)$ to $SU(3)$ and from $SO(4) = SU(2) \times SO(3)$ to $U(2)$.

This interpretation gives a justification for the earlier proposal that the descriptions provided by the old-fashioned low energy hadron physics assuming $SU(2)_L \times SU(2)_R$ and acting acting as covering group for isometries $SO(4)$ of E^4 and by high energy hadron physics relying on color group $SU(3)$ are dual to each other.

Skyrmions and $M^8 - H$ duality

I received a link (https://tinyurl.com/ycathr3u) to an article telling about research (https://tinyurl.com/yddwhr2o) carried out for skyrmions, which are very general condensed matter quasiparticles. They were found to replicate like DNA and cells. I realized that I have not clarified myself the possibility of skyrmions on TGD world and decided to clarify my thoughts.

1. What skyrmions are?

Consider first what skyrmions are.

1. Skyrmions are topological entities. One has some order parameter having values in some compact space S. This parameter is defined in say 3-ball such that the parameter is constant at the boundary meaning that one has effectively 3-sphere. If the 3rd homotopy group of S characterizing topology equivalence classes of maps from 3-sphere to S is non-trivial, you get soliton-llike entities, stable field configurations not deformable to trivial ones (constant value). Skyrmions can be assigned to space S which is coset space $SU(2)_L \times SU(2)_R / SU(2)_V$, essentially S^3 and are labelled by conserved integer-valued topological quantum number.
2. One can imagine variants of this. For instance, one can replace 3-ball with disk. $SO(3) = S^3$ with 2-sphere S^2. The example considered in the article corresponds to discretized situation in which one has magnetic dipoles/spins at points of say discretized disk such that spins have same direction about boundary circle. The distribution of directions of spin can give rise to skyrmion-like entity. Second option is distribution of molecules which do not have symmetry axis so that as rigid bodies the space of their orientations is discretized version of $SO(3)$. The field would be the orientation of a molecule of lattice and one has also now discrete analogs of skyrmions.
3. More generally, skyrmions emerge naturally in old-fashioned hadron physics, where $SU(2)_L \times SU(2)_R / SU(2)_V$ involves left-handed, right-handed and vectorial subgroups of $SO(4) = SU(2)_L \times SU(2)_R$. The realization would be in terms of 4-component field (π, σ), where π is charged pion with 3 components - axial vector - and σ which is scalar. The additional constraint $\pi \cdot \pi + \sigma^2 = constant$ defines 3-sphere so that one has field with values in S^3. There are models assigning this kind of skyrmion with nucleon, atomic nuclei, and also in the bag model of hadrons bag can be thought of as a hole inside skyrmion. These models seem to have something to do with reality so that a natural question is whether skyrmions might appear in TGD.

2. Skyrmion number as winding number

In TGD framework one can regard space-time as 4-surface in either octonionic M^8_c, c refers here to complexification by an imaginary unit i commuting with octonions, or in $M^4 \times CP_2$. For the solution surfaces M^8 has natural decomposition $M^8 = M^2 \times E^6$ and E^6 has $SO(6)$ as isometry group containing subgroup $SU(3)$ having automorphisms of octonions as subgroup leaving M^2 invariant. $SO(6) = SU(4)$ contains $SU(3)$ as subgroup, which has interpretation as isometries of CP_2 and counterpart of color gauge group. This supports $M^8 - H$ duality, whose most recent form is discussed in [L101].

The map $S^3 \rightarrow S^3$ defining skyrmion could be taken as a phenomenological consequence of $M^8 - H$ duality implying the old-fashioned description of hadrons involving broken $SO(4)$ symmetry (PCAC) and unbroken symmetry for diagonal group $SO(3)_V$ (CCV). The analog of $(\pi, sigma)$ field could correspond to a B-E condensate of pions $(\pi, sigma)$.

The obvious question is whether the map $S^3 \to S^3$ defining skyrmion could have a deeper interpretation in TGD framework. I failed to find any elegant formulation. One could however generalize and ask whether skyrmion like entities characterize by winding number are predicted by basic TGD.

1. In the models of nucleon and nuclei the interpretation of conserved topological skyrmion number is as baryon number. This number should correspond to the homotopy class of the map in question, essentially winding number. For polynomials of complex number degree corresponds to winding number. Could the degree $n = h_{eff}/h_0$ of polynomial P having interpretation as effective Planck constant and measure of complexity - kind of number theoretic IQ - be identifiable as skyrmion number? Could it be interpreted as baryon number too?
2. For leptons regarded as local 3 anti-quark composites in TGD based view about SUSY [L81] the same interpretation would make sense. It seems however that the winding number must have both signs. Degree is n is however non-negative.

   Here complexification of M^8 to M^8_c is essential. One an allow both holomorphic and anti-holomorphic continuations of real polynomials P (with rational coefficients) using complexification defined by commutative imaginary unit i in M^8_c so that one has polynomials $P(z)$ *resp.* $P(\overline{z})$ in turn algebraically continued to complexified octonionic polynomials $P(z,o)$ *resp.* $P(\overline{z},o)$.

   Particles *resp.* antiparticles would correspond to the roots of octonionic polynomial $P(z,o)$ resp. $P(\overline{z},o)$ meaning space-time geometrization of the particle-antiparticle dichotomy and would be conjugates of each other. This could give a nice physical interpretation to the somewhat mysterious complex roots of P.

3. More detailed formulation

To make this formulation more detailed on must ask how 4-D space-time surfaces correspond to 8-D "roots" for the "imaginary" ("real") part of complexified octonionic polynomial as surfaces in M^8_c.

1. Equations state the simultaneous vanishing of the 4 components of complexified quaternion valued polynomial having degree n and with coefficients depending on the components of O_c, which are regarded as complex numbers $x + iy$, where i commutes with octonionic units. The coefficients of polynomials depend on complex coordinates associated with non-vanishing "real" ("imaginary") part of the O_c valued polynomial.
2. To get perspective, one can compare the situation with that in catastrophe theory in which one considers roots for the gradient of potential function of behavior variables x^i. Potential function is polynomial having control variables as parameters. Now behavior variable correspond "imaginary" ("real") part and control variables to "real" ("imaginary") of octonionic polynomial.

   For a polynomial with real coefficients the solution divides to regions in which some roots are real and some roots are complex. In the case of cusp catastrophe one has cusp region with 3-D region of the parameter defined by behavior variable x and 2 control parameters with 3 real roots, the region in which one has one real root. The boundaries for the projection of 3-sheeted cusp to the plane defined by control variables correspond to degeneration of two complex roots to one real root.

   In the recent case it is not clear whether one cannot require the M^8_c coordinates for space-time surface to be real but to be in $M^8 = M^1 + iE^7$.
3. Allowing complex roots gives 8-D space-time surfaces. How to obtain real 4-D space-time surfaces?
   (a) One could project space-time surfaces to real M^8 to obtain 4-D real space-time surfaces. For M^8 this would mean projection to $M^1 + iE^7$ and in time direction the real part of root is accepted and is same for the root and its conjugate. For E^7 this would mean that imaginary part is accepted and means that conjugate roots correspond to different space-time surfaces and the notion of baryon number is realized at space-time level.
   (b) If one allows only real roots, the complex conjugation proposed to relate fermions and anti-fermions would be lost.

4. One can select for 4 complex M^8_c coordinates X^k of the surface and the remaining 4 coordinates Y^k can be formally solved as roots of n:th degree polynomial with dynamical coefficients depending on X^k and the remaining Y^k. This is expected to give rise to preferred extremals with varying dimension of M^4 and CP_2 projections.

5. It seems that all roots must be complex.

   (a) The holomorphy of the polynomials with respect to the complex M^8_c coordinates implies that the coefficients are complex in the generic point M^8_c. If so, all 4 roots are in general complex but do not appear as conjugate pairs. The naïve guess is that the maximal number of solutions would be n^4 for a given choice of M^8 coordinates solved as roots. An open question is whether one can select subset of roots and what happens at $t = r_n$ surfaces: could different solutions be glued together at them.

   (b) Just for completeness one can consider also the case that the dynamical coefficients are real - this is true in the E^8 sector and whether it has physical meaning is not clear. In this case the roots come as real roots and pairs formed by complex root and its conjugate. The solution surface can be divided into regions depending on the character of 4 roots. The n roots consist of complex root pairs and real roots. The members or complex root pairs are mapped to same point in E^8.

4. Could skyrmions in TGD sense replicate?

What about the observation that condensed matter skyrmions replicate? Could this have analog at fundamental level?

1. The assignment of conserved topological quantum number to the skyrmion is not consistent with replication unless the skyrmion numbers of outgoing states sum up to that of the initial state. If the system is open one can circumvent this objection. The replication would be like replication of DNA in which nucleotides of new DNA strands are brought to the system to form new strands.

2. It would be fascinating if all skyrmions would correspond to space-time surfaces at fundamental M^8 level. If so, skyrmion property also in magnetic sense could be induced by from a deeper geometric skyrmion property of the MB of the system. The openness of the system would be essential to guarantee conservation of baryon number. Here the fact that leptons and baryons have opposite baryon numbers helps in TGD framework. Note also ordinary DNA replication could correspond to replication of MB and thus of skyrmion sequences.

6.2.3 About p-adic length scale hypothesis and dark matter hierarchy

It is good to introduce first some background related to p-adic length scale hypothesis discussed in chapters of [K68] and dark matter hierarchy discussed in chapters [K50, K51], in particular in chatper [?].

General form of p-adic length scale hypothesis

The most general form of p-adic length scale hypothesis does not pose conditions on allowed p-adic primes and emerges from p-adic mass calculations [K28, K60, K70]. It has two forms corresponding to massive particles and massless particles.

1. For massive particles the preferred p-adic mass calculations based on p-adic thermodynamics predicts the p-adic mass squared m^2 to be proportional to p or its power- the real counterpart of m^2 is proportional to $1/p$ or its power. In the simplest case one has

$$m^2 = \frac{X}{p}\frac{\hbar}{L_0} \quad ,$$

   where L_0 is apart from numerical constant the length R of CP_2 geodesic circle. X is a numerical constant not far from unity. $X \geq 1$ is small integer in good approximation. For instance for electron one has $x = 5$.

By Uncertainty Principle the Compton length of particle is characterizing the size of 3-surfaces assignable to particle are proportional to $\sqrt{p}$:

$$L_c(m) = \frac{\hbar}{m} = \sqrt{\frac{1}{X}} L_p \ , \quad L_p = \sqrt{p} L_0 = \ .$$

Here L_p is p-adic length scale and corresponds to minimal mass for given p-adic prime. p-Adic length scale would be would characterize the size of the 3-surface assignable to the particle and would correspond to Compton length.

2. For massless particles mass vanishes and the above picture is not possible unless there is very small mass coming from p-adic thermodynamics and determined by the size scale of CD - this is quite possible. The preferred time/spatial scales p-adic energy- equivalently 3-momentum are proportional to p-adic prime p or its power. The real energy is proportional to $1/p$. At the embedding space level the size of scale causal diamond (CD) [L80] would be proportional to p: $L = T = pL_0$, $L_0 = T_0$ for $c = 1$. The interpretation in terms of Uncertainty Principle is possible.

   There would be therefore two levels: space-time level and embedding space level . At the space-time level the primary p-adic length scale would be proportional to $\sqrt{p}$ whereas the p-adic length scale at embedding space-time would correspond to secondary p-adic length scale proportional to p. The secondary p-adic length scales would assign to elementary new physics in macroscopic scales. For electron the size scale of CD would be about .1 seconds, the time scale associated with the fundamental bio-rhythm of about 10 Hz.

3. A third piece in the picture is adelic physics [L52, L53] inspiring the hypothesis that effective Planck constant h_{eff} given by $h_{eff}/h_0 = n$, $h = 6h_0$, labels the phases of ordinary matter identified as dark matter. n would correspond to the dimension of extension of rationals.

   The connection between preferred primes and the value of $n = h_{eff}/h_0$ is interesting. One proposal is that preferred primes p in p-adic length scale hypothesis determining the mass scale of particle correspond to so called ramified primes, which characterize the extensions. The p-adic variant of the polynomial defining space-time surfaces in M^8 picture would have vanishing discriminant in order $O(p)$. Since discriminant is proportional to the product of differences of different roots of the polynomial, two roots would be very near to each other p-adically. This would be mathematical correlate for criticality in p-adic sense.

   $M^8 - H$ duality [L76, L73] leads to the prediction that the roots r_n of polynomial defining the space-time region in M^8 correspond to preferred time values $t = t_n =\propto r_n$- I have called $t = t_n$ "special moments in the life of self". Since the squares for the differences for the roots are proportional to ramified primes, these time differences would code for ramified primes assignable to the space-time surface. There would be several p-adic time scales involved and they would be coded by $t_{ij} = r_i - r_j$, whose moduli squared are divided by so called ramified primes defining excellent candidates for preferred p-adic primes. p-Adic physics would make itself visible at the level of space-time surface in terms of "special moments in the life of self".

4. p-Adic length scales emerge naturally from $M^8 - H$ duality [L76, L73]. Ramified primes would in M^8 picture appear as factors of time differences associated with "special moments in the life of self" associated with CD [L73]. One has $|t_i - t_j| \propto \sqrt{p_{ij}}$, p_{ij} ramified prime. It is essential that square root of ramified prime appears here.

   This suggests strongly that p-adic length scale hypothesis is realized at the level of space-time surface and there are several p-adic length scales present coded to the time differences. Knowing of the polynomial would give information about p-adic physics involved. If dark scales correlate with p-adic length scales as proposed, the definition of dark scale should assume the dependence of ramified primes quite generally rather than as a result of number theoretic survival of fittest as one might also think.

   The factors $t_i - t_j$ are proportional - not only to the typically very large p-adic prime p_{max} charactering the system - but also smaller primes or their powers. Could the scales in question be of form $l_p = \sqrt{X}\sqrt{p_{max}} L_0$ rather than p-adic length scales $L_{p_{ram}}$ defined by various ramified primes. Here X would be integer consisting of small ramified primes.

   p-Adic mass calculations predict in an excellent approximation the mass of the particle is given by $m = (\sqrt{X}/\sqrt{p})m_0$, X small integer and $m_0 = 1/L_0$. Compton length would be given by $L_c(p) = \sqrt{p}/\sqrt{X})L_0$. The identification $l_p = L_c(p)$ would be attractive but is not possible unless one has $X = 1$. In this case one would be considering p-adic length scale L_p. the interpretation in terms of multi-p-adicity seems to be the realistic option.

About more detailed form of p-adic length scale hypothesis

More specific form of p-adic length scale hypothesis poses conditions on physically preferred p-adic primes. There are several guesses for preferred primes. They could be primes near to integer powers 2^k, where k could be positive integer, which could satisfy additional conditions such as being odd, prime or be associated with Mersenne prime or Gaussian Mersenne. One can consider also powers of other small primes such as $p = 2, 3, 5$. p-Adic length scale hypothesis in is basic form would generalize the notion of period doubling. For odd values of k one would indeed obtain period doubling, tripling, etc... suggesting strongly chaos theoretic origin.

1. p-Adic length scale hypothesis in its basic form

Consider first p-adic length scale hypothesis in its basic form.

1. In its basic form states that primes $p \simeq 2^k$ are preferred p-adic primes and correspond by p-adic mass calculations p-adic length scales $L_p \equiv L(k) \propto \sqrt{p} = 2^{k/2}$. Mersenne primes and primes associated with Gaussian Mersennes as especially favored primes and charged leptons ($k \in \{127, 113, 107\}$) and Higgs boson ($k = 89$) correspond to them. Also hadron physics ($k = 107$) and nuclear physics ($k = 113$) correspond to these scales. One can assign also to hadron physics Mersenne prime and the conjecture is that Mersennes and Gaussian Mersennes define scaled variants of hadron physics and electroweak physics. In the length scale between cell membrane thickness fo 10 nm and nuclear size about 2.5 μm there are as many as 4 Gaussian Mersennes corresponding to $k \in \{151, 157, 163, 167\}$.

   Mersenne primes correspond to prime values of k and I have proposed that k is prime for fundamental p-adic length scales quite generally. There are also however also other p-adic length scales - for instance, for quarks k need not be prime - and it has remained unclear what criterion could select the preferred exponents k. One can consider also the option that odd values of k defined fundamental p-adic length scales.

2. What makes p-adic length scale hypothesis powerful is that masses of say scaled up variant of hadron physics can be estimated by simple scaling arguments. It is convenient to use electron's p-adic length scale and calculate other p-adic length scales by scaling $L(k) = 2^{(k-127)/2}L(127)$.

Here one must make clear that there has been a confusion in the definitions, which was originally due to a calculational error.

1. I identified the p-adic length scale $L(151)$ mistakenly as $L(151) = 2^{(k-127)/2}L_e(127)$ by using instead of $L(127)$ electron Compton length $L_e \simeq L(127/\sqrt{5}$. The notation for these scales would be therefore $L_e(k)$ identified as $L_e(k) = 2^{(k-127)/2}L_e(127)$ and I have tried to use it systematically but failed to use the wrong notation in informal discussions.
2. This mistake might reflect highly non-trivial physics. It is scaled up variants of L_e which seem to appear in physics. For instance, $L_e(151) \simeq 10$ nm corresponds to basic scale in living matter. Why the biological important scales should correspond to scaled up Compton lengths for electron? Could dark electrons with scaled up Compton scales equal to $L_e(k)$ be important in these scales? And what about the real p-adic length scales relate to these scales by a scaling factor $\sqrt{5} \simeq 2.23$?

2. Possible modifications of the p-adic length scale hypothesis

One can consider also possible modifications of the p-adic length scale hypothesis. In an attempt to understand the scales associated with INW structures in terms of p-adic length scale hypothesis it occurred to me that the scales which do not correspond to Mersenne primes or Gaussian Mersennes might be generated somehow from the these scales.

1. Geometric mean $L = \sqrt{L(k_1)L(k_2)}$ would length scale which would correspond to L_p with $p \simeq 2^{(k_1+k_2)/2}$. This is of the required form only if $k = k_1 + k_2$ is even so that k_1 and k_2

are both even or odd. If one starts from Mersennes and Gaussian Mersennes the condition is satisfied. The value of $k = (k_1 + k_2)/2$ can be also even.

Remark: The geometric mean $(127 + 107)/2 = 117$ of electronic and hadronic Mersennes corresponding to mass 16 MeV rather near to the mass of so called X boson [L32] (`https://tinyurl.com/ya3yuzeb`).

2. One can also consider the formula $L = (L(k_1)L(k_2)..L(k_n))^{1/n}$ but in this case the scale would correspond to prime $p \simeq 2^{k_1+...k_n)/n}$. Since $(k_1 + ..k_n)/n$ is integer only if $k_1 + ...k_n$ is proportional to n.

What about the allowed values of fundamental integers k? It seems that one must allow all odd integers.

1. If only prime values of k are allowed, one can obtain obtain for twin prime pair $(k-1, k+1)$ even integer k as geometric mean $\sqrt{k}$ if k is square. If prime k is not a member of this kind of pair, it is not possible to get integers $k-1$ and $k+1$. If only prime values of k are fundamental, one could assign to $k = 89$ characterizing Higgs boson weak bosons $k = 90$ possibly characterizing weak bosons. Therefore it seems that one must allow all odd integers with the additional condition already explained.

2. Just for fun one can check whether $k = 161$ forced by the argument related to electroweak scale and h_{eff} corresponds to a geometric mean of two Gaussian Mersennes. One has $k(k_1, k_2) = (k_2 + k_2)/2$ giving the list $k(151, 157) = 154)$, $k(151, 163) = 157$ Gaussian Mersenne itself, $k(151, 167) = 159$, $k(157, 163) = 160$, $k(157, 167) = 162$, $k(163, 167) = 165$. Unfortunately, $k = 161$ does not belong to this set. If one allows all odd values of k as fundamental, the problem disappears.

One can also consider refinements of p-adic length scale hypothesis in its basic form.

1. One can consider also a generalization of p-adic length scale hypothesis to allow length scales coming as powers of small primes. The small primes $p = 2, 3, 5$ assignable to Platonic solids would be especially interesting. $p = 2, 3, 5$ and also Fermat primes and Mersenne primes are maximally near to powers of two and their powers would define secondary and higher p-adic length scales. In this sense the extension would not actually bring anything new.

   There is evidence for the occurrence of long p-adic time scales coming as powers of 3 [I10, I11] (`http://tinyurl.com/ycesc5mq`) and [K71] (`https://tinyurl.com/y8camqlt`. Furthermore, prime 5 and Golden Mean are related closely to DNA helical structure. Portion of DNA with L(151) contains 10 DNA codons and is the minimal length containing an integer number of codons.

2. The presence of length scales associated with 1 nm and 2 nm thick structures encourage to consider the possibility of p-adic primes near integers $2^k3^l5^m$ defining generators of multiplicative ideals of integers. They do not satisfy the maximal nearness criterion anymore but would be near to integers representable as products of powers of primes maximally near to powers of two.

What could be the interpretation of the integer k appearing in $p \simeq 2^k$? Elementary particle quantum numbers would be associated with wormhole contacts with size scale of CP_2 whereas elementary particles correspond to p-adic size scale about Compton length. What could determine the size scale of wormhole contact? I have proposed that to p-adic length scale there is associated a scale characterizing wormhole contact and depending logarithmically on it and corresponds to $L_k = (1/2)log(p)L_0 = (k/2)log(2)L_0$. The generalization of this hypothesis to the case of $p \simeq 2^k3^l5^m...$ be straightforward and be $L_{k,l,m} = (1/2)(klog(2) + llog(3) + mlog(5) + ..)$.

Dark scales and scales of CDs and their relation to p-adic length scale hierarchy

There are two length scale hierarchies. p-Adic length scale hierarchy assignable to space-time surfaces and the dark hierarchy assignable to CDs. One should find an identification of dark scales and understand their relationship to p-adic length scales.

1. *Identification of dark scales*

The dimension n of the extension provides the roughest measure for its complexity via the formula $h_{eff}/h_0 = n$. The basic - rather ad hoc - assumption has been that n as dimension of extension defines not only h_{eff} but also the size scale of CD via $L = nL_0$.

This assumption need not be true generally and already the attempt to understand gravitational constant [L102] as a prediction of TGD led to the proposal that gravitational Planck constant $h_{gr} = n_{gr}h_0 = GMm/v_0$ [E18] could be coded by the data relating to a normal subgroup of Galois group appearing as a factor of n.

The most general option is that dark scale is coded by a data related to extension of its sub-extension and this data involves ramified primes. Ramified primes depend on the polynomial defining the extension and there is large number polynomials defining the same extension. Therefore ramified ramifies code information also about polynomial and dynamics of space-time surface.

First some observations.

1. For Galois extension the order n has a natural decomposition to a product of orders n_i of its normal subgroups serving also as dimensions of corresponding extensions: $n = \prod_i n_i$. This implies a decomposition of the group algebra of Galois group to a tensor product of state spaces with dimensions n_i [L109].
2. Could one actually identify several dark scales as the proposed identifications of gravitational, electromagnetic, etc variants of h_{eff} suggest? The hierarchy of normal subgroups of Galois group of rationals corresponds to sub-groups with orders given by $N(i,1) = n_i n_{i-1}...n_{i-1}$ of n define orders for the normal subgroups of Galois group. For extensions of $k-1$:th extension of rationals one has $N(i,k) = n_i n_{i-1}...n_{i-k}$. The most general option is that these normal subgroups provide only the data allowing to associate dark scales to each of them. The spectrum of h_{eff} could correspond to the $\{N_{i,k}\}$ or at least the set $\{N_{i,1}\}$.
3. The extensions with prime dimension $n = p$ have no non-trivial normal subgroups and $n = p$ would hold for them. For these extensions the state space of group algebra is prime as Hilbert space and does not decompose to tensor product so that it would represent fundamental system. Could these extensions be of special interest physically? SSFRs would naturally involve state function reduction cascades proceeding downwards along hierarchy of normal subgroups and would represent cognitive measurements [L109].

The original guess was that dark scale $L_D = nL_p$, where n is the order n for the extensions and p is a ramified prime for the extension. A generalized form would allow $L_D = N(i,1)L_{p_k}$ for the sub-extension such that p_k is ramified prime for the sub-extension.

2. Can one identify the size scale of CD as dark scale?

It would be natural if the scale of CD would be determined by the extension of rationals. Or more generally, the scales of CD and hierarchy of sub-CDs associated with the extension would be determined by the inclusion hierarchy of extensions and thus correspond to the hierarchy of normal sub-groups of Galois group.

The simplest option would be $L_{CD} = L_D$ so that the size scales of sub-CD would correspond dark scales for sub-extension given by $L_{CD,i} = N(i,1)L_{p_k}$, p_k ramified prime of sub-extension.

1. The differences $|r_i - r_j|$ would correspond to differences for Minkowski time of CD. CD need not contain all values of hyperplanes $t = r_i$ and the evolution by SSFR would gradually bring in day-light all roots r_n of the polynomial P defining space-time surface as "very special moments in the life of self". If the size scale of CD is so large that also the largest value of $|r_i|$ is inside the upper or lower half of CD, the size scale of CD would correspond roughly to the largest p-adic length scale.

   CD contains sub-CDs and these could correspond to normal subgroups of Galois extension as extension of extension of ....
2. One can ask what happens when all special moments $t = r_n$ have been experienced? Does BSFR meaning death of conscious entity take place or is there some other option? In [L103] I considered a proposal for how chaos could emerge via iterations of P during the sequence of SSFRs.

   One could argue that when CD has reached by SSFRs following unitary evolutions a size for which all roots r_n have become visible, the evolution could continues by the replacement of

P with $P \circ P$, and so on. This would give rise to iteration and space-time analog for the approach to chaos.

3. Eventually the evolution by SSFRs must stop. Biological arguments suggests that metabolic limitations cause the death of self since the metabolic energy feed is not enough to preserve the distribution of values of h_{eff} (energies increase with $h_{eff} \propto Nn$, for N:th iteration and h_{eff} is reduced spontaneously) [L110].

6.3 Fermionic variant of $M^8 - H$ duality

The topics of this section is $M^8 - H$ duality for fermions. Consider first the bosonic counterpart of $M^8 - H$ duality.

1. The octonionic polynomial giving rise to space-time surface X^4 as its "root" is obtained from ordinary real polynomial P with rational coefficients by algebraic continuation. The conjecture is that the identification in terms of roots of polynomials of even real analytic functions guarantees associativity and one can formulate this as rather convincing argument [L47, L48, L49]. Space-time surface X^4_c is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. These options correspond to complexified-quaternionic tangent- or normal spaces. For $P(x) = x^n + ..$ ordinary roots are algebraic integers. The real 4-D space-time surface is projection of this surface from M^8_c to M^8. One could drop the subscripts "$_c$" but in the sequel they will be kept.

   M^4_c appears as a special solution for any polynomial P. M^4_c seems to be like a universal reference solution with which to compare other solutions.

   One obtains also brane-like 6-surfaces as 6-spheres as universal solutions. They have M^4 projection, which is a piece of hyper-surface for which Minkowski time as time coordinate of CD corresponds to a root $t = r_n$ of P. For monic polynomials these time values are algebraic integers and Galois group permutes them.

2. One cannot exclude rational functions or even real analytic functions in the sense that Taylor coefficients are octonionically real (proportional to octonionic real unit). Number theoretical vision - adelic physics [L52], suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

   The generalization of the relationship between reals, extensions of p-adic number fields, and algebraic numbers in their intersection is suggestive. The "world of classical worlds" (WCW) would contain the space-time surfaces defined by polynomials with general real coefficients. Real WCW would be continuous space in real topology. The surfaces defined by rational or perhaps even algebraic coefficients for given extension would represent the intersection of real WCW with the p-adic variants of WCW labelled by the extension.

3. $M^8 - H$ duality requires additional condition realized as condition that also space-time surface itself contains 2-surfaces having commutative (complex) tangent or normal space. These surfaces can be 2-D also in metric sense that is light-like 3-D surfaces. The number of these surfaces is finite in generic case and they do not define a slicing of X^4 as was the first expectation. Strong form of holography (SH) makes it possible to map these surfaces and their tangent/normal spaces to 2-D surfaces $M4 \times CP_2$ and to serve as boundary values for the partial differential equations for variational principle defined by twistor lift. Space-time surfaces in H would be minimal surface apart from singularities.

Concerning $M^8 - H$ duality for fermions, there are strong guidelines: also fermionic dynamics should be algebraic and number theoretical.

1. Spinors should be octonionic. I have already earlier considered their possible physical interpretation. [L23].

2. Dirac equation as linear partial differential equation should be replaced with a linear algebraic equation for octonionic spinors which are complexified octonions. The momentum space

variant of the ordinary Dirac equation is an algebraic equation and the proposal is obvious: $P\Psi = 0$, where P is the octonionic continuation of the polynomial defining the space-time surface and multiplication is in octonionic sense. The conjugation in O_c is induced by the conjugation of the commuting imaginary unit i. The square of the Dirac operator is real if the space-time surface corresponds to the projection $O_c \to M^8 \to M^4$ with real time coordinate and imaginary spatial coordinates so that the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

The masslessness condition restricts the solutions to light-like 3-surfaces $m_{kl}P^kP^l = 0$ in Minkowskian sector analogous to mass shells in momentum space - just as in the case of ordinary massless Dirac equation. $P(o)$ rather than octonionic coordinate o would define momentum. These mass shells should be mapped to light-like partonic orbits in H.

3. This picture leads to the earlier phenomenological picture about induced spinors in H. Twistor Grassmann approach suggests the localization of the induced spinor fields at light-like partonic orbits in H. If the induced spinor field allows a continuation from 3-D partonic orbits to the interior of X^4, it would serve as a counterpart of virtual particle in accordance with quantum field theoretical picture.

6.3.1 $M^8 - H$ duality for space-time surfaces

It is good to explain $M^8 - H$ duality for space-time surfaces before discussing it in fermionic sector.

Space-time as 4-surface in $M^8_c = O_c$

One can regard real space-time surface $X^4 \subset M^8$ as a M^8--projection of $X^4_c \subset M^8_c = O_c$. M^4_c is identified as complexified quaternions H_c [L76, L101]. The dynamics is purely algebraic and therefore local an associativity is the basic dynamical principle.

1. The basic condition is associativity of $X^4 \subset M^8$ in the sense that either the tangent space or normal space is associative - that is quaternionic. This would be realized if X^4_c as a root for the quaternion-valued "real" or "imaginary part" for the O_c algebraic continuation of real analytic function $P(x)$ in octonionic sense. Number theoretical universality requires that the Taylor coefficients are rational numbers and that only polynomials are considered.

   The 4-surfaces with associative normal space could correspond to elementary particle like entities with Euclidian signature (CP_2 type extremals) and those with associative tangent space to their interaction regions with Minkowskian signature. These two kinds space-time surfaces could meet along these 6-branes suggesting that interaction vertices are located at these branes.

2. The conditions allow also exceptional solutions for any polynomial for which both "real" and "imaginary" parts of the octonionic polynomial vanish. Brane-like solutions correspond to 6-spheres S^6 having $t = r_n$ 3-ball B^3 of light-cone as M^4 projection: here r_n is a root of the real polynomial with rational coefficients and can be also complex - one reason for complexification by commuting imaginary unit i. For scattering amplitudes the topological vertices as 2-surfaces would be located at the intersections of X^4_c with 6-brane. Also Minkowski space M^4 is a universal solution appearing for any polynomial and would provide a universal reference space-time surface.

3. Polynomials with rational coefficients define EQs and these extensions form a hierarchy realized at the level of physics as evolutionary hierarchy. Given extension induces extensions of p-adic number fields and adeles and one obtains a hierarchy of adelic physics. The dimension n of extension allows interpretation in terms of effective Planck constant $h_{eff} = n \times h_0$. The phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ behave like dark matter and galactic dark matter could correspond to classical energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes. It is not completely clear whether number galactic dark matter must have $h_{eff} > h$. Dark energy in would correspond to the volume part of the energy of the flux tubes.

   There are good arguments in favor of the identification $h = 6h_0$ [L58]. "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining X^4 as a covering space locally. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

The ramified primes of extension in turn are identified as preferrred p-adic primes. The moduli for the time differences $|t_r - t_s|$ have identification as p-adic time scales assignable to ramified primes [L101]. For ramified primes the p-adic variants of polynomials have degenerate zeros in $O(p) = 0$ approximation having interpretation in terms of quantum criticality central in TGD inspired biology.

4. During the preparation of this article I made a trivial but overall important observation. Standard Minkowski signature emerges as a prediction if conjugation in O_c corresponds to the conjugation with respect to commuting imaginary unit i rather than octonionic imaginary units as though earlier. If the space-time surface corresponds to the projection $O_c \rightarrow M^8 \rightarrow M^4$ with real time coordinate and imaginary spatial coordinates the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

Realization of $M^8 - H$ duality

$M^8 - H$ duality allows to $X^4 \subset M^8$ to $X^4 \subset H$ so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D preferred 2-surfaces defining holography making possible $M^8 - H$ duality and possibly appearing as singularities in H. The dynamics of minimal surfaces, which are also extremals of Kähler action, reduces for known extremals to purely algebraic conditions analogous to holomorphy conditions in string models and thus involving only gradients of coordinates. This condition should hold generally and should induce the required huge reduction of degrees of freedom proposed to be realized also in terms of the vanishing of super-symplectic Noether charges already mentioned [K85].

Twistor lift allows several variants of this basic duality [L88]. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of EQs defining an evolutionary hierarchy. This forms the basics for the number theoretical vision about TGD.

As already noticed, $X^4 \subset M^8$ would satisfy an infinite number of additional conditions stating vanishing of Noether charges for a sub-algebra $SSA_n \subset SSA$ of super-symplectic algebra SSA actings as isometries of WCW.

$M^8 - H$ duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions [L76].

1. Associativity condition for tangent-/normal spaces is the first essential condition for the existence of $M^8 - H$ duality and means that tangent - or normal space is associative/quaternionic.
2. Each tangent space of X^4 at x must contain a preferred $M^2_c(x) \subset M^4_c$ such that $M^2_c(x)$ define an integrable distribution and therefore complexified string world sheet in M^4_c. This gives similar distribution for their orthogonal complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense. $E^2_c(x)$ would correspond to partonic 2-surface. This condition generalizes for X^4 with quaternionic normal space. A possible interpretation is as a space-time correlate for the selection of quantization axes for energy (rest system) and spin.

One can imagine two realizations for the additional condition.

Option I: Global option states that the distributions $M^2_c(x)$ and $E^2_c(x)$ define a slicing of X^4_c.

Option II: Only a discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce $X^4 \subset H$. This would be the minimal option.

It seems that only **Option II** can be realized.

1. The basic observation is that X^2_c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c-valued "real" or "imaginary" part in C_c

sense for P vanishes. M_c^2 would be the simplest solution but also more general complex sub-manifolds $X_c^2 \subset M_c^4$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. SH makes possible $M^8 - H$ duality assuming that associativity conditions hold true only at 2-surfaces including partonic 2-surfaces or string world sheets or perhaps both. Thus one can give up the conjecture that the polynomial ansatz implies the additional condition globally.

   SH indeed states that PEs are determined by data at 2-D surfaces of X^4. Even if the conditions defining X_c^2 have only a discrete set of solutions, SH at the level of H could allow to deduce the PEs from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. The existence of $M^2(x)$ would be required only at the 2-D surfaces.

3. There is however a delicacy involved: X^2 might be 2-D only metrically but not topologically! The 3-D light-like surfaces X_L^3 indeed have metric dimension $D = 2$ since the induced 4-metric degenerates to 2-D metric at them. Therefore their pre-images in M^8 would be natural candidates for the singularities at which the dimension of the quaternionic tangent or normal space reduces to $D = 2$ [L75] [K10]. If this happens, SH would not be quite so strong as expected. The study of fermionic variant of $M^8 - H$-duality supports this conclusion.

One can generalize the condition selecting X_c^2 so that it selects 1-D surface inside X_c^2. By assuming that R_c-valued "real" or "imaginary" part of complex part of P sense at this 2-surface vanishes. One obtains preferred M_c^1 or E_c^1 containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as a complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

6.3.2 What about $M^8 - H$ duality in the fermionic sector?

During the preparation of this article I become aware of the fact that the realization $M^8 - H$ duality in the fermionic sector has remained poorly understood. This led to a considerable integration of the ideas about $M^8 - H$ duality also in the bosonic sector and the existing phenomenological picture follows now from $M^8 - H$ duality. There are powerful mathematical guidelines available.

Octonionic spinors

By supersymmetry, octonionicity should have also fermionic counterpart.

1. The interpretation of M_c^8 as complexified octonions suggests that one should use complexified octonionic spinors in M_c^8. This is also suggested by SO(1,7) triality unique for dimension $d = 8$ and stating that the dimensions of vector representation, spinor representation and its conjugate are same and equal to $D = 8$. I have already earlier considered the possibility to interpret M^8 spinors as octonionic [L23]. Both octonionic gamma matrices and spinors have interpretation as octonions and gamma matrices satisfy the usual anti-commutation rules. The product for gamma matrices and gamma matrices and spinors is replaced with non-associative octonionic product.

2. Octonionic spinors allow only one M^8-chirality, which conforms with the assumption of TGD inspired SUSY that only quarks are fundamental fermions and leptons are their local composites [L81].

3. The decomposition of $X^2 \subset X^4 \subset M^8$ corresponding to $R \subset C \subset Q \subset O$ should have analog for the O_c spinors as a tensor product decomposition. The special feature of dimension $D = 8$ is that the dimensions of spinor spaces associated with these factors are indeed 1, 2, 4, and 8 and correspond to dimensions for the surfaces!

   One can define for octonionic spinors associative/co-associative sub-spaces as quaternionic/co-quaternionic spinors by posing chirality conditions. For $X^4 \subset M_c^8$ one could define the analogs of projection operators $P_\pm = (1 \pm \gamma_5)/2$ as projection operators to either factor of the spinor space as tensor product of spinor space associated with the tangent and normal spaces of

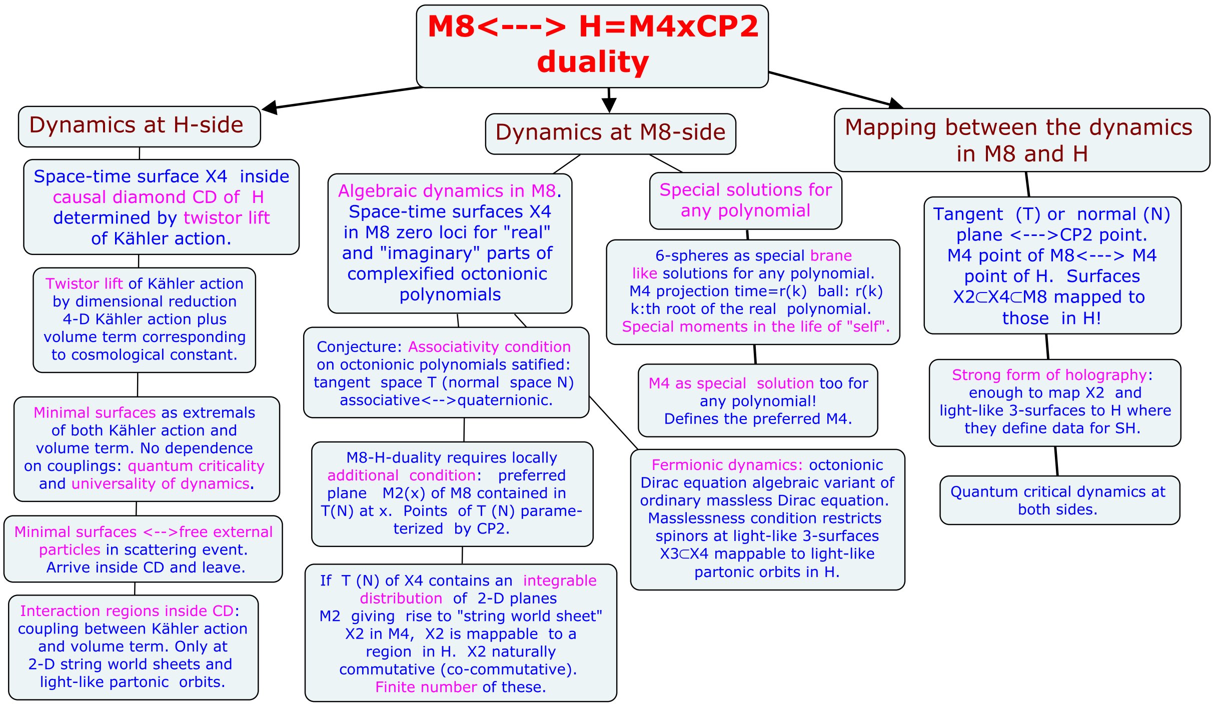

Figure 6.1: M^8-H duality.

X^4: the analog of γ_5 would correspond to tangent or normal space depending on whether tangent or normal space is associative. For the spinors with definite chirality there would be no entanglement between the tensor factors. The condition would generalize the chirality condition for massless M^4 spinors to a condition holding for the local M^4 appearing as tangent/normal space of X^4.

4. The chirality condition makes sense also for $X^2 \subset X^4$ identified as complex/co-complex surface of X^4. Now γ_5 is replaced with γ_3 and states that the spinor has well-defined spin in the direction of axis defined by the decomposition of X^2 tangent space to $M^1 \times E^1$ with M^1 defining real octonion axis and selecting rest frame. Interpretation in terms of quantum measurement theory is suggestive.

What about tangent space quantum numbers in M^8 picture. In H-picture they correspond to spin and electroweak quantum numbers. In M^8 picture the geometric tangent space group for a rest system is product $SU(2) \times SU(2)$ with possible modifications due to octonionicity reducing tangent space group to those respecting octonionic automorphisms.

What about the sigma matrices for the octonionic gamma matrices? The surprise is that the commutators of M^4 sigma matries and those of E^4 sigma matrices close to the sama $SO(3)$ algebra allowing interpretation as representation for quaternionic automorphisms. Lorentz boosts are represented trivially, which conforms with the fact that octonion structure fixes unique rest system. Analogous result holds in E^4 degrees of freedom. Besides this one has unit matrix assignable to the generalize spinor structure of CP_2 so that also electroweak $U(1)$ factor is obtained.

One can understand this result by noticing that octonionic spinors correspond to 2 copies of a tensor products of the spinor doublets associated with spin and weak isospin. One has $2 \otimes 2 = 3 \oplus 1$

so that one must have $1 \oplus 3 \oplus 1 \oplus 3$. The octonionic spinors indeed decompose like $1+1+3+\overline{3}$ under $SU(3)$ representing automophisms of the octonions. $SO(3)$ could be interpreted as $SO(3) \subset SU(3)$. $SU(3)$ would be represented as tangent space rotations.

Dirac equation as partial differential equation must be replaced by an algebraic equation

Algebraization of dynamics should be also supersymmetric. The modified Dirac equation in H is linear partial differential equation and should correspond to a linear algebraic equation in M^8.

1. The key observation is that for the ordinary Dirac equation the momentum space variant of Dirac equation for momentum eigenstates is algebraic! Could the interpretation for $M^8 - H$ duality as an analog of momentum-position duality of wave mechanics considered already earlier make sense! This could also have something to do with the dual descriptions of twistorial scattering amplitudes in terms of either twistor and momentum twistors. Already the earlier work excludes the interpretation of the octonionic coordinate o as 8-momentum. Rather, $P(o)$ has this interpretation and o corrresponds to embedding space coordinate.

2. The first guess for the counterpart of the modified Dirac equation at the level of $X^4 \subset M^8$ is $P\Psi = 0$, where Ψ is octonionic spinor and the octonionic polynomial P defining the space-time surface can be seen as a generalization of momentum space Dirac operator with octonion units representing gamma matrices. If associativity/co-associativity holds true, the equation becomes quaternionic/co-quaternionic and reduces to the 4-D analog of massless Dirac equation and of modified Dirac equation in H. Associativity hols true if also Ψ satisfies associativity/co-associativity condition as proposed above.

3. What about the square of the Dirac operator? There are 3 conjugations involved: quaternionic conjugation assumed in the earlier work, conjugation with respect to i, and their combination. The analog of octonionic norm squared defined as the product $o_c o_c^*$ with conjugation with respect to i only, gives Minkowskian metric $m_{kl} o^k \overline{o}^l$ as its real part. The imaginary part of the norm squared is vanishing for the projection $O_c \rightarrow M^8 \rightarrow M^4$ so that time coordinate is real and spatial coordinates imaginary. Therefore Dirac equation allows solutions only for the M^4 projection X^4 and M^4 ($M8$) signature of the metric can be said to be an outcome of quaternionicity (octonionicity) alone in accordance with the duality between metric and algebraic pictures.

   Both $P^\dagger P$ and PP should annihilate Ψ. $P^\dagger P\Psi = 0$ gives $m_{kl} P^k \overline{P}^l = 0$ as the analog of vanishing mass squared in M^4 signature in both associative and co-associative cases. $PP\Psi = 0$ reduces to $P\Psi = 0$ by masslessness condition. One could perhaps interpret the projection $X^4_c \rightarrow M^8 \rightarrow M^4$ in terms of Uncertainty Principle.

   There is a $U(1)$ symmetry involved: instead of the plane M^8 one can choose any plane obtained by a rotation $exp(i\phi)$ from it. Could it realize quark number conservation in M^8 picture?

   For $P = o$ having only $o = 0$ as root $Po = 0$ reduces to $o^\dagger o = 0$ and o takes the role of momentum, which is however vanishing. 6-D brane like solutions S^6 having $t = r_n$ balls $B^3 \subset CD_4$ as M^4 projections one has $P = 0$ so that the Dirac equation trivializes and does not pose conditions on Ψ. o would have interpretation as space-time coordinates and $P(o)$ as position dependent momentum components P^k.

   The variation of P at mass shell of M^8_c (to be precise) could be interpreted in terms of the width of the wave packet representing particle. Since the light-like curve at partonic 2-surface for fermion at X^3_L is not a geodesic, mass squared in M^4 sense is not vanishing. Could one understand mass squared and the decay width of the particle geometrically? Note that mass squared is predicted also by p-adic thermodynamics [K60].

4. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. $M^8 - H$ duality [L76] suggests that this boundary is mapped to $X^3_L \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_k P^k = 0$ surfaces as X^3_L gives a very powerful constraint on SH and $M^8 - H$ duality.

5. Also at 2-surfaces $X^2 \subset X^4$ an the variant Dirac equation would hold true and should commute with the corresponding chirality condition. Now $D^\dagger D\Psi = 0$ gives 2-D variant

of masslessness condition with 2-momentum components represented by those of P. 2-D masslessness locates the spinor to a 1-D curve X^1_L. Its H-image would naturally contain the boundary of the string word sheet at X^3_L assumed to carry fermion quantum numbers and also the boundary of string world sheet at the light-like boundary of CD_4. The interior of string world sheet in H would not carry induced spinor field.

6. The general solution for both 4-D and 2-D cases can be written as $\Psi = P\Psi_0$, Ψ_0 a constant spinor - this in a complete analogy with the solution of modified Dirac equation in H. P depends on position: the WKB approximation using plane waves with position dependent momentum seems to be nearer to reality than one might expect.

The phenomenological picture at H-level follows from the M^8-picture

Remarkably, the partly phenomenological picture developed at the level of H is reproduced at the level of M^8. Whether the induced spinor fields in the interior of X^4 are present or not, has been long standing question since they do not seem to have any role in the physical picture. The proposed picture answers this question.

Consider now the explicit realization of $M^8 - H$-duality for fermions.

1. SH and the expected analogy with the bosonic variant of $M^8 - H$ duality lead to the first guess. The spinor modes in $X^4 \subset M^8$ restricted to X^2 can be mapped by $M^8 - H$-duality to those at their images $X^2 \subset H$, and define boundary conditions allowing to deduce the solution of the modified Dirac equation at $X^4 \subset H$. X^2 would correspond to string world sheets having boundaries X^1_L at X^3_L.

   The guess is not quite correct. Algebraic Dirac equation requires that the solutions are restricted to the 3-D and 1-D mass shells $P_kP^k = 0$ in M^8. This should remain true also in H and X^3_L and their 1-D intersections X^1_L with string world sheets remain. Fermions would live at boundaries. This is just the picture proposed for the TGD counterparts of the twistor amplitudes and corresponds to that used in twistor Grassmann approach!

   For 2-D case constant octonionic spinors Ψ_0 and gamma matrix algebra are equivalent with the ordinary Weyl spinors and gamma matrix algebra and can be mapped as such to H. This gives one additional reason for why SH must be involved.

2. At the level of H the first guess is that the modified Dirac equation $D\Psi = 0$ is true for D based on the modified gamma matrices associated with both volume action and Kähler action. This would select preferred solutions of modified Dirac equation and conform with the vanishing of super-symplectic Noether charges for SSA_n for the spinor modes. The guess is not quite correct. The restriction of the induced spinors to X^3_L requires that Chern-Simons action at X^3_L defines the modified Drac action.

3. The question has been whether the 2-D modified Dirac action emerges as a singular part of 4-D modified Dirac action assignable to singular 2-surface or can one assign an independent 2-D Dirac action assignable to 2-surfaces selected by some other criterion. For singular surfaces $M^8 - H$ duality fails since tangent space would reduce to 2-D space so that only their images can appear in SH at the level of H.

   This supports the view that singular surfaces are actually 3-D mass shells M^8 mapped to X^3_L for which 4-D tangent space is 2-D by the vanishing of $\sqrt{g_4}$ and light-likeness. String world sheets would correspond to non-singular $X^2 \subset M^8$ mapped to H and defining data for SH and their boundaries $X^1_L \subset X^3_L$ and $X^1_L \subset CD_4$ would define fermionic variant of SH.

What about the modified Dirac operator D in H?

1. For X^3_L modified Dirac equation $D\Psi = 0$ based on 4-D action S containing volume and Kähler term is problematic since the induced metric fails to have inverse at X^3_L. The only possible action is Chern-Simons action S_{CS} used in topological quantum field theories and now defined as sum of C-S terms for Kähler actions in M^4 and CP_2 degrees of freedom. The presence of M^4 part of Kähler form of M^8 is forced by the twistor lift, and would give rise to small CP breaking effects explaining matter antimatter asymmetry [L81]. S_{C-S} could emerge as a limit of 4-D action.

The modified Dirac operator D_{C-S} uses modified gamma matrices identified as contractions $\Gamma^{\alpha}_{CS} = T^{\alpha k}\gamma_k$, where $T^{\alpha k} = \partial L_{CS}/\partial(\partial_\alpha h^k)$ are canonical momentum currents for S_{C-S} defined by a standard formula.

2. CP_2 part would give conserved Noether currents for color in and M^4 part Poincare quantum numbers: the apparently small CP breaking term would give masses for quarks and leptons! The bosonic Noether current $J_{B,A}$ for Killing vector j^k_A would be proportional to $J^{\alpha}_{B,A} = T^{\alpha}_k j_A k$ and given by $J_{B,A} = \epsilon^{\alpha\beta\gamma}[J_{\beta\gamma}A_k + A_\beta J_{\gamma k}]j^k_A$.
   Fermionic Noether current would be $J_{F,A} = \overline{\Psi}J^{\alpha}\Psi$ 3-D Riemann spaces allow coordinates in which the metric tensor is a direct sum of 1-D and 2-D contributions and are analogous to expectation values of bosonic Noether currents. One can also identify also finite number of Noether super currents by replacing $\overline{\Psi}$ or Ψ by its modes.

3. In the case of X^3_L the 1-D part light-like part would vanish. If also induced Kähler form is non-vanishing only in 2-D degrees of freedom, the Noether charge densities J^t reduce to $J^t = JA_k j^k_A$, $J = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma}$ defining magnetic flux. Modified Dirac operator would reduce to $D = JA_k\gamma^k D_t$ and 3-D solutions would be covariantly constant spinors along the light-like geodesics parameterized by the points 2-D cross section. One could say that the number of solutions is finite and corresponds to covariantly constant modes continued from X^1_L to X^3_L. This picture is just what twistor Grassmannian approach led to [L64].

A comment inspired by the ZEO based quantum measurement theory

I cannot resist the temptation to make a comment relating to quantum measurement theory inspired by zero energy ontology (ZEO) extending to a theory of consciousness [L80, L109, L110].

I have proposed [L101, L103] that the time evolution by "big" state function reductions (BSFRs) could be induced by iteration of real polynomial P - at least in some special cases. The foots of the real polynomial P would define a fractal at the limit of larger number of iterations. The roots of n-fold iterate $\circ^n P$ would contain the inverse images under $\circ^{-n+1}P$ of roots of P and for $P(0) = 0$ the inverse image $\circ^n P$ would consist of inverse images under $\circ^{-k}P$, $k = 0, ..., n-1$, of roots of P.

Also the mass shells for $\circ^n P$ would be unions of inverses images under $\circ^{-k}P$, $k = 0, , ..., n-1$, of roots of P. This gives rather concrete view about evolution of M^4 projections of the partonic orbits. A rough approximate expression for the largest root of real P approximated as $P(x) \simeq a_n x^n + an - 1ix^{n-1}$ for large x is $x_{max} \sim a_n/a_{n-1}$. For $\circ^n P$ one obtains the same estimate. This suggests that the size scales of the partonic orbits are same for the iterates. The mass shells would not differ dramatically: could they have an interpretation in terms of mass splitting?

The evolution by iteration would add new partonic orbits and preserve the existing ones: this brings in mind conservation of genes in biological evolution. This is true also for a more general evolution allowing general functional decomposition $Q \rightarrow Q \circ P$ to occur in BSFR.

What next in TGD?

The construction of scattering amplitudes has been the dream impossible that has driven me for decades. Maybe the understanding of fermionic $M^8 - H$ duality provides the needed additional conceptual tools. The key observation is utterly trivial but far reaching: there are 3 possible conjugations for octonions corresponding to the conjugation of commutative imaginary unit or of octonionic imaginary units or both of them. 1st norm gives a real valued norm squared in Minkowski signature natural at M^8 level! Second one gives a complex valued norm squared in Euclidian signature. 1st and 2nd norms are equivalent for octonions light-like with respect to the first norm. The 3rd conjugation gives a real-valued Euclidian norm natural at the level of Hilbert space.

1. M^8 picture looks simple. Space-time surfaces in M^8 can be constructed from real polynomials with real (rational) coefficients, actually knowledge of their roots is enough. Discrete data - roots of the polynomial!- determine space-time surface as associative or co-associative region! Besides this one must pose additional condition selecting 2-D string world sheets and 3-D light-like surfaces as orbits of partonic 2-surfaces. These would define strong form of holography (SH) allowing to map space-time surfaces in M^8 to $M^4 \times CP_2$.

2. Could SH generalize to the level of scattering amplitudes expressible in terms of n-point functions of CFT?! Could the n points correspond to the roots of the polynomial defining space-time region!

   Algebraic continuation to quaternion valued scattering amplitudes analogous to that giving space-time sheets from the data coded SH should be the key idea. Their moduli squared are real - this led to the emergence of Minkowski metric for complexified octonions/quaternions) would give the real scattering rates: this is enough! This would mean a number theoretic generalization of quantum theory.

3. One can start from complex numbers and string world sheets/partonic 2-surfaces. Conformal field theories (CFTs) in 2-D play fundamental role in the construction of scattering string theories and in modelling 2-D statistical systems. In TGD 2-D surfaces (2-D at least metrically) code for information about space-time surface by strong holography (SH) .

   Are CFTs at partonic 2-surfaces and string world sheets the basic building bricks? Could 2-D conformal invariance dictate the data needed to construct the scattering amplitudes for given space-time region defined by causal diamond (CD) taking the role of sphere S^2 in CFTs. Could the generalization for metrically 2-D light-like 3-surfaces be needed at the level of "world of classical worlds" (WCW) when states are superpositions of space-time surfaces, preferred extremals?

The challenge is to develop a concrete number theoretic hierarchy for scattering amplitudes: $R \rightarrow C \rightarrow H \rightarrow O$ - actually their complexifications.

1. In the case of fermions one can start from 1-D data at light-like boundaries LB of string world sheets at light-like orbits of partonic 2-surfaces. Fermionic propagators assignable to LB would be coded by 2-D Minkowskian QFT in manner analogous to that in twistor Grassmann approach. n-point vertices would be expressible in terms of Euclidian n-point functions for partonic 2-surfaces: the latter element would be new as compared to QFTs since point-like vertex is replaced with partonic 2-surface.

2. The fusion (product?) of these Minkowskian and Euclidian CFT entities corresponding to different realization of complex numbers as sub-field of quaternions would give rise to 4-D quaternionic valued scattering amplitudes for given space-time sheet. Most importantly: there moduli squared are real for both norms.

   It is not quite clear whether one must use the 1st Minkowskian norm requiring "time-like" scattering amplitudes to achieve non-negative probabilities or use the 3rd norm to get the ordinary positive-definite Hilbert space norm. A generalization of quantum theory (CFT) from complex numbers to quaternions (quaternionic "CFT") would be in question.

3. What about several space-time sheets? Could one allow fusion of different quaternionic scattering amplitudes corresponding to different quaternionic sub-spaces of complexified octonions to get octonion-valued non-associative scattering amplitudes. Again scattering rates would be real. This would be a further generalization of quantum theory.

There is also the challenge to relate M^8- and H-pictures at the level of WCW. The formulation of physics in terms of WCW geometry [K85, L87] leads to the hypothesis that WCW Kähler geometry is determined by Kähler function identified as the 4-D action resulting by dimensional reduction of 6-D surfaces in the product of twistor spaces of M^4 and CP_2 to twistor bundles having S^2 as fiber and space-time surface $X^4 \subset H$ as base. The 6-D Kähler action reduces to the sum of 4-D Kähler action and volume term having interpretation in terms of cosmological constant.

The question is whether the Kähler function - an essentially geometric notion - can have a counterpart at the level of M^8.

1. SH suggests that the Kähler function identified in the proposed manner can be expressed by using 2-D data or at least metrically 2-D data (light-like partonic orbits and light-like boundaries of CD). Note that each WCW would correspond to a particular CD.

2. Since 2-D conformal symmetry is involved, one expects also modular invariance meaning that WCW Kähler function is modular invariant, so that they have the same value for $X^4 \subset H$ for which partonic 2-surfaces have induced metric in the same conformal equivalence class.

3. Also the analogs of Kac-Moody type symmetries would be realized as symmetries of Kähler function. The algebra of super-symplectic symmetries of the light-cone boundary can be regarded as an analog of Kac-Moody algebra. Light-cone boundary has topology $S^2 \times R_+$ where R_+ corresponds to radial light-like ray parameterized by radial light-like coordinate r. Super symplectic transformations of $S^2 \times CP_2$ depend on the light-like radial coordinate r, which is analogous to the complex coordinate z for he Kac-Moody algebras.

   The infinitesimal super-symplectic transformations form algebra SSA with generators proportional to powers r^n . The Kac-Moody invariance for physical states generalizes to a hierarchy of similar invariances. There is infinite fractal hierarchy of sub-algebras $SSA_n \subset SSA$ with conformal weights coming as n-multiples of those for SSA. For physical states SSA_n and $[SSA_n, SSA]$ would act as gauge symmetries. They would leave invariant also Kähler function in the sector WCW_n defined by n. This would define a hierarchy of sub- WCWs of the WCW assignable to given CD.

   The sector WCW_n could correspond to extensions of rationals with dimension n, and one would have inclusion hierarchies consisting of sequences of n_i with n_i dividing n_{i+1}. These inclusion hierarchies would naturally correspond to those for hyper-finite factors of type II_1 [K112].

6.4 Cognitive representations and algebraic geometry

The general vision about cognition is realized in terms of adelic physics as physics of sensory experience and cognition [L52, L51]. Rational points and their generalization as ratios of algebraic integers for geometric objects would define cognitive representations as points common for real and various p-adic variants of the space-time surface. The finite-dimensionality for induced p-adic extensions allows also extensions of rationals involving root of e and its powers. This picture applies both at space-time level, embedding space level, and at the level of space-time surfaces but basically reduces to embedding space level. Hence counting of the (generalized) rational points for geometric objects would be determination of the cognitive representability.

6.4.1 Cognitive representations as sets of generalized rational points

The set of rational points depends on the coordinates chosen and one can argue that one must allow different cognitive representations and classify them according to their effectiveness.

How uniquely the M^8_c coordinates can be chosen?

1. Polynomial property allows only linear transformations of the complex octonionic coordinates with coefficients which belong to the extension of rationals used. This poses extremely strong restrictions on the allowed representations once the quaternionic moduli defining a foliation of M^4_0 is chosen. One has therefore moduli space of quaternionic structures. One must also fix the time axis in M^4 assignable to real octonions.
2. One can also define several inequivalent octonionic structures and associate a moduli space to these. The moduli space for octonionic structures would correspond to the space of $M^4_0 \subset M^8$s as quaternionic planes containing fixed M^2_0. One can allow even allow Lorentz transforms mixing real and imaginary octonionic coordinates. It seems that these moduli are not relevant at the level of H.

What could the precise definition of rationality?

1. The coordinates of point are rational in the sense defined by the extension of rationals used. Suppose that one considers parametric representations of surfaces as maps from space-time surface to embedding space. Suppose that one uses as space-time coordinates subset of preferred coordinates for embedding space. These coordinate changes cannot be global and one space-time surface decomposes to regions in which different coordinates apply.
2. The coordinate transformations between over-lapping regions are birational in the sense that both the map and its inverse are in terms of rational functions. This makes the notion of rationality global.
3. When cognitively easy rational parametric representations are possible? For algebraic curves with $g \geq 2$ in CP_2 represented as zeros of polynomials this cannot the case since the number

of rational points is finite for instance for $g \geq 2$ surfaces. There is simple explanation for this. Solving second complex coordinate in terms of the other one gives it as an algebraic function for $g \geq 2$: this must be the reason for the loss of dense set of rational points. For elliptic surfaces $y^2 - x^3 - ax - b = 0$ y^2 is however polynomial of x and one can find rational parametric representation by taking y^2 as coordinate [L43]. For $g = 0$ one has linear equations and one obtains dense set of rational points. For conic sections one can also have dense set of rational points but not always. Generalizing from this it would seem that the failure to have rational parametric representation is the basic reason for the loss of dense set of rational points.

This picture does not work for general surfaces but generalizes for algebraic varieties defined by several polynomial equations. The co-dimension $d_c = 1$ case is however unique and the most studied one since for several polynomial equations one encounters technical difficulties when the intersection of the surfaces defined by the d_c polynomials need not be complete for $d_c > 1$. In the recent situation one has $d_c = 4$ but octonion analyticity could be powerful enough symmetry to solve the problem of non-complete intersections by eliminating them or providing a physical interpretation for them.

6.4.2 Cognitive representations assuming $M^8 - H$ duality

Many questions should be answered.

1. Can one generalize the results applying to algebraic varieties? Could the general vision about rational and potentially dense set of rational points generalize?. At M^8 side the description of space-time surfaces as algebraic varieties indeed conforms with this picture. Could one understand SH from the fact that real analyticity octonionic polynomials are determined by ordinary polynomial real coordinate completely? In information theoretic sense sense SH reduces to 1-D holography and the polynomial property makes the situation effectively discrete since finite number of points of real axis allows to determine the octonionic polynomial completely! It is a pity that one cannot measure octonionic polynomial directly!
2. Also the notion of Zariski dimension should make sense in TGD at M^8 side. Preferred extremals define the notion of closed set for given CD at M^8 side? It would indeed seem that one define Zariski topology at the level of M^8_c. Zariski topology would require 4-surfaces, string world sheets, or partonic 2-surfaces and even 1-D curves. This picture conforms with the recent view about TGD and resembles the M-theory picture, where one has branes. SH suggests that the analog of Zariski dimension of space-time surface reduces to that for strings world sheets and partonic 2-surfaces and that even these are analogous to 1-D curves by complex analyticity. Integrability of TGD and preferred extremal property would indeed suggest simplicity.

   $M^8 - H$ hypothesis suggests that these conjectures make sense also at H side. String world sheets, partonic 2-surface, space-like 3-surfaces at the ends of space-time surface at boundaries of CD, and light-like 3-surfaces correspond to closed sets also at the level of WCW in the topology most natural for WCW.
3. Also the problems related to Minkowskian signature could be solved. String world sheets are problematic because of the Minkowskian signature. They however have the topology of disk plus handles suggesting immediately a vision about cognitive representations in terms of rational points. One can can complexify string world sheets and it seems possible to apply the results of algebraic geometry holding true in Euclidian signature. This would be analogous to the Wick rotation used in QFTs and also in twistor Grassmann approach.
4. What about algebraic geometrization of the twistor lift? How complex are twistor spaces of M^4, CP_2 and space-time surface? How can one generalize twistor lift to the level of M^8. S^2 bundle structure and the fact that S^2 allows a dense set of rational suggests that the complexity of twistor space is that of space-time surface itself so that the situation actually reduces to the level of space-time surfaces.

Suppose one accepts $M^8 - H$ duality requiring that the tangent space of space-time surface at given point x contains $M^2(x)$ such that $M^2(x)$ define an integrable distribution giving rise to string world sheets and their orthogonal complements give rise to partonic 2-surfaces. This would give rise to a foliation of the space-time surface by string world sheets and partonic 2-surface conjecture on basis of the properties of extremals of Kähler action. As found these foliations could correspond to quaternion structures that is allowed choices of quaterionic coordinates.

Should one define cognitive representations at the level of M^8 or at the level of $M^4 \times CP_2$? Or both? For M^8 option the condition that space-time point belongs to an extension of rationals applies at the level of M^8 coordinates. For $M^4 \times CP_2$ option cognitive representations are at the level of M^4 and CP_2 parameterizing the points of M^4 and their tangent spaces. The formal study of partial differential equations alone does not help much in counting the number of rational points. One can define cognitive representation in very many ways, and some cognitive representation could be preferred only because they are more efficient than others. Hence both cognitive representations seems to be acceptable.

Some cognitive representations are more efficient than others. General coordinate invariance (GCI) at the level of cognition is broken. The precise determination of cognitive efficiency is a challenge in itself. For instance, the use of coordinates for which coordinate lines are orbits of subgroups of the symmetry group should be highly efficient. Only coordinate transformations mediated by bi-rational maps can take polynomial representations to polynomial representations. It might well be that only a rational (in generalized sense) sub-group G_2 of octonionic automorphisms is allowed. For rational surfaces allowing parametric representation in terms of polynomial functions the rational points form a dense set.

The cognitive resolution for a dense set of rational points is unrealistically high since cognitive representation would contain infinite number of points. Hence one must tighten the notion of cognitive representation. The rational points must contain a fermion. Fermions are indeed are identified as correlates for Boolean cognition [K27]. This would suggests a view in which cognitive representations are realized at the light-like orbits of partonic 2-surfaces at which Minkowskian associative and Euclidian co-associative space-time surfaces meet. The general wisdom is that rational points are localized to lower-dimensional sub-varieties (Bombieri-Lang conjecture): this conforms with the view that fermion lines reside at the orbits of partonic 2-surfaces.

6.4.3 Are the known extremals in H easily cognitively representable?

Suppose that one takes TGD inspired adelic view about cognition seriously. If cognitive representations correspond to rational points for an extension of rationals, then the surfaces allowing large number of this kind of points are easily representable cognitively by adding fermions to these points. One could even speculate that mathematical cognition invents those geometric objects, which are easily cognitively representable and thus have a large number of rational points.

Could the known extremals of twistor lift be cognitively easy?

Also TGD is outcome of mathematical cognition. Could the known extremals of the twistor lift of Kähler action be cognitively easy? This is suggested by the fact that even such a pariah class theoretician as I am have managed to discover then! Positive answer could be seen as support for the proposed description of cognition!

1. If one believes in $M^8 - H$ duality and the proposed identification of associative and co-associative space-time surfaces in terms of algebraic surfaces in octonionic space M^8_c, the generalization of the results of algebraic geometry should give overall view about the cognitive representations at the level of M^8. In particular, surfaces allowing rational parametric representation (polynomials would have rational coefficients) would allow dense set or rational points since the images of rational points are rational. Rationals are understood here as ratios of algebraic integers in extension of rationals.
2. Also for H the existence of parameter representation using preferred H-coordinates and rational functions with rational coefficients implies that rational points are dense. If $M^8 - H$ correspondence maps the parametric representations in terms of rational functions to similar representations, dense set of rational points is preserved in the correspondence. There is however no obvious reason why $M^8 - H$ duality should have this nice property.
   One can even play with the idea that the surfaces, which are cognitively difficult at the M^8 side, might be cognitively easy at H-side or vice versa. Of course, if the explicit representation as algebraic functions makes sense at M^8 side, this side looks cognitively ridiculously easy

as compared to H side. The preferred extremal property and SH can however change the situation.

3. At M^8 side and for a given point of M^4 there are several points of E^4 (or vice versa) if the degree of the polynomial is larger than $n = 1$ so that for the image of the surface H there are several CP_2 points for a given point of M^4 (or vice versa) depending on the choice of coordinates. This is what the notion of the many-sheeted space-time predicts.
4. The equations for the surface at H side are obtained by a composite map assigning first to the coordinates of $X^4 \subset M^8$ point of $M^4 \times E^4$, and then assigning to the points of $X^4 \subset M^8$ CP_2 coordinates of the tangent space of the point. At this step the slightly non-local tangent space information is fed in and the surfaces in $M^4 \times CP_2$ cannot be given by zeros of polynomials. The indeed satisfy instead of algebraic equations partial differential equations given by the Kähler action for the twistor lift TGD. Algebraic equations instead of partial differential equations suggests that the M^8 representation is much simpler than H-representation. On the other hand, reduction to algebraic equations at M^8 side could have interpretation in terms of the conjectured complete integrability of TGD [K10, K100].

Testing the idea about self-reference

In any case, it is possible to test the idea about self-reference by looking whether the known extremals in H are cognitively easy and even have a dense set of rational points in natural coordinates. Here I will consider the situation at the level of $M^4 \times CP_2$. It was already found that the known extremals can have inverse images in M^8.

1. Canonically imbedded M^4 with linear coordinates and constant CP_2 coordinates rational is the simple example about preferred extremal and it seems that TGD based cosmology at microscopic relies on these extremals. In this case it is obvious that one has a dense set of rational points at both sides. Could this somehow relate to the fact that physics as physics M^4 was discovered before general relativity?
   Canonically imbedded M^4 corresponds to a first order octonionic polynomial for which imaginary part is put to constant so that tangent space is same everywhere and corresponds to a constant CP_2 coordinate.
2. CP_2 type extremals have 4-D CP_2 projection and light-like geodesic line of M^4 as M^4 projection. One can choose the time parameter as a function of CP_2 coordinates in infinitely many ways. Clearly the rational points are dense in any CP_2 coordinates.
3. Massless extremals (MEs) are given as zeros of arbitrary functions of CP_2 coordinates and 2 M^4 coordinates representing local light-like direction and polarization direction orthogonal to it. In the simplest situation these directions are constant. In the general case light-like direction would define tangent space of string world sheet giving rise also to a distribution of ortogonal polarization planes. This is consistent with the general properties of the M^8 representation and corresponds to the decomposition of quaternionic tangent plane to complex plane and its complement. One can ask whether one should allow only polynomials with rational coefficients as octonionic polynomials.
4. String like objects $X^2 \times Y^2$ with $X^2 \subset M^4$ a minimal surface and Y^2 complex or Lagrangian surface of CP_2 are also basic extremals and their deformations in M^4 directions are expected to give rise to magnetic flux tubes.
   If Y^2 is complex surface with genus $g = 0$ rational points are dense. Also for $g = 1$ one obtains a dense set of rational points in some extension of rationals. For elliptic curves one has lattice of rational points. What happens for Lagrangian surfaces Y^2? In this case one does not have complex curves but real co-dimension 2 surfaces. There is no obvious objection why these surfaces would not be possible.
5. What about string world sheets? If the string world is static $M^2 \subset M^4$ one has a dense set of rational points. One however expects something more complex. If the string world sheet is rational map M^2 to its orthogonal complement E^2 one has rational surface. For rotating strings this does not make sense except for certain period of time. If the choice of the quaternion structure corresponds to a choice of minimal surface in M^4 as integrable distribution for $M^2(x)$, the coordinates associated with the Hamilton-Jacobi structure could make the situation simple.

If one restricts the consideration the intersections of partonic 2-surfaces and string world sheets at two boundaries of CD the situation simplifies and the question is only about the rationality of the M^4 coordinates at rational points of $Y^2 \subset CP_2$. This would simplify the situation enormously and might even allow to use existing knowledge.

6. The slicing of of space-time surfaces by string world sheets and partonic 2-surfaces required by Hamilton-Jacobi structure could be seen as a fibering analogous to that possessed by elliptic surfaces. This suggest that M^8 counterparts of spacetime surfaces are not of general type in Kodaira classification and that the number of rational points can be large. If the existence of Hamilton-Jacobi structure does not allow handles, this factor would be cognitively simple. This would however suggests that fermion number is not localized at the ends of strings only - as assumed in the construction of scattering amplitudes inspired by twistor Grassmann approach [L30] - but also to the interior of the light-like curves inside string world sheets.

6.4.4 Twistor lift and cognitive representations

What about twistor lift of TGD replacing space-time surfaces with their twistor spaces. Consider first M^8 side.

1. At M^8 side S^2 seems to introduce nothing new. One might expect that the situation does not change at H-side since space-time surfaces are obtained essentially by dimensional reduction and the possible problem relates to the choice of base space as section of is twistor bundle and the embedding of space-time as base space could have singularities at the boundary of Euclidian and Minkowskian space-time regions as discussed in [L43].
At the side of M^8 the proposed induction of twistor structure is just a projection of the twistor sphere S^6 to its geodesic sphere and one has 4-D moduli space for geodesic spheres $S^2 \subset S^6$. If one interprets the choice of $S^2 \subset S^6$ as as a section in the moduli space, the moduli of S^2 can depend on the point of space-time surface. Note that there are is also a position dependent choice of preferred point of S^2 representing Kähler form, and this choice is good candidate for giving rise to Hamilton-Jacobi structures with position dependent M^2.
2. The notion of Kodaira dimension is defined also for co-dimension 4 algebraic varieties in M^8_c. The cognitively easiest spacetime surfaces would allow rational parametric representation with complex coordinates serving as parameters. If this is not possible, one has algebraic functions, which makes the situation much more complex so that the number of rational points would be small.
3. For some complex enough extensions of rationals the set of rational points can be dense. $g \geq 2$ genera are basic example and one expects also in more general case that polynomials involving powers larger than $n = 4$ make the situation problematic. The condition that real or imaginary part of real analytic octonionic polynomial is in question is a strong symmetry expected to faciliate cognitive representability.
4. The general intuitive wisdom from algebraic geometry is that the rational points are dense only in lower-dimensional sub-varieties (Bombieri-Lang and Vojta conjectures mentioned in the first section). The general vision inspired by SH and the proposal for the construction of twistor amplitudes indeed is that the algebraic points (rational in generalized sense) defining cognitive representations are associated with the intersections of string world sheets and partonic 2-surfaces to which fermions are assigned. This would suggest that partonic 2-surfaces and string world sheets contain the cognitive representation, which under additional conditions can contain very many points.
5. An interesting question concerns the M^8 counterparts of partonic 2-surfaces as space-time regions with Minkowskian and Euclidian signature. The partonic orbits representing the boundaries between these regions should be mapped to each other by M^8-H duality. This conforms with the fact that induced metric must have degenerate signature $(0,-1,-1,-1)$ at partonic orbits. Can one assume that the topologies of partonic 2-surfaces at two sides are identical? Consider partonic 2-surface of genus g in $M^4 \times CP_2$ - say at the boundary of CD. It should be inverse image of a 2-surface in $M^4 \times E^4$ such that the tangent space of this surface labelled by CP_2 coordinates is mapped to a 2-surface in $M^4 \times CP_2$. If the inverse of $M^8 - H$ correspondence is continuous one expects that g is preserved.

Consider next the H-side. There is a conjecture that for Cartesian product the Kodaira dimension is sum $d_K = \sum_i d_{K,i}$ of the Kodaira dimensions for factors. Suppose that CP_1 fiber as surface in the 12-D twistor bundle $T(M^4) \times T(CP_2)$ has Kodaira dimension $d_K(CP_1) = -\infty$ (it is expected to be rational surface) then the fact that the bundle decomposes to Cartesian product locally and rational points are pairs of rational points in the factors, is indeed consistent with the proposal. S^2 would give dense set of rational points in S^2 and the bundle would have infinite number of rational points.

In TGD context, it is however space-time surface which matters. Space-time surface as section of the bundle would not however have a dense set of points in the general case and the relevant Kodaira dimension be $d_K = d_K(X^4)$. One can of course ask whether the space-time surface as an algebraic section (not many of them) of the twistor bundle could chosen to be cognitively simple.

6.4.5 What does cognitive representability really mean?

The following considerations reflect the ideas inspired by Face Book debate with Santeri Satama (SS) relating to the notion of number and the notion of cognitive representation.

SS wants to accept only those numbers that are constructible, and SS mentioned the notion of demonstrability due to Gödel. According to my impression demonstrability means that number can be constructed by a finite algorithm or at least that the information needed to construct the number can be constructed by a finite algorithm although the construction itself would not be possible as digit sequence in finite time. If the constructibility condition is taken to extreme, one is left only with rationals.

As a physicists, I cannot consider starting to do physics armed only with rationals: for instance, continuous symmetries and the notion of Riemann manifold would be lost. My basic view is that we should identify the limitations of cognitive representability as limitations for what can exist. I talked about cognitive representability of numbers central in the adelic physics approach to TGD. Not all real numbers are cognitively representable and need not be so.

Numbers in the extensions of rationals would be cognitively representable as points with coordinates in an extension of rationals. The coordinates themselves are highly unique in the octonionic approach to TGD and different coordinates choices for complexified octonionic M^8 are related by transformations changing the moduli of the octonion structure. Hence one avoids problems with general coordinate invariance). Not only algebraic extensions of rationals are allowed. Neper number e is an exceptional transcendental in that e^p is p-adic number and finite-D extensions of p-adic numbers by powers for root of e are possible.

My own basic interest is to find a deeper intuitive justification for why algebraic numbers shoud be cognitively representable. The naïve view about cognitive representability is that the number can be produced in a finite number of steps using an algorithm. This would leave only rationals under consideration and would mean intellectual time travel to ancient Greece.

Situation changes if one requires that only the information about the construction of number can be produced in a finite number of steps using an algorithm. This would replace construction with the recipe for construction and lead to a higher abstraction level. The concrete construction itself need not be possible in a finite time as bit sequence but could be possible physically ($\sqrt{2}$ as a diagonal of unit square, one can of course wonder where to buy ideal unit squares). Both number theory and geometry would be needed.

Stern-Brocot tree associated with partial fractions indeed allows to identify rationals as finite paths connecting the root of S-B tree to the rational in question. Algebraic numbers can be identified as infinite periodic paths so that finite amount of information specifies the path. Transcendental numbers would correspond to infinite non-periodic paths. A very close analogy with chaos theory suggests itself.

Demonstrability viz. cognitive representability

SS talked about demonstrable numbers. According to Gödel demonstrable number would be representable by a formula G, which is provable in some axiom system. I understand this that G would give a recipe for constructing that number. In computer programs this can even mean infinite loop, which is easy to write but impossible to realize in practice. Here comes the possibility that demonstrability does not mean constructibility in finite number of steps but only a finite recipe for this.

The requirement that all numbers are demonstrable looks strange to me. I would talk about cognitive representability and reals and p-adic number fields emerge unavoidably as prerequisites for this notion: cognitive representation must be about something in order to be a representation.

About precise construction of reals or something bigger - such as surreals - containing them, there are many views and I am not mathematician enough to take strong stance here. Note however that if one accepts surreals as being demonstrable (I do not really understand what this could mean) one also accept reals as such. These delicacies are not very interesting for the formulation of physics as it is now.

The algorithm defining G defines a proof. But what does proof mean? Proof in mathematical sense would reduce in TGD framework be a purely cognitive act and assignable to the p-adic sectors of adele. Mathematicians however tend to forget that for physicist the demonstration is also experimental. Physicist does not believe unless he sees: sensory perception is needed. Experimental proofs are what physicists want. The existence of $\sqrt{2}$ as a diagonal of unit square is experimentally demonstrable in the sense of being cognitively representable but not deducible from the axioms for rational numbers. As a physicist I cannot but accept both sensory and cognitive aspects of existence.

Instead of demonstrable numbers I prefer to talk about cognitively representable numbers.

1. All numbers are cognizable (p-adic) or sensorily perceivable (real). These must form continua if one wants to avoid problems in the construction of physical theories, where continuous symmetries are in a key role.
   Some numbers but not all are also *cognitively representable* that is being in the intersection reals and p-adics - that is in extension of rationals if one allows extensions of p-adics induced by extensions of rationals. This generalizes to intersection of space-time surfaces with real/p-adic coordinates, which are highly unique linear coordinates at octonionic level so that objections relating to a loss of general coordinate invariance are circumvented. General coordinate transformations reduce to automorphisms of octonions.
   The relationship to the axiom of choice is interesting. Should axiom of choice be restricted to the points of complexified octonions with coordinates in extensions of rationals? Only points in the extensions could be selected and this selection process would be physical in the sense that fermions providing realization of quantum Boolean algebra would reside at these points [K27]. In preferred octonionic coordinates the M^8 coordinates of these points would be in given extension of rationals. At the limit of algebraic numbers these points would form a dense set of reals.
   Remark: The spinor structure of "world of classical worlds" (WCW) gives rise to WCW spinors as fermionic Fock states at given 3-surface. In ZEO many-fermion states have interpretation in terms of superpositions of pairs of Boolean statements $A \rightarrow B$ with A and B represented as many-fermion states at the ends of space-time surface located at the opposite light-like boundaries of causal diamond (CD). One could say that quantum Boolean logic emerges as square root of Kähler geometry of WCW.
   At partonic 2-surfaces these special points correspond to points at which fermions can be localized so that the representation is physical. Universe itself would come in rescue to make representability possible. One would not anymore try to construct mathematics and physics as distinct independent disciplines.
   Even observer as conscious entity is necessarily brought into both mathematics and physics. TGD Universe as a spinor field in WCW is re-created state function reduction by reduction and evolves: evolution for given CD corresponds to the increase of the size of extension of rationals in statistical sense. Hence also mathematics with fixed axioms is replaced with a q dynamical structure adding to itself new axioms discovery by discovery [L53, L52].

2. Rationals as cognitively representable numbers conforms with naïve intuition. One can however criticize the assumption that also algebraic numbers are such. Consider $\sqrt{2}$: one can simply define it as length of diagonal of unit square and this gives a meter stick of length $\sqrt{2}$: one can represent any algebraic number of form $m + n\sqrt{2}$ by using meter stricks with length of 1 and $\sqrt{2}$. Cognitive representation is also sensory representation and would bring in additional manner to represent numbers.

Note that algebraic numbers in n-dimensional extension are points of n-dimensional space and their cognitive representations as points on real axis obtained by using the meter sticks assignable to the algebraic numbers defining base vectors. This should generalize to the roots arbitrary polynomials with rational or even algebraic coefficients. Essentially projection form n-D extension to 1-D real line is in question. This kind of projection might be important in number theoretical dynamics. For instance, quasi-periodic quasi-crystals are obtained from higher-D periodic crystals as projections.

n-D algebraic extensions of p-adics induced by those of rationals might also related to our ability to imagine higher-dimensional spaces.

3. In TGD Universe cognitive representability would emerge from fundamental physics. Extensions of rationals define a hierarchy of adeles and octonionic surfaces are defined as zero loci for real or imaginary parts (in quaternionic sense) of polynomials of real argument with coefficients in extension continued to octonionic polynomials [L46]. The zeros of real polynomial have a direct physical interpretation and would represent algebraic numbers physically. They would give the temporal positions of partonic 2-surfaces representing particles at light-like boundary of CD.
4. Note that all calculations with algebraic numbers can be done without using approximations for the genuinely algebraic numbers defining the basis for the extension. This actually simplifies enormously the calculation and one avoids accumulating errors. Only at the end one represents the algebraic units concretely and is forced to use rational approximation unless one uses above kind of cognitive representation.

For these reasons I do not feel any need to get rid of algebraics or even transcendentals. Sensory aspects of experience require reals and cognitive aspects of experience require p-adic numbers fields and one ends up with adelic physics. Cognitive representations are in the intersection of reality and various p-adicities, something expressible as formulas and concrete physical realizations or at least finite recipes for them.

What the cognitive representability of algebraic numbers could mean?

Algebraic numbers should be in some sense simple in order to be cognitively representable.

1. For rationals representation as partial fractions produces the rational number by using a finite number of steps. One starts from the top of Stern-Brocot (S-B) tree (see `http://tinyurl.com/yb6ldekq`) and moves to right or left at each step and ends up to the rational number appearing only once in S-B tree.
2. Algebraic numbers cannot be produced in a finite number of steps. During the discussion I however realized that one can produce the information needed to construct the algebraic number in a finite number of steps. One steps to a new level of abstraction by replacing the object with the information allowing to construct the object using infinite number of steps but repeating the same sub-algorithm with finite number of steps: infinite loop would be in question.

   Similar abstraction takes place as one makes a step from the level of space-time surface to the level of WCW. Space-time surface with a continuum of points is represented by a finite number of WCW coordinates, in the octonionic representation of space-time surface by the coefficients of polynomial of finite degree belonging to an extension of rationals [L46]. Criticality conditions pose additional conditions on the coefficients. Finite number of algebraic points at space-time surface determines the entire space-time surface under these conditions! Simple names for complex things replacing the complex things is the essence of cognition!
3. The interpretation for expansions of numbers in given base suggests an analog with complexity theory and symbolic dynamics associated with division. For cognitively representable numbers the information about this dynamics should be coded by an algorithm with finite steps. Periodic orbit or fixed point orbit would be the dynamical analog for simplicity. Non-periodic orbit would correspond to complexity and possibly also chaos.

These ideas led to two approaches in attempt to understand the cognitive representability of algebraic numbers.

1. Generalized rationals in extensions of rationals as periodic orbits for the dynamics of division

The first approach allows to represent ratios of algebraic integers for given extension using periodic expansion in the base so that a finite amount of information is needed to code the number if one accepts the numbers defining the basis of the algebraic extension as given.

1. Rationals allow periodic expansion with respect to any base. For p-adic numbers the base is naturally prime. Therefore the information about rational is finite. One can see the expansion as a periodic orbit in dynamics determining the expansion by division m/n in given base. Periodicity follows from the fact that the output of the division algorithm for a given digit has only a finite number of outcomes so that the process begins to repeat itself sooner or later.
2. This generalizes to generalized rationals in given extension of rationals defined as ratios of algebraic integers. One can reduce the division to the construction of the expansion of ordinary rational identified as number theoretic norm $|N|$ of the denominator in the extension of rationals considered.
   The norm $|N|$ of N is the determinant $|N| = det(N)$ for the linear map of extension induced by multiplication with N. $det(N)$ is ordinary (possibly p-adic) integer. This is achieved by multiplying $1/N$ by $n-1$ conjugates of N both in numerator and denominator so that one obtains product of $n-1$ conjugates in the numerator and $det(N)$ in the denominator. The computation of $1/N$ as series in the base used reduces to that in the case of rationals.
3. One has now periodic orbits in n-dimensional space defined by algebraic extensions which for ordinary rationals reduced to periodic orbits in 1-D space. This supports the interpretation of numbers as orbits of number theoretic dynamics determining the next digit of the generalized rational for given base. This picture also suggests that transcendentals correspond to non-periodic orbits. Some transcendentals could still allow a finite algorithm: in this case the dynamics would be still deterministic. Some transcendentals would be chaotic.
4. Given expansion of algebraic number is same for all extensions of rationals containing the extension in question and the ultimate extension corresponds to algebraic numbers.

The problem of this approach is that the algebraic numbers defining the extension do not have representation and must be accepted as irreducibles.

2. Algebraic numbers as infinite periodic orbits in the dynamics of partial fractions

Second approach is based on partial fractions and Stern-Brocot tree (see `http://tinyurl.com/yb6ldekq`, see also `http://tinyurl.com/yc6hhboo`) and indeed allows to see information about algebraic numbers as constructible by using an algorithm with finite number of steps, which is allowed if one accepts abstraction as basic aspect of cognition. I had managed to not become aware of this possibility and am grateful for SS for mentioning the representation of algebraics in terms of S-B tree.

1. The definition S-B tree is simple: if m/n and m'/n' are any neighboring rationals at given level in the tree, one adds $(m+m')/(n+n')$ between them and obtains in this manner the next level in the tree. By starting from $(0/1)$ and $(1/0)$ as representations of zero and ∞ one obtains $(0/1)(1/1)(1/0)$ as the next level. One can continue in this manner ad infinitum. The nodes of S-B tree represent rational points and it can be shown that given rational appears only once in the tree.
   Given rational can be represented as a finite path beginning from $1/1$ at the top of tree consisting of left moves L and right moves R and ending to the rational which appears only once in S-B tree. Rational can be thus constructured by a sequences $R^{a_0}L^{a_1}L^{a_2}....$ characterized by the sequence $a_0; a_1, a_2....$ For instance, $4/11 = 0 + 1/(2+x)$, $x = 1/(1+1/3)$ corresponds to $R^0L^2R^1L^{3-1}$ labelled by $0; 2, 1, 3$.
2. Algebraic numbers correspond to infinite but periodic paths in S-B tree in the sense that some sequence of L:s and R:s characterized by sequences of non-negative integers starts to repeat itself. Periodicity means that the information needed to construct the number is finite.
   The actual construction as a digit sequence representing algebraic number requires infinite amount of time. In TGD framework octonionic physics would come in rescue and construct algebraic numbers as roots of polynomials having concrete interpretations as coordinate values assignable to fermions at partonic 2-surfaces.
3. Transcendentals would correspond to non-periodic infinite sequences of L:s and R:s. This does not exclude the possibility that these sequences are expressible in terms of some rule involving

finite number of steps so that the amount of information would be also now finite. Information about number would be replaced by information about rule.

This picture conforms with the ideas about transition to chaos. Rationals have finite paths. A possible dynamical analog is particle coming at rest due to the dissipation. Algebraic numbers would correspond to periodic orbits possible in presence of dissipation if there is external feed of energy. They would correspond to dynamical self-organization patterns.

Remark: If one interprets the situation in terms of conservative dynamics, rationals would correspond to potential minima and algebraic numbers closed orbits around them.

The assignment of period doubling and p-pling to this dynamics as the dimension of extension increases is an attractive idea. One would expect that the complexity of periodic orbits increases as the degree of the defining irreducible polynomial increases. Algebraic numbers as maximal extension of rationals possibly also containing extension containing all rational roots of e and transcendentals would correspond to chaos.

Transcendentals would correspond to non-periodic orbits. These orbits need not be always chaotic in the sense of being non-predictable. For instance, Neper number e can be said to be p-adically algebraic number (e^p is p-adic integer albeit infinite as real integer). Does the sequence of L:s and R:s allow a formula for the powers of L and R in this case?

4. TGD should be an integrable theory. This suggests that scattering amplitudes involve only cognitive representations as number theoretic vision indeed strongly suggests [L46]. Cognitively representable numbers would correspond to the integrable sub-dynamics [L56]. Also in chaotic systems both periodic and chaotic orbits are present. Complexity theory for characterization of real numbers exists. The basic idea is that complexity is measured by the length of the shortest program needed to code the bit sequences coding for the number.

Surreals and ZEO

The following comment is not directly related to cognitive representability but since it emerged during discussion, I will include it. SS favors surreals (see `http://tinyurl.com/86jatas`) as ultimate number field containing reals as sub-field. I must admit that my knowledge and understanding of surreals is rather fragmentary.

I am agnostic in these issues and see no conflict between TGD view about numbers and surreals. Personally I however like very much infinite primes, integers, and rationals over surreals since they allow infinite numbers to have number theoretical anatomy [K94]. A further reason is that the construction of infinite primes resembles structurally repeated second quantization of the arithmetic number field theory and could have direct space-time correlate at the level of many-sheeted space-time. One ends up also to a generalization of real number. Infinity can be seen as something related to real norm: everything is finite with respect to various p-adic norms.

Infinite rationals with unit real norm and various p-adic norms bring in infinitely complex number theoretic anatomy, which could be even able to represent even the huge WCW and the space of WCW spinor fields. One could speak of number theoretical holography or algebraic Brahman=Atman principle. One would have just complexified octonions with infinitely richly structure points.

Surreals are represented in terms of pairs of sets. One starts the recursive construction from empty set identified as 0. The definition says that the pairs (.|.) of sets defining surreals x and y satisfy $x \leq y$ if the left hand part of x as set is to left from the pair defining y and the right hand part of y is to the right from the pair defining x. This does not imply that one has always $x < y$, $y < x$ or $x = y$ as for reals.

What is interesting that the pair of sets defining surreal x is analogous to a pair of states at boundaries of CD defining zero energy state. Is there a connection with zero energy ontology (ZEO)? One could perhaps say at the level of CD - forgetting everything related to zero energy states - following. The number represented by CD_1 - say represented as the distance between its tip - is smaller than than the number represented by CD_2, if CD_1 is inside CD_2. This conforms with the left and righ rule if left and right correspond to the opposite boundaries of CD. A more detailed definition would presumably say that CD_1 can be moved so that it is inside CD_2.

What makes this also interesting is that CD is the geometric correlate for self, conscious entity, also mathematical mental image about number.

6.5 Galois groups and genes

In an article discussing a TGD inspired model for possible variations of G_{eff} [L61], I ended up with an old idea that subgroups of Galois group could be analogous to conserved genes in that they could be conserved in number theoretic evolution. In small variations such as above variation Galois subgroups as genes would change only a little bit. For instance, the dimension of Galois subgroup would change.

The analogy between subgoups of Galois groups and genes goes also in other direction. I have proposed long time ago that genes (or maybe even DNA codons) could be labelled by $h_{eff}/h = n$. This would mean that genes (or even codons) are labelled by a Galois group of Galois extension (see `http://tinyurl.com/zu5ey96`) of rationals with dimension n defining the number of sheets of space-time surface as covering space. This could give a concrete dynamical and geometric meaning for the notion of gene and it might be possible some day to understand why given gene correlates with particular function. This is of course one of the big problems of biology.

6.5.1 Could DNA sequence define an inclusion hierarchy of Galois extensions?

One should have some kind of procedure giving rise to hierarchies of Galois groups assignable to genes. One would also like to assign to letter, codon and gene and extension of rationals and its Galois group. The natural starting point would be a sequence of so called intermediate Galois extensions E^H leading from rationals or some extension K of rationals to the final extension E. Galois extension has the property that if a polynomial with coefficients in K has single root in E, also other roots are in E meaning that the polynomial with coefficients K factorizes into a product of linear polynomials. For Galois extensions the defining polynomials are irreducible so that they do not reduce to a product of polynomials.

Any sub-group $H \subset Gal(E/K))$ leaves the intermediate extension E^H invariant in element-wise manner as a sub-field of E (see `http://tinyurl.com/y958drcy`). Any subgroup $H \subset Gal(E/K))$ defines an intermediate extension E^H and subgroup $H_1 \subset H_2 \subset ...$ define a hierarchy of extensions $E^{H_1} > E^{H_2} > E^{H_3}...$ with decreasing dimension. The subgroups H are normal - in other words $Gal(E)$ leaves them invariant and $Gal(E)/H$ is group. The order $|H|$ is the dimension of E as an extension of E^H. This is a highly non-trivial piece of information. The dimension of E factorizes to a product $\prod_i |H_i|$ of dimensions for a sequence of groups H_i.

Could a sequence of DNA letters/codons somehow define a sequence of extensions? Could one assign to a given letter/codon a definite group H_i so that a sequence of letters/codons would correspond a product of some kind for these groups or should one be satisfied only with the assignment of a standard kind of extension to a letter/codon?

Irreducible polynomials define Galois extensions and one should understand what happens to an irreducible polynomial of an extension E^H in a further extension to E. The degree of E^H increases by a factor, which is dimension of E/E^H and also the dimension of H. Is there a standard manner to construct irreducible extensions of this kind?

1. What comes into mathematically uneducated mind of physicist is the functional decomposition $P^{m+n}(x) = P^m(P^n(x))$ of polynomials assignable to sub-units (letters/codons/genes) with coefficients in K for a algebraic counterpart for the product of sub-units. $P^m(P^n(x))$ would be a polynomial of degree $n+m$ in K and polynomial of degree m in E^H and one could assign to a given gene a fixed polynomial obtained as an iterated function composition. Intuitively it seems clear that in the generic case $P^m(P^n(x))$ does not decompose to a product of lower order polynomials. One could use also polynomials assignable to codons or letters as basic units. Also polynomials of genes could be fused in the same manner.
2. If this indeed gives a Galois extension, the dimension m of the intermediate extension should be same as the order of its Galois group. Composition would be non-commutative but associative as the physical picture demands. The longer the gene, the higher the algebraic complexity would be. Could functional decomposition define the rule for who extensions and Galois groups correspond to genes? Very naïvely, functional decomposition in mathematical sense would correspond to composition of functions in biological sense.
3. This picture would conform with $M^8 - M^4 \times CP_2$ correspondence [L46] in which the construction of space-time surface at level of M^8 reduces to the construction of zero loci of polynomials

of octonions, with rational coefficients. DNA letters, codons, and genes would correspond to polynomials of this kind.

6.5.2 Could one say anything about the Galois groups of DNA letters?

A fascinating possibility is that this picture could allow to say something non-trivial about the Galois groups of DNA letters.

1. Since $n = h_{eff}/h$ serves as a kind of quantum IQ, and since molecular structures consisting of large number of particles are very complex, one could argue that n for DNA or its dark variant realized as dark proton sequences can be rather large and depend on the evolutionary level of organism and even the type of cell (neuron viz. soma cell). On the other, hand one could argue that in some sense DNA, which is often thought as information processor, could be analogous to an integrable quantum field theory and be solvable in some sense. Notice also that one can start from a background defined by given extension K of rationals and consider polynomials with coefficients in K. Under some conditions situation could be like that for rationals.
2. The simplest guess would be that the 4 DNA letters correspond to 4 non-trivial finite groups with smaller possible orders: the cyclic groups Z_2, Z_3 with orders 2 and 3 plus 2 finite groups of order 4 (see the table of finite groups in `http://tinyurl.com/j8d5uyh`). The groups of order 4 are cyclic group $Z_4 = Z_2 \times Z_2$ and Klein group $Z_2 \oplus Z_2$ acting as a symmetry group of rectangle that is not square - its elements have square equal to unit element. All these 4 groups are Abelian. Polynomial equations of degree not larger than 4 can be solved exactly in the sense that one can write their roots in terms of radicals.
3. Could there exist some kind of connection between the number 4 of DNA letters and 4 polynomials of degree less than 5 for whose roots one an write closed expressions in terms of radicals as Galois found? Could it be that the polynomials obtained by a a repeated functional composition of the polynomials of DNA letters have also this solvability property?
   This could be the case! Galois theory states that the roots of polynomial are solvable by radicals if and only if the Galois group is solvable meaning that it can be constructed from abelian groups using Abelian extensions (see `https://cutt.ly/4RuXmGo`).
   Solvability translates to a statement that the group allows so called sub-normal series $1 < G_0 < G_1 ... < G_k$ such that G_{j-1} is normal subgroup of G_j and G_j/G_{j-1} is an abelian group. An equivalent condition is that the derived series $G \triangleright G^{(1)} \triangleright G^{(2)} \triangleright ...$ in which $j+1$:th group is commutator group of G_j ends to trivial group. If one constructs the iterated polynomials by using only the 4 polynomials with Abelian Galois groups, the intuition of physicist suggests that the solvability condition is guaranteed! Wikipedia article also informs that for finite groups solvable group is a group whose composition series has only factors which are cyclic groups of prime order.
   Abelian groups are trivially solvable, nilpotent groups are solvable, p-groups (having order, which is power prime) are solvable and all finite p-groups are nilpotent. Every group with order less than 60 elements is solvable. Fourth order polynomials can have at most S_4 with 24 elements as Galois groups and are thus solvable. Fifth order polynomials can have the smallest non-solvable group, which is alternating group A_5 with 60 elements as Galois group and in this case are not solvable. S_n is not solvable for $n > 4$ and by the finding that S_n as Galois group is favored by its special properties (see `https://arxiv.org/pdf/1511.06446.pdf`).
   A_5 acts as the group icosahedral orientation preserving isometries (rotations). Icosahedron and tetrahedron glued to it along one triangular face play a key role in TGD inspired model of bio-harmony and of genetic code [L24, L62]. The gluing of tetrahedron increases the number of codons from 60 to 64. The gluing of tetrahedron to icosahedron also reduces the order of isometry group to the rotations leaving the common face fixed and makes it solvable: could this explain why the ugly looking gluing of tetrahedron to icosahedron is needed? Could the smallest solvable groups and smallest non-solvable group be crucial for understanding the number theory of the genetic code.

An interesting question inspired by $M^8 - H$-duality [L46] is whether the solvability could be posed on octonionic polynomials as a condition guaranteeing that TGD is integrable theory in number theoretical sense or perhaps following from the conditions posed on the octonionic polynomials. Space-time surfaces in M^8 would correspond to zero loci of real/imaginary parts (in quaternionic sense) for octonionic polynomials obtained from rational polynomials by analytic continuation. Could solvability relate to the condition guaranteeing M^8 duality boiling down to the condition that the tangent spaces of space-time surface are labelled by points of CP_2. This requires that tangent or normal space is associative (quaternionic) and that it contains fixed complex subspace of octonions or perhaps more generally, there exists an integrable distribution of complex subspaces of octonions defining an analog of string world sheet.

What could the interpretation for the events in which the dimension of the extension of rationals increases? Galois extension is extensions of an extension with relative Galois group $Gal(rel) = Gal(new)/Gal(old)$. Here $Gal(old)$ is a normal subgroup of $Gal(new)$. A highly attractive possibility is that evolutionary sequences quite generally (not only in biology) correspond to this kind of sequences of Galois extensions. The relative Galois groups in the sequence would be analogous to conserved genes, and genes could indeed correspond to Galois groups [K33] [L46]. To my best understanding this corresponds to a situation in which the new polynomial P_{m+n} defining the new extension is a polynomial P_m having as argument the old polynomial $P_n(x)$: $P_{m+n}(x) = P_m(P_n(x))$.

What about the interpretation at the level of conscious experience? A possible interpretation is that the quantum jump leading to an extension of an extension corresponds to an emergence of a reflective level of consciousness giving rise to a conscious experience about experience. The abstraction level of the system becomes higher as is natural since number theoretic evolution as an increase of algebraic complexity is in question.

This picture could have a counterpart also in terms of the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs). The included factor M and including factor N would correspond to extensions of rationals labelled by Galois groups $Gal(M)$ and $Gal(N)$ having $Gal(M) \subset Gal(M)$ as normal subgroup so that the factor group $Gal(N)/Gal(M)$ would be the relative Galois group for the larger extension as extension of the smaller extension. I have indeed proposed [L63] that the inclusions for which included and including factor consist of operators which are invariant under discrete subgroup of $SU(2)$ generalizes so that all Galois groups are possible. One would have Galois confinement analogous to color confinement: the operators generating physical states could have Galois quantum numbers but the physical states would be Galois singlets.

6.6 Could the precursors of perfectoids emerge in TGD?

In algebraic-geometry community the work of Peter Scholze [A72] (see `http://tinyurl.com/y7h2sms7`) introducing the notion of perfectoid related to p-adic geometry has raised a lot of interest. There are two excellent popular articles about perfectoids: the first article in AMS (see `http://tinyurl.com/ydx38vk4`) and second one in Quanta Magazine (see `http://tinyurl.com/yc2mxxqh`). I had heard already earlier about the work of Scholze but was too lazy to even attempt to understand what is buried under the horrible technicalities of modern mathematical prose. Rachel Francon re-directed my attention to the work of Scholze (see `http://tinyurl.com/yb46oza6`). The work of Scholze is interesting also from TGD point of view since the construction of p-adic geometry is a highly non-trivial challenge in TGD.

1. One should define first the notion of continuous manifold but compact-open characteristic of p-adic topology makes the definition of open set essential for the definition of topology problematic. Even single point is open so that hopes about p-adic manifold seem to decay to dust. One should pose restrictions on the allowed open sets and p-adic balls with radii coming as powers of p are the natural candidates. p-Adic balls are either disjoint or nested: note that also this is in conflict with intuitive picture about covering of manifold with open sets. All this strangeness originates in the special features of p-adic distance function known as ultra-metricity. Note however that for extensions of p-adic numbers one can say that the Cartesian products of p-adic 1-balls at different genuinely algebraic points of extension along particular axis of extension are disjoint.
2. At level of M^8 the p-adic variants of algebraic varieties defined as zero loci of polynomials do not seem to be a problem. Equations are algebraic conditions and do not involve derivatives like partial differential equations naturally encountered if Taylor series instead of polynomials

are allowed. Analytic functions might be encountered at level of $H = M^4 \times CP_2$ and here p-adic geometry might well be needed.
The idea is to define the generalization of p-adic algebraic geometry in terms of p-adic function fields using definitions very similar to those used in algebraic geometry. For instance, generalization of variety corresponds to zero locus for an ideal of p-adic valued function field. p-Adic ball of say unit radius is taken as the basic structure taking the role of open ball in the topology of ordinary manifolds. This kind of analytic geometry allowing all power series with suitable restrictions to function field rather than allowing only polynomials is something different from algebraic geometry making sense for p-adic numbers and even for finite fields.

3. One would like to generalize the notion of analytic geometry even to the case of number fields with characteristic p (p-multiple of element vanishes), in particular for finite fields F_p and for function fields $F_p[t]$. Here one encounters difficulties. For instance, the factorial $1/n!$ appearing as normalization factor of forms diverges if p divides it. Also the failure of Frobenius homomorphism to be automorphism for $F_p[t]$ causes difficulties in the understanding of Galois groups.

The work of Scholze has led to a breakthrough in unifying the existing ideas in the new framework provided by the notion of perfectoid. The work is highly technical and involves infinite-D extension of ordinary p-adic numbers adding all powers of all roots p^{1/p^m}, $m = 1, 2....$ Formally, an extension by powers of p^{1/p^∞} is in question.

This looks strange at first but it guarantees that all p-adic numbers in the extension have p:th roots, one might say that one forms a p-fold covering/wrapping of extension somewhat analogous to complex numbers. This number field is called perfectoid since it is perfect meaning that Frobenius homomorphism $a \to a^p$ is automorphism by construction. $Frob$ is injection always and by requiring that p:th roots exist always, it becomes also a surjection.

This number field has same Galois groups for all of its extensions as the function field $G[t]$ associated with the union of function fields $G = F_p[t^{1/p^m}]$. Automorphism property of $Frob$ saves from the difficulties with the factorization of polynomials and p-adic arithmetics involving remainders is replaced with purely local modulo p arithmetics.

6.6.1 About motivations of Scholze

Scholze has several motivations for this work. Since I am not a mathematician, I am unable to really understand all of this at deep level but feel that my duty as user of this mathematics is at least to try!

1. Diophantine equations is a study of polynomial equations in several variables, say $x^2+2xy+y = 0$. The solutions are required to be integer valued: in the example considered $x = y = 0$ and $x = -y = -1$ is such a solution. For integers the study of the solution is very difficult and one approach is to study these equations modulo p that is reduced the equations to finite field G_p for any p. The equations simplify enormously since ane has $a^p = a$ in F_p. This identity in fact defines so called Frobenius homomorphism acting as automorphism for finite fields. This holds true also for more complex fields with characteristic p say the ring $F_p[t]$ of power series of t with coefficients in F_p.
The powers of variables, say x, appearing in the equation is reduced to at most x^{p-1}. One can study the solutions also in p-adic number fields. The idea is to find first whether finite field solution, that is solution modulo p, does exist. If this is the case, one can calculate higher powers in p. If the series contains finite number of terms, one has solution also in the sense of ordinary integers.

2. One of the related challenges is the generalization of the notion of variety to a geometry defined in arbitrary number field. One would like to have the notion of geometry also for finite fields, and for their generalizations such as $F_p[t]$ characterized by characteristic p ($px = 0$ holds true for any element of the field). For fields of characteristic 1 - extensions of rationals, real, and p-adic number fields) $xp = 0$ not hold true for any $x \neq 0$. Any field containing rationals as sub-field, being thus local field, is said to have characteristic equal to 1. For local fields the challenge is relatively easy.

3. The situation becomes more difficult if one wants a generalization of differential geometry. In differential geometry differential forms are in a key role. One wants to define the notion

of differential form in fields of characteristic p and construct a generalization of cohomology theory. This would generalize the notion of topology to p-adic context and even for finite fields of finite character. A lot of work has been indeed done and Grothendieck has been the leading pioneer.
The analogs of cohomology groups have values in the field of p-adic numbers instead of ordinary integers and provide representations for Galois groups for the extensions of rationals inducing extensions of p-adic numbers and finite fields.
In ordinary homology theory non-contractible sub-manifolds of various dimensions correspond to direct summands Z (group of integers) for homology groups and by Poincare duality those for cohomology groups. For Galois groups Z is replaced with Z_N. N depends on extension to which Galois group is associated and if N is divisible by p one encounters technical problems.
There are many characteristic p- and p-adic cohomologies such as etale cohomology, chrystalline cohomology, algebraic de-Rham cohomology. Also Hodge theory for complex differential forms generalizes. These cohomologies should be related by homomorphism and category theoretic thinking the proof of the homomorphism requires the construction of appropriate functor between them.
The integrals of forms over sub-varieties define the elements of cohomology groups in ordinary cohomology and should have p-adic counterparts. Since p-adic numbers are not well-ordered, definite integral has no straightforward generalization to p-adic context. One might however be able to define integrals analogous to those associated with differential forms and depending only on the topology of sub-manifold over which they are taken. These integrals would be analogous to multiple residue integrals, which are the crux of the twistor approach to scattering amplitudes in super-symmetric gauge theories. One technical difficulty is that for a field of finite characteristic the derivative of X^p is pX^{p-1} and vanishes. This does not allow to define what integral $\int X^{p-1}dX$ could mean. Also $1/n!$ appears as natural normalization factor of forms but if p divides it, it becomes infinite.

6.6.2 Attempt to understand the notion of perfectoid

Consider now the basic ideas behind the notion of perfectoid.

1. For finite finite fields F_p Frobenius homomorphism $a \rightarrow a^p$ is automorphism since one has $a^p = a$ in modulo p arithmetics. A field with this property is called perfect and all local fields are perfect. Perfectness means that an algebraic number in any extension L of perfect field K is a root of a separable minimal polynomial. Separability means that the number of roots in the algebraic closure of K of the polynomial is maximal and the roots are distinct.
2. All fields containing rationals as sub-fields are perfect. For fields of characteristic p $Frob$ need not be a surjection so that perfectness is lost. For instance, for $F_p[t]$ $Frob$ is trivially injection but surjective property is lost: $t^{1/p}$ is not integer power of t.
One can however extend the field to make it perfect. The trick is simple: add to $F_p[t]$ all fractional powers t^{1/p^n} so that all p:th roots exist and $Frob$ becomes and automorphism. The automorphism property of $Frob$ allows to get rid of technical problems related to a factorization of polynomials. The resulting extension is infinite-dimensional but satisfies the perfectness property allowing to understand Galois groups, which play key role in various cohomology theories in characteristic p.
3. Let $K = Q_p[p^{1/p^\infty}]$ denote the infinite-dimensional extension of p-adic number field Q_p by adding all powers of p^m:th roots for all all $m = 1, 2, ...$ This is not the most general option: K could be also only a ring. The outcome is perfect field although it does not of course have Frobenius automorphism since characteristic equals to 1.
One can divide K by p to get K/p as the analog of finite field F_p as its infinite-dimensional extension. K/p allows all p:th roots by construction and $Frob$ is automorphism so that K/p is perfect by construction.
The structure obtained in this manner is closely related to a perfect field with characteristic p having same Galois groups for all its extensions. This object is computationally much more attractive and allows to prove theorems in p-adic geometry. This motivates the term perfectoid.
4. One can assign to K another object, which is also perfectoid but has characteristic p. The correspondence is as follows.

(a) Let F_p be finite field. F_p is perfect since it allows trivially all p:th roots by $a^p=a$. The ring $F_p[t]$ is however not prefect since t^{1/p^m} is not integer power of t. One must modify $F_p[t]$ to obtain a perfect field. Let $G_m=F_p[t^{1/p^m}]$ be the ring of formal series in powers of t^{1/p^m} defining also function field. These serious are called t-adic and one can define t-adic norm.

(b) Define t-adic function field K_b called the **tilt** of K as

$$K_b=\cup_{m=1,...}(K/p)[t^{1/pm}][t] \ .$$

One has all possible power series with coefficients in K/p involving all roots t^{1/p^m}, $m=1,2,...$, besides powers of positive integer powers of t. This function field has characteristic p and all roots exist by construction and $Frob$ is automorphism. K_b/t is perfect meaning that the minimal polynomials for the for given analog of algebraic number in any of its extensions allows separable polynomial with maximal number of roots in its closure.

This sounds rather complicated! In any case, K_b/t has same number theoretical structure as $Q_p[p^{1/p^\infty}]/p$ meaning that Galois groups for all of its extensions are canonically isomorphic to those for extensions of K. Arithmetics modulo p is much simpler than p-adic arithmetic since products are purely local and there is no need to take care about remainders in arithmetic operations, this object is much easier to handle.
Note that also p-adic number fields fields Q_p as also $F_p=Q_p/p$ are perfect but the analog of $K_b=F_b[t]$ fails to be perfect.

6.6.3 Second attempt to understand the notions of perfectoid and its tilt

This subsection is written roughly year after the first version of the text. I hope that it reflects a genuine increase in my understanding.

1. Scholze introduces first the notion of perfectoid. This requires some background notions. The characteristic p for field is defined as the integer p (prime) for which $px=0$ for all elements x. Frobenius homomorphism (Frob familiarly) is defined as $Frob: x\rightarrow x^p$. For a field of characteristic p $Frob$ is an algebra homomorphism mapping product to product and sum to sum: this is very nice and relatively easy to show even by a layman like me.
2. Perfectoid is a field having either characteristic $p=0$ (reals, p-adics for instance) or for which $Frob$ is a surjection meaning that $Frob$ maps at least one number to a given number x.
3. For finite fields $Frob$ is identity: $x^p=x$ as proved already by Fermat. For reals and p-adic number fields with characteristic p=0 it maps all elements to unit element and is not a surjection. Field is perfect if it has either $p=0$ (reals, p-adics) or if Frobenius is surjection. Finite fields are obviously perfectoids too.

Scholze introduces besides perfectoids K also what he calls tilt K_b of the perfectoid. K_b is infinite-D extension of p-adic numbers by iterated p:th roots p-adic numbers: the units of the extension correspond to the roots p^{1/p^k}. They are something between p-adic number fields and reals and leads to theorems giving totally new insights to arithmetic geometry. Unfortunately, my technical skills in mathematics are hopelessly limited to say anything about these theorems.

1. As we learned during the first student year of mathematics, real numbers can be defined as Cauchy sequences of rationals converging to a real number, which can be also algebraic number or transcendental. The elements in the tilt K_b would be this kind of sequences.
2. Scholze starts from (say) p-adic numbers and considers infinite sequence of iterates of $1/p$:th roots. At given step $x\rightarrow x^{1/p}$. This gives the sequence $(x,x^{1/p},x^{1/p^2},x^{1/p^3},...)$ identified as an element of the tilt K_b. At the limit one obtains $1/p^\infty$ root of x.
Remark: For finite fields each step is trivial ($x^p=x$) so that nothing interesting results: one has $(x,x,x,x,...)$
(a) For p-adic number fields the situation is non-trivial. $x^{1/p}$ exists as p-adic number for all p-adic numbers with unit norm having $x=x_0+x_1p+....$ In the lowest order $x\simeq x_0$ the root is just x since x is effectively an element of finite field in this approximation. One can develop the $x^{1/p}$ to a power series in p and continue the iteration. The sequence obtained defines an element of tilt K_b of field K, now p-adic numbers.

(b) If the p-adic number x has norm p^n, $n \neq 0$ and is therefore not p-adic unit, the root operation makes sense only if one performs an extension of p-adic numbers containing all the roots p^{1/p^k} . These roots define one particular kind of extension of p-adic numbers and the extension is infinite-dimensional since all roots are needed. One can approximate K_b by taking only finite number iterated roots.

3. The tilt is said to be fractal: this is easy to understand from the presence of the iterated p:th root. Each step in the sequence is like zooming. One might say that p-adic scale becomes p:th root of itself. In TGD the p-adic length scale L_p is proportional to $p^{1/2}$: does the scaling mean that the p-adic length scale would defined hierarchy of scales proportional to $p^{1/2kp}$: root of itself and approach the CP_2 scale since the root of p approaches unity. Tilts as extensions by iterated roots would improve the length scale resolution.

One day later after writing this I got the feeling that I might have vaguely understood one more important thing about the tilt of p-adic number field: changing of the characteristic 0 of p-adic number field to characteristics $p > 0$ of the corresponding finite field for its tilt. What could this mean?

1. Characteristic p (p is the prime labelling p-adic number field) means $px = 0$. This property makes the mathematics of finite fields extremely simple: in the summation one need not take care of the residue as in the case of reals and p-adics. The tilt of the p-adic number field would have the same property! In the infinite sequence of the p-adic numbers coming as iterated p:th roots of the starting point p-adic number one can sum each p-adic number separately. This is really cute if true!
2. It seems that one can formulate the arithmetics problem in the tilt where it becomes in principle as simple as in finite field with only p elements! Does the existence of solution in this case imply its existence in the case of p-adic numbers? But doesn't the situation remain the same concerning the existence of the solution in the case of rational numbers? The infinite series defining p-adic number must correspond a sequence in which binary digits repeat with some period to give a rational number: rational solution is like a periodic solution of a dynamical system whereas non-rational solution is like chaotic orbit having no periodicity? In the tilt one can also have solutions in which some iterated root of p appears: these cannot belong to rationals but to their extension by an iterated root of p.

The results of Scholze could be highly relevant for the number theoretic view about TGD in which octonionic generalization of arithmetic geometry plays a key role since the points of space-time surface with coordinates in extension of rationals defining adele and also what I call cognitive representations determining the entire space-time surface if $M^8 - H$ duality holds true (space-time surfaces would be analogous to roots of polynomials). Unfortunately, my technical skills in mathematics needed are hopelessly limited.

TGD inspires the question is whether this kind of extensions could be interesting physically. At the limit of infinite dimension one would get an ideal situation not realizable physically if one believes that finite-dimensionality is basic property of extensions of p-adic numbers appearing in number theoretical quantum physics (they would related to cognitive representations in TGD). Adelic physics [L52] involves all finite-D extensions of rationals and the extensions of p-adic number fields induced by them and thus also cutoffs of extensions of type K_b- which I have called precursors of K_b.

How this relates to Witt vectors?

Witt vectors provide an alternative representation of p-adic arithmetics of p-adic integers in which the sum and product are reduced to purely local digit-wise operations for each power of p for the components of Witt vector so that one need not worry about carry pinary digit.

1. The idea is to consider the sequence consisting pinary cutoffs to p-adic number $xmodp^n$ and identify p-adic integer as this kind of sequence as n approaches infinity. This is natural approach when one identifies finite measurement resolution or cognitive resolution as a cutoff in some power of p^n. One simply forms the numbers $X_n = x\ mod\ p^{n+1}$: for numbers $1, ..., p-1$ they are called Teichmueller representatives and only they are needed to construct the sequences for general x. One codes this sequence of pinary cutoffs to Witt vector.

2. The non-trivial observation made by studying sums of p-adic numbers is that the sequence $X_0, X_1, X_2, ...$ of approximations define a sequence of components of Witt vector as $W_0 = X_0$, $W_1 = X_0^p + pX_1$, $W_2 = X_0^{(}p^2) + pX_1^p + p^2X_2$, ... or more formally $W_n = Sum_{i<n} p^i Z X_i^{[} p^{(} n-i)]$.
3. The non-trivial point is that Witt vectors form a commutative ring with local digit-wise multiplication and sum modulo p: there no carry digits. Effectively one obtains infinite Cartesian power of finite field F_p. This means a great simplification in arithmetics. One can do the arithmetics using Witt vectors and deduce the sum and product from their product.
4. Witt vectors are universal. In particular, they generalize to any extension of p-adic numbers. Could Witt vectors bring in something new from physics point of view? Could they allow a formulation for the hierarchy of pinary cutoffs giving some new insights? For instance, neuro-computationalist might ask whether brain could perform p-adic arithmetics using a linear array of modules (neurons or neuron groups) labelled by $n = 1, 2, ...$ calculates sum or product for component W_n of Witt vector? No transfer of carry bits between modules would be needed. There is of course the problem of transforming p-adic integers to Witt vectors and back - it is not easy to imagine a natural realization for a module performing this transformation. Is there any practical formulation for say p-adic differential calculus in terms of Witt vectors?

I would seem that Witt vectors might relate in an interesting manner to the notion of perfectoid. The basic result proved by Petter Scholtze is that the completion $\cup_n Q_p(p^{1/p^n})$ of p-adic numbers by adding p^n:th roots and the completion of Laurent series $F_p((t))$ to $\cup_n F_p((t^{1/p^n}))$ have isomorphic absolute Galois groups and in this sense are one and same thing. On the other hand, p-adic integers can be mapped to a subring of $F_p(t)$ consisting of Taylor series with elements allowing interpretation as Witt vectors.

6.6.4 TGD view about p-adic geometries

As already mentioned, it is possible to define p-adic counterparts of n-forms and also various p-adic cohomologies with coefficient field taken as p-adic numbers and these constructions presumably make sense in TGD framework too. The so called rigid analytic geometry is the standard proposal for what p-adic geometry might be.

The very close correspondence between real space-time surfaces and their p-adic variants plays realized in terms of cognitive representations [L54, L53, L46] plays a key role in TGD framework and distinguishes it from approaches trying to formulate p-adic geometry as a notion independent of real geometry.

Ordinary approaches to p-adic geometry concentrate the attention to single p-adic prime. In the adelic approach of TGD one considers both reals and all p-adic number fields simultaneously.

Also in TGD framework Galois groups take key role in this framework and effectively replace homotopy groups and act on points of cognitive representations consisting of points with coordinates in extension of rationals shared by real and p-adic space-time surfaces. One could say that homotopy groups at level of sensory experience are replaced by Galois at the level of cognition. It also seems that there is very close connection between Galois groups and various symmetry groups. Galois groups would provide representations for discrete subgroups of symmetry groups.

In TGD framework there is strong motivation for formulating the analog of Riemannian geometry of $H = M^4 \times CP_2$ for p-adic variants of H. This would mean p-adic variant of Kähler geometry. The same challenge is encountered even at the level of "World of Classical Worlds" (WCW) having Kähler geometry with maximal isometries. p-Adic Riemann geometry and n-forms make sense locally as tensors but integrals defining distances do not make sense p-adically and it seems that the dream about global geometry in p-adic context is not realizable. This makes sense: p-adic physics is a correlate for cognition and one cannot put thoughts in weigh or measure their length.

Formulation of adelic geometry in terms of cognitive representations

Consider now the key ideas of adelic geometry and of cognitive representations.

1. The king idea is that p-adic geometries in TGD framework consists of p-adic balls of possibly varying radii p^n assignable to points of space-time surface for which the preferred embedding space coordinates are in the extension of rationals. At level of M^8 octonion property

fixes preferred coordinates highly uniquely. At level of H preferred coordinates come from symmetries.

These points define a cognitive representation and inside p-adic points the solution of field equations is p-adic variant of real solution in some sense. At M^8 level the field equations would be algebraic equations and real-p-adic correspondence would be very straightforward. Cognitive representations would make sense at both M^8 level and H level.

Remark: In ordinary homology theory the decomposition of real manifold to simplexes reduces topology to homology theory. One forgets completely the interiors of simplices. Could the cognitive representations with points labelling the p-adic balls could be seen as analogous to decompositions to simplices. If so, homology would emerge as something number theoretically universal. The larger the extension of rationals, the more precise the resolution of homology would be. Therefore p-adic homology and cohomology as its Poincare dual would reduce to their real counterparts in the cognitive resolution used.

2. M^8-H correspondence would play a key role in mapping the associative regions of space-time varieties in M^8 to those in H. There are two kinds of regions. Associative regions in which polynomials defining the surfaces satisfy criticality conditions and non-associative regions. Associative regions represent external particles arriving in CDs and non-associative regions interaction regions within CDs.

3. In associative regions one has minimal surface dynamics (geodesic motion) at level of H and coupling parameters disappear from the field equations in accordance with quantum criticality. The challenge is to prove that M^8-H correspondence is consistent with the minimal surface dynamics n H. The dynamics in these regions is determined in M^8 as zero loci of polynomials satisfying quantum criticality conditions guaranteeing associativity and is deterministic also in p-adic sectors since derivatives are not involved and pseudo constants depending on finite number of pinary digits and having vanishing derivative do not appear. M^8-H correspondence guarantees determinism in p-adic sectors also in H.

4. In non-associative regions M^8-H correspondence does not make sense since the tangent space of space-time variety cannot be labelled by CP_2 point and the real and p-adic H counterparts of these regions would be constructed from boundary data and using field equations of a variational principle (sum of the volume term and Kähler action term), which in non-associative regions gives a dynamics completely analogous to that of charged particle in induced Kähler field. Now however the field characterizes extended particle itself.

Boundary data would correspond to partonic 2-surfaces and string world sheets and possibly also the 3-surfaces at the ends of space-time surface at boundaries of CD and the light-like orbits of partonic 2-surfaces. At these surfaces the 4-D (!) tangent/normal space of space-time surface would be associative and could be mapped by M^8-H correspondence from M^8 to H and give rise to boundary conditions.

Due to the existence of p-adic pseudo-constants the p-adic dynamics determined by the action principle in non-associative regions inside CD would not be deterministic in p-adic sectors. The interpretation would be in terms of freedom of imagination. It could even happen that boundary values are consistent with the existence of space-time surface in p-adic sense but not with the existence of real space-time surfaces. Not all that can be imagined is realizable.

At the level of M^8 this vision seems to have no obvious problems. Inside each ball the same algebraic equations stating vanishing of $IM(P)$ (imaginary part of P in quaternionic sense) hold true. At the level of H one has second order partial differential equations, which also make sense also p-adically. Besides this one has infinite number of boundary conditions stating the vanishing of Noether charges assignable to sub-algebra super-symplectic algebra and its commutator with the entire algebra at the 3-surfaces at the boundaries of CD. Are these two descriptions really equivalent?

During writing I discovered an argument, which skeptic might see as an objection against M^8-H correspondence.

1. M^8 correspondence maps the space-time varieties in M^8 in non-local manner to those in $H=M^4\times CP_2$. CP_2 coordinates characterize the tangent space of space-time variety in M^8 and this might produce technical problems. One can map the real variety to H and find the points of the image variety satisfying the condition and demand that they define the "spine" of the p-adic surface in p-adic H.

2. The points in extensions of rationals in H need not be images of those in M^8 but should this be the case? Is this really possible? M^4 point in $M^4 \times E^4$ would be mapped to $M^4 \subset M^4 \times CP_2$: this is trivial. 4-D associative tangent/normal space at m containing preferred M^2 would be characterized by CP_2 coordinates: this is the essence of $M^8 - H$ correspondence. How could one guarantee that the CP_2 coordinates characterizing the tangent space are really in the extension of rationals considered? If not, then the points of cognitive representation in H are not images of points of cognitive representation in M^8. Does this matter?

Are almost-perfectoids evolutionary winners in TGD Universe?

One could take perfectoids and perfectoid spaces as a mere technical tool of highly refiner mathematical cognition. Since cognition is basic aspect of TGD Universe, one could also ask perfectoids or more realistically, almost-perfectoids, could be an outcome of cognitive evolution in TGD Universe?

1. p-Adic algebraic varieties are defined as zero loci of polynomials. In the octonionic M^8 approach identifying space-time varieties as zero loci for RE or IM of octonionic polynomial (RE and IM in quaternionic sense) this allows to define p-adic variants of space-time surfaces as varieties obeying same polynomial equations as their real counterparts provided the coefficients of octonion polynomials obtainable from real polynomials by analytic continuation are in an extension of rationals inducing also extension of p-adic numbers.
   The points with coordinates in the extension of rationals common to real and p-adic variants of M^8 identified as cognitive representations are in key role. One can see p-adic space-time surfaces as collections of "monads" labelled by these points at which Cartesian product of 1-D p-adic balls in each coordinate degree. The radius of the p-adic ball can vary. Inside each ball the same polynomial equations are satisfied so that the monads indeed reflect other monads. Kind of algebraic hologram would be in question consisting of the monads. The points in extension allow to define ordinary real distance between monads. Only finite number of monads would be involved since the number of points in extension tends to be finite. As the extension increases, this number increases. Cognitive representations become more complex: evolution as increase of algebraic complexity takes place.
2. Finite-dimensionality for the allowed extensions of p-adic number fields is motivated by the idea about finiteness of cognition. Perfectoids are however infinite-dimensional. Number theoretical universality demands that on only extensions of p-adics induced by those of rationals are allowed and defined extension of the entire adele. Extensions should be therefore be induced by the same extension of rationals for all p-adic number fields.
   Perfectoids correspond to an extension of Q_p apparently depending on p. This dependence is in conflict with number theoretical universality if real. This extension could be induced by corresponding extension of rationals for all p-adic number fields. For p-adic numbers Q_q $q \neq p$ all equation $a^{p^n} = x$ reduces to $a^n = x \ mod \ p$ and this in term to $a^m = x \ mod \ p$, $m = n \ mod \ p$. Finite-dimensional extension is needed to have all roots of required kind! This extension is therefore finite-D for all $q \neq p$ and infinite-D for p.
3. What about infinite-dimensionality of the extension. The real world is rarely perfect and our thoughts about it even less so, and one could argue that we should be happy with almost-perfectoids! "Almost" would mean extension induced by powers of p^{1/p^m} for large enough m, which is however not infinite. A finite-dimensional extension approaching perfectoid asymptotically is quite possible!
4. One could see the almost perfectoid as an outcome of evolution and perfectoid as the asymptotic states. High dimension of extension means that p-adic numbers and extension of rationals have large number of common numbers so that also cognitive representations contain a large number of common points. Maybe the p-adic number fields, which are evolutionary winners, have managed to evolve to especially high-dimensional almost-perfectoids! Note however that also the roots of e can be considered as extensions of rationals since corresponding p-adic extensions are finite-dimensional. Similar evolution can be considered also now.
   To get some perspective mote that for large primes such as $M_{127} = 2^{127} - 1$ characterizing electron the lowest almost perfectoid would give powers of $M_{127}^{1/M_{127}} = (2^{127} - 1)^{1/(2^{127}-1)} \sim 1 + log(2)2^{-120}$! The lattice of points in extension is extremely dense near real unit. The

density of of points in cognitive representations near this point would be huge. Note that the length scales comes as negative powers of two, which brings in mind p-adic length scale hypothesis [K68].

Although the octonionic formulation in terms of polynomials (or rational functions identifying space-time varieties as zeros or poles of $RE(P)$ or $IM(P)$ is attractive in its simplicity, one can also consider the possibility of allowing analytic functions of octonion coordinate obtained from real analytic functions. These define complex analytic functions with commutative imaginary unit used to complexify octonions. Could meromorphic functions real analytic at real axis having only zeros and poles be allowed? The condition that all p-adic variants of these functions exist simultaneously is non-trivial. Coefficients must be in the extension of rationals considered and convergence poses restrictions. For instance, e^x converges only for $|x|_p < 1$. These functions might appear at the level of H.

6.7 Secret Link Uncovered Between Pure Math and Physics

I learned about a possible existence of a very interesting link between pure mathematics and physics (see `http://tinyurl.com/y86bckmo`). The article told about ideas of number theorist Minhyong Kim working at the University of Oxford. As I read the popular article, I realized it is something very familiar to me but from totally different view point.

Number theoretician encounters the problem of finding rational points of an algebraic curve defined as real or complex variant in which case the curve is 2-D surface and 1-D in complex sense. The curve is defined as root of polynomials polynomials or several of them. The polynomial have typically rational coefficients but also coefficients in extension of rationals are possible.

For instance, Fermat's theorem is about whether $x^n + y^n = 1$, $n = 1, 2, 3, ...$ has rational solutions for $n \geq 1$. For $n = 1$, and $n = 2$ it has, and these solutions can be found. It is now known that for $n > 2$ no solutions do exist. Quite generally, it is known that the number is finite rather than infinite in the generic case.

A more general problem is that of finding points in some algebraic extension of rationals. Also the coefficients of polynomials can be numbers in the extension of rationals. A less demanding problem is mere counting of rational points or points in the extension of rationals and a lot of progress has been achieved in this problem. One can also dream of classifying the surfaces by the character of the set of the points in extension.

I have consider the identification problem earlier in [L46] and I glue here a piece of text summarizing some basic results. The generic properties of sets of rational points for algebraic curves are rather well understood. Mordelli conjecture proved by Falting as a theorem (see `http://tinyurl.com/y9oq37ce`) states that a curve over Q with genus $g = (d-1)(d-2)/2 > 1$ (degree $d > 3$) has only finitely many rational points.

1. Sphere CP_1 in CP_2 has rational points as a dense set. Quite generally rational surfaces, which by definition allow parametric representation using polynomials with rational coefficients (encountered in context of Du Val singularities characterized by the extended Dynkin diagrams for finite subgroups of $SU(2)$) allow dense set of rational points [A61, A69]).
   $g = 0$ does not yet guarantee that there is dense set of rational points. It is possible to have complex conics (quadratic surface) in CP_2 with no rational points. Note however that this depends on the choice of the coordinates: if origin belongs to the surface, there is at least one rational point
2. Elliptic curve $y^2 - x^3 - ax - b$ in CP_2 (see `http://tinyurl.com/lovksny`) has genus $g = 1$ and has a union of lattices of rational points and of finite cyclic groups of them since it has origin as a rational point. This lattice of points are generated by translations. Note that elliptic curve has no singularities that is self intersections or cusps (for $a = 0, b = 0$ origin is a singularity).
   $g = 1$ does not guarantee that there is infinite number of rational points. Fermat's last theorem and CP_2 as example. $x^d + y^d = z^d$ is projectively invariant statement and therefore defines a curve with genus $g = (d-1)(d-2)/2$ in CP_2 (one has $g = 0, 0, 2, 3, 6, 10, ...$). For $d > 2$, in particular $d = 3$, there are no rational points.
3. $g \geq 2$ curves do not allow a dense set of rational points nor even potentially dense set of rational points.

In my article [L46] providing TGD perspective about the role of algebraic geometry in physics, one can find basic results related to the identification problem including web links and references to literature.

6.7.1 Connection with TGD and physics of cognition

The identification problem is extremely difficult even for mathematicians - to say nothing about humble physicist like me with hopelessly limited mathematical skills. It is however just this problem which I encounter in TGD inspired vision about adelic physics [L53, L52, L46]. Recall that in TGD space-times are 4-surfaces in $H = M^4 \times CP_2$, preferred extremals of the variational principle defining the theory [K85, L64].

1. In this approach p-adic physics for various primes p provide the correlates for cognition: there are several motivations for this vision. Ordinary physics describing sensory experience and the new p-adic physics describing cognition for various primes p are fused to what I called adelic physics. The adelic physics is characterized by extension of rationals inducing extensions of various p-adic number fields. The dimension n of extension characterizes kind of intelligence quotient and evolutionary level since algebraic complexity is the larger, the larger the value of n is. The connection with quantum physics comes from the conjecture that n is essentially effective Planck constant $h_{eff}/h_0 = n$ characterizing a hierarchy of dark matters. The larger the value of n the longer the scale of quantum coherence and the higher the evolutionary level, the more refined the cognition.
2. An essential notion is that of cognitive representation [K71] [L52, L46]. It has several realizations. One of them is the representation as a set of points common to reals and extensions of various p-adic number fields induced by the extension of rationals. These space-time points have points in the extension of rationals considered defining the adele. The coordinates are the embedding space coordinates of a point of the space-time surface. The symmetries of embedding space provide highly unique embedding space coordinates.
3. The gigantic challenge is to find these points common to real number field and extensions of various p-adic number fields appearing in the adele.
4. If this were not enough, one must solve an even tougher problem. In TGD the notion of "world of classical worlds" (WCW) is also a central notion [K85]. It consists of space-time surfaces in embedding space $H = M^4 \times CP_2$, which are so called preferred extremals of the action principle of theory. Quantum physics would reduce to geometrization of WCW and construction of classical spinor fields in WCW and representing basically many-fermion states: only the quantum jump would be genuinely quantal in quantum theory.

   There are good reasons to expect that space-time surfaces are minimal surfaces with 2-D singularities, which are string world sheets - also minimal surfaces [L64, L75]. This gives nice geometrization of gauge theories since minimal surfaces equations are geometric counterparts for massless field equations.

   One must find the algebraic points, the cognitive representation, for all these preferred extremals representing points of WCW (one must have preferred coordinates for H - the symmetries of embedding space crucial for TGD and making it unique, provide the preferred coordinates)!
5. What is so beautiful is that in given cognitive resolution defined by the extension of rationals inducing the discretization of space-time surface, the cognitive representation defines the coordinates of the space-time surfaces as a point of WCW. In finite cognitive and measurement resolution this huge infinite-dimensional space WCW discretizes and the situation can be handled using finite mathematics.

6.7.2 Connection with Kim's work

So: what is then the connection with the work and ideas of Kim. There has been a lot of progress in understanding the problem: here I an only refer to the popular article.

1. One step of progress has been the realization that if one uses the fact that the solutions are common to both reals and various p-adic number fields helps a lot. The reason is that for rational points the rationality implies that the solution of equation representable as infinite

power series of p contains only finite number powers of p. If one manages to prove the this happens for even single prime, a rational solution has been found.

The use of reals and all p-adic numbers fields is nothing but adelic physics. Real surfaces and all its p-adic variants form pages of a book like structure with infinite number of pages. The rational points or points in extension of rationals are the cognitive representation and are points common to all pages in the back of the book.

This generalizes also to algebraic extensions of rationals. Solving the number theoretic problem is in TGD framework nothing but finding the points of the cognitive representation. The surprise for me was that this viewpoint helps in the problem rather than making it more complex.

There are however problematic situations in some cases the hypothesis about finite set of algebraic points need not make sense. A good example is Fermat for $x+y=1$. All rational points and also algebraic points are solutions. For $x^2+y^2=1$ the set of Pythagorean triangles characterizing the solutions is infinite. How to cope with these situations in which one has accidental symmetries as one might say?

2. Kim argues that one can make even further progress by considering the situation from even wider perspective by making the problem even bigger. Introduce what the popular article (see `http://tinyurl.com/y86bckmo`) calls the space of spaces. The space of string world sheets is what string models suggests. WCW is what TGD suggests. One can get a wider perspective of the problem of finding algebraic points of a surface by considering the problem in the space of surfaces and at this level it might be possible to gain much more understanding. The notion of WCW would not mean horrible complication of a horribly complex problem but possible manner to understand the problem!

   The popular article mentioned in the beginning mentions so called Selmer varieties as a possible candidate for the space of spaces. From the Wikipedia article (see `http://tinyurl.com/y27so3f2`) telling about Kim one can find a link to an article [A59] related to Selmer varieties. This article goes over my physicist's head but might give for a more mathematically oriented reader some grasp about what is involved. One can find also a list of publications of Kim (see `http://people.maths.ox.ac.uk/kimm/`.

   Kim also suggests that the spaces of gauge field configurations could provide the spaces of spaces. The list contains an article [A68] with title *Arithmetic Gauge Theory: A Brief Introduction* (see `http://tinyurl.com/y66mphkh`) , which might help physicist to understand the ideas. An arithmetic variant of gauge theory could provide the needed space of spaces.

6.7.3 Can one make Kim's idea about the role of symmetries more concrete in TGD framework?

The crux of the Kim's idea is that somehow symmetries of space of spaces could come in rescue in the attempts to understand the rational points of surface. The notion of WCW suggest in TGD framework rather concrete realization of this idea that I have discussed from the point of view of construction of quantum states.

1. A little bit more of zero energy ontology (ZEO) is needed to follow the argument. In ZEO causal diamonds (CDs) are central. CDs are defined as intersections of future and past directed light-cones with points replaced with CP_2 and forming a scale hierarchy are central. Space-time surfaces are preferred extremals with ends at the opposite boundaries of CD indeed looking like diamond. Symplectic group for the boundaries of causal diamond (CD) is the group of isometries of WCW [K85] [L64]. Maximal isometry group is required to guarantee that the WCW Kähler geometry has Riemann connection - this was discovered for loop spaces by Dan Freed [A44]. Its Lie algebra has structure of Kac- Moody algebra with respect to the light-like radial coordinate of the light-like boundary of CD, which is piece of light-cone boundary. This infinite-D group plays central role in quantum TGD: it acts as maximal group of WCW isometries and zero energy states are invariant under its action at opposite boundaries.
2. As one replaces space-time surface with a cognitive representation associated with an extension of rationals, WCW isometries are replaced with their infinite discrete subgroup acting in the number field define by the extension of rationals defining the adele. These discrete isometries do not leave the cognitive representation invariant but replace with it new one having the same

number of points and one obtains entire orbit of cognitive representations. This is what the emergence of symmetries in wider conceptual framework would mean.

3. One can in fact construct invariants of the symplectic group. Symplectic transformations leave invariant the Kähler magnetic fluxes associated with geodesic polygons with edges identified as geodesic lines of H. There are also higher-D symplectic invariants. The simplest polygons are geodesic triangles. The symplectic fluxes associated with the geodesic triangles define symplectic invariants characterizing the cognitive representation. For the twistor lift one must allow also M^4 to have analog of Kähler form and it would be responsible for CP violation and matter antimatter asymmetry [L41]. Also this defines symplectic invariants so that one obtains them for both M^4 and CP_2 projections and can characterize the cognitive representations in terms of these invariants. Note that the existence of twistor lift fixes the choice of H uniquely since M^4 and CP_2 are the only 4-D spaces allowing twistor space with Kähler structure [A57] necessary for defining the twistor lift of Kähler action.
More complex cognitive representations in an extension containing the given extension are obtained by adding points with coordinates in the larger extension and this gives rise to new geodesic triangles and new invariants. A natural restriction could be that the polynomial defining the extension characterizing the preferred extremal via $M^8 - H$ duality defines the maximal extension involved.

4. Also in this framework one can have accidental symmetries. For instance, M^4 with CP_2 coordinates taken to be constant is a minimal surface, and all rational and algebraic points for given extension belong to the cognitive representation so that they are infinite. Could this has something to do with the fact that we understand M^4 so well and have even identified space-time with Minkowski space! Linear structure would be cognitively easy for the same reason and this could explain why we must linearize.
CP_2 type extremals with light-like M^4 geodesic as M^4 projection is second example of accidental symmetries. The number of rational or algebraic points with rational M^4 coordinates at light-like curve is infinite - the situation is very similar to $x + y = 1$ for Fermat. Simplest cosmic strings are geodesic sub-manifolds, that is products of plane $M^2 \subset M^4$ and CP_2 geodesic sphere. Also they have exceptional symmetries.
What is interesting from the point of view of proposed model of cognition is that these cognitively easy objects play a central role in TGD: their deformations represent more complex dynamical situations. For instance, replacing planar string with string world sheet replaces cognitive representation with a discrete or perhaps even finite one in M^4 degrees of freedom.

5. A further TGD based simplification would be $M^8 - H$ ($H = M^4 \times CP_2$) duality in which space-time surfaces at the level of M^8 are algebraic surfaces, which are mapped to surfaces in H identified as preferred extremals of action principle by the $M^8 - H$ duality [L46]. Algebraic surfaces satisfying algebraic equations are very simple as compared to preferred extremals satisfying partial differential equations but "preferred" is what makes possible the duality. This huge simplification of the solution space of field equations guarantees holography necessitated by general coordinate invariance implying that space-time surfaces are analogous to Bohr orbits. It would also guarantee the huge symmetries of WCW making it possible to have Kähler geometry.
This suggests in TGD framework that one finds the cognitive representation at the level of M^8 using methods of algebraic geometry and maps the points to H by using the $M^8 - H$ duality. TGD and octonionic variant of algebraic geometry would meet each other.
It must be made clear that now solutions are not points but 4-D surfaces and this probably means also that points in extension of rationals are replaced with surfaces with embedding space coordinates defining function in extensions of rational functions rather than rationals. This would bring in algebraic functions. This might provide also a simplification by providing a more general perspective. Also octonionic analyticity is extremely powerful constraint that might help.

6.8 Cognitive representations for partonic 2-surfaces, string world sheets, and string like objects

Cognitive representations are identified as points of space-time surface $X^4 \subset M^4 \times CP_2$ having embedding space coordinates in the extension of of rationals defined by the polynomial defined by the M^8 pre-image of X^4 under $M^8 - H$ correspondence [L47, L48, L84, L76, L74, L68]. Cognitive representations have become key piece in the formulation of scattering amplitudes [L78] . One might argue that number theoretic evolution as increase of the dimension of the extension of rationals favors space-time surfaces with especially large cognitive representations since the larger the number of points in the representation is, the more faithful the representation is.

One can pose several questions if one accepts the idea that space-time surfaces with large cognitive representations are survivors.

1. Preferred p-adic primes are proposed to correspond to the ramified primes of the extension [L86]. The proposal is that the p-adic counterparts of space-time surfaces are identifiable as imaginations whereas real space-time surfaces correspond to realities. p-Adic space-time surfaces would have the embedding space points in extension of rationals as common with real surfaces and large number of these points would make the representation realistic. Note that the number of points in extension does not depend on p-adic prime.
Could some extensions have an especially high number of points in the cognitive representation so that the corresponding ramified primes could be seen as survivors in number theoretical fight for survival, so to say? Galois group of the extension acts on cognitive representation. Galois extension of an extension has the Galois group of the original extension as normal subgroup so that ormal Galois group is analogous to a conserved gene.
2. Also the type of extremal matters. For instance, for instance canonically imbedded M^4 and CP_2 contain all points of extension. These surfaces correspond to the vanishing of real or imaginary part (in quaternionic sense) for a linear octonionic polynomial $P(o) = o$! As a matter of fact, this is true for all known preferred extremals under rather mild additional conditions. Boundary conditions posed at both ends of CD in ZEO exclude these surfaces and the actual space-time surfaces are expected to be their deformations.
3. Could the surfaces for which the number of points in cognitive representation is high, be the ones most easily discovered by mathematical mind? The experience with TGD supports positive answer: in TGD the known extremals [K10] are examples of such mathematical objects! If so, one should try to identify mathematical objects with high symmetries and look whether they allow TGD realization.
4. One must also specify more precisely what cognitive representation means. Strong form of holography (SH) states that the information gives at 2-D surfaces - string world sheets and partonic 2-surfaces - is enough to determine the space-time surfaces. This suggests that it is enough to consider cognitive representation restricted to these 2-surfaces. What kind of 2-surfaces are the cognitively fittest one? It would not be surprising if surfaces with large symmetries acting in extension were favored and elliptic curves with discrete 2-D translation group indeed turn out to be assigable string world sheets as singularities and string like objects. In the case of partonic 2-surfaces geodesic sphere of CP_2 is similar object.

All known extremals, in particular preferred extremals, are good candidates in this respect because of their high symmetries. By strong form of holography (SH) partonic 2-surfaces and string world sheets are expected to give rise to cognitive representations. Also cosmic strings are expected to carry them. Under what conditions these representations are large?

6.8.1 Partonic 2-surfaces as seats of cognitive representations

One can start from SH and look the situation more concretely. The situation for partonic 2-surfaces has been considered already earlier [L85, L73] but deserves a separate discussion.

1. Octonionic polynomials allow special solutions for which the entire polynomial vanishes. This happens at 6-sphere S^6 at the boundary of 8-D light-cone. S^6 is analogous to brane and has radius $R = r_n$, which is a root of the real polynomial with rational coefficients algebraically continued to the octonionic polynomial.

S^6 has the ball B^3 of radius r_n of the light-cone M^4_+ with time coordinate $t = r_n$ as analog of base space and sphere S^3 of E^4 with radius $R = \sqrt{r_n^2 - r^2}$, r the radial coordinate of B^3 as an analog of fiber. The analog of the fiber contracts to a point at the boundary of the light-cone. The points with B^3 projection and E^4 coordinates in extension of rationals belong to the cognitive representation. The condition that $R^2 = x_i x^i = r_n^2 - r^2$ is square of a number of extension is rather mild and allows infinite number of solutions.

2. The 4-D space-time surfaces X^4 are obtained as generic solutions of $Im(P(o)) = 0$ or $Re(P(o)) = 0$. Their intersection with S^6 - partonic 2-surface X^2 - is 2-D. The assumption is that the incoming and outgoing 4-D space-time surfaces representing orbits of particles in topological sense are glued together at X^2 and possibly also in their interiors. X^2 serves as an analog of vertex for 3-D particles. This gives rise to topological analogs of Feynman diagrams.
   In the generic case the number of points in cognitive representation restricted to X^2 is finite unless the partonic 2-surface X^2 is special - say correspond to a geodesic spere of S^6.
3. The discrete isometries and conformal symmetries of the cognitive representation restricted to X^2 possibly represented as elements of Galois group might play a role. For $X^2 = S^2$ the finite discrete subgroups of $SO(3)$ giving rise to finite tessellations and appearing in ADE correspondence might be relevant. For genera $g = 01, 2$ conformal symmetry Z_2 is always possible but for higher genera only in the case of hyper-elliptic surfaces- this used to explain why only $g = 0, 1, 2$ correspond to observed particles [K28] whereas higher genera could be regarded as many-particle states of handles having continuous mass spectrum. Torus is an exceptional case and one can ask whether discrete subgroup of its isometries could be realized.
4. In TGD inspired theory of consciousness [L54, L73] the moments $t = r_n$ corresponds to "very special moments in the life of self". They would be also cognitively very special - kind of eureka moments with a very large number of points in cognitive representation. The question is whether these surfaces might be relevant for understanding the nature of mathematical consciousness and how the mathematical notions emerge at space-time level.

6.8.2 Ellipticity

Surfaces with discrete translational symmetries is a natural candidate for a surface with very large cognitive representation. Are their analogs possible? The notions of elliptic function, curve, and surface suggest themselves as a starting point.

1. Elliptic functions (`http://tinyurl.com/gpugcnh`) have 2-D discrete group of translations as symmetries and are therefore doubly periodic and thus identifiable as functions on torus.
   Weierstrass elliptic functions $\mathcal{P}(z; \omega_1, \omega_2)$ (`http://tinyurl.com/ycu8oa4r`) are defined on torus and labelled by the conformal equivalence class $\lambda = \omega_1/\omega_2$ of torus identified as the ratio $\lambda = \omega_1/\omega_2$ of the complex numbers ω_i defining the periodicities of the lattice involved. Functions $\mathcal{P}(z; \omega_1, \omega_2)$ are of special interest as far as elliptic curves are considered and defines an embedding of elliptic curve to CP_2 as will be found.
   If the periods are in extension of rationals then values in the extension appear infinitely many times. Elliptic functions are not polynomials. Although the polynomials giving rise to octonionic polynomials could be replaced by analytic functions it seems that elliptic functions are not the case of primary interest. Note however that the roots r_n could be also complex and could correspond to values of elliptic function forming a lattice.
2. Elliptic curves (`http://tinyurl.com/lovksny`) are defined by the polynomial equation

$$y^2 = P(x) = x^3 + ax + b \ . \tag{6.8.1}$$

   An algebraic curve of genus 1 allowing 2-D discrete translations as symmetries is in question. If a point of elliptic curve has coordinates in extension of rationals then 2-D discrete translation acting in extension give rise to infinite number of points in the cognitive representation. Clearly, the 2-D vectors spanning the lattice defining the group must be in extension of rationals.

One can indeed define commutative sum $P + Q$ for the points of the elliptic curve. The detailed definition of the group law and its geometric illustration can be found in Wikipedia article (`http://tinyurl.com/lovksny`).

1. Consider real case for simplicity so that elliptic curve is planar curve. $y^2 = P(x) = x^3 + ax + b$ must be non-negative to guarantee that y is real. $P(x) \geq 0$ defines a curve in upper (x, y)

plane extending from some negative value x_{min} corresponding to $y^2 = P(x_{min}) = 0$ to the right. Given value of y can correspond to 3 real roots or 1 real root of $P_y(x) = y^2 - P(x)$. At the two extrema of $P_y(x)$ 2 real roots co-incide. The graph of $y = \pm\sqrt{P(x)}$ is reflection symmetric having two branches beginning from $(x_{min}, y = 0)$.

2. The negative $-P$ is obtained by reflection with respect to x-axis taking y_P to $-y_P$. Neutral element O is identified as point a infinity (assuming compactification of the plane to a sphere) which goes to itself under reflection $y \rightarrow -y$.
3. One assigns to the points P and Q of the elliptic curve a line $y = sx+d$ containing them so that one has $s = (y_p - y_Q)/(x_P - x_Q)$. In the generic case the line intersects the elliptic curve also at third point R since $P_{y=sx+d}(x)$ is third order polynomial having three roots (x_P, x_Q, x_R). It can happen that 2 roots are complex and one has 1 real root. At criticalityfor the transiton from 3 to 1 real roots one has $x_Q = x_R$.
   Geometrically one can distinguish between 4 cases.
   - The roots P, Q, R of $P_{y=sx+d}(x)$ are different and finite: one defines the sum as $P + Q = -R$.
   - $P \neq Q$ and $Q = R$ (roots Q and R are degenerate): $P + Q + Q = O$ giving $R = -P/2$.
   - P and Q are at a line parallel to y-axis and one has $R = O$: $P + Q + O = O$ and $P = -Q$.
   - P is double root of $P_{y=sx+d}(x)$ with tangent parallel to y-axis at the point $(x_{min}, y = 0)$ at which the elliptic curve begins so that one has $R = O$: $P + P + O = O$ gives $P = -P$. This corresponds to torsion.
4. Elliptic surfaces (see `http://tinyurl.com/yc33a6dg`) define a generalization of elliptic curves and are defined for 4-D complex manifolds. Fiber is required to be smooth and has genus 1.

6.8.3 String world sheets and elliptic curves

In twistor lift of TGD space-time surfaces identifiable as minimal surfaces with singularities, which are string world sheets and partonic 2-surfaces. Preferred extremal property means that space-time surfaces are extremals of both Kähler action and volume action except at singularities.

Are string world sheets with very large number of points in cognitive representation possible? One has right to expect that string world sheets allow special kind of symmetries allowing large, even infinite number of points at the limit of large sheet and related by symmetries acting in the extension of rationals. If one of the points is in the extension, also other symmetry related points are in the extension. For a non-compact group, say translation one would have infinite number of points in the representation but the finite size of CD would pose a limitation to the number of points.

String world sheets are good candidates for the realization of elliptic curves.

1. The general conjecture is that preferred extremals allow what I call Hamilton-Jacobi structure for M^4 [K85]. The distribution of tangent spaces having decomposition $M^4(x) = M^2(x) \times E^2(x)$ would be integrable giving rise to a family of string world sheets Y^2 and partonic 2-surfaces X^2 more general than those defined above. X^2 and Y^2 are orthogonal to each other at each point of X^4. One can introduce local light-cone coordinates (u, v) for Y^2 and local E^2 complex coordinate w for X^2.
2. Space-time surface itself would be a deformation of M^4 with Hamilton-Jacobi structure in CP_2 direction. w coordinate as function $w(z)$ of CP_2 complex coordinate z or vice versa would define the string world sheet. This would be a transversal deformation of the basic string world sheet Y^2: stringy dynamics is indeed transversal.
3. The idea about maximal cognitive representation suggests that $w \leftrightarrow z$ correspondence defines elliptic curve. One would have $y^2 = P(x) = x^3 + ax + b$ with either $(y = w, x = z)$ or $(y = z, x = w)$. A natural conjecture is that for the space-time surface corresponding to a given extension K of rationals the coefficients a an b belong to K so that the algebraic complexity of string world sheet would increase in number theoretic evolution [L83]. The orbit of a algebraic point at string world sheet would be lattice made finite by the size of CD. Elliptic curves would define very special deformed string world sheets in space-time.
4. It is interesting to consider the pre-image of given point y ($y = w$ or $y = z$) covering point x. One has $y = \pm\sqrt{u}$, $u = P(x)$ corresponding to group element and its negative: there are

two points of covering given value of u. $u = P(x)$ covers 3 values of x. The values of x would belong to 6-fold covering of rationals. The number theoretic interpretation for the effective Planck constant $h_{eff} = nh_0$ states that n is the number of sheets for space-time surface as covering.
There is evidence that $h_{eff} = h$ corresponds to $n = 6$ [L31]. Could 6-fold covering of rationals be fundamental since it gives very large cognitive representation at the level of string world sheets?
For extensions K of rationals the x coordinates for the points of cognitive representation would belong to 6-D extension of K.

5. Ellipticity condition would apply on the string world sheets themselves. In the number theoretic vision string world sheets would correspond at M^8 level to singularities at which the quaternionic tangent space degenerates to 2-D complex space. Are these conditions consistent with each other? It would seem that the two conditions would select cognitively very special string world sheets and partonic 2-surfaces defining by strong form of holography (SH) space-time surface as a hologram in SH. Consciousness theorist interested in mathematical cognition might ask whether the notion of elliptic surfaces have been discovered just because it is cognitively very special. In the case of partonic 2-surfaces geodesic sphere of CP_2 is similar object.

6.8.4 String like objects and elliptic curves

String like objects - cosmic strings - and their deformations, are fundamental entities in TGD based cosmology and astrophysics and also in TGD inspired quantum biology. One can assign elliptic curves also to string like objects.

1. Quite generally, the products $X^2 \times Y^2 \subset M^4$ of string world sheets X^2 and complex surfaces Y^2 of CP_2 define extremals that I have called cosmic strings [K10].
2. Elliptic curves allow a standard embedding to CP_2 as complex surfaces constructible in terms of Weierstrass elliptic function $\mathcal{P}(z)$ (`http://tinyurl.com/ycu8oa4r`) satisfying the identity

$$[\mathcal{P}'(z)]^2 = [\mathcal{P}(z)]^3 - g_2\mathcal{P}(z) - g_3 \quad . \tag{6.8.2}$$

Here g_2 and g_3 are modular invariants. This identity is of the same form as the condition $y^2 = x^3 + ax + b$ with identifications $y = \mathcal{P}'(z), x = \mathcal{P}(z)$ and $(a = -g_2, b = -g_3)$. From the expression

$$y^2 = x(x-1)(x-\lambda) \tag{6.8.3}$$

in terms of the modular invariant $\lambda = \omega_1/\omega_2$ of torus one obtains

$$g_2 = \frac{4^{1/3}}{3}(\lambda^2 - \lambda + 1 \ , \quad g_3 = \frac{1}{27}(\lambda+1)(2\lambda^2 - 5\lambda + 2) \quad . \tag{6.8.4}$$

Note that third root of a appears in the formula. The so called modular discriminant

$$\Delta = g_2^3 - 27g_3^2 = \lambda^2(\lambda-1)^2 \quad . \tag{6.8.5}$$

vanishes for $\lambda = 0$ and $\lambda = 1$ for which the lattice degenerates.

3. The embedding of the elliptic curve to CP_2 can be expressed in projective coordinates of CP_2 as

$$(z^1, z^2, z^3) = (\xi^1, \xi^2, 1) = (\frac{\mathcal{P}'(w)}{2}, \mathcal{P}(w), 1) \quad . \tag{6.8.6}$$

6.9 Are fundamental entities discrete or continuous and what discretization at fundamental level could mean?

There was an interesting FB discussion about discrete and continuum. I decided to write down my thoughts and emphasize those points that I see as important.

6.9.1 Is discretization fundamental or not?

The conversation inspired the question whether discreteness is something fundamental or not. If it is assumed to be fundamental, one encounters problems. The discrete structures are not unique. One has deep problem with the known space-time symmetries. Symmetries are reduced to discrete subgroup or totally lost. A further problem is the fact that in order to do physics, one must bring in topology and length measurements.

In discrete situation topology, in particular space-time dimension, must be put in via homology effectively already meaning use of embedding to Euclidian space. Length measurement remains completely ad hoc. The construction of discrete metric is highly non-unique procedure and the discrete analog of of say Einstein's theory (Regge calculus) is rather clumsy. One feeds in information, which was not there by using hand weaving arguments like infrared limit. It is possible to approximate continuum by discretization but discrete to continuum won't go.

In hype physics these hand weaving arguments are general. For instance, the emergence of 3-space from discrete Hilbert space is one attempt to get continuum. One puts in what is factually a discretization of 3-space and then gets 3-space back at IR limit and shouts "Eureka!".

6.9.2 Can one make discretizations unique?

Then discussion went to numerics. Numerics is for mathematicians same as eating for poets. One cannot avoid it but luckily you can find people doing the necessary programming if you are a professor. Finite discretization is necessary in numerics and is highly unique.

I do not have anything personal against discretization as a numerical tool. Just the opposite, I see finite discretization as absolutely essential element of adelic physics as an attempt to describe also the correlates of cognition in terms of p-adic physics with p-adic space-time sheets as correlates of "thought bubbles" [L52, L53]. Cognition is discrete and finite and uses rational numbers: this is the basic clue.

1. Cognitive representations are discretizations of (for instance) space-time surface. One can say that physics itself builds its cognitive representation in all scales using p-adic space-time sheets. They should be unique once measurement resolution is characterized if one is really talking about fundamental physics.
   The idea abou tp-adic physics as physics of cognition indeed led to powerful calculational recipes. In p-adic thermodynamics the predictions come in power series of p-adic prime p and for the values of p assignable to elementary particles the two lowest terms give practically exact result [K60]. Corrections are of order 10^{-76} for electron characterized by Mersenne prime $M_{127} = 2^{127} - 1 \sim 10^{38}$.
2. Adelic physics [L52] provides the formulation of p-adic physics: it is assumed that cognition is universal. Adele is a book like structure having as pages reals and extensions of various p-adic number fields induced by given extension of rationals. Each extension of rationals defines its own extension of the rational adele by inducing extensions of p-adic number fields. Common points between pages consist of points in extension of rationals. The books associated with the adeles give rise to an infinite library.
   At space-time level the points with coordinates in extension define what I call cognitive representation. In the generic case it is discrete and has finite number of points. The loss of general coordinate invariance is the obvious objection. In TGD however the symmetries of the embedding space fix the coordinates used highly uniquely. $M^8 - H$ duality ($H = M^4 \times CP_2$) and octonionic interpretation implies that M^8 octonionic linear coordinates are highly unique [L46, L76]. Note that M^8 must be complexified. Different coordinatizations correspond to different octonionic structures- to different moduli - related by Poincare transformations of M^8. Only rational time translations as transformations of octonionic real coordinate are allowed as coordinate changes respecting octonionic structure.
3. Discretization by cognitive representation is unique for given extension of rationals defining the measurement resolution. At the limit of algebraic numbers algebraic points form a dense set of real space-time surface and p-adic space-time surfaces so that the measurement resolution is ideal. One avoids the usual infinities of quantum field theories induced by continuous delta functions, which for cognitive representations are replaced with Kronecker deltas. This

seems to be the best that one can achieve with algebraic extensions of rationals. Also for transcendental extensions the situation is discrete.
This leads to a number theoretic vision about second quantization of induced spinor fields central for the construction of gamma matrices defining the spinor structure of "world of classical worlds" (WCW) providing the arena of quantum dynamics in TGD analogous to the super-space of Wheeler [K85]. One ends up to a construction allowing to understand TGD view about SUSY as necessary aspect of second quantization of fermions and leads to the conclusions that in the simplest scenario only quarks are elementary fermions and leptons can be seen as their local composites analogous to super partners.

4. Given polynomial defining space-time surfaces in M^8 defines via its roots extension of rationals. The hierarchy of extensions defines an evolutionary hierarchy. The dimension n of extension defines kind of IQ measuring algebraic complexity and n corresponds also to effective Planck constant labelling phases of dark matter in TGD sense so that a direct connection with physics emerges.
Embedding space assigns to a discretization a natural metric. Distances between points of metric are geodesic distances computed at the level of embedding space.

5. An unexpected finding was that the equations defining space-time surfaces as roots of real or imaginary parts of octonionic polynomials have also 6-D brane like entities with topology of S^6 as solutions [L73, L84]. These entities intersect space-time surfaces at 3-D sections for which linear M^4 time is constant. 4-D roots can be glued together along these branes. These solutions turn out to have an interpretation in TGD based theory of quantum measurement extending to a theory of consciousness. The interpretation as moments of "small" state function reductions as counterparts of so called weak measurements. They could correspond to special moments in the life of conscious entity.

6.9.3 Can discretization be performed without lattices?

For a systems obeying dynamics defined by partial differential equations, the introduction of lattices seems to be necessary aspect of discretization. The problem is that the replacement of derivatives with discrete approximations however means that there is no hope about exact results. In the general case the discretization for partial differential equations involving derivatives forces to introduce lattice like structures. This is not needed in TGD.

1. At the level of M^8 ordinary polynomials give rise to octonionic polynomials and space-time surfaces are algebraic surfaces for which imaginary or real part of octonionic polynomial in quaternionic sense vanishes. The equations are purely algebraic involving no partial derivatives and there is no need for lattice discretization.
For surfaces defined by polynomials the roots of polynomial are enough to fix the polynomials and therefore also the space-time surface uniquely: discretization is not an approximation but gives an exact result! This could be called number theoretical holography and generalizes the ordinary holography. Space-time surfaces are coded by the roots of polynomials with rational coefficients.

2. What about the field equations at the level of $H = M^4 \times CP_2$? $M^8 - H$ duality maps these surfaces to preferred extremals as 4-surfaces in H analogous to Bohr orbits. Twistor lift of TGD predicts that they should be minimal surfaces with 2-D singularities being also extremals of 4-D Kähler action. The field equations would reduce locally to purely algebraic conditions. In properly chosen coordinates for H they are expected to be determined in terms of polynomials coding for the same extension of rationals as their M^8 counterparts so that the degree should be same [L76]. This would allow to deduce the partial derivatives of embedding space for the image surfaces without lattice approximation.

3. The simplest assumption is that the polynomials have rational coefficients. Number theoretic universality allows to consider also algebraic coefficients. In both cases also WCW is discretized and given point -space-time surface in QCD has coordinates given by the points of the number theoretically universal cognitive representation of the space-time surface. Even real coefficients are possible. This would allow to obtain WCW as a continuum central for the construction of WCW metric but is not consistent with number theoretical universality.

Can one have polynomial/functions with rational coefficients and discretization of WCW without lattice but without losing WCW metric? Maybe the same trick that works at space-time level works also in WCW!

(a) The group WCW isometries is identified as symplectic transformations of $\delta M^4_\pm \times CP_2$ ($\delta M^4_\pm$ denotes light-cone boundary) containing the boundary of causal diamond CD. The Lie algebra *Sympl* of this group is analogous half-Kac Moody algebra having symplectic transformations of $S^2 \times CP_2$ as counterpart of finite-D Lie group has fractal structure containing infinite number of sub-algebras $Sympl_n$ isomorphic to algebra itself: the conformal weights assignable to radial light-like coordinate are n-multiples of those for the entire algebra. Note that conformal weights of *Sympl* are non-negative.

(b) One formulation for the preferred extremal property is in terms of infinite number of analogs of gauge conditions stating the vanishing of classical and also Noether charges for $Sympl_n$ and $[Sympl_n, Sympl]$. The conditions generalize to the super-counterpart of *Sympl* and apply also to quantum states rather than only space-time surfaces. In fact, while writing this I realized that - contrary to the original claim - also the vanishing of the Noether charges of higher commutators is required so that effectively $Sympl_n$ would define normal subgroup of *Sympl*. These conditions does not follow automatically.

The Hamiltonians of $Sympl(S^2 \times CP_2)$ are also labelled by the representations of the product of the rotation group $SO(3) \subset SO(3,1)$ of S^2 and color group $SU(3)$ together forming the analog of the Lie group defining Kac-Moody group. This group does not have have the fractal hierarchy of subgroups. The strongest condition is that the algebra corresponding to Hamiltonian isometries does not annihilate the physical states.

The space of states satisfying the gauge conditions is finite-D and that WCW becomes effectively finite-dimensional. A coset space associated with *Sympl* would be in question and it would have maximal symmetries as also WCW. The geometry of the reduced WCW, WCW_{red} could be deduced from symmetry considerations alone.

(c) Number theoretic discretization would correspond to a selection of points of this subspace with the coordinates in the extension of rationals.The metric of $\mathrm{WCW}_{red,n}$ at the points of discretization would be known and no lattice discretization would be needed. The gauge conditions are analogous to massless Dirac equation in WCW and could be solved in the points of discretization without introducing the lattice to approximate derivatives. As a matter fact, Dirac equation can be formulated solely in terms of the generators of *Sympl*.

(d) This effectively restricts WCW to $WCW_{red,n}$ in turn reduced to its discrete subset - since infinite number of WCW coordinates are fixed. If this sub-space can be regarded as realization of infinite number of algebraic conditions by polynomials with rational coefficients one can assign to it extension of rationals defining naturally the discretization of $\mathrm{WCW}_{red,n}$. This extension is naturally the same as for space-time surfaces involved so that the degree of polynomials defining $\mathrm{WCW}_{red,n}$ would be naturally n and same as that for the polynomial defining the space-time surface. $\mathrm{WCW}_{red,n}$ would decompose to union of spaces WCW_{red,E_n} labelled by extensions E_n of rationals with same dimension n.

There is analogy with gauge fixing. WCW_{red,E_n} is a coset space of WCW defined by the gauge conditions. One can represent this coset space as a sub-manifold of WCW by taking one representative point from each coset. This choice is not unique but one can hope finding a gauge choice realized by an infinite number of polynomials of degree n defining same extension of rationals as the polynomial defining the space-time surfaces in question.

(e) WCW spinor fields would be always restricted to finite-D algebraic surface of WCW_{red,E_n} expressible in terms of algebraic equations. Finite measurement resolution indeed strongly suggests that WCW spinor field mode is non-vanishing only in a region parameterized in WCW by finite number of parameters. There is also a second manner to see this. WCW_{red,E_n} could be also seen as $n+4$-dimensional surface in WCW.

(f) One can make this more concrete. Cognitive representation by points of space-time surface with coordinates in the extension - possibly satisfying additional conditions such as belonging to the 2-D vertices at which space-time surfaces representing different roots meet - provides WCW coordinates of given space-time surface. Minimum number of points corresponds to the dimension of extension so that the selection of coordinate can be redundant. As the values of these coordinates vary, one obtains coordinatization for the

sector of WCW_{red,E_n}. An interesting question is whether one could represent the distances of space-time surfaces in this space in terms of the data provided by the points of discretization.
An interesting question is whether one can represent the distances of space-time surfaces in this space in terms of the data provided by the points of cognitive representation. One can define distance between two disjoint surfaces as the minimum of distance between the points of 2-surfaces. Could something like this work now? The points would be restricted to the cognitive representations. Could one define the distance between two cognitive representations with same number N of points in the following manner.
Consider all bipartitions formed by the cognitive representations obtained by connecting their points together in 1-1 manner. There are N! bipartitions of this kind if the number of points is N. Calculate the sum of the squares of the embedding space distances between paired points. Find the bipartition for which this distance squared is minimum and define the distance between cognitive representations as this distance. This definition works also when the numbers of points are different.

(g) If there quantum states are the basic objects and there is nothing "physical" behind them one can ask how we can imagine mathematical structures which different from basic structure of TGD. Could quantum states of TGD Universe in some sense represent all mathematical objects which are internally consistent. One could indeed say that at the level of WCW all $n+4$-D manifolds can be represented concretely in terms of WCW spinor fields localized to n-D subspaces of WCW. WCW spinor fields can represent concept of 4-surface of $\text{WCW}_{red,n}$ as a quantum superposition of its instance and define at the same time $n+4$-D surfaces [L87] [L75, L79, L78, L87].

6.9.4 Simple extensions of rationals as codons of space-time genetic code

A fascinating idea is that extensions of rationals define the analog of genetic code for space-time surfaces, which would therefore represent number theory and also finite groups.

(a) The extensions of rationals define an infinite hierarchy: the proposal is that the dimension of extensions corresponds to the integer n characterizing subalgebra $Sympl_n$. This would give direct correspondence between the inclusions of HFFs assigned to the hierarchy of algebras $Sympl_n$ and hierarchy of extensions of rationals with dimension n.
Galois group for a extension of extension contains Galois group of extension as normal subgroup and is therefore *not simple*. Extension hierarchies correspond to inclusion hierarchies for normal subgroups. Simple Galois groups are in very special position and associated with what one might call simple extensions serving as fundamental building bricks of inclusion hierarchies. They would be like elementary particles and define fundamental space-time regions. Their Galois groups would act as groups of physical symmetries.

(b) One can therefore talk about elementary space-time surfaces in M^8 and their compositions by function composition of octonionic polynomials. Simple groups would label elementary space-time regions. They have been classified: (see `http://tinyurl.com/y3xh4hrh`). The famous Monster groups are well-known examples about simple finite groups and would have also space-time counterparts. Also the finite subgroups of Lie groups are special and those of $SU(2)$ are associated with Platonic solids and seem to play key role in TGD inspired quantum biology. In particular, vertebrate genetic code can be assigned to icosahedral group.

(c) There is also an analogy with genes. Extensions with simple Galois groups could be seen as codons and sequences of extension obtained by functional composition as analogs of genes. I have even conjectured that the space-time surfaces associated with genes could quite concretely correspond to extensions of extensions of ...

6.9.5 Are octonionic polynomials enough or are also analytic functions needed?

I already touched the question whether also analytic functions with rational coefficients (number theoretical universality) might be needed.

(a) The roots of analytic functions generate extension of rationals. If the roots involve transcendental numbers they define infinite extensions of rationals. Neper number e is very special in this sense since e^p is ordinary p-adic number for all primes p so that the induced extension is finite-dimensional. One could thus allow it without losing number theoretical universality. The addition of π gives infinite-D extension but one could do by adding only roots of unity to achieve finite-D extensions with finite accuracy of phase measurement. Phases would be number theoretically universal but not angles.

(b) One could of course consider only transcendental functions with rational roots. Trigonometric function $sin(x/2\pi)$ serves as a simple example. One can also argue that since physics involves in an essential manner trigonometric functions via Fourier analysis, the inclusion of analytic functions with algebraic roots must be allowed.

(c) What about analytic functions as limits of polynomials with rational coefficients such that the number of roots becomes infinite at the limit? Also their imaginary and real part can vanish in quaternionic sense and could define space-time surfaces - analogs of transcendentals as space-time surfaces. It is not clear whether these could be allowed or not.

Could one have a universal polynomial like function giving algebraic numbers as the extension of rationals defined by its algebraic roots? Could Riemann zeta (see `http://tinyurl.com/nfbkrsx`) code algebraic numbers as an extension via its roots. I have conjectured that roots of Riemann zeta are algebraic numbers: could they span all algebraic numbers?

It is known that the real or imaginary part of Riemann zeta along $s = 1/2$ critical line can approximate any function to arbitrary accuracy: also this would fit with universality. Could one think that the space-time surface defined as root of octonionic continuation of zeta could be universal entity analogous to a fixed point of iteration in the construction of fractals? This does not look plausible.

4. One can construct iterates of Riemann zeta having at least the same roots as zeta by the rule

$$\begin{aligned} &f_0(s) = \zeta(s) \ , \\ &f_n(s) = \zeta(f_{n-1}(s)) - \zeta(0), \ \ \zeta(0) = -1/2 \ . \end{aligned} \tag{6.9.1}$$

ζ is not a fixed point of this iteration as the fractal universality would suggest. The set of roots however is. Should one be happy with this.

5. Riemann zeta has also counterpart in all extensions of rationals known as Dedekind zeta (see `http://tinyurl.com/y5grktv`) [L50, L86, L77]. Riemann zeta is therefore not unique. One can ask whether Dedekind zetas associated with simple Galois groups are special and whether Dedekind zetas associated with extensions of extensions of .... can be constructed by using the Dedekind zetas of simple extensions. How do the roots of Dedekind zeta depend on the associated extension of rationals? How the roots of Dedekind zeta for extension of extension defined by composite of two polynomials depend on extensions involved? Are the roots union for the roots associated with the composites?

6. What about forming composites of Dedekind zetas? Categorical according to my primitive understanding raises the question whether a composition of extensions could correspond to a composition of functions. Could Dedekind zeta for a composite of extensions be obtained from a composite of Dedekind zetas for extensions? Requiring that roots of extension E_1 are preserved would give formula

$$\zeta_{D,E_1E_2} = \zeta_{D,E_1} \circ \zeta_{D,E_2} - \zeta_{D,E_1}(0) \ . \tag{6.9.2}$$

The zeta function would be obtained by an iteration of simple zeta functions labelled by simple extensions. The inverse image for the set of roots of ζ_{D,E_1} under ζ_{D,E_2} that is the set $\zeta^{-1}_{D,E_2}(roots(\zeta_{D,E_1})$ would define also roots of ζ_{D,E_1E_2}. This looks rather sensible.

But what about iteration of Riemann zeta, which corresponds to trivial extension? Riemann ζ is not invariant under iteration although its roots are. Should one accept this and say that it is the set of roots which defines the invariant. Could one say that the iterates of various Dedekind zetas define entities which are somehow universal.

Chapter 7

Could quantum randomness have something to do with classical chaos?

7.1 Introduction

There was an interesting guest post by Tim Palmer in the blog of Sabine Hosssenfelder (http://tinyurl.com/yx7htn3u).

7.1.1 Palmer's idea

Consider first what was said in the post "Undecidability, Uncomputability and the Unity of Physics. Part 1" by Tim Palmer.

1. I understood (perhaps mis-) that the idea is to reduce quantum randomness to classical chaos. If this is taken to mean that quantum theory reduces to chaos theory, I will not follow. The precise rules of quantum measurement having interpretation as measurements performed for the observables - typically generators of symmetries - are very restrictive and it is extremely difficult to image that classical physics could explain them. Quantum theory is much more than probability theory. Probabilities are essentially moduli squared for probability amplitudes and this gives rise to interference and entanglement. Therefore the idea of reducing state function reduction (SFR) and quantum randomness to classical chaos does not look promising. One could however consider the possibility classical chaos is in some sense as a correlate for quantum randomness or associated with state function reductions.
2. The difficulty to combine general relativity (GRT) to quantum gravity was mentioned. The difficulty is basically due to the loss of Poincare symmetries in curved space-time. Already string models solve the problem by assuming that strings live in M^{10} or its spontaneous compactification. Strings are however 2-D, not 4-D, and this leads to a catastrophe. In TGD $H = M^4 \times CP_2$ allows to have Poincare invariance and conservation laws are not lost. In QFT picture this means that the existence of energy guarantees existence of Hamiltonian defining time evolution operator and S-matrix.
3. It was noticed that chaos in quantum theory cannot be assigned to Schrödinger equation. This is true and applies quite generally to unitary time evolution generated by unitary S-matrix acting linearly. It as also noticed that in statistical mechanism Liouville operator defines a linear equation for phase space probability distribution analogous to Schrödinger equation. Liouville equation allows the classical system to be non-linear and chaotic. Could Schrödinger equation in some sense replace Liouville equation in in quantum theory since phase space ceases to make sense by Uncertainty Principle.
   Could Schrödinger equation allow in some sense non-linear chaotic classical systems? In Copenhagen interpretation no classical system exists except at macroscopic limit as an approximation. One has only wave function coding for the knowledge about physical system changing in

quantum measurement. There is no classical reality and there are no classical orbits of particle since one gives up the notion of Bohr orbit. Could Bohr orbit be more than approximation?

The author considers also the question about definition of chaos.

1. Chaos is difficult to define in GRT. The replacement time coordinate with its logarithm exponentially growing difference becomes linearly growing and one does not have chaos. By general coordinate invariance this definition of chaos does not therefore make sense.
2. Strange attractors are typical asymptotic situations in chaotic systems and can make sense also in general relativity (GRT). They represent lower dimensional manifolds to which the dynamics of the system is restricted asymptotically. It is not possible to predict to which strange attractor the chaotic dynamical system ends up. This definition of chaos makes sense also in GRT.

Remark: One must remember that the notion of chaos is often used in misleading sense. The increase of complexity looks like chaos for external observer but need not have anything to do with genuine chaos.

7.1.2 Could TGD allow realization of Palmer's idea in some form?

It came as a surprise to me that these to notions could a have deep relationship in TGD framework.

1. Strong form of Palmer's idea stating that quantum randomness reduces to classical chaos certainly fails but one can consider weaker forms of the idea. Even these variants fail in Copenhagen interpretation since strictly speaking there is no classical reality, only wave function coding for the knowledge about the system. Bohr orbits should be more than approximation and in TGD framework space-time surface as preferred extremal of action is analogous to Bohr orbit and classical physics defined by Bohr orbits is an exact part of quantum theory.
2. In the zero energy ontology (ZEO) of TGD the idea works in weaker form and has very strong implications for the more detailed understanding of ZEO and $M^8-M^4\times CP_2$ duality. Ordinary ("big") state functions (BSFRs) meaning the death of the system in a universal sense and re-incarnation with opposite arrow of time would involve quantum criticality accompanied by classical chaos assignable to the correspondence between geometric time and subjective time identified as sequence of "small" state function reductions (SSFRs) as analogs of weak measurements. The findings of Minev *et al* [L69] give strong support for this view [L69] and Libet's findings about active aspects of consciousness [J3] can be understood if the act of free will corresponds to BSFR.

M^8 picture identifies 4-D space-time surfaces X^4 as roots for "imaginary" or "real" part of octonionic polynomial P_2P_1 obtained as a continuation of real polynomial $P_2(L-r)P_1(r)$, whose arguments have origin at the the tips of B and A and roots a the light-cone boundaries associated with tips. Causal diamond (CD) is identified intersection of future and past directed light-cones light-cones A and B. In the sequences of SSFRs $P_2(L-r)$ assigned to B varies and $P_1(r)$ assigned to A is unaffected. L defines the size of CD as distance $\tau = 2L$ between its tips.

Besides 4-S space-time surfaces there are also brane-like 6-surfaces corresponding to roots $r_{i,k}$ of $P_i(r)$ and defining "special moments in the life of self" having $t_i = r_{i,k}$ ball as M^4_+ projection. The number of roots and their density increases rapidly in the sequence of SSFRs. The condition that the largest root belongs to CD gives a lower bound to it size L as largest root. Note that L increases.

Concerning the approach to chaos, one can consider three options.

Option I: The sequence of steps consisting of unitary evolutions followed by SSFR corresponds to a functional factorization at the level of polynomials as sequence $P_2 = Q_1 \circ Q_2 \circ ...Q_n$. The size L of CD increases if it corresponds to the largest root, also the tip of active boundary of CD must shift so that the argument of P_2 $L-r$ is replaced in each iteration step to with updated argument with larger value of L identifiable as the largest root of P_2.

Option II: A completely unexpected connection with the iteration of analytic functions and Julia sets, which are fractals assigned also with chaos interpreted as complexity emerges. In a reasonable approximation quantum time evolution by SSFRs could be induced by an iteration of a polynomial or even an analytic function: $P_2 = P_2 \rightarrow P_2^{\circ 2} \rightarrow ....$ For $P_2(0) = 0$ the roots of the iterate consists of inverse images of roots of P_2 by $P_2^{\circ -k}$ for $k = 0, ..., N-1$.

Suppose that M^8 and X^4 are complexified and thus also $t = r$ and "real" X^4 is the projection of X^4_c to real M^8. Complexify also the coefficients of polynomials P. If so, the Mandelbrot and Julia sets (http://tinyurl.com/cplj9pe and http://tinyurl.com/cvmr83g) characterizing fractals would have a physical interpretation in ZEO.

Chaos is approached in the sense that the inverse images of the roots of P_2 assumed to belong to filled Julia set approaching to points of Julia set of P_2 as the number N of iterations increases in statistical sense. The size L as largest root of $P_2^{\circ N}$ would increase with N if CD is assumed to contain all roots. The density of the roots in Julia set increases near L since the size of CD is bounded by the size Julia set. One could perhaps say that near the $t = L$ in the middle of CD the life of self when the size of CD has become almost stationary, is the most intensive.

Option III: A conservative option is to consider only real polynomials $P_2(r)$ with real argument r. Only non-negative real roots r_n are of interest whereas in the general case one considers all values of r. For a large N the inverse iterates of the roots of P_2 would approach to the real Julia set obtained as a real projection of Julia set for complex iteration.

How the size L of CD is determined and when can BSFR occur?

Option I: If L is minimal and thus given by the largest root of $P_2^{\circ N}$ in Julia set, it is bound to increase in the iteration (this option is perhaps too deterministic). Should L be smaller than the sizes of Julia sets of both$A$ and B if the iteration gives no roots outside Julia set.

Could BSFR become probable when L as the largest allowed root for $P_2^{\circ N}$ is larger than the size of Julia set of A? There would be no more new "special moments in the life of self" and this would make death and re-incarnation with opposite arrow of time probable. The size of CD could decrease dramatically in the first iteration for P_1 if it is determined as the largest allowed root of P_1: the re-incarnated self would have childhood.

Option II: The size of CD could be determined in SSFR statistically as an allowed root of P_2. Since the density of roots increases, one would have a lot of choices and quantum criticality and fluctuations of the order of clock time $\tau = 2L$: the order of subjective time would not anymore correspond to that for clock time. BSFR would occur for the same reason as for the first option.

The fact that fractals quite generally assignable to iteration (http://tinyurl.com/ctmcdx5) appear everywhere gives direct support for the ZEO based view about consciousness and self-organization and would give a completely new meaning for "self" in "self-organization" [L77]. Fractals, quantum measurement theory, theory of self-organization, and theory of consciousness would be closely related.

7.2 Could classical chaos and state function reduction relate to each other in TGD Universe?

In the sequel the idea about connection between chaos in some sense and state function reductions as they are understood in ZEO is discussed.

7.2.1 Classical physics is an exact part of quantum physics in TGD

Concerning the relation between classical and quantum the situation changes in TGD framework. Classical physics becomes an exact part of quantum theory. In zero energy ontology (ZEO) quantum states are superpositions of space-time surfaces preferred extremals of basic variational principle connecting 3-surfaces at opposite boundaries of causal diamond (CD). This solves the well-known basic problem of quantum measurement theory. Unitary time evolution operator or its generalization are totally different things from classical time evolution defined by highly non-linear field equations. There is nothing preventing quantum counterpart of chaos - it need not be classical chaos at space-time level but could correspond to some other form of chaos. Ordinary state function reduction in ZEO involves naturally quantum criticality involving long range quantum fluctuations assignable to chaotic systems so that the correlation between classical chaos defined in proper manner and state function reduction might make sense.

7.2.2 TGD space-time and $M^8 - H$ duality

M^8-H duality combined with zero energy ontology (ZEO) is central for the TGD inspired proposal for the connection between chaos and quantum.

Basic vision

Consider first what TGD space-time is.

1. In TGD framework space-times can be regarded 4-surfaces in $H = M^4 \times CP_2$ or in complexifiation of octonionic M^8. Linear Minkowski coordinates or Robertson-Walker coordinates for light-cone (used in TGD based cosmology) provide highly unique coordinate choice and this problem disappears. Exponential divergence in M^4 coordinates could be used as a symptom for a chaotic behavior.
2. The solutions of field equations are preferred extremals satisfying extremely powerful additional conditions giving rise to a huge generalization of the ordinary 2-D conformal symmetry to 4-D context. In fact, twistor twist of TGD predicts that one has minimal surfaces, which are also extremals of 4-D Kähler action apart from 2-D singularities identifiable as string world sheets and partonic 2-surfaces having a number theoretical interpretation. The huge symmetries act as maximal isometry group of "world of classical worlds" (WCW) consisting of preferred extremals connecting pair of 3-surfaces, whose members are located at boundaries of causal diamond (CD). These symmetries strongly suggest that TGD represents completely integrable system and thus non-chaotic and diametrical opposite of a chaotic system. Therefore the chaos - if present - would be something different.

M^8-H duality suggests an analogous picture at the level of M^8. M^8-H duality in itse most restrictive form states that space-time surfaces are characterized by "roots" of rational polynomials extended to complexified octonionic ones by replacing the real coordinate by octonionic coordinate o [L47, L48, L49].

1. One can define the imaginary and real parts $IM(P)$ and $RE(P)$ of $P(o)$ in octonionic sense by using the decomposition of octonions $o = q_1 + I_4 q_2$ to two quaternions so that $IM(P)$ and $RE(P)$ are quaternion valued. For 4-D space-time surfaces one has either $IM(P) = 0$ or $RE(P) = 0$ in the generic case. The curve defined by the vanishing of imaginary or real part of complex function serves as the analog.
2. If the condition $P(0) = 0$ is satisfied, the boundary of δM^8_+ of M^8 light-cone is special. By the light-likeness of δM^8_+ points the polynomial $P(o)$ at δM^8_+ reduces to ordinary real polynomial $P(r)$ of the radial M^4 coordinate r identifiable as linear M^4 time coordinate t: $r = t$.
   Octonionic roots $P(o) = 0$ at M^8 light-cone reduce to roots $t = r_n$ of the real polynomial $P(r)$ and give rise to 6-D exceptional solutions with $IM(P) = RE(P) = 0$ vanish. The solutions are located to δM^8_+ and have topology of 6-sphere S^6 having 3-balls B^3 with $t = r_n$ as of M^4_+ projections. The "fiber" at point of B^3 with radial M^4 coordinate $r_M \leq r_n$ is 3-sphere $S^3 \subset E^4 \subset M^8 = M^4 \times E^4$ contracting to point at the δM^4_+.
   These 6-D objects are analogous to 5-branes in string theory and define "special moments in the life of self". At these surfaces the 4-D "roots" for $IM(P)$ or $RE(P)$ intersect and intersection is 2-D partonic surface having interpretation as a generalization of vertex for particles generalized to 3-D surfaces (instead of strings). In string theory string world sheets have boundaries at branes. Strings are replaced with space-time surfaces and branes with "special moments in the life of self".
   Quite generally, one can consider gluing 4-D "roots" for different polynomials P_1 and P_2 at surface $t = r_n$ when r_n is common root. For instance, P and its iterates $P^{\circ N}$ having r_n and the lower inverse iterates as common roots can be glued in this manner.
3. It is possible complexify M^8 and thus also r. Complexification is natural since the roots of P are in general complex. Also 4- space-time surface is complexified to 8-D surface and real space-time surface can be identified as its real projection.

To sum up, space-time surfaces would be coded a polynomial with rational or at most algebraic coefficients. Essentially the discrete data provided by the roots r_n of P would dictate the space-time surface so that one would have extremely powerful form of holography.

One can consider generalizations of the simplest picture.

1. One can also consider a generalization of polynomials to general analytic functions F of octonions obtained as octonionic continuation of a real function with rational Taylor coefficients: the identification of space-time surfaces as "roots" of $IM(F)$ or $RE(F)$ makes sense.
2. What is intriguing that for space-time surfaces for which $IM(F_1) = 0$ and $IM(F_2) = 0$, one has $IM(F_1F_2) = RE(F_1)IM(F_2)+IM(F_1)RE(F_2) = 0$. One can multiply space-time surfaces by multiplying the polynomials. Multiplication is possible also when one has $RE(F_1) = 0$ and $IM(F_2) = 0$ or $RE(F_2) = 0$ or $IM(F_1) = 0$ since one has $RE(F_1F_2) = RE(F_1)RE(F_2) - IM(F_1)IM(F_2) = 0$.
For $IM(F) = 0$ type space-time surfaces one can even define polynomials analytic functions of the space-time surface with rational Taylor coefficients. One could speak of functions having space-time surface as argument, space-time surface itself would behave like number.
3. One can also form functional composites $P \circ Q$ (also for analytic functions with complex coefficients). Since $P \circ Q$ at $IM(Q) = 0$ surface is quaternionic, its image by P is quaterionic and satisfies $IM(P \circ Q) = 0$ so that one obtains a new solution. One can iterate space-time surfaces defined by $Im(P) = 0$ condition by iterating these polynomials to give $P, P^{circ2}, ..., P^{\circ N}$... From $IM(P) = 0$ solutions one obtains a solutions with $RE(Q) = 0$ by multiplying the M^8 coordinates with I_4 appearing in $o = q_1 + I_4q_2$.
The $Im(P) = 0$ solutions can be iterated to give $P \rightarrow P \circ P \rightarrow ..$, which suggests that the sequence of SSFRs could at least approximately correspond to the dynamics of iterations and generalizations of Mandelbrot and Julia sets and other complex fractals and also their space-time counterparts. Chaos (or rather, complexity theory) including also these fractals could be naturally part of TGD!

Building many-particle states at the level of M^8

The polynomials defining surfaces in M^8 are defined in preferred M^8 coordinates with preferred selection of M^8 time axis M^1 as real octonionic axis and one octonionic imaginary axes characterizing subspace $M^2 \subset M^8$. $M^4 \subset M^8$ is quaternionic subspace containing M^2. Different choices of $M^4 \sup M^2$ are labelled by points of CP_2 and $M^8 - H$ duality maps these choices to points of CP_2.

The origin of $M8$ coordinates coordinates must be at M^1 so that the 8-D Poincare symmetry reduces to time translations and rotations of around spatial coordinate axis M^2 respecting the rationality of polynomial coefficients or in more general case the extension of rationals associated with the coefficients. This corresponds to a selection of quantization axis for energy and angular momentum and could have a deeper meaning in quantum measurement theory.

The Lorentz transformations of M^4 change the direction of time axis and also M^2 in the general case and generate new octonionic structure and quaternonic structure. One should understand how space-time regions as roots of octonionic polynomials with different rest frames relate to each other.

The intuitive picture is that each particle as a region determined by octonionic polynomial corresponds to its own CD and rest frame determined by its 4-momentum in fixed coordinate frame for M^4. Also quantization axis of spin fixed. One can assign CD for to interacting many particle system with common rest frame. One can speak of external (incoming and outgoing) free particles with their own CDs characterizing their rest systems. The challenges is to related the polynomials P_n associated with the external particles to the polynomial characterizing the interacting system.

1. Assume that the polynomial defining the CD is product P_1P_2 of polynomials P_1 and P_2 assignable to its active and passive boundaries with origins of octonionic coordinates at the tips $t = 0$ and $t = \tau$ of CD. If the space-time surface reduces to the root of P_1 at passive boundary and root of $IM(P_2)$ at active boundary, one could say that the 3-surfaces at these boundaries correspond to P_1 and P_2 asymptotically. If these conditions are true everywhere, one has two un-correlated space-time surfaces, which does not make sense. $IM(P_1)RE(P_2) + RE(P_1)IM(P_2) = 0$ indeed allows more general solutions than $IM(P_1) = 0$ and $IM(P_2) = 0$ everywhere. The fact that the boundaries correspond to special 6-D brane like solutions in M^8 suggests that it is possible to pose the boundary condition $IM(P_1) = 0$ *resp.* $IM(P_2) = 0$ at the boundaries.
2. The formation of products is possible also at the boundaries so that one can assume that P_i at the boundary of many-particle CD is with product $P_i = \prod_k P_{ik}$. The boundary conditions

would read read $P_{ik} = 0$ at active *resp.* passive boundary of many-particle CD respectively. The interpretation would be that P_{ik} corresponds to an external particle which is in interacting state at active boundary. In the interior of many-particle CD only the condition $Im(P_1P_2) = 0$ would hold true so that interactions of particles would have algebraic description.

3. One should also understand how the external particles characterized by CDs with different rest frame are glued to the boundary of many-particle CD. Assume that M^4 is same for all these particles so that CP_2 coordinates are same. The boundaries of 4-D CDs are 3-D light-cones with different origins so that their M^4 intersection is 2-D defining a 2-D surface at the boundary of CD. The interpretation in terms of partonic 2-surface suggests itself. The partonic 2-surfaces of free particle and its interacting variant would be same at the intersection.
The gluing should correspond to a root $t = r_n$ of polynomial defining a "special moment in the life of self". The roots of P_1 and its Lorentz boots as values of coordinates at light-radial geodesic are related by Lorentz boost and are not same in general. One could require that the root r_n and its Lorentz boost belong to the 2-D interaction of two light-cones and thus define two points of partonic 2-surface. These points would not be identical and the interaction would be non-local in the scale of partonic 2-surface. It seems that the condition that root r_n and Lorentz boost $L(rm)$ co-incide would pose too strong constraints on external momenta.

7.2.3 In what sense chaos/complexity could emerge in TGD Universe?

Consider now in what sense chaos (or complexity, one must be precise here) could emerge in TGD framework?

1. Chaos (or complexity) could be an approximate property emerging in number theoretical discretization for cognitive representations labelled by extensions of rationals as the dimension of extension and therefore algebraic complexity increases ad the number of points in cognitive representation as points of M^8 with coordinates in the extension of rationals increases. The minimal number of points corresponds to the degree of the polynomial determining the extension. At the limit of maximal complexity the extension would consists of algebraic numbers and the cognitive representation would be dense subset of space-time surface. It is not clear whether the roots r_n are also dense along time axis.
2. Also transcendental extensions of rationals can be considered. Typically they are infinite-D in both real and p-adic sectors. Exponential function is however number-theoretically completely unique. Neper number e and its roots define infinite-D extensions of rationals but - rather remarkably - finite-dimensional extensions of p-adic numbers since e^p is ordinary p-adic number. Extension of rationals would become infinite-D but the induced extensions of rationals would remain finite-D in accordance with the idea that cognition is always finite-D.
Could one allow e and its roots and thus exponential functions besides polynomials? Could exponential divergence be the hallmark of chaos or perhaps the first step in the transition to transcendental chaos (or rather, complexity)? Could chaos (complexity) in real sense be possible for extensions of rationals generated by a root of e? One can however argue that the finite dimension of induced p-adic extensions means that cognitive chaos is not yet present.
For general transcendentals the dimensions of p-adic extensions are infinite and one would have also cognitive chaos (infinite complexity). Could the transition to chaos means the emergence of analytic functions with rational coefficients having also roots, which are transcendentals. Chaos would mean that one can only approximated f analytic function as a polynomial giving approximation for the roots. By M^8-H duality these roots would correspond to values of M^4 time inside light-cone, preferred moments of time [L73]. These would become transcendental and in general p-adic extension would become infinite-D.
3. An interesting analogy with real numbers emerges. Real numbers have expansion in powers of any integer, in particular any prime p. The sequence defined by the coefficients of the expansion are analogous to an orbit of a discrete dynamical system. For transcendentals the expansion is unpredictable and analogous to a chaotic orbit.
For rationals this expansion is periodic so that one has analog of a periodic orbit. This applies also to expansion of rationals formed from the integers in finite-D extensions of rationals. One must of course accept that the algebraic numbers defining the roots do not allow periodic expansion but one can do all calculations in extension and perform approximation only at

end of computation. Therefore the extensions of rationals represent also islands of order in the ocean of trancendental chaos. Could one see he gradual increase of the dimension of extension of rationals as a transition to chaos: of course, chaos would be wrong term since increase in algebraic complexity, which corresponds to evolution in TGD Universe is in question. Cognition becomes more and more refined.

4. As found, space-time surfaces behave like numbers and one can have functions having space-time surface as argument. Could the picture about emergence of chaos for reals be translated to the level of space-time surfaces identified as "roots" of octonion analytic function in M^8? The polynomial space-time surfaces would represent islands of order in chaos defined by general analytic functions with rational Taylor coefficients.

Can one imagine a connection between quantum randomness and chaos?

To my view, the reduction of quantum randomness to classical chaos is definitely excluded. Quantum classical correspondence allows however to consider a looser connection between quantum randomness and chaos.

1. The following considerations lead to a formulation of a more precise view about the sequence of steps consisting of a unitary evolution followed by SSFR as a a model of self. M^8-H duality involving representation of space-time surface in terms of a polymial with rational coefficients leads to an approximate model of the quantal time evolution by SSFRs as quantum counterpart for an iteration of a polynomial map, and gives a direct connection with chaos as algebraic complexity in the sense of generalization of Mandelbrot and Julia sets (`http://tinyurl.com/cplj9pe` and `http://tinyurl.com/cvmr83g`).
   The identification of time evolution as iteration $P \rightarrow P^{\circ 2} \rightarrow ...$ is very probably only an approximation. More general picture would assume that the corresponds to a functional factorization of P as $P = P_1 \circ P_2 \circ ... \circ P_n$. Even this assumption can be only approximate.
2. The fixed points of iteration would correspond to asymptotics for the evolution of space-time surface defined by iteration and approach of CD to a fixed point CD. This conforms with the idea that fixed points of iteration as representations of fractals, criticality and chaos. Chaos understood as genuine chaos could correspond to a fluctuation of the arrow of time in the sequence of SSFRs as a fixed point of iteration is reached.

It must be of course made clear that the view about M^8-H duality already considered and the view about the emergence of fractals to be discussed are only one of the many options that one can imagine and involve many poorly understood aspecs. Only time will tell whether the proposals work and how they must be improved.

Chaos and time

TGD Universe has gigantic symmeries [K31, K85] and looks like a completely integrable system and the idea about genuine chaos at space-time level does not look attractive. M^8-H duality suggests that chaos - actually complexity - in the sense of Mandelbrot fractals looks more promising idea. ZEO int turn suggests that chaos could be associated with the relationship between geometric and subjective time in the sense that the orderings of the two times would not be strictly identical.

1. Often the chaos is taken to mean increase of complexity (Mandelbrot and Julia sets), which actually means a diametric opposite of chaos. In TGD framework a more promising connection is between finite measurement resolution and complexity as that for extension of rationals. For trivial extensions of rationals the points of cognitive representation have rational M^8 (and becase also H-) coordinates. All other points fail to have a cognitive representation. For extensions of rationals the number of points in cognitive representations increases: the increase of cognitive complexity has actually nothing to do with emergence of a genuine chaos. Here one must be however very cautious and one must consider ZEO view about state function reduction in detail to see what happens.
2. M^8-H duality allows to consider a concrete example. The roots r_n of real rational polynomials P or even analytic functions correspond "special moments in the life of self". Could the increase of complexity be understood in terms of what happens for the roots. The number of these moments equals to the degree n of P and cognitive representation more and more complex

since the dimension of extension equals to n: this could occur in BSFRs at least. The clock defined by the moments roots $t = r_k$ could become more precise. It will be found that in presence of quantum criticality the emerging complexity could also correspond to a genuine chaos.

3. One can define clock time as a temporal distance τ between tips of CD after "small" state function reduction (SSFR), which corresponds to weak measurement in standard picture. Passive boundary and the states at the passive boundary of CD remain unchanged (generalized Zeno effect) and the states at active boundary is change. Also the distance between tips of CD changes but increases in statistical sense.
The statistical nature of the change implies that the ordering for subjective time as sequence of SSFRs is not quite the same as that for τ (one could of course assume that only increase of the CD size is possible in BSFR but this would be an ad hoc assumption). This corresponds to a kind of quantum randomness due to the state function reductions. If the number of roots is large and the average time chronon is small, the changes of time order could occur often. Could this have interpretation as a genuine chaos in short time scales due to SSFRs? This need not correspond to a genuine chaos at the level of space-surfaces as preferred extremals. Chaos as algebraic complexity could however increases and would be consistent with complete integrability: this happends in n increases in BSFRs.

Chaos in death according to ZEO

The assignment of a genuine chaos to death looks natural from what we know about biological death. Could this assignment make sense in ZEO where BSFR corresponds in a well-defined sense to death?

1. Recall that BSFR corresponds to ordinary state function reduction in which the arrow of geometric time identifiable as distance between the tips of CD changes: self dies and re-incarnates with an opposite arrow of time. The active boundary of CD becomes passive. The passive boundary becomes active and the size of CD starts to statistically increase in opposite time direction in SSFRs. The former passive boundary CD can remain at the critical moment but could also shift towards the former active boundary - the re-incarnated self would have small CD and could have "childhood."
The continual increase of CD looks strange. Also our mental images would increase in size and unless one makes special assumptions (say that the average change of the size of CD is proportional to its size (scaling)) one ends up with difficulties. Time evolution as stepwise scaling would be indeed natural.
2. Under what conditions does BSFR - death and reincarnation - occur? A quantum criticality implying instability against BSFR should be involved. The size scales of CD as temporal distances τ between its tips would have critical values τ_{cr} at which death of self in this universal sense could take place. τ_{cr} could be integer multiple of CP_2 length scale with allowed integers being primes of preferred primes allowed by p-adic length scale hypothesis. Criticality indeed involves long range fluctuations assigned with chaotic behavior: the simplest example is the transition to chaos in convection as energy feed to the system increases.
3. A concrete model for SSFRs [L80] suggests that one can assign to CD temperature T satisfying $T \propto 1/\tau$ so that the evolution of self would correspond to T as analog of cosmic temperature. Death could correspond to a critical temperature T_{cr} (τ_{cr}) and would be unavoidable. The quantum criticality assignable to death could correspond to the emergence of a genuine temporal chaos. The time order would become more and more ill-defined, and time τ would go forth and back so that eventually one would $\tau = \tau_{crit}$ as size of CD and death would occur. This however requires that the number of roots r_n increases so that also their density increases. This requires that the degree of the polynomial P defining the extension increases. Can this be consistent with the assumption that passive boundary does not change?
Remark: Why I take this seriously is that I have had near death experience being in clinically unconscious but actually conscious state and I experienced quite literally the flow of time forth and back and was fighting to preserve the usual arrow of time.
4. This picture applies to all BSFRs and SSFRs and therefore to ordinary state functions reductions in all scales: the findings of Minev *et al* [L69] can be understood if the arrow of time

indeed changes [L69]. There would be a connection between state function reductions and chaos understood as genuine chaos. The idea that this chaos corresponds to a strange attractor at space-time level is not plausible. Rather it could be analogous to chaos in the sense of an attractor of iteration of complex function by functional decomposition. Fixed point is also a fractal and corresponds to criticality.

What gives rise to the lethal quantum criticality, BSFR, and death?

What could give rise to quantum criticality leading to death and reincarnation of self as BSFR?

1. If P remains the same during SSFRs, one could think that once the CD size is so large that all "special moments in the life of self" have been experienced as time values $\tau = r_n$, the system is ready to die. But how could this give rise to quantum criticality?
2. Assume that CD is defined as the intersection of future and past light-cones and the polynomial P corresponds to a product $P_1(r)P_2(L-r)$ of polynomials associated with these two light-cones such that P_i vanishes at the tip of its light-cone corresponding to $r=0$ *resp.* $L-r=0$. P_1 associated with the passive boundary of CD would not change in SSFRs but P_2 associated with the active boundary would change. Most importantly its degree would increase and the number of roots and their density would increase too. Eventually the density of active roots would become so high that death as BSFR is bound to occur as event $\tau = \tau_{cr}$.
   Remark: One can consider two options: real M^8 and real r or complexified M^8 and complex r.
3. As already noticed, if the space-time surface reduces to the root of P_1 at passive boundary and root of P_2 at active boundary, one could say that the 3-surfaces at these boundaries correspond to P_1 and P_2 asymptotically. The fact that the boundaries correspond to special 6-D brane like solutions in M^8 sugests that the boundary conditions are possible.
4. The statistically increasing extension of rationals would correspond to "personal" evolution for the changing part of self during life cycle. Note that $n = h_{eff}/h_0$ corresponds to the scale of quantum coherence thus increasing. This extension would define the evolutionary level of the unchanging part ("soul") during the next re-incarnation.

Could polynomial iteration approximate quantum time evolution by SSFRs in statistical sense?

I have considered rather concrete models for the counterpart of S-matrix for given space-time surface [L63, L64, L81] but the deeper understanding of the sequence of SSFRs is still lacking although quite concrete proposals already exists.

Number theoretical vision suggests that also the time evolution by SSFRs should reduce to number theory being induced by some natural number theoretical dynamics.

1. The most general option is that in each SSFR a superposition over extensions defined by various polynomials with varying rational coefficients is generated. The idea about the correspondence of the sequence of SSFRs with a functional decomposition of polynomials is however attractive.
2. The sequence of unitary evolutions brings strongly in mind the iteration $U \rightarrow U^2 \rightarrow U^3$.... One can however consider also the possibly $U \rightarrow U_1U \rightarrow U_2U_1U$.... The obvious guess for the iteration of U is that it is induced by a functional iteration of polynomial P_2 assigned to the active boundary of CD $P_2 \rightarrow P_2 \circ P_2 \rightarrow$ .... The more general option would not be iteration anymore but a composition of form $P_2 \rightarrow P_3 \circ P_2 \rightarrow$ .....
   The boundary conditions at the boundary of CD and at gluing points - possibly $t = r_n$ surfaces to which 6-branes are assignable as special solutions and identified as "special moments in the life of self" could make the superpositions of functional composites more probable contributions in the superposition. The polynomial $P \circ Q$ has same roots as Q (for $P(0) = Q(0) = 0$) and this favors conservative state function reductions preserving the state already achieved.
   Iteration would be even more conservative option. If the solutions assignable to P and Q are to be glued together along brane with $t = r_n$ they must share r_n as root. This would favor iterations if one has superposition over different rational coefficient values for P and Q with fixed degree.
   Remark: Also critical points of Q as zeros of derivative are preserved in $Q \rightarrow P \rightarrow Q$ as one finds by applying chain rule. For iteration both the new critical points/roots of $P \circ P^{\circ k}$

are inverse images of critical points/roots of $P^{\circ k}$. Only roots are of significance in the picture considered.

3. Superpositions of different iterates generated in the unitary time evolution preceding SSFR are required by the model of temporal chaos. SSFR selects extension of rationals and thus fixed iteration. In statistical sense the degree of iteration is bound to increase so that in statistical sense quantum iteration reduces to classical one. At the limit of fixed point of iteration the number of critical points $t = p_n$ and roots $t = r_n$ of the iterate increases as also their density along time axis and temporal chaos emerges leading to fluctuation of CD size τ.
4. Iteration of the real polynomial P satisfying $P(0) = 0$ would mean that one would have a series extensions obtained as powers of generating extension: E, $E \circ E$, $E \circ E \circ E$,... conserving the roots of E provided the polynomials involved vanish at origin: $P(0) = 0$. The proposal has been that biological evolution corresponds to a more general series of extensions $E_1, E_2 \circ E_1, E_3 \circ E_2 \circ E_1, ...$ Also now Galois groups in the series of them would be conserved. I have proposed that Galois groups are analogs of conserved genes [L46, L49].

The proposed picture is only one possibility to interpret evolution of self as iteration leading to chaos in the proposed sense.

1. One could argue that the polynomial $P_{nk} = P_n \circ .... \circ P_n$ associated with the active boundary remains the same during SSFRs as long as possible. This because the increase of degree from nk to $n(k+1)$ in $P_{nk} \to P_{nk} \circ P_n$ increases h_{eff} by factor $(k+1)/k$ so that the metabolic feed needed to preserve the value of h_{eff} increases.
   Rather, when all roots of the polynomials P assignable to the active boundary of CD are revealed in the gradual increase of CD preserving P_{nk}, the transition $P_{nk} \to P_{nk} \circ P_n$ could occur provided the metabolic resources allow this. Otherwise BSFR occurs and self dies and re-incarnates. The idea that BSFR occurs when metabolic resources are not available is discussed in [L110].
2. Could $P_{nk} \to P_{nk} \circ P_n$ occur only in BSFRs so that the degree n of P would be preserved during single life cycle of self - that n can increase only in BSFRs was indeed the original guess.

7.2.4 Basic facts about iteration of real polynomials

The iteration of real polynomials and also more general functions can be understood graphically. Assign to a x point $y = f(x)$ of the graph and reflect through the line $y = x$ and project to the graph to obtain the image point $x_1 = f(x)$. Fixed points $x = f(x)$ correspond to the intersections of the line $y = x$ and graph $y = f(x)$. The magnitude $|df/dx|$ at the intersection point determines whether it is attractor ($|df/dx| < 1$ or repellor ($|df/dx| \geq 1$) in which case large jumps in the value of x can occur, as one can easily check. Quite generally iteration in the part of the graph below (above) $y = x$ decreases (increases) x. Real polynomial $c - x^2$ provides a simple example.

Feigenbaum discovered by iterating logistic map numerically (`http://tinyurl.com/u3zwmar`) that the approach to chaos - not only for logistic map but - for real functions $f(x)$ with one quadratic maximum and depending on a varied parameter a is universal. Period-doubling bifurcations occur at parameter values satisfying at the limit $n \to \infty$

$$\frac{a_{N-1} - a_{N-2}}{a_N - a_{N-1}} \to 4.669201609... \ .$$

Second universality relates to the widths of tines - distances between the branches of bifurcation - appearing in the sequence of bifurcations. The ratio between width of the tine to widths of its sub-tines approaches at the limit $N \to \infty$ to constant given by

$$\alpha = 2.502907875095892822283902873218... \ .$$

.

In TGD framework conservative option would correspond to real M^4 so that the coordinates t and r would be real and the polynomials P_1 an P_2 would have real coefficients. The time evolution by iterations of P_2 would reduce to an iteration of a real polynomial P_2.

The number of real roots is in general smaller than the degree n of the polynomial. Only non-negative roots can be considered since one as $r \geq 0$ and $r = 0$ is a root. This condition could generalize to complex polynomials of complexified r as a condition $Re(r_c) \geq 0$ guaranteeing that roots are in the upper half plane for the variable $z = ir_c$.

The real polynomial $P(x)$ of degree n one has either positive or negative values between neighboring roots and at least one extremum between them. The n roots of $P_n(x)$ gives rise to Nn roots in N:th iteration and only non-negative ones are allowed. Since the roots are below the axis $y = s$, the root is obtained from the inverse of the roots by reflecting with respect to $y = x$ and projecting to the graph. The inverse of this operation increases the root. One has special case of complex iteration.

7.2.5 What about TGD analogs of Mandelbrot -, Julia-, and Fatou sets?

What about the interpretation of Mandelbrot -, Julia-, and Fatou sets (http://tinyurl.com/cplj9pe and http://tinyurl.com/cvmr83g) in the proposed picture? Could the iteration of P_2 define analogs of Mandelbrot and Julia fractals? This would give the long-sought-for connection between quantum physics and Mandelbrot and Julias sets, which are simply too beautiful objects to lack a physical application. Period-doubling bifurcations (http://tinyurl.com/t2swmdg) are involved with the iteration of real functions and relate closely to the complex fractals when the polynomials considered have real coefficients.

1. In the simplest situation both Mandelbrot and Julia sets are fractals associated with the iteration of complex polynomial $P_c(z) = z^2 + c$ where z and c are complex numbers (note that in TGD would have $c = 0$ in this case). One can consider also more general polynomials and even rational functions, in particular polynomial $f = P_2$ defined earlier, and replace $z = 0$ with any critical point satisfying $df/dz = 0$. Even meromorphic transcendental functions can be considered: what is required that the image contains the domain.
2. Mandelbrot set M is defined as the region of the plane spanned by the values of c for which the iteration starting from the critical point z_{cr} does not lead to infinity. Physically the restriction to Mandelbrot set looks natural.
3. For rational functions Julia set J_c (http://tinyurl.com/cplj9pe corresponds to a fixed value of c, and is defined as points z for which are unstable in the sense that for an arbitrary small perturbation of z iteration can lead to infinity. Inside J_c the iteration is repelling: $|f(w) - f(z)| > |w - z|$ for all w in neighbourhood of z within J_c. One can say that the behavior is chaotic within J_c and regular in its complement - Fatou set. Julia set can contain also cycles and iteration in J_c leads to these cycles. These cycles are analogs of the limit cycles appearing in the iteration of real-valued function discovered by Feigenbaum (http://tinyurl.com/u3zwmar).
   For polynomials Julia set can be identified as the boundary of the filled Julia set consisting of points for which iterates remain bounded. Also the inverse iterates in this set remain bounded. The filled J_c - denote it by $J_{c,in}$ - can be regarded as a set of points, which are inverse images of fixed points of the polynomial. All points except at most two points of J_c can be regarded as points in the limiting set for the union $\cup_n f^{-n}(z)$ of the inverse images for the points z in filled Julia set. Julia set and its complement Fatou set are invariant under both P and P^{-1} and therefore also under their functional powers. Julia set is the set of pre-images for practically any point of J_c: this can be used for computational purposes. If I have understood correctly there can be single exceptional point for which this is not the case. J_c can be regarded as a fractal curve. For parameter values inside M J_c is connected, which seems counter intuitive. For c outside the M, Julia set is a discrete Cantor space, Fatou dust.
   What is remarkable from TGD point of view is that the new roots obtained in N:th step of iteration are $N-1$:th inverse images of the roots of P. Since polynomial iteration takes sufficiently distant points to ∞, its inverse does the opposite so that the roots of $P^{\circ N}$ are bounded: this strongly suggests that the roots of $P^{\circ N}$ are in J_c if those of P_2 are. One can say that the situation becomes chaotic at the large N limit since the number of roots increases without bound.
4. Fatou set F_c can be identified as the complement of Julia set. Fatou set fills the complex plane densely and has disjoint components, which contain at least one point with $df/dz = 0$ unless Fatou set contains $z = \infty$. Note however that critical point is ot fixed point as in gradient dynamics. This allows to deduce the number or at least upper bound for the number

of components of Fatou set, which equals to the degree n of polynomial in the generic case. All components have entire J_c accumulation points. Since the points of J_c are infinitely near to more than 2 disjoint sets for Fatou set with more than 2 components, J_c cannot be a smooth curve in this case being thus fractal. However, the Julia set of $P = z^2 + c$ is also fractal although Fatou set has only two components corresponding to the critical point $z = 0$ and $z = \infty$.

A couple of examples are in order: for $P(z) = z^2$ Julia set is unit circle S^1 and Fatou set has interior and exterior of S^1 as its components. The cycles in Julia set correspond to roots of unity and the orbits of other points form dense sets of unit circle. For $P(z) = z^2 - 2$ Julia set is the interval $(-2, 2)$ having fixed points as its ends. Fatou set has only one component as the complement of Julia set. For $P(z) = z^2 + c$, c complex Julia set is in general fractal. Hence the roots of the polynomial need not belong to Julia set.

Emergence of Mandelbrot and Julia sets from ZEO assuming $M^8 - H$ duality

Consider now the application to TGD assuming $M^8 - H$ duality [L47, L48, L49, L76] .

1. In TGD framework complex numbers $x + iy$ emerge in the complexification of M^8 and i commutes with octonionic units. If space-time surfaces are identified as real projection of their complexified variants obtained as roots of polynomials one can consider also polynomials with complexified coefficients c. Note that c would be complex rational but one can also consider complex algebraic numbers. The most general situation corresponds to analytic functions with complex rational Taylor coefficients. Complex argument with complex coefficients is possible space-time surface is identified by projection the complex space-time surface to real part of complexified M^8 [L47, L48, L48].
2. The complexified light-like coordinate r at the active boundary CD defines the analog of z plane in which iterates of P_2 act. r corresponds directly to the complexified linear time coordinate t of M^8 (time-axis connects tips of CD) and the roots r_n of P_2 correspond to the "special moments in the life of self" as time values $t = r_n$. Assume that $P_2(0)$ vanishes so that r_n are also roots of iterates.
3. Julia set J_c bounds filled Julia set $J_{c,in}$ of the complexified r-plane, whose interior points remain inside $J_{c,in}$ in the iterations by fixed P_2. Julia set J_c is connected but the Fatou set as its complement has several components labelled by the $n-1$ points p_k satisfying $dP_2(z)/dz = 0$ and by $z = \infty$ so that Fatou set has n components. The inverse iterates of roots need not belong to Fatou sets not containing ∞ or to the filled Julia set.
4. There are several Mandelbrot sets and the extrema of P_2 satisfying $dP_2/dr = 0$ label them. The extrema of P_2 are also extrema of its iterates. There are $n-1$ extrema p_k. In the real case they can be classified as either attractors or repellors but in complex situation they correspond to saddle points. Denote by $M(p_n)$ the region of parameter space of polynomial coefficients c for which the iteration starting starting at $p(n)$ does not lead outside it.
In the real case the iteration of P_2 leads to the attractors $t = p_k$. In complex case the situation is not so simple and the basic of attraction is replaced with the Fatou set $F_c(p_k)$.
Since c parameterizes points in the space of polynomials characterizing space-time surfaces in TGD, Mandelbrot set can be defined as a sub-space of "world of classical worlds" (WCW). Inside $M(p_n)$ the iteration maps r_n to a point $M_{in}(r_n)$. Note that also new roots emerge in each iteration and the Mandelbrot set for the iterates contains more components.

Remark: In TGD only the roots of P_2 are interesting. The roots of iterates are inverse iterates of roots of P_2.

Could one understand the size of CD and its evolution during the iteration of P_2?

1. Consider first the situation for real time $t = r$ and real polynomials. Since the boundary of CD contains only the roots $t = r_n$, the simplest guess is that the size of CD corresponds to the largest root of $P_2^{\circ N}$. The size of CD would increase in the iterations. The inverse images of the roots approach to Julia set so that the real counterpart of Julia set is important for understanding the asymptotic situation. Mandelbrot set defines the coefficient values for which iteration does not lead to infinity.

2. The situation is essentially the same for complexified time. The size of CD would correspond to the modulus for the largest of the iterate root and increases during iteration. The size of CD approaches to that for a point in Julia set.

Could the iteration lead to a stationary size of CD?

One can represent an objection to the idea that quantum iteration of P_2 could be more than an approximation.

1. Suppose that the size of CD is determined by the maximum for the iterates of the roots of P_2. Suppose that the parameters c are fixed and belong to Mandelbrot set $M(p_k)$. For given c there is therefore an upper for $\tau = 2r$ given by $r = r_{max}(c, p_k)$ for the Fatou set $F_c(p_k)$. One gets stuck to fixed τ since maximal root cannot become larger than $r_{max}(c)$ in the iteration. Note that in this situation the number of roots of $P_2^{\circ k}$ increases and if they corresponds to "special moments in the life of self", this could lead to quantum criticality and occurrence of BSFR.
2. Fluctuations of τ in the sequences of SSFRs is possible if superpositions of iterates are allowed. This could cause BSFR would occur and eventually second BSFR would eventually lead to the original situation. If P_2 is not modified, the iteration continues and one is still at criticality. BSFR soon occurs and same repeats itself.

Is this situation acceptable? Maybe - I have considered the possibility that the size of CD remains below some upper bound [L80, L70]. The selves such as our mental images could continue to live in the geometric past and memories would be communications with them. Or should one get rid of this situation? How?

1. Assume that SSFR creates a superposition of iterates with varying values of parameters c belonging to the Mandelbrot set $M(P_2)$. The value of $r_{max}(c, p_k)$ depends on c and it is possible to increase the value of τ in statistical sense if SSFR selects the values of c suitably. The value of L would be however given by maximal root and would remain below the maximum r_{max} of $r_{max}(c, p_k)$ in $M(P_2)$ if c belongs to $M(P_2)$. $\tau = 2L$ would remain below the maximum for the size of $J_c(P_2)$ in $M(P_2)$. One would get stuck if this size is finite, which is the case if $r_{max}(c, p_k)$ is bounded as function of c and p_k?
   Is $r_{max}(c, p_k)$ bounded? The polynomials with given degree of can have arbitrarily large roots and critical points in the same extension of rationals. Therefore it might be possible to avoid getting stuck if there is no restriction on the size of the roots of P_2 in the superposition over different values of c.

When death occurs and can self have a childhood?

I hope that talking about death and reincarnation does not irritate the reader too much. I use these terms as precisely defined technical terms applying universally. There are two extreme options for what happens to the former passive boundary in BSFR. The real situation could be between these two.

1. The first shift after reincarnation is to geometric past so that CD size increases.
2. The first shift is towards the former active boundary so that the size of CD decreases at least to the size of CD when the iteration of P_2 began. The reincarnated self would have "childhood" and would start from scratch so to say.

Consider P_1P_2 option. Suppose that time evolution is induced by iteration of either polynomial and maximal root defines the size of the size of CD. What happens to P_1?

1. Could the new functional iteration start from where it stopped in previous re-incarnation: if P_1 is n:th functional power of Q ($P_1 = Q^{\circ n}$), the first step would corresponds to $P_1 \to Q \circ P_1$. This conservative option does not quite correspond to the idea that one starts from scratch.
2. If P_1 can change, could one require that P_1 is replaced with a polynomial, which is minimal in the sense that it is not functional power of form $P_{1,new} = Q_{new}^{\circ n}$. Or could one even require that it is functional prime having prime valued degree: $n = p$. This would mean starting from scratch except that the algebraic extension of P_2 is fixed.

Probably these options represent only extreme situations. The most general option is that BSFR generates a state, which corresponds to a superposition of extensions of rationals characterized by polynomials P_2P_1, P_2 fixed, and from these one is selected.

Suppose that L as the size of CD is minimal and thus given by the largest root of $P_2^{\circ N}$ in the filled Julia set, it is bound to increase in the iteration (this option is perhaps too deterministic). Under what conditions can BSFR occur? Can the re-incarnated self have childhood?

1. One can argue that L should be smaller than the sizes of Julia sets of both A and B since the iteration gives no roots outside Julia set. This would require iteration to stop when the largest root of P_2^{circN} exceeds the size of the Julia set of A. When applied to B this condition would prevent BSFRs in the opposite time direction would prevent the growth of CD and it would become stationary. This condition looks too deterministic.
2. This picture suggests that the unitary evolution preceding SSFR creates a superposition of iterates $P_2^{\circ N}$ and that the size of CD as outcome of SSFR is determined statistically as a maximal root for $P_2^{\circ N}$ selected in the iteration. N could also decrease. Since the density of roots increases, one would have a lot of choices for the maximal root and quantum criticality and fluctuations of the order of clock time $\tau = 2L$: the order of subjective time would not anymore correspond to that for clock time.
3. Could BSFR become only probable as L as the largest root for the iterate $P_2^{\circ N}$ has exceeded the size of Julia set of A? A quantum analogy with super-cooling comes in mind. The size of CD boundary at side A would contain more volume than needed to store the information provided by the roots r_n and bring no new "special moments in the life of self" at A side. At B side the density of these moments would eventually become large enough so that the reduction of the size of CD destroying part of these moments would mean only a loss of precision. Could this make death and re-incarnation with an opposite arrow of time probable?
   If $P_2^{\circ N}$ is achieved during the life cycle, the reduction in the size of CD in BSFR would reduce N to $N_1 < N$. For $P_1 = Q_1^M$ similar reduction of M to $M_1 < M$ would take place. If one returns to the situation when the iterated started, all new "special moments" are lost. Nothing would have been learned but one could start from scratch and live a childhood, as one might say.
   In the proposed picture - one of many - the opposite boundaries of CD would correspond to both short and long range quantum fluctuations. Could this observation be raised to a guiding principle: could one even say that the opposite boundaries of CD give holistic and reductionistic representations.
4. Do the roots of $P_2^{\circ N}$ belonging to filled Julia set approach the Julia set as N increases? Or are they located randomly inside Julia set? Indeed, the inverse iterate of a root of P_2 is larger than the root as one finds graphically. The $P_2^{\circ N}$ does the same for the roots $P_2^{\circ N}$. If this argument is correct, the density of the roots is largest near Julia set and near the maximum $L - t = L - r$ near the corner of CD.
5. The proposed picture is interesting from the point of view of consciousness theory. Action would be near the corner of CD in the sense that conscious experience would gain most of its content in Minkowskian sense here whereas larger smaller values of $L - r$.
   This does not mean paradox since the size of CD inreases and special moments already experienced are shifted to the future direction and would define the unchanging part - "soul" - of the next re-incarnation. This could be seen as wisdom gained in the previous life [L80].
6. Suppose that the approach to chaos in the iteration of P_2 indeed leads to death and re-incarnation. Can one avoid this or at least increase the span of life cycle? Could one start a new life by replacing P_2 with some polynomial Q_2 in the iteration so that the new iterates would be of form $Q_2^{N_2} \circ P_2^{\circ N_1}$. If the replacement is done sufficiently early, the development of chaos might be delayed since reaching the boundary of Julia set of Q would require quite a many iterations if its largest root is larger than that for P_2. This is also true if the degree of Q_2 is small enough.

Unexpected observations about memories

Some comments about memories in the model of self based on iteration.

1. The conscious activity is at the corner of CD in middle of CD if the new roots define "special moments in the life of self" as conscious experiences. The roots r_n of $P_2^{\circ N}$ defining already experienced special moments shift to Minkowskian geometric future as CD increases in size. Subjective memories are in Minkowskian future and become in re-incarnation stable memories about previous life!
   Subjective memories from recent and previous life could be obtained by communications with geometric future and past involving time reflection of the signal so that the constraints due to the finite light velocity can be overcome.
   One can ask whether self can have "remember" or "anticipate" also external world. If this is possible then the "memories" are indeed from geometric past and "anticipations" from geometric future.
2. The view about subjective memories raises interesting speculations (to be made with tongue in cheek). Consider an unlucky theoretician who believes that he has discovered wonderful theory and has used his lifetime to develop it. Unfortunately, colleagues have not shown a slightest to his theory. Although personal fame might not matter for him, he might be interested in knowing during his lifetime whether his life work will ever gain recognition. Is this possible in TGD Universe?
   Suppose that dreams involve sub-selves representing signals to Minkowskian future and their time reflection inside CD (re-incarnation). If sub-selves near the boundary of CD are able to send time signals to geometric future they might get information about the external world, maybe even about what colleagues think about the theory of unlucky theoretician. Dreams might allow to receive this information indirectly. Dreams might even involve meetings with colleagues of geometricfuture and if their behavior is very respectful, unlucky theoretician might wonder whether his work might have been recognized or is this only wishful thinking!
3. Usually it is thought the recollection of past is not good idea. One can however argue that it communication not only with subjective past but also with objective future (the world external to personal CD). This would give information about the external world of geometric future and also increase the span the time scale of conscious experience and of temporal quantum coherence. This might helpful or a theoretician not interested in fashionable thinking only.

7.3 Can one define the analogs of Mandelbrot and Julia sets in TGD framework?

The stimulus to this contribution came from the question related to possible higher-dimensional analogs of Mandelbrot and Julia sets (see this). The notion complex analyticity plays a key role in the definition of these notions and it is not all clear whether one can define these analogs.

I have already earlier considered the iteration of polynomials in the TGD framework [**?**] suggesting the TGD counterparts of these notions. These considerations however rely on a view of $M^8 - H$ duality which is replaced with dramatically simpler variant and utilizing the holography=holomorphy principle [L138] so that it is time to update these ideas.

This principle states that space-time surfaces are analogous to Bohr orbits for particles which are 3-D surfaces rather than point-like particles. Holography is realized in terms of space-time surfaces which can be regarded as complex surfaces in $H = M^4 \times CP_2$ in the generalized sense. This means that one can give H 4 generalized complex coordinates and 3 such generalized complex coordinates can be used for the 4-surface. These surfaces are always minimal surfaces irrespective of the action defining them as its externals and the action makes itself visible only at the singularities of the space-time surface.

7.3.1 Ordinary Mandelbrot and Julia sets

Consider first the ordinary Mandelbrot and Julia sets.

1. The simplest example of the situation is the map $f : z \to z^2 + c$. One can consider the iteration of f by starting from a selected point z and look for various values of complex parameter c whether the iteration converges or diverges to infinity. The interface between the sets of the complex c-plane is 1-D Mandelbrot set and is a fractal. One can generalize the iteration to an arbitrary rational function f, in particular polynomials.

2. For polynomials of degree n also consider $n-1$ parameters c_i, $i = 1, ..., n$, to obtain $n-1$ complex-dimensional analog of Mandelbrot set as boundaries of between regions where the iteration lead or does not lead to infinity. For $n = 2$ one obtains a 4-D set.
3. One can also fix the parameter c and consider the iteration of f. Now the complex z-plane decomposes to two a finite region with a finite number of components and its complement, Fatou set. The iteration does not lead out from the finite region but diverges in the complement. The 1-D fractal boundary between these regions is the Julia set.

7.3.2 Holography= holomorphy principle

The generalization to the TGD framework relies heavily on holography=holomorphy principle.

1. In the recent formulation of TGD, holography required by the realization of General Coordinate Invariance is realized in terms of two functions f_1, f_2 of 4 analogs of generalized complex coordinates, one of them corresponds to the light-like (hypercomplex) M^4 coordinate for a surface $X^2 \subset M^4$ and the 3 complex coordinates to those of Y^2 orthogonal to X^2 and the two complex coordinates of CP_2.
   Space-time surfaces are defined by requiring the vanishing of these two functions: $(f_1, f_2) = (0, 0)$. They are minimal surfaces irrespective of the action as long it is general coordinate invariant and constructible in terms of the induced geometry.
2. In the number theoretic vision of TGD, $M^8 - H$-duality [L138] maps the space-time as a holomorphic surface $X^4 \subset H$ is mapped to an associative 4-surface $Y^4 \subset M^8$. The condition for holography in M^8 is that the normal space of Y^4 is quaternionic.
   In the number theoretic vision, the functions f_i are naturally rational functions or polynomials of the 4 generalized complex coordinates. I have proposed that the coefficients of polynomials are rationals or even integers, which in the most stringent approach are smaller than the degree of the polynomial. In the most general situation one could have analytic functions with rational Taylor coefficients.
   The polynomials $f_i = P_i$ form a hierarchy with respect to the degree of P_i, and the iteration defined is analogous to that appearing in the 2-D situation. The iteration of P_i gives a hierarchy of algebraic extensions, which are central in the TGD view of evolution as an increase of algebraic complexity. The iteratikon would also give a hierarchy of increasingly complex space-time surface and the approach to chaos at the level of space-time would correspond to approach of Mandelbrot or Julia set.
3. In the TGD context, there are 4-complex coordinates instead of 1 complex coordinate z. The iteration occurs in H and the vanishing conditions for the iterates define a sequence of 4-surfaces. The initial surface is defined by the conditions $(f_1, f_2) = 0$. This set is analogous to the set $f(z) = 0$ for ordinary Julia sets.
   One could consider the iteration as $(f_1, f_2) \rightarrow (f_1 \circ f_1, f_2 \circ f_2)$ continued indefinitely. One could also iterate only f_1 or f_2. Each step defines by the vanishing conditions a 4-D surface, which would be analogous to the image of the $z = 0$ in the 2-D iteration. The iterates form a sequence of 4-surfaces of H analogous to a sequence of iterates of z in the complex plane.
   The sequence of 4-surfaces also defines a sequence of points in the "world of classical worlds" (WCW) analogous to the sequence of points $z, f(z), ...$ This conforms with the idea that 3-surface is a generalization of point-like particles, which by holography can be replaced by a Bohr orbit-like 4-surface.
4. Also in this case, one can see whether the iteration converges to a finite result or not. In the zero energy ontology (ZEO), convergence could mean that the iterates of X^4 stay within a causal diamond CD having a finite volume.

7.3.3 The counterparts of Mandelbrot and Julia sets at the level of WCW

What the WCW analogy of the Mandelbrot and Julia sets could look like?

1. Consider first the Mandelbrot set. One could start from a set of roots of $(f_1, f_2) = (c_1, c_2)$ equivalent with the roots of $(f_1 - c_1, f_2 - c_2) = (0, 0)$. Here c_1 and c_2 define complex parameters analogous to the parameter c of the Mandelbrot sent. One can iterate the two functions for all

pairs (c_1, c_2). One can look whether the iteration converges or not and identify the Mandelbrot set as the critical set of parameters (c_1, c_2). The naive expectation is that this set is 3-D dimensional fractal.

2. The definition of Julia set requires a complex plane as possible initial points of the iteration. Now the iteration of $(f_1, f_2) = 0$ fixes the starting point (not necessarily uniquely since 3-D surface does not fix the Bohr orbit uniquely: this is the basic motivation for ZEO). The analogy with the initial point of iteration suggests that we can assume $(f_1, f_2) = (c_1, c_2)$ but this leads to the analog of the Mandelbrot set. The notions coincide at the level of WCW.

3. Mandelbrot and Julia sets and their generalizations are critical in a well-defined sense. Whether iteration could be relevant for quantum dynamics is of course an open question. Certainly it could correspond to number theoretic evolution in which the dimension of the algebraic extension rapidly increases. For instance, one could one consider a WCW spinor field as a wave function in the set of converging iterates. Quantum criticality would correspond to WCW spinor fields restricted to the Mandelbrot or Julia sets.

Could the 3-D analogs of Mandelbrot and Julia sets correspond to the light-like partonic orbits defining boundaries between Euclidean and Minkowskian regions of the space-time surface and space-time boundaries? Can the extremely complex fractal structure as sub-manifold be consistent with the differentiability essential for the induced geometry? Could light-likeness help here.

7.3.4 Do the analogs of Mandelbrot and Julia sets exist at the level of space-time?

Could one identify the 3-D analogs of Mandelbrot and Julia sets for a given space-time surface? There are two approaches.

1. The parameter space (c_1, c_2) for a given initial point h of H for iterations of $f_1 - c_1, f_2 - c_2)$ defines a 4-D complex subspace of WCW. Could one identify this subset as a space-time surface and interpret the coordinates of H as parameters? If so, there would be a duality, which would represent the complement of the Fatou set (the thick Julia set) defined as a subset of WCW as a space-time surface!

2. One could also consider fixed points of iteration for which iteration defines a holomorphic map of space-time surface to itself. One can consider generalized holomorphic transformations of H leaving X^4 invariant locally. If they are 1-1 maps they have interpretation as general coordinate transformations. Otherwise they have a non-trivial physical effect so that the analog of the Julia set has a physical meaning. For these transformations one can indeed find the 3-D analog of Julia set as a subset of the space-time surface. This set could define singular surface or boundary of the space-time surface.

7.3.5 Could Mandelbrot and Julia sets have 2-D analogs in TGD?

What about the 2-D analogs of the ordinary Julia sets? Could one identify the counterparts of the 2-D complex plane (coordinate z) and parameter space (coordinate c).

1. Hamilton-Jacobi structure defines what the generalized complex structure is [L134] and defines a slicing of M^4 in terms of integrable distributions of string world sheets and partonic 2-surfaces transversal or even orthogonal to each other. Partonic 2-surface could play the role of complex plane and string world sheet the role of the parameter space or vice versa.
   Partonic 2-surfaces *resp.* and string world sheet having complex *resp.* hyper-complex structures would therefore be in a key role. $M^8 - H$ duality maps these surfaces to complex *resp.* co-complex surfaces of octonions having Minkowskian norm defined as number theoretically as $Re(o^2)$.

2. In the case of Julia sets, one could consider generalized holomorphic transformations of H mapping X^4 to itself as a 4-surface but not reducing to 1-1 maps. If f_2 (f_1) acts trivially at the partonic 2-surface Y^2 (string world sheet X^2), the iteration reduces to that for f_1 (f_2). Within string world sheets and partonic 2-surfaces the iteration defines Julia set and its hyperbolic analog in the standard way. One can argue that string world sheets and partonic

2-surfaces should correspond to singularities in some sense. Singularity could mean this fixed point property.

The natural proposal is that the light-like 3-surfaces defining boundaries between Euclidean and Minkowskian regions of the space-time surface define light-like orbits of the partonic 2-surface. And string world sheets are minimal surfaces having light-like 1-D boundaries at the partonic 2-surface having physical interpretation as world-lines of fermions.

One could also iterate only f_1 or f_2 allow the parameter c of the initial value of f_1 to vary. This would give the analog of Mandelbrot set as a set of 2-D surfaces of H and it might have dual representation as a 2-surface.

3. The 2-D analog of the Mandelbrot set could correspond to a set of 2-surfaces obtained by fixing a point of the string world sheet X^2. Also now one could consider holomorphic maps leaving the space-time surface locally but not acting 1-1 way. The points of Y^2 would define the values of the complex parameter c remaining invariant under these maps. The convergence of the iteration of f_1 in the same sense as for the Mandelbrot fractal would define the Mandelbrot set as a critical set. For the dual of the Mandelbrot set X^2 and Y^2 would change their roles.

Chapter 8

TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors

8.1 Introduction

There are two mysterious looking correspondences involving ADE groups. McKay correspondence between McKay graphs characterizing tensor products for finite subgroups of $SU(2)$ and Dynkin diagrams of affine ADE groups is the first one. The correspondence between principal diagrams characterizing inclusions of hyper-finite factors of type II_1 (HFFs) with Dynkin diagrams for a subset of ADE groups and Dynkin diagrams for affine ADE groups is the second one.

I have considered the interpretation of McKay correspondence in TGD framework already earlier [K112, K43] but the decision to look it again led to a discovery of a bundle of new ideas allowing to answer several key questions of TGD.

1. Asking questions about $M^8 - H$ duality at the level of 8-D momentum space [L46] led to a realization that the notion of mass is relative as already the existence of alternative QFT descriptions in terms of massless and massive fields suggests (electric-magnetic duality). Depending on choice $M^4 \subset M^8$, one can describe particles as massless states in $M^4 \times CP_2$ picture (the choice is M^4_L depending on state) and as massive states (the choice is fixed M^4_T) in M^8 picture. p-Adic thermal massivation of massless states in M^4_L picture can be seen as a universal dynamics independent mechanism implied by ZEO. Also a revised view about zero energy ontology (ZEO) based quantum measurement theory as theory of consciousness suggests itself.
2. Hyperfinite factors of type II_1 (HFFs) [K112, K43] and number theoretic discretization in terms of what I call cognitive representations [L63] provide two alternative approaches to the notion of finite measurement resolution in TGD framework. One obtains rather concrete view about how these descriptions relate to each other at the level of 8-D space of light-like momenta. Also ADE hierarchy can be understood concretely.
3. The description of 8-D twistors at momentum space-level is also a challenge of TGD. 8-D twistorializations in terms of octo-twistors (M^4_T description) and $M^4 \times CP_2$ twistors (M^4_L description) emerge at embedding space level. Quantum twistors could serve as a twistor description at the level of space-time surfaces.

8.1.1 McKay correspondence in TGD framework

Consider first McKay correspondence in more detail.

1. McKay correspondence states that the McKay graphs characterizing the tensor product decomposition rules for representations of discrete and finite sub-groups of $SU(2)$ are Dynkin diagrams for the affine ADE groups obtained by adding one node to the Dynkin diagram of ADE group. Could this correspondence make sense for any finite group G rather than only discrete subgroups of $SU(2)$? In TGD Galois group of extensions K of rationals can be any finite group G. Could Galois group take the role of G?
2. Why the subgroups of $SU(2)$ should be in so special role? In TGD framework quaternions and octonions play a fundamental role at M^8 side of $M^8 - H$ duality [L46]. Complexified M^8 represents complexified octonions and space-time surfaces X^4 have quaternionic tangent or normal spaces. $SO(3)$ is the automorphism group of quaternions and for number theoretical discretizations induced by extension K of rationals it reduces to its discrete subgroup $SO(3)_K$ having $SU(2)_K$ as a covering. In certain special cases corresponding to McKay correspondence this group is finite discrete group acting as symmetries of Platonic solids. Could this make the Platonic groups so special? Could the semi-direct products $Gal(K) \triangleleft SU(2)_K$ take the role of discrete subgroups of $SU(2)$?

8.1.2 HFFs and TGD

The notion of measurement resolution is definable in terms of inclusions of HFFs and using number theoretic discretization of X^4. These definitions should be closely related.

1. The inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs with index $\mathcal{M} : \mathcal{N} < 4$ are characterized by Dynkin diagrams for a subset of ADE groups. The TGD inspired conjecture is that the inclusion hierarchies of extensions of rationals and of corresponding Galois groups could correspond to the hierarchies for the inclusions of HFFs. The natural realization would be in terms of HFFs with coefficient field of Hilbert space in extension K of rationals involved.
   Could the physical triviality of the action of unitary operators $\mathcal{N}$ define measurement resolution? If so, quantum groups assignable to the inclusion would act in quantum spaces associated with the coset spaces $\mathcal{M}/\mathcal{N}$ of operators with quantum dimension $d = \mathcal{M} : \mathcal{N}$. The degrees of freedom below measurement resolution would correspond to gauge symmetries assignable to $\mathcal{N}$.
2. Adelic approach [L52] provides an alternative approach to the notion of finite measurement resolution. The cognitive representation identified as a discretization of X^4 defined by the set of points with points having H (or at least M^8 coordinates) in K would be common to all number fields (reals and extensions of various p-adic number fields induced by K). This approach should be equivalent with that based on inclusions. Therefore the Galois groups of extensions should play a key role in the understanding of the inclusions.

How HFFs could emerge from TGD?

1. The huge symmetries of "world of classical words" (WCW) could explain why the ADE diagrams appearing as McKay graphs and principal diagrams of inclusions correspond to affine ADE algebras or quantum groups. WCW consists of space-time surfaces X^4, which are preferred extremals of the action principle of the theory defining classical TGD connecting the 3-surfaces at the opposite light-like boundaries of causal diamond $CD = cd \times CP_2$, where cd is the intersection of future and past directed light-cones of M^4 and contain part of $\delta M^4_\pm \times CP_2$. The symplectic transformations of $\delta M^4_+ \times CP_2$ are assumed to act as isometries of WCW. A natural guess is that physical states correspond to the representations of the super-symplectic algebra SSA.
2. The sub-algebras SSA_n of SSA isomorphic to SSA form a fractal hierarchy with conformal weights in sub-algebra being n-multiples of those in SSA. SSA_n and the commutator $[SSA_n, SSA]$ would act as gauge transformations. Therefore the classical Noether charges for these sub-algebras would vanish. Also the action of these two sub-algebras would annihilate the quantum states. Could the inclusion hierarchies labelled by integers $.. < n_1 < n_2 < n_3....$ with n_{i+1} divisible by n_i would correspond hierarchies of HFFs and to the hierarchies of extensions of rationals and corresponding Galois groups? Could n correspond to the dimension of Galois group of K.
3. Finite measurement resolution defined in terms of cognitive representations suggests a reduction of the symplectic group SG to a discrete subgroup SG_K, whose linear action is char-

acterized by matrix elements in the extension K of rationals defining the extension. The representations of discrete subgroup are infinite-D and the infinite value of the trace of unit operator is problematic concerning the definition of characters in terms of traces. One can however replace normal trace with quantum trace equal to one for unit operator. This implies HFFs and the hierarchies of inclusions of HFFs [K112, K43]. Could inclusion hierarchies for extensions of rationals correspond to inclusion hierarchies of HFFs and of isomorphic subalgebras of SSA?

Quantum spinors are central for HFFs. A possible alternative interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group. This has also application in TGD inspired theory of consciousness [K43]: the idea is that the truth value of Boolean statement is fuzzy. At the level of quantum measurement theory this would mean that the outcome of quantum measurement is not anymore precise eigenstate but that one obtains only probabilities for the appearance of different eigenstate. One might say that probability of eigenstates becomes a fundamental observable and measures the strength of belief.

8.1.3 New aspects of $M^8 - H$ duality

M^8-H duality ($H = M^4 \times CP_2$) [L46] has become one of central elements of TGD. M^8-H duality implies two descriptons for the states.

1. $M^8 - H$ duality assumes that space-time surfaces in M^8 have associative tangent- or normal space M^4 and that these spaces share a common sub-space $M^2 \subset M^4$, which corresponds to complex subspace of octonions (also integrable distribution of $M^2(x)$ can be considered). This makes possible the mapping of space-time surfaces $X^4 \subset M^8$ to $X^4 \subset H = M^4 \times CP_2$) giving rise to $M^8 - H$ duality.
2. M^8-H duality makes sense also at the level of 8-D momentum space in one-one correspondence with light-like octonions. In $M^8 = M^4 \times E^4$ picture light-like 8-momenta are projected to a fixed quaternionic $M^4_T \subset M^8$. The projections to $M^4_T \supset M^2$ momenta are in general massive. The group of symmetries is for E^4 parts of momenta is $Spin(SO(4)) = SU(2)_L \times SU(2)_R$ and identified as the symmetries of low energy hadron physics.
$M^4 \supset M^2$ can be also chosen so that the light-like 8-momentum is parallel to $M^4_L \subset M^8$. Now CP_2 codes for the E^4 parts of 8-momenta and the choice of M^4_L and color group $SU(3)$ as a subgroup of automorphism group of octonions acts as symmetries. This correspond to the usual description of quarks and other elementary particles. This leads to an improved understanding of $SO(4) - SU(3)$ duality. A weaker form of this duality $S^3 - CP_2$ duality: the 3-spheres S^3 with various radii parameterizing the E^4 parts of 8-momenta with various lengths correspond to discrete set of 3-spheres S^3 of CP_2 having discrete subgroup of $U(2)$ isometries.
3. The key challenge is to understand why the MacKay graphs in McKay correspondence and principal diagrams for the inclusions of HFFs correspond to ADE Lie groups or their affine variants. It turns out that a possible concrete interpretation for the hierarchy of finite subgroups of $SU(2)$ appears as discretizations of 3-sphere S^3 appearing naturally at M^8 side of $M^8 - H$ duality. Second interpretation is as covering of quaternionic Galois group. Also the coordinate patches of CP_2 can be regarded as piles of 3-spheres and finite measurement resolution. The discrete groups of $SU(2)$ define in a natural way a hierarchy of measurement resolutions realized as the set of light-like M^8 momenta. Also a concrete interpretation for Jones inclusions as inclusions for these discretizations emerges.
4. A radically new view is that descriptions in terms of massive and massless states are alternative options leads to the interpretation of p-adic thermodynamics as a completely universal massivation mechanism having nothing to do with dynamics. The problem is the paradoxical looking fact that particles are massive in H picture although they should be massless by definition. The massivation is unavoidable if zero energy states are superposition of massive states with varying masses. The M^4_L in this case most naturally corresponds to that associated with the dominating part of the state so that higher mass contributions can be described by using

p-adic thermodynamics and mass squared can be regarded as thermal mass squared calculable by p-adic thermodynamics.

5. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory. 4-D space-time surfaces correspond to roots of octonionic polynomials $P(o)$ with real coefficients corresponding to the vanishing of the real or imaginary part of $P(o)$.
   These polynomials however allow universal roots, which are not 4-D but analogs of 6-D branes and having topology of S^6. Their M^4 projections are time =constant snapshots $t = r_n, r_M \leq r_n$ 3-balls of M^4 light-cone (r_n is root of $P(x)$). At each point the ball there is a sphere S^3 shrinking to a point about boundaries of the 3-ball.
   What suggests itself is following "braney" picture. 4-D space-time surfaces intersect the 6-spheres at 2-D surfaces identifiable as partonic 2-surfaces serving as generalized vertices at which 4-D space-time surfaces representing particle orbits meet along their ends. Partonic 2-surfacew would define the space-time regions at which one can pose analogs of boundary values fixing the space-time surface by preferred extremal property. This would realize strong form of holography (SH): 3-D holography is implied already by ZEO.
   This picture forces to consider a modification of the recent view about ZEO based theory of consciousness. Should one replace causal diamond (CD) with light-cone, which can be however either future or past directed. "Big" state function reductions (BSR) meaning the death and re-incarnation of self with opposite arrow of time could be still present. An attractive interpretation for the moments $t = r_n$ would be as moments assignable to "small" state function reductions (SSR) identifiable as "weak" measurements giving rise to sensory input of conscious entity in ZEO based theory of consciousness. One might say that conscious entity becomes gradually conscious about its roots in increasing order. The famous question "What it feels to be a bat" would reduce to "What it feels to be a polynomial?"! One must be however very cautious here.

8.1.4 What twistors are in TGD framework?

The basic problem of the ordinary twistor approach is that the states must be massless in 4-D sense. In TGD framework particles would be massless in 8-D sense. The meaning of 8-D twistorialization at space-time level is relatively well understood but at the level of momentum space the situation is not at all so clear.

1. In TGD particles are massless in 8-D sense. For M^4_L description particles are massless in 4-D sense and the description at momentum space level would be in terms of products of ordinary M^4 twistors and CP_2 twistors. For M^4_T description particles are massive in 4-D sense. How to generalize the twistor description to 8-D case?
   The incidence relation for twistors and the need to have index raising and lowering operation in 8-D situation suggest the replacement of the ordinary l twistors with either with octo-twistors or non-commutative quantum twistors.
2. I have assumed that what I call geometric twistor space of M^4 is simply $M^4 \times S^2$. It however turned out that one can consider standard twistor space CP_3 with metric signature (3,-3) as an alternative. This option reproduces the nice results of the earlier approach but the philosophy is different: there is no fundamental length scale but the hierarchy of causal diamonds (CDs) predicted by zero energy ontology (ZEO) gives rise to the breaking of the exact scaling invariance of M^8 picture. This forces to modify $M^8 - H$ correspondence so that it involves map from M^4 to CP_3 followed by a projection to hyperbolic variant $CP_{2,h}$ of CP_2. Note that also the original form of $M^8 - H$ duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$.
   M^4 in H would not be be replaced with conformally compactified M^4 (M^4_{conf}) but conformally compactified cd (cd_{conf}) for which a natural identification is as CP_2 with second complex coordinate replaced with hypercomplex coordinate. The sizes of twistor spaces of cd_{conf} using CP_2 size as unit would reflect the hierarchy of size scales for CDs. The consideration on the twistor space of M^8 in similar picture leads to the identification of corresponding twistor space as HP_3 - quaternionic variant of CP_3: the counterpart of CD_8 would be HP_2.
3. Octotwistors can be expressed as pairs of quaternionic twistors. Octotwistor approach sug-

gests a generalization of twistor Grassmannian approach obtained by replacing the bi-spinors with complexified quaternions and complex Grassmannians with their quaternionic counterparts. Although TGD is not a quantum field theory, this proposal makes sense for cognitive representations identified as discrete sets of spacetime points with coordinates in the extension of rationals defining the adele [L52] implying effective reduction of particles to point-like particles.

4. The outcome of octo-twistor approach together with M^8-H duality leads to a nice picture view about twistorial description of massive states based on quaternionic generalization of twistor Grassmannian approach. A radically new view is that descriptions in terms of massive and massless states are alternative options, and correspond to two different alternative twistorial descriptions and leads to the interpretation of p-adic thermodynamics as completely universal massivation mechanism having nothing to do with dynamics. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory relying on the universal roots of octonionic polynomials of M^8, which are not 4-D but analogs of 6-D branes. By M^8-H duality the finite sub-groups of $SU(2)$ of McKay correspondence appear quite concretely in the description of the measurement resolution of 8-momentum.

What about super-twistors in TGD framework?

1. The parallel progress in the understanding SUSY in TGD framework [L81] in turn led to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.
2. The great surprise from physics point of view is that in fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites identifiable as spartners of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of embedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.
3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

What about the interpretation of quantum twistors? They could make sense as 4-D space-time description analogous to description at space-time level. Now one can consider generalization of the twistor Grassmannian approach in terms of quantum Grassmannians.

8.2 McKay correspondence

Consider first McKay correspondence from TGD point of view.

8.2.1 McKay graphs

McKay graps are defined in the following manner. Consider group G which is discrete but not necessarily finite. If the group is finite there is a finite number of irreducible representations χ_I. Select preferred representation V - usually V is taken to be the fundamental representation of G and form tensor products $\chi_I \otimes V$. Suppose irrep χ_J appears n_{ij} times in the tensor product $\chi_I \otimes \chi_0$. Assign to each representation χ_I dot and connect the dots of χ_I and χ_J by n_{ij} arrows. This gives rise to MacKay graph.

Consider now the situation for finite-D groups of $SU(2)$. 2-D $SU(2)$ spinor representation as a fundamental representation is essential for obtaining the identification of McKay graphs as Dynkin diagrams of simply laced affine algebras having only single line connecting the roots (the angle between positive roots is 120 degrees) (see `http://tinyurl.com/z48d92t`).

1. For $SU(2)$ representations one has the basic rule $j_1 - 1/2 \leq j \leq j_1 + 1/2$ for the tensor product $j_1 \otimes 1/2$. This rule must be broken for finite subgroups of $SU(2)$ since the number of

representations if finite so that branching point appears in McKay graph. In branching point the decomposition of $j_1 \otimes 1/2$ decomposes to 3 lower-dimensional representations of the finite subgroup takes place.

2. Simply lacedness means that given representation appears only once in $chi_I \otimes V$, when V is 2-D representation as it can be for a subgroup of $SU(2)$. Additional exceptional properties is the absence of loops ($n_{ii} = 0$) and connectedness of McKay graph.
3. One can consider binary icosahedral group (double covering of icosahedral group A_5 with order 60) as an example (for the McKay graph see `http://tinyurl.com/y2h55jwp`). The representations of A_5 are $1_A, 3_A, 3'_B, 4_A, 5_A$, where integer tells the dimension. Note that $SO(3)$ does not allow 4-D representation. For binary icosahedral group one obtains also the representations $2_A, 2'_B, 4_B, 6_A$. The McKay graph of E_8 contains one branching point in which one has the tensor product of 6-D and 2-D representations 6_A and 2_A giving rise to $5_A \oplus 3_C \oplus 4_B$.

McKay graphs can be defined for any finite group and they are not even unions of simply laced diagrams unless one has subgroups of $SU(2)$. Still one can wonder whether McKay correspondence generalizes from subgroups of $SU(2)$ to all finite groups. At first glance this does not seem possible but there might be some clever manner to bring in all finite groups.

The proposal has been that this McKay correspondence has a deeper meaning. Could simply laced affine ADE algebras, ADE type quantum algebras, and/or corresponding finite groups act as symmetry algebras in TGD framework?

8.2.2 Number theoretic view about McKay correspondence

Could the physical picture provided by TGD help to answer the above posed questions?

1. Could one identify discrete subgroups of $SU(2)$ with those of the covering group $SU(2)$ of $SO(3)$ of quaternionic automorphisms defining the continuous analog of Galois group and reducing to a discrete subgroup for a finite resolution characterized by extension K of rationals. The tensor products of 2-D spinor representation of these discrete subgroups $SU(2)_K$ would give rise to irreps appearing in the McKay graph.
2. In adelic physics [L52] extensions K of rationals define an evolutionary hierarchy with effective Planck constant $h_{eff}/h_0 = n$ identified as the dimension of K. The action of discrete and finite subgroups of various symmetry groups can be represented as Galois group action making sense at the level of X^4 [L46] for what I have called cognitive representations. By $M^8 - H$ duality also the Galois group of quaternions and its discrete subgroups appear naturally.
This suggests a possible generalization of McKay correspondence so that it would apply to all finite groups G. Any finite group G can appear as Galois group. The Galois group $Gal(K)$ characterizing the extension of rationals induces in turn extensions of p-adic number fields appearing in the adele. Could the representation of G as Galois group of extension of rationals allow to generalize McKay correspondence?

Could the following argument inspired by these observations make sense?

1. $SU(2)$ is identified as spin covering of the quaternionic automorphism group. One can define the subgroups in matrix representation of $SU(2)$ based on complex numbers. One can replace complex numbers with the extension of rationals and speak of group $SU(2)_K$ identified as a discrete subgroup of $SU(2)$ having in general infinite order.
The discrete finite subgroups $G \subset SU(2)$ appearing in the standard McKay correspondence correspond to extensions K of rationals for which one has $G = SU(2)_K$. These special extensions can be identified by studying the matrix elements of the representation of G and include the discrete groups Z_n acting as rotation symmetries of the Platonic solids. For instance, for icosahedral group Z_2,Z_3 and Z_5 are involved and correspond to roots of unity.
2. The semi-direct product $Gal \triangleleft SU(2)_K$ with group action

$$(gal_1, g_1)(gal_2, g_2) = (gal_1 \circ gal_2, g_1(gal_1 g_2))$$

makes sense. The action of $Gal \triangleleft SU(2)_K$ in the representation is therefore well-defined. Since all finite groups G can appear as Galois groups, it seems that one obtains extension of the McKay correspondence to semi-direct products involving all finite groups G representable as Galois groups.

3. A good guess is that the number of representations of $SU(2)_K$ involved is infinite if $SU(2)_K$ has infinite order. For $\tilde{A}_n$ and $\tilde{D}_n$ the branching occurs only at the ends of the McKay graph. For E_k, $k = 6, 7, 8$ the branching occurs in middle of the graph (see `http://tinyurl.com/y2h55jwp`). What happens for arbitrary G. Does the branching occur at all? One could argue that if the discrete subgroup has infinite order, the representation can be completed to a representation of $SU(2)$ in terms of real numbers so that the McKay graphs must be identical.
4. A concrete realization of ADE type Kac-Moody algebras is proposed. It relies on the group algebra of $Gal(K) \triangleleft SU(2)_K$ and free field representation of ADE type Kac-Moody algebra identifying the free scalar fields in Kac-Moody Cartan algebra as group algebra elements defined by the traces of representation matrices (characters).
5. A possible interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group [K43]. TGD inspired theory of consciousness is a possible application.
   Also the notion of quantum twistor [L88] can be considered. In TGD particles are massless in 8-D sense and in general massive in 4-D sense but 4-D twistors are needed also now so that a modification of twistor approach is needed. The incidence relation for twistors suggests the replacement of the usual twistors with non-commutative quantum twistors.

8.3 ADE diagrams and principal graphs of inclusions of hyperfinite factors of type II_1

Dynkin diagrams for ADE groups and corresponding affine groups characterize also the inclusions of hyperfinite factors of type II_1 (HFFs) [K43].

8.3.1 Principal graphs and Dynkin diagrams for ADE groups

1. If the index $\beta = \mathcal{M} : \mathcal{N}$ of the Jones inclusion satisfies $\beta < 4$, the affine Dynkin diagrams of $SU(n)$ (discrete symmetry groups of n-polygons) and E_7 (symmetry group of octahedron and cube) and $D(2n+1)$ (symmetries of double 2n+1-polygons) are not allowed.
2. Vaughan Jones [A85] (see `http://tinyurl.com/y8jzvogn`) has speculated that these finite subgroups could correspond to quantum groups as kind of degenerations of Kac-Moody groups. Modulo arithmetics defined by the integer n defining the quantum phase suggests itself strongly. For $\beta = 4$ one can construct inclusions characterized by extended Dynkin diagram and any finite sub-group of $SU(2)$. In this case affine ADE hierarchy appear as principal graphs characterizing the inclusions. For $\beta < 4$ the finite measurement resolution could reduce affine algebra to quantum algebra.
3. The rule is that for odd values of n defining the quantum phase the Dynkin diagram does not appear. If Dynkin diagrams correspond to quantum groups, one can ask whether they allow only quantum group representations with quantum phase $q = exp(i\pi/n)$ equal to even root of unity.

8.3.2 Number theoretic view about inclusions of HFFs and preferred role of $SU(2)$

Consider next the TGD inspired interpretation.

1. TGD suggests the interpretation in terms of representations of $Gal(K(G)) \triangleleft G$ for finite subgroups G of $SU(2)$, where $K(G)$ would be an extension associated with G. This would generalize to subgroups of $SU(2)$ with infinite order in the case of arbitrary Galois group. Quantum groups have finite number of representations in 1-1-correspondence with terms of finite-D representations of G. Could the representations of $Gal(K(G)) \triangleleft G$ correspond to the representations of quantum group defined by G?
   This would conform with the vision that there are two ways to realize finite measurement resolution. The first one would be in terms of inclusions of hyper-finite factors. Second would

be in terms cognitive representations defining a number theoretic discretization of X^4 with embedding space coordinates in the extension of rationals in which Galois group acts.
In fact, also the discrete subgroup of infinite-D group of symplectic transformations of $\Delta M^4_+ \times CP_2$ would act in the cognitive representations and this suggests a far reaching implications concerning the understanding of the cognitive representations, which pose a formidable looking challenge of finding the set of points of X^4 in given extension of rationals [L78].

2. This brings in mind also the model for bio-harmony in which genetic code is defined in terms of Hamiltonian cycles associated with icosahedral and tetrahedral geometries [L24, L67]. One can wonder why the Hamiltonian cycles for cubic/octahedral geometry do not appear. On the other hand, according to Vaughan the Dynkin diagram for E_7 is missing from the hierarchy of so principal graphs characterizing the inclusions of HFFs for $\beta < 4$ (a fact that I failed to understand). Could the genetic code directly reflect the properties of the inclusion hierarchy?

How would the hierarchies of inclusions of HFFs and extensions of rationals relate to each other?

1. I have proposed that the inclusion hierarchies of extensions K of rationals accompanied by similar hierarchies of Galois groups $Gal(K)$ could correspond to a fractal hierarchy of sub-algebras of hyperfinite factor of type II_1. Quantum group representations in ADE hierarchy would somehow correspond to these inclusions. The analogs of coset spaces for two algebras in the hierarchy define would quantum group representations with quantum dimension characterizing the inclusion.
2. The quantum group in question would correspond to a quantum analog of finite-D group of $SU(2)$ which would be in completely unique role mathematically and physically. The infinite-D group in question could be the symplectic group of $\delta M^4_+ \times CP_2$ assumed to act as isometries of WCW. In the hierarchy of Galois groups the quantum group of finite group $G \subset SU(2)$ would correspond to Galois group of an extension K.
3. The trace of unit matrix defining the character associated with unit element is infinite for these representations for factors of type I. Therefore it is natural to assume that hyper-finite factor of type II_1 for which the trace of unit matrix can be normalized to 1. Sub-factors would have trace of projector with trace smaller than 1.
4. Do the ADE diagrams for groups $Gal(K(G)) \triangleleft G$ indeed correspond to quantum groups and affine algebras? Why the finite groups should give rise to affine/Kac-Moody algebras? In number theoretic vision a possible answer would be that depending on the value of the index β of inclusion the symplectic algebra reduces in the number theoretic discretization by gauge conditions specifying the inclusion either to Kac-Moody group ($\beta = 4$) or to quantum group ($\beta < 4$).

What about subgroups of groups other than $SU(2)$? According to Vaughan there has been work about inclusion hierarchies of $SU(3)$ and other groups and it seems that the results generalize and finite subgroups of say $SU(3)$ appear. In this case the tensor products of finite sub-groups McKay graphs do not however correspond to the principal graphs for inclusions. Could the number theoretic vision come in rescue with the replacement of discrete subgroup with Galois group and the identification of $SU(2)$ as the covering for the Galois group of quaternions?

8.3.3 How could ADE type quantum groups and affine algebras be concretely realized?

The questions discussed are following. How to understand the correspondence between the McKay graph for finite group $G \subset SU(2)$ and ADE (affine) group Dynkin diagram for $\beta < 4$ ($\beta = 4$)? How the nodes of McKay grap representing the irreps of finite group can correspond to the positive roots of a Dynkin diagram, which are essentially vectors defined by eigenvalues of Cartan algebra generators for complexified Lie-algebra basis.

The first thing that comes in mind is the construction of representation of Kac-Moody algebra using scalar fields labelled by Cartan algebra generators (see `http://tinyurl.com/y9lkeelk`): these representations are discussed by Edward Frenkel [A45]. The charged generators of Kac-Moody algebra in the complement of Cartan algebra are obtained by exponentiating the contractions of the vector formed by these scalar fields with roots to get $\alpha \cdot \Phi = \alpha_i \Phi^i$. The charged field is represented as a normal ordered product $: exp(i\alpha \cdot \Phi) :$. If one can assign to each irrep of G a scalar field in a natural manner one could achieve this.

Neglect first the presence of the group algebra of $Gal(K(G)) \triangleleft G$. The standard rule for the dimensions of the representations of finite groups reads as $\sum_i d_I^2 = n(G)$. For double covering of G one obtains this rule separately for integer spin representations and half-odd integers spin representations. An interesting possibility is that this could be interpreted in terms of supersymmetry at the level of group algebra in which representation of dimension d_I appears d_I times.

The group algebra of G and its covering provide a universal manner to realize these representations in terms of a basis for complex valued functions in group (for extensions of rationals also the values of the functions must belong to the extension).

1. Representation with dimension d_I occurs d_I times and corresponds to $d_I \times d_I$ representation matrices D^I_{mn} of representation χ_I, whose columns and rows provide representations for left- and right-sided action of G. The tensor product rules for the representations χ_I can be formulated as double tensor products. For basis states $|J, n\rangle$ in χ_I and $|J, n\rangle$ in χ_J one has

$$|I, m\rangle_\otimes |J, n\rangle = c^{K,p}_{I,m|J,n} |K, p\rangle \ ,$$

where $c^{K,p}_{J,n|J,n}$ are Glebch-Gordan coefficients.

2. For the tensor product of matrices D^I_{mn} and D^J_{mn} one must apply this rule to both indices. The orthogonality properties of Glebsch-Gordan coefficients guarantee that the tensor product contains only terms in which one has same representation at left- and right-hand side. The orthogonality rule is

$$\sum_{m,n} c^{K,p}_{I,m|J,n} c^{K,q}_{I,r|J,s} \propto \delta_{K,L} \ .$$

3. The number of states is $n(G)$ whereas the number $I(G)$ of irreps corresponds to the dimension of Cartan algebra of Kac-Moody algebra or of quantum group is smaller. One should be able to pick only one state from each representation D^I.
The condition that the state X of group algebra is invariant under automorphism gXg^{-1} implies that the allowed states as function in group algebra are traces $Tr(D^I)(g)$ of the representation matrices. The traces of representation matrices indeed play fundamental role as automorphism invariants. This suggests that the scalar fields Φ_I in Kac-Moody algebra correspond to Hilbert space coefficients of $Tr(D^I)(g)$ as elements of group algebra labelled by the representation. The exponentiation of $\alpha \cdot \Phi$ would give the charged Kac-Moody algebra generators as free field representation.

4. For infinite sub-groups $G \subset SU(2)$ $d(G)$ is infinite. The traces are finite also in this case if the dimensions of the representations involved are finite. If one interprets the unit matrix as a function having value 1 in entire group $Tr(Id)$ diverges. Unit dimension for HFFs provide a more natural notion of dimension $d = n(G)$ of group algebra $n(G)$ as $d = n(G) = 1$. Therefore HFFs would emerge naturally.

It is easy to take into account $Gal(K(G))$. One can represent the elements of semi-direct product $Gal(K(G)) \triangleleft G$ as functions in $Gal(K(G)) \times G$ and the proposed construction brings in also the tensor products in the group algebra of $Gal(K(G))$. It is however essential that group algebra elements have values in K. This brings in tensor products of representations Gal and G and the number of representations is $n(Gal) \times n(G)$. The number of fields Φ_I as also the number of Cartan algebra generators of ADE Lie algebra increases from $I(G)$ to $I(Gal) \times I(G)$. The reduction of the extension of coefficient field for the Kac-Moody algebra from complex numbers to K splits the Hilbert space to sectors with smaller number of states.

8.4 $M^8 - H$ duality

The generalization of the standard twistor Grassmannian approach to TGD remains a challenge for TGD and one can imagine several approaches. $M^8 - H$ duality is essential for these approaches and will be discussed in the sequel.

The original form of $M^8 - H$ duality assumed $H = M^4 \times CP_2$ but quite recently it turned out that one could replace the twistor space of M^4 identified as $M^4 \times S^2$ with $CP_{3,h}$, which is hyperbolic variant of CP_3. This option forces to replace H with $H = CP_{2,h} \times CP_2$. $M^8 - H$ duality would consist of a map of M^4 point to corresponding twistor sphere in $CP_{3,h}$ and its projection to $CP_{2,h}$. This option will be discussed in the section about twistor lift of TGD.

8.4.1 $M^8 - H$ duality at the level of space-time surfaces

$M^8 - H$ duality [L46] relates two views about space-time surfaces X^4: as algebraic surfaces in complexified octonionic M^8 and as minimal surfaces with singularities in $H = M^4 \times CP_2$.

1. Octonion structure at the level of M^8 means a selection of a suitable decomposition $M^8 = M^4 \times E^4$ in turn determining $H = M^4 \times CP_2$. Choices of M^4 share a preferred 2-plane $M^2 \subset M^4$ belonging to the tangent space of allowed space-time surfaces $X^4 \subset M^8$ at various points. One can parameterize the tangent space of $X^4 \subset M^8$ with this property by a point of CP_2. Therefore X^4 can be mapped to a surface in $H = M^4 \times CP_2$: one M^8-duality. One can consider also the possibility that the choice of M^2 is local but that the distribution of $M^2(x)$ is integrable and defines string world sheet in M^4. In this case $M^2(x)$ is mapped to same $M^2 \subset H$.
2. Since 8-momenta p_8 are light-like one can always find a choice of $M^4_L \subset M^8$ such that p_8 belongs to M^4_L and is thus light-like. The momentum has in the general case a component orthogonal to M^2 so that M^4_L is unique by quaternionicity: quaternionic cross product for tangent space quaternions gives the third imaginary quaternionic unit. For a fixed M^4, call it M^4_T, the M^4 projections of momenta are time-like. When momentum belongs to M^2 the choices is non-unique and any $M^4 \subset M^2$ is allowed.
3. Space-time surfaces $X^4 \subset M^8$ have either quaternionic tangent- or normal spaces. Quantum classical correspondence (QCC) requires that charges in Cartan algebra co-incide with their classical counters parts determined as Noether charges by the action principle determining X^4 as preferred extremal. Parallelity of 8-momentum currents with tangent space of X^4 would conform with the naïve view about QCC. It does not however hold true for the contributions to four-momentum coming from string world sheet singularities (string world sheet boundaries are identified as carriers of quantum numbers), where minimal surface property fails.

An important aspect of $M^8 - H$ duality is the description of space-time surfaces $X^4_c \subset M^8_c$ as roots for the "real" or "imaginary" part in quaternionic sense of complexified-octonionic polynomial with real coefficients: these options correspond to complexified-quaternionic tangent - or normal spaces. The real space-time surfaces would be naturally obtained as "real" parts with respect to i of their complexified counterparts by projection from M^8_c to M^4_c. One could drop the subscripts "$_c$" but in the sequel they are kept.

Remark:O_c,O_c,C_c,R_c will be used in the sequel for complexifications of octonions, quaternions, etc.. number fields using commuting imaginary unit i appearing naturally via the roots of real polynomials.

$M^8 - H$ duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions.

1. Space-time surface is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. For $P(x) = x^n + ..$ ordinary roots are algebraic integers. The 4-D space-time surface is projection of this surface from M^8_c to M^8.
   The tangent space of space-time surface and thus space-time surface itself contains a preferred $M^2_c \subset M^4_c$ or more generally, an integrable distribution of tangent spaces $M^2_c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense.
   X^2c can be fixed by posing to the non-vanishing Q_c-valued part of octonionic polynomial condition that the C_c valued "real" or "imaginary" part in C_c sense for this polynomial vanishes. M^2_c would be the simplest solution but also more general complex sub-manifolds $X^2_c \subset M^4_c$ are possible. In general one would obtain book like structures as collections of several string world sheets having real axis as back.
   By assuming that R_c-valued "real" or "imaginary" part of the polynomial at this 2-surface vanishes. one obtains preferred M^1_c or E^1_c containing octonionic real and preferred imaginary

unit or distribution of the imaginary unit having interpretation as complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_{\rightarrow}C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

Remark: Also M_c^4 appears as a special solution for any polynomial P. M_c^4 seems to be like a universal reference solution with which to compare other solutions. M_c^4 would intersect all other solutions along string world sheets X_c^2. Also this would give rise to a book like structures with 2-D string world sheet representing the back of given book. The physical interpretation of these book like structures remains open in both cases.

I have proposed that string world sheets as singularities correspond to 2-D folds of space-time surfaces at which the dimension of the quaternionic tangent space degenerates from 4 to 2 [L75] [K10]. This interpretation is consistent with the identification as a book like structure with 2-pages. Also 1-D real and imaginary manifols could be interpreted as folds or equivalently books with 2 pages.

2. Associativity condition for tangent-/normal space is second essential condition and means that tangent - or normal space is quaternionic. The conjecture is that the identification in terms of roots of polynomials guarantees this and one can formulate this as rather convincing argument [L47, L48, L49].

One cannot exclude rational functions and or even real analytic functions in the sense that Taylor coefficients are octonionically real (propotional to octonionic real unit). Number theoretical vision - adelic physics [L52], suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

1. In general the zero loci for imaginary or real part are 4-D but the 7-D light-cone δM_+^8 of M^8 with tip at the origin of coordinates is an exception [L46]. At δM_+^8 the octonionic coordinate o is light-like and one can write $o = re$, where 8-D time coordinate and radial coordinate are related by $t = r$ and one has $e = (1 + e_r)/\sqrt{2}$ such that one as $e^2 = e$.

   Polynomial $P(o)$ can be written at δM_+^8 as $P(o) = P(r)e$ and its roots correspond to 6-spheres S^6 represented as surfaces $t_M = t = r_N$, $r_M = \sqrt{r_N^2 - r_E^2} \leq r_N$, $r_E \leq r_N$, where the value of Minkowski time $t = r = r_N$ is a root of $P(r)$ and r_M denotes radial Minkowski coordinate. The points with distance r_M from origin of $t = r_N$ ball of M^4 has as fiber 3-sphere with radius $r = \sqrt{r_N^2 - r_E^2}$. At the boundary of S^3 contracts to a point.

2. These 6-spheres are analogous to 6-D branes in that the 4-D solutions would intersect them in the generic case along 2-D surfaces X^2. The boundaries $r_M = r_N$ of balls belong to the boundary of M^4 light-cone. In this case the intersection would be that of 4-D and 3-D surface, and empty in the generic case (it is however quite not clear whether topological notion of "genericity" applies to octonionic polynomials with very special symmetry properties).

3. The 6-spheres $t_M = r_N$ would be very special. At these 6-spheres the 4-D space-time surfaces X^4 as usual roots of $P(o)$ could meet. Brane picture suggests that the 4-D solutions connect the 6-D branes with different values of r_n.

   The basic assumption has been that particle vertices are 2-D partonic 2-surfaces and light-like 3-D surfaces - partonic orbits identified as boundaries between Minkowskian and Euclidian regions of space-time surface in the induced metric (at least at H level) - meet along their 2-D ends X^2 at these partonic 2-surfaces. This would generalize the vertices of ordinary Feynman diagrams. Obviously this would make the definition of the generalized vertices mathematically elegant and simple.

   Note that this does not require that space-time surfaces X^4 meet along 3-D surfaces at S^6. The interpretation of the times t_n as moments of phase transition like phenomena is suggestive. ZEO based theory of consciousness suggests interpretation as moments for state function reductions analogous to weak measurements ad giving rise to the flow of experienced time.

4. One could perhaps interpret the free selection of 2-D partonic surfaces at the 6-D roots as initial data fixing the 4-D roots of polynomials. This would give precise content to strong form of holography (SH), which is one of the central ideas of TGD and strengthens the 3-D holography coded by ZEO alone in the sense that pairs of 3-surfaces at boundaries of CD define unique preferred extremals. The reduction to 2-D holography would be due to preferred extremal

property realizing the huge symplectic symmetries and making M^8-H duality possible as also classical twistor lift.

I have also considered the possibility that 2-D string world sheets in M^8 could correspond to intersections $X^4 \cap S^6$? This is not possible since time coordinate t_M constant at the roots and varies at string world sheets.

Note that the compexification of M^8 (or equivalently octonionic E^8) allows to consider also different variants for the signature of the 6-D roots and hyperbolic spaces would appear for $(\epsilon_1, \epsilon_i, .., \epsilon_8)$, $epsilon_i = \pm 1$ signatures. Their physical interpretation - if any - remains open at this moment.

5. The universal 6-D brane-like solutions S^6_c have also lower-D counterparts. The condition determining X^2 states that the C_c-valued "real" or "imaginary" for the non-vanishing Q_c-valued "real" or "imaginary" for P vanishes. This condition allows universal brane-like solution as a restriction of O_c to M^4_c (that is CD_c) and corresponds to the complexified time=constant hyperplanes defined by the roots $t = r_n$ of P defining "special moments in the life of self" assignable to CD. The condition for reality in R_c sense in turn gives roots of $t = r_n$ a hyper-surfaces in M^2_c.

8.4.2 $M^8 - H$ duality at the level of momentum space

$M^8 - H$ duality occurs also at the level of momentum space and has different meaning now.

1. At M^8 level 8-momenta are quaternionic and light-like. The choices of $M^4_L \supset M^2$ for which 8-momentum in M^4_L, are parameterized by CP_2 parameterizing also the choices of tangent or normal spaces of $X^4 \subset M^8$ at space-time level. This maps M^8 light-like momenta to light-like M^4_L momenta and to CP_2 point characterizing the M^4 and depending on 8-momentum. One can introduce CP_2 wave functions expressible in terms of spinor harmonics and generators of of a tensor product of Super-Virasoro algebras.
2. For a fixed choice M^4_T momenta in general time-like and the E^4 component of 8-momentum has value equal to mass squared. E^4 momenta are points of 3-sphere so that $SO(3)$ harmonics with $SO(4)$ symmetry could parametrize the states. The quantum numbers are $M^4_T \supset M^2$ momenta with fixed mass and the two angular momenta with identical values for S^3 harmonics, which correspond to the quantum states of a spherical quantum mechanical rigid body, and are given by the matrix elements $D^j_{m,n}$ $SU(2)$ group elements ($SO(4)$ decomposes to $SU(2)_L) \times SU(2)_R$ acting from left and right).

   This picture suggests what one might call $SO(4) - SU(3)$ duality at the level of momentum space. There would be two descriptions of states: as massless states with $SU(3)$ symmetry and massive states with $SO(4)$ symmetry.
3. What about the space formed by the choices of the space of the light-like 8-momenta? This space is the space for the choices of preferred M^2 and parameterized by the 6-D (symmetric space $G_2/SU(3)$, where $SU(3) \subset G_2$ leaving complex plane M^2 invariant is subgroup of quaternionic automorphic group $G(2)$ leaving octonionic real unit defining the rest system invariant. This space is moduli space for octonionic structures each of which defines family of space-time surfaces. 8-D Lorent transformations produce even more general octonionic structures. The space for the choices of color quantization axes is $SU(3)/U(1) \times U(1)$, the twistor space of CP_2.

Do M^4_L and M^4_T have analogs at the space-time level?

As found, the solutions of octonionic polynomials consisting of 4-D roots and special 6-D roots coming as 6-sphere S^6 s at 7-D light-cone of M^8. The roots at $t = r$ light-cone boundary are given by the roots $r = r_N$ of the polynomial $P(t)$ and correspond to M^4 slices $t_M = r_N, r_M \leq r_N$. At point r_M S^3 fiber as radius $r(S^3) = \sqrt{r_N^2 - r_M^2}$ and contracts to a point at its boundaries.

Could M^4_L and M_T have analogies at the space-time level?

1. The sphere S^3 associated M^4_T could have counterpart at the level of space-time description. The momenta in M^4_T would naturally be mapped to momenta in the section $t = r_n$ in this case the S^3:s of different mass squared values would naturally correspond to S^3:s assignable to the points of the balls $t = r_n$ and code for mass squared value.

The counterpart of M^4_L should correspond to light-cone boundary but what does CP_2 correspond? Could the pile of S^3 associated with $t = r_n$ correspond also to CP_2. Could this be the case if there is wormhole contact carrying monopole flux at the origin so that the analog for the replacement of 3-sphere at $r_{CP_2} = \infty$ with homologically non-trivial 2-sphere would be realized?

2. Does the 6-sphere as a root polynomial have counterpart in H? The image should be consistent with $M^8 - H$ duality and correspond to a fixed structure depending on the root r_n only. Since S^3 associated with the E^4 momenta reduces to a point for M^4_L, the natural guess is that S^6 reduces to $t = r_n, 0 \leq r_M \leq r_n$ surface in H.

$S^3 - CP_2$ duality

$S^3 - CP_2$ duality at the level of quantum numbers suggest strongly itself. What does this require? One can approach the problem from two different perspectives.

1. The first approach would be that the representations of $SU(3)$ and $SO(4)$ groups somehow correspond to each other: one could speak of $SU(3) - SO(4)$ duality [K96, K111]. The original form of this duality was this. The color symmetries of quark physics at high energies would be dual to the $SO(4) = SU(2)_L \times SU(2)_R$ symmetries of the low energy hadron physics. Since the physical objects are partons and hadrons formed from the one cannot expect the duality to hold true at the level of details for the representations, and the comparison of the representations makes this clear.
2. The second approach relies on the notion of cognitive representation meaning discretization of CP_2 and S^3 and counting of points of cognitive representations providing discretization in terms of M^8 or H points belonging to the extension of rationals considered. In this case it is more natural to talk about $S^3 - CP_2$ duality.
   The basic observation is that the open region $0 \leq r < \infty$ of CP_2 in Eguchi-Hanson coordinates with r labeling 3-spheres $S^3(r)$ with finite radius can be regarded as pile of $S^3(r)$. In discretization one would have discrete pile of these 3-spheres with finite number of points in the extension of rationals. They points of given S^3 could be related by isometries in special cases.

How $S^3 - CP_2$ duality at the level of light-like M^8 momenta could emerge?

1. Consider first the situation in which one chooses $M^4 \supset M^2$ sub-spaces so that momentum projection to it is light-like. For cognitive representation the choices of $M^4 \supset M^2$ correspond to ad discrete set of points of CP_2 and thus points in the pile of S^3 with discrete radii since all E^4 parts of momenta with fixed length squared to zero in this choice and each E^4 momentum with fixed lengthand thus identifiable as discrete point of S^3 would correspond to one choice. All these choices would give rise to a pile of S^3:s identifiable as set $0 \leq r < \infty$ of CP_2. The number of CP_2 points would be same as total number of points in the pile of discrete S^3s. This is what $S^3 - CP_2$ duality would say.
   Remark: The volumes of CP_2 and S^3 with unit radius are $8\pi^2$ and $2\pi^2$ so that ration is rational number.
2. Consider now the situation for M^4_T so that one has non-vanishing M^4 mass squared equal to E^4 mass squared, having discretized values. The E^4 would momenta correspond to points for a pile of discretized S^3 and thus to the points of CP_2 by above argument. One would have $S^3 - CP_2$ correspondence also now as one indeed expects since the two ways to see the situation should be equivalent.
3. In the space of light-like M^8 momenta E^8 momenta could naturally organize into representations of finite discrete subgroups of $SU(2)$ appearing in McKay correspondence with ADE groups: the groups are cyclic groups, dihedral groups, and the isometry groups associated with tetrahedron, octahedron (cube) and icosahedron (dodecahedron) (see `http://tinyurl.com/yyyn9p95`).
4. Could a concrete connection with the inclusion hierarchy of HFFs be based on increasing momentum resolution realized in terms of these groups at sphere S^3. Notice however that for cyclic and dihedral groups the orbits are circles and pairs of circles for dihedral groups so that the discretization looks too simple and is rotationally asymmetric. Discretization should improve as n increases.

One can of course ask why C_n and D_n with single direction of rotation axes would appear? Could it be that the directions of rotation axis correspond to the directions defined by the vertices of the 5 Platonic solids. Or could the orbits of fixed axis under the 5 Platonic orbits be allowed. Also this looks still too simple.
Could the discretization labelled by n_{max} be determined by the product of the groups up to n_{max} and define a group with infinite order. One can consider also groups defined by subsets $\{n_1, n_2...n_3\}$ and these a pair of sequences with larger sequence containing the smaller one could perhaps define an inclusion. The groups C_n and D_n allow prime decomposition in obvious manner and it seems enough to include to the product only the groups C_p and D_p, where p is prime as generators so that one would have set $\{p_1, ...p_n\}$ of primes labelling these groups besides the Platonic groups. The extension of rationals used poses a cutoff on the number of groups involved and on the group elements representable since since too high roots of unity resulting in the multiplication of C_{p_i} and D_{p_j} do not belong to the extension.
At the level of momentum space the hierarchy of finite discrete groups of $SU(2)$ would define the notion measurement resolution. The discrete orbits of $SU(2) \times U(1)$ at S^3 would be analogous to tessellations of sphere S^2 known as Platonic solids at sphere S^2 and appearing in the ADE correspondence assignable to Jones inclusions as description of measurement resolution. This would also explain also why Z_2 coverings of the subgroups of $SO(3)$ appear in ADE sequence.
This picture is probably not enough for the needs of adelic physics [L52] allowing all extensions of rationals. Besides roots of unity only the roots of small integers $2, 3, 5$ associated with the geometry of Platonic solids would be included in these discretizations. One could interpret these discretizations in terms of subgroups of discrete automorphism groups of quaternions. Also the extensions of rationals are probably needed.

Could $S^3 - CP_2$ duality make sense at space-time level? Consider the space-time analog for the projection of M^8 momenta to fixed M^4_T.

1. Suppose that the 3-surfaces determining the space-time surfaces as algebraic surfaces in $X^4 \subset M^8$ are given at the surfaces $t = r_N, r_M \leq r_N$ and have a 3-D fiber which should be surface in CP_2. On can assign to each point of this ball $S^3(r_M)$ with radius going to zero at $r_M = r_N$. One has pile of $S^3(r_M)$ which corresponds to the region $0 \leq r < \infty$ of CP_2. This set is discretized. Suppose that the discretization is like momentum discretization. If so, the points would correspond to points of CP_2. It is not however clear why the discretization should be so symmetric as in momentum discretization.
2. The initial values could be chosen by choosing discrete set of points in this pile of S^3:s and this would give rise to a discrete set of points of CP_2 fixing tangent or normal plane of X^4 at these points. One should show that the selection of a point of S^6 at each point indeed determines quaternionic tangent or normal plane plane for a given polynomial $P(o)$ in M^8.

It would seem that this correspondence need not hold true.

8.4.3 $M^8 - H$ duality and the two ways to describe particles

The isometry groups for $M^4 \times CP_2$ is $P \times SU(3)$ (P for Poincare group). The isometry group for $M^8 = M^4 \times E^4$ with a fixed choice of M^4 breaks down to $P \times SO(4)$. A further breaking by selection $M^4 \subset M^2$ of preferred octonionic complex plane M^2 necessary in the algebraic approach to space-time surfaces $X^4 \subset M^8$ brings in preferred rest system and reduces the Poincare symmetry further. At the space-time level the assumption that the tangent space of X^4 contains fixed M^2 or at least integral distribution of $M^2(x) \subset M^4$ is necessary for $M^8 - H$ duality [L46].

The representations $SO(4)$ and $SU(3)$ could provide alternative description of physics so that one would have what I have called $SO(4)-SU(3)$ duality [K96]. This duality could manifest in the description of strong interaction physics in terms of hadrons and quarks respectively (conserved vector current hypothesis and partially conserved axial current hypothesis based on $Spin(SO(4)) = SU(2) \times SU(2)_R$. The challenge is to understand in more detail this duality. This could allow also to understand better how the two twistor descriptions might relate.

$SO(4) - SU(3)$ duality implies two descriptions for the states and scattering amplitudes.

Option I: One uses projection of 8-momenta to a fixed $M^4_T \supset M^2$.

Option II: One assumes that $M^4_L \supset M^2$ is defines the frame in which quaternionic octonion momentum is parallel to M^4_L: this M^4_L depends on particle state and describes this dependence in terms of wave function in CP_2.

Option I: fixed $M^4_T \supset M^2$

For Option I the description would be in terms of a *fixed* $M^4_T \subset M^8 = M^4_T \times E^4$ and $M^2 \subset M^4_T$ fixed for both options. For given quaternionic light-like M^8 momentum one would have projection to M^4_T, which is in general massive. E^4 momentum would have same the length squared by light-likeness.

De-localization M^4_T mass squared equal to $p^2(M^4_T) = m^2$ in E^4 can be described in terms of $SO(4)$ harmonics at sphere having $p^2(E^4) = m^2$. This would be the description applied to hadrons and leptons and particles treated as massive particles. Particle mass would be due to the fixed choice of M^4_T. What dictates this choice is an interesting question. An interesting question is how these descriptions relate to QFT Higgs mechanism as (in principle) alternative descriptions: the choice of fixed M^4_T could be seen as analog for the generation of vacuum expectation of Higgs selecting preferred direction in the space of Higgs fields.

Option II: varying $M^4_L \supset M^2$

For Option II the description would use $M^4_L \supset M^2$, which is *not fixed* but chosen so that it contains light-like M^8 momentum. This would give light-like momentum in M^4_L identifiable as quaternionic sub-space of complexified octonions.

1. One could assign to the state wave function function for the choices of M^4 and by quaternionicity of 8-momenta this would correspond to a state in super-conformal representation with vanishing M^4_L mass: CP_2 point would code the information about E^4 component light-like 8-momentum. This description would apply to the partonic description of hadrons in terms of massless quarks and gluons.
2. For this option one could use the product of ordinary M^4 twistors and CP_2 twistors. One challenge would be the generalization of the twistor description to the case of CP_2 twistors.

p-Adic particle massivation and ZEO

The two pictures about description of light-like M^8 momenta do not seem to be quite consistent with the recent view about TGD in which H-harmonics describe massivation of massless particles. What looks like a problem is following.

1. The resulting particles are massive in M^4. But they should be massless in $M^4 \times CP_2$ description. The non-vanishing mass would suggest the correct description in terms of Option I for which the description in terms of E^4 momenta with length equal to mass and thus identifiable as points of S^3. Momentum space wave functions at S^3 - essentially rigid body wave functions given by representation matrices of $SU(2)$ could characterize the states rather than CP_2 harmonic.
2. The description based on CP_2 color partial waves however works and this would favor Option II with vanishing M^4 mass. What goes wrong?

To understand what might be involved, consider p-adic mass calculations.

1. The massivation of physical fermion states includes also the action of super-conformal generators changing the mass. The particles are originally massless and p-adic mass squared is generated thermally and mapped to real mass squared by canonical identification map.
   For CP_2 spinor harmonics mass squared is of order CP_2 mass squared and thus gigantic. Therefore the mass squared is assumed to contain negative tachyonic ground state contribution due to the negative half-odd integer valued conformal weight $h_{vac} < 0$ of vacuum. The origin of this contribution has remained a mystery in p-adic thermodynamics but it makes possible to construct massless states. h_{vac} cancels the spinorial contributions and the contribution from positive conformal weights of super-conformal generators so that the particle states are massless before thermalization. This would conform with the idea of using varying M^4_L and thus CP_2 description.

2. What does the choice of M^4 mean in terms of super-conformal representations? Could the mysterious vacuum conformal weight h_{vac} provide a description for the effect of the needed $SU(3)$ rotation of M^4 from standard orientation on super-conformal representation. The effect would be very simple and in certain sense reversal to the effect of Higgs vacuum expectation value in that it would cancel mass rather than generate it.
   An important prediction would be that heavy states should be absent from the spectrum in the sense that mass squared would be p-adically of order $O(p)$ or $O(p^2)$ (in real sense of order $O(1/p)$ or $O(1/p^2)$). The trick would be that the generation of h_0 as a representation of SU(3) rotation of M^4 makes always the dominating contribution to the mass of the state vanishing.
   Remark: If the canonical identification I mapping the p-adic mass integers to their real numbers is of the simplest form $m = \sum_n x_n p^n \rightarrow I(m) = \sum_n x_n p^{-n}$, it can happen that the image of rational m/n with p-adic norm not larger than 1 represented as p-adic integer by expanding it in powers of p, can be near to the maximal value of p and the mass of the state can be of order CP_2 mass - about 10^{-4} Planck masses. If the canonical identification is defined as $m/n \rightarrow I/(m)/I(n)$ the image of the mass is small for small values of m and n.
3. Unfortunately, it is easy to get convinced that this explanation of h_{vac} is not physically attractive. Identical mass spectra at the level of M^8 and H looks like a natural implication of $M^8 - H$-duality. $SU(3)$ rotation of M^4 in M^8 cannot however preserve the general form of $M^4 \times CP_2$ mass squared spectrum for the M^4 projections of M^8 momenta at level of M^8.
   Remark: For $H = M^4 \times CP_2$ the mass squared in given representation of Super-conformal symmetries is given as a sum of CP_2 mass squared for the spinor harmonic determining the ground state and of a Virasoro contribution proportional to a non-negative integer. The masses are required to independent of electroweak quantum numbers.

One can imagine two further identifications for the origin of h_{vac}.

1. Take seriously the possibility of complex momenta allowed by the complexification of M^8 by commuting imagine unit i and also suggested by the generalization of the twistorialization. The real and imaginary parts of light-like complex 8-momenta $p_8 = p_{8,Re} + ip_{8,Im}$ are orthogonal to each other: $p_{8,Re} \cdot p_{8,Im} = 0$ so that both real and imaginary parts of p_8 are light-like: $p_{8,Re}^2 = p_{8,Im}^2 = 0$. The M^4 mass squared can be written has $m^2 = m_{Re}^2 - m_{Im}^2$ with $h_{vac} \propto -m_{Im}^2$. The representations of Super-conformal algebra would be labelled by $h_{vac} \propto m_{Im}^2$.
   Could the needed wrong sign contribution to CP_2 mass squared mass make sense? CP_2 type extremals having light-like geodesic as M^4 projection are locally identical with CP_2 but because of light-like projection they can be regarded as CP_2 with a hole and thus non-compact. Boundary conditions allow analogs of CP_2 harmonics for which spinor d'Alembertian would have complex eigenvalues.
   Does quantum-classical correspondence allow complex momenta: can the classical four-momenta assignable to 6-D Kähler action be complex? The value of Kähler coupling strength allows the action to have complex phase but parts with different phases are not allowed. Could the imaginary part to 4-momentum could come from the CP_2 type extremal with Euclidian signature of metric?
2. Second - not necessarily independent - idea relies on the observation that in TGD one has besides the usual conformal algebra acting on complex coordinate z also its analog acting on the light-like radial coordinate r of light-cone boundary. At light-cone boundary one has both super-symplectic symmetries of $\Delta M^4_+ \times CP_2$ and extension of super-conformal symmetries of sphere S^2 to analogs of conformal symmetries depending on z and r and it seems that one must chose between these two options. Also the extension of ordinary Kac-Moody algebra acts at the light-like orbits of partonic 2-surfaces.
   There are two scaling generators: the usual $L_0 = zd/dz$ and the second generator $L_{0,1} = ird/dr$. For $L_{0,1}$ at light-cone boundary powers of z^n are replaced with $(r/r_0)^{ik} = exp(iku)$, $u = log(r/r_0))$. Could it be that mass squared operator is proportional to $L_0 + L_{0,1}$ having eigenvalues $h = n - k$? The absence of tachyons requires $h \geq 0$. Could k characterize given Super-Virasoro representation? Could $k \geq 0$ serve as an analog of positive energy condition allowing to analytically continue $exp(iku)$ to upper u-plane? How to interpret this continuation?
   The 3-D generalization of super-symplectic symmetries at light-cone boundary and extended Ka-Moody symmetries at partonic 2-surfaces should be possible in some sense. Could the

continuation to the upper u-plane correspond to the continuation of the extended conformal symmetries from light-cone boundary to future light-one and from light-partonic 2-surfaces to space-time interior?

Why p-adic massivation should occur at all? Here ZEO comes in rescue.

1. In ZEO one can have superposition of states with different 4-momenta, mass values and also other charges: this does not break conservation laws. How to fix M^4 in this case? One cannot do it separately for the states in superposition since they have different masses. The most natural choices is as the M^4 associated with the dominating contribution to the zero energy state. The outcome would be thermal massivation described excellently by p-adic thermodynamics [K60]. Recently a considerable increase in the understanding of hadron and weak boson masses took place [L89].
2. In ZEO quantum theory is square root of thermodynamics in a well-defined formal sense, and one can indeed assign to p-adic partition function a complex square root as a genuine zero energy state. Since mass varies, one must describe the presence of higher mass excitations in zero energy state as particles in M^4 assigned with the dominating part of the state so that the observed particle mass squared is essentially p-adic thermal expectation value over thermal excitations. p-Adic thermodynamics would thus describe the fact that the choice of M^4_L cannot not ideal in ZEO and massivation would be possible only in ZEO.
3. Current quarks and constituent quarks are basic notions of hadron physics. Constituent quarks with rather large masses appear in the low energy description of hadrons and current quarks in high energy description of hadronic reactions. That both notions work looks rather paradoxical. Could massive quarks correspond to M_T picture and current quarks to M^4_L picture but with p-adic thermodynamics forced by the superposition of mass eigenstates with different masses.
The massivation of ordinary massless fermion involves mixing of fermion chiralities. This means that the $SU(3)$ rotation determined by the dominating component in zero energy state must induce this mixing. This should be understood.

8.4.4 $M^8 - H$ duality and consciousness

$M^8 - H$ duality is one of the key ideas of TGD and one can ask whether it has implications for TGD inspired theory of consciousness and it indeed forces to challenge the recent ZEO based view about consciousness [L54] .

Objections against ZEO based theory of consciousness

Consider first objections against ZEO based view about consciousness.

1. ZEO (zero energy ontology) based view about conscious entity can be regarded as a sequence of "small" state function reductions (SSRs) identifiable as analogs of so called weak measurements at the active boundary of causal diamond (CD) receding reduction by reduction farther away from the passive boundary, which is unchanged as also the members of state pairs at it. One can say that weak measurements commute with the observables, whose eigenstates the states at passive boundary are. This asymmetry assigns arrow of time to the self having CD as embedding space correlate. "Big" state function reductions (BSRs) would change the roles of boundaries of CD and the arrow of time. The interpretation is as death and re-incarnation of the conscious entity with opposite arrow of time.
The question is whether quantum classical correspondence (QCC) could allow to say something about the time intervals between subsequent values of temporal distance between weak state function reductions.
2. The questionable aspect of this view is that $t_M = constant$ sections look intuitively more natural as seats of quantum states than light-cone boundaries forming part of CD boundaries. The boundaries of CD are however favoured by the huge symplectic symmetries assignable to the boundary of M^4 light-cone with points replaced with CP_2 at level of H. These symmetries are crucial or the existence of the geometry of WCW ("world of classical worlds").
3. Second objection is that the size of CD increases steadily: this nice from the point of view of cosmology but the idea that CD as correlate for a conscious entity increases from CP_2

size to cosmological scales looks rather weird. For instance, the average energy of the state assignable to either boundary of CD would increase. Since zero energy state is a superposition of states with different energies classical conservation law for energy does not prevent this [L82]: essentially quantal effect due to the fact that the zero energy states are not exact eigenstates of energy could be in question. In BSRs the energy would gradually increase. Admittedly this looks strange and one must be keen for finding more conventional options.

4. Third objection is that re-incarnated self would not have any "childhood" since CD would increase all the time.

One can ask whether $M^8 - H$ duality and this braney picture has implications for ZEO based theory of consciousness. Certain aspects of $M^8 - H$ duality indeed challenge the recent view about consciousness based on ZEO (zero energy ontology) and ZEO itself.

1. The moments $t = r_n$ defining the 6-branes correspond classically to special moments for which phase transition like phenomena occur. Could $t = r_n$ have a special role in consciousness theory?
   (a) For some SSRs the increase of the size of CD reveals new $t = r_n$ plane inside CD. One can argue that these SSRS define very special events in the life of self. This would not modify the original ZEO considerably but could give a classical signature for how many ver special moments of consciousness have occurred: the number of the roots of P would be a measure for the lifetime of self and there would be the largest root after which BSR would occur.
   (b) Second possibility is more radical. One could one think of replacing CD with single truncated future- or past-directed light-cone containing the 6-D universal roots of P up to some r_n defining the upper boundary of the truncated cone? Could $t = r_n$ define a sequence of moments of consciousness? To me it looks more natural to assume that they are associated with very special moments of consciousness.
2. For both options SSRs increase the number of roots r_n inside CD/truncated light-one gradually and thus its size? When all roots of $P(o)$ would have been measured - meaning that the largest value r_{max} of r_n is reached -, BSR would be unavoidable.
   BSR could replace $P(o)$ with $P_1(r_1 - o)$: r_1 must be real and one should have $r_1 > r_{max}$. The new CD/truncated light-cone would be in opposite direction and time evolution would be reversed. Note that the new CD could have much smaller size size if it contains only the smallest root r_0. One important modification of ZEO becomes indeed possible. The size of CD after BSR could be much smaller than before it. This would mean that the re-incarnated self would have "childhood" rather than beginning its life at the age of previous self - kind of fresh start wiping the slate clean.
   One can consider also a less radical BSR preserving the arrow of time and replacing the polynomial with a new one, say a polynomial having higher degree (certainly in statistical sense so that algebraic complexity would increase).

Could one give up the notion of CD?

A possible alternative view could be that one the boundaries of CD are replaced by a pair of two $t = r_N$ snapshots $t = r_0$ and $t = r_N$. Or at least that these surfaces somehow serve as correlates for mental images. The theory might allow reformulation also in this case, and I have actually used this formulation in popular lectures since it is easier to understand by laymen.

1. Single truncated light-cone, whose size would increase in each SSR would be present now since the spheres correspond to balls of radius r_n at times r_n. If $r_0 = 0$, which is the case for $P(o) \propto o$, the tip of the light-cone boundary is one root. One cannot avoid association with big bang cosmology. For $P(0) \neq r_0$ the first conscious moment of the cosmology corresponds to $t = r_0$. One can wonder whether the emergence of consciousness in various scales could be described in terms of the varying value of the smallest root r_0 of $P(o)$.
   If one allows BSR:s this picture differs from the earlier one in that CDs are replaced with alternation of light-cones with opposite directions and their intersections would define CD.
2. For this option the preferred values of t for SSRs would naturally correspond to the roots of the polynomial defining $X^4 \subset M^8$. Moments of consciousness as state function reductions would be due to collisions of 4-D space-time surfaces X^4 with 6-D branes! They would replace

the sequence of scaled CD sizes. CD could be replaced with light-one and with the increasing sequence $(r_0, ...r_n)$ of roots defining the ticks of clock and having positive and negative energy states at the boundaries r_0 and r_n.

3. What could be the interpretation for BSRs representing death of a conscious entity in the new variant of ZEO? Why the arrow of time would change? Could it be because there are no further roots of $P(o)$? The number of roots of $P(o)$ would give the number of small state function reductions?
What would happen to $P(o)$ in BSR? The vision about algebraic evolution as increase of the dimension for the extension of rationals would suggest that the degree of $P(o)$ increases as also the number of roots if all complex roots are allowed. Could the evolution continue in the same direction or would it start to shift the part of boundary corresponding to the lowest root in opposite direction of time. Now one would have more roots and more algebraic complexity so that evolutionary step would occur.
In the time reversal one would have naturally $t_{max} \geq r_{n_{max}}$ for the new polynomial $P(t-t_{max})$ having $r_{n_{max}}$ as its smallest root. The light-cone in M^8 with tip at $t = t_{max}$ would be in opposite direction now and also the slices $t - t_{max} = r'_n$ would increase in opposite direction! One would have two light-cones with opposite directions and the $t = r_n$ sections would replace boundaries of CDs. The reborn conscious entity would start from the lowest root so that also it would experience childhood.

This option could solve the argued problems of the previous scenario and give concrete connection with the classical physics in accordance with QCC. On the other hand, a minimal modification of original scenario combined with $M^8 - H$ duality with moments $t = r_n$ as special moments in the life of conscious entity allows also to solve these problems if the active boundary of CD is interpreted as boundary beyond which classical signals cannot contribute to perceptions.

What could be the minimal modification of ZEO based view about consciousness?

What would be the minimal modification of the earlier picture? Could one *assume* that CDs serve as embedding space correlates for the perceptive field?

1. Zero energy states would be defined as before that is in terms of 3-surfaces at boundaries of CD: this would allow a realization of huge symmetries of WCW and the active boundary A of CD would define the boundary of the region from which self can receive classical information about environment. The passive boundary P of CD would define the boundary of the region providing classical information about the state of self. Also now BSR would mean death and reincarnation with an opposite arrow of time. Now however CD would shrink in BSR before starting to grow in opposite time direction. Conscious entity would have "childhood".

2. If the geometry of CD were fixed, the size scale of the $t = r_n$ balls of M^4 would first increase and then start to decrease and contract to a point eventually at the tip of CD. One must however remember that the size of $t = r_n$ planes increases all the time as also the size of CD in the sequences of SSRs. Moments $t = r_n$ could represent special moments in the life of conscious entity taking place in SSRs in which $t = r_n$ hyperplane emerges inside CD with increased size. The recent surprising findings challenging the Bohrian view about quantum jumps [L69] can be understood in this picture [L69].

3. $t = r_n$ planes could also serve as correlates for memories. As CD increases at active boundary new events as $t = r_n$ planes would take place and give rise to memories. The states at $t = r_n$ planes are analogous to seats of boundary conditions in strong holography and the states at these planes might change in state function reductions - this would conform with the observations that our memories are not absolute.

To sum up, the original view about ZEO seems to be essentially correct. The introduction of moments $t = r_n$ as special moments in the life of self looks highly attractive as also the possibility of wiping the slate clear by reduction of the size of CD in BSR.

8.5 Could standard view about twistors work at space-time level after all?

While asking what super-twistors in TGD might be, I became critical about the recent view concerning what I have called geometric twistor space of M^4 identified as $M^4 \times S^2$ rather than CP_3 with hyperbolic metric. The basic motivations for the identification come from M^8 picture in which there is number theoretical breaking of Poincare and Lorentz symmetries. Second motivation was that M^4_{conf} - the conformally compactified M^4 - identified as group $U(2)$ [B7] (see http://tinyurl.com/y35k5wwo) assigned as base space to the standard twistor space CP_3 of M^4, and having metric signature (3,-3) is compact and is stated to have metric defined only modulo conformal equivalence class.

As found in the previous section, TGD strongly suggests that M^4 in $H = M^4 \times CP_2$ should be replaced with hyperbolic variant of CP_2, and it seems to me that these spaces are not identical. Amusingly, $U(2)$ and CP_2 are fiber and base in the representation of $SU(3)$ as fiber space so that the their identification does not seem plausible.

On can however ask whether the selection of a representative metric from the conformal equivalence class could be seen as breaking of the scaling invariance implied also by ZEO introducing the hierarchy of CDs in M^8. Could it be enough to have M^4 only at the level of M^8 and conformally compactified M^4 at the level of H? Should one have $H = cd_{conf} \times CP_2$? What cd_{conf} would be: is it hyperbolic variant of CP_2?

8.5.1 Getting critical

The only way to make progress is to become very critical now and then. These moments of almost despair usually give rise to a progress. At this time I got very critical about the TGD inspired identification of twistor spaces of M^4 and CP_2 and their properties.

Getting critical about geometric twistor space of M^4

Let us first discuss the recent picture and how to modify it so that it is consistent with the hierarchy of CDs. The key idea is that the twistor space and its base space represents CD so that one obtains scale hierarchy of twistor spaces as a realization of broken scale invariance giving rise to the p-adic length scale hierarchy.

1. I have identified the twistor space of M^4 simply as $T(M^4) = M^4 \times S^2$. The interpretation would be at the level of octonions as a product of M^4 and choices of M^2 as preferred complex sub-space of octonions with S^2 parameterizing the directions of spin quantization axes. Real octonion axis would correspond to time coordinate. One could talk about the space of of light-like directions. Light-like vector indeed defines M^2. This view could be defended by the breaking of both translation and Lorentz invariance in the octonionic approach due to the choice of M^2 and by the fact that it seems to work.
   Remark: $M^8 = M^4 \times E^4$ is complexified to M^8_c by adding a commuting imaginary unit i appearing in the extensions of rationals and ordinary M^8 represents its particular sub-space. Also in twistor approach one uses often complexified M^4.
2. The objection is that it is ordinary twistor space identifiable as CP_3 with (3,-3) signature of metric is what works in the construction of twistorial amplitudes. CP_3 has metric as compact space and coset space. Could this choice of twistor space make sense after all as geometric twistor space?
   Here one must pause and recall that the original key idea was that Poincare invariance is symmetry of TGD for $X^4 \subset M^4 \times CP_2$. Now Poincare symmetry has been transformed to a symmetry acting at the level of M^8 in the moduli space of octonion structures defined by the choice of the direction of octonionic real axis reducing Poincare group to $T \times SO(3)$ consisting of time translations and rotations. Fixing of M^2 reducrs the group to $T \times SO(2)$ and twistor space can be seen as the space for selections of quantization axis of energy and spin.
3. But what about the space H? The first guess is $H = M^4_{conf} \times CP_2$. According to [B7] (see http://tinyurl.com/y35k5wwo) one has $M^4_{conf} = U(2)$ such that $U(1)$ factor is time- like and $SU(2)$ factor is space-like. One could understand $M^4_{conf} = U(2)$ as resulting by addition

and identification of metrically 2-D light-cone boundaries at $t = \pm\infty$. This is topologically like compactifying E^3 to S^3 and gluing the ends of cylinder $S^3 \times D^1$ together to the $S^3 \times S^1$. The conformally compactified Minkowski space M^4_{conf} should be analogous to base space of CP_3 regarded as bundle with fiber S^2. The problem is that one cannot imagine an analog of fiber bundle structure in CP_3 having $U(2)$ as base. The identification $H = M^4_{conf} \times CP_2$ does not make sense.

4. In ZEO based breaking of scaling symmetry it is CD that should be mapped to the analog of M^4_{conf} - call it cd_{conf}. The only candidate is $cd_{conf} = CP_2$ with one hypercomplex coordinate. To understand why one can start from the following picture. The light-like boundaries of CD are metrically equivalent to spheres. The light-like boundaries at $t = \pm\infty$ are identified as in the case of M^4_{conf}. In the case of CP_2 one has 3 homologically trivial spheres defining coordinate patches. This suggests that cd_{conf} is simply CP_2 with second complex coordinate made hypercomplex. M^4 and E^4 differ only by the signature and so would do cd_{conf} and CP_2. The twistor spheres of CP_3 associated with points of M^4 intersect at point if the points differ by light-like vector so that one has and singular bundle structure. This structure should have analog for the compactification of CD. CP_3 has also bundle structure $CP_3 \to CP_2$. The S^2 fibers and base are homologically non-trivial and complex analogs of mutually orthogonal line and plane and intersect at single point. This defines the desired singular bundle structure via the assignment of S^2 to each point of CP_2.
   The M^4 points must belong to the interior of cd and this poses constraints on the distance of M^4 points from the tips of cd. One expects similar hierarchy of cds at the level of momentum space.

5. In this picture $M^4_{conf} = U(2)$ could be interpreted as a base space for the space of CDs with fixed direction of time axis identified as direction of octonionic real axis associated with various points of M^4 and therefore of M^4_{conf}. For Euclidian signature one would have base and fiber of the automorphism sub-group $SU(3)$ regarded as $U(2)$ bundle over CP_2: now one would have CP_2 bundle over $U(2)$. This is perhaps not an accident, and one can ask whether these spaces could be interpreted as representing local trivialization of $SU(3)$ as $U(2) \times CP_2$. This would give to metric cross terms between $U(2)$ and CP_2.

6. The proposed identification can be tested by looking whether it generalizes. What the twistor space for entire M^8 would be? $cd = CD_4$ is replaced with CD_8 and the discussion of the preceding chapter demonstrated that the only possible identification of the twistor space is now is as the 12-D hyperbolic variant of HP_3 whereas $CD_{8,conf}$ would correspond to 8-D hyperbolic variant of HP_2 analogous to hyperbolic variant of CP_2.

The outcome of these considerations is surprising.

1. One would have $T(H) = CP_3 \times F$ and $H = CP_{2,H} \times CP_2$ where $CP_{2,H}$ has hyperbolic metric with metric signature $(1, -3)$ having M^4 as tangent space so that the earlier picture can be understood as an approximation. This would reduce the construction of preferred extremals of 6-D Kähler action in $T(H)$ to a construction of polynomial holomorphic surfaces and also the minimal surfaces with singularities at string world sheets should result as bundle projection. Since $M^8 - H$ duality must respect algebraic dynamics the maximal degree of the polynomials involved must be same as the degree of the octonionic polynomial in M^8.

2. The hyperbolic variant Kähler form and also spinor connection of hyperbolic CP_2 brings in new physics beyond standard model. This Kähler form would serve as the analog of Kähler form assigned to M^4 earlier, and suggested to explain the observed CP breaking effects and matter antimatter asymmetry for which there are two explanations [L81].

Some comments about the Minkowskian signature of the hyperbolic counterparts of CP_3 and CP_2 are in order.

1. Why the metric of CP_3 could not be Euclidian just as the metric of F? The basic objection is that propagation of fields is not possible in Euclidian signature and one completely loses the earlier picture provided by $M^4 \times CP_2$. The algebraic dynamics in M^8 picture can hardly replace it.

2. The map assigning to the point M^4 a point of CP_3 involves Minkowskian sigma matrices but it seems that the Minkowskian metric of CP_3 is not explicitly involved in the construction of scattering amplitudes. Note however that the antisymmetric bi-spinor metric for the spin

1/2 representation of Lorentz group and its conjugate bring in the signature. $U(2,2)$ as representation of conformal symmetries suggests $(2,2)$ signature for 8-D complex twistor space with 2+2 complex coordinates representing twistors.
The signature of CP_3 metric is not explicitly visible in the construction of twistor amplitudes but analytic continuations are carried out routinely. One has also complexified M^4 and M^8 and one could argue that the problems disappear. In the geometric situation the signatures of the subspaces differ dramatically. As already found, analytic continuation could allow to define the variants of twistor spaces elegantly by replacing a complex coordinate with a hyperbolic one.
Remark: For E^4 CP_3 is Euclidian and if one has $E^4_{conf} = U(2)$, one could think of replacing the Cartesian product of twistor spaces with $SU(3)$ group having $M^4_{conf} = U(2)$ as fiber and CP_2 as base. The metric of $SU(3)$ appearing as subgroup of quaternionic automorphisms leaving $M^4 \subset M^8$ invariant would decompose to a sum of M^4_{conf} metric and CP_2 metric plus cross terms representing correlations between the metrics of M^4_{conf} and CP_2. This is probably mere accident.

$M^8 - H$ duality and twistor space counterparts of space-time surfaces

It seems that by identifying $CP_{3,h}$ as the twistor space of M^4, one could develop $M^8 - H$ duality to a surprisingly detailed level from the conditions that the dimensional reduction guaranteed by the identification of the twistor spheres takes place and the extensions of rationals associated with the polynomials defining the space-time surfaces at M^8- and twistor space sides are the same. The reason is that minimal surface conditions reduce to holomorphy meaning algebraic conditions involving first partial derivatives in analogy with algebraic conditions at M^8 side but involving no derivatives.

1. The simplest identification of twistor spheres is by $z_1 = z_2$ for the complex coordinates of the spheres. One can consider replacing z_i by its Möbius transform but by a coordinate change the condition reduces to $z_1 = z_2$.
2. At M^8 side one has either $RE(P) = 0$ or $IM(P) = 0$ for octonionic polynomial obtained as continuation of a real polynomial P with rational coefficients giving 4 conditions (RE/IM denotes real/imaginary part in quaternionic sense). The condition guarantees that tangent/normal space is associative.
Since quaternion can be decomposed to a sum of two complex numbers: $q = z_1 + Jz_2$ $RE(P) = 0$ correspond to the conditions $Re(RE(P)) = 0$ and $Im(RE(P)) = 0$. $IM(P) = 0$ in turn reduces to the conditions $Re(IM(P)) = 0$ and $Im(IM(P)) = 0$.
3. The extensions of rationals defined by these polynomial conditions must be the same as at the octonionic side. Also algebraic points must be mapped to algebraic points so that cognitive representations are mapped to cognitive representations. The counterparts of both $RE(P) = 0$ and $IM(P) = 0$ should be satisfied for the polynomials at twistor side defining the same extension of rationals.
4. $M^8 - H$ duality must map the complex coordinates $z_{11} = Re(RE)$ and $z_{12} = Im(RE)$ ($z_{21} = Re(IM)$ and $z_{22} = Im(IM)$) at M^8 side to complex coordinates u_{i1} and u_{i2} with $u_{i1}(0) = 0$ and $u_{i2}(0) = 0$ for $i = 1$ or $i = 2$, at twistor side.
Roots must be mapped to roots in the same extension of rationals, and no new roots are allowed at the twistor side. Hence the map must be linear: $u_{i1} = a_i z_{i1} + b_i z_{i2}$ and $u_{i2} = c_i z_{i1} + d_i z_{i2}$ so that the map for given value of i is characterized by SL(2,Q) matrix $(a_i, b_i; c_i, d_i)$.
5. These conditions do not yet specify the choices of the coordinates (u_{i1}, u_{i2}) at twistor side. At CP_2 side the complex coordinates would naturally correspond to Eguchi-Hanson complex coordinates (w_1, w_2) determined apart from color $SU(3)$ rotation as a counterpart of $SU(3)$ as sub-group of automorphisms of octonions.
If the base space of the twistor space $CP_{3,h}$ of M^4 is identified as $CP_{2,h}$, the hyper-complex counterpart of CP_2, the analogs of complex coordinates would be (w_3, w_4) with w_3 hypercomplex and w_4 complex. A priori one could select the pair (u_{i1}, u_{i2}) as any pair $(w_{k(i)}, w_{l(i)})$, $k(i) \neq l(i)$. These choices should give different kinds of extremals: such as CP_2 type extremals, string like objects, massless extremals, and their deformations.

String world sheet singularitees and world-line singularities as their light-like boundaries at the light-like orbits of partonic 2-surfaces are conjectured to characterize preferred extremals as surfaces of H at which there is a transfer of canonical momentum currents between Kähler and volume degrees of freedom so that the extremal is not simultaneously an extremal of both Kähler action and volume term as elsewhere. What could be the counteparts of these surfaces in M^8?

1. The interpretation of the pre-images of these singularities in M^8 should be number theoretic and related to the identification of quaternionic imaginary units. One must specify two non-parallel octonionic imaginary units e^1 and e^2 to determine the third one as their cross product $e^3 = e^1 \times e^2$. If e^1 and e^2 are parallel at a point of octonionic surface, the cross product vanishes and the dimension of the quaternionic tangent/normal space reduces from $D = 4$ to $D = 2$.
2. Could string world sheets/partonic 2-surfaces be images of 2-D surfaces in M^8 at which this takes place? The parallelity of the tangent/normal vectors defining imaginary units e_i, $i = 1, 2$ states that the component of e_2 orthogonal to e_1 vanishes. This indeed gives 2 conditions in the space of quaternionic units. Effectively the 4-D space-time surface would degenerate into 2-D at string world sheets and partonic 2-surfacesa as their duals. Note that this condition makes sense in both Euclidian and Minkowskian regions.
3. Partonic orbits in turn would correspond surfaces at which the dimension reduces to D=3 by light-likeness - this condition involves signature in an essential manner - and string world sheets would have 1-D boundaries at partonic orbits.

Getting critical about implicit assumptions related to the twistor space of CP_2

One can also criticize the earlier picture about implicit assumptions related the twistor spaces of CP_2.

1. The possibly singular decomposition of F to a product of S^2 and CP_2 would has a description similar to that for CP_3. One could assign to each point of CP_2 base homologically non-trivial sphere intersecting it orthogonally.
2. I have assumed that the twistor space $T(CP_2) = F = SU(3)/U(1) \times U(1)$ allows Kaluza-Klein type metric meaning that the metric decomposes to a sum of the metrics assignable to the base CP_2 and fiber S^2 plus cross terms representing interaction between these degrees of freedom. It is easy to check that this assumption holds true for Hopf fibration $S^3 \rightarrow S^2$ having circle $U(1)$ as fiber (see `http://tinyurl.com/qbvktsx`). If Kaluza-Klein picture holds true, the metric of F would decompose to a sum of CP_2 metric and S^2 metric plus cross terms representing correlations between the metrics of CP_2 and S^2.
3. One should demonstrate that $F = SU(3)/U(1) \times U(1)$ has metric with the expected Kaluza-Klein property. One can represent $SU(3)$ matrices as products XYZ of 3 matrices. X represents a point of base space CP_2 as matrix, Y represents the point of the fiber $S^2 = U(2)/U(1) \times U(1)$ of F in similar manner as $U(2)$ matrix, and the Z represents $U(1) \times U(1)$ element as diagonal matrix [B7](see `http://tinyurl.com/y6c3pp2g`).
By dropping $U(1) \times U(1)$ matrix one obtains a coordinatization of F. To get the line element of F in these coordinates one could put the coordinate differentials of $U(1) \times U(1)$ to zero in an expression of $SU(3)$ line element. This should leave sum of the metrics of CP_2 and S^2 with constant scales plus cross terms. One might guess that the left- and righ-invariance of the $SU(3)$ metric under $SU(3)$ implies KK property.

Also CP_3 should have the KK structure if one wants to realize the breaking of scaling invariance as a selection of the scale of the conformally compactified M^4. In absence of KK structure the space-time surface would depend parametrically on the point of the twistor sphere S^2.

8.5.2 The nice results of the earlier approach to M^4 twistorialization

The basic nice results of the earlier picture should survive in the new picture.

1. Central for the entire approach is twistor lift of TGD replacing space-time surfaces with 6-D surfaces in 12-D $T(M^4) \times T(CP_2)$ having space-time surfaces as base and twistor sphere S^2

as fiber. Dimensional reduction identifying twistor spheres of $T(M^4)$ an $T(CP_2)$ and makes these degrees of freedom non-dynamical.

2. Dimensionally reduced action 6-D Kähler action is sum of 4-D Kähler action and a volume term coming from S^2 contribution to the induced Kähler form. On interpretation is as a generalization of Maxwell action for point like charge by making particle a 3-surface.

   The interpretation of volume term is in terms of cosmological constant. I have proposed that a hierarchy of length scale dependent cosmological constants emerges. The hierarchy of cosmological constants would define the running length scale in coupling constant evolution and would correspond to a hierarchy of preferred p-aic length scales with preferred p-adic primes identified as ramified primes of extension of rationals.

3. The twistor spheres associated $M^4 \times S^2$ and F were assumed to have same radii and most naturally same Euclidian signature: this looks very nice since there would be only single fundamental length equal to CP_2 radius determining the radius of its twistor sphere. The vision to be discussed would be different. There would be no fundamental scale and length scales would emerge through the length scale hierarchy assignable to CDs in M^8 and mapped to length scales for twistor spaces.

   The identification of twistor spheres with same radius would give only single value of cosmological constant and the problem of understanding the huge discrepancy between empirical value and its naïve estimate would remain. I have argued that the Kähler forms and metrics of the two twistor spheres can be rotated with respect to each other so that the induced metric and Kähler form are rotated with respect to each other, and the magnetic energy density assignable to the sum of the induced Kähler forms is not maximal.

   The definition of Kähler forms involving preferred coordinate frame would gives rise to symmetry breaking. The essential element is interference of real Kähler forms. If the signatures of twistor spheres were opposite, the Kähler forms differ by imaginary unit and the interference would not be possible.

   Interference could give rise to a hierarchy of values of cosmological constant emerging as coefficient of the Kähler magnetic action assignable to $S^2(X^4)$ and predict length scale dependent value of cosmological constant and resolve the basic problem related to the extremely small value of cosmological constant.

4. One could criticize the allowance of relative rotation as adhoc: note that the resulting cosmological constant becomes a function depending on S^2 point. For instance, does the rotation really produce preferred extremals as minimal surfaces extremizing also Kähler action except at string world sheets? Each point of S^2 would correspond to space-time surface X^4 with different value of cosmological constant appearing as a parameter. Moreover, non-trivial relative rotation spoils the covariant constancy and $J^2(S^2) = -g(S^2)$ property for the S^2 part of Kähler form, and that this does not conform with the very idea of twistor space.

5. One nice implication would be that space-time surfaces would be minimal surfaces apart from 2-D string world sheet singularities at which there is a transfer of canonical momentum currents between Kähler and volume degrees of freedom. One can also consider the possibility that the minimal surfaces correspond to surfaces give as roots of 3 polynomials of hypercomplex coordinate of M^2 and remaining complex coordinates.

   Minimal surface property would be direct translation of masslessness and conform with the twistor view. Singular surfaces would represent analogs of Abelian currents. The universal dynamics for minimal surfaces would be a counterpart for the quantum criticality. At M^8 level the preferred complex plane M^2 of complexified octonions would represent the singular string world sheets and would be forced by number theory.

   Masslessness would be realized as generalized holomorphy (allowing hyper-complexity in M^2 plane) as proposed in the original twistor approach but replacing holomorphic fields in twistor space with 6-D twistor spaces realized as holomorphic 6-surfaces.

8.5.3 ZEO and twistorialization as ways to introduce scales in M^8 physics

M^8 physics as such has no scales. One motivation for ZEO is that it brings in the scales as sizes of causal diamonds (CDs).

ZEO generates scales in M^8 physics

Scales are certainly present in physics and must be present also in TGD Universe.

1. In TGD Universe CP_2 scale plays the role of fundamental length scale, there is also the length scale defined by cosmological constant and the geometric mean of these two length scales defining a scale of order 10^{-4} meters emerging in the earlier picture and suggesting a biological interpretation.
   The fact that conformal inversion $m^k \rightarrow R^2m^k/a^2$, $a^2 = m^km_k$ is a conformal transformation mapping hyperboloids with $a \geq R$ and $a \leq R$ to each other, suggests that one can relate CP_2 scale and cosmological scale defined by Λ by inversion so that cell length scale would define one possible radius of cd_{conf}.
2. In fact, if one has $R(cd_{conf}) = x \times R(CP_2)$ one obtains by repeated inversions a hierarchy $R(k) = x^kR$ and for $x = \sqrt{p}$ one obtains p-adic length scale hierarchy coming as powers of $\sqrt{p}$, which can be also negative. This suggests a connection with p-adic length scale hypothesis and connections between long length scale and short length scale physics: they could be related by inversion. One could perhaps see Universe as a kind of Leibnizian monadic system in which monads reflect each other with respect to hyperbolic surfaces $a = constant$. This would conform with the holography.
3. Without additional assumptions there is a complete scaling invariance at the level of M^8. The scales could come from the choice of 8-D causal diamond CD_8 as intersection of 8-D future and past directed light-cones inducing choice of cd in M^4. CD serves as a correlate for the perceptive field of a conscious entity in TGD inspired theory of consciousness and is crucial element of zero energy ontology (ZEO) allowing to solve the basic problem of quantum measurement theory.

Twistorial description of CDs

Could the map of the surfaces of 4-surfaces of M^8 to $cd_{conf} \times CP_2$ by a modification of $M^8 - H$ correspondence allow to describe these scales? If so, compactification via twistorialization and $M^8 - H$ correspondence would be the manner to describe these scales as something emergent rather than fundamental.

1. The simplest option is that the scale of cd_{conf} corresponds to that of CD_8 and CD_4. One should also understand what CP_2 scale corresponds. The simplest option is that CP_2 scale defines just length unit since it is difficult to imagine how this scale could appear at M^8 level. cd_{conf} scale squared would be multiple or CP_2 scale squared, say prime multiple of it, and assignable to ramified primes of extension of rationals. Inversions would produce further scales. Inversion would allow kind of hologram like representation of physics in long length scales in arbitrary short length scales and vice versa.
2. The compactness of cd_{conf} corresponds to periodic time assignable to over-critical cosmologies starting with big bang and ending with big crunch. Also CD brings in mind over-critical cosmology, and one can argue that the dynamics at the level of cd_{conf} reflects the dynamics of ZEO at the level of M^8.

Modification of H and $M^8 - H$ correspondence

It is often said that the metric of M^4_{conf} is defined only modulo conformal scaling factor. This would reflect projectivity. One can however endow projective space CP_3 with a metric with isometry group $SU(2,2)$ and the fixing of the metric is like gauge choice by choosing representative in the projective equivalence class. Thus CP_3 with signature (3,-3) might perhaps define geometric twistor space with base cd_{conf} rather than M^4_{conf} very much like the twistor space $T(CP_2) = F = SU(3)/U(1) \times U(1)$ at the level. Second projection would be to M^4 and map twistor sphere to a point of M^4. The latter bundle structure would be singular since for points of M^4 with light-like separation the twistor spheres have a common point: this is an essential feature in the construction of twistor amplitudes.

New picture requires a modification of the view about H and about $M^8 - H$ correspondence.

1. H would be replaced with $cd_{conf} \times CP_2$ and the corresponding twistor space with $CP_3 \times F$. $M^8 - H$ duality involves the decomposition $M^2 \subset M^4 \subset M^8 = M^4 \times CP_2$, where M^4 is

quaternionic sub-space containing preferred place M^2. The tangent or normal space of X^4 would be characterized by a point of CP_2 and would be mapped to a point of CP_2 and the point of CP_2 - or rather point plus the space S^2 or light-like vectors characterizing the choices of M^2 - would mapped to the twistor sphere S^2 of CP_3 by the standard formulas.

$S^2(cd_{conf})$ would correspond to the choices of the direction of preferred octonionic imaginary unit fixing M^2 as quantization axis of spin and $S^2(CP_2)$ would correspond to the choice of isospin quantization axis: the quantization axis for color hyperspin would be fixed by the choice of quaternionic $M^4 \subset M^8$. Hence one would have a nice information theoretic interpretation.

2. The M^4 point mapped to twistor sphere $S^2(CP_3)$ would be projected to a point of cd_{conf} and define M^8-H correspondence at the level of M^4. This would define compactification and associate two scales with it. Only the ratio $R(cd_{conf})/R(CP_2)$ matters by the scaling invariance at M^8 level and one can just fixe the scale assignable to $T(CP_2)$ and call it CP_2 length scale.

One should have a concrete construction for the hyperbolic variants of CP_n.

1. One can represent Minkowski space and its variants with varying signatures as sub-spaces of complexified quaternions, and it would seem that the structure of sub-space must be lifted to the level of the twistor space. One could imagine variants of projective spaces CP_n, $n=2,3$ as and HP_n, $n=2,3$. They would be obtained by multiplying imaginary quaternionic unit I_k with the imaginary unit i commuting with quaternionic units. If the quaternions λ involved with the projectivization $(q_1,...,q_n) \equiv \lambda(q_1,...,q_n)$ are ordinary quaternions, the multiplication respects the signature of the subspace. By non-commutativity of quaternions one can talk about left- and right projective spaces.
2. One would have extremely close correspondence between M^4 and CP_2 degrees of freedom reflecting the M^8-H correspondence. The projection $CP_3 \rightarrow CP_2$ for E^4 would be replaced with the projection for the hyperbolic analogs of these spaces in the case of M^4. The twistor space of M^4 identified as hyperbolic variant of CP_3 would give hyperbolic variant of CP_2 as conformally compactified cd. The flag manifold $F=SU(3)/U(1)\times U(1)$ as twistor space of CP_2 would also give CP_2 as base space.

The general solution of field equations at the level of $T(H)$ would correspond to holomorphy in general sense for the 6-surfaces defined by 3 vanishing conditions for holomorphic functions - 6 real conditions. Effectively this would mean the knowledge of the exact solutions of field equations also at the level of H: TGD would be an integrable theory. Scattering amplitudes would in turn constructible from these solutions using ordinary partial differential equations [L81].

1. The first condition would identify the complex coordinates of $S^2(cd_{conf})$ and $S^2(CP_2)$: here one cannot exclude relative rotation represented as a holomorphic transformation but for $R(cd_{conf}) \gg R(CP_2)$ the effect of the rotation is small.
2. Besides this there would be vanishing conditions for 2 holomorphic polynomials. The coordinate pairs corresponding to $M^2 \subset M^4$ would correspond to hypercomplex behavior with hyper complex coordinate $u=\pm t-z$. t and z could be assigned with $U(1)$ fibers of Hopf fibrations $SU(2)\rightarrow S^2$.
3. The octonionic polynomial $P(o)$ of degree $n=h_{eff}/h_0$ with rational coefficients fixes the extension of rationals and since the algebraic extension should be same at both sides, the polynomials in twistor space should have same degree. This would give enormous boos concerning the understanding of the proposed cancellation of fermionic Wick contractions in SUSY scattering amplitudes forced by number theoretic vision [L81].

Possible problems related to the signatures

The different signatures for the metrics of the twistor spheres of cd_{conf} and CP_2 can pose technical problems.

1. Twistor lift would replace X^4 with 6-D twistor space X^6 represented as a 6-surface in $T(M^4)\times T(CP_2)$. X^6 is defined by dimensional reduction in which the twistor spheres $S^2(cd_{conf})$ and $S^2(CP_2)$ are identified and define the twistor sphere $S^2(X^4)$ of X^6 serving as a fiber whereas space-time surface X^4 serves as a base. The simplest identification is as $(\theta,\phi)_{S^2(M^4)} = (\theta,\phi)_{S^2(CP_2)}$: the same can be done for the complex coordinates $z_{S^2(M^4_{conf})} = z_{S^2(CP_2))})$. An

open question is whether a Möbius transformation could relate the complex coordinates. The metrics of the spheres are of opposite sign and differ only by the scaling factors $R^2(cd_{conf})$ and $R^2(CP_2)$.

2. For cd_{conf} option the signatures of the 2 twistor spheres would be opposite (time-like for cd_{conf}). For $R(cd_{conf})/R(CP_2) = 1$. $J^2 = -g$ is the only consistent option unless the signature of space is not totally positive or negative and implies that the Kähler forms of the two twistor spheres differ by i. The magnetic contribution from $S^2(X^4)$ would give rise to an infinite value of cosmological constant proportional to $1/\sqrt{g_2}$, which would diverge $R(cd_{conf})/R(CP_2) = 1$. There is however no need to assume this condition as in the original approach.

8.5.4 Hierarchy of length scale dependent cosmological constants in twistorial description

At the level of M^8 the hierarchy of CDs defines a hierarchy of length scales and must correspond to a hierarchy of length scale dependent cosmological constants. Even fundamental scales would emerge.

1. If one has $R(cd_{conf})/R(CP_2) >> 1$ as the idea about macroscopic cd_{conf} would suggest, the contribution of $S^2(cd_{conf})$ to the cosmological constant dominates and the relative rotation of metrics and Kähler form cannot affect the outcome considerably. Therefore different mechanism producing the hierarchy of cosmological constants is needed and the freedom to choose rather freely the ratio $R(cd_{conf})/R(CP_2)$ would provide the mechanism. What looked like a weakness would become a strength.
2. $S^2(cd_{conf}$ would have time-like metric and could have large scale. Is this really acceptable? Dimensional reduction essential for the twistor induction however makes $S^2(cd_{conf})$ non-dynamical so that time-likeness would not be visible even for large radii of $S^2(cd_{conf})$ expected if the size of cd_{conf} can be even macroscopic. The corresponding contribution to the action as cosmological constant has the sign of magnetic action and also Kähler magnetic energy is positive. If the scales are identical so that twistor spheres have same radius, the contributions to the induced metric cancel each other and the twistor space becomes metrically 4-D.
3. At the limit $R(cd_{conf}) \rightarrow RCP_2)$ cosmological constant coming from magnetic energy density diverges for $J^2 = -G$ option since it is proportional to $1/\sqrt{g_2}$. Hence the scaling factors must be different. The interpretation is that cosmological constant has arbitrarily large values near CP_2 length scale. Note however that time dependence is replaced with scale dependence and space-time sheets with different scales have only wormhole contacts.

It would seem that this approach could produce the nice results of the earlier approach. The view about how the hierarchy of cosmological constants emerges would change but the idea about reducing coupling constant evolution to that for cosmological constant would survive. The interpretation would be in terms of the breaking of scale invariance manifesting as the scales of CDs defining the scales for the twistor spaces involved. New insights about p-adic coupling constant evolution emerge and one finds a new "must" for ZEO. $H = M^4 \times CP_2$ picture would emerge as an approximation when cd_{conf} is replaced with its tangent space M^4. The consideration of the quaternionic generalization of twistor space suggests natural identification of the conformally compactified twistor space as being obtained from CP_2 by making second complex coordinate hyperbolic. This need not conform with the identification as $U(2)$.

8.6 How to generalize twistor Grassmannian approach in TGD framework?

One should be able to generalize twistor Grassmannian approach in TGD framework. The basic modification is replacement of 4-D light-like momenta with their 8-D counterparts. The octonionic interpretation encourages the idea that twistor approach could generalize to 8-D context. Higher-dimensional generalizations of twistors have been proposed but the basic problem is that the index raising and lifting operations for twistors do not generalize (see http://tinyurl.com/y24lkwce).

1. For octonionic twistors as pairs of quaternionic twistors index raising would not be lost working for M_T option and light-like M^8 momenta can be regarded sums of M^4_T and E^4 parts as also twistors. Quaternionic twistor components do not commute and this is essential for incidence relation requiring also the possibility to raise or lower the indices of twistors. Ordinary complex twistor Grassmannians would be replaced with their quaternionic countparts. The twistor space as a generalization of CP_3 would be 3-D quaternionic projective space $T(M^8) = HP_3$ with Minkowskian signature (6,6) of metric and having real dimension 12 as one might expect. Another option realizing non-commutativity could be based on the notion of quantum twistor to be also discussed.
2. Second approach would rely on the identification of $M^4 \times CP_2$ twistor space as a Cartesian product of twistor spaces of M^4 and CP_2. For this symmetries are not broken, $M^4_L \subset M^8$ depends on the state and is chosen so that the projection of M^8 momentum is light-like so that ordinary twistors and CP_2 twistors should be enough. $M^8 - H$ relates varying M^4_L based and M^4_T based descriptions.
3. The identification of the twistor space of M^4 as $T(M^4) = M^4 \times S^2$ can be motivated by octonionic considerations but might be criticized as non-standard one. The fact that quaternionic twistor space HP_3 looks natural for M^8 forces to ask whether $T(M^4) = CP_3$ endowed with metric having signature (3,3) could work in the case of M^4. In the sequel also a vision based on the identification $T(M^4) = CP_3$ endowed with metric having signature (3,3) will be discussed.

8.6.1 Twistor lift of TGD at classical level

In TGD framework twistor structure is generalized [K100, K87, K13, L64]. The inspiration for TGD approach to twistorialization has come from the work of Nima Arkani-Hamed and colleagues [B25, B17, B18, B21, B43, B26, B12]. The new element is the formulation of twistor lift also at the level of classical dynamics rather than for the scattering amplitudes only [K100, K13, K87, L64].

1. The 4-D light-like momenta in ordinary twistor approach are replaced by 8-D light-like momenta so that massive particles in 4-D sense become possible. Twistor lift of TGD takes places also at the space-time level and is geometric counterpart for the Penrose's replacement of massless fields with twistors. Roughly, space-time surfaces are replaced with their 6-D twistor spaces represented as 6-surfaces. Space-time surfaces as preferred extremals are minimal surfaces with 2-D string world sheets as singularities. This is the geometric manner to express masslessness. X^4 is simultaneously also extremal of 4-D Kähler action outside singularities: this realizes preferred extremal property.
2. One can say that twistor structure of X^4 is induced from the twistor structure of $H = M^4 \times CP_2$, whose twistor space $T(H)$ is the Cartesian product of geometric twistor space $T(M^4) = M^4 \times CP_1$ and $T(CP_2) = SU(3)/U(1) \times U(1)$. The twistor space of M^4 assigned to momenta is usually taken as a variant of CP_3 with metric having Minkowski signature and both CP_1 fibrations appear in the more precise definition of $T(M^4)$. Double fibration [B42] (see `http://tinyurl.com/yb4bt74l`) means that one has fibration from $M^4 \times CP_1$ - the trivial CP_1 bundle defining the geometric twistor space to the twistors space identified as complex projective space defining conformal compactification of M^4. Double fibration is essential in the twistorialization of TGD [L30].
3. The basic objects in the twistor lift of classical TGD are 6-D surfaces in $T(H)$ having the structure of twistor space in the sense that they are CP_1 bundles having X^4 as base space. Dimensional reduction to CP_1 bundle effectively eliminates the dynamics in CP_1 degrees of freedom and its only remnant is the value of cosmological constant appearing as coefficient of volume term of the dimensionally reduced action containing also 4-D Kähler action. Cosmological term depends on p-adic length scales and has a discrete spectrum [L64, L63].

CP_1 has also an interpretation as a projective space constructed from 2-D complex spinors. Could the replacement of these 2-spinors with their quantum counterparts defining in turn quantum CP_1 realize finite quantum measurement resolution in M^4 degrees of freedom? Projective invariance for the complex 2-spinors would mean that one indeed has effectively CP_1.

8.6.2 Octonionic twistors or quantum twistors as twistor description of massive particles

For M^4_T option the particles are massive and the one encounters the problem whether and how to generalize the ordinary twistor description.

8.6.3 Basic facts about twistors and bi-spinors

It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda^a\tilde{\lambda}^{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality (see `http://tinyurl.com/y6bnznyn`).

1. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned} \langle\lambda,\mu\rangle &= \epsilon_{ab}\lambda^a\mu^b \ , \\ \left[\tilde{\lambda},\tilde{\mu}\right] &= \epsilon_{a'b'}\tilde{\lambda}^{a'}\tilde{\mu}^{b'} \ , \\ p\cdot q &= \langle\lambda,\mu\rangle\left[\tilde{\lambda},\tilde{\mu}\right] \ , \ (q_{aa'} = \mu_a\tilde{\mu}_{a'}) \ . \end{aligned} \quad (8.6.1)$$

2. Spinor indices are lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$. If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned} \epsilon_{aa'} &= \frac{\mu_a\tilde{\lambda}_{a'}}{\langle\mu,\lambda\rangle} \ , \ \text{positive helicity} \ , \\ \epsilon_{aa'} &= \frac{\lambda_a\tilde{\mu}_{a'}}{\left[\tilde{\mu},\tilde{\lambda}\right]} \ , \ \text{negative helicity} \ . \end{aligned} \quad (8.6.2)$$

In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

3. What makes 4-D twistors unique is the existence of the index raising and lifting operations using ϵ tensors. In higher dimensions they do not exist and this causes difficulties. For octonionic twistors with quaternionic components possibly only in $D = 8$ the situation changes.

To get a very rough idea about twistor Grassmannian approach idea, consider tree amplitudes of $\mathcal{N} = 4$ SUSY as example and it is convenient to drop the group theory factor $Tr(T_1T_2\cdots T_n)$. The starting point is the observation that tree amplitude for which more than $n-2$ gluons have the same helicity vanish. MHV amplitudes have exactly $n-2$ gluons of same helicity- taken by a convention to be negative- have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle\lambda_x,\lambda_y\rangle^4}{\prod_{i=1}^n\langle\lambda_i,\lambda_{i+1}\rangle} \quad (8.6.3)$$

When the sign of the helicities is changed $\langle..\rangle$ is replaced with [..].

An essential point in what follows is that the amplitudes are expressible in terms of the antisymmetric bi-linears $\langle\lambda_i,\lambda_j\rangle$ making sense also for octotwistors and identifiable as quaternions rather than octonions.

$M^8 - H$ duality and two alternative twistorializations of TGD

$M^8 - H$ duality suggests two alternative twistorializations of TGD.

1. The first approach would be in terms of M^8 twistors suggested by quaternionic light-lineness of 8-momenta. M^8 twistors would be Cartesian products of M^4 and E^4 twistors. One can imagine a straightforward generalization of twistor scattering amplitudes in terms of generalized Grassmannian approach replacing complex Grassmannian with quaternionic Grassmannian, which is a mathematically well-defined notion.

2. Second approach would rely on $M^4 \times CP_2$ twistors, which are products of M^4 twistors and CP_2 twistors: this description works nicely at classical space-time level but at the level of momentum space the problem is how to describe massivation of M^4 momenta using twistors.

Why the components of twistors must be non-commutative?

How to modify the 4-D twistor description of light-like 4-momenta so that it applies to massive 4-momenta?

1. Twistor consists of a pair $(\mu_{\dot{\alpha}}, \lambda^{\alpha})$ of bi-spinors in conjugate representations of $SU(2)$. One can start from the 4-D incidence relations for twistors

$$\mu_{\dot{\alpha}} = p_{\alpha\dot{\alpha}}\lambda^{\alpha} \ .$$

Here $p_{\alpha\dot{\alpha}}$ denotes the representation of four-momentum $p^k\sigma_k$. The antisymmetric permutation symbols $\epsilon^{\alpha\beta}$ and its dotted version define antisymmetric "inner product" in twistor space. By taking the inner product of μ with itself, one obtains the commutation relation $\mu_1\mu_2-\mu_2\mu_1=0$, which is consistent with right-hand side for massless particles with $p_kp^k=0$.

2. In TGD framework particles are massless only in 8-D sense so that the right hand side in the contraction is in general non-vanishing. In massive case one can replace four-momentum with unit vector. This requires

$$\langle \mu 1, \mu_2\rangle = \mu_1\mu_2 - \mu_2\mu_1 \neq 0 \ .$$

The components of 2-spinor become non-commutative.

This raises two questions.

1. Could the replacement of complex twistors by quaternionic twistors make them non-commutative and allow massive states?
2. Could non-commutative quantum twistors solve the problem caused by the light-likeness of momenta allowing 4-D twistor description?

Octotwistors or quantum twistors?

One should be able to generalize twistor amplitudes and twistor Grassmannian approach to TGD framework, where particles are massless in 8-D sense and massive in 4-D sense. Could twistors be replaced by octonionic or quantum twistors.

1. One can express mass squared as a product of commutators of components of the twistors λ and $\tilde{\lambda}$, which is essentially the conjugate of λ:

$$p\cdot p \quad = \quad \langle\lambda,\lambda\rangle \left[\tilde{\lambda},\tilde{\lambda}\right] \ . \tag{8.6.4}$$

This operator should be non-vanishing for non-vanishing mass squared. Both terms in the product vanish unless commutativity fails so that mass vanishes. The commutators should have the quantum state as its eigenstate.

2. Also 4-momentum components should have well-defined values. Four-momentum has expression $p^{aa'} = \lambda^a\tilde{\lambda}^{a'}$ in massless case. This expression should generalized to massive case as such. Eigenvalue condition and reality of the momentum components requires that the components $p^{aa'}$ are commuting Hermitian operators.
In twistor Grassmannian approach complex but light-like momenta are possible as analogs of virtual momenta. Also in TGD framework the complexity of Kähler coupling strength allows to consider complex momenta. For twistor lift they however differ from real momenta only by a phase factor associated with the $1/\alpha_K$ associated with 6-D Kähler action.
Remark: I have considered also the possibility that states are eigenstates only for the longitudinal M^2 projection of 4-momentum with quark model of hadrons serving as a motivation.
   (a) Could this equation be obtained in massive case by regarding λ^a and $\tilde{\lambda}^{a'}$ as commuting octo-spinors and their complex conjugates? Octotwistors would naturally emerge in the description at embedding space level. I have already earlier considered the notion of octotwistor [K95] [L46]).

(b) Or could it be obtained for quantum bi-spinors having same states as eigenstates. Could quantum twistors as generalization of the ordinary twistors correspond to the reduction of the description from the level of M^8 or H to at space-time level so that one would have 4-D twistors and massive particles with 4-momentum identifiable as Noether charge for the action principle determining preferred extremals? I have considered also the notion of quantum spinor earlier [K43, K67, K62, K4, K110].

3. In the case of quantum twistors the generalization of the product of the quantities $\langle\lambda_i, \lambda_{i+1}\rangle$ appearing in the formula should give rise to c-number in the case of quantum spinors. Can one require that the quantities $\langle\lambda_i, \lambda_{i+1}\rangle$ or even $\langle\lambda_i, \lambda_j\rangle$ are c-numbers simultaneously? This would also require that $\langle\lambda, \lambda\rangle$ is non-vanishing c-number in massive case: also incidence relation suggest this condition. Could one think λ as an operator such that $\langle\lambda, \lambda\rangle$ has eigenvalue spectrum corresponding to the quantities $\langle\lambda_i, \lambda_{i+1}\rangle$ appearing in the scattering amplitude?

8.6.4 The description for M^4_T option using octo-twistors?

For option I with massive M^4_T projection of 8-momentum one could imagine twistorial description by using M^8 twistors as products of M^4_T and E^4 twistors, and a rather straightforward generalization of standard twistor Grassmann approach can be considered.

Could twistor Grassmannians be replaced with their quaternionic variants?

The first guess would simply replace $Gr(k, n)$ with $Gr(2k, 2n)$ 4-D twistors 8-D twistors. From twistor amplitudes with quaternionic M^8-momenta one could construct physical amplitudes by going from 8-momentum basis to the 4-momentum- basis with wave functions in irreps of $SO(3)$. Life is however not so simple.

1. The notion of ordinary twistor involves in an essential manner Pauli matrices σ_i satisfying the well-known anti-commutation relations. They should be generalized. In fact, σ_0 and $\sqrt{-1}\sigma_i$ can be regarded as a matrix representation for quaternionic units. They should have analogs in 8-D case.
Octonionic units ie_i indeed provide this analog of sigma matrices. Octonionic units for the complexification of octonions allow to define incidence relation and representation of 8-momenta in terms of octo-spinors. They do not however allow matrix representation whereas time-like octonions allow interpretation as quaternion in suitable bases and thus matrix representation. Index raising operation is essential for twistors and makes dimension $D = 4$ very special. For naïve generalizations of twistors to higher dimensions this operation is lost (see `http://tinyurl.com/y24lkwce`).
2. Could one avoid multiplication of more than two octo-twistors in Grassmann amplitudes leading to difficulties with associativity. An important observation is that in the expressions for the twistorial scattering amplitudes only products $\langle\lambda_i, \lambda_j\rangle$ or $[\tilde{\lambda}_i, \tilde{\lambda}_{i+1}]$ but not both occur. These products are associative even if the spinors are replaced by quaternionic spinors.
These operations are antisymmetric in the arguments, which suggests cross product for quaternions giving rise to imaginary quaternion so that the product of objects would give rise to a product of imaginary quaternions. This might be a problem since a large number of terms in the product would approach to zero for random imaginary quaternions.
An ad hoc guess would be that scattering probability is proportional to the product of amplitude as product $\langle\lambda_i, \lambda_j\rangle$ and its "hermitian conjugate" with the conjugates $[\tilde{\lambda}_i, \tilde{\lambda}_{i+1}]$ in the reverse order (this does not affect the outcome) so that the result would be real. Scattering amplitude would be more like quaternion valued operator. Could one have a formulation of quantum theory or at least TGD view about quantum theory allowing this?
3. If ordinary massless 4-momenta correspond to quaternionic sigma matrices, twistors can be regarded as pairs of 2-spinors in matrix representation. Octonionic 8-momenta should correspond to pairs of 4-spinors. As already noticed, octonions do not however allow matrix representation! Octonions for a fixed decomposition $M^8 = M^4 \times E^4$ can be however decomposed to linear combination of two quaternions just like complex numbers to a combination of real numbers. These quaternions would have matrix representation and quaternionic analogs of twistor pair $(\mu, \tilde{\lambda})$. One could perhaps formulate the generalization of twistor Grassmann

amplitudes using these pairs. This would suggest replacement of complex bi-spinors with complexified quaternions in the ordinary formalism. This might allow to solve problems with associativity if only $\langle\lambda_i, \lambda_j\rangle$ or $[\tilde{\lambda}_i, \tilde{\lambda}_{i+1}]$ appear in the amplitudes.

4. The argument in the momentum conserving delta function $\delta(\lambda_i\tilde{\lambda}_i)$ should be real so that the conjugation with respect to i would not change the argument and non-commutativity would not be problem. In twistor Grassmann amplitudes the argument $C \cdot Z$ of delta momentum conserving function is linear in the components of complex twistor Z. If complex twistor is replaced with quaternionic twistor, the Grassmannian coordinates C in delta functions $\delta(C \cdot Z)$ must be replaced with quaternionic one.

The replacement of complex Grassmannians $Gr_C(k,n)$ with quaternionic Grassmannians $Gr_H(k,n)$ is therefore highly suggestive. Quaternionic Grassmannians (see `http://tinyurl.com/y23jsffn`) are quotients of symplectic Lie groups $Gr_H(k,n) = U_n(H)/(U_r(H) \times U_{n-r}(H))$ and thus well-defined. In the description using $Gl_H(k,n)$ matrices the matrix elements would be quaternions and $k \times k$ minors would be quaternionic determinants.

Remark: Higher-D projective spaces of octonions do not exist so that in this sense dimension $D = 8$ for embedding space would be maximal.

Twistor space of M^8 as quaternionic projective space HP_3?

The simplest Grassmannian corresponds to twistor space and one can look what one obtains in this case. One can also try to understand how to cope with the problems caused by Minkowskian signature.

1. In previous section it was found that the modification of H to $H = cd_{conf} \times CP_2$ with $cd_{conf} = CP_{2,h}$ identifiable as CP_2 with Minkowskian signature of metric is strongly suggestive.
2. For E^8 quaternionic twistor space as analog of CP_3 would be its quaternionic variant HP_3 with expected dimension $D = 16 - 4 = 12$. Twistor sphere would be replaced with its quaternionic counterpart $SU(2)_H/U(1)_H$ having dimension 4 as expected. $CD_{8,conf}$ as conformally compactified CD_8 must be 8-D. The space HP_2 has dimension 8 and is analog of CP_2 appearing as analog of base space of CP_3 identified as conformally compactified 4-D causal diamond cd_{conf}. The quaternionic analogy of $M^4_{conf} = U(2)$ identified as conformally compactified M^4 would be $U(2)_H$ having dimension $D = 10$ rather than 8.
   HP_3 and HP_2 might work for E^8 but it seems that the 4-D analog of twistor sphere should have signature (2,-2) whereas base space should have signature (1,-7). Some kind of hyperbolic analogs of these spaces obtained by replacing quaternions with their hypercomplex variant seem to be needed. The same receipe in the twistorialization of M^4 would give cd_{conf} as analog of CP_2 with second complex coordinate made hyperbolic. I have already considered the construction of hyperbolic analogs of CP_2 and CP_3 as projective spaces. These results apply to HP_2 and HP_3.
3. What about octonions? Could one define octonionic projective plane OP_2 and its hyperbolic variants corresponding to various sub-spaces of M^8? Euclidian OP_2 known as Cayley plane exists as discovered by Ruth Moufang in 1933. Octonionic higher-D projective spaces and Grassmannians do not however exist so that one cannot assign OP_3 as twistor spaces.

Can one obtain scattering amplitudes as quaternionic analogs of residue integrals?

Can one obtain complex valued scattering amplitudes (i commuting with octonionic units) in this framework?

1. The residue integral over quaternionic C-coordinates should make sense, and pick up the poles as vanishing points of minors. The outcome of repeated residue integrations should give a sum over poles with complex residues.
2. Residue calculus requires analyticity. The problem is that quaternion analyticity based on a generalization of Cauchy-Riemann equations allows only linear functions. One could define quaternion (and octonion) analyticity in restricted sense using powers series with real coefficients (or in extension involving i commuting with octonion units). The quaternion/octonion analytic functions with real coefficients are closed with respect to sum and product. I have used this definition in the proposed construction of algebraic dynamics for in $X^4 \subset M^8$ [L46].

3. Could one define the residue integral purely algebraically? Could complexity of the coefficients (i) force complex outcome: if pole q_0 is not quaternionically real the function would not allow decompose to $f(q)/(q-q_0)$ with f allowing similar Taylor series at pole. If so, then the formulas of Grassmannian formalism could generalize more or less as such at M^8 level and one could map the predictions to predictions of $M^4 \times CP_2$ approach by analog of Fourier transform transforming these quantum state basis to each other.

This option looks rather interesting and involves the key number theoretic aspects of TGD in a crucial manner.

8.6.5 Do super-twistors make sense at the level of M^8?

By $M^8 - H$ duality [L46] there are two levels involved: M^8 and H. These levels are encountered both at the space-time level and momentum space level. Do super-octonions and super-twistors make sense at M^8 level?

1. At the level of M^8 the high uniqueness and linearity of octonion coordinates makes the notion of super-octonion natural. By $SO(8)$ triality octonionic coordinates (bosonic octet 8_0), octonionic spinors (fermionic octet 8_1), and their conjugates (anti-fermionic octet 8_{-1}) would for triplet related by triality. A possible problem is caused by the presence of separately conserved B and L. Together with fermion number conservation this would require $\mathcal{N}=4$ or even $\mathcal{N}=4$ SUSY, which is indeed the simplest and most beautiful SUSY.
2. At the level of the 8-D momentum space octonionic twistors would be pairs of two quaternionic spinors as a generalization of ordinary twistors. Super octo-twistors would be obtained as generalization of these.

The progress in the understanding of the TGD version of SUSY [L81] led to a dramatic progress in the understanding of super-twistors.

1. In non-twistorial description using space-time surfaces and Dirac spinors in H, embedding space coordinates are replaced with super-coordinates and spinors with super-spinors. Theta parameters are replaced with quark creation and annihilation operators. Super-coordinate is a super-polynomial consisting of monomials with vanishing total quark number and appearing in pairs of monomial and its conjugate to guarantee hermiticity.
   Dirac spinor is a polynomial consisting of powers of quark creation operators multiplied by monomials similar to those appearing in the super-coordinate. Anti-leptons are identified as spartners ofquarks identified as local 3-quark states. The multi-spinors appearing in the expansions describe as such local many-quark-antiquark states so that super-symmetrization means also second quantization. Fermionic and bosonic states assignable to H-geometry interact since super-Dirac action contains induced metric and couplings to induced gauge potentials.
2. The same recipe works at the level of twistor space. One introduces twistor super-coordinates analogous to super-coordinates of H and M^8. The super YM field of $\mathcal{N}=4$ SUSY is replaced with super-Dirac spinor in twistor space. The spin degrees of freedom associated with twistor spheres S^2 would bring in 2 additional spin-like degrees of freedom.
   The most plausible option is that the new spin degrees are frozen just like the geometric S^2 degrees of freedom. The freezing of bosonic degrees of freedom is implied by the construction of twistor space of X^4 by dimensional reduction as a 6-D surface in the product of twistor spaces of M^4 and CP_2. Chirality conditions would allow only single spin state for both spheres.
3. Number theoretical vision implies that the number of Wick contractions of quarks and antiquarks cannot be larger than the degree of the octonionic polynomial, which in turn should be same as that of the polynomials of twistor space giving rise to the twistor space of space-time surface as 6-surface. The resulting conditions correspond to conserved currents identifiable as Noether currents assignable to symmetries.

Also Grassmannian is replaced with super-Grassmannian and super-coordinates as matrix elements of super matrices are introduced.

1. The integrand of the Grassmannian integral defining the amplitude can be expanded in Taylor series with respect to theta parameters associated with the super coordinates C as rows of super $G(k,n)$ matrix.

2. The delta function $\delta(C, Z)$ factorizing into a product of delta functions is also expanded in Taylor series to get derivatives of delta function in which only coordinates appear. By partial integration the derivatives acting on delta function are transformed to derivatives acting on integrand already expanded in Taylor series in theta parameters. The integration over the theta parameters using the standard rules gives the amplitudes associated with different powers of theta parameters associated with Z and from this expression one can pick up the scattering amplitudes for various helicities of external particles.

The super-Grassmannian formalism is extremely beautiful but one must remember that one is dealing with quantum field theory. It is not at all clear whether this kind of formalism generalizes to TGD framework, where particle are 3-surfaces [L46]. The notion of cognitive representation effectively reducing 3-surfaces to a set of point-like particles strongly suggests that the generalization exists.

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It seems now clear that sparticles are predicted and SUSY remains in the simplest scenario exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The increased undestanding of what twistorialization leads to an improved understanding of what twistor space in TGD could be. It turns out that the hyperbolic variant $CP_{3,h}$ of the standard twistor space CP_3 is a more natural identification than the earlier $M^4 \times S^2$ also in TGD framework but with a scale corresponding to the scale of CD at the level of M^8 so that one obtains a scale hierarchy of twistor spaces. Twistor space has besides the projection to M^4 also a bundle projection to the hyperbolic variant $CP_{2,h}$ of CP_2 so that a remarkable analogy between M^4 and CP_2 emerges. One can formulate super-twistor approach to TGD using the same formalism as will be discussed in this article for the formulation at the level of H. This requires introducing besides 6-D Kähler action and its super-variant also spinors and their super-variants in super-twistor space. The two formulations are equivalent apart from the hierarchy of scales for the twistor space. Also M^8 allows analog of twistor space as quaternionic Grassmannian HP_3 with signature (6,6). What about super- variant of twistor lift of TGD? consider first the situation before the twistorialization.

1. The parallel progress in the understanding SUSY in TGD framework [L81] leads to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.
2. In fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites identifiable as spartners of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of embedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.
3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

Super-counterpart of twistor lift using the proposed formalism

The construction of super-coordinates and super-spinors [L81] suggests a straightforward formulation of the super variant of twistor lift . One should only replace the super-embedding space and super-spinors with super-twistor space and corresponding super-spinors and formulate the theory using 6-D super-Kähler action and super-Dirac equation and the same general prescription for constructing S-matrix. Dimensional reduction should give essentially the 4-D theory apart from the variation of the radius of the twistor space predicting variation of cosmological constant. The size scale of CD would correspond to the size scale of the twistor space for M^4 and for CP_2 the size scale would serve as unit and would not vary.

The first step is the construction of ordinary variant of Kähler action and modified Dirac action for 6-D surfaces in 12-D twistor space.

1. Replace the spinors of H with the spinors of 12-D twistor space and assume only quark chirality. By the bundle property of the twistor space one can express the spinors as tensor products of spinors of the twistor spaces $T(M^4)$ and $T(CP_2)$. One can express the spinors of $T(M^4)$ tensor products of spinors of M^4 - and S^2 spinors locally and spinors of $T(CP_2)$ as tensor products of CP_2 - and S^2 spinors locally. Chirality conditions should reduce the number of 2 spin components for both $T(M^4)$ and $T(CP_2)$ to one so that there are no additional spin degrees of freedom.
   The dimensional reduction can be generalized by identifying the two S^2 fibers for the preferred extremals so that one obtains induced twistor structure. In spinorial sector the dimensional reduction must identify spinorial degrees of freedom of the two S^2s by the proposed chirality conditions also make them non-dynamical. The S^2 spinors covariantly constant in S^2 degrees of freedom.

2. Define the spinor structure of 12-D twistor space, define induced spinor structure at 6-D surfaces defining the twistor space of space-time surface. Define the twistor counterpart of the analog of modified Dirac action using same general formulas as in case of H.

Construct next the super-variant of this structure.

1. Introduce second quark oscillator operators labelled by the points of cognitive representation in 12-D twistor space effectively replacing 6-D surface with its discretization and having quantized quark field q as its continuum counterpart. Replace the coordinates of the 12-D twistor space with super coordinates h_s expressed in terms of quark and anti-quark oscillator operators labelled by points of cognitive representation, and having interpretation as quantized quark field q restricted to the points of representation.

2. Express 6-D Kähler action and Dirac action density in terms of super-coordinates h_s. The local monomials of q appear in h_s and therefore also in the expansion of super-variants of modified gamma matrices defined by 6-D ähler action as contractions of canonical momentum currents of the action density L_K with the gamma matrices of 12-D twistor space. In super-Kähler action also the local composites of q giving rise to currents formed from the local composites of 3-quarks and antiquarks and having interpretation as leptons and anti-leptons occur - leptons would be therefore spartners of squarks.

3. Perform super-expansion also for the induced spinor field q_s in terms of monomials of q. $q_s(q)$ obeys super-Dirac equation non-linear in q. But also q should satisfy super-Dirac action as an analog of quantized quark field and non-linearity indeed forces also q to have has super-expansion. Thus both quark field q and super-quark field q_s both satisfy super-Dirac equation. The only possibility is $q_s = q$ stating fixed point property under $q \to q_s$ having interpretation in terms of quantum criticality fixing the values of the coefficients of various terms in q_s and in the super-coordinate h_s having interpretation as coupling constants. One has quantum criticality and discrete coupling constant evolution with respect to extension of rationals characterizing adelic physics.

4. Super-Dirac action vanishes for its solutions and the exponent of super-action reduces to exponent of super-Kähler action, whose matrix elements between positive and negative energy parts of zero energy states give S-matrix elements.
   Super-Dirac action has however an important function: the derivatives of quark currents appearing in the super-Kähler action can be transformed to a linear strictly local action of super spinor connection ($\partial_\alpha \to A_{\alpha,s}$ effectively). Without this lattice discretization would be needed and cognitive representation would not be enough.

To sum up, the super variants of modified gamma matrices of the 6-surface would satisfy the condition $D_{\alpha,s}\Gamma^\alpha_s = 0$ expressing preferred extremal property and guaranteeing super-hermicity of D_s. q_s would obey super-Dirac equation $D_s q_s = 0$. The self-referential identification $q = q_s$ would express quantum criticality of TGD.

8.7 Could one describe massive particles using 4-D quantum twistors?

The quaternionic generalization of twistors looks almost must. But before this I considered also the possibility that ordinary twistors could be generalized to quantum twistors to describe particle massivation. Quantum twistors could provide space-time level description, which requires 4-D twistors, which cannot be ordinary M^4 twistors. Also the classical 4-momenta, which by QCC would be equal to M^8 momenta, are in general massive so that the ordinary twistor approach cannot work. One cannot of course exclude the possibility that octo-twistors are enough or that M^8_L description is equivalent with space-time description using quantum twistors.

8.7.1 How to define quantum Grassmannian?

The approach to twistor amplitude relies on twistor Grassmann approach [B19, B15, B14, B23, B25, B12] (see `http://tinyurl.com/yxllwcsn`). This approach should be replaced by replacing Grassmannian $GR(K,N) = Gl(n,C)/Gl(n-m,C) \times Gl(m,C)$ with quantum Grassmannian.

naïve approach to the definition of quantum Grassmannian

Quantum Grassmannian is a notion studied in mathematics and the approach of [A64] (see `http://tinyurl.com/y5q6kv6b`) looks reasonably comprehensible even for physicist. I have already earlier tried to understand quantum algebras and their possible role in TGD [K15]. It is however better to start as ignorant physicist and proceed by trial and error and find whether mathematicians have ended up with something similar.

1. Twistor Grassmannian scattering amplitudes involving k negative helicity gluons involve product of $k \times k$ minors of an $k \times n$ matrix C taken in cyclic order. C defines $k \times n$ coordinates for Grassmannian $Gr(k,n)$ of which part is redundant by the analogs of gauge symmetries $Gl(n-m,C) \times Gl(m,C)$. Here n is the number of external gluons and k the number of negative helicity gluons. The $k \times k$ determinants taken in cyclic order appear in the integrand over Grassmannian. Also the quantum variants of these determinants and integral over quantum Grassmannian should be well-defined and residue calculus gives hopes for achieving this.
2. One should define quantum Grassmannian as algebra according to my physicist's understanding algebra can be defined by starting from a free algebra generated by a set of elements - now the matrix elements of quantum matrix. One poses on these elements relations to get the algebra considered. What could these conditions be in the recent case.
3. A natural condition is that the definition allows induction in the sense that its restriction to quantum sub-matrices is consistent with the general definition of $k \times n$ quantum matrices. In particular, one can identify the columns and rows of quantum matrices as instances of quantum vectors.
4. How to generalize from 2×2 case to $k \times n$ case? The commutation relations for neighboring elements of rows and columns are fixed by induction. In 4×4 corresponding to M^4 twistors one would obtain for $(a_1, ..., a_4)$. $a_i a_{i+1} = q a_{i+1} a_i$ cyclically ($k = 1$ follows $k = 4$).
What about commutations of a_i and a_{i+k}, $k > 1$. Is there need to say anything about these commutators? In twistor Grassmann approach only connected $k \times k$ minors in cyclic order appear. Without additional relations the algebra might be too large. One could argue that the simplest option is that one has $a_i a_{i+k} = q a_{i+k} a_i$ for k odd $a_i a_{i+k} = q^{-1} a_{i+k} a_i$ for k even. This is required from the consistency with cyclicity. These conditions would allow to define also sub-determinants, which do not correspond to connected $k \times k$ squares by moving the elements to a a connected patch by permutations of rows and columns.
5. What about elements along diagonal? The induction from 2×2 would require the commutativity of elements along right-left diagonals. Only commutativity of the elements along left-right diagonal be modified. Or is the commutativity lost only along directions parallel to left-right diagonal? The problem is that the left-right and right-left directions are transformed to each other in odd permutations. This would suggest that only even permutations are allowed in the definition of determinant

6. Could one proceed inductively and require that one obtains the algebra for 2×2 matrices for all 2×2 minors? Does this apply to all 2×2 minors or only to connected 2×2 minors with cyclic ordering of rows and columns so that top and bottom row are nearest neighbors as also right and left column. Also in the definition of 3×3 determinant only the connected developed along the top row or left column only 2×2 determinants involving nearest neighbor matrix elements appear. This generalizes to $k \times k$ case.

It is time to check how wrong the naïve intuition has been. Consider 2×2 matrices as simple example. In this case this gives only 1 condition ($ad - bc = -da + cb$) corresponding to the permutation of rows or columns. Stronger condition suggested by higher-D case would be $ad = da$ and $bc = cb$. The definition of 2×2 in [A64] however gives for quantum 2-matrices $(a, b; c, d)$ the conditions

$$\begin{array}{ll} ac = qca \ , & bd = qda \ , \\ ab = qba \ , cd = qdc \ , & \\ bc = cb \ , & ad - da = (q - q^{-1})bc \ . \end{array} \tag{8.7.1}$$

The commutativity along left-right diagonal is however lost for $q \neq 1$ so that quantum determinant depends on what row or column is used to expand it. The modification of the commutation relations along rows and columns is what one might expect and wants in order to achieve non-commutativity of twistor components making possible massivation in M^4 sense.

The limit $q \rightarrow 1$ corresponds to non-trivial algebra in general and would correspond to $\beta = 4$ for inclusions of HFFs expected to give representations of Kac-Moody algebras. At this limit only massless particles in 4-D sense are allowed. This suggests that the reduction of Kac-Moody algebras to quantum groups corresponds to symmetry breaking associated with massivation in 4-D sense.

Mathematical definition of quantum Grassmannian

It would seem that the proposed approach is reasonable. The article [A84] (see `http://tinyurl.com/yycflgrd`) proposing a definition of quantum determinant explains also the basic interpretation of what the non-commutativity of elements of quantum matrices does mean.

1. The first observation is that the commutation of the elements of quantum matrix corresponds to braiding rather than permutation and this operation is represented by R-matrix. The formula for the action of braiding is

$$R^{ab}_{cd} t^c_e t^d_f = t^a_d t^b_c R^{cd}_{ef} \ . \tag{8.7.2}$$

Here R-matrix is a solution of Yang-Baxter equaion and characterizes completely the commutation relations between the elements of quantum matrix. The action of braiding is obtained by applying the inverse of R-matrix from left to the equation. By iterating the braidings of nearest neighbors one can deduce what happens in the braiding exchanging quantum matrix elements which are not nearest neighbors. What is nice that the R-matrix would fix the quantum algebra, in particular quantum Grassmannian completely.

2. In the article the notion of quantum determinant is discussed and usually the definition of quantum determinant involves also the introduction of metric g^{ab} allowing the raising of the indices of the permutation symbol. One obtains formulas relating metric and R-matrix and restricting the choice of the metric. Note however that if ordinary permutation symbol is used there is no need to introduce the metric.

The definition quantum Grassmannian proposed does not involve hermitian conjugates of the matrices involved. One can define the elements of Grassmannian and Grassmannian residue integrals without reference to complex conjugation: could one do without hermitian conjugates? On the other hand, Grassmannians have complex structure and Kähler structure: could this require hermitian conjugates and commutation relations for these?

8.7.2 Two views about quantum determinant

If one wants to define quantum matrices in $Gr(k, n)$ so that quantal twistor-Grassmann amplitudes make sense, the first challenge is to generalize the notion of $k \times k$ determinant.

One can consider two approaches concerning the definition of quantum determinant.

1. The first guess is that determinant should not depend on the ordering of rows or columns apart from the standard sign factor. This option fails unless one modifies the definition of permutation symbol.
2. The alternative view is that permutation symbol is ordinary and there is dependence on the row or column with respect to which one develops. This dependence would however disappear in the scattering amplitudes. If the poles and corresponding residues associated with the $k \times k$-minors of the twistor amplitude remain invariant under the permutation, this is not a problem. In other words, the scattering amplitudes are invariant under braid group. This is what twistor Grassmann approach implies and also TGD predict.

For the first option quantum determinant would be braiding invariant. The standard definition of quantum determinant is discussed in detail in [A84] (see `http://tinyurl.com/yycflgrd`).

1. The commutation of the elements of quantum matrix corresponds to braiding rather than permutation and as found, this operation is represented by R-matrix.
2. Quantum determinant would change only by sign under the braidings of neighboring rows and columns. The braiding for the elements of quantum matrix would compensate the braiding for quantum permutation symbol. Permutation symbol is assumed to be q-antisymmetric under braiding of any adjacent indices. This requires that permutation $i_k \leftrightarrow i_{k+1}$ regarded as braiding gives a contraction of quantum permutation symbol $\epsilon_{i_1,...1_k}$ with $R^{ij}_{i_k i_{k+1}}$ plus scaling by some normalization factor λ besides the change of sign.

$$\epsilon_{a_1...a_k a_{k+1}...a_n} = -\lambda \epsilon_{a_1...ij...a_n} R^{ji}_{a_k a_{k+1}} \quad . \tag{8.7.3}$$

The value of λ can be calculated.
3. The calculation however leads to the result that quantum determinant $\mathcal{D}$ satisfies $\mathcal{D}^2 = 1$! If the result generalizes for sub-determinants defined by $k \times k$-minors (, which need not be the case) would have determinants satisfying $\mathcal{D}^2 = 1$, and the idea about vanishing of $k \times k$-minor essential for getting non-trivial twistor scattering amplitude as residue would not make sense.

It seems that the braiding invariant definition of quantum determinant, which of course involves technical assumptions) is too restrictive. Does this mean that the usual definition requiring development with respect to preferred row is the physically acceptable option? This makes sense if only the integral but not integrand is invariant under braidings. Braiding symmetry would be analogous to gauge invariance.

8.7.3 How to understand the Grassmannian integrals defining the scattering amplitudes?

The beauty of the twistor Grassmannian approach is that the residue integrals over quantum $Gr(k,n)$ would reduce to sum over poles (or possibly integrals over higher-D poles). Could residue calculus provide a manner to integrate q-number valued functions of q-numbers? What would be the minimal assumptions allowing to obtain scattering amplitudes as c-numbers?

Consider first what the integrand to be replaced with its quantum version looks like.

1. Twistor scattering amplitudes involve also momentum conserving delta function expressible as $\delta(\lambda_a \tilde{\lambda}^a)$. This sum and - as it seems - also the summands should be c-numbers - in other words one has eigenstates of the operators defining the summands.
2. By introducing Grassmannian space $Gr(k,n)$ with coordinates $C_{\alpha,i}$ (see `http://tinyurl.com/yxllwcsn`), one can linearize $\delta(\lambda_a \tilde{\lambda}^a)$ to a product of delta functions $\delta(C \cdot Z) = \delta(C \cdot \tilde{\lambda}) \times \delta(C^{\perp} \cdot \lambda)$ (I have not written the delta function is Grassmann parameters related to super coordinates). Z is the n-vector formed by the twistors associated with incoming particles.
The $4 \times k$ components of $C_{\alpha,k} Z^k$ should be c-numbers at least when they vanish. One should define quantum twistors and quantum Grassmannian and pose the constraints on the poles.

How to achieve the goal? Before proceeding it is good to recall the notion of non-commutative geometry (see `http://tinyurl.com/yxrcr8xv`). Ordinary Riemann geometry can be obtained from exterior algebra bundle, call it E. The Hilbert space of square integrable sections in E carries a representation of the space of continuous functions $C(M)$ by multiplication operators. Besides this there is unbounded differential operator D, which so called signature operator and defined in terms of exterior derivative and its dual: $D = d + d^*$. This spectral triple of algebra, Hilbert space, and operator D allows to deduce the Riemann geometry.

The dream is that one could assign to non-commutative algebras non-commutative spaces using this spectral triple. The standard q-p quantization is example of this: one obtains now Lagrange manifolds as ordinary commutative manifolds.

Consider now the situation in the case of quantum Grassmannian.

1. In the recent case the points defining the poles of the function - it might be that the eventual poles are not a set of discrete points but a higher-dimensional object - would form the commutative part of non-commutative quantum space. In this space the product of quantum minors would become ordinary number as also the argument $C \cdot Z$ of the delta function. This commutative sub-space would correspond to a space in which maximum number of minors vanish and residues reduce to c-numbers.

   Thus poles of the integrand of twistor amplitude would correspond to eigenstates for some $k \times k$ minors of Grassmannian with a vanishing eigenvalue. The residue at the pole at given step in the recursion pole by pole need not be c-number but the further residue integrals should eventually lead to a c-number or c-number valued integrand.

2. The most general option would be that the conditions hold true only in the sense that some $k \times k$ minors for $k \geq 2$ are c-numbers and have a vanishing eigenvalue but that smaller minors need not have this property. Also $C_{\alpha,k}Z^k$ should be c-numbers and vanish. Residue calculus would give rise to lower-D integrals in step-wise manner.

   The simplest and most general option is that one can speak only about eigenvalues of $k \times k$ minors. At pole it is enough to have one minor for which eigenvalue vanishes whereas other minors could remain quantal. In the final reduction the product of all non-vanishing $k \times k$ minors appearing in cyclic order in the integrand should have a well-defined c-number as eigenvalue. Does this allow the appearance of only cyclic minors.

   A stronger condition would be that all non-vanishing minors reduce to their eigenvalues. Could it be that only the n cyclic minors can commute simultaneously and serve as analogs of q-coordinates in phase space? The complex dimension of $G_C(n,k)$ is $d = (n-k)k$. If the space spaced by minors corresponds to Lagrangian manifold with real dimension not larger than d, one has $k \leq d = (n-k)k$. This gives $k \leq n/2(1 + \sqrt{1-2/n})$ For $k = 2$ this gives $k \leq n/2$. For $n \to \infty$ one has $k \leq n/2 + 1$. For $k > n/2$ one can change the roles of positive and negative helicities. It has been found that in certain sense the Grassmannian contributing to the twistor amplitude is positive.

   The notion of positivity found to characterize the part of Grassmannian contributing to the residue integral and also the minors and the argument of delta function [B22](see `http://tinyurl.com/yd9tf2ya`) would suggest that it is also real sub-space in some sense and this finding supports this picture.

   The delta function constraint forcing $C \cdot Z$ to zero must also make sense. $C \cdot Z$ defines $k \times 6$ matrix and also now one must consider eigenvalues of $C \cdot Z$. Positivity suggest reality also now. Z adds $4 \times n$ degrees of freedom and the number $6 \times k$ of additional conditions is smaller than $4 \times n$. $6k \leq 4 \times n$ combined with $k \leq n/2$ gives $k \leq n/2$ so that the conditions seems to be consistent.

3. The c-number property for the cyclic minors could define the analog of Lagrangian manifold for the phase space or Kähler manifold. One can of course ask, whether Kähler structure of $Gr(k,n)$ could generalize to quantum context and give the integration region as a sub-manifold of Lagrangian manifold of $Gr(k,n)$ and whether the twistor amplitudes could reduce to integral over sub-manifold of Lagrangian manifold of ordinary $Gr(k,n)$.

To sum up, I have hitherto thought that TGD allows to get rid of the idea of quantization of coordinates. Now I have encountered this idea from totally unexpected perspective in an attempt to understand how 8-D masslessness and its twistor description could relate to 4-D one. Grassmannians are however very simple and symmetric objects and have natural coordinates as $k \times n$ matrices interpretable as quantum matrices. The notion of quantum group could find very concrete application as a solution to the basic problem of the standard twistor approach. Therefore one can consider the possibility that they have quantum counterparts and at least the residue integrals reducing to c-numbers make sense for quantum Grassmannians in algebraic sense.

Chapter 9

The Recent View about SUSY in TGD Universe

What SUSY is in TGD framework is a longstanding question, which found a rather convincing answer rather recently. In twistor Grassmannian approach to $\mathcal{N}=4$ SYM [B25, B17, B18, B21, B43, B26, B12] twistors are replaced with supertwistors and the extreme elegance of the description of various helicity states using twistor space wave functions suggests that super-twistors are realized both at the level of M^8 geometry and momentum space.

In TGD framework M^8-H duality allows to geometrize the notion of super-twistor in the sense that at the level of M^8 different components of super-field correspond to components of super-octonion each of which corresponds to a space-time surfaces satisfying minimal surface equations with string world sheets as singularities - this is geometric counterpart for masslessness.

9.0.1 New view about SUSY

The progress in understanding of M^8-H duality [L76] throws also light to the problem whether SUSY is realized in TGD [L81] and what SUSY breaking cold mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them. Super-octonion components of polynomials have different orders so that also the extension of rational assignable to them is different and therefore also the ramified primes so that p-adic prime as one them can be different for the members of SUSY multiplet and mass splitting is obtained.

The question how to realize super-field formalism at the level of $H=M^4\times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and perhaps also hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge could appear as a space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with an improved understanding of quantum criticality and the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials: the criticality is criticality for the polynomials interpreted as p-adic polynomials in $O(p) = 0$ approximation meaning the presence of multiple roots in this approximation.

9.0.2 Connection of SUSY and second quantization

The linear combinations monomials of theta parameters appearing in super-fields are replaced in case of hermitian H super coordinates consisting of combinations of monomials with vanishing quark number. For super-spinors of H the monomials carry odd quark number with quark number 1. Monomials of theta parameters are replaced by local monomials of quark oscillator operators labelled besides spin and weak isospin also by points of cognitive representation with embedding space coordinates in an extension of rationals defining the adele. Discretization allows anti-commutators which are Kronecker deltas rather than delta functions. If continuum limit makes sense, normal ordering must be assumed to avoid delta functions at zero coming from the contractions. The monomials (not only the coefficients appearing in them) are solved from generalized classical field equations and are linearly related to the monomials at boundary of CD playing the role of quantum fields and classical field equations determine the analogs of propagators.

The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals. The polynomial composition hierarchies correspond to inclusion hierarchies for isomorphic sub-algebras of super-symplectic algebra having interpretation in terms of inclusions of hyper-finite factors of type II_1.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

9.0.3 Proposal for S-matrix

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces.

1. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.
2. Super-Dirac equation implies that super-Dirac action vanishes on-mass-shell. The proposed construction however allows to get also scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation however makes possible to express derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution follows from the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.
3. S-matrix is trivial unless CD contains the images of 6-D analogs of branes as universal special solutions of the algebraic equations determining space-time surfaces at the level of M^8. 4-D

space-time surfaces representing particle orbits meet at the partonic 2-surfaces associated with the 3-D surfaces at $t = r_n$ hyper-surfaces of M^4. The values of $t = r_n$ correspond to the roots of the real polynomial with rational coefficients determining the space-time surface. These transitions are analogs of weak measurements, and in TGD theory of consciousness they give rise to the experience flow of time and can be said to represent "very special moments" in the life of self [L73].

4. The creation and annihilation operators at vertices associated with the monomials would be connected to the points assignable to cognitive representations at opposite boundaries of CD and also to partonic 2-surfaces in the interior of CD possibly accompanied by sub-CDs. This would give analogs of twistor Grassmannian diagrams containing finite number of partonic 2-surfaces as topological vertices containing in turn finite number ordinary vertices defined by the monomials. The diagrams would be completely classical objects in accordance with the fact that quantum TGD is completely classical theory apart from state function reduction.
5. This view allows also a formulation of continuum theory since the monomials appearing in the action density in the interior of CD are linear superposition of the monomials at the points of boundary of CD involving 3-D integral so that contractions of oscillator operators only reduce one integration without introducing divergence. One can also normal order the monomials at boundary of CD serving as initial values. If preferred extremals are analogs of Bohr orbits, one can express extremals using either boundary as the seat of initial data.

9.1 How to formulate SUSY at the level of $H = M^4 \times CP_2$?

In the following I will represent the recent trial for constructing SUSY at the level of $H = M^4 \times CP_2$. The first trial replaced theta parameters of SUSY with quark oscillator operators labelled by spin and isospin and had rather obvious shortcomings: in particular, one did not obtain many-quark states with large quark numbers. The second trial allows quark oscillator operators to have as labels also the points of space-time surface in cognitive representation and thus having coordinates of H belonging to an extension of rationals defining the adele [**?**]

9.1.1 First trial

If SUSY is realized at the level of M^8, it should have a formulation also at the level of H. The basic elements of the first trial form part of also second trial. The basic modification made in the second trial is that finite number of theta parameters replaced with the fermionic oscillator operators labelled by the points of cognitive representations so that they are analogous to fermion fields in lattice, and only local composites of the oscillator operators appear in the super coordinates and super-spinors. This means that SUSY is essential element of the second quantization of fermions in TGD.

1. $M^8 - H$ duality is non-local and means that the dynamics at the level of H is not strictly local but dictated by partial differential equations for super-fields having interpretation as describing purely local many-fermion states made of fundamental fermions with quantum numbers of leptons and quarks (quarks do not possess color as spin like quantum number) ad their antiparticles.
2. Classical field equations and modified Dirac equation must result from this picture. Induction procedure for the spinors of H must generalize so that spinors are replaced by super-spinors Ψ_s having multi-spinors as components multiplying monomials of theta parameters θ. The determinant of metric and modified gamma matrices depend on embedding space coordinates h replaced with super coordinates h_s so that monomials of θ appear in two different ways. Hermiticity requires that sums of monomial and its hermitian conjugate appear in h_s. Monomials must also have vanishing fermion numbers. Otherwise one can obtain fermionic states propagating like bosons. For Dirac action one must assume that Ψ_s involves only odd monomials of θ with quark number 1 involving monomials appearing in h_s to get only states with quark number 1 and correct kind of propagators.
3. One Taylor expands both bosonic action density (6-D Kähler action dimensionally reducing to 4-D Kähler action plus volume term) and Super-Dirac action with respect to the super-coordinates h_s. In Super-Dirac action one has also the expansion of super-spinor in odd

monomials with total quark number 1. The coefficients of the monomials of θ:s are obtained are partial derivatives of the action. Since the number of θ parameters is finite and corresponds to the number of spin-weak-isopin states of quarks and leptons, the number of terms is finite if the θ parameters anti-commute to zero. If not, one can get an infinite number of terms from the Taylor series for the action to the coefficient given monomial. Number theoretical considerations do not favor this and there should exist a cancellation mechanism for the radiative corrections coming from fermionic Wick contractions if thetas correspond to fermionic oscillator operators as it seems to be.

4. One can interpret the superspace as the exterior algebra of the spinors of H. This reminds of the result that the sections of the exterior algebra of Riemann manifold codes for the Riemann geometry (see `http://tinyurl.com/yxrcr8xv`). This generalizes the observation that one can hear the shape of a drum since the sound spectrum is determined by its frequency spectrum defined by Laplacian.
   Super-fields define a Clifford algebra generated by θ parameters as a kind of square root of exterior algebra which corresponds to the Clifford algebra of gamma matrices. Maybe this algebra could code also for the spinor structure of embedding space or even that of space-time surface so that the super-fields could be seen as carriers of geometric information about space-time surface as a preferred extremal. In 8-D case there is also $SO(1,8)$ triality suggesting that corresponding three Clifford algebras correspond to exterior algebra fermionic and anti-fermionic algebras.

What about the situation at the level of M^8?

1. At M^8 level the components of super-octonion correspond to various derivatives of the basic polynomial $P(t)$ so that space-time geometry correlates with the quantum numbers assignable to super-octonion components - this is in accordance with QCC (quantum-classical correspondence). This is highly desirable at the level of H too.
   Could the space-time surface in M^8 be same for super-field components with degree $d < d_{max}$ in some special cases? The polynomial associated with super octonion components are determined by the derivatives of the basic polynomial $P(t)$ with order determined by the degree of the super-monomial. If they have decomposition $P(t) = P_1^k(t)$, the monomials with degree $d < k$ the roots corresponding to the roots $P_1(t)$ co-incide. Besides this there are additional roots of d^rP_1/dt^r for super-octonion component with r θ parameters.
   A possible interpretation could be as quantum criticality in which there is no SUSY breaking for components having $d < k$ (masses in p-adic thermodynamics could be the same since the extension defined by P_1 and corresponding ramified primes would be same). This would conform with the general vision about quantum criticality.
2. Usual super-field formalism involves Grassmann integration over θ parameters to give the action. M^8 formalism does not involve the θ integral at all. Should this be the case also at the level of H? This would guarantee that different components of H- coordinates as super-field would give rise to different space-time surface and QCC would be realized. θ integration produces SUSY invariants naturally involved with the definition of vertices involving components of super-fields. Also vertices involving fermionic and bosonic states emerge since bosonic super-field components appear in super-coordinates in super-Dirac action.

This approach does not say anything about second quantization. There is a strong temptation to replace the theta parameters with fermionic oscillator operators. One cannot however obtain second quantization of fermions in this manner since the maximal quark number (and lepton number if leptons are present as fundamental fermions) of the states is 4. To achieve second quantization, one must replace the theta parameters with fermionic oscillator operators labelled besides spin and weak isospin by the coordinates of points of 3-surface, most naturally the points belonging to a cognitive representation characterizing space-time surface for given extension of rationals.

9.1.2 Second trial

I have already earlier considered a proposal for how SUSY could be realized in TGD framework. As it often happens, the original proposal was not quite correct. The following discussion gives a formulation solving the problems of the first proposal and suggests a concrete formulas for the scattering amplitudes in ZEO based on super-counterparts of preferred extremals. In the sequel I will talk about super Kähler function as functional of 3-surfaces and - super Kähler function action. By holography allowing to identify 3-surfaces with corresponding space-time surfaces as analogs of Bohr orbits, these notions have the same meaning.

Could the exponent of super-Kähler function as vacuum functional define S-matrix as its matrix elements

Consider first the key ideas - some of them new - formulated as questions.

1. Could one see SUSY in TGD sense as a counterpart for the quantization in the sense of QFT so that oscillator operators replace theta parameters and would become fermionic oscillator operators labelled by spin and electroweak spin - as proposed originally - and by selected points of 3-surface of light-cone boundary with embedding space coordinates in extension of rationals? One would have analog of fermion field in lattice identified as a number theoretic cognitive representation for given extension of rationals. The new thing would be allowance of local composites of oscillator operators having interpretation in terms of analogs for the components of super-field.

   SUSY in TGD sense would be realized by allowing local composites of oscillator operators containing 4+4 quark oscillator operators at most. At continuum limit normal ordering would produce delta functions at origin unless one assumes normal ordering from beginning. For cognitive representations one would have only Kronecker deltas and one can consider the possibility that normal ordering is not present. The vanishing of normal ordering terms above some number of them suggested to be the dimension for the extension of rationals would give rise to a discrete coupling constant evolution due to the contractions of fermionic oscillator operators.

2. What is dynamical in the superpositions of oscillator operator monomials? Are the coefficients dynamical? Or are the oscillator operators themselves dynamical - this would mean a QFT type reduction to single particle level? The latter option seems to be correct. Oscillator operators are labelled by points of cognitive representation and in continuum case define an analog of quantum spinor field, call it q. This suggests that this field satisfies the super counter part of modified Dirac equation and must involve also super part formed from the monomials of q and $\overline{q}$. This however requires the replacement of q with q_s in super-Dirac operator and super-coordinates h_s and one ends up with an iteration $q \rightarrow q_s \rightarrow ...$

   The only solution to the paradoxical situation is that one has self-referential equation $q = q_s$ having interpretation in terms of quantum criticality fixing the coefficients of terms in $q = q_s$. Analogous condition $h_s = (h_s)_s$ must be satisfied by h_s under substitution $h_s \rightarrow (h_s)_s$. These conditions fix coefficients of terms in H super-coordinate h_s and q_s interpreted as coupling constants so that quantum criticality implying a discrete coupling constant evolution as function of extension of rationals follows. Also super-Dirac equation $D_s q_s = 0$ and field equations $D_{s,\alpha}\Gamma^{\alpha,s} = 0$ for Kähler action guaranteeing hermiticity are satisfied.

3. Could one interpret the time reversal operation taking creation- and annihilation operators to each other as time reflection permuting the points at the opposite boundaries of CD? The positive *resp.* negative energy parts of zero energy states would be created by creation *resp.* annihilation operators from respective vacuums assigned to the opposite boundaries of CD.

4. Could one regard preferred extremal regarded as 4-surface in super embedding space parameterized by the hermitian embedding coordinates plus the coefficients of the monomials of quarks and antiquarks with vanishing quark number, whose time evolution follows from dimensionally reduced 6-D super-Kähler action? Could one assume similar interpretation for super spinors consisting of monomials with total quark number equal to 1 and appearing in super-Dirac action?

5. In WKB approximation the exponent of action defines wave function. In QFTs path integral is defined by an exponent of action and scattering operator can be formally defined as action exponential. Could the matrix elements for the exponent of the super counterpart of Kähler function plus super Dirac action between states at opposite boundaries of CD between positive and negative energy parts of zero energy states define S-matrix? Could the positive and negative energy parts of zero energy states be identified as many particles states formed from the monomials associated with embedding space super-coordinates and super-spinors?

6. Could the construction of S-matrix elements as matrix elements of super-action exponential reduce to classical theory? Super-field monomials in the interior of CD would be linear superpositions of super-field monomials at boundary of CD. Note that oscillator operator monomials rather than their coefficients would be the basic entities and the dynamics would reduce to that for oscillator operators as in QFTs. The analogs of propagators would relate the monomials to those at boundary ly to the monomials at the boundary of CD, and would be determined by classical field equations so that in this sense everything would be classical. Note however that the fixed point condition $q = q_s$ and super counterpart of modified Dirac equation are non-linear.
   Vertices would be defined by monomials appearing in super-coordinate and super-spinor field appearing in terms of those at boundary of CD. If two vertices at interior points x and y of CD are connected there is line leading from x to a point z at boundary of CD and back to y and one would have sum over points z in cognitive representation. This applies also to self energy corrections with $x = y$. At the possibly existing continuum limit integral would smoothen the delta function singularities and in presence of normal ordering at continuum would eliminate them.
   In the expressions for the elements of S-matrix annihilation operators appearing in the monomials would be connected to the passive boundary P of CD and creation operators to the active boundary. If no partonic 2-surfaces appear as topological vertices in the interior of CD, this would give trivial S-matrix!
   $M^8 - H$ duality however predicts the existence of brane like entities as universal 6-D surfaces as solutions of equations determining space-time surfaces. Their M^4 projection is $t = r_n$ hyperplane, where r_n corresponds to a root of a real polynomial with algebraic coefficients giving rise to octonion polynomial, and is mapped to similar surface in H. 4-D space-time surfaces representing incoming and outgoing lines would meet along their ends at these partonic 2-surfaces.
   Partonic 2-surfaces at these hyper-surfaces would contain ordinary vertices as points in cognitive representation. Given vertex would have at most 4+4 incoming and outgoing lines assignable to the monomial defining the vertex. This picture resembles strongly the picture suggested by twistor Grassmannian approach. In particular the number of vertices is finite and their seems to be no superposition over different diagrams. In this proposal, the lines connecting vertices would correspond to 1-D singularities of the space-time surfaces as minimal surfaces in H. Also stringy singularities can be considered but also these should be discretized.
   By fixing the set of monomials possibly defining orthonormal state basis at both boundaries one would obtain given S-matrix element. S-matrix elements would be matrix elements of the super-action exponential between states formed by monomials of quark oscillator operators. Also entanglement between the monomials defining initial and finals states can be allowed. Note that this in principle allows also initial and final states not expressible using monomials but that monomials are natural building bricks as analogs of field operators in QFTs.

7. The monomials associated with embedding space coordinates are embedding space vectors constructible from Dirac currents (left- or right-handed) with oscillator operators replacing the induced spinor field and its conjugate. The proposed rules for constructing S-matrix would give also scattering amplitudes with odd quark number at boundaries of CD. Could the super counterpart of the bosonic action (super Kähler function) be all that is needed to construct the S-matrix?
   In fact, classically Dirac action vanishes on mass shell: if this is true also for super-Dirac action then the addition of Dirac action would not be needed. The super-Taylor expansion of super-Kähler action gives rise to the analogs of perturbation theoretic interaction terms so that one has perturbation theory without perturbation theory as Wheeler might state it. The detailed study of the structure of the monomials appearing in the super-Kähler action shows that they have interpretation as currents assignable to gauge bosons and scalar and pseudo-scalar Higgs. Super Dirac action is however needed. Super-Dirac equation for q and $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ allow to reduce ordinary divergences $\partial_\alpha j^\alpha$ of fermionic currents appearing in super-Kähler action to commutators $[A_{\alpha,s} j^\alpha]$. Therefore no information about q at nearby points is needed and one avoids lattice discretization: cognitive representation is enough.

8. Topological vertices represent discontinuities of the space-time surface bringing strongly in

mind the non-determinism of quantum measurement, and one can ask whether the 3-branes and associated partonic 2-surfaces. Could the state function reductions analogous to weak measurements correspond to these discontinuities? Ordinary state function reductions would change the arrow of time and the roles of active and passive boundaries of CD [L69]. In TGD inspired theory of consciousness these time values would correspond to "very special moments" in the life of self [L73].

9. The unitarity of S-matrix can be understood from the structure of the exponent of Kähler action. The exponent decomposes to a sum of real and purely imaginary parts. The exponent of the hermitian imaginary part is a unitary operator for a given space-time surface. Real exponent containing also radiative corrections from the normal ordering gives exponent of Kähler function as vacuum functional in WCW (sum in the case of cognitive representations) and by choosing the normalization factor of the state appropriately one obtains unitary S-matrix.

9.1.3 More explicit picture

The following sketch tries to make the picture of the second trial more explicit.

1. The construction of S-matrix should reduce to super-geometry coded by super Kähler function determined by the 6-D Kähler action for twistor lift by dimensional reduction. This might be possible since zero energy states have vanishing total conserved charges and exponent of super-Kähler function has matrix elements only between states at opposite boundaries of CD having same total charges.
2. Construction should reduce to preferred extremals and their super-deformations determined by variational principle with boundary conditions. The boundary values of super-deformations at either boundary could be also interpreted as initial values for preferred extremals analogous to Bohr orbits. The expectations for the super action with fixed initial values between positive and negative energy parts would give the scattering amplitudes assignable to a given space-time surface. There would be functional integral over space-time surfaces using exponent of Kähler function as weight. In number theoretic vision this would reduce to sum over preferred extremals labelled by cognitive representations serving as WCW coordinates.
3. Number theoretic vision suggests a discretization in terms of cognitive representation consisting of points with coordinates in extension of rationals defining the adele. This representation could be associated with the boundaries of CD and possibly with M^4 time=constant hyperplanes assignable with the universal special solutions in M^8. At the partonic 2-surfaces associated with these hyper-planes 4-D extremals would meet along their ends: topological particle vertices would be in question. Is string world sheets and partonic 2-surfaces correspond to singularities, the boundaries of strings world sheets as intersections of the string world sheets and orbits of partonic 2-surfaces should represent fermion lines.
4. Creation operators would be assigned with the passive boundary of CD - call it P - and annihilation operators as their conjugates would act as creation operators at the opposite boundary, active boundary - call it A. Time reversal symmetry of CD suggests that annihilation operator as conjugate of creation operator labelled by the a point of boundary of CD corresponds to the same point in common coordinates for light-cone boundary. This would conform also with the basic character of the half-algebras associated with super-symplectic symmetries.

   The original proposal was that oscillator operators have only spin and electroweak spin as indices but the standard view about spin and statistics requires that also the points of the 3-surface must label them. Also the fact that the total quark number can be larger than 4 of course requires this too. Algebraically the only difference with respect to this proposal is that one allows also the points of 3-surface at the boundary of CD as labels.
5. Number theoretical vision requires that only points of 3-surface having embedding space coordinates in the extension of rationals defining the adelic physics are allowed. In the generic case the number of points in the cognitive representation would be finite and would increase with the dimension of extension so that at the limit of algebraic numbers they form a dense set of 3-surface.

   Since action has infinite expansion in powers of super coordinates the contractions of oscillator operators would give rise to a renormalization of the coefficients of the monomials and of classical action. For cognitive representations one would avoid normal ordering problems sine the number of contractions is limited by the number of points in cognitive representation. This would give rise to discrete coupling constant evolution as function of the extension of rationals.

6. In continuum theory all points of 3-D boundary would label quark oscillator operators and one must normal order the oscillator operators in given local monomial. Also now the idea about connecting creation and annihilation operators to opposite boundaries of CD would allow to get rid of infinities due to contractions.

The action exponential would lead to a rather concrete proposal for the coefficients of the monomials appearing in super-fields.

1. The deformations of embedding space coordinates would be expressible as WCW-local superpositions of isometry generators or as WCW-global superpositions of Hamiltonian currents contracted with the coordinate deformations. The latter would conform with super-symplectic symmetries of WCW. CP_2 Hamiltonian currents would give color quantum numbers. S^2 Hamiltonian currents would be also present. One could see space-time local Kac-Moody symmetries assignable to light-like partonic orbits and string world sheets as a dual representations at space-time level of symplectic symmetries at embedding space level.
2. Spinor modes would be expressible as superpositions of embedding space spinor modes having expansion as super-Taylor series at the boundaries of CD. This would give spin and electroweak quantum numbers.

Does one really obtain description of gauge bosons and gravitons by using the exponent?

1. Could the coefficients of super-monomials at boundary of CD allow interpretation in terms of gauge bosons? These entities could have well-defined quantum numbers so that this might be possible. Quark spin and isospin would represent additional spin degrees of freedom. The Hamiltonians of H of CP_2 expressible for given 3-surface as local superpositions of SU(3) Killing vector fields would represent color degrees of freedom.
   For string world sheets one would naturally have transversal M^4 super-coordinates and CP_2 super-coordinates as analogs of fields. Could this allow to get gauge bosons as excitations of strings as in string theories.
2. Gauge bosons could be also bi-local composites of fermion and anti-fermion at opposite boundaries of wormhole contact or at opposite wormhole contacts of wormhole flux tube. Gravitons could be 4-local composites. Baryons and mesons could be this kind of non-local composites. One can consider also the analog of monopole phase of QFTs in which particles would be multilocal composites.
3. The bosonic action is for induced metric and induced Kähler form. QFT wisdom would suggest that their super-analogs could correspond to external particles. One could indeed take the induced gauge potentials or -fields at boundary and form their contractions with Killing vectors of isometries to obtain general coordinate invariant quantities and form their super-analogs as normal ordered local composites. One can consider the same idea for induced gravitational field or its deviation from Minkowski metric.
   Formally this would correspond to an addition to the action exponential of perturbative terms of type jA appearing in QFTs representing coupling to external currents and take the limit $j \to 0$. In QFT picture this works since various gauge fields are functionally independent but in TGD framework this is not the case. Second problem is to to construct a complete orthonormalized set of states in this manner. Therefore it seems this description can make sense only at QFT limit of TGD.

Dimensionally reduced 6-D Kähler action as an analog of SYM action

The 6-D dimensionally reduced Kähler action reduces to a sum of 4-D Kähler action and volume term and will be simply referred to as Kähler action. The super variant of this action is obtained by replacing embedding space coordinates with their super counterparts. Super-Kähler action is analogous to pure SYM action.

1. Space-time would be super-surface in super counterpart of $H = M^4 \times CP_2$ with coordinates h^k having super components proportional to multi-spinors multiplying the monomials of oscillator operators. The ocillator operator monomials rather than only the multi-spinor coefficients of

the oscillator monomials transforming like vectors of H are regarded as analogs of quantum fields expressible by classical field equations as linear superpositions of their values at the boundary of CD for preferred extremals. The dynamics of monomials would reduce to that for oscillator operators labelled by points of cognitive representation and having interpretation as restriction of quantized quark field satisfying super-Dirac equation and the quantum criticality condition $q = q_s$.

2. Fermionic creation operators and annihilation operators labelled not only by spin and weak isospin as in the original proposal but also by the finite number of points of the cognitive representation. Therefore oscillator operators are analogous to the values of fermion field in discretization obeying super variant of modified Dirac equation. Both leptonic and quark like oscillator operators corresponding to two different H-chiralities and having different couplings to Kähler gauge potential could be present but octonionic triality allows only quarks. The vacuum expectation value of the action action exponentials contains only monomials with vanishing B (and L if leptons are present as fundamental fields). The matrix elements between positive and negative energy parts of zero energy states gives S-matrix.
   Real super-coordinates can be assumed to be hermitian and thus contain only sums of monomials and their conjugates having vanishing fermion numbers. This guarantees super-symmetrization respecting bosonic statistics at the level of propagators since all kinetic terms involve two covariant derivatives - one can indeed transform ordinary derivatives of monomials coming from the Taylor expansion to covariant derivatives involving also the coupling to Kähler form since the total Kähler charge of terms vanishes.

The lack of anti-commutativity of fermionic oscillator operators implies the presence of terms resulting in contractions.

1. The super-Taylors series would involve a finite number of partial derivatives of action. Wick contractions of oscillator operators would give rise to an infinite number of terms in continuum case. The appearance of infinite Taylor series defining the coefficients of super-polynomial is however troublesome from the point of view of number theoretic vision since there is no guarantee that the coefficients are rational functions. The finite number of points in the cognitive representation implying finite number of oscillator operators however allows only finite number of terms in the super-Taylor expansion.
   The monomials appearing in action in the interior of CD can be expressed as linear superpositions of those at boundary also in continuum case. Therefore each monomial is 3-D integral over the monomials at the boundary of CD. As a consequence, the contractions giving delta functions only eliminate one integration but do not give rise to infinities. A general solution to the divergence problems emerges.
   This is actually nothing new: one of the key ideas behind the notion of WCW is that path integral over space-time surfaces is replaced by a functional integral over 3-surfaces in WCW holographically equivalent with preferred extremals as analogs of Bohr orbits. The non-locality of the theory due to the replacement of point-like particles with 3-surfaces would solve the divergence problems.

An interesting possibility in line with the speculations of Nima-Arkani Hamed and others is that the action defining space-time as a 4-surface of embedding space could emerge from the anti-commutators of the oscillator operator monomials as radiative corrections so that the bosonic action would vanish when the super-part of h_s vanishes.

Super-Dirac action

Before doing anything one can recall what happens in the case of modified Dirac action.

1. One has separate modified Dirac actions $\overline{\Psi}D\Psi$, $D = \Gamma^\alpha D_\alpha$ for quarks and leptons (later it will be found that modified Dirac action for quarks might be enough) and the covariant derivatives differ since there is a coupling to n-ple of included Kähler potential. For leptons one has $n = -3$ and for quarks $n = 1$. This guarantees that em charges come out correctly. This coupling appears in the covariant derivative D_α of fermionic super field.
2. One obtains modified Dirac equations for quarks and leptons by variation with respect to spinors. The variation with respect to the embedding space coordinates gives quantized ver-

sions of classical conservation laws with respect to isometries. One also obtains and infinite number of super-currents as contractions of modes of the modified Dirac operator with Ψ.

3. Classical field equations for the space-time surface emerge as a consistency condition guaranteeing that the modified Dirac operator is hermitian: canonical momentum currents of classical action must be conserved and define conserved quantum when contracted with Killing vectors of isometries. Quantum-classical correspondence (QQC) requires than for Cartan algebra of symmetry algebra the classical Noether charges are same as the fermionic Noether charges.
It turns out that the super-symmetrization of modified Dirac equation gives only fermions and they fermionic superpartners in this manner if one requires that propagators are consistent with statistics.

Consider first the situation without the quantum criticality condition $q = q_s = \Psi_s$. H coordinates are super-symmetrized and induced spinor field becomes a super-spinor $\Psi_s = \Psi^N O_N(q,\overline{q})$ with Ψ_N depending on h_s (summation over N is understood).

1. As in the case of bosonic action the vacuum expectation value gives modified Dirac action conserving fermion numbers but one could assume that the monomials in the leptonic (quark) modified Dirac action have either non-vanishing L (B) and vanishing B (L). It seems that the lepton (baryon -) number of monomials can vary from 1 to maximum value. A more restrictive condition would be that the value is 1 for all terms.

2. Super-Dirac spinor is expanded in monomials $O_N(q,\overline{q})$ of q and its conjugate $\overline{q}$, whose anti-commutator is non-trivial. One can equally well talk about quark like oscillator operators. The sum $\Psi = \Psi^N O_N$ defining super-spinor field. The multi-spinors Ψ_N are functions of space-time coordinates, which are ordinary numbers. Quark oscillator operators are same as appearing in the embedding space super-coordinates. Only monomials O_N having total quark number equal to 1 are allowed. Super-spinor field however contains terms involving quark pairs giving rise to spartners of multiquark states with fixed quark number. The conjugate of super-spinor is defined in an obvious manner.

3. The metric determinant and modified gamma matrices appearing in the Dirac action are expanded as Taylor series in hermitian super-coordinate $h_s + \overline{h}_s$ with $h = h^N O_N$. This as as in the case of bosonic action.
There are also couplings to gauge potentials defined by the spinor connection of CP_2 and the expansion of them with respect to the embedding space coordinates gives at the first step rise covariant derivatives of gauge potentials giving spinor curvature. At next steps one obtains covariant derivatives of spinor curvature, which however vanish so that the number of terms coming from the dependence of spinor connection on CP_2 coordinates is expected to be finite. Constant curvature property of CP_2 is therefore be essential (not that also M^4 would have covariantly constant spinor curvature in twistor lift and give rise to CP breaking).
The super-coordinate expansion of the metric determinant $\sqrt{g}$ and modified gamma matrices Γ^α and covariant derivatives D_α involving dependence on H coordinates give additional monomials of q parameters appear as hermitian monomials. Classical field equations correspond to $D_\alpha \Gamma^\alpha = 0$ guaranteeing the hermiticity of $D = \Gamma^\alpha D_\alpha$.

4. When super-coordinates of H are replaced with ordinary embedding space coordinates the only Wick contractions are between O^N and $\overline{O}^N$ in the vacuum expectation of Dirac action, and the action reduces to super-Dirac action with components satisfying modified Dirac equation. Propagator is Dirac propagator for all terms and the presence of only odd components in Ψ with quark number 1 and even components in h^s guarantees that Fermi statistics is not violated at the level of propagators. The dependence on h_s induces coupling between different components of the super-spinor. The components of super-spinor are interpreted as second quantized objects.

5. The terms in the action would typically involve n-tuples of partial derivatives $L_{k_1\alpha_1...k_n 1\alpha_n}$ defined earlier for $L = \sqrt{g}$ coming from super-Taylor expansions. Similar derivatives come from the modified gamma matrices Γ^α.
Also now one obtains loops from the self contractions in the terms coming from the expression of action and gamma matrices. These terms should vanish and as already found this would requires vanishing of currents perhaps identifiable as Noether currents of symmetries. This guarantees that the Taylor expansion contains only finite number of terms as required by number theoretic vision.

The multi-fermion vertices defined by the action would be non-trivial but involve always contraction of all fermion indices between monomials formed from oscillator operators in Ψ and their conjugates in $\overline{\Psi}$ if the loop contractions sum up to zero. One could interpret these super-symmetric vertices as a redistribution of fermions of a local many-fermion state between external local many-fermion states particles represented by the monomials appearing in the vertices. The fermions making the initial state would be same as in final state and all distributions of fermion number between sfermion lines would be allowed. The action obtained by contraction would has SUSY as symmetry but the propagation of different sfermions is fermionic and does not look like that for ordinary spartners.

The quantum criticality condition $q = q_s$ makes the situation non-linear and should fix the coefficients of various terms in super-Taylor expansions as fixed point values of coupling constants.

Could super-Kähler action alone give fermionic scattering amplitudes?

The concrete study of the super-counterpart of Kähler action led to a realization of an astonishing possibility: super-Kähler action alone could give also fermionic scattering amplitudes.

1. In principle this is possible if in S-marix one has contractions of quark creation operator and annihilation operator appearing in quark-antiquark bilinear with different partonic 2-surfaces. This would give fermionic line connecting the points of the cognitive representation at the boundary of CD with points at partonic 2-surfaces in $t = r_n$ hyper-planes in the interior of CD or at the opposite boundary of CD.
   As a matter of fact, this must be the case if the exponent for the sum of super-Kähler and super-Dirac action gives the scattering amplitudes as its matrix elements! The reason is that super-Dirac action vanishes or its solutions.
   The super-Dirac equation must be however present and corresponding variational principle must be satisfied. The hermiticity of the modified Dirac operator requires the vanishing of the covariant derivatives of the modified gamma matrices meaning that bosonic field equations are satisfied. This must be true also for the super variants of the modified gamma matrices.
   If super-Dirac equation is satisfied, the action of modified Dirac operator without connection (ordinary rather than covariant derivative) terms on the discretized quark fields can be expressed in terms of spinor connection as $\Gamma^\alpha - s\partial_\alpha\Psi = \Gamma^\alpha_s A_{\alpha,s}\Psi$ and there is no need for explicit information about the behavior of quark field in the nearby points so that cognitive representation is enough. Otherwise one must have the usual lattice type discretization.
2. The super expansion of super-Kähler action contains only ordinary derivatives of 4-currents defined by quark bi-linears. If the quark field operators with continuous arguments are behind those with discretized arguments and satisfy modified Dirac equation, one can transform the action on quark and antiquark fields to a multiplication with induced gauge potential. This gives nothing but the coupling terms to the gauge potentials in the standard perturbation theory, where one assumes free solutions of Dirac action as approximate solutions. One therefore obtains on mass shell variant of the perturbation theory! Perturbation theory without perturbation theory, might Wheeler say. Or more concretely: the fact that one can treat super-coordinates only perturbatively.
3. The natural guess is that all terms in the expansion of super-Kähler can be transformed to interaction terms and super-Kähler action gives the analog of perturbation theory as a discretized version. The leptonic terms associated with $(3,3)$ term in super-Kähler action should transfrom to the analog of interaction terms for leptonic Dirac action. Whether Kähler gauge potential and spinor connection are developed in super-Taylor series in ordinary manner or remains an open questions.

9.1.4 What super-Dirac equation could mean and does one need super-Dirac action at all?

What does super-Dirac equation actually mean? Super Dirac action vanishes on mass shell and super-Kähler action would give all scattering amplitudes. Are super-Dirac action and super-spinor field needed at all? Should one interpret the oscillator operators defining analog of quark field q as the super-Dirac field Ψ_s as conceptual economy suggests. But doesn't this imply $q = q_s$?

One can consider 3 options as an attempt to answer these questions. Options I and II are not promising. Option III leads to very nice concrete realization of quantum criticality.

Option I: No super-Dirac action and constant oscillator operators

1. If oscillator operators can be regarded as constant, the super Taylor expansion for super Kähler action would give ordinary divergences of the fermionic currents and the action of derivative would be on modified gamma matrices and charge matrix A commutator of $[A_\alpha, \Gamma^\alpha Q]$ and the outcome would be non-vanishing so that one would obtain the coupling terms also now. Could the commutator $[A_\alpha, \Gamma^\alpha]$ be interpreted in terms of gravitational interaction and the commutator $[A_\alpha, Q]$ as electro-weak interaction? In any case, there would be no need for super-Dirac action!
2. There is however an objection. Quark oscillator operators are labelled by the points of cognitive representation and in continuum case they are analogous to the values of quantized spinor field. Should one identify this spinor field with super-spinor field and solve it using a generalization of modified Dirac equation to super-Dirac equation? Can one argue that oscillator operators labelled by points represent superpositions of constant oscillator operators involving integration over 3-D surface at light-cone boundary and are indeed constant?

   This option does not look promising.

Option II: q satisfies ordinary Dirac equation

1. Could one assume that the solution q_0 of ordinary Dirac equation defines the solution to be used as q in the super-Kähler action. The coupling terms of super-Kähler action obtained using $D_0 q_0 = 0$ would be proportional to the classical spinor connection. Classical Kähler action does not involve gauge potentials so that internal consistency would not be lost at this level. The super-variant of Kähler action however involves derivatives of the analogs of fermion currents and there transformation to purely local objects requires the introduction of electroweak gauge potentials so that the symmetry between super-Kähler and super-Dirac would be lost.
2. This would save from developing gauge potentials A_k to super Taylor series - as found this would give only 2 terms by the covariant constancy of spinor curvature. The divergence would reduce to a term involving only a commutator $[A_{alpha}, Q]$, where A_α is purely classical. If Q is Kähler charge, this commutator would vanish, which looks strange since electroweak hypercharge is proportional to Q_K. This could be seen as a failure. If Kähler gauge potential is replaced with its super-variant $A_\alpha + J_{\alpha l}\delta h^l_s$ the commutator is non-vanishing as it should be.
3. Leptons would not appear in $q = q_0$ but since the exponent of super-Kähler action would define the scattering amplitudes by the vanishing of (super-)Dirac action, one could say that leptons emerge as 3-quark composites. SUSY would be dynamical after all!

   Mathematically this option looks awkward and must be dropped from consideration.

Option III: q is a solution of super-Dirac equation

It is best to start from an objection.

1. Assume that q is given Super-Dirac equation

$$D_s(q)q = 0 \ .$$

   This non-linear equation involves powers of q and its conjugate. The problem is that super-Dirac equation is non-linear in q and there are actually 7 separate equations for the part of q with quark number one. 7 equations is too much. The only manner to solve the problem is to replace q with q_s to get $D_s q_s = 0$. But this would require replacing q with q_s in $D_s(q)$ and it would seem that one has an infinite recursion.
2. Could q be self-referential in the sense that one has

$$q_s = q \ . \tag{9.1.1}$$

q would be invariant under iteration $q \to q_s$. This would give excellent hopes of fixing q uniquely. This allows also physical interpretation. The fixed points of iteration give typically fractals and quantum criticality means indeed fractality. This condition could therefore realize quantum criticality, and would give hopes about unique solution for $q = q_s$ for given extension of rationals.

Also h_s should satisfy similar self-referentiality condition expressing quantum criticality:

$$h_s = (h_s)_s \quad . \tag{9.1.2}$$

The general ansatz for h_s involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

The general ansatz for h_s^k involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

3. Many consciousness theorists love the idea of self-referentiality described by Douglas Hofstadter in fascinating manner in his book "Gödel, Escher, Bach". They might get enthusiastic about the naïve identification of q_s and h_s with field of consciousness. In TGD inspired theory of consciousness the self-referentiality of consciousness is understood in different manner but $q = q_s$ and $h_s = (h_s)_s$ as quantum correlated for the self-referentiality is certainly a fascinating possibility.

Consider now a more detailed picture.

1. What does one really mean with q_s? q_s could contain parts with quark number 1 and 3 but a very natural requirement is that it has well-defined fermion number and thus has only a part with quark number 1. The part with quark number 3 is not needed since super-Kähler action would contain it: leptons would emerge as local 3-quark composites from super-Kähler action.
2. Super-Dirac equation would be given by

$$\begin{aligned} D_s(q)q &= 0 \ , \\ D_s(q) &= \Gamma^{\alpha,s}(q)D_{\alpha,s}(q) \ . \end{aligned} \tag{9.1.3}$$

$D_s(q)$ is super-Dirac operator and

$$\Gamma_s^\alpha = T_s^{\alpha k}\gamma_k \tag{9.1.4}$$

are super counterparts of the modified gamma matrices $\Gamma^\alpha = T^{\alpha k}\gamma_k$ defined by the contractions of canonical momentum currents of Kähler action with the gamma matrices γ_k of H:

$$T_k^\alpha = \frac{\partial L_K}{\partial(\partial_\alpha h^k)} \quad . \tag{9.1.5}$$

One would have $\gamma_{k,s} = \gamma_k$ by covariant constancy. L_K denotes Kähler action density, which is sum of 4-D Kähler action and volume term. The field equations of super Kähler action give

$$D_{\alpha,s}\Gamma_s^\alpha = 0 \tag{9.1.6}$$

guaranteeing the hermiticity of the super Dirac operator.

3. The basic equations would thus reduce to

$$\begin{aligned} q &= q_s \ , \\ D_{\alpha,s}\Gamma_s^\alpha &= 0 \ , \\ D_s(q)q &= 0 \ . \end{aligned} \tag{9.1.7}$$

In the continuum case one could think of solving the field equations iteratively.

1. One would first by solve $q = q_0$ for classical modified Dirac operator $D(h_0)$ defined by the ordinary coordinates h_0 of H. Next one would solve $q_1 = q_0 + \Delta q_1$ for the super version $D_1 = D(q_0)$. This would allow to solve next iterate $h_1 = h_0 + \Delta h_1$ using $D(q_1)$. One could continue this process in the hope that the iteration converges. At each step one have group of equations $D_n q_n = 0$ for q_n and for h_{n+1}.
2. An objection is that the iteration could lead outside the extension of rationals if it involves infinite number of iterates. This could occur for space-time surface itself if the normal ordering terms affect the classical action and force to modify the preferred extremal and also cognitive representation at each step. Remaining inside the extension of rationals could also mean that the coefficients of the monomials at points of cognitive representation belong to the extension. It is not of course completely clear whether these equations make sense in the interior of CD or can be solved unlike the lowest equation. It however seems that for each independent monomial m_n the equation would be of form $D_0 m_n = ...$ so that other terms would define kind of sources term and the equation super-Dirac equation could be written as non-linear equation $D_0 q = -\Delta D(q) q$.
3. Each order of bosonic monomials would give its own group of equations making sense also for the cognitive representations and the same would be true for quark monomials and monomials of different orders would be coupled but different quark numbers in q (quarks and leptons) would decouple. These equations are analogous to those appearing in QFT in a gauge theory involving gauge fields and fermion fields.

For cognitive representations the situation is much simpler.

1. All that is needed is the transformation of the ordinary divergences of fermionic currents to a form in which derivative ∂_α is replaced with the linear action of super-gauge potential $A_{\alpha,s}$. Therefore there is no need to solve the non-linear modified Dirac equation in this case and it would become necessary only at the continuum limit. The full solution of non-linear super-Dirac equation would be necessary only in the continuum theory.
2. Could one think that q has vanishing derivatives at the points of cognitive representation: $\partial_\alpha q = 0$ implying $\Gamma^\alpha A_\alpha q = 0$ If the condition holds true then q would be effectively constant for cognitive representations and the situation would effectively reduce to that for option I. This condition is is diffeo-invariant but not gauge invariant. If the points of cognitive representation correspond to singularities of the space-time surface at which several roots of the octonionic polynomial co-incide, the tangent space at the level of M^8 parameterized by a point of CP_2 is not unique and the singular point is mapped to several points in H, and the conditions $\partial_\alpha q = 0$ would make sense at the level of M^8 at least.
3. If one assumes that the quarks correspond to singular points defined by intersections of roots also in the continuum case, one obtains discretization also in this case irrespective of whether one assumes $\partial_\alpha q = 0$ at singularities. Allowing analytic functions with rational Taylor coefficients one obtains also now roots which can be however transcendental and one can identify intersections of roots in the similar manner.

To sum up, there are many uncertainties involved but to my opinion the most satisfactory option is Option III. If one assumes that condition at continuum case, one would obtain also now the discretization.

What information is needed to solve the scattering amplitudes?

One can look the situation also from a more practical point of view. Are there any hopes of actually calculating something? Is it possible to have the information needed?

1. The condition that super-Dirac equation is satisfied would remove the need to have a lattice and cognitive representation would be enough. If the condition $\partial_\alpha q = 0$ holds true, the situation simplifies even more but this condition is not essential. The condition that the points of the cognitive representation assignable to quark oscillator operators correspond to singularities of space-time surface at which several space-time sheets intersect, would make the identification of these points of cognitive representation easier. Note that the notion of singular point makes sense also at the continuum limit giving cognitive representation even in this case in terms of possibly transcendental roots of octonion analytic functions.

If the singular points correspond to solution to 4 polynomial conditions on octonionic polynomials besides the 4 conditions giving rise to the space-time surfaces. The intersections for two branches representing two roots of polynomial equation for space-time surface indeed involve 4 additional polynomial conditions so that the points would have coordinates in an extension of rationals, which is however larger than for the roots $t = r_n$. One could of course consider an additional condition requiring that the points belong to the extension defined by r_n but this seems un-necessary.
The octonionic coordinates used at M^8-side are unique apart from a translation of real coordinate and value of the radial light-like coordinate $t = r_n$ corresponds to a root of the polynomial defining the octonionic polynomial as its algebraic continuation. At this plane the space-time surfaces corresponding to polynomials defining external particles as space-time surfaces would intersect at partonic 2-surfaces containing the shared singular points defined as intersections.

2. The identification of cognitive representations goes beyond the recent knowhow in algebraic geometry. I have considered this problem in [L79] in light of some recent number theoretic ideas. If the preferred extremals are images of octonionic polynomial surfaces and $M^8 - H$ duality the situation improves, and one might hope of having explicit representation of the images surfaces in H-side as minimal surfaces defined by polynomials.

9.1.5 About super-Taylor expansion of super-Kähler and super-Dirac actions

The study of the details of of the general vision reveals several new rather elegant features and clarifies the connections with QFT picture.

About the structure of bosonic and fermionic monomials

The super part of the embedding space coordinates is H-vector and this allows to pose strong conditions on the form of the monomials.

1. One can construct the simplest monomials as bilinears of quarks and anti-quarks. Since oscillator operators are analogs of quark fields, one can construct analogs of left- and right-handed electroweak currents $\overline{q}(1 \pm \gamma_5)\gamma^k Qq$ involving charge matrix Q naturally assignable to electroweak interactions. The charge matrices Q should reflect the structure of CP_2 spinor connection so that analogs of electroweak currents would be in question. One can multiply the objects Hamiltonians HA_A of the isometries and even symplectic transformations at the boundary of CD.

2. One can obtain higher monomials of q and $\overline{q}$ by multiplying these vectorial currents by bilinears, which are scalars and pseudo-scalars obtained by contracting some symmetry related vector field j^k_A of H with gamma matrices of H to give $\overline{q}(1 \pm \gamma_5) j^k_A Q \gamma_k q$ giving rise to analogs of scalar and pseudoscalar Higgs. The Killing vector fields of isometries of H and symplectic vector fields assignable to the Hamiltonians of $\delta CD \times CP_2$ are a natural choice for j^k_A.
One can construct also scalar currents for which gamma matrices contract with gradient of Hamiltonian to give $\overline{q}(1 \pm \gamma_5)\gamma^k \partial_k H_A Q \gamma_k q$ as kind of duals of symplectic currents. These do not define symplectic transformations.
These vector fields make sense at the boundaries of CD and this is enough (they could make sense also at shifted boundaries) since the field equations would allow to express monomials as linear superpositions of the monomials at boundary of CD. Oscillator would always be assigned with the boundaries of CD.

3. If the spin of graviton is assigned with spinor indices, the vector nature of the monomials excludes the analog of graviton. One can however consider also the possibility that the second spin index of graviton like state corresponds to the Hamilton of a symplectic isometry of S^2: for small enough size scales of CD this angular momentum would look like spin. In CP_2 degrees this would give rise to an analog of gluon. Also gluon with spin zero would be obtained.
An alternative option is to assume that graviton corresponds to a non-local state with vectorial excitations at opposite throats of wormhole contact or at different wormhole contacts of closed flux tube. All these states are in principle possible and the question is which of them correspond to ordinary gravitons.

The super counterpart of Dirac spinor consists of odd monomials of quark spinor. Well-defined fermion number allows only monomials with quark number 1 and with definite H-chirality. Quark spinors allow leptons like stats as local 3-quark composites appearing in the super-Kähler action determining the scattering amplitudes since super-Dirac action vanishes at mass shell.

1. In the bosonic case one has vectorial entities and now it is natural to require that one has an object transforming like spinor of H. This poses strong conditions on the monomials since one should have spin 1/2-isospin 1/2 representation.
2. The lowest monomial correponds to quark-antiquark current. What about leptonic analog. The number of oscillator operators at given point is 4+4=8. Leptonic part of super-Kähler action must have 3+3 indices. Therefore also leptonic bilinear seems to be possible and pairs of quarks and lepton like states are possible.
   Intuitively it is clear that leptonic term exists and corresponds to an entity completely anti-symmetric in spin-isospin index pairs (s_3, i_3) of quark spinors. The construction of baryons without color symmetry indeed gives proton and neutron. In order to obtain Δ resonance from u and d quarks, one must have color degrees of freedom and perform anti-symmetrization in these.
   The general condition is that the tensor product of 3 8-D spin representation of $SO(1,7)$ contains 8-D representation in its decomposition. The existence of lepton representation is clear from the fact that the completely antisymmetric representation formed from 4 quarks is $SO(1,7)$ singlet and is product of lepton representation with 3 fold tensor product which must therefore contain spin-isospin 4-plet . The coupling to Kähler gauge potential would correspond to leptonic coupling, which is 3 times the quark coupling.
3. Quarks and lepton monomials have also satellites obtained by adding scalars and pseudo-scalars constructible as quark-anti-quark bi-linears in the manner already discussed. The interpretation as analogs of Higgs fields might make sense.

Normal ordering terms from contractions of oscillator operators

Normal ordering terms from contractions of oscillator operators is a potential problem. In the discretization based on cognitive representations this problem disappears.

1. Contraction terms could induce discrete coupling constant evolution by renormalizing the local monomials. Infinite number these terms would spoil number theoretical vision since a sum over infinite number of terms in general leads outside the extension of rationals involved. If the number of contractions is finite, there are no problems. This is the case in the number theoretical vision since contraction involves always a pair of points. If the rule for construction of S-matrix holds true these points are at opposite boundaries of CD. In the general case they can be at the same boundary. The number of contracted points cannot be larger than the number of points in cognitive representation, which is finite in the generic situation.
   This would give discrete coupling constant evolution as function of extension of rationals since the contractions renormalize the coefficients of the 4+4 terms in the local composites of oscillator operators. The original proposal that additional symmetries are needed to obtain discrete coupling constant evolution is not needed.
2. One could argue that algebraic numbers as a limit for extension is enough to get the continuum limit since the points of cognitive representation would be dense subset of 3-surface. For continuum theory 3-D delta functions would replace Kronecker deltas in anti-commutators implying in ordinary QFT divergences coming as powers of 3-D delta function at zero.
   In the proposed vision one can allow contractions even in the continuum case. The monomials in the interior are linear multilocal composites of those at either boundary of CD involving 3-D integration over boundary points. Contractions associated with two monomials in the interior means an appearance of delta function cancelling the second integration so that there is no divergence.

About the super-Taylor expansions of spinor connection and -curvature

There are also questions related to the details of the expansion of of spinor connection and -curvature in powers of monomials of quark oscillator operators.

1. The rule is that one develops Kähler function as Taylor series with argument shifted by super-part of the super-coordinate. This involves expansion in powers of coordinate gradients and also the expansion of Kähler gauge potential. In the case of modified Dirac action one must expand also the spinor connection of CP_2.
   A potential problem is that the Taylor expansions of Kähler gauge potential and spinor connection have infinite number of terms. Since the monomials in the interior can be expressed linearly in terms of those at boundary of CD by classical field equations, number theoretic discretization based on cognitive representation implies that only a finite number of terms are obtained by using normal ordering and the fact that the number of oscillator operators at same point is 4+4=8. Normal ordering terms would represent radiative corrections giving rise to renormalization depending on the extension of rationals.
2. Is this enough or should one modify the Taylor expansion of Kähler gauge potential A? The idea that $A_k dh^k$ is the basic entity suggests that one must form super Taylor series for both A_k and dh^k. This would give $A_k dh^k \rightarrow A_k \partial_k \delta h^k + A_l \partial_(\delta h^l))dh^k$. By performing an infinitesimal super gauge transformation $A_l \rightarrow A_l + \partial_l (A_l \delta h^k)$ one obtains $A_k \rightarrow A_k + J_{kl} \Delta h_s^k$, where Δh_s^k denotes super part of super-coordinate. The next term would vanish by covariant constancy of J_{kl}.
   The same trick could be applied to spinor connection and since also spinor curvature is covariantly constant, one would obtain only 2 terms in the expansion also in the continuum case. This provides an additional reason for why S $(= CP_2)$ must be constant curvature space.
   This applies also to M^4: in fact, twistor approach strongly suggests that also M^4 has the analog of covariantly constant Kähler form. This conforms with the breakdown of Poincare symmetry at M^8 level forced by the selection of the octonion structure. Poincare invariance is gained by integrating over the moduli space of octonion structures in the construction of scattering amplitudes. What is remarkable that one could use the irreps of Lorentz group at boundaries of CD, which for obvious reasons are much more natural than than those of Poincare group.
3. In the case of embedding metric the same trick would give only the c-number term and only the gradients of embedding space coordinates would contribute to the super counterpart of the induced metric. In this case general gauge super-coordinate transformation would allow to treat the components of metric as constants.

What is the role of super-symplectic algebra?

This picture is not the whole story yet. Super-symplectic approach predicts that the super-symplectic algebra (SSA) generated essentially by the Hamiltonians of $S^2 \times CP_2$ assignable to the representations of $SO(3) \times SU(3)$ localized with the respect to the light-like radial coordinate of light-cone boundary characterize the states besides electro-weak quantum numbers. Color quantum numbers would correspond to Hamiltonians in octet representation. This would predict huge number of additional states.

There are however gauge conditions stating that sub-algebra of SSA having radial conformal weights coming as n-ples of SSA and isomorphic to SSA and its commutator with SSA annihilate physical states. This reduces the degrees of freedom considerably but the number of symplectic Hamiltonians is still infinite: measurement resolution very probably makes this number to finite.

9.2 Other aspects of SUSY according to TGD

In this section other aspects of SUSY according to the present proposal are discussed.

9.2.1 $M^8 - H$ duality and SUSY

$M^8 - H$ duality and $h_{eff}/h_0 = n$ hypothesis pose strong constraints on SUSY in TGD sense.

1. $h_{eff}/h_0 = n$ interpreted as dimension of extension of rationals gives constraints. Galois extensions are defined by irreducible monic polynomials $P(t)$ extended to octonionic polynomials, whose roots correspond to 4-D space-surfaces and in special case 6-spheres at 7-D light-cones of M^8 taking the role of branes.

The condition that the roots of extension defined by Q are preserved for larger extension $P \circ Q$ is satisfied if P has zero as root:

$$P(0) = 0 \ .$$

This simple observation is of crucial importance, and suggests an evolutionary hierarchy $P \circ Q$ with simplest possible polynomials Q at the bottom of the hierarchy are very naturally assignable to elementary particles. These polynomials have degree two and are of form $Q = x^2 \pm n$. Discriminant equals to $D = 2n$ and has the prime factors of n as divisors defining ramified primes identified as p-adic primes assignable to particles.

Remark: Also polynomials $P(t) = t - c$ are in principle possible. The corresponding space-time surfaces at the level of H would be M^4 and CP_2 and they are extremals of Kähler action but do not have particle interpretation.

It turns out the normal ordering of oscillator operators renormalizes the coefficients of P. In particular P can be shifted by a constant term and this deforms the roots of the real polynomial. Also the action principle to be discussed allows $RE(P) = c$ and $IM(P) = c$ surfaces as solutions.

2. The key idea is that the powers o^n of octonion are associative. If the coefficients of $P(o)$ are real or possibly even complex rationals $m + in$ commuting with octonions, associativity is not lost. Octonion o would be replaced by super-octions o_s with (possibly complex-) rational coefficients. o_s is octonion shifted by oscillator operator polynomial analogous to a real number. The conjugate octonion $\overline{o}$ would be treated analogously. Associativity would be preserved.

3. One could assign oscillator operators to both leptons and quarks but the option identifying leptons as local 3-quark local composites and in this sense spartners of quarks allows only baryon number zero composites of quarks and anti-quarks to appear in the octonionic polynomial, which is also hermitian. This would conform with $SO(1,7)$ triality.

   Remark: Anti-leptons are spartners of quarks in the sense of being their local composites but not in the sense that they would appear as local composites in q_s. Leptonic currents can appear in super-Kähler action so that anti-leptons are spartners of quarks in this sense.

   Oscillator operators would transform like components of 8-D spinor *resp.* its conjugate and have interpretation as quark *resp.* anti-quark like spinors. $SO(1,7)$ triality allows only leptonic or quark-like spinors and quark-like spinors are the only physical choice. Also the super-quark q_s which must satisfy self-referential condition $q_s = q$ must have components behaving like $8-D$ spinors with quark number 1. o_s should satisfy analogous condition $o_s = (o_s)_s$.

4. Super-polynomial $P_s(o)$ would be defined by super-analytic continuation as $P(o_s)$ by Taylor expanding it with respect to the super-part of o_s. The outcome is super-polynomial with coefficients of oscillator operator monomials containing k quark-antiquark pairs given by ordinary octonionic polynomials $P_{n-k}(o)$. Each $P_{n-k}(o)$ obtained by algebraically continuing the k:th derivative of the real polynomial $P(t)$ would define 4-surface by requiring that the imaginary or real part of $P_{n-k}(o)$ (in quaternionic sense) vanishes or is constant. Normal ordering of oscillator operators renormalizes the coefficients of P_{n-k}. The interpretation would be as radiative corrections.

   Octonionic super-polynomials obtained from octonionic polynomials of degree n as super-Taylor series decompose to a sum of products of octonionic polynomials $P_k(o)$ with degree $k = n-d$ with oscillator operator monomials consisting of d quark-antiquark pairs. If the degree n of the octonionic polynomial is smaller than the maximal number $N = 4$ of oscillator operator pairs in super-polynomial, only a fraction of spartners are possible. SUSY is realized only partially and one can say that part of spartners are absent at the lowest levels of evolutionary hierarchy. At the lowest level of hierarchy corresponding to $n = 2$ only fermions (quarks) would be present as local states and would form non-local states such as baryons and mesons. Gauge bosons and Higgs like state would be bi-local states and graviton 4-local state.

   Remark: Gauge bosons and Higgs like states as local fermion-anti-fermion composites at level $n = 2 \times 2$. For the option involving only quarks (color is not spin like quantum number). Note that the value of $n_0 = 3 \times 2 = 6$ in $h = n_0 \times h_0$ suggested by the findings of Randel Mills [L31, L60] would allow the known elementary particles.

5. The geometric description of SUSY would be in terms of super-octonions and polynomials and the components of SUSY multiplet would correspond to components of a real polynomial

continued to that of super-octonion and would in general give rise to minimal space-time surfaces as their roots: one space-time sheet for each component of the super-polynomial. The components would have different degrees so that the minimal extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles could be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected. The components of super-polynomial would have different degrees so that the extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles would be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected.

9.2.2 Can one construct S-matrix at the level of M^8 using exponent of super-action?

The construction of S-matrix in H picture in terms of exponential of action defining Kähler function of WCW forces to ask whether M^8 really is an alternative picture as the term "duality" would suggest or is it only part of a description necessitating both M^8 and H. If the duality holds true in strict sense the proposed construction of S-matrix at the level of H should make sense also at the level of M^8. Is this possible at all or could it be that S-matrix emerges the level of H and that M^8 level provides only a tool to describe preferred extremals in H by using what I have called M^8 duality? In the sequel I will look what one obtains if the duality holds true in strict sense.

1. The original idea was to identify space-time-surfaces in M^8 as roots of polynomial equations generalizing ordinary polynomial conditions. Could this makes sense also when octonions are replaced by super-octonions and what super-octonions and quark oscillator operators could mean?
2. The oscillator operators are interpreted as a discretized version of second quantized quark field q allowing local composites of q defining analogs of SUSY multiplets. One can indeed define second quantization for cognitive representations also now. Quark oscillator operators would be analogs of complex coefficients commuting with octonionic units ($i = \sqrt{-1}$ commute with them). The gamma matrices appearing in the quark-antiquark bi-linears would be ordinary gamma matrices of M^8.
   Remark: I have also considered the possibility that M^8 spinors correspond to octonionic spinors with octonionic units defining sigma matrices.
3. One could define simplest contribution the octonionic super-coordinate o_s as sum of M^8 octonion and super-part defined as contraction of 8-component quark current $\overline{q}\gamma^k q$ with contracted with octonionic units e_k to give $\Delta o_s = \overline{q}\gamma^k Qqe_k$. Charge matrices Q are linear combinations of sigma matrices of M^8 in the currents. Gamma matrices should be ordinary gamma matrices and q would transform like ordinary M^8 spinor. The entity $o_s = o + \Delta o_s$ would replace octonionic coordinate o in polynomial equations expressing the vanishing of the real or imaginary part (in quaternionic sense) for $P(0_s)$.
   The contractions of Killing vector fields of translations with gamma matrices would give scalars $j^k\gamma_k$ giving in turn scalars $S = \overline{q}j^k\gamma_k Qq$ and these could be used to build higher monomials. Octonion analyticity in the proposed sense does not allow to use Killing vector fields of rotations and symplectic currents. On the other hand, for cognitive representations these vector fields are restricted to single point of cognitive representation: could this mean that one can allow also the more general scalars.
   Leptons should emerge from o_s. This is the case if one allows also higher monomials in o_s. Also leptonic tri-linears and their conjugate could be built and these would give leptonic bi-linears $\overline{L}\gamma^k QL$. Therefore all (covariantly) constant contributions to super-octonion are possible. The coefficients of various monomials in o_s would be derivatives of polynomial P since they are obtained as super-Taylor series and the coefficients of these polynomials would have interpretation as coupling constants.

4. At the level of H one can construct much larger number of monomials of quark oscillator operators transforming like vector in H. The scalars and pseudo-scalars constructed from the Killing vector fields and symplectic currents can be used to build higher monomials. At the level of H the super-symplectic Hamiltonian currents except those associated with isometries could however annihilate physical states.
The quark currents defined by symplectic isometries are however not constant so that there seems to be a slight inconsistency. Could one assume that also color isometries at the level of H annihilate states quite generally as also S^2 isometries associated with the "heavenly" sphere S^2 in the decomposition $\delta M^4_+ = S^2 \times R_+$? Or can one argue that the restriction to translations is enough because one considers only points of cognitive representation?

5. What about quantum super-spinors q_s (analog of quantized quark field). q would be ordinary rather than octonionic spinor. q_s would be constructed using q and the scalars already discussed. These monomials would carry information about couplings constants. If they are identifiable as the spinors appearing in o_s, one must have $q = q_s$ realizing quantum criticality in quark sector. This would pose strong conditions on the coefficients of the monomials appearing in q interpreted as coupling constants. The conditions would depend on the extension of rationals defined by the polynomia $P(o)$.
The discretization by cognitive representations at the level of H is made possible by super-Dirac equation. At M^8 level there is no need to get rid of partial derivatives acting on currents and super-Dirac equation is not needed.

6. The polynomial equations are purely local algebraic equations and the notions of propagation and boundary value problem do not make sense at the level of M^8. $M^8 - H$ correspondence should lead to the emergence of these notions by mapping surfaces to minimal surfaces natural by quantum criticality. Octonion analyticity and associativity of tangent or normal space inducing dynamics should induce M^8 analog of propagation.

Could one imagine a counterpart for the action exponential and a construction of S-matrix similar to that in the case of H?

1. The action principle should be purely local involving no derivatives of the super-octonionic polynomial $P(o_s)$. It should produce $RE(P) = 0$ and $IM(P) = 0$ as solutions. One might allow also solution $RE(P) = c$, where c is rational number. This would shift of the real polynomial continued algebraically to octonionic polynomial modifying the roots. One should obtain also 6-spheres as universal solutions and identifiable as subsets of 7-D light cones. Now one would have $IM(P) = 0$, $RE(P) = c$ modifying the roots $t = r_n$ defining hyper-surfaces in M^4.

2. Action should be sum over contributions over the points of cognitive representation, perhaps identifiable as the set of singular points at which two roots co-incide.
   (a) Could one minimize the action with respect to the components of $RE(P)$ or $IM(P)$? If this were the case one obtains one would have either $RE(P) = 0$ or $IM(P) = 0$. Surfaces with associative tangent and normal space should have different action and this does not look nice.
   (b) Could one require stationarity of the action with respect to the small deformations of the points of cognitive representation so that they would represent local extrema of action density? These points indeed change, when the polynomial is modified. Since only the deformations of these points are the visible trace of variation for cognitive representations, one could require that the value of action is stationary against these variations rather than variations of the values of $RE(P)$ and or $IM(P)$. This would give rise a condition involving derivatives of $RE(P)$ and $IM(P)$ at singular points with respect to space-time components of octonion. This option will be considered in the sequel.

3. The action density should be finite, and allow both solution types. One can imagine two options.
Option I: If one requires that the action density is dimensionless, the simplest guess for the "action density" L is

$$L = \frac{(RE, IM)}{[(RE, RE) + (IM, IM)]} ,$$

where one has $RE \equiv RE(P(o))$ and $IM \equiv IM(P(o))$ and the inner product is quaternionic inner product. The problem is that denominator gives infinite series giving rise to infinite number of normal ordering terms which may lead out of extension. For exceptional solutions $RE = 0, IM = 0$ the denominator also diverges.

Option II: The alternative avoiding these problems is analogous to the action density of completely local free field theory given by

$$L = K(RE, IM) \ . \tag{9.2.1}$$

K is constant with dimensions of inverse length squared and should relate to the CP_2 length squared. This is not dimensionless but can remain bounded if the quantity (RE, IM) remains bounded for large values of $(RE, RE) + (IM, IM)$.

4. For **Option I** L is a generalization of conformally invariant action from 2-D complex case, in which L reduces to $L = w_1w_2/(w_1^2 + w_2^2) = sin(\phi)cos(\phi)$, $w_1 = Re(w(z))$,$w_2 = Im(w(z))$. (ϕ) is the conformally invariant direction angle associated with w.
The variation of 2-D action with respect to position of the point of cognitive representation gives

$$\frac{[(\partial_u w_1 w_2 + w_1 \partial_u w_2)(w_1^2 + w_2^2) + w_1 w_2 (w_1 \partial_u w_1 + w_2 \partial_u w_2)]}{(w_1^2 + w_2^2)^2} \ , \ u \in \{x, y\} \ .$$

The general solutions are $w_i = c_i \neq 0$, where c_i are constant rational numbers.
The criticality of the action density (maybe it could be seen as a manifestation of quantum criticality) is essential and means that the graph of L as function of w_1 and w_2 is analogous to saddle $w_1w_2/((w_1^2 + w_2^2)$. The condition that L is well-defined requires $c_1 \neq 0$. c_1 could in principle depend on point of cognitive representation. **Option II** gives the same equations in complex case.

5. For **Option II** one obtains 8 equations in the octonionic case and the outcome is that the derivatives of RE or IM or both with respect to components of o vanish. One can have $RE(P(o)) = c_1 \neq 0$ or $IM(P(o)) = c_2 \neq 0$, where c_i is rational. Both conditions are true for the special 6-D solution at 7-D light-cone boundary. Also now both options give the same equations.

What about the super variant of the variational principle?

1. Super-Taylor expansion must be carried out and normal ordering reduces the action to 5 independent terms according to the number $k \in \{0, ..., 4\}$ of quark pairs involved. It seems that only **Option II** is free of number theoretical problems due to normal ordering. Also in this case one has renormalization corrections to various terms in RE and IM. Inner product does not however give rise to additional terms. The degree of the polynomial $P_{n-k}(o_s)$ is equal to $n - k$ and decreases as the degree h of the monomial increases and normal ordering terms are present.

2. One can decompose action action density as $L = \sum L_k$ corresponding to different numbers k of quark pairs. The stationarity conditions hold true for the polynomial coefficient $P_{n-k}(o)$ of each oscillator operator monomial appearing in RE and IM. One has both $RE(P_{n-k}) = c_k \neq$ and $IM(P_{n-k}) = c_k \neq 0$ options. Both conditions are true for the special solutions. Without further conditions the option can depend on k and on the point of cognitive representation. $c_k \neq 0$ for some values of k guarantees that L to be non-vanishing so that the exponential of S can define a non-trivial S-matrix.
Since an approximation of continuous case should be in question, the options should be same all points of the cognitive representation. In the lowest order approximation one obtains $k = 0$ solution obtained without super-symmetry. Normal ordering terms however modify the coefficients of $P(o)$ so that this solution is not exact.

3. Each monomial $P_{n-k}(o)$ defines its own space-time surface and conditions should hold true independently for each super-component L_k. Second option would be to consider vacuum expectation value of the action in which case one would have only single surface.

4. One would have purely local free field theory and the construction of S-matrix would be extremely simple. One could introduce CDs and the identification of hermitian conjugates of fermionic oscillator operators labelled by points at given boundary of CD as creation operators

at time reflected points at opposite boundary. If one can talk about sub-CDs assignable to partonic 2-surfaces in M^8 picture one obtains similar identification for them. Also leptons would emerge from S-matrix.

To sum up, the second trial has a generalization although octonionic picture allows only the Killing vectors of translations of E^8 in the construction of o_s and q_s. The action principle replaces the earlier ansatz with solution in which one has roots of polynomials of $RE(P)$ and $IM(P)$ shifted by rational number. Also a renormalization of P takes place.

9.2.3 How the earlier vision about coupling constant evolution would be modified?

In [L71, L63] I have considered a vision about coupling constant evolution assuming twistor space $T(M^4) = M^4 \times S^2$. In this model the interference of the Kähler form made possible by the same signature of $S^2(M^4)$ and $S^2(CP_2)$ gives rise to a length scale dependent cosmological constant appearing defining the running mass squared scale of coupling constant evolution.

For $T(M^4)$ identified as $CP_3(3,h)$ the signatures of twistor spheres are opposite and Kähler forms differ by factor i (imaginary unit commuting with octonion units) so that the induced Kähler forms do not interfere anymore. The evolution of cosmological constant must come from the evolution of the ratio of the radii of twistor spaces (twistor spheres). This forces to modify the earlier picture.

1. M^8-H duality has two alternative forms with $H = CP_{2,h} \times CP_2$ or $H = M^4 \times CP_2$ depending on whether one projects the twistor spheres of $CP_{3,h}$ to $CP_{2,h}$ or M^4. Let us denote the twistor space $SU(3)/U(1) \times U(1)$ of CP_2 by F.
2. The key idea is that the p-adic length scale hierarchy for the size of 8-D CDs and their 4-D counterparts is mapped to a corresponding hierarchy for the sizes of twistor spaces $CP_{3,h}$ assignable to M^4 by M^8-H-duality. By scaling invariance broken only by discrete size scales of CDs one can take the size scale of CP_2 as a unit so that $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ becomes an evolution parameter.
   Coupling constant evolution must correspond to a variation for the ratio of $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ and a reduction to p-adic length scale evolution is expected. A simple argument shows that Λ is inversely proportional to constant magnetic energy assignable to $S^2(X^4)$ divided by $1/\sqrt{g_2(S^2)}$ in dimensional reduction needed to induce twistor structure. Thus one has $\Lambda \propto 1/r^2 \propto 1/L_p^2$. Preferred p-adic primes would be identified as ramified primes of extension of rationals defining the adele so that coupling constant evolution would reduce to number theory.
3. The induced metric would vanish for $R(S^2(CP_{3,h}) = R(S^2(F))$. Λ would be infinite at this limit so that one must have $R(S^2(CP_{3,h}) \neq R(S^2(F))$. The most natural assumption is that one $R(S^2(CP_{3,h}) > R(S^2(F))$ but one cannot exclude the alternative option. Λ behaves like $1/L_p^2$. Inversions of CDs with respect to the values of the cosmological time parameter $a = L_p$ would produce hierarchies of length scales, in particular p-adic length scales coming as powers of $\sqrt{p}$. CP_2 scale and the scale assignable to cosmological constant could be seen as inversions of each other with respect to a scale which is of order 10^{-4} meters defined by the density of dark energy in the recent Universe and thus biological length scale.
4. The original model for the length scale evolution of coupling parameters [L71] would reduce to that along paths at $S^2(CP_2)$ and would depend on the ends points of the path only. This picture survives as such. Also in the modified picture the zeros of Riemann zeta could naturally correspond to the quantum critical points as fixed points of evolution defining the coupling constants for a given extension of rationals.
   Space-time surfaces the level of M^8 would be determined by octonionic polynomials determined by real polynomials with rational coefficients. The non-critical values of couplings might correspond to the values of the couplings for space-time surfaces associated with octonion analytic functions determined by real analytic functions with rational Taylor coefficients.

9.2.4 How is the p-adic mass scale determined?

p-Adic prime identified as a ramified prime of extension of rationals is assumed to determine the p-adic mass scale. There are however several ramified primes and somehow the quantum numbers of particle should dictate with ramified prime is chosen. There are two options to consider depending on whether both the extension and ramified prime are same for all spartners Option 1) or whether spartners can have different ramified primes (Option 2)). There also options depending on whether both leptons and quarks appear in their own super-Dirac actions (Option a) or whether only quarks appear in super-Dirac action (Option b implied by quark number conservation) . Call the 4 composite options Option 1a), 2a), 1b), 2b) respectively.

1. Consider first Options 1a) and 1b). The ramified prime is same for all states corresponding to the same degree of θ monomial and thus same value of $F + \overline{F}$. At the lowest $k = 2$ level containing only fermions as local states the p-adic thermal masses of quarks and leptons are same for Option 1a) at least for single generation and for all generations if Q_2 does not depend on the genus g of the partonic 2-surface. For Option 1b) the masses would *not* be same for leptons and quarks since they would correspond to different degrees of super-octonionic polymials. For both options would have $n = n(g)$.
2. For Option 2 ramified prime depends on the state of the SUSY multiplet. This would require that for fermions with $k = 2$ the integer n in $Q_2(x) = x^2 \pm n$ has the p-adic primes assignable to leptons and quarks as factors.
   There are 6 different quarks and 6 different leptons with different p-adic mass scales. For Option 2a) n should have 12 prime factors which are near to power of 2. For leptons the factors correspond to Mersenne primes M_k, $k \in \{107, 127\}$ and Gaussian Mersenne $k = 113$. Gaussian Mersenne is complex integer. TGD requires complexification of octonions with imaginary unit i commuting with octonionic units so that also Gaussian primes are possible. This would resolve the question whether $P(t)$ can have complex coefficients $m + in$.
   For option 2b) quarks and leptons as local proton and neutron would have different extensions since the polynomials would be different. The p-adic primes for 6 quark states quarks would depend on genus. The value of n need not depend on genus g since the ramified primes p depends on g: $p = p(g)$.
   Since the polynomials describing higher levels of the dark hierarchy would be composites $P \circ Q_2$ with $P(0) = 0$, Q_2 would be a really fundamental polynomial in TGD Universe. For Option 2b) it would be associated with quarks and would code for the elementary particles physics. The higher levels such as leptons would represent dark matter levels.
3. The crucial test is whether the mass scales of gauge bosons can be understood. If one assumes additivity of p-adic mass squares so that the masses for 2-local bosons would be p-adically sums of mass squared at the "ends" of the flux tube. If the discriminant $D = 2n$ of Q_2 contains high enough number of factors this is possible. The value of the factor p for photon would be rather larger from the limits on photon mass. For graviton the value p would be even larger.

To sum up, the vision about dark phases suggests that the monopole phase is possible already for the minimal value $n = 2$ involving only fundamental quarks for Option 2b), which is the simplest one and could solve the probelm of matter antimatter asymmetry. Bosons and leptons as purely local composites of quarks are possible for $n = 6$. Rather remarkably, also empirical constraints [L31, L60] led to the conclusion $h = 6h_0$. The condition is actually weaker: $h/h_0 \ mod \ 6 = 0$.

9.2.5 Super counterpart for the twistor lift of TGD

Twistor lift of TGD is now relatively well understood. I have made somewhat adhoc attempts to construct TGD analog of the Grassmannian approach so super-twistors. The proposed formalism for constructing scattering amplitudes seems to generalize as such to the twistor lift of TGD.

Could twistor Grassmannian approach make sense in TGD?

By $M^8 - H$ duality [L46] there are two levels involved: M^8 and H. These levels are encountered both at the space-time level and momentum space level. Do super-octonions and super-twistors make sense at M^8 level?

1. At the level of M^8 the high uniqueness and linearity of octonion coordinates makes the notion of super-octonion natural. By $SO(8)$ triality octonionic coordinates (bosonic octet 8_0), octonionic spinors (fermionic octet 8_1), and their conjugates (anti-fermionic octet 8_{-1}) would for triplet

related by triality. A possible problem is caused by the presence of separately conserved B and L. Together with fermion number conservation this would require $\mathcal{N}=4$ or even $\mathcal{N}=4$ SUSY, which is indeed the simplest and most beautiful SUSY.

2. At the level of the 8-D momentum space octonionic twistors would be pairs of two quaternionic spinors as a generalization of ordinary twistors. Super octo-twistors would be obtained as generalization of these.

Also Grassmannian is replaced with super-Grassmannian and super-coordinates as matrix elements of super matrices are introduced.

1. The integrand of the Grassmannian integral defining the amplitude can be expanded in Taylor series with respect to θ parameters associated with the super coordinates C as rows of super $G(k,n)$ matrix.
2. The delta function $\delta(C,Z)$ factorizing into a product of delta functions is also expanded in Taylor series to get derivatives of delta function in which only coordinates appear. By partial integration the derivatives acting on delta function are transformed to derivatives acting on integrand already expanded in Taylor series in θ parameters. The integration over the θ parameters using the standard rules gives the amplitudes associated with different powers of θ parameters associated with Z and from this expression one can pick up the scattering amplitudes for various helicities of external particles.

The super-Grassmannian formalism is extremely beautiful but one must remember that one is dealing with quantum field theory. It is not at all clear whether this kind of formalism generalizes to TGD framework, where particle are 3-surfaces [L46]. The notion of cognitive representation effectively reducing 3-surfaces to a set of point-like particles strongly suggests that the generalization exists.

The progress in understanding of M^8-H duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It seems now clear that sparticles are predicted and SUSY remains in the simplest scenario exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The increased undestanding of what twistorialization leads to an improved understanding of what twistor space in TGD could be. It turns out that the hyperbolic variant $CP_{3,h}$ of the standard twistor space CP_3 is a more natural identification than the earlier $M^4\times S^2$ also in TGD framework but with a scale corresponding to the scale of CD at the level of M^8 so that one obtains a scale hierarchy of twistor spaces [L88]. Twistor space has besides the projection to M^4 also a bundle projection to the hyperbolic variant $CP_{2,h}$ of CP_2 so that a remarkable analogy between M^4 and CP_2 emerges. One can formulate super-twistor approach to TGD using the same formalism as will be discussed in this article for the formulation at the level of H. This requires introducing besides 6-D Kähler action and its super-variant also spinors and their super-variants in super-twistor space. The two formulations are equivalent apart from the hierarchy of scales for the twistor space. Also M^8 allows analog of twistor space as quaternionic Grassmannian HP_3 with signature (6,6). What about super- variant of twistor lift of TGD? consider first the situation before the twistorialization.

1. The parallel progress in the understanding SUSY in TGD framework [L81] leads to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.
2. In fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of embedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.
3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic

prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

Super-counterpart of twistor lift using the proposed formalism

The construction of super-coordinates and super-spinors suggests a straightforward twistorialization. One would only replace the super-embedding space and super-spinors with super-twistor space and corresponding super-spinors. Dimensional reduction should give essentially the 4-D theory apart from the variation of the radius of the twistor space predicting variation of cosmological constant. The size scale of CD would correspond to the size scale of the twistor space for M^4 and for CP_2 the size scale would serve as unit and would not vary.

1. Replace the coordinates of twistor space with superspinors expressed in terms of quark and anti-quark spinors lifted to the corresponding spinors of twistor space. Express 6-D Kähler action in terms of super-coordinates.
2. Replace H-spinors with the spinors of 12-D twistor space and assume only quark chirality. By the bundle property of the twistor space one can express the spinors as tensor products of spinors of the twistor spaces $T(M^4)$ and $T(CP_2)$. One can express the spinors of $T(M^4)$ tensor products of spinors of M^4 - and S^2 spinors locally and spinors of $T(CP_2)$ as tensor products of CP_2 - and S^2 spinors locally. Chirality conditions should reduce the number of 2 spin components for both $T(M^4)$ and $T(CP_2)$ to one so that there are no additional spin degrees of freedom.

   The dimensional reduction can be generalized by identifying the two S^2 fibers for the preferred extremals so that one obtains induced twistor structure. In spinorial sector the dimensional reduction must identify spinorial degrees of freedom of the two S^2s by the proposed chirality conditions also make them non-dynamical. The S^2 spinors covariantly constant in S^2 degrees of freedom.

   Define the twistor counterpart of the analog of modified Dirac action using same general formulas as in case of H.
3. Identify super spinors as sum of odd monomials of theta parameters with quark number 1 identified as oscillator operators. Identify super-Dirac action for twistor space by replacing $T(H)$ coordinates with their super variants and Dirac spinors with their super variants.

9.3 Are quarks enough to explain elementary particle spectrum?

TGD based SUSY involves super-spinors and super-coordinates. Suppose that one has a cognitive representation defined by the points of space-time surface with coordinates in an extension of rationals defining adele and belonging to the partonic 2-surfaces defined by the intersections of 6-D roots of octonionic polynomials with 4-D roots. This representation has H counterpart.

Cognitive representation gives rise to a tensor product of these algebras and the oscillator operators define a discretized version of fermionic oscillator operator algebra of quantum field theories. One would have interpretation as many-fermion states but the local many-fermion states would have particle interpretation. This would replace fermions of the earlier identification of elementary particles with SUSY multiplets in the proposed sense. This brings in large number of new particles. One can however ask whether the return to the original picture in which single partonic 2-surface corresponds to elementary particle could be possible. Certainly it would simplify the picture dramatically.

Could this picture explain elementary particle spectrum and how it would modify the recent picture?: these are the questions.

9.3.1 Attempt to gain bird's eye of view

Rather general arguments suggest that SYM action plus Super-Dirac action could explain elementary particle spectrum. Some general observations help to get a bird's eye of view about the situation.

1. The antisymmetric tensor products for fermions and anti-fermions produce states with same spectrum of electro-weak quantum numbers irrespectively of whether the fermion and anti-fermion are at same point or at different points. Which option is correct or are these options correspond analogous to two different phases of lattice gauge theory in which nodes *resp.* links determine the states? Only multi-local states containing fermions with identical spin and weak isospin at different points are not possible as local states.
   There is no point in denying the existence of either kind of states. What suggests itself is the generalization of electric-magnetic duality relating perturbative Coulomb phase in which ordinary particles dominate and the non-perturbative phase in which magnetic monopoles dominate. I have considered what I have called weak form of electic-magnetic duality already earlier [K113] but as a kind of self-duality stating that for homologically charged partonic 2-surfaces electric and magnetic fluxes are identical. The new picture would conform with the view of ordinary QFT about this duality.
2. The basic distinction between TGD and standard model is that color is not spin-like quantum number but represented as color partial waves basically reducing to the spinor harmonics plus super-symplectic generators carrying color quantum numbers. Spinor harmonics as such have non-physical correlation between color and electro-weak quantum numbers [K60] although quarks and leptons correspond to triality $t = 1$ and triality $t = 0$ states.
3. It turns out that one could understand quarks, leptons, and electro-weak gauge bosons and their spartners as states involving only single partonic 2-surface [K28]: this would give essentially the original topological model for family replication in which partonic 2-surfaces were identified as boundary components of 3-surface. In principle one can allow also quarks and gluons with unit charge matrix with color partial waves defining Lie-algebra generator as bosonic states. Could these states correspond to free partons for which perturbative QCD applies at high energies?
   Also color octet partial waves of electro-weak bosons and Higgs and the predicted additional pseudo-scalar - something totally new - are possible as both local and bi-local states. There would be no mixing of $U(1)_Y$ state and neutral $SU(2)_w$ states for color octet gluon. In this sense electro-weak symmetry breaking would be absent.
4. Electro-weak group as holonomy group of CP_2 can be mapped to the Cartan group of color group, and electro-weak and color quantum numbers would relate like spin and angular momentum to each other. This encourages to think that there are deep connections between electro-weak physics and color physics, which have remained hidden in standard model.
   The conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) of hadron physics suggests a strong connection between color physics and electro-weak physics. There is also evidence for so called X bosons with mass 16.7 MeV [C39] [L32] suggesting in TGD framework that weak physics could have fractally scaled down copy in hadronic and even nuclear scales.
   Could ordinary gluons be responsible for CVC whereas colored variants of weak bosons and Higgs/pseudo-scalar Higgs would be responsible for PCAC? Usually strong force in hadronic sense is assigned with pion exchange. This approach does not work perturbatively. Could one assign strong force with the exchange of pseudo-scalar, and colored variants of gluons, pseudo-scalar, and Higgs?
5. Hitherto it has been assumed that homology charges (Kähler magnetic charges) characterize flux tubes connecting the two wormhole throats associated with the monopole flux of elementary particle. Could one understand the bi-local or multi-local objects of this kind as exotic phase analogous to magnetic monopole dominated phase of gauge theories as dual of Coulomb phase?
   Hadrons would certainly be excellent candidates for monopole dominated phase. Gluons would be pairs of quarks associated with homologically charged partonic 2-surfaces with opposite homology charges. Gluons would literally serve as "glue" in the spirit of lattice QCD. Gluons and hadrons would be multi-local states made from quarks and gluons as homologically trivial configurations with vanishing total homology charge.
6. Is there a correlation between color hyper-charge and homology charge forcing quarks and gluons to be always in this phase and forcing leptons to be homologically neutral? This could provide topological realization of color confinement. The simplest option is that valence quarks

have homology charges $2, -1, -1$ summing up to zero. This was one of the first ideas in TGD about 38 years ago.

One can also imagine that the homological quark charges $(3, -2, -1)$ summing up to zero define a classical correlate for the color triplet of quarks, a realization of Fermi statistics, and allow to understand color confinement topologically. The color partial waves in H would emerge at the embedding space level and characterize the ground states of super-symplectic representations. Color triplets of quarks and antiquarks could thus correspond to homology charges $(3, -2, -1)$ and $(-3, 2, 1)$ and neutral gluons could be superpositions of pairs of form $(q, -q)$, $q = 3, -1, -1$. Charged gluons as flux tubes would not be possible in the confined phase.

7. Is monopole phase possible also for leptons as general QFT wisdom suggests? For instance, could Cooper pairs could be flux tubes having members of Cooper pair - say electrons - at its ends and photons in this phase be superposition of fermion and anti-fermion at the ends of the flux tube and monopole confinement would make the length of flux tube short and photon massive in superconducting phase.

9.3.2 Comparing the new and older picture about elementary particles

The speculative view held hitherto about elementary particles in TGD Universe correspond to the TGD analog of the magnetic monopole dominated phase of QFTs. This view is considerably more complicated than the new view and involves unproven assumptions.

1. Identification of elementary particles

   Old picture: Ordinary bosons (and also fermions) are identified as multilocal many-fermion states. The fermions and anti-fermions would reside at different throats of the 2 wormhole contacts associated with a closed monopole flux tube associated with the elementary particle and going through wormhole contact to second space-time sheet. All elementary particles are analogous to hadron-like entities involving closed monopole flux tubes.

   One can raise objections against this idea. Leptons are known to be very point-like. One must also assume that the topologies of monopole throats are same for given genus in order that p-adic mass calculations make sense. The assumption that quarks correspond to monopole pairs makes things unnecessarily complex: it would would be enough to assume that they correspond to partonic 2-surfaces with monopole charge at the "ends" of flux tubes at given space-time sheet.

   One must assume that the genus of the 4 throats is same for known elementary particles: this assumption looks rather natural but can be criticized. The correlations forced by preferred extremal property should of course force the genera of wormhole throats to be identical.

   New picture: Elementary fermions would be partonic 2-surfaces. Leptons would have vanishing homology charge. Elementary bosons could be simply pairs of fermion anti-fermion located at the opposite ends of flux tubes. This would dramatically simplify the topological description of particle reactions. In the case of quarks however the homological space-time correlate of color confinement is attractive and would force monopole flux tubes. It turns out that this picture corresponds to the simplest level in the $h_{eff} = nh_0$ hierarchy. One could also see leptons and quarks as analogs of perturbative and non-perturbative monopole dominated phases of gauge theories.

   Flux tubes could allow to understand phases like super-conductivity involving massivation of photons (Meissner effect). For instance, Cooper pairs could correspond closed flux tubes involving charged fermions at their "ends". In high Tc super-conductivity Cooper pairs in this sense would be formed at higher critical temperature and at lower critical temperature they would form quantum coherent phase [K80, K81]. Flux tube picture could also allow to understand strongly interacting phases of electrons.

2. Electroweak massivation

   Old picture: Electro-weak massivation has been assumed to involve screening of electro-weak isospin by a neutrino pair at the second wormhole contact. The screening is not actually necessary in p-adic thermodynamics in its recent form since the thermal massivation is due to the mixing of different mass eigenstates.

   New picture: There is no need to add pairs of right- and left-handed neutrino to screen the weak charges in the scale of flux tube.

3. Identification of vertices
Old picture: In old picture one could do almost without vertices: in the simplest proposal particle reactions would correspond to re-arrangements of fermions and antifermions so that fermion and antifermion number would be conserved separately. Therefore one needs an analog of vertex in which partonic 2-surface turns back in time in order to describe creation of particle pairs and emission of bosons identified as fermion-antifermion pairs.
New picture: In vertices fermions and antifermions assignable to super spinor component would be redistributed between different orbits of partonic 2-surfaces meeting along their ends at the 6-D braney object in M^8 picture or turn backwards in time - the interpretation for this might be in terms of interaction with classical induce gauge field. What is new are the new vertices corresponding to the monomials of oscillator operators in the super-spinor. The original identification of particles (given up later) as single partonic 2-surface predicts genus-generation correspondence without additional assumptions. Both old and new picture predict also higher gauge boson genera for which some evidence exists: TGD predictions for the masses are correct [K64].

9.3.3 Are quarks enough as fundamental fermions?

For the first option - call it Option a) - quarks and leptons would define their own super-spinors. Whether only quark or lepton-like spinors are enough remains still an open question.

1. I have also considered the possibility that quarks are actually anti-leptons carrying homology charge and have anomalous em charge equal to $-1/3$ units. One might perhaps say that quarks are kind of anyonic states [K77]. It is however difficult to understand how the coupling to Kähler form could be dynamical and have values $n = -3$ and $n = 1$ for homologically neutral and charged states respectively. This would mean that only lepton like θ parameters appear in super-coordinates and only leptonic Dirac action is needed.
2. For this option proton would be bound state of homologically charged leptons. This in principle allows decays of type $p \rightarrow e^+...$ and $p \rightarrow e^+ + e^+ + \overline{\nu}$ requiring that the 3 partonic 2-surfaces fused with non-trivial homology charges fuse to single homologically trivial 2-surface. This form of proton instability would be different from that of GUTs. The topology changing process is expected to be slow. Is the introduction of two super-octonionic θ parameters natural assignable to B and L or is single parameter enough?
3. The coupling to Kähler form is not explicitly visible on the bosonic action but is visible in modified Dirac action. Could leptonic modified Dirac action transform to quark type modified Dirac action? This does not seem plausible.

The super-Dirac action for quarks however suggests another option, call it Option b). Leptons could be local 3-quark states.

1. Could one identify leptons as local 3 quark composites - essentially anti-baryons as far as quantum numbers are considered - but with different p-adic scale and emerging from the super-Dirac action for quarks as purely local states with super-degree $d = 3$? Could one imagine totally new approach to the matter antimatter asymmetry?
Leptons would be purely local 3-quark composites and baryons non-local 3-quark composites so that charge neutrality alone would would guarantee matter-antimatter symmetry at fundamental level. Anti-quark matter would slightly prefer to be purely local and quark matter 3-local. The small CP violation due to the M^4 part of Kähler action forced by twistor lift should explain this asymmetry.
Leptons and anti-leptons would drop from thermal equilibrium with quarks at some stage in very early cosmology. The reason would be the slowness of the reactions producing local 3-quark composites from quarks. This slowness is required also by the stability of proton. Opposite matter anti-matter asymmetries at the level of both leptons and quarks would have been generated at this stage by CP violation and would have become visible after annihilation.
2. The local baryons would have much simpler spectrum and would correspond for given genus g (lepton generation) to the baryons formed from u and d quarks having however no color. There would be no counterparts for higher quarks. This would suggests that (L, ν_L) could be local analog of (p, n).

For ordinary baryons statistics is a problem and this led to the introduction of quark color absent for local states. The isospin structure of the local analogs of p and n is not a problem. In uud (udd) type states allowed by statistics the spins of the u (d) quarks must have opposite spin. The analogs of Δ resonances are not possible so that one would obtain only the analogs of p and n!

3. The widely different mass scales for leptons and quarks would be due to locality making possible different ramified primes for the extension of rationals. The widely differing p-adic length scales of leptons and neutrinos could be undersood if the ramified prime for given extension can be different for the particles super-multiplets with same degree of octonionic polynomial. This could be caused by electroweak symmetry breaking. The vanishing electroweak quantum numbers of right-handed neutrino implies a dynamics in sharp contrast with that of neutron, whose dynamics would be dictated by non-locality.

   Also local pions are possible. The lepto-pions of lepto-hadron hypothesis [K104] could correspond to either local pions or to pion-like bound states of lepton and anti-leptons. There is evidence also for the muon- and tau-pions.

4. This idea might provide a mathematically extremely attractive solution to the matter antimatter asymmetry: matter and antimatter would be staring us directly into eyes. The alternative TGD inspired solution would be that small CP breaking would induce opposite matter-antimatter asymmetries inside long cosmic strings and in their exteriors so that annihilation period would lead to the observed asymmetry.

The decay $p \to e^+ + X$ could in principle take place and also the reverse decay $e^+ \to p + X$ can be considered in higher energy collisions of electron. The life-time for the decay modes predicted by GUTs is extremely long - longer than 1.67×10^{34} years (see `http://tinyurl.com/nqco2j7`). This fact provides a killer test for the proposal.

One should estimate the life-time of proton in number theoretic approach. The corresponding SUSY vertex corresponds to a Wick contraction involving 4 terms in super-Dirac action: the trilinear term for quarks and 3 linear terms.

1. The vertex would associated with a partonic 2-surface at which 3 incoming quark space-time sheets and outgoing electron space-time sheet meet. At quark level the vertex means an emanation of 3 quark lines from single 3-quark line at a point of partonic 2-surface in the intersection of the ends of 4 space-time surfaces with 6-sphere $t = r_n$ defining a universal root of octononic polynomial $P(o)$. t is M^4 time coordinate [L76]. The vertex itself does not seem to be small.

2. A fusion of 3 homologically non-trivial partonic 2-surfaces to single partonic 2-surface with trivial homology charge cannot occur since partonic 2-surfaces with different homology charge cannot co-incide.

   The reaction $p \to e^+ + ..$ can occur only if the quark-like partonic 2-surface fuse first to single homologically trivial partonic 2-surface: this would correspond to de-confinement phase transition for quarks. After that the 3 quark lines would fuse to single e^+ line.

   (a) To gain some intuition consider two oppositely oriented circles around a puncture of a plane with opposite homology charges. The circles can reconnect to homologically trivial circle. Instead of circles one would now have 3 homologically trivial quark-like 2-surfaces at three light-like boundaries between Minkowskian and Euclidian regions of the space-time surface representing proton. First 2 quark-like 2-surfaces would touch and develop a wormhole contact connecting them. After that the resulting di-quark 2-surface and third quark 2-surface would fuse. The 3 quarks would be now analogous to de-confined quarks.

   (b) At the next step the 3 separate quark lines would fuse to single one. This process must occur in single step since di-quark cannot correspond to single point because the Dirac super-polynomial is odd in oscillator operators and has quark number 1. The fusion point would correspond to 3 degenerate roots of the octonionic polynomial associated with the partonic 2-surface. This partonic 2-surface would be associated with $t = r_n$ hyperplane of M^4 and it would become leptonic 3-surface.

   (c) 3 4-D sheets defined by the roots of the octonionic polynomial should meet at the vertex assignable to $t = r_n$ hyper-plane. This gives 2 additional conditions besides the conditions defining space-time sheets. This for both the protonic and positronic space-time sheets.

One would have double quantum criticality. The tip of a cusp catastrophe serves as an analog. Since the coefficients of the octonionic polynomial are rational numbers, it might be possible to estimate the probability for this to occur: the probability could be proportional to the ratio N_2/N_0 of the number N_2 of doubly critical points to the number N_0 of all points with coordinates in the extension. This could make the process very rare.

It must be however emphasized that also the option in which also leptons are fundamental fermions cannot be excluded.

9.3.4 What bosons the super counterpart of bosonic action predicts?

It has been already noticed that the spectra of fermion-antifermion states are identical for local and bi-local states if one assumes that the wave function in the relative coordinate of fermion and anti-fermion is symmetric. This does not yet imply that the particle spectrum is realistic in the case of the bosonic action.

The situation is simplified considerably by the facts that color is not spin-like quantum number but analogous to momentum and can therefore be forgotten, family replication can be explained topologically, and depending B and L are separately conserved for Option a) but for Option b) L reduces to B since leptons would be local 3-quark composites. Let us restrict first the considered to Option b).

1. What kind of spectrum would be predicted? Consider first quark Clifford algebra formed by the oscillator operators defining the spartners of quark without any conditions on total quark number of the monomial Forgetting color, one has 8 states coming from left and right handed weak doublet and their anti-doublets. The numbers of elements $N(k)$ in Clifford algebra with given quark number $B = k = N(q) - N(\overline{q})$ is given by $N(k) = \sum 0 \leq q \leq 4 - kB(4, q + k) \times B(4, q)$ in terms of binomial coefficients.
For $B = 0$ one obtains $N(0) = \sum 0 \leq q \leq 4B(4, q)^2 = 70$ states. The states corresponding to the same degree of oscillator operator polynomial and therefore having fixed $q+\overline{q} = B+\overline{B}$ have same masses. For $q - \overline{q} = 0$ bosonic state having $q = \overline{q} = 0$ with fixed k one has $q + \overline{q} = 4 + k$ so that one has $N(k) = B(4, k)^2$ ($N(k)$ states with same mass even after p-adic massivation). The numbers $N(k)$ are $(1, 4^2 = 16, 6^2 = 36, 4^2 = 16, 1)$.

2. The number of $q\overline{q}$ type states in super-Kähler action is 16. If one considers super-symmetrization of the bosonic action, these states would correspond to bosons. Could these states allow an interpretation in terms of the known gauge bosons and Higgs? Weak bosons correspond to 4 helicity doublets giving 8 states. Higgs doublet corresponds to doublet and its conjugate. There is also a pseudo-scalar doublet and its conjugate.
Gluon cannot belong to this set of states, which actually conforms with the fact that gluon corresponds to CP_2 isometries rather than holonomies and gluon corresponds to CP_2 partial wave since color is not spin-like quantum number. Known particle would give 8+2+2=12 states and pseudo-scalar doublets the remaining 4. This kind of pseudo-scalar states are predicted both as local and the bi-local states. As already explained, one can however also understand gluons in this picture as octet color partial waves. Also color octet variants of $SU(2)_w$ weak bosons are predicted.

3. There are actually some indications for a Higgs like state with mass 96 GeV (see `http://tinyurl.com/yxnmy8c7`) . Could this be the pseudo-scalar state. Higgs mass 125 GeV is very nearly the minimal mass for $k = 89$. The minimal mass for $k = 90$ would be 88 GeV so that the interpretation as pseudo-scalar with $k = 90$ might make sense. The proposal that gluons could have also weak counterparts suggests that also the pseudo-scalar could have this kind of counterpart. The scaling of the mass of the Higgs like state with $k = 90$ to $k = 112$ ($k = 113$ corresponds to nuclear p-adic scale) would give mass $m(107) = 37.5$ MeV. Kh.U. Abraamyan *et al* have found evidence for pion like boson with mass 38 MeV [C11, C12, C27] (see `http://tinyurl.com/y7zer8dw`).

4. For Option b) only monomials with $N(q) - N(\overline{q}) = k = 1$ are allowed in q_s and leptons would be local 3-quark states and currents formed from them would appear in super-Kähler action. One would obtain $N(k = 1) = \sum 0 \leq q \leq 3B(4, q + 1) \times B(4, q) = 56$ statesi quark multiplet. There would be no doubling gauge bosons since only one H-chirality would be present. The

observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.

Option b) involving only quarks as fundamental fermions does not predict unobserved gauge bosons whereas Option a) involving both leptons and quarks as fundamental fermions does so.

1. For Option a) taking into account quarks and restricting to electro-weak bosonic states to those with ($B = L = 0$) leads to a doubling of bosonic states at $k = 2$ level. The couplings of gauge bosons require that the states are superpositions of quark and lepton pairs with coefficients proportional to the coupling parameters. There are two orthogonal superpositions of quark and lepton pairs having orthogonal charge matrices with inner product defined by trace for the product. Ordinary gauge bosons correspond to the first combination.
   The orthogonality of charge matrices gives a condition on them. The charged matrices having vanishing trace can be chosen that they have opposite signs for opposite H-chiralities. For charge matrices involving unit matrix one must have charge matrices proportional to (-3,1) for (L,q) one must have (1,3) for second state. For gluons there is no condition if one treats color octet as Lie algebra generator with vanishing trace. The problem is that there is no experimental evidence for these bosons.
2. For Option b) leptons would be local 3-quark states and spartners of quarks. There would be no doubling gauge bosons since only one H-chirality would be present. The observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.
3. Option b) predicts that given quark with given isospin and M^4 helicity L or R), say u_L, has 5 spartners with same quantum numbers given by $u_L u_R \overline{u}_L$, $u_L d_R \overline{d}_L$, $u_L d_L \overline{d}_R$; $u_R d_L \overline{d}_L$; and $d_L d_R \overline{u}_L$. These 6 states cannot correspond to quark families and SUSY breaking due to the possibility of having different p-adic scale (ramified prime) making the mass scale of the spartners large is suggestive.

There would be two phases of matter corresponding to local and bi-local states (baryons would be 3-local states).

1. For both phases electro-weak bosons and also gluons with electro-weak charge matrix 1 to bosonic super action as states involving only single partonic 2-surface. As already mentioned, also color counterparts of $SU(2)_w$ bosons are possible. Also graviton could correspond to spartner for bosonic super-action. This would give essentially the original model for family replication. 2-surfaces would be homologically trivial in this phase analogous to Coulomb phase.
2. In the dual phase the bi-local states would correspond to non-vanishing homology charges for quarks at least. In this phase one should assign also to leptons 2 wormhole contacts. In super-conducting phase it could the second electron of Cooper pair. Massive photons in this phase would consist of homologically charged fermion pairs. Lepton could also involve screening lepton-neutrino pair at second wormhole contact.

The universality of gauge boson couplings provides a test for the model.

1. In bi-local model gauge bosons would correspond to representations of a dynamical symmetry group $SU(3)_g$ associated with the 3 genera [K28]. Bosons would correspond to octet and singlet representations and one expects that the 3 color neutral states are light. This would give 3 gauge boson generations. Only the couplings of the singlet representation of $SU(3)_g$ would be universal and higher generations would break universality both for both gluons and electro-weak bosons. There is evidence the breaking of universality as also for second and third generation of some weak bosons and the mass scales assigned with Mersenne primes above M^{89} are correct [K64].
2. If also fermions correspond to closed flux tubes with 2 wormhole contacts, the fermion boson couplings would correspond to the gluing of two closed flux tube strings along their both "ends" defined by wormhole contacts. A pair of 3-vertices for Feynman diagrams would be in question. If fermions are associated with single wormhole contact, its is not so easy to imagine how the closed bosonic flux tube could transform to single wormhole contact in the process. The wormhole contacts that meet and have opposite fermion numbers should disappear. This is allowed in the scenario involving 6-branes if the magnetic flux is trivial as it must be. For quarks and gluons the homology charges must be opposite if wormhole contact is to disappear.

3. If gauge bosons correspond to local fermion pairs, the most natural boson states have fixed value of g apart from topological mixing giving rise to CKM mixing just like fermions and universality is not natural. One can of course assume topological mixing guaranteeing it. Ordinary gauge bosons should be totally de-localized in the space of 3 lowest genera [K28] (analogous to constant plane waves) in order to have universality. The vertices could be understood as a fusion of partonic 2-surfaces. One should however understand why the mixing is so different for fermions and bosons. SUSY would suggest identical mixings.

The simplest model corresponds to quarks as fundamental fermions. Leptons and various bosons would be local composites in perturbative phase. In monopole dominate phase hadronic quarks would have homology charges and gluons would be pairs of quark and anti-quark at opposite throats of closed monopole flux tube. Basically particle reaction vertices would correspond to gluing of 3-surfaces along partonic 2-surfaces at 3-spheres defining $t = r_n$ hyperplanes of M^4.

9.4 Is it possible to have leptons as (effectively) local 3-quark composites?

The idea about leptons as composites of 3 quarks is strongly suggested by the mathematical structure of TGD. In [L81] a proposal that leptons are local composites of quarks. In [L111, L99, L100] a more general idea that leptons look like local composites of quarks in scale longer than CP_2 scale defining the scale of partonic 2-surface assignable to the particle.

A strong mathematical motivation for the proposal is that quark oscillator operators are enough to construct the gamma matrices of the "world of classical worlds" (WCW) and leptonic oscillator operators corresponding to opposite chirality for $H = M^4 \times CP_2$ spinors are somehow superfluous.

The proposal has profound consequences. One might say that SUSY in the TGD sense has been below our nose for more than a century. The proposal could also solve matter-antimatter asymmetry since the twistor-lift of TGD predicts the analog of Kähler structure for Minkowski space and a small CP breaking, which could make possible a cosmological evolution in which quarks prefer to form baryons and antiquarks to form leptons.

The objection against the proposal is that the leptonic analog of Δ might emerge. One must explain why this state is at least experimentally absent. In [L81] I did not develop a detailed argument for the intuition that one indeed avoids the leptonic analog of Δ. In this article the construction of leptons as effectively local 3 quark states allowing effective description in terms of the modes of leptonic spinor field in $H = M^4 \times CP_2$ having H-chirality opposite to quark spinors is discussed in detail.

9.4.1 Some background

Some background is necessary.

1. In TGD color is not spin-like quantum number but corresponds to color partial waves in CP_2 for H-spinors describing fundamental fermions distinguished from fermions as elementary particles.
   Different chiralities of H-spinors were identified in the original model as leptons and quarks. If quarks couple to $n = 1$ Kähler gauge potential of CP_2 and leptons to its $n = 3$ multiple, ew quantum numbers of quarks and leptons come out correctly and lepton and quark numbers are separately conserved.
2. Few years ago emerged the idea that fundamental leptons to be distinguished from physical leptons are bound states of 3-quarks. They could be either local composites or look like local composites in scales larger than CP_2 size scale assignable to partonic 2-surface associated with the lepton.
3. The spin, ew quantum numbers associated with $SU(2)_L \times U(1)_R$ are additive and these quantum numbers should come out correctly for states with leptonic spin and ew numbers.
   Fundamental leptons/quarks are not color singlets/triplets although have vanishing triality. The color quantum numbers also correlate with ew quantum numbers and M^4 helicity/handedness. Only the right-handed neutrino ν_R is a color singlet. The mass squared values

of the resulting states deducible from the massless Dirac equation in H are non-vanishing since CP_2 partial waves carry mass of order CP_2 mass.
The application of color octet generators of super-symplectic algebra (SSA) of super-Kac-Moody algebra (SKMA) with non-vanishing conformal weight contributing to mass squared can guarantee that color quantum numbers are those of physical leptons and quarks. In p-adic mass calculations one must assume negative half-integer valued ground state conformal weight $h_{vac} < 0$.

There are two challenges.

1. One must construct leptons as local of the effectively local 3-quark composites. The challenge is to prove that the resulting states with spin and ew quantum numbers possess the color quantum numbers of fundamental leptons.
2. A priori one cannot exclude leptonic analog of Δ resonance obtained in the quark model of baryons as states for which the wave functions in spin and ew spin degrees of freedom are completely symmetric. The color wave function would be indeed completely antisymmetric also for the leptonic Δ. The challenge is to explain why they do not exist or are not observed.

9.4.2 Color representations and masses for quarks and leptons as modes of $M^4 \times CP_2$ spinor field

It would be also highly desirable to obtain for the masses of 3-quark states the same expressions as embedding space Dirac operator predicts for leptonic masses. The masses depend on ew spin but are same for right and left-handed modes except in the case of right-handed neutrino. This could fixes the value of h_{vac} for leptons if it is assumed to be representable as 3-quark state. Empirical data are consistent with its absence from the spectrum.

The color representations associated with quark and lepton modes of $M^4 \times CP_2$ spinor fields were originally discussed by Hawking and Pope [A56] and are considered from TGD point of view in [K60].

Consider first quarks. For U_R the representations $(p+1, p)$ with triality 1 are obtained and $p = 0$ corresponds to color triplet 3. For D_R the representations $(p, p+2)$ are obtained and color triplet is missing from the spectrum ($p = 0$corresponds to $\overline{6}$). The representations and masses are the same for the left handed representations in both cases since the left handed modes are obtained by applying CP_2 Dirac operator to the right-handed modes.

The CP_2 contributions to the quark masses are given by the formula

$$\begin{aligned} m^2(U,p) &= \frac{m_1^2}{3}\left[p^2+3p+2\right] \ , \ p \geq 0 \ , \\ m^2(D,p) &= \frac{m_1^2}{3}\left[p^2+4p+4\right] = \frac{m_1^2}{3}(p+2)^2 \ , \ p \geq 0 \ , \\ m_1^2 &\equiv 2\Lambda \ . \end{aligned} \tag{9.4.1}$$

Here Λ is cosmoloigal constant characterizing the CP_2 metric. The mass squared splitting between U and D type states is given by

$$\Delta m^2(D,U) = m^2(D,p) - m^2(U,p) = \frac{m_1^2}{3}(p+2) \ . \tag{9.4.2}$$

Consider next leptons. Right handed neutrino ν_R corresponds to (p, p) states with $p \geq 0$ with mass spectrum

$$m^2(\nu) = \frac{m_1^2}{3}\left[p^2+2p\right] \ , \ p \geq 0 \ . \tag{9.4.3}$$

Charged handed charged leptons L correspond to $(p, p+3)$ states with mass spectrum

$$m^2(L) = \frac{m_1^2}{3}\left[p^2+5p+6\right] \ , \ p\geq 0 \ . \tag{9.4.4}$$

$(p, p+3)$ instead of (p, p) reflects the fact that leptons couple to 3-multiple of Kähler gauge potential. Right-handed neutrino has however vanishing total coupling.

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p, p) = (0, 0)$ state) annihilates it.

The mass splitting between charged leptons and neutrinos is given by

$$\Delta m^2(L,\nu) = m^2(L,p) - m^2(\nu,p) = m_1^2(p+2) = 3\Delta m^2(D,U) \ , \tag{9.4.5}$$

and is 3 times larger than the corresponding mass splitting. The mass splitting for leptons as states of type UUD and UDD is however different. If mass squared is additive as assumed in p-adic mass calculations one has $\Delta m^2(UDD, UUD) = \Delta m^2(D, U)$. The condition that the mass splitting for lepton states is the same as predicted by the identification as 3-quark states requires that the scale factor m_1^2 for 3 quarks states is 3 times larger than for quarks:

$$m_1^2(L) = 3m_1^2(q) \ . \tag{9.4.6}$$

9.4.3 Additivity of mass squared for quarks does not give masses of lepton modes

It would be natural that the same values for the leptons as 3-quark composites are same as for leptons as fundamental fermions. It is interesting to see whether the additivity of the mass squared values conforms with this hypothesis.

The sums of mass squared values for UUD (charged lepton) and UDD (neutrino) type states are given by

$$\begin{aligned} m^2(UUD) &= 2m(U)^2 + m(D)^2 = 3p^2 + 10p + 8 \ , \\ m^2(UDD) &= 2m(D)^2 + m(U)^2 = 3p^2 + 11p + 10 \ . \end{aligned} \tag{9.4.7}$$

These mass squared values are not consistent with the values proportional to the mass squared values proportional to $p^2 + 5p + 6$ for L and to $p(p + 2)$ for neutrinos. Covariantly constant right handed neutrino is not possible as a 3-quark state and this conforms with empirical facts.

The working hypothesis that mass squared is additive can be of course given up and a more general condition could be formulated in terms of four-momenta:

$$\begin{aligned} & p_1(U) + p_2(U) + p(D))^2 \\ & = 2m(U)^2 + m(D)^2 + 2\sum[p_1(U)\cdot p_2(U) + (p_1(U)+p_2(U))\cdot p(D)] = km(L)^2 \ , \\ & (p(U) + p_1(D) + p_2(D))^2 \\ & = m(U)^2 + 2m(D)^2 + 2\sum[p_1(D)\cdot p_2(D) + (p_1(D)+p_2(D))\cdot p(U)] = km(\nu)^2 \ . \end{aligned} \tag{9.4.8}$$

k is proportionality constant. These condition give single constraint in the 9-dimensional 3-fold Cartesian power of 3-D mass shells. The constraint is rather mild.

9.4.4 Can one obtain observed leptons and avoid leptonic Δ?

The antisymmetry of the wave function under exchange of quark states gives a strong constraint and fixes the allowed states. Does one obtain states with the quantum numbers of observed leptons as color singlets, and can one avoid the leptonic analogue of Δ?

1. For ordinary leptons complete color antisymmetry would require a complete symmetry under permutations of spin-ew quantum numbers: there are four states altogether. Antisymmetrization would be completely analogous to that occurring for baryons as 3-quark states and would require that fundamental leptons are antisymmetric color singlets.

2. The standard quark model picture natural for strong isospin does not conform with spin-ew symmetries and the resulting states need not allow an interpretation as effective modes of fundamental leptonic spinors. For $SU(2)_L \times U(1)_R$ the situation changes since right-handed helicities are $SU(2)_L$ singlets. The states of form $U_L D_R U_R$ (L_R) and $D_L D_R U_R$ (ν_R) could correspond to right-handed leptons and states of form $U_L D_R U_R$ (L_L) and $D_L D_R U_R$ (ν_L) to left-handed leptons.

3. The manipulation of Yang Tableaux (`https://cutt.ly/Ik9SGuU`) allow to see when a color singlet is contained in all 3-fold tensor products - that is $3 \otimes 3 \times 3$, $3 \times 3 \times \overline{6}$, $3 \times \overline{6} \times \overline{6}$, and $\overline{6} \times \overline{6} \times \overline{6}$ - formed from the representations 3 and $\overline{6}$.
One has $3 \otimes 3 = \overline{3} + 6$ and $\overline{6} \otimes \overline{6} = 6 + 15_1 + 15_2$. Both $\overline{3} \otimes 3 = 1 \oplus 2 \times 8 \oplus 10$ and $\overline{6} \otimes 6 = 1 \oplus 8 \oplus 27$ contain singlet and octet.
Therefore both $3 \otimes 3 \times 3$ (UUU) and $\overline{6} \times \overline{6} \times \overline{6}$ (DDD) contain 1 and 8. $3 \otimes 3 \otimes \overline{6}$ (UUD corresponding to charged lepton) contains $6 \otimes \overline{6}$ and therefore both 1 and 8. However, $3 \otimes \overline{6} \otimes \overline{6}$ (neutrino as UDD) contains neither singlet nor octet.

4. The singlet contained in $\overline{6} \otimes 6$ should be also antisymmetric under the permutations of the color partial waves of quarks in 6. The singlet state has representation of the form $B_{KLM} A^K A^L A^M$, where $A^K = A^K_{rs} q^r q^s$ is the representation of $\overline{6}$ in terms of color triplet q^i. The tensor G_{KLM} should be antisymmetric. Since the singlet comes from Yang diagram as a vertical column, which corresponds to an anti-symmetric representation of S_3t, it seems that it is indeed antisymmetric.
If this is the case, UUU and DDD singlets are indeed antisymmetric with respect to the exchange of quarks, and the state in spin-ew degrees of freedom can be totally symmetric.

5. As found, $\overline{6} \otimes 3 \times 3$) (charged lepton as UUD) contains both 1 and 8 and 1 is antisymmetric as a full vertical column in the Yang diagram. If charged lepton corresponds to 1 it is analogous to proton in these degrees of freedom.
$\overline{6} \otimes \overline{6} \times 3$) (neutrino as DDU) contains neither 1 nor 8. In both cases an entanglement between color and spin-ew degrees of freedom is implied.
Remark: Baryonic quarks reside at distinct partonic 2-surfaces and allow separate color neutralization by SSA or SKMA generators and are color triplets so that the standard picture about color confinement prevails in the baryonic sector.

6. If the 3-quark state is not a color octet, the operators needed to cancel the negative conformal weight must consist of at least two SSA or SKMA operators, which are color octets. UUD contains 8 and 1 but UDD does not. For neutrinos which cannot be color octets or singlets, at least 2 color octet generators are required to neutralize the color. For color singlet charged lepton this is not needed since p-adic thermodynamics allows a massless ground state. The difference charged leptons and neutrinos might relate to the fact that the long p-adic length scales for neutrinos are so long as compared to those for charged leptons.
As has become clear, the neutral Δ type state UDD is not possible since color singlet and octet are not allowed and the neutralization of the negative conformal weight using at least two color generators as in the case of neutrino. Also for other components of Δ color singlet-ness requires at least two generators whereas octet requires only one generator. For color octets a complete symmetry in spin-ew degrees of freedom is not possible.

The conclusion is that charged lepton and charged components of Δ allow for color singlet completely symmetric wave function in spin-ew degrees of freedom unentangled from color. Neutrino and neutral Δ require entanglement between color and spin-ew degrees of freedom.

9.4.5 Are both quarks and leptons or only quarks fundamental fermions?

One of the longstanding open problems of TGD has been which of the following options is the correct one.

1. Quarks and leptons are fundamental fermions having opposite H-chiralities. This predicts separate conservation of baryon and lepton numbers in accordance with observations.
2. Leptons correspond to bound states of 3 quarks in CP_2 scale. This option is simple but an obvious objection is that they are expected to have mass of order CP_2 mass. Baryons could decay to 3 leptons. Also GUTs have this problem. This scenario also allows the existence of exotic leptons as analogs of Delta resonances for baryons.

I haven't been able to answer this question yet and several arguments supporting the quarks + leptons option have emerged.

Consider first what is known.

1. Color is real and baryons are color singlets like leptons.
2. In QCD, it is assumed that quarks are color triplets and that color does not correlate with electroweak quantum numbers, but this is only an assumption of QCD. Because of quark confinement, we cannot be sure of this.

The TGD picture has two deviations from the QCD picture, which could also cause problems.

1. The fundamental difference is that color and electroweak quantum numbers are correlated for the spinor harmonics of H in both the leptonic and quark sector. In QCD, they are not assumed to be correlated. Both u and d quarks are assumed to be color triplets in QCD, and charged lepton L and ν_L are color singlets.
   (a) Could the QCD picture be wrong? If so, the quark confinement model should be generalized. Color confinement would still apply, but now the color singlet baryons would not be made up of color triplet quark states, but would be more general irreducible representations of the color group. This is possible in principle, but I haven't checked the details.
   (b) Or can one assume, as I have indeed done, that the accompanying color-Kac Moody algebra allows the construction of "observed" quarks as color triplet states. In the case of leptons, one would get color singlets. I have regarded this as obvious. One should carefully check out which option works or whether both might work.
2. The second problem concerns the identification of leptons. Are they fundamental fermions with opposite H-chirality as compared to quarks or are they composites of three antiquarks in the CP_2 scale (wormhole contact). In this case, the proton would not be completely stable since it could decay into three antileptons.
   (a) If leptons are fundamental, color singlet states must be obtained using color-Kac-Moody. It must be admitted that I am not absolutely sure that this is the case.
   (b) If leptons are states of three antiquarks, then first of all, other electroweak multiplets than spin and isospin doublets are predicted. There are 2 spin-isospin doublets (spin and isospin 1/2) and 1 spin-isospin quartets (spin and isospin 3/2). This is a potential problem. Only one duplicate has been detected.
   (c) Limitations are brought by the antisymmetrization due to Fermi statistics, which drops a large number of states from consideration. In addition, masses are very sensitive to quantum numbers, so it will probably happen that the mass scale is the CP_2 mass scale for the majority of states, perhaps precisely for the unwanted states.

It is good to start by taking a closer look at the tensor product of the irreducible representations (irreps) of the color group [K60].

1. The irreps are labeled by two integers (n_1, n_2) by the maximal values of color isospin and hypercharge. The integer pairs (n_1, n_2) are not additive in the tensor product, which splits into a direct sum of irreducible representations. There is however a representation for which the weights are obtained as the sum of the integer pairs (n_1, n_2) for the representations appearing in the tensor product.
   Rotation group presentations simplified example. We get the impulse moment $j_1+j_2, ...|j_1-j_2|$. Further, three quarks make a singlet.

2. On basis of the triality symmetry, one expects that, by adding Kac-Moody octet gluons, the states corresponding to (p,p+3)-type and (p,p)-type representations can be converted to each other and even the conversion to color singlet (0,0) is possible. This is the previous assumption that I took for granted and there is no need to give it up.

   Let's look at quarks and baryons first.

1. U type spinor harmonics correspond to (p+1,p) type color multiplets, while D type spinor harmonics correspond to (p,p+2) type representations. From these, quark triplets can be obtained by adding Kac-Moody gluons and the QCD picture would emerge. But is this necessary? Could one think of using only quark spinor harmonics?

2. The three-quark state UUD corresponds to irreducible representations in the decomposed tensor product. The maximum weight pair is (3p+2,3p+2) if p is the same for all quarks, while UDD with this assumption corresponds to the maximum weights (3p+1,3p+1+3). The value of p may depend on the quark, but even then we get (P,P) and (P,P+3) as maximal weight pairs. UUU and DDD states can also be viewed.
   Besides these, there are other pairs with the same triallity and an interesting question is whether color singlets can be obtained without adding gluons. This would change the QCD picture because the fundamental quarks would no longer be color triplets and the color would depend on the weak isospin.

3. The tensor product of a (p,p+3)-type representation and (possibly more) gluon octets yields also (p,p)-type representations. In particular, it should be possible to get (0,0) type representation.

   Consider next the identification of leptons.

1. For leptons, neutrino nu$_L correspondstoa(p,p)-typerepresentationandchargedlepton$L$toa(p+3,p)-typerepresentation$.

2. Could the charged antilepton correspond to a representation of the type UDD and antineutrino to a representation of the type UUD?
Here comes the cold shower! This assumption is inconsistent with charge additivity! UDD is neutral and corresponds to (p,p+3) rather than (p,p). You would expect the charge to be 1 if the correspondence for color and electroweak quantum numbers is the same as for the lepton + quark option!
UUD corresponds to (p,p) rather than (p,p+3) and the charge is 1. You would expect it to be zero. Lepton charges cannot be obtained correctly by adding charge +1 or -1 to the system.
In other words, the 3-quark state does not behave for its quantum numbers like a lepton, i.e. an opposite spinor with H-chirality as a spinor harmonic.
Therefore bound states of quarks cannot be approximated in terms of spinor modes of H for purely group-theoretic reasons. The reason might be that leptonic and quark spinors correspond to opposite H-chiralities. Of course, it could be argued that since the physical leptons are color singlets, this kind of option could be imagined. Aesthetically it is an unsatisfactory option.

   To sum up, the answers to the questions posed above would therefore be the following:

1. Quark spinor harmonics can be converted into color triplets by adding gluons to the state (Kac-Moody). Even if this is not done, states built from three non-singlet quarks can be converted into singlets by adding gluons.

2. The states of the fundamental leptons can be converted into color singlets by adding Kac-Moody gluons. Therefore the original scenario, where the baryon and lepton numbers are preserved separately, is group-theoretically consistent.

3. Building of analogs of leptonic spinor harmonics from antiquarks is not possible since the correlation between color and electroweak quantum numbers is not correct. I should have noticed this a long time ago, but I didn't. In any case, there are also other arguments that support the lepton + quark option. For example, symplectic *resp.* conformal symmetry representations could involve only quarks *resp.* leptons.

9.5 Appendix: Still about the topology of elementary particles and hadrons

In its recent form TGD allows several options for the model of elementary particles [L81]. I wrote this piece of text because I got worried about details of the definition of wormhole contact appearing as basic building brick of elementary particle.

1. Wormhole contacts in 4-D sense (having Euclidian signature of induced metric) modellable as deformed pieces of CP_2 type extremals connecting Minkowskian space-time sheets (representable as graphs of a map $M^4 \rightarrow CP_2$) are identified basic building bricks of elementary particles. 3-D light-like orbits of 2-D wormhole throats- partonic 2-surfaces - at which the signature of induced metric changes from Euclidian to Minkowskian - partonic orbits - are assumed to be carriers of elementary particle quantum numbers localized at points representing intersections of fermionics string world sheets with the partonic 2-surfaces.
2. One can identify simplest wormhole contact as topological sum: two surfaces touch each other. Remove 3-D regions from both space-time sheets and connecting the topologically identical boundaries with a cylinder $X^2 \times D^1$, where X^2 has the topology of the boundary characterized by genus. The assumption that X^2 is boundary requires that its projection to CP_2 is homologically trivial.
   This is not consistent with the assumption that the flux tube carries monopole flux. These wormhole contacts are unstable and must be distinguished from wormhole contacts mediating monopole flux. I have not however defined the notion precisely enough.
3. One can consider two situations in which homologically non-trivial wormhole contact appears.
   Option I: Assume that the 3-D time=constant sections of two Minkowskian space-time sheets are glued together along their boundaries to form a closed 2-sheeted surface and the throats of wormhole contact - partonic 2-surfaces - serve as magnetic charges creating opposite fluxes. One can say that the two throats have opposite homology charges and therefore form a homologically trivial 2-surface to which one can glue the wormhole contact along its boundaries. The flux at sheet B could be seen as return flux from sheet A and the throat could be seen as very short monopole flux tube.
   Option II: Assume no gluing along boundaries for the 3-D time=constant sections of two Minkowskian space-time sheets. In this case one must assume at least two wormhole contacts to get vanishing homology charges at both sheets. At both space-time sheets the throats of the contacts with opposite homology charges would be connected by monopole fluxes flowing through the wormhole contacts identifiable as a very short monopole flux tube. This makes sese also for the Option I and might be required since is not clear whether space-time having boundaries carrying monopole flux can be glued together.
   Remark: One can also consider the light-like orbit of partonic 2-surface connecting its ends (the minimal distance between partonic 2-surfaces vanishes). The homology charges of ends are opposite in ZEO.

The proper identification of the model of elementary particles remains still open [L81] [K64]. What relevance do these two options this picture have to the model of elementary particles?

1. For Option I leptons and gauge bosons could be identified as single wormhole contact carrying non-trivial homology flux. The size scale of the closed space-time sheet would correspond to the Compton wavelength of the particle. This model is the simplest one at the level of scattering diagrams and was re-considered in [L81].
   Even Euclidian regions of single space-time sheet with vanishing homology charge can be considered as a model for leptons and gauge bosons. In this case it is however not clear how to understand how the size scale of the particle as Compton length could be understood at space-time level. This model was one of the first models. I have also considered the identification of the particle as boundary component of Minkowskian space-time surface.
2. Option II was assumed in the model following the original model for leptons and gauge bosons. It was also proposed that electroweak confinement as dual description of massivation takes place in the sense that the weak charges associated with the two wormhole contacts cancel each other. The size scale of flux tube at given sheet would correspond to the Compton length assignable to the particle. In this case scattering amplitudes are more complex topologically.

What about baryons?

1. The simplest model assumes that quarks do not differ from leptons and gauge bosons in any manner. The contribution of the quarks to masses of hadrons is very small fraction of total mass, which suggests that color flux tubes carrying also homology charge are present and give the dominating contribution.
   One can also consider a structure formed by color magnetic monopole flux tubes carrying most of the hadron mass with Minkowskian signature carrying flux of 2 units branching to two flux tubes carrying 1 unit each. The flux tubes would have length given by hadronic p-adic length scale. The ends of flux tubes would be wormhole throats connected by wormhole contacts to the mirror image of this structure. One can say that homology charges 2,-1,-1 assignable to the throats of single space-time sheet sum up to zero. This brings in mind color hypercharge. Could color confinement have vanishing of homology charge as classical space-time correlate?
2. In this article I have considered two alternative identification of leptons. Leptons and quarks could correspond to the different chiralities of $M^4 \times CP_2$ spinors and lepton and baryon numbers would be separately conserved. For second option leptons would b local 3-quark composites and therefore analogous to spartners of quarks: this option is possible only in TGD framework and the reason is that color is not spin-like quantum number in TGD framework. Baryon and lepton numbers would not be separately conserved.
   One can ask what could be the simplest mechanism inducing the decay of baryon as 3-quark composite involving only 3 wormhole contacts and giving lepton as a local 3-quark composite plus something. Wormhole throats of 3 quarks carrying the quark quantum numbers should fuse together to form a leptonic wormhole throat, and the 3 quark lines representing boundaries of string world sheets should fuse to single line. If the sum of quark homology charges is vanishing, lepton must have a vanishing homology charge unless the reaction involves also a step taking care of the conservation of homology charge as a decay of the resulting wormhole contact with vanishing monopole flux to two wormhole contacts with opposite monopole fluxes. Already the first step of the decay process is quite complex, and one can hope that the rate for the reaction is slow enough.

Chapter 10

Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory

10.1 Introduction

I have worked with the problem of understanding the construction of scattering amplitudes in the framework provided by Topological Geometrodynamics (TGD) for about four decades. It soon became clear that the naïve generalization of the path integral approach to a path integral over space-time surfaces did not work because of the horrible non-linearities involved. Around 1985 I started to work with the notion that I later called the "world of classical worlds" (WCW). Eventually I apprehended that the realization of general coordinate invariance (GCI) forces to assign to a 3-surface possibly unique space-time surface (X^4) at which the general coordinate transformations act [K52, K31]. Holography would reduce to GCI. The intuitive expectation is that either space-like 3-surfaces or light-like partonic orbits defining boundaries between Minkowskian and Euclidian space-time regions should be enough to determine X^4 as an analog of Bohr orbit. This leads to strong form of holography (SH) stating that data at partonic 2-surfaces and string world sheets code for X^4.

It should be possible to geometrize the entire quantum physics in terms of WCW geometry and associated spinor structure identifying WCW spinors as fermionic Fock states. A geometrization of the hermitian conjugation essential in quantum theory is needed. This fixed the WCW geometry to be Kähler geometry determined by Kähler function and defining Kähler form providing a realization of the imaginary unit as an antisymmetric tensor [K52]. The existence of Riemann connection fixes the Kähler geometry uniquely already in the case of loop spaces [A44]: maximal isometry group is required. In TGD framework it would correspond supersymplectic transformations of $\delta M^4_\pm \times CP_2$, where $\delta M^4_\pm$,denotes future or past light-cone [K31].

Classical physics becomes an exact part of quantum physics if the space-time surfaces are preferred extremals for some action and therefore analogous to Bohr orbits. Spinor fields should obey the modified Dirac equation (MDE). Modified Dirac action (MDA) is determined by the bosonic action via supersymmetry condition. Kähler function identified as the action for the preferred extremal associated with the 3-surface defines in complex coordinates the Kähler metric and Kähler form via its second derivatives of type $(1,1)$.

The natural looking identification of the action was as Kähler action - a non-linear generalization of Maxwell action replacing Maxwell field and metric with induced Kähler form and metric. It possessed a huge vacuum degeneracy interpreted as spin glass degeneracy and for a long time I looked this feature as something positive despite the fact that the WCW metric becomes degenerate at the vacuum extremals and classical determinism is lost. The addition of volume term having interpretation in terms of cosmological constant would have been a possible cure but would have broken conformal invariance bringing in an *ad hoc* dimensional coupling.

Decades later the proposal for a twistor lift of TGD led to the identification of fundamental action as an analog of Kähler action for 6-D twistor spaces having X^4 as base space and S^2 as fiber [L88]. The induction of the twistor structure from that for the 6+6-D product of twistor spaces of M^4 and CP_2 (these spaces are the only 4-spaces allowing twistor space with Kähler structure [A57] so that TGD is unique) to the 6-surface forces a dimensional reduction reducing 6-D Kähler action to a sum of 4-D Kähler action and volume them. The counterpart of the cosmological constant emerges dynamically. Λ depends on the p-adic length scale characterizing space-time surfaces and approaches to zero in long length scales [L88].

The ontology of standard quantum theory in which 3-D $t = constant$ slice of space-time contains the quantum states, does not fit nicely with TGD framework. Space-time surfaces in 1-1 correspondence with 3-surfaces are more natural objects to consider. This conforms also with the notion of holography implied by GCI: actually SH is highly suggestive and means that 2-D data at partonic 2-surfaces and string world sheets determined the X^4 as a preferred extremal. In particular, various anomalies suggest that the arrow of time need not be fixed.

Eventually this led to zero energy ontology (ZEO) [L80] in which quantum states are essentially superpositions of preferred extremals inside causal diamond (CD): space-time surfaces have ends at the boundaries of CD and these pairs of 3-surfaces or equivalently the 4-surfaces are the basic objects. CDs form a hierarchy: there are CDs with CDs and CDs can also intersect. They would form an analog of atlas of coordinate charts. Each CD would serve as a correlate for a conscious entity so that the charts can be said to be conscious.

ZEO leads to a quantum measurement theory and allows avoiding the basic problems of the standard quantum measurement theory. Zero energy states correspond to state pairs at opposite boundaries of CD or equivalently, superpositions of deterministic time evolutions. In state function reduction (SFR) as a superposition of classical deterministic time evolutions is replaced with a new one.

"Big" and "small" state function reduction - BSFR and SSFR - are the basic notions. In SSFRs as analogs of "weak" measurements following a unitary time evolution, the size of CD increases in statistical sense. The members of the state pairs associated with the passive boundary of CD do not change during SSFRs: this gives rise to the analog of Zeno effect. The active boundary and the states at it change. Active boundary also shifts farther from the passive one. BSFRs correspond to ordinary state function reductions and in BSFRs the arrow of time changes. One could speak of a death of a conscious entity in universal sense and reincarnation with an opposite arrow of time. For instance, the findings of Minev *et al* [L69] provide support for the time reversal [L69].

10.1.1 How to construct the TGD counterpart of unitary S-matrix?

The concrete construction of scattering amplitudes remained a challenge from very beginning. During years I have proposed several proposals and many important aspects of the problem are understood but simple rules are still lacking.

1. The time evolutions assignable to SSFRs should be describable by a unitary S-matrix or its analog.
2. The counterpart of S-matrix should have the huge super-symplectic algebra (SSA) and Kac-Moody algebras related to isometries of H as symmetries. These symmetries, extended further to Yangian symmetries and quantum groups with both algebra and co-algebra structure, are expected to be a key element in the construction of the counterpart of S-matrix. In particular, product and co-product in the super-symplectic algebra define excellent candidates for vertices. What has been missing was a concrete guiding principle.
3. Feynman (or twistor) diagrammatics should generalize. Point-like particles are replaced with 3-surfaces and topologically incoming and outgoing many-particle states correspond to disjoint unions of 3-surfaces at the boundaries of CD. The first guess is that the vertices correspond to 3-surfaces at which 4-D lines of the analog of Feynman diagram meet. SH and $M^8 - H$

duality [L76] however suggest that the lines of the diagrams should correspond to 3-D light-like orbits of partonic 2-surfaces defining boundaries between space-time regions with Euclidian and Minkowskian signature of the induced metric. Also string world sheets connecting them and also serving as carriers of information in SH should be considered. The 1-D light-like intersections of strings world sheets with partonic orbits would define carriers of fermion number.

4. The identification of fermionic anti-commutation relations was a longstanding challenge. It turned out that the induction of second quantized free fermion fields from H to X^4 fixes the anti-commutations of the induced spinor fields and allows to calculate fermionic propagators. Therefore quantum algebra would give what is needed to calculate scattering amplitudes: the interaction vertices assignable to partonic 2-surfaces and fermionic propagators would result from the induction procedure. 8-D fermions have however 7-D delta functions as anti-commutators and normal ordering of fermions can produce divergences already at the level of the MDA.

   The problem disappears if the MDA is made bilocal [L104]: in this article a more detailed discussion is given and leads to a rather detailed picture about MDA.

5. M^8-H duality [L76, L47, L48, L49] allows to concretize this picture. One can regard X^4 either as a surface in the complexified M^8 or in H. M^8-H duality maps space-time surfaces from M^8 to H. Space-time surfaces in the complexified M^8 correspond to algebraic 4-surfaces determined by real polynomials with real (rational if one requires p-adicization) coefficients. Also rational and even analytic functions can be considered, in which case polynomials could be seen as approximations. The roots of the real polynomial dictate the space-time surfaces as quaternionic/associative 4-surfaces in complexified octonionic M^8. Holography becomes discrete.

   The algebraic equations defining space-time surfaces also have special solutions, in particular 6-spheres. These analogs of 6-branes have as M^4 projections in both M^8 and $H=M^4\times CP_2 t=r_n$ hyperplanes, where r_n corresponds to a root of a real polynomial defining X^4 in complexified M^8 The interpretation of these hyper-planes is in TGD inspired consciousness is as "very special moments in the life of self".

   The solutions of the analog of Dirac equation in M^8 as algebraic equation [L105] are localized to 3-D light-like surfaces and mapped to light-like 3-surfaces in H identifiable as orbits of partonic 2-surfaces. Partonic 2-surfaces serving as vertices of topological analogs of Feynman diagrams would reside at the above described $t=r_n$ hyperplanes of $H=M^4\times CP_2$. Scattering amplitudes would have partonic 2-surfaces as vertices and their 3-D light-like orbits as lines. The intersections of string world sheets with the partonic orbits would be 1-D lines and could be interpreted as fermion lines so that also the point particle description would be part of the picture.

   CDs inside CD would define the regions inside which particle reactions occur and this suggests a fractal hierarchy of CDs within CDs as a counterpart for the hierarchy of radiative corrections.

What is still missing is the general principle allowing a bird's eye of view about the counterpart of S-matrix. Wheeler was the first to introduce the notion of unitary S-matrix, which generalizes probability conservation to an infinite number of conditions. Could one challenge the unitary principle and consider something else instead of it?

1. Unitary time evolution is natural in non-relativistic quantum mechanics but is already problematic in quantum field theory (QFT), in particular in twistor Grassmannian approach [B24]. The idea about the reduction of physics to Kähler geometry inspires the question whether Kähler geometry of WCW could provide a general principle for the construction of the scattering amplitudes and perhaps even an explicit formulas for them.

   Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurements. Could the Kähler metric of state space code the counterpart of S-matrix and even unitary S-matrix? Also the Kähler metric satisfies conditions analogous to unitarity conditions.

   An amazingly simple argument demonstrates that one could construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric the analog of a unitary S-matrix by assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied.

(a) If the probabilities are identified as the real parts of complex analogs $p^c_{i,j} = g_{i,\bar{j}} g^{\bar{j},i}$ of probabilities, it is enough to require $Re(p^c_{i,j}) \geq 0$. The complex analogs of $ip^c_{i,j})$ would define the analog of Teichmueller matrix [A33, A51, A42] (https://en.wikipedia.org/wiki/Teichm\unhbox\voidb@x\bgroup\let\unhbox\voidb@x\setbox\@tempboxa\hbox{u\global\mathchardef\accent@spacefactor\spacefactor}\let\begingroup\endgroup\relax\let\ignorespaces\relax\accent127u\egroup\spacefactor\accent@spacefactorller_space) for which imaginary parts of matrix elements are non-negative. Teichmueller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By SH, the most natural candidate would be Cartesian product of Teichmueller spaces of partonic 2 surfaces with punctures and string world sheets.

(b) By positing the condition that $g_{i,\bar{j}}$ and $g^{\bar{j},i}$ have opposite phases, one can assign to Kähler metric a unitary S-matrix but this does not seem to be necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique. These solutions would be special case of Teichmueller solutions: Teichmueller matrix would be purely imaginary. The condition looks too restrictive. For instance, for torus, this would correspond to a metric conformally equivalent with a flat metric.

2. This inspires the idea that quantum physics could be geometrized by the same way as Einstein geometrized gravitation. Take a flat Hilbert space bundle (in the case of TGD) and replace its flat Kähler metric both base space and fiber with a non-flat Kähler metric. The replacement of flat metric with a curved one would lead from a non-interacting quantum theory to an interacting one. Quantum theory would be gravitation at the level of this Hilbert bundle! This replacement is completely universal.

   In the TGD framework the world of classical worlds (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at X^4 and induced from second quantized spinors of the embedding space. Scattering amplitudes would be determined by the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in ZEO and satisfying Teichmueller condition guaranteeing non-negative probabilities. WCW geometry is also characterized by zero modes corresponding to non-complex coordinates for WCW giving no contribution to WCW metric. This is self-evident from SH. The zero modes would be in 1-1 correspondence with Teichmueller parameters and WCW Kähler metrics.

   Equivalence Principle (EP) generalizes to level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give a possible interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic QFT [K94].

There is also challenge of constructing the Kähler metric and associated spinor structure for the spinor bundle of WCW. This would mean a specification of the analogs of Feynman rules so that instead of two problems one would have only one problem.

1. WCW gamma matrices can be identified as superpositions of fermionic oscillator operators associated with quark spinors [L81]. One can consider two approaches to the quantization of these spinors: one studies induced spinor fields obeying MDE and quantizes this or one generalizes the induction of spinors from H to the induction of second quantized spinor fields in H: this would mean simply projecting the spinor fields to X^4. The latter option is extremely simple. It seems possible to avoid divergence problems if the anti-commutators are assigned to different 3-surfaces at different boundaries of CD. This would allow the identification of the Dirac propagator. As a matter of fact, the two approaches are equivalent.
2. WCW gamma matrices would allow the identification as super generators of SSA identified as contractions of gamma matrices SSA with Killing vectors. Quantum states would be created by bosonic and fermionic SSA generators.
3. I have proposed a further supersymmetrization of both H coordinates and spinors by replacing them with expansions in powers of local composites of oscillator operators for quarks and antiquarks [L81]. This however requires Kronecker delta type anti-commutators natural for

cognitive representations defining unique discretization of X^4: this allows to avoid normal ordering divergences. Induction of the H spinor fields would lead to 8-D delta function type divergences. This suggests that local composites are not quite local but states consisting of quarks and antiquarks at opposite throats of wormhole contacts identifiable as partonic 2-surfaces. One would obtain leptons as 3-quark states with quarks at the same partonic 2-surface but not at the same point anymore as in the proposal of [L81].

4. The matrix elements of the Kähler metric of WCW Hilbert bundle correspond to scattering amplitudes analogous to Feynman diagrams. What are the Feynman rules? Partonic two surfaces and their orbits correspond to vertices and propagators topologically. The TGD counterpart for $F\overline{F}B$ vertex would correspond to a bosonic wormhole contact with a fermion and antifermion at opposite wormhole throats and representing SCA generator which decomposes to two partonic 2-surfaces carrying fermions at opposite throats representing fermionic SCA generators. This allows avoiding of normal ordering divergences.
   The vertex would correspond to a product or co-product, which can be said to be time reversals of each other. The structure constants of SCA extended to quantum algebra would fix the vertices and thus the analogs of Feynman diagrams completely. Their number is presumably finite for a X^4 with fixed 3-surfaces at its ends and summation over Feynman diagrams would correspond to integration in WCW.

Before discussing them current proposal in detail, the complementary way to overview TGD as either WCW geometry or as number theory are discussed below. Readers might skip these sections at their first reading and choose to read the section discussing the basic idea in more detail.

In the sequel the basic idea about representation of scattering amplitudes as elements of Kähler metric satisfying what I call "Teichmueller condition", is discussed in TGD framework.

The detailed formulation allows a formulation of conditions for the cancellation of normal ordering divergences and also other divergences. The induction of the second quantized free spinor field from H to space-time surface fixes the propagators at the space-time level. If the creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts, divergences are avoided. ZEO suggests an alternative but not exclusive option that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

The fact that the Dirac propagators for massive particles in the TGD sense reduce in a good approximation to massless propagators when the propagation takes place along light-like distances, allows to considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

Of course, one must not forget that this proposal is only an interesting thought game. It is quite possible that zero energy ontology allows to define a natural way a unitary S-matrix or a more general isometric map between the states spaces associated with the extensions of rationals with different algebraic dimensions assignable naturally to to space-time regions inside causal diamonds. The huge symmetries of WCW generalized to Yangian symmetries could lead to a unique S-matrix and number theoretic conditions pose extremely powerful constraints. In [L133], a proposal along these lines was developed 3 years after writing this.

10.2 Physics as geometry

One can end up with TGD in two ways (see **Fig. 10.2**). Either as a solution of energy problem of GRT realizing Einstein's dream about geometrization of classical physicsor as a generalization of hadronic string model or of superstring theory [B32]. In case of hadronic string model the generalization of string to 3-surface would allow to get rid of spontaneous compactification and the landscape catastrophe implied by it.

At fundamental level TGD could be seen as a hybrid of GRT and SRT: the notion of force does not disappear and can be defined as rate for an exchange of conserved quantity which can be Poincare or color charge. This connection with Newtonian limit is more clear than in GRT, where the conservation laws are lost.

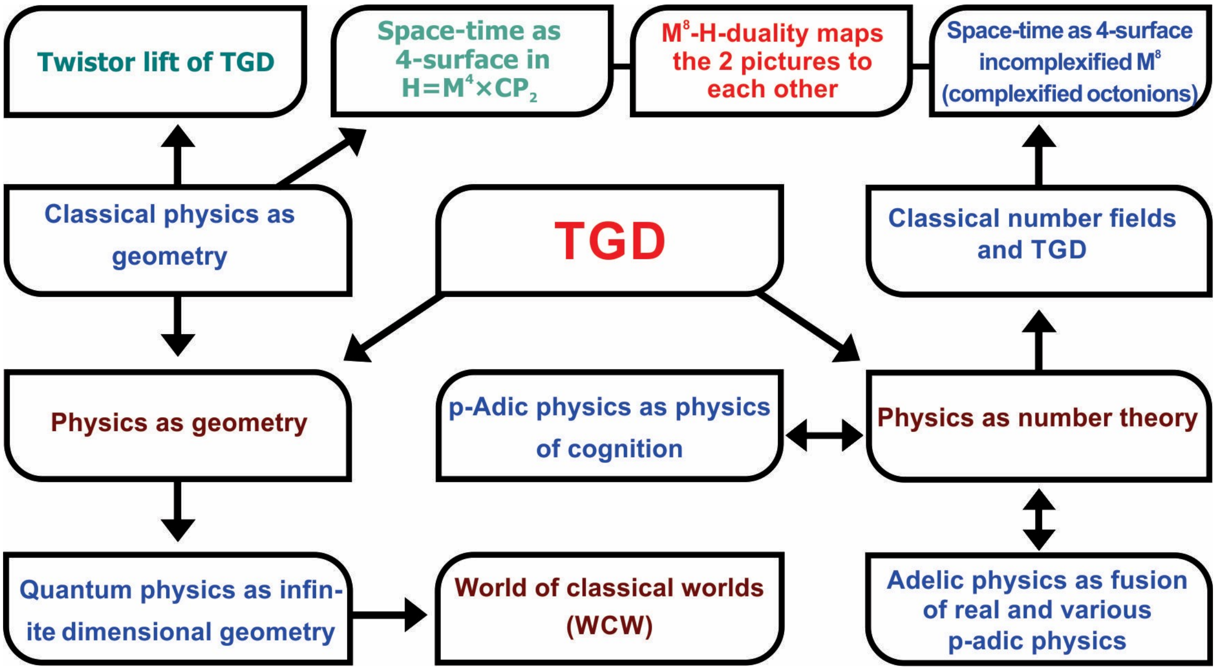

Figure 10.1: TGD is based on two complementary visions: physcs as geometry and physics as number theory.

10.2.1 Classical physics as sub-manifold geometry

The new elements are many-sheeted space-time topologically non-trivial in all scales, and topological field quantization implying that physical systems have field identity, field body, in particular magnetic body (MB) central in applications [L14, L13] (see **Fig. 10.3**).

Induction procedure

One ends up to a geometrization of gravitational field and gauge fields of the standard model as induced fields. Induction means induction of bundle structure is in question. Parallel translation at X^4 is carried out by using spinor connection of H and distances are measured using the metric of H. The components of induced gauge potentials and metric are projections to X^4. Color gauge potentials are identified as projections of Killing vector fields of CP_2 and one can define for them gauge algebra structure. The components of the induced color field are proportional to $H_A J$, where H_A is the Hamiltonian of color isometry and J induced Kähler form. For details see [L15] or the material at my homepage.

The induction of spinor structure allows to avoid the problems related to the definition of spinor structure for general 4-geometry encountered in GRT. For the induced spinor structure induction means projection of gamma matrices to X^4. The definition of gamma matrices is modified when classical action defining the space-time dynamics contains besides volume term also Kähler action with the projection of CP_2 Kähler form defining the analog of Maxwell field. Modified gamma matrices are contractions $T^{\alpha k}\gamma_k$ of the embedding space gamma matrices γ_k with canonical momentum currents $T^{\alpha k}$ associated with the action: this is required by the hermiticity of the modified Dirac action and means existence of infinite number of super currents labelled by the

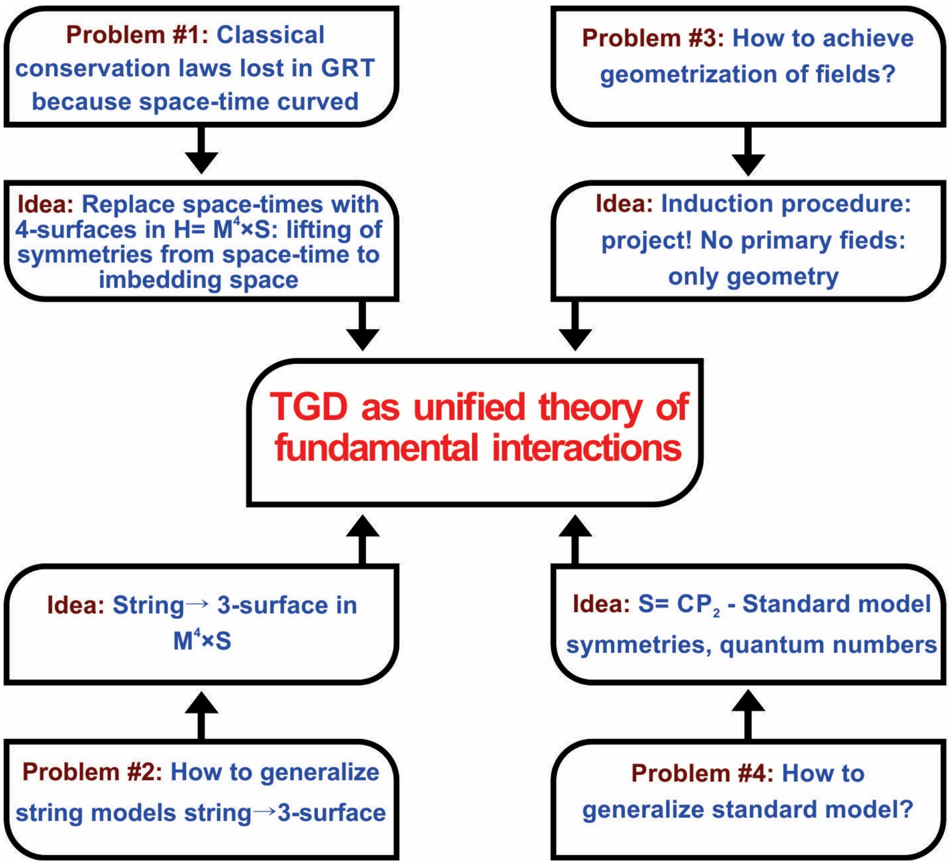

Figure 10.2: The problems leading to TGD as their solution.

modes of the modified Dirac action.

Spacetime is topologically complex

Locally the theory is extremely simple: by GCI there are only 4 field-like variables corresponding to a suitable identification of embedding space coordinates as space-time coordinates. The possibility to choose the coordinates in this manner means enormous simplification since the problems caused by GCI in GRT disappear. It is however obvious that 4 field-like variables does not conform with standard model and GRT. This simplicity is compensated by topological complexity in all scales implied by the many-sheeted space-time. The QFT-GRT limit explained in introduction gives the space-time of gauge theories and GRT.

Geometrically the QFT limit for space-time surfaces having 4-D M^4 projection is obtained by replacing the sheets of many-sheeted space-time with slightly curved region of M^4 and identifying gauge potentials and gravitational field (deviation of the metric from M^4 metric) as superpositions of induced fields at various space-time sheets. Einstein's equations hold true as a remnant of the Poincare invariance.

The presence of space-time regions with M^4 projection of dimension $D < 4$ must be described at QFT limit as particle- or string-like entities. Particle-like entities correspond to CP_2 type extremals having Euclidian signature of induced metric and light-like M^4 projection. 3-D light-like surfaces serve as boundaries between them and Minkowskian space-time regions: the identification is as partonic orbits carrying fermion number serving as building bricks of elementary particles [L57].

The topology of partonic 2-surface is characterized by its genus (number of handless attached to sphere) and is propose to explain family replication for fermions. Also for bosons 3 families are

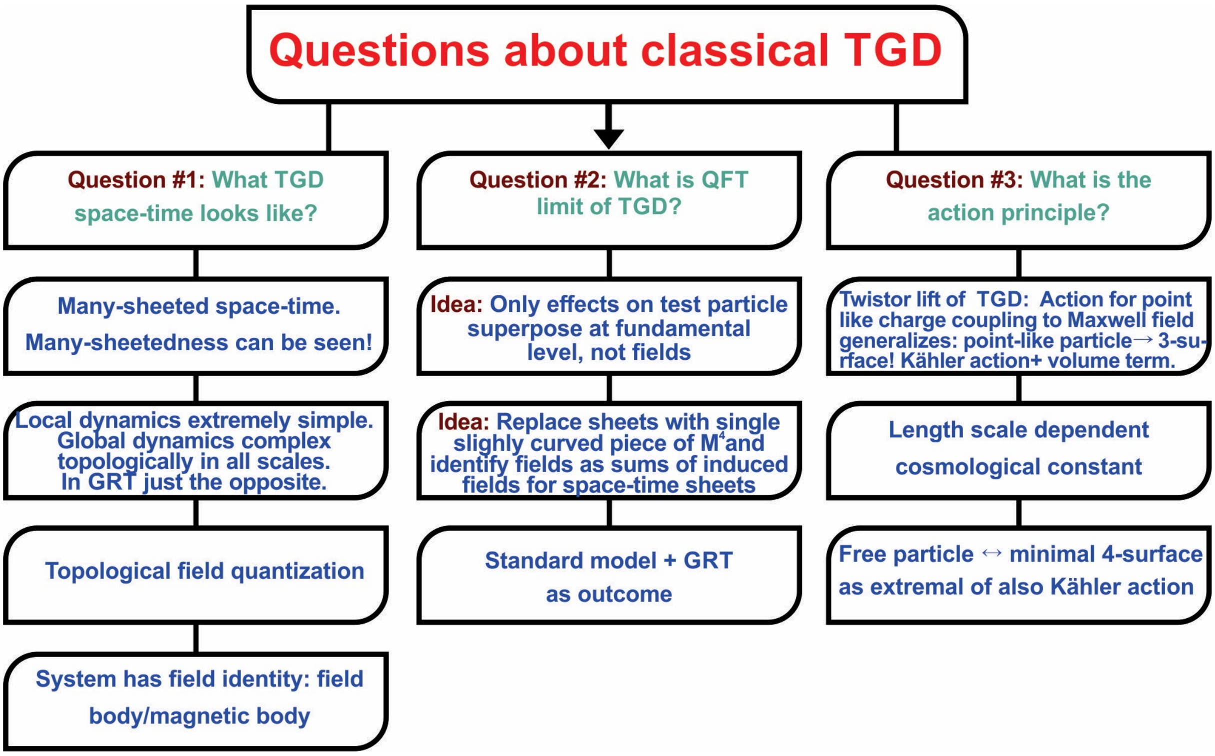

Figure 10.3: Questions about classical TGD.

predicted. The existence of 3 light fermion families is understood in terms of the fact that only 3 lowest genera have global Z_2 as conformal symmetry making possible bound state of 2 handles. For the higher genera handles would behave like particles and mass spectrum would be continuum.

Cosmic strings are fundamental objects of this kind and appear as two different species. Those carrying monopole flux mean deviation from Maxwell's theory. They are unstable against perturbations making their M^4 projection 4-D and transforming them to magnetic flux tubes playing a key role in TGD inspired cosmology.

Twistor lift

One could end up with the twistor lift of TGD from problems of twistor Grassmannian approach originally due to Penrose [B40] and developed to a powerful computational tool in $\mathcal{N} = 4$ SYM [B20, B14, B27, B11, B21].

Twistor lift of TGD [L33, L96, L97] generalizes the ordinary twistor approach [L65, L66] (see **Fig. 10.4**). The 4-D masslessness implying problems in twistor approach is replaced with 8-D masslessness so that masses can be non-vanishing in 4-D sense.

The basic recipe is simple: replaced fields with surfaces. Twistors as field configurations are replaced with 6-D surfaces in the 12-D product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having the structure of S^2 bundle and analogous to twistor space $T(X^4)$. Bundle structure requires dimensional reduction. The induction of twistor structure allows to avoid the problems with the non-existence of twistor structure for arbitrary 4-geometry encountered in GRT.

The pleasant surprise is that twistor space has Kähler structure only for M^4 and CP_2 [A57]: this had been discovered already when started to develop TGD! Since Kähler structure is necessary for the twistor lift of TGD, TGD is unique. One outcome is length scale dependent cosmological constant Λ assignable to any system - even hadron - taking a central role in the theory. At long length scales Λ approaches zero and this solves the basic problem associated with it. At this limit action reduces to Kähler action, which for a long time was the proposal for the variational principle.

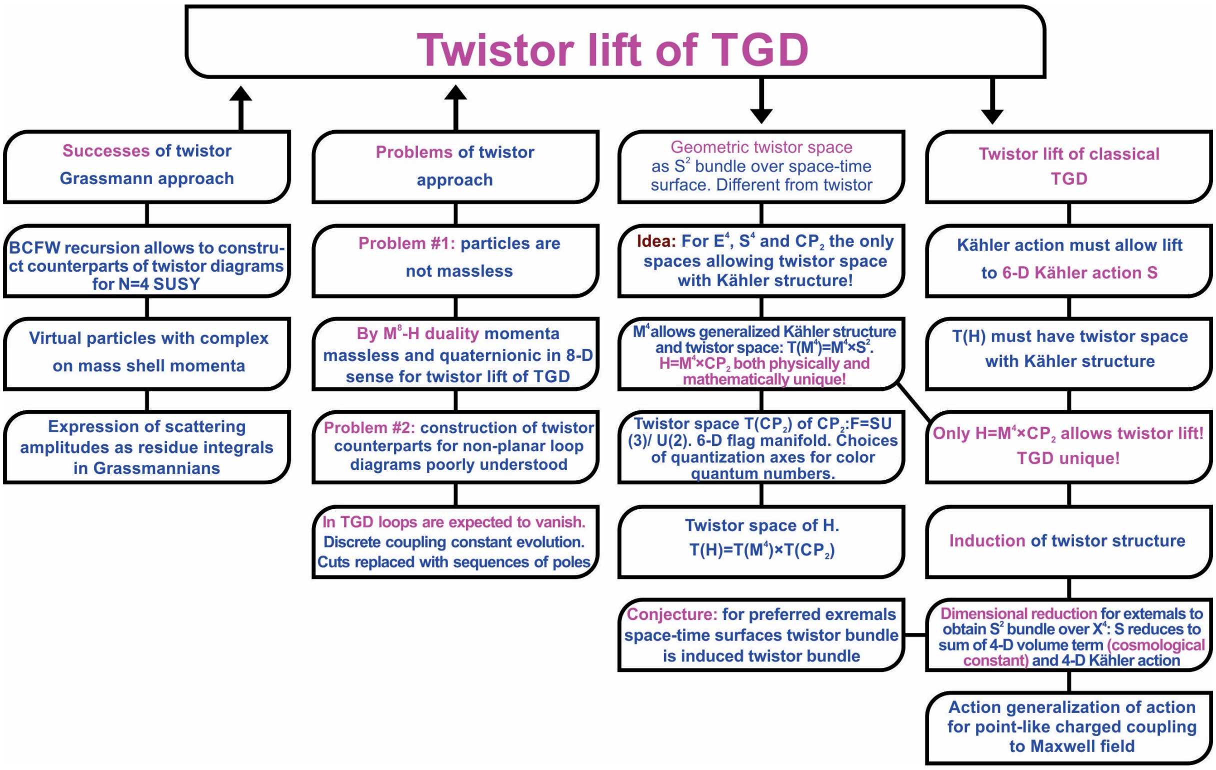

Figure 10.4: Twistor lift

10.2.2 Quantum physics as WCW geometry

WCW as an analog of Wheeler's superspace

Quantum TGD replaces Wheeler's superspace of 3-geometries with the "World of Classical Worlds" (WCW) as the space of 3-surfaces (see **Fig. 10.5**). The holography forced by general coordinate invariance (GCI) implies their 1-1 correspondence with space-time surfaces identified as preferred extremals (PEs) of the basic variational principle analogous to Bohr orbits. Classical physics becomes an exact part of quantum physics [L18, L17]. Einstein's geometrization of classical physics extends to that of quantum physics.

The geometry of infinite-D WCW (see **Fig. 10.5**) and physics is highly unique from its mere existence requiring maximal group of isometries: a result proved by Freed for loop spaces [A44]. The group of WCW isometries is identified as the group of symplectic (contact) transformations of $\delta M^3_+ \times CP_2$ having the light-like radial coordinate in the role of complex variable z in conformal field theories

Remark: The geometric properties of boundary of 4-D light-cone are unique by its metric 2-dimensionality. In particular, the ordinary 2-D conformal symmetries involving local scaling of the radial light-like coordinate give rise to isometries).

The assumption that space-time surfaces as preferred extremals (PEs) are fundamental entities leads to zero energy ontology (ZEO) in which quantum superpositions of pairs (X^3_1, X^3_2) of

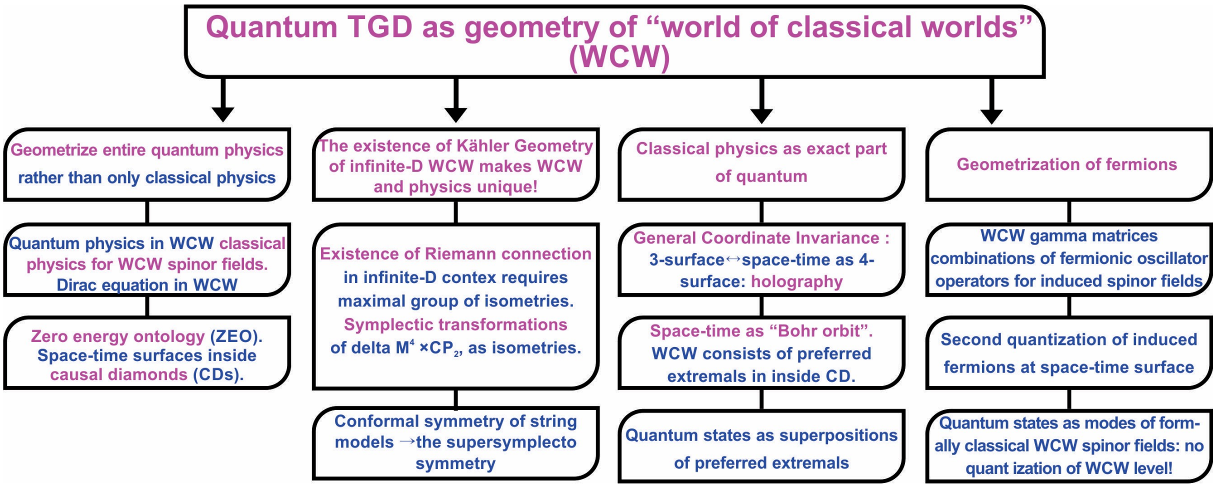

Figure 10.5: Geometrization of quantum physics in terms of WCW

3-surfaces at opposite boundaries of causal diamond (CD) and connected by PE represent quantum states [L115]. This leads to a solution of the basic problem of quantum measurement theory due to the conflict between the determinism of field equations and non-determinism of state function reduction (SFR) and quantum measurement theory extends to a theory of consciousness bringing observer a part of the physisal system.

Quantum states are identified as modes of classical WCW spinor fields so that apart from quantum jump the theory is formally classical. WCW spinor structure involves complexified gamma matrices expressible as superpositions of second quantized oscillator operators of the induced spinor fields at space-time so that a geometrization of fermionic statistics is achieved [L26, L98, L104]. The simplest formulation assumes only quark spinors and would predict that lepton are local composites of 3 quarks.

10.2.3 Super-symplectic group as isometries of WCW

The work of Freed related to the geometrization of loop spaces [A44] demonstrated that the Kähler metric allows awell-defined Riemann connection onlyif it has a maximal group of isometries. This fixes the metric completely. The natural conjecture isthat this is true also in 3-D case and that the group consists of symplectic (contact) transformations at $\delta M^4_\pm \times CP_2$. Here $\delta M^4_\pm$ is future/past directed lightcone boundary containing the "upper"/"lower" boundary of a causal diamond of M^4.

WCW allows as infinitesimal isometries huge super-symplectic algebra (SSA) [K52, K31] acting on space-like 3-surfaces at the ends of space-time surfaces inside causal diamond (CD) and also generalization of Kac-Moody and conformal symmetries acting on the 3-D light-like orbits of partonic 2-surfaces (partonic super-conformal algebra (PSCA)). These symmetry algebras have a fractal structure containing a hierarchy of sub-algebras isomorphic to the full algebra. Even ordinary conformal algebras with non-negative conformal weights have similar fractal structure as also Yangian. In fact, quantum algebras are formulated in terms of these half algebras.

The proposal is that physical states are annihilated by a sub-algebra SSA_n of SSA (with non-negative conformal weights), $n = 1, 2, ...,$ with conformal weights coming as n-multiples of those for SSA and thus isomorphic to the entire SSA, and by the commutator $[SSA_n, SSA]$. What remains seems to be a finite-D Kac-Moody algebra as an effective "coset" algebra obtained. Note that the resulting analog of a normal sub-group could actually be a quantum group.s There is a direct analogy with the decomposition of the Galois group *Gal* to a product of sub-group and normal subgroup H. If the normal subgroup H acts trivially on the representation of *Gal* reduces to that of the group Gal/H. Now one works at Lie algebra level: *Gal* is replaced with SSA and H with its sub-algebra with conformal weights multiples of those for SSA. These two hierarchies of subgroups could correspond to each other and to the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs) [K112, K43]. These conditions would guarantee preferred extremal property of the space-time surface and holography or even its strong form.

Holography from GCI

Gravitational holography has been one of the dominating themes in recent day theoretical physics. It was originally proposed by Susskind [B36], and formulated by Maldacena as AdS/CFT correspondence [B34]. One application is by Preskill *et al* to quantum error correcting codes [B29].

By holography implied by GCI the basic variational problem can be seen either as boundary value problem with 3-surfaces at opposite boundaries of CD or as initial value problem caused by PE property. Ordinary 3-D holography is thus forced by general coordinate invariance (GCI) and loosely states that the data at 3-surface at either boundary of CD allows to determine $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

Holography poses additional strong conditions on X^4.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SSA_n with radial conformal weights coming as n-multiples of those for the entire algebra SSA and its commutator $[SSA_n, SSSA]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations. An open question is whether this hierarchy corresponds to the hierarchy of EQs with n identifiable as dimension of the extension.
2. Second conjecture is that PEs are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor structure from the product of twistor spaces of M^4 and CP_2 to 6-D S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [A57] so that TGD is unique.

Strong form of holography

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated

1. either with light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian,
2. or the space-like 3-surfaces at the ends of CD to determine X^4 as PE (in case that it exists),

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken the strong form of $M^8 - H$ duality [L91] mapping $X^4 \subset M^8$ to $X4 \subset H = M^4 \times CP_2$ that it allows to map only certain 2-D sub-manifolds $X^2 \subset X^4 \subset M^8$: SH allows to determine $X^4 \subset H$ from this 2-D data.

Further generalizations

This picture about WCW is not general enough.

1. $M^8 - H$ duality [L91] suggests that the notion of WCW applies also M^8 picture. The parameters of polynomials defining $X^4 \subset M^8$ are assumed to be rational. The points of M^8 counterpart of WCW have the rational coefficients of these polynomials as coordinates so that WCW should be discrete in real topology. This should be the case also for H counterpart of WCW. Could one see real and p-adic variants of WCW as completions of this discrete WCW.
2. Adelic physics inspires the question whether p-adic and adelic variants of WCW make sense or is it enough to have number theoretically universal cognitive representations to define unique discretized variants of X^4 and correspondingly discretized WCW.
3. For TGD variant of SUSY [L95, L94] super coordinates for H correspond to hermitian local composites of quark oscillator operators. For super-quarks they correspond to local components with fixed quark number. Leptons can be understand as local composites of quarks - super field components [L104]. SUSY replaces modes of super-field with super-surfaces so that the components of super-field correspond to sets of disjoint 4-surfaces. This is true also for the points of super WCW.

10.3 Physics as number theory

Number theoretical vision is second thread of TGD. It decomposes to 3 threads corresponding to various p-adic physics [L19] fusing to adelic physics [L57], classical number fields [L20], and infinite primes [L21] (not discussed in the sequel).

10.3.1 p-Adic and adelic physics and extensions of rationals (EQs)

p-Adic number fields would serve as correlates of cognition and imagination (see **Fig. 10.6**) . Space-time is replaced with a book like structure having both real and various p-adic space-time sheets as pages. The outcome is adelic physics as fusion of various p-adic physics [L53, L57] (see `http://tinyurl.com/ycbhse5c`). The EQ induces extensions of p-adic numbers fields and of adele giving rise to a hierarchy of physics having interpretation in terms of evolution induced by the increase of the complexity of the EQ.

Adelic physics leads also the hierarchy of Planck constants $h_{eff}/h_0 = n$ with n identified as dimension of EQ labelling phases of ordinary matter behaving like dark mater, and making possible quantum coherence in arbitrarily long time scales essential for understanding living matter.

EQs are characterized by discriminant D assignable to a polynomial giving rise to the extension (for second order polynomials D has expression familiar from school days). Now polynomials with rationals (equivalently integer) valued coefficients are interesting. The primes dividing the discriminant are known as ramified primes and they have a property that for p-adic variant of polynomial degenerate roots appear in $O(p) = 0$ approximation [L93]. The interpretation could be in terms of quantum criticality and physically preferred p-adic primes are identified as ramified primes of extension [L101].

Remark: One can also consider polynomials with algebraic coefficients. The notion of Galois group make sense also for real coefficients.

The hierarchy of EQs labelling levels of dark matter hierarchy and of hierarchy of adelic physics follows from $M^8 - H$ duality allowing to identify $X^4 \subset M^8$ as a projection of $X^4_c \subset M^8_c$ - identified as complexified octonions O_c - and satisfying algebraic equations associated with a polynomial of degree n.

Real and p-adic physics are strongly correlated and mass calculations represent the most important application of p-adic physics [K60]. Elementary particles seem to correspond to p-adic primes near powers 2^k (there are also indicatons for powers of 3). Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of period doubling in which prime $p = 2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p = 3$ (period tripling as approach to chaos) [I10, I11]. Many elementary particles and also hadron physics and electroweak physics seem

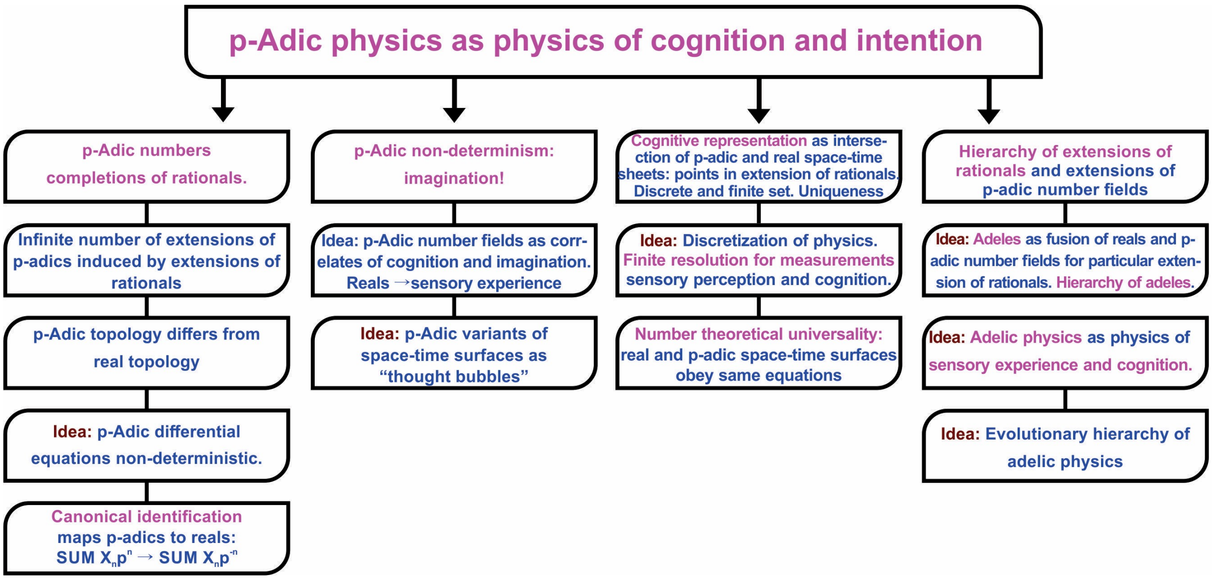

Figure 10.6: p-Adic physics as physics of cognition and imagination.

to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2 and the challenge is to understand this [L34].

10.3.2 Classical number fields

Second aspect of number theoretical vision are classical number fields: reals, complex numbers, quaternions and octonions and their complexifications by a commuting imaginary unit i (see **Fig. 10.7**).

Space-time as 4-surface in $M^8_c = O_c$

One can regard real space-time surface $X^4 \subset M^8$ as a M^8–-projection of $X^4_c \subset M^8_c = O_c$. M^4_c is identified as complexified quaternions H_c [L91, L101]. The dynamics is purely algebraic and therefore local.

1. The basic condition is associativity of $X^4 \subset M^8$ in the sense that either the tangent space or normal space is associative - that is quaternionic. This would be realized if X^4_c as a root for the quaternion-valued "real" or "imaginary part" for the O_c algebraic continuation of real analytic function $P(x)$ in octonionic sense. Number theoretical universality requires that the Taylor coefficients are rational numbers and that only polynomials are considered.
   The 4-surfaces with associative normal space could correspond to elementary particle like entities with Euclidian signature (CP_2 type extremals) and those with associative tangent space to their interaction regions with Minkowskian signature. These two kinds space-time surfaces could meet along these 6-branes suggesting that interaction vertices are located at these branes.

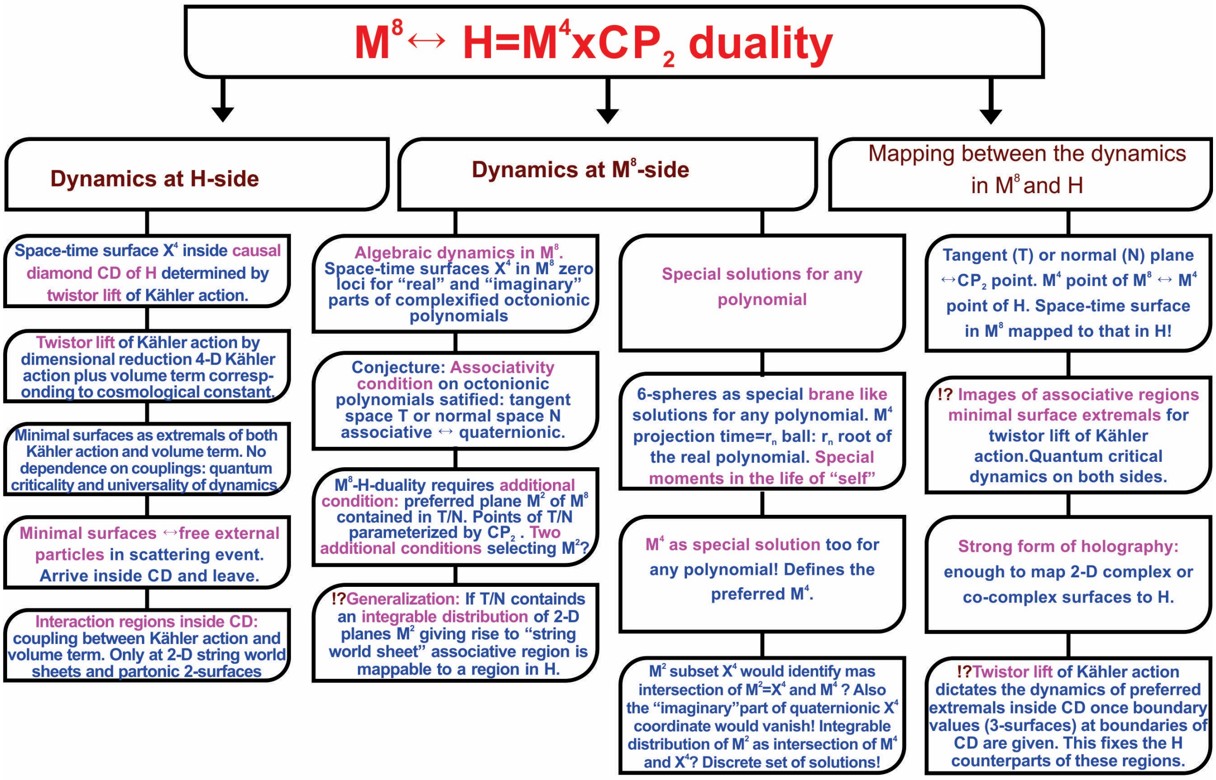

Figure 10.7: $M^8 - H$ duality

2. The conditions allow also exceptional solutions for any polynomial for which both "real" and "imaginary" parts of the octonionic polynomial vanish. Brane-like solutions correspond to 6-spheres S^6 having $t = r_n$ 3-ball B^3 of light-cone as M^4 projection: here r_n is a root of the real polynomial with rational coefficients and can be also complex - one reason for complexification by commuting imaginary unit i. For scattering amplitudes the topological vertices as 2-surfaces would be located at the intersections of X^4_c with 6-brane. Also Minkowski space M^4 is a universal solution appearing for any polynomial and would provide a universal reference space-time surface.

3. Polynomials with rational coefficients define EQs and these extensions form a hierarchy realized at the level of physics as evolutionary hierarchy. Given extension induces extensions of p-adic number fields and adeles and one obtains a hierarchy of adelic physics. The dimension n of extension allows interpretation in terms of effective Planck constant $h_{eff} = n \times h_0$. The phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ behave like dark matter and galactic dark matter could correspond to classical energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes. It is not completely clear whether number galactic dark matter must have $h_{eff} > h$. Dark energy in would correspond to the volume part of the energy of the flux tubes.

   There are good arguments in favor of the identification $h = 6h_0$ [**?**] "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining X^4 as a covering space locally. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0. The ramified primes of extension in turn are identified as preferrred p-adic primes. The moduli for the time differences $|t_r - t_s|$ have identification as p-adic time scales assignable to ramified primes [L101]. For ramified primes the p-adic variants of polynomials have degenerate zeros in

$O(p) = 0$ approximation having interpretation in terms of quantum criticality central in TGD inspired biology.

4. During the preparation of this article I made a trivial but overall important observation. Standard Minkowski signature emerges as a prediction if conjugation in O_c corresponds to the conjugation with respect to commuting imaginary unit i rather than octonionic imaginary units as though earlier. If X^4 corresponds to the projection $O_c \rightarrow M^8 \rightarrow M^4$ with real time coordinate and imaginary spatial coordinates the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

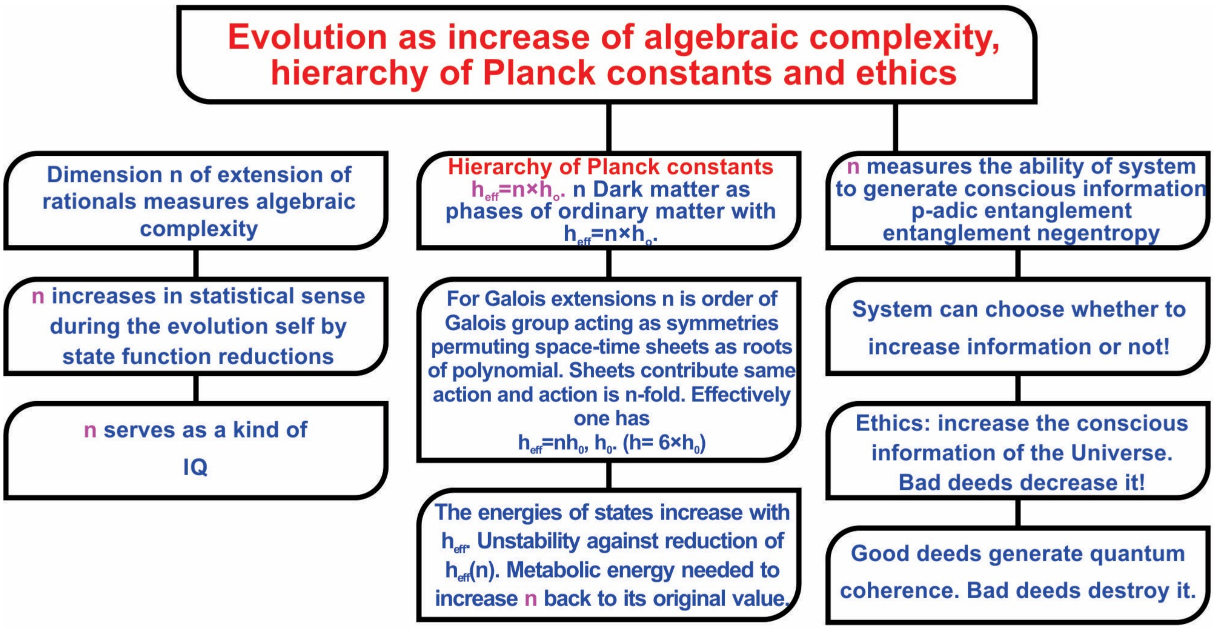

Figure 10.8: Number theoretic view about evolution

How to realize $M^8 - H$ duality?

$M^8 - H$ duality (see **Fig. 10.7**) allows to $X^4 \subset M^8$ to $X^4 \subset H$ so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D preferred 2-surfaces defining holography making possible $M^8 - H$ duality and possibly appearing as singularities in H. The dynamics of minimal surfaces, which are also extremals of Kähler action, reduces for known extremals to purely algebraic conditions analogous to holomorphy conditions in string models and thus involving only gradients of coordinates. This condition should hold generally and should induce the required huge reduction of degrees of freedom proposed to be realized also in terms of the vanishing of super-symplectic Noether charges already mentioned [L98].

Twistor lift allows several variants of this basic duality [L96, L97]. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of EQs defining an evolutionary hierarchy. This forms the basics for the number theoretical vision about TGD.

As already noticed, $X^4 \subset M^8$ would satisfy an infinite number of additional conditions stating vanishing of Noether charges for a sub-algebra $SSA_n \subset SSA$ of super-symplectic algebra SSA actings as isometries of WCW.

M^8-H duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions [L91].

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of M^8-H duality and means that tangent - or normal space is associative/quaternionic.
2. Each tangent space of X^4 at x must contain a preferred $M_c^2(x) \subset M_c^4$ such that $M_c^2(x)$ define an integrable distribution and therefore complexified string world sheet in M_c^4. This gives similar distribution for their orthogonal complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X_c^2 \subset X_c^4$ in R_c sense. $E_c^2(x)$ would correspond to partonic 2-surface. This condition generalizes for X^4 with quaternionic normal space.

   One can imagine two realizations for this condition.

Option I: Global option states that the distributions $M_c^2(x)$ and $E_c^2(x)$ define a slicing of X_c^4.

Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce $X^4 \subset H$. This would be the minimal option.

It seems that only **Option II** can be realized.

1. The basic observation is that X_c^2 can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M_c^2 would be the simplest solution but also more general complex sub-manifolds $X_c^2 \subset M_c^4$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.
   These surfaces would be like the families of curves in complex plane defined by $u=0$ an $v=0$ curves of analytic function $f(z)=u+iv$. One should have family of polynomials differing by a constant term, which should be real so that $v=0$ surfaces would form a discrete set.
2. SH makes possible M^8-H duality assuming that associativity conditions hold true only at 2-surfaces including partonic 2-surfaces or string world sheets or perhaps both. Thus one can give up the conjecture that the polynomial ansatz implies the additional condition globally.
   SH indeed states that PEs are determined by data at 2-D surfaces of X^4. Even if the conditions defining X_c^2 have only a discrete set of solutions, SH at the level of H could allow to deduce the PEs from the data provided by the images of these 2-surfaces under M^8-H duality. The existence of $M^2(x)$ would be required only at the 2-D surfaces.
3. There is however a delicacy involved: the X^2 might be only metrically 2-D but not topologically. The partonic orbits are 3-D light-like surfaces with metric dimension $D=2$. The 4-metric degenerates to 2-D metric at them. Therefore their pre-images would be natural candidates for the singularities at which the dimension of the quaternionic tangent or normal space reduces to 2 [L92] . If this happens, SH would not be quite so strong as expected. The study of fermionic variant of M^8-H correspondence indeed leads to this conclusion.

One can generalize the condition selecting X_c^2 so that it selects 1-D surface inside X_c^2. By assuming that R_c-valued "real" or "imaginary" part of complex part of P at this 2-surface vanishes. One obtains preferred M_c^1 or E_c^1 containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as a complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

What about M^8-H duality in the fermionic sector?

During the preparation of this article I became aware of the fact that the realization M^8-H duality in the fermionic sector has remained poorly understood. This led to a considerable integration of the ideas about M^8-H duality also in the bosonic sector and the existing phenomenological picture follows now from M^8-H duality. There are powerful mathematical guidelines available.

1. Octonionic spinors

By supersymmetry, octonionicity should have also a fermionic counterpart.

1. The interpretation of M_c^8 as complexified octonions suggests that one should use complexified octonionic spinors in M_c^8. This is also suggested by SO(1,7) triality unique for dimension $d = 8$ and stating that the dimensions of vector representation, spinor representation and its conjugate are same and equal to $D = 8$. I have already earlier considered the possibility to interpret M^8 spinors as octonionic [L27]. Both octonionic gamma matrices and spinors have interpretation as octonions and gamma matrices satisfy the usual anti-commutation rules. The product for gamma matrices and spinors is replaced with the non-associative octonionic product.
2. Octonionic spinors allow only one M^8-chirality, which conforms with the assumption of TGD inspired SUSY that only quarks are fundamental fermions and leptons are their local composites [L95, L94].
3. The decomposition of $X^2 \subset X^4 \subset M^8$ corresponding to $R \subset C \subset Q \subset O$ should have an analog for the O_c spinors as a tensor product decomposition. The special feature of dimension $D = 8$ is that the dimensions of spinor spaces associated with these factors are indeed 1, 2, 4, and 8 and correspond to dimensions for the surfaces!
One can define for the octonionic spinors associative/co-associative sub-spaces as quaternionic/co-quaternionic spinors by posing chirality conditions. For $X^4 \subset M_c^8$ one could define the analogs of projection operators $P_\pm = (1 \pm \gamma_5)/2$ as projection operators to either factor of the spinor space as tensor product of spinor space associated with the tangent and normal spaces of X^4: the analog of γ_5 would correspond to tangent or normal space depending on whether tangent or normal space is associative. For the spinors with definite chirality there would be no entanglement between the tensor factors. The condition would generalize the chirality condition for massless M^4 spinors to a condition holding for the local M^4 appearing as tangent/normal space of X^4.
4. The chirality condition makes sense also for $X^2 \subset X^4$ identified as a complex/co-complex surface of X^4. Now γ_5 is replaced with γ_3 and states that the spinor has well-defined spin in the direction of axis defined by the decomposition of X^2 tangent space to $M^1 \times E^1$ with M^1 defining real octonion axis and selecting rest frame. Interpretation in terms of quantum measurement theory is suggestive.

What about the sigma matrices associated with the octonionic gamma matrices? The surprise is that the commutators of M^4 sigma matrices and those of E^4 sigma matrices close to the same $SO(3)$ algebra allowing interpretation as representation for quaternionic automorphisms. Lorentz boosts are represented trivially, which conforms with the fact that octonion structure fixes unique rest system. Analogous result holds in E^4 degrees of freedom. Besides this one has unit matrix assignable to the generalized spinor structure of CP_2 so that also electroweak $U(1)$ factor is obtained.

One can understand this result by noticing that octonionic spinors correspond to 2 copies of a tensor products of the spinor doublets associated with spin and weak isospin. One has $2 \otimes 2 = 3 \oplus 1$ so that one must have $1 \oplus 3 \oplus 1 \oplus 3$. The octonionic spinors indeed decompose like $1 + 1 + 3 + \overline{3}$ under $SU(3)$ representing automorphisms of the octonions. $SO(3)$ could be interpreted as $SO(3) \subset SU(3)$. $SU(3)$ would be represented as tangent space rotations.

2. Dirac equation as partial differential equation must be replaced by an algebraic equation

Algebraization of the dynamics should be supersymmetric. The modified Dirac equation in H is linear partial differential equation and should correspond to a linear algebraic equation in M^8.

1. The key observation is that for the ordinary Dirac equation the momentum space variant of Dirac equation for momentum eigenstates is algebraic! Could the interpretation for $M^8 - H$ duality as an analog of momentum-position duality of wave mechanics considered already earlier make sense! This could also have something to do with the dual descriptions of twistorial scattering amplitudes in terms of either twistor and momentum twistors. Already the earlier work excludes the interpretation of the octonionic coordinate o as 8-momentum. Rather, $P(o)$ has this interpretation and o corresponds to the embedding space coordinates.

2. The first guess for the counterpart of the modified Dirac equation at the level of $X^4 \subset M^8$ is $P\Psi = 0$, where Ψ is octonionic spinor and the octonionic polynomial P defining X^4 can be seen as a generalization of momentum space Dirac operator with octonion units representing gamma matrices. If associativity/co-associativity holds true, the equation becomes quaternionic/co-quaternionic and reduces to the 4-D analog of massless Dirac equation and of modified Dirac equation in H. Associativity holds true if also Ψ satisfies associativity/co-associativity condition as proposed above.
3. What about the square of the Dirac operator? There are 3 conjugations involved: quaternionic conjugation assumed in the earlier work, conjugation with respect to i, and their combination. The analog of octonionic norm squared defined as the product $o_c o_c^*$ with conjugation with respect to i only, gives Minkowskian metric $m_{kl}o^k\overline{o}^l$ as its real part. The imaginary part of the norm squared is vanishing for the projection $O_c \to M^8 \to M^4$ so that time coordinate is real and spatial coordinates imaginary. Therefore Dirac equation allows solutions only for the M^4 projection X^4 and M^4 ($M8$) signature of the metric can be said to be an outcome of quaternionicity (octonionicity) alone in accordance with the duality between metric and algebraic pictures.

   Both $P^\dagger P$ and PP should annihilate Ψ. $P^\dagger P\Psi = 0$ gives $m_{kl}P^k\overline{P}^l = 0$ as the analog of vanishing mass squared in M^4 signature in both associative and co-associative cases. $PP\Psi = 0$ reduces to $P\Psi = 0$ by masslessness condition. One could perhaps interpret the projection $X_c^4 \to M^8 \to M^4$ in terms of Uncertainty Principle.

   There is a $U(1)$ symmetry involved: instead of the plane M^8 one can choose any plane obtained by a rotation $exp(i\phi)$ from it. Could it realize quark number conservation in the M^8 picture? For $P = o$ having only $o = 0$ as root $Po = 0$ reduces to $o^\dagger o = 0$ and o takes the role of momentum, which is however vanishing. 6-D brane like solutions S^6 having $t = r_n$ balls $B^3 \subset CD_4$ as M^4 projections one has $P = 0$ so that the Dirac equation trivializes and does not pose conditions on Ψ. o would have interpretation as space-time coordinates and $P(o)$ as position dependent momentum components P^k.

   The variation of P at the mass shell of M_c^8 (to be precise) could be interpreted in terms of the width of the wave packet associated with a particle. Since the light-like curve at partonic 2-surface for fermion at X_L^3 is not a geodesic, mass squared in M^4 sense is not vanishing. Could one understand mass squared and the decay width of the particle geometrically? Note that mass squared is predicted also by p-adic thermodynamics [K60].
4. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. M^8-H duality [L91] suggests that this boundary is mapped to $X_L^3 \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X_L^3 gives a very powerful constraint on SH and $M^8 - H$ duality.
5. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. M^8-H duality [L91] suggests that this boundary is mapped to $X_L^3 \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X_L^3 gives a very powerful constraint on SH and $M^8 - H$ duality.
6. The variant Dirac equation would hold true also at 2-surfaces $X^2 \subset X^4$ and should commute with the corresponding chirality condition. Now $D^\dagger D\Psi = 0$ defines a 2-D variant of masslessness condition with 2-momentum components represented by those of P. 2-D masslessness locates the spinor to a 1-D curve X_L^1. Its H-image would naturally contain the boundary of the string word sheet at X_L^3 assumed to carry fermion quantum numbers and also the boundary of string world sheet at the light-like boundary of CD_4. The interior of the string world sheet in H would not carry an induced spinor field.
7. The general solution for both 4-D and 2-D cases can be written as $\Psi = P\Psi_0$, Ψ_0 a constant spinor - this is in a complete analogy with the solution of modified Dirac equation in H. P depends on position: the WKB approximation using plane waves with position dependent momentum seems to be nearer to reality than one might expect.

3. The phenomenological picture at H-level follows from the M^8-picture

Remarkably, the partly phenomenological picture developed at the level of H is reproduced at the level of M^8. Whether the induced spinor fields in the interior of X^4 are present or not, has been a long standing question since they do not seem to have any role in the physical picture. The proposed picture answers this question.

Consider now the explicit realization of $M^8 - H$-duality for fermions.

1. SH and the expected analogy with the bosonic variant of $M^8 - H$ duality lead to the first guess. The spinor modes in $X^4 \subset M^8$ restricted to X^2 can be mapped by $M^8 - H$-duality to those at their images $X^2 \subset H$, and define boundary conditions allowing to deduce the solution of the modified Dirac equation at $X^4 \subset H$. X^2 would correspond to string world sheets having boundaries X^1_L at X^3_L.
The guess is not quite correct. Algebraic Dirac equation requires that the solutions are restricted to the 3-D and 1-D mass shells $P_k P^k = 0$ in M^8. This should remain true also in H and X^3_L and their 1-D intersections X^1_L with string world sheets remain. Fermions would live at boundaries. This is just the picture proposed for the TGD counterparts of the twistor amplitudes and corresponds to that used in the twistor Grassmann approach!
For 2-D case constant octonionic spinors Ψ_0 and gamma matrix algebra are equivalent with the ordinary Weyl spinors and gamma matrix algebra and can be mapped as such to H. This gives one additional reason for why SH must be involved.

2. At the level of H the first guess is that the modified Dirac equation $D\Psi = 0$ is true for D based on the modified gamma matrices associated with both volume action and Kähler action. This would select preferred solutions of modified Dirac equation and conform with the vanishing of super-symplectic Noether charges for SSA_n for the spinor modes. The guess is not quite correct. The restriction of the induced spinors to X^3_L requires that Chern-Simons action at X^3_L defines the modified Drac action.

3. The question has been whether the 2-D modified Dirac action emerges as a singular part of 4-D modified Dirac action assignable to singular 2-surface or can one assign an independent 2-D Dirac action assignable to 2-surfaces selected by some other criterion. For singular surfaces $M^8 - H$ duality fails since tangent space would reduce to 2-D space so that only their images can appear in SH at the level of H.
This supports the view that singular surfaces are actually 3-D mass shells M^8 mapped to X^3_L for which 4-D tangent space is 2-D by the vanishing of $\sqrt{g_4}$ and light-likeness. String world sheets would correspond to non-singular $X^2 \subset M^8$ mapped to H and defining data for SH and their boundaries $X^1_L \subset X^3_L$ and $X^1_L \subset CD_4$ would define fermionic variant of SH.

What about the modified Dirac operator D in H?

1. For X^3_L modified Dirac equation $D\Psi = 0$ based on 4-D action S containing volume and Kähler term is problematic since the induced metric fails to have inverse at X^3_L. The only possible action is Chern-Simons action S_{CS} used in topological quantum field theories and now defined as sum of C-S terms for Kähler actions in M^4 and CP_2 degrees of freedom. The presence of M^4 part of Kähler form of M^8 is forced by the twistor lift, and would give rise to small CP breaking effects explaining matter antimatter asymmetry [L95, L94]. S_{C-S} could emerge as a limit of 4-D action.
The modified Dirac operator D_{C-S} uses modified gamma matrices identified as contractions $\Gamma^\alpha_{CS} = T^{\alpha k}\gamma_k$, where $T^{\alpha k} = \partial L_{CS}/\partial(\partial_\alpha h^k)$ are canonical momentum currents for S_{C-S} defined by a standard formula.

2. CP_2 part would give conserved Noether currents for color in and M^4 part Poincare quantum numbers: the apparently small CP breaking term would give masses for quarks and leptons! The bosonic Noether current $J_{B,A}$ for Killing vector j^k_A would be proportional to $J^\alpha_{B,A} = T^\alpha_k j_A k$ and given by $J_{B,A} = \epsilon^{\alpha\beta\gamma}[J_{\beta\gamma}A_k + A_\beta J_{\gamma k}]j^k_A$.
Fermionic Noether current would be $J_{F,A} = \overline{\Psi} J^\alpha \Psi$ 3-D Riemann spaces allow coordinates in which the metric tensor is a direct sum of 1-D and 2-D contributions and are analogous to expectation values of bosonic Noether currents. One can also identify also finite number of Noether super currents by replacing $\overline{\Psi}$ or Ψ by its modes.

3. In the case of X^3_L the 1-D part light-like part would vanish. If also induced Kähler form is non-vanishing only in 2-D degrees of freedom, the Noether charge densities J^t reduce to $J^t = JA_k j^k_A$, $J = \epsilon^{\alpha\beta\gamma} J_{\beta\gamma}$ defining magnetic flux. The modified Dirac operator would reduce to $D = JA_k\gamma^k D_t$ and 3-D solutions would be covariantly constant spinors along the light-like geodesics parameterized by the points 2-D cross section. One could say that the number of

solutions is finite and corresponds to covariantly constant modes continued from X^1_L to X^3_L. This picture is just what the twistor Grassmannian approach led to [L65, L66].

10.4 Could Kähler metric of state space replace S-matrix?

In the sequel a more detailed view about the reduction of S-matrix to a non-flat Kähler geometry of Hilbert space consisting of WCW spinor fields is considered. The proposal is novel in the sense that the state space would codes interactions to its geometry just like space-time geometry codes gravitational interaction in general relativity.

10.4.1 About WCW spinor fields

Induction of second quantized spinor fields from H

There are two approaches to the quantization of induced spinors at space-time surfaces, and these approaches are equivalent.

1. Induction means that gamma matrices are determined by Kähler action as analogs for projections of embedding space gamma matrices and space-time spinor field χ is simply the restriction of H spinor field Ψ. For a given action determining X^4, supersymmetry allows the identification of the modified Dirac operator D and finding of the modes of the induced H spinor field as solutions of the modified Dirac equation (MDE) $D\chi = 0$. Second quantization would replace their coefficients with oscillator operators. However, it is not clear what the anti-commutation relations for the oscillator operators are.
2. One can generalize the classical induction of spinors Ψ to an induction of second quantized spinor fields in H as a restriction of the second quantized Ψ in H to the X^4. One must however get rid of normal ordering diverges due the fact that the anti-commutators for coinciding points give 7-D delta functions. One gets rid of them, if the Ψ and $\overline{\Psi}$ are assigned to disjoin space-time regions. This leads to bi-local modified Dirac action (MDA), implying automatically the classical field equations for the action determining D.
What does $D\Psi = 0$ really mean when Ψ is quantum field? One can develop the restrictions of the c-valued modes of Ψ in terms of modes of χ satisfying $D\chi = 0$, and obtain an expression for Ψ at X^4 in terms of these modes each satisfying MDE. The operator valued coefficients of Ψ modes contributing to a given mode of χ would define the corresponding oscillator value fermionic oscillator operators at X^4.
Also the generalizations of the variants of MDE restricted to sub-manifolds of X^4 make sense and are needed. The beauty is that there is no need to introduce spinor fields at lower-D surfaces as independent dynamical degrees of freedom. For instance, one only a variant of a modified Dirac action defined by Cherns-Simons analog of Kähler action makes sense at light-like partonic orbits so that one has an analog of a topological quantum field theory (TQFT).

How to avoid normal ordering divergences from fermionic oscillator operators?

The normal ordering divergences due to the anti-commutators of fermionic fields at the same point are really serious since induce spinor fields of 8-D $H = M^4 \times CP_2$ so that normal ordering singularities are proportional to 7-D delta function $\delta^7(0)$. They are encountered already for the ordinary MDA giving rise to bosonic SCA charges as Noether charges, which also are plagued by these divergences. Normal ordering for the oscillators in the Noether charges associated with MDA would allow to get rid of the divergences but is a mere trick. The proposal considered in [L104] is to make MDA bi-local at the space-time level.

Consider the general constraints on bi-locality coming from the cancellation of the normal ordering divergences.

1. Consider first 4-D variant of MDA. The most general option for MDA is that there is an integral over the entire X^4 for both Ψ and $\overline{\Psi}$ separately sothat one has 2 4-D integrations. One obtains potential normal ordering divergences proportional to $\delta^7(0)d^8x$. If one has two space-time sheets which in the generic case intersect transversally at discrete set of points, one obtains a vanishing result. However, the self-pairing of a given space-time sheet gives a

divergence as a 4-D volume integral of $\delta^3(0)$. The definition of the self-pairing as a limit of separate space-time sheets approaching each other to get rid of the divergences looks like a trick.

This suggests that the pairing can occur only between disjoint space-time regions, most naturally space-time sheets. For instance, parallel space-time sheets with overlapping M^4 projections. Allowing pairing only between disjoint regions eliminates also the divergences associated with the bosonic Noether charges deduced from MDA and involving 3+4-D integral instead of 3-D integral.

What could be the precise definition for these disjoint regions? M^8-H duality suggests that they correspond to different roots of the octonionic polynomial defined by real polynomials. When 2 roots coincide, one obtains a term of type $\delta^7(0)d^7x$ giving a finite result. What if the number of coinciding roots is higher than 2? This case will be discussed later in number theoretic context.

What about space-like regions, in particular the wormhole contacts expected to be small deformations of a warped embedding of CP_2 having light-like M^4 projection but having same Kähler metric and Kähler form as CP_2 [K10]? There is no pairing with a parallel space-time sheet now. It seems that the pairing must be between different wormhole contacts. This pairing could be essential for the understanding of string like entities as paired wormhole contacts providing a model for elementary particles.

2. For the bilinear MDA, the variation of the 4+4-D modified Dirac action with respect to $\overline{\Psi}$ and Ψ yields both the modified Dirac equation $D\Psi = 0$ plus the field equations for the preferred extremal. This gives the modes of the induced spinor field. In the standard picture the hermiticity condition for the Dirac action yields the same outcome and has interpretation as a supersymmetry between classical and fermionic degrees of freedom.

3. Both the phenomenological picture developed during years and M^8-H duality strongly suggest that spinors can be restricted also to lower-D surfaces. For the lower-D variants of MDA the normal ordering divergences appear already for tranversal intersections. For instance, for 3-D variant of MDA one has $\delta^7(0)d^7x$ type divergences. The only possible manner to avoid them is to require that paired regions are disjoint. For the 3-D Chern-Simons-Kähler action associated with the light-like partonic orbits the paired space-time sheets are very naturally the opposite wormhole throats so that fermions and antifermions would reside at opposite wormhole throats.

   Physical picture also suggests the assignment of actions to 2-D string world sheets and 1-D light-boundaries defining their intersections with partonic orbits.

4. Also 6-D brane-like solutions having the topology of S^6 and $t = r_n$ hyper-plane as intersection with M^4 are of physical interest. Different 4-D space-time surfaces could be glued together along 3-surfaces or 2-D partonic 2-surfaces at S^6. Arguments similar to those already discussed exclude the pairing of various objects with these 6-branes as also their self-pairing.

   Also M^4 and CP_2 define special solutions to the algebraic equations in M^8. MDA reduces to ordinary massless Dirac equation in M^4. In the case of CP_2 one has a massless Dirac equation in CP_2 and only the right-handed neutrino ν_R is possible as a solution. If only quarks are allowed, this solution is excluded. What happens for the deformations of CP_2? Could it be that quarks cannot reside inside wormhole contacts as 4-D entities? Or could one allow solutions of $D\Psi = 0$ as analytic functions of CP_2 coordinates finite in the region in which they are defined - wormhole contact does not span the entire CP_2?

Cognitive representations provide additional insights to the problem of normal ordering divergences, and it could be even argued that they are the only possible manner to define scattering amplitudes as a sequences of improving approximation natural in the approach based on hyperfinite factors of type II_1 (HFFs).

1. For a given extension of rationals determined by the polynomial defining the space-time region in M^8, the space-time surfaces inside CD are replaced with their discretizations consisting of points of M^8 in the extension considered. This surface and cognitive representation are mapped to H by M^8-H correspondence [L76]. For cognitive representations one can perform discretization by replacing the integrals defining SCA generators with discrete sums over points of the cognitive representation. This replacement is very natural since in the p-adic context the counterpart of the Riemann integral does not exist.

2. The Galois group of extension serves as a symmetry group and one can form analogs of group algebra elements - wave functions in discrete Galois group - acting on the cognitive representation and giving rise to discrete representation of quantum states. This state space has as its dimension the dimension n of the Galois group which for Galois extensions coincides with the dimension of extension [L36, L109]. This group algebra-like structure can be given Kähler metric and also spinor structure and this spinor structure could discretize the spinor structure of WCW if gamma matrices are identified as fermionic oscillator operators.

3. Also now one can avoid divergences if the paired space-time regions, say space-time sheets, in MDA are disjoint. It can however happen that n separate points at the orbit of the Galois group approach each other and coincide: this would correspond to the touching of space-time sheets meaning coinciding roots of the octonionic polynomial. In this situation a subgroup of the Galois group would leave the intersection point invariant.
The possible normal ordering divergence comes from different pairs of the m points, which coincide. In 4-D case, the situation corresponds to transversal space-time sheets so that the divergence vanishes. For lower-dimensional surfaces, say partonic orbits, the intersections do not occur in the generic situation but if they occur, the divergence is multiplied by a sum over the values of wave function at coinciding branches and vanishes if the representation is *non-singlet*. It would thus seem that the non-singlet character of Galois representations must be posed as an additional condition.

4. This cancellation mechanism works even without discretization since the notions of Galois group and its representations make sense for arbitrary polynomial surfaces without a restriction to rational or algebraic polynomial coefficients so that the cancellation occurs for non-singlet representations when the space-time sheets intersect.

Are fermions 4-D in H but 3-D in M^8?

M^8-H duality suggests the restriction of the induced spinor fields to light-like 3-surfaces having 2-D partonic surfaces as ends. M^8-H duality reduces space-time surfaces in M^8 to algebraic surfaces defined by polynomials of real variable. The coefficients can be complex. Concerning p-adicization real rationals defines the most attractive option. This leads to a picture in which a hierarchy of extensions of rationals defines evolutionary and cognitive hierarchies. The extensions provide cognitive representations as unique discretizations of the X^4 with embedding space coordinates in extension of rationals and the one can formulate quantum TGD in finite measurement resolution at least using these representations.

The fermionic variant of M^8-H duality [L105] leads to the conclusion that spinor modes in M^8 are restricted at 3-D light-like surfaces obeying an algebraic equations analogs to the momentum space variant of massless Dirac equation. Are H fermions also always restricted to the 3-D light-like orbits of the partonic 2-surfaces at which the signature of the induced metric changes?

On the other hand, the picture deduced at the level of H from the cancellation of the normal ordering divergences allows 4-D fermions, and also implies field equations for X^4 itself. Can one say that free fermions can reside in 4-D space-time but reside only at the 3-D mass shell in momentum space. M^8-H duality would be analogous to the duality between space-time and momentum space descriptions of particles.

Even more, string world sheets have light-like boundaries at the parton orbits. Also fermions in H would be naturally located at string boundaries and behave like point-like particles. One would obtain a picture resembling that provided by twistor Grassmannian approach. Also the cancellation of normal ordering divergences supports this picture and leads to a detailed form of bi-linear modified Dirac action. Also strong form of holography (SH) stating that 2-surfaces carry all information needed to construct the X^4 supports this view. This is actually the same as the phenomenological picture that has been applied.

M^8-H duality predicts also "very special moments in the life self" to have as correlates 6-branes with M^4 time defining in M^8 octonionic real axis (unique rest system) having as values roots of the polynomial defining the space-time surfaces. These surfaces should contain the partonic 2-surfaces defining the reaction vertices. If there is a non-determinism associated with these surfaces it should preserve classical charges and also SSA charge.

Is the proposed counterpart of QFT supersymmetry only an approximate symmetry?

The proposal for the cancellation of the normal ordering divergences allows overviewing leptons as three quark composites with 3 quarks at the same wormhole throat. This option is strongly suggested by the conceptual economy since quarks are enough for WCW spinor structure.

An interesting question is whether TGD allows a counterpart of QFT supersymmetry (SUSY). This was proposed in [L81]. The idea was that both embedding space coordinates and spinors can be expanded as polynomials in the local compositesof quark and antiquark oscillator operators - rather than anticommuting hermitian theta parameters leading to problems with fermion number conservation - with a well-defined quark number.

The proposal was that leptons are purely local 3-quark-composite analogous to a superpartner of quark: note however that quark superspinor would have quark number one so that precise spartner interpretation fails. This option and only its slightly local variant is possible only for the TGD view about color as angular momentum rather than spin-like quantum number.

This proposal was based on discrete cognitive representations as unique discretizations of the X^4 and on the crucial assumption that fermionic oscillator operators obey Kronecker delta type anticommutations rather than the 8-D anticommutations giving $\delta^7(0)$ anti-commutator singularities for the induced second quantized quark field in H. Can the notion of super-field based on local composites of quarks and antiquarks with a definite fermion number avoid normal ordering divergences for the induced anticommutation relations? One can of course think of a normal ordering of monomials but one expects problems with vertices.

This suggests that the super coordinates of H and superspinors can be only approximate notions. Superfield components would correspond to states with a fixed quark number but quarks and antiquarks would reside at opposite wormhole throats rather than forming exactly local composites. Since the throat is expected to have CP_2 size, these states would be for all practical purposes strictly local composites.

10.4.2 Kähler metric as the analog of S-matrix

Kähler metric defines a complex inner product. Complex inner products also define scattering amplitudes. Usually metric is regarded as defining length and angle measurement. Could the Kähler metric define unitary S-matrix? Under simple additional conditions this is true!

The analogs of unitarity conditions

The following little arguments show that given Kähler metric defines an analog of unitary S-matrix giving rise positive transition probabilities, and under additional conditions also a unitary S-matrix between states with quantum numbers labeling basis of complex vectors or of complexified gamma matrices. This defines an S-matrix like entity and under some additional conditions even an unitary S-matrix.

1. The defining conditions for unitary S-matrix and Kähler metric are very similar. S and $S^\dagger$ would correspond to the covariant metric $g_{m\overline{n}}$ and contravariant metric $g_{\overline{m}n}$. Unitary for S-matrix corresponds to the conditions

$$S_{mr}S^\dagger_{rn} = S_{mr}S_{nr} = \delta_{m,n} \quad .$$

(there is summation over repeated indices). The rows of S-matrix are orthonormalized. The definition of the contravariant metriccorresponds the conditions

$$g_{m\overline{r}}g^{\overline{r}n} = \delta_{m,n} \quad .$$

The complex rows of metric tensor and contravariant metric are orthonormalized also now and rows are orthonormal

2. For S-matrix the probabilities are given by $p_{mn} = S_{mn}S^\dagger_{nm} = S_{mn}S_{mn}*$ and are real and non-negative and their sum is equal to one. Also for the Kähler metric the complex analogs of probabilities defined by

$$p^c_{mn} = g_{m\overline{r}}g^{\overline{r}n}$$

sum up to unity. Hence the real parts $Re(p^c_{mn})$ of p^c_{mn} sum up to unity whereas the imaginary parts sum up to zero.

3. p^c_{mn} are not however automatically real and non-negative and it is not clear how to interpret complex or even real but negative probabilities physically. One can however pose the positivity of the real parts of p^c_{mn} as an additional condition on the phase factors $U_{m\overline{n}} = exp(\Phi_{m\overline{n}})$ and $V_{m\overline{n}} = exp(\Psi_{m\overline{n}})$ associated with $g_{m\overline{n}} = R_{mn}U_{m\overline{n}}$ and $g^{\overline{n}m} = S_{nm}V_{\overline{n}m}$. The condition for positivity is

$$U_{m\overline{n}}V_{\overline{n}m} = cos(\Phi_{\overline{n}m} - \Psi_{\overline{n}m} \geq 0$$

and is rather mild requiring the angle difference to be in the range $(-\pi/2, \pi/2)$. This is true of the angles are in the range $(\pi/4, \pi/4)$. The condition $Re(p^c_{mn}) \geq 0$ is equivalent with the condition $Im(ip^c_{mn}) \geq 0$, and characterizes the coefficients of Teichmueller matrices [A33, A51, A42] [K28]: the meaning of this condition will be discussed below.

4. Under what conditions p^c_{mn} reduce to non-negative real numbers? One can express the probabilities as $p_{mn} = g_{m\overline{n}} \times cof(g_{m\overline{n}})/det(g)$. Note that $Z = det(g)$ is constant depending only on the point of the Kähler manifold. One can express g_{mn} as $g_{m\overline{n}} = A_{mn}U_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ as $cof(g_{m\overline{n}}) = B_{mn}V_{m\overline{n}}$. The reality condition implies

$$U_{m\overline{n}} = \overline{V_{m\overline{n}}} \ .$$

The phases of $g_{m\overline{n}}$ and $cof(g_{m\overline{n}})$ are opposite.
This gives additional conditions. Kähler metric involves $N_{tot} = 2N^2$ real parameters There are $(N^2 - N)/2$ elements in say upper diagonal and by hermiticity they are complex conjugates of the lower diagonal. This is the number N_{cond} of conditions coming from the reality. There is also one additional condition due to the fact that the probabilities do not depend on the normalization of g. The total number of real parameters is

$$N_{param} = N_{tot} - N_{cond} - 1 = N(N-1) - 1 \ .$$

For instance, for $N \in \{2, 3, 4\}$ one has $N_{param} =\in \{1, 5, 11\}$. Unitary matrix allows $N_{unit} = N^2$ real parameters and the ratio $N_{param}/N_{unit} = (N(N-1)-1)/N^2$ approaches unity for large values of N. Note that a unitary matrix with real diagonals has $N^2 - N$ parameters so that the number of parameters is the same as for a hermitian metric with unit determinant.

5. Could one transform the metric defining non-negative probabilities to a unitary matrix by a suitable scaling? One can indeed define a matrix S as a matrix $S_{mn} = \sqrt{A_{mn}B_{mn}/Z}U_{mn}$. One has $S_{mn}S_{mn}* = A_{mn}B_{mn}/Z$ given also by the product of $g_{m\overline{n}}g^{\overline{n}m}$ so that the probabilities are the same. The unitarity conditions reduce to $g_{m\overline{r}}g^{\overline{r}n} = \delta^n_m$.
In infinite dimensional case problems might be produced by the appearance of the square root of determinant expected to be infinite. However, also the cofactors are expected to diverge, and one can express them as partial derivatives of the metric determinant with respect to the corresponding element of the metric. This is expected to give a finite value for the elements of the contravariant metric. Note that the ratios of the probabilities do not depend on the metric determinant.

Can one distinguish between the descriptions based on Kähler metric and S-matrix?

For the Teichmueller option the proposed analog for S-matrix involves imaginary part. Does it have some physically observable consequences?

Could one imagine a physical situation allowing to test whether the S-matrix description or its TGD variant is nearer to truth? One can indeed imagine an analog of a Markov process characterized by a matrix p of transition probabilities p_{mn} at a given step. For a two-step process the transition matrix would be p^2_{mn}.

In the TGD context one would have $p_{mn} = Re(p^c_{mn})$. What happens in a two-step process? Should one use use p^2_{mn} or $Re((p^c)^2)_{mn} = Re((p^c)2)_{mn} - Im(p^c)^2_{mn}$? If both options are possible, what could distinguish physically between them?

Could the correct interpretation be that p^2_{mn} describes the process when the outcome is measured in both steps, and $Re((p^c)^2)_{mn}$ the process in which only initial and final states are measured? This picture would generalize to n-step processes and predict a deviations from the ordinary Markov process and perhaps allow to compare the predictions of the TGD view and standard view and deduce $Im(p^c)$.

S-matrix and its Hermitian conjugate correspond in standard physics to situations related by CPT symmetry defined as the product of charge conjugation C, spatial reflection P and time reversal T. The transition probabilities would remain invariant in this transformation although transition amplitudes are replaced with their complex conjugates.

What happens to CPT in TGD framework? In TGD framework CPT induces a hermitian conjugation $g_{m\overline{n}} \rightarrow g_{\overline{n}m} =$

10.5 The role of fermions

In this section the role of fermions (quarks as it seems) is discussed in more detail. In particular, the conditions on the scattering amplitudes from the cancellation of normal ordering divergences and co-associative octonionic spinors at the level of M^8 are discussed. Also the formulation of scattering amplitudes the level of M^8 is briefly considered.

10.5.1 Some observations about Feynman propagator for fundamental quark field

In the sequel the divergence cancellation mechanism and the properties of Dirac propagator are discussed in detail. The surprise is that the massive propagators with CP_2 mass scale reduce essentially to massless propagators for light-like separations. This allows understanding of why quarks can give rise to light elementary particles.

The second quantized free quark field Ψ in H defines fundamental fermions appearing as a building brick of elementary particles. The Feynman propagator for Ψ appears in the analogs of Feynman diagrams. Apart from the right handed neutrino (present only as a 3 quark composite at partonic 2-surface if only quarks are involved) the modes of Ψ are extremely massive. Elementary particles are light. How can one understand this?

In p-adic thermodynamics the generation of small mass was assumed to involve a generation of a negative, "tachyonic", ground state conformal weight encountered also in string models. M^8-H correspondence allows a more sophisticated description based on the choice of $M^4 \subset M^8$ mapped to $M^4 \subset H$. By 8-D Lorentz invariance the 4-D mass squared of ground state massless in 8-D sense, depends on the choice of $M^4 \subset H$, and with a proper re-choice of M^4 the particle having large M^4 mass becomes massless.

The action of the generators of super-conformal algebra creates states with a well-defined conformal weight, which are massless for a proper choice of $M^4 \subset M^8$. In p-adic thermodynamics the choice of $M^4 \subset M^8$ would correspond to a generation of negative ground state conformal weight.

The states can however mix slightly with states having higher value of conformal weight, and since one cannot choose M^4 separately for these states, a small mass is generated and described by p-adic thermodynamics. The classical space-time correlate for the almost masslessness is minimal surface property, which provides a non-linear geometrization for massless fields as surfaces. The non-linearity at the classical level leads to a generation of small mass in 4-D sense for which p-adic thermodynamics provides a model.

The propagators for the fundamental quarks in H correspond to CP_2 mass scale. Can this be consistent with the proposed picture? The following simple observations about the properties of predicted fermion propagator and anticommutator for the induced spinor fields lead to a result, which was a surprise to me. The propagators and anti-commutators of massive quarks at light cone boundary are in excellent approximation massless for light-like distances. This makes it possible to understand why elementary fermions are light.

This mechanism does not work in QFT defined in M^4 since inverse propagator is $\gamma^k p_k+m$ so that M^4 chiralities mix for massive states. In TGD picture H-chirality is fixed by 8-D masslessnes and the product of M^4 and CP_2 chiralities for spinors equals to the H chirality. The inverse progator is proportional to the operator $p^k\gamma + D_{CP_2}$, where D_{CP2} is CP_2 part of Dirac operator.

General form of the Dirac propagator in H

Second quantized quark field Ψ restricted to the space-time surface determines the Feynman propagator fundamental quark. The propagator can be expressed as a sum of left- and right-handed propagators as

$$S_F = S_{F,L} + S_{F,R} = D_L G_{F,L} + D_R G_{F,R} \ .$$

Here D_L and D_R are the left- and right-handed parts of a massless (in 8-D sense) Dirac operator D in H involving couplings to CP_2 spinor connection depending on CP_2 chirality in accordance with electroweak parity breaking. $G_{F,L}$ *resp.* $G_{F,R}$ is the propagator for a massless (in 8-D sense) scalar Laplacian in H coupling to the spinor connection assignable to left *resp.* right handed modes. G_F can be expressed by generalizing the formula from 4-D case

$$G_{F,I} \sum_n \int d^4p \frac{1}{p^2 - M_{n,I}^2} exp(ip\cdot(m_1 - m_2))\Phi^*_{n,I}(s_1)\Phi_{n,I}(s_2) \ .$$

Here one has $I \in \{L,R\}$ and the mass spectra are different for these modes. Here m_i denote points of M^4 and s_i points of CP_2. n,I, $I \in \{L,R\}$, labels the modes $\Phi_{n,I}$ of a scalar field in CP_2 associated with right and left handed modes having mass squared $M_{n,R}$. Since H-chirality is fixed to be quark chirality, there is a correlation between M^4 - and CP_2 chiralities. Apart from ν_R all modes are massive (ν_R is need not be present as a fundamental fermion) and the mass M_n, which is of order CP_2 mass about 10^{-4} Planck masses, is determined by the CP_2 length scale and depends on CP_2 chirality.

$G_{F,I}$ reduces to a superposition over massive propagators with mass $M_{n,I}$:

$$G_{F,I} = \sum_n G_F(m_1 - m_2|M_n)\Phi^*_{n,I}(s_1)\Phi_{n,I}(s_2)P_I \ .$$

Here P_I, $I \in \{L,R\}$ is a projector to the left/right handed spinors. One can express $S_{F,I}$ as a sum of the free M^4 part and interaction term proportional to the left - or right-handed part of CP_2 spinor connection:

$$S_{F,I} = D(M^4)G_{F,I} + A_I G_{F,I} \ .$$

A_I, $I \inf\{L,R\}$ acts either on s_1 or s_2 but the outcome should be the same. The first term gives sum over terms proportional to massive free Dirac propagator in M^4 allowing to get a good idea about the behavior of the propagator.

About the behavior of the quark propagator

The quark propagator reduces to left- and right-handed contributions corresponding to various mass values $M_{n,I}$. To get view about the behaviour of the quark propagator it is useful to study the behavior of $G_F(x,y|M)$ for a given mass as well as the behaviors of free and interacting parts of S_F its free part

From the explicit expression of $G_F(m_1 - m_2|M_n)$ one can deduce the behavior of the corresponding contribution to the Feynman propagator. Only ν_R could give a massless contribution to the progator. Explicit formula for G_F can be found from Wikipedia [A4] (`https://en.wikipedia.org/wiki/Propagator#Feynman_propagator`):

$$G_F(x,y|m) = \begin{cases} -\frac{1}{4\pi}\delta(s) + \frac{m}{8\pi\sqrt{s}}H_1^{(1)}(m\sqrt{s}), & s \geq 0 \\ -\frac{im}{4\pi^2\sqrt{-s}}K_1^{(1)}(m\sqrt{-s}), & s \leq 0 \ . \end{cases}$$

Here $H_1^{(1)}(x)$ is Hankel function of first kind and $K_1^{(1)}$ is modified Bessel function [A2](`https://en.wikipedia.org/wiki/Bessel_function`). Note that for massless case the Hankel term vanishes.

Consider first Hankel function.

1. Hankel function $H_\alpha^{(1)}(x)$ [A2, A4] obeys the defining formula

$$H_\alpha^{(1)}(x) = \frac{J_{-\alpha}(x) - exp(i\alpha\pi i)J_\alpha(x)}{isin(\alpha\pi)} \quad .$$

For integer values of α one has $J_{-n}(x) = (-1)^n J_n(x)$ so that $\alpha = n$ case gives formally 0/0 and the limit must be obtained using Hospital's rule.

2. Hankel function $H_1^{(1)}(x)$ can be expressed as sum of Bessel functions of first and second kind

$$H_1^{(1)}(x) = J_1(x) + iY_1(x) \quad .$$

J_1 vanishes at origin whereas Y_1 diverges like $1/x$ at origin.

3. The behaviors of Bessel functions and their variants near origin and asymptotically are easy to understand by utilizing Schrödinger equation inside a cylinder as a physical analogy. The asymptotic behaviour of Hankel function for large values of x is

$$H_\alpha^{(1)}(x) = \frac{2}{\pi x} exp(i(x - 3\pi/4)) \quad ,$$

4. The asymptotic behavior of Hankel function implies that the massive Feynman propagator an oscillatory behavior as a function of $m\sqrt{s}$. Modulus decreases like $1/\sqrt{m\sqrt{s}}$. The asymptotic behavior for the real and imaginary parts corresponds to that for Bessel functions of first kind (J_1) and second kind (Y_1). At origin $H_\alpha^{(1)}(x)$ diverges like $Y_1(x) \sim \frac{(x/2)^{-n}}{\pi}$ near origin. For large values of x $K_1(x)$ decreases exponentially like $exp(-x)\frac{\sqrt{\pi}}{2x}$. At origin $K_1(x)$ diverges.

5. In the recent case the quark propagator would oscillate extremely rapidly leaving only the $\delta(s)$ part so that the propagator behaves like massless propagator!
The localization of quarks to the partonic surfaces with a size scale of CP_2 radius implies that that the oscillation does not lead to a vanishing of the Hankel contribution to the scattering amplitudes. For induced spinor fields in the interior of space-time surfaces destructive interference is however expected to occur so that behavior is like that for a massless particle. This should explain why the observed particles are light although the fundamental fermions are extremely massive. The classical propagation would be essentially along light-like rays.
The long range correlations between quarks would come from the $\delta(s)$ part of the propagator, and would not depend on quark mass so that it would effectively behave like a massless particle. Also the action of Dirac operator on $G_F(x, y)$ in M^4 degrees of freedom is that of a massless Dirac propagator coupling to induced gauge potentials. The quarks inside hadrons and also elementary particles associated with the wormhole throats of flux tubes could be understood as quarks at different partonic 2-surfaces at the boundary of CD having light-like distance in an excellent approximation.

6. The above argument is for the Feynman propagator but should generalize also for anticommutator. The anticommutator for Dirac operator D in M^4 can be expressed as $D\Delta(x, y)$, where D is a scalar field propagator.

$$\Delta(x, y|m) \propto \begin{cases} \frac{m}{8\pi\sqrt{s}} H_1^{(1)}(m\sqrt{s}), & s \geq 0 \\ -\frac{m}{\sqrt{-s}} K_1^{(1)}(m\sqrt{-s}), & s \leq 0 \end{cases} .$$

Apart from possible proportionality constants the behavior is very similar to that for Feynman propagator except that the crucial $\delta(s)$ term making possible effectively massless propagation is absent. At light-cone boundary however $\sqrt{s}$ is zero along light rays, and this gives long range correlations between fermions at different partonic 2-surfaces intersected by light rays from the origin. Hence one could have a non-vanishing Hermitian inner product for 3-D states at boundaries of CD.
Rather remarkably, these results provide a justification for twistor-diagrams identified as polygons consisting of light-like segments.

Possible normal ordering divergences

Concerning the cancellation of normal ordering divergences the singularities of the propagators G_F are crucial. The bi-linearity of the modified Dirac action forcing anticommuting quark and antiquark oscillator operators at different throats of wormhole contacts but this need not guarantee the absence of the divergence since the free quark propagator in M^4 contains mass independent $\delta(s)$ part plus the divergent part from Hankel function behaving like $1/\sqrt{sm}$. For the massless propagator assignable to ν_R the propagator would reduce to M^4 propagator and only the $\delta(s)$ would contribute.

$s = 0$ condition tells that the distance between fermion and anti-fermion is light-like and is possible to satisfy at the light-like boundary of CD. Paired quark and antiquark at the wormhole throats must reside at the same light-like radial ray from the tip of cd (cd corresponds to causal diamond in M^4). Since partonic surfaces are 2-D this condition selects discrete pairs of points at the pair of the partonic surfaces. The integration over the position of the end of the propagator line over paired partonic 2-surfaces should smooth out the divergences and yield a finite result. This would be crucial for having an inner product for states at the boundary of the light-cone.

This applies also to the point pairs at opposite throats of wormhole contact. Time-ordered product vanishing for $t_1 = t_2$ so that the points must have different values of t and this is possible. The two 2-D integrations are expected to smooth out the singularities and eliminate divergences also now.

10.6 Conclusions

TGD predicts revolution in quantum theory based on three new principles.

1. ZEO solving the basic paradox of quantum measurement theory. Ordinary ("big") state function reduction involves time reversal forcing a generalization of thermodynamics and leading to a theory of quantum self-organization and self-organized quantum criticality (homeostasis in living matter).
2. Phases of ordinary matter labelled by effective Planck constant $h_{eff} = nh_0$ identified as dark matter and explaining the coherence of living matter in terms of dark matter at magnetic body serving as a master, and predicting quantum coherence in all scales at the level of magnetic bodies. $h_{eff}/h_0 = n$ has interpretation as the dimension for an extension of rationals and is a measure of algebraic complexity. Evolution corresponds to the increase of n.
   Extensions of rationals are associated with adelic physics providing description of sensory experience in terms of real physics and of cognition in terms of p-adic physics. Central notion is cognition representation providing unique discretization of X^4 in terms of points with embedding space coordinates in the extension of rationals considered $M^8 - H$ duality realizes the hierarchy of rational extensions and assigns them to polynomials defining space-time regions at the level of M^8 and mapped to minimal surfaces in H by $M8 - H$ duality.
3. The replacement of the unitary S-matrix with the Kähler metric of the Kähler space defined by WCW spinor fields satisfying the analog of unitarity and predicting positive definite transition probabilities defining matrix in Teichmueller space. Einstein's geometrization of classical physics extends to the level of state space, Equivalence Principle generalizes, and interactions are coded by the geometry of the state space rather than by an *ad hoc* unitary matrix. Kähler geometry for the spinor bundle of WCW has Riemann connection only for a maximal group of isometries identified as super-symplectic transformations (SS). This makes the theory unique and leads to explicit analogs of Feynman rules and to a proof that theory is free of divergences.

In this work the third principle, which is new, is formulated and some of its consequences are discussed. The detailed formulation allows understanding of how normal ordering divergences and other divergences cancel. The key idea is to induce the second quantized free spinor field from H to space-time surface. This determines the propagators at the space-time level. The condition that creation and annihilation operators are at different space-time sheets - say at throats of wormhole contacts is enough. An alternative but not exclusive option suggested by ZEO is that the annihilation operators correspond to creation operators for conjugated Dirac vacuum associated with the opposite half-cone of CD or sub-CD.

A further observation is that the Dirac propagators for particles reduce in a good approximation to massless propagators when the propagation takes place along light-like distances: this provides a considerable insight to why physical particles are so light although the spinor harmonics for CP_2 correspond to CP_2 mass scale.

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Chapter 11

Breakthrough in understanding of $M^8 - H$ duality

11.1 Introduction

$M^8 - H$ duality [L76, L73, L74, L105] has become a cornerstone of quantum TGD but several aspects of this duality are still poorly understood.

11.1.1 Development of the idea about $M^8 - H$ duality

A brief summary about the development of the idea is in order.

1. The original version of M^8-H duality assumed that space-time surfaces in M^8 can be identified as associative or co-associative surfaces. If the surface has associative tangent/normal space and contains a complex co-complex surface, it can be mapped to a 4-surface in $M^4 \times CP_2$.
2. Later emerged the idea that octonionic analyticity realized in terms of a real polynomials P algebraically continued to polynomials of complexified octonion might realize the dream [L47, L48, L49]. The original idea was that the vanishing condition for the real/imaginary part of P in quaternion sense could give rise to co-assocative/associative sense.
   $M^8 - H$ duality concretizes number theoretic vision [L53, L52] summarized as adelic physics fusing ordinary real number based physics for the correlates of sensory experience and various p-adic physics ($p = 2, 3, ...$) as physics for the correlates of cognition. The polynomials of real variable restricted to be rational valued defines an extension or rationals via the roots of the polynomials and one obtains an evolutionary hierachy associated with these extensions increasing in algebraic complexity. These extensions induce extensions of p-adic numbers and the points of space-time surface in M^8 with coordinates in the extension of rationals define cognitive representations as unique discretizations of the space-time surface.
3. The realization of the general coordinate invariance in TGD framework [K52, K31, K85, L112] [L108] motivated the idea that strong form of holography (SH) in H could allow realizing $M^8 - H$ duality by assuming associativity/co-associativity conditions only at 2-D string world sheet and partonic 2-surfaces and possibly also at their light-like 3-orbits at which the signature of the induced metric changes from Minkowskian to Euclidian.

11.1.2 Critical re-examination of the notion

In this article $M^8 - H$ duality is reconsidered critically.

1. The healthy cold shower was the learning that quaternion (associative) sub-spaces of quaternionic spaces are geodesic manifolds [A34]. The distributions of quaternionic normal spaces are however always integrable. Hence, co-associativity remains the only interesting option. Also the existence of co-commutative sub-manifolds of space-time surface demanding the existence of a 2-D integrable sub-distribution of subspaces is possible. This learning experience motivated a critical examination of the $M^8 - H$ duality hypothesis.

2. The basic objection is that for the conjectured associative option, one must assign to each state of motion of a particle its own octonionic structure since the time axis would correspond to the octonionic real axis. It was however clear from the beginning that there is an infinite number of different 4-D planes O_c in which the number theoretical complex valued octonion inner product reduces to real - the number theoretic counterpart for Riemann metric. In the co-associative case this is the only option. Also the Minkowski signature for the real projection turns out to be the only physically acceptable option. The mistake was to assume that Euclidian regions are co-associative and Minkowskian regions associative: both must be co-associative.
3. The concrete calculation of the octonion polynomial was the most recent step - carried already earlier [L47, L48, L49] but without realizing the implications of the extremely simple outcome. The imaginary part of the polynomial is proportional to the imaginary part of octonion itself. It turned out that the roots $P = 0$ of the octonion polynomial P are 12-D complex surfaces in O_c rather than being discrete set of points defined as zeros $X = 0, Y = 0$ of two complex functions of 2 complex arguments. The analogs of branes are in question. Already earlier 6-D real branes assignable to the roots of the real polynomial P at the light-like boundary of 8-D light-cone were discovered: also their complex continuations are 12-D [L73, L80].
4. P has quaternionic de-composition $P = Re_Q(P) + I_4 Im_Q(P)$ to real and imaginary parts in a quaternionic sense. The naive expectation was that the condition $X = 0$ implies that the resulting surface is a 4-D complex surface X^4_c with a 4-D real projection X^4_r, which could be co-associative.
   The expectation was wrong! The equations $X = 0$ and $Y = 0$ involve the same(!) complex argument o_c^2 as a complex analog for the Lorentz invariant distance squared from the tip of the light-cone. This implies a cold shower. Without any additional conditions, $X = 0$ conditions have as solutions 7-D complex mass shells H^7_c determined by the roots of P. The explanation comes from the symmetries of the octonionic polynomial.
   There are solutions $X = 0$ and $Y = 0$ only if the two polynomials considered have a common a_c^2 as a root! Also now the solutions are complex mass shells H^7_c.
5. How could one obtain 4-D surfaces X^4_c as sub-manifolds of H^7_c? One should pose a condition eliminating 4 complex coordinates: after that a projection to M^4 would produce a real 4-surface X^4 .
   A co-associative X^4_c is obtained by acting with a local SU_3 transformation g to a co-associative plane $M^4 \subset M^8_c$. If the image point $g(p)$ is invariant under $U(2)$, the transformation corresponds to a local CP_2 element and the map defines $M^8 - H$ duality even if the co-associativity in geometric sense were not satisfied.
   The co-associativity of the plane M^4 is preserved in the map because G_2 acts as an automorphism group of the octonions. If this map also preserves the value of 4-D complex mass squared, one can require that the intersections of X^4_c with H^7_c correspond to 3-D complex mass shells. One obtains holography with mass shells defined by the roots of P giving boundary data. The condition H images are analogous to Bohr orbits, corresponds to number theoretic holography.

It this, still speculative, picture is correct, it would fulfil the original dream about solving classical TGD exactly in terms of roots for real/imaginary parts of octonionic polynomials in M^8 and by mapping the resulting space-time surfaces to H by $M^8 - H$ duality. In particular, strong form of holography (SH) would not be needed at the level of H, and would be replaced with a dramatically stronger number theoretic holography.

Octonionic Dirac equation, which is purely algebraic equation and the counterpart for ordinary Dirac equation in momentum space, serves as a second source of information.

1. The first implication is that O_c has interpretation as an analog of momentum space for quarks: this has profound implications concerning the interpretation. The space-time surface in M^8 would be analog of Fermi ball. The octonionic Dirac equation reduces to the mass shell condition $m^2 = r_n$, where r_n is a root of the polynomial P defining the 4-surface but only in the co-associative case.
2. Cognitive representations are defined by points of M^8 with coordinates having values in the extensions of rational defined by P and allowing an interpretation as 4-momenta of quarks. In the generic case the cognitive representations are finite. If the points of M^8 correspond to quark momenta, momentum conservation is therefore expected to make the scattering trivial.

However, a dramatic implication of the reduction of the co-associativity conditions to the vanishing of ordinary polynomials Y is that by the Lorentz invariance of roots of P, the 3-D mass shells ofd M^4 have an infinite number of points in a cognitive representation defined by points with coordinates having values in the extensions of rational defined by P and allowing an interpretation as 4-momenta. This is what makes interesting scattering amplitudes for massive quarks possible.

3. What is the situation for the images of M^4 points under the effective local CP_2 element defined by local $SU(3)$ element g preserving the mass squared and mapping H^3 to $g(H^3)$? If g is expressible in terms of rational functions with rational coefficients, algebraic points are mapped to algebraic points. This is true also in the interior of M^4.
   This would mean a kind of cognitive explosion for massive quark momenta. Without the symmetry one might have only forward scattering in the interior of X^4_r. Note that massless quarks can however arrive at the boundary of CD which also allows cognitive representation with an infinite number of points.
4. In the number theoretic approach, kinematics becomes a highly non-trivial part of the scattering. The physically allowed momenta would naturally correspond to algebraic integers in the extension E of rationals defined by P. Momentum conservation and on-mass-shell conditions together with the condition that momenta are algebraic integers in E are rather strong. The construction of Pythagorean squared generalize to the case of quaternions provides a general solutions to the conditions: the solutions to the conditions are combinations of momenta which correspond to squares of quaternions having algebraic integers as components.
5. The original proposal was that local $G_{2,c}$ element $g(x)$ defines a vanishing holomorphic gauge field and its restriction to string world sheet or partonic 2-surface defines conserved current. $M^8 - H$ duality however requires that local $SU(3)$ element with the property that image point is invariant under $U(2)$ is required by $M^8 - H$ duality defines $X^4 \subset M^8$.
   In any case, these properties suggest a Yangian symmetry assignable to string world sheets and partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The generators of the Yangian algebra have a representation as Hamiltonians which are in involution. They define conserved charges at the orbits for a Hamiltonian evolution defined by any combination of these the Hamiltonians. ZEO suggests a concrete representation of this algebra in terms of quark and antiquark oscillator operators. This algebra extends also to super-algebra. The co-product of the associated Yangian would give rise to zero energy states defining as such the scattering amplitudes.

11.1.3 Octonionic Dirac equation

The octonionic Dirac equation allows a second perspective on associativity. Everything is algebraic at the level of M^8 and therefore also the octonionic Dirac equation should be algebraic. The octonionic Dirac equation is an analog of the momentum space variant of the ordinary Dirac equation and also this forces the interpretation of M^8 as momentum space.

Fermions are massless in the 8-D sense and massive in 4-D sense. This suggests that octonionic Dirac equation reduces to a mass shell condition for massive particle with $q \cdot q = m^2 = r_n$, where $q \cdot q$ is octonionic norm squared for quaternion q defined by the expression of momentum p as $p = I_4 q$, where I_4 is octonion unit orthogonal to q. r_n represents mass shell as a root of P.

For the co-associative option the co-associative octonion p representing the momentum is given in terms of quaternion q as $p = I_4 q$. One obtains $p \cdot p = qq = m^2 = r_n$ at the mass shell defined as a root of P. Note that for M^4 subspace the space-like components of $p\,p$ are proportional to i and the time-like component is real. All signatures of the number theoretic metric are possible.

For associative option one would obtain $qq = m^2$, which cannot be satisfied: q reduces to a complex number $zx + Iy$ and one has analog of equation $z^2 = z^2 - y^2 + 2Ixy = m_n^2$, which cannot be true. Hence co-associativity is forced by the octonionic Dirac equation.

Before continuing, I must apologize for the still fuzzy organization of the material related to $M^8 - H$ duality. The understanding of its details has been a long and tedious process, which still continues, and there are unavoidably inaccuracies and even logical inconsistencies caused by the presence of archeological layers present.

11.2 The situation before the cold shower

The view about $M^8 - H$ duality before the cold shower - leading to what I dare to call a breakthrough - helps to gain idea about the phenomenological side of $M^8 - H$ duality. Most of the phenomenology survives the transition to a more precise picture. This section is however not absolutely necessary for what follows it.

11.2.1 Can one deduce the partonic picture from $M^8 - H$ duality?

The M^8 counterparts for partons and their light like orbits in H can be understood in terms of octonionic Dirac equation in M^8 as an analog for the algebraic variant of ordinary Dirac equation at the level of momentum space [L105, L104] but what about the identification of partonic 2-surfaces as interaction vertices at which several partonic orbits meet? Can one deduce the phenomenological view about elementary particles as pairs of wormhole contacts connected by magnetic flux tubes from $M^8 - H$ duality? There is also the question whether partonic orbits correspond to their own sub-CDs as the fact that their rest systems correspond to different octonionic real axes suggests.

There are also some questions which have become obsolote. For instance: qhy should the partonic vertices reside at $t = r_n$ branes? This became obsolste with the realization that M^8 is analogous to momentum space so that the identification as real octonionic coordinate corresponds now to a component of 8-momentum identifiable as energy. Furthermore, the assumption the associativity of the 4-surface in M^8 had to be replaced with-co-associtivity and octonionic real coordinate does not have interpretation as time coordinate is associative surface

$M^8 - H$ duality indeed conforms with the phenomenological picture about scattering diagrams in terms of partonic orbits [L112, L111] [L111, L112] [L112], and leads to the view about elementary particles as pairs of Euclidian wormhole contacts associated with flux tubes carrying monopole flux.

11.2.2 What happens to the "very special moments in the life of self"?

The original title was "What happens at the "very special moments in the life of self?" but it turned out that "at" must be replaced with "to". The answer to the new question would be "They disappear from the glossary".

The notion of "very special moments in the life of self" (VPM) [L73, L80] makes sense if M^8 has interpretation as an 8-D space-time. M^4 projections of VPMs were originally identified as hyperplanes $t = r_n$, where t is time coordinate and r_n is a root of the real polynomial defining octonionic polynomial as its algebraic continuation.

The interpretation of M^8 as cotangent space of H was considered from the beginning but would suggest the interpretation of M^8 as the analog of momentum space. It is now clear that this interpretation is probably correct and that $M^8 - H$ duality generalizes the momentum-position duality of wave mechanics. Therefore one should speak of $E = r_n$ plane and simply forget the misleading term VMP. VPMs would correspond to constant values of the M^8 energy assignable to M^4 time coordinate.

The identification of space-time surface as co-associative surface with quaternionic normal space containing integrable distribution of 2-D commutative planes essential for $M^8 - H$ duality is also in conflict with the original interpretation. Also the modification of $M^8 - H$ duality in M^4 degrees of freedom forced by Uncertainty Principle [L121] has led to the conclusion that VMPs need not have a well-defined images in H.

11.2.3 What does SH mean and its it really needed?

SH has been assumed hitherto but what is its precise meaning?

1. Hitherto, SH at the level of H is believed to be needed: it assumes that partonic 2-surfaces and/or string world sheets serve as causal determinants determining X^4 via boundary conditions.
   (a) The normal or tangent space of X^4 at partonic 2-surfaces and possibly also at string world sheets has been assumed to be associative that is quaternionic. This condition should be true at the entire X^4.

(b) Tangent or normal space has been assumed to contain preferred M^2 which could be replaced by an integrable distribution of $M^2(x) \subset M^4$. At string world sheets only the tangent space can be associative. At partonic 2-surfaces also normal space could be associative. This condition would be true only at string world sheets and partonic 2-surfaces so that only these can be mapped to H by $M^8 - H$ duality and continued to space-time surfaces as preferred extremals satisfying SH.

The current work demonstrates that although SH could be used at the level of SH, this is not necessary. Co-associativity together with co-commutativity for string world sheets allows the mapping of the real space-time surfaces in M^8 to H implying exact solvability of the classical TGD.

11.2.4 Questions related to partonic 2-surfaces

There are several questions related to partonic 2-surfaces.

Q1: What are the M^8 pre-images of partons and their light-like partonic orbits in H?
It will be found that the octonionic Dirac equation in M^8 implies that octo-spinors are located to 3-D light-like surfaces Y^3_r - actually light-cone boundary and its 3-D analogs at which number theoretic norm squared is real and vanishes - or to the intersections of X^3_r with the 6-D roots of P in which case Dirac equation trivializes and massive states are allowed. They are mapped to H by $M^8 - H$ duality.
Remark: One can ask whether the same is true in H in the sense that modified Dirac action would be localized to 3-D light-like orbits and 3-D ends of the space-time surfaces at the light-like boundaries of CD having space-like induced metric. Modified Dirac action would be defined by Cherm-Simons term and would force the classical field equations for the bosonic Chern-Simons term. If the interior part of the modified Dirac action is absent, the bosonic action is needed to define the space-time surfaces as extremals. They would be minimal surfaces and universal by their holomorphy and would not depend on coupling parameters so that very general actions can have them as preferred extremals. This issue remains still open.
The naïve - and as it turned out, wrong - guess was that the images of the light-like surfaces should be light-like surfaces in H at the boundaries of Minkowskian and Euclidian regions (wormhole contacts). In the light-like case Y^3_r corresponds to the light-cone boundary so that this would be the case. X^3_r however turns out to correspond to a hyperboloid in M^4 as an analog of a mass shell and is not identifiable as a partonic orbit.
It turned out that the complex surface X^4_c allows real sections in the sense that the number theoretic complex valued metric defined as a complex continuation of Minkowski norm is real at 4-D surfaces: call them Z^4_r. They are bounded by a 3-D region at Z^3_r at which the value of norm squared vanishes. This surface is an excellent candidate for the pre-image of the light-like orbit of partonic 2-surface serving as a topological vertex. One has therefore strings worlds sheets, partonic 2-surfaces and their light-like orbits and they would connect the "mass shells" at X^4_r. All ingredients for SH would be present.
The intersections of Z^3_r with X^3_r identifiable as the section of X^4_r $a = constant$ hyperboloid would give rise to partonic 2-surfaces appearing as topological reaction vertices.
The assumption that the 4-D tangent space at these light-like 3-surfaces is co-associative, would give an additional condition determining the image of this surface in H, so that the boundary conditions for SH would become stronger. One would have boundary conditions at light-like partonic orbits. Note that string world sheets are assumed to have light-like boundaries at partonic orbits.

Q2: Why should partonic 2-surfaces appear as throats of wormhole contact in H? Wormhole contacts do not appear in M^8.

1. In M^8 light-like orbits are places where the Minkowskian signature changes to Euclidian. Does M^8-H duality map the images of these coinciding roots for Euclidian and Minkowskian branches to different throats of the wormhole contact in H so that the intersection would disappear?
2. This is indeed the case. The intersection of Euclidian and Minkowskian branches defines a single 3-surface but the tangent and normal spaces of branches are different. Therefore

their H images under $M^8 - H$ duality for the partonic 2-surface are different since normal spaces correspond to different CP_2 coordinates. These images would correspond to the two throats of wormhole contact so that the H-image by SH is 2-sheeted. One would have wormhole contacts in H whereas in M^8 the wormhole contact would reduce to a single partonic 2-surface.

3. The wormhole contact in H can have only Euclidian signature of the induced metric. The reason is that the M^4 projections of the partonic surfaces in H are identical so that the points with same M^4 coordinates have different CP_2 coordinates and their distance is space-like.

Q3: In H picture the interpretation of space-time surfaces as analogs of Feynman graphs assumes that several partonic orbits intersect at partonic 2-surfaces. This assumption could be of course wrong.This raises questions.

What the pre-images of partonic 2-surfaces are in M^8? Why should several partonic orbits meet at a given partonic 2-surface? Is this needed at all?

The space-time surface X^4_r associated intersects the surface X^6_r associated with different particle - say with different value of mass along 2-D surface. Could this surface be identified as partonic 2-surface X^2_r? This occurs symmetrically so that one has a pair of 2-surfaces X^2_r. What does this mean? Could these surface map to the throats of wormhole contact in H?

Why several partonic surfaces would co-incide in topological reaction vertex at the level of H? At this moment is is not clear whether this is forced by M^8 picture.

Octonionic Dirac equation implies that M^8 has interpretation as analog of momentum space so that interaction vertices are replaced by multilocal vertices representing momenta and propagators become local being in this sense analogous to vertices of QFT. One could of course argue that without the gluing along ends there would be no interactions since the interactions in X^6_r for two 3-surfaces consist in the generic case of a discrete set of points. One could also ask whether the surfaces Y^3_r associated with the space-time surfaces X^4_r associated with incoming particles must intersect along partonic 2-surface rather than at discrete set of points.

The meeting along ends need not be true at the level of M^8 since the momentum space interpretation would imply that momenta do not differ much so that particles should have identical masses: for this to make sense one should assume that the exchanged virtual particles are massless. One other hand, if momenta are light-like for Y^3_r, this might be the case.

Q4: Why two wormhole contacts and monopole flux tubes connecting them at the level of H? Why monopole flux?

1. The tangent spaces of the light-like orbits have different light-like direction. Intuitively, this corresponds to different directions of light-like momenta. Momentum conservation requires more than one partonic orbit changing its direction meeting at partonic 2-surface. By light-likeness, the minimum is 2 incoming and two outgoing lines giving a 4-vertex. This allows the basic vertices involving Ψ and $\overline{Psi}$ at opposite throats of wormhole contacts. Also a higher number of partonic orbits is possible.
2. A two-sheeted closed monopole flux tube having wormhole contacts as its "ends" is suggested by elementary particle phenomenology. Since M^8 homology is trivial, there is no monopole field in M^8. If M^8-H duality is continuous it maps homologically trivial partonic 2-surfaces to homologically trivial 2-surfaces in H. This allows the wormhole throats in H to have opposite homology charges. Since the throats cannot correspond to boundaries there must be second wormhole contact and closed flux tube.
3. What does the monopole flux for a partonic 2-surface mean at the level of M^8? The distribution of quaternionic 4-D tangent/normal planes containing preferred M^2 and associated with partonic 2-surface in M^8 would define a homologically on-trivial 2-surface in CP_2. The situation is analogous to a distribution of tangent planes or equivalently normal vectors in S^2.

Q4: What is the precise form of $M^8 - H$ duality: does it apply only to partonic 2-surfaces and string world sheets or to the entire space-time surfaces?

$M^8 - H$ duality is possible if the X^4 in M^8 contains also integrable distribution of complex tangent or normal 2-planes at which 4-D tangent space is quaternionic/associative. String world sheets and partonic 2-surfaces define these distributions.

The minimum condition allowed by SH in H is that string world sheets and there is a finite number of partonic 2-surfaces and string world sheets. In this case only these 2-surfaces can be mapped to H and SH assigns to them a 4-D space-time surface. The original hypothesis was that these surfaces define global orthogonal slicings of the X^4 so that $M^8 - H$ duality could be applied to the entire X^4. This condition is probably too strong.

11.3 Challenging $M^8 - H$ duality

$M^8 - H$ duality involves several alternative options and in the following arguments possibly leading to a unique choice are discuses.

1. Are both associativity and co-associativity possible or is only either of these options allowed? Is it also possible to pose the condition guaranteeing the existence of 2-D complex sub-manifolds identifiable as string world sheets necessary to map the entire space-time surface from M^8 to H? In other words, is the strong form of holography (SH) needed in M^8 and/or H or is it needed at all?
2. The assignment of the space-time surface at the level of M^8 to the roots of real or imaginary part (in quaternionic sense) of octonionic polynomial P defined as an algebraic continuation of real polynomial is an extremely powerful hypothesis in adelic physics [L52, L53] and would mean a revolution in biology and consciousness theory.
   Does P fix the space-time surface with the properties needed to realize $M^8 - H$ duality or is something more needed? Does the polynomial fix the space-time surface uniquely - one would have extremely strong number theoretic holography - so that one would have number theoretic holography with coefficients of a real polynomial determining the space-time surface?
3. $M^8 - H$ duality involves mapping of $M^4 \subset M^8$ to $M^4 \subset H$. Hitherto it has been assumed that this map is direct identification. The form of map should however depend on the interpretation of M^8. In octonionic Dirac equation M^8 coordinates are in the role of momenta [L105]. This suggests the interpretation of M^8 as an analog of 8-D momentum space. If this interpretation is correct, Uncertainty Principles demands that the map $M^4 \subset M^8 \rightarrow M^4 \subset H$ is analogous to inversion mapping large momenta to small distances.
4. Twistor lift of TGD [K100] is an essential part of the TGD picture. Ttwistors and momentum twistors provide dual approaches to twistor Grassmann amplitudes. Octonionic Dirac equation suggests that M^8 and H are in a similar dual relation. Could $M^8 - H$ duality allow a generalization of twistorial duality to TGD framework?

11.3.1 Explicit form of the octonionic polynomial

What does the identification of the octonionic polynomial P as an octonionic continuation of a polynomial with real or complexified coefficients imply? In the following I regard M^8_c as O^8_c and consider products for complexified octonions.

Remark: In adelic vision the coefficients of P must be rationals (or at most algebraic numbers in some extension of rationals).

One interesting situation corresponds to the real subspace of O_c spanned by $\{I_0, iI_k\}, = 1, ..7$, with a number theoretic metric signature $(1, -1, -1..., -1)$ of M^8 which is complex valued except at in various reals subspaces.This subspace is associative. The original proposal was that Minkowskian space-time regions as projections to this signature are associative whereas Euclidian regions are co-associative. It however turned out that associative space-time surfaces are physically uninteresting.

The canonical choice $(iI_0, I_1, I_2, iI_3, I_4, iI_5, I_6, iI_7)$ defining the complexification of the tangent space represents a co-associative sub-space realizing Minkowski signature. It turns out that both Minkowskian and Euclidian space-time regions must be co-associative .

Surprises

The explicit calculation of the octonionic polynomial yielded a chilling result. If one poses (co-)associativity conditions as vanishing of the imaginary or real part in quaterionic sense: $Im_Q(P) = 0$ or $Re_Q(P) = 0$, the outcome is that the space-time surface is just M^4 or E^4. Second chilling result is that quaternionic sub-manifolds are geodesic sub-manifolds. This led to the question how to modify the (co-)associativity hypothesis.

The vision has been that space-time surfaces can be identified as roots for the imaginary (co-associative) part $Im_Q(O)$ or real part $Im_Q(O)$ of octonionic polynomial using the standard decomposition $(1, e_1, e_2, e_3)$.

1. The naïve counting of dimensions suggests that one obtains 4-D surfaces. The surprise was that also 6-D brane like entities located at the boundary of M^8 light-cone and with topology of 6-sphere S^6 are possible. They correspond to the roots of a real polynomial $P(o)$ for the choice $(1, iI_1, ..., iI_7)$. The roots correspond to the values of the real octonion coordinate interpreted as values of linear M^4 time in the proposal considered. Also for the canonical proposal one obtains a similar result. In O_c they correspond to 12-D complex surfaces X_c^6 satisfying the same condition conditions $x_0^2 + r^2 = 0$ and $P(x_0) = 0$.
2. There was also another surprise. As already described, the general form for the octonionic polynomial $P(o)$ induced from a real polynomial is extremely simple and reduces to $X(t^2, r^2)I_0 + iY(t^2, r^2)Im(o)$. There are only two complex variables t and r^2 involved and the solutions of $P = 0$ are 12-D complex surfaces X_c^6 in O_c. Also the special solutions have the same dimension.
3. In the case of co-associativity 8 conditions are needed for $Re_Q(P) = 0$: note that $X = 0$ is required. The naive expectation is that this gives a complex manifold X_c^4 with 4-D real projection X_r^4 as an excellent candidate for a co-associative surface.
   The expectation turned out to be wrong: in absence of any additional conditions the solutions are complex 7-dimensional mass shells! This is due to the symmetries of the octonionic polynomials as algebraic continuation of a real polynomial.
4. The solution of the problem is to change the interpretation completely. One must assign to the 7-D complex mass shell H_c^7 a 3-D complex mass shell H_c^3.
   One can do this by assuming space-time surface is surface intersecting the7-D mass shell obtained as a deformation of $M_c^4 \subset M_c^8$ by acting with local $SU(3)$ gauge transformation and requiring that the image point is invariant under $U(2)$. If the 4-D complex mass squared remains invariant in this transformation, X_c^4 intersects H_c^7.
   With these assumptions, a local CP_2 element defines X_c^4 and X_r^4 is obtained as its real projection in M^4. This definition assigns to each point of M^4 a point of CP_2 so that $M^8 - H$ duality is well-defined.
   One obtains holography in which the fixing of 3-D mass shells fixes the 4-surface and also assigns causal diamond with the pair of mass shells with opposite energies. If the space-time surface is analog of Bohr orbit, also its preimage under $M^8 - H$ duality should be such and P would determine 4-surface highly uniquely [L123] and one would have number theoretic holography.

General form of P and of the solutions to $P = 0$, $Re_Q(P) = 0$, and $Im_Q(P) = 0$

It is convenient to introduce complex coordinates for O_c since the formulas obtained allow projections to various real sections of O_c.

1. To see what happens, one can calculate o_c^2. Denote o_c by $o_c = tI_0 + \overline{o_c}$ and the norm squared of $\overline{o}$ by r^2, where $r^2 = \sum o_k^2$ where o_k are the complex coordinates of octonion. Number theoretic norm squared for o_c is $t^2 + r^2$ and reduces to a real number in the real sections of O_c. For instance, in the section $(I_1, iI3, iI_5, iI_7)$ the norm squared is $-x_1^2 + x_3^2 + x_5^2 + x_7^2$ and defines Minkowskian norm squared.
   For o^2 one has:
   $$o^2 = t^2 - r^2 + 2t\overline{o} \equiv X_2 + \overline{Y}_2 \ .$$
   For o^3 one obtains
   $$o^3 = tX_2 - \overline{o} \cdot \overline{Y}_2 + t\overline{Y}_2 + X_2\overline{o} \ .$$
   Clearly, $Im_Q(o^n)$ has always the same direction as $Im_Q(o)$. Hence one can write in the general case
   $$o^n = X + Y\overline{o} \ . \quad (11.3.1)$$

This trivial result was obtained years ago but its full implications became evident only while preparing the current article. The point is that the solutions to associativity/co-associativity conditions by putting $Re(Q(P) = 0$ or $Im_Q(P) = 0$ are trivial: just M^4 or E^4. What goes wrong with basic assumptions, will be discussed later.
Remark: In M^8 sub-space one has imaginary $\overline{o}$ is proportional to the commuting imaginary unit.

2. It is easy to deduce a recursion formula for the coefficients for X and Y for n:th power of o_c. Denote by t the coordinate associated with the real octonion unit (not time coordinate). One obtains

$$\begin{array}{l} o_c^n = X_n I_0 + Y_n \overline{o} \ , \\ X_n = tX_{n-1} - rY_{n-1} \ , \\ Y_n = tY_{n-1} + rX_{n-1} \ . \end{array} \tag{11.3.2}$$

In the co-associative case one has $t = 0$ or possibly constant $t = T$ (note that in the recent interpretation t does not have interpretation as time coordinate). The reason is that the choice of octonionic coordinates is unique apart from translation along the real axis from the condition that the coefficients of P remain complex numbers in powers of the new variable.

3. The simplest option correspond to $t = 0$. One can criticize this option since the quaternionicity of normal space should not be affected if t is constant different from zero. In any case, for $t = 0$ the recursion formula gives for the polynomial $P(o_c)$ the expression

$$P(o_c) = \sum (-1)^n r^{2n} (p_{2n-1} I_0 + p_{2n} \overline{o}) \ . \tag{11.3.3}$$

Denoting the even and of odd parts of P by P_{even} and P_{odd}, the roots $r_{k,odd}$ of $X = Re(P(o_c))$ are roots P_{odd} and roots $r_{k,even}$ of $Y = Im(P(o_c))$ are roots of P_{even}. Co-associativity gives roots of X and the roots of P as simultaneous roots of P_{odd} and P_{even}. The interpretation of roots is as in general complex mass squared values.
In the general case, the recursion relation would give the solution

$$\begin{array}{c} \left(\begin{array}{c} X_n \\ Y_n \end{array} \right) = A^n \left(\begin{array}{c} t \\ r \end{array} \right) \\ A = \left(\begin{array}{cc} t & -r \\ r & t \end{array} \right) \end{array} \tag{11.3.4}$$

One can diagonalize the matrix appearing in the iteration by solving the eigenvalues $\lambda_\pm = t \pm ir$ and eigenvectors $X_\pm = (\pm i, 1)$ and by expressing $(X_1, Y_1) = (t, r)$ in terms of the eigenvectors as $(t, r) = ((it + r)X_+ + (r - it)X_-)/2$. This gives

$$\left(\begin{array}{c} X_n \\ Y_n \end{array} \right) = \frac{1}{2} \left(\begin{array}{c} (t+ir)^n i - (t-ir)^n i \\ (t+ir)^n + (t-ir)^n \end{array} \right) \tag{11.3.5}$$

This gives

$$\begin{array}{l} P(o_c) = P_1 I_0 + P_2 \overline{o} \ , \\ P_1(r) = \sum X_n p_n r^{2n} \ , \\ P_2(r) = \sum Y_n p_n r^{2n} \ . \end{array} \tag{11.3.6}$$

For the restriction to M_c^4, r^2 reduces to complex 4-D mass squared given by the root r_n. In general case r^2 corresponds to complex 8-D mass squared. All possible signatures are obtained by assuming M_c^8 coordinates to be either real or imaginary (the number theoretical norm squared is real with this restriction).

How does one obtain 4-D space-time surfaces?

Contrary to the naive expections, the solutions of the vanishing conditions for the $Re_Q(P)$ ($Im_Q(P)$) (real (imaginary) part in quaternionic sense) are 7-D complex mass shells $r^2 = r_{n,1}$ as roots of $P_1(r) = 0$ or $r^2 = r_{n,2}$ of $P_2(r) = 0$ rather than 4-D complex surfaces (for a detailed discussion see [K23]) A solution of both conditions requires that P_1 and P_2 have a common root but the solution remains a 7-D complex mass shell! This was one of the many cold showers during the development of the ideas about $M^8 - H$ duality! It seems that the adopted interpretation is somehow badly wrong. Here zero energy ontology (ZEO) and holography come to the rescue.

1. Could the roots of P_1 or P_2 define only complex mass shells of the 4-D complex momentum space identifiable as M_c^4? ZEO inspires the question whether a proper interpretation of mass shells could be as pre-images of boundaries of cd:s (intersections of future and past directed light-cones) as pairs of mass shells with opposite energies. If this is the case, the challenge would be to understand how X_c^4 is determined if P does not determine it.
   Here holography, considered already earlier, suggests itself: the complex 3-D mass shells belonging to X_c^4 would only define the 3-D boundary conditions for holography and the real mass shells would be mapped to the boundaries of cds. This holography can be restricted to X_R^4. Bohr orbit property at the level of H suggests that the polynomial P defines the 4-surface more or less uniquely.
2. Let us take the holographic interpretation as a starting point. In order to obtain an X_c^4 mass shell from a complex 7-D light-cone, 4 complex degrees of freedom must be eliminated. $M^8 - H$ duality requires that X_c^4 allows M_c^4 coordinates.
   Note that if one has $X_c^4 = M_c^4$, the solution is trivial since the normal space is the same for all points and the H image under $M^8 - H$ duality has constant $CP_2 = SU(3)/U(2)$ coordinates. X_c^4 should have interpretation as a non-trivial deformation of M_c^4 in M^8.
3. By $M^8 - H$ duality, the normal spaces should be labelled by $CP_2 = SU(3)/U(2)$ coordinates. $M^8 - H$ duality suggests that the image $g(p)$ of a momentum $p \in M_c^4$ is determined essentially by a point $s(p)$ of the coset space $SU(3)/U(2)$. This is achieved if M_c^4 is deformed by a local $SU(3)$ transformation $p \to g(p)$ in such a way that each image point is invariant under $U(2)$ and the mass value remains the same: $g(p)^2 = p^2$ so that the point represents a root of P_1 or P_2.
   Remark: I have earlier considered the possibility of G_2 and even $G_{2,c}$ local gauge transformation. It however seems that that local $SU(3)$ transformation is the only possibility since G_2 and $G_{2,c}$ would not respect $M^8 - H$ duality. One can also argue that only real $SU(3)$ maps the real and imaginary parts of the normal space in the same manner: this is indeed an essential element of $M^8 - H$ duality.
4. This option defines automatically $M^8 - H$ duality and also defines causal diamonds as images of mass shells $m^2 = r_n$. The real mass shells in H correspond to the real parts of r_n. The local $SU(3)$ transformation g would have interpretation as an analog of a color gauge field. Since the H image depends on g, it does not correspond physically to a local gauge transformation but is more akin to an element of Kac-Moody algebra or Yangian algebra which is in well-defined half-algebra of Kac-Moody with non-negative conformal weights.

The following summarizes the still somewhat puzzling situation as it is now.

1. The most elegant interpretation achieved hitherto is that the polynomial P defines only the mass shells so that mass quantization would reduce to number theory. Amusingly, I started to think about particle physics with a short lived idea that the d'Alembert equation for a scalar field could somehow give the mass spectrum of elementary particles so that the issue comes full circle!
2. Holography assigns to the complex mass shells complex 4-surfaces for which $M^8 - H$ duality is well-defined even if these surfaces would fail to be 4-D co-associative. These surfaces are expected to be highly non-unique unless holography makes them unique. The Bohr orbit property of their images in H indeed suggests this apart from a finite non-determinism [L123]. Bohr orbit property could therefore mean extremely powerful number theoretical duality for which the roots of the polynomial determine the space-time surface almost uniquely. $SU(3)$ as color symmetry emerges at the level of M^8. By $M^8 - H$ duality, the mass shells are mapped to the boundaries of CDs in H.
3. Do we really know that X_r^4 co-associative and has distribution of 2-D commuting subspaces of normal space making possible $M^8 - H$ duality? The intuitive expectation is that the answer is affirmative [A34]. In any case, $M^8 - H$ duality is well-defined even without this condition.

4. The special solutions to $P = 0$, discovered already earlier, are restricted to the boundary of CD_8 and correspond to the values of energy (rather than mass or mass squared) coming as roots of the real polynomial P. These mass values are mapped by inversion to "very special moments in the life of self" (a misleading term) at the level of H as special values of light-cone proper time rather than linear Minkowski time as in the earlier interpretation [L73]. The new picture is Lorenz invariant.

Octonionic Dirac equation as analog of momentum space variant of ordinary Dirac equation forces the interpretation of M^8 as an analog of momentum space and Uncertainty Principle forces to modify the map $M^4 \subset M^8 \to M^4 \subset H$ from identification to inversion. The equations for $Re_Q(P) = 0$ reduce to simultaneous roots of the real polynomials defined by the odd and even parts of P having interpretation as complex values of mass squared mapped to light-cone proper time constant surfaces in H. This leads to the idea that the formulation of scattering amplitudes at M^8 levels provides the counterpart of momentum space description of scattering whereas the formulation at the level of H provides the counterpart of space-time description.

This picture combined with zero energy ontology (ZEO) leads also to a view about quantum TGD at the level of M^8. Local $SU(3)$ element has properties suggesting a Yangian symmetry assignable to string world sheets and possibly also partonic 2-surfaces. The representation of Yangian algebra using quark oscillator operators would allow to construct zero energy states at representing the scattering amplitudes. The physically allowed momenta would naturally correspond to algebraic integers in the extension of rationals defined by P. The co-associative space-time surfaces (unlike generic ones) allow infinite-cognitive representations making possible the realization of momentum conservation and on-mass-shell conditions.

11.3.2 The input from octonionic Dirac equation

The octonionic Dirac equation allows a second perspective on associativity. Everything is algebraic at the level of M^8 and therefore also the octonionic Dirac equation should be algebraic. The octonionic Dirac equation is an analog of the momentum space variant of ordinary Dirac equation and also this forces the interpretation of M^8 as momentum space.

Fermions are massless in the 8-D sense and massive in 4-D sense. This suggests that octonionic Dirac equation reduces to a mass shell condition for massive particle with $q \cdot q = m^2 = r_n$, where $q \cdot q$ is octonionic norm squared for quaternion q defined by the expression of momentum p as $p = I_4 q$, where I_4 is octonion unit orthogonal to q. r_n represents mass shell as a root of P.

For the co-associative option the co-associative octonion p representing the momentum is given in terms of quaternion q as $p = I_4 q$. One obtains $p \cdot p = q\overline{q} = m^2 = r_n$ at the mass shell defined as a root of P. Note that for M^4 subspace the space-like components of $p\,p$ are proportional to i and the time-like component is real. All signatures of the number theoretic metric are possible.

For associative option one would obtain $qq = m^2$, which cannot be satisfied: q reduces to a complex number $zx + Iy$ and one has analog of equation $z^2 = z^2 - y^2 + 2Ixy = m_n^2$, which cannot be true. Hence co-associativity is forced by the octonionic Dirac equation.

One of the big surprises was that the cognitive representations for both light-like boundary and X^4_r are not generic meaning that they would consist of a finite set of points but infinite due to the Lorentz symmetry: a kind of cognitive explosion would happen by the Lorentz symmetry. The natural assumption is that for a suitable momentum unit, physical momenta satisfying mass shell conditions are algebraic integers in the extension of rationals defined by P. Periodic boundary conditions in turn suggest that for the physical states the total momenta are ordinary integers and this leads to Galois confinement as a universal mechanism for the formation of bound states.

Hamilton-Jacobi structure and Kähler structure of $M^4 \subset H$ and their counterparts in $M^4 \subset M^8$

The Kähler structure of $M^4 \subset H$, forced by the twistor lift of TGD, has deep physical implications and seems to be necessary. It implies that for Dirac equation in H, modes are eigenstates of only the longitudinal momentum and in the 2 transversal degrees of freedom one has essentially harmonic oscillator states [L121, L119], that is Gaussians determined by the 2 longitudinal momentum components. For real longitudinal momentum the exponents of Gaussians are purely imaginary or purely real.

The longitudinal momentum space $M^2 \subset M^4$ and its orthogonal complement E^2 is in a preferred role in gauge theories, string models, and TGD. The localization of this decomposition leads to the notion of Hamilton-Jacobi (HJ) structure of M^4 and the natural question is how this relates to Kähler structures of M^4. At the level of H spinors fields only the Kähler structure corresponding to constant decomposition $M^2 \oplus E^2$ seems to make sense and this raises the question how the H-J structure and Kähler structure relate. TGD suggests the existence of two geometric structure in M^4: HJ structure and Kähler structure. It has remained unclear whether HJ structure and Kähler structure with covariantly constant self-dual Kähler form are equivalent notions or whether there several H-J structures accompaning the Kähler structure.

In the following I argue that H-J structures correspond to different choices of symplectic coordinates for M^4 and that the properties of $X^4 \subset H$ determined bt $M^- H$ duality make it natural to to choose particular symplectic coordinates for M^4.

Consider first what H-J structure and Kähler structure could mean in H.

1. The H-J structure of $M^4 \subset H$ would correspond to an integrable distribution of 2-D Minkowskian sub-spaces of M^4 defining a distribution of string world sheets $X^2(x)$ and orthogonal distribution of partonic 2-surfaces $Y^2(x)$. Could this decomposition correspond to self-dual covariantly Kähler form in M^4?
   What do we mean with covariant constancy now? Does it mean a separate covariant constancy for the choices of $M^2(x)$ and $Y^2(x)$ or only of their sum, which in Minkowski coordinates could correspond to a constant electric and magnetic fields orthogonal to each other?
2. The non-constant choice of $M^2(x)$ ($E^2(x)$) cannot be covariantly constant. One can write $J(M^4) = J(M^2(x)) \oplus J(E^2(x)$ corresponding to decomposition to electric and magnetic parts. Constancy of $J(M^2(x)$ would require that the gradient of $J(M^2(x)$ is compensated by the gradient of an antisymmetric tensor with square equal to the projector to $M^2(x)$. Same condition holds true for $J(E^2(x))$. The gradient of the antisymmetric tensor would be parallel to itself implying that the tensor is constant.
3. H-J structure can only correspond to a transformation acting on J but leaving $J_{kl}dm^kdm^l$ invariant. One should find analogs of local gauge transformations leaving J invariant. In the case of CP_2, these correspond to symplectic transformations and now one has a generalization of the notion. The M^4 analog of the symplectic group would parameterize various decompositions of $J(M^4)$.
   Physically the symplectic transformations define local choices of 2-D space $E^2(x)$ of transversal polarization directions and longitudinal momentum space M^2 emerging in the construction of extremals of Kähler action.
4. For the simplest Kähler form for $M^4 \subset H$, this decomposition in Minkowski coordinates would be constant: orthogonal constant electric and magnetic fields. This Kähler form extends to its number theoretical analog in M^8. The local $SU(3)$ element g would deform M^4 to $g(M^4)$ and define an element of local CP_2 defining $M^8 - H$ duality. g should correspond to a symplectic transformation of M^4.

Consider next the number theoretic counterparts of H-J- and Kähler structures of $M^4 \subset H$ in $M^4 \subset M^8$.

1. In M^4 coordinates H-J structure would correspond to a constant $M^2 \times E^2$ decomposition. In M^4 coordinates Kähler structure would correspond to constant E and B orthogonal to each other. Symplectic transformations give various representations of this structure as H-J structures.
2. The number theoretic analog of H-J structure makes sense also for $X^4 \subset M^8$ as obtained from the distribution of quaternionic normal spaces containing 2-D commutative sub-space at each point by multiplying then by local unit $I_4(x)$ orthogonal to the quaternionic units $\{1, I_1 = I_2 = I_3\}$ with respect to octonionic inner product. There is a hierarchy of CDs and the choices of these structures would be naturally parameterized by G_2.
   This would give rise to a number theoretically defined slicing of $X^4_c \subset M^8_c$ by complexified string world sheets X^2_c and partonic 2-surfaces Y^2_c orthogonal with respect to the octonionic inner product for complexified octonions.
3. In $M^8 - H$ duality defined by $g(p) \subset SU(3)$ assigns a point of CP_2 to a given point of M^4. $g(p)$ maps the number theoretic H-J to H-J in $M^4 \subset M^8$. The space-time surface itself - that

is $g(p)$ - defines these symplectic coordinates and the local $SU(3)$ element g would naturally define this symplectic transformation.

4. For $X^4 \subset M^8$ g reduces to a constant color rotation satisfying the condition that the image point is $U(2)$ invariant. Unit element is the most natural option. This would mean that g is constant at the mass and energy shells corresponding to the roots of P and the mass shell is a mass shell of M^4 rather than some deformed mass shell associated with images under $g(p)$. This alone does not yet guarantee that the 4-D tangent space corresponds to M^4. The additional physically very natural condition on g is that the 4-D momentum space at these mass shells is the same. $M^8 - H$ duality maps these mass shells to the boundaries of these cd:s in M^4 (CD= $cd \times CP_2$). This conforms with the identification of zero energy states as pairs of 3-D states at the boundaries of CD.

This generalizes the original intuitive but wrong interpretation of the roots r_n of P as "very special moments in the life of self" [L73].

1. Since the roots correspond to mass squared values, they are mapped to the boundaries of cd with size $L = \hbar_{eff}/m$ by $M^8 - H$ duality in M^4 degrees of freedom. During the sequence of SSFRs the passive boundary of CD remains does not shift only changes in size, and states at it remain unaffected. Active boundary is shifted due to scaling of cd.
   The hyperplane at which upper and lower half-cones of CD meet, is shifted to the direction of geometric future. This defines a geometric correlate for the flow of experienced time.
2. A natural proposal is that the moments for SSFRs have as geometric correlates the roots of P defined as intersections of geodesic lines with the direction of 4-momentum p from the tip of CD to its opposite boundary (here one can also consider the possibility that the geodesic lines start from the center of cd). Also energy shells as roots $E = r_n$ of P are predicted. They decompose to a set of mass shells $m_{n.,k}$ with the same $E = r_n$: similar interpretation applies to them.
3. What makes these moments very special is that the mass and energy shells correspond to surfaces in M^4 defining the Lorentz quantum numbers. SSFRs correspond to quantum measurements in this basis and are not possible without this condition. At $X^4 \subset M^8$ the mass squared would remain constant but the local momentum frame would vary. This is analogous to the conservation of momentum squared in general relativistic kinematics of point particle involving however the loss of momentum conservation.
4. These conditions, together with the assumption that g is a rational function with real coefficients, strongly suggest what I have referred to as preferred extremal property, Bohr orbitology, strong form of holography, and number theoretical holography.

In principle, by a suitable choice of M^4 one can make the momentum of the system light-like: the light-like 8-momentum would be parallel to M^4. I have asked whether this could be behind the fact that elementary particles are in a good approximation massless and whether the small mass of elementary particles is due to the presence of states with different mass squares in the zero state allowed by Lorentz invariance.

The recent understanding of the nature of right-handed neutrinos based on M^4 Kähler structure [L119] makes this mechanism un-necessary but poses the question about the mechanism choosing some particular M^4. The conditions that $g(p)$ leaves mass shells and their 4-D tangent spaces invariant provides this kind of mechanism. Holography would be forced by the condition that the 4-D tangent space is same for all mass shels representing inverse images for very special moments of time.

What about string world sheets and partonic 2-surfaces?

One can apply the above arguments also to the identification of 2-D string world sheets and partonic 2-surfaces.

1. One has two kinds of solutions: M^2 and 3-D surfaces of X^4 as analogs of 6-brane. The interpretation for 3-D *resp.* 2-D branes as a light-like 3-surface associated with the octonionic Dirac equation representing mass shell condition *resp.* string world sheet is attractive.
2. M^2 would be replaced with an integrable distribution of $M^2(x)$ in local tangent space $M^4(x)$. The space for the choices of $M^2(x)$ would be S^3 corresponding to the selection of a preferred quaternion imaginary unit equal to the choices of preferred octonion imaginary unit.

The choices of the preferred complex subspace $M^2(x)$ at a given point would be characterized by its normal vector and parameterized by sphere S^2: the interpretation as a quantization axis of angular momentum is suggestive. One would have space $S^3 \times S^2$. Also now the integrability conditions $de_A = 0$ would hold true.

3. String world sheets could be regarded as analogs of superstrings connecting 3-D brane like entities defined by the light-like partonic orbits. The partonic 2-surfaces at the ends of light-like orbits defining also vertices could correspond to the 3-surfaces at which quaternionic 4-surfaces intersect 6-branes.

11.3.3 Is (co-)associativity possible?

The number theoretic vision relying on the assumption that space-time surfaces are 8-D complex 4-surfaces in o_c^8 determined as algebraic surfaces for octonionic continuations of real polynomials, which for adelic physics would have coefficients which are rational or belong to an extension of rationals. The projections to subspaces Re^8 of o_c^8 defined as space for which given coordinate is purely real or imaginary so that complexified octonionic norm is real would give rise to real 4-D space-time surfaces. $M^8 - H$ duality would map these surfaces to geometric objects in $M^4 \times CP_2$. This vision involves several poorly understood aspects and it is good to start by analyzing them.

Challenging the notions of associativity and co-associativity

Consider first the notions of associativity *resp.* co-associativity equivalent with quaternionicity *resp.* co-quaternionicity. The original hope was that both options are possible for surfaces of real sub-spaces of O_c ("real" means here that complexified octonionic metric is real).

1. The original idea was that the associativity of the tangent space or normal space of a real space-time surface X^4 reduces the classical physics at the level of M^8 to associativity. Associativity/co-associativity of the space-time surface states that at each point of the tangent-/normal space of the real space-time surface in O is quaternionic. The notion generalizes also to $X_c^4 \subset O_c^8$. (Co-)associativity makes sense also for the real subspaces space of O with Minkowskian signature.
2. It has been however unclear whether (co-)associativity is possible. The cold shower came as I learned that associativity allows only for geodesic sub-manifolds of quaternionic spaces about which octonions provide an example [A34]. The good news was that the distribution of co-associative tangent spaces always defines an integrable distribution in the sense that one can find sub-manifold for which the associative normal space at a given point has tangent space as an orthogonal complement. Should the number theoretic dynamics rely on co-associativity rather than associativity?
3. Minkowskian space-time regions have been assumed to be associative and to correspond to the projection to the standard choice for basis as $\{1, iI_1, iI_2, iI_3\}$. The octonionic units $\{1, I_1, I_2, I_3\}$ define quaternionic units and associative subspace and their products with unit I_4 define the orthogonal co-associative subspace as $\{I_4, I_5 = I4I_1, I_6 = I_4I_2, I_7 = I_4I_3\}$. This result forces either to weaken the notion of associativity or to consider alternative identifications of Minkowskian regions, which can be co-associative: fortunately, there exists a large number of candidates.

The article [A34] indeed kills the idea about the associativity of the space-time surface. The article starts from a rather disappointing observation that associative sub-manifolds are geodesic sub-manifolds and therefore trivial. Co-associative quaternion sub-manifolds are however possible. With a motivation coming from this observation, the article discusses what the author calls RC quaternionic sub-manifolds of quaternion manifolds. For a quaternion manifold the tangent space allows a realization of quaternionic units as antisymmetric tensors. These manifolds are constant curvature spaces and typically homogeneous spaces.

1. Quaternion sub-manifold allows a 4-D integrable distribution of quaternion units. The normal complement of this distribution is expressible in terms of the second fundamental form and the condition that it is trivial implies that the second fundamental form is vanishing so that one has a geodesic submanifold. Quaternionic sub-manifolds are thus too trivial to be interesting. As a diametric opposite, one can also define totally real submanifolds for which the normal space

contains a distribution of quaternion units. In this case the distribution is always integrable. This case is much more interesting from the TGD point of view.

2. Author introduces the notion of CR quaternion sub-manifold $N \subset M$, where M is quaternion manifold with constant sectional curvatures. N has quaternion distribution D in its tangent spaces if the action of quaternion units takes D to itself. $D^\perp$ is the co-quaternionic orthogonal complement D in the normal space N. D would take also $D^\perp$ to itself. $D^\perp$ can be expressed in terms of the components of the second fundamental form and vanishes for quaternion sub-manifolds.
3. Author deduces results about CR quaternion sub-manifolds, which are very interesting from the TGD point of view.
   (a) Sub-manifold is CR quaternion sub-manifold only if the curvature tensor of R_M of the embedding space satisfies $R_M(D, D, D^\perp, D\ \ ) = 0$. The condition is trivial if the quaternion manifold is flat. In the case of octonions this would be the case.
   (b) D is integrable only if the second fundamental form restricted to it vanishes meaning that one has a geodesic manifold. Totally real distribution $D^\perp$ is always integrable to a co-associative surface.
   (c) If $D^\perp$ integrates to a minimal surface then N itself is a minimal surface.

Could one consider RC quaternion sub-manifolds in TGD framework? Both octonions and their complexifixation can be regarded as quaternionic spaces. Consider the real case.

1. If the entire D is quaternionic then N is a geodesic sub-manifold. This would leave only E^4 and its Minkowskian variants with various signatures. One could have however 4-D totally real (co-associative) space-time surfaces. Simple arguments will show that the intersections of the conjectured quaternionic and co-quaternionic 4-surfaces have 2- and 3-D intersections with 6-branes.
   Should one replace associative space-time surfaces with CR sub-manifolds with $d \leq 3$ integrable distribution D whereas the co-quaternionic surfaces would be completely real having 4-D integrable $D^\perp$? Could one have 4-D co-associative surfaces for which $D^\perp$ integrates to $n \geq 1$-dimensional minimal surface (geodesic line) and the X^4 itself is a minimal surface?
   Partially associative CR manifold do not allow M^8H duality. Only co-associative surfaces allow it and also their signature must be Minkowskian: the original idea [L76, L47, L48, L49] about Euclidian (Minkowskian) signature for co-associative (associative) surfaces was wrong.
2. The integrable 2-D sub-distributions D defining a distribution of normal planes could define foliations of the X^4 by 2-D surfaces. What springs to mind is foliations by string world sheets and partonic 2 surfaces orthogonal to them and light-like 3-surfaces and strings transversal to them. This expectation is realized.

How to identify the Minkowskian sub-space of O_c?

There are several identifications of subspaces of O_c with Minkowskian signature. What is the correct choice has been far from obvious. Here symmetries come in rescue.

1. Any subspace of O^c with 3 (1) imaginary coordinates and 1 (3) real coordinates has Minkowskian signature in octonionic norm algebraically continued to O_c (complex valued continuation of real octonion norm instead of real valued Hilbert space norm for O_c). Minkowskian regions should have local tangent space basis consisting of octonion units which in the canonical case would be $\{I_1, iI_3, iI_5, iI_7\}$, where i is commutative imaginary unit. This particular basis is co-associative having whereas its complement $\{iI_0, I_2, I_4, I_6\}$ is associative and has also Minkowskian signature.
2. The size of the isometry group of the subspace of M^8_c depends on whether the tangent basis contains real octonion unit 1 or not. The isometry group for the basis containing I_0 is $SO(3)$ acting as automorphisms of quaternions and $SO(k, 3-k)$ when $3-k$ units are proportional to i. The reason is that G_2 (and its complexification $G_{2,c}$) and its subgroups do not affect I_0. For the tangent spaces built from 4 imaginary units I_k and iI_l the isometry group is $SO(k, 4-k) \subset G_{2,c}$.
   The choice therefore allows larger isometry groups and also co-associativity is possible for a suitable choice of the basis. The choice $\{I_1, iI_3, iI_5, iI_7\}$ is a representative example, which

will be called canonical basis. For these options the isometry group is the desired $SO(1,3)$ as an algebraic continuation of $SO(4) \subset G_2$ acting in $\{I_1, I_3, I_5, I_7\}$, to $SO(1,3) \subset G_{2,c}$.
Also Minkowskian signature - for instance for the original canonical choice $\{I_0, iI_1, iI_2, iI_3\}$ - can have only $SO(k, 3-k)$ as isometries. This is the basic objection against the original choice $\{I_0, iI_1, iI_2, iI_3\}$. This identification would force the realization of $SO(1,3)$ as a subgroup of $SO(1,7)$. Different states of motion for a particle require different octonion structure with different direction of the octonion real axis in M^8. The introduction of the notion of moduli space for octonion structures does not look elegant. For the option $\{I_1, iI_3, iI_5, iI_7\}$ only a single octonion structure is needed and $G_{2,c}$ contains $SO(1,3)$.
Note that also the signatures $(4,0)$, $(0,4)$ and $(2,2)$ are possible and the challenge is to understand why only the signature (1,3) is realized physically.

Co-associative option is definitely the only physical alternative. The original proposal for the interpretation of the Minkowski space in terms of an associative real sub-space of M^4 had a serious problem. Since time axis was identified as octonionic real axis, one had to assign different octonion structure to particles with non-parallel moment: $SO(1,7)$ would relate these structures: how to glue the space-time surfaces with different octonion structures together was the problem. This problem disappears now. One can simply assign to particles with different state of motion real space-time surface defined related to each other by a transformation in $SO(1,3) \subset G_{2,c}$.

The condition that $M^8 - H$ duality makes sense

The condition that $M^8 - H$ duality makes sense poses strong conditions on the choice of the real sub-space of M^8_c.

1. The condition that tangent space of O_c has a complexified basis allowing a decomposition to representations of $SU(3) \subset G_2$ is essential for the map to $M^8 \rightarrow H$ although it is not enough. The standard representation of this kind has basis $\{\pm iI_0 + I_1\}$ behaving like $SU(3)$ singlets $\{I_2 + \epsilon iI_3, I_4 + \epsilon iI_5, \epsilon I_6 \pm iI_7\}$ behaves like SU(3) triplet 3 for $\epsilon = 1$ and its conjugate $\overline{3}$ for $\epsilon = -1$. $G_{2,c}$ provides new choices of the tangent space basis consistent with this choice. $SU(3)$ leaves the direction I_1 unaffected but more general transformations act as Lorentz transformation changing its direction but not leaving the M^4 plane. Even more general $G_{2,c}$ transformations changing M^4 itself are in principle possible.
Interestingly, for the canonical choice the co-associative choice has $SO(1,3)$ as isometry group whereas the complementary choice failing to be associative correspond to a smaller isometry group $SO(3)$. The choice with M^4 signature and co-associativity would provide the highest symmetries. For the real projections with signature $(2,2)$ neither consistent with color structure, neither full associativity nor co-associativity is possible.
2. The second essential prerequisite of $M^8 - H$ duality is that the tangent space is not only (co-)associative but contains also (co-)complex - and thus (co-)commutative - plane. A more general assumption would be that a co-associative space-time surface contains an integrable distribution of planes $M^2(x)$, which could as a special case reduce to M^2.
The proposal has been that this integrable distribution of $M^2(x)$ could correspond to string sheets and possibly also integrable orthogonal distribution of their co-complex orthogonal complements as tangent spaces of partonic 2-surfaces defining a slicings of the space-time surface. It is now clear that this dream cannot be realized since the space-time surface cannot be even associative unless it is just E^4 or its Minkowskian variants.
3. As already noticed, any distribution of the associative normal spaces integrates to a co-associative space-time surface. Could the normal spaces also contain an integrable distribution of co-complex planes defined by octonionic real unit 1 and real unit $I_k(x)$, most naturally I_1 in the canonical example? This would give co-commutative string world sheet. Commutativity would be realized at the 2-D level and associativity at space-time level. The signature of this plane could be Minkowskian or Euclidian. For the canonical example $\{I_1, iI_3, iI_5, iI_7\}$ the 2-D complex plane in quaternionic sense would correspond to $(a \times 1, +n_2I_2 + n_4I_6 + n_6I_6$, where the unit vector n_i has real components and one has $a = 1$ or $a = i$ is forced by the complexification as in the canonical example.
Since the distribution of normal planes integrates to a 4-surface, one expects that its sub-distribution consting of commutative planes integrates to 2-D surface inside space-time surface

and defines the counterpart of string worlds sheet. Also its normal complement could integrate to a counterpart of partonic 2-surface and a slicing of space-time surface by these surfaces would be obtained.

4. The simplest option is that the commutative space does not depend on position at X^4. This means a choice of a fixed octonionic imaginary unit, most naturally I_1 for the canonical option. This would make $SU(3)$ and its sub-group $U(2)$ independent of position. In this case the identification of the point of $CP_2 = SU(3)/U(2)$ labelling the normal space at a given point is unique.
   For a position dependent choice $SU(3)(x)$ it is not clear how to make the specification of $U(2)(x)$ unique: it would seem that one must specify a unique element of $G_2(x)$ relating $SU(3)(x)$ to a choice at special point x_0 and defining the conjugation of both $SU(3)(x)$ and $U(2)(x)$. Otherwise one can have problems. This would also mean a unique choice for the direction of time axis in O and fixing of $SO(1,3)$ as a subgroup of $G_{2,c}$. Also this distribution of associative normal spaces is integrable. Physically this option is attractive but an open question is whether it is consistent with the identification of space-time surfaces as roots $Re_Q(P) = 0$ of P.

Co-associativity from octonion analyticity or/and from G_2 holography?

Candidates for co-associative space-time surfaces X^4_r are defined as restrictions X^4_r for the roots X^4_c of the octonionic polynomials such that the O_c coordinates in the complement of a real co-associative sub-space of O_c vanish or are constant. Could the surfaces X^4_r or even X^4_c be co-associative?

1. X^4_r is analogous to the image of real or imaginary axis under a holomorphic map and defines a curve in complex plane preserving angles. The tangent vectors of X^4_r and X^4_c involve gradients of all coordinates of O_c and are expressible in terms of all octonionic unit vectors. It is not obvious that their products would belong to the normal space of X^4_r a strong condition would be that this is the case for X^4_c.
2. Could octonion analyticity in the proposed sense guarantee this? The products of octonion units also in the tangent space of the image would be orthogonal to the tangent space. Ordinary complex functions preserve angles, in particular, the angle between x- and y-axis is preserved since the images of coordinate curves are orthogonal. Octonion analyticity would preserve the orthogonality between tangent space vectors and their products.
3. This idea could be killed if one could apply the same approach to associative case but this is not possible! The point is that when the real tangent space of O_c contains the real octonion unit, the candidate for the 4-D space-time surface is a complex surface X^2_c. The number theoretic metric is real only for 2-D X^2_r so that one obtains string theory with co-associativity replaced with co-commutativity and $M^4 \times CP_2$ with $M^2 \times S^2$. One could of course ask whether this option could be regarded as a "sub-theory" of the full theory.

My luck was that I did not realize the meaning of the difference between the two cases first and realized that one can imagine an alternative approach.

1. G_2 as an automorphism group of octonions preserves co-associativity. Could the image of a co-associative sub-space of O_c defined by an octonion analytic map be regarded as an image under a local G_2 gauge transformation. $SU(3) \subset G_2$ is an especially interesting subgroup since it could have a physical interpretation as a color gauge group. This would also give a direct connection with $M^8 - H$ duality since $SU(3)$ corresponds to the gauge group of the color gauge field in H.
2. One can counter-argue that an analog of pure gauge field configuration is in question at the level of M^8. But is a pure gauge configuration for $G_{2,c}$ a pure gauge configuration for G_2? The point is that the $G_{2,c}$ connection $g^{-1}\partial_\mu g$ trivial for $G_{2,c}$ contains by non-linearity cross terms from $g_2g, c = g_{2,1} + ig_{2,2}$, which are of type $Re = X[g_{2,1}, g_{2,1}] - X[g_{2,2}, g_{2,2}] = 0$ and $Im = iZ[g_{2,1}, g_{2,2}] = 0$. If one puts $g_{2,2}$ contributions to zero, one obtains $Re = X[g_{2,1}, g_{2,1}]$, which does not vanish so that $SU(3)$ gauge field is non-trivial.
3. X^4_r could be also obtained as a map of the co-associative M^4 plane by a local $G_{2,c}$ element. It will turn out that $G_{2,c}$ could give rise to the speculated Yangian symmetry [L42] at string

world sheets analogous to Kac-Moody symmetry and gauge symmetry and crucial for the construction of scattering amplitudes in $M8$.

4. The decomposition of the co-associative real plane of O_c should contain a preferred complex plane for $M^8 - H$ duality to make sense. $G_{2,c}$ transformation should trivially preserve this property so that SH would not be necessary at H side anymore.

There is a strong motivation to guess that the two options are equivalent so that $G_{2,c}$ holography would be equivalent with octonion analyticity. The original dream was that octonion analyticity would realize both associative and co-associative dynamics but was exaggeration!

Does one obtain partonic 2-surfaces and strings at boundaries of ΔCD_8?

It is interesting to look for the dimensions of the intersections of the light-like branes at the boundary of CD_8 giving rise to the boundary of CD_4 in M^4 to see whether it gives justification for the existing phenomenological picture involving light-like orbits of partonic 2-surfaces connected by string world sheets.

1. Complex light-cone boundary has dimension $D = 14$. $P = 0$ as an additional condition at δCD_8 gives 2 complex conditions and defines a 10-D surface having 5-D real projections.
2. The condition $Im_Q(P) = 0$ gives 8 conditions and gives a 2-D complex surface with 1-D real projection. The condition $Re_Q(P) = 0$ gives 3 complex conditions since $X = 0$ is already satisfied and the solution is a 4-D surface having 2-D real projection. Could the interpretation be in terms of the intersection of the orbit of a light-like partonic surface with the boundary of CD_8?
3. Associativity is however not a working option. If only co-associative Minkowskian surfaces allowing mapping to H without SH are present then only 4-D space-time surfaces with Minkowskian signature, only partonic 2-surfaces and their light-like orbits would emerge from co-associativity. This option would not allow string world sheets for which there is a strong intuitive support. What could a co-complex 2-surface of a co-associative manifold mean? In the co-associative case the products of octonion imaginary units are in the normal space of space-time surface. Could co-complex surface $X_c^2 \subset X_c^4$ be defined by an integrable co-complex sub-distribution of co-associative distribution. The 4-D distribution of normal planes is always integrable.
   Could the 2-D sub-distributions of co-associative distribution integrate trivially and define slicings by string world sheets or partonic 2-surfaces. Could the distribution of string distributions and its orthogonal complement be both integrable and provide orthogonal slicings by string world sheets and partonic 2-surfaces? String world sheets with Minkowskian signature should intersect the partonic orbits with Euclidian signature along light-like lines. This brings in mind the orthogonal grid of flow lines defined by the $Re(f) = 0$ and $Im(f) = 0$ lines of an analytic function in plane.
4. In this picture the partonic 2-surfaces associated with light-like 3-surface would be physically unique and could serve as boundary values for the distributions of partonic 2-surfaces. But what about string world sheets connecting them? Why would some string world sheets be exceptional? String world sheets would have a light-like curve as an intersection with the partonic orbit but this is not enough.
   Could the physically special string world sheets connect two partonic surfaces? Could the string associated with a generic string world sheet be like a flow line in a hydrodynamic flow past an obstacle - the partonic 2-surface? The string as a flowline would go around the obstacle along either side but there would be one line which ends up to the object.

Interactions would correspond geometrically to the intersections of co-associative space-time surfaces X_r^4 associated with particles and corresponding to different real sub-spaces of O_c related by Lorentz boost in $SO(1,3) \subset G_{2,c}$. In the generic case the intersection would be discrete. In the case that X and Y have a common root the real surfaces $X_r^4 \subset X_r^6$ associated with quarks and depending on their state of motion would reside inside the same 6-D surface X_r^6 and have a 2-D surface X_r^2 as intersection. Could this surface be interpreted as a partonic 2-surface? One must however bear in mind that partonic 2-surfaces as topological vertices are assumed to be non-generic in the sense that the light-like partonic orbits meet at them. At the level of H, the intersections would be partonic 2-surfaces X^2 at which the four 3-D partonic orbits would meet along their ends. Does this hold true at the level of M^8? Or can it hold true even at the level H?

The simplest situation corresponds to 4 external quarks. There are 6 different intersections. Not all of them are realized since a given quark can belong only to a single intersection. One must have two disjoint pairs -say 12 and 34. Most naturally positive *resp.* negative energy quarks form a pair. These pairs are located in different half-cones. The intersections would give two partonic 2-surfaces and this situation would be generic. This suggests a modification of the description of particle reaction in M^8 . $M^8 - H$ duality suggests a similar description in H.

What could be the counterparts of wormhole contacts at the level of M^8?

The experience with H, in particular the presence of extremals with Euclidian signature of the induced metric and identified as building bricks of elementary particles, suggest that also the light-like 3-surfaces in M^8_c could have a continuation with an Euclidian signature of the number theoretic metric with norm having real values only for the projections to planes allowing real coordinates.

The earlier picture has been that the wormhole contacts as CP_2 type extremals correspond to co-associative regions and their exteriors to associative regions. If one wants $M^8 - H$ duality in strong form and thus without need for SH, one should assume that both these regions are co-associative.

1. The simplest option is that the real Minkowskian time coordinate becomes imaginary. Instead of the canonical (I_1, iI_3, iI_5, iI_7) the basis would be (iI_1, iI_3, iI_5, iI_7) having Euclidian signature and SO(4) as isometry group. The signature would naturally change at light-like 3-surface the time coordinate along light-like curves becomes zero - proper time for photon vanishes - and can ransforms continuously from real to imaginary.
2. Wormhole contacts in H behave like pairs of magnetic monopoles with monopole charges at throats. If one does not allow point-like singularity, the monopole flux must go to a parallel Minkowskian space-time sheet through the opposite wormhole throat. Wormhole contact with effective magnetic charge would correspond in M^8_c to a distribution of normal 4-planes at the partonic 2-surfaces analogous to the radial magnetic field of monopole at a sphere surrounding it. To avoid singularity of the distribution, there must be another light-like 3-surface M^8 such that its partonic throat has a topologically similar distribution of normal planes.

In the case of X^3_c dimension does not allow co-quaternion structure: could they allow 4-D co-associative sub-manifolds? It will be found that this option is not included since co-associative tangent space distributions in a quaternion manifold (now O) are always integrable.

11.3.4 Octonionic Dirac equation and co-associativity

Also the role of associativity concerning octonionic Dirac equation in M^8 must be understood. It is found that co-associativity allows very elegant formulation and suggests the identification of the points appearing as the ends of quark propagator lines in H as points of boundary of CD representing light-like momenta of quarks. Partonic vertices would involve sub-CDs and momentum conservation would have purely geometric meaning bringing strongly in mind twistor Grassmannian approach [B23, B21, ?]. I have discussed the twistor lift of TGD replacing twistors as fields with surfaces in twistor space having induced twistor structure in [K100, K87, L64] [L87, L88].

Octonionic Dirac equation

The following arguments lead to the understanding of co-associativity in the case of octonion spinors. The constant spinor basis includes all spinors but the gamma matrices appearing in the octonionic Dirac equation correspond to co-associative octonion units.

1. At the level of O_c the idea about massless Dirac equation as partial differential equation does not make sense. Dirac equation must be algebraic and the obvious idea is that it corresponds to the on mass shell condition for a mode of ordinary Dirac equation with well-define momentum: $p^k\gamma_k\Psi = 0$ satisfying $p^kp_k = 0$. This suggests that octonionic polynomial P defines the counterpart of $p^k\gamma_k$ so that gamma matrices γ_k would be represented as octonion components. Does this make sense?

2. Can one construct octonionic counterparts of gamma matrices? The imaginary octonion units I_k indeed define the analogs of gamma matrices as $\gamma_k \equiv iI_k$ satisfying the conditions $\{\gamma_k, \gamma_l\} = 2\delta_{kl}$ defining Euclidian gamma matrices. The problem is that one has $I_0 I_l k + I_k I_0 = 2I_k$. One manner to solve the problem would be to consider tensor products $I_0\sigma_3$ and $I_k\sigma_2$ where σ_3 and $sigma_2$ are Pauli's sigma matrices with anti-commutation relations $\{\sigma_i, \sigma_j\} = \delta_{i,j}$. Note that I_k do not allow a matrix representation.
   Co-associativity condition suggests an alternative solution. The restriction of momenta to be co-associative and therefore vanishing component p^0 as octonion, would selects a sub-space spanned by say the canonical choice $\{I_2, iI3, iI_5, iI_7\}$ satisfying the anticommutation relations of Minkowskian gamma matrices. Octonion units do not allow a matrix representation because they are not associative. The products for a co-associative subset of octonion units are however associative $(a(bc) = (ab)c$ so that they can be mapped to standard gamma matrices in Minkowski space. Co-associativity would allow the representation of 4-D gamma matrices as a maximal associative subset of octonion units.
3. What about octonionic spinors. The modes of the ordinary Dirac equation with a well-defined momentum are obtained by applying the Dirac operator to an orthogonal basis of constant spinors u_i to give $\Psi = p^k\gamma_k u_i$. Now the counterparts of constant spinors u_i would naturally be octonion units $\{I_0, I_k\}$: this would give the needed number 8 of real spinor components as one has for quark spinors.
   Dirac equation reduces to light-likeness conditions $p^k p_k = 0$ and p_k must be chosen to be real - if pk are complex, the real and imaginary parts of momentum are parallel. One would obtain an entire 3-D mass shell of solution and a single mode of Dirac equation would correspond to a point of this mass shell.
   Remark: Octonionic Dirac equation is associative since one has a product of form $(p_k\gamma_k)^2 u_i$ and octonion products of type $x^2 y$ are associative.
4. p^k would correspond to the restriction of $P(o_c)$ to M^4 as sub-space of octonions. Since co-associativity implies $P(o_c) = Y(o_c)o_c$ restricted to counterpart of M^4 (say subspace spanned by $\{I_2, iI3, iI_5, iI_7\}$), Dirac equation reduces to the condition $o^k o_k = 0$ in M^4 defining a light-cone of M^4. This light-cone is mapped to a curved light-like 3-surface X^3 in o_c as $o_c \rightarrow P(o_c) = Y o_c$. $M^8 - H$ duality maps points of space-time surface on M^8 H and therefore the light-cone of M^4 corresponds to either light-like boundary of CD. It seems that the image of X^3 in H has M^4 projection to the light-like boundary of CD.
   Co-associative space-time surfaces have 3-D intersections X^3 with the surface $P = 0$: the conjecture is that X^3 corresponds to a light-like orbit of partonic 2-surfaces in H at which the induced metric signature changes. At X^3 one has besides $X = 0$ also $Y = 0$ so that octonionic Dirac equation $P(o_c)\Psi = P^k I_k\Psi = Y p^k I_k \Psi = 0$ is trivially satisfied for all momenta $p^k = o^k$ defined by the M^4 projections of points of X^3 and one would have $P^k = Y p^k = 0$ so that the identification of P^k as 4-momentum would not allow to assign non-vanishing momenta to X^3. The direction of p^k is constrained only by the condition of belonging to X^3 and the momentum would be in general time-like since X^3 is inside future light-cone.
   $Y = 0$ condition conforms with the proposal that X^3 defines a boundary of Minkowskian and Euclidian region: Euclidian mass shell condition for real P^k requires $P^k = 0$. The general complex solution to $P^2 = 0$ condition is $P = P_1 + iP_2$ with $P_1^2 = P_2^2$.

A single mode of Dirac equation with a well-defined value of p^k as the analog of 4-momentum would correspond to a selection of single time-like point at X^3 or light-like point at the light-like boundary of CD. X^3 intersects light-cone boundary as part of boundary of 7-D light-cone. The picture about scattering amplitudes - consistent with the view about cognitive representations as a unique discretization of space-time surface - is that quarks are located at discrete points of partonic 2-surfaces representing the ends of fermionic propagator lines in H and that one can assign to them light-like momenta.

Challenging the form of $M^8 - H$ duality for the map $M^4 \subset M^8$ to $M^4 \subset H$

The assumption that the map $M^4 \subset M^8$ to $M^4 \subset H$ in $M^8 - H$ duality is a simple identification map has not been challenged hitherto.

1. Octonionic Dirac equation forces the identification of M^8 as analog of 8-D momentum space and the earlier simple identification is in conflict with Uncertainty Principle. Inversion al-

lowed by conformal invariance is highly suggestive: what comes first in mind is a map $m^k \to \hbar_{eff} m^k/m^k m_k$.
At the light-cone boundary the map is ill-defined. Here on must take as coordinate the linear time coordinate m^0 or equivalently radial coordinate $r_M = m^0$. In this case the map would be of fporm $t \to \hbar_{eff}/m^0$: m^0 has interpretation as energy of massless particle.
The map would give a surprisingly precise mathematical realization for the intuitive arguments assigning to mass a length scale by Uncertainty Principle.

2. Additional constraints on $M^8 - H$ duality in M^4 degrees of freedom comes from the following argument. The two half-cones of CD contain space-time surfaces in M^8 as roots of polynomials $P_1(o)$ and $P_2(2T - o)$ which need not be identical. The simplest solution is $P_2(o) = P_1(2T - o)$: the space-time surfaces at half-cones would be mirror images of each other. This gives $P_1(T, Im_R(o)) = P_1(T - Im_R(o))$ Since P_1 depends on $t^2 - \overline{o}^2$ only, the condition is identically satisfied for both options.
There are two options for the identification of the coordinate t.
Option a): t is identified as octonionic real coordinate o_R identified and also time coordinate as in the original option. In the recent option octonion o_R would correspond to the Euclidian analog of time coordinate. The breaking of symmetry from $SO(4)$ to $SO(3)$ would distinguish t as a Newtonian time.
At the level of M^8, The M^4 projection of CD_8 is a union of future and past directed light-cones with a common tip rather than CD_4. Both incoming and outgoing momenta have the same origin automatically. This identification is the natural one at the level of M^8.
Option b): t is identified as a Minkowski time coordinate associated with the imaginary unit I_1 in the canonical decomposition $\{I_1, iI_3, iI_5, iI_7\}$. The half-cone at $o = 0$ would be shifted to $O = (0, 2T, 0...0)$ and reverted. M^4 projection would give CD_4 so that this option is consistent with ZEO. This option is natural at the level of Hbut not at the level of M^8.
If **Option a)** is realized at the level of M^8 and **Option b)** at the level of H, as seems natural, a time translation $m^0 \to m^0 + 2T$ of the past directed light-cone in $M^4 \subset H$ is required in order to to give upper half-cone of CD_4.

3. The map of the momenta to embedding space points does not prevent the interpretation of the points of M^8 as momenta also at the level of H since this information is not lost. One cannot identify p^k as such as four-momentum neither at the level of M^8 nor H as suggested by the naïve identification of the Cartesian factors M^4 for M^8 and H. This problem is circumvented by a conjugation in M^8_c changing the sign of 3-momentum. The light-like momenta along the light-cone boundary are non-physical but transform to light-like momenta arriving into light-cone as the physical intuition requires.
Therefore the map would have in the interior of light-cone roughly the above form but there is still a question about the precise form of the map. Does one perform inversion for the M^4 projection or does one take M^4 projection for the inversion of complex octonion. The inversion of M^4 projection seems to be the more plausible option. Denoting by $P(o_c)$ the real M^4 projection of X^4 point one therefore has:
$$P(o_c) \to \hbar_{eff} \frac{\overline{P(o_c)}}{P(o_c) \cdot P(o_c)} \quad . \tag{11.3.7}$$
Note that the conjugation changes the direction of 3-momentum.
At the light-cone boundary the inversion is ill-defined but Uncertainty Principle comes in rescue, and one can invert the M^4 time coordinate:
$$Re(m^0) = t \to \hbar_{eff} \frac{1}{t} \quad . \tag{11.3.8}$$

A couple of remarks are in order.

1. The presence of $\hbar_{eff}$ instead of $\hbar$ is required by the vision about dark matter. The value of $\hbar_{eff}/h_0$ is given by the dimension of extension of rationals identifiable as the degree of P.
2. The image points $\overline{p}^k$ in H would naturally correspond to the ends of the propagator lines in the space-time representation of scattering amplitudes.

The information about momenta is not lost in the map. What could be the interpretation of the momenta $\overline{p}^k$ at the level of H?

1. Super-symplectic generators at the partonic vertices in H do not involve momenta as labels. The modes of the embedding space spinor field assignable to the ground states of super-symplectic representations at the boundaries of CD have 4-momentum and color as labels. The identification of $\overline{p}^k$ as this momentum label would provide a connection with the classical picture about scattering events.
   At the partonic 2-surfaces appearing as vertices, one would have a sum over the ground states (spinor harmonics). This would give integral over momenta but $M^8 - H$ duality and number theoretic discretization would select a finite subset and the momentum integral would reduce to a discrete sum. The number of M^8 points with coordinates in a given extension of rationals is indeed finite.
2. $M^4 \subset M^8$ could be interpreted as the space of 4-momenta labeling the spinor harmonics of M^8. Same would apply at the level of H: spinor harmonics would correspond to the ground states of super-symplectic representations.
3. The interpretation of the points of M^4_c as complex 4-momenta inspires the question whether the interpretation of the imaginary part of the momentum squared in terms of decay decay width so that M^8 picture would code even information about the dynamics of the particles.

11.4 How to achieve periodic dynamics at the level of $M^4 \times CP_2$?

Assuming $M^8 - H$ duality, how could one achieve typical periodic dynamics at the level of H - at least effectively?

It seems that one cannot have an "easy" solution to the problem?

1. Irreducible polynomials which are products of monomials corresponding to roots r_n which are in good approximation evenly spaced $r_n = r_0 + nr_1\Delta r_n$ would give "very special moments in the life of self" as values of M^4 time which are evenly spaced [L76, L73]. This could give rise to an effective periodicity but it would be at the level of M^8, not H, where it is required.
2. Is it enough that the periodic functions are *only* associated with the spinor harmonics of H involved with the construction of scattering amplitudes in H [L111]? For the modified Dirac equation [K113] the periodic behavior is possible. Note also that the induced spinors defining ground states of super-symplectic representations are restrictions of second quantized spinors of H proportional to plane waves in M^4. These solutions do not guarantee quantum classical correspondence.

11.4.1 The unique aspects of Neper number and number theoretical universality of Fourier analysis

Could one assume more general functions than polynomials at the level of H? Discrete Fourier basis is certainly an excellent candidate in this respect but does it allow number theoretical universality?

1. Discrete Fourier analysis involves in the Euclidian geometry periodic functions $exp(2\pi x)$, n integer and in hyperbolic geometry exponential functions $exp(kx)$.
   Roots of unity $exp(i2\pi/n)$ allow to generalize Fourier analysis. The p-adic variants of $exp(ix)$ exist for rational values of $x = k2\pi/n$ for $n = K$ if $exp(i2\pi/K)$ belongs to the extension of rationals. $x = k = 2pi/n$ does not exist as a p-adic number but $exp(x) = exp(i2\pi/n)$ can exist as phase replacing x as coordinate in extension of p-adics. One can therefore define Fourier basis $\{exp(inx)|n \in Z\}$ which exist at discrete set of rational points $x = k/n$
   Neper number e is also p-adically exceptional in that e^p exists as a p-adic number for all primes p. One has a hierarchy of finite-D extensions of p-adic numbers spanned by the roots $e^{1/n}$. Finiteness of cognition might allow them. Hyperbolic functions $exp(nx)$, $n = 1, 2...$ would have values in extension of p-adic number field containing $exp(1/N)$ in a discrete set of points $\{x = k/N|k \in Z\}$.
2. (Complex) rationality guarantees number theoretical universality and is natural since CP_2 geometry is complex. This would correspond to the replacement $x \to exp(ix)$ or $x \to exp(x)$ for powers x^n. The change of the signature by replacing real coordinate x with ix would automatically induce this change.

3. Exponential functions are in a preferred position also group theoretically. Exponential map maps $g \rightarrow exp(itg)$ the points of Lie algebra to the points of the Lie group so that the tangent space of the Lie algebra defines local coordinates for the Lie group. One can say that tangent space is mapped to space itself. M^4 defines an Abelian group and the exponential map would mean replacing of the M^4 coordinates with their exponential, which are p-adically more natural. Ordinary Minkowski coordinates have both signs so that they would correspond to the Lie algebra level.
4. CP_2 is a coset space and its points are obtained as selected points of $SU(3)$ using exponentiation of a commutative subalgebra t in the decomposition $g = h + t + \bar{t}$ in the Lie-algebra of $SU(3)$. One could interpret the CP_2 points as exponentials and the emergence of exponential basis as a basis satisfying number theoretical universality.

11.4.2 Are CP_2 coordinates as functions of M^4 coordinates expressible as Fourier expansion

Exponential basis is not natural at the level of M^8. Exponential functions belong to dynamics, not algebraic geometry, and the level H represents dynamics.

It is the dependence of CP_2 coordinates on M^4 coordinates, where the periodicity is needed. The map of the tangent spaces of $X^4 \subset M^8$ to points of CP_2 is slightly local since it depends on the first derivatives crucial for dynamics. Could this bring in dynamics and exponential functions at the level of H?

These observations inspire the working hypothesis that CP_2 points as functions of M^4 coordinates are expressible as polynomials of hyperbolic and trigonometric exponentials of M^4 coordinates.

Consider now the situation in more detail.

1. The basis for roots of e would be characterized by integer K in $e^{1/K}$. This brings in a new parameter characterizing the extension of rationals inducing finite extensions of p-adic numbers. K is analogous to the dimension of extension of rationals: the p-adic extension has dimension $d = Kp$ depending on the p-adic prime explicitly.
2. If CD size T is given, $e^{-T/K}$ defines temporal and spatial resolution in H. K or possibly Kp could naturally correspond to the gravitational Planck constant [L59] [K12] [E18] $K = n_{gr} = \hbar_{gr}/h_0$.
3. In [L113] many-sheetedness with respect to CP_2 was proposed to correspond to flux tubebundles in M^4 forming quantum coherent structures. A given CP_2 point corresponds to several M^4 points with the same tangent space and their number would correspond to the number of the flux tubes in the bundle.
   Does the number of these points relate to K or Kp? p-Adic extension would have finite dimension $d = Kp$. Could $d = Kp$ be analogous to a degree of polynomial defining the dimension of extension of rationals? Could this be true in p-adic length scale resolution $O(p^2) = 0$ The number of points would be Kp and very large. For electron one has $p = M_{127} = 2^{127} - 1$.
4. The dimension n_A Abelian extension associated with EQ would naturally satisfy $n_A = K$ since the trigonometric and hyperbolic exponentials are obtained from each other by replacing a real coordinate with an imaginary one.
5. There would be two effective Planck constants. $h_{eff} = nh_0$ would be defined by the degree n of the polynomial P defining $X^4 \subset M^8$. $\hbar_{gr} = n_{gr}h_0$ would define infra-red cutoff in M^4 as the size scale of CD in $H = M^4 \times CP_2$. n *resp.* $n_{gr} = Kp$ would characterize many-sheetedness in M^4 *resp.* CP_2 degrees of freedom.

11.4.3 Connection with cognitive measurements as analogs of particle reactions

There is an interesting connection to the notion of cognitive measurement [L113, L114, L118].

1. The dimension n of the extension of rationals as the degree of the polynomial $P = P_{n_1} \circ P_{n_2} \circ ...$ is the product of degrees of degrees n_i: $n = \prod_i n_i$ and one has a hierarchy of Galois groups G_i

associated with $P_{n_i} \circ ....$ G_{i+1} is a normal subgroup of G_i so that the coset space $H_i = G_i/G_{i+1}$ is a group of order n_i. The groups H_i are simple and do not have this kind of decomposition: simple finite groups appearing as building bricks of finite groups are classified. Simple groups are primes for finite groups.

2. The wave function in group algebra $L(G)$ of Galois group G of P has a representation as an entangled state in the product of simple group algebras $L(H_i)$. Since the Galois groups act on the space-time surfaces in M^8 they do so also in H. One obtains wave functions in the space of space-time surfaces. G has decomposition to a product (not Cartesian in general) of simple groups. In the same manner, $L(G)$ has a representation of entangled states assignable to $L(H_i)$ [L113, L118].

This picture leads to a model of analysis as a cognitive process identified as a cascade of "small state function reductions" (SSFRs) analogous to "weak" measurements.

1. Cognitive measurement would reduce the entanglement between $L(H_1)$ and $L(H_2)$, the between $L(H_2)$ and $L(H_3)$ and so on. The outcome would be an unentangled product of wave functions in $L(H_i)$ in the product $L(H_1) \times L(H_2) \times ....$ This cascade of cognitive measurements has an interpretation as a quantum correlate for analysis as factorization of a Galois group to its prime factors. Similar interpretation applies in M^4 degrees of freedom.
2. This decomposition could correspond to a replacement of P with a product $\prod_i P_i$ of polynomials with degrees $n = n_1 n_2 ...$, which is irreducible and defines a union of separate surfaces without any correlations. This process is indeed analogous to analysis.
3. The analysis cannot occur for simple Galois groups associated with extensions having no decomposition to simpler extensions. They could be regarded as correlates for irreducible primal ideas. In Eastern philosophies the notion of state empty of thoughts could correspondto these cognitive states in which SSFRs cannot occur.
4. An analogous process should make sense also in the gravitational sector and would mean the splitting of $K = n_A$ appearing as a factor $n_{gr} = Kp$ to prime factors so that the sizes of CDs involved with the resulting structure would be reduced. This process would reduce to a simultaneous measurement cascade in hyperbolic and trigonometric Abelian extensions. The IR cutoffs having interpretation as coherence lengths would decrease in the process as expected. Nature would be performing ordinary prime factorization in the gravitational degrees of freedom.

Cognitive process would also have a geometric description.

1. For the algebraic EQs, the geometric description would be as a decay of n-sheeted 4-surface with respect to M^4 to a union of n_i-sheeted 4-surfaces by SSFRs. This would take place for flux tubes mediating all kinds of interactions.
   In gravitational degrees of freedom, that is for trascendental EQs, the states with $n_{gr} = Kp$ having bundles of Kp flux tubes would deca to flux tubes bundles of $n_{gr,i} = K_i p$, where K_i is a prime dividing K. The quantity $log(K)$ would be conserved in the process and is analogous to the corresponding conserved quantity in arithmetic quantum field theories (QFTs) and relates to the notion of infinite prime inspired by TGD [K94].
2. This picture leads to ask whether one could speak of cognitive analogs of particle reactions representing interactions of "thought bubbles" i.e. space-time surfaces as correlates of cognition. The incoming and outgoing states would correspond to a Cartesian product of simple subgroups: $G = \prod_i^{\times} H_i$. In this composition the order of factors does not matter and the situation is analogous to a many particle system without interactions. The non-commutativity in general case leads to ask whether quantum groups might provide a natural description of the situation.
3. Interestingly, Equivalence Principle is consistent with the splitting of gravitational flux tube structures to smaller ones since gravitational binding energies given by Bohr model in $1/r$ gravitational potential do not depend on the value of $\hbar_{gr}$ if given by Nottale formula $\hbar_{gr} = GMm/v_0$ [L122]. The interpretation would be in terms of spontaneous quantum decoherence taking place as a decay of gravitational flux tube bundles as the distance from the source increases.

11.4.4 Still some questions about $M^8 - H$ duality

There are still on questions to be answered.

1. The map $p^k \rightarrow m^k = \hbar_{eff} p^k / p \cdot p$ defining $M^8 - H$ duality is consistent with Uncertainty Principle but this is not quite enough. Momenta in M^8 should correspond to plane waves in H.

   Should one demand that the momentum eigenstate as a point of cognitive representation associated with $X^4 \subset M^8$ carrying quark number should correspond to a plane wave with momentum at the level of $H = M^4 \times CP_2$? This does not make sense since $X^4 \subset CD$ contains a large number of momenta assignable to fundamental fermions and one does not know which of them to select.

2. One can however weaken the condition by assigning to CD a 4-momentum, call it P. Could one identify P as

   (a) the total momentum assignable to either half-cone of CD

   (b) or the sum of the total momenta assignable to the half-cones?

The first option does not seem to be realistic. The problem with the latter option is that the sum of total momenta is assumed to vanish in ZEO. One would have automatically zero momentum planewave. What goes wrong?

1. Momentum conservation for a single CD is an ad hoc assumption in conflict with Uncertainty Principle, and does not follow from Poincare invariance. However, the sum of momenta vanishes for non-vanishing planewave when defined in the entire M^4 as in QFT, not for planewaves inside finite CDs. Number theoretic discretization allows vanishing in finite volumes but this involves finite measurement resolution.
2. Zero energy states represent scattering amplitudes and at the limit of infinite size for the large CD zero energy state is proportional to momentum conserving delta function just as S-matrix elements are in QFT. If the planewave is restricted within a large CD defining the measurement volume of observer, four-momentum is conserved in resolution defined by the large CD in accordance with Uncertainty Principle.
3. Note that the momenta of fundamental fermions inside half-cones of CD in H should be determined at the level of H by the state of a super-symplectic representation as a sum of the momenta of fundamental fermions assignable to discrete images of momenta in $X^4 \subset H$.

$M^8 - H$-duality as a generalized Fourier transform

This picture provides an interpretation for $M^8 - H$ duality as a generalization of Fourier transform.

1. The map would be essentially Fourier transform mapping momenta of zero energy as points of $X^4 \subset CD \subset M^8$ to plane waves in H with position interpreted as position of CD in H. CD and the superposition of space-time surfaces inside it would generalize the ordinary Fourier transform . A wave function localized to a point would be replaced with a superposition of space-time surfaces inside the CD having interpretation as a perceptive field of a conscious entity.
2. $M^8 - H$ duality would realize momentum-position duality of wave mechanics. In QFT this duality is lost since space-time coordinates become parameters and quantum fields replace position and momentum as fundamental observables. Momentum-position duality would have much deeper content than believed since its realization in TGD would bring number theory to physics.

How to describe interactions of CDs?

Any quantum coherent system corresponds to a CD. How can one describe the interactions of CDs? The overlap of CDs is a natural candidate for the interaction region.

1. CD represents the perceptive field of a conscious entity and CDs form a kind of conscious atlas for M^8 and H. CDs can have CDs within CDs and CDs can also intersect. CDs can have shared sub-CDs identifiable as shared mental images.
2. The intuitive guess is that the interactions occur only when the CDs intersect. A milder assumption is that interactions are observed only when CDs intersect.

3. How to describe the interactions between overlapping CDs? The fact the quark fields are induced from second quantized spinor fields in in H *resp.* M^8 solves this problem. At the level of H, the propagators between the points of space-time surfaces belonging to different CDs are well defined and the systems associated with overlapping CDs have well-defined quark interactions in the intersection region. At the level of M^8 the momenta as discrete quark carrying points in the intersection of CDs can interact.

Zero energy states as scattering amplitudes and subjective time evolution as sequence of SSFRs

This is not yet the whole story. Zero energy states code for the ordinary time evolution in the QFT sense described by the S-matrix. What about subjective time evolution defined by a sequence of "small" state function reductions (SSFRs) as analogs of "weak" measurements followed now and then by BSFRs? How does the subjective time evolution fit with the QFT picture in which single particle zero energy states are planewaves associated with a fixed CD.

1. The size of CD increases at least in statistical sense during the sequence of SSFRs. This increase cannot correspond to M^4 time translation in the sense of QFTs. Single unitary step followed by SSFR can be identified as a scaling of CD leaving the passive boundary of the CD invariant. One can assume a formation of an intermediate state which is quantum superposition over different size scales of CD: SSFR means localization selecting single size for CD. The subjective time evolution would correspond to a sequence of scalings of CD.
2. The view about subjective time evolution conforms with the picture of string models in which the Lorentz invariant scaling generator L_0 takes the role of Hamiltonian identifiable in terms of mass squared operator allowing to overcome the problems with Poincare invariance. This view about subjective time evolution also conforms with super-symplectic and Kac-Moody symmetries of TGD.

   One could perhaps say that the Minkowski time T as distance between the tips of CDs corresponds to exponentiated scaling: $T = exp(L_0 t)$. If t has constant ticks, the ticks of T increase exponentially.

The precise dynamics of the unitary time evolutions preceding SSFRs has remained open.

1. The intuitive picture that the scalings of CDs gradually reveal the entire 4-surface determined by polynomial P in M^8: the roots of P as "very special moments in the life of self" would correspond to the values of time coordinate for which SSFRs occur as one new root emerges. These moments as roots of the polynomial defining the space-time surface would correspond to scalings of the size of both half-cones for which the space-time surfaces are mirror images. Only the upper half-cone would be dynamical in the sense that mental images as sub-CDs appear at "geometric now" and drift to the geometric future.
2. The scaling for the size of CD does *not* affect the momenta associated with fermions at the points of cognitive representation in $X^4 \subset M^8$ so that the scaling is not a genuine scaling of M^4 coordinates which does not commute with momenta. Also the fact that L_0 for super symplectic representations corresponds to mass squared operator means that it commutes with Poincare algebra so that M^4 scaling cannot be in question.
3. The Hamiltonian defining the time evolution preceding SSFR could correspond to an exponentiation of the sum of the generators L_0 for super-symplectic and super-Kac Moody representations and the parameter t in exponential corresponds to the scaling of CD assignable to the replaced of root r_n with root r_{n+1} as value of M^4 linear time (or energy in M^8). L_0 has a natural representation at light cone boundaries of CD as scalings of light-like radial coordinate.
4. Does the unitary evolution create a superposition over all over all scalings of CD and does SSFR measure the scale parameter and select just a single CD?

   Or does the time evolution correspond to scaling? Is it perhaps determined by the increase of CD from the size determined by the root r_n as "geometric now" to the root r_{n+1} so that one would have a complete analogy with Hamiltonian evolution? The scaling would be the ratio r_{n+1}/r_n which is an algebraic number.

   Hamiltonian time evolution is certainly the simplest option and predicts a fixed arrow of time during SSFR sequence. L_0 identifiable essentially as a mass squared operator acts like conjugate for the logarithm of the logarithm of light-cone proper time for a given half-cone. One can assume that L_0 as the sum of generators associated with upper and lower half-cones if the fixed state at the lower half-cone is eigenstate of L_0.

How does this picture relate to p-adic thermodynamics in which thermodynamics isdetermined by partition function which would in real sector be regarded as a vacuum expectation value of an exponential $exp(iL_0t)$ of a Hamiltonian for imaginary time $t = i\beta$ $\beta = 1/T$ defined by temperature. L_0 is proportional to mass squared operator.

1. In p-adic thermodynamics temperature T is dimensionless parameter and $\beta = 1/T$ is integer valued. The partition function as exponential $exp(-H/T)$ is replaced with $p^{\beta L_0)}$, $\beta = n$, which has the desired behavior if L_0 has integer spectrum. The exponential form $e^{L_0/T_R)}$, $\beta_R = nlog(p)$ equivalent in the real sector does not make sense p-adically since the p-adic exponential function has p-adic norm 1 if it exists p-adically.
2. The time evolution operator $exp(-iL_0t)$ for SSFRs (t would be the scaling parameter) makes sense for the extensions of p-adic numbers if the phase factors for eigenstates are roots of unity belonging to the extension. $t = 2\pi k/n$ since L_0 has integer spectrum. SSFRs would define a clock. The scaling $exp(t) = exp(2\pi k/n)$ is however not consistent with the scaling by r_{n-1}/r_n.
   Both the temperature and scaling parameter for time evolution by SSFRs would be quantized by number theoretical universality. p-Adic thermodynamics could have its origins in the subjective time evolution by SSFRs.
3. In the standard thermodynamics it is possible to unify temperature and time by introducing a complex time variable $\tau = t + i\beta$, where $\beta = 1/T$ is inverse temperature. For the space-time surface in complexified M^8, M^4 time is complex and the real projection defines the 4-surface mapped to H. Could thermodynamics correspond to the imaginary part of the time coordinate?
   Could one unify thermodynamics and quantum theory as I have indeed proposed: this proposal states that quantum TGD can be seen as a "complex square root" of thermodynamics. The exponentials $U = exp(\tau L_0/2)$ would define this complex square root and thermodynamical partition function would be given by $UU^\dagger = exp(-\beta L_0)$.

11.5 Can one construct scattering amplitudes also at the level of M^8?

$M^8 - H$ duality suggests that the construction is possible both at the level of H and M^8. These pictures would be based on differential geometry on one hand and algebraic geometry and number theory on the other hand. The challenge is to understand their relationship.

11.5.1 Intuitive picture

H picture is phenomenological but rather detailed and M^8 picture should be its pre-image under $M^8 - H$ duality. The following general questions can be raised.

1. Can one construct the counterparts of the scattering amplitudes also at the level of M^8?
2. Can one use $M^8 - H$ duality to map scattering diagrams in M^8 to the level of H?

Consider first the notions of CD and sub-CD.

1. The intuitive picture is that at the level of H that one must surround partonic vertices with sub-CDs, and assign the external light-like momenta with the ends of propagator lines from the boundaries of CD and other sub-CDs. The incoming momenta $\overline{p}^k$ would be assigned to the boundary of sub-CD.
2. What about the situation in M^8? Sub-CDs must have different origin in the general case since the momentum spectrum would be shifted. Therefore the sub-CDs have the same tip - either upper or lower tip, and have as their boundary part of either boundary of CD. A hierarchy of CDs associated with the same upper or lower tip is suggestive and the finite maximal size of CD in H gives IR cutoff and the finite maximal size of CD in M^8 gives UV cutoff.

3. Momentum conservation at the vertices in M^8 could decompose the diagram to sub-diagrams for which the momentum conservation is satisfied. On the basis of QFT experience, one expects that there are some minimal diagrams from which one can construct the diagram: in the TGD framework this diagram would describe 4-quark scattering. The condition that the momenta belong to the extension of rationals gives extremely strong constraints and it is not clear that one obtains any solutions to the conditions unless one poses some conditions on the polynomials assigned with the two boundaries of CD.
   The two half-cones (HCs) of CD contain space-time surfaces in M^8 as roots of polynomials $P_1(o)$ and $P_2(2T-o)$ which need not be identical. The simplest solution is $P_2(o) = P_1(2T-o)$: the space-time surfaces at HCs would be mirror images of each other. This gives $P_1(T, Im_R(o)) = P_1(T - Im_R(o))$ Since P_1 depends on $t^2 - r^2$ only, the condition is identically satisfied for both options.
   There are two options for the identification of the coordinate t.
   Option (a): t is identified as octonionic real coordinate o_R identified and also time coordinate as in the original option. In the recent option octonion o_R would correspond to the Euclidian analog of time coordinate. The breaking of symmetry from $SO(4)$ to $SO(3)$ would distinguish t as a Newtonian time. The M^4 projection of CD_8 gives a union of future and past directed light-cones with a common tip rather than CD_4 in M^4 at the level of M^8 . Both incoming and outgoing momenta have the same origin automatically. This identification seems to be the natural one at the level of M^8.
   Option (b): t is identified as a Minkowski time coordinate associated with the imaginary unit I_1 in the canonical decomposition $\{I_1, iI_3, iI_5, iI_7\}$. The HC at $o = 0$ would be shifted to $O = (0, 2T, 0...0)$ and reverted. M^4 projection would give CD_4 so that this option is consistent with ZEO. This option is natural at the level of Hbut not at the level of M^8.
   If Option (a) is realized at the level of M^8 and Option b) at the level of H, as seems natural, a time translation of the past directed light-cone by T in $M^4 \subset H$ is required to give CD_4. The momentum spectra of the two HCs differ only by sign and at least a scattering diagram in which all points are involved is possible. In fact all the pairs of subsets with opposite momenta are allowed. These however correspond to a trivial scattering. The decomposition to say 4-vertices with common points involving momentum space propagator suggests a decomposition into sub-CDs. The smaller the sub-CDs at the tips of the CD, the smaller the momenta are and the better is the IR resolution.

4. The proposal has been that one has a hierarchy of discrete size scales for the CDs. Momentum conservation gives a constraint on the positions of quarks at the ends of propagator lines in M^8 mapped to a constraint for their images in H: the sum of image points in H is however not vanishing since inversion is not a linear map.

5. QFT intuition would suggest that at the level of M^8 the scattering diagrams decompose to sub-diagrams for which momentum conservation is separately satisfied. If two such sub-diagrams A and B have common momenta, they correspond to internal lines of the diagram involving local propagator D_p, whose non-local counterpart at the level of H connects the image point to corresponding point of all copies of B.
   The usual integral over the endpoint of the propagator line $D(x, y)$ at space-time level should correspond to a sum in which the H image of B is shifted in M^4. Introduction of a large number of copies of H image of the sub-diagram looks however extremely ugly and challenges the idea of starting from the QFT picture.
   What comes in mind is that all momenta allowed by cognitive representation and summing up to zero define the scattering amplitude as a kind of super-vertex and that Yanigian approach allows this construction.

11.5.2 How do the algebraic geometry in M^8 and the sub-manifold geometry in H relate?

Space-time surfaces in H have also Euclidian regions - in particular wormhole contacts - with induced metric having Euclidian signature due to the large CP_2 contribution to the induced metric. They are separated from Minkowskian regions by a light-like 3-surfaces identifiable as partonic orbits at which the induced metric becomes degenerate.

1. The possible M^8 counterparts of these regions are expected to have Euclidian signature of the number theoretic metric defined by complexified octonion inner product, which must be real in these regions so that the coordinates for the canonical basis $\{I_1, iI_3, iI_5, iI_7\}$ are either imaginary or real. This allows several signatures.
2. The first guess is that the energy p^0 assignable to I_1 becomes imaginary. This gives tachyonic p^2. The second guess is that all components of 3-momentum $\{iI_3, iI_5, iI_7\}$ become imaginary meaning that the length of 3-momentum becomes imaginary.
3. One cannot exclude the other signatures, for instance the situation in which 1 or 2 components of the 3-momentum become imaginary. Hence the transition could occur in 3 steps as $(1, -1, -1-, 1) \rightarrow (1, 1, -1, -1) \rightarrow (1, 1, 1, -1) \rightarrow (1, 1, 1, 1)$. The values of $p^2 \equiv Re(p_c^2)$ would be non-negative and also their images in $M^4 \subset H$ would be inside future light-cone. This could relate to the fact that all these signatures are possible inthe twistor Grassmannian approach.
4. These regions belong to the complex mass shell $p_c^2 = r_n = m_0^2 = r_n$ appearing as a root to the co-associativity condition $X = 0$. This gives the conditions

$$\begin{aligned} Re(p_c) \cdot Im(p_c^2) &= Im(r_n) \ , \\ Re(p_c^2) \equiv p^2 &= Im(p_c^2) + m_n^2 \ , \\ m_n^2 &\equiv Re(r_n) \geq 0 \ . \end{aligned} \tag{11.5.1}$$

Consider first the case $(1, 1, 1, 1)$.

1. The components of p_c are either real or imaginary. Using the canonical basis $\{I_1, iI_3, iI_5, iI_7\}$ the components of p_c are real in the Minkowskian region and imaginary in the totally time-like Euclidian region. One has for the totally time-like momentum $p = (p_0, iIm(p_3))$ in the canonical basis.
   This would give
   $$Re(p_c^2) \equiv p^2 = p_0^2 = -Im(p_3)^2 + m_n^2 \ . \tag{11.5.2}$$
   The number theoretic metric is Euclidian and totally time-like but one has $p^2 \geq 0$ in the range $[m_0^2, 0]$. This region is a natural counterpart for an Euclidian space-time region in H. The region $p^2 \geq m_0^2$ has Minkowskian signature and counterpart for Minkowskian regions in H. The region $0 \leq p^2 < m_0^2$ is a natural candidate for an Euclidian region in M^4.
   Remark: A possible objection is that Euclidian regions in O_c are totally time-like and totally space-like in H.
2. The image of these regions under the map $Re(p^k) \rightarrow M^k$ under inversion plus octonionic conjugation defined as $p^k \rightarrow \hbar_{eff}\overline{p^k}/p^2$ (to be discussed in more detail in the sequel) consists of points M^k in the future light-cone of $M^4 \subset H$. The image of the real Euclidian region of O_c with $p^2 \in [0, m_0^2)$ is mapped to the region $M^k M_k < \hbar_{eff}^2/m_0^2$ of $M^4 \subset H$.
3. The contribution of CP_2 metric to the induced metric is space-like so that it can become Euclidian. This would naturally occur in the image of a totally time-like Euclidian region and this region would correspond to small scales $M^k M_k < \hbar_{eff}^2/m_0^2$. The change of the signature should take place at the orbits of partonic 2-surfaces and the argument does not say anything about this. The boundary of between the two regions corresponds to momenta $p = (p_0, 0)$ which is is a time-like line perhaps identifiable as the analog of the light-like geodesic defining the M^4 projection of CP_2 type extremal, which is an idealized solution to actual field equations.

This transition does not explain the M^8 counterpart of the 3-D light-like partonic orbit to which the light-light geodesic thickens in the real situation?

The above argument works also for the other signatures of co-associative real sub-spaces and provides additional insights. Besides the Minkowskian signature, 3 different situations with signatures $(1, 1, 1, 1)$, $(1, -1, 1, 1)$, and $(1, -1 - 1, 1)$ with non-space-like momentum squared are possible.

The following formulas list the signatures, the expressions of real momentum squared, and dimension D of the transition transition $Im(p_c^2) = 0$ as generalization of partonic orbit and the possible identification of the transition region.

$$\begin{array}{llll}
\textbf{Signature} & p^2, & D & \\
(+,-,-,+): & (p^0)^2-(p^1)^2-(p^2)^2=-Im(p^3)^2+m_n^2 & 3 & , \\
\textbf{Identification} & \text{partonic orbit} & & . \\
 & & & \\
\textbf{Signature} & p^2 & D & \\
(+,-,+,+): & (p^0)^2-(p^1)^2=-Im(p^2)^2-Im(p^3)^2+m_n^2 \ , & 2 & , \\
\textbf{Identification} & \text{string world sheet} & & . \\
 & & & \\
\textbf{Signature} & p^2 & D & \\
(+,+,+,+): & (p^0)^2=-Im(p^1)^2-Im(p^2)^2-Im(p^3)^2+m_n^2 \ , & 1 & . \\
\textbf{Identification} & \text{string boundary} & & .
\end{array} \quad (11.5.3)$$

Since the map of the co-associative normal space to CP_2 does not depend on the signature, M^8-H duality is well defined for all these signatures. One can ask whether a single transition creates partonic orbit, two transitions a string world sheet and 3 transitions ends of string world sheet inside partonic orbit or even outside it.

11.5.3 Quantization of octonionic spinors

There are questions related to the quantization of octonionic spinors.

1. Co-associative gamma matrices identified as octonion units are associative with respect to their octonionic product so that matrix representation is possible. Do second quantized octonionic spinors in M^8 make sense? Is it enough to second quantize them in M^4 as induced octonionic spinors? Are the anti-commutators of oscillator operators Kronecker deltas or delta functions in which case divergence difficulties might be encountered? This is not needed since the momentum space propagators can be identified as those for E^8_c restricted to X^4_r as a subspace with real octonion norm.
   The propagators are just massless Dirac propagators for the choice of M^4 for which light-like M^8 momentum reduces to M^4 momentum. Could one formulate the scattering amplitudes using only massless inverse propagators as in the twistor Grassmannian approach?This does not seem to be the case.
2. Could the counterpart of quark propagator as inverse propagator in M^8 as the idea about defining momentum space integrals as residue integrals would suggest? This would allow on-mass-shell propagation like in twistor diagrams and would conform with the idea that inversion relates M^8 and H descriptions. This is suggested by the fact that no integration over intermediate virtual momenta appears in the graphs defined by the algebraic points of the pre-images of the partonic 2-surfaces X^2_r .

How to identify external quarks? Note that bosons would consist of correlated quark-antiquark pairs with the propagator obtained as a convolution of quark propagators. The correlation would be present for the external states and possibly also for the states in the diagram and produced by topologically.

1. The polynomial P and the $P=0$ surface with 6-D real projection X^6_r is not affected by octonion automorphisms. Quarks with different states of motion would correspond to the same P but to different choices of M^4 as co-associative subspace for M^8_c. P could be seen as defining a class of scattering diagrams. P determines the vertices.
2. The space-time surface associated with a quark carrying given 4-momentum should be obtainable by a Lorenz transformation in $SO(3,1)\subset G_{2,c}$ to give it light-like M^4 so that complexified octonionic automorphisms would generate 3-surfaces representing particles. If $M^4\subset M^8$ and thus the CD associated with the quark is chosen suitably, the quark is massless. Any incoming particle would be massless in this frame.
   Lorentz invariance however requires a common Lorentz frame provided by the CD. The momentum of a quark in CD would be obtained by $G_{2,c}$ transformation. In the frame of CD the external quark momenta arriving to the interior of CD at vertices associated with $X^3_r\cap Y^3_r$ are time-like. Momentum conservation would hold in this frame. The difference between massive constituent quarks and massless current quarks could be understood as reflecting M^8 picture.

To sum up, the resulting picture is similar to that at the level of H these diagrammatic structures would be mapped to H by momentum inversion. Quantum classical correspondence would be very detailed providing both configuration space and momentum space pictures.

11.5.4 Does $M^8 - H$ duality relate momentum space and space-time representations of scattering amplitudes?

It would seem that the construction of the scattering amplitudes is possible also at the level of M^8 [L111]. M^8 picture would provide momentum representation of scattering diagrams whereas H picture would provide the space-time representation.

Consider first a possible generalization of QFT picture involving propagators and vertices.

1. At first it seems that it is not possible to talk about propagation at the level of momentum space: in positive energy ontology nothing propagates in momentum space if the propagator is a free propagator D_p! In ZEO this is not quite so. One can regard annihilation operators as creation operators for the fermionic vacuum associated with the opposite HC of CD (or sub-CD): one has momentum space propagation from p to $-p$! The expressions of bosonic charges would be indeed bi-local with annihilation and creation operators associated with the mirror paired points in the two HCs of CD forming pairs. The momentum space propagator D_p would actually result from the pairing of creation creation operators with the opposite values of p and the notation $D(p,-p)$ would be more appropriate.
2. In QFT interaction vertices are local in space-time but non-local in momentum space. The n-vertex conserves the total momentum. Therefore one should just select points of M^8 and they are indeed selected by cognitive representation and assign scattering amplitude to this set of points. To each point one could assign momentum space propagator of quark in M^8_c but it would not represent propagation! The vertex would be a multilocal entity defined by the vertices defining the masses involved at light cone boundary and mass shells.
The challenge would be to identify these vertices as poly-local entities. In the QFT picture there would be a set of n-vertices with some momenta common. What could this mean now? One would have subset sets of momenta summing up to zero as vertices. If two subsets have a common momentum this would correspond to a propagator line connecting them. Should one decompose the points of cognitive representation so that it represents momentum space variant of Feynman graph? How unique this decomposition is and do this kind of decompositions exist unless one poses the condition that the total momenta associated with opposite boundaries sum up to zero as done in ZEO. A given n-vertex in the decomposition means the presence of sub-CDs for which the external momenta sum up to zero. This poses very tight constraints on the cognitive representation, and one can wonder they can be satisfied if the cognitive representation is finite as it is in the generic case.
3. Note that for given a polynomial P allowing only points in cognitive representation, one would *not* have momentum space integrations as in QFT: they could however come from integrations over the polynomial coefficients and would correspond to integration of WCW. In adelic picture one allows only rational coefficients for the polynomials. This strongly suggests that the twistor Grasmmannian picture [B21, ?, B43, B12] in which residue integral in the momentum space gives as residues inverse quark propagators at the poles. M^8 picture would represent the end result of this integration and only on mass shell quarks would be involved. One could even challenge the picture based on propagators and vertices and start from Yangian algebra based on the generalization of local symmetries to multilocal symmetries [A29, A71] [B17] [L42].
4. In the case of H restriction of the second quantized free quark field of H to space-time surface defines the propagators. In the recent case one would have a second quantized octonionic spinor field in M^8. The allowed modes of H spinor field are just the co-associative modes for fixed selection of M^4 analogous to momentum space spinors and restricted to Y^3_r. One could speak of wave functions at Y^3_r, which is very natural since they correspond to mass shells.
The induced spinor field would have massless part corresponding to wave functions at the M^4 light-cone boundary and part corresponding to X^3 at which the modes would have definite mass. $P = 0$ would select a discrete set of masses. Could second quantization have the standard meaning in terms of anti-commutation relations posed on a free M^8 spinor field. In the case of M^8_c one avoids normal ordering problems since there is no Dirac action. The anti-

commutators however have singularities of type 7-D delta function. The anti-commutators of oscillator operators at the same point are the problem. If only a single quark oscillator operator at a given point of M^8 is allowed since there is no local action in coordinate space with the interaction part producing the usual troubles.

5. Could one perform a second quantization for E^8 spinor field using free Dirac action? Could one restrict the expansion of the spinor field to co-associative space-time surfaces giving oscillator operators at the points of cognitive representation with the additional restriction to the pre-image of given partonic 2-surface, whose identification was already considered. Scattering amplitudes would involve n-vertices consisting of momenta summing up to zero and connected to opposite incoming momenta at the opposite sides of the HCs with the same tip in M^8. Scattering amplitude would decompose to sub-diagrams defining a cluster decomposition, and would correspond to sub-CDs. The simplest option is that there are no internal propagator lines. The vanishing of the total momenta poses stringent conditions on the points of cognitive representation.

   Normal ordering divergences can however produce problems for this option in the case of bosonic charges bilear in oscillator operators. At the level of H the solution came from a bilocal modified Dirac action leading to bilocal expressions for conserved charges. Now Yangian symmetry suggests a different approach: local vertices in momentum space can involve only commuting oscillator operators.

   Indeed, in ZEO one can regard annihilation operators as creation operators for the fermionic vacuum associated with the opposite HC of CD (or sub-CD). The expressions of bosonic charges would be indeed bi-local with annihilation and creation operators associated with the mirror paired points in the two HCs of CD forming pairs. As already noticed, also the momentum space propagator $D_p = D(p, -p)$ would be also a bi-local object.

6. This is not enough yet. If there is only a single quark at given momentum, genuine particle creation is not possible and the particle reactions are only re-arrangements of quarks but already allowing formation of bosons as bound states of quarks and antiquarks. Genuine particle creation demands local composites of several quarks at the same point p having interpretation as a state with collinear momenta summing up to p and able to decay to states with the total momentum p. This suggests the analog of SUSY proposed in [L81]. Also Yangian approach is highly suggestive.

   To sum up, momentum conservation together with the assumption of finite cognitive representations is the basic obstacle requiring new thinking.

11.5.5 Is the decomposition to propagators and vertices needed?

One can challenge the QFT inspired picture.

1. As already noticed, the relationship $P_1(t) = P(2T-t)$ makes it possible to satisfy this condition at least for the entire set of momenta. This does not yet allow non-trivial interactions without posing additional conditions on the momentum spectrum. This does not look nice. One can ask whether there is a kind of natural selection leading to polynomials defining space-time surfaces allowing cognitive representations with vertex decompositions and polynomials $P(t)$ and $P_r(t)$ without this symmetry? This idea looks ugly. Or could evolution start from simplest surfaces allowing 4 vertices and lead to an engineering of more complex scattering diagrams from these?
2. The map of momentum space propagators regarded as completely local objects in M^8 to H propagators is second ugly feature. The beauty and simplicity of the original picture would be lost by introducing copies of sub-diagrams mapped to the various translations in H.
3. The Noether charges of the Dirac action in H fail to give rise to 4-fermion vertex operator. The theory would be naturally just free field theory if one assumes cognitive representations.

The first heretic question is whether the propagators are really needed at the level of momentum space. This seems to be the case.

1. In ZEO the propagators pair creation and operators with opposite 4-momenta assignable to the opposite HCs of CD having conjugate fermionic vacua (Dirac sea of negative energy fermions and Dirac sea of positive energy fermions) so that momentum space propagators $D(p, -p)$ are non-local objects. The propagators would connect positive and negative energy fermions

at the opposite HCs and this should be essential in the formulation of scattering amplitudes. They cannot be avoided.

2. The propagators would result from the contractions of fermion oscillator operators giving a 7-D delta function at origin in continuum theory. This catastrophe is avoided in the number theoretic picture. Since one allows only points with M^8 coordinates in an extension of rationals, one can assume Kronecker delta type anti-commutators. Besides cognitive representations, this would reflect the profound difference between momentum space and space-time.
This would also mean that the earlier picture about the TGD analog of SUSY based on local composites of oscillator operators [L81] makes sense at the level of M^8. The composites could be however local only for oscillator operators associated with the HC of CD. With the same restriction they could be local also in the$H$ picture.

What about vertices? Could Yangian algebra give directly the scattering amplitudes? This would simplify dramatically the $M^8 - H$ duality for transition amplitudes. For this option the $P_1(t) = P(2T - t)$ option required by continuity would be ideal.

1. Without vertices the theory would be a free field theory. The propagators would connect opposite momenta in opposite HCs of CD. Vertices are necessary and they should be associated with sub-CDs. Unless sub-CDs can have different numbers of positive and negative energy quarks at the opposite HCs, the total quark number is the same in the initial and final states if quarks and antiquarks associated with bosons as bound states of fermion and antiquark are counted. This option would require minimally 4-quark vertex having 2 fermions of opposite energies at the two hemi-spheres of the CD. A more general option looks more plausible. One obtains non-trivial scattering amplitudes by contracting fermions assigned to the boundary P (F) past (future) HC of CD to the past (future) boundary P_{sub} (F_{sub}) of a sub-CD. Sub-CD and CD must have an opposite arrow of time to get the signs of energies correctly.
Sub-CDs would thus make particle creation and non-trivial scattering possible. There could be an arbitrary number of sub-CDs and they should be assignable to the pre-images of the partonic 2-surfaces X^2_r if the earlier picture is correct. The precise identification of the partonic 2-surfaces is still unclear as also the question whether light-like orbits of partonic 2-surfaces meet along their ends in the vertices.

2. As in the case of H, one could assign the analogs of n-vertices at pre-images of partonic 2-surfaces at X^2_r representing the momenta of massive modes of the octonionic Dirac equation and belonging to the cognitive representations. The idea is to use generators of super-Yangian algebra to be discussed later which are both bosonic and fermionic. The simplest construction would assign these generators to the vertices as points in cognitive representation.
An important point is that Yangian symmetry would be a local symmetry at the level of momentum space and correspond to non-local symmetry at the level of space-time rather than vice versa as usually. The conserved currents would be local composites of quark oscillator operators with same momentum just as they are in QFTs at space-time level representing parallelly propagating quarks and antiquarks.
The simplest but not necessary assumption is that they are linear and bilinear in oscillator operators associated with the same point of M^8 and thus carrying 8-momenta assignable to the modes of E^8 spinor field and restricted to the co-associative 4-surface. Their number of local composites is finite and corresponds to the number 8 of different states of 8-spinors of given chirality.
Also a higher number of quarks is possible, and this was indeed suggested in [L81]. The proposal was that instance leptons would correspond to local composites of 3 quarks. The TGD based view about color allows this. These states would be analogous to the monomials of theta parameters in the expansion of super-field. The H picture allows milder assumptions: leptonic quarks reside at partonic 2-surface at different points but this is not necessary.

3. Instead of super-symplectic generators one has $G_{2,c}$ as the complexified automorphism group. Also the Galois group of the extension acts as an automorphism group and is proposed to have a central role in quantum TGD with applications to quantum biology [L36, L109]. As found, $G_{2,c}$ acts as an analog of gauge or Kac-Moody group. Yangian has analogous structure but the analogs of conformal weights are non-negative.

4. The identification of the analogs of the poly-local vertex operators as produces of charges generators associated with FHC anbd PHC is the basic challenge. They should consist of

quark creation operators (annihilation operators being associated as creation operators at the opposite HC) and be generators of infinitesimal symmetries which in number theoretic physics would correspond instead of isometries of WCW to the octonionic automorphism group G_2 complexified to $G_{2,c}$ containing also the generators of $SO(4) \subset G_2$ and thus also those of Lorentz group $SO(1,3) \subset G_{2,c}$.
The construction Noether charges of E^8 second quantized spinor field at momentum space representation gives bilinear expressions in creation and annihilation operators associated with opposite 3-momenta and would have a single fermion in a given HC. This is not enough: there should be at least 4 fermions.

What strongly suggests itself are Yangian algebras [A29] [L42] having poly-local generators and considered already earlier and appearing in the twistor Grassmannian approach [B21, **?**]. The sums of various quantum numbers would vanish for the vertex operators. These algebras are quantum algebras and the construction of n-vertices could involve co-algebra operation. What is new as compared to Lie algebras is that Yangian algebras are quantum algebras having co-algebra structure allowing to construct n-local generators representing scattering amplitudes. It might be possible replace oscillator operators with the quantum group counterparts.

11.5.6 Does the condition that momenta belong to cognitive representations make scattering amplitudes trivial?

Yangian symmetry is associated with 2-D integrable QFTs which tend to be physically rather uninteresting. The scattering is in the forward direction and only phase shifts are induced. There is no particle creation. If the relationship $P_1(t) = P(2T - t)$ is applied the momentum spectra for FHC and PHC differ only by the sign. If all momenta are involved and the cognitive representations are finite, the situation would be the same! Also the existence of cluster compositions involving summations of subsets of momenta to zero is implausible. Something seems to go wrong!

The basic reason for the problem is the assumption that the momenta belong to cognitive representations assumed to be finite as they indeed are in the generic case. But are they finite in the recent situation involving symmetries?

1. The assumption that all possible momenta allowed by cognitive representation are involved, allows only forward scattering unless there are several subsets of momenta associated with either HC such that the momenta sum-up to the same total momentum. This would allow the change of the particle number. The subsets S_i with same total momentum p_{tot} in the final state could save as final states of subsets S_j with the same total momentum p in the initial state. What could be the number theoretical origin of this degeneracy?

2. In the generic case the cognitive representation contains only a finite set of points (Fermat theorem, in which one considers rational roots of $x^n + y^n = z^n$, $n > 2$ is a basic example of this) . There are however special cases in which this is not true. In particular, M^4 and its geodesic sub-manifolds provide a good example: all points in the extension of rationals are allowed in M^4 coordinates (note that there are preferred coordinates in the number theoretic context).
The recent situation is indeed highly symmetric due to the Lorentz invariance of space-time surfaces as roots reducing the equations to ordinary algebraic equations for a single complex variable. $X = 0$ condition gives as a result $a_c^2 = constant$ complex hyperboloid with a real mass hyperboloid as a real projection. $a_c^2 = r_n$ is in the extension of rationals as a root of n:th order polynomial. One has the condition $Re(m^2)^2 - Im(m^2) = Re(r_n)$ giving X_r^4 a slicing by real mass hyperboloids. If $Im(m)$ and the spatial part of $Re(m)$ belongs to the extension, one has for real time coordinate $t = \sqrt{r_M^2 + Im(m^2) + r_n}$. If $r_M^2 + Im(m)^2 + r_n$ is a square in the extension also t belongs to the extension. Cognitive representation would contain an infinite number of points and the it would be possible to have non-trivial cluster decompositions. Scattering amplitude would be a sum over different choices of the momenta of the external particles satisfying momentum conservation condition.
As found, the intersection of X_r^4 and X_r^6 is either empty or X_r^4 belongs to X_r^6, Cognitive representations would have an infinite number of points also now by the previous argument. Partonic 2-surfaces at X_r^3 would be replaced with 3-D surfaces in X_r^4 in this situation and would contain a large number of roots. The partonic 2-surfaces would be still present and

correspond to the intersections of incoming space-time surfaces of quarks inside X^6_r. These surfaces would also contain the vertices.

3. Could number theoretic evolution gradually select space-time surfaces for which the number theoretic dynamics involving massive quarks is possible? First would be generic polynomials for which X^3_r would be empty and only massless quarks arriving at the light-cone boundary would be possible. After that surfaces allowing non-empty X^3_r and massive quarks would appear. There is a strong resemblance with the view about cosmological evolution starting from massless phases and proceeding as a sequence of symmetry breakings causing particle massivation. Now the massivation would not be caused by Higgs like fields but have purely number theoretic interpretation and conform with the p-adic mass calculations [K60].
Also a cognitive explosion would occur since these space-time surfaces would be cognitively superior after the emergence of massive quarks. If this picture has something to do with reality, the space-time surfaces contributing to the scattering amplitudes would be very special and interactions could be seen as a kind of number theoretical resonance phenomenon.
4. Even is not enough to obtain genuine particle reaction instead of re-arrangements: one must have also local composites of collinear quarks at the same momentum p identifiable as the sum of parallel momenta discussed in [L81]. This kind of situation is also encountered for on-mass-shell vertices in twistor Grassmannian approach. The local composites could decay to local composites with a smaller number of quarks but respecting momentum conservation. Here the representations of Yangian algebra would come in rescue.

11.5.7 Momentum conservation and on-mass-shell conditions for cognitive representations

Momentum conservation and on-mass shell-conditions are very powerful for cognitive representations, which in the generic case are finite. At mass shells the cognitive representations consist of momenta in the extension of rationals satisfying the condition $p^2 = Re(r_n)$, r_n a complex root of X, which is polynomial of degree n in p^2 defined by the odd part of P. If $\sqrt{Re(r_n)}$ does not belong to the extension defined by P, it can be extended to contain also $\sqrt{Re(r_n)}$.

For Pythagorean triangles in the field of rationals, mass shell condition gives for the momentum components in extension an equation analogous to the equation $k^2 + l^2 = m^2$, which can be most easily solved by noticing that the equation has rotation group $SO(2)$ consisting of rational rotation matrices as symmetries. The solutions are of form $(k = r^2 - s^2, l = 2rs, m = r^2 + s^2)$. By $SO(2)$ invariance, one can choose the coordinate frame so that one has $(k, l) = (r^2 + s^2, 0)$. By applying to this root a rational rotation with $cos(\phi) = (r^2 - s^2)/(r^2 + s^2), sin(\phi) = 2rs/(r^2 + s^2)$ to obtain the general solution $(k = r^2 - s^2, l = 2rs, n = r^2 + s^2)$. The expressions for k and l can be permuted, which means replacing ϕ with $\phi - pi/2$. For a more general case $k^2 + l^2 = n$ one can replace n with $\sqrt{n}$ so that one has an extension of rationals.

For the hyperbolic variants of Pythagorean triangles, one has $k^2 - l^2 = m^2$ or equivalently $l^2 + m^2 = k^2$ giving a Pythagorean triangle. The solution is $k = r^2 + s^2, l = r^2 - s^2, m^2 = 2rs$. The expressions for l and ma can be permuted. Rotation is replaced with 2-D Lorentz boost $cosh(\eta) = (r^2 + s^2)/(r^2 - s^2)$ and $sinh(\eta) = 2rs/(r^2 - s^2)$ with rational matrix elements.

Consider now the 4-D case.

1. The algebra behind the solution depends in no manner on the number field considered and makes sense even for the non-commutative case if m and n commute. Hence one can apply the Pythagorean recipe also in 4-D case to the extension of rationals defined by P by adding to it $\sqrt{r_n}$.
2. Assume that a Lorentz frame can be chosen to be the rest frame in which one has $p = (E = \sqrt{Rer_n}, 0)$ (this might not be possible always). As in the Pythagorean case, there must be a consistency condition. Now it would be of form $E = \sqrt{r_n} = p_0^2 - p_1^2 - p_2^2 - p_3^2$ in the extension defined by $\sqrt{r_n}$. It is not clear whether this condition can be solved for all choices of momentum components in the extension or assuming that algebraic integers of extension are in question. One can also consider an option in which one has algebraic integer divided by some integer N. p-Adic considerations would suggest that prime powers $N = p^k$ might be interesting.

The solutions $\sqrt{r_n} = p_1^2 - p_2^2$ represent a special case. The general solution is obtained by making Lorenz transformation with a matrix with elements in the discrete subgroup of Lorentz group with matrix elements in the extension of rationals.

3. The solutions would define a discretization of the mass shell (3-D hyperbolic space) defined as the orbit of the infinite discrete subgroup of $SO(1,3)$ considered - perhaps the subgroup of $SL(2,C)$ with matrix elements identified as algebraic integers.

If the entire subgroup of SL(2,C) with matrix elements in the extension of rationals is realized, the situation would correspond effectively to a continuous momentum spectrum for infinite cognitive representations. The quantization of momenta is however physically a more realistic option.

1. An interesting situation corresponds to momenta with the same time component, in which case the group would be a discrete subgroup of $SO(3)$. The finite discrete symmetry subgroups act as symmetries of Platonic solids and polygons forming the ADE hierarchy associated to the inclusions of hyperfinite factors of type II_1 and proposed to provide description of finite measurement resolution in TGD framework.
2. The scattering would be analogous to diffraction and only to the directions specified by the vertices of the Platonic solid. Platonic solids, in particular, icosahedron appear also in TGD inspired quantum biology [L24, L106], and also in Nature. Could their origin be traced to $M^8 - H$ duality mapping the Platonic momentum solids to H by inversion?

A more general situation would correspond to the restriction to a discrete non-compact sub-group $\Gamma \subset SL(2,C)$ with matrix elements in the extension of rationals. $Sl(2,C)$ has a representation as Möbius transformations of upper half-plane H^2 of complex plane acting as conformal transformations whereas the action in H^3 is as isometries. The Möbius transformation acting as isometries of H^2 corresponds to $SL(2,Z)$ having also various interesting subgroups, in particular congruence subgroups.

1. Subgroups Γ of the modular group $SL(2,Z)$ define tessellations (analogs of orindary lattices in a curved space) of both H^2 and H^4. The fundamental domain [A7] (`https://cutt.ly/ahBrtT5`) of the tessellation defined by $\Gamma \subset SL(2,C)$ contains exactly one point at from each orbit of Γ. The fundamental domain is analogous to lattice cell for an Euclidian 3-D lattice.
   Γ must be small enough since the orbits would be otherwise dense just like rationals are a dense sub-set of reals. In the case of rationals this leaves into consideration tje modular subgroup $SL(2,Z)$ or its subgroups. In the recent situation an extension of the modular group allowing matrix elements to be algebraic integers of the extension is natural. Physically this would correspond to the quantization of momentum components as algebraic integers. The tessellation in M^8 and its image in H would correspond to reciprocal lattice and lattice in condensed matter physics.
2. So called uniform honeycombs [A14, A9, A27] (see `https://cutt.ly/xhBwTph`, `https://cutt.ly/lhBwPRc`, and `https://cutt.ly/OhBwUOO`) in H^3 assignable to $SL(2,Z)$ can be regarded as polygons in 4-D space and H^3 takes the roles of sphere S^2 for platonic solids for which the tessellation defined by faces is finite.
   The four regular compact honeycombs in H^3 for which the faces and vertex figures (the faces meeting the vertex) are finite are of special interest physically. In the Schönflies notation characterizing polytopes (tessellations are infinite variants of them) they are labelled by (p,q,r), where p is the number of vertices of face, q is the number of faces meeting at vertex, and s is the number of cells meeting at edge.
   The regular compact honeycombs are listed by (5,3,4), (4,3,5), (3,5,3), (5,3,5). For Platonic solids (5,3) characterizes dodecahedron, (4,3) cube, and (3,5) for icosahedron so that these Platonic solids serve as basic building bricks of these tessellations. Rather remarkably, icosahedral symmetries central in the TGD based model of genetic code [L24, L106], characterize cells for 3 uniform honeycombs.

Consider now the momentum conservation conditions explicitly assuming momenta to be algebraic integers. It is natural to restrict the momenta to algebraic integers in the extension of rationals defined by the polynomial P. This allows linearization of the constraints from momentum conservation quite generally.

Pythagorean case allows to guess what happens in 4-D case.

1. One can start from momentum conservation in the Pythagorean case having interpretation in terms of complex integers $p = (r + is)^2 = r^2 - s^2 + 2irs$. The momenta in the complex plane are squares of complex integers $z = r + is$ obtained by map $z \rightarrow w = z^2$ and complex integers. One picks up in the w-plane integer momenta for the incoming and outgoing states satisfying the conservation conditions $\sum_i P_{out,i} = \sum_k P_{in,k}$: what is nice is that the conditions are linear in w-plane. After this one checks whether the inverse images $\sqrt{P_{out,i}}$ and $\sqrt{P_{in,i}}$ are also complex integers.
2. To get some idea about constraints, one can check what CM system for a 2-particle system means (it is not obvious whether it is always possible to find a CM system: one could have massive particles which cannot form a rest system). One must have opposite spatial momenta for $P_1 = (r_1 + is_1)^2$ and $P_2 = (r_2 + is_2)^2$. This gives $r_{s1} = r_2 s_2$. The products $r_i s_i$ correspond to different compositions of the same integer N to factors. The values of $r_i^2 + si^2$ are different.
3. In hyperbolic case one obtains the same conditions since the roles of $r^2 - s^2$ and $r^2 + s^2$ in the conditions are changed so that $r^2 - s^2$ corresponds now to mass mass mass and differs for different decomposition of N to factors. The linearization of the conservation conditions generalizes also to the algebraic extensions of rationals with integers replaced by algebraic integers.

The generalization to the 4-D case is possible in terms of octonions.

1. Replace complex numbers by quaternions $q = q_0 + \overline{q}$. The square of quaternion is $q^2 = q_0^2 - \overline{q} \cdot \overline{q} + 2iq_0\overline{q}$. Allowed momenta for given mass correspond to points in q^2-plane. Conservation conditions in the q^2 plane are linear and satisfied by quaternionic integers, which are squares. So that in the q^2 plane the allowed momenta form an integer lattice and the identification as a square selects a subset of this lattice. This generalizes also to the algebraic integers in the extension of rationals.

2. What about the co-associative case corresponding to the canonical basis $\{I_1, iI_3, iI_5, iI_7\}$? Momenta would be as co-associative octonion o but o^2 is a quaternion in the plane defined by $\{I_0, iI_2, iI_4, iI_6\}$. o representable in terms of a complexified quaternion $q = q_0 + i\overline{q}$ as $o = I_4 q$ and the in general complex values norm squared is give by $o\overline{o}$ with conjugation of octonionic imaginary units but not i: this gives Minkowskian norm squared. This reduces the situation to the quaternionic case.
3. In this case the CM system for two-particle case corresponds to the conditions $q_{1,0}\overline{q}_1 = q_{2,0}\overline{q}_2$ implying that q_1 and q_2 have opposite directions and $q_{1,0}|\overline{q}_1| = q_{2,0}|\overline{q}_2|$. The ratio of the lengths of the momenta is integer. Now the squares $q_{i,0}|\overline{q}_i|^2$, $i = 1, 2$ are factorizations of the same integer N. Masses are in general different.
4. The situation generalizes also to complexified quaternions - the interpretation of the imaginary part of momentum might be in terms of a decay width - and even to general octonions since associativity is not involved with the conditions.

11.5.8 Further objections

The view about scattering amplitudes has developed rather painfully by objections creating little shocks. The representation of scattering amplitudes is based on quark oscillator operator algebra. This raises two further objections.

The non-vanishing contractions of the oscillator operators are necessary for obtaining non-trivial scattering amplitudes but is this condition possible to satisfy.

1. One of the basic deviations of TGD from quantum field theories (QFTs) is the hypothesis that all elementary particles, in particular bosons, can be described as bound states of fermions, perhaps only quarks. In TGD framework the exchange of boson in QFT would mean an emission of a virtual quark pair and its subsequent absorption. In ZEO in its basic form this seems to be impossible.
2. If scattering corresponds to algebra morphism mapping products to products of co-products - the number of quarks in say future HC is higher than in the past HC as required. But how to obtain non-vanishing scattering amplitudes? There should be non-vanishing counterparts

of propagators between points of FHC but this is not possible if only creation operators are present in a given HC as ZEO requires. All particle reactions would be re-arrangements of quarks and antiquarks to elementary fermions and bosons (OZI rule of the hadronic string model: `https://en.wikipedia.org/wiki/OZI_rule`). The emission of virtual or real bosons requires the creation of quark antiquark pairs and seems to be in conflict with the OZI rule.

3. It would be natural to assign to quarks and bosons constructed as their bound states non-trivial inner product in a given HC of CD. Is this possible if the counterparts of annihilation operators act as creation operators in the opposite HC? Can one assign inner product to a given boundary of CD by assuming that hermitian conjugates of quark oscillator operators act in the dual Hilbert space of the quark Fock space? Could this dual Hilbert space relate to the Drinfeld's double?

How could one avoid the OZI rule?

1. Is it enough to also allow annihilation operators in given HC? Bosonic $G_{2,c}$ generators could involve them. The decay of boson to quark pair would still correspond to re-arrangement but one would have inner product for states at given HC. The creation of bosons would still be a problem. Needless to say, this option is not attractive.
2. A more plausible solution for this problem is suggested by the phenomenological picture in which quarks at the level of H are assigned with partonic 2-surfaces and their orbits, string world sheets, and their boundaries at the orbits of partonic 2-surfaces. By the discussion in the beginning of this section, these surfaces could correspond at the level of M^8 to space-time regions of complexified space-time surface with real number theoretic metric having signature (+,+,-,-), (+,+,+,-), (+,+,+,+) having 2,3, or 4 time-like dimensions. They would allow non-negative values of mass squared and would be separated from the region of Minkowskian signature by a transition region space-time region with dimension $D \in \{3,2,1\}$ mapped to CP_2.
In these regions one would have 1, 2, or 3 additional energy like momentum components $p_i = E_i$. E_i. Could the change of sign for E_i transform creation operator to annihilation operator as would look natural. This would give bosonic states with a non-vanishing norm and also genuine boson creation. What forces to take this rather radical proposal seriously that it conforms with the phenomenological picture.
In this region one could have a non-trivial causal diamond CD with signature (+,+,-,-), (+,+,+,-). For the signature (+,+,+,+) CD reduces to a point with a vanishing four-momentum and would correspond to CP_2 type extremals (wormhole contacts). Elementary fermions and bosons would consist of quarks in regions with signature (+,+,-,-) and (+,+,+,-). It would seem that the freedom to select signature in twistorial amplitude is not mere luxury but has very deep physical content.

One can invent a further objection. Suppose that the above proposal makes sense and allows to assign propagators to a given HC. Does Yangian co-product allow a construction of zero energy states giving rise to scattering amplitudes, which typically have a larger number of particles in the future HC (FHC) than in past HC (PHC) and represent a genuine creation of quark pairs?

1. One can add to the PHC quarks and bosons one-by-one by forming the product super $G(2,c)$ generators assignable to the added particles. To the FHC one would add the product of co-products of these super $G(2,c)$ generators (co-product of product is product of co-products as an algebra morphism).
2. By the basic formula of co-product each addition would correspond to a superposition of two states in FHC. The first state would be the particle itself having suffered a forward scattering. Second state would involve 2 generators of super $G_{2,c}$ at different momenta summing up to that for the initial state, and represent a scattering $q \rightarrow q+b$ for a quark in the initial state and scattering $b \rightarrow 2b$, $b \rightarrow 2b$, or $b \rightarrow 2q$ for a boson in the initial state.
Number theoretic momentum conservation assuming momenta to be algebraic integers should allow processes in which quark oscillator operators are contracted between the states in FHC and PHC or between quarks in the FHC.
3. Now comes the objection. Suppose that the state in PC consists of fundamental quarks. Also the FC containing the product of co-products of quarks must contain these quarks

with the same momenta. But momentum conservation does not allow anything else in FC! The stability of quarks is a desirable property in QFTs but something goes wrong! How to solve the problem?
Also now phenomenological picture comes to the rescue and tells that elementary particles - as opposed to fundamental fermions - are composites of fundamental fermions assignable to flux tubes like structures involving 2 wormhole contacts. In particular, quarks as elementary particles would involve quark at either throat of the first wormhole contact and quark-antiquark pair associated with the second wormhole contact. The state would correspond to a quantum superposition of different multilocal momentum configurations defining multi-local states at M^8 level. The momentum conservation constraint could be satisfied without trivializing the scattering amplitudes since the contractions could occur between different components of the superposition - this would be essential.
Note also that at H level there can be several quarks at a given wormhole throat defining a multilocal state in M^8: one could have a superposition of these states with different momenta and again different components of the wave function could contract. By Uncertainty Principle the almost locality in H would correspond to strong non-locality in M^8. This could be seen as an approximate variant of the TGD variant of H variant of SUSY considered in [L81].
Could the TGD variant of SUSY proposed in [L81] but realized at the level of momentum space help to circumvent the objection? Suppose that the SUSY multiplet in M^8 can be created by a local algebraic product possessing a co-product delocalizing the local product of oscillator operators at point p in PC and therefore represents the decay of the local composite to factors with momenta at p_1 and $p - p_1$ in FC. This would not help to circumvent the objection. Non-locality and wave functions in momentum space is needed.

11.6 Symmetries in M^8 picture

11.6.1 Standard model symmetries

Can one understand standard model symmetries in M^8 picture?

1. $SU(3) \subset G_2$ would respect a given choice of time axis as preferred co-associative set of imaginary units ($I_2 \subset \{I_2, iI_3, iIb, iI_7\}$ for the canonical choice). The labels would therefore correspond to the group $SU(3)$. $SU(3)_c$ would be analogous to the local color gauge group in the sense that the element of local $SU(3)_c$ would generate a complecofied space-time surface from the flat and real M^4. The real part of pure $SU(3)_c$ gauge potential would not however reduce to pure $SU(3)$ gauge potential. Could the vertex factors be simply generators of $SU(3)$ or $SU(3)_c$?
2. What about electroweak quantum numbers in M^8 picture? Octonionic spinors have spin and isospin as quantum numbers and can be mapped to H spinors. Bosons would be bound states of quarks and antiquarks at both sides.
How could electroweak interactions emerge at the level of M^8? At the level of H an analogous problem is met: spinor connection gives only electroweak spinor connection but color symmetries are isometries and become manifest via color partial waves. Classical color gauge potentials can be identified as projections of color isometry generators to the space-time surface.
Could electroweak gauge symmetries at the level of M^8 be assigned with the subgroup $U(2) \subset SU(3)$ of $CP_2 = SU(3)/U(2)$ indeed playing the role of gauge group? There is a large number of space-time surfaces mapped to the same surface in H and related by a local $U(2)$ transformation. If this transformation acted on the octonionic spinor basis, it would be a gauge transformation but this is not the case: constant octonion basis serves as a gauge fixing. Also the space-time surface in M^8 changes but preserves its "algebraic shape".

11.6.2 How the Yangian symmetry could emerge in TGD?

Yangian symmetry [A29, A71] appears in completely 2-D systems. The article [B30] (`https://arxiv.org/pdf/1606.02947.pdf`) gives a representation which is easy to understand by a physicist like me whereas the Wikipedia article remains completely incomprehensible to me.

Yangian symmetry is associated with 2-D QFTs which tend to be physically rather uninteresting. The scattering is in forward direction and only phase shifts are induced. There is no particle creation. Yangian symmetry appears in 4-D super gauge theories [B17] and in the twistor approach to scattering amplitudes [B18, B25, B21, ?]. I have tried to understand the role of Yangian symmetry in TGD [L42].

Yangian symmetry from octonionic automorphisms

An attractive idea is that the Yangian algebra having co-algebra structure could allow to construct poly-local conserved charges and that these could define vertex operators in M^8.

1. Yangian symmetry appears in 2-D systems only. In TGD framework strings world sheets could be these systems as co-commutative 2-surfaces of co-associative space-time surface.
2. What is required is that there exists a conserved current which can be also regarded as a flat connection. In TGD the flat connection would a connection for $G_{2,c}$ or its subgroup associated with the map taking standard co-associative sub-space of O_c for which the number theoretic norm squared is real and has Minkowski signature (M^4 defined by the canonical choice $\{I_2, iI_3, iI_5, iI_7\}$.
   The recent picture about the solution of co-associativity conditions fixes the subgroup of G_2 to $SU(3)$. X^4 corresponds to element g of the local $SU(3)$ acting on preferred $M^4 \subset M^8_c$ with the additional condition that the 4-surface $X^4 \subset M^8$ is invariant under $U(2) \subset SU(3)$ so that each point of X^4 corresponds to a CP_2 point. At the mas shells as roots of a polynomial P, g reduces to unity and the 4-D tangent space is parallel to the preferred M^4 on which g acts. One can induce this flat connection to string world sheet and holomorphy of g at this surface would guarantee the conservation of the current given by $j_{0)} = g^{-1}dg$.
3. Under these conditions the integral of the time component of current along a space-like curve at string world sheets with varying end point is well-defined and the current
   $$j_{1)}(x) = \epsilon\mu\nu j_{0),\nu}(x) - \frac{1}{2}[j^{\mu}_{0)}(x,t), \int^x j^0_{0)}(t,y)dy]$$
   is conserved. This is called the current at first level. Note that the currents have values in the Lie algebra considered. It is essential that the integration volume is 1-D and its boundary is characterized by a value of single coordinate x.
4. One can continue the construction by replacing j_0 with j_1 in the above formula and one obtains an infinite hierarchy of conserved currents $j_{n)}$ defined by the formula
   $$j_{n+1)}(x) = \epsilon\mu\nu j_{n),\nu}(x) - \frac{1}{2}[j^{\mu}_{n)}(x,t), \int^x j^0_{n)}(t,y)dy] \quad (11.6.1)$$
   The corresponding conserved charges Q_n define the generators of Yangian algebra.
5. 2-D metric appears in the formulas. In the TGD framework one does not have Riemann metric - only the number theoretic metric which is real only at real space-time surfaces already discussed. Is the (effective) 2-dimensionality and holomorphy enough to avoid the possible problems? Holomorphy makes sense also number theoretically and implies that the metric disappears from the formulas for currents. Also current conservation reduces to the statement of that current is equivalent to complex differential form.
6. Conserved charges would however require a 1-D integral and number theory does not favor this. The solution of the problem comes from the observation that one can construct a slicing of string world sheet to time-like curves as Hamiltonian orbits with Hamiltonian belonging to the Yangian algebra and defined by the conserved current by standard formula $j^{\alpha} = J^{\alpha\beta}\partial_{\beta}H$ in terms of Kähler form defined by the 2-D Kähler metric of string world sheet. This generalizes to Minkowskian signature and also makes sense for partonic 2-surfaces. Hamiltonians become the classical conserved charges constant along the Hamiltonian orbit. This gives an infinite hierarchy of conserved Hamiltonian charges in involution. Hamiltonian can be any combination of the Hamiltonians in the hierarchy and labelled by a non-negative integer and the label of $G_{2,c}$ generator. This is just what integrability implied by Yangian algebra means. Co-associativity and co-commutativity would be the deeper number theoretic principles implying the Yangian symmetry.

7. Could one formulate this argument in dimension $D = 4$? Could one consider instead of local current the integral of conserved currents over 2-D surfaces labelled by single coordinate x for a given value of t? If the space-time surface in M^8 (analog of Fermi sphere) allows a slicing by orthogonal strings sheets and partonic 2-surfaces, one might consider the fluxes of the currents $g^{-1}dg$ over the 2-D partonic 2-surfaces labelled by string coordinates (t, x) as effectively 2-D currents, whose integrals over x would give the conserved charge. Induced metric should disappear from the expressions so that fluxes of holomorphic differential forms over partonic 2-surface at (t, x) should be in question. Whether this works is not clear.

One should interpret the above picture at the level of momentum space instead of ordinary space-time. The roles of momentum space and space-time are changed. At this point, one can proceed by making questions.

1. One should find a representation for the algebra of the Hamiltonians associated with $g(x)$ defining the space-time surface. The charges are associated with the slicings of string world sheets or partonic 2-surfaces by the orbits of Hamiltonian dynamics defined by a combination of conserved currents so that current conservation becomes charge conservation. These charges are labelled by the coordinate x characterizing the slices defined by the Hamiltonian orbits and from these one can construct a non-local basis discrete basis using Fourier transform.
2. What the quantization of these classical charges - perhaps using fermionic oscillator operators in ZEO picture for which the local commutators vanish - could mean (only the anti-commutators of creation operators associated with the opposite half-cones of CD with opposite momenta are non-vanishing)? Do the Yangian charges involve only creation operators of either type with the same 8-momentum as locality at M^8 level suggests? Locality is natural l since these Yangian charges are analogous to charges constructed from local currents at space-time level.
3. Could the Yangian currents give rise to poly-local charges assignable to the set of vertices in a cognitive representation and labelled by momenta? Could the level n somehow correspond to the number n of the vertices and could the co-product Δ generate the charges? What does the tensor product appearing in the co-product really mean: do the sector correspond to different total quark numbers for the generators? Is it a purely local operation in M^8 producing higher monomials of creation operators with the same momentum label or is superposition over Hamiltonian slices by Fourier transform possibly involved ?

How to construct quantum charges

One should construct quantum charges. In the TGD framework the quantization of $g(x)$ is not an attractive idea. Could one represent the charges associated with g it in terms of quark oscillator operators induced from the second quantized E^8 spinors so that propagators would emerge in the second quantization? Analogs of Kac Moody representations but with a non-negative spectrum of conformal weights would be in question. Also super-symplectic algebra would have this property making the formulation of the analogs of gauge conditions possible, and realizing finite measurement resolution in terms of hierarchy of inclusions of hyper-finite factors of type II_1 [K112, K43]. The Yangian algebra for $G_{2,c}$ or its subgroup could be the counterpart for these symmetries at the level of H.

The following proposal for the construction for the charges and super-charges of Yangian algebra in terms of quark oscillator operators is the first attempt.

1. One knows the Lie-algebra part of Yangian from the Poisson brackets of Hamiltonians associated with string world sheet slicing and possibly also for a similar slicing for partonic 2-surfaces. One should construct a representation in terms of quark ocillator operators in ZEO framework for both Lie-algebra generators and their super-counterparts. Also co-product should be needed.
2. The oscillator operators of E^8 spinor field located at the points of X^4 are available. The charges must be local and describe states with non-linear quarks and antiquarks.
   One must construct conserved charges as currents associated with the Hamiltonian orbits. Bosonic currents are bilinear in quark and antiquark oscillator operators and their super counterparts linear in quark or antiquark oscillator operators.

3. Since the system is 2-D one can formally assume in Euclidian signature (partonic 2-surface) Kähler metric $g^{z\overline{z}}$ and Kähler form $J^{z\overline{z}} = igz\overline{z}$, which is antisymmetric and real in real coordinates ($J^{kl} = -J^{lk}$) knowing that they actually disappear from the formulas. One can also define gamma matrices $\Gamma_\alpha = \gamma_k \partial_\alpha p^k$ as projections of embedding space gamma matrices to the string world sheet. In the case of string world sheet one can introduce light-like coordinates (u, v) as analogous of complex coordinates and the only non-vanishing component of the metric is g^{uv}.
4. The claim is that the time components J^u_n the bosonic currents

$$J^\alpha_n = b^\dagger_p \overline{v}(p) \Gamma^\alpha H_n u(p) a^\dagger \tag{11.6.2}$$

at the Hamiltonian curves with time coordinate t define conserved charges ($\alpha \in \{u, v\}$ at the string world sheet).
Remark: v_p corresponds to momentum $-p$ for the corresponding plane wave in the Fourier expansion of quark field but the physical momentum is p and the point of M^8 that this state corresponds.
Therefore one should have

$$\frac{J^u_n}{du} = 0 \tag{11.6.3}$$

One can check by a direct calculation what additional conditions are possibly required by this condition.
5. The first point is that H_n is constant if $v = constant$ coordinate line is a Hamiltonian orbit. Also oscillator operators creating fermions and antifermions are constant. The derivative of $u(p)$ is

$$\frac{du(p)}{du} = \frac{\partial u(p)}{\partial_{p^k}} \frac{dp^k}{du} \quad .$$

. u_p is expressible as $u_p = Du_a$, where D is a massless Dirac operator in M^8 and u_a is a constant 8-D quark spinor with fixed chirality. D is sum of M^4- and E^4 parts and M^4 part is given by $D(M^4) = \gamma^k p_k$ so that one has $dp^k/dt = \gamma_r dp^r/dt$.
This gives

$$\frac{d(\Gamma^u H_n u(p))}{du} = g^{uv} \gamma_k \partial_v p^k \frac{du(p)}{du} = g^{uv} \partial_u p \cdot \partial_v p \quad .$$

If the tangent curves of u and v are orthogonal in the induced metric and $v = 0$ constant lines are Hamiltonian orbits the bosonic charges are conserved.
One can perform a similar calculation for $d\frac{d\overline{v}(p)}{du}$ and the result is vanishing.
One must also have $dg^{uv}/du = 0$. This should reduce to the covariant constancy of g^{uv}. If the square root of the metric determinant for string world sheet is included it cancels g^{uv}.
6. From the bosonic charges one construct corresponding fermionic super charges by replacing the fermionic or anti-quark oscillator operator part with a constant spinor.

The simplest option is that partonic 2-surfaces contain these operators at points of cognitive representation. One can ask whether co-product could forces local operators having a higher quark number. What is clear that this number is limited to the number $n = 0$ of spin degrees of $n = 8$.

1. The commutators of bosonic and fermionic charges are fermionic charges and co-product would in this case be a superposition of tensor products of bosonic and fermionic charges, whose commutator gives bosonic charge. Now however the bosonic and fermionic charges commute in the same half-cone of CD. Does this mean that the tensor product in question must be tensor product for the upper and lower half-cones of CD?
For instance, in the fermionic case one would obtain superposition over pairs of fermions at say lower half-cone and bosons at the upper half-cone. The momenta would be opposite meaning that a local bosonic generator would have total momentum $2p$ at point p and fermionic generator at opposite cone would have momentum $-p$. The commutator would have momentum p as required. In this manner one could create bosons in either half-cone.

2. One can also assign to the bosonic generators a co-product as a pair of bosonic generators in opposite half-cones commuting to the bosonic generator. Assume that bosonic generator is at lower half-cone. Co-product must have a local composite of 4 oscillator operators in the lower half-cone and composite of 2 oscillator operators in the upper half-cone. Their anti-commutator contracts two pairs and leaves an operator of desired form. It therefore seems.
   Statistics allows only generators with a finite number of oscillator operators corresponding to 8 spin indices, which suggests an interpretation in terms of the proposed SUSY [L81]. The roots of P are many-sheeted coverings of M^4 and this means that there are several 8-momenta with the same M^4 projection. This degree of freedom corresponds to Galois degrees of freedom.
3. Only momenta in cognitive representation are allowed and momentum is conserved. The products of generators appearing in the sum defining the co-product of a given generator T, which is a local composite of quarks, would commute or anti-commute to T, and their momenta would sum-up to the momentum associated with T. The co-product would be poly-local and receive contributions from the points of the cognitive representation. Also other quantum numbers are conserved.

About the physical picture behind Yangian and definition of co-product

The physical picture behind the definition of Yangian in the TGD framework differs from that adopted by Drinfeld, who has proposed - besides a general definition of the notion of quantum algebra - also a definition of Yangian. In the Appendix Drinfeld's definition is discussed in detail: this discussion appears almost as such in [L42].

1. Drinfeld proposes a definition in terms of a representation in terms of generators of a free algebra to which one poses relations [B37]. Yangian can be seen as an analog of Kac-Moody algebra but with generators labelled by integer $n \geq 0$ as an analog of non-negative conformal weight. Also super-symplectic algebra has this property and its Yangianization is highly suggestive. The generators of Yangian as algebra are elements J_n^A, $n \geq 0$, with $n = 0$ and $n = 1$. Elements J_0^A define the Lie algebra and elements J_1^A transform like Lie-algebra elements so that commutators at this level are fixed.
   Remark: I have normally used generator as synonym for the element of Lie algebra: I hope that this does not cause confusion
   The challenge is to construct higher level generators J_n^A. Their commutators with JA^0 with J^An are fixed and also the higher level commutators can be guessed from the additivity of n and the transformation properties of generators J_n^A. The commutators are very similar to those for Kac-Moody algebra. In the TGD picture the representation as Hamiltonians fixes these commutation relations as being induced by a Poisson bracket. The Lie-algebra part of Yangian can be therefore expressed explicitly.
2. The challenge is to understand the co-product Δ. The first thing to notice is that Δ is a Lie algebra homomorphism so that one has $\Delta(XY) = \Delta(X)\Delta(Y)$ plus formulas expressing linearity. The intuitive picture is that Δ adds a tensor factor and is a kind of time reversal of the product conserving total charges and the total value of the weight n. Already this gives a good overall view about the general structure of the co-commutation relations.
   The multiplication of generators by the unit element Id of algebra gives the generator itself so that $\Delta(J_A)$ should involve part $Id \otimes J^A \oplus J^A \otimes Id$. Generators are indeed additive in the ordinary tensor product for Lie-algebra generators - for instance, rotation generators are sums of those for the two systems. However, one speaks of interaction energy: could the notion of "interaction quantum numbers" make sense quite generally. Could this notion provide some insights to proton spin puzzle [C28] meaning that quark spins do not seem to contribute considerably to proton spin? A possible TGD based explanation is in terms of angular momentum associated with the color magnetic flux tubes [K64], and the formulation of this notion at M^8 level could rely on the notion of "interaction angular momentum".
   The time reversal rule applied to $[J_A^m, J_B^n] \propto f_{ABC}J_C^{m+n}$ suggests that $\Delta(T_A^n)$ contains a term proportional to $f_{CBA}J_C^m \otimes J_B^{n-m}$. This would suggest that co-product as a time reversal involves also in the case of J_A^0 the term $k_1 f_{CBA}J_C^0 \otimes J_B^0$, where k_1 as an analog of interaction energy.
   Drinfeld's proposal does not involve this term in accordance with Drinfeld's intuition that co-product represents a deformation of Lie-algebra proportional to a parameter denoted by

$\hbar$, which need not (and cannot!) actually correspond to $\hbar$. This view could be also defended by the fact that J_0^A do not create physical states but only measures the quantum numbers generated by J_A^n, $n > 0$. TGD suggests interpretation as the analog of the interaction energy.

3. In Drinfeld's proposal, the Lie-algebra commutator is taken to be $[J_A^0, J_B^0] = kf_{ABC}J_C^0$, $k = 1$. Usually one thinks that generators have the dimension of $\hbar$ so that dimensional consistency requires $k = \hbar$. It seems that Drinfeld puts $\hbar = 1$ and the $\hbar$ appearing in the co-product has nothing to do with the actual $\hbar$.
   The conservation of dimension applied to the co-product would give $k_1 = 1/\hbar$! What could be the interpretation? The scattering amplitudes in QFTs are expanded in powers of gauge coupling strengths $\alpha = g^2/4\pi\hbar$. In ZEO co-product would be essential for obtaining non-trivial scattering amplitudes and the expansion in terms of $1/\hbar$ would emerge automatically from the corrections involving co-products - in path integral formalism this expansion emerges from propagors
   This view would also conform with the vision that Mother Nature loves her theoreticians. The increase of $h_{eff}/h_0 = n$ as dimension of extension of rationals would be Mother Nature's way to make perturbation theory convergent [K42]. The increase of the degree of P defining the space-time surface increases the algebraic complexity of the space-time surface but reduces the value of α as a compensation.

4. Drinfeld gives the definition of Yangian in terms of relations for the generating elements with weight $n = 0$ and $n = 1$. From these one can construct the generators by applying Δ repeatedly. Explicit commutation relations are easier to understand by a physicist like me, and I do not know whether the really nasty looking representation relations - Drinfeld himself calls "horrible" [B30] - are the only manner to define the algebra. In the TGD framework the definition based on the idea about co-product as a strict time reversal of product would mean deviation in the $n = 0$ sector giving rise to an interaction term having natural interpretation as analog of interaction energy.

5. Drinfeld proposes also what is known as Drinfeld's double [A73] (see `http://tinyurl.com/y7tpshkp`) as a fusion of two Hopf algebras and allowing to see product and co-product as duals of each other. The algebra involves slight breaking of associativity characterized by Drinfeld's associator. ZEO suggests [K53] that the members of Drinfeld's double correspond to algebra and co-algebra located at the opposite half-cones and there are two different options. Time reversal occurring in "big" state functions reductions (BSFRs) would transform the members to each other and change the roles of algebra and co-algebra (fusion would become decay).

In the TGD framework there is also an additional degree of freedom related to the momenta in cognitive representation, which could be regarded also as a label of generators. The idea that commutators and co-commutators respect conservation of momentum allows the fixing of the general form of Δ. Co-product of a generator at momentum p ina given half-cone would be in the opposite half-cone and involve sum over all momentum pairs of generators at p_1 and p_2 with the constraint $p_1 + p_2 + p = 0$.

Summation does not make sense for momenta in the entire extension of rationals. The situation changes if the momenta are algebraic integers for the extension of rationals considered: quarks would be particles in a number theoretic box. In the generic case, very few terms - if any - would appear in the sum but for space-time surfaces as roots of octonionic polynomials this is not the case. The co-products would as such define the basic building bricks of the scattering amplitudes obtained as vacuum expectation reducing the pairs of fermions in opposite half-cones to propagators.

11.7 Appendix: Some mathematical background about Yangians

In the following necessary mathematical background about Yangians are summarized.

11.7.1 Yang-Baxter equation (YBE)

YBE has been used for more than four decades in integrable models of statistical mechanics of condensed matter physics and of 2-D quantum field theories (QFTs) [A71]. It appears also in topological quantum field theories (TQFTs) used to classify braids and knots [B17] (see http://tinyurl.com/mcvvcqp) and in conformal field theories and models for anyons. Yangian symmetry appears also in the twistor Grassmann approach to scattering amplitudes [B18, B25] and thus involves YBE. At the same time new invariants for links were discovered and a new braid-type relation was found. YBEs emerged also in 2-D conformal field theories.

Yang-Baxter equation (YBE) has a long history described in the excellent introduction to YBE by Jimbo [B37] (see http://tinyurl.com/l4z6zyr, where one can also find a list of references). YBE was first discovered by McGuire (1964) and 3 years later by Yang in a quantum mechanical many-body problem involving a delta function potential $\sum_{i<j}\delta(x_i-x_j)$. Using Bethe's Ansatz for building wave functions they found that the scattering matrix factorized that it could be constructed using as a building brick 2-particle scattering matrix - R-matrix. YBE emerged for the R-matrix as a consistency condition for factorization. Baxter discovered in 1972 a solution of the eight vertex model in terms of YBE. Zamolodchikov pointed out that the algebraic mechanism behind factorization of 2-D QFTs is the same as in condensed matter models.

1978-1979 Faddeev, Sklyanin, and Takhtajan proposed a quantum inverse scattering method as a unification of classical and quantum integrable models. Eventually the work with YBE led to the discovery of the notion of quantum group by Drinfeld. Quantum group can be regarded as a deformation $U_q(g)$ of the universal enveloping algebra $U(g)$ of Lie algebra. Drinfeld also introduced the universal R-matrix, which does not depend on the representation of algebra used.

R-matrix satisfying YBE is now the common aspect of all quantum algebras. I am not a specialist in YBE and can only list the basic points of Jimbo's article. The interested reader can look for details and references in the article of Jimbo.

In 2-D quantum field theories R-matrix $R(u)$ depends on one parameter u identifiable as hyperbolic angle characterizing the velocity of the particle. $R(u)$ characterizes the interaction experienced by two particles having delta function potential passing each other (see the figure of http://tinyurl.com/kyw6xu6). In 2-D quantum field theories and in models for basic gate in topological quantum computation the R-matrix is unitary. R-matrix can be regarded as an endomorphism mapping $V_1\otimes V_2$ to $V_2\otimes V_1$ representing permutation of the particles.

YBE

R-matrix satisfies Yang-Baxter equation (YBE)

$$R_{23}(u)R_{13}(u+v)R_{12}(v) = R_{12}(v)R_{13}(u+v)R_{23}(u) \tag{11.7.1}$$

having interpretation as associativity condition for quantum algebras.

At the limit $u,v\rightarrow\infty$ one obtains R-matrix characterizing braiding operation of braid strands. Replacement of permutation of the strands with braiding operation replaces permutation group for n strands with its covering group. YBE states that the braided variants of identical permutations (23)(13)(12) and (12)(13)(23) are identical.

The equations represent n^6 equations for n^4 unknowns and are highly over-determined so that solving YBE is a difficult challenge. Equations have symmetries, which are obvious on the basis of the topological interpretation. Scaling and automorphism induced by linear transformations of V act as symmetries, and the exchange of tensor factors in $V\otimes V$ and transposition are symmetries as also shift of all indices by a constant amount (using modulo N arithmetics).

One can pose to the R-matrix some boundary condition. For $V\otimes V$ the condition states that $R(0)$ is proportional to the permutation matrix P for the factors.

General results about YBE

The following lists general results about YBE.

1. Belavin and Drinfeld proved that the solutions of YBE can be continued to meromorphic functions in the complex plane with poles forming an Abelian group. R-matrices can be

classified to rational, trigonometric, and elliptic R-matrices existing only for $sl(n)$. Rational and trigonometric solutions have a pole at origin and elliptic solutions have a lattice of poles. In [B37] (see http://tinyurl.com/l4z6zyr) simplest examples about R-matrices for $V_1 = V_2 = C^2$ are discussed, one of each type.

2. In [B37] it is described how the notions of R-matrix can be generalized to apply to a collection of vector spaces, which need not be identical. The interpretation is as commutation relations of abstract algebra with co-product Δ - say quantum algebra or Yangian algebra. YBE guarantees the associativity of the algebra.
3. One can define quasi-classical R-matrices as R-matrices depending on Planck constant like parameter $\hbar$ (which need have anythingto do with Planck constant) such that small values of u one has $R = constant \times (I + \hbar r(u) + O(\hbar^2))$. $r(u)$ is called classical r-matrix and satisfies CYBE conditions

$$[r_{12}(u), r_{13}(u+v)] + [r_{12}(u), r_{23}(v)] + [r_{13}(u+v), r_{23}(v)] = 0$$

   obtained by linearizing YBE. $r(u)$ defines a deformation of Lie-algebra respecting Jacobi-identities. There are also non-quasi-classical solutions. The universal solution for r-matrix is formulated in terms of Lie-algebra so that the representation spaces V_i can be any representation spaces of the Lie-algebra.

4. Drinfeld constructed quantum algebras $U_q(g)$ as quantized universal enveloping algebras $U_q(g)$ of a Lie algebra g. One starts from a classical r-matrix r and Lie algebra g. The idea is to perform a "quantization" of the Lie-algebra as a deformation of the universal enveloping algebra $U_q(g)$ of $U(g)$ by r. Drinfeld introduces a universal R-matrix independent of the representation used. This construction will not be discussed here since it does not seem to be as interesting as Yangian: in this case co-product Δ does not seem to have a natural interpretation as a description of interaction. The quantum groups are characterized by parameter $q \in C$. For a generic value the representation theory of q-groups does not differ from the ordinary one. For roots of unity situation changes due to degeneracy caused by the fact $q^N = 1$ for some N.
5. The article of Jimbo discusses also a fusion procedure initiated by Kulish, Restetikhin, and Sklyanin allowing to construct new R-matrices from existing one. Fusion generalizes the method used to construct group representation as powers of fundamental representation. Fusion procedure constructs the R-matrix in $W \otimes V^2$, where one has $W = W_1 \otimes W_2 \subset V \otimes V^1$. Picking W is analogous to picking a subspace of tensor product representation $V \otimes V^1$.

11.7.2 Yangian

Yangian algebra $Y(g(u))$ is associative Hopf algebra (see http://tinyurl.com/qfl8dwu) that is bi-algebra consisting of associative algebra characterized by product μ: $A \otimes A \to A$ with unit element 1 satisfying $\mu(1, a) = a$ and co-associative co-algebra consisting of co-product $\Delta A \in A \otimes A$ and co-unit $\epsilon : A \to C$ satisfying $\epsilon \circ \Delta(a) = a$. Product and co-product are "time reversals" of each other. Besides this one has antipode S as algebra anti-homomorphism $S(ab) = S(b)S(a)$. YBE has interpretation as an associativity condition for co-algebra $(\Delta \otimes 1) \circ \Delta = (1 \otimes \Delta) \circ \Delta$. Also ϵ satisfies associativity condition $(\epsilon \otimes 1) \circ \Delta = (1 \otimes \epsilon) \circ \Delta$.

There are many alternative formulations for Yangian and twisted Yangian listed in the slides of Vidas Regelskis at http://tinyurl.com/ms9q8u4. Drinfeld has given two formulations and there is FRT formulation of Faddeev, Restetikhin and Takhtajan.

Drinfeld's formulation [B37] (see http://tinyurl.com/qfl8dwu) involves the notions of Lie bi-algebra and Manin triple, which corresponds to the triplet formed by half-loop algebras with positive and negative conformal weights, and full loop algebra. There is isomorphism mapping the generating elements of positive weight and negative weight loop algebra to the elements of loop algebra with conformal weights 0 and 1. The integer label n for positive half loop algebra corresponds in the formulation based on Manin triple to conformal weight. The alternative interpretation for $n + 1$ would be as the number of factors in the tensor power of algebra and would in TGD framework correspond to the number of partonic 2-surfaces. In this interpretation the isomorphism becomes confusing.

In any case, one has two interpretations for $n+1 \geq 1$: either as parton number or as occupation number for harmonic oscillator having interpretation as bosonic occupation number in quantum field theories. The relationship between Fock space description and classical description for n-particle states has remained somewhat mysterious and one can wonder whether these two interpretations improve the understanding of classical correspondence (QCC).

Witten's formulation of Yangian

The following summarizes my understanding about Witten's formulation of Yangian for $\mathcal{N}=4$ SUSY [B17], which does not mention explicitly the connection with half loop algebras and loop algebra and considers only the generators of Yangian and the relations between them. This formulation gives the explicit form of Δ and looks natural, when n corresponds to parton number. Also Witten's formulation for Super Yangian will be discussed.

However, it must be emphasized that Witten's approach is not general enough for the purposes of TGD. Witten uses the identification $\Delta(J_1^A) = f^A_{BC}J_0^B \times J_0^C$ instead of the general expression $\Delta(J_1^A) = J_1^A \otimes 1 + 1 \times J_1^A + f^A_{BC}J_0^B \times J_0^C$ needed in TGD strongly suggested by the dual roles of the super-symplectic conformal algebra and super-conformal algebra associated with the light-like partonic orbits realizing generalized EP. There is also a nice analogy with the conformal symmetry and its dual twistor Grassmann approach.

The elements of Yangian algebra are labelled by non-negative integers so that there is a close analogy with the algebra spanned by the generators of Virasoro algebra with non-negative conformal weight. The Yangian symmetry algebra is defined by the following relations for the generators labeled by integers $n=0$ and $n=1$. The first half of these relations discussed in very clear manner in [B17] follows uniquely from the fact that adjoint representation of the Lie algebra is in question

$$\left[J^A, J^B\right] = f^{AB}_C J^C \quad , \quad \left[J^A, J^{(1)B}\right] = f^{AB}_C J^{(1)C} \quad . \tag{11.7.2}$$

Besides this Serre relations are satisfied. These have more complex form and read as

$$\begin{aligned}
&\left[J^{(1)A},\left[J^{(1)B},J^C\right]\right] + \left[J^{(1)B},\left[J^{(1)C},\ J^A\right]\right] + \left[J^{(1)C},\left[J^{(1)A},\ J^B\right]\right] \\
&\qquad = \frac{1}{24}\ f^{ADK}f^{BEL}f^{CFM}f_{KLM}\{J_D,J_E,J_F\} \quad , \\
&\left[\left[J^{(1)A},J^{(1)B}\right],\ \left[J^C,J^{(1)D}\right]\right] + \left[\left[J^{(1)C},J^{(1)D}\right],\ \left[J^A,J^{(1)B}\right]\right] \\
&\qquad = \frac{1}{24}(f^{AGL}f^{BEM}f^{CD}_K \\
&\qquad + f^{CGL}f^{DEM}f^{AB}_K)f^{KFN}f_{LMN}\{J_G,J_E,J_F\} \quad .
\end{aligned} \tag{11.7.3}$$

The indices of the Lie algebra generators are raised by invariant, non-degenerate metric tensor g_{AB} or g^{AB}. $\{A,B,C\}$ denotes the symmetrized product of three generators.

The right hand side often has coefficient $\hbar^2$ instead of $1/24$. $\hbar$ need not have anything to do with Planck constant and as noticed in the main text has dimension of $1/\hbar$. The Serre relations give constraints on the commutation relations of $J^{(1)A}$. For $J^{(1)A}=J^A$ the first Serre relation reduces to Jacobi identity and second to the antisymmetry of the Lie bracket. The right hand side involved completely symmetrized trilinears $\{J_D,J_E,J_F\}$ making sense in the universal covering of the Lie algebra defined by J^A.

Repeated commutators allow to generate the entire algebra, whose elements are labeled by a non-negative integer n. The generators obtained in this manner are n-local operators arising in $(n-1)$-commutator of $J^{(1)}$: s. For $SU(2)$ the Serre relations are trivial. For other cases the first Serre relation implies the second one so the relations are redundant. Why Witten includes it is for the purpose of demonstrating the conditions for the existence of Yangians associated with discrete one-dimensional lattices (Yangians exist also for continuum one-dimensional index).

Under certain consistency conditions, a discrete one-dimensional lattice provides a representation for the Yangian algebra. One assumes that each lattice point allows a representation

R of J^A so that one has $J^A = \sum_i J_i^A$ acting on the infinite tensor power of the representation considered. The expressions for the generators J^{1A} in Witten's approach are given as

$$J^{(1)A} = f^A_{BC} \sum_{i<j} J_i^B J_j^C \quad . \tag{11.7.4}$$

This formula gives the generators in the case of conformal algebra. This representation exists if the adjoint representation of G appears only one in the decomposition of $R \otimes R$. This is the case for SU(N) if R is the fundamental representation or is the representation of by k^{th} rank completely antisymmetric tensors.

This discussion does not apply as such to $\mathcal{N} = 4$ case the number of lattice points is finite and corresponds to the number of external particles so that cyclic boundary conditions are needed guarantee that the number of lattice points reduces effectively to a finite number. Note that the Yangian in color degrees of freedom does not exist for SU(N) SYM.

As noticed, Yangian algebra is a Hopf algebra and therefore allows co-product. The co-product Δ is given by

$$\begin{aligned} \Delta(J^A) &= J^A \otimes 1 + 1 \otimes J^A \quad , \\ \Delta(J^{(1)A}) &= J^{(1)A} \otimes 1 + 1 \otimes J^{(1)A} + f^A_{BC} J^B \otimes J^C \end{aligned} \tag{11.7.5}$$

Δ allows to imbed Lie algebra into the tensor product in a non-trivial manner and the non-triviality comes from the addition of the dual generator to the trivial co-product. In the case that the single spin representation of $J^{(1)A}$ is trivial, the co-product gives just the expression of the dual generator using the ordinary generators as a non-local generator. This is assumed in the recent case and also for the generators of the conformal Yangian.

Super-Yangian

Also the Yangian extensions of Lie super-algebras make sense. From the point of physics especially interesting Lie super-algebras are $SU(m|m)$ and $U(m|m)$. The reason is that $PSU(2,2|4)$ (P refers to "projective") acting as super-conformal symmetries of $\mathcal{N} = 4$ SYM and this super group is a real form of $PSU(4|4)$. The main point of interest is whether this algebra allows Yangian representation and Witten demonstrated that this is indeed the case [B17].

These algebras are Z_2 graded and decompose to bosonic and fermionic parts which in general correspond to n- and m-dimensional representations of $U(n)$. The representation associated with the fermionic part dictates the commutation relations between bosonic and fermionic generators. The anti-commutator of fermionic generators can involve besides the unit operator also bosonic generators if the symmetrized tensor product in question contains adjoint representation. This is the case if fermions are in the fundamental representation and its conjugate. For $SU(3)$ the symmetrized tensor product of adjoint representations contains adjoint (the completely symmetric structure constants d_{abc}) and this might have some relevance for the super $SU(3)$ symmetry.

The elements of these algebras in the matrix representation (no Grassmann parameters involved) can be written in the following form

$$x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad .$$

a and d representing the bosonic part of the algebra are $n \times n$ matrices and $m \times m$ matrices corresponding to the dimensions of bosonic and fermionic representations. b and c are fermionic matrices are $n \times m$ and $m \times n$ matrices, whose anti-commutator is the direct sum of $n \times n$ and $n \times n$ matrices. For $n = m$ bosonic generators transform like Lie algebra generators of $SU(n) \times SU(n)$ whereas fermionic generators transform like $n \otimes \overline{n} \oplus \overline{n} \otimes n$ under $SU(n) \times SU(n)$. Supertrace is defined as $Str(x) = Tr(a) - Tr(b)$. The vanishing of Str defines $SU(n|m)$. For $n \neq m$ the super trace condition removes the identity matrix and $PU(n|m)$ and $SU(n|m)$ are the same. This does not happen for $n = m$: this is an important delicacy since this case corresponds to $\mathcal{N} = 4$ SYM. If any two matrices differing by an additive scalar are identified (projective scaling as a new physical effect) one obtains $PSU(n|n)$ and this is what one is interested in.

Witten shows that the condition that adjoint is contained only once in the tensor product $R \otimes \overline{R}$ holds true for the physically interesting representations of $PSU(2,2|4)$ so that the generalization of the bilinear formula can be used to define the generators of $J^{(1)A}$ of super Yangian of $PU(2,2|4)$. The defining formula for the generators of the Super Yangian reads as

$$\begin{aligned} J^{(1)}_C &= g_{CC'} J^{(1)C'} = g_{CC'} f^{C'}_{AB} \sum_{i<j} J^A_i J^B_j \\ &= g_{CC'} f^{C'}_{AB} g^{AA'} g^{BB'} \sum_{i<j} J^i_{A'} J^j_{B'} \quad . \end{aligned} \tag{11.7.6}$$

Here $g_{AB} = Str(J_A J_B)$ is the metric defined by super trace and distinguishes between $PSU(4|4)$ and $PSU(2,2|4)$. In this formula both generators and super generators appear.

11.8 Conclusions

$M^8 - H$ duality plays a crucial role in quantum TGD and this motivated a critical study of the basic assumptions involved.

11.8.1 Co-associativity is the only viable option

The notion of associativity of the tangent or normal space as a number theoretical counterpart of a variational principle. This is not enough in order to have $M^8 - H$ duality. The first guess was that the tangent space is associative and contains a commutative 2-D sub-manifold to guarantee $M^8 - H$ duality.

1. The cold shower came as I learned that 4-D associative sub-manifolds of quaternion spaces are geodesic manifolds and thus trivial. Co-associativity is however possible since any distribution of associative normal spaces integrates to a sub-manifold. Typically these sub-manifolds are minimal surfaces, which conforms with the physical intuitions. Therefore the surface X^4_r given by holography should be co-associative. By the same argument space-time surface contains string world sheets and partonic 2-surfaces as co-complex surfaces.
2. $X = Re_Q(o) = 0$ and $Y = Im_Q(P) = 0$ allow M^4 and its complement as associative/co-associative subspaces of O_c. The roots $P = 0$ for the complexified octonionic polynomials satisfy two conditions $X = 0$ and $Y = 0$.
Surprisingly, universal solutions are obtained as brane-like entities X^6_c with real dimension 12, having real projection X^6_r ("real" means that the number theoretic complex valued octonion norm squared is real valued).
Equally surprisingly, the non-universal solutions to the conditions to $X = 0$ correspond complex mass shells with real dimension 6 rather than 8. The solutions to $X = Y = 0$ correspond to common roots of the two polynomials involved and are also 6-D complex mass shells.
The reason for the completely unexpected behavior is that the equations $X = 0$ and $Y = 0$ are reduced by Lorentz invariance to equations for the ordinary roots of polynomials for the complexified mass squared type variable. The intersection is empty unless X and Y have a common root and X^4_r belongs to X^6_r for a common root.

How to associate to the polynomial P a real 4-surface satisfying the conditions making $M8 - H$-duality?

1. P would fix complex mass shells in terms of its roots but not the 4-surfaces, contrary to the original expectations. The fact that the 3-D mass shells belong to the same M^4 and also their tangent spaces are parallel to M^4 together with rationality conditions for local $SU(3)$ element suggests number theoretical holography.
2. The key observation is that G_2 as the automorphism group of octonions respects the co-associativity of the 4-D real sub-basis of octonions. Therefore a local G_2 gauge transformation applied to a 4-D co-associative sub-space $M^c \subset O_c$ gives a co-associative four-surface as a real projection. Also octonion analyticity allows G_2 gauge transformation. If X^4 is the image M^4

by a local $SU(3)$ element such that it also remains invariant under $SU(2)$ at each point, one obtains automatically $M^8 - H$ duality.
The image of X^4 under $M^8 - H$ duality depends on g so that gauge invariance is not in question. The plausible interpretation in case of $SU(3)$ is in terms of Kac-Moody - or even Yangian symmetry. Note that at QFT limit the gauge potentials defined at H level as projections of Killing vector fields of $SU(3)$ are replaced by their sums over parallel space-time sheets to give gauge fields as the space-time sheets are approximated with a single region of Minkowski space.

The study of octonionic Dirac equation shows that the solutions correspond to momenta at mass shells $m^2 = r_n$ obtained as roots of the polynomial P and that co-associativity is an essential for the octonionic Dirac equation. This conforms with the reduction of everything to algebraic conditions at the level of M^8.

11.8.2 Construction of the momentum space counter parts of scattering amplitudes in M^8

The construction of scattering amplitudes in M^8 was the main topic of this article. ZEO and the interpretation of M^8 as a momentum space analogous to the interior of the Fermi sphere give powerful constraints on the scattering amplitudes. 0

1. The fact that $SU(3)$ gauge transformation with boundary conditions defined by the mass shells as roots of polynomial P defines space-time surface and the corresponding gauge field vanishes plus the fact that at string world sheets the gauge potential defines a conserved current by holomorphy strongly suggest Yangian symmetry differing from Kac-Moody symmetry in that the analogs of conformal weights are non-negative. This leads to a proposal for how vertex operators can be constructed in terms of co-product using fermionic oscillator operators but with Kronecker delta anti-commutators since the cognitive representation is discrete.

2. The main objection is that the scattering amplitudes are trivial if quark momenta belong to cognitive representations, which are finite in the generic case. This would be the case also in 2-D integrable theories. The objection can be circumvented. First, the huge symmetries imply that cognitive representations can contain a very large - even an infinite - number of points. At partonic 2-surface this number could reduce to finite. Equally importantly, local composites of quark oscillation operators with collinear quark momenta are possible and would be realized in terms of representations of Yangian algebra for $G_{2,c}$ serving as the counterpart for super-symplectic and Kac-Moody algebras at the level of H.
3. ZEO leads to a concrete proposal for the construction of zero energy states - equivalently scattering amplitudes - by using a representation of Yangian algebra realized in terms of positive and negative energy quarks in opposite half-cones. Co-product plays a key role in the construction. Also the proposed local composites of quarks proposed in [L81] make sense.
4. Momentum conservation conditions and mass shell conditions combined with the requirement that the momenta are algebraic integers in the extension of rationals determined by the polynomial P look rather difficult to solve. These conditions however linearize in the sense that one can express the allowed momenta as squares of integer quaternions.

Also the construction of scattering amplitudes in M^8 is considered. ZEO and the interpretation of M^8 as a momentum space analogous to the interior of the Fermi sphere give powerful constraints on the scattering amplitudes. The fact that $G_{2,c}$ gauge transformation defines space-time surface and the corresponding gauge field vanishes plus the fact that at string world sheets the gauge potential defines a conserved current by holomorphy strongly suggest Yangian symmetry differering from Kac-Moody symmetry in that the analogs of conformal weights are non-negative. This leads to a proposal for how vertex operators can be constructed in terms of co-product using fermionic oscillator operators but with Kronecker delta anticommutators since the cognitive representation is discrete.

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Part II

APPLICATIONS

Chapter 12

Cosmology and Astrophysics in Many-Sheeted Space-Time

12.1 Introduction

This chapter is devoted to the applications of TGD to astrophysics and cosmology are discussed. It must be admitted that the development of the proper interpretation of the theory has been rather slow and involved rather weird twists motivated by conformist attitudes. Typically these attempts have brought into theory ad hoc identifications of say gravitational four-momentum although theory itself has from very beginning provided completely general formulas.

Perhaps the real problem has been that radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology (ZEO) states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

12.1.1 Zero Energy Ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state.

Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K45, K30] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

Zero energy states decompose into positive and negative energy parts having identification as initial and final states of particle reaction in time scales of perception longer than the geometrotemporal separation T of positive and negative energy parts of the state. If the time scale of perception is smaller than T, the usual positive energy ontology applies.

In zero energy ontology inertial four-momentum is a quantity depending on the temporal time scale T used and in time scales longer than T the contribution of zero energy states with parameter $T_1 < T$ to four-momentum vanishes. This scale dependence alone implies that it does not make sense to speak about conservation of inertial four-momentum in cosmological scales. Hence it would be in principle possible to identify inertial and gravitational four-momenta and achieve strong form of Equivalence Principle. It however seems that this is not the correct approach to follow.

The the relationship between TGD and GRT was understood quite recently (2014). GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical form of Equivalence Principle (EP) for the GRT limit in long length scales at least expressed in terms of Einstein's equations in given resolution scale with space-time sheets with size smaller than resolution scale represented as external currents.

One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

The vacuum extremals are absolutely essential for the TGD based view about long length scale limit about gravitation. Effective GRT space time would be imbeddable as a vacuum extremal to H. This is just assumption albeit coming first in mind - especially so when one has not yet understood how GRT space-time emerges from TGD!

Already the Kähler action defined by CP_2 Kähler form J allows enormous vacuum degeneracy: any four-surface having Lagrangian sub-manifold of CP_2 as its CP_2 projection is a vacuum extremal. The dimension of these sub-manifolds is at most two. Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K45, K30] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

In TGD framework topological field quantization leads to the hypothesis that quantum concepts should have geometric counterparts and also potential energy should have precise correlate at the level of description based on topological field quanta. This indeed seems to be the case. As already explained, TGD allows space-time sheets to have both positive and negative time orientations. This in turn implies that also the sign of energy can be also negative. This suggests that the generation of negative energy space-time sheets representing virtual gravitons together with energy conservation makes possible the generation of huge gravitationally induced kinetic energies and gravitational collapse. In this process inertial energy would be conserved since instead, of positive energy gravitons, the inertial energy would go to the energy of matter.

This picture has a direct correlate in quantum field theory where the exchange negative energy virtual bosons gives rise to the interaction potential. The gravitational red-shift of microwave background photons is the strongest support for the non-conservation of energy in General Relativity. In TGD it could have concrete explanation in terms of absorption of negative energy virtual gravitons by photons leading to gradual reduction of their energies. This explanation is consistent with the classical geometry based explanation of the red-shift based on the stretching of electromagnetic wave lengths. This explanation is also consistent with the intuition based on Feynman diagram description of gravitational acceleration in terms of graviton exchanges.

12.1.2 Dark Matter Hierarchy And Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the embedding space was inspired both by empirical input (Bohr quantization of planetary orbits and anomalies of biology) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence. Consider first the mathematical structure in question.

1. The Clifford algebra of World of Classical Worlds (WCW) creating many fermion states is a standard example of an algebra expressible as a direct integral of copies of von Neumann algebras known as hyper-finite factor of type II_1 (HFFs). The inclusions of HFFs relate very intimately to the notion of finite measurement resolution. There is a canonical hierarchy of Jones inclusions [A1] labeled by finite subgroups of SU(2) [A85]. Quantum classical correspondence suggests that these inclusions have space-time correlates [K112, K42] and the generalization of embedding space would provide these correlates.
2. The space $CD \times CP_2$, where $CD \subset M^4$ is so called causal diamond identified as the intersection of future and past directed light-cones defines the basic geometric structure in zero energy ontology. The positive (negative) energy part of the zero energy state is located to the lower (upper) light-like boundaries of $CD \times CP_2$ and has interpretation as the initial (final) state of the physical event in standard positive energy ontology. p-Adic length scale hypothesis follows if one assumes that the temporal distance between the tips of CD comes as an octave of fundamental time scale defined by the size of CP_2. The "world of classical worlds" (WCW) is union of sub-WCWs associated with spaces $CD \times CP_2$ with different locations in $M^4 \times CP_2$.
3. One can say that causal diamond CD and the space CP_2 appearing as factors in $CD \times CP_2$ forms the basic geometric structure in zero energy ontology, is replaced with a book like structure obtained by gluing together infinite number of singular coverings and factor spaces of CD *resp.* CP_2 together. The copies are glued together along a common "back" $M^2 \subset M^2$ of the book in the case of CD. In the case of CP_2 the most general option allows two backs corresponding to the two non-isometric geodesic spheres S^2_i, $i = I, II$, represented as sub-manifolds $\xi^1 = \overline{\xi}^2$ and $\xi^1 = \xi^2$ in complex coordinates transforming linearly under $U(2) \subset SU(3)$. Color rotations in CP_2 produce different choices of this pair.
4. The selection of S^2 and M^2 is an imbedding space correlate for the fixing of quantization axes and means symmetry breaking at the level of embedding space geometry. WCW is union over all possible choices of CD and pairs of geodesic spheres so that at the level no symmetry breaking takes place. The points of M^2 and S^2 have a physical interpretation in terms of quantum criticality with respect to the phase transition changing Planck constant (leakage to another page of the book through the back of the book).
5. The pages of the singular coverings are characterized by finite subgroups G_a and G_b of $SU(2)$ and these groups act in covering or leave the points of factor space invariant. The pages are labeled by Planck constants $\hbar(CD) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$, where n_a and n_b are integers characterizing the orders of maximal cyclic subgroups of G_a and G_b. For singular factor spaces one has $\hbar(CD) = \hbar_0/n_a$ and $\hbar(CP_2) = \hbar_0/n_b$. The observed Planck constant corresponds to $\hbar = (\hbar(CD)/\hbar(CP_2)) \times \hbar_0$. What is also important is that $(\hbar/\hbar_0)^2$ appears as a scaling factor of M^4 covariant metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

The interpretation in terms of dark matter comes as follows.

1. Large values of $\hbar$ make possible macroscopic quantum phase since all quantum scales are scaled upwards by $\hbar/\hbar_0$. Anyonic and charge fractionization effects allow to "measure" $\hbar(CD)$ and $\hbar(CP_2)$ rather than only their ratio. $\hbar(CD) = \hbar(CP_2) = \hbar_0$ corresponds to what might be called standard physics without any anyonic effects and visible matter is identified as this phase.
2. Particle states belonging to different pages of the book can interact via classical fields and by exchanging particles, such as photons, which leak between the pages of the book. This leakage means a scaling of frequency and wavelength in such a way that energy and momentum of photon are conserved. Direct interactions in which particles from different pages appear in the same vertex of generalized Feynman diagram are impossible. This seems to be enough to explain what is known about dark matter. This picture differs in many respects from more conventional models of dark matter making much stronger assumptions and has far reaching implications for quantum biology, which also provides support for this view about dark matter.

This is the basic picture. One can imagine large number of speculative applications.

1. The number theoretically simple ruler-and-compass integers n having as factors only first powers of Fermat primes and power of 2 would define a physically preferred values of n_a and n_b and thus a sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself. Ruler and compass hypothesis implies that besides p-adic length scales also their 3- and 5-multiples should be important.
2. G_a could correspond directly to the observed symmetries of visible matter induced by the underlying dark matter if singular factor space is in question [K42]. For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [K42, K21] and recently reported strange properties of graphene [D12]. Also the tetrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [K39]. [D21].
3. A further fascinating possibility is that the evidence for Bohr orbit quantization of planetary orbits [E18] could have interpretation in terms of gigantic Planck constant for underlying dark matter [K89] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many way: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.
4. Since the gravitational Planck constant $\hbar_{gr} = GM_1m/v_0$, $v_0 = 2^{-11}$ for the inner planets, is proportional to the product of the gravitational masses of interacting systems, it must be assigned to the field body of the two systems and characterizes the interaction between systems rather than systems themselves. This observation applies quite generally and each field body of the system (em, weak, color, gravitational) is characterized by its own Planck constant.

12.1.3 Many-Sheeted Cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the vapor phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

The most important differences are following.

1. Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.
2. TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.
3. The new view about energy means that inertial energy is negative for space-time sheets with negative time orientation and that the density of inertial energy vanishes in cosmological length scales. Therefore any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.
4. Dark matter hierarchy with dynamical quantized Planck constant implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4. Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.
5. Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations

of the microwave background reflect the quantum criticality of the critical period rather than amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

12.1.4 Cosmic Strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically.

TGD predicts two basic types of strings.

1. The analogs of hadronic strings correspond to deformations of vacuum extremals carrying non-vanishing induced Kähler fields. p-Adic thermodynamics for super-symplectic quanta condensed on them with additivity of mass squared yields without further assumptions stringy mass formula. These strings are associated with various fractally scaled up variants of hadron physics.
2. Cosmic strings correspond to homologically non-trivial geodesic sphere of CP_2 (more generally to complex sub-manifolds of CP_2) and have a huge string tension. These strings are expected to have deformations with smaller string tension which look like magnetic flux tubes with finite thickness in M^4 degrees of freedom. The signature of these strings would be the homological non-triviality of the CP_2 projection of the transverse section of the string.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L22].

12.2 Basic Principles Of General Relativity From TGD Point Of View

General Coordinate Invariance, Equivalence Principle are corner stones of general relativity and one expects that they hold true also in TGD some sense. The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$ instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time. This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

Objections against TGD have turned out to be the best route to the correct interpretation of the theory. A very general objection against TGD relies on the notion of induced gauge fields and metric implying extremely strong constraints between classical gauge fields for preferred extremals. These constraints cannot hold true for gauge fields in the usual sense. Also linear superposition is lost. The solution of the problem comes from simple observation: it is not fields which superpose but their effects on test particle topologically condensed to space-time sheets carrying the classical fields. Superposition is replaced with set theoretic union. This leads also naturally to explicit identification of the effective metric and gauge potentials defined in M^4 and defining GRT limit of TGD.

Finite length scale resolution is central notion in TGD and implies that the topological inhomogenities (space-time sheets and other topological inhomogenities) are treated as point-like objects and described in terms of energy momentum tensor of matter and various currents coupling to effective YM fields and effective metric important in length scales above the resolution scale. Einstein's equations with coupling to gauge fields and matter relate these currents to the Einstein tensor and metric tensor of the effective metric of M^4. The topological inhomogenities below cutoff scale serve determine the curvature of the effective metric.

The original proposal, which I called smoothed out space-time, took into account the topological inhomogenities but neglected many-sheetedness in length scales above resolution scale. I also identified the effective metric can be identified as induced metric: this is very strong assumption although the properties of vacuum extremals support this identification at least in some important special cases.

The attempts to understand Kähler-Dirac (or Kähler-Dirac-) action has provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions: for instance, if is far from clear how em charge can be well-defined for the modes of the induced spinor field and how the effective absence of weak bosons above weak scale is realized at classical level for Kähler-Dirac action.

12.2.1 General Coordinate Invariance

General Coordinate Invariance plays in the formulation of quantum TGD even deeper role than in that of GRT. Since the fundamental objects are 3-D surfaces, the construction of the geometry of the configuration space of 3-surfaces (the world of classical worlds, WCW) requires that the definition of the geometry assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$. This space-time surface is completely analogous to Bohr orbit, which means a completely unexpected connection with quantum theory.

General Coordinate Invariance is analogous to gauge symmetry and requires gauge fixing. The definition assigning $X^4(X^3)$ to given X^3 must be such that the outcome is same for all 4-diffeomorphs of X^3. This condition is highly non-trivial since $X^4(X^3) = X^4(Y^3)$ must hold true if X^3 and Y^3 are 4-diffeomorphs. One manner to satisfy this condition is by assuming quantum holography and weakened form of General Coordinate Invariance: there exists physically preferred 3-surfaces X^3 defining $X^4(X^3)$, and the 4-diffeomorphs Y^3 of X^3 at $X^4(X^3)$ provide classical holograms of X^3: $X^4(Y^3) = X^4(X^3)$ is trivially true. Zero energy ontology allows to realize this form of General Coordinate Invariance.

1. In ZEO WCW decomposes into a union of sub-WCWs associated with causal diamonds $CD \times CP_2$ (CD denotes the intersection of future and past directed light-cones of M^4), and the intersections of space-time surface with the light-light boundaries of $CD \times CP_2$ are excellent candidates for preferred space-like 3-surfaces X^3. The 3-surfaces at $\delta CD \times CP_2$ are indeed physically special since they carry the quantum numbers of positive and negative energy parts of the zero energy state.
2. Preferred 3-surfaces could be also identified as light-like 3-surfaces X_l^3 at which the Euclidian signature of the induced space-time metric changes to Minkowskian. Also light-like boundaries of X^4 can be considered. These 3-surfaces are assumed to carry elementary particle quantum numbers and their intersections with the space-like 3-surfaces X^3 are 2-dimensional partonic surfaces so that effective 2-dimensionality consistent with the conformal symmetries of X_l^3 results if the identifications of 3-surfaces are physically equivalent. Light-like 3-surfaces are identified as generalized Feynman diagrams and due to the presence of 2-D partonic 2-surfaces representing vertices fail to be 3-manifolds. Generalized Feynman diagrams could be also identified as Euclidian regions of space-time surface.
3. General Coordinate Invariance in minimal form requires that the slicing of $X^4(X_l^3)$ by light light 3-surfaces Y_l^3 "parallel" to X_l^3 predicted by number theoretic compactification gives rise to quantum holography in the sense that the data associated with any Y_l^3 allows an equivalent formulation of quantum TGD. This poses a strong condition on the spectra of the Kähler-Dirac operator at Y_l^3 and thus to the preferred extremals of Kähler action since the WCW Kähler functions defined by various choices of Y_l^3 can differ only by a sum of a holomorphic function and its conjugate [K113, K30] .

12.2.2 The Basic Objection Against TGD

The basic objection against TGD is that induced metrics for space-time surfaces in $M^4 \times CP_2$ form an extremely limited set in the space of all space-time metrics appearing in the path integral formulation of General Relativity. Even special metrics like the metric of a rotating black hole fail to be imbeddable as an induced metric. For instance, one can argue that TGD cannot reproduce the post-Newtonian approximation to General Relativity since it involves linear superposition of gravitational fields of massive objects. As a matter fact, Holger B. Nielsen- one of the very few colleagues who has shown interest in my work - made this objection for at least two decades ago in some conference and I remember vividly the discussion in which I tried to defend TGD with my poor English.

The objection generalizes also to induced gauge fields expressible solely in terms of CP_2 coordinates and their gradients. This argument is not so strong as one might think first since in standard model only classical electromagnetic field plays an important role.

1. Any electromagnetic gauge potential has in principle a local embedding in some region. Preferred extremal property poses strong additional constraints and the linear superposition of massless modes possible in Maxwell's electrodynamics is not possible.
2. There are also global constraints leading to topological quantization playing a central role in the interpretation of TGD and leads to the notions of field body and magnetic body having non-trivial application even in non-perturbative hadron physics. For a very large class of preferred extremals space-time sheets decompose into regions having interpretation as geometric counterparts for massless quanta characterized by local polarization and momentum directions. Therefore it seems that TGD space-time is very quantal. Is it possible to obtain from TGD what we have used to call classical physics at all?

The imbeddability constraint has actually highly desirable implications in cosmology. The enormously tight constraints from imbeddability imply that imbeddable Robertson-Walker cosmologies with infinite duration are sub-critical so that the most pressing problem of General Relativity disappears. Critical and over-critical cosmologies are unique apart from a parameter characterizing their duration and critical cosmology replaces both inflationary cosmology and cosmology characterized by accelerating expansion. In inflationary theories the situation is just the opposite of this: one ends up with fine tuning of inflaton potential in order to obtain recent day cosmology.

Despite these and many other nice implications of the induced field concept and of sub-manifold gravity the basic question remains. Is the imbeddability condition too strong physically? What about linear superposition of fields which is exact for Maxwell's electrodynamics in vacuum and a good approximation central also in gauge theories. Can one obtain linear superposition in some sense?

1. Linear superposition for small deformations of gauge fields makes sense also in TGD but for space-time sheets the field variables would be the deformations of CP_2 coordinates which are scalar fields. One could use preferred complex coordinates determined about SU(3) rotation to do perturbation theory but the idea about perturbations of metric and gauge fields would be lost. This does not look promising. Could linear superposition for fields be replaced with something more general but physically equivalent?
2. This is indeed possible. The basic observation is utterly simple: what we know is that the *effects* of gauge fields superpose. The assumption that fields superpose is un-necessary! This is a highly non-trivial lesson in what operationalism means for theoreticians tending to take these kind of considerations as mere "philosophy".
3. The hypothesis is that the superposition of effects of gauge fields occurs when the M^4 projections of space-time sheets carrying gauge and gravitational fields intersect so that the sheets are extremely near to each other and can touch each other (CP_2 size is the relevant scale).

A more detailed formulation goes as follows.

1. One can introduce common M^4 coordinates for the space-time sheets. A test particle (or real particle) is identifiable as a wormhole contact and is therefore point-like in excellent approximation. In the intersection region for M^4 projections of space-time sheets the particle forms topological sum contacts with all the space-time sheets for which M^4 projections intersect.

2. The test particle experiences the sum of various gauge potentials of space-time sheets involved. For Maxwellian gauge fields linear superposition is obtained. For non-Abelian gauge fields gauge fields contain interaction terms between gauge potentials associated with different space-time sheets. Also the quantum generalization is obvious. The sum of the fields induces quantum transitions for states of individual space time sheets in some sense stationary in their internal gauge potentials.
3. The linear superposition applies also in the case of gravitation. The induced metric for each space-time sheet can be expressed as a sum of Minkowski metric and CP_2 part having interpretation as gravitational field. The natural hypothesis that in the above kind of situation the effective metric is sum of Minkowski metric with the sum of the CP_2 contributions from various sheets. The effective metric for the system is well-defined and one can calculate a curvature tensor for it among other things and it contains naturally the interaction terms between different space-time sheets. At the Newtonian limit one obtains linear superposition of gravitational potentials. One can also postulate that test particles moving along geodesics in the effective metric. These geodesics are not geodesics in the metrics of the space-time sheets.
4. This picture makes it possible to interpret classical physics as the physics based on effective gauge and gravitational fields and applying in the regions where there are many space-time sheets which M^4 intersections are non-empty. The loss of quantum coherence would be due to the effective superposition of very many modes having random phases.

The effective superposition of the CP_2 parts of the induced metrics gives rise to an effective metric which is not in general imbeddable to $M^4 \times CP_2$. Therefore many-sheeted space-time makes possible a rather wide repertoire of 4-metrics realized as effective metrics as one might have expected and the basic objection can be circumvented In asymptotic regions where one can expect single sheetedness, only a rather narrow repertoire of "archetypal" field patterns of gauge fields and gravitational fields defined by topological field quanta is possible.

The skeptic can argue that this still need not make possible the embedding of a rotating black hole metric as induced metric in any physically natural manner. This might be the case but need of course not be a catastrophe. We do not really know whether rotating blackhole metric is realized in Nature. I have indeed proposed that TGD predicts new physics [K106]. Unfortunately, gravity probe B could not check whether this new physics is there since it was located at equator where the new effects vanish.

12.2.3 How GRT And Equivalence Principle Emerge From TGD?

The question how TGD relates to General Relativity Theory (GRT) has been a rich source of problems during last 37 years. In the light of after-wisdom the problems have been due to my too limited perspective. I have tried to understand GRT limit in the TGD framework instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book) . This resolves also the worries related to Equivalence Principle.

TGD itself gains the status of "microscopic" theory of gravity and the experimental challenges relate to how make the microscopy of gravitation experimentally visible. This involves questions such as "How to make the presence of Euclidian space-time regions visible?",

> How to reveal many-sheeted character of space-time, topological field quantization, and the presence of magnetic flux tubes?,"How to reveal quantum gravity as understood in TGD involving in an essential manner gravitational Planck constant h_{gr} identifiable as h_{eff} inspired by anomalies of bio-electromagnetism?

[K81].

More technical questions relate to the Kähler-Dirac action, in particular to how conservation laws are realized. During all these years several questions have been lurking at the boarder of conscious and sub-conscious. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

TGD and GRT

Concerning GRT limit the basic questions are the following ones.

1. Is it really possible to obtain a realistic theory of gravitation if general space-time metric is replaced with induced metric depending on 8 embedding space coordinates (actually only 4 by general coordinate invariance?
2. What happens to Einstein equations?
3. What about breaking of Poincare invariance, which seems to be real in cosmological scales? Can TGD cope with it?
4. What about Equivalence Principle (EP)
5. Can one predict the value of gravitational constant?
6. What about TGD counterpart of blackhole, which certainly represents the boundary of realm in which GRT applies?

Consider first possible answers to the first three questions.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **??** in the appendix of this book).
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same manner and one expects that classical electroweak fields vanish in the length scales above weak scale.
3. This picture brings in mind the old intuitive notion of smoothed out quantum average space-time thought to be realized as surface in $M^4 \times CP_2$ rather than in terms of averages metric and gauge potentials in M^4. The problem of this approach was that it was not possible to imagine any quantitative recipe for the averaging and this was essentially dur to the sub-manifold assumption.
4. One could generalize this picture and consider effective metrics for CP_2 and $M^2 \times CP_2$ corresponding to CP_2 type vacuum extremals describing elementary particles and cosmic strings respectively.
5. Einstein's equations could hold true for the effective metric. The vanishing of the covariant divergence of energy momentum tensor would be a remnant of Poincare invariance actually still present in the sense of Zero Energy Ontology (ZEO) but having realization as global conservation laws.
6. The breaking of Poincare invariance at the level of effective metric could have interpretation as effective breaking due to zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

The following considerations are about answers to the fourth and fifth questions.

1. EP at classical level would hold true in local sense if Einstein's equations hold true for the effective metric. Underlying Poincare invariance suggests local covariant conservation laws.
2. The value of gravitational constant is in principle a prediction of theory containing only radius as fundamental scale and Kähler coupling strength as only coupling constant analogous to critical temperature. In GRT inspired quantum theory of gravitation Planck length scale given by $L_P = \sqrt{\hbar_{eff} \times G}$ is the fundamental length scale. In TGD size R defines it and it is independent of h_{eff}. The prediction for gravitational constant is prediction for the TGD counterpart of L_P: $L_P^2 = R^2/n$, n dimensionless constant. The prediction for G would be $G = R^2/(n \times \hbar_{eff})$ or $G = R^2/(n \times \hbar_{eff,min})$. The latter option is the natural one.

Interesting questions relate to the fate of blackholes in TGD framework.

1. Blackhole metric as such is quite possible as effective metric since there is no need to imbed it to embedding space. One could however argue that blackhole metric is so simple that it must be realizable as single-sheeted space-time surface. This is indeed possible above some radius which can be smaller than Schwarschild radius. This is due to the compactness of CP_2 . A general result is that the embedding carriers non-vanishing gauge charge say em charge. This need not have physical significance if the metric of GRT corresponds to the effective metric obtained by the proposed recipe.
2. TGD forces to challenge the standard view about black holes. For instance, could it be that blackhole interior corresponds microscopically to Euclidian space time regions? For these CP_2 endowed with effective metric would be appropriate GRT type description. Reissner-Nordström metric with cosmological constant indeed allows CP_2 as solution [K106]. M^4 region and CP_2 region would be joined along boundaries at which determinant of four-metric vanishes. If the radial component of R-N metric is required to be finite, one indeed obtains metric with vanishing determinant at horizon and it is natural to assume that the metric inside is Euclidian. Similar picture would applied to the cosmic strings as spaces $M^2 \times S^2$ with effective metric.
3. Could holography hold true in the sense that blackhole horizon is replaced with a partonic 2-surface with astrophysical size and having light-like orbit as also black-hole horizon has.
4. The notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0$, where v_0 is typical rotation velocity in the system consisting of masses M and m, has been one of the speculative aspects of TGD. h_{gr} would be assigned with "gravitational" magnetic flux tube connecting the systems in question and it has turned out that the identification $h_{gr} = h_{eff}$ makes sense in particle length scales. The gravitational Compton length is universal and given $\lambda_{gr} = GM/v_0$. This strongly suggests that quantum gravity becomes important already above Schwarschild radius $r_S = 2GMm$. The critical velocity at which gravitational Compton length becomes smaller than r_S is $v_0/c = 1/\sqrt{2}$. All astrophysical objects would be genuinely quantal objects in TGD Universe point and blackholes would lose their unique role. An experimental support for these findings comes from experiments of Tajmar *et al* [E24, E34] [K81].

For few ago entropic gravity [B9, B41] was a buzzword in blogs. The idea was that gravity would have a purely thermodynamical origin. I have commented the notion of entropic gravity from the point of view of TGD earlier [K106].

The basic objection is standard QM against the entropic gravity is that gravitational interaction of neutrons with Earth's gravitational field is describable by Schrödinger equation and this does not fit with thermodynamical description.

Although the idea as such does not look promising TGD indeed suggests that the correlates for thermodynamical quantities at space-time level make sense in ZEO leading to the view that quantum TGD is square root of thermodynamics.

Th interesting question is whether temperature has space-time correlate.

1. In Zero Energy Ontology quantum theory can be seen as a square root of thermodynamics formally and this raises the question whether ordinary temperature could parametrize wave functions having interpretation as square roots of thermal distributions in ZEO. The quantum model for cell membrane [K38] having the usual thermodynamical model as limit gives support for this idea. If this were the case, temperature would have by quantum classical correspondence direct space-time correlate.
2. A less radical view is that temperature can be assigned with the effective space-time metric only. The effective metric associated with M^4 defining GRT limit of TGD is defined statistically in terms of metric of many-sheeted space-time and would naturally contain in its geometry thermodynamical parameters. The averaging over the WCW spinors fields involving integral over 3-surfaces is also involved.

Equivalence Principle

Equivalence Principle has several interpretations.

1. The global form form of Equivalence Principle (EP) realized in Newtonian gravity states that inertial mass = gravitational mass (mass is replaces with four-momentum in the possible

relativistic generalization). This form does not make sense in general relativity since four-momentum is not well-defined: this problem is the starting point TGD.

2. The local form of EP can be expressed in terms of Einstein's equations. Local covariant conservation law does not imply global conservation law since energy momentum tensor is indeed tensor. One can try to define gravitational mass as something making sense in special cases. The basic problem is that there is no unique identification of empty space Minkowski coordinates. Gravitational mass could be identified as a a parameter appearing in asymptotic expression of solutions of Einstein's equations.

In TGD framework EP need not be problem of principle.

1. In TGD gravitational interaction couples to inertial four-momentum, which is well-defined as classical Noether charge associated with Kähler action. The very close analogy of TGD with string models suggest the same.
2. Only if one assumes that gravitational and inertial exist separately and are forced to be identical, one ends up with potential problems in TGD. This procedure might have sound physical basis in TGD but one should identify it in convincing manner.
3. In cosmology mass is not conserved, which in positive energy ontology would suggests breaking of Poincare invariance. In Zero Energy Ontology (ZEO) this is not the case. The conserved four-momentum assignable to either positive or negative energy part of the states in the basis of zero energy states depends on the scale of causal diamond (CD). Note that in ZEO zero energy states can be also superpositions of states with different four-momenta and even fermion numbers as in case of coherent state formed by Cooper pairs.

Consider now EP in quantum TGD.

1. Inertial momentum is defined as Noether charge for Kähler action.
2. One can assign to Kähler-Dirac action quantal four-momentum (I will use "Kähler-Dirac" instead of "modified" used in earlier work) [K113]. Its conservation is however not at all all trivial since embedding space coordinates appear in KD action like external fields. It however seems that at least for the modes localized at string world sheets the four-momentum conservation could be guaranteed by an assumption motivated by holomorphy [K113]. The assumption states that the variation of holomorphic/antiholomorphic Kähler-Dirac gamma matrices induced by isometry is superposition of K-D gamma matrices of same type.
3. Quantum Classical Correspondence (QCC) suggests that the eigenvalues of quantal four-momentum are equal to those of Kähler four-momentum. If this is the case, QCC would imply EP and force conservation of antal four-momenta even if the assumption about variations of gamma matrices fails! This could be realized in terms of Lagrange multiplier terms added to Kähler action and localized at the ends of CD and analogous to constraint terms in ordinary thermodynamics.
4. QCC generalizes to Cartan sub-algebra of symmetries and would give a correlation between geometry of space-time sheet and conserved quantum numbers. One can consider even stronger form of QCC stating that classical correlation functions at space-time surface are same as the quantal once.

The understanding of EP at classical level has been a long standing head-ache in TGD framework. What seems to be the eventual solution looks disappointingly trivial in the sense that its discovery requires only some common sense.

The trivial but important observation is that the GRT limit of TGD does *not* require that the space-times of GRT limit are imbeddable to the embedding space $M^4 \times CP_2$. The most elegant understanding of EP at classical level relies on following argument suggesting how GRT space-time emerges from TGD as an effective notion.

1. Particle experiences the sum of the effects caused by gravitational forces. The linear superposition for gravitational fields is replaced with the sum of effects describable in terms of effective metric in GRT framework. Hence it is natural to identify the metric of the effective space-time as the sum of M^4 metric and the deviations of various space-time sheets to which particle has topological sum contacts. This metric is defined for the M^4 serving as coordinate space and is not in general expressible as induced metric.

2. Underlying Poincare invariance is not lost but global conservation laws are lost for the effective space-time. A natural assumption is that global energy-momentum conservation translates to the vanishing of covariant divergence of energy momentum tensor.
3. By standard argument this implies Einstein's equations with cosmological constant Λ: this at least in statistical sense. Λ would parametrize the presence of topologically condensed magnetic flux tubes. Both gravitational constant and cosmological constant would come out as predictions.

This picture is in principle all that is needed. TGD is in this framework a "microscopic" theory of gravitation and GRT describes statistically the many-sheetedness in terms of single sheeted space-time identified as M^4 as manifold. All notions related to many-sheeted space-time - such as cosmic strings, magnetic flux tubes, generalized Feynman diagrams representing deviations from GRT. The theoretical and experimental challenge is discover what these deviations are and how to make them experimentally visible.

One can of course ask whether EP or something akin to it could be realized for preferred extremals of Kähler action.

1. In cosmological and astrophysical models vacuum extremals play a key role. Could small deformations of them provide realistic enough models for astrophysical and cosmological scales in statistical sense?
2. Could preferred extremals satisfy something akin to Einstein's equations? Maybe! The mere condition that the covariant divergence of energy momentum tensor for Kähler action vanishes, is satisfied if Einsteins equations with cosmological terms are satisfied. One can however consider also argue that this condition can be satisfied also in other ways. For instance, four-momentum currents associated with them be given by Einstein's equations involving several cosmological "constants". The vanishing of covariant divergence would however give a justification for why energy momentum tensor is locally conserved for the effective metric and thus gives rise to Einstein's equations.

EP as quantum classical correspondence

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.
2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This view might be equivalent with coset space view. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K106].

   A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and

QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

12.2.4 The Recent View About Kähler-Dirac Action

The understanding of Kähler-Dirac action and equation have provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of howe EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions.

The understanding of Kähler Dirac action has been second long term project. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

Kähler-Dirac action

12.2.5 Kähler-Dirac Action

Kähler-Dirac equation

12.2.6 Kähler-Dirac Equation In The Interior Of Space-Time Surface

The solution of K-D equation at string world sheets is very much analogous to that in string models and holomorphy (actually, its Minkowskian counterpart) plays a key role. Note however the K-D gamma matrices might not necessarily define effective metric with Minkowskian signature even for string world sheets. Second point to notice is that one can consider also solutions restricted to partonic 2-surfaces. Physical intuition suggests that they are very important because wormhole throats carry particle quantum numbers and because wormhole contacts mediat the interaction between space-time sheets. Whether partonic 2-surfaces are somehow dual to string world sheets remains an open question.

1. Conformal invariance/its Minkowskian variant based on hyper-complex numbers realized at string world sheets suggests a general solution of Kähler-Dirac equation. The solution ansatz is essentially similar to that in string models.
2. Second half of complexified Kähler-Dirac gamma matrices annihilates the spinors which are either holomorphic or anti-holomorphic functions of complex (hyper-complex) coordinate.
3. What about possible modes delocalized into entire 4-D space-time sheet possible if there are preferred extremals for which induced gauge field has only em part. What suggests itself is global slicing by string world sheets and obtain the solutions as integrals over localized modes over the slices.

The understanding of symmetries (isometries of embedding space) of K-D equation has turned out to be highly non-trivial challenge. The problem is that embedding space coordinates appear in the role of external fields in K-D equation. One cannot require the vanishing of the variations of the K-D action with respect to the embedding space-time coordinates since the action itself is second quantized object. Is it possible to have conservation laws associated with the embedding space isometries?

1. Quantum classical correspondence (QCC) suggests the conserved Noether charges for Kähler action are equal to the eigenvalues of the Noether charges for Kähler-Dirac action. The quantal charge conservation would be forced by hand. This condition would realize also Equivalence Principle.
2. Second possibility is that the current following from the vanishing of second variation of Kähler action and the modification of Kähler gamma matrices defined by the deformation are linear combinations of holomorphic or anti-holomorphic gammas just like the gamma matrix itself so that K-D remains true. Conformal symmetry would therefore play a fundamental role. Isometry currents would be conserved although variations with respect to embedding space coordinates would not vanish in general.
3. The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig. ??** also in the Appendix).

12.2.7 Boundary Terms For Kähler-Dirac Action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naïve guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

What one wants?

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$$

at the space-like ends of space-time surface. This condition makes sense also at partonic orbits although they are not boundaries in the usual sense of the word. Here however delicacies since g_4 vanishes at them. The localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.
The general idea is that the space-time geometry near the fermion line would *define* the four-momentum propagating along the line and quantum classical correspondence would be realized. The integral over four-momenta would be included to the functional integral over 3-surfaces.
The basic condition is that $\sqrt{g_4}\Gamma^n$ is constant at the boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.
2. If p^k is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to

articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification as virtual momenta is correct, continuous mass spectrum suggests itself. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation. Note however that the string curves along the space-like ends of space-time surface are also internal lines and expected to carry virtual momentum: classical picture suggests that p^k tends to be space-like.

Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of embedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{12.2.1}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = \gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. If Chern-Simons term is present one obtains also Chern-Simons term in this condition but also now fermion would be massless in global sense. The physical interpretation would be as incoming massless fermions.

12.2.8 About The Notion Of Four-Momentum In TGD Framework

The starting point of TGD was the energy problem of General Relativity [K106]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K106]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of ZEO (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K68] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K100]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian [A29] [B25, B17, B18] variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [B12]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

Scale dependent notion of four-momentum in zero energy ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K90], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of ZEO (ZEO) [K9, K110], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K110]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K113].

Are the classical and quantal four-momenta identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

What Equivalence Principle (EP) means in quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

1. Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

2. QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.
   Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant.It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.
   Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset

construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

The objection against the identification h in the decomposition $g = t + h$ of the symplectic algebra as Kac-Moody algebra is that this does not make sense mathematically. The strong form of holography implied by strong form of General Coordinate Invariance however implies that the action of Kac-Moody algebra for the maxima of Kähler function induces unique action of sub-algebra of symplectic algebra so that the identification makes sense after all [K31].

3. Does EP reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

   Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K68] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors would be extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep, and it seems that the coset option is definitely wrong: the reason is that for H in G/H decomposition the four-momentum vanishes.

TGD-GRT correspondence and Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig. ??** in the Appendix).
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.
3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the

effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in ZEO (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

How translations are represented at the level of WCW?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

1. Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n -- > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

2. The action of translations at space-time sheets

The action of embedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred etxremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

Yangian and four-momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [B12]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K100], where also references to the work of pioneers can be found.

1. *Yangian symmetry*

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K100] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant andconcrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B17] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

2. How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A11] and Virasoro algebras [A23] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.
2. The formal generalization looks surprisingly straightforward at the formal level. In ZEO one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in ZEO and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.
3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.
2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.
3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.
4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about

polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

12.3 TGD Inspired Cosmology

TGD Universe consists of quantum counterparts of a statistical system at critical temperature. As a consequence, topological condensate is expected to possess hierarchical, fractal like structure containing topologically condensed 3-surfaces with all possible sizes. Both Kähler magnetized and Kähler electric 3-surfaces ought to be important and string like objects indeed provide a good example of Kähler magnetic structures important in TGD inspired cosmology. In particular space-time is expected to be many-sheeted even at cosmological scales and ordinary cosmology must be replaced with many-sheeted cosmology. The presence of vapor phase consisting of free cosmic strings and possibly also elementary particles is second crucial aspects of TGD inspired cosmology.

It should be made clear from beginning that many-sheeted cosmology involves a vulnerable asumption. It is assumed that single-sheeted space-time surface is enough to model the cosmology. This need not to be the case. GRT limit of TGD is obtained by lumping together the sheets of many-sheeted space-time to a piece of Minkowski space and endowing it with an effective metric, which is sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Hence the proposed models make sense only if GRT limits allowing embedding as a vacuum extremal of Kähler action have special physical role.

Quantum criticality of TGD Universe (Kähler coupling strength is analogous to critical temperature) supports the view that many-sheeted cosmology is in some sense critical. Criticality in turn suggests fractality. Phase transitions, in particular the topological phase transitions giving rise to new space-time sheets, are (quantum) critical phenomena involving no scales. If the curvature of the 3-space does not vanish, it defines scale: hence the flatness of the cosmic time=constant section of the cosmology implied by the criticality is consistent with the scale invariance of the critical phenomena. This motivates the assumption that the new space-time sheets created in topological phase transitions are in good approximation modellable as critical Robertson-Walker cosmologies for some period of time at least.

Any one-dimensional sub-manifold allows global embeddings of subcritical cosmologies whereas for a given 2-dimensional Lagrange manifold of CP_2 critical and overcritical cosmologies allow only one-parameter family of partial embeddings. The infinite size of the horizon for the imbeddable critical cosmologies is in accordance with the presence of arbitrarily long range quantum fluctuations at criticality and guarantees the average *isotropy* of the cosmology. Embedding is possible for some critical duration of time. The parameter labelling these cosmologies is a scale factor characterizing the duration of the critical period. These cosmologies have the same optical properties as inflationary cosmologies but exponential expansion is replaced with logarithmic one. Critical cosmology can be regarded as a "Silent Whisper amplified to Bang" rather than "Big Bang" and transformed to hyperbolic cosmology before its embedding fails. Split strings decay to elementary particles in this transition and give rise to seeds of galaxies. In some later stage the hyperbolic cosmology can decompose to disjoint 3-surfaces. Thus each sub-cosmology is analogous to biological growth process leading eventually to death.

The critical cosmologies can be used as a building blocks of a fractal cosmology containing cosmologies containing ... cosmologies. p-Adic length scale hypothesis allows a quantitative formulation of the fractality [K89] . Fractal cosmology predicts cosmos to have essentially same optical properties as inflationary scenario. Fractal cosmology explains the paradoxical result that the observed density of the matter is much lower than the critical density associated with the largest space-time sheet of the fractal cosmology. Also the observation that some astrophysical objects seem to be older than the Universe, finds a nice explanation.

Absolutely essential element of the considerations (and longstanding puzzle of TGD inspired cosmology) is the conservation of energy implied by Poincare invariance which seems to be in conflict with the non-conservation of gravitational energy. It took long time to discover the natural resolution of the paradox. In TGD Universe matter and antimatter have opposite energies and gravitational four-momentum is identified as difference of the four momenta of matter and antimatter (or vice versa, so that gravitational energy is positive). The assumption that the net inertial energy density vanishes in cosmological length scales is the proper interpretation for the fact that Robertson-Walker cosmologies correspond to vacuum extremals of Kähler action.

Tightly bound, possibly coiled pairs of cosmic strings are the basic building block of TGD inspired cosmology and all al structures including large voids, galaxies, stars, and even planets can be seen as pearls in a cosmic fractal necklace consisting of cosmic strings containing smaller cosmic strings linked around them containing... During cosmological evolution the cosmic strings are transformed to magnetic flux tubes and these structures are also key players in TGD inspired quantum biology.

Negative energy virtual gravitons represented by topological quanta having negative time orientation and hence also negative energy. The absorption of negative energy gravitons by photons could explain gradual red-shifting of the microwave background radiation at particle level. Negative energy virtual gravitons give also rise to a negative gravitational potential energy. Quite generally, negative energy virtual bosons build up the negative interaction potential energy. An important constraint to TGD inspired cosmology is the requirement that Hagedorn temperature $T_H \sim 1/R$, where R is CP_2 size, is the limiting temperature of radiation dominated phase.

12.3.1 Robertson-Walker Cosmologies

Robertson-Walker cosmologies are the basic building block of standard cosmologies and sub-critical R-W cosmologies have a very natural place in TGD framework as Lorentz invariant cosmologies. Inflationary cosmologies are replaced with critical cosmologies being parameterized by a single parameter telling the duration of the critical cosmology. Over-critical cosmologies are not possible at all.

Why Robertson-Walker cosmologies?

One can hope Robertson Walker cosmology represented as a vacuum extremal of the Kähler action to be a reasonable idealization only in the length scales, where the density of the Kähler charge vanishes. Since (visible) matter and antimatter carry Kähler charges of opposite sign this means that Kähler charge density vanishes in length scales, where matter-antimatter asymmetry disappears on the average. This length scale is certainly very large in present day cosmology: in the proposed model for cosmology its present value is of the order of 10^8 light years: the size of the observed regions containing visible matter predominantly on their boundaries [E40] . That only matter is observed can be understood from the fact that fermions reside dominantly at future oriented space-time sheets and anti-fermions on past-oriented space-time sheets.

Robertson Walker cosmology is expected to apply in the description of the condensate locally at each condensate level and it is assumed that the GRT based criteria for the formation of "structures" apply. In particular, the Jeans criterion stating that density fluctuations with size between Jeans length and horizon size can lead to the development of the "structures" will be applied.

Imbeddability requirement for RW cosmologies

Standard Robertson-Walker cosmology is characterized by the line element [E33]

$$ds^2 = f(a)da^2 - a^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \ , \qquad (12.3.1)$$

where the values $k = 0, \pm 1$ of k are possible.

The line element of the light cone is given by the expression

$$ds^2 = da^2 - a^2(\frac{dr^2}{1+r^2} + r^2 d\Omega^2) \ . \qquad (12.3.2)$$

Here the variables a and r are defined in terms of standard Minkowksi coordinates as

$$\begin{aligned} a &= \sqrt{(m^0)^2 - r_M^2} \ , \\ r_M &= ar \ . \end{aligned} \qquad (12.3.3)$$

Light cone clearly corresponds to mass density zero cosmology with $k = -1$ and this makes the case $k = -1$ is rather special as far embeddings are considered since any Lorentz invariant map $M^4_+ \to CP_2$ defines embedding

$$s^k = f^k(a) \ . \qquad (12.3.4)$$

Here f^k are arbitrary functions of a.

$k = -1$ requirement guarantees imbeddability if the matter density is positive as is easy to see. The matter density is given by the expression

$$\rho = \frac{3}{8\pi G a^2}(\frac{1}{g_{aa}} + k) \ . \qquad (12.3.5)$$

A typical embedding of $k = -1$ cosmology is given by

$$\begin{aligned} \phi &= f(a) \ , \\ g_{aa} &= 1 - \frac{R^2}{4}(\partial_a f)^2 \ . \end{aligned} \qquad (12.3.6)$$

where ϕ can be chosen to be the angular coordinate associated with a geodesic sphere of CP_2 (any one-dimensional sub-manifold of CP_2 works equally well). The square root term is always positive by the positivity of the mass density and the embedding is indeed well defined. Since g_{aa} is smaller than one, the matter density is necessarily positive.

Critical and over-critical cosmologies

TGD allows vacuum extremal embeddings of a one-parameter family of critical over-critical cosmologies. Critical cosmologies are however not inflationary in the sense that they would involve the presence of scalar fields. Exponential expansion is replaced with a logarithmic one so that the cosmologies are in this sense exact opposites of each other. Critical cosmology has been used hitherto as a possible model for the very early cosmology. What is remarkable that this cosmology becomes vacuum at the moment of "Big Bang" since mass density behaves as $1/a^2$ as function of the light cone proper time. Instead of "Big Bang" one could talk about "Small Whisper" amplified to bang gradually. This is consistent with the idea that space-time sheet begins as a vacuum space-time sheet for some moment of cosmic time. As an imbedded 4-surface this cosmology would correspond to a deformed future light cone having its tip inside the future light cone. The interpretation of the tip as a seed of a phase transition is possible. The embedding makes sense up to some moment of cosmic time after which the cosmology becomes necessarily hyperbolic. At later time hyperbolic cosmology stops expanding and decomposes to disjoint 3-surfaces behaving as particle like objects co-moving at larger cosmological space-time sheet. These 3-surfaces topologically condense on larger space-time sheets representing new critical cosmologies.

Consider now in more detail the embeddings of the critical and overcritical cosmologies. For $k = 0, 1$ the imbeddability requirement fixes the cosmology almost uniquely. To see this, consider as an example of $k = 0/1$ embedding the map from the light cone to S^2, where S^2 is a geodesic sphere of CP_2 with a vanishing Kähler form (any Lagrage manifold of CP_2 would do instead of S^2). In the standard coordinates (Θ, Φ) for S^2 and Robertson-Walker coordinates (a, r, θ, ϕ) for future light cone (, which can be regarded as empty hyperbolic cosmology), the embedding is given as

$$\begin{aligned} sin(\Theta) &= \frac{a}{a_1} \ , \\ (\partial_r \Phi)^2 &= \frac{1}{K_0} \left[\frac{1}{1-kr^2} - \frac{1}{1+r^2} \right] \ , \\ K_0 &= \frac{R^2}{4a_1^2} \ , \ k = 0, 1 \ , \end{aligned} \tag{12.3.7}$$

when Robertson-Walker coordinates are used for both the future light cone and space-time surface.

The differential equation for Φ can be written as

$$\partial_r \Phi = \pm \sqrt{\frac{1}{K_0} \left[\frac{1}{1-kr^2} - \frac{1}{1+r^2} \right]} \ . \tag{12.3.8}$$

For $k = 0$ case the solution exists for all values of r. For $k = 1$ the solution extends only to $r = 1$, which corresponds to a 4-surface $r_M = m^0/\sqrt{2}$ identifiable as a ball expanding with the velocity $v = c/\sqrt{2}$. For $r \to 1$ Φ approaches constant Φ_0 as $\Phi - \Phi_0 \propto \sqrt{1-r}$. The space-time sheets corresponding to the two signs in the previous equation can be glued together at $r = 1$ to obtain sphere S^3.

The expression of the induced metric follows from the line element of future light cone

$$ds^2 = da^2 - a^2 (\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \ . \tag{12.3.9}$$

The imbeddability requirement fixes almost uniquely the dependence of the S^2 coordinates a and r and the g_{aa} component of the metric is given by the same expression for both $k = 0$ and $k = 1$.

$$\begin{aligned} g_{aa} &= 1 - K \ , \\ K &\equiv K_0 \frac{1}{(1-u^2)} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \tag{12.3.10}$$

The embedding fails for $a \geq a_1$. For $a_1 \gg R$ the cosmology is essentially flat up to immediate vicinity of $a = a_1$. Energy density and "pressure" follow from the general equation of Einstein tensor and are given by the expressions

$$\begin{aligned} \rho &= \frac{3}{8\pi G a^2} (\frac{1}{g_{aa}} + k) \ , \ k = 0, 1 \ , \\ \frac{1}{g_{aa}} &= \frac{1}{1-K} \ , \\ p &= -(\rho + \frac{a \partial_a \rho}{3}) = -\frac{\rho}{3} + \frac{2}{3} K_0 u^2 \frac{1}{(1-K)(1-u^2)^2} \rho_{cr} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \tag{12.3.11}$$

Here the subscript "cr" refers to $k = 0$ case. Since the time component g_{aa} of the metric approaches constant for very small values of the cosmic time, there are no horizons associated with this metric. This is clear from the formula

$$r(a) = \int_0^a \sqrt{g_{aa}} \frac{da}{a}$$

for the horizon radius.

The mass density associated with these cosmologies behaves as $\rho \propto 1/a^2$ for very small values of the M^4_+ proper time. The mass in a co-moving volume is proportional to $a/(1-K)$ and goes to zero at the limit $a \to 0$. Thus, instead of Big Bang one has "Silent Whisper" gradually amplifying to Big Bang. The embedding fails at the limit $a \to a_1$. At this limit energy density becomes infinite. This cosmology can be regarded as a cosmology for which co-moving strings ($\rho \propto 1/a^2$) dominate the mass density as is clear also from the fact that the "pressure" becomes negative at big bang ($p \to -\rho/3$) reflecting the presence of the string tension. The natural interpretation is that cosmic strings condense on the space-time sheet which is originally empty.

The facts that the embedding fails and gravitational energy density diverges for $a = a_1$ necessitates a transition to a hyperbolic cosmology. For instance, a transition to radiation or matter dominated hyperbolic cosmology can occur at the limit $\theta \to \pi/2$. At this limit $\phi(r)$ must transform to a function $\phi(a)$. The fact, that vacuum extremals of Kähler action are in question, allows large flexibility for the modelling of what happens in this transition. Quantum criticality and p-adic fractality suggest the presence of an entire fractal hierarchy of space-time sheets representing critical cosmologies created at certain values of cosmic time and having as their light cone projection sub-light cone with its tip at some a=constant hyperboloid.

More general embeddings of critical and over-critical cosmologies as vacuum extremals

In order to obtain embeddings as more general vacuum extremals, one must pose the condition guaranteeing the vanishing of corresponding the induced Kähler form (see the Appendix of this book). Using coordinates $(r, u = cos(\Theta), \Psi, \Phi)$ for CP_2 the surfaces in question can be expressed as

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D|k+u| \ , \\ u &\equiv cos(\Theta) \ , \ D = \frac{r_0^2}{1+r_0^2} \times \frac{1}{C} \ , \quad C = |k + cos(\Theta_0)| \ . \end{aligned} \tag{12.3.12}$$

Here C and D are integration constants.

These embeddings generalize to embeddings to $M^4 \times Y^2$, where Y^2 belongs to a family of Lagrange manifolds described in the Appendix of this book with induced metric

$$\begin{aligned} ds^2_{eff} &= \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ . \end{aligned} \tag{12.3.13}$$

For $k \neq 1$ $u = \pm 1$ corresponds in general to circle rather than single point as is clear from the fact that $s^{eff}_{\Phi\Phi}$ is non-vanishing at $u = \pm 1$ so that u and Φ parameterize a piece of cylinder. The generalization of the previous embedding is as

$$sin(\Theta) = ka \quad \to \quad \sqrt{s^{eff}_{\Phi\Phi}} = ka \ . \tag{12.3.14}$$

For Φ the expression is as in the previous case and determined by the requirement that g_{rr} corresponds to $k = 0, 1$.

The time component of the metric can be expressed as

$$g_{aa} = 1 - \frac{R^2k^2}{4} \frac{s^{eff}_{\Theta\Theta}}{\frac{d\sqrt{s^{eff}_{\Phi\Phi}}}{d\Theta}} \tag{12.3.15}$$

In this case the $1/(1-k^2a^2)$ singularity of the density of gravitational mass at $\Theta = \pi/2$ is shifted to the maximum of $s^{eff}_{\Phi\Phi}$ as function of Θ defining the maximal value a_{max} of a for which the embedding exists at all. Already for $a_0 < a_{max}$ the vanishing of g_{aa} implies the non-physicality of the embedding since gravitational mass density becomes infinite.

The geometric properties of critical cosmology change radically in the transition to the radiation dominated cosmology: before the transition the CP_2 projection of the critical cosmology is two-dimensional. After the transition it is one-dimensional. Also the isometry group of the cosmology changes from $SO(3) \times E^3$ to $SO(3,1)$ in the transition. One could say that critical cosmology represents Galilean Universe whereas hyperbolic cosmology represents Lorentzian Universe.

String dominated cosmology

A particularly interesting cosmology is string dominated cosmology with very nearly critical mass density. Assuming that strings are co-moving the mass density of this cosmology is proportional to $1/a^2$ instead of the $1/a^3$ behavior characteristic to the standard matter dominated cosmology. The line element of this metric is very simple: the time component of the metric is simply constant smaller than 1:

$$g_{aa} = K < 1 \ . \tag{12.3.16}$$

The Hubble constant for this cosmology is given by

$$H = \frac{1}{\sqrt{K}a} \ , \tag{12.3.17}$$

and the so called acceleration parameter [E33] k_0 proportional to the second derivative $\ddot{a}$ therefore vanishes. Mass density and pressure are given by the expression

$$\rho = \frac{3}{8\pi G K a^2}(1-K) = -3p \ . \tag{12.3.18}$$

What makes this cosmology so interesting is the absence of the horizons. The comparison with the critical cosmology shows that these two cosmologies resemble each other very closely and both could be used as a model for the very early cosmology.

Stationary cosmology

An interesting candidate for the asymptotic cosmology is stationary cosmology for which gravitational four-momentum currents (and also gravitational color currents) are conserved. This cosmology extremizes the Einstein-Hilbert action with cosmological term given by $\int (kR+\lambda)\sqrt{g}d^4x + \lambda$ and is obtained as a sub-manifold $X^4 \subset M^4_+ \times S^1$, where S^1 is the geodesic circle of CP_2 (note that embedding is now unique apart from isometries by variational principle).

For a vanishing cosmological constant, field equations reduce to the conservation law for the isometry associated with S^1 and read

$$\partial_a(G^{aa}\partial_a\phi\sqrt{g}) = 0 \ , \tag{12.3.19}$$

where ϕ denotes the angle coordinate associated with S^1. From this one finds for the relevant component of the metric the expression

$$\begin{aligned} g_{aa} &= \frac{(1-2x)}{(1-x)} \ , \\ x &= (\frac{C}{a})^{2/3} \ . \end{aligned} \tag{12.3.20}$$

The mass density and "pressure" of this cosmology are given by the expressions

$$\begin{aligned} \rho &= \frac{3}{8\pi G a^2}\frac{x}{(1-2x)} \ , \\ p &= -(\rho + \frac{a\partial_a \rho}{3}) = -\frac{\rho}{9}\left[3 - \frac{2}{(1-2x)}\right] \ . \end{aligned} \tag{12.3.21}$$

The asymptotic behavior of the energy density is $\rho \propto a^{-8/3}$. "Pressure" becomes negative indicating that this cosmology is dominated by the string like objects, whose string tension gives negative contribution to the "pressure". Also this cosmology is horizon free as are all string dominated cosmologies: this is of crucial importance in TGD inspired cosmology.

It should be noticed that energy density for this cosmology becomes infinite for $x = (C/a)^{2/3} = 1/2$ implying that this cosmology doesn't make sense at very early times so that the non-conservation of gravitational energy is necessary during the early stages of the cosmology.

Non-conservation of gravitational energy in RW cosmologies

In RW cosmology the gravitational energy in a given co-moving sphere of radius r in local light cone coordinates (a, r, θ, ϕ) is given by

$$E = \int \rho g^{aa}\partial_a m^0 \sqrt{g} dV \ . \tag{12.3.22}$$

The rate characterizing the non-conservation of gravitational energy is determined by the parameter X defined as

$$X \equiv \frac{(dE/da)_{vap}}{E} = \frac{(dE/da + \int |g^{rr}| p \partial_r m^0 \sqrt{g} d\Omega)}{E} \ , \tag{12.3.23}$$

where p denotes the pressure and $d\Omega$ denotes angular integration over a sphere with radius r. The latter term subtracts the energy flow through the boundary of the sphere.

The generation of the pairs of positive and negative (inertial) energy space-time sheets leads to non-conservation of gravitational energy. The generation of pairs of positive and negative energy cosmic strings would be involved with the generation of a critical sub-cosmology.

For RW cosmology with subcritical mass density the calculation gives

$$X = \frac{\partial_a(\rho a^3/\sqrt{g_{aa}})}{(\rho a^3/\sqrt{g_{aa}})} + \frac{3p g_{aa}}{\rho a} \ . \tag{12.3.24}$$

This formula applies to any infinitesimal volume. The rate doesn't depend on the details of the embedding (recall that practically any one-dimensional sub-manifold of CP_2 defines a huge family of subcritical cosmologies). Apart from the numerical factors, the rate behaves as $1/a$ in the most physically interesting RW cosmologies. In the radiation dominated and matter dominated cosmologies one has $X = -1/a$ and $X = -1/2a$ respectively so that gravitational energy decreases in radiation and matter dominated cosmologies. For the string dominated cosmology with $k = -1$ having $g_{aa} = K$ one has $X = 2/a$ so that gravitational energy increases: this might be due to the generation of dark matter due to pairs of cosmic strings with vanishing net inertial energy.

For the cosmology with exactly critical mass density Lorentz invariance is broken and the contribution of the rate from 3-volume depends on the position of the co-moving volume. Taking the limit of infinitesimal volume one obtains for the parameter X the expression

$$\begin{aligned} X &= X_1 + X_2 \ , \\ X_1 &= \frac{\partial_a(\rho a^3/\sqrt{g_{aa}})}{(\rho a^3/\sqrt{g_{aa}})} \ , \\ X_2 &= \frac{p g_{aa}}{\rho a} \times \frac{3+2r^2}{(1+r^2)^{3/2}} \ . \end{aligned} \tag{12.3.25}$$

Here r refers to the position of the infinitesimal volume. Simple calculation gives

$$\begin{aligned} & X = X_1 + X_2 \ , \\ & X_1 = \frac{1}{a}\left[1 + 3K_0 u^2 \frac{1}{1-K}\right] \ , \\ & X_2 = -\frac{1}{3a}\left[1 - K - \frac{2K_0 u^2}{(1-u^2)^2}\right] \times \frac{3+2r^2}{(1+r^2)^{3/2}} \ , \\ & \quad K = \frac{K_0}{1-u^2} \ , \quad u = \frac{a}{a_0} \ , \quad K_0 = \frac{R^2}{4a_0^2} \ . \end{aligned} \tag{12.3.26}$$

The positive density term X_1 corresponds to increase of gravitational energy which is gradually amplified whereas pressure term ($p < 0$) corresponds to a decrease of gravitational energy changing however its sign at the limit $a \rightarrow a_0$.

The interpretation is in terms of creation of pairs of positive and negative energy particles contributing nothing to the inertial energy. Also pairs of positive energy gravitons and negative anti-gravitons are involved. The contributions of all particle species are determined by thermal arguments so that gravitons should not play any special role as thought originally.

Pressure term is negligible at the limit $r \rightarrow \infty$ so that topological condensation occurs all the time at this limit. For $a \rightarrow 0, r \rightarrow 0$ one has $X > 0 \rightarrow 0$ so that condensation starts from zero at $r = 0$. For $a \rightarrow 0, r \rightarrow \infty$ one has $X = 1/a$ which means that topological condensation is present already at the limit $a \rightarrow 0$.

Both the existence of the finite limiting temperature and of the critical mass density imply separately finite energy per co-moving volume for the condensate at the very early stages of the cosmic evolution. In fact, the mere requirement that the energy per co-moving volume in the vapor phase remains finite and non-vanishing at the limit $a \rightarrow 0$ implies string dominance as the following argument shows.

Assuming that the mass density of the condensate behaves as $\rho \propto 1/a^{2(1+\alpha)}$ one finds from the expression

$$\rho \propto \frac{(\frac{1}{g_{aa}} - 1)}{a^2} \ ,$$

that the time component of the metric behaves as $g_{aa} \propto a^\alpha$. Unless the condition $\alpha < 1/3$ is satisfied or equivalently the condition

$$\rho < \frac{k}{a^{2+2/3}} \tag{12.3.27}$$

is satisfied, gravitational energy density is reduced. In fact, the limiting behavior corresponds to the stationary cosmology, which is not imbeddable for the small values of the cosmic time. For stationary cosmology gravitational energy density is conserved which suggests that the reduction of the density of cosmic strings is solely due to the cosmic expansion.

12.3.2 Free Cosmic Strings

The free cosmic strings correspond to four-surfaces of type $X^2 \times S^2$, where S^2 is the homologically nontrivial geodesic sphere of CP_2 [L2] , [L2] and X^2 is minimal surface in M^4_+. As a matter fact, any complex manifold $Y^2 \subset CP_2$ is possible. In this section, a co-moving cosmic string solution inside the light cone $M^4_+(m)$ associated with a given m point of M^4_+ will be constructed.

Recall that the line element of the light cone in co-moving coordinates inside the light cone is given by

$$ds^2 = da^2 - a^2(\frac{dr^2}{1+r^2} + r^2 d\Omega^2) \ . \tag{12.3.28}$$

Outside the light cone the line element is given

$$ds^2 = -da^2 - a^2(-\frac{dr^2}{1-r^2} + r^2 d\Omega^2) \ , \tag{12.3.29}$$

and is obtained from the line element inside the light cone by replacements $a \rightarrow ia$ and $r \rightarrow -ir$.

Simplest solutions

Using the coordinates $(a = \sqrt{(m^0)^2 - r_M^2}, ar = r_M)$ for X^2 the orbit of the cosmic string is given by

$$\begin{aligned} \theta &= \frac{\pi}{2} \ , \\ \phi &= f(r) \ . \end{aligned} \tag{12.3.30}$$

Inside the light cone the line element of the induced metric of X^2 is given by

$$ds^2 = da^2 - a^2(\frac{1}{1+r^2} + r^2 f_{,r}^2)dr^2 \ . \tag{12.3.31}$$

The equations stating the minimal surface property of X^2 can be expressed as a differential conservation law for energy or equivalently for the component of the angular momentum in the direction orthogonal to the plane of the string. The conservation of the energy current T^α gives

$$\begin{aligned} T^\alpha_{,\alpha} &= 0 \ , \\ T^\alpha &= T g^{\alpha\beta} m^0_{,\beta}\sqrt{g} \ , \\ T &= \frac{1}{8\alpha_K R^2} \simeq .52 \times 10^{-6}\frac{1}{G} \ . \end{aligned} \tag{12.3.32}$$

The numerical estimate $TG \simeq .52 \times 10^{-6}$ for the string tension is upper bound and corresponds to a situation in which the entire area of S^2 contributes to the tension. It has been obtained using $\alpha_K/104$ and $R^2/G = 2.5 \times 10^7$ given by the most recent version of p-adic mass calculations (the earlier estimate was roughly by a factor 1/2 too small due to error in the calculation [K45, L63]). The string tension belongs to the range $TG \in [10^{-6} - 10^{-7}]$ predicted for GUT strings [E36] . WMAP data give the upper bound $TG \in [10^{-6} - 10^{-7}]$, which does not however hold true in the recent case since criticality predicts adiabatic spectrum of perturbations as in the inflationary scenarios.

The non-vanishing components of energy current are given by

$$\begin{aligned} T^a &= TUa \ , \\ T^r &= -T\frac{r}{U} \ , \\ U &= \sqrt{1 + r^2(1+r^2)f_{,r}^2} \ . \end{aligned} \tag{12.3.33}$$

The equations of motion give

$$U = \frac{r}{\sqrt{r^2 - r_0^2}} \ , \tag{12.3.34}$$

or equivalently

$$\phi_{,r} = \frac{r_0}{r\sqrt{(r^2 - r_0^2)(1 + r^2)}} \ , \tag{12.3.35}$$

where r_0 is an integration constant to be determined later. Outside the light cone the solution has the form

$$\phi_{,r} = \frac{r_0}{\sqrt{r^2 + r_0^2}r\sqrt{1 - r^2}} \ . \tag{12.3.36}$$

In the region inside the light cone, where the conditions

$$r_0 << r << 1 \tag{12.3.37}$$

hold, the solution has the form

$$\begin{aligned} \phi(r) &\simeq \phi_0 + \frac{v}{r} \ , \\ v &= \frac{r_0}{\sqrt{1 + r_0^2}} \ , \end{aligned} \tag{12.3.38}$$

corresponding to the linearized equations of motion

$$f_{,rr} + \frac{2f_{,r}}{r} = 0 \ , \tag{12.3.39}$$

obtained most nicely from the angular momentum conservation condition.

Cosmic string is stationary in comoving coordinates

In co-moving coordinates (in general the co-moving coordinates of sub-light-cone M^4_+!) the string is stationary. In Minkowski coordinates string rotates with an angular velocity inversely proportional to the distance from the origin

$$\omega \simeq \frac{v}{r_M} \tag{12.3.40}$$

so that the orbital velocity of the string becomes essentially constant in this region. For very large values of r the orbital velocity of the string vanishes as $1/r$. Outside the light cone the variable r is in the role of time and for a given value of the time variable r strings are straight and one can regard the string as a rigidly rotating straight string in this region.

Inside the light cone, the solution becomes ill defined for the values of r smaller than the critical value r_0. Although the derivative $\phi_{,r}$ becomes infinite at this limit, the limiting value of ϕ is finite so that strings winds through a finite angle. The normal component T^r of the energy momentum current vanishes at $r = r_0$ identically, which means that no energy flows out at the end of the string. The coordinate variable r becomes however bad at $r = r_0$ (string resembles a circle at r_0) and this conclusion must be checked using ϕ as coordinate instead of r. The result is that the normal component of the energy current indeed vanishes.

Field equations are not however satisfied at the end of the string since the normal component of the angular momentum current (in z- direction) is non-vanishing at the boundary and given by

$$J^r = Tr^2a \ . \tag{12.3.41}$$

This means that the string loses angular momentum through its ends although the angular momentum density of the string is vanishing. The angular momentum lost at moment a is given by

$$J = \frac{Tr^2a^2}{2} = \frac{Tr_M^2}{2} \ . \tag{12.3.42}$$

This angular momentum is of the same order of magnitude as the angular momentum of a typical galaxy [E38] .

In M^4 coordinates singularity corresponds to a disk in the plane of string growing with a constant velocity, when the coordinate m^0 is positive

$$\begin{aligned} r_M &= vm^0 \ , \\ v &= \frac{r_0}{\sqrt{1+r_0^2}} \ . \end{aligned} \tag{12.3.43}$$

From the expression of the energy density of the string

$$\begin{aligned} T^a &= T\frac{ar}{\sqrt{r^2-r_0^2}} \ , \\ T &= \frac{1}{8\alpha_K R^2} \ , \end{aligned} \tag{12.3.44}$$

it is clear that energy density diverges at the singularity.

Energy of the cosmic string

As already noticed, the string tension is by a factor of order 10^{-6} smaller than the critical string tension $T_{cr} = 1/4G$ implying angle deficit of 2π in GRT so that there seems to be no conflict with General Relativity (unlike in the original scenario, in which the CP_2 radius was of order Planck length).

The energy of the string portion ranging from r_0 to r_1 is given by

$$E = T\sqrt{(r_1^2-r_0^2)a} = T\sqrt{\delta r_M^2} \ . \tag{12.3.45}$$

It should be noticed that M^4 time development of the string can be regarded as a scaling: each point of the string moves to radial direction with a constant velocity v.

One can calculate the total change of the angle ϕ from the integral

$$\Delta\phi = \sqrt{\frac{r_0^2}{1+r_0^2}}\int_{r_0}^{\infty} dr\frac{1}{r\sqrt{(r^2-r_0^2)(1+r^2)}} \ . \tag{12.3.46}$$

The upper bound of this quantity is obtained at the limit $r_0 \rightarrow 0$ and equals to $\Delta\phi = \pi/2$.

12.3.3 Cosmic Strings And Cosmology

The model for cosmic strings has forced to question all cherished assumptions including positive energy ontology, Equivalence Principle, and positivity of gravitational mass. The final outcome turned out to be rather conservative. ZEO is unavoidable, Equivalence Principle holds true universally but its general relativistic formulation makes sense only in long length scales, and gravitational mass has definite sign for positive/negative energy states. As a matter fact, all problems were created by the failure to realize that the expression of gravitational energy in terms of Einstein's tensor does not hold true in short length scales and must be replaced with the stringy expression resulting naturally by dimensional reduction of quantum TGD to string model like theory [K113, K45, L63].

The realization that GRT is only an effective description of many-sheeted space-time as Minkowski space M^4 endowed with effective metric whose deviation from flat metric is the sum of the corresponding deviations for space-time sheets in the region of M^4 considered resolved finally the problems and allowed to reduced Equivalence Principle to its form in GRT. Similar description applies also to gauge interactions.

TGD is therefore a microscopic theory and the physics for single space-time sheet is expected to be extremely simpler, much simpler than in gauge theory and general relativity already due to the fact that only four bosonic variables (4 embedding space coordinates) defined the dynamics at this level.

ZEO and cosmic strings

There are two kinds of cosmic strings: free and topological condensed ones and both are important in TGD inspired cosmology.

1. Free cosmic strings are not absolute minima of the Kähler action (the action has wrong sign). In the original identification of preferred extremals as absolute minima of Kähler action this was a problem. In the new formulation preferred extremals correspond to quantum criticality identified as the vanishing of the second variation of Kähler action at least for the deformations defining symmetries of Kähler action [K113, K45]. The symmetries very probably correspond to conformal symmetries acting as or almost as gauge symmetries. The number of conformal equivalence classes of space-time sheets with same Kähler action and conserved charges is expected to be finite and correspond to n in $h_{eff} = n \times h$ defining the hierarchy of Planck constants labelling phases of dark matter (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??**) in the appendix of this book).
Criticality guarantees the conservation of the Noether charges assignable to the Kähler-Dirac action. Ideal cosmic strings are excluded because they fail to satisfy the conditions characterizing the preferred extremal as a space-time surface containing regions with both Euclidian and Minkowskian signature of the induced metric with light-like 3-surface separating them identified as orbits of partonic 2-surfaces carrying elementary particle quantum numbers. The topological condensation of CP_2 type vacuum extremals representing fermons generates negative contribution to the action and reduces the string tension and leaves cosmic strings still free.
2. If the topologically condensate of fermions has net Kähler charges as the model for matter antimatter asymmetry suggests, the repulsive interaction of the particles tends to thicken the cosmic string by increasing the thickness of its infinitely thin M^4 projection so that Kähler magnetic flux tubes result. These flux tubes are ideal candidates for the carriers of dark matter with a large value of Planck constant. The criterion for the phase transition increasing $\hbar$ is indeed the presence of a sufficiently dense plasma implying that perturbation theory in terms of $Z^2\alpha_{em}$ (Z is the effective number of charges with interacting with each other without screening effects) fails for the standard value of Planck constant. The phase transition $h \rightarrow h_{eff}$ reduces the value of $\alpha_{em} = e^2/2 \times h_{eff}$ so that perturbation theory works. This phase transition scales up also the transversal size of the cosmic string. Similar criterion works also for other charges. The resulting phase is anyonic if the resulting 2-surfaces containing almost spherical portions connected by flux tubes to each other encloses the tip of the causal diamond (CD). The proposal is that dark matter resides on complex anyonic 2-surfaces surrounding the tips of CDs.

3. The topological condensation of cosmic strings generates wormhole contacts represented as pieces of CP_2 type vacuum extremals identified as bosons composed of fermion-anti-fermion pairs. Also this generates negative action and can make cosmic string a preferred extremal of Kähler action. The earliest picture was based on dynamical cancelation mechanism involving generation of strong Kähler electric fields in the condensation whose action compensated for Kähler magnetic action. Also this mechanism might be at work. Cosmic strings could also form bound states by the formation graviton like flux tubes connecting them and having wormhole contacts at their ends so that again action is reduced.
4. One can argue that in long enough length and time scales Kähler action per volume must vanish so that the idealization of cosmology as a vacuum extremal becomes possible and there must be some mechanism compensating the positive action of the free cosmic strings. The general mechanism could be topological condensation of fermions and creation of bosons by topological condensation of cosmic strings to space-time sheets.

In this framework zero energy states correspond to cosmologies leading from big bang to big crunch separated by some time interval T of geometric time. Quantum jumps can gradually increase the value T and TGD inspired theory of consciousness suggests that the increase of T might relate to the shift for the contents of conscious experience towards geometric future. In particular, what is usually regarded as cosmology could have started from zero energy state with a small value of T.

Topological condensation of cosmic strings

In the original vision about topological condensation of cosmic strings I assumed that large voids represented by space-time sheets contain "big" cosmic string in their interior and galactic strings near their boundaries. The recent much simpler view is that there are just galactic strings which carry net fermion numbers (matter antimatter asymmetry). If they have also net em charge they have a repulsive interaction and tend to end up to the boundaries of the large void. Since this slows down the expansive motion of strings, the repulsive interaction energy increases and a phase transition increasing Planck constant and scaling up the size of the void occurs after which cosmic strings are again driven towards the boundary of the resulting larger void.

One cannot assume that the exterior metric of the galactic strings is the one predicted by assuming General Relativity in the exterior region. This would mean that metric decomposes as $g = g_2(X^2) + g_2(Y^2)$. $g(X^2)$ would be flat as also $g_2(Y^2)$ expect at the position of string. The resulting angle defect due to the replacement of plane Y^2 with cone would be large and give rise to lense effect of same magnitude as in the case of GUT cosmic strings. Lense effect has not been observed.

This suggests that General Relativity fails in the length scale of large void as far as the description of topologically condensed cosmic strings is considered. The constant velocity spectrum for distant stars of galaxies and the fact that galaxies are organized along strings suggests that these string generate in a good approximation Newtonian potential. This potential predicts constant velocity spectrum with a correct value velocity.

In the stationary situation one expects that the exterior metric of galactic string corresponds to a small deformation of vacuum extremal of Kähler action which is also extremal of the curvature scalar in the induced metric. This allows a solution ansatz which conforms with Newtonian intuitions and for which metric decomposes as $g = g_1 + g_3$, where g_1 corresponds to axis in the direction of string and g_3 remaining $1 + 2$ directions.

Dark energy is replaced with dark matter in TGD framework

The observed accelerating expansion of the Universe has forced to introduce the notion of cosmological constant in the GRT based cosmology. In TGD framework the situation is different.

1. The gigantic value of gravitational Planck constant implies that dark matter makes TGD Universe a macroscopic quantum system even in cosmological length scales. Astrophysical systems become stationary quantum systems which participate in cosmic expansion only via quantum phase transitions increasing the value of gravitational Planck constant.
2. Critical cosmologies, which are determined apart from a single parameter in TGD Universe, are natural during all quantum phase transitions, in particular the phase transition periods

increasing the size of large voids and having interpretation in terms of an increase of gravitational Planck constant. Cosmic expansion is predicted to be accelerating during these periods. The mere criticality requires that besides ordinary matter there is a contribution $\Omega_\Lambda \simeq .74$ to the mass density besides visible matter and dark matter. In fact, also for the over-critical cosmologies expansion is accelerating.

3. In GRT framework the essential characteristic of dark energy is its negative pressure. In TGD framework critical and over-critical cosmologies have automatically effective negative pressure. This is essentially due to the constraint that Lorentz invariant vacuum extremal of Kähler action is in question. The mysterious negative pressure would be thus a signal about the representability of space-time as 4-surface in H and there is no need for any microscopic description in terms of exotic thermodynamics.

The values for the TGD counterpart of cosmological constant

One can introduce a parameter characterizing the contribution of dark mass to the mass density during critical periods and call it cosmological constant recalling however that the contribution does not correspond to dark energy. The value of this parameter is same as in the standard cosmology from mere criticality assumption.

What is new that p-adic fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing Λ. The order of magnitude for the recent value of the cosmological constant comes out correctly. The gravitational energy density assignable to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

The naïve expectation would be the density of cosmic strings would behave as $1/a^2$ as function of M^4_+ proper time. The vision about dark matter as a phase characterized by gigantic Planck constant however implies that large voids do not expand in continuous manner during cosmic evolution but in discrete quantum jumps increasing the value of the gravitational Planck constant and thus increasing the size of the large void as a quantum state. Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids.

TGD cosmic strings are consistent with the fluctuations of CMB

GUT cosmic strings were excluded by the fluctuation spectrum of the CMB background [E2] . In GRT framework these fluctuations can be classified to adiabatic density perturbations and isocurvature density perturbations. Adiabatic density perturbations correspond to overall scaling of various densities and do not affect the vanishing curvature scalar. For isocurvature density fluctuations the net energy density remains invariant. GUT cosmic strings predict isocurvature density perturbations while inflationary scenario predicts adiabatic density fluctuations.

In TGD framework inflation is replaced with quantum criticality of the phase transition period leading from the cosmic string dominated phase to matter dominated phase. Since curvature scalar vanishes during this period, the density perturbations are indeed adiabatic.

Matter-antimatter asymmetry and cosmic strings

Despite huge amount of work done during last decades (during the GUT era the problem was regarded as being solved!) matter-antimatter asymmetry remains still an unresolved problem of cosmology. A possible resolution of the problem is matter-antimatter asymmetry in the sense that cosmic strings contain antimatter and their exteriors matter. The challenge would be to understand the mechanism generating this asymmetry. The vanishing of the net gauge charges of cosmic string allows this symmetry since electro-weak charges of quarks and leptons can cancel each other.

The challenge is to identify the mechanism inducing the CP breaking necessary for the matter-antimatter asymmetry. Quite a small CP breaking inside cosmic strings would be enough.

1. The key observation is that vacuum extremals as such are not physically acceptable: small deformations of vacuum extremals to non-vacua are required. This applies also to cosmic strings since as such they do note present preferred extremals. The reason is that the preferred

extremals involve necessary regions with Euclidian signature providing four-dimensional representations of generalized Feynman diagrams with particle quantum numbers at the light-like 3-surfaces at which the induced metric is degenerate.

2. The simplest deformation of vacuum extremals and cosmic strings would be induced by the topological condensation of CP_2 type vacuum extremals representing fermions. The topological condensation at larger space-time surface in turn creates bosons as wormhole contacts.
3. This process induces a Kähler electric fields and could induce a small Kähler electric charge inside cosmic string. This in turn would induce CP breaking inside cosmic string inducing matter antimatter asymmetry by the minimization of the ground state energy. Conservation of Kähler charge in turn would induce asymmetry outside cosmic string and the annihilation of matter and antimatter would then lead to a situation in which there is only matter.
4. Either galactic cosmic strings or big cosmic strings (in the sense of having large string tension) at the centers of galactic voids or both could generate the asymmetry and in the recent scenario big strings are not necessary. One might argue that the photon to baryon ratio $r \sim 10^{-9}$ characterizing matter asymmetry quantitatively must be expressible in terms of some fundamental constant possibly characterizing cosmic strings. The ratio $\epsilon = G/\hbar R^2 \simeq 4 \times 10^{-8}$ is certainly a fundamental constant in TGD Universe. By replacing R with $2\pi R$ would give $\epsilon = G/(2\pi R)^2 \simeq 1.0 \times 10^{-9}$. It would not be surprising if this parameter would determine the value of r.

The model can be criticized.

1. The model suggest only a mechanism and one can argue that the Kähler electric fields created by topological condensates could be random and would not generate any Kähler electric charge. Also the sign of the asymmetry could depend on cosmic string. A CP breaking at the fundamental level might be necessary to fix the sign of the breaking locally.
2. The model is not the only one that one can imagine. It is only required that antimatter is somewhere else. Antimatter could reside also at other p-adic space-time sheets and at the dark space-time sheets with different values of Planck constant.

The needed CP breaking is indeed predicted by the fundamental formulation of quantum TGD in terms of the Kähler-Dirac action associated with Kähler action and its generalization allowing include instanton term as imaginary part of Kähler action inducing CP breaking [K113, K77] .

1. The key idea in the formulation of quantum TGD in terms of modified Dirac equation associated with Kähler action is that the Dirac determinant defined by the generalized eigenvalues assignable to the Dirac operator D_K equals to the vacuum functional defined as the exponent of Kähler function in turn identifiable as Kähler action for a preferred extremal, whose proper identification becomes a challenge. In ZEO (ZEO) 3-surfaces are pairs of space-like 3-surfaces assignable to the boundaries of causal diamond (CD) and for deterministic action principle this suggests that the extremals are unique. In presence of non-determinism the situation changes.
2. The huge vacuum degeneracy of Kähler action suggests that for given pair of 3-surfaces at the boundaries of CD there is a continuum of extremals with the same Kähler action and conserved charges obtained from each other by conformal transformations acting as gauge symmetries and respecting the light-likeness of wormhole throats (as well as the vanishing of the determinant of space-time metric at them). The interpretation is in terms of quantum criticality with the hierarchy of symmetries defining a hierarchy of criticalities analogous to the hierarchy defined by the rank of the matrix defined by the second derivatives of potential function in Thom's catastrophe theory.
3. The number of gauge equivalence classes is expected to be finite integer n and the proposal is that it corresponds to the value of the effective Planck constant $h_{eff} = n \times h$ so that a connection with dark matter hierarchy labelled by values of n emerges [K42].
4. This representation generalizes - at least formally. One could add an imaginary instanton term to the Kähler function and corresponding Kähler-Dirac operator D_K so that the generalized eigenvalues assignable to D_K become complex. The generalized eigenvalues correspond to the square roots of the eigenvalues of the operator $DD^\dagger = (p^k\gamma_k + \Gamma^n)(p^k\gamma_k + \Gamma^n)^\dagger$ acting at

the boundaries of string world sheets carrying fermion modes and it seems that only space-like 3-surfaces contribute. Γ^n is the normal component of the vector defined by Kähler-Dirac gamma matrices. One can define Dirac determinant formally as the product of the eigenvalues of $DD^\dagger$.
The conjecture is that the resulting Dirac determinant equals to the exponent of Kähler action and imaginary instanton term for the preferred extremal. The instanton term does not contribute to the WCW metric but could provide a first principle description for CP breaking and anyonic effects. It also predicts the dependence of these effects on the page of the book like structure defined by the generalized embedding space realizing the dark matter hierarchy with levels labeled by the value of Planck constant.

5. In the case of cosmic strings CP breaking could be especially significant and force the generation of Kähler electric charge. Instanton term is proportional to $1/h_{eff}$ so that CP breaking would be small for the gigantic values of h_{eff} characterizing dark matter. For small values of h_{eff} the breaking is large provided that the topological condensation is able to make the CP_2 projection of cosmic string four-dimensional so that the instanton contribution to the complexified Kähler action is non-vanishing and large enough. Since instanton contribution as a local divergence reduces to the contributions assignable to the light-like 3-surfaces X_l^3 representing topologically condensed particles, CP breaking is large if the density of topologically condensed fermions and wormhole contacts generated by the condensation of cosmic strings is high enough.

CP breaking at the level of CKM matrix

The CKM matrix for quarks contains CP breaking phase factors and this could lead to different evaporation rates for baryons and anti-baryons are different (quark cannot appear as vapor phase particle since vapor phase particle must have vanishing color gauge charges and in the recent vision about quantum TGD CP_2 type vacuum extremal which has not suffered topological condensation represents vacuum). The CP breaking at the level of CKM matrix would be implied by the instanton term present in the complexified Kähler action and Kähler-Dirac operator. The mechanism might rely on hadronic Kähler electric fields which are accompanied by color electric gauge fields proportional to induced Kähler form.

The topological condensation of quarks on hadronic strings containing weak color electric fields proportional to Kähler electric fields should be responsible for its string tension and this should in turn generate CP breaking. At the parton level the presence of CP breaking phase factor $exp(ikS_{CS})$, where $S_{CS} = \int_{X^4} J \wedge J + boundary\ term$ is purely topological Chern Simons term and naturally associated with the boundaries of space-time sheets with at most $D = 3$-dimensional CP_2 projection, could have something to do with the matter antimatter asymmetry. Note however that TGD predicts no strong CP breaking as QCD does [L63] .

Development of strings in the string dominated cosmology

The development of the string perturbations in the Robertson Walker cosmology has been studied [E12] and the general conclusion seems to be that all the details smaller than horizon are rapidly smoothed out. One must of course take very cautiously the application of these result in TGD framework.

In present case, the horizon has an infinite size so that details in all scales should die away. To see what actually happens consider small perturbations of a static string along z-axis. Restrict the consideration to a perturbation in the y-direction. Using instead of the proper time coordinate t the "conformal time coordinate" τ defined by $d\tau = dt/a$ the equations of motion read [E12]

$$\begin{aligned} (\partial_\tau + \frac{2\dot{a}}{a})(\dot{y}U) &= \partial_z(y'U) \ , \\ U &= \frac{1}{\sqrt{1+(y')^2 - \dot{y}^2}} \ . \end{aligned} \quad (12.3.47)$$

Rest Restrict the consideration to small perturbations for which the condition $U \simeq 1$ holds. For the string dominated cosmology the quantity $\dot{a}/a = 1/\sqrt{K}$ is constant and the equations of motion reduce to a very simple approximate form

$$\ddot{y} + \frac{2}{\sqrt{K}}\dot{y} - y'' = 0 \ . \quad (12.3.48)$$

The separable solutions of this equation are of type

$$\begin{aligned} y &= g(a)(C\sin(kz) + D\cos(kz)) \ , \\ g(a) &= (\frac{a}{a_0})^r \ . \end{aligned} \quad (12.3.49)$$

where r is a solution of the characteristic equation $r^2 + 2r/\sqrt{K} + k^2 = 0$:

$$r = -\frac{1}{\sqrt{K}}(1 \pm \sqrt{1 - k^2 K}) \ . \quad (12.3.50)$$

For perturbations of small wavelength $k > 1/\sqrt{K}$, an extremely rapid attenuation occurs; $1/\sqrt{K} \simeq 10^{27}$! For the long wavelength perturbations with $k << 1/\sqrt{K}$ (physical wavelength is larger than t) the attenuation is milder for the second root of above equation: attenuation takes place as $(a/a_0)^{\sqrt{K}k^2/2}$. The conclusion is that irregularities in all scales are smoothed away but that attenuation is much slower for the long wave length perturbations.

The absence of horizons in the string dominated phase has a rather interesting consequence. According to the well known Jeans criterion the size L of density fluctuations leading to the formation of structures [E12] must satisfy the following conditions

$$l_J < L < l_H \ , \quad (12.3.51)$$

where l_H denotes the size of horizon and l_J denotes the Jeans length related to the sound velocity v_s and cosmic proper time as [E12]

$$l_J \simeq 10 v_s t \ . \quad (12.3.52)$$

For a string dominated cosmology the size of the horizon is infinite so that no upper bound for the size of the possible structures results. These structures of course, correspond to string like objects of various sizes in the microscopic description. This suggests that primordial fluctuations create structures of arbitrary large size, which become visible at much later time, when cosmology becomes string dominated again.

Limiting temperature

Since particles are extended objects in TGD, one expects the existence of the limiting temperature T_H (Hagedorn temperature as it is called in string models) so that the primordial cosmology is in Hagedorn temperature. A special consequence is that the contribution of the light particles to the energy density becomes negligible: this is in accordance with the string dominance of the critical mass cosmology. The value of T_H is of order $T_H \sim \hbar/R$, where R is CP_2 radius of order $R \sim 10^{3.5}\sqrt{G}$ and thus considerably smaller than Planck temperature. Note that T_H increases with Planck constant and one can wonder whether this increase continues only up to $T_H = \hbar_{cr}/R = \sqrt{\hbar_{cr}/G}$, which corresponds to the critical value $\hbar_{cr} = R^2/G$. The value $R^2/G = 3 \times 20^{23}\hbar_0$ is consistent with p-adic mass calculations and is favored by by number theoretical arguments [K45, L63] .

The existence of limiting temperature gives strong constraint to the value of the light cone proper time a_F when radiation dominance must have established itself in the critical cosmology which gave rise to our sub-cosmology. Before the moment of transition to hyperbolic cosmology critical cosmology is string dominated and the generation of negative energy virtual gravitons builds up gradually the huge energy density density, which can lead to gravitational collapse, splitting of the strings and establishment of thermal equilibrium with gradually rising temperature. This temperature cannot however become higher than Hagedorn temperature T_H, which serves thus as the highest possible temperature of the effectively radiation dominated cosmology following the critical period. The decay of the split strings generates elementary particles providing the seeds of galaxies.

If most strings decay to light particles then energy density is certainly of the form $1/a^4$ of radiation dominated cosmology. This is not the only manner to obtain effective radiation dominance. Part of the thermal energy goes to the kinetic energy of the vibrational motion of strings and energy density $\rho \propto 1/a^2$ cannot hold anymore. The strings of the condensate is expected to obey the scaling law $\rho \propto 1/a^4$, $p = \rho/3$ [E12] . The simulations with string networks suggest that the energy density of the string network behaves as $\rho \propto 1/a^{2(1+v^2)}$, where v^2 is the mean square velocity of the point of the string [E15] . Therefore, if the value of the mean square velocity approaches light velocity, effective radiation dominance results even when strings dominate [E35] . In radiation dominated cosmology the velocity of sound is $v = 1/\sqrt{3}$. When v lowers to sound velocity one obtains stationary cosmology which is string dominated.

An estimate for a_F is obtained from the requirement that the temperature of the radiation dominated cosmology, when extrapolated from its value $T_R \simeq .3\text{eV}$ at the time about $a_R \sim 3 \times 10^7$ years for the decoupling of radiation and matter to $a = a_F$ using the scaling law $T \propto 1/a$, corresponds to Hagedorn temperature. This gives

$$\begin{aligned} a_F &= a_R \frac{T_R}{T_H} \ , \\ T_H &= \frac{n}{R} \ , \qquad a_R \sim 3 \times 10^7 \ y \ , \quad T_R = .27 \ eV \ . \end{aligned} \tag{12.3.53}$$

This gives a rough estimate $a_F \sim 3 \times 10^{-10}$ seconds, which corresponds to length scale of order 7.7×10^{-2} meters. The value of a_F is quite large.

The result does not mean that radiation dominated sub-cosmologies might have not developed before $a = a_F$. In fact, entire series of critical sub-cosmologies could have developed to radiation dominated phase before the final one leading to our sub-cosmology is actually possible. The contribution of sub-cosmology i to the total energy density of recent cosmology is in the first approximation equal to the fraction $(a_F(i)/a_F)^4$. This ratio is multiplied by a ratio of numerical factors telling the number of effectively massless particle species present in the condensate if elementary particles dominate the mass density. If strings dominate the mass density (as expected) the numerical factor is absent.

For some reason the later critical cosmologies have not evolved to the radiation dominated phase. This might be due to the reduced density of cosmic strings in the vapor phase caused by the formation of the earlier cosmologies which does not allow sufficiently strong gravitational collapse to develop and implies that critical cosmology transforms directly to stationary cosmology without the intervening effectively radiation dominated phase. Indeed, condensed cosmic strings develop Kähler electric field compensating the huge positive Kähler action of free string and can survive the decay to light particles if they are not split. The density of split strings yielding light particles is presumably the proper parameter in this respect.

p-Adic length scale hypothesis allows rather predictive quantitative model for the series of sub-cosmologies [K89] predicting the number of them and allowing to estimate the moments of their birth, the durations of the critical periods and also the durations of radiation dominated phases. p-Adic length scale hypothesis allows also to estimate the maximum temperature achieved during the critical period: this temperature depends on the duration of the critical period a_1 as $T \sim n/a_1$, where n turns out to be of order 10^{30}. This means that if the duration of the critical period is long enough, transition to string dominated asymptotic cosmology occurs with the intervening decay of cosmic strings leading to the radiation dominated phase.

The existence of the limiting temperature has radical consequences concerning the properties of the very early cosmology. The contribution of a given massless particle to the energy density becomes constant. So, unless the number of the effectively massless particle families $N(a)$ increases too fast the contribution of the effectively massless particles to the energy density becomes negligible. The massive excitations of large size (string like objects) are indeed expected to become dominant in the mass density.

What about thermodynamical implications of dark matter hierarchy?

The previous discussion has not mentioned dark matter hierarchy labeled by increasing values of Planck constants and predicted macroscopic quantum coherence in arbitrarily long scales. In TGD Universe dark matter hierarchy means also a hierarchy of conscious entities with increasingly long span of memory and higher intelligence [K98, K38] .

This forces to ask whether the second law is really a fundamental law and whether it could reflect a wrong view about existence resulting resulting when all these dark matter levels and information associated with conscious experiences at these levels is neglected. For instance, biological evolution difficult to understand in a universe obeying second law relies crucially on evolution as gradual progress in which sudden leaps occur as new dark matter levels emerge.

TGD inspired consciousness suggests that Second Law holds true only for the mental images of a given self (a system able to avoid bound state entanglement with environment [K98]) rather than being a universal physical law. Besides these mental images there is irreducible basic awareness of self and second law does not apply to it. Also the hierarchy of higher level conscious entities is there. In this framework second law would basically reflect the exclusion of conscious observers from the physical model of the Universe.

12.3.4 Mechanism Of Accelerated Expansion In TGD Universe

In TGD framework the most plausible identification for the accelerated periods of cosmic expansion is in terms of phase transitions increasing gravitational Planck constant. These phase transitions would in average sense provide quantum counterpart for smooth cosmic expansion. These phase transitions might be initiated by the repulsive Coulomb interaction between cosmic strings driven to the boundaries of the large voids. It is interesting to see how this view relates with the assumption of positive cosmological constant.

How accelerated expansion results in standard cosmology?

The accelerated of cosmic expansion means that the deceleration parameter

$$q = -(ad^2a/ds^2)/(da/ds)^2$$

is negative. For Robertson-Walker cosmologies one has

$$\begin{aligned} H^2 &\equiv (\frac{da/ds}{a})^2 = \frac{8\pi G\rho + \Lambda}{3} - K/a^2, \quad K = 0, \pm 1 \ , \\ 3\frac{d^2a/ds^2}{a} &= \Lambda - 4\pi G(\rho + 3p) \equiv -4\pi G(1+3w)\rho \ . \end{aligned} \tag{12.3.54}$$

It is clear that the accelerated expansion requires positive value of Λ.

The deceleration parameter can be expressed as $q = \frac{1}{2}(1+3w)(1+K/(aH)^2)$. $K =, 0, 1, -1$ tells whether the cosmology is flat, hyper-spherical, or hyperbolic. The rate for the change of Hubble constant can be expressed as $(dH/ds)/H^2 = (1+q)$ and the acceleration of cosmic expansion means $q < -1$. All particle models predict $q \geq -1$.

On basis of modified Einstein's equations written for the recent metric convention (+,-,-.-) (note that opposite signature changes the sign of the left hand side)

$$-G^{\alpha\beta} - \Lambda g^{\alpha\beta} = 8\pi G T^{\alpha\beta} \tag{12.3.55}$$

it is clear that the introduction of a positive cosmological constant could be interpreted by saying that for gravitational vacuum carries energy density equal to $\Lambda/8\pi$ and negative pressure. The negative gravitational pressure would induce the acceleration.

Cosmological term at the level of field equations could be also interpreted by saying that Einstein's equations hold true in the original sense but that energy momentum tensor contains besides the density of inertial mass also a positive density of purely gravitational mass: $T \rightarrow T + \Lambda g$ so that Equivalence Principle fails. Since cosmological constant means effectively negative pressure $p = -\Lambda/8\pi$ the introduction of the cosmological constant means the effective replacement $\rho+3p \rightarrow \rho+3p-2\Lambda/8\pi$). In the so called $\Lambda-CDM$ model [E5] the densities of dark energy, ordinary matter, and dark matter are assumed to sum up to critical mass density $\rho_{cr} = 3/(8\pi g_{aa} G a^2)$. The fraction of dark matter density is deduced to be $\Omega_\Lambda = .74$ from mere criticality.

Critical cosmology predicts accelerated expansion

In order to get clue about the mechanism of accelerated cosmic expansion in TGD framework it is useful to study the deceleration parameter for various cosmologies in TGD framework.

In standard Friedmann cosmology with non-vanishing cosmological constant one has

$$3\frac{d^2a/ds^2}{a} = \Lambda - 4\pi G(\rho + 3p) \ . \tag{12.3.56}$$

From this form it is obvious why $\Lambda > 0$ is required in order to obtain accelerating expansion.

Deceleration parameter is a purely geometric property of cosmology and defined as

$$q \equiv -a\frac{d^2a/ds^2}{(da/ds)^2} \ . \tag{12.3.57}$$

During radiation and matter dominated phases the value of q is positive. In TGD framework there are several metrics which are independent of details of dynamics.

1. *String dominated cosmology*

String dominated cosmology is hyperbolic cosmology and might serve as a model for very early cosmology corresponds to the metric

$$g_{aa} \equiv (ds/da)^2 = 1 - K_0 \ . \tag{12.3.58}$$

In this case one has $q = 0$.

2. *Critical cosmology*

Critical cosmology with flat 3-space corresponds to

$$\begin{aligned} g_{aa} &= 1 - K \ , \\ K &\equiv \frac{K_0}{1-u^2} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \tag{12.3.59}$$

g_{aa} has the same form also for over-critical cosmologies. Both cosmologies have finite duration. In this case q is given by

$$q = -K_0\frac{K_0u^2}{1-u^2-K_0} < 0 \ , \tag{12.3.60}$$

and is negative. The rate of change for Hubble constant is

$$\frac{dH/ds}{H^2} = -(1+q) \ , \tag{12.3.61}$$

so that one must have $q < -1$ in order to have acceleration. This holds true for $a > \sqrt{(1-K_0)/(1+K_0)}a_1$.

Quantum critical cosmology could be seen as a universal characteristic of quantum critical phases associated with phase transition like phenomena. No assumptions about the mechanism behind the transition are made. There is great temptation to assign this cosmology to the phase transitions increasing the size of large voids occurring during late cosmology. The observed jerk assumed to lead from de-accelerated to accelerated expansion for about 13 billion years ago might have interpretation as a transition of this kind.

3. Stationary cosmology

TGD predicts a one-parameter family of stationary cosmologies from the requirement that the density of gravitational 4-momentum is conserved. This is guaranteed if curvature scalar is extremized. These cosmologies are expected to define asymptotic cosmologies or at least characterize the stationary phases between quantum phase transitions. The metric is given by

$$\begin{aligned} g_{aa} &= \frac{1-2x}{1-x} \ , \\ x &= (\frac{a_0}{a})^{2/3} \ . \end{aligned} \tag{12.3.62}$$

The deceleration parameter

$$q = \frac{1}{3}\frac{x}{(1-2x)(1-x)} \ . \tag{12.3.63}$$

is positive so that it seems that TGD does not lead to a continual acceleration which might be regarded as tearing galaxies into pieces.

If quantum critical phases correspond to the expansion of large voids induced by the accelerated radial motion of galactic strings as they reach the boundaries of the voids, one can consider a series of phase transitions between stationary cosmologies in which the value of gravitational Planck constant and the parameter a_0 characterizing the stationary cosmology increase by some even power of two as the ruler-and-compass integer hypothesis [K45, K42] and p-adic length scale hypothesis suggests.

4. Summary

One can safely conclude that TGD predict accelerated cosmic expansion during critical periods and that dark energy is replaced with dark matter in TGD framework. There is also a rather clear view about detailed mechanism leading to the accelerated expansion at "microscopic" level. Some summarizing remarks are in order.

1. Accelerated expansion is predicted only during periods of over-critical and critical cosmologies parameterized essentially by their duration. The microscopic description would be in terms of phase transitions increasing the size scale of large void. This phase transition is basically a quantum jump increasing gravitational Planck constant and thus the size of the large void. p-Adic length scales are favored sizes of the large voids. A large piece of 4-D cosmological history would be replaced by a new one in this transition so that quite a dramatic event would be in question.
2. p-Adic fractality forces to ask whether there is a fractal hierarchy of time scales in which Equivalence Principle in the formulation provided by General Relativity sense fails locally (no failure in stringy sense). This would predict a fractal hierarchy of large voids and phase transitions during which accelerated expansion occurs.
3. Cosmological constant can be said to be vanishing in TGD framework and the description of accelerated expansion in terms of a positive cosmological constant is not equivalent with TGD description since only effective pressure is negative. TGD description has some resemblance to the description in terms of quintessence [E8] , a hypothetical form of matter for which equation of state is of form $p = -w\rho$, $w < -1/3$, so that one has $\rho + 3p = 1 - w < 0$ and deceleration parameter can be negative. The energy density of quintessence is however positive. TGD does not predict endlessly accelerated acceleration tearing galaxies into pieces if the total purely gravitational energy of large voids is assumed to vanish so that Equivalence Principle holds above this length scale.

TGD counterpart of Λ as a density of dark matter rather than dark energy

The value of Λ is expressed usually as a fraction of vacuum energy density from the critical mass density. Combining the data about acceleration of cosmic expansion with the data about cosmic microwave background gives $\Omega_\Lambda \simeq .74$.

1. Critical mass density requires also in TGD framework the presence of dark contribution since visible matter contribute only a few percent of the total mass density and $\Omega_\Lambda \simeq . - 74$ characterizes this contribution. Since the acceleration mechanism has nothing to do with dark energy, dark energy can be replaced with dark matter in TGD framework.
2. The dark matter hierarchy labeled by the values of Planck constant suggests itself. The $1/a^2$ behavior of dark matter density suggests an interpretation as dark matter topologically condensed on cosmic strings. Besides ordinary particles also super-symplectic bosons and their super partners playing a key role in the model of hadrons and black holes suggest themselves.
3. Stationary cosmology predicts that the density of stringy matter and thus dark matter decreases like $1/a^2$ as a function of M^4_+ proper time. This behavior is very natural in cosmic string dominated cosmology and one expects that the TGD counterpart of cosmological constant should behave as $\Lambda \propto 1/a^2$ in average sense. At primordial period cosmological constant would be gigantic but its recent value would be extremely small and naturally of correct order of magnitude if the fraction of positive gravitational energy is few per cent about negative gravitational energy. Hence the basic problem of the standard cosmology would find an elegant solution.

Piecewise constancy of TGD counterpart of Λ and p-adic length scale hypothesis

There are good reasons to believe that TGD counterpart of Λ is piecewise constant. Classical picture suggests that the sizes of large voids increase in discrete jumps. The transitions increasing the size of the void would occur when the galactic strings end up to the boundary of the large void and large repulsive Coulomb energy forces the phase transition increasing Planck constant.

Also the quantum astrophysics based on the notion of gravitational Planck constant strongly suggests that astrophysical systems are analogous to stationary states of atoms so that the sizes of astrophysical systems remain constant during the cosmological expansion, and can change only in quantum jumps increasing the value of Planck constant and therefore increasing the radius of the large void regarded as dark matter bound state.

Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids with phase transitions increasing k by even integer. What values of k are realized depends on the time scale of the dynamics driving the galactic strings to the boundaries of expanded large void. Even if all values of k are realized the transitions becomes very rare for large values of a.

p-Adic fractality predicts that the effective cosmological constant Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing the value of effective cosmological constant Λ. As noticed, the allowed values of k would be of form $k = k_0+2n$, where however all integer value need not be realized. By p-adic length scale hypothesis primes are candidates for k. The recent value of the effective cosmological constant can be understood. The gravitational energy density usually assigned to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic prediction is consistent with the recent study [E37] according to which cosmological constant has not changed during the last 8 billion years: the conclusion comes from the reshifts of supernovae of type I_a. If p-adic length scales $L_e(k) = p \simeq 2^k$, k any positive integer, are allowed, the finding gives the lower bound $T_N > \sqrt(2)/(\sqrt{2}-1)) \times 8 = 27.3$ billion years for the recent age of the universe.

Brad Shaefer from Lousiana University has studied the red shifts of gamma ray bursters up to a red shift $z = 6.3$, which corresponds to a distance of 13 billion light years [E14] , and claims that the fit to the data is not consistent with the time independence of the cosmological constant. In TGD framework this would mean that a phase transition changing the value of the cosmological constant must have occurred during last 13 billion years. In principle the phase transitions increasing the size of large voids could be observed as sudden changes of sign for the deceleration parameter.

The reported cosmic jerk as an accelerated period of cosmic expansion

There is an objection against the hypothesis that cosmological constant has been gradually decreasing during the cosmic evolution. Type Ia supernovae at red shift $z \sim .45$ are fainter than expected, and the interpretation is in terms of an accelerated cosmic expansion [E13] . If a period of an accelerated expansion has been preceded by a decelerated one, one would naïvely expect that for older supernovae from the period of decelerating expansion, say at redshifts about $z > 1$, the effect should be opposite. The team led by Adam Riess [E23] has identified 16 type Ia supernovae at redshifts $z > 1.25$ and concluded that these supernovae are indeed brighter. The conclusion is that about about 5 billion years ago corresponding to $z \simeq .48$, the expansion of the Universe has suffered a cosmic jerk and transformed from a decelerated to an accelerated expansion.

The apparent dimming/brightening of supernovae at the period of accelerated/decelerated expansion the follows from the luminosity distance relation

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2} , \tag{12.3.64}$$

where $\mathcal{L}$ is actual luminosity and $\mathcal{F}$ measured luminosity, and from the expression for the distance d_L in flat cosmology in terms of red shift z in a flat Universe

$$\begin{aligned} d_L &= (1+z)\int_0^z \frac{du}{H(u)} \\ &= (1+z)H_0^{-1}\int_0^z exp\left[-\int_0^u du\,[1+q(u)]\,d(ln(1+u)\right] du , \end{aligned} \tag{12.3.65}$$

where one has

$$\begin{aligned} H(z) &= \frac{dln(a)}{ds} , \\ q &\equiv -\frac{d^2a/ds^2}{aH^2} = \frac{dH^{-1}}{ds} - 1 . \end{aligned} \tag{12.3.66}$$

In TGD framework a corresponds to the light-cone proper time and s to the proper time of Robertson-Walker cosmology. Depending on the sign of the deceleration parameter q, the distance d_L is larger or smaller and accordingly the object looks dimmer or brighter.

The natural interpretation for the jerk would be as a period of accelerated cosmic expansion due to a phase transition increasing the value of gravitational Planck constant.

12.4 Microscopic Description Of Black-Holes In TGD Universe

In TGD framework the embedding of the metric for the interior of Schwartshild black-hole fails below some critical radius. This strongly suggests that only the exterior metric of black-hole makes sense in TGD framework and that TGD must provide a microscopic description of black-holes. Somewhat unexpectedly, I ended up with this description from a model of hadrons.

Super-symplectic algebra is a generalization of Kac-Moody algebra obtained by replacing the finite-dimensional group G with the group of symplectic transformations of $\delta M^4_\pm \times CP_2$. This algebra defines the group of isometries for the “world of classical worlds” and together with the Kac-Moody algebra assignable to the deformations of light-like 3-surfaces representing orbits of 2-D partonic surfaces it defines the mathematical backbone of quantum TGD as almost topological QFT.

From the point of view of experimentalist the basic question is how these super-symplectic degrees of freedom reflect themselves in existing physics and the pleasant surprise was that super-symplectic bosons explain what might be called the missing hadronic mass and spin. The point is that quarks explain only about 170 MeV of proton mass. Also the spin puzzle of proton is known for years. Also precise mass formulas for hadrons emerge.

Super-symplectic degrees of freedom represent dark matter in electro-weak sense and highly entangled hadronic strings in Hagedorn temperature are very much analogous to black-holes. This indeed generalizes to a microscopic model for black-holes created when hadronic strings fuse together in high density.

12.4.1 Super-Symplectic Bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-symplectic algebra [K31, K113] , whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry. If one assumes that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5,6,58) for the three lowest generations. The application of super-symplectic bosons in TGD based model of hadron masses is discussed in [K70] and here only a brief summary is given.

As explained in [K70] , the assignment of these bosons to hadronic space-time sheet is an attractive idea.

1. Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD. A possible identification of this contribution is in terms of super-symplectic gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.
2. Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.
3. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.
4. Super-symplectic bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [C32] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-symplectic matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [C38]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the

penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

12.4.2 Are Ordinary Black-Holes Replaced With Super-Symplectic Black-Holes In TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-hole like states associated with super-symplectic gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-symplectic black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-symplectic black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-symplectic black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-symplectic black-hole and ordinary black-holes would be naturally replaced with super-symplectic black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

1. Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = (\frac{M}{m(CP_2)})^2 \times log(p) \ , \quad (12.4.1)$$

where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-symplectic particles it gives the entire mass.

2. If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = klog(2) \times (\frac{M}{m(CP_2)})^2, \quad (12.4.2)$$

$m(CP_2) = \hbar/R$, R the "radius" of CP_2, corresponds to the standard value of h for all values of h_{eff}.

3. Hawking-Bekenstein area law gives in the case of Schwartschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi G M^2 \times \hbar \ . \quad (12.4.3)$$

For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $klog(2) \simeq log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p. For macroscopic super-symplectic black-holes modified area law results if the radius of the large wormhole throat equals to Schwartschild radius. Schwartschild radius is indeed natural: a simple deformation of the Schwartschild exterior metric to a metric representing rotating star transforms Schwartschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

4. The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [K89] reads as

$$\hbar_{gr} = \frac{GMm}{v_0}\hbar_0 \ .$$

$v_0 = 2^{-11}$ is the preferred value of v_0. One could argue that the value of gravitational Planck constant is such that the Compton length $\hbar_{gr}/M$ of the black-hole equals to its Schwartshild radius. This would give

$$\hbar_{gr} \ = \ \frac{GM^2}{v_0}\hbar_0 \ , \ v_0 = 1/2 \ . \qquad (12.4.4)$$

The requirement that $\hbar_{gr}$ is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [K89] . Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

5. The gravitational collapse of a star would correspond to a process in which the initial value of v_0 , say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0. If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.
6. Black-hole evaporation could be seen as means for the super-symplectic black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-symplectic black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-symplectic black-hole would have only angular momentum and right handed neutrino number.
7. In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-symplectic particles so that super-symplectic black-hole would be single gigantic super-symplectic parton. The interior of super-symplectic black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

12.4.3 Anyonic View About Blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy - being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic. $\hbar = GM^2/v_0$, $v_0 = 1/4$, would hold true for an ideal black hole with Planck length $(\hbar G)^{1/2}$ equal to Schwartshild radius $2GM$. Since black hole entropy is inversely proportional to $\hbar$, this would predict black hole entropy to be of order single bit. This of course looks totally non-sensible if one believes in standard thermodynamics. For the star with mass equal to 10^{40} Planck masses the entropy associated with the initial state of the star would be roughly the number of atoms in star equal to about 10^{60}. Black hole entropy proportional to $GM^2/\hbar$ would be of order 10^{80} provided the standard value of $\hbar$ is used as unit. This stimulates some questions.

1. Does second law pose an upper bound on the value of $\hbar$ of dark black hole from the requirement that black hole has at least the entropy of the initial state. The maximum value of $\hbar$ would be given by the ratio of black hole entropy to the entropy of the initial state and about 10^{20} in the example consider to be compared with $GM^2/v_0 \sim 10^{80}$.
2. Or should one generalize thermodynamics in a way suggested by ZEO by making explicit distinction between subjective time (sequence of quantum jumps) and geometric time? The arrow of geometric time would correlate with that of subjective time. One can argue that the geometric time has opposite direction for the positive and negative energy parts of the zero energy state interpreted in standard ontology as initial and final states of quantum event. If second law would hold true with respect to subjective time, the formation of ideal dark black hole would destroy entropy only from the point of view of observer with standard arrow of geometric time. The behavior of phase conjugate laser light would be a more mundane example. Do self assembly processes serve as example of non-standard arrow of geometric time in biological systems? In fact, zero energy state is geometrically analogous to a big bang followed by big crunch. One can however criticize the basic assumption as ad hoc guess. One should really understand the the arrow of geometric time. This is discussed in detail in [L4] .

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

12.5 A Quantum Model For The Formation Of Astrophysical Structures And Dark Matter?

D. Da Rocha and Laurent Nottale, the developer of Scale Relativity, have ended up with an highly interesting quantum theory like model for the evolution of astrophysical systems [E18] (I am grateful for Victor Christianito for informing me about the article). In particular, this model applies to planetary orbits. I learned later that also A. Rubric and J. Rubric have proposed a Bohr model for planetary orbits [E32] already 1998.

The model is simply Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant

$$\hbar \quad \rightarrow \quad \hbar_{gr} = \frac{GmM}{v_0} \quad . \tag{12.5.1}$$

Here I have used units $\hbar = c = 1$. v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. The peak orbital velocity of stars in galactic halos is 142 ± 2 km/s whereas the average velocity is 156 ± 2 km/s. Also sub-harmonics and harmonics of v_0 seem to appear.

The model makes fascinating predictions which hold true. For instance, the radii of planetary orbits fit nicely with the prediction of the hydrogen atom like model. The inner solar system (Mercury,Venus,Earth, Mars) corresponds to v_0 and outer solar system to $v_0/5$.

The predictions for the distribution of major axis and eccentricities have been tested successfully also for exoplanets. Also the periods of 3 planets around pulsar PSR B1257+12 fit with the predictions with a relative accuracy of few hours/per several months. Also predictions for the distribution of stars in the regions where morphogenesis occurs follow from the gravitational Schödinger equation.

What is important is that there are no free parameters besides v_0. In [E18] a wide variety of astrophysical data is discussed and it seem that the model works and has already now made predictions which have been later verified. In the following I shall discuss Nottale's model from the point of view of TGD.

12.5.1 TGD Prediction For The Parameter v_0

One of the basic questions is the origin of the parameter v_0, which according to a rich amount of experimental data discussed in [E18] seems to play a role of a constant of Nature. One of the first applications of cosmic strings in TGD sense was an explanation of the velocity spectrum of stars in the galactic halo in terms of dark matter which could consists of cosmic strings. Cosmic strings could be orthogonal to the galactic plane going through the nucleus (jets) or they could be in galactic plane in which case the strings and their decay products would explain dark matter assuming that the length of cosmic string inside a sphere of radius R is or has been roughly R [K32] . The predicted value of the string tension is determined by the CP_2 radius whose ratio to Planck length is fixed by electron mass via p-adic mass calculations. The resulting prediction for the v_0 is correct and provides a working model for the constant orbital velocity of stars in the galactic halo.

The parameter $v_0 \simeq 2^{-11}$, which has actually the dimension of velocity unless on puts $c = 1$, and also its harmonics and sub-harmonics appear in the scaling of $\hbar$. v_0 corresponds to the velocity of distant stars in the model of galactic dark matter. TGD allows to identify this parameter as the parameter

$$\begin{aligned} v_0 &= 2\sqrt{TG} = \sqrt{\frac{1}{2\alpha_K}}\sqrt{\frac{G}{R^2}} \ , \\ T &= \frac{1}{8\alpha_K}\frac{\hbar_0}{R^2} \ . \end{aligned} \tag{12.5.2}$$

Here T is the string tension of cosmic strings, R denotes the "radius" of CP_2 ($2R$ is the radius of geodesic sphere of CP_2). α_K is Kähler coupling strength, the basic coupling constant strength of TGD, whose evolution as a function of p-adic length scale is fixed by quantum criticality. The condition that G is invariant in the p-adic coupling constant evolution and number theoretical arguments predict

$$\begin{aligned} \alpha_K(p) &= k\frac{1}{log(p)+log(K)} \ , \\ K &= \frac{R^2}{\hbar_0 G} = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23 \ , \quad k\simeq \pi/4 \ . \end{aligned} \tag{12.5.3}$$

The predicted value of v_0 depends logarithmically on the p-adic length scale and for $p \simeq 2^{127}-1$ (electron's p-adic length scale) one has $v_0 \simeq 2^{-11}$.

12.5.2 Model ror planetary orbits without $v_0 \rightarrow v_0/5$ scaling

Also harmonics and sub-harmonics of v_0 appear in the model of Nottale and Da Rocha. For instance, the outer planets (Jupiter, Saturn,...) correspond to $v_0/5$ whereas the 4 inner planets correspond to v_0. Quite generally, it is found that the values seem to come as harmonics and sub-harmonics of v_0: $v_n = nv_0$ and v_0/n, and the argument [E18] is that the different values of n relate to fractality. This scaling is not necessary for the planetary orbits in TGD based model.

Effectively a multiplication $n \to 5n$ of the principal quantum number is in question in the case of outer planets. If one accepts the interpretation that visible matter has concentrated around dark matter, which is in macroscopic quantum phase around Bohr orbits, this allows to consider also the possibility that $\hbar_{gr}$ has the same value for all planets.

1. Some gravitational perturbation has kicked dark matter from the region of the asteroid belt to $n \simeq 5k$, $k = 2, .., 6$, orbits. The best fit is obtained by using values of n deviating somewhat from multiples of 5 which suggests that the scaling of v_0 is not needed. Gravitational perturbations might have caused the same for the visible matter. The fact that the tilt angles of Earth and outer planets other than Pluto are nearly the same suggests that the orbits of these planets might be an outcome of some violent quantum process for dark matter preserving the orbital plane in a good approximation. Pluto might in turn have experienced some violent collision changing its orbital plane.
2. There could exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter.

Table 12.1 gives the radii of planet orbits predicted by Bohr orbit model and by Titius-Bode law.

	Exp.	T-B	Bohr_1	Bohr_2
Planet	R/R_M	R/R_M	$[n, R/R_M]$	$[n, R/R_M]$
Mercury	1	1	[3, 1]	
Venus	1.89	1.75	[4, 1.8]	
Earth	2.6	2.5	[5, 2.8]	
Mars	3.9	4	[6, 4]	
Asteroids	6.1-8.7	7	[(7, 8, 9), (5.4, 7.1, 9)]	
Jupiter	13.7	13	[11, 13.4]	[2×5,11.1]
Saturn	25.0	25	[3×5, 25]	[3×5, 25]
Uranus	51.5	49	[22, 53.8]	[4×5,44.4]
Neptune	78.9	97	[27 , 81]	[5×5, 69.4]
Pluto	105.2	97	[31, 106.7]	[6×5,100]

Table 12.1: Table represents the experimental average orbital radii of planets, the predictions of Titius-Bode law (note the failure for Neptune), and the predictions of Bohr orbit model assuming a) that the principal quantum number n corresponds to best possible fit, b) the scaling $v_0 \to v_0/5$ for outer planets. Option a) gives the best fit with errors being considerably smaller than the maximal error $|\Delta R|/R \simeq 1/n$ except for Uranus. R_M denotes the orbital radius of Mercury. T-B refers to Titius-Bode law.

How to understand the harmonics and sub-harmonics of v_0 in TGD framework?

Also harmonics and sub-harmonics of v_0 appear in the model of Nottale and Da Rocha. In particular, the outer planets (Jupiter, Saturn,...) correspond to $v_0/5$ whereas the 4 inner planets correspond to v_0 in this model. As already found, TGD allows also an alternative explanation.

Quite generally, it is found that the values seem to come as harmonics and sub-harmonics of v_0: $v_n = nv_0$ and v_0/n, and the argument [E18] is that the different values of n relate to fractality. This quantization is a challenge for TGD since v_0 certainly defines a fundamental constant in TGD Universe.

1. Consider first the harmonics of v_0. Besides cosmic strings of type $X^2 \times S^2 \subset M^4 \times CP_2$ one can consider also deformations of these strings defining their multiple coverings so that the deformation is n-valued as a function of S^2-coordinates (Θ, Φ) and the projection to S^2 is thus an $n \to 1$ map. The solutions are higher dimensional analogs of originally closed orbits which after perturbation close only after n turns. This kind of surfaces emerge in the TGD inspired

model of quantum Hall effect naturally [K7] and $n \to \infty$ limit has an interpretation as an approach to chaos [K102] .
Using the coordinates (x, y, θ, ϕ) of $X^2 \times S^2$ and coordinates m^k for M^4 of the unperturbed solution the space-time surface the deformation can be expressed as

$$\begin{aligned} m^k &= m^k(x,y,\theta,\phi) \ , \\ (\Theta,\Phi) &= (\theta, n\phi) \ . \end{aligned} \quad (12.5.4)$$

The value of the string tension would be indeed n^2-fold in the first approximation since the induced Kähler form defining the Kähler magnetic field would be $J_{\theta\phi} = nsin(\Theta)$ and one would have $v_n = nv_0$. At the limit $m^k = m^k(x, y)$ different branches for these solutions collapse together.

2. Consider next how sub-harmonics appear in TGD framework. Suppose that cosmic strings decay to magnetic flux tube structures. This could the counterpart for cosmic expansion. The Kähler magnetic flux $\Phi = BS$ is conserved in the process but the thickness of the M^4 projection of the cosmic string increases field strength is reduced. This means that string tension, which is proportional to B^2S, is reduced (so that also Kähler action is reduced). The fact that space-time surface is Bohr orbit in generalized sense means that the reduced string tension (magnetic energy per unit length) is quantized.

   The task is to guess how the quantization occurs. There are two options.

1. The simplest explanation for the reduction of v_0 is based on the decay of a flux tube resembling a disk with a hole to n identical flux tubes so that $v_0 \to v_0/n$ results for the resulting flux tubes. It turns out that this mechanism is favored and explains elegantly the value of $\hbar_{gr}$ for outer planetary system. One can also consider small-p p-adicity so that n would be prime.
2. Second explanation is more intricate. Consider a magnetic flux tube. Since magnetic flux is quantized, the magnetic field strengths are quantized in integer multiples of basic strength: $B = nB_0$ and would rather naturally correspond to the multiple coverings of the original magnetic flux tube with magnetic energy quantized in multiples of n^2. The idea is to require internal consistency in the sense that the allowed reduced field strengths are such that the spectrum associated with B_0 is contained to the spectrum associated with the quantized field strengths $B_1 > B_0$. This would allow only field strengths $B = B_S/n^2$, where B_S denotes the field strength of the fundamental cosmic string and one would have $v_n = v_0/n$. Flux conservation requires that the area of the flux tube scales as n^2.

Sub-harmonics might appear in the outer planetary system and there are indications for the higher harmonics below the inner planetary system [E18] : for instance, solar radius corresponds to $n = 1$ orbital for $v_3 = 3v_0$. This would suggest that Sun and also planets have an onion like structure with highest harmonics of v_0 and strongest string tensions appearing in the solar core and highest sub-harmonics appearing in the outer regions. If the matter results as decay remnants of cosmic strings this means that the mass density inside Sun should correlate strongly with the local value of n characterizing the multiple covering of cosmic strings.

One can ask whether the very process of the formation of the structures could have excited the higher values of n just like closed orbits in a perturbed system become closed only after n turns. The energy density of the cosmic string is about one Planck mass per $\sim 10^7$ Planck lengths so that $n > 1$ excitation increasing this density by a factor of n^2 is obviously impossible except under the primordial cosmic string dominated period of cosmology during which the net inertial energy density must have vanished. The structure of the future solar system would have been dictated already during the primordial phase of cosmology when negative energy cosmic string suffered a time reflection to positive energy cosmic strings.

Nottale equation is consistent with the TGD based model for dark matter

TGD allows two models of dark matter. The first one is spherically symmetric and the second one cylindrically symmetric. The first thing to do is to check whether these models are consistent with the gravitational Schrödinger equation/Bohr quantization.

1. Spherically symmetric model for the dark matter

The following argument based on Bohr orbit quantization demonstrates that this is indeed the case for the spherically symmetric model for dark matter. The argument generalizes in a trivial manner to the cylindrically symmetric case.

1. The gravitational potential energy $V(r)$ for a mass distribution $M(r) = xTr$ (T denotes string tension) is given by

$$V(r) = Gm\int_r^{R_0} \frac{M(r)}{r^2}dr = GmxTlog(\frac{r}{R_0}) \ . \tag{12.5.5}$$

   Here R_0 corresponds to a large radius so that the potential is negative as it should in the region where binding energy is negative.

2. The Newton equation $\frac{mv^2}{r} = \frac{GmxT}{r}$ for circular orbits gives

$$v = xGT \ . \tag{12.5.6}$$

3. Bohr quantization condition for angular momentum by replacing $\hbar$ with $\hbar_{gr}$ reads as $mvr = n\hbar_{gr}$ and gives

$$\begin{aligned} r_n &= \frac{n\hbar_{gr}}{mv} = nr_1 \ , \\ r_1 &= \frac{GM}{vv_0} \ . \end{aligned} \tag{12.5.7}$$

   Here v is rather near to v_0.

4. Bound state energies are given by

$$E_n = \frac{mv^2}{2} - xTlog(\frac{r_1}{R_0}) + xTlog(n) \ . \tag{12.5.8}$$

   The energies depend only weakly on the radius of the orbit.

5. The centrifugal potential $l(l+1)/r^2$ in the Schrödinger equation is negligible as compared to the potential term at large distances so that one expects that degeneracies of orbits with small values of l do not depend on the radius. This would mean that each orbit is occupied with same probability irrespective of value of its radius. If the mass distribution for the starts does not depend on r, the number of stars rotating around galactic nucleus is simply the number of orbits inside sphere of radius R and thus given by $N(R) \propto R/r_0$ so that one has $M(R) \propto R$. Hence the model is self consistent in the sense that one can regard the orbiting stars as remnants of cosmic strings and thus obeying same mass distribution.

2. Cylindrically symmetric model for the galactic dark matter

TGD allows also a model of the dark matter based on cylindrical symmetry. In this case the dark matter would correspond to the mass of a cosmic string orthogonal to the galactic plane and traversing through the galactic nucleus. The string tension would the one predicted by TGD. In the directions orthogonal to the plane of galaxy the motion would be free motion so that the orbits would be helical, and this should make it possible to test the model. The quantization of radii of the orbits would be exactly the same as in the spherically symmetric model. Also the quantization of inclinations predicted by the spherically symmetric model could serve as a sensitive test. In this kind of situation general theory of relativity would predict only an angle deficit giving rise to a lens effect. TGD predicts a Newtonian $1/\rho$ potential in a good approximation.

Spiral galaxies are accompanied by jets orthogonal to the galactic plane and a good guess is that they are associated with the cosmic strings. The two models need not exclude each other. The vision about astrophysical structures as pearls of a fractal necklace would suggest that the visible matter has resulted in the decay of cosmic strings originally linked around the cosmic string going through the galactic plane and creating $M(R) \propto R$ for the density of the visible matter in the galactic bulge. The finding that galaxies are organized along linear structures [E40] fits nicely with this picture.

MOND and TGD

TGD based model explains also the MOND (Modified Newton Dynamics) model of Milgrom [E29] for the dark matter. Instead of dark matter, the model assumes a modification of Newton's law of gravitation. The model is based on the observation that the transition to a constant velocity spectrum seems in the galactic halos seems to occur at a constant value of the stellar acceleration equal to $a_0 \simeq 10^{-11}g$, where g is the gravitational acceleration at the Earth. MOND theory assumes that Newtonian laws are modified below a_0.

The explanation relies on Bohr quantization. Since the stellar radii in the halo are quantized in integer multiples of a basic radius and since also rotation velocity v_0 is constant, the values of the acceleration are quantized as $a(n) = v_0^2/r(n)$ and a_0 correspond to the radius $r(n)$ of the smallest Bohr orbit for which the velocity is still constant. For larger orbital radii the acceleration would indeed be below a_0. a_0 would correspond to the distance above which the density of the visible matter does not appreciably perturb the gravitational potential of the straight string. This of course requires that gravitational potential is that given by Newton's theory and is indeed allowed by TGD.

The MOND theory (see http://tinyurl.com/qt875) [E29] and its variants predict that there is a critical acceleration below which Newtonian gravity fails. This would mean that Newtonian gravitation is modified at large distances. String models and also TGD predict just the opposite since in this regime General Relativity should be a good approximation.

1. The $1/r^2$ force would transform to $1/r$ force at some critical acceleration of about $a = 10^{-10}$ m/s^2: this is a fraction of 10^{-11} about the gravitational acceleration at the Earth's surface.
2. The recent empirical study (see http://tinyurl.com/ychyy3z3) [E25] giving support for this kind of transition in the dynamics of stars at large distances and therefore breakdown of Newtonian gravity in MOND like theories.

In TGD framework critical acceleration is predicted but the recent experiment does not force to modify Newton's laws. Since Big Science is like market economy in the sense that funding is more important than truth, the attempts to communicate TGD based view about dark matter [K42, K89, K75, K90, K32] have turned out to be hopeless. Serious Scientist does not read anything not written on silk paper.

1. One manner to produce this spectrum is to assume density of dark matter such that the mass inside sphere of radius R is proportional to R at last distances [K32]. Decay products of and ideal cosmic strings (see http://tinyurl.com/y8wbeo4q) would predict this. The value of the string tension predicted correctly by TGD using the constraint that p-adic mass calculations give electron mass correctly [K60].
2. One could also assume that galaxies are distributed along cosmic string like pearls in necklace. The mass of the cosmic string would predict correct value for the velocity of distant stars. In the ideal case there would be no dark matter outside these cosmic strings.
   (a) The difference with respect to the first mechanism is that this case gravitational acceleration would vanish along the direction of string and motion would be free motion. The prediction is that this kind of motions take place along observed linear structures formed by galaxies and also along larger structures.
   (b) An attractive assumption is that dark matter corresponds to phases with large value of Planck constant is concentrated on magnetic flux tubes. Holography would suggest that the density of the magnetic energy is just the density of the matter condensed at wormhole throats associated with the topologically condensed cosmic string.
   (c) Cosmic evolution modifies the ideal cosmic strings and their Minkowski space projection gets gradually thicker and thicker and their energy density - magnetic energy - characterized by string tension could be affected

TGD option differs from MOND in some respects and it is possible to test empirically which option is nearer to the truth.

1. The transition at same critical acceleration is predicted universally by this option for all systems-now stars- with given mass scale if they are distributed along cosmic strings like like pearls in necklace. The gravitational acceleration due the necklace simply wins the gravitational acceleration due to the pearl. Fractality encourages to think like this.
2. The critical acceleration predicted would correspond to acceleration of the same order of magnitude as the acceleration caused by cosmic string. From $M^2/R_{cr} = GM/R_{cr}^2 = TG/R_{cr}$ (assuming that dark matter dominates) one obtains the estimate $R_{cr} = M/T$ and $a_{cr} = GT^2/M$, where M is the visible mass of the object - for instance the ordinary matter of a

galaxy. If critical acceleration is always the same, one would have $T = (a_{cr}M/G)^{1/2}$ so that the visible mass would scale like $M \propto T^2$ if a_{cr} is constant of Nature.

3. If $1/r^2$ changes to $1/r$ in the MOND model, one obtains the same predictions as in TGD for the planar orbits orthogonal to the long string along which galaxies correspond to flux tube tangles. The models are not equivalent. In TGD, general orbit of the star corresponds to a helical motion of the star in the plane orthogonal to the cosmic string and along the cosmic string so that the obser concentration of visible matter on a preferred plane is predicted. This concentration of orbits in a single plane has been recently reported as an anomaly of dark matter models [L124].

TGD option explains also other strange findings of cosmology.

1. The basic prediction is the large scale motions of dark matter along cosmic strings. The characteristic length and time scale of dynamics is scaled up by the scaling factor of $\hbar$. This could explain the observed large scale motion of galaxy clusters - dark flow (see `http://tinyurl.com/ckfg25`) [E3] - assigned with dark matter in conflict with the expectations of standard cosmology.
2. Cosmic strings could also relate to the strange relativistic jet like structures (see `http://tinyurl.com/2x5od6`) [E9] meaning correlations between very distant objects. Universe would be a spaghetti of cosmic strings around which matter is concentrated.
3. The TGD based model for the final state of star (see `http://tinyurl.com/yantmeot`) [K106] actually predicts the presence of string like object defining preferred rotation axis. The beams of light emerging from supernovae would be preferentially directed along this lines- actually magnetic flux tubes. Same would apply to the gamma ray bursts (see `http://tinyurl.com/csd2an`) [E4] from quasars, which would not be distributed evenly in all directions but would be like laser beams along cosmic strings.

12.5.3 The Interpretation Of $\hbar_{gr}$ And Pre-Planetary Period

$\hbar_{gr}$ could corresponds to a unit of angular momentum for quantum coherent states at magnetic flux tubes or walls containing macroscopic quantum states. Quantitative estimate demonstrates that $\hbar_{gr}$ for astrophysical objects cannot correspond to spin angular momentum. For Sun-Earth system one would have $\hbar_{gr} \simeq 10^{77}\hbar$. This amount of angular momentum realized as a mere spin would require 10^{77} particles! Hence the only possible interpretation is as a unit of orbital angular momentum. The linear dependence of $\hbar_{gr}$ on m is consistent with the additivity of angular momenta in the fusion of magnetic flux tubes to larger units if the angular momentum associated with the tubes is proportional to both m and M.

Just as the gravitational acceleration is a more natural concept than gravitational force, also $\hbar_{gr}/m = GM/v_0$ could be more natural unit than $\hbar_{gr}$. It would define a universal unit for the circulation $\oint v \cdot dl$, which is apart from $1/m$-factor equal to the phase integral $\oint p_\phi d\phi$ appearing in Bohr rules for angular momentum. The circulation could be associated with the flow associated with outer boundaries of magnetic flux tubes surrounding the orbit of mass m around the central mass $M \gg m$ and defining light like 3-D CDs analogous to black hole horizons.

The expression of $\hbar_{gr}$ depends on masses M and m and can apply only in space-time regions carrying information about the space-time sheets of Mandthe orbit of m. Quantum gravitational holography suggests that the formula applies at 3-D light like causal determinant (CD) X_l^3 defined by the wormhole contacts gluing the space-time sheet X_l^3 of the planet to that of Sun. More generally, X_l^3 could be the space-time sheet containing the planet, most naturally the magnetic flux tube surrounding the orbit of the planet and possibly containing dark matter in super-conducting state. This would give a precise meaning for $\hbar_{gr}$ and explain why $\hbar_{gr}$ does not depend on the masses of other planets.

The simplest option consistent with the quantization rules and with the explanatory role of magnetic flux structures is perhaps the following one.

1. X_l^3 is a torus like surface around the orbit of the planet containing de-localized dark matter. The key role of magnetic flux quantization in understanding the values of v_0 suggests the interpretation of the torus as a magnetic or Z^0 magnetic flux tube. At pre-planetary period the dark matter formed a torus like quantum object. The conditions defining the radii of Bohr orbits follow from the requirement that the torus-like object is in an eigen state of angular

momentum in the center of mass rotational degrees of freedom. The requirement that rotations do not leave the torus-like object invariant is obviously satisfied. Newton's law required by the quantum-classical correspondence stating that the orbit corresponds to a geodesic line in general relativistic framework gives the additional condition implying Bohr quantization.

2. A simple mechanism leading to the localization of the matter would have been the pinching of the torus causing kind of a traffic jam leading to the formation of the planet. This process could quite well have involved a flow of matter to a smaller planet space-time sheet Y^3_l topologically condensed at X^3_l. Most of the angular momentum associated with torus like object would have transformed to that of planet and situation would have become effectively classical.
3. The conservation of magnetic flux means that the splitting of the orbital torus would generate a pair of Kähler magnetic charges. It is not clear whether this is possible dynamically and hence the torus could still be there. In fact, TGD explanation for the tritium beta decay anomaly citeTroitsk,Mainz in terms of classical Z^0 force [K93] requires the existence of this kind of torus containing neutrino cloud whose density varies along the torus. This picture suggests that the lacking $n = 1$ and $n = 2$ orbits in the region between Sun and Mercury are still in magnetic flux tube state containing mostly dark matter.
4. The fact that $\hbar_{gr}$ is proportional to m means that it could have varied continuously during the accumulation of the planetary mass without any effect in the planetary motion: this is of course nothing but a manifestation of Equivalence Principle.
5. It is interesting to look for the scaled up versions of Planck mass $m_{Pl} = \sqrt{\hbar_{gr}/\hbar} \times \sqrt{\hbar/G} = \sqrt{M_1M_2/v_0}$ and Planck length $L_{Pl} = \sqrt{\hbar_{gr}/\hbar} \times \sqrt{\hbar/G} = G\sqrt{M_1M_2/v_0}$. For $M_1 = M_2 = M$ this gives $m_{Pl} = M/\sqrt{v_0} \simeq 45.6\times M$ and $L_{Pl} = r_S/2\sqrt{v_0} \simeq 22.8\times r_S$, where r_S is Schwartshild radius. For Sun r_S is about 2.9 km so that one has $L_{Pl} \simeq 66$ km. For a few years ago it was found that Sun contains "inner-inner" core of radius about $R = 300$ km [F7] which is about $4.5 \times L_{Pl}$.

12.5.4 Inclinations For The Planetary Orbits And The Quantum Evolution Of The Planetary System

The inclinations of planetary orbits provide a test bed for the theory. The semiclassical quantization of angular momentum gives the directions of angular momentum from the formula

$$cos(\theta) = \frac{m}{\sqrt{j(j+1)}} \quad , \quad |m| \leq j \ . \tag{12.5.9}$$

where θ is the angle between angular momentum and quantization axis and thus also that between orbital plane and (x,y)-plane. This angle defines the angle of tilt between the orbital plane and (x,y)-plane.

$m = j = n$ gives minimal value of angle of tilt for a given value of n of the principal quantum number as

$$cos(\theta) = \frac{n}{\sqrt{n(n+1)}} \ . \tag{12.5.10}$$

For $n = 3, 4, 5$ (Mercury, Venus, Earth) this gives $\theta = 30.0$, 26.6, and 24.0 degrees respectively.

Only the relative tilt angles can be compared with the experimental data. Taking as usual the Earth's orbital plane as the reference the relative tilt angles give what are known as inclinations. The predicted inclinations are 6 degrees for Mercury and 2.6 degrees for Venus. The observed values [E10] are 7.0 and 3.4 degrees so that the agreement is satisfactory. If one allows half-odd integer spin the fit is improved. For $j = m = n - 1/2$ the predictions are 7.1 and 2.9 degrees for Mercury and Venus respectively. For Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto the inclinations are 1.9, 1.3, 2.5, 0.8, 1.8, 17.1 degrees. For Mars and outer planets the tilt angles are predicted to have wrong sign for $m = j$. In a good approximation the inclinations vanish for outer planets except Pluto and this would allow to determine m as $m \simeq \sqrt{5n(n+1)/6}$: the fit is not good.

The assumption that matter has condensed from a matter rotating in (x,y)-plane orthogonal to the quantization axis suggests that the directions of the planetary rotation axes are more or less the same and by angular momentum conservation have not changed appreciably. The prediction for the tilt of the rotation axis of the Earth is 24 degrees of freedom in the limit that the Earth's spin can be treated completely classically, that is for $m = j >> 1$ in the units used for the quantization of the Earth's angular momentum. What is the value of $\hbar_{gr}$ for Earth is not obvious (using the unit $\hbar_{gr} = GM^2/v_0$ the Earth's angular momentum would be much smaller than one). The tilt of the rotation axis of Earth with respect to the orbit plane is 23.5 degrees so that the agreement is again satisfactory. This prediction is essentially quantal: in purely classical theory the most natural guess for the tilt angle for planetary spins is 0 degrees.

The observation that the 4 inner planets Mercury, Venus, Earth, and Mars have in a reasonable approximation the predicted inclinations suggest that they originate from a primordial period during which they formed spherical cells of dark matter and had thus full rotational degrees of freedom and were in eigen states of angular momentum corresponding to a full rotational symmetry. The subsequent $SO(3) \rightarrow SO(2)$ symmetry breaking leading to the formation of torus like configurations did not destroy the information about this period since the information about the value of j and m was coded by the inclination of the planetary orbit.

In contrast to this, the dark matter associated with Earth and outer planets up to Neptune formed a flattened magnetic or Z^0 magnetic flux tube resembling a disk with a hole and the subsequent symmetry breaking broke it to separate flux tubes. Earth's spherical disk was joined to the disk formed by the outer planets. The spherical disk could be still present and contain super-conducting dark matter. The presence of this "heavenly sphere" might closely relate to the fact that Earth is a living planet. The time scale $T = 2\pi R/c$ is very nearly equal to 5 minutes and defines a candidate for a bio-rhythm.

If this flux tube carried the same magnetic flux as the flux tubes associated with the inner planets, the decomposition of the disk with a hole to 5 flux tubes corresponding to Earth and to the outer planets Mars, Jupiter, Saturn and Neptune, would explain the value of v_0 correctly and also the small inclinations of outer planets. That Pluto would not originate from this structure, is consistent with its anomalously large values of inclination $i = 17.1$ degrees, small value of eccentricity $e = .248$, and anomalously large value of inclination of equator to orbit about 122 degrees as compared to 23.5 degrees in the case of Earth [E10] .

12.5.5 Eccentricities And Comets

Bohr-Sommerfeld quantization allows also to deduce the eccentricities of the planetary and comet orbits. One can write the quantization of energy as

$$\begin{aligned} \frac{p_r^2}{2m_1} + \frac{p_\theta^2}{2m_1 r^2} + \frac{p_\phi^2}{2m_1 r^2 sin^2(\theta)} - \frac{k}{r} &= -\frac{E_1}{n^2} \ , \\ E_1 = \frac{k^2}{2\hbar_{gr}^2} \times m_1 &= \frac{v_0^2}{2} \times m_1 \ . \end{aligned} \tag{12.5.11}$$

Here one has $k = GMm_1$. E_1 is the binding energy of $n = 1$ state. In the orbital plane ($\theta = \pi/2, p_\theta = 0$) the conditions are simplified. Bohr quantization gives $p_\phi = m\hbar_{gr}$ implying

$$\frac{p_r^2}{2m_1} + \frac{k^2\hbar_{gr}^2}{2m_1 r^2} - \frac{k}{r} = -\frac{E_1}{n^2} \ . \tag{12.5.12}$$

For $p_r = 0$ the formula gives maximum and minimum radii $r_\pm$ and eccentricity is given by

$$e^2 = \frac{r_+ - r_-}{r_+} = \frac{2\sqrt{1 - \frac{m^2}{n^2}}}{1 + \sqrt{1 - \frac{m^2}{n^2}}} \ . \tag{12.5.13}$$

For small values of n the eccentricities are very large except for $m = n$. For instance, for ($m = n - 1, n$) for $n = 3, 4, 5$ gives $e = (.93, .89, .86)$ to be compared with the experimental values (.206,

.007, .0167). Thus the planetary eccentricities with Pluto included ($e = .248$) must vanish in the lowest order approximation and must result as a perturbation of the magnetic flux tube.

The large eccentricities of comet orbits might however have an interpretation in terms of $m < n$ states. The prediction is that comets with small eccentricities have very large orbital radius. Oort's cloud is a system weakly bound to a solar system extending up to 3 light years. This gives the upper bound $n \leq 700$ if the comets of the cloud belong to the same family as Mercury, otherwise the bound is smaller. This gives a lower bound to the eccentricity of not nearly circular orbits in the Oort cloud as $e > .32$.

12.5.6 Why The Quantum Coherent Dark Matter Is Not Visible?

The obvious objection against quantal astrophysics is that astrophysical systems look extremely classical. Quantal dark matter in many-sheeted space-time resolves this counter argument. As already explained, the sequence of symmetry breakings of the rotational symmetry would explain nicely why astral Bohr rules work. The prediction is however that de-localized quantal dark matter is probably still present at (the boundaries of) magnetic flux tubes and spherical shells. It is however the entire structure defined by the orbit which behaves like a single extended particle so that the localization in quantum measurement does not mean a localization to a point of the orbit. Planet itself corresponds to a smaller localized space-time sheet condensed at the flux tube.

One should however understand why this dark matter with a gigantic Planck constant is not visible. The simplest explanation is that there cannot be any direct quantum interactions between ordinary and dark matter in the sense that particles with different values of Planck constant could appear in the same particle vertex. This would allow also a fractal hierarchy copies of standard model physics to exist with different p-adic mass scales.

There is also second argument. The inability to observe dark matter could mean inability to perform state function reduction localizing the dark matter. The probability for this should be proportional to the strength of the measurement interaction. For photons the strength of the interaction is characterized by the fine structure constant. In the case of dark matter the fine structure constant is replaced with

$$\alpha_{em,gr} = \alpha_{em} \times \frac{\hbar}{\hbar_{gr}} = \alpha_{em} \times \frac{v_0}{GMm} \ . \tag{12.5.14}$$

For $M = m = m_{Pl} \simeq 10^{-8}$ kg the value of the fine structure constant is smaller than $\alpha_{em} v_0$ and completely negligible for astrophysical masses. However, for processes for which the lowest order classical rates are non-vanishing, rates are not affected in the lowest order since the increase of the Compton length compensates the reduction of α. Higher order corrections become however small. What makes dark matter invisible is not the smallness of α_{em} but the fact that the binding energies of say hydrogen atom proportional to $\alpha^2 m_e$ are scaled as $1/\hbar^2$ so that the spectrum is scaled down.

12.5.7 Quantum Interpretation Of Gravitational Schrödinger Equation

Schrödinger equation - or even Bohr rules - in astrophysical length scales with a gigantic value of Planck constant looks sheer madness from the standard physics point of view. In TGD Universe situation is different. TGD predicts infinite hierarchy of effective values of Planck constants $h_{eff} = n \times h$ and $h_{gr} = h_{eff}$ is a natural assumption. The high values of Planck constant is effective but it implies macroscopic quantum coherence in scales proportional to h_{eff}. The hierarchy of effective Planck constants labels the levels of a hierarchy of quantum criticalities, which is basic prediction of TGD. The hierarchy of Planck constants is associated with dark matter.

The special feature of gravitational interaction is that h_{gr} characterizing its strength is proportional to the product of the interacting masses. Hence gravitational Compton length $\hbar_{gr}/m = GM/v_0$ is independent of the smaller mass and same for all particles. The predictions for the quantal behavior of massive bodies follow from the mere assumption that microscopic particles couple to the large central mass via magnetic flux tubes with large value of h_{gr}. What the situation actually is remains open. Interestingly, in the model of bio-photons as decay products of dark photons with $h_{gr} = h_{eff}$ the energy spectrum of dark cyclotron photons is universal and co-incides with the spectrum of bio-photons [K76, **?**].

Bohr quantization of planetary orbits and prediction for Planck constant

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. Generalization of the p-adic length scale hypothesis

The evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only square roots of rationals are number theoretically special since they correspond to algebraic extensions of p-adic numbers involving only square roots which should emerge first and therefore systems involving these values of q should be especially abundant in Nature.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K71] .

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constants assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution. The conjecture would be that $h_{gr}/h = n_F$ holds true.

2. Can one really identify gravitational and inertial Planck constants?

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_{eff} = n \times h$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v \cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This suggests that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.
2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of h_{eff}. Due to the large masses the identification $h_{gr} = h_{eff} = n \times h$ can be made without experimental uncertainties.

The recent view about the quantization of Planck constant in terms of coverings of space-time seems to resolve these problems.

1. One can also make the identification $\hbar_{gr} = \hbar_{eff} = n \times \hbar_0$ and associate it with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets would correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets correspond to n-fold covering of M^4, one can understand $\hbar_{gr} = n \times \hbar_0$ as a particular instance of the $\hbar_{eff}$. Note that v_0 could depend on planet in this case.

2. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant $\hbar_{gr}=\hbar_{eff} = n \times h$ within experimental resolution. A stronger prediction would follow from that v_0 is constant for inner *resp.* outer planets and $\hbar_{gr}/\hbar_0 = n_F$. The ratios of planetary masses would be ratios of Fermat integers in this case. The accuracy is about 10 per cent and the discrepancy could be explained in terms of the variation of v_0. One can imagine also other preferred values of n. In particular, $n = p^k$, p prime, is favored by the generalized p-adic length scale hypothesis following from number theoretical arguments and NMP [K111].

Quantization as a means of avoiding gravitational collapse

Schrödinger equation provided a solution to the infrared catastrophe of the classical model of atom: the classical prediction was that electron would radiate its energy as brehmstrahlung and would be captured by the nucleus. The gravitational variant of this process would be the capture of the planet by a black hole, and more generally, a collapse of the star to a black hole. Gravitational Schrödinger equation could obviously prevent the catastrophe.

For $1/r$ gravitation potential the Bohr radius is given by $a_{gr} = GM/v_0^2 = r_S/2v_0^2$, where $r_S = 2GM$ is the Schwartchild radius of the mass creating the gravitational potential: obviously Bohr radius is much larger than the Schwartschild radius. That the gravitational Bohr radius does not depend on m conforms with Equivalence Principle, and the proportionality $\hbar_{gr} \propto Mm$ can be deduced from it. Gravitational Bohr radius is by a factor $1/2v_0^2$ larger than black hole radius so that black hole can swallow the piece of matter with a considerable rate only if it is in the ground state and also in this state the rate is proportional to the black hole volume to the volume defined by the black hole radius given by $2^3v_0^6 \sim 10^{-20}$.

The $\hbar_{gr} \to \infty$ limit for $1/r$ gravitational potential means that the exponential factor $exp(-r/a_0)$ of the wave function becomes constant: on the other hand, also Schwartshild and Bohr radii become infinite at this limit. The gravitational Compton length associated with mass m does not depend on m and is given by GM/v_0 and the time $T = E_{gr}/\hbar_{gr}$ defined by the gravitational binding energy is twice the time taken to travel a distance defined by the radius of the orbit with velocity v_0 which suggests that signals travelling with a maximal velocity v_0 are involved with the quantum dynamics.

In the case of planetary system the proportionality $\hbar_{gr} \propto mM$ creates problems of principle since the influence of the other planets is not taken account. One might argue that the generalization of the formula should be such that M is determined by the gravitational field experienced by mass m and thus contains also the effect of other planets. The problem is that this field depends on the position of m which would mean that $\hbar_{gr}$ itself would become kind of field quantity.

Does the transition to non-perturbative phase correspond to a change in the value of $\hbar$?

Nature is populated by systems for which perturbative quantum theory does not work. Examples are atoms with $Z_1Z_2e^2/4\pi\hbar > 1$ for which the binding energy becomes larger than rest mass, non-perturbative QCD resulting for $Q_{s,1}Q_{s,2}g_s^2/4\pi\hbar > 1$, and gravitational systems satisfying $GM_1M_2/4\pi\hbar > 1$. Quite generally, the condition guaranteeing troubles is of the form $Q_1Q_2g^2/4\pi\hbar > 1$. There is no general mathematical approach for solving the quantum physics of these systems but it is believed that a phase transition to a new phase of some kind occurs.

The gravitational Schrödinger equation forces to ask whether Nature herself takes care of the problem so that this phase transition would involve a change of the value of the Planck constant to guarantee that the perturbative approach works. The values of $\hbar$ would vary in a stepwise manner from $\hbar(\infty)$ to $\hbar(3) = \hbar(\infty)/4$. The non-perturbative phase transition would correspond to transition to the value of

$$\frac{\hbar}{\hbar_0} \rightarrow \left[\frac{Q_1Q_2g^2}{v}\right] \tag{12.5.15}$$

where $[x]$ is the integer nearest to x, inducing

$$\frac{Q_1Q_2g^2}{4\pi\hbar} \rightarrow \frac{v}{4\pi} \quad . \tag{12.5.16}$$

The simplest (and of course ad hoc) assumption making sense in TGD Universe is that v is a harmonic or subharmonic of v_0 appearing in the gravitational Schrödinger equation. For instance, for the Kepler problem the spectrum of binding energies would be universal (independent of the values of charges) and given by $E_n = v^2m/2n^2$ with v playing the role of small coupling. Bohr radius would be g^2Q_2/v^2 for $Q_2 \gg Q_1$.

This provides a new insight to the problems encountered in quantizing gravity. QED started from the model of atom solving the infrared catastrophe. In quantum gravity theories one has started directly from the quantum field theory level and the recent decline of the M-theory shows that we are still practically where we started. If the gravitational Schrödinger equation indeed allows quantum interpretation, one could be more modest and start from the solution of the gravitational IR catastrophe by assuming a dynamical spectrum of $\hbar$ comes as integer multiples of ordinary Planck constant. The implications would be profound: the whole program of quantum gravity would have been misled as far as the quantization of systems with $GM_1M_2/\hbar > 1$ is considered. In practice, these systems are the most interesting ones and the prejudice that their quantization is a mere academic exercise would have been completely wrong.

An alternative formulation for the occurrence of a transition increasing the value of $\hbar$ could rely on the requirement that classical bound states have reasonable quantum counterparts. In the gravitational case one would have $r_n = n^2\hbar_{gr}^2/GM_1^2M$, for $M_1 \ll M$, which is extremely small distance for $\hbar_{gr} = \hbar$ and reasonable values of n. Hence, either n is so large that the system is classical or $\hbar_{gr}/\hbar$ is very large. Equivalence Principle requires the independence of r_n on M_1, which gives $\hbar = kGM_1M_2$ giving $r_n = n^2kGM$. The requirement that the radius is above Schwartshild radius gives $k \geq 2$. In the case of Dirac equation the solutions cease to exist for $Z \geq 137$ and which suggests that $\hbar$ is large for hypothetical atoms having $Z \geq 137$.

12.5.8 How Do The Magnetic Flux Tube Structures And Quantum Gravitational Bound States Relate?

In the case of stars in galactic halo the appearance of the parameter v_0 characterizing cosmic strings as orbital rotation velocity can be understood classically. That v_0 appears also in the gravitational dynamics of planetary orbits could relate to the dark matter at magnetic flux tubes. The argument explaining the harmonics and sub-harmonics of v_0 in terms of properties of cosmic strings and magnetic flux tubes identifiable as their descendants strengthens this expectation.

The notion of magnetic body

In TGD inspired theory of consciousness the notion of magnetic body plays a key role: magnetic body is the ultimate intentional agent, experiencer, and performer of bio-control and can have astrophysical size: this does not sound so counter-intuitive if one takes seriously the idea that cognition has p-adic space-time sheets as space-time correlates and that rational points are common to real and p-adic number fields. The point is that infinitesimal in p-adic topology corresponds to infinite in real sense so that cognitive structures would have literally infinite size.

The magnetic flux tubes carrying various supra phases can be interpreted as special instance of dark energy and dark matter. This suggests a correlation between gravitational self-organization and quantum phases at the magnetic flux tubes and that the gravitational Schrödinger equation somehow relates to the ordinary Schrödinger equation satisfied by the macroscopic quantum phases at magnetic flux tubes. Interestingly, the transition to large Planck constant phase should occur when the masses of interacting is above Planck mass since gravitational self-interaction energy is $V \sim GM^2/R$. For the density of water about 10^3 kg/m^3 the volume carrying a Planck mass correspond to a cube with side 2.8×10^{-4} meters. This corresponds to a volume of a large neuron, which suggests that this phase transition might play an important role in neuronal dynamics.

Could gravitational Schrödinger equation relate to a quantum control at magnetic flux tubes?

An infinite self hierarchy is the basic prediction of TGD inspired theory of consciousness ("everything is conscious and consciousness can be only lost"). Topological quantization allows to assign to any material system a field body as the topologically quantized field pattern created by the system [?, ?] . This field body can have an astrophysical size and would utilize the material body as a sensory receptor and motor instrument.

Magnetic flux tube and flux wall structures are natural candidates for the field bodies. Various empirical inputs have led to the hypothesis that the magnetic flux tube structures define a hierarchy of magnetic bodies, and that even Earth and larger astrophysical systems possess magnetic body which makes them conscious self-organizing living systems. In particular, life at Earth would have developed first as a self-organization of the super-conducting dark matter at magnetic flux tubes [?] .

For instance, EEG frequencies corresponds to wavelengths of order Earth size scale and the strange findings of Libet about time delays of conscious experience [J8, J6] find an elegant explanation in terms of time taken for signals propagate from brain to the magnetic body [?] . Cyclotron frequencies, various cavity frequencies, and the frequencies associated with various p-adic frequency scales are in a key role in the model of bio-control performed by the magnetic body. The cyclotron frequency scale is given by $f = eB/m$ and rather low as are also cavity frequencies such as Schumann frequencies: the lowest Schumann frequency is in a good approximation given by $f = 1/2\pi R$ for Earth and equals to 7.8 Hz.

1. Quantum time scales as "bio-rhythms" in solar system?

To get some idea about the possible connection of the quantum control possibly performed by the dark matter with gravitational Schrödinger equation, it is useful to look for the values of the periods defined by the gravitational binding energies of test particles in the fields of Sun and Earth and look whether they correspond to some natural time scales. For instance, the period $T = 2GM_S n^2/v_0^3$ defined by the energy of n^{th} planetary orbit depends only on the mass of Sun and defines thus an ideal candidate for a universal "bio-rhythm".

For Sun black hole radius is about 2.9 km. The period defined by the binding energy of lowest state in the gravitational field of Sun is given $T_S = 2GM_S/v_0^3$ and equals to 23.979 hours for $v_0/c = 4.8233 \times 10^{-4}$. Within experimental limits for v_0/c the prediction is consistent with 24 hours! The value of v_0 corresponding to exactly 24 hours would be $v_0 = 144.6578$ km/s (as a matter fact, the rotational period of Earth is 23.9345 hours). As if as the frequency defined by the lowest energy state would define a "biological" clock at Earth! Mars is now a strong candidate for a seat of life and the day in Mars lasts 24hr 37m 23s! $n = 1$ and $n = 2$ are orbitals are not realized in solar system as planets but there is evidence for the $n = 1$ orbital as being realized as a peak in the density of IR-dust [E18] . One can of course consider the possibility that these levels are populated by small dark matter planets with matter at larger space-time sheets. Bet as it may, the result supports the notion of quantum gravitational entrainment in the solar system.

The slower rhythms would become as n^2 sub-harmonics of this time scale. Earth itself corresponds to $n = 5$ state and to a rhythm of .96 hours: perhaps the choice of 1 hour to serve as a fundamental time unit is not merely accidental. The magnetic field with a typical ionic cyclotron frequency around 24 hours would be very weak: for 10 Hz cyclotron frequency in Earth's magnetic field the field strength would about 10^{-11} T. However, $T = 24$ hours corresponds with 6 per cent accuracy to the p-adic time scale $T(k = 280) = 2^{13}T(2, 127)$, where $T(2, 127)$ corresponds to the secondary p-adic time scale of .1 s associated with the Mersenne prime $M_{127} = 2^{127} - 1$ characterizing electron and defining a fundamental bio-rhythm and the duration of memetic codon [K46] .

Comorosan effect [K114] , [I13, I4] demonstrates rather peculiar looking facts about the interaction of organic molecules with visible laser light at wavelength $\lambda = 546\ nm$. As a result of irradiation molecules seem to undergo a transition $S \rightarrow S^*$. S^* state has anomalously long lifetime and stability in solution. $S \rightarrow S^*$ transition has been detected through the interaction of S^* molecules with different biological macromolecules, like enzymes and cellular receptors. Later Comorosan found that the effect occurs also in non-living matter. The basic time scale is $\tau = 5$ seconds. p-Adic length scale hypothesis does not explain τ, and it does not correspond to any obvious astrophysical time scale and has remained a mystery.

The idea about astro-quantal dark matter as a fundamental bio-controller inspires the guess that τ could correspond to some Bohr radius R for a solar system via the correspondence $\tau = R/c$. As observed by Nottale, $n = 1$ orbit for $v_0 \rightarrow 3v_0$ corresponds in a good approximation to the solar radius and to $\tau = 2.18$ seconds. For $v_0 \rightarrow 2v_0$ $n = 1$ orbit corresponds to $\tau = AU/(4 \times 25) = 4.992$ seconds: here $R = AU$ is the astronomical unit equal to the average distance of Earth from Sun.

The deviation from τ_C is only one per cent and of the same order of magnitude as the variation of the radius for the orbit due to orbital eccentricity $(a-b)/a = .0167$ [E10] .

2. Earth-Moon system

For Earth serving as the central mass the Bohr radius is about 18.7 km, much smaller than Earth radius so that Moon would correspond to $n = 147.47$ for v_0 and $n = 1.02$ for the sub-harmonic $v_0/12$ of v_0. For an afficionado of cosmic jokes or a numerologist the presence of the number of months in this formula might be of some interest. Those knowing that the Mayan calendar had 11 months and that Moon is receding from Earth might rush to check whether a transition from $v/11$ to $v/12$ state has occurred after the Mayan culture ceased to exist: the increase of the orbital radius by about 3 per cent would be required! Returning to a more serious mode, an interesting question is whether light satellites of Earth consisting of dark matter at larger space-time sheets could be present. For instance, in [?] I have discussed the possibility that the larger space-time sheets of Earth could carry some kind of intelligent life crucial for the bio-control in the Earth's length scale.

The period corresponding to the lowest energy state is from the ratio of the masses of Earth and Sun given by $M_E/M_S = (5.974/1.989) \times 10^{-6}$ given by $T_E = (M_E/M_S) \times T_S = .2595$ s. The corresponding frequency $f_E = 3.8535$ Hz frequency is at the lower end of the theta band in EEG and is by 10 per cent higher than the p-adic frequency $f(251) = 3.5355$ Hz associated with the p-adic prime $p \simeq 2^k$, $k = 251$. The corresponding wavelength is 2.02 times Earth's circumference. Note that the cyclotron frequencies of Nn, Fe, Co, Ni, and Cu are $5.5, 5.0, 5.2, 4.8$ Hz in the magnetic field of $.5 \times 10^{-4}$ Tesla, which is the nominal value of the Earth's magnetic field. In [K83] I have proposed that the cyclotron frequencies of Fe and Co could define biological rhythms important for brain functioning. For $v_0/12$ associated with Moon orbit the period would be 7.47 s: I do not know whether this corresponds to some bio-rhythm.

It is better to leave for the reader to decide whether these findings support the idea that the super conducting cold dark matter at the magnetic flux tubes could perform bio-control and whether the gravitational quantum states and ordinary quantum states associated with the magnetic flux tubes couple to each other and are synchronized.

12.5.9 About The Interpretation Of The Parameter v_0

The formula for the gravitational Planck constant contains the parameter $v_0/c = 2^{-11}$. This velocity defines the rotation velocities of distant stars around galaxies. It can be seen also as a charasteristic velocity scale for inner planets. The presence of a parameter with dimensions of velocity should carry some important information about the geometry of dark matter space-time sheets.

Velocity like parameters appear also in other contexts. There is evidence for the Tifft's quantization of cosmic redshifts in multiples of $v_0/c = 2.68 \times 10^{-5}/3$: also other units of quantization have been proposed but they are multiples of v_0 [E39] .

The strange behavior of graphene includes high conductivity with conduction electrons behaving like massless particles with light velocity replaced with $v_0/c = 1/300$. The TGD inspired model [K21] explains the high conductivity as being due to the Planck constant $\hbar(M^4) = 6\hbar_0$ increasing the de-localization length scale of electron pairs associated with hexagonal rings of mono-atomic graphene layer by a factor 6 and thus making possible overlap of electron orbitals. This explains also the anomalous conductivity of DNA containing 5- and 6-cycles [K21] .

p-Adic length scale hypothesis and $v_0 \to v_0/5$ transition at inner-outer border for planetary system

$v_0 \to v_0/5$ transition would allow to interpret the orbits of outer planets as $n \geq 1$ orbits. The obvious question is whether inner to outer zone as $v_0 \to v_0/5$ transition could be interpreted in terms of the p-adic length scale hierarchy.

1. The most important p-adic length scale are given by primary p-adic length scales $L_e(k) = 2^{(k-151)/2} \times 10$ nm and secondary p-adic length scales $L_e(2, k) = 2^{k-151} \times 10$ nm, k prime.
2. The p-adic scale $L_e(2, 139) = 114$ Mkm is slightly above the orbital radius 109.4 Mkm of Venus. The p-adic length scale $L_e(2, 137) \simeq 28.5$ Mkm is roughly one half of Mercury's orbital

radius 57.9 Mkm. Thus strong form of p-adic length scale hypothesis could explain why the transition $v_0 \to v_0/5$ occurs in the region between Venus and Earth ($n = 5$ orbit for v_0 layer and $n = 1$ orbit for $v_0/5$ layer).

3. Interestingly, the *primary* p-adic length scales $L_e(137)$ and $L_e(139)$ correspond to fundamental atomic length scales which suggests that solar system be seen as a fractally scaled up "secondary" version of atomic system.
4. Planetary radii have been fitted also using Titius-Bode law predicting $r(n) = r_0 + r_1 \times 2^n$. Hence on can ask whether planets are in one-one correspondence with primary and secondary p-adic length scales $L_e(k)$. For the orbital radii $58, 110, 150, 228$ Mkm of Mercury, Venus, Earth, and Mars indeed correspond approximately to k= $276, 278, 279, 281$: note the special position of Earth with respect to its precedessor. For Jupiter, Saturn, Uranus, Neptune, and Pluto the radii are 52,95,191,301,395 Mkm and would correspond to p-adic length scales $L_e(280 + 2n))$, $n = 0, ..., 3$. Obviously the transition $v_0 \to v_0/5$ could occur in order to make the planet–p-adic length scale one-one correspondence possible.
5. It is interesting to look whether the p-adic length scale hierarchy applies also to the solar structure. In a good approximation solar radius .696 Mkm corresponds to $L_e(270)$, the lower radius .496 Mkm of the convective zone corresponds to $L_e(269)$, and the lower radius .174 Mkm of the radiative zone (radius of the solar core) corresponds to $L_e(266)$. This encourages the hypothesis that solar core has an onion like sub-structure corresponding to various p-adic length scales. In particular, $L_e(2, 127)$ ($L_e(127)$ corresponds to electron) would correspond to 28 Mm. The core is believed to contain a structure with radius of about 10 km: this would correspond to $L_e(231)$. This picture would suggest universality of star structure in the sense that stars would differ basically by the number of the onion like shells having standard sizes.

Quite generally, in TGD Universe the formation of join along boundaries bonds is the space-time correlate for the formation of bound states. This encourages to think that (Z^0) magnetic flux tubes are involved with the formation of gravitational bound states and that for $v_0 \to v_0/k$ corresponds either to a splitting of a flux tube resembling a disk with a whole to k pieces, or to the scaling down $B \to B/k^2$ so that the magnetic energy for the flux tube thickened and stretched by the same factor k^2 would not change.

After decade of developing this model, it has become clear that TGD favors generalization of p-adic length scale hypothesis: primes near but below powers of prime are favored. This could explain the factor five scaling of $1/v_0$

Is dark matter warped?

The reduced light velocity could be due to the warping of the space-time sheet associated with dark electrons. TGD predicts besides gravitational red-shift a non-gravitational red-shift due to the warping of space-time sheets possible because space-time is 4-surface rather than abstract 4-manifold. A simple example of everyday life is the warping of a paper sheet: it bends but is not stretched, which means that the induced metric remains flat although one of its component scales (distance becomes longer along direction of bending). For instance, empty Minkowski space represented canonically as a surface of $M^4 \times CP_2$ with constant CP_2 coordinates can become periodically warped in time direction because of the bending in CP_2 direction. As a consequence, the distance in time direction shortens and effective light-velocity decreases when determined from the comparison of the time taken for signal to propagate from A to B along warped space-time sheet with propagation time along a non-warped space-time sheet.

The simplest warped embedding defined by the map $M^4 \to S^1$, S^1 a geodesic circle of CP_2. Let the angle coordinate of S^1 depend linearly on time: $\Phi = \omega t$. g_{tt} component of metric becomes $1 - R^2\omega^2$ so that the light velocity is reduced to $v_0/c = \sqrt{1 - R^2\omega^2}$. No gravitational field is present.

The fact that M^4 Planck constant $n_a\hbar_0$ defines the scaling factor n_a^2 of CP_2 metric could explain why dark matter resides around strongly warped embeddings of M^4. The quantization of the scaling factor of CP_2 by $R^2 \to n_a^2 R^2$ implies that the initial small warping in the time direction given by $g_{tt} = 1-\epsilon$, $\epsilon = R^2\omega^2$, will be amplified to $g_{tt} = 1-n_a^2\epsilon$ if ω is not affected in the transition to dark matter phase. $n_a = 6$ in the case of graphene would give $1 - x \simeq 1 - 1/36$ so that only a one per cent reduction of light velocity is enough to explain the strong reduction of light velocity for dark matter.

Is c/v_0 quantized in terms of ruler and compass rationals?

The known cases suggests that c/v_0 is always a rational number expressible as a ratio of integers associated with n-polygons constructible using only ruler and compass.

1. $c/v_0 = 300$ would explain graphene. The nearest rational satisfying the ruler and compass constraint would be $q = 5 \times 2^{10}/17 \simeq 301.18$.
2. If dark matter space-time sheets are warped with $c_0/v = 2^{11}$ one can understand Nottale's quantization for the radii of the inner planets. For dark matter space-time sheets associated with outer planets one would have $c/v_0 = 5 \times 2^{11}$.
3. If Tifft's red-shifts relate to the warping of dark matter space-time sheets, warping would correspond to $v_0/c = 2.68 \times 10^{-5}/3$. $c/v_0 = 2^5 \times 17 \times 257/5$ holds true with an error smaller than .1 per cent.

Tifft's quantization and cosmic quantum coherence

An explanation for Tifft's quantization in terms of Jones inclusions could be that the subgroup G of Lorentz group defining the inclusion consists of boosts defined by multiples $\eta = n\eta_0$ of the hyperbolic angle $\eta_0 \simeq v_0/c$. This would give $v/c = sinh(n\eta_0) \simeq nv_0/c$. Thus the dark matter systems around which visible matter is condensed would be exact copies of each other in cosmic length scales since G would be an exact symmetry. The property of being an exact copy applies of course only in single level in the dark matter hierarchy. This would mean a de-localization of elementary particles in cosmological length scales made possible by the huge values of Planck constant. A precise cosmic analog for the de-localization of electron pairs in benzene ring would be in question.

Why then η_0 should be quantized as ruler and compass rationals? In the case of Planck constants the quantum phases $q = exp(im\pi/n_F)$ are number theoretically simple for n_F a ruler and compass integer. If the boost $exp(\eta)$ is represented as a unitary phase $exp(im\eta)$ at the level of discretely de-localized dark matter wave functions, the quantization $\eta_0 = n/n_F$ would give rise to number theoretically simple phases. Note that this quantization is more general than $\eta_0 = n_{F,1}/n_{F,2}$.

12.6 Some Examples About Gravitational Anomalies In TGD Universe

The many-sheeted space-time and the hierarchy of Planck constants predict new physics which should be seen as anomalies in the models based on general relativity. In the following some examples about these anomalies are discussed.

12.6.1 SN1987A And Many-Sheeted Space-Time

Lubos Motl has written a highly rhetoric, polemic, and adrenaline rich posting (see `http://tinyurl.com/px4hzdc`) about the media buzz related to supernova SN1987A. The target of Lubos Motl is the explanation proposed by James Franson from the University of Maryland for the findings discussed in Physics Archive Blog (see `http://tinyurl.com/mde7jat`). I do not have any strong attitude to Franson's explanation but the buzz is about very real thing: unfortunately Lubos Motl tends to forget the facts in his extreme orthodoxy.

What happened was following. Two separate neutrino bursts arrived from SN 1987 A. At 7.35 AM Kamionakande detected 11 antineutrons, IMB 8 antineutrinos, and Baksan 5 antineutrinos. Approximately 3 hours later Mont Blanc liquid scintillator detected 5 antineutrinos. Optical signal came 4.7 hours later.

The are several very real problems as one can get convinced by going to Wikipedia (`http://tinyurl.com/mglkm4`):

1. If neutrinos and photons are emitted simultaneously and propagate with the same speed, they should arrive simultaneously. I am not specialist enough to try to explain this difference in terms of standard astrophysics. Franson however sees this difference as something not easy to explain and tries to explain it in his own model.

2. There are two neutrino bursts rather than one. A modification of the model of supernova explosion allowing two bursts of neutrinos would be needed but this would suggest also two photon bursts.

These problems have been put under the carpet. Those who are labelled as crackpots often are much more aware about real problems than the academic career builders.

In TGD framework the explanation would be in terms of many-sheeted space-time. In GRT limit of TGD the sheets of the many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. A-6.1** in the appendix of this book) are lumped to single sheet: Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the metrics of the various sheets from Minkowski metric. The same recipe gives effective gauge potentials in terms of induced gauge potentials.

Different arrival times for neutrinos and photons would be however a direct signature of the many-sheeted space-time since the propagation velocity along space-time sheets depends on the induced metric. The larger the deviation from the flat metric is, the slower the propagation velocity and thus longer the arrival time is. Two neutrino bursts would have explanation as arrivals along two different space-time sheets. Different velocity for photons and neutrinos could be explained if they arrive along different space-time sheets. I proposed for more than two decades ago this mechanism as an explanation for the finding of cosmologists that there are two different Hubble constants: they would correspond to different space-time sheets.

The distance of SN1987A is 168, 000 light- years. This means that the difference between velocities is $\Delta c/c \simeq \Delta T/T \simeq 3hours/168 \times 10^3 \simeq 2 \times 10^{-9}$. The long distance is what makes the effect visible.

I proposed earlier sub-manifold gravity as an explanation for the claimed super-luminality of the neutrinos coming to Gran Sasso from CERN. In this case the effect would have been $\Delta c/c \simeq 2.5 \times 10^{-5}$ and thus four orders of magnitude larger than four supernova neutrinos. It however turned out that the effect was not real.

Towards the end of 2014 Lubos Motl Motl had a posting about galactic blackhole Sagittarius A as neutrino factory (see `http://tinyurl.com/pvzrqoz`). Chandra X-ray observatory (see `http://tinyurl.com/6jdp7es`) and also Nustar (`http://tinyurl.com/89b8r96`) and Swift Gamma-Ray Burst Mission (see `http://tinyurl.com/ybmrpuu6`) detected some X-ray flares from Sagittarius A. 2-3 hours earlier IceCube (see `http://tinyurl.com/lg7mko`) detected high energy neutrinos by IceCube on the South Pole.

Could neutrinos arrive from the galactic center? If they move with the same (actually somewhat lower) velocity than photons, this cannot be the case. The neutrinos did the same trick as SN1987A neutrinos and arrived 2-3 hours before the X-rays! What if one takes TGD seriously and estimates $\Delta c/c$ for this event? The result is $\Delta c/c \sim (1.25-1.40) \times 10^{-8}$ for 3 hours lapse using the estimate $r = 25,900 \pm 1,400$ light years (see `http://tinyurl.com/5vexvq`). $\Delta c/c$ is by a factor 4 larger than for SN1987A at distance about 168, 000 light years (see `http://tinyurl.com/mglkm4`). This distance is roughly 8 times longer. This would suggests that the smaller the space-time sheets the nearer the velocity of neutrinos is to its maximal value. For photons the reduction from the maximal signal velocity is larger.

12.6.2 Pioneer And Flyby Anomalies For Almost Decade Later

The article [E19] (see `http://tinyurl.com/avmndwa`) is about two old anomalies discovered in the solar system: Pioneer anomaly [E7] and Flyby anomaly [E21, E20, E17, E27] with which I worked for years ago.

I remember only the general idea that dark matter concentrations at orbits of planets or at spheres with radii equal that of orbit could cause the anomalies. So I try to reconstruct all from scratch and during reconstruction become aware of something new and elegant that I could not discover for years ago.

The popular article [E19] claims that Pioneer anomaly is understood. I am not at all convinced about the solution of Pioneer anomaly. Several "no new physics" solutions have been tailored during years but later it has been found that they do not work.

Suppose that dark matter is at the surface of sphere so that by a well-known text book theorem it does not create gravitational force inside it. This is an overall important fact, which

I did not use earlier. The model explains both anomalies and also allow to calculate the total amount of dark matter at the sphere.

1. Consider first the Pioneer anomaly.
   (a) Inside the dark matter sphere with radius of Jupiter's orbit the gravitational force caused by dark matter vanishes. Outside the sphere also dark matter contributes to the gravitational attraction and Pioneer's acceleration becomes a little bit smaller since the dark matter at the sphere containing the orbit radius of Jupiter or Saturn also attracts the space-craft after the passby. A simple test for spherical model is the prediction that the mass of Jupiter effectively increases by the amount of dark matter at the sphere after passby.
   (b) The magnitude of the Pioneer anomaly is about $\Delta a/a = 1.3 \times 10^{-4}$ [K89] and translates to $M_{dark}/M \simeq 1.3 \times 10^{-4}$. What is highly non-trivial is that the anomalous acceleration is given by Hubble constant suggesting that there is a connection with cosmology fixing the value of dark mass once the area of the sphere containing it is fixed. This follows as a prediction if the surface mass density is universal and proportional to the Hubble constant. Could one interpret the equality of the two accelerations as an equilibrium condition? The Hubble acceleration H associated with the cosmic expansion (expansion velocity increases with distance) would be compensated by the acceleration due to the gravitational force of dark matter. The formula for surface density of dark matter is from Newton's law $GM_{dark} = H$ given by $\sigma_{dark} = H/4\pi G$. The approximate value of dark matter surface density is from $Hc = 6.7 \times 10^{-10}$ m/s^2 equal to $\sigma = .8$ kg/m^2 and surprisingly large.
   (c) The value of acceleration is $a = .8 \times 10^{-10} \times g$, $g = 9.81$ m/s^2 whereas the MOND model (see `http://tinyurl.com/32t9wt`) finds the optimal value for the postulated minimal gravitational acceleration to be $a_0 = 1.2 \times 10^{-10}$ m/s^2. In TGD framework it would be assignable to the traversal through the dark matter shell. The ratio of the two accelerations is $a/a_0 = 6.54$.
   (d) TGD inspired quantum biology requiring that the universal cyclotron energy spectrum of dark photons $h_{eff} = h_{gr}$ transforming to to bio-photons is in visible and UV range for charged particles gives the estimate $M_{dark}/M_E \simeq 2 \times 10^{-4}$ [K76] and is of the same order of magnitude smaller than for Jupiter. The minimum value of the magnetic field at flux tubes has been assumed to be $B_E = .2$ Gauss, which is the value of endogenous magnetic field explaining the effects of ELF em radiation on vertebrate brain. The two estimates are clearly consistent.
2. In Flyby anomaly spacecraft goes past Earth to gain momentum (Earth acts as a sling) for its travel towards Jupiter. During flyby a sudden acceleration occurs but this force is on only during the flyby but not before or after that. The basic point is that the spacecraft visits near Earth, and this is enough to explain the anomaly.
   The space-craft enters from a region outside the orbit of Earth containing dark matter and thus experiences also the dark force created by the sphere. After that the space craft enters inside the dark matter region, and sees a weaker gravitational force since the dark matter sphere is outside it and does not contribute. This causes a change in its velocity. After flyby the spacecraft experiences the forces caused by both Earth and dark matter sphere and the situation is the same as before flyby. The net effect is a change in the velocity as observed. From this the total amount of dark matter can be estimated. Also biology based argument gives an estimate for the fraction of dark matter in Earth.

This model supports the option in which the dark matter is concentrated on sphere. The other option is that it is concentrated at flux tube around orbit: quantitative calculations would be required to see whether this option can work. One can consider of course also more complex distributions: say $1/r$ distribution outside the sphere giving rise to constant change in acceleration outside the sphere.

A possible very simple TGD model for the sphere containing dark matter could be in terms of a boundary defined by a gigantic wormhole contact with large $h_{eff} = h_{gr}$ (at its space-time sheet representing "line of generalized Feynman diagram" one has deformation of CP_2 type vacuum extremal with Euclidian signature of induced metric) with radius given by the radius of Bohr orbit with gravitational Planck constant equal to $\hbar_{gr} = GMm/v_0$, where v_0 is a parameter with dimensions of velocity. This radius does not depend on the mass of the particle involved and is given by $r_n = GM/v_0^3$ where $r_S = 2GM$ is Schwartschild radius equal to 3 km for Sun [K89]. One has $v_0/c \simeq 2^{-11}$ for 4 inner planets. For outer planets v_0 is scaled down by a factor 1/5.

The sphere should also correspond to a magnetic flux sheet with field line topology of dipole field. By flux conservation the flux must arrive along flux tube parallel to a preferred axis presumably orthogonal to the plane of planets and flux conservation should must true. This kind of structure is predicted also by the TGD model in terms of cylindrically symmetric candidate for an extremal of Kähler action representing astrophysical object [K17].

An interesting possibility is that also Earth-Moon system contains a spherical shell of dark matter at distance given by the radius of Moon's orbit (about 60 Earth's radii). If so the analogs of the two effects could be observed also in Earth Moon system and the testing of the effects would become much easier. This would also mean understanding of the formation of Moon. Also interior of Earth (and also Sun) could contain spherical shells containing dark matter as the TGD inspired model for the spherically symmetric orbit constructed for more than two decades ago [K17] suggests. One can raise interesting questions. Could also the matter in small scale systems be accompanied by dark matter shells at radii equal to Bohr radii in the first approximation and could these effects be tested? Note that a universal surface density for dark matter predicts that the change of acceleration universally be given by Hubble constant H.

12.6.3 Further Progress In The Understanding Of Dark Matter And Energy In TGD Framework

The remarks below were inspired by an extremely interesting link to a popular article (see `http://tinyurl.com/ybjox4zb`) about a possible explanation of dark matter in terms of vacuum polarization associated with gravitation. The model can make sense only if the sign of the gravitational energy of antimatter is opposite to that of matter and whether this is the case is not known. Since the inertial energies of matter and antimatter are positive, one might expect that this is the case also for gravitational energies by Equivalence Principle but one might also consider alternative and also I have done this in TGD framework.

The popular article lists four observations related to dark matter that neither cold dark matter (CMD) model nor modified gravitation model (MOND) can explain, and the claim is that the vacuum energy model is able to cope with them.

Consider first the TGD based model.

1. The model assumes that galaxies are like pearls along strings defined by cosmic strings expended to flux tubes during cosmic expansion survives also these tests. This is true also in longer scales due to the fractality if TGD inspired cosmology: for instance, galaxy clusters would be organized in a similar manner.
2. The dark magnetic energy of the string like object (flux tube) is identifiable as dark energy and the pearls would correspond to dark matter shells with a universal mass density of.8 kg/m^2 estimated from Pioneer and Flyby anomalies assuming to be caused by spherical dark matter shells assignable to the orbits of planets. This value follows from the condition that the anomalous acceleration is identical with Hubble acceleration. Even Moon could be accompanied by this kind of shell: if so, the analog of Pioneer anomaly is predicted.
3. The dark matter shell around galactic core could have decayed to smaller shells by h_{eff} reducing phase transition. This phase transition would have created smaller surfaces with smaller values of $h_{eff} = h_{gr}$. One can consider also the possibility that it contains all the galactic matter as dark matter. There would be nothing inside the surface of the gigantic wormhole throat: this would conform with holography oriented thinking.

I checked the four observations listed in the popular article (see `http://tinyurl.com/ybjox4zb`) some of which CMD (cold dark matter) scenario and MOND fail to explain. TGD explains all of them.

1. It has been found that the effective surface mass density $\sigma = \rho_0 R_0/3$ (volume density times volume of ball equals to effective surface density times surface area of the ball for constant volume density) of galactic core region containing possible halo is universal and its value is .9 kg/m^2 (see the article (see `http://tinyurl.com/y864lfyx`). Pioneer and Flyby anomalies fix the surface density to.8 kg/m^2. The difference is about 10 per cent! One must of course

be cautious here: even the correct order of magnitude would be fine since Hubble acceleration parameter might be different for the cluster than for the solar system now.
Note that in the article the effective surface density is defined as $\sigma = \rho_0 r_0$, where r_0 is the radius of the region and ρ_0 is density in its center. The correct definition for a constant 3-D density inside ball is $\sigma = \rho_0 r_0/3$.

2. The dark matter has been found to be inside core region within few hundred parsecs. This is just what TGD predicts since the velocity spectrum of distant stars is due to the gravitational field created by dark energy identifiable as magnetic energy of cosmic string like object - the thread containing galaxies as pearls.
3. It has been observed that there is no dark matter halo in the galactic disk. Also this is an obvious prediction of TGD model.
4. The separation of matter - now plasma clouds between galaxies - and dark matter in the collisions of galaxy clusters (observed for instance for bullet cluster consisting of two colliding clusters) is also explained qualitatively by TGD. The explanation is qualitatively similar to that in the CMD model of the phenomenon. Stars of galaxies are not affected except from gravitational slow-down much but the plasma phase interacts electromagnetically and is slowed down much more in the collision. The dominating dark matter component making itself visible by gravitational lensing separates from the plasma phase and this is indeed observed: the explanation in TGD framework would be that it is macroscopically quantum coherent ($h_{eff} = h_{gr}$) and does not dissipate so that the thermodynamical description does not apply. In the case of galaxy clusters also the dark energy of cosmic strings is involved besides the galactic matter and this complicates the situation but the basic point is that dark matter component does not slow down as plasma phase does.
CMD model has the problem that the velocity of dark matter bullet (smaller cluster of bullet cluster) is higher than predicted by CMD scenario. Attractive fifth force acting between dark matter particles becoming effective at short distances has been proposed as an explanation: intuitively this adds to the potential energy negative component so that kinetic energy is increased. I have proposed that gravitational constant might vary and be roughly twice the standard value: I do not believe this explanation now.
The most feasible explanation is that the anomaly relates to the presence of thickened cosmic strings carrying dark energy as magnetic energy and dark matter shells instead of 3-D cold dark matter halos. This additional component would contribute to gravitational potential experienced by the smaller cluster and explain the higher velocity.

12.6.4 Variation Of Newston's Constant And Of Length Of Day

J. D. Anderson *et al* [E22] have published an article discussing the observations suggesting a periodic variation of the measured value of Newton constant and variation of length of day.

According to the article, about a dozen measurements of Newton's gravitational constant, G, since 1962 have yielded values that differ by far more than their reported random plus systematic errors. Authors find that these values for G are oscillatory in nature, with a period of $P = 5.899 \pm 0.062$ yr , an amplitude of , $S = 1.619 \pm 0.103 \times 10^{-14}$ m^{3}kg^{-1} s^{-2} and mean-value crossings in 1994 and 1997. The relative variation $\Delta G/G \sim 2.4 \times 10^{-4}$. Authors suggest that the actual values of G does not vary but some unidentified factor in the measurement process is responsible for an apparent variations.

According to the article, of other recently reported results, the only measurement with the same period and phase is the Length of Day (LOD —defined as a frequency measurement such that a positive increase in LOD values means slower Earth rotation rates and therefore longer days). The period is also about half of a solar activity cycle, but the correlation is far less convincing. The 5.9 year periodic signal in LOD has previously been interpreted as due to fluid core motions and inner-core coupling. We report the G/LOD correlation, whose statistical significance is 0.99764 assuming no difference in phase, without claiming to have any satisfactory explanation for it. Least unlikely, perhaps, are currents in the Earth's fluid core that change both its moment of inertia (affecting LOD) and the circumstances in which the Earth-based experiments measure G. In this case, there might be correlations with terrestrial-magnetic-field measurements.

In the popular article "Why do measurements of the gravitational constant vary so much?"

(see http://tinyurl.com/k5onwoe) Anderson states that there is also a possible connection with Flyby anomaly [E21], which also shows periodic variation.

In the following TGD inspired model for the findings is developed. The gravitational coupling would be in radial scaling degree of freedom and rigid body rotational degrees of freedom. In rotational degrees of freedom the model is in the lowest order approximation mathematically equivalent with Kepler model. The model for the formation of planets around Sun suggests that the dark matter shell has radius equal to that of Moon's orbit. This leads to a prediction for the oscillation period of Earth radius: the prediction is consistent with the observed 5.9 years period. The dark matter shell would correspond to $n = 1$ Bohr orbit in the earlier model for quantum gravitational bound states based on large value of Planck constant if the velocity parameter v_0 appearing in $\hbar_{gr} = GM_EM_D/v_0$ equals to the rotation velocity of Moon. Also $n > 1$ orbits are suggestive and their existence would provide additional support for TGD view about quantum gravitation. There are further amazing co-incidences. The gravitational Compton length GM/v_0 of particle is very near to the Earth's radius in case Earth if central mass is Earth mass. For the mass of dark matter shell it is the variation ΔR_E. This strongly suggest that quantum coherence in astrophysical scales has been and perhaps still is present.

Coupled oscillations of radii of Earth and dark matter shell as an explanation for the variations

A possible TGD explanation for the variation emerges from the following arguments.

1. By angular momentum conservation requiring $I\omega = L = constant$ the oscillation of the length of day (LOD) can be explained by the variation of the radius R_E of Earth since the moment of inertia is proportional to R_E^2. This gives $\Delta LOD/LOD = 2\Delta R/R$. This explains also the apparent variation of G since the gravitational acceleration at the surface of Earth is $g = GM/R_E^2$ so that one has $\Delta g/g = 2\Delta R/R$. Note that the variations have opposite phase.
2. Flyby and Pioneer anomalies [K2] relies on the existience of dark matter shell with a universal surface mass density, whose value is such that in the case of Earth the total mass in the shell would be $M_D \sim 10^{-4} M_E$. The value $M_D/M_E \simeq 1.3 \times 10^{-4}$ suggested by TGD is of the same order of magnitude as $\Delta R/R$. Even galactic dark matter around galactic core could correspond to a shell with this surfaces density of mass [K2]. This plus the claim that also Flyby anomaly has oscillatory character suggest a connection. Earth and dark mass shell are in a collective pulsation with a frequency of Earth pulsation about 6 years and the interaction is gravitational attraction. Note that the frequencies need not be the same. Momentum conservation in radial direction indeed requires that both of them participate in oscillation.

A detailed model

One can construct a model for the situation.

1. Earth and dark matter shell are modelled as rigid bodies with spatially constant density except that their radii can change. Earth and dark matter shell are characterized by moments of inertia $I_E = (3/5) \times M_E r_E^3$ and $I_D = (2/3) \times M_D r_D^2$. If one restricts the consideration to a rigid body rotation around fixed axis (call it z-axis), one has effective point masses $M_1 = 3M_E/5$ and $M_2 = 2M_D/3$ and the problem is mathematically very similar to a motion point like particles with these effective masses in plane subject to the mutual gravitational force obtained by averaging the gravitational $1/r$ potential over the volumes of the two mass distributions. In the lowest order the problem is very similar to a central force problem with $1/r$-potential plus corrections coming as series in r_E/r_D. This problem can be solved by using angular momentum conservation and energy conservation.
2. In the lowest order approximation $r_E/r_D = 0$ one has just Kepler problem in $1/r_D$ force between masses M_1 and M_2 for M_D and one obtains the analogs of elliptic orbit in the analog of plane defined by r_D and ϕ. Kepler's law $T_D^2 \propto r_D^3$ fixes the average value of r_D, call this value R_D.
3. In the next approximation one feeds this solution to the equations for r_E by replacing r_D with its average value R_D to obtain the interaction potential depending on the radius r_E. It must be harmonic oscillator potential and the elastic constant determines the oscillation period of r_E. The value of this period should be about 6 yr.

The Lagrangian is sum of kinetic terms plus potential term

$$L = T_E + T_D + V_{gr} \ ,$$
$$T_E = \frac{1}{2}M_E(\frac{dR_E}{dt})^2 + \frac{1}{2}I_E(\frac{d\Phi_E}{dt})^2 \ , \quad T_D = \frac{1}{2}M_D(\frac{dR_D}{dt})^2 + \frac{1}{2}I_D(\frac{d\Phi_D}{dt})^2 \ . \tag{12.6.1}$$

One could criticize the choice of the coefficients of the kinetic terms for radial coordinates R_E and R_D as masses and one could indeed consider a more general choices. One can also argue, that the rigid bodies cannot be completely spherically since in this case it would not be possible to talk about rotation - at least in quantum mechanical sense.

Gravitational interaction potential is given by

$$V_{gr} = -G\int dV_E \int dA_D \rho_E \sigma_D \frac{1}{r_{D,E}} \ , \quad r_{D,E} = |\overline{r}_D - \overline{r_E}| \ ,$$
$$dA_D = r_D^2 d\Omega_D \qquad dV_E = r_E^2 dr_E d\Omega_E \ ,$$
$$\rho_E = \frac{3M_E}{4\pi R_E^3} \ , \qquad \sigma_D = \frac{M_D}{4\pi R_D^2} \ . \tag{12.6.2}$$

The integration measures are the standard integration measures in spherical coordinates.

One can extract the r_D factor from $r_{D,E}$ (completely standard step) to get

$$\frac{1}{r_{D,E}} = \frac{1}{r_D}X \ ,$$
$$X = \frac{1}{|\overline{n}_D - x\overline{n}_E|} = \frac{1}{[1+x^2-2xcos(\theta)]^{1/2}} = \frac{1}{(1+x^2)^{1/2}}\frac{1}{(1-2xcos(\theta)/(1+x^2))^{1/2}} \ ,$$
$$x = \frac{r_E}{r_D} \ , \quad cos(\theta) = \overline{n}_D \cdot \overline{n}_E \ . \tag{12.6.3}$$

Angular integration over θ is trivial and only the integration over r_E remains.

$$V_{gr} = -GM_DM_E\frac{3r_D^2}{r_E^3}\int_0^{r_E/r_D} F(\epsilon(x))\frac{x^2}{(1-x^2)^{1/2}}dx \ ,$$
$$F(\epsilon) = \frac{(1+\epsilon)^{1/2}-(1-\epsilon)^{1/2})}{\epsilon} \simeq 1 - \frac{\epsilon}{8} \ ,$$
$$\epsilon = \frac{2x}{1+x^2} \ , \quad x = \frac{r_E}{r_D} \ . \tag{12.6.4}$$

In the approximation $F(\epsilon) = 1$ introducing error of few per cent the outcome is

$$V_{gr} = -\frac{3GM_DM_E}{r_D} \times \left[arcsin(x) - x\sqrt{1-x^2}\right] = \frac{3GM_DM_E}{r_D}\left[\frac{2}{3} + \frac{x^2}{5} + O(x^3) + ...\right] \ ,$$
$$x = \frac{r_E}{r_D} \ . \tag{12.6.5}$$

The physical interpretation of the outcome is clear.

1. The first term in the series gives the gravitational potential between point like particles depending on r_D only giving rise to the Kepler problem. The orbit is closed - an ellipse whose eccentricity determines the amplitude of $\Delta R_D/R_D$. In higher orders one expects that the strict periodicity is lost in the general case. From the central force condition $M_2\omega_d^2 r_D = GM_D M_E/r_D^2$ one has

$$T_D = \sqrt{\frac{2}{3}} \times \sqrt{\frac{R_D}{r_{S,E}}}\frac{2\pi R_D}{c} \quad , \quad r_{S,E} = 2GM_E \quad . \tag{12.6.6}$$

$r_{S,E} \simeq 8.87$ mm is the Earth's Schwartschild radius. The first guess is that the dark matter shell has the radius of Moon orbit $R_{Moon} \simeq 60.33 \times R_E$, $R_E = 6.731 \times 10^6$ m. This would give $T_D = T_{Moon} \simeq 30$ days.

2. Second term gives harmonic oscillator potential $k_E R_E^2/2$, $k_E = 6GM_D M_E/5R_D^3$ in the approximation that r_D is constant. Oscillator frequency is

$$T\omega_E^2 = \frac{k_E}{M_E} \times \frac{6GM_D}{5R_D^3} \quad . \tag{12.6.7}$$

The oscillator period is given by

$$T_E \quad = \quad 2\pi \times \sqrt{\frac{5R_D^3}{6GM_D}} = 2\pi \times \sqrt{5}3 \times \sqrt{\frac{R_D}{R_{S,D}}} \times \frac{R_D}{c} \quad . \tag{12.6.8}$$

In this approximation the amplitude of oscillation cannot be fixed but the non-linearity relates the amplitude to the amplitude of r_D.

3. One can estimate the period of oscillation by feeding in the basic numbers. One has $R_D \sim R_{Moon} = 60.34R_E$, $R_E = 6.371 \times 10^6$ m. A rough earlier estimate for M_D is given by $M_D/M_E \simeq 1.3 \times 10^{-4}$. The relative amplitude of the oscillation is $\Delta G/G = 2\Delta R/R \simeq 2.4 \times 10^{-4}$, which suggests $\Delta R/R \simeq M_D/M_E$.
The outcome is $T_E \simeq 6.1$ yr whereas the observed period is $T_E \simeq 5.9$ yr. The discrepancy could be due to non-linear effects making the frequency continuous classically.

An interesting question is whether macroscopic quantal effects might be involved.

1. The applicability of Bohr rules to the planetary motion [K89] first proposed by Nottale [E18] encourages to ask whether one could apply also to the effective Kepler problem Bohr rules with gravitational Planck constant $\hbar_{gr} = GM_E M_D/v_0$, where v_0 is a parameter with dimensions of velocity. The rotation velocity of Moon $v_0/c = 10^{-5}/3$ is the first order of magnitude guess. Also one can ask whether also $n > 1$ other dark matter layers are possible at Bohr orbits so that one would have the analog of atomic spectroscopy.
2. From angular momentum quantization requires $L = m\omega^2 R = n\hbar_{gr}$ and from central force condition one obtains the standard formula for the radius of Bohr orbit $r_n = n^2 GM_E/v_0^2$. For $n = 1$ the radius of the orbit would be radius of the orbit of Moon with accuracy of 3 per cent. Note that the mass of Moon is about 1 per cent of the Earth's mass and thus roughly by a factor 100 higher than the mass of the spherical dark matter shell.

Clearly, the model might have caught something essential about the situation. What remains to be understood is the amplitude $\Delta R/R$. It seems that $\Delta R/R \simeq M_D/M_E$ holds true. This is not too surprising but one should understand how this follows from the basic equations.

Chapter 13

Overall View About TGD from Particle Physics Perspective

13.1 Introduction

Topological Geometrodynamics is able to make rather precise and often testable predictions. In the following I want to describe the recent over all view about the aspects of quantum TGD relevant for particle physics.

During these 37 years TGD has become quite an extensive theory involving also applications to quantum biology and quantum consciousness theory. Therefore it is difficult to decide in which order to proceed. Should one represent first the purely mathematical theory as done in the articles in Prespace-time Journal [L6, L7, L11, L12, L9, L5, L10, L1]? Or should one start from the TGD inspired heuristic view about space-time and particle physics and represent the vision about construction of quantum TGD briefly after that? In this and other two chapters I have chosen the latter approach since the emphasis is on the applications on particle physics.

Second problem is to decide about how much material one should cover. If the representation is too brief no-one understands and if it is too detailed no-one bothers to read. I do not know whether the outcome was a success or whether there is any way to success but in any case I have been sweating a lot in trying to decide what would be the optimum dose of details.

The third problem are the unavoidable mammoth bones and redundancy as one deals with are extensive material as TGD is. The attempts to get rid of them have turned out to be a Sisyfian task but I have done my best!

In the first chapter I concentrate the heuristic picture about TGD with emphasis on particle physics.

- First I represent briefly the basic ontology: the motivations for TGD and the notion of many-sheeted space-time, the concept of zero energy ontology, the identification of dark matter in terms of hierarchy of Planck constant which now seems to follow as a prediction of quantum TGD, the motivations for p-adic physics and its basic implications, and the identification of space-time surfaces as generalized Feynman diagrams and the basic implications of this identification.
- Symmetries of quantum TGD are discussed. Besides the basic symmetries of the embedding space geometry allowing to geometrize standard model quantum numbers and classical fields there are many other symmetries. General Coordinate Invariance is especially powerful in TGD framework allowing to realize quantum classical correspondence and implies effective 2-dimensionality realizing strong form of holography. Super-conformal symmetries of super string models generalize to conformal symmetries of 3-D light-like 3-surfaces associated with light-like boundaries of so called causal diamonds defined as intersections of future and past directed light-cones (CDs) and with light-like 3-surfaces. Whether super-conformal symmetries imply space-time SUSY is far from a trivial question. What is suggested is a generalization of the space-time supersymmetry analogous to $\mathcal{N} = 2$ SUSY and not involving Majorana spinors since fermion numbers are conserved in TGD. Twistorial approach to gauge theories has gradually become part of quantum TGD and the natural generalization of the Yangian

symmetry identified originally as symmetry of $\mathcal{N} = 4$ SYMs is postulated as basic symmetry of quantum TGD.

- The understanding of the relationship between TGD and GRT and quantum and classical variants of Equivalence Principle (EP) in TGD have develope rather slowly but the recent picture is rather feasible.
  1. The recent view is that EP at quantum level reduces to Quantum Classical Correspondence (QCC) in the sense that Cartan algebra Noether charges assignable to 3-surface in case of Kähler action (inertial charges) are identical with eigenvalues of the quantal variants of Noether charges for Kähler-Dirac action (gravitational charges). The well-definedness of the latter charges is due to the conformal invariance assignable to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) at which the spinor modes are localized in generic case. This localization follows from the condition that em charge has well defined value for the spinor modes. The localization is possibly only for the Kähler-Dirac action and key role is played by the modification of gamma matrices to Kähler-Dirac gamma matrices. The gravitational four-momentum is thus completely analogous to stringy four-momentum.
  2. At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. Similar procedure applies to induced gauge fields.
     The classical four-momentum assignable to the light-like boundaries of string world sheets at partonic orbits can be identified as gravitational momentum naturally identifiable as inertial momentum assignable to embedding space spinor harmonics defined a ground state of super-conformal representation.
- The so called weak form of electric-magnetic duality has turned out to have extremely far reaching consequences and is responsible for the recent progress in the understanding of the physics predicted by TGD. The duality leads to a detailed identification of elementary particles as composite objects of massless particles and predicts new electro-weak physics at LHC. Together with a simple postulate about the properties of preferred extremals of Kähler action the duality allows also to realized quantum TGD as almost topological quantum field theory giving excellent hopes about integrability of quantum TGD.
- There are two basic visions about the construction of quantum TGD. Physics as infinite-dimensional Kähler geometry of world of classical worlds (WCW) endowed with spinor structure and physics as generalized number theory. These visions are briefly summarized as also the practical construction involving the concept of Dirac operator. As a matter fact, the construction of TGD involves several Dirac operators.
  1. The Kähler Dirac equation holds true in the interior of space-time surface and its solutions localized at string world sheets have a natural interpretation in terms of fundamental fermions forming building bricks of all particles.
  2. A very natural boundary condition at the light-like boundaries of string world sheets is that induced 1-D Dirac operator annihilates the spinor modes so that they are characterized by light-like 8-momentum crucial for 8-D tangent space twistorialization.
  3. Third Dirac operator is associated with embedding space spinor harmonics defining ground states of super-conformal representations.
  4. The fourth Dirac operator is associated with super Virasoro generators and super Virasoro conditions define Dirac equation in WCW. These conditions characterize zero energy states as modes of WCW spinor fields and code for the generalization of S-matrix to a collection of what I call M-matrices defining the rows of unitary U-matrix defining unitary process.
- Twistor approach has inspired several ideas in quantum TGD during the last years and it seems that the Yangian symmetry and the construction of scattering amplitudes in terms of Grassmannian integrals generalizes to TGD framework. This is due to ZEO allowing to assume that all particles have massless fermions as basic building blocks. ZEO inspires the hypothesis that incoming and outgoing particles are bound states of fundamental fermions associated with

wormhole throats. Virtual particles would also consist of on mass shell massless particles but without bound state constraint. This implies very powerful constraints on loop diagrams and there are excellent hopes about their finiteness.

The discussion of this chapter is rather sketchy and the reader interesting in details can consult the books about TGD [K109, K101, K26, K68, K84, K73, K102] .

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L22].

13.2 Some Aspects Of Quantum TGD

In the following I summarize very briefly those basic notions of TGD which are especially relevant for the applications to particle physics. The representation will be practically formula free. The article series published in Prespace-time Journal [L6, L7, L11, L12, L9, L5, L10, L16] describes the mathematical theory behind TGD. The seven books about TGD [K109, K101, K26, K84, K73, K68, K49, K92] provide a detailed summary about the recent state of TGD.

13.2.1 New Space-Time Concept

The physical motivation for TGD was what I have christened the energy problem of General Relativity. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The way out is based on assumption that space-times are imbeddable as 4-surfaces to certain 8-dimensional space by replacing the points of 4-D empty Minkowski space with 4-D very small internal space. This space -call it S- is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [L16] , is the unique choice.

The replacement of the abstract manifold geometry of general relativity with the geometry of surfaces brings the shape of surface as seen from the perspective of 8-D space-time and this means additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any general coordinate invariant variational principle led soon to the realization that the notion space-time in this framework is much more richer than in general relativity quite contrary to what one might expect on basis of representability as a surface in 8-D embedding space.

1. Space-time decomposes into space-time sheets (see **Fig. ??** in the appendix of this book) with finite size: this lead to the identification of physical objects that we perceive around us as space-time sheets. For instance, the outer boundary of the table is where that particular space-time sheet ends. Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.
2. Elementary particles are identified as topological inhomogenuities glued to these space-time sheets (see figs. `http://tgdtheory.fi/appfigures/particletgd.jpg`, `http://tgdtheory.fi/appfigures/elparticletgd.jpg`, which are also in the appendix of this book). In this conceptual framework material structures and shapes are not due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in general relativity.
3. Also the view about classical fields changes. One can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K74] . One can speak about field body or magnetic body of the system.

Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C2] and the color magnetic body of u quark whose size is somethat larger than the Bohr radius could explain the anomaly [K64] .

13.2.2 ZEO

In standard ontology of quantum physics physical states are assumed to have positive energy. In ZEO physical states decompose to pairs of positive and negative energy states such that all net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events. By quantum classical correspondences zero energy states must have space-time and embedding space correlates.

1. Positive and negative energy parts reside at future and past light-like boundaries of causal diamond (CD) defined as intersection of future and past directed light-cones and visualizable as double cone (see ig. **??** in the appendix of this book)). The analog of CD in cosmology is big bang followed by big crunch. CDs for a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs can also intersect.
2. p-Adic length scale hypothesis [K69] motivates the hypothesis that the temporal distances between the tips of the intersecting light-cones come as octaves $T = 2^n T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K11] . This means a direct coupling between microscopic and macroscopic scales.

ZEO conforms with the crossing symmetry of quantum field theories meaning that the final states of the quantum scattering event are effectively negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem which results in any theory assuming symmetries giving rise to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in general relativity based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must be speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

For thermodynamical states this is indeed the case and this leads to the idea that quantum theory in ZEO can be regarded as a "complex square root" of thermodynamics obtained as a product of positive diagonal square root of density matrix and unitary S-matrix. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and replaces S-matrix as the fundamental observable. In standard quantum measurement theory this time-like entanglement would be reduced in quantum measurement and regenerated in the next quantum jump if one accepts Negentropy Maximization Principle (NMP) [K63] as the fundamental variational principle. Various M-matrices define the rows of the unitary U matrix characterizing the unitary process part of quantum jump. From the point of view of consciousness theory the importance of ZEO is that conservation laws in principle pose no restrictions for the new realities created in quantum jumps: free will is maximal.

The most dramatic implications of ZEO are to the modelling of living matter since the basic unit is now a pair of space-like 3-surfaces at the opposite boundaries of CD rather than single 3-surface at either boundary. By holography the space-time surface connecting them can be taken as basic units and define space-time correlates for behavioral patterns. This modifies dramatically the views about self-organization and morphogenesis.

13.2.3 The Hierarchy Of Planck Constants

The motivations for the hierarchy of Planck constants come from both astrophysics [K89] and biology [K82, K38] . In astrophysics the observation of Nottale [E18] that planetary orbits in solar system seem to correspond to Bohr orbits with a gigantic gravitational Planck constant motivated the proposal that Planck constant might not be constant after all [K89, K75] .

This led to the introduction of the quantization of Planck constant as an independent postulate. It has however turned that quantized Planck constant in effective sense could emerge from the basic structure of TGD alone. Canonical momentum densities and time derivatives of the embedding space coordinates are the field theory analogs of momenta and velocities in classical mechanics. The extreme non-linearity and vacuum degeneracy of Kähler action imply that the correspondence between canonical momentum densities and time derivatives of the embedding space coordinates is 1-to-many: for vacuum extremals themselves 1-to-infinite (see **Fig. ??** in the appendix of this book).

A convenient technical manner to treat the situation is to replace embedding space with its n-fold singular covering. Canonical momentum densities to which conserved quantities are proportional would be same at the sheets corresponding to different values of the time derivatives. At each sheet of the covering Planck constant is effectively $h_{eff} = n \times h$. This splitting to multi-sheeted structure can be seen as a phase transition reducing the densities of various charges by factor $1/n$ and making it possible to have perturbative phase at each sheet (gauge coupling strengths are proportional to $1/h_{eff}$ and scaled down by $1/n$). The connection with fractional quantum Hall effect [D2] is suggestive [K77] .

This has many profound implications, which are welcome from the point of view of quantum biology but the implications would be profound also from particle physics perspective and one could say that living matter represents zoome up version of quantum world at elementary particle length scales.

1. Quantum coherence and quantum superposition become possible in arbitrary long length scales. One can speak about zoomed up variants of elementary particles and zoomed up sizes make it possible to satisfy the overlap condition for quantum length parameters used as a criterion for the presence of macroscopic quantum phases. In the case of quantum gravitation the length scale involved are astrophysical. This would conform with Penrose's intuition that quantum gravity is fundamental for the understanding of consciousness and also with the idea that consciousness cannot be localized to brain.
2. Photons with given frequency can in principle have arbitrarily high energies by $E = hf$ formula, and this would explain the strange anomalies associated with the interaction of ELF em fields with living matter [J4] . Quite generally the cyclotron frequencies which correspond to energies much below the thermal energy for ordinary value of Planck constant could correspond to energies above thermal threshold.
3. The value of Planck constant is a natural characterizer of the evolutionary level and biological evolution would mean a gradual increase of the largest Planck constant in the hierarchy characterizing given quantum system. Evolutionary leaps would have interpretation as phase transitions increasing the maximal value of Planck constant for evolving species. The space-time correlate would be the increase of both the number and the size of the sheets of the covering associated with the system so that its complexity would increase.
4. The phase transitions changing Planck constant change also the length of the magnetic flux tubes. The natural conjecture is that biomolecules form a kind of Indra's net connected by the flux tubes and $\hbar$ changing phase transitions are at the core of the quantum bio-dynamics. The contraction of the magnetic flux tube connecting distant biomolecules would force them near to each other making possible for the bio-catalysis to proceed. This mechanism could be central for DNA replication and other basic biological processes. Magnetic Indra's net could be also responsible for the coherence of gel phase and the phase transitions affecting flux tube lengths could induce the contractions and expansions of the intracellular gel phase. The reconnection of flux tubes would allow the restructuring of the signal pathways between biomolecules and other subsystems and would be also involved with ADP-ATP transformation inducing a transfer of negentropic entanglement [**?**] (see **Fig. ??** in the appendix of this book). The braiding of the magnetic flux tubes could make possible topological quantum computation like processes and analog of computer memory realized in terms of braiding patterns [K6] .
5. p-Adic length scale hypothesis and hierarchy of Planck constants suggest entire hierarchy of zoomed up copies of standard model physics with range of weak interactions and color forces scaling like $\hbar$. This is not conflict with the known physics for the simple reason that we know

very little about dark matter (partly because we might be making misleading assumptions about its nature). One implication is that it might be someday to study zoomed up variants particle physics at low energies using dark matter.
Dark matter would make possible the large parity breaking effects manifested as chiral selection of bio-molecules [C52] . The classical Z^0 and possibly also W fields responsible for parity breaking effects must be experienced by fundamental fermions in cellular length scale. This is not possible for ordinary value of Planck constant above weak scale since the induced spinor modes are restricted on string world sheets at which W and Z^0 fields vanish: this follows from the well-definedness of em charge. If the value of Planck constant is so large that weak scale is some biological length scale, weak fields are effectively massless below this scale and large parity breaking effects become possible.
For the solutions of field equations which are almost vacuum extremals Z^0 field is non-vanishing and proportional to electromagnetic field. The hypothesis that cell membrane corresponds to a space-time sheet near a vacuum extremal (this corresponds to criticality very natural if the cell membrane is to serve as an ideal sensory receptor) leads to a rather successful model for cell membrane as sensory receptor with lipids representing the pixels of sensory qualia chart. The surprising prediction is that bio-photons [I7] and bundles of EEG photons can be identified as different decay products of dark photons with energies of visible photons. Also the peak frequencies of sensitivity for photoreceptors are predicted correctly [K82] .

The hierarchy of Planck constants has become key part of TGD and is actually forced by the condition that strings connecting partonic 2-surfaces are correlates for the formation of bound states. The basic problem of both QFTs and string theories is the failure to describe bound states, and the generalization of quantum theory by introducing the hierarchy of Planck constant solves this problem.

13.2.4 P-Adic Physics And Number Theoretic Universality

p-Adic physics [K68, K96] has become gradually a key piece of TGD inspired biophysics. Basic quantitative predictions relate to p-adic length scale hypothesis and to the notion of number theoretic entropy. Basic ontological ideas are that life resides in the intersection of real and p-adic worlds and that p-adic space-time sheets serve as correlates for cognition. Number theoretical universality requires the fusion of real physics and various p-adic physics to single coherent whole analogous to adeles. On implication is the generalization of the notion of number obtained by fusing real and p-adic numbers to a larger structure.

p-Adic number fields

p-Adic number fields Q_p [A36] -one for each prime p- are analogous to reals in the sense that one can speak about p-adic continuum and that also p-adic numbers are obtained as completions of the field of rational numbers. One can say that rational numbers belong to the intersection of real and p-adic numbers. p-Adic number field Q_p allows also an infinite number of its algebraic extensions. Also transcendental extensions are possible. For reals the only extension is complex numbers.

p-Adic topology defining the notions of nearness and continuity differs dramatically from the real topology. An integer which is infinite as a real number can be completely well defined and finite as a p-adic number. In particular, powers p^n of prime p have p-adic norm (magnitude) equal to p^{-n} in Q_p so that at the limit of very large n real magnitude becomes infinite and p-adic magnitude vanishes.

p-Adic topology is rough since p-adic distance $d(x,y) = d(x-y)$ depends on the lowest pinary digit of $x-y$ only and is analogous to the distance between real points when approximated by taking into account only the lowest digit in the decimal expansion of $x-y$. A possible interpretation is in terms of a finite measurement resolution and resolution of sensory perception. p-Adic topology looks somewhat strange. For instance, p-adic spherical surface is not infinitely thin but has a finite thickness and p-adic surfaces possess no boundary in the topological sense. Ultra-metricity is the technical term characterizing the basic properties of p-adic topology and is coded by the inequality $d(x-y) \leq Min\{d(x), d(y)\}$. p-Adic topology brings in mind the decomposition of perceptive field to objects.

Motivations for p-adic number fields

The physical motivations for p-adic physics came from the observation that p-adic thermodynamics -not for energy but infinitesimal scaling generator of so called super-conformal algebra [A23] acting as symmetries of quantum TGD [K101] - predicts elementary particle mass scales and also masses correctly under very general assumptions [K68] . The calculations are discussed in more detail in the second article of the series. In particular, the ratio of proton mass to Planck mass, the basic mystery number of physics, is predicted correctly. The basic assumption is that the preferred primes characterizing the p-adic number fields involved are near powers of two: $p \simeq 2^k$, k positive integer. Those nearest to power of two correspond to Mersenne primes $M_n = 2^n - 1$. One can also consider complex primes known as Gaussian primes, in particular Gaussian Mersennes $M_{G,n} = (1+i)^n - 1$.

It turns out that Mersennes and Gaussian Mersennes are in a preferred position physically in TGD based world order. What is especially interesting that the length scale range 10 nm-5 μm contains as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ assignable to Gaussian Mersennes $M_k = (1+i)^k - 1$, $k = 151, 157, 163, 167$, [K82] . This number theoretical miracle supports the view that p-adic physics is especially important for the understanding of living matter.

The philosophical for p-adic numbers fields come from the question about the possible physical correlates of cognition [K71]. Cognition forms representations of the external world which have finite cognitive resolution and the decomposition of the perceptive field to objects is an essential element of these representations. Therefore p-adic space-time sheets could be seen as candidates of thought bubbles, the mind stuff of Descartes.

Rational numbers belong to the intersection of real and p-adic continua. An obvious generalization of this statement applies to real manifolds and their p-adic variants. When extensions of p-adic numbers are allowed, also some algebraic numbers can belong to the intersection of p-adic and real worlds. The notion of intersection of real and p-adic worlds has actually two meanings.

1. The intersection could consist of the rational and possibly some algebraic points in the intersection of real and p-adic partonic 2-surfaces at the ends of CD. This set is in general discrete. The interpretation could be as discrete cognitive representations.
2. The intersection could also have a more abstract meaning. For instance, the surfaces defined by rational functions with rational coefficients have a well-defined meaning in both real and p-adic context and could be interpreted as belonging to this intersection. There is strong temptation to assume that intentions are transformed to actions only in this intersection. One could say that life resides in the intersection of real and p-adic worlds in this abstract sense.

Additional support for the idea comes from the observation that Shannon entropy $S = -\sum p_n log(p_n)$ allows a p-adic generalization if the probabilities are rational numbers by replacing $log(p_n)$ with $-log(|p_n|_p)$, where $|x|_p$ is p-adic norm. Also algebraic numbers in some extension of p-adic numbers can be allowed. The unexpected property of the number theoretic Shannon entropy is that it can be negative and its unique minimum value as a function of the p-adic prime p it is always negative. Entropy transforms to information!

In the case of number theoretic entanglement entropy there is a natural interpretation for this. Number theoretic entanglement entropy would measure the information carried by the entanglement whereas ordinary entanglement entropy would characterize the uncertainty about the state of either entangled system. For instance, for p maximally entangled states both ordinary entanglement entropy and number theoretic entanglement negentropy are maximal with respect to R_p norm. Negentropic entanglement carries maximal information. The information would be about the relationship between the systems, a rule. Schrödinger cat would be dead enough to know that it is better to not open the bottle completely (see **Fig. ??** in the appendix of this book).

Negentropy Maximization Principle [K63] coding the basic rules of quantum measurement theory implies that negentropic entanglement can be stable against the effects of quantum jumps unlike entropic entanglement. Therefore living matter could be distinguished from inanimate matter also by negentropic entanglement possible in the intersection of real and p-adic worlds. In consciousness theory negentropic entanglement could be seen as a correlate for the experience of understanding or any other positively colored experience, say love.

Negentropically entangled states are stable but binding energy and effective loss of relative translational degrees of freedom is not responsible for the stability. Therefore bound states are not in question. The distinction between negentropic and bound state entanglement could be compared to the difference between unhappy and happy marriage. The first one is a social jail but in the latter case both parties are free to leave but do not want to. The special characterics of negentropic entanglement raise the question whether the problematic notion of high energy phosphate bond [I2] central for metabolism could be understood in terms of negentropic entanglement. This would also allow an information theoretic interpretation of metabolism since the transfer of metabolic energy would mean a transfer of negentropy [?] .

13.3 Symmetries Of TGD

Symmetry principles play key role in the construction of WCW geometry have become and deserve a separate explicit treatment even at the risk of repetitions. Symmetries of course manifest themselves also at space-time level and space-time supersymmetry - possibly present also in TGD - is the most non-trivial example of this.

13.3.1 General Coordinate Invariance

General coordinate invariance is certainly of the most important guidelines and is much more powerful in TGD framework thanin GRT context.

1. General coordinate transformations as a gauge symmetries so that the diffeomorphic slices of space-time surface equivalent physically. 3-D light-like 3-surfaces defined by wormhole throats define preferred slices and allows to fix the gauge partially apart from the remaining 3-D variant of general coordinate invariance and possible gauge degeneracy related to the choice of the light-like 3-surface due to the Kac-Moody invariance. This would mean that the random light-likeness represents gauge degree of freedom except at the ends of the light-like 3-surfaces.
2. GCI can be strengthened so that the pairs of space-like ends of space-like 3-surfaces at CDs are equivalent with light-like 3-surfaces connecting them. The outcome is effective 2-dimensionality because their intersections at the boundaries of CDs must carry the physically relevant information. One must however notice also the presence of string world sheets emerging from number theoretic vision and from the condition that spinor modes have well-defined cm charge. Partonic 2-surfaces (plus 4-D tangent space data) and string world sheets would carry the data about quantum states and the interpretation would be in terms of strong holography. The role of string world sheets in TGD is very much analogous to their role in AdS/CFT duality.

13.3.2 Generalized Conformal Symmetries

One can assign Kac-Moody type conformal symmetries to light-like 3-surfaces as isometries of H localized with respect to light-like 3-surfaces. Kac Moody algebra essentially the Lie algebra of gauge group with central extension meaning that projective representation in which representation matrices are defined only modulo a phase factor. Kac-Moody symmetry is not quite a pure gauge symmetry.

One can assign a generalization of Kac-Moody symmetries to the boundaries of CD by replacing Lie-group of Kac-Moody algebra with the group of symplectic (contact-) transformations [A43, A26, A25] of H_+ provided with a degenerate Kähler structure made possible by the effective 2-dimensionality of δM^4_+. The light-like radial coordinate of δM^4_+ plays the role of the complex coordinate of conformal transformations or their hyper-complex analogs. The basic hypothesis is that these transformations define the isometry algebra of WCW.

p-Adic mass calculations require also second super-conformal symmetry. It is defined by Kac-Moody algebra assignable to the isometries of the embedding space or possibly those of δCD. This algebra must appear together with symplectic algebra as a direct sum. The original guess was that Kac-Moody algebra is associated with light-like 3-surfaces as a local algebra localized by hand with respect to the internal coordinates. A more elegant identification emerged in light of the wisdom gained from the solutions of the Kähler-Dirac equation. Neutrino modes and symplectic Hamiltonians generate symplectic algebra and the remaining fermion modes and Hamiltonians of symplectic isometries generate the Kac-Moody algebra and the direct sum of these algebras acts naturally on physical states.

A further physically well-motivated hypothesis inspired by holography and extended GCI is that these symmetries extend so that they apply at the entire space-time sheet and also at the level of embedding space.

1. The extension to the entire space-time surface requires the slicing of space-time surface by partonic 2- surfaces and by stringy world sheets such that each point of stringy world sheet defines a partonic 2-surface and vice versa. This slicing has deep physical motivations since it realizes geometrically standard facts about gauge invariance (partonic 2-surface defines the space of physical polarizations and stringy space-time sheet corresponds to non-physical polarizations) and its existence is a hypothesis about the properties of the preferred extremals of Kähler action.
   There is a similar decomposition also at the level of CD and so called Hamilton-Jacobi coordinates for M^4_+ [K17] define this kind of slicings. This slicing can induced the slicing of the space-time sheet. The number theoretic vision gives a further justification for this hypothesis and also strengthens it by postulating the presence of the preferred time direction having interpretation in terms of real unit of octonions. In ZEO this time direction corresponds to the time-like vector connecting the tips of CD.

2. The simplest extension of the symplectic algebra at the level of embedding space is by parallel translating the light-cone boundary. This would imply duality of the formulations using light-like and space-like 3-surfaces and Equivalence Principle (EP) might correspond to this duality in turn implied by strong form of general coordinate invariance (GCI).

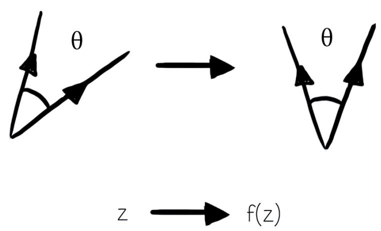

Figure 13.1: Conformal symmetry preserves angles in complex plane

Conformal symmetries (see **Fig. 13.1**) would provide the realization of WCW as a union of symmetric spaces. Symmetric spaces are coset spaces of form G/H. The natural identification of G and H is as groups of symplectic transformations and its subroup leaving preferred 3-surface invariant (acting as diffeomorphisms for it). Quantum fluctuating (metrically non-trivial) degrees of freedom would correspond to symplectic transformations of H_+ and fluxes of the induced Kähler form would define a local representation for zero modes: not necessarily all of them.

A highly attractive hypothesis motivated by fractality is that the algebras of conformal symmetries represent broken conformal symmetries in the sense that the sub-algebras with conformal weights coming as integer multiples of fixed integer n annihilate the physical states and corresponding Noether charges associated with Kähler and Kähler-Dirac action vanish. The hierarchies of symmetry breakings defined by the sequences $n_{i+1} = \prod_{k<i+1} m_k$ would correspond to hierarchies of Planck constants h_{eff} and hierarchies of CDs with increasing sizes characterized by the distance between the tips of CD. The transformation of generators from those of gauge symmetries to real physical symmetries would bring in new degrees of freedom increasing measurement resolution. The hierarchies would define also inclusion hierarchies of hyper-finite factors of type II_1 [K112]. The level of Kähler action n would tell the number of conformal equivalence classes connecting the 3-surfaces at the boundaries of CD.

13.3.3 Equivalence Principle And Super-Conformal Symmetries

Equivalence Principle (EP) is a second corner stone of General Relativity and together with GCI leads to Einstein's equations. What EP states is that inertial and gravitational masses are identical. In this form it is not well-defined even in GRT since the definition of gravitational and inertial four-momenta is highly problematic because Noether theorem is not available. Therefore the realization is in terms of local equations identifying energy momentum tensor with Einstein tensor.

Thinking EP in terms of scattering amplitudes for graviton exchange, it seems obvious that EP is true in TGD since all particles are string like objects (monopole flux tubes connecting pairs of wormhole contacts accompanied by fermionic strings). How EP is realized in TGD has been a longstanding open question [K106]. The problem has been that at the classical level EP in its GRT form can hold true only in long enough length scales and it took long to time to realize that only the stringy form of this principle is required. The first question is how to identify the gravitational and inertial four-momenta. I have considered very many proposals in this regard!

One could argue that Equivalence Principle (EP) reduces to a mere tautology in TGD framework since stringy picture implies stringy scattering amplitudes for graviton exchanges. This might be the case at quantum level. There are however problems: how the exact Poincare invariance can be consistent with the non-conservation of four-momentum in GRT based cosmologies? What EP could mean at quantum level? Does EP reduce at classical level to Einstein's equations in some sense. How to take into account the many-sheetedness of TGD space-time? The following represents the latest vision about EP in TGD.

1. ZEO and non-conservation of Poincare charges in Poincare invariant theory of gravitation

In positive energy ontology the Poincare invarance of TGD is in sharpt contrast with the fact that GRT based cosmology predicts non-conservation of Poincare charges (as a matter fact, the definition of Poincare charges is very questionable for general solutions of field equations).

In zero energy ontology (ZEO) all conserved (that is Noether-) charges of the Universe vanish identically and their densities should vanish in scales below the scale defining the scale for observations and assignable to causal diamond (CD). This observation allows to imagine a ways out of what seems to be a conflict of Poincare invariance with cosmological facts.

ZEO would explain the local non-conservation of average energies and other conserved quantum numbers in terms of the contributions of sub-CDs analogous to quantum fluctuations. Classical gravitation should have a thermodynamical description if this interpretation is correct. The average values of the quantum numbers assignable to a space-time sheet would depend on the size of CD and possibly also its location in M^4. If the temporal distance between the tips of CD is interpreted as a quantized variant of cosmic time, the non-conservation of energy-momentum defined in this manner follows. One can say that conservation laws hold only true in given scale defined by the largest CD involved.

2. Equivalence Principle at quantum level

The interpretation of EP at quantum level has developed slowly and the recent view is that it reduces to quantum classical correspondence meaning that the classical charges of Kähler action can be identified with eigen values of quantal charges associated with Kähler-Dirac action.

1. At quantum level I have proposed coset representations for the pair of super-symplectic algebras assignable to the light-like boundaries of CD and the Super Kac-Moody algebra assignable to the light-like 3-surfaces defining the orbits of partonic 2-surfaces as realization of EP. For coset representation the differences of super-conformal generators would annihilate the physical states so that one can argue that the corresponding four-momenta are identical. One could even say that one obtains coset representation for the "vibrational" parts of the super-conformal algebras in question. It is now clear that this idea does not work. Note however that coset representations occur naturally for the subalgebras of symplectic algebra and Super Kac-Moody algebra and are naturally induced by finite measurement resolution.
2. The most recent view (2014) about understanding how EP emerges in TGD is described in [K106] and relies heavily on superconformal invariance and a detailed realisation of ZEO at quantum level. In this approach EP corresponds to quantum classical correspondence (QCC): four-momentum identified as classical conserved Noether charge for space-time sheets

associated with Käbler action is identical with quantal four-momentum assignable to the representations of super-symplectic and super Kac-Moody algebras as in string models and having a realisation in ZEO in terms of wave functions in the space of causal diamonds (CDs).

3. The latest realization is that the eigenvalues of quantal four-momentum can be identified as eigenvalues of the four-momentum operator assignable to the Kähler-Dirac equation. This realisation seems to be consistent with the p-adic mass calculations requiring that the super-conformal algebra acts in the tensor product of 5 tensor factors.

3. Equivalence Principle at classical level

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD.

The first proposal making sense even when one does not assume ZEO is that vacuum extremals are only approximate representations of the physical situation and that small fluctuations around them give rise to an inertial four-momentum identifiable as gravitational four-momentum identifiable in terms of Einstein tensor. EP would hold true in the sense that the average gravitational four-momentum would be determined by the Einstein tensor assignable to the vacuum extremal. This interpretation does not however take into account the many-sheeted character of TGD spacetime and is therefore questionable.

The resolution of the problem came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets.
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.
3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.
4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

One can of course consider the possibility that Einstein's equations generalize for preferred extremals of Kähbler action. This would actually represent at space-time level the notion of QCC rather than realise QCC interpreted as EP. The condition that the energy momentum tensor for Kähler action has vanishing covariant divergence would be satisfied in GRT if Einstein's equations with cosmological term hold true. This is the case.

An interesting question is whether inertial-gravitational duality generalizes to the case of color gauge charges so that color gauge fluxes would correspond to "gravitational" color charges and the charges defined by the conserved currents associated with color isometries would define "inertial" color charges. Since the induced color fields are proportional to color Hamiltonians multiplied by Kähler form they vanish identically for vacuum extremals in accordance with "gravitational" color confinement.

The latest clarification related to EP comes from the natural boundary condition that the boundaries of string world sheets at light-like orbits of the partonic 2-surfaces are light-like (if the boundary curve is not light-like, it is necessarily space-like). These orbits correspond to light-like embedding space 8-momenta classically, which leads to a generalization of 4-D twistors to 8-D ones at the level of the tangent space M^8 by introducing octonion structure and allowing to generalize twistor formalism so that it applies to particles massive in M^4 sense [K100]. If the light-like curve is light-like geodesic, the 8-momentum is conserved and its M^4 and CP_2 parts have constant length. In E^4 degrees of freedom this means $SO(4)$ symmetry, which might allow an interpretation as the symmetry of strong interactions in the description applying at hadron level. The particle states would not be eigenstates of E^4 momentum but characterized by wave functions in S^3 assignable to irreducible $SO(4)$ representations. At quark and gluon level the harmonics of CP_2 would describe color. At the level of generalized Feynman diagrams the natural identification of M^4 part of the 8-momentum would be as incoming M^4 momentum labelling the harmonics of the embedding space and this identification would provide a concrete realization of EP. In CP_2 degrees of freedom $CP_2 - E^4$ duality relating hadrons and quarks and gluons would be a more abstract realization of EP.

13.3.4 Extension Of Super-Conformal Symmetries

The original idea behind the extension of conformal symmetries to super-conformal symmetries was the observation that isometry currents defining infinitesimal isometries of WCW have natural super-counterparts obtained by contracting the Killing vector fields with the complexified gamma matrices of the embedding space.

This vision has generalized considerably as the construction of WCW spinor structure in terms of Kähler-Dirac action has developed. The basic philosophy behind this idea is that WCW spinor structure must relate directly to the fermionic sector of quantum physics. In particular, Kähler-Dirac gamma matrices should be expressible in terms of the fermionic oscillator operators associated with the second quantized induced spinor fields.

The explicit realization of this program leads to an identification of rich spectrum of super-conformal symmetries and generalization of the ordinary notion of space-time supersymmetry. What happens that all fermionic oscillator operator generate broken super-conformal gauge symmetries whereas in SUSYs there is only finite number of them.

One can however identify sub-algebra of super-conformal symmetries associated with right handed neutrino and this suggests $\mathcal{N} = 2$ super-symmetry respecting conservation of fermion numbers as the least broken SUSY [B6] [K88].

One must be however extremely cautious here since one can imagine several variants for space-time SUSY. The sparticles predicted by a typical supersymmetric extension of standard model have not been observed at LHC. A possible explanation is that supersymmetric matter corresponds to a non-standard value of h_{eff} and thus dark matter and does not appear in the vertices of Feynman diagrams involving ordinary matter. If this is the case, the mass scales of sparticles and particles could be same.

13.3.5 Does TGD Allow The Counterpart Of Space-Time Super-Symmetry?

It has been clear from the beginning that the notion of super-conformal symmetry crucial for the successes of super-string models generalizes in TGD framework. The answer to the question whether space-time SUSY makes sense in TGD framework has not been obvious at all but it seems now that the answer is affirmative. The evolution of the ideas relevant for the formulation of SUSY in TGD framework is summarized in the chapters of [K84] . The chapters devoted to the SUSY QFT limit of TGD [?], to twistor approach to TGD [K100] , and to the generalization of Yangian symmetry of $\mathcal{N} = 4$ SYM manifest in the Grassmannian twistor approach [B25] to a multi-local variant of super-conformal symmetries [K100] represent a gradual development of the ideas about how super-symmetric M-matrix could be constructed in TGD framework.

Before continuing a warning to the reader is in order. In their recent form the above listed chapters do not represent the final outcome but just an evolution of ideas proceeding by trial and error.

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the Kähler-Dirac action [K113, K30] . It is possible to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation $\mathcal{N} = 2N$ SUSY algebra (an inherent cutoff on the number of fermionic modes at light-like wormhole throat) or fermionic part of super-conformal algebra with infinite number of oscillator operators results. The addition of fermion in particular mode would define particular super-symmetry. This super-symmetry is badly broken due to the dynamics of the Kähler-Dirac operator which also mixes M^4 chiralities inducing massivation. Since right-handed neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

ZEO combined with the analog of the twistor approach to $\mathcal{N} = 4$ SYMs and weak form of electric-magnetic duality has actually led to this kind of formulation [K100] . What is new that also virtual particles have massless fermions as their building blocks. This implies manifest finiteness of loop integrals so that the situation simplifies dramatically. What is also new element that physical particles and also string like objects correspond to bound states of massless fermions.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of a fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.
2. The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The general classification of super-symmetric theories indeed demonstrates that for $D = 8$ Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in $D = 10$ and $D = 11$ as the only possible candidates for TOE after it turned out that chiral anomalies cancel. It indeed turns out that TGD view about space-time SUSY is internally consistent. Even more, the separate conservation of quark and lepton number is essential for the internal consistency of this view [**?**] .
3. The massivation of particles is the basic problem of both SUSYs and twistor approach. I have discussed several solutions to this problem [K100]. Twistor Grassmannian approach to $\mathcal{N} = 4$ SYM and the generalization of the Yangian symmetry of this theory inspires two approaches to the problem.
   (a) In ZEO one can construct physical particles as bound states of massless particles associated with the opposite wormhole throats. If the particles have opposite 3-momenta the resulting state is automatically massive. In fact, this forces massivation of also spin one bosons since the fermion and anti-fermion must move in opposite directions for their spins to be parallel so that the net mass is non-vanishing: note that this means that even photon, gluons, and graviton have small mass.
   This mechanism makes topologically condensed fermions massive and padic thermodynamics allows to describe the massivation in terms of zero energy states and M-matrix. Bosons would receive to their mass besides the small mass coming from thermodynamics also a stringy contribution which would be the counterpart of the contribution coming from Higgs vacuum expectation value and Higgs gives rise to longitudinal polarizations. No Higgs potential is however needed. The cancellation of infrared divergences necessary for exact Yangian symmetry and the observation that even photon receives small mass suggest that scalar Higgs would disappear completely from the spectrum.
   (b) Second approach relies on the generalization of twistor approach. 4-D twistors become 8-dimensional when quaternionic sigma matrices are replaced by octonionic ones. Light-likeness in 8-D sense would allow massive particles in 4-D sense [K100]. The classical 8-momentum associated with the light-like boundary of string world sheet would realize M^8 octonionic twistoriality concretely. This approach is very elegant and allows the 4-momenta of fermions decomposing particles to be massive and there are no problems with the massivation and emergence of the third polarization. Infrared problems are automatically absent in this framework. Encouragingly, M^4 and CP_2 are indeed the unique four-manifolds allowing twistor space which is Kähler manifold. It seems that this option is the only physically plausible one.

Basic data bits

Let us first summarize the data bits about possible relevance of super-symmetry for TGD before the addition of the 3-D measurement interaction term to the Kähler-Dirac action [K113] .

1. Right-handed covariantly constant neutrino spinor ν_R defines a super-symmetry in CP_2 degrees of freedom in the sense that Dirac equation is satisfied by covariant constancy and there is no need for the usual ansatz $\Psi = D\Psi_0$ giving $D^2\Psi = 0$. This super-symmetry allows to construct solutions of Dirac equation in CP_2 [A48, A56, A39, A53].
2. In $M^4 \times CP_2$ this means the existence of massless modes $\Psi = \not{p}\Psi_0$, where Ψ_0 is the tensor product of M^4 and CP_2 spinors. For these solutions M^4 chiralities are not mixed unlike for all other modes which are massive and carry color quantum numbers depending on the CP_2 chirality and charge. As matter fact, covariantly constant right-handed neutrino spinor mode is the only color singlet. The mechanism leading to non-colored states for fermions is based on super-conformal representations for which the color is neutralized [K60, K60] . The negative conformal weight of the vacuum (assumption) also cancels the enormous contribution to mass squared coming from mass in CP_2 degrees of freedom.
3. The massless right-handed neutrinos would be associated with string boundaries light-like M^4 - rather than only M^8 sense. They would satisfy massless Dirac equation. What this Dirac equation is, is far from obvious and I have considered almost all possibilities that one can imagine.
   The minimal option is that the gamma matrix associated with the fermion line is the light-like Kähler-Dirac gamma matrix since the K-D gamma matrix in normal direction should vanish by natural boundary conditions for the extremal of Kähler action. This gamma matrix should have a vanishing covariant divergence by field equations.
   This would allow a light-like M^4 momentum with varying direction: light-likeness of M^4 momentum gives just Virasoro conditions in the same manner as for CP_2 type vacuum extremals. For general M^8 type orbits a mixing with left handed neutrino would take place but if string world sheets do not carry induced W boson fields, the mixing with charged spinor components does not occur (W gauge potential is present but can be gauge transformed away). This mixing would induce breaking of SUSY and give mass for the right-handed neutrino.
4. Space-time super-symmetry in the conventional sense of the word is impossible in TGD framework since it would require Majorana spinors. In 8-D space-time with Minkowski signature of metric Majorana spinors are definitely ruled out by the standard argument leading to super string model. Majorana spinors would also break separate conservation of lepton and baryon numbers in TGD framework.

Could one generalize super-symmetry?

Could one then consider a more general space-time super-symmetry with "space-time" identified as space-time surface rather than Minkowski space?

1. The TGD variant of the super-symmetry could correspond quite concretely to the addition of right-handed neutrinos to fermion and boson states at partonic 2-surfaces. Since right-handed neutrinos do not have electro-weak interactions, the addition might not appreciably affect the mass formula although it could affect the p-adic prime defining the mass scale.
2. The problem is to understand what this addition of the right-handed neutrino means. To begin with, notice that in TGD Universe fermions reside at light-like 3-surfaces at which the signature of induced metric changes. Bosons correspond to pairs of light-like wormhole throats with wormhole contact having Euclidian signature of the induced metric.
   The long standing head ache has been that for bosons with parallel light-like four-momenta with same sign of energy the spins of fermion and anti-fermion are opposite so that one would obtain only scalar bosons! The problem disappears when 4-D light-likeness is replaced with 8-D light-likeness. The massless Dirac equation using induced gamma matrices at the light-like boundary of string world sheet indeed allows momenta which are light-like in 8-D sense and massive in M^4 sense so that a mixing of M^4 chiralities occurs. This allows to have both spin one bosonic states.
3. The super-symmetry as an addition of a fermion carrying right handed neutrino quantum numbers to the wormhole throat opposite to that carrying many-fermion state does not make sense since the resulting state cannot be distinguished from gauge boson or Higgs type particle. The light-like 3-surfaces can however carry fermion numbers up to the number of modes of the induced spinor field, which is expected to be infinite inside string like objects having wormhole

throats at ends and finite when one has space time sheets containing the throats [K113]. In very general sense one could say that each mode defines a very large broken N-super-symmetry with the value of N depending on state and light-like 3-surface. The breaking of this super-symmetry would come from electro-weak - , color - , and gravitational interactions. Right-handed neutrino would by its electro-weak and color inertness define a minimally broken super-symmetry.

4. What this addition of the right handed neutrinos or more general fermion modes could precisely mean? One cannot assign fermionic oscillator operators to right handed neutrinos which are covariantly constant in both M^4 and CP_2 degrees of freedom since the modes with vanishing energy (frequency) cannot correspond to fermionic oscillator operator creating a physical state since one would have $a = a^\dagger$. The intuitive view is that all the spinor modes move in an exactly collinear manner - somewhat like quarks inside hadron do approximately. This would suggest right-handed neutrinos have a non-vanishing but massless four-momentum so that there is an unavoidable breaking of SUSY.

TGD counterpart of space-time super-symmetry

This picture allows to define more precisely what one means with the approximate super-symmetries in TGD framework.

1. One can in principle construct many-fermion states containing both fermions and anti-fermions at given light-like 3-surface. The four-momenta of states related by super-symmetry need not be same. Super-symmetry breaking is present and has as the space-time correlate the deviation of the Kähler-Dirac gamma matrices from the ordinary M^4 gamma matrices. In particular, the fact that $\hat{\Gamma}^\alpha$ possesses CP_2 part in general means that different M^4 chiralities are mixed: a space-time correlate for the massivation of the elementary particles.
2. For right-handed neutrino super-symmetry breaking is expected to be smallest but also in the case of the right-handed neutrino mode mixing of M^4 chiralities takes place and breaks the TGD counterpart of super-symmetry.
3. The fact that all helicities in the state are physical for a given light-like 3-surface has important implications. For instance, the addition of a right-handed antineutrino to right-handed (left-handed) electron state gives scalar (spin 1) state. Also states with fermion number two are obtained from fermions. For instance, for e_R one obtains the states $\{e_R, e_R\nu_R\overline{\nu}_R, e_R\overline{\nu}_R, e_R\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,0,1)$. For e_L one obtains the states $\{e_L, e_L\nu_R\overline{\nu}_R, e_L\overline{\nu}_R, e_L\nu_R\}$ with lepton numbers $(1,1,0,2)$ and spins $(1/2,1/2,1,0)$. In the case of gauge boson and Higgs type particles -allowed by TGD but not required by p-adic mass calculations- gauge boson has 15 super partners with fermion numbers $[2,1,0,-1,-2]$.

The cautious conclusion is that the recent view about quantum TGD allows the analog of super-symmetry which is necessary broken and for which the multiplets are much more general than for the ordinary super-symmetry. Right-handed neutrinos might however define something resembling ordinary super-symmetry to a high extent. The question is how strong prediction one can deduce using quantum TGD and proposed super-symmetry.

1. For a minimal breaking of super-symmetry only the p-adic length scale characterizing the super-partner differs from that for partner but the mass of the state is same. This would allow only a discrete set of masses for various super-partners coming as half octaves of the mass of the particle in question. A highly predictive model results.
2. The quantum field theoretic description should be based on QFT limit of TGD formulated in terms of bosonic emergence . This formulation should allow to calculate the propagators of the super-partners in terms of fermionic loops.
3. This TGD variant of space-time super-symmetry resembles ordinary super-symmetry in the sense that selection rules due to the right-handed neutrino number conservation and analogous to the conservation of R-parity hold true. The states inside super-multiplets have identical electro-weak and color quantum numbers but their p-adic mass scales can be different. It should be possible to estimate reaction reaction rates using rules very similar to those of super-symmetric gauge theories.
4. It might be even possible to find some simple generalization of standard super-symmetric gauge theory to get rough estimates for the reaction rates. There are however problems. The

fact that spins $J = 0, 1, 2, 3/2, 2$ are possible for super-partners of gauge bosons forces to ask whether these additional states define an analog of non-stringy strong gravitation. Note that graviton in TGD framework corresponds to a pair of wormhole throats connected by flux tube (counterpart of string) and for gravitons one obtains 2^8-fold degeneracy.

To sum up, this approach does not suggest that particles and sparticles should have different p-adic mass scales. A possible way out of the problem is that the p-adic mass scales are same but sparticles have different h_{eff} and dark relative to particles so that they are not observable in particle physics experiments. The breaking of super-conformal symmetry indeed occurs and could mean a transformation of super-conformal gauge degrees of freedom to dynamical ones and increase of $h_{eff}/h = n$ characterizing the breaking of the conformal symmetry.

13.3.6 What Could Be The Generalization Of Yangian Symmetry Of $\mathcal{N} = 4$ SUSY In TGD Framework?

There has been impressive steps in the understanding of $\mathcal{N} = 4$ maximally sypersymmetric YM theory possessing 4-D super-conformal symmetry. This theory is related by AdS/CFT duality to certain string theory in $AdS_5 \times S^5$ background. Second stringy representation was discovered by Witten and is based on 6-D Calabi-Yau manifold defined by twistors. The unifying proposal is that so called Yangian symmetry is behind the mathematical miracles involved.

The notion of Yangian symmetry would have a generalization in TGD framework obtained by replacing conformal algebra with appropriate super-conformal algebras. Also a possible realization of twistor approach and the construction of scattering amplitudes in terms of Yangian invariants defined by Grassmannian integrals is considered in TGD framework and based on the idea that in zero energy ontology one can represent massive states as bound states of massless particles. There is also a proposal for a physical interpretation of the Cartan algebra of Yangian algebra allowing to understand at the fundamental level how the mass spectrum of n-particle bound states could be understood in terms of the n-local charges of the Yangian algebra.

Twistors were originally introduced by Penrose to characterize the solutions of Maxwell's equations. Kähler action is Maxwell action for the induced Kähler form of CP_2. The preferred extremals allow a very concrete interpretation in terms of modes of massless non-linear field. Both conformally compactified Minkowski space identifiable as so called causal diamond and CP_2 allow a description in terms of twistors. These observations inspire the proposal that a generalization of Witten's twistor string theory relying on the identification of twistor string world sheets with certain holomorphic surfaces assigned with Feynman diagrams could allow a formulation of quantum TGD in terms of 3-dimensional holomorphic surfaces of $CP_3 \times CP_3$ mapped to 6-surfaces dual $CP_3 \times CP_3$, which are sphere bundles so that they are projected in a natural manner to 4-D space-time surfaces. Very general physical and mathematical arguments lead to a highly unique proposal for the holomorphic differential equations defining the complex 3-surfaces conjectured to correspond to the preferred extremals of Kähler action.

Background

I am outsider as far as concrete calculations in $\mathcal{N} = 4$ SUSY are considered and the following discussion of the background probably makes this obvious. My hope is that the reader had patience to not care about this and try to see the big pattern.

The developments began from the observation of Parke and Taylor [B38] that n-gluon tree amplitudes with less than two negative helicities vanish and those with two negative helicities have unexpectedly simple form when expressed in terms of spinor variables used to represent light-like momentum. In fact, in the formalism based on Grassmanian integrals the reduced tree amplitude for two negative helicities is just "1" and defines Yangian invariant. The article *Perturbative Gauge Theory As a String Theory In Twistor Space* [B19] by Witten led to so called Britto-Cachazo-Feng-Witten (BCFW) recursion relations for tree level amplitudes [B14, B15, B14] allowing to construct tree amplitudes using the analogs of Feynman rules in which vertices correspond to maximally helicity violating tree amplitudes (2 negative helicity gluons) and propagator is massless Feynman propagator for boson. The progress inspired the idea that the theory might be completely integrable meaning the existence of infinite-dimensional un-usual symmetry. This symmetry would be so called Yangian symmetry [K100] assigned to the super counterpart of the conformal group of 4-D Minkowski space.

Drumond, Henn, and Plefka represent in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory* [B18] an argument suggesting that the Yangian invariance of the scattering amplitudes ins an intrinsic property of planar $\mathcal{N} = 4$ super Yang Mills at least at tree level.

The latest step in the progress was taken by Arkani-Hamed, Bourjaily, Cachazo, Carot-Huot, and Trnka and represented in the article *Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory* [B25]. At the same day there was also the article of Rutger Boels entitled *On BCFW shifts of integrands and integrals* [B39] in the archive. Arkani-Hamed *et al* argue that a full Yangian symmetry of the theory allows to generalize the BCFW recursion relation for tree amplitudes to all loop orders at planar limit (planar means that Feynman diagram allows embedding to plane without intersecting lines). On mass shell scattering amplitudes are in question.

Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K100]. Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant andconcrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B17]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, I have nothing to say. I am just perplexed. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A11] and Virasoro algebras [A23] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.
2. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.
3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras ontains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.
2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of partonic 2-surface. A coset construction is applied to these two Virasoro algebras so that the differences of the corresponding Super-Virasoro generators and Kac-Moody generators annihilate physical states. Contrary to the original belief, this construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.
3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.
4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about

polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

Is there any hope about description in terms of Grassmannians?

At technical level the successes of the twistor approach rely on the observation that the amplitudes can be expressed in terms of very simple integrals over sub-manifolds of the space consisting of k-dimensional planes of n-dimensional space defined by delta function appearing in the integrand. These integrals define super-conformal Yangian invariants appearing in twistorial amplitudes and the belief is that by a proper choice of the surfaces of the twistor space one can construct all invariants. One can construct also the counterparts of loop corrections by starting from tree diagrams and annihilating pair of particles by connecting the lines and quantum entangling the states at the ends in the manner dictated by the integration over loop momentum. These operations can be defined as operations for Grassmannian integrals in general changing the values of n and k. This description looks extremely powerful and elegant and nosta importantly involves only the external momenta.

The obvious question is whether one could use similar invariants in TGD framework to construct the momentum dependence of amplitudes.

1. The first thing to notice is that the super algebras in question act on infinite-dimensional representations and basically in the world of classical worlds assigned to the partonic 2-surfaces correlated by the fact that they are associated with the same space-time surface. This does not promise anything very practical. On the other hand, one can hope that everything related to other than M^4 degrees of freedom could be treated like color degrees of freedom in $\mathcal{N}=4$ SYM and would boil down to indices labeling the quantum states. The Yangian conditions coming from isometry quantum numbers, color quantum numbers, and electroweak quantum numbers are of course expected to be highly non-trivial and could fix the coefficients of various singlets resulting in the tensor product of incoming and outgoing states.
2. The fact that incoming particles can be also massive seems to exclude the use of the twistor space. The following observation however raises hopes. The Dirac propagator for wormhole throat is massless propagator but for what I call pseudo momentum. It is still unclear how this momentum relates to the actual four-momentum. Could it be actually equal to it? The recent view about pseudo-momentum does not support this view but it is better to keep mind open. In any case this finding suggests that twistorial approach could work in in more or less standard form. What would be needed is a representation for massive incoming particles as bound states of massless partons. In particular, the massive states of super-conformal representations should allow this kind of description.

Could zero energy ontology allow to achieve this dream?

1. As far as divergence cancellation is considered, zero energy ontology suggests a totally new approach producing the basic nice aspects of QFT approach, in particular unitarity and coupling constant evolution. The big idea related to zero energy ontology is that all virtual particle particles correspond to wormhole throats, which are pairs of on mass shell particles. If their momentum directions are different, one obtains time-like continuum of virtual momenta and if the signs of energy are opposite one obtains also space-like virtual momenta. The on mass shell property for virtual partons (massive in general) implies extremely strong constraints on loops and one expect that only very few loops remain and that they are finite since loop integration reduces to integration over much lower-dimensional space than in the QFT approach. There are also excellent hopes about Cutkoski rules.
2. Could zero energy ontology make also possible to construct massive incoming particles from massless ones? Could one construct the representations of the super conformal algebras using only massless states so that at the fundamental level incoming particles would be massless and one could apply twistor formalism and build the momentum dependence of amplitudes using Grassmannian integrals.

   One could indeed construct on mass shell massive states from massless states with momenta along the same line but with three-momenta at opposite directions. Mass squared is given by $M^2 = 4E^2$ in the coordinate frame, where the momenta are opposite and of same magnitude. One could also argue that partonic 2-surfaces carrying quantum numbers of fermions and their superpartners serve as the analogs of point like massless particles and that topologically condensed fermions and gauge bosons plus their superpartners correspond to pairs of wormhole throats. Stringy objects would correspond to pairs of wormhole throats at the same space-time sheet in accordance with the fact that space-time sheet allows a slicing by string worlds sheets with ends at different wormhole throats and defining time like braiding.

The weak form of electric magnetic duality indeed supports this picture. To understand how, one must explain a little bit what the weak form of electric magnetic duality means.

1. Elementary particles correspond to light-like orbits of partonic 2-surfaces identified as 3-D surfaces at which the signature of the induced metric of space-time surface changes from Euclidian to Minkowskian and 4-D metric is therefore degenerate. The analogy with black hole horizon is obvious but only partial. Weak form of electric-magnetic duality states that the Kähler electric field at the wormhole throat and also at space-like 3-surfaces defining the ends of the space-time surface at the upper and lower light-like boundaries of the causal diamond is proportonial to Kähler magnetic field so that Kähler electric flux is proportional Kähler magnetic flux. This implies classical quantization of Kähler electric charge and fixes the value of the proportionality constant.
2. There are also much more profound implications. The vision about TGD as almost topological QFT suggests that Kähler function defining the Kähler geometry of the "world of classical worlds" (WCW) and identified as Kähler action for its preferred extremal reduces to the 3-D Chern-Simons action evaluated at wormhole throats and possible boundary components. Chern-Simons action would be subject to constraints. Wormhole throats and space-like 3-surfaces would represent extremals of Chern-Simons action restricted by the constraint force stating electric-magnetic duality (and realized in terms of Lagrange multipliers as usual).
   If one assumes that Kähler current and other conserved currents are proportional to current defining Beltrami flow whose flow lines by definition define coordinate curves of a globally defined coordinate, the Coulombic term of Kähler action vanishes and it reduces to Chern-Simons action if the weak form of electric-magnetic duality holds true. One obtains almost topological QFT. The absolutely essential attribute "almost" comes from the fact that Chern-Simons action is subject to constraints. As a consequence, one obtains non-vanishing four-momenta and WCW geometry is non-trivial in M^4 degrees of freedom. Otherwise one would have only topological QFT not terribly interesting physically.

Consider now the question how one could understand stringy objects as bound states of massless particles.

1. The observed elementary particles are not Kähler monopoles and there much exist a mechanism neutralizing the monopole charge. The only possibility seems to be that there is opposite Kähler magnetic charge at second wormhole throat. The assumption is that in the case of color neutral particles this throat is at a distance of order intermediate gauge boson Compton length. This throat would carry weak isospin neutralizing that of the fermion and only electromagnetic charge would be visible at longer length scales. One could speak of electro-weak confinement. Also color confinement could be realized in analogous manner by requiring the cancellation of monopole charge for many-parton states only. What comes out are string like objects defined by Kähler magnetic fluxes and having magnetic monopoles at ends. Also more general objects with three strings branching from the vertex appear in the case of baryons. The natural guess is that the partons at the ends of strings and more general objects are massless for incoming particles but that the 3-momenta are in opposite directions so that stringy mass spectrum and representations of relevant super-conformal algebras are obtained. This description brings in mind the description of hadrons in terms of partons moving in parallel apart from transversal momentum about which only momentum squared is taken as observable.
2. Quite generally, one expects for the preferred extremals of Kähler action the slicing of space-time surface with string world sheets with stringy curves connecting wormhole throats. The ends of the stringy curves can be identified as light-like braid strands. Note that the strings themselves define a space-like braiding and the two braidings are in some sense dual. This has a concrete application in TGD inspired quantum biology, where time-like braiding defines topological quantum computer programs and the space-like braidings induced by it its storage into memory. Stringlike objects defining representations of super-conformal algebras must correspond to states involving at least two wormhole throats. Magnetic flux tubes connecting

the ends of magnetically charged throats provide a particular realization of stringy on mass shell states. This would give rise to massless propagation at the parton level. The stringy quantization condition for mass squared would read as $4E^2 = n$ in suitable units for the representations of super-conformal algebra associated with the isometries. For pairs of throats of the same wormhole contact stringy spectrum does not seem plausible since the wormhole contact is in the direction of CP_2. One can however expect generation of small mass as deviation of vacuum conformal weight from half integer in the case of gauge bosons.

If this picture is correct, one might be able to determine the momentum dependence of the scattering amplitudes by replacing free fermions with pairs of monopoles at the ends of string and topologically condensed fermions gauge bosons with pairs of this kind of objects with wormhole throat replaced by a pair of wormhole throats. This would mean suitable number of doublings of the Grassmannian integrations with additional constraints on the incoming momenta posed by the mass shell conditions for massive states.

Could zero energy ontology make possible full Yangian symmetry?

The partons in the loops are on mass shell particles have a discrete mass spectrum but both signs of energy are possible for opposite wormhole throats. This implies that in the rules for constructing loop amplitudes from tree amplitudes, propagator entanglement is restricted to that corresponding to pairs of partonic on mass shell states with both signs of energy. As emphasized in [B25], it is the Grassmannian integrands and leading order singularities of $\mathcal{N} = 4$ SYM, which possess the full Yangian symmetry. The full integral over the loop momenta breaks the Yangian symmetry and brings in IR singularities. Zero energy ontologist finds it natural to ask whether QFT approach shows its inadequacy both via the UV divergences and via the loss of full Yangian symmetry. The restriction of virtual partons to discrete mass shells with positive or negative sign of energy imposes extremely powerful restrictions on loop integrals and resembles the restriction to leading order singularities. Could this restriction guarantee full Yangian symmetry and remove also IR singularities?

Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

13.4 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B4] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K31] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.
2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.
3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.
4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.
5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current. Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

13.4.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the embedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.
2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.
3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.
4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{13.4.1}$$

A more general form of this duality is suggested by the considerations of [K52] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{13.4.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.
5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \tag{13.4.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on

Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L2] , [L2] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \tag{13.4.4}$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \tag{13.4.5}$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \tag{13.4.6}$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.
The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \tag{13.4.7}$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.
2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K77] supports this interpretation.
3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \to \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.
4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{13.4.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{13.4.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \to 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{13.4.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L2]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.
3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K82]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.
2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K106]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.
3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.
4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

13.4.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.
2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D11] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [**?**] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_\pm$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies ZEO. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.
2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.
3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K63] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.
4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K64] .

13.4.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^{\alpha} A_{\alpha}$ plus and integral of the boundary term $J^{n\beta} A_{\beta} \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_{\beta} \sqrt{g_4}$ over the ends of the 3-surface.
2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $h \to n \times h$ would effectively describe this. Boundary conditions would however

give $1/n$ factor so that $\hbar$ would disappear from the Kähler function! It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K17]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.
2. The original naïve conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \ . \tag{13.4.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.
3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.
4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{13.4.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_\beta \partial_\gamma j^K_{delta} = 0 \ . \tag{13.4.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B8] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K17] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 \ . \tag{13.4.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

13.5 Quantum TGD Very Briefly

13.5.1 Two Approaches To Quantum TGD

There are two basic approaches to the construction of quantum TGD. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry [A12] for the "world of classical worlds" (WCW) identified as the space of 3-surfaces in in certain 8-dimensional space. Essentially a generalization of the Einstein's geometrization of physics program is in question. The second vision is the identification of physics as a generalized number theory involving p-adic number fields and the fusion of real numbers and p-adic numbers to a larger structure, classical number fields, and the notion of infinite prime.

With a better resolution one can distinguish also other visions crucial for quantum TGD. Indeed, the notion of finite measurement resolution realized in terms of hyper-finite factors, TGD as almost topological quantum field theory, twistor approach, ZEO, and weak form of electric-magnetic duality play a decisive role in the actual construction and interpretation of the theory. One can however argue that these visions are not so fundamental for the formulation of the theory than the first two.

Physics as infinite-dimensional geometry

It is good to start with an attempt to give overall view about what the dream about physics as infinite-dimensional geometry is. The basic vision is generalization of the Einstein's program for the geometrization of classical physics so that entire quantum physics would be geometrized. Finite-dimensional geometry is certainly not enough for this purposed but physics as infinite-dimensional geometry of what might be called world of classical worlds (WCW) -or more neutrally WCW of some higher-dimensional imbeddign space- might make sense. The requirement that the Hermitian conjugation of quantum theories has a geometric realization forces Kähler geometry for WCW. WCW defines the fixed arena of quantum physics and physical states are identified as spinor fields in WCW. These spinor fields are classical and no second quantization is needed at this level. The justification comes from the observation that infinite-dimensional Clifford algebra [A3] generated by gamma matrices allows a natural identification as fermionic oscillator algebra.

The basic challenges are following.

1. Identify WCW.
2. Provide WCW with Kähler metric and spinor structure
3. Define what spinors and spinor fields in WCW are.

There is huge variety of finite-dimensional geometries and one might think that in infinite-dimensional case one might be drowned with the multitude of possibilities. The situation is however exactly opposite. The loop spaces associated with groups have a unique Kähler geometry due to the simple condition that Riemann connection exists mathematically [A44]. This condition requires that the metric possesses maximal symmetries. Thus raises the vision that infinite-dimensional Kähler geometric existence is unique once one poses the additional condition that the resulting geometry satisfies some basic constraints forced by physical considerations.

The observation about the uniqueness of loop geometries leads also to a concrete vision about what this geometry could be. Perhaps WCW could be regarded as a union of symmetric spaces

[A24] for which every point is equivalent with any other. This would simplify the construction of the geometry immensely and would mean a generalization of cosmological principle to infinite-D context [K52, K85], [L7].

This still requires an answer to the question why $H = M^4 \times CP_2$ is so unique. Something in the structure of this space must distinguish it in a unique manner from any other candidate.

1. The uniqueness of M^4 factor can be understood from the miraculous conformal symmetries of the light-cone boundary but in the case of CP_2 there is no obvious mathematical argument of this kind although physically CP_2 is unique [L16].
2. The observation that $M^4 \times CP_2$ has dimension 8, the space-time surfaces have dimension 4, and partonic 2-surfaces, which are the fundamental objects by holography have dimension 2, suggests that classical number fields [A16, A5, A21] are involved and one can indeed end up to the choice $M^4 \times CP_2$ from physics as generalized number theory vision by simple arguments [K96], [L9]. In particular, the choices M^8 -a subspace of complexified octonions (for octonions see [A16]), which I have used to call hyper-octonions- and $M^4 \times CP_2$ can be regarded as physically equivalent: this "number theoretical compactification" is analogous to spontaneous compactification in M-theory. No dynamical compactification takes place so that $M^8 - H$ duality is a more appropriate term. Octonionic spinor structure required to be equivalent with the ordinary one makes also possible to generalize the twistors from 4-D to 8-D context and replaced 4-D light-likeness with 8-D one.
3. A further powerful argument in favor of H is that M^4 and CP_2 are the only twistor spaces with Kähler structure. The twistor lift of space-time surfaces to their twistor spaces with twistor structure induced from that of $M^4 \times CP_2$ indeed provides a new approach to TGD allowing to utilize powerful tools of algebraic geometry [K100].

Physics as generalized number theory

Physics as a generalized number theory (for an overview about number theory see [A15]) program consists of three separate threads: various p-adic physics and their fusion together with real number based physics to a larger structure [K95] , [L12], the attempt to understand basic physics in terms of classical number fields [K96], [L9] (in particular, identifying associativity condition as the basic dynamical principle), and infinite primes [K94] , [L5], whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory. In this article a summary of the philosophical ideas behind this dream and a summary of the technical challenges and proposed means to meet them are discussed.

The construction of p-adic physics and real physics poses formidable looking technical challenges: p-adic physics should make sense both at the level of the embedding space, the "world of classical worlds" (WCW), and space-time and these physics should allow a fusion to a larger coherent whole. This forces to generalize the notion of number by fusing reals and p-adics along rationals and common algebraic numbers. The basic problem that one encounters is definition of the definite integrals and harmonic analysis [A8] in the p-adic context [K69]. It turns out that the representability of WCW as a union of symmetric spaces [A24] provides a universal group theoretic solution not only to the construction of the Kähler geometry of WCW but also to this problem. The p-adic counterpart of a symmetric space is obtained from its discrete invariant by replacing discrete points with p-adic variants of the continuous symmetric space. Fourier analysis [A8] reduces integration to summation. If one wants to define also integrals at space-time level, one must pose additional strong constraints which effectively reduce the partonic 2-surfaces and perhaps even space-time surfaces to finite geometries and allow assign to a given partonic 2-surface a unique power of a unique p-adic prime characterizing the measurement resolution in angle variables. These integrals might make sense in the intersection of real and p-adic worlds defined by algebraic surfaces.

The dimensions of partonic 2-surface, space-time surface, and embedding space suggest that classical number fields might be highly relevant for quantum TGD. The recent view about the connection is based on hyper-octonionic representation of the embedding space gamma matrices, and the notions of associative and co-associative space-time regions defined as regions for which the Kähler-Dirac gamma matrices span quaternionic or co-quaternionic plane at each point of the region. A further condition is that the tangent space at each point of space-time surface contains a preferred hyper-complex (and thus commutative) plane identifiable as the plane of non-physical polarizations so that gauge invariance has a purely number theoretic interpretation. WCW can be regarded as the space of sub-algebras of the local octonionic Clifford algebra [A3] of the embedding space defined by space-time surfaces with the property that the local sub-Clifford algebra spanned by Clifford algebra valued functions restricted at them is associative or co-associative in a given region.

The recipe for constructing infinite primes is structurally equivalent with a repeated second quantization of an arithmetic super-symmetric quantum field theory. At the lowest level one has fermionic and bosonic states labeled by finite primes and infinite primes correspond to many particle states of this theory. Also infinite primes analogous to bound states are predicted. This hierarchy of quantizations can be continued indefinitely by taking the many particle states of the previous level as elementary particles at the next level. Construction could make sense also for hyper-quaternionic and hyper-octonionic primes although non-commutativity and non-associativity pose technical challenges. One can also construct infinite number of real units as ratios of infinite integers with a precise number theoretic anatomy. The fascinating finding is that the quantum states labeled by standard model quantum numbers allow a representation as wave functions in the discrete space of these units. Space-time point becomes infinitely richly structured in the sense that one can associate to it a wave function in the space of real (or octonionic) units allowing to represent the WCW spinor fields. One can speak about algebraic holography or number theoretic Brahman=Atman identity and one can also say that the points of embedding space and space-time surface are subject to a number theoretic evolution.

One fascinating aspect of infinite primes is that besides the simplest infinite primes analogous to Fock states of a supersymmetric arithmetic QFT constructed from single particle states labelled by primes, also infinite primes having interpretation as bound states emerge. They correspond to polynomials characterized by degree n. Since the formation of bound states in TGD framework corresponds to a hierarchy of conformal symmetry breakings labelled by integer $n = h_{eff}/h$, the natural question is whether these two integers correspond to each other.

Questions

The experience has shown repeatedly that a correct question and identification of some weakness of existing vision is what can only lead to a genuine progress. In the following I discuss the basic questions, which have stimulated progress in the challenge of constructing WCW geometry.

1. What is WCW?

Concerning the identification of WCW I have made several guesses and the progress has been basically due to the gradual realization of various physical constraints and the fact that standard physics ontology is not enough in TGD framework.

1. The first guess was that WCW corresponds to all possible space-like 3-surfaces in $H = M^4 \times CP_2$, where M^4 denotes Minkowski space and CP_2 denotes complex projective space of two complex dimensions having also representation as coset space $SU(3)/U(2)$ (see the separate article summarizing the basic facts about CP_2 and how it codes for standard model symmetries [L2], [L10, L2]). What led to the this particular choice H was the observation that the geometry of H codes for standard model quantum numbers and that the generalization of particle from point like particle to 3-surface allows to understand also remaining quantum numbers having no obvious explanation in standard model (family replication phenomenon). What is important to notice is that Poincare symmetries act as exact symmetries of M^4 rather than space-time surface itself: this realizes the basic vision about Poincare invariant theory of gravitation. This lifting of symmetries to the level of embedding space and the new dynamical degrees of freedom brought by the sub-manifold geometry of space-time surface are absolutely essential for entire quantum TGD and distinguish it from general relativity and string models. There is however a problem: it is not obvious how to get cosmology.
2. The second guess was that WCW consists of space-like 3-surfaces in $H_+ = M^4_+ \times CP2$, where M^4_+ future light-cone having interpretation as Big Bang cosmology at the limit of vanishing mass density with light-cone property time identified as the cosmic time. One obtains cosmology but loses exact Poincare invariance in cosmological scales since translations lead out of future light-cone. This as such has no practical significance but due to the metric 2-dimensionality of light-cone boundary δM^4_+ the conformal symmetries of string model

assignable to finite-dimensional Lie group generalize to conformal symmetries assignable to an infinite-dimensional symplectic group of $S^2 \times CP_2$ and also localized with respect to the coordinates of 3-surface. These symmetries are simply too beautiful to be important only at the moment of Big Bang and must be present also in elementary particle length scales. Note that these symmetries are present only for 4-D Minkowski space so that a partial resolution of the old conundrum about why space-time dimension is just four emerges.

3. The third guess was that the light-like 3-surfaces inside CD are more attractive than space-like 3-surfaces. The reason is that the infinite-D conformal symmetries characterize also light-like 3-surfaces because they are metrically 2-dimensional. This leads to a generalization of Kac-Moody symmetries [A11] of super string models with finite-dimensional Lie group replaced with the group of isometries of H. The natural identification of light-like 3-surfaces is as 3-D surfaces defining the regions at which the signature of the induced metric changes from Minkowskian $(1,-1,-1,-1)$ to Euclidian $(-1-1-1-1)$- I will refer these surfaces as throats or wormhole throats in the sequel. Light-like 3-surfaces are analogous to blackhole horizons and are static because strong gravity makes them light-like. Therefore also the dimension 4 for the space-time surface is unique.
This identification leads also to a rather unexpected physical interpretation. Single light-like wormhole throat carriers elementary particle quantum numbers. Fermions and their super-partners are obtained by glueing Euclidian regions (deformations of so called CP_2 type vacuum extremals of Kähhler action) to the background with Minkowskian signature. Bosons are identified as wormhole contacts with two throats carrying fermion *resp.* anti-fermionic quantum numbers. These can be identified as deformations of CP_2 vacuum extremals between between two parallel Minkowskian space-time sheets. One can say that bosons and their superpartners emerge. This has dramatic implications for quantum TGD [K29] and QFT limit of TGD .
The question is whether one obtains also a generalization of Feynman diagrams. The answer is affirmative. Light-like 3-surfaces or corresponding Euclidian regions of space-time are analogous to the lines of Feynman diagram and vertices are replaced by 2-D surface at which these surfaces glued together. One can speak about Feynman diagrams with lines thickened to light-like 3-surfaces and vertices to 2-surfaces. The generalized Feynman diagrams are singular as 3-manifolds but the vertices are non-singular as 2-manifolds. Same applies to the corresponding space-time surfaces and space-like 3-surfaces. Therefore one can say that WCW consists of generalized Feynman diagrams- something rather different from the original identification as space-like 3-surfaces and one can wonder whether these identification could be equivalent.

4. The fourth guess was a generalization of the WCW combining the nice aspects of the identifications $H = M^4 \times CP_2$ (exact Poincare invariance) and $H = M^4_+ \times CP_2$ (Big Bang cosmology). The idea was to generalize WCW to a union of basic building bricks -causal diamonds (CDs) - which themselves are analogous to Big Bang-Big Crunch cosmologies breaking Poincare invariance, which is however regained by the allowance of union of Poincare transforms of the causal diamonds.
The starting point is General Coordinate Invariance (GCI). It does not matter, which 3-D slice of the space-time surface one choose to represent physical data as long as slices are related by a diffeomorphism of the space-time surface. This condition implies holography in the sense that 3-D slices define holograms about 4-D reality.
The question is whether one could generalize GCI in the sense that the descriptions using space-like and light-like 3-surfaces would be equivalent physically. This requires that finite-sized space-like 3-surfaces are somehow equivalent with light-like 3-surfaces. This suggests that the light-like 3-surfaces must have ends. Same must be true for the space-time surfaces and must define preferred space-like 3-surfaces just like wormhole throats do. This makes sense only if the 2-D intersections of these two kinds of 3-surfaces -call them partonic 2-surfaces- and their 4-D tangent spaces carry the information about quantum physics. A strengthening of holography principle would be the outcome. The challenge is to understand, where the intersections defining the partonic 2-surfaces are located.
ZEO (ZEO) allows to meet this challenge.

   (a) Assume that WCW is union of sub-WCWs identified as the space of light-like 3-surfaces assignable to $CD \times CP_2$ with given CD defined as an intersection of future and past directed light-cones of M^4. The tips of CDs have localization in M^4 and one can perform for CD both translations and Lorentz boost for CDs. Space-time surfaces inside CD define the basic building brick of WCW. Also unions of CDs allowed and the CDs belonging to the union can intersect. One can of course consider the possibility of intersections and analogy with the set theoretic realization of topology.

(b) ZEO property means that the light-like boundaries of these objects carry positive and negative energy states, whose quantum numbers are opposite. Everything can be created from vacuum and can be regarded as quantum fluctuations in the standard vocabulary of quantum field theories.

(c) Space-time surfaces inside CDs begin from the lower boundary and end to the upper boundary and in ZEO it is natural to identify space-like 3-surfaces as pairs of space-like 3-surfaces at these boundaries. Light-like 3-surfaces connect these boundaries.

(d) The generalization of GCI states that the descriptions based on space-like 3-surfaces must be equivalent with that based on light-like 3-surfaces. Therefore only the 2-D intersections of light-like and space-like 3-surfaces - partonic 2-surfaces- and their 4-D tangent spaces (4-surface is there!) matter. Effective 2-dimensionality means a strengthened form of holography but does not imply exact 2-dimensionality, which would reduce the theory to a mere string model like theory. Once these data are given, the 4-D space-time surface is fixed and is analogous to a generalization of Bohr orbit to infinite-D context. This is the first guess. The situation is actually more delicate due to the non-determinism of Kähler action motivating the interaction of the hierarchy of CDs within CDs.

In this framework one obtains cosmology: CDs represent a fractal hierarchy of big bang-big crunch cosmologies. One obtains also Poincare invariance. One can also interpret the non-conservation of gravitational energy in cosmology which is an empirical fact but in conflict with exact Poincare invariance as it is realized in positive energy ontology [K106, K90]. The reason is that energy and four-momentum in ZEO correspond to those assignable to the positive energy part of the zero energy state of a particular CD. The density of energy as cosmologist defines it is the statistical average for given CD: this includes the contributions of sub-CDs. This average density is expected to depend on the size scale of CD density is should therefore change as quantum dispersion in the moduli space of CDs takes place and leads to large time scale for any fixed sub-CD.

Even more, one obtains actually quantum cosmology! There is large variety of CDs since they have position in M^4 and Lorentz transformations change their shape. The first question is whether the M^4 positions of both tips of CD can be free so that one could assign to both tips of CD momentum eigenstates with opposite signs of four-momentum. The proposal, which might look somewhat strange, is that this not the case and that the proper time distance between the tips is quantized as integer multiples of a fundamental time scale $T = R/c$ defined by CP_2 size R.

A stronger - maybe un-necessarily strong - condition would be that the quantization is in octaves. This would explain p-adic length scale hypothesis, which is behind most quantitative predictions of TGD. That the time scales assignable to the CD of elementary particles correspond to biologically important time scales [K38] forces to take this hypothesis very seriously. The interpretation for T could be as a cosmic time. Even more general quantization is proposed to take place. The relative position of the second tip with respect to the first defines a point of the proper time constant hyperboloid of the future light cone. The hypothesis is that one must replace this hyperboloid with a lattice like structure. This implies very powerful cosmological predictions finding experimental support from the quantization of redshifts for instance [K90]. For quite recent further empirical support see [E30].

One should not take this argument without a grain of salt. Can one really realize ZEO in this framework? The geometric picture is that translations correspond to translations of CDs. Translations should be done independently for the upper and lower tip of CD if one wants to speak about zero energy states but this is not possible if the proper time distance is quantized. If the relative M^4_+ coordinate is discrete, this pessimistic conclusion is strengthened further.

The manner to get rid of problem is to assume that translations are represented by quantum operators acting on states at the light-like boundaries. This is just what standard quantum theory assumes. An alternative- purely geometric- way out of difficulty is the Kac-Moody symmetry associated with light-like 3-surfaces meaning that local M^4 translations depending on the point of partonic 2-surface are gauge symmetries. For a given translation leading out of CD this gauge symmetry allows to make a compensating transformation which allows to satisfy the constraint.

This picture is roughly the recent view about WCW. What deserves to be emphasized is that a very concrete connection with basic structures of quantum field theory emerges already at the level of basic objects of the theory and GCI implies a strong form of holography and almost stringy picture.

2.Some Why's

In the following I try to summarize the basic motivations behind quantum TGD in form of various Why's.

1. Why WCW?
   Einstein's program has been extremely successful at the level of classical physics. Fusion of general relativity and quantum theory has however failed. The generalization of Einstein's geometrization program of physics from classical physics to quantum physics gives excellent hopes about the success in this project. Infinite-dimensional geometries are highly unique and this gives hopes about fixing the physics completely from the uniqueness of the infinite-dimensional Kähler geometric existence.
2. Why spinor structure in WCW?
   Gamma matrices defining the Clifford algebra [A3] of WCW are expressible in terms of fermionic oscillator operators. This is obviously something new as compared to the view about gamma matrices as bosonic objects. There is however no deep reason denying this kind of identification. As a consequence, a geometrization of fermionic oscillator operator algebra and fermionic statistics follows as also geometrization of super-conformal symmetries [A23, A11] since gamma matrices define super-generators of the algebra of WCW isometries extended to a super-algebra.
3. Why Kähler geometry?
   Geometrization of the bosonic oscillator operators in terms of WCW vector fields and fermionic oscillator operators in terms of gamma matrices spanning Clifford algebra. Gamma matrices span hyper-finite factor of type II_1 and the extremely beautiful properties of these von Neuman algebras [A60] (one of the three von Neumann algebras that von Neumann suggests as possible mathematical frameworks behind quantum theory) lead to a direct connection with the basic structures of modern physics (quantum groups, non-commutative geometries, .. [A37]).
   A further reason why is the finiteness of the theory.
   (a) In standard QFTs there are two kinds of infinities. Action is a local functional of fields in 4-D sense and one performs path integral over **all** 4-surfaces to construct S-matrix. Mathematically path integration is a poorly defined procedure and one obtains diverging Gaussian determinants and divergences due to the local interaction vertices. Regularization provides the manner to get rid of the infinities but makes the theory very ugly.
   (b) Kähler function defining the Kähler geometry is a expected to be non-local functional of the space-like 3-surfaces at the ends of space-time surface reducing by strong form of holography to a functional of partonic 2-surfaces and their 4-D tangent space data (Kähler action for the Euclidian regions of the preferred extremal and having as interpretation in terms of generalized Feynman diagram).
   Path integral is replaced with a functional integral, which is mathematically well-defined procedure and one performs functional integral only over the unions of 3-surfaces at opposite boundaries of CD and having vanishing super-conformal charges for a sub-algebras of conformal algebras with conformal weights coming as multiples of integer $h = h_{eff}/h$. This realizes the strong form of holography. The exponent of Kähler function - Kähler action for the Euclidian space-time regions - defines a unique vacuum functional whereas Minkowskian contribution to Kähler action gives the analog of ordinary imaginary exponent of action.
   The local divergences of local quantum field theories are expected t be absent since there are no local interaction vertices. Also the divergences associated with the Gaussian determinant and metric determinant cancel since these two determinants cancel each other in the integration over WCW. As a matter fact, symmetric space property suggest a much

more elegant manner to perform the functional integral by reducing it to harmonic analysis in infinite-dimensional symmetric space [K113].

(c) One can imagine also the possibility of divergences in fermionic degrees of freedom but the generalization of the twistor approach to 8-D context [K100] suggests that the generalized Feynman diagrams in ZEO are manifestly finite: in particular IR divergences plaguing ordinary twistor approach should be absent by 8-D masslessness. The only fermionic interaction vertex is 2- vertex associated with the discontinuity of K-D operator assignable to string world sheet boundary at partonic 2-surfaces serving as geometric vertices. At fermionic level scattering amplitudes describe braiding and OZI rule is satisfied so that the analog of topological QFT is obtained. The topological vertices describing the joining of incoming light-like orbits of partonic 2-surface at the vertices imply the non-triviality of the scattering amplitudes.

4. Why infinite-dimensional symmetries?
WCW must be a union of symmetric spaces in order that the Riemann connection exists (this generalizes the finding of Freed for loop groups [A44]). Since the points of symmetric spaces are metrically equivalent, the geometrization becomes tractable although the dimension is infinite. A union of symmetric spaces is required because 3-surfaces with a size of galaxy and electron cannot be metrically equivalent. Zero modes distinguish these surfaces and can be regarded as purely classical degrees of freedom whereas the degrees of freedom contributing to the WCW line element are quantum fluctuating degrees of freedom.
One immediate implication of the symmetric space property is constant curvature space property meaning that the Ricci tensor proportional to metric tensor. Infinite-dimensionality means that Ricci scalar either vanishes or is infinite. This implies vanishing of Ricci tensor and vacuum Einstein equations for WCW.

5. Why ZEO and why causal diamonds?
The consistency between Poincare invariance and GRT requires ZEO. In positive energy ontology only one of the infinite number of classical solutions is realized and partially fixed by the values of conserved quantum numbers so that the theory becomes obsolete. Even in quantum theory conservation laws mean that only those solutions of field equations with the quantum numbers of the initial state of the Universe are interesting and one faces the problem of understanding what the the initial state of the universe was. In ZEO these problems disappear. Everything is creatable from vacuum: if the physical state is mathematically realizable it is in principle reachable by a sequence of quantum jumps. There are no physically non-reachable entities in the theory. ZEO leads also to a fusion of thermodynamics with quantum theory. Zero energy states ae defined as entangled states of positive and negative energy states and entanglement coefficients define what I call M-matrix identified as "complex square root" of density matrix expressible as a product of diagonal real and positive density matrix and unitary S-matrix [K29].
There are several good reasons why for causal diamonds. ZEO requires CDs, the generalized form of GCI and strong form of holography (light-like and space-like 3-surfaces are physically equivalent representations) require CDs, and also the view about light-like 3-surfaces as generalized Feynman diagrams requires CDs. Also the classical non-determinism of Kähler action can be understood using the hierarchy CDs and the addition of CDs inside CDs to obtain a fractal hierarchy of them provides an elegant manner to undersand radiative corrections and coupling constant evolution in TGD framework.
A strong physical argument in favor of CDs is the finding that the quantized proper time distance between the tips of CD fixed to be an octave of a fundamental time scale defined by CP_2 happens to define fundamental biological time scale for electron, u quark and d quark [K38]: there would be a deep connection between elementary particle physics and living matter leading to testable predictions.

13.5.2 Overall View About Kähler Action And Kähler Dirac Action

In the following the most recent view about Kähler action and the Kähler-Dirac action (Kähler-Dirac action) is explained in more detail. The proposal is one of the many that I have considered.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action. The action could

contain also Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term could be chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD). For Euclidian wormhole contacts Chern-Simons term need not reduce to a mere boundary terms since the gauge potential is not globally defined. One can also consider the possibility that only Minkowskian regions involve the Chern-Simons boundary term. One can also argue that Chern-Simons term is actually an un-necessary complication not needed in the recent interpretation of TGD.

There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.

The vanishing of conformal Noether charges for sub-algebras of various conformal algebras are also posed. They could be also realized as Lagrange multiplied terms at the ends of 3-surface.

2. By supersymmetry requirement the Kähler-Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with embedding space gamma matrices to obtain K-D gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. As explained, it is assumed that localiztion to 2-D string world sheets occurs. At the light-like boundaries the limit of K-D equation gives K-D equation at the ferminonic liness expressing 8-D light-likeness or 4-D light-likeness in effective metric.

Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.
2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.
3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface. This however leads to an unphysical outcome.

Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naïve guess has been that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This is however a mere guess and need not be correct. The outcome is actually that the limit of K-D equation at string world sheets defines the Dirac equation at the boundaries of string world sheets.

One should try to make first clear what one really wants.

1. What one wants are generalized Feynman diagrams demanding massless Dirac propagators in 8-D sense at the light-like boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that 8-D generalization of the twistor Grassmannian approach works. The localization of spinors at string world sheets is crucial for achieving this.
   In ordinary QFT fermionic propagator results from the kinetic term in Dirac action. Could the situation be same also now at the boundary of string world sheet associated with parton orbit? One can consider the Dirac action
   $$L_{ind} = \int \overline{\Psi}\Gamma^t_{ind}\partial_t\Psi\sqrt{g_1}dt$$
   defined by the induced gamma matrix Γ^t_{ind} and induced 1-metric. This action need to be associated only to the Minkowskian side of the space-surface. By supersymmetry Dirac action must be accompanied by a bosonic action $\int\sqrt{g_1}dt$. It forces the boundary line to be a geodesic line. Dirac equation gives
   $$\Gamma^t_{ind}D_t\Psi = ip^k(M^8)\gamma_k\Psi = 0 \ .$$
   The square of the Dirac operator gives $(\Gamma^t_{ind})^2 = 0$ for geodesic lines (the components of the second fundamental form vanish) so that one obtains 8-D light-likeness.
   Boundary line would behave like point-like elementary particle for which conserved 8-momentum is conserved and light-like: just as twistor diagrammatics suggests. 8-momentum must be real since otherwise the particle orbit would belong to the complexification of H. These conditions can be regarded as boundary conditions on the string world sheet and spinor modes. There would be no additional contribution to the Kähler action.
2. The special points are the ends of the fermion lines at incoming and outgoing partonic 2-surfaces and at these points M^4 mass squared is assigned to the embedding space spinor harmonic associated with the incoming fermion. CP_2 mass squared corresponds to the eigenvalue of CP_2 spinor d'Alembertian for the spinor harmonic.
   At the end of the fermion line $p(M^4)^k$ corresponds to the incoming fermionic four-momentum. The direction of $p(E^4)^k$ is not fixed and one has $SO(4)$ harmonic at the mass shell $p(E^4)^2 = m^2$, m the mass of the incoming particle. At embedding space level color partial waves correspond to $SO(4)$ partial waves ($SO(4)$ could be seen as the symmetry group of low energy hadron physics giving rise to vectorial and axial isospin).

Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n\Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

Associativity (co-associativity) and quantum criticality

Quantum criticality is one of the basic notions of TGD. It was originally introduced to fix the value(s) of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current: this current vanishes for Cartan algebra of isometries. A clearer formulation of criticality is as a condition that the various conformal charges vanish for 3-surfaces at the ends of space-time surface for conformal weights coming as multiples of integer n. The natural expectation is that the numbers of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations is finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The conjecture is that quantum critical space-time surfaces are associative (co-associative) in the sense that the tangent vectors span a associative (co-associative) subspace of complexified octonions at each point of the space-time surface is consistent with what is known about preferred extremals. The notion of octonionic tangent space can be expressed by introducing octonionic structure realized in terms of vielbein in manner completely analogous to that for the realization of gamma matrices.

One can also introduce octonionic representations of gamma matrices but this is not absolutely necessarily. The condition that both the Kähler-Dirac gamma matrices and spinors are quaternionic at each point of the space-time surface leads to a precise ansatz for the general solution of the Kähler-Dirac equation making sense also in the real context. The octonionic version of the Kähler-Dirac equation is very simple since $SO(7,1)$ as vielbein group is replaced with G_2 acting as automorphisms of octonions so that only the neutral Abelian part of the classical electro-weak gauge fields survives the map.

This condition is analogous to what happens for the spinor modes when they are restricted at string worlds sheets carrying vanishing induced W fields (and also Z^0 fields above weak length scale) to guarantee well-definedness of em charge and it might be that this strange looking condition makes sense. The possibility to define G_2 structure would thus be due to the well-definedness of em charge and in the generic case possible only for string world sheets and possibly also partonic 2-surfaces.

Octonionic gamma matrices provide also a non-associative representation for the 8-D version of Pauli sigma matrices and encourage the identification of 8-D tangent space twistors as pairs of octonionic spinors conjectured to be highly relevant also for quantum TGD. Quaternionicity condition implies that octo-twistors reduce to something closely related to ordinary twistors.

The sigma matrices are however an obvious problem since their commutators are proportional to M^4 sigma matrices. This raises the question whether the equivalence with ordinary Kähler-Dirac equation should be assumed. This assumption very strongly suggests a localization string world sheets implied also by the condition that electromagnetic charge is well-defined for the spinor modes. The weakest manner to satisfy the equivalence would be for Dirac equation restricted to the light-like boundaries of string world sheets and giving just 8-D light-likeness condition but with random direction of light-like momentum.

The analog AdS/CFT duality

Although quantum criticality in principle predicts the possible values of Kähler coupling strength coming as a series of critical temperatures $\alpha_K = g_K^2/4\pi\hbar_{eff}$, $\hbar_{eff}/h = n$ characterizing quantum criticalities, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

Since WCW Kähler metric can be defined as anti-commutators of WCW gamma matrices identified as super-conformal super-charges for the K-D action, one would have the analog of AdS/CFT duality between bosonic definition of Kähler metric in terms of Kähler function defined by Euclidian contribution to Kähler action and fermionic definition in terms of anti-commutator of conformal supercharges.

This encourages to ask whether Dirac determinant - if it can be defined - could be identified as exponent of Kähler function or Kähler action. This might be of course un-necessary and highly unpractical outcome: it seems Kähler function is easy to obtain as Kähher action and Kähler metric as anti-commutators of super-charges. This is discussed in [K3].

13.5.3 Various Dirac Operators And Their Interpretation

The physical interpretation of Kähler Dirac equation is not at all straightforward. The following arguments inspired by effective 2-dimensionality suggest that the Kähler-Dirac gamma matrices and corresponding effective metric could allow dual gravitational description of the physics associated with wormhole throats. This applies in particular to condensed matter physics.

Four Dirac equations

To begin with, Dirac equation appears in four forms in TGD.

1. The Dirac equation in the world of classical worlds codes (WCW) for the super Virasoro conditions for the super Kac-Moody and similar representations formed by the states of wormhole contacts forming the counterpart of string like objects (throats correspond to the ends of the string. WCW Dirac operator generalizes the Dirac operator of 8-D embedding space by bringing in vibrational degrees of freedom. This Dirac equation should give as its solutions zero energy states and corresponding M-matrices generalizing S-matrix.
   The unitary U-matrix realizing discrete time evolution in the moduli space of CDs can be constructed as an operator in the space of zero energy states relating M-matrices [K67]. The natural application of U-matrix appears in consciousness theory as a coder of what Penrose calls U-process. The ground states to which super-conformal algebras act correspond to embedding space spinor modes in accordance with the idea that point like limit gives QFT in embedding space.
2. The analog of massless Dirac equation at the level of 8-D embedding space and satisfied by fermionic ground states of super-conformal representations.
3. Kähler Dirac equation is satisfied in the interior of space-time. In this equation the gamma matrices are replaced with Kähler-Dirac gamma matrices defined by the contractions of canonical momentum currents $T^{\alpha}_k = \partial L/\partial_{\alpha} h^k$ with embedding space gamma matrices Γ_k. This replacement is required by internal consistency and by super-conformal symmetries. The well-definedness of em charge implies that the modes of induced spinor field are localized at 2-D surfaces so that a connection with string theory type approach emerges.
4. At the light-like boundaries of string world sheets K-D equation gives rise to an analog of 4-D massless Dirac equation also one has light-like 8-momentum corresponding to the light-like tangent vector of the fermion carrying line. This equation is equivalent with its octonionic counterpart.

Kähler-Dirac equation defines Dirac equation at space-time level. Consider first K-D equation in the interior of space-time surface.

1. The condition that electromagnetic charge operator defined in terms of em charge expressed in terms of Clifford algebra is well defined for spinor modes (completely analogous to spin defined in terms of sigma matrices) leads to the proposal that induced spinor fields are necessarily localized at 2-dimensional string worlds sheets [K113]. Only the covariantly constant right handed neutrino and its modes assignable to massless extremals (at least) generating supersymmetry (super-conformal symmetries) would form an exception since electroweak couplings would vanish. Note that the Kähler-Dirac gamma matrices possess CP_2 and this must vanish in order to have de-localization.
2. This picture implies stringy realization of super Kac-Moody symmetry elementary particles can be identified as string like objects albeit in different sense than in string models. At light-like 3-surfaces defining the orbits of partonic 2-surfaces spinor fields carrying electroweak quantum numbers would be located at braid strands as also the notion of finite measurement resolution requires.
3. Could Kähler Dirac equation provide a first principle justification for the light-hearted use of effective mass and the analog of Dirac equation in condensed manner physics? This would conform with the holographic philosophy. Partonic 2-surfaces with tangent space data and their light-like orbits would give hologram like representation of physics and the interior of space-time the 4-D representation of physics. Holography would have in the recent situation interpretation also as quantum classical correspondence between representations of physics in terms of quantized spinor fields at the light-like 3-surfaces on one hand and in terms of classical fields on the other hand.

4. The resulting dispersion relation for the square of the Kähler-Dirac operator assuming that induced like metric, Kähler field, etc. are very slowly varying contains quadratic and linear terms in momentum components plus a term corresponding to magnetic moment coupling. In general massive dispersion relation is obtained as is also clear from the fact that Kähler Dirac gamma matrices are combinations of M^4 and CP_2 gammas so that modified Dirac mixes different M^4 chiralities (basic signal for massivation). If one takes into account the dependence of the induced geometric quantities on space-time point dispersion relations become non-local.
5. Sound as a concept is usually assigned with a rather high level of description. Stringy world sheets could however dramatically raise the status of sound in this respect. The oscillations of string world sheets connecting wormhole throats describe non-local 2-particle interactions. Holography suggests that this interaction just "gravitational" dual for electroweak and color interactions. Could these oscillations inducing the oscillation of the distance between wormhole throats be interpreted at the limit of weak "gravitational" coupling as analogs of sound waves, and could sound velocity correspond to maximal signal velocity assignable to the effective metric?
6. The latest progress in the understanding of quantum TGD imply that the area of string world sheet in the effective metric defined by the K-D gamma matrices indeed plays a fundamental role in quantum TGD (of course, WCW Kähler metric also involves this effective metric). By conformal invariance this metric could be equivalent with the induced metric. The string tension would be dynamical and the conjecture is that one can express Kähler action as total effective area of string world sheets. The hierarchy of Planck constants is essential in making possible to understand the description of not only gravitational but all bound states in terms of strings connecting partonic 2-surfaces. This description is analogous to AdS/CFT correspondence. That the string tension is defined by the Kähler action rather than assumed to be determined by Newton's constants allows to avoid divergences.

The status of the Chern-Simons counterpart of K-D action has remained unclear. K-D action reduces to Chern-Simons boundary terms in Minkowskian space-time regions at least. I have considered Chern-Simons boundary term as an additional term in Kähler action and considered also Chern-Simons-Dirac operator. The localization of spinors to string world sheets however suggests that its introduction produces more problems than solves them. One reason is that C-S-D action involves only CP_2 gamma matrices so that one cannot realize 8-D masslessness for the spinor localized at fermion line defining the boundary of string world sheet.

Does energy metric provide the gravitational dual for condensed matter systems?

The Kähler-Dirac gamma matrices define an effective metric via their anti-commutators quadratic in components of energy momentum tensor (canonical momentum densities). This effective metric vanishes for vacuum extremals. Note that the use of the Kähler-Dirac gamma matrices guarantees among other things internal consistency and super-conformal symmetries of the theory.

If the above argument is on the right track, this effective metric should have applications in condensed matter theory. The energy metric has a natural interpretation in terms of effective light velocities which depend on direction of propagation. One can diagonalize the energy metric $g_e^{\alpha\beta}$ (contravariant form results from the anti-commutators) and one can denote its eigenvalues by (v_0, v_i) in the case that the signature of the effective metric is $(1, -1, -1, -1)$. The 3-vector v_i/v_0 has interpretation as components of effective light velocity in various directions as becomes clear by thinking the d'Alembert equation for the energy metric. This velocity field could be interpreted as that of hydrodynamic flow. The study of the extremals of Kähler action shows that if this flow is actually Beltrami flow so that the flow parameter associated with the flow lines extends to global coordinate, Kähler action reduces to a 3-D Chern-Simons action and one obtains effective topological QFT. The conserved fermion current $\overline{\Psi}\Gamma_e^\alpha\Psi$ has interpretation as incompressible hydrodynamical flow.

This would give also a nice analogy with AdS/CFT correspondence allowing to describe various kinds of physical systems in terms of higher-dimensional gravitation and black holes are introduced quite routinely to describe condensed matter systems. In TGD framework one would have an analogous situation but with 10-D space-time replaced with the interior of 4-D space-time and the boundary of AdS representing Minkowski space with the light-like 3-surfaces carrying matter. The effective gravitation would correspond to the "energy metric". One can associate with it analogs of curvature tensor, Ricci tensor and Einstein tensor using standard formulas and identify effective energy momentum tensor associated as Einstein tensor with effective Newton's constant appearing as constant of proportionality. Note however that the besides ordinary metric and "energy" metric one would have also the induced classical gauge fields having purely geometric interpretation and action would be Kähler action. This 4-D holography could provide a precise, dramatically simpler, and also a very concrete dual description. This cannot be said about model of graphene based on the introduction of 10-dimensional black holes, branes, and strings chosen in more or less ad hoc manner.

This raises questions. Could this give a general dual gravitational description of dissipative effects in terms of the "energy" metric and induced gauge fields? Does one obtain the analogs of black holes? Do the general theorems of general relativity about the irreversible evolution leading to black holes generalize to describe analogous fate of condensed matter systems caused by dissipation? Can one describe non-equilibrium thermodynamics and self-organization in this manner?

One might argue that the incompressible Beltrami flow defined by the dynamics of the preferred extremals is dissipationless and viscosity must therefore vanish locally. The failure of complete determinism for Kähler action however means generation of entropy since the knowledge about the state decreases gradually. This in turn should have a phenomenological local description in terms of viscosity, which characterizes the transfer of energy to shorter scales and eventually to radiation. The deeper description should be non-local and basically topological and might lead to quantization rules. For instance, one can imagine the quantization of the ratio η/s of the viscosity to entropy density as multiples of a basic unit defined by its lower bound (note that this would be analogous to Quantum Hall effect). For the first M-theory inspired derivation of the lower bound of η/s [D7] . The lower bound for η/s is satisfied in good approximation by what should have been QCD plasma but found to be something different (RHIC and the first evidence for new physics from LHC [K64]).

An encouraring sign comes from the observation that for so called massless extremals representing classically arbitrarily shaped pulses of radiation propagating without dissipation and dispersion along single direction the canonical momentum currents are light-like. The effective contravariant metric vanishes identically so that fermions cannot propate in the interior of massless extremals! This is of course the case also for vacuum extremals. Massless extremals are purely bosonic and represent bosonic radiation. Many-sheeted space-time decomposes into matter containing regions and radiation containing regions. Note that when wormhole contact (particle) is glued to a massless extremal, it is deformed so that CP_2 projection becomes 4-D guaranteeing that the weak form of electric magnetic duality can be satisfied. Therefore massless extremals can be seen as asymptotic regions. Perhaps one could say that dissipation corresponds to a de-coherence process creating space-time sheets consisting of matter and radiation. Those containing matter might be even seen as analogs blackholes as far as energy metric is considered.

Preferred extremals as perfect fluids

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional conform the discovery [C14]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [D6] . In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [D3] .

1. The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gradient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3}\partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{13.5.1}$$

The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{13.5.2}$$

From $dF^i = T^{ij}S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This term is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

2. The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} \ . \tag{13.5.3}$$

The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p)u^\alpha u^\beta + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{13.5.4}$$

Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentrations in the definition of Minkowski metric ($(1,-1,-1,-1)$).

3. If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parameter. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p)g^{tt}\delta^\alpha_t \delta^\beta_t + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{13.5.5}$$

In this case the vanishing of the viscous term means that one has perfect fluid in strong sense. The existence of a global flow parameter means that one has

$$v_i = \Psi \partial_i \Phi \ . \tag{13.5.6}$$

Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \ . \tag{13.5.7}$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in appropriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

1. Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the "topological" half of Maxwell's equations (Faraday's induction law and the statement that no non-topological magnetic are possible) is satisfied.

2. Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The Euclidian regions (or wormhole throats, depends on one's tastes) define what I identify as generalized Feynman diagrams.
   Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x. What causes the failure of the exact perfect fluid property?

1. Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At embedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).
2. The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of superconductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.
   The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper} at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is fed into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.
   This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.
3. The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of h_{eff} can be large in some situations so that the quantal character of dissipation could become

visible even macroscopically. Whether this a situation with large h_{eff} is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

1. The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by $v_{\parallel}$ *resp.* $v_{\perp}$ the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.
2. The flow of momentum component parallel to the main flow due to the absorbed at S is

$$n_{abs} m v_{\parallel} v_{\perp} S \quad . \tag{13.5.8}$$

This momentum flow must be equal to the viscous force

$$F_{visc} = \eta \frac{v_{\parallel}}{d} \times S \quad . \tag{13.5.9}$$

From this one obtains

$$\eta = n_{abs} m v_{\perp} d \quad . \tag{13.5.10}$$

If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

so that one has

$$\frac{\eta}{s} = m v_{\perp} d \quad . \tag{13.5.11}$$

This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x, Angular momentum quantization suggests strongly itself as origin of the quantization.
3. Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities $v_{\perp}$. Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.
Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv \ \ D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon} \tag{13.5.12}$$

Quantization condition would give

$$\epsilon = 4\pi \quad . \tag{13.5.13}$$

One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increases suggesting that also the radius of the vortices scales like d.

4. One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

Is the effective metric one- or two-dimensional?

The following argument suggests that the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices is effectively one- or two-dimensional. Effective one-dimensionality would conform with the observation that the solutions of the modified Dirac equations can be localized to one-dimensional world lines in accordance with the vision that finite measurement resolution implies discretization reducing partonic many-particle states to quantum superpositions of braids. The localization to 1-D curves occurs always at the 3-D orbits of the partonic 2-surfaces. Note that the localization of induced spinor fields to string world sheets with 2-D CP_2 projection and carrying vanishing classical W fields would require only 2-D property.

The localization requires that the embedding space 1-forms associated with the K-D gamma matrices define lower-dimensional linearly independent set with elements proportional to gradients of embedding space coordinates defining coordinates for the lower-dimensional manifold. Therefore Frobenius conditions would be satisfied.

The argument is based on the following assumptions.

1. The Kähler-Dirac gamma matrices for Kähler action are contractions of the canonical momentum densities T^α_k with the gamma matrices of H.
2. The strongest assumption is that the isometry currents

$$J^{A\alpha} = T^\alpha_k j^{Ak} \quad (13.5.14)$$

for the preferred extremals of Kähler action are of form

$$J^{A\alpha} = \Psi^A(\nabla\Phi)^\alpha \quad (13.5.15)$$

with a common function Φ guaranteeing that the flow lines of the currents integrate to coordinate lines of single global coordinate variables (Beltrami property). Index raising is carried out by using the ordinary induced metric.

3. A weaker assumption is that one has two functions Φ_1 and Φ_2 assignable to the isometry currents of M^4 and CP_2 respectively.:

$$\begin{aligned} J_1^{A\alpha} &= \Psi_1^A(\nabla\Phi_1)^\alpha \ , \\ J_2^{A\alpha} &= \Psi_2^A(\nabla\Phi_2)^\alpha \ . \end{aligned} \quad (13.5.16)$$

The two functions Φ_1 and Φ_2 could define dual light-like curves spanning string world sheet. In this case one would have effective 2-dimensionality and decomposition to string world sheets [K54]. Isometry invariance does not allow more that two independent scalar functions Φ_i.

Consider now the argument.

1. One can multiply both sides of this equation with j^{Ak} and sum over the index A labeling isometry currents for translations of M^4 and $SU(3)$ currents for CP_2. The tensor quantity $\sum_A j^{Ak}j^{Al}$ is invariant under isometries and must therefore satisfy

$$\sum_A \eta_{AB} j^{Ak} j^{Al} = h^{kl} \ , \quad (13.5.17)$$

where η_{AB} denotes the flat tangent space metric of H. In M^4 degrees of freedom this statement becomes obvious by using linear Minkowski coordinates. In the case of CP_2 one can first consider the simpler case $S^2 = CP_1 = SU(2)/U(1)$. The coset space property implies in standard complex coordinate transforming linearly under $U(1)$ that only the isometry currents belonging to the complement of $U(1)$ in the sum contribute at the origin and the identity holds true at the origin and by the symmetric space property everywhere. Identity can be verified also directly in standard spherical coordinates. The argument generalizes to the case of $CP_2 = SU(3)/U(2)$ in an obvious manner.

2. In the most general case one obtains

$$\begin{aligned} T_1^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_1)^\alpha \equiv f_1^k (\nabla\Phi_1)^\alpha \ , \\ T_2^{\alpha k} &= \sum_A \Psi_1^A j^{Ak} \times (\nabla\Phi_2)^\alpha \equiv f_2^k (\nabla\Phi_2)^\alpha \ . \end{aligned} \quad (13.5.18)$$

3. The effective metric given by the anti-commutator of the modified gamma matrices is in turn is given by

$$G^{\alpha\beta} = m_{kl} f_1^k f_1^l (\nabla\Phi_1)^\alpha (\nabla\Phi_1)^\beta + s_{kl} f_2^k f_2^l (\nabla\Phi_2)^\alpha (\nabla\Phi_2)^\beta \ . \quad (13.5.19)$$

The covariant form of the effective metric is effectively 1-dimensional for $\Phi_1 = \Phi_2$ in the sense that the only non-vanishing component of the covariant metric $G_{\alpha\beta}$ is diagonal component along the coordinate line defined by $\Phi \equiv \Phi_1 = \Phi_2$. Also the contravariant metric is effectively 1-dimensional since the index raising does not affect the rank of the tensor but depends on the other space-time coordinates. This would correspond to an effective reduction to a dynamics of point-like particles for given selection of braid points. For $\Phi_1 \neq \Phi_2$ the metric is effectively 2-dimensional and would correspond to stringy dynamics.

One can also develop an objection to effective 1- or 2-dimensionality. The proposal for what preferred extremals of Kähler action as deformations of the known extremals of Kähler action could be leads to a beautiful ansatz relying on generalization of conformal invariance and minimal surface equations of string model [K17]. The field equations of TGD reduce to those of classical string model generalized to 4-D context.

If the proposed picture is correct, field equations reduce to purely algebraically conditions stating that the Maxwellian energy momentum tensor for the Kähler action has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1, 1) and second fundamental form H^k a tensor of type (2, 0) and (0, 2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

There is however an important consistency condition involved. The Maxwell energy momentum tensor for Kähler action must have vanishing covariant divergence. This is satisfied if it is linear combination of Einstein tensor and metric. This gives Einstein's equations with cosmological term in the general case. By the algebraic character of field equations also minimal surface equations are satisfied and Einstein's General Relativity would be exact part of TGD.

In the case of Kähler-Dirac equation the result means that modified gamma matrices are contractions of linear combination of Einstein tensor and metric tensor with the induced gamma matrices so that the TGD counterpart of ordinary Dirac equation would be modified by the addition of a term proportional to Einstein tensor. The condition of effective 1- or 2-dimensionality seems to pose too strong conditions on this combination.

13.6 Summary Of Generalized Feynman Diagrammatics

This section gives a summary about the recent view abot generalized Feynman diagrammatics, which can be seen as a hybrid of Feynman diagrammatics and stringy diagrammatics. The analogs of Feynman diagrams are realized at the level of space-time topology and geometry and the lines of these diagrams are Euclidian space-time regions identifiable as wormhole contacts. For fundamental fermions one has the usual 1-D propagator lines.

Physical particles can be seen as bound state of massless fundamental fermions and involve two wormhole contacts forming parts of closed Kähler magnetic flux tubes carrying monopole flux. The orbits of wormhole throats are connected by fermionic string world sheets whose boundaries correspond to massless fermion lines defining strands of braids. String world sheets in turn can form 2-braids.

It is a little bit matter of taste whether one refers to these diagrams generalized Feynman diagrams, generalized stringy diagrams, generalized Wilson loops or generalized twistor diagrams. All these labels are partly misleading.

In the sequel the basic action principles - Kähler action and Kähler-Dirac action are discussed first, and then a proposal for the diagrams describing M-matrix elements is discussed.

13.6.1 The Basic Action Principle

In the following the most recent view about Kähler action and the Kähler-Dirac action (Kähler-Dirac action) is explained in more detail. The proposal is one of the many that I have considered.

1. The minimal formulation involves in the bosonic case only 4-D Kähler action. The action could contain also Chern-Simons boundary term localized to partonic orbits at which the signature of the induced metric changes. The coefficient of Chern-Simons term could be chosen so that this contribution to bosonic action cancels the Chern-Simons term coming from Kähler action (by weak form of electric-magnetic duality) so that for preferred extremals Kähler action reduces to Chern-Simons terms at the ends of space-time surface at boundaries of causal diamond (CD). For Euclidian wormhole contacts Chern-Simons term need not reduce to a mere boundary terms since the gauge potential is not globally defined. One can also consider the possibility that only Minkowskian regions involve the Chern-Simons boundary term. One can also argue that Chern-Simons term is actually an un-necessary complication not needed in the recent interpretation of TGD.
   There are constraint terms expressing weak form of electric-magnetic duality and constraints forcing the total quantal charges for Kähler-Dirac action in Cartan algebra to be identical with total classical charges for Kähler action. This realizes quantum classical correspondence. The constraints do not affect quantum fluctuating degrees of freedom if classical charges parametrize zero modes so that the localization to a quantum superposition of space-time surfaces with same classical charges is possible.
   The vanishing of conformal Noether charges for sub-algebras of various conformal algebras are also posed. They could be also realized as Lagrange multiplied terms at the ends of 3-surface.
2. By supersymmetry requirement the Kähler-Dirac action corresponding to the bosonic action is obtained by associating to the various pieces in the bosonic action canonical momentum densities and contracting them with embedding space gamma matrices to obtain K-D gamma matrices. This gives rise to Kähler-Dirac equation in the interior of space-time surface. As explained, it is assumed that localiztion to 2-D string world sheets occurs. At the light-like boundaries the limit of K-D equation gives K-D equation at the ferminonic liness expressing 8-D light-likeness or 4-D light-likeness in effective metric.

Lagrange multiplier terms in Kähler action

Weak form of E-M duality can be realized by adding to Kähler action 3-D constraint terms realized in terms of Lagrange multipliers. These contribute to the Chern-Simons Dirac action too by modifying the definition of the modified gamma matrices.

Quantum classical correspondence (QCC) is the principle motivating further additional terms in Kähler action.

1. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones assignable to Kähler-Dirac action. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.
2. The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the

boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in zero energy ontology (ZEO), the procedure looks logically sound.

3. The consistency with Kähler-Dirac equation for which Chern-Simons boundary term at parton orbits (not genuine boundaries) seems necessary suggests that also Kähler action has Chern-Simons term as a boundary term at partonic orbits. Kähler action would thus reduce to contributions from the space-like ends of the space-time surface. This however leads to an unphysical outcome.

Boundary terms for Kähler-Dirac action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naïve guess has been that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This is however a mere guess and need not be correct. The outcome is actually that the limit of K-D equation at string world sheets defines the Dirac equation at the boundaries of string world sheets.

One should try to make first clear what one really wants.

1. What one wants are generalized Feynman diagrams demanding massless Dirac propagators in 8-D sense at the light-like boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that 8-D generalization of the twistor Grassmannian approach works. The localization of spinors at string world sheets is crucial for achieving this.
   In ordinary QFT fermionic propagator results from the kinetic term in Dirac action. Could the situation be same also now at the boundary of string world sheet associated with parton orbit? One can consider the Dirac action

$$L_{ind} = \int \overline{\Psi} \Gamma^t_{ind} \partial_t \Psi \sqrt{g_1} dt$$

   defined by the induced gamma matrix Γ^t_{ind} and induced 1-metric. This action need to be associated only to the Minkowskian side of the space-surface. By supersymmetry Dirac action must be accompanied by a bosonic action $\int \sqrt{g_1} dt$. It forces the boundary line to be a geodesic line. Dirac equation gives

$$\Gamma^t_{ind} D_t \Psi = ip^k(M^8)\gamma_k \Psi = 0 \ .$$

   The square of the Dirac operator gives $(\Gamma^t_{ind})^2 = 0$ for geodesic lines (the components of the second fundamental form vanish) so that one obtains 8-D light-likeness.
   Boundary line would behave like point-like elementary particle for which conserved 8-momentum is conserved and light-like: just as twistor diagrammatics suggests. 8-momentum must be real since otherwise the particle orbit would belong to the complexification of H. These conditions can be regarded as boundary conditions on the string world sheet and spinor modes. There would be no additional contribution to the Kähler action.

2. The special points are the ends of the fermion lines at incoming and outgoing partonic 2-surfaces and at these points M^4 mass squared is assigned to the embedding space spinor harmonic associated with the incoming fermion. CP_2 mass squared corresponds to the eigenvalue of CP_2 spinor d'Alembertian for the spinor harmonic.
   At the end of the fermion line $p(M^4)^k$ corresponds to the incoming fermionic four-momentum. The direction of $p(E^4)^k$ is not fixed and one has $SO(4)$ harmonic at the mass shell $p(E^4)^2 = m^2$, m the mass of the incoming particle. At embedding space level color partial waves correspond to $SO(4)$ partial waves ($SO(4)$ could be seen as the symmetry group of low energy hadron physics giving rise to vectorial and axial isospin).

Constraint terms at space-like ends of space-time surface

There are constraint terms coming from the condition that weak form of electric-magnetic duality holds true and also from the condition that classical charges for the space-time sheets in the superposition are identical with quantal charges which are net fermionic charges assignable to the strings.

These terms give additional contribution to the algebraic equation $\Gamma^n \Psi = 0$ making in partial differential equation reducing to ordinary differential equation if induced spinor fields are localized at 2-D surfaces. These terms vanish if Ψ is covariantly constant along the boundary of the string world sheet so that fundamental fermions remain massless. By 1-dimensionality covariant constancy can be always achieved.

13.6.2 A Proposal For M-Matrix

The proposed general picture reduces the core of U-matrix to the construction of S-matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.
2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized eigenvalue $p^k\gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-light geodesics of embedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.
3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.
4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.
   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K100].

The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface the vertices would be represented by partonic 2-surfaces. In [K100] the interpretation of scattering ampiltudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are "free". At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

   The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K15] is a remnant of an "idea that came too early". The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond topartial wave in the space S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

   The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures ??, ?? (http://tgdtheory.fi/appfigures/elparticletgd.jpg http://tgdtheory.fi/appfigures/tgdgrpahs.jpg) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K100] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

Chapter 14

Particle Massivation in TGD Universe

14.1 Introduction

This chapter represents the most recent view about particle massivation in TGD framework. This topic is necessarily quite extended since many several notions and new mathematics is involved. Therefore the calculation of particle masses involves five chapters [K28, K60, K70, K64, K65] of [K68]. In the following my goal is to provide an up-to-date summary whereas the chapters are unavoidably a story about evolution of ideas.

The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics. The latter task is relatively straightforward. The thorough understanding of the massless spectrum requires however a real understanding of quantum TGD. It would be also highly desirable to understand why p-adic thermodynamics combined with p-adic length scale hypothesis works. A lot of progress has taken place in these respects during last years.

Zero energy ontology providing a detailed geometric view about bosons and fermions, the generalization of S-matrix to what I call M-matrix, the notion of finite measurement resolution characterized in terms of inclusions of von Neumann algebras, the derivation of p-adic coupling constant evolution and p-adic length scale hypothesis from the first principles, the realization that the counterpart of Higgs mechanism involves generalized eigenvalues of the Kähler-Dirac operator: these are represent important steps of progress during last years with a direct relevance for the understanding of particle spectrum and massivation although the predictions of p-adic thermodynamics are not affected.

Since 2010 a further progress took place. These steps of progress relate closely to ZEO, bosonic emergence, the discovery of the weak form of electric-magnetic duality, the realization of the importance of twistors in TGD, and the discovery that the well-definedness of em charge forces the modes of Kähler-Dirac operator to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. This allows to assign to elementary particle closed string with pieces at two parallel space-time sheets and accompanying a Kähler magnetic flux tube carrying monopole flux.

Twistor approach and the understanding of the solutions of Kähler-Dirac Dirac operator served as a midwife in the process giving rise to the birth of the idea that all fundamental fermions are massless and that both ordinary elementary particles and string like objects emerge from them. Even more, one can interpret virtual particles as being composed of these massless on mass shell particles assignable to wormhole throats. Four-momentum conservation poses extremely powerful constraints on loop integrals but does not make them manifestly finite as believed first. String picture is necessary for getting rid of logarithmic divergences.

The weak form of electric-magnetic duality led to the realization that elementary particles correspond to bound states of two wormhole throats with opposite Kähler magnetic charges with second throat carrying weak isospin compensating that of the fermion state at second wormhole throat. Both fermions and bosons correspond to wormhole contacts: in the case of fermions topological condensation generates the second wormhole throat. This means that altogether four wormhole throats are involved with both fermions, gauge bosons, and gravitons (for gravitons this is unavoidable in any case). For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N}=4$ SYM. Besides this there is weak "stringy" contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets.

One cannot avoid the question about the relation between p-adic mass calculations and Higgs mechanism. Higgs is predicted but does the analog of Higgs vacuum expectation emerge as the existence of QFT limit would suggest? Boundary conditions for Kähler-Dirac action with measurement interaction term for four-momentum lead to what looks like an as algebraic variant of masssless Dirac equation in Minkowski space coupled to the analog of Higgs vacuum expectation value restricted at fermionic strings. This equation does not however provide an analog of Higgs mechanism but a space-time correlate for the stringy mass formula coming from the vanishing of the scaling generator L_0 of superconformal algbra. It could also give a first principle explanation for the necessarily tachyonic ground state with half integer conformal weight.

For p-adic thermodynamics the mathematical counterpart of string corresponds to a wormhole contact with size of order CP_2 size with the role of its ends played by wormhole throats at which the signature of the induced 4-metric changes. The key observation is that for massless states the throats of spin 1 particle must have opposite three-momenta so that gauge bosons are necessarily massive, even photon and other particles usually regarded as massless must have small mass which in turn cancels infrared divergences and give hopes about exact Yangian symmetry generalizing that of $\mathcal{N}=4$ SYM.

Besides this there is weak "stringy" contribution to the mass assignable to the magnetic flux tubes connecting the two wormhole throats at the two space-time sheets. In fact, this contribution can be assigned to the additional conformal weight assignable to the stringy curve. The extension of this conformal algebra to Yangian brings in third integer characterizing the poly-locality of the Yangian generator (n-local generator acts on n partonic 2-surfaces simultaneously. Therefore three integers would characterize the generators of the full symmetry algebra as the very naive expectation on basis of 3-dimensionality of the fundamental objects would suggest. p-Adic mass calculations should be carried out for Yangian generalization of p-adic thermodynamics.

14.1.1 Physical States As Representations Of Super-Symplectic And Super Kac-Moody Algebras

Physical states belong to the representations of super-symplectic algebra and Super Kac-Moody algebran. The precise identification of the two algebras has been rather tedious task but the recent progress in the construction of WCW geometry and spinor structure led to a considerable progress in this respect [K85].

1. In the generic case the generators of both algebras receive information from 1-D ends of 2-D string world sheets at which the modes of induced spinor fields are localized by the condition that the modes are eigenstates of electromagnetic charge. Right-handed neutrino is an exception since it has no electroweak couplings. One must however require that right-handed neutrino does not mix with the left-handed one if the mode is de-localized at entire space-time sheet.
   Either the preferred extremal is such that Kähler-Dirac gamma matrices defined in terms of canonical momentum currents of Kähler action consist of only M^4 or CP_2 type flat space gammas so that there is no mixing with the left-handed neutrino. Or the CP_2 and M^4 parts of the Kähler Dirac operator annihilate the right-handed neutrino mode separately. One can of course have also modes which are mixtures of right- and left handed neutrinos but these are necessarily localized at string world sheets.
2. The definition of super generator involves integration of string curve at the boundary of causal diamond (CD) so that the generators are labelled by *two* conformal weights: that associated

with the radial light-like coordinate and that assignable with the string curve. This strongly suggests that the algebra extends to a 4-D Yangian involving multi-local generators (locus means partonic surface now) assignable to various partonic surfaces at the boundaries of CD - as indeed suggested [K100].

3. As before, the symplectic algebra corresponds to a super-symplectic algebra assignable to symplectic transformations of $\delta M^4_\pm \times CP_2$. One can regard this algebra as a symplectic algebra of $S^2 \times CP_2$ localized with respect to the light-like radial coordinate r_M taking the role of complex variable z in conformal field theories. Super-generators are linear in the modes of right-handed neutrino. Covariantly constant mode and modes decoupling from left-handed neutrino define the most important modes.

4. Second algebra corresponds to the Super Kac-Moody algebra. The corresponding Lie algebra generates symplectic isometries of $\delta M^4_\pm \times CP_2$. Fermionic generators are linear in the modes of induced spinor field with non-vanishing electroweak quantum numbers: that is left-hand neutrinos, charged leptons, and quarks.

5. The overall important conclusion is that overall Super Virasoro algebra has five tensor factors corresponding to one tensor factor for super-symplectic algebra, and 4 tensor factors for Super Kac-Moody algebra $SO(2) \times SU(3) \times SU(2)_{rot} \times U(2)_{ew}$ (CP_2 isometries, S^2 isometries, electroweak $SU(2)_{ew} \times U(1)$). This is essential for mass calculations.

What looks like the most plausible option relies on the generalization of a coset construction proposed already for years ago but badly mis-interpreted. The construction itself is strongly supported and perhaps even forced by the vision that WCW is union of homogenous or even symmetric spaces of form G/H [K85], where G is the isometry group of WCW and H its subgroup leaving invariant the chosen point of WCW (say the 3-surface corresponding to a maximum of Kähler function in Euclidian regions and stationary point of the Morse function defined by Kähler action for Minkowskian space-time regions). It seems clear that only the Super Virasoro associated with G can involve four-momentum so that the original idea that there are two identical four-momenta identifiable as gravitational and inertial four-momenta must be given up. This boils dow to the following picture.

1. Assume a generalization of the coset construction so that the differences of G and H super-conformal generators O_n annihilate the physical states: $(O_n(G) - O_n(H))|phys\rangle = 0$.

2. In zero energy ontology (ZEO) p-adic thermodynamics must be replaced with its square root so that one consideres genuine quantum states rather than thermodynamical states. Hence the system is quantum coherent. In the simplest situation this implies only that thermodynamical weights are replaced by their square roots possibly multiplied by square roots irrelevant for the mass squared expectation value.

3. Construct first ground states with negative conformal weight annihilated by G and H generators G_n, L_n, $n < 0$. Apply to these states generators of tensor factors of Super Viraroso algebras to obtain states with vanishing G and H conformal weights. After this construct thermal states as superpositions of states obtained by applying H generators and corresponding G generators G_n,L_n, $n > 0$. Assume that these states are annihilated by G and H generators G_n, L_n,$n > 0$ and by the differences of *all* G and H generators.

4. Super-symplectic algebra represents a completely new element and in the case of hadrons the non-perturbative contribution to the mass spectrum is easiest to understand in terms of super-symplectic thermal excitations contributing roughly 70 per cent to the p-adic thermal mass of the hadron.

Yangian algebras associated with the super-conformal algebras and motivated by twistorial approach generalize the already generalized super-conformal symmetry and make it multi-local in the sense that generators can act on several partonic 2-surfaces simultaneously. These partonic 2-surfaces generalize the vertices for the external massless particles in twistor Grassmann diagrams [K100]. The implications of this symmetry are yet to be deduced but one thing is clear: Yangians are tailor made for the description of massive bound states formed from several partons identified as partonic 2-surfaces. The preliminary discussion of what is involved can be found in [K100].

14.1.2 Particle Massivation

Particle massivation can be regarded as a generation of thermal conformal weight identified as mass squared and due to a thermal mixing of a state with vanishing conformal weight with those having higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

One can imagine several microscopic mechanisms of massivation. The following proposal is the winner in the fight for survival between several competing scenarios.

The original observation was that the pieces of CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Light-like randomness gives rise to classical Virasoro conditions. This picture generalizes since the basic dynamical objects are light-like but otherwise random 3-surfaces. The identification of elementary particles developed in three steps.

1. Originally germions were identified as light-like 3-surfaces at which the signature of induced metric of deformed CP_2 type extremals changes from Euclidian to the Minkowskian signature of the background space-time sheet. Gauge bosons and Higgs were identified as wormhole contacts with light-like throats carrying fermion and anti-fermion quantum numbers. Gravitons were identified as pairs of wormhole contacts bound to string like object by the fluxes connecting the wormhole contacts. The randomness of the light-like 3-surfaces and associated super-conformal symmetries justify the use of thermodynamics and the question remains why this thermodynamics can be taken to be p-adic. The proposed identification of bosons means enormous simplification in thermodynamical description since all calculations reduced to the calculations to fermion level. This picture generalizes to include super-symmetry. The fermionic oscillator operators associated with the partonic 2-surfaces act as generators of badly broken SUSY and right-handed neutrino gives to the not so badly broken $\mathcal{N}=1$ SUSY consistent with empirical facts.
   Of course, "badly" is relative notion. It is quite possible that the mixing of right-handed neutrino with left-handed one becomes important only in CP_2 scale and causes massivation. Hence spartners might well have mass of order CP_2 mass scale. The question about the mass scale of right-handed neutrino remains open.
2. The next step was to realize that the topological condensation of fermion generates second wormhole throat which carries momentum and symplectic quantum numbers but no fermionic quantum numbers. This is also needed to the massivation by p-adic thermodynamics applied to the analogs of string like objects defined by wormhole throats with throats taking the role of string ends. p-Adic thermodynamics did not however allow a satisfactory understanding of the gauge bosons masses and it became clear that some additional contribution - maybe Higgsy or stringy contribution - dominates for weak gauge bosons. Gauge bosons should also somehow obtain their longitudinal polarizations and here Higgs like particles indeed predicted by the basic picture suggests itself strongly.
3. A further step was the discovery of the weak form of electric-magnetic duality, which led to the realization that wormhole throats possess Kähler magnetic charge so that a wormole throat with opposite magnetic charge is needed to compensate this charge. This wormhole throat can also compensate the weak isospin of the second wormhole throat so that weak confinement and massivation results. In the case of quarks magnetic confinement might take place in hadronic rather than weak length scale. Second crucial observation was that gauge bosons are necessarily massive since the light-like momenta at two throats must correspond to opposite three-momenta so that no Higgs potential is needed. This leads to a picture in which gauge bosons eat the Higgs scalars and also photon, gluons, and gravitons develop small mass.
4. A further step was the realization that although the existence of Higgs is established, it need not contribute to neither fermion or gauge boson masses. CP_2 geometry does not even allow covariantly constant holomorphic vector field as a representation for the vacuum expectation value of Higgs. Elementary particles are string like objects and string tension can give additional contribution to the mass squared. This would explain the large masses of weak bosons as compared to the mass of photon predicted also to be non-vanishing in principle. Also a small contribution to fermion masses is expected.
   Higgs vacuum expectation would be replaced with the stringy contribution to the mass squared, which by perturbative argument should apart from normalization factor have the form $\Delta m^2 \propto$

g^2T, where g is the gauge coupling assignable to the weak boson, and T is the analog of hadronic string tension but in weak scale. This predicts correctly the ratio of W and Z boson masses in terms of Weinberg angle.

5. The conformal weight characterizing fermionic masses in p-adic thermodynamics can be assigned to the very short piece of string connecting the opposite throats of wormhole contact. The conformal weight associated with the long string connecting the throats of two wormhole contacts should give the dominant contribution to the masses of weak gauge bosons. Five tensor factors are needed in super-conformal algebra and super-symplectic and super-Kac Moody contributions assignable to symplectic isometries give five factors.

   One can assign conformal weights to both the light-like radial coordinate r_M of $\delta M^4_\pm$ and string. A third integer-valued quantum number comes from the extension of the extended super-conformal algebra to multi-local Yangian algebra. Yangian extension should take place for quark wormhole contacts inside hadrons and give non-perturbative multi-local contributions to hadron masses and might explain most of hadronic mass since quark contribution is very small. That three integers classify states conforms with the very naive first guess inspired by 3-dimensionality of the basic objects.

   The details of the picture are however still fuzzy. Are the light-like radial and stringy conformal weights really independent quantum numbers as it seems? These conformal weights however must be additive in the expression for mass squared to get five tensor factors. Could one identify stringy coordinate with the light-like radial coordinate r_M in Minkowskian space-time regions to explain the additivity? The dominating contribution to the vacuum conformal weight must be negative and half-integer valued. What is the origin of this tachyonic contribution?

The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces.

1. Dynamics is constrained by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. Chern-Simons action and its Dirac counterpart result as boundary terms of Kähler action and its Dirac counterpart for preferred extremals. This requires that $j \cdot A$ contribution to Kähler action vanishes for preferred extremals plus weak form of electric-magnetic duality.

   The addition of 3-D measurement interaction term - essentially Dirac action associated with 3-D light-like orbits of partonic 2-surfaces implies that Chern-Simons Dirac operator plus Lagrangian multiplier term realizing the weak form of electric magnetic duality acts like massless M^4 Dirac operator assignable to the four-momentum propagating along the line of generalized Feynman diagram. This simplifies enormously the definition of the Dirac propagator needed in twistor Grassmannian approach [K100].

2. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is besides Loretnz invariance suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

   Mass squared is identified as the p-adic thermal expectation value of mass squared operator m^2 appearing as M^4 contribution in the scaling generator $L_0(G)$ in the superposition of states with vanishing total conformal weight but with varying mass squared eigenvalues associated with the difference $L_0(G) - L_0(H)$ annihilating the physical state. This definition does not break Lorentz invariance in zero energy ontology. The states appearing in the superposition of different states with vanishing total conformal weight give different contribution to the p-adic thermodynamical expectation defining mass squared and the ability to physically observe this as massivation might be perhaps interpreted as breaking of conformal invariance.

3. There is also a modular contribution to the mass squared, which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. It dominates for higher genus partonic 2-surfaces. For bosons both Virasoro and modular contributions seem to be negligible and could be due to the smallness of the p-adic temperature.

4. A long standing problem has been whether coupling to Higgs boson is needed to explain gauge boson masses via a generation of Higgs vacuum expectation having possibly interpretation in terms of a coherent state. Before the detailed model for elementary particles in terms of

pairs of wormhole contacts at the ends of flux tubes the picture about the situation was as follows. From the beginning it was clear that is that ground state conformal weight must be negative. Then it became clear that the ground state conformal weight need not be a negative integer. The deviation Δh of the total ground state conformal weight from negative integer gives rise to stringy contribution to the thermal mass squared and dominates in case of gauge bosons for which p-adic temperature is small. In the case of fermions this contribution to the mass squared is small. The possible Higgs vacuum expectation makes sense only at QFT limit perhaps allowing to describe the Yangian aspects, and would be naturally proportional to Δh so that the coupling to Higgs would only apparently cause gauge boson massivation.

5. A natural identification of the non-integer contribution to the conformal weight is as stringy contribution to the vacuum conformal weight. In twistor approach the generalized eigenvalues of Chern-Simons Dirac operator for external particles indeed correspond to light-like momenta and when the three-momenta are opposite this gives rise to non-vanishing mass. Higgs is necessary to give longitudinal polarizations for weak gauge bosons.

An important question concerns the justification of p-adic thermodynamics.

1. The underlying philosophy is that real number based TGD can be algebraically continued to various p-adic number fields. This gives justification for the use of p-adic thermodynamics although the mapping of p-adic thermal expectations to real counterparts is not completely unique. The physical justification for p-adic thermodynamics is effective p-adic topology characterizing the 3-surface: this is the case if real variant of light-like 3-surface has large number of common algebraic points with its p-adic counterpart obeying same algebraic equations but in different number field. In fact, there is a theorem stating that for rational surfaces the number of rational points is finite and rational (more generally algebraic points) would naturally define the notion of number theoretic braid essential for the realization of number theoretic universality.
2. The most natural option is that the descriptions in terms of both real and p-adic thermodynamics make sense and are consistent. This option indeed makes if the number of generalized eigen modes of Kähler-Dirac operator is finite. This is indeed the case if one accepts periodic boundary conditions for the Chern-Simons Dirac operator. In fact, the solutions are localized at the strands of braids. This makes sense because the theory has hydrodynamic interpretation. This reduces $\mathcal{N} = \infty$ to finite SUSY and realizes finite measurement resolution as an inherent property of dynamics.
   The finite number of fermionic oscillator operators implies an effective cutoff in the number conformal weights so that conformal algebras reduce to finite-dimensional algebras. The first guess would be that integer label for oscillator operators becomes a number in finite field for some prime. This means that one can calculate mass squared also by using real thermodynamics but the consistency with p-adic thermodynamics gives extremely strong number theoretical constraints on mass scale. This consistency condition allows also to solve the problem how to map a negative ground state conformal weight to its p-adic counterpart. Negative conformal weight is divided into a negative half odd integer part plus positive part Δh, and negative part corresponds as such to p-adic integer whereas positive part is mapped to p-adic number by canonical identification.

p-Adic thermodynamics is what gives to this approach its predictive power.

1. p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/n$, $n > 1$, seems to be the only reasonable choice for gauge bosons.
2. p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^{3.5}\sqrt{\hbar G}$ and therefore roughly $10^{3.5}$ times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order $10^{-3.5}$ Planck mass.

14.1.3 What Next?

The successes of p-adic mass calculations are basically due to the power of super-conformal symmetries and of number theory. One cannot deny that the description of the gauge boson and hadron massivation involves phenomenological elements. There are however excellent hopes that it might be possible some day to calculate everything from first principles. The non-local Yangian symmetry generalizing the super-conformal algebras suggests itself strongly as a fundamental symmetry of quantum TGD. The generalized of the Yangian symmetry replaces points with partonic 2-surfaces being multi-local with respect to them, and leads to general formulas for multi-local operators representing four-momenta and other conserved charges of composite states.

In TGD framework even elementary particles involve two wormhole contacts having each two wormhole throats identified as the fundamental partonic entities. Therefore Yangian approach would naturally define the first principle approach to the understanding of masses of elementary particles and their bound states (say hadrons). The power of this extended symmetry might be enough to deduce universal mass formulas. One of the future challenges would therefore be the mathematical and physical understanding of Yangian symmetry. This would however require the contributions of professional mathematicians.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L22].

14.2 Identification Of Elementary Particles

14.2.1 Partons As Wormhole Throats And Particles As Bound States Of Wormhole Contacts

The assumption that partonic 2-surfaces correspond to representations of Super Virasoro algebra has been an unchallenged assumption of the p-adic mass calculations for a long time although one might argue that these objects do not possess stringy characteristics, in particular they do not possess two ends. The progress in the understanding of the Kähler-Dirac equation and the introduction of the weak form of electric magnetic duality [K113] however forces to modify the picture about the origin of the string mass spectrum.

1. The weak form of electric-magnetic duality, the basic facts about Kähler-Dirac equation and the proposed twistorialization of quantum TGD [K100] force to conclude that both strings and bosons and their super-counterparts emerge from massless fermions moving collinearly at partonic two-surfaces. Stringy mass spectrum is consistent with this only if p-adic thermodynamics describes wormhole contacts as analogs of stringy objects having quantum numbers at the throats playing the role of string ends. For instance, the three-momenta of massless wormhole throats could be in opposite direction so that wormhole contact would become massive. The fundamental string like objects would therefore correspond to the wormhole contacts with size scale of order CP_2 length. Already these objects must have a correct correlation between color and electroweak quantum numbers. The colored super-generators taking care that anomalous color is compensated can be assigned with purely bosonic quanta associated with the wormhole throats which carry no fermion number.
2. Second modification comes from the necessity to assume weak confinement in the sense that each wormhole throat carrying fermionic numbers is accompanied by a second wormhole throat carrying neutrino pair cancelling the net weak isospin so that only electromagnetic charge remains unscreened. This screening must take place in weak length scale so that ordinary elementar particles are predicted to be string like objects. This string tension has however nothing to do with the fundamental string tension responsible for the mass spectrum. This picture is forced also by the fact that fermionic wormhole throats necessarily carry Kähler magnetic charge [K113] so that in the case of leptons the second wormhole throat must carry a compensating Kähler magnetic charge. In the case of quarks one can consider the possibility that magnetic charges are not neutralized completely in weak scale and that the compensation occurs in QCD length scale so that Kähler magnetic confinement would accompany color confinement. This means color magnetic confinement since classical color gauge fields are proportional to induced Kähler field.

These modifications do not seem to appreciably affect the results of calculations, which depend only on the number of tensor factors in super Virasoro representation, they are not taken explicitly into account in the calculations. The predictions of the general theory are consistent with the earliest mass calculations, and the earlier ad hoc parameters disappear. In particular, optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. What is new is the possibility to describe the massivation of gauge bosons by including the contribution from the string tension of weak string like objects: weak boson masses have indeed been the trouble makers and have forced to conclude that Higgs expectation might be needed unless some other mechanism contributes to the conformal vacuum weight of the ground state.

14.2.2 Family Replication Phenomenon Topologically

One of the basic ideas of TGD approach has been genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies.

With the advent of ZEO this picture changed somewhat. It is the wormhole throats identified as light-like 3-surfaces at with the induced metric of the space-time surface changes its signature from Minkowskian to Euclidian, which correspond to the light-like orbits of partonic 2-surfaces. One cannot of course exclude the possibility that also boundary components could allow to satisfy boundary conditions without assuming vacuum extremal property of nearby space-time surface. The intersections of the wormhole throats with the light-like boundaries of causal diamonds (CDs) identified as intersections of future and past directed light cones ($CD \times CP_2$ is actually in question but I will speak about CDs) define special partonic 2-surfaces and it is the moduli of these partonic 2-surfaces which appear in the elementary particle vacuum functionals naturally.

The first modification of the original simple picture comes from the identification of physical particles as bound states of pairs of wormhole contacts and from the assumption that for generalized Feynman diagrams stringy trouser vertices are replaced with vertices at which the ends of light-like wormhole throats meet. In this picture the interpretation of the analog of trouser vertex is in terms of propagation of same particle along two different paths. This interpretation is mathematically natural since vertices correspond to 2-manifolds rather than singular 2-manifolds which are just splitting to two disjoint components. Second complication comes from the weak form of electric-magnetic duality forcing to identify physical particles as weak strings with magnetic monopoles at their ends and one should understand also the possible complications caused by this generalization.

These modifications force to consider several options concerning the identification of light fermions and bosons and one can end up with a unique identification only by making some assumptions. Masslessness of all wormhole throats- also those appearing in internal lines- and dynamical $SU(3)$ symmetry for particle generations are attractive general enough assumptions of this kind. This means that bosons and their super-partners correspond to wormhole contacts with fermion and anti-fermion at the throats of the contact. Free fermions and their superpartners could correspond to CP_2 type vacuum extremals with single wormhole throat. It turns however that dynamical $SU(3)$ symmetry forces to identify massive (and possibly topologically condensed) fermions as (g, g) type wormhole contacts.

Do free fermions correspond to single wormhole throat or (g, g) wormhole?

The original interpretation of genus-generation correspondence was that free fermions correspond to wormhole throats characterized by genus. The idea of $SU(3)$ as a dynamical symmetry suggested that gauge bosons correspond to octet and singlet representations of $SU(3)$. The further idea that all lines of generalized Feynman diagrams are massless poses a strong additional constraint and it is not clear whether this proposal as such survives.

1. Twistorial program assumes that fundamental objects are massless wormhole throats carrying collinearly moving many-fermion states and also bosonic excitations generated by super-symplectic algebra. In the following consideration only purely bosonic and single fermion throats are considered since they are the basic building blocks of physical particles. The reason is that propagators for high excitations behave like p^{-n}, n the number of fermions associated with the wormhole throat. Therefore single throat allows only spins 0,1/2,1 as elementary particles in the usual sense of the word.
2. The identification of massive fermions (as opposed to free massless fermions) as wormhole contacts follows if one requires that fundamental building blocks are massless since at least

two massless throats are required to have a massive state. Therefore the conformal excitations with CP_2 mass scale should be assignable to wormhole contacts also in the case of fermions. As already noticed this is not the end of the story: weak strings are required by the weak form of electric-magnetic duality.

3. If free fermions corresponding to single wormhole throat, topological condensation is an essential element of the formation of stringy states. The topological condensation of fermions by topological sum (fermionic CP_2 type vacuum extremal touches another space-time sheet) suggest $(g, 0)$ wormhole contact. Note however that the identification of wormhole throat is as 3-surface at which the signature of the induced metric changes so that this conclusion might be wrong. One can indeed consider also the possibility of (g, g) pairs as an outcome of topological conensation. This is suggested also by the idea that wormhole throats are analogous to string like objects and only this option turns out to be consistent with the BFF vertex based on the requirement of dynamical $SU(3)$ symmetry to be discussed later. The structure of reaction vertices makes it possible to interpret (g, g) pairs as $SU(3)$ triplet. If bosons are obtained as fusion of fermionic and anti-fermionic throats (touching of corresponding CP_2 type vacuum extremals) they correspond naturally to (g_1, g_2) pairs.
4. p-Adic mass calculations distinguish between fermions and bosons and the identification of fermions and bosons should be consistent with this difference. The maximal p-adic temperature $T = 1$ for fermions could relate to the weakness of the interaction of the fermionic wormhole throat with the wormhole throat resulting in topological condensation. This wormhole throat would however carry momentum and 3-momentum would in general be non-parallel to that of the fermion, most naturally in the opposite direction.
   p-Adic mass calculations suggest strongly that for bosons p-adic temperature $T = 1/n$, $n > 1$, so that thermodynamical contribution to the mass squared is negligible. The low p-adic temperature could be due to the strong interaction between fermionic and anti-fermionic wormhole throat leading to the "freezing" of the conformal degrees of freedom related to the relative motion of wormhole throats.
5. The weak form of electric-magnetic duality forces second wormhole throat with opposite magnetic charge and the light-like momenta could sum up to massive momentum. In this case string tension corresponds to electroweak length scale. Therefore p-adic thermodynamics must be assigned to wormhole contacts and these appear as basic units connected by Kähler magnetic flux tube pairs at the two space-time sheets involved. Weak stringy degrees of freedom are however expected to give additional contribution to the mass, perhaps by modifying the ground state conformal weight.

Dynamical $SU(3)$ fixes the identification of fermions and bosons and fundamental interaction vertices

For 3 light fermion families $SU(3)$ suggests itself as a dynamical symmetry with fermions in fundamental $N = 3$-dimensional representation and $N \times N = 9$ bosons in the adjoint representation and singlet representation. The known gauge bosons have same couplings to fermionic families so that they must correspond to the singlet representation. The first challenge is to understand whether it is possible to have dynamical $SU(3)$ at the level of fundamental reaction vertices.

This is a highly non-trivial constraint. For instance, the vertices in which n wormhole throats with same (g_1, g_2) glued along the ends of lines are not consistent with this symmetry. The splitting of the fermionic worm-hole contacts before the proper vertices for throats might however allow the realization of dynamical $SU(3)$. The condition of $SU(3)$ symmetry combined with the requirement that virtual lines resulting also in the splitting of wormhole contacts are always massless, leads to the conclusion that massive fermions correspond to (g, g) type wormhole contacts transforming naturally like $SU(3)$ triplet. This picture conformsl with the identification of free fermions as throats but not with the naïve expectation that their topological condensation gives rise to $(g, 0)$ wormhole contact.

The argument leading to these conclusions runs as follows.

1. The question is what basic reaction vertices are allowed by dynamical $SU(3)$ symmetry. FFB vertices are in principle all that is needed and they should obey the dynamical symmetry. The meeting of entire wormhole contacts along their ends is certainly not possible. The splitting

of fermionic wormhole contacts before the vertices might be however consistent with $SU(3)$ symmetry. This would give two a pair of 3-vertices at which three wormhole lines meet along partonic 2-surfaces (rather than along 3-D wormhole contacts).

2. Note first that crossing gives all possible reaction vertices of this kind from $F(g_1)\overline{F}(g_2) \rightarrow B(g_1, g_2)$ annihilation vertex, which is relatively easy to visualize. In this reaction $F(g_1)$ and $\overline{F}(g_2)$ wormhole contacts split first. If one requires that all wormhole throats involved are massless, the two wormhole throats resulting in splitting and carrying no fermion number must carry light-like momentum so that they cannot just disappear. The ends of the wormhole throats of the boson must glued together with the end of the fermionic wormhole throat and its companion generated in the splitting of the wormhole. This means that fermionic wormhole first splits and the resulting throats meet at the partonic 2-surface.
   his requires that topologically condensed fermions correspond to (g, g) pairs rather than $(g, 0)$ pairs. The reaction mechanism allows the interpretation of (g, g) pairs as a triplet of dynamical $SU(3)$. The fundamental vertices would be just the splitting of wormhole contact and 3-vertices for throats since $SU(3)$ symmetry would exclude more complex reaction vertices such as n-boson vertices corresponding the gluing of n wormhole contact lines along their 3-dimensional ends. The couplings of singlet representation for bosons would have same coupling to all fermion families so that the basic experimental constraint would be satisfied.
3. Both fermions and bosons cannot correspond to octet and singlet of $SU(3)$. In this case reaction vertices should correspond algebraically to the multiplication of matrix elements e_{ij}: $e_{ij}e_{kl} = \delta_{jk}e_{il}$ allowing for instance $F(g_1, g_2) + \overline{F}(g_2, g_3) \rightarrow B(g_1, g_3)$. Neither the fusion of entire wormhole contacts along their ends nor the splitting of wormhole throats before the fusion of partonic 2-surfaces allows this kind of vertices so that BFF vertex is the only possible one. Also the construction of QFT limit starting from bosonic emergence led to the formulation of perturbation theory in terms of Dirac action allowing only BFF vertex as fundamental vertex [**?**] .
4. Weak electric-magnetic duality brings in an additional complication. $SU(3)$ symmetry poses also now strong constraints and it would seem that the reactions must involve copies of basic BFF vertices for the pairs of ends of weak strings. The string ends with the same Kähler magnetic charge should meet at the vertex and give rise to BFF vertices. For instance, $F\overline{F}B$ annihilation vertex would in this manner give rise to the analog of stringy diagram in which strings join along ends since two string ends disappear in the process.

If one accepts this picture the remaining question is why the number of genera is just three. Could this relate to the fact that $g \leq 2$ Riemann surfaces are always hyper-elliptic (have global Z_2 conformal symmetry) unlike $g > 2$ surfaces? Why the complete bosonic de-localization of the light families should be restricted inside the hyper-elliptic sector? Does the Z_2 conformal symmetry make these states light and make possible de-localization and dynamical $SU(3)$ symmetry? Could it be that for $g > 2$ elementary particle vacuum functionals vanish for hyper-elliptic surfaces? If this the case and if the time evolution for partonic 2-surfaces changing g commutes with Z_2 symmetry then the vacuum functionals localized to $g \leq 2$ surfaces do not disperse to $g > 2$ sectors.

The notion of elementary particle vacuum functional

Obviously one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals, is made.

The basic assumptions underlying the construction are the following ones:

1. Elementary particle vacuum functionals depend on the geometric properties of the two-surface X^2 representing elementary particle.
2. Vacuum functionals possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting preferred extremal of the Kähler action as a generalized Bohr orbit [K52]) and determined in $Diff^3$ invariant manner.
3. ZEO allows to select uniquely the partonic two surface as the intersection of the wormhole throat at which the signature of the induced 4-metric changes with either the upper or lower

boundary of $CD \times CP_2$. This is essential since otherwise one one could not specify the vacuum functional uniquely.

4. Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.
5. Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.
6. Elementary particle vacuum functionals are stable against the decay $g \rightarrow g_1 + g_2$ and one particle decay $g \rightarrow g - 1$. This process corresponds to genuine particle decay only for stringy diagrams. For generalized Feynman diagrams the interpretation is in terms of propagation along two different paths simultaneously.

In [K28] the construction of elementary particle vacuum functionals is described in more detail. This requires some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces and the concept of hyper-ellipticity. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are needed. Also possible explanations for the experimental absence of the higher fermion families are considered.

14.2.3 Critizing the view about elementary particles

The concrete model for elementary particles has developed gradually during years and is by no means final. In the recent model elementary particle corresponds to a pair of wormhole contacts and monopole flux runs between the throats of of the two contacts at the two space-time sheets and through the contacts between space-time sheets.

The first criticism relates to twistor lift of TGD [L30]. In the case of Kähler action the wormhole contacts correspond to deformations for pieces of CP_2 type vacuum extremals for which the 1-D M^4 projection is light-like random curve. Twistor lift adds to Kähler action a volume term proportional to cosmological constant and forces the vacuum extremal to be a minimal surface carrying non-vanishing light-like momentum (this is of course very natural): one could call this surface CP_2 type extremal. This implies that M^4 projection is light-like geodesic: this is physically rather natural.

Twistor lift leads to a loss of the proposed space-time correlate of massivation used also to justify p-adic thermodynamics: the average velocity for a light-like random curve is smaller than maximal signal velocity - this would be a clear classical signal for massivation. One could however conjecture that the M^4 projection for the light-like boundaries of string world sheets becomes light-like geodesic of $M^4 \times CP_2$ instead light-like geodesic of M^4 and that this serves as the correlate for the massivation in 4-D sense.

Second criticism is that I have not considered in detail what the monopole flux hypothesis really means at the level of detail. Since the monopole flux is due to the CP_2 topology, there must be a closed 2-surface which carries this flux. This implies that the flux tube cannot have boundaries at larger space-time surface but one has just the flux tube with closed cross section obtained as a deformation of a cosmic string like object $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a complex surface of CP_2 characterized by genus. Deformation would have 4-D M^4 projection instead of 2-D string world sheet.

Note: One can also consider objects for which the flux is not monopole flux: in this case one would have deformations of surfaces of type $X^2 \times S^2$, S^2 homologically trivial geodesic sphere: these are non-vacuum extremals for the twistor lift of Kähler action (volume term). The net magnetic flux would vanish - as a matter fact, the induced Kähler form would vanish identically for the simplest situation. These objects might serve as correlates for gravitons since the induced metric is the only field degree of freedom. One could also have non-vanishing fluxes for flux tubes with disk-like cross section.

If this is the case, the elementary particles would be much simpler than I have though hitherto.

1. Elementary particles would be simply closed flux tubes which look like very long flattened squares. Short sides with length of order CP_2 radius would be identifiable as pieces of deformed

CP_2 type extremals having Euclidian signature of the induced metric. Long sides would be deformed cosmic strings with Minkowskian signature with apparent ends, which are light-like 3-surfaces at which the induced 4-metric is degenerate. Both Minkowskian and Euclidian regions of closed flux tubes would be accompanied by fermionic strings. These objects would topologically condense at larger space-time sheets with wormhole contacts that do not carry monopole flux: touching the larger space-time surface but not sticking to it.

2. One could understand why the genus for all wormhole throats must be the same for the simplest states as the TGD explanation of family replication phenomenon demands. Of course, the change of the topology along string like object cannot be excluded but very probably corresponds to an unstable higher mass excitation.
3. The basic particle reactions would include re-connections of closed string like objects and their reversals. The replication of 3-surfaces would remain a new element brought by TGD. The basic processes at fermionic level would be reconnections of closed fermionic strings. The new element would be the presence of Euclidian regions allowing to talk about effective boundaries of strings as boundaries between the Minkowskian or Euclidian regions. This would simplify enormously the description of particle reactions by bringing in description topologically highly analogous to that provided by closed strings.
4. The original picture need not of course be wrong: it is only slightly more complex than the above proposal. One would have two space-time sheets connected by a pair of wormhole contacts between, which most of the magnetic flux would flow like in flux tube. The flux from the throat could emerges more or less spherically but eventually end up to the second wormhole throat. The sheets would be connected along their boundaries so that 3-space would be connected. The absence of boundary terms in the action implies this. The monopole fluxes would sum up to a vanishing flux at the boundary, where gluing of the sheets of the covering takes place.

There is a further question to be answered. Are the fermionic strings closed or not? Fermionic strings have certainly the Minkowskian portions ending at the light-like partonic orbits at Minkowskian-Euclidian boundaries. But do the fermionic strings have also Euclidian portions so that the signature of particle would be 2+2 kinks of a closed fermionic string? If strong for of holography is true in both Euclidian and Minkowskian regions, this is highly suggestive option.

If only Minkowskian portions are present, particles could be seen as pairs of open fermionic strings and the counterparts of open string vertices would be possible besides reconnection of closed strings. For this option one can also consider single fermionic open strings connecting wormhole contacts: now possible flux tube would not carry monopole flux.

14.2.4 Basic Facts About Riemann Surfaces

In the following some basic aspects about Riemann surfaces will be summarized. The basic topological concepts, in particular the concept of the mapping class group, are introduced, and the Teichmueller parameters are defined as conformal invariants of the Riemann surface, which in fact specify the conformal equivalence class of the Riemann surface completely.

Mapping class group

The first homology group $H_1(X^2)$ of a Riemann surface of genus g contains $2g$ generators [A33, A51, A42] : this is easy to understand geometrically since each handle contributes two homology generators. The so called canonical homology basis can be identified (see **Fig. 14.1**).

One can define the so called intersection $J(a,b)$ for two elements a and b of the homology group as the number of intersection points for the curves a and b counting the orientation. Since $J(a,b)$ depends on the homology classes of a and b only, it defines an antisymmetric quadratic form in $H_1(X^2)$. In the canonical homology basis the non-vanishing elements of the intersection matrix are:

$$J(a_i,b_j) = -J(b_j,a_i) = \delta_{i,j} \quad . \tag{14.2.1}$$

J clearly defines symplectic structure in the homology group.

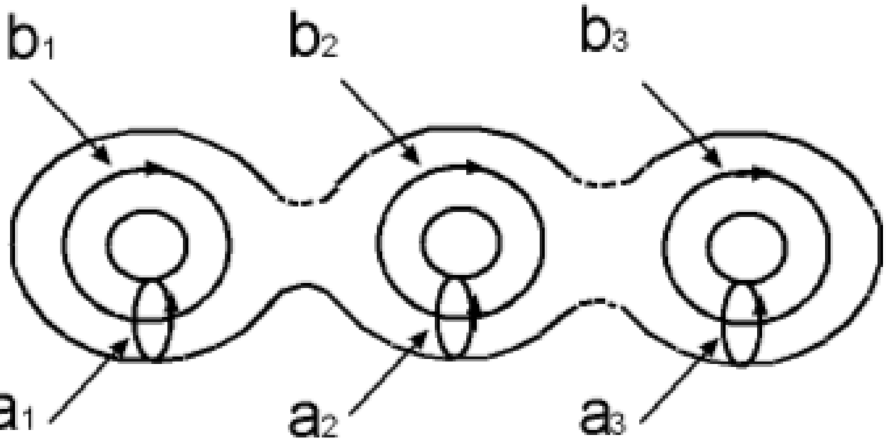

Figure 14.1: Definition of the canonical homology basis

The dual to the canonical homology basis consists of the harmonic one-forms $\alpha_i, \beta_i, i = 1, .., g$ on X^2. These 1-forms satisfy the defining conditions

$$\begin{array}{ll} \int_{a_i} \alpha_j = \delta_{i,j} & \int_{b_i} \alpha_j = 0 \ , \\ \int_{a_i} \beta_j = 0 & \int_{b_i} \beta_j = \delta_{i,j} \ . \end{array} \quad (14.2.2)$$

The following identity helps to understand the basic properties of the Teichmueller parameters

$$\int_{X^2} \theta \wedge \eta = \sum_{i=1,..,g} [\int_{a_i} \theta \int_{b_i} \eta - \int_{b_i} \theta \int_{a_i} \eta] \ . \quad (14.2.3)$$

The existence of topologically nontrivial diffeomorphisms, when X^2 has genus $g > 0$, plays an important role in the sequel. Denoting by $Diff$ the group of the diffeomorphisms of X^2 and by $Diff_0$ the normal subgroup of the diffeomorphisms homotopic to identity, one can define the mapping class group M as the coset group

$$M = Diff/Diff_0 \ . \quad (14.2.4)$$

The generators of M are so called Dehn twists along closed curves a of X^2. Dehn twist is defined by excising a small tubular neighborhood of a, twisting one boundary of the resulting tube by 2π and gluing the tube back into the surface: see **Fig. 14.2**.

It can be shown that a minimal set of generators is defined by the following curves

$$a_1, b_1, a_1^{-1}a_2^{-1}, a_2, b_2, a_2^{-1}a_3^{-11}, ..., a_g, b_g \ . \quad (14.2.5)$$

The action of these transformations in the homology group can be regarded as a symplectic linear transformation preserving the symplectic form defined by the intersection matrix. Therefore

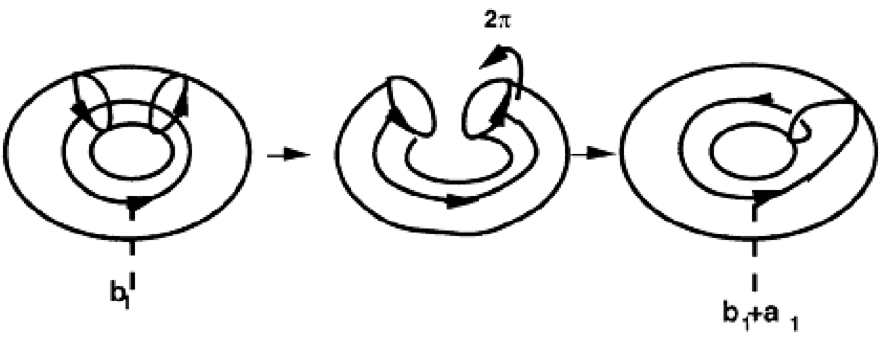

Figure 14.2: Definition of the Dehn twist

the matrix representing the action of $Diff$ on $H_1(X^2)$ is $2g\times 2g$ matrix M with integer entries leaving J invariant: $MJM^T=J$. Mapping class group is often referred also and denoted by $Sp(2g,Z)$. The matrix representing the action of M in the canonical homology basis decomposes into four $g\times g$ blocks A,B,C and D

$$M=\left(\begin{array}{cc} A & B\\ C & D\end{array}\right) \ , \tag{14.2.6}$$

where A and D operate in the subspaces spanned by the homology generators a_i and b_i respectively and C and D map these spaces to each other. The notation $D=[A,B;C,D]$ will be used in the sequel: in this notation the representation of the symplectic form J is $J=[0,1;-1,0]$.

Teichmueller parameters

The induced metric on the two-surface X^2 defines a unique complex structure. Locally the metric can always be written in the form

$$ds^2 \ = \ e^{2\phi}dzd\bar{z} \ . \tag{14.2.7}$$

where z is local complex coordinate. When one covers X^2 by coordinate patches, where the line element has the above described form, the transition functions between coordinate patches are holomorphic and therefore define a complex structure.

The conformal transformations ξ of X^2 are defined as the transformations leaving invariant the angles between the vectors of X^2 tangent space invariant: the angle between the vectors X and Y at point x is same as the angle between the images of the vectors under Jacobian map at the image point $\xi(x)$. These transformations need not be globally defined and in each coordinate patch they correspond to holomorphic (anti-holomorphic) mappings as is clear from the diagonal form of the metric in the local complex coordinates. A distinction should be made between local conformal transformations and globally defined conformal transformations, which will be referred to as conformal symmetries: for instance, for hyper-elliptic surfaces the group of the conformal symmetries contains two-element group Z_2.

Using the complex structure one can decompose one-forms to linear combinations of one-forms of type $(1,0)$ $(f(z,\bar{z})dz)$ and $(0,1)$ $(f(z,\bar{z})d\bar{z})$. $(1,0)$ form ω is holomorphic if the function f is holomorphic: $\omega=f(z)dz$ on each coordinate patch.

There are g independent holomorphic one forms ω_i known also as Abelian differentials Alvarez,Farkas,Mumford and one can fix their normalization by the condition

$$\int_{a_i}\omega_j \ = \ \delta_{ij} \ . \tag{14.2.8}$$

This condition completely specifies ω_i.

Teichmueller parameters Ω_{ij} are defined as the values of the forms ω_i for the homology generators b_j

$$\Omega_{ij} = \int_{b_j} \omega_i \ . \tag{14.2.9}$$

The basic properties of Teichmueller parameters are the following:

1. The $g \times g$ matrix Ω is symmetric: this is seen by applying the formula (14.2.3) for $\theta = \omega_i$ and $\eta = \omega_j$.
2. The imaginary part of Ω is positive: $Im(\Omega) > 0$. This is seen by the application of the same formula for $\theta = \eta$. The space of the matrices satisfying these conditions is known as Siegel upper half plane.
3. The space of Teichmueller parameters can be regarded as a coset space $Sp(2g,R)/U(g)$ [A42] : the action of $Sp(2g,R)$ is of the same form as the action of $Sp(2g,Z)$ and $U(g) \subset Sp(2g,R)$ is the isotropy group of a given point of Teichmueller space.
4. Teichmueller parameters are conformal invariants as is clear from the holomorphy of the defining one-forms.
5. Teichmueller parameters specify completely the conformal structure of Riemann surface [A51] .

Although Teichmueller parameters fix the conformal structure of the 2-surface completely, they are not in one-to-one correspondence with the conformal equivalence classes of the two-surfaces:
i) The dimension for the space of the conformal equivalence classes is $D = 3(g-1)$, when $g > 1$ and smaller than the dimension of Teichmueller space given by $d = (g \times g + g)/2$ for $g > 3$: all Teichmueller matrices do not correspond to a Riemann surface. Note that for $g = 2$ the two dimensions are same so that the 3 lowest genera are special. In TGD approach this does not produce any problems as will be found later.
ii) The action of the topologically nontrivial diffeomorphisms on Teichmueller parameters is nontrivial and can be deduced from the action of the diffeomorphisms on the homology ($Sp(2g,Z)$ transformation) and from the defining condition $\int_{a_i} \omega_j = \delta_{i,j}$: diffeomorphisms correspond to elements $[A,B;C,D]$ of $Sp(2g,Z)$ and act as generalized Möbius transformations

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \ . \tag{14.2.10}$$

All Teichmueller parameters related by $Sp(2g,Z)$ transformations correspond to the same Riemann surface.
iii) The definition of the Teichmueller parameters is not unique since the definition of the canonical homology basis involves an arbitrary numbering of the homology basis. The permutation S of the handles is represented by same $g \times g$ orthogonal matrix both in the basis $\{a_i\}$ and $\{b_i\}$ and induces a similarity transformation in the space of the Teichmueller parameters

$$\Omega \rightarrow S\Omega S^{-1} \ . \tag{14.2.11}$$

Clearly, the Teichmueller matrices related by a similarity transformations correspond to the same conformal equivalence class. It is easy to show that handle permutations in fact correspond to $Sp(2g,Z)$ transformations.

Hyper-ellipticity

The motivation for considering hyper-elliptic surfaces comes from the fact, that $g > 2$ elementary particle vacuum functionals turn out to be vanishing for hyper-elliptic surfaces and this in turn will be later used to provide a possible explanation the non-observability of $g > 2$ particles.

Hyper-elliptic surface X can be defined abstractly as two-fold branched cover of the sphere having the group Z_2 as the group of conformal symmetries (see [A74, A51, A42] . Thus there exists a map $\pi : X \rightarrow S^2$ so that the inverse image $\pi^{-1}(z)$ for a given point z of S^2 contains two points except at a finite number (say p) of points z_i (branch points) for which the inverse image contains only one point. Z_2 acts as conformal symmetries permuting the two points in $\pi^{-1}(z)$ and branch points are fixed points of the involution.

The concept can be generalized [A74] : g-hyper-elliptic surface can be defined as a 2-fold covering of genus g surface with a finite number of branch points. One can consider also p-fold coverings instead of 2-fold coverings: a common feature of these Riemann surfaces is the existence of a discrete group of conformal symmetries.

A concrete representation for the hyper-elliptic surfaces [A42] is obtained by studying the surface of C^2 determined by the algebraic equation

$$w^2 - P_n(z) \quad = \quad 0 \ , \qquad (14.2.12)$$

where w and z are complex variables and $P_n(z)$ is a complex polynomial. One can solve w from the above equation

$$w_{\pm} \quad = \quad \pm\sqrt{P_n(z)} \ , \qquad (14.2.13)$$

where the square root is determined so that it has a cut along the positive real axis. What happens that w has in general two roots (two-fold covering property), which coincide at the roots z_i of $P_n(z)$ and if n is odd, also at $z = \infty$: these points correspond to branch points of the hyper-elliptic surface and their number r is always even: $r = 2k$. w is discontinuous at the cuts associated with the square root in general joining two roots of $P_n(z)$ or if n is odd, also some root of P_n and the point $z = \infty$. The representation of the hyper-elliptic surface is obtained by identifying the two branches of w along the cuts. From the construction it is clear that the surface obtained in this manner has genus $k - 1$. Also it is clear that Z_2 permutes the different roots $w_{\pm}$ with each other and that $r = 2k$ branch points correspond to fixed points of the involution.

The following facts about the hyper-elliptic surfaces [A51, A42] turn out to be important in the sequel:

i) All $g < 3$ surfaces are hyper-elliptic.

ii) $g \geq 3$ hyper-elliptic surfaces are not in general hyper-elliptic and form a set of codimension 2 in the space of the conformal equivalence classes [A42] .

Theta functions

An extensive and detailed account of the theta functions and their applications can be found in the book of Mumford [A42] . Theta functions appear also in the loop calculations of string [J5] [A33] . In the following the so called Riemann theta function and theta functions with half integer characteristics will be defined as sections (not strictly speaking functions) of the so called Jacobian variety.

For a given Teichmueller matrix Ω, Jacobian variety is defined as the $2g$-dimensional torus obtained by identifying the points z of C^g (vectors with g complex components) under the equivalence

$$z \sim z + \Omega m + n \ , \qquad (14.2.14)$$

where m and n are points of Z^g (vectors with g integer valued components) and Ω acts in Z^g by matrix multiplication.

The definition of Riemann theta function reads as

$$\Theta(z|\Omega) \quad = \quad \sum_n exp(i\pi n \cdot \Omega \cdot n + i2\pi n \cdot z) \ . \qquad (14.2.15)$$

Here $\cdot$ denotes standard inner product in C^g. Theta functions with half integer characteristics are defined in the following manner. Let a and b denote vectors of C^g with half integer components (component either vanishes or equals to 1/2). Theta function with characteristics $[a, b]$ is defined through the following formula

$$\Theta[a,b](z|\Omega) \;=\; \sum_n exp\left[i\pi(n+a)\cdot\Omega\cdot(n+a)+i2\pi(n+a)\cdot(z+b)\right] \quad . \tag{14.2.16}$$

A brief calculation shows that the following identity is satisfied

$$\Theta[a,b](z|\Omega) \;=\; exp(i\pi a\cdot\Omega\cdot a+i2\pi a\cdot b)\times\Theta(z+\Omega a+b|\Omega) \tag{14.2.17}$$

Theta functions are not strictly speaking functions in the Jacobian variety but rather sections in an appropriate bundle as can be seen from the identities

$$\begin{aligned} \Theta[a,b](z+m|\Omega) &= exp(i2\pi a\cdot m)\Theta[a,b](z\Omega) \ , \\ \Theta[a,b](z+\Omega m|\Omega) &= exp(\alpha)\Theta[a,b](z|\Omega) \ , \\ exp(\alpha) &= exp(-i2\pi b\cdot m)exp(-i\pi m\cdot\Omega\cdot m-2\pi m\cdot z) \ . \end{aligned} \tag{14.2.18}$$

The number of theta functions is 2^{2g} and same as the number of nonequivalent spinor structures defined on two-surfaces. This is not an accident [A33] : theta functions with given characteristics turn out to be in a close relation to the functional determinants associated with the Dirac operators defined on the two-surface. It is useful to divide the theta functions to even and odd theta functions according to whether the inner product $4a\cdot b$ is even or odd integer. The numbers of even and odd theta functions are $2^{g-1}(2^g+1)$ and $2^{g-1}(2^g-1)$ respectively.

The values of the theta functions at the origin of the Jacobian variety understood as functions of Teichmueller parameters turn out to be of special interest in the following and the following notation will be used:

$$\Theta[a,b](\Omega) \;\equiv\; \Theta[a,b](0|\Omega) \ , \tag{14.2.19}$$

$\Theta[a,b](\Omega)$ will be referred to as theta functions in the sequel. From the defining properties of odd theta functions it can be found that they are odd functions of z and therefore vanish at the origin of the Jacobian variety so that only even theta functions will be of interest in the sequel.

An important result is that also some *even* theta functions vanish for $g>2$ hyper-elliptic surfaces : in fact one can characterize $g>2$ hyper-elliptic surfaces by the vanishing properties of the theta functions [A51, A42] . The vanishing property derives from conformal symmetry (Z_2 in the case of hyper-elliptic surfaces) and the vanishing phenomenon is rather general [A74] : theta functions tend to vanish for Riemann surfaces possessing discrete conformal symmetries. It is not clear (to the author) whether the presence of a conformal symmetry is in fact equivalent with the vanishing of some theta functions. As already noticed, spinor structures and the theta functions with half integer characteristics are in one-to-one correspondence and the vanishing of theta function with given half integer characteristics is equivalent with the vanishing of the Dirac determinant associated with the corresponding spinor structure or equivalently: with the existence of a zero mode for the Dirac operator Alvarez . For odd characteristics zero mode exists always: for even characteristics zero modes exist, when the surface is hyper-elliptic or possesses more general conformal symmetries.

14.2.5 Elementary Particle Vacuum Functionals

The basic assumption is that elementary particle families correspond to various elementary particle vacuum functionals associated with the 2-dimensional boundary components of the 3-surface.

These functionals need not be localized to a single boundary topology. Neither need their dependence on the boundary component be local. An important role in the following considerations is played by the fact that the minimization requirement of the Kähler action associates a unique 3-surface to each boundary component, the "Bohr orbit" of the boundary and this surface provides a considerable (and necessarily needed) flexibility in the definition of the elementary particle vacuum functionals. There are several natural constraints to be satisfied by elementary particle vacuum functionals.

Extended Diff invariance and Lorentz invariance

Extended Diff invariance is completely analogous to the extension of 3-dimensional Diff invariance to four-dimensional Diff invariance in the interior of the 3-surface. Vacuum functional must be invariant not only under diffeomorphisms of the boundary component but also under the diffeomorphisms of the 3- dimensional "orbit" Y^3 of the boundary component. In other words: the value of the vacuum functional must be same for any time slice on the orbit the boundary component. This is guaranteed if vacuum functional is functional of some two-surface Y^2 belonging to the orbit and defined in $Diff^3$ invariant manner.

An additional natural requirement is Poincare invariance. In the original formulation of the theory only Lorentz transformations of the light cone were exact symmetries of the theory. In this framework the definition of Y^2 as the intersection of the orbit with the hyperboloid $\sqrt{m_{kl}m^k m^l} = a$ is $Diff^3$ and Lorentz invariant.

1. Interaction vertices as generalization of stringy vertices

For stringy diagrams Poincare invariance of conformal equivalence class and general coordinate invariance are far from being a trivial issues. Vertices are now not completely unique since there is an infinite number of singular 3-manifolds which can be identified as vertices even if one assumes space-likeness. One should be able to select a unique singular 3-manifold to fix the conformal equivalence class.

One might hope that Lorentz invariant invariant and general coordinate invariant definition of Y^2 results by introducing light cone proper time a as a height function specifying uniquely the point at which 3-surface is singular (stringy diagrams help to visualize what is involved), and by restricting the singular 3-surface to be the intersection of $a = constant$ hyperboloid of M^4 containing the singular point with the space-time surface. There would be non-uniqueness of the conformal equivalence class due to the choice of the origin of the light cone but the decomposition of the configuration space of 3-surfaces to a union of WCWs characterized by unions of future and past light cones could resolve this difficulty.

2. Interaction vertices as generalization of ordinary ones

If the interaction vertices are identified as intersections for the ends of space-time sheets representing particles, the conformal equivalence class is naturally identified as the one associated with the intersection of the boundary component or light like causal determinant with the vertex. Poincare invariance of the conformal equivalence class and generalized general coordinate invariance follow trivially in this case.

Conformal invariance

Conformal invariance implies that vacuum functionals depend on the conformal equivalence class of the surface Y^2 only. What makes this idea so attractive is that for a given genus g WCW becomes effectively finite-dimensional. A second nice feature is that instead of trying to find coordinates for the space of the conformal equivalence classes one can construct vacuum functionals as functions of the Teichmueller parameters.

That one can construct this kind of functions as suitable functions of the Teichmueller parameters is not trivial. The essential point is that the boundary components can be regarded as sub-manifolds of $M^4_+ \times CP_2$: as a consequence vacuum functional can be regarded as a composite function:

2-surface $\rightarrow$ Teichmueller matrix Ω determined by the induced metric $\rightarrow \Omega_{vac}(\Omega)$

Therefore the fact that there are Teichmueller parameters which do not correspond to any Riemann surface, doesn't produce any trouble. It should be noticed that the situation differs from that in the Polyakov formulation of string models, where one doesn't assume that the metric of the two-surface is induced metric (although classical equations of motion imply this).

Diff invariance

Since several values of the Teichmueller parameters correspond to the same conformal equivalence class, one must pose additional conditions on the functions of the Teichmueller parameters in order to obtain single valued functions of the conformal equivalence class.

The first requirement of this kind is the invariance under topologically nontrivial Diff transformations inducing $Sp(2g, Z)$ transformation $(A, B; C, D)$ in the homology basis. The action of these transformations on Teichmueller parameters is deduced by requiring that holomorphic one-forms satisfy the defining conditions in the transformed homology basis. It turns out that the action of the topologically nontrivial diffeomorphism on Teichmueller parameters can be regarded as a generalized Möbius transformation:

$$\Omega \rightarrow (A\Omega + B)(C\Omega + D)^{-1} \ . \tag{14.2.20}$$

Vacuum functional must be invariant under these transformations. It should be noticed that the situation differs from that encountered in the string models. In TGD the integration measure over WCW is Diff invariant: in string models the integration measure is the integration measure of the Teichmueller space and this is not invariant under $Sp(2g, Z)$ but transforms like a density: as a consequence the counterpart of the vacuum functional must be also modular covariant since it is the product of vacuum functional and integration measure, which must be modular invariant.

It is possible to show that the quantities

$$(\Theta[a, b]/\Theta[c, d])^4 \ . \tag{14.2.21}$$

and their complex conjugates are $Sp(2g, Z)$ invariants [A42] and therefore can be regarded as basic building blocks of the vacuum functionals.

Teichmueller parameters are not uniquely determined since one can always perform a permutation of the g handles of the Riemann surface inducing a redefinition of the canonical homology basis (permutation of g generators). These transformations act as similarities of the Teichmueller matrix:

$$\Omega \rightarrow S\Omega S^{-1} \ , \tag{14.2.22}$$

where S is the $g \times g$ matrix representing the permutation of the homology generators understood as orthonormal vectors in the g- dimensional vector space. Therefore the Teichmueller parameters related by these similarity transformations correspond to the same conformal equivalence class of the Riemann surfaces and vacuum functionals must be invariant under these similarities.

It is easy to find out that these similarities permute the components of the theta characteristics: $[a, b] \rightarrow [S(a), S(b)]$. Therefore the invariance requirement states that the handles of the Riemann surface behave like bosons: the vacuum functional constructed from the theta functions is invariant under the permutations of the theta characteristics. In fact, this requirement brings in nothing new. Handle permutations can be regarded as $Sp(2g, Z)$ transformations so that the modular invariance alone guarantees invariance under handle permutations.

Cluster decomposition property

Consider next the behavior of the vacuum functional in the limit, when boundary component with genus g splits to two separate boundary components of genera g_1 and g_2 respectively. The splitting into two separate boundary components corresponds to the reduction of the Teichmueller matrix Ω^g to a direct sum of $g_1 \times g_1$ and $g_2 \times g_2$ matrices ($g_1 + g_2 = g$):

$$\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2} \ , \tag{14.2.23}$$

when a suitable definition of the Teichmueller parameters is adopted. The splitting can also take place without a reduction to a direct sum: the Teichmueller parameters obtained via $Sp(2g, Z)$ transformation from $\Omega^g = \Omega^{g_1} \oplus \Omega^{g_2}$ do not possess direct sum property in general.

The physical interpretation is obvious: the non-diagonal elements of the Teichmueller matrix describe the geometric interaction between handles and at this limit the interaction between the handles belonging to the separate surfaces vanishes. On the physical grounds it is natural to require that vacuum functionals satisfy cluster decomposition property at this limit: that is they reduce to the product of appropriate vacuum functionals associated with the composite surfaces.

Theta functions satisfy cluster decomposition property [A33, A42] . Theta characteristics reduce to the direct sums of the theta characteristics associated with g_1 and g_2 ($a = a_1 \oplus a_2$, $b = b_1 \oplus b_2$) and the dependence on the Teichmueller parameters is essentially exponential so that the cluster decomposition property indeed results:

$$\Theta[a,b](\Omega^g) \quad = \quad \Theta[a_1,b_1](\Omega^{g_1})\Theta[a_2,b_2](\Omega^{g_2}) \ . \tag{14.2.24}$$

Cluster decomposition property holds also true for the products of theta functions. This property is also satisfied by suitable homogenous polynomials of thetas. In particular, the following quantity playing central role in the construction of the vacuum functional obeys this property

$$Q_0 \quad = \quad \sum_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 \ , \tag{14.2.25}$$

where the summation is over all even theta characteristics (recall that odd theta functions vanish at the origin of C^g).

Together with the $Sp(2g, Z)$ invariance the requirement of cluster decomposition property implies that the vacuum functional must be representable in the form

$$\Omega_{vac} \quad = \quad P_{M,N}(\Theta^4, \bar{\Theta}^4)/Q_{MN}(\Theta^4, \bar{\Theta}^4) \tag{14.2.26}$$

where the homogenous polynomials $P_{M,N}$ and $Q_{M,N}$ have same degrees (M and N as polynomials of $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$.

Finiteness requirement

Vacuum functional should be finite. Finiteness requirement is satisfied provided the numerator $Q_{M,N}$ of the vacuum functional is real and positive definite. The simplest quantity of this type is the quantity Q_0 defined previously and its various powers. $Sp(2g, Z)$ invariance and finiteness requirement are satisfied provided vacuum functionals are of the following general form

$$\Omega_{vac} \quad = \quad \frac{P_{N,N}(\Theta^4, \bar{\Theta}^4)}{Q_0^N} \ , \tag{14.2.27}$$

where $P_{N,N}$ is homogenous polynomial of degree N with respect to $\Theta[a,b]^4$ and $\bar{\Theta}[a,b]^4$. In addition $P_{N,N}$ is invariant under the permutations of the theta characteristics and satisfies cluster decomposition property.

Stability against the decay $g \rightarrow g_1 + g_2$

Elementary particle vacuum functionals must be stable against the genus conserving decays $g \rightarrow g_1 + g_2$. This decay corresponds to the limit at which Teichmueller matrix reduces to a direct sum of the matrices associated with g_1 and g_2 (note however the presence of $Sp(2g, Z)$ degeneracy). In accordance with the topological description of the particle reactions one expects that this decay doesn't occur if the vacuum functional in question vanishes at this limit.

In general the theta functions are non-vanishing at this limit and vanish provided the theta characteristics reduce to a direct sum of the odd theta characteristics. For $g < 2$ surfaces this condition is trivial and gives no constraints on the form of the vacuum functional. For $g = 2$ surfaces the theta function $\Theta(a, b)$, with $a = b = (1/2, 1/2)$ satisfies the stability criterion identically (odd theta functions vanish identically), when Teichmueller parameters separate into a direct sum. One can however perform $Sp(2g, Z)$ transformations giving new points of Teichmueller space describing the decay. Since these transformations transform theta characteristics in a nontrivial manner to each other and since all even theta characteristics belong to same $Sp(2g, Z)$ orbit [A33, A42] , the conclusion is that stability condition is satisfied provided $g = 2$ vacuum functional is proportional to the product of fourth powers of all even theta functions multiplied by its complex conjugate.

If $g > 2$ there always exists some theta functions, which vanish at this limit and the minimal vacuum functional satisfying this stability condition is of the same form as in $g = 2$ case, that is proportional to the product of the fourth powers of all even Theta functions multiplied by its complex conjugate:

$$\Omega_{vac} = \prod_{[a,b]} \Theta[a,b]^4 \bar{\Theta}[a,b]^4 / Q_0^N \ , \tag{14.2.28}$$

where N is the number of even theta functions. The results obtained imply that genus-generation correspondence is one to one for $g > 1$ for the minimal vacuum functionals. Of course, the multiplication of the minimal vacuum functionals with functionals satisfying all criteria except stability criterion gives new elementary particle vacuum functionals: a possible physical identification of these vacuum functionals is most naturally as some kind of excited states.

One of the questions posed in the beginning was related to the experimental absence of $g > 0$, possibly massless, elementary bosons. The proposed stability criterion suggests a nice explanation. The point is that elementary particles are stable against decays $g \rightarrow g_1 + g_2$ but not with respect to the decay $g \rightarrow g +$ *sphere*. As a consequence the direct emission of $g > 0$ gauge bosons is impossible unlike the emission of $g = 0$ bosons: for instance the decay muon $\rightarrow$ electron $+(g = 1)$ photon is forbidden.

Stability against the decay $g \rightarrow g - 1$

This stability criterion states that the vacuum functional is stable against single particle decay $g \rightarrow g - 1$ and, if satisfied, implies that vacuum functional vanishes, when the genus of the surface is smaller than g. In stringy framework this criterion is equivalent to a separate conservation of various lepton numbers: for instance, the spontaneous transformation of muon to electron is forbidden. Notice that this condition doesn't imply that that the vacuum functional is localized to a single genus: rather the vacuum functional of genus g vanishes for all surfaces with genus smaller than g. This hierarchical structure should have a close relationship to Cabibbo-Kobayashi-Maskawa mixing of the quarks.

The stability criterion implies that the vacuum functional must vanish at the limit, when one of the handles of the Riemann surface suffers a pinch. To deduce the behavior of the theta functions at this limit, one must find the behavior of Teichmueller parameters, when i:th handle suffers a pinch. Pinch implies that a suitable representative of the homology generator a_i or b_i contracts to a point.

Consider first the case, when a_i contracts to a point. The normalization of the holomorphic one-form ω_i must be preserved so that that ω_i must behaves as $1/z$, where z is the complex coordinate vanishing at pinch. Since the homology generator b_i goes through the pinch it seems obvious that the imaginary part of the Teichmueller parameter $\Omega_{ii} = \int_{b_i} \omega_i$ diverges at this limit (this conclusion is made also in [A42]): $Im(\Omega_{ii}) \rightarrow \infty$.

Of course, this criterion doesn't cover all possible ways the pinch can occur: pinch might take place also, when the components of the Teichmueller matrix remain finite. In the case of torus topology one finds that $Sp(2g,Z)$ element $(A,B;C,D)$ takes $Im(\Omega)=\infty$ to the point C/D of real axis. This suggests that pinch occurs always at the boundary of the Teichmueller space: the imaginary part of Ω_{ij} either vanishes or some matrix element of $Im(\Omega)$ diverges.

Consider next the situation, when b_i contracts to a point. From the definition of the Teichmueller parameters it is clear that the matrix elements Ω_{kl}, with $k,l\neq i$ suffer no change. The matrix element Ω_{ki} obviously vanishes at this limit. The conclusion is that i:th row of Teichmueller matrix vanishes at this limit. This result is obtained also by deriving the $Sp(2g,Z)$ transformation permuting a_i and b_i with each other: in case of torus this transformation reads $\Omega\rightarrow -1/\Omega$.

Consider now the behavior of the theta functions, when pinch occurs. Consider first the limit, when $Im(\Omega_{ii})$ diverges. Using the general definition of $\Theta[a,b]$ it is easy to find out that all theta functions for which the i:th component a_i of the theta characteristic is non-vanishing (that is $a_i=1/2$) are proportional to the exponent $exp(-\pi\Omega_{ii}/4)$ and therefore vanish at the limit. The theta functions with $a_i=0$ reduce to $g-1$ dimensional theta functions with theta characteristic obtained by dropping i:th components of a_i and b_i and replacing Teichmueller matrix with Teichmueller matrix obtained by dropping i:th row and column. The conclusion is that all theta functions of type $\Theta(a,b)$ with $a=(1/2,1/2,....,1/2)$ satisfy the stability criterion in this case.

What happens for the $Sp(2g,Z)$ transformed points on the real axis? The transformation formula for theta function is given by [A33, A42]

$$\Theta[a,b]((A\Omega+B)(C\Omega+D)^{-1}) \;=\; exp(i\phi)det(C\Omega+D)^{1/2}\Theta[c,d](\Omega) \;, \tag{14.2.29}$$

where

$$\left(\begin{array}{c} c\\ d\end{array}\right)=\left(\begin{array}{cc} A & B\\ C & D\end{array}\right)\left(\left(\begin{array}{c} a\\ b\end{array}\right)-\left(\begin{array}{c} (CD^T)_d/2\\ (AB^T)_d/2\end{array}\right)\right) \;. \tag{14.2.30}$$

Here ϕ is a phase factor irrelevant for the recent purposes and the index d refers to the diagonal part of the matrix in question.

The first thing to notice is the appearance of the diverging square root factor, which however disappears from the vacuum functionals (P and Q have same degree with respect to thetas). The essential point is that theta characteristics transform to each other: as already noticed all even theta characteristics belong to the same $Sp(2g,Z)$ orbit. Therefore the theta functions vanishing at $Im(\Omega_{ii})=\infty$ do not vanish at the transformed points. It is however clear that for a given Teichmueller parameterization of pinch some theta functions vanish always.

Similar considerations in the case $\Omega_{ik}=0$, i fixed, show that all theta functions with $b=(1/2,....,1/2)$ vanish identically at the pinch. Also it is clear that for $Sp(2g,Z)$ transformed points one can always find some vanishing theta functions. The overall conclusion is that the elementary particle vacuum functionals obtained by using $g\rightarrow g_1+g_2$ stability criterion satisfy also $g\rightarrow g-1$ stability criterion since they are proportional to the product of all even theta functions. Therefore the only nontrivial consequence of $g\rightarrow g-1$ criterion is that also $g=1$ vacuum functionals are of the same general form as $g>1$ vacuum functionals.

A second manner to deduce the same result is by restricting the consideration to the hyper-elliptic surfaces and using the representation of the theta functions in terms of the roots of the polynomial appearing in the definition of the hyper-elliptic surface [A42] . When the genus of the surface is smaller than three (the interesting case), this representation is all what is needed since all surfaces of genus $g<3$ are hyper-elliptic.

Since hyper-elliptic surfaces can be regarded as surfaces obtained by gluing two compactified complex planes along the cuts connecting various roots of the defining polynomial it is obvious that the process $g\rightarrow g-1$ corresponds to the limit, when two roots of the defining polynomial coincide. This limit corresponds either to disappearance of a cut or the fusion of two cuts to a single cut. Theta functions are expressible as the products of differences of various roots (Thomae's formula [A42])

$$\Theta[a,b]^4 \propto \prod_{i<j\in T}(z_i - z_j)\prod_{k<l\in CT}(z_k - z_l) \ , \tag{14.2.31}$$

where T denotes some subset of $\{1, 2, ..., 2g\}$ containing $g + 1$ elements and CT its complement. Hence the product of all even theta functions vanishes, when two roots coincide. Furthermore, stability criterion is satisfied only by the product of the theta functions.

Lowest dimensional vacuum functionals are worth of more detailed consideration.

i) $g = 0$ particle family corresponds to a constant vacuum functional: by continuity this vacuum functional is constant for all topologies.

ii) For $g = 1$ the degree of P and Q as polynomials of the theta functions is 24: the critical number of transversal degrees of freedom in bosonic string model! Probably this result is not an accident.

ii) For $g = 2$ the corresponding degree is 80 since there are 10 even genus 2 theta functions.

There are large numbers of vacuum functionals satisfying the relevant criteria, which do not satisfy the proposed stability criteria. These vacuum functionals correspond either to many particle states or to unstable single particle states.

Continuation of the vacuum functionals to higher genus topologies

From continuity it follows that vacuum functionals cannot be localized to single boundary topology. Besides continuity and the requirements listed above, a natural requirement is that the continuation of the vacuum functional from the sector g to the sector $g+k$ reduces to the product of the original vacuum functional associated with genus g and $g = 0$ vacuum functional at the limit when the surface with genus $g + k$ decays to surfaces with genus g and k: this requirement should guarantee the conservation of separate lepton numbers although different boundary topologies suffer mixing in the vacuum functional. These requirements are satisfied provided the continuation is constructed using the following rule:

Perform the replacement

$$\Theta[a,b]^4 \quad \rightarrow \quad \sum_{c,d}\Theta[a\oplus c, b\oplus d]^4 \tag{14.2.32}$$

for each fourth power of the theta function. Here c and d are Theta characteristics associated with a surface with genus k. The same replacement is performed for the complex conjugates of the theta function. It is straightforward to check that the continuations of elementary particle vacuum functionals indeed satisfy the cluster decomposition property and are continuous.

To summarize, the construction has provided hoped for answers to some questions stated in the beginning: stability requirements explain the separate conservation of lepton numbers and the experimental absence of $g > 0$ elementary bosons. What has not not been explained is the experimental absence of $g > 2$ fermion families. The vanishing of the $g > 2$ elementary particle vacuum functionals for the hyper-elliptic surfaces however suggest a possible explanation: under some conditions on the surface X^2 the surfaces Y^2 are hyper-elliptic or possess some conformal symmetry so that elementary particle vacuum functionals vanish for them. This conjecture indeed might make sense since the surfaces Y^2 are determined by the asymptotic dynamics and one might hope that the surfaces Y^2 are analogous to the final states of a dissipative system.

14.2.6 Explanations For The Absence Of The $g > 2$ ElementaryParticles From Spectrum

The decay properties of the intermediate gauge bosons [C37] are consistent with the assumption that the number of the light neutrinos is $N = 3$. Also cosmological considerations pose upper bounds on the number of the light neutrino families and $N = 3$ seems to be favored [C37]. It must be however emphasized that p-adic considerations [K64] encourage the consideration the existence of higher genera with neutrino masses such that they are not produced in the laboratory at present energies. In any case, for TGD approach the finite number of light fermion families is a potential difficulty since genus-generation correspondence suggests that the number of the fermion (and possibly also boson) families is infinite. Therefore one had better to find a good argument showing that the number of the observed neutrino families, or more generally, of the observed elementary particle families, is small also in the world described by TGD.

It will be later found that also TGD inspired cosmology requires that the number of the effectively massless fermion families must be small after Planck time. This suggests that boundary topologies with handle number $g > 2$ are unstable and/or very massive so that they, if present in the spectrum, disappear from it after Planck time, which correspond to the value of the light cone proper time $a \simeq 10^{-11}$ seconds.

In accordance with the spirit of TGD approach it is natural to wonder whether some geometric property differentiating between $g > 2$ and $g < 3$ boundary topologies might explain why only $g < 3$ boundary components are observable. One can indeed find a good candidate for this kind of property: namely hyper-ellipticity, which states that Riemann surface is a two-fold branched covering of sphere possessing two-element group Z_2 as conformal automorphisms. All $g < 3$ Riemann surfaces are hyper-elliptic unlike $g > 2$ Riemann surfaces, which in general do not posses this property. Thus it is natural to consider the possibility that hyper-ellipticity or more general conformal symmetries might explain why only $g < 2$ topologies correspond to the observed elementary particles.

As regards to the present problem the crucial observation is that some even theta functions vanish for the hyper-elliptic surfaces with genus $g > 2$ [A42] . What is essential is that these surfaces have the group Z_2 as conformal symmetries. Indeed, the vanishing phenomenon is more general. Theta functions tend to vanish for $g > 2$ two-surfaces possessing discrete group of conformal symmetries [A74] : for instance, instead of sphere one can consider branched coverings of higher genus surfaces.

From the general expression of the elementary particle vacuum functional it is clear that elementary particle vacuum functionals vanish, when Y^2 is hyper-elliptic surface with genus $g > 2$ and one might hope that this is enough to explain why the number of elementary particle families is three.

Hyper-ellipticity implies the separation of $g \leq 2$ and $g > 2$ sectors to separate worlds

If the vertices are defined as intersections of space-time sheets of elementary particles and if elementary particle vacuum functionals are required to have Z_2 symmetry, the localization of elementary particle vacuum functionals to $g \leq 2$ topologies occurs automatically. Even if one allows as limiting case vertices for which 2-manifolds are pinched to topologies intermediate between $g > 2$ and $g \leq 2$ topologies, Z_2 symmetry present for both topological interpretations implies the vanishing of this kind of vertices. This applies also in the case of stringy vertices so that also particle propagation would respect the effective number of particle families. $g > 2$ and $g \leq 2$ topologies would behave much like their own worlds in this approach. This is enough to explain the experimental findings if one can understand why the $g > 2$ particle families are absent as incoming and outgoing states or are very heavy.

What about $g > 2$ vacuum functionals which do not vanish for hyper-elliptic surfaces?

The vanishing of all $g \geq 2$ vacuum functionals for hyper-elliptic surfaces cannot hold true generally. There must exist vacuum functionals which do satisfy this condition. This suggest that elementary particle vacuum functionals for $g > 2$ states have interpretation as bound states of g handles and that the more general states which do not vanish for hyper-elliptic surfaces correspond to many-particle states composed of bound states $g \leq 2$ handles and cannot thus appear as incoming and outgoing states. Thus $g > 2$ elementary particles would decouple from $g \leq 2$ states.

Should higher elementary particle families be heavy?

TGD predicts an entire hierarchy of scaled up variants of standard model physics for which particles do not appear in the vertices containing the known elementary particles and thus behave like dark matter [K112] . Also $g > 2$ elementary particles would behave like dark matter and in principle there is no absolute need for them to be heavy.

The safest option would be that $g > 2$ elementary particles are heavy and the breaking of Z_2 symmetry for $g \geq 2$ states could guarantee this. p-Adic considerations lead to a general mass formula for elementary particles such that the mass of the particle is proportional to $\frac{1}{\sqrt{p}}$ [K68] . Also the dependence of the mass on particle genus is completely fixed by this formula. What remains however open is what determines the p-adic prime associated with a particle with given quantum numbers. Of course, it could quite well occur that p is much smaller for $g > 2$ genera than for $g \leq 2$ genera.

14.3 Non-Topological Contributions To Particle masses From P-Adic Thermodynamics

In TGD framework p-adic thermodynamics provides a microscopic theory of particle massivation in the case of fermions. The idea is very simple. The mass of the particle results from a thermal mixing of the massless states with CP_2 mass excitations of super-conformal algebra. In p-adic thermodynamics the Boltzmann weight $exp(-E/T)$ does not exist in general and must be replaced with p^{L_0/T_p} which exists for Virasoro generator L_0 if the inverse of the p-adic temperature is integer valued $T_p = 1/n$. The expansion in powers of p converges extremely rapidly for physical values of p, which are rather large. Therefore the three lowest terms in expansion give practically exact results. Thermal massivation does not not necessarily lead to light states and this drops a large number of exotic states from the spectrum of light particles. The partition functions of N-S and Ramond type representations are not changed in TGD framework despite the fact that fermionic super generators carry fermion numbers and are not Hermitian. Thus the practical calculations are relatively straightforward albeit tedious.

In free fermion picture the p-adic thermodynamics in the boson sector is for fermion-anti-fermion states associated with the two throats of the bosonic wormhole. The question is whether the thermodynamical mass squared is just the sum of the two independent fermionic contributions for Ramond representations or should one use N-S type representation resulting as a tensor product of Ramond representations.

The overall conclusion about p-adic mass calculations is that fermionic mass spectrum is predicted in an excellent accuracy but that the thermal masses of the intermediate gauge bosons come 20-30 per cent to large for $T_p = 1$ and are completely negligible for $T_p = 1/2$. The bound state character of the boson states could be responsible for $T_p < 1$ and for extremely small thermodynamical contribution to the masses (present also for photon).

This forces to consider seriously the possibility that thermal contribution to the bosonic mass is negligible and that TGD can, contrary to the original expectations, provide dynamical Higgs field as a fundamental field and that even Higgs mechanism could contribute to the particle masses.

Higgs mechanism is probably the only viable description of Higgs mechanism in QFT approach, where particles are point-like but not in TGD, where particles are replaced by string like objects consisting of two wormhole contacts with monopole Kähler magnetic flux flowing between "upper" throats and returning back along "lower" space-time sheets. In this framework the assumption that fermion masses would result from p-adic thermodynamics but boson masses from Higgs couplings looks like an ugly idea. A more plausible vision is that the dominating contribution to gauge boson masses comes from the two flux tubes connecting the two wormhole contacts defining boson. This contribution would be present also for fermions but would be small. The correct W/Z mass ratio is obtained if the string tension is proportional to weak gauge coupling squared. The nice feature of this scenario is that naturalness is not lost: the dimensional gradient coupling of fermion to Higgs is same for all fermions.

The stringy contribution to mass squared could be expressed in terms of the deviation of the ground state conformal weight from negative half integer.

The problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro generators is necessary for the success of p-adic mass calculations. The intuitive expectation is that the solution of this problem must relate to the Euclidian signature of the regions representing lines of generalized Feynman diagrams.

14.3.1 Partition Functions Are Not Changed

One must write Super Virasoro conditions for L_n and *both* G_n and $G_n^\dagger$ rather than for L_n and G_n as in the case of the ordinary Super Virasoro algebra, and it is a priori not at all clear whether the partition functions for the Super Virasoro representations remain unchanged. This requirement is however crucial for the construction to work at all in the fermionic sector, since even the slightest changes for the degeneracies of the excited states can change light state to a state with mass of order m_0 in the p-adic thermodynamics.

Super conformal algebra

Super Virasoro algebra is generated by the bosonic the generators L_n (n is an integer valued index) and by the fermionic generators G_r, where r can be either integer (Ramond) or half odd integer (NS). G_r creates quark/lepton for $r > 0$ and antiquark/antilepton for $r < 0$. For $r = 0$, G_0 creates lepton and its Hermitian conjugate anti-lepton. The defining commutation and anti-commutation relations are the following:

$$
\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{2}m(m^2-1)\delta_{m,-n} \ , \\
[L_m, G_r] &= (\frac{m}{2}-r)G_{m+r} \ , \\
\left[L_m, G_r^\dagger\right] &= (\frac{m}{2}-r)G_{m+r}^\dagger \ , \\
\{G_r, G_s^\dagger\} &= 2L_{r+s} + \frac{c}{3}(r^2-\frac{1}{4})\delta_{m,-n} \ , \\
\{G_r, G_s\} &= 0 \ , \\
\{G_r^\dagger, G_s^\dagger\} &= 0 \ .
\end{aligned} \tag{14.3.1}
$$

By the inspection of these relations one finds some results of a great practical importance.

1. For the Ramond algebra G_0, G_1 and their Hermitian conjugates generate the $r \geq 0, n \geq 0$ part of the algebra via anti-commutations and commutations. Therefore all what is needed is to assume that Super Virasoro conditions are satisfied for these generators in case that G_0 and $G_0^\dagger$ annihilate the ground state. Situation changes if the states are *not* annihilated by G_0 and $G_0^\dagger$ since then one must assume the gauge conditions for both L_1, G_1 and $G_1^\dagger$ besides the mass shell conditions associated with G_0 and $G_0^\dagger$, which however do not affect the number of the Super Virasoro excitations but give mass shell condition and constraints on the state in the cm spin degrees of freedom. This will be assumed in the following. Note that for the ordinary Super Virasoro only the gauge conditions for L_1 and G_1 are needed.
2. NS algebra is generated by $G_{1/2}$ and $G_{3/2}$ and their Hermitian conjugates (note that $G_{3/2}$ cannot be expressed as the commutator of L_1 and $G_{1/2}$) so that only the gauge conditions associated with these generators are needed. For the ordinary Super Virasoro only the conditions for $G_{1/2}$ and $G_{3/2}$ are needed.

Conditions guaranteeing that partition functions are not changed

The conditions guaranteeing the invariance of the partition functions in the transition to the modified algebra must be such that they reduce the number of the excitations and gauge conditions for a given conformal weight to the same number as in the case of the ordinary Super Virasoro.

1. The requirement that physical states are invariant under $G \leftrightarrow G^\dagger$ corresponds to the charge conjugation symmetry and is very natural. As a consequence, the gauge conditions for G and $G^\dagger$ are not independent and their number reduces by a factor of one half and is the same as in the case of the ordinary Super Virasoro.
2. As far as the number of the thermal excitations for a given conformal weight is considered, the only remaining problem are the operators $G_n G_n^\dagger$, which for the ordinary Super Virasoro reduce to $G_n G_n = L_{2n}$ and do not therefore correspond to independent degrees of freedom. In present case this situation is achieved only if one requires

$$(G_nG_n^\dagger - G_n^\dagger G_n)|phys\rangle = 0 \ . \quad (14.3.2)$$

It is not clear whether this condition must be posed separately or whether it actually follows from the representation of the Super Virasoro algebra automatically.

Partition function for Ramond algebra

Under the assumptions just stated, the partition function for the Ramond states not satisfying any gauge conditions

$$Z(t) = 1 + 2t + 4t^2 + 8t^3 + 14t^4 + ... \ , \quad (14.3.3)$$

which is identical to that associated with the ordinary Ramond type Super Virasoro.

For a Super Virasoro representation with $N = 5$ sectors, of main interest in TGD, one has

$$\begin{aligned} Z_N(t) &= Z^{N=5}(t) = \sum D(n)t^n \\ &= 1 + 10t + 60t^2 + 280t^3 + ... \ . \end{aligned} \quad (14.3.4)$$

The degeneracies for the states satisfying gauge conditions are given by

$$d(n) = D(n) - 2D(n-1) \ . \quad (14.3.5)$$

corresponding to the gauge conditions for L_1 and G_1. Applying this formula one obtains for $N = 5$ sectors

$$d(0) = 1 \ , \quad d(1) = 8 \ , \quad d(2) = 40 \ , \quad d(3) = 160 \ . \quad (14.3.6)$$

The lowest order contribution to the p-adic mass squared is determined by the ratio

$$r(n) = \frac{D(n+1)}{D(n)} \ ,$$

where the value of n depends on the effective vacuum weight of the ground state fermion. Light state is obtained only provided the ratio is integer. The remarkable result is that for lowest lying states the ratio is integer and given by

$$r(1) = 8 \ , \quad r(2) = 5 \ , \quad r(3) = 4 \ . \quad (14.3.7)$$

It turns out that $r(2) = 5$ gives the best possible lowest order prediction for the charged lepton masses and in this manner one ends up with the condition $h_{vac} = -3$ for the tachyonic vacuum weight of Super Virasoro.

Partition function for NS algebra

For NS representations the calculation of the degeneracies of the physical states reduces to the calculation of the partition function for a single particle Super Virasoro

$$Z_{NS}(t) = \sum_n z(n/2)t^{n/2} \ . \quad (14.3.8)$$

Here $z(n/2)$ gives the number of Super Virasoro generators having conformal weight $n/2$. For a state with N active sectors (the sectors with a non-vanishing weight for a given ground state) the degeneracies can be read from the N-particle partition function expressible as

$$Z_N(t) = Z^N(t) . \tag{14.3.9}$$

Single particle partition function is given by the expression

$$Z(t) = 1 + t^{1/2} + t + 2t^{3/2} + 3t^2 + 4t^{5/2} + 5t^3 + ... \tag{14.3.10}$$

Using this representation it is an easy task to calculate the degeneracies for the operators of conformal weight Δ acting on a state having N active sectors.

One can also derive explicit formulas for the degeneracies and calculation gives

$$\begin{array}{ll} D(0,N) = 1 \ , & D(1/2,N) = N \ , \\ D(1,N) = \frac{N(N+1)}{2} \ , & D(3/2,N) = \frac{N}{6}(N^2+3N+8) \ , \\ D(2,N) = \frac{N}{2}(N^2+2N+3) \ , & D(5/2,N) = 9N(N-1) \ , \\ D(3,N) = 12N(N-1)+2N(N-1) & . \end{array} \tag{14.3.11}$$

as a function of the conformal weight $\Delta = 0, 1/2, ..., 3$.

The number of states satisfying Super Virasoro gauge conditions created by the operators of a conformal weight Δ, when the number of the active sectors is N, is given by

$$d(\Delta, N) = D(\Delta, N) - D(\Delta - 1/2, N) - D(\Delta - 3/2, N) . \tag{14.3.12}$$

The expression derives from the observation that the physical states satisfying gauge conditions for $G^{1/2}$, $G^{3/2}$ satisfy the conditions for all Super Virasoro generators. For $T_p = 1$ light bosons correspond to the integer values of $d(\Delta + 1, N)/d(\Delta, N)$ in case that massless states correspond to thermal excitations of conformal weight Δ: they are obtained for $\Delta = 0$ only (massless ground state). This is what is required since the thermal degeneracy of the light boson ground state would imply a corresponding factor in the energy density of the black body radiation at very high temperatures. For the physically most interesting nontrivial case with $N = 2$ two active sectors the degeneracies are

$$d(0,2) = 1 \ , \quad d(1,2) = 1 \ , \quad d(2,2) = 3 \ , \quad d(3,2) = 4 \ . \tag{14.3.13}$$

N, Δ	0	1/2	1	3/2	2	5/2	3
2	1	1	1	3	3	4	4
3	1	2	3	9	11		
4	1	3	5	19	26		
5	1	4	10	24	150		

Table 14.1: Degeneracies $d(\Delta, N)$ of the operators satisfying NS type gauge conditions as a function of the number N of the active sectors and of the conformal weight Δ of the operator. Only those degeneracies, which are needed in the mass calculation for bosons assuming that they correspond to N-S representations are listed.

14.3.2 Fundamental Length And Mass Scales

The basic difference between quantum TGD and super-string models is that the size of CP_2 is not of order Planck length but much larger: of order $10^{3.5}$ Planck lengths. This conclusion is forced by several consistency arguments, the mass scale of electron, and by the cosmological data allowing to fix the string tension of the cosmic strings which are basic structures in TGD inspired cosmology.

The relationship between CP_2 radius and fundamental p-adic length scale

One can relate CP_2 "cosmological constant" to the p-adic mass scale: for $k_L = 1$ one has

$$m_0^2 = \frac{m_1^2}{k_L} = m_1^2 = 2\Lambda \ . \quad (14.3.14)$$

$k_L = 1$ results also by requiring that p-adic thermodynamics leaves charged leptons light and leads to optimal lowest order prediction for the charged lepton masses. Λ denotes the "cosmological constant" of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$ with cosmological term).

The real counterpart of the p-adic thermal expectation for the mass squared is sensitive to the choice of the unit of p-adic mass squared which is by definition mapped as such to the real unit in canonical identification. Thus an important factor in the p-adic mass calculations is the correct identification of the p-adic mass squared scale, which corresponds to the mass squared unit and hence to the unit of the p-adic numbers. This choice does not affect the spectrum of massless states but can affect the spectrum of light states in case of intermediate gauge bosons.

1. For the choice

$$M^2 = m_0^2 \leftrightarrow 1 \quad (14.3.15)$$

the spectrum of L_0 is integer valued.

2. The requirement that all sufficiently small mass squared values for the color partial waves are mapped to real integers, would fix the value of p-adic mass squared unit to

$$M^2 = \frac{m_0^2}{3} \leftrightarrow 1 \ . \quad (14.3.16)$$

For this choice the spectrum of L_0 comes in multiples of 3 and it is possible to have a first order contribution to the mass which cannot be of thermal origin (say $m^2 = p$). This indeed seems to happen for electro-weak gauge bosons.

p-Adic mass calculations allow to relate m_0 to electron mass and to Planck mass by the formula

$$\begin{aligned} \frac{m_0}{m_{Pl}} &= \frac{1}{\sqrt{5+Y_e}} \times 2^{127/2} \times \frac{m_e}{m_{Pl}} \ , \\ m_{Pl} &= \frac{1}{\sqrt{\hbar G}} \ . \end{aligned} \quad (14.3.17)$$

For $Y_e = 0$ this gives $m_0 = .2437 \times 10^{-3} m_{Pl}$.

This means that CP_2 radius R defined by the length $L = 2\pi R$ of CP_2 geodesic is roughly $10^{3.5}$ times the Planck length. More precisely, using the relationship

$$\Lambda = \frac{3}{2R^2} = M^2 = m_0^2 \ ,$$

one obtains for

$$L = 2\pi R = 2\pi\sqrt{\frac{3}{2}}\frac{1}{m_0} \simeq 3.1167 \times 10^4 \sqrt{\hbar G} \ \text{for} \ Y_e = 0 \ . \quad (14.3.18)$$

The result came as a surprise: the first belief was that CP_2 radius is of order Planck length. It has however turned out that the new identification solved elegantly some long standing problems of TGD. **Table 14.2** gives the value of the scale parameter K_R.

The value of top quark mass favors $Y_e = 0$ and $Y_e = .5$ is largest value of Y_e marginally consistent with the limits on the value of top quark mass.

Y_e	0	.5	.7798
$(m_0/m_{Pl})10^3$	.2437	.2323	.2266
$K_R \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{\hbar G}) \times 10^{-4}$	3.1580	3.3122	3.3954
$K \times 10^{-7}$	2.4606	2.4606	2.4606
$(L/\sqrt{\hbar G}) \times 10^{-4}$	3.1167	3.1167	3.1167
K_R/K	1.0267	1.1293	1.1868

Table 14.2: Table gives the values of the ratio $K_R = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$. Also the ratio of K_R/K, where $K = 2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \times 2^{-3} * (15/17)$ is rational number producing R^2/G approximately is given.

CP_2 radius as the fundamental p-adic length scale

The identification of CP_2 radius as the fundamental p-adic length scale is forced by the Super Virasoro invariance. The pleasant surprise was that the identification of the CP_2 size as the fundamental p-adic length scale rather than Planck length solved many long standing problems of older TGD.

1. The earliest formulation predicted cosmic strings with a string tension larger than the critical value giving the angle deficit 2π in Einstein's equations and thus excluded by General Relativity. The corrected value of CP_2 radius predicts the value k/G for the cosmic string tension with k in the range $10^{-7} - 10^{-6}$ as required by the TGD inspired model for the galaxy formation solving the galactic dark matter problem.
2. In the earlier formulation there was no idea as how to derive the p-adic length scale $L \sim 10^{3.5}\sqrt{\hbar G}$ from the basic theory. Now this problem becomes trivial and one has to predict gravitational constant in terms of the p-adic length scale. This follows in principle as a prediction of quantum TGD. In fact, one can deduce G in terms of the p-adic length scale and the action exponential associated with the CP_2 type extremal and gets a correct value if α_K approaches fine structure constant at electron length scale (due to the fact that electromagnetic field equals to the Kähler field if Z^0 field vanishes).
   Besides this, one obtains a precise prediction for the dependence of the Kähler coupling strength on the p-adic length scale by requiring that the gravitational coupling does not depend on the p-adic length scale. p-Adic prime p in turn has a nice physical interpretation: the critical value of α_K is same for the zero modes with given p. As already found, the construction of graviton state allows to understand the small value of the gravitational constant in terms of a de-coherence caused by multi-p fractality reducing the value of the gravitational constant from L_p^2 to G.
3. p-Adic length scale is also the length scale at which super-symmetry should be restored in standard super-symmetric theories. In TGD this scale corresponds to the transition to Euclidian field theory for CP_2 type extremals. There are strong reasons to believe that sparticles are however absent and that super-symmetry is present only in the sense that super-generators have complex conformal weights with $Re(h) = \pm 1/2$ rather than $h = 0$. The action of this super-symmetry changes the mass of the state by an amount of order CP_2 mass.

14.4 Color Degrees Of Freedom

The ground states for the Super Virasoro representations correspond to spinor harmonics in $M^4 \times CP_2$ characterized by momentum and color quantum numbers. The correlation between color and electro-weak quantum numbers is wrong for the spinor harmonics and these states would be also hyper-massive. The super-symplectic generators allow to build color triplet states having negative vacuum conformal weights, and their values are such that p-adic massivation is consistent with the predictions of the earlier model differing from the recent one in the quark sector. In the following the construction and the properties of the color partial waves for fermions and bosons are considered. The discussion follows closely to the discussion of [A39] .

14.4.1 SKM Algebra And Counterpart Of Super Virasoro Conditions

There have been a considerable progress also in the understanding of super-conformal symmetries [K113, K30].

1. Super-symplectic algebra corresponds to the isometries of WCW constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW.
2. One expects also gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one. Note that the geometric part of SKM algebra must respect the light-likeness of the partonic 3-surface.
3. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

The requirement that symplectic generators have well defined radial conformal weight with respect to the light-like coordinate r of X^3 restricts M^4 conformal transformations to the group $SO(3) \times E^3$. This involves choice of preferred time coordinate. If the preferred M^4 coordinate is chosen to correspond to a preferred light-like direction in $\delta M^4_\pm$ characterizing the theory, a reduction to $SO(2) \times E^2$ more familiar from string models occurs. SKM algebra contains also $U(2)_{ew}$ Kac-Moody algebra acting as holonomies of CP_2 and having no bosonic counterpart.

p-Adic mass calculations require $N = 5$ sectors of super-conformal algebra. These sectors correspond to the 5 tensor factors for the $SO(3) \times E^3 \times SU(3) \times U(2)_{ew}$ (or $SO(2) \times E^2 \times SU(3) \times U(2)_{ew}$) decomposition of the SKM algebra to gauge symmetries of gravitation, color and electroweak interactions.

For symplectic isometries (Super-Kac-Moody algebra) fermionic algebra is realized in terms second quantized induced spinor field Ψ and spinor modes with well-defined em charge restricted to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. The full symplectic algebra is realized in terms of Ψ and covariantly constant right handed neutrino mode. One can consider also the possiblity of extended the symplectic isometries of $\delta M^4_\pm = S^2 \times R_+$ to include all isometries which act as conformal transformations of S^2 and for which conformal scaling of the metric is compensated by S^2 local scaling of the light-like radial coordinate r_M of R_+.

The algebra differs from the standard one in that super generators $G(z)$ carry lepton and quark numbers are not Hermitian as in super-string models (Majorana conditions are not satisfied). The counterparts of Ramond representations correspond to zero modes of a second quantized spinor field with vanishing radial conformal weight.

The Ramond or N-S type Virasoro conditions satisfied by the physical states in string model approach are replaced by the formulas expressing mass squared as a conformal weight. The condition is not equivalent with super Virasoro conditions since four-momentum does not appear in super Virasoro generators. It seems possible to assume that the commutator algebra $[SKM, SC]$ and the commutator of $[SKMV, SSV]$ of corresponding Super Virasoro algebras annihilate physical states. This would give rise to the analog of Super Virasoro conditions which could be seen as a Dirac equation in the world of classical worlds.

CP_2 CM degrees of freedom

Important element in the discussion are center of mass degrees of freedom parameterized by embedding space coordinates. By the effective 2-dimensionality it is indeed possible to assign to partons momenta and color partial waves and they behave effectively as free particles. In fact, the technical problem of the earlier scenario was that it was not possible to assign symmetry transformations acting only on the light-like 3-surfaces at which the signature of the induced metric transforms from Minkowskian to Euclidian.

The original assumption was that 3-surface has boundary components to which elementary particle quantum numbers were assigned. It however became clear that boundary conditions at boundaries probably fail to be satisfied. Hence the above described light-like 3-surfaces took the role the boundary components. Space-time sheets were replaced with surfaces looking like double-sheeted (at least) structures from M^4 perspective with sheets meeting along 3-D surfaces. Sphere in Euclidian 3-space is the simplest analog for this kind of structure.

One can assign to each eigen state of color quantum numbers a color partial wave in CP_2 degrees of freedom. Thus color quantum numbers are not spin like quantum numbers in TGD framework except effectively in the length scales much longer than CP_2 length scale. The correlation between color partial waves and electro-weak quantum numbers is not physical in general: only the covariantly constant right handed neutrino has vanishing color.

Mass formula, and condition determining the effective string tension

Mass squared eigenvalues are given by

$$M^2 = m^2_{CP_2} + kL_0 \ . \tag{14.4.1}$$

The contribution of CP_2 spinor Laplacian to the mass squared operator is in general not integer valued.

The requirement that mass squared spectrum is integer valued for color partial waves possibly representing light states fixes the possible values of k determining the effective string tension modulo integer. The value $k = 1$ is the only possible choice. The earlier choice $k_L = 1$ and $k_q = 2/3$, $k_B = 1$ gave integer conformal weights for the lowest possible color partial waves. The assumption that the total vacuum weight h_{vac} is conserved in particle vertices implied $k_B = 1$.

14.4.2 General Construction Of Solutions Of Dirac Operator Of H

The construction of the solutions of massless spinor and other d'Alembertians in $M^4_+ \times CP_2$ is based on the following observations.

1. d'Alembertian corresponds to a massless wave equation $M^4 \times CP_2$ and thus Kaluza-Klein picture applies, that is M^4_+ mass is generated from the momentum in CP_2 degrees of freedom. This implies mass quantization:

$$M^2 = M_n^2 \ ,$$

   where M_n^2 are eigenvalues of CP_2 Laplacian. Here of course, ordinary field theory is considered. In TGD the vacuum weight changes mass squared spectrum.
2. In order to get a respectable spinor structure in CP_2 one must couple CP_2 spinors to an odd integer multiple of the Kähler potential. Leptons and quarks correspond to $n = 3$ and $n = 1$ couplings respectively. The spectrum of the electromagnetic charge comes out correctly for leptons and quarks.
3. Right handed neutrino is covariantly constant solution of CP_2 Laplacian for $n = 3$ coupling to Kähler potential whereas right handed "electron" corresponds to the covariantly constant solution for $n = -3$. From the covariant constancy it follows that all solutions of the spinor Laplacian are obtained from these two basic solutions by multiplying with an appropriate solution of the scalar Laplacian coupled to Kähler potential with such a coupling that a correct total Kähler charge results. Left handed solutions of spinor Laplacian are obtained simply by multiplying right handed solutions with CP_2 Dirac operator: in this operation the eigenvalues of the mass squared operator are obviously preserved.
4. The remaining task is to solve scalar Laplacian coupled to an arbitrary integer multiple of Kähler potential. This can be achieved by noticing that the solutions of the massive CP_2 Laplacian can be regarded as solutions of S^5 scalar Laplacian. S^5 can indeed be regarded as a circle bundle over CP_2 and massive solutions of CP_2 Laplacian correspond to the solutions

of S^5 Laplacian with $exp(is\tau)$ dependence on S^1 coordinate such that s corresponds to the coupling to the Kähler potential:

$$s = n/2 \ .$$

Thus one obtains

$$D_5^2 = (D_\mu - iA_\mu\partial_\tau)(D^\mu - iA^\mu\partial_\tau) + \partial_\tau^2 \quad (14.4.2)$$

so that the eigen values of CP_2 scalar Laplacian are

$$m^2(s) = m_5^2 + s^2 \quad (14.4.3)$$

for the assumed dependence on τ.

5. What remains to do, is to find the spectrum of S^5 Laplacian and this is an easy task. All solutions of S^5 Laplacian can be written as homogenous polynomial functions of C^3 complex coordinates Z^k and their complex conjugates and have a decomposition into the representations of $SU(3)$ acting in natural manner in C^3.
6. The solutions of the scalar Laplacian belong to the representations $(p, p+s)$ for $s \geq 0$ and to the representations $(p+|s|, p)$ of $SU(3)$ for $s \leq 0$. The eigenvalues $m^2(s)$ and degeneracies d are

$$\begin{aligned} m^2(s) &= \frac{2\Lambda}{3}[p^2 + (|s|+2)p + |s|] \ , \ p > 0 \ , \\ d &= \frac{1}{2}(p+1)(p+|s|+1)(2p+|s|+2) \ . \end{aligned} \quad (14.4.4)$$

Λ denotes the "cosmological constant" of CP_2 ($R_{ij} = \Lambda s_{ij}$).

14.4.3 Solutions Of The Leptonic Spinor Laplacian

Right handed solutions of the leptonic spinor Laplacian are obtained from the asatz of form

$$\nu_R = \Phi_{s=0}\nu_R^0 \ ,$$

where u_R is covariantly constant right handed neutrino and Φ scalar with vanishing Kähler charge. Right handed "electron" is obtained from the ansats

$$e_R = \Phi_{s=3}e_R^0 \ ,$$

where e_R^0 is covariantly constant for $n = -3$ coupling to Kähler potential so that scalar function must have Kähler coupling $s = n/2 = 3$ a in order to get a correct Kähler charge. The d'Alembert equation reduces to

$$\begin{aligned} &(D_\mu D^\mu - (1-\epsilon)\Lambda)\Phi = -m^2\Phi \ , \\ &\quad \epsilon(\nu) = 1 \ , \quad \epsilon(e) = -1 \ . \end{aligned} \quad (14.4.5)$$

The two additional terms correspond to the curvature scalar term and $J_{kl}\Sigma^{kl}$ terms in spinor Laplacian. The latter term is proportional to Kähler coupling and of different sign for ν and e, which explains the presence of the sign factor ϵ in the formula.

Right handed neutrinos correspond to (p, p) states with $p \geq 0$ with mass spectrum

$$\begin{aligned} m^2(\nu) &= \frac{m_1^2}{3}\left[p^2 + 2p\right] \ , \ p \geq 0 \ , \\ m_1^2 &\equiv 2\Lambda \ . \end{aligned} \quad (14.4.6)$$

Right handed "electrons" correspond to $(p, p+3)$ states with mass spectrum

$$m^2(e) = \frac{m_1^2}{3}\left[p^2 + 5p + 6\right] \ , \ p \geq 0 \ . \quad (14.4.7)$$

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p = 0, p = 0)$ state) annihilates it.

14.4.4 Quark Spectrum

Quarks correspond to the second conserved H-chirality of H-spinors. The construction of the color partial waves for quarks proceeds along similar lines as for leptons. The Kähler coupling corresponds to $n = 1$ (and $s = 1/2$) and right handed U type quark corresponds to a right handed neutrino. U quark type solutions are constructed as solutions of form

$$U_R = u_R \Phi_{s==1} \ ,$$

where u_R possesses the quantum numbers of covariantly constant right handed neutrino with Kähler charge $n = 3$ ($s = 3/2$). Hence Φ_s has $s = -1$. For D_R one has

$$D_R = d_r \Phi_{s=2} \ .$$

d_R has $s = -3/2$ so that one must have $s = 2$. For U_R the representations $(p + 1, p)$ with triality one are obtained and $p = 0$ corresponds to color triplet. For D_R the representations $(p, p + 2)$ are obtained and color triplet is missing from the spectrum ($p = 0$ corresponds to $\bar{6}$).

The CP_2 contributions to masses are given by the formula

$$\begin{aligned} m^2(U,p) &= \frac{m_1^2}{3}\left[p^2 + 3p + 2\right] \ , \ p \geq 0 \ , \\ m^2(D,p) &= \frac{m_1^2}{3}\left[p^2 + 4p + 4\right] \ , \ p \geq 0 \ . \end{aligned} \tag{14.4.8}$$

Left handed quarks are obtained by applying Dirac operator to right handed quark states and mass formulas and color partial wave spectrum are the same as for right handed quarks.

The color contributions to p-adic mass squared are integer valued if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since canonical identification does not commute with a division by integer. More precisely, the images of number xp in canonical identification has a value of order 1 when x is a non-trivial rational whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes. This choice does not however affect the spectrum of massless states but can affect the spectrum of light states in case of electro-weak gauge bosons.

14.4.5 Spectrum Of Elementary Particles

The assumption that $k = 1$ holds true for all particles forces to modify the earlier construction of quark states. This turns out to be possible without affecting the p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (which can be negative).

Leptonic spectrum

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \ mod \ 3 \neq 2 \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p, p) representations with $p \geq 1$ whereas charged leptons correspond to $(p, p + 3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2 + 2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2 + 5p + 6)/3$ gives $h_c = 2$ for $p = 0$ (decouplet). In both cases super-symplectic operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-symplectic conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-symplectic conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-symplectic conformal weights contains both octet and decouplet so that singlets are obtained. What strengthens the hopes that the construction is not ad hoc is that the same operator appears in the construction of quark states too.

Right handed neutrino remains essentially massless. $p = 0$ right handed neutrino does not however generate $N = 1$ space-time (or rather, embedding space) super symmetry so that no sparticles are predicted. The breaking of the electro-weak symmetry at the level of the masses comes out basically from the anomalous color electro-weak correlation for the Kaluza-Klein partial waves implying that the weights for the ground states of the fermions depend on the electromagnetic charge of the fermion. Interestingly, TGD predicts lepto-hadron physics based on color excitations of leptons and color bound states of these excitations could correspond topologically condensed on string like objects but not fundamental string like objects.

Spectrum of quarks

Earlier arguments [K70] related to a model of CKM matrix as a rational unitary matrix suggested that the string tension parameter k is different for quarks, leptons, and bosons. The basic mass formula read as

$$M^2 = m^2_{CP_2} + kL_0 \quad .$$

The values of k were $k_q = 2/3$ and $k_L = k_B = 1$. The general theory however predicts that $k = 1$ for all particles.

1. By earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as unit, color conformal weight $h_c \equiv m^2_{CP_2}$ is integer for $p \ mod \ = \pm 1$ for U type quark belonging to $(p+1, p)$ type representation and obeying $h_c(U) = (p^2 + 3p + 2)/3$ and for $p \ mod \ 3 = 1$ for D type quark belonging $(p, p+2)$ type representation and obeying $h_c(D) = (p^2 + 4p + 4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.
2. In the recent case $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This forces the super-symplectic operator O to compensate the anomalous color to have a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SSV: this would also explain why h_{sc} would be same for all fermions.
3. It would seem that the tensor product of the spinor harmonic of quarks (as also leptons) with Hamiltonians gives rise to a large number of exotic colored states which have same thermodynamical mass as ordinary quarks (and leptons). Why these states have smaller values of p-adic prime that ordinary quarks and leptons, remains a challenge for the theory. Note that the decay widths of intermediate gauge bosons pose strong restrictions on the possible color excitations of quarks. On the other hand, the large number of fermionic color exotics can spoil the asymptotic freedom, and it is possible to have and entire p-adic length scale hierarchy of QCDs existing only in a finite length scale range without affecting the decay widths of gauge bosons.

Table 14.3 summarizes the color conformal weights and super-symplectic vacuum conformal weights for the elementary particles.

Photon, graviton and gluon

For photon, gluon and graviton the conformal weight of the $p = 0$ ground state is $h_{gr} = h_{vac} = 0$. The crucial condition is that $h = 0$ ground state is non-degenerate: otherwise one would obtain

	L	ν_L	U	D	W	γ, G, g
h_{vac}	-3	-3	-3	-3	-2	0
h_c	2	1	2	3	2	0

Table 14.3: The values of the parameters h_{vac} and h_c assuming that $k = 1$. The value of $h_{vac} \leq -h_c$ is determined from the requirement that p-adic mass calculations give best possible fit to the mass spectrum.

several physically more or less identical photons and this would be seen in the spectrum of black-body radiation. This occurs if one can construct several ground states not expressible in terms of the action of the Super Virasoro generators.

Masslessness or approximate masslessness requires low enough temperature $T_p = 1/n$, $n > 1$ at least and small enough value of the possible contribution coming from the ground state conformal weight.

In NS thermodynamics the only possibility to get exactly massless states in thermal sense is to have $\Delta = 0$ state with one active sector so that NS thermodynamics becomes trivial due to the absence of the thermodynamical excitations satisfying the gauge conditions. For neutral gauge bosons this is indeed achieved. For $T_p = 1/2$, which is required by the mass spectrum of intermediate gauge bosons, the thermal contribution to the mass squared is however extremely small even for W boson.

14.5 Modular Contribution To The Mass Squared

The success of the p-adic mass calculations gives convincing support for the generation-genus correspondence. The basic physical picture is following.

1. Fermionic mass squared is dominated by partonic contribution, which is sum of cm and modular contributions: $M^2 = M^2(cm) + M^2(mod)$. Here "cm" refers to the thermal contribution. Modular contribution can be assumed to depend on the genus of the boundary component only.
2. If Higgs contribution for diagonal (g, g) bosons (singlets with respect to "topological" $SU(3)$) dominates, the genus dependent contribution can be assumed to be negligible. This should be due to the bound state character of the wormhole contacts reducing thermal motion and thus the p-adic temperature.
3. Modular contribution to the mass squared can be estimated apart from an overall proportionality constant. The mass scale of the contribution is fixed by the p-adic length scale hypothesis. Elementary particle vacuum functionals are proportional to a product of all even theta functions and their conjugates, the number of even theta functions and their conjugates being $2N(g) = 2^g(2^g + 1)$. Also the thermal partition function must also be proportional to $2N(g)$:th power of some elementary partition function. This implies that thermal/ quantum expectation $M^2(mod)$ must be proportional to $2N(g)$. Since single handle behaves effectively as particle, the contribution must be proportional to genus g also. The success of the resulting mass formula encourages the belief that the argument is essentially correct.

The challenge is to construct theoretical framework reproducing the modular contribution to mass squared. There are two alternative ways to understand the origin modular contribution.

1. The realization that super-symplectic algebra is relevant for elementary particle physics leads to the idea that two thermodynamics are involved with the calculation of the vacuum conformal weight as a thermal expectation. The first thermodynamics corresponds to Super Kac-Moody algebra and second thermodynamics to super-symplectic algebra. This approach allows a first principle understanding of the origin and general form of the modular contribution without any need to introduce additional structures in modular degrees of freedom. The very fact that super-symplectic algebra does not commute with the modular degrees of freedom explains the dependence of the super-symplectic contribution on moduli.

2. The earlier approach was based on the idea that he modular contribution could be regarded as a quantum mechanical expectation value of the Virasoro generator L_0 for the elementary particle vacuum functional. Quantum treatment would require generalization the concepts of the moduli space and theta function to the p-adic context and finding an acceptable definition of the Virasoro generator L_0 in modular degrees of freedom. The problem with this interpretation is that it forces to introduce, not only Virasoro generator L_0, but the entire super Virasoro algebra in modular degrees of freedom. One could also consider of interpreting the contribution of modular degrees of freedom to vacuum conformal weight as being analogous to that of CP_2 Laplacian but also this would raise the challenge of constructing corresponding Dirac operator. Obviously this approach has become obsolete.

The thermodynamical treatment taking into account the constraints from that p-adicization is possible might go along following lines.

1. In the real case the basic quantity is the thermal expectation value $h(M)$ of the conformal weight as a function of moduli. The average value of the deviation $\Delta h(M) = h(M) - h(M_0)$ over moduli space $\mathcal{M}$ must be calculated using elementary particle vacuum functional as a modular invariant partition function. Modular invariance is achieved if this function is proportional to the logarithm of elementary particle vacuum functional: this reproduces the qualitative features basic formula for the modular contribution to the conformal weight. p-Adicization leads to a slight modification of this formula.
2. The challenge of algebraically continuing this calculation to the p-adic context involves several sub-tasks. The notions of moduli space $\mathcal{M}_p$ and theta function must be defined in the p-adic context. An appropriately defined logarithm of the p-adic elementary particle vacuum functional should determine $\Delta h(M)$. The average of $\Delta h(M)$ requires an integration over $\mathcal{M}_p$. The problems related to the definition of this integral could be circumvented if the integral in the real case could be reduced to an algebraic expression, or if the moduli space is discrete in which case integral could be replaced by a sum.
3. The number theoretic existence of the p-adic Θ function leads to the quantization of the moduli so that the p-adic moduli space is discretized. Accepting the sharpened form of Riemann hypothesis [K86] , the quantization means that the imaginary *resp.* real parts of the moduli are proportional to integers *resp.* combinations of imaginary parts of zeros of Riemann Zeta. This quantization could occur also for the real moduli for the maxima of Kähler function. This reduces the problematic p-adic integration to a sum and the resulting sum defining $\langle \Delta h \rangle$ converges extremely rapidly for physically interesting primes so that only the few lowest terms are needed.

14.5.1 Conformal Symmetries And Modular Invariance

The full SKM invariance means that the super-conformal fields depend only on the conformal moduli of 2-surface characterizing the conformal equivalence class of the 2-surface. This means that all induced metrics differing by a mere Weyl scaling have same moduli. This symmetry is extremely powerful since the space of moduli is finite-dimensional and means that the entire infinite-dimensional space of deformations of parton 2-surface X^2 degenerates to a finite-dimensional moduli spaces under conformal equivalence. Obviously, the configurations of given parton correspond to a fiber space having moduli space as a base space. Super-symplectic degrees of freedom could break conformal invariance in some appropriate sense.

Conformal and SKM symmetries leave moduli invariant

Conformal transformations and super Kac Moody symmetries must leave the moduli invariant. This means that they induce a mere Weyl scaling of the induced metric of X^2 and thus preserve its non-diagonal character $ds^2 = g_{z\bar{z}}dzd\bar{z}$. This is indeed true if

1. the Super Kac Moody symmetries are holomorphic isometries of $X^7 = \delta M^4_{\pm} \times CP_2$ made local with respect to the complex coordinate z of X^2, and
2. the complex coordinates of X^7 are holomorphic functions of z.

Using complex coordinates for X^7 the infinitesimal generators can be written in the form

$$J^{An} = z^n j^{Ak} D_k + \overline{z}^n j^{A\overline{k}} D_{\overline{k}} \ . \quad (14.5.1)$$

The intuitive picture is that it should be possible to choose X^2 freely. It is however not always possible to choose the coordinate z of X^2 in such a way that X^7 coordinates are holomorphic functions of z since a consistency of inherent complex structure of X^2 with that induced from X^7 is required. Geometrically this is like meeting of two points in the space of moduli.

Lorentz boosts produce new inequivalent choices of S^2 with their own complex coordinate: this set of complex structures is parameterized by the hyperboloid of future light cone (Lobatchevski space or mass shell), but even this is not enough. The most plausible manner to circumvent the problem is that only the maxima of Kähler function correspond to the holomorphic situation so that super-symplectic algebra representing quantum fluctuations would induce conformal anomaly.

The isometries of δM^4_+ are in one-one correspondence with conformal transformations

For CP_2 factor the isometries reduce to $SU(3)$ group acting also as symplectic transformations. For $\delta M^4_+ = S^2 \times R_+$ one might expect that isometries reduce to Lorentz group containing rotation group of $SO(3)$ as conformal isometries. If r_M corresponds to a macroscopic length scale, then X^2 has a finite sized S^2 projection which spans a rather small solid angle so that group $SO(3)$ reduces in a good approximation to the group $E^2 \times SO(2)$ of translations and rotations of plane.

This expectation is however wrong! The light-likeness of δM^4_+ allows a dramatic generalization of the notion of isometry. The point is that the conformal transformations of S^2 induce a conformal factor $|df/dw|^2$ to the metric of δM^4_+ and the local radial scaling $r_M \rightarrow r_M/|df/dw|$ compensates it. Hence the group of conformal isometries consists of conformal transformations of S^2 with compensating radial scalings. This compensation of two kinds of conformal transformations is the deep geometric phenomenon which translates to the condition $L_{SC} - L_{SKM} = 0$ in the sub-space of physical states. Note that an analogous phenomenon occurs also for the light-like CDs X^3_l with respect to the metrically 2-dimensional induced metric.

The X^2-local radial scalings $r_M \rightarrow r_M(z, \overline{z})$ respect the conditions $g_{zz} = g_{\overline{zz}} = 0$ so that a mere Weyl scaling leaving moduli invariant results. By multiplying the conformal isometries of δM^4_+ by z^n (z is used as a complex coordinate for X^2 and w as a complex coordinate for S^2) a conformal localization of conformal isometries would result. Kind of double conformal transformations would be in question. Note however that this requires that X^7 coordinates are holomorphic functions of X^2 coordinate. These transformations deform X^2 unlike the conformal transformations of X^2. For X^3_l similar local scalings of the light like coordinate leave the moduli invariant but lead out of X^7.

Symplectic transformations break the conformal invariance

In general, infinitesimal symplectic transformations induce non-vanishing components $g_{zz}, g_{\overline{zz}}$ of the induced metric and can thus change the moduli of X^2. Thus the quantum fluctuations represented by super-symplectic algebra and contributing to the WCW metric are in general moduli changing. It would be interesting to know explicitly the conditions (the number of which is the dimension of moduli space for a given genus), which guarantee that the infinitesimal symplectic transformation is moduli preserving.

14.5.2 The Physical Origin Of The Genus Dependent Contribution To The Mass Squared

Different p-adic length scales are not enough to explain the charged lepton mass ratios and an additional genus dependent contribution in the fermionic mass formula is required. The general form of this contribution can be guessed by regarding elementary particle vacuum functionals in the modular degrees of freedom as an analog of partition function and the modular contribution to the conformal weight as an analog of thermal energy obtained by averaging over moduli. p-Adic length scale hypothesis determines the overall scale of the contribution.

The exact physical origin of this contribution has remained mysterious but super-symplectic degrees of freedom represent a good candidate for the physical origin of this contribution. This would mean a sigh of relief since there would be no need to assign conformal weights, super-algebra, Dirac operators, Laplacians, etc.. with these degrees of freedom.

Thermodynamics in super-symplectic degrees of freedom as the origin of the modular contribution to the mass squared

The following general picture is the simplest found hitherto.

1. Elementary particle vacuum functionals are defined in the space of moduli of surfaces X^2 corresponding to the maxima of Kähler function. There some restrictions on X^2. In particular, p-adic length scale poses restrictions on the size of X^2. There is an infinite hierarchy of elementary particle vacuum functionals satisfying the general constraints but only the lowest elementary particle vacuum functionals are assumed to contribute significantly to the vacuum expectation value of conformal weight determining the mass squared value.
2. The contribution of Super-Kac Moody thermodynamics to the vacuum conformal weight h coming from Virasoro excitations of the $h=0$ massless state is estimated in the previous calculations and does not depend on moduli. The new element is that for a partonic 2-surface X^2 with given moduli, Virasoro thermodynamics is present also in super-symplectic degrees of freedom.
   Super-symplectic thermodynamics means that, besides the ground state with $h_{gr}=-h_{SC}$ with minimal value of super-symplectic conformal weight h_{SC}, also thermal excitations of this state by super-symplectic Virasoro algebra having $h_{gr}=-h_{SC}-n$ are possible. For these ground states the SKM Virasoro generators creating states with net conformal weight $h=h_{SKM}-h_{SC}-n\geq 0$ have larger conformal weight so that the SKM thermal average h depends on n. It depends also on the moduli M of X^2 since the Beltrami differentials representing a tangent space basis for the moduli space $\mathcal{M}$ do not commute with the super-symplectic algebra. Hence the thermally averaged SKM conformal weight h_{SKM} for given values of moduli satisfies

$$h_{SKM} = h(n,M) \ . \tag{14.5.2}$$

3. The average conformal weight induced by this double thermodynamics can be expressed as a super-symplectic thermal average $\langle\cdot\rangle_{SC}$ of the SKM thermal average $h(n,M)$:

$$h(M) = \langle h(n,M)\rangle_{SC} = \sum p_n(M)h(n) \ , \tag{14.5.3}$$

   where the moduli dependent probability $p_n(M)$ of the super-symplectic Virasoro excitation with conformal weight n should be consistent with the p-adic thermodynamics. It is convenient to write $h(M)$ as

$$h(M) = h_0+\Delta h(M) \ , \tag{14.5.4}$$

   where h_0 is the minimum value of $h(M)$ in the space of moduli. The form of the elementary particle vacuum functionals suggest that h_0 corresponds to moduli with $Im(\Omega_{ij})=0$ and thus to singular configurations for which handles degenerate to one-dimensional lines attached to a sphere.
4. There is a further averaging of $\Delta h(M)$ over the moduli space $\mathcal{M}$ by using the modulus squared of elementary particle vacuum functional so that one has

$$h = h_0+\langle\Delta h(M)\rangle_{\mathcal{M}} \ . \tag{14.5.5}$$

   Modular invariance allows to pose very strong conditions on the functional form of $\Delta h(M)$. The simplest assumption guaranteeing this and thermodynamical interpretation is that $\Delta h(M)$ is proportional to the logarithm of the vacuum functional Ω:

$$\Delta h(M) \propto -log(\frac{\Omega(M)}{\Omega_{max}}) \ . \tag{14.5.6}$$

   Here Ω_{max} corresponds to the maximum of Ω for which $\Delta h(M)$ vanishes.

Justification for the general form of the mass formula

The proposed general ansatz for $\Delta h(M)$ provides a justification for the general form of the mass formula deduced by intuitive arguments.

1. The factorization of the elementary particle vacuum functional Ω into a product of $2N(g) = 2^g(2^g+1)$ terms and the logarithmic expression for $\Delta h(M)$ imply that the thermal expectation values is a sum over thermal expectation values over $2N(g)$ terms associated with various even characteristics (a,b), where a and b are g-dimensional vectors with components equal to $1/2$ or 0 and the inner product $4a\cdot b$ is an even integer. If each term gives the same result in the averaging using Ω_{vac} as a partition function, the proportionality to $2N_g$ follows.
2. For genus $g\geq 2$ the partition function defines an average in $3g-3$ complex-dimensional space of moduli. The analogy of $\langle\Delta h\rangle$ and thermal energy suggests that the contribution is proportional to the complex dimension $3g-3$ of this space. For $g\leq 1$ the contribution the complex dimension of moduli space is g and the contribution would be proportional to g.

$$\begin{aligned}\langle\Delta h\rangle &\propto g\times X(g) \text{ for } g\leq 1 \ ,\\ \langle\Delta h\rangle &\propto (3g-3)\times X(g) \text{ for } g\geq 2 \ ,\\ X(g) &= 2^g(2^g+1) \ . \end{aligned} \tag{14.5.7}$$

   If X^2 is hyper-elliptic for the maxima of Kähler function, this expression makes sense only for $g\leq 2$ since vacuum functionals vanish for hyper-elliptic surfaces.
3. The earlier argument, inspired by the interpretation of elementary particle vacuum functional as a partition function, was that each factor of the elementary particle vacuum functional gives the same contribution to $\langle\Delta h\rangle$, and that this contribution is proportional to g since each handle behaves like a particle:

$$\langle\Delta h\rangle \propto g\times X(g) \ . \tag{14.5.8}$$

   The prediction following from the previous differs by a factor $(3g-3)/g$ for $g\geq 2$. This would scale up the dominant modular contribution to the masses of the third $g=2$ fermionic generation by a factor $\sqrt{3/2}\simeq 1.22$. One must of course remember, that these rough arguments allow $g-$ dependent numerical factors of order one so that it is not possible to exclude either argument.

14.5.3 Generalization Of Θ Functions And Quantization Of P-Adic Moduli

The task is to find p-adic counterparts for theta functions and elementary particle vacuum functionals. The constraints come from the p-adic existence of the exponentials appearing as the summands of the theta functions and from the convergence of the sum. The exponentials must be proportional to powers of p just as the Boltzmann weights defining the p-adic partition function. The outcome is a quantization of moduli so that integration can be replaced with a summation and the average of $\Delta h(M)$ over moduli is well defined.

It is instructive to study the problem for torus in parallel with the general case. The ordinary moduli space of torus is parameterized by single complex number τ. The points related by $SL(2,Z)$ are equivalent, which means that the transformation $\tau\rightarrow(A\tau+B)/(C\tau+D)$ produces a point equivalent with τ. These transformations are generated by the shift $\tau\rightarrow\tau+1$ and $\tau\rightarrow -1/\tau$. One can choose the fundamental domain of moduli space to be the intersection of the slice $Re(\tau)\in[-1/2,1/2]$ with the exterior of unit circle $|\tau|=1$. The idea is to start directly from physics and to look whether one might some define p-adic version of elementary particle vacuum functionals in the p-adic counterpart of this set or in some modular invariant subset of this set.

Elementary particle vacuum functionals are expressible in terms of theta functions using the functions $\Theta^4[a,b]\overline{\Theta}^4[a,b]$ as a building block. The general expression for the theta function reads as

$$\Theta[a,b](\Omega) = \sum_n exp(i\pi(n+a)\cdot\Omega\cdot(n+a))exp(2i\pi(n+a)\cdot b) \ . \tag{14.5.9}$$

The latter exponential phase gives only a factor $\pm i$ or ± 1 since $4a \cdot b$ is integer. For $p\ mod\ 4 = 3$ imaginary unit exists in an algebraic extension of p-adic numbers. In the case of torus (a, b) has the values $(0, 0)$, $(1/2, 0)$ and $(0, 1/2)$ for torus since only even characteristics are allowed.

Concerning the p-adicization of the first exponential appearing in the summands in Eq. 14.5.9, the obvious problem is that π does not exists p-adically unless one allows infinite-dimensional extension.

1. Consider first the real part of Ω. In this case the proper manner to treat the situation is to introduce and algebraic extension involving roots of unity so that $Re(\Omega)$ rational. This approach is proposed as a general approach to the p-adicization of quantum TGD in terms of harmonic analysis in symmetric spaces allowing to define integration also in p-adic context in a physically acceptable manner by reducing it to Fourier analysis. The simplest situation corresponds to integer values for $Re(\Omega)$ and in this case the phase are equal to $\pm i$ or ± 1 since a is half-integer valued. One can consider a hierarchy of variants of moduli space characterized by the allowed roots of unity. The physical interpretation for this hierarchy would be in terms of a hierarchy of measurement resolutions. Note that the real parts of Ω can be assumed to be rationals of form m/n where n is constructed as a product of finite number of primes and therefore the allowed rationals are linear combinations of inverses $1/p_i$ for a subset $\{p_i\}$ of primes.
2. For the imaginary part of Ω different approach is required. One wants a rapid convergence of the sum formula and this requires that the exponents reduces in this case to positive powers of p. This is achieved if one has

$$Im(\Omega) = -n\frac{log(p)}{\pi)} \ , \tag{14.5.10}$$

Unfortunately this condition is not consistent with the condition $Im(\Omega) > 0$. a way to circumvent the difficulty is to replace Ω with its complex conjugate. Second approach is to define the real discretized variant of theta function first and then map it by canonical identification to its p-adic counterpart: this would map phase to phases and powers of p to their inverses. Note that a similar change of sign must be performed in p-adic thermodynamics for powers of p to map p-adic probabilities to real ones. By rescaling $Im(\Omega) \rightarrow \frac{log(p)}{\pi)} Im(\Omega)$ one has non-negative integer valued spectrum for $Im(\Omega)$ making possible to reduce integration in moduli space to a summation over finite number of rationals associated with the real part of Ω and powers of p associated with the imaginary part of Ω.
3. Since the exponents appearing in

$$p^{(n+a)\cdot Im(\Omega_{ij,p})\cdot(n+a)} = p^{a\cdot Im(\Omega)\cdot a} \times p^{2a\cdot Im(\Omega\cdot n} \times p^{+n\cdot Im(\Omega_{ij,p})\cdot n}$$

are positive integers valued, $\Theta_{[a,b]}$ exist in R_p and converges. The problematic factor is the first exponent since the components of the vector a can have values $1/2$ and 0 and its existence implies a quantization of $Im(\Omega_{ij})$ as

$$Im(\Omega) = -Kn\frac{log(p)}{p} \ , \ n \in Z \ , \ n \geq 1 \ , \tag{14.5.11}$$

In p-adic context this condition must be formulated for the exponent of Ω defining the natural coordinate. $K = 4$ guarantees the existence of Θ functions and $K = 1$ the existence of the building blocks $\Theta^4[a, b]\overline{\Theta}^4[a, b]$ of elementary particle vacuum functionals in R_p. The extension to higher genera means only replacement of Ω with the elements of a matrix.
4. One can criticize this approach for the loss of the full modular covariance in the definition of theta functions. The modular transformations $\Omega \rightarrow \Omega + n$ are consistent with the number theoretic constraints but the transformations $\Omega \rightarrow -1/\Omega$ do not respect them. It seem that one can circumvent the difficulty by restricting the consideration to a fundamental domain satisfying the number theoretic constraints.

This variant of moduli space is discrete and p-adicity is reflected only in the sense that the moduli space makes sense also p-adically. One can consider also a continuum variant of the p-adic moduli space using the same prescription as in the construction of p-adic symmetric spaces [K95] .

1. One can introduce $exp(i\pi Re(\Omega))$ as the counterpart of $Re(\Omega)$ as a coordinate of the Teichmueller space. This coordinate makes sense only as a local coordinate since it does not differentiate between $Re(\Omega)$ and $Re(\Omega + 2n)$. On the other hand, modular invariance states that Ω abd $\Omega + n$ correspond to the same moduli so that nothing is lost. In the similar manner one can introduce $exp^(\pi Im(\Omega)) \in \{p^n, n > 0\}$ as the counterpart of discretized version of $Im(\Omega)$.
2. The extension to continuum would mean in the case of $Re(\Omega)$ the extension of the phase $exp(i\pi Re(\Omega))$ to a product $exp(i\pi Re(\Omega))exp(ipx) = exp(i\pi Re(\Omega) + exp(ipx)$, where x is p-adic integer which can be also infinite as a real integer. This would mean that each root of unity representing allowed value $Re(\Omega)$ would have a p-adic neighborhood consisting of p-adic integers. This neighborhood would be the p-adic counterpart for the angular integral $\Delta\phi$ for a given root of unity and would not make itself visible in p-adic integration.
3. For the imaginary part one can also consider the extension of $exp(\pi Im(\Omega))$ to $p^n \times exp(npx)$ where x is a p-adic integer. This would assign to each point p^n a p-adic neighborhood defined by p-adic integers. This neighborhood is same all integers n with same p-adic norm. When n is proportional to p^k one has $exp(npx) - 1 \propto p^k$.

The quantization of moduli characterizes precisely the conformal properties of the partonic 2-surfaces corresponding to different p-adic primes. In the real context -that is in the intersection of real and p-adic worlds- the quantization of moduli of torus would correspond to

$$\tau = K\left[\sum q + i \times n\frac{log(p)}{\pi}\right] , \quad (14.5.12)$$

where q is a rational number expressible as linear combination of inverses of a finite fixed set of primes defining the allowed roots of unity. $K = 1$ guarantees the existence of elementary particle vacuum functionals and $K = 4$ the existence of Theta functions. The ratio for the complex vectors defining the sides of the plane parallelogram defining torus via the identification of the parallel sides is quantized. In other words, the angles Φ between the sides and the ratios of the sides given by $|\tau|$ have quantized values.

The quantization rules for the moduli of the higher genera is of exactly same form

$$\Omega_{ij} = K\left[\sum q_{ij} + i \times n_{ij} \times \frac{log(p)}{\pi}\right] , \quad (14.5.13)$$

If the quantization rules hold true also for the maxima of Kähler function in the real context or more precisely- in the intersection of real and p-adic variants of the "world of classical worlds" identified as partonic 2-surfaces at the boundaries of causal diamond plus the data about their 4-D tangent space, there are good hopes that the p-adicized expression for Δh is obtained by a simple algebraic continuation of the real formula. Thus p-adic length scale would characterize partonic surface X^2 rather than the light like causal determinant X_l^3 containing X^2. Therefore the idea that various p-adic primes label various X_l^3 connecting fixed partonic surfaces X_i^2 would not be correct.

Quite generally, the quantization of moduli means that the allowed 2-dimensional shapes form a lattice and are thus additive. It also means that the maxima of Kähler function would obey a linear superposition in an extreme abstract sense. The proposed number theoretical quantization is expected to apply for any complex space allowing some preferred complex coordinates. In particular, WCW of 2-surfaces could allow this kind of quantization in the complex coordinates naturally associated with isometries and this could allow to define WCW integration, at least the counterpart of integration in zero mode degrees of freedom, as a summation.

Number theoretic vision leads to the notion of multi-p-p-adicity in the sense that the same partonic 2-surface can correspond to several p-adic primes and that infinite primes code for these primes [K113, K94] . At the level of the moduli space this corresponds to the replacement of p with an integer in the formulas so that one can interpret the formulas both in real sense and p-adic sense for the primes p dividing the integer. Also the exponent of given prime in the integer matters.

14.5.4 The Calculation Of The Modular Contribution $\langle \Delta H \rangle$ To The Conformal Weight

The quantization of the moduli implies that the integral over moduli can be defined as a sum over moduli. The theta function $\Theta[a,b](\Omega)_p(\tau_p)$ is proportional to $p^{a\cdot a Im(\Omega_{ij,p})} = p^{Kn_{ij}m(a)/4}$ for $a \cdot a = m(a)/4$, where $K = 1$ *resp.* $K = 4$ corresponds to the existence existence of elementary particle vacuum functionals *resp.* theta functions in R_p. These powers of p can be extracted from the thetas defining the vacuum functional. The numerator of the vacuum functional gives $(p^n)^{2K\sum_{a,b} m(a)}$. The numerator gives $(p^n)^{2K\sum_{a,b} m(a_0)}$, where a_0 corresponds to the minimum value of $m(a)$. $a_0 = (0,0,..,0)$ is allowed and gives $m(a_0) = 0$ so that the p-adic norm of the denominator equals to one. Hence one has

$$|\Omega_{vac}(\Omega_p)|_p = p^{-2nK\sum_{a,b} m(a)} \quad (14.5.14)$$

The sum converges extremely rapidly for large values of p as function of n so that in practice only few moduli contribute.

The definition of $log(\Omega_{vac})$ poses however problems since in $log(p)$ does not exist as a p-adic number in any p-adic number field. The argument of the logarithm should have a unit p-adic norm. The simplest manner to circumvent the difficulty is to use the fact that the p-adic norm $|\Omega_p|_p$ is also a modular invariant, and assume that the contribution to conformal weight depends on moduli as

$$\Delta h_p(\Omega_p) \propto log(\frac{\Omega_{vac}}{|\Omega_{vac}|_p}) . \quad (14.5.15)$$

The sum defining $\langle \Delta h_p \rangle$ converges extremely rapidly and gives a result of order $O(p)$ p-adically as required.

The p-adic expression for $\langle \Delta h_p \rangle$ should result from the corresponding real expression by an algebraic continuation. This encourages the conjecture that the allowed moduli are quantized for the maxima of Kähler function, so that the integral over the moduli space is replaced with a sum also in the real case, and that Δh given by the double thermodynamics as a function of moduli can be defined as in the p-adic case. The positive power of p multiplying the numerator could be interpreted as a degeneracy factor. In fact, the moduli are not primary dynamical variables in the case of the induced metric, and there must be a modular invariant weight factor telling how many 2-surfaces correspond to given values of moduli. The power of p could correspond to this factor.

14.6 The Contributions Of P-Adic Thermodynamics To Particle Masses

In the sequel various contributions to the mass squared are discussed.

14.6.1 General Mass Squared Formula

The thermal independence of Super Virasoro and modular degrees of freedom implies that mass squared for elementary particle is the sum of Super Virasoro, modular and Higgsy contributions:

$$M^2 = M^2(color) + M^2(SV) + M^2(mod) + M^2(Higgsy) . \quad (14.6.1)$$

Also small renormalization correction contributions might be possible.

14.6.2 Color Contribution To The Mass Squared

The mass squared contains a non-thermal color contribution to the ground state conformal weight coming from the mass squared of CP_2 spinor harmonic. The color contribution is an integer multiple of $m_0^2/3$, where $m_0^2 = 2\Lambda$ denotes the "cosmological constant" of CP_2 (CP_2 satisfies Einstein equations $G^{\alpha\beta} = \Lambda g^{\alpha\beta}$).

The color contribution to the p-adic mass squared is integer valued only if $m_0^2/3$ is taken as a fundamental p-adic unit of mass squared. This choice has an obvious relevance for p-adic mass calculations since the simplest form of the canonical identification does not commute with a division by integer. More precisely, the image of number xp in canonical identification has a value of order 1 when x is a non-trivial rational number whereas for $x = np$ the value is n/p and extremely is small for physically interesting primes.

The choice of the p-adic mass squared unit are no effects on zeroth order contribution which must vanish for light states: this requirement eliminates quark and lepton states for which the CP_2 contribution to the mass squared is not integer valued using m_0^2 as a unit. There can be a dramatic effect on the first order contribution. The mass squared $m^2 = p/3$ using $m_0^2/3$ means that the particle is light. The mass squared becomes $m^2 = p/3$ when m_0^2 is used as a unit and the particle has mass of order 10^{-4} Planck masses. In the case of W and Z^0 bosons this problem is actually encountered. For light states using $m_0^2/3$ as a unit only the second order contribution to the mass squared is affected by this choice.

14.6.3 Modular Contribution To The Mass Of Elementary Particle

The general form of the modular contribution is derivable from p-adic partition function for conformally invariant degrees of freedom associated with the boundary components. The general form of the vacuum functionals as modular invariant functions of Teichmueller parameters was derived in [K28] and the square of the elementary particle vacuum functional can be identified as a partition function. Even theta functions serve as basic building blocks and the functionals are proportional to the product of all even theta functions and their complex conjugates. The number of theta functions for genus $g > 0$ is given by

$$N(g) = 2^{g-1}(2^g+1) \ . \tag{14.6.2}$$

One has $N(1) = 3$ for muon and $N(2) = 10$ for τ.

1. Single theta function is analogous to a partition function. This implies that the modular contribution to the mass squared must be proportional to $2N(g)$. The factor two follows from the presence of both theta functions and their conjugates in the partition function.
2. The factorization properties of the vacuum functionals imply that handles behave effectively as particles. For example, at the limit, when the surface splits into two pieces with g_1 and $g-g_1$ handles, the partition function reduces to a product of g_1 and $g-g_1$ partition functions. This implies that the contribution to the mass squared is proportional to the genus of the surface. Altogether one has

$$\begin{aligned} M^2(mod,g) &= 2k(mod)N(g)g\frac{m_0^2}{p} \ , \\ k(mod) &= 1 \ . \end{aligned} \tag{14.6.3}$$

Here $k(mod)$ is some integer valued constant (in order to avoid ultra heavy mass) to be determined. $k(mod) = 1$ turns out to be the correct choice for this parameter.

Summarizing, the real counterpart of the modular contribution to the mass of a particle belonging to $g+1$:th generation reads as

$$\begin{aligned} M^2(mod) &= 0 \ for \ e,\nu_e,u,d \ , \\ M^2(mod) &= 9\frac{m_0^2}{p(X))} \ for \ X=\mu,\nu_\mu,c,s \ , \\ M^2(mod) &= 60\frac{m_0^2}{p(X)} \ for \ X=\tau,\nu_\tau,t,b \ . \end{aligned} \tag{14.6.4}$$

The requirement that hadronic mass spectrum and CKM matrix are sensible however forces the modular contribution to be the same for quarks, leptons and bosons. The higher order modular contributions to the mass squared are completely negligible if the degeneracy of massless state is $D(0, mod, g) = 1$ in the modular degrees of freedom as is in fact required by $k(mod) = 1$.

14.6.4 Thermal Contribution To The Mass Squared

One can deduce the value of the thermal mass squared in order $O(p^2)$ (an excellent approximation) using the general mass formula given by p-adic thermodynamics. Assuming maximal p-adic temperature $T_p = 1$ one has

$$
\begin{aligned}
M^2 &= k(sp + Xp^2 + O(p^3)) \ , \\
s_\Delta &= \frac{D(\Delta+1)}{D(\Delta)} \ , \\
X_\Delta &= 2\frac{D(\Delta+2)}{D(\Delta)} - \frac{D^2(\Delta+1)}{D^2(\Delta)} \ , \\
k &= 1 \ .
\end{aligned} \tag{14.6.5}
$$

Δ is the conformal weight of the operator creating massless state from the ground state.

The ratios $r_n = D(n+1)/D(n)$ allowing to deduce the values of s and X have been deduced from p-adic thermodynamics in [K60] . Light state is obtained only provided $r(\Delta)$ is an integer. The remarkable result is that for lowest lying states this is the case. For instance, for Ramond representations the values of r_n are given by

$$
(r_0, r_1, r_2, r_3) = (8, 5, 4, \frac{55}{16}) \ . \tag{14.6.6}
$$

The values of s and X are

$$
\begin{aligned}
(s_0, s_1, s_2) &= (8, 5, 4) \ , \\
(X_0, X_1, X_2) &= (16, 15, 11 + 1/2)) \ .
\end{aligned} \tag{14.6.7}
$$

The result means that second order contribution is extremely small for quarks and charged leptons having $\Delta < 2$. For neutrinos having $\Delta = 2$ the second order contribution is non-vanishing.

14.6.5 The Contribution From The Deviation Of Ground StateConformal Weight From Negative Integer

The interpretation inspired by p-adic mass calculations is that the squares λ_i^2 of the eigenvalues of the Kähler-Dirac operator correspond to the conformal weights of ground states. Another natural physical interpretation of λ is as an analog of the Higgs vacuum expectation. The instability of the Higgs=0 phase would corresponds to the fact that $\lambda = 0$ mode is not localized to any region in which ew magnetic field or induced Kähler field is non-vanishing. A good guess is that induced Kähler magnetic field B_K dictates the magnitude of the eigenvalues which is thus of order $h_0 = \sqrt{B_K R}$, R CP_2 radius. The first guess is that eigenvalues in the first approximation come as $(n+1/2)h_0$. Each region where induced Kähler field is non-vanishing would correspond to different scale mass scale h_0.

1. The vacuum expectation value of Higgs is only proportional to an eigenvalue λ, not equal to it. Indeed, Higgs and gauge bosons as elementary particles correspond to wormhole contacts carrying fermion and anti-fermion at the two wormhole throats and must be distinguished from the space-time correlate of its vacuum expectation as something proportional to λ. In the fermionic case the vacuum expectation value of Higgs does not seem to be even possible since fermions do not correspond to wormhole contacts between two space-time sheets but possess only single wormhole throat (p-adic mass calculations are consistent with this).

2. Physical considerations suggest that the vacuum expectation of Higgs field corresponds to a particular eigenvalue λ_i of Kähler-Dirac operator so that the eigenvalues λ_i would define TGD counterparts for the minima of Higgs potential. Since the vacuum expectation of Higgs corresponds to a condensate of wormhole contacts giving rise to a coherent state, the vacuum expectation cannot be present for topologically condensed CP_2 type vacuum extremals representing fermions since only single wormhole throat is involved. This raises a hen-egg question about whether Higgs contributes to the mass or whether Higgs is only a correlate for massivation having description using more profound concepts. From TGD point of view the most elegant option is that Higgs does not give rise to mass but Higgs vacuum expectation value accompanies bosonic states and is naturally proportional to λ_i. With this interpretation λ_i could give a contribution to both fermionic and bosonic masses.

3. p-Adic mass calculations require negative ground state conformal weight compensated by Super Virasoro generators in order to obtain massless states. The tachyonicity of the ground states would mean a close analogy with both string models and Higgs mechanism. λ_i^2 is very natural candidate for the ground state conformal weights identified but would have wrong sign if the effective metric of X_l^3 defined by the inner products $T_K^{k\alpha}T_K^{l\beta}h_{kl}$ of the Kähler energy momentum tensor $T^{k\alpha} = h^{kl}\partial L_K/\partial h_\alpha^l$ and appearing in the Kähler-Dirac operator D_K has Minkowskian signature.
The situation changes if the effective metric has Euclidian signature. This seems to be the case for the light-like surfaces assignable to the known extremals such as MEs and cosmic strings. In this kind of situation light-like coordinate possesses Euclidian signature and real eigenvalue spectrum is replaced with a purely imaginary one. Since Dirac operator is in question both signs for eigenvalues are possible and one obtains both exponentially increasing and decreasing solutions. This is essential for having solutions extending from the past end of X_l^3 to its future end. Non-unitary time evolution is possible because X_l^3 does not strictly speaking represent the time evolution of 2-D dynamical object but actual dynamical objects (by light-likeness both interpretation as dynamical evolution and dynamical object are present). The Euclidian signature of the effective metric would be a direct analog for the tachyonicity of the Higgs in unstable minimum and the generation of Higgs vacuum expectation would correspond to the compensation of ground state conformal weight by conformal weights of Super Virasoro generators.

4. In accordance with this λ_i^2 would give constant contribution to the ground state conformal weight. What contributes to the thermal mass squared is the deviation of the ground state conformal weight from half-odd integer since the negative integer part of the total conformal weight can be compensated by applying Virasoro generators to the ground state. The first guess motivated by cyclotron energy analogy is that the lowest conformal weights are of form $h_c = \lambda_i^2 = -1/2-n+\Delta h_c$ so that lowest ground state conformal weight would be $h_c = -1/2$ in the first approximation. The negative integer part of the net conformal weight can be canceled using Super Virasoro generators but Δh_c would give to mass squared a contribution analogous to Higgs contribution. The mapping of the real ground state conformal weight to a p-adic number by canonical identification involves some delicacies.

5. p-Adic mass calculations are consistent with the assumption that Higgs type contribution is vanishing (that is small) for fermions and dominates for gauge bosons. This requires that the deviation of λ_i^2 with smallest magnitude from half-odd integer value in the case of fermions is considerably smaller than in the case of gauge bosons in the scale defined by p-adic mass scale $1/L(k)$ in question. Somehow this difference could relate to the fact that bosons correspond to pairs of wormhole throats.

14.6.6 General Mass Formula For Ramond Representations

By taking the modular contribution from the boundaries into account the general p-adic mass formulas for the Ramond type states read for states for which the color contribution to the conformal weight is integer valued as

$$\begin{aligned}
\frac{m^2(\Delta=0)}{m_0^2} &= (8+n(g))p + Yp^2 \ , \\
\frac{m^2(\Delta=1)}{m_0^2} &= (5+n(g)p + Yp^2 \ , \\
\frac{m^2(\Delta=2)}{m_0^2} &= (4+n(g))p + (Y+\frac{23}{2})p^2 \ , \\
n(g) &= 3g\cdot 2^{g-1}(2^g+1) \ .
\end{aligned} \tag{14.6.8}$$

Here Δ denotes the conformal weight of the operators creating massless states from the ground state and g denotes the genus of the boundary component. The values of $n(g)$ for the three lowest generations are $n(0)=0$, $n(1)=9$ and $n(2)=60$. The value of second order thermal contribution is nontrivial for neutrinos only. The value of the rational number Y can, which corresponds to the renormalization correction to the mass, can be determined using experimental inputs.

Using m_0^2 as a unit, the expression for the mass of a Ramond type state reads in terms of the electron mass as

$$\begin{aligned}
M(\Delta,g,p)_R &= K(\Delta,g,p)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0,g,p) &= \sqrt{\frac{n(g)+8+Y_R}{X}} \\
K(1,g,p) &= \sqrt{\frac{n(g)+5+Y_R}{X}} \\
K(2,g,p) &= \sqrt{\frac{n(g)+4+Y_R}{X}} \ , \\
X &= \sqrt{5+Y(e)_R} \ .
\end{aligned} \tag{14.6.9}$$

Y can be assumed to depend on the electromagnetic charge and color representation of the state and is therefore same for all fermion families. Mathematica provides modules for calculating the real counterpart of the second order contribution and for finding realistic values of Y.

14.6.7 General Mass Formulas For NS Representations

Using $m_0^2/3$ as a unit, the expression for the mass of a light NS type state for $T_p=1$ ad $k_B=1$ reads in terms of the electron mass as

$$\begin{aligned}
M(\Delta,g,p,N)_R &= K(\Delta,g,p,N)\sqrt{\frac{M_{127}}{p}}m_e \\
K(0,g,p,1) &= \sqrt{\frac{n(g)+Y_R}{X}} \ , \\
K(0,g,p,2) &= \sqrt{\frac{n(g)+1+Y_R}{X}} \ , \\
K(1,g,p,3) &= \sqrt{\frac{n(g)+3+Y_R}{X}} \ , \\
K(2,g,p,4) &= \sqrt{\frac{n(g)+5+Y_R}{X}} \ , \\
K(2,g,p,5) &= \sqrt{\frac{n(g)+10+Y_R}{X}} \ , \\
X &= \sqrt{5+Y(e)_R} \ .
\end{aligned} \tag{14.6.10}$$

Here N is the number of the "active" NS sectors (sectors for which the conformal weight of the massless state is non-vanishing). Y denotes the renormalization correction to the boson mass and in general depends on the electro-weak and color quantum numbers of the boson.

The thermal contribution to the mass of W boson is too large by roughly a factor $\sqrt{3}$ for $T_p = 1$. Hence $T_p = 1/2$ must hold true for gauge bosons and their masses must have a non-thermal origin perhaps analogous to Higgs mechanism. Alternatively, the non-covariant constancy of charge matrices could induce the boson mass [K60] .

It is interesting to notice that the minimum mass squared for gauge boson corresponds to the p-adic mass unit $M^2 = m_0^2 p/3$ and this just what is needed in the case of W boson. This forces to ask whether $m_0^2/3$ is the correct choice for the mass squared unit so that non-thermally induced W mass would be the minimal $m_W^2 = p$ in the lowest order. This choice would mean the replacement

$$Y_R \rightarrow \frac{(3Y)_R}{3}$$

in the preceding formulas and would affect only neutrino mass in the fermionic sector. $m_0^2/3$ option is excluded by charged lepton mass calculation. This point will be discussed later.

14.6.8 Primary Condensation Levels From P-Adic Length ScaleHypothesis

p-Adic length scale hypothesis states that the primary condensation levels correspond to primes near prime powers of two $p \simeq 2^k$, k integer with prime values preferred. Black hole-elementary particle analogy [K72] suggests a generalization of this hypothesis by allowing k to be a power of prime. The general number theoretical vision discussed in [K95] provides a first principle justification for p-adic length scale hypothesis in its most general form. The best fit for the neutrino mass squared differences is obtained for $k = 13^2 = 169$ so that the generalization of the hypothesis might be necessary.

A particle primarily condensed on the level k can suffer secondary condensation on a level with the same value of k: for instance, electron ($k = 127$) suffers secondary condensation on $k = 127$ level. u, d, s quarks ($k = 107$) suffer secondary condensation on nuclear space-time sheet having $k = 113$). All quarks feed their color gauge fluxes at $k = 107$ space-time sheet. There is no deep reason forbidding the condensation of p on p. Primary and secondary condensation levels could also correspond to different but nearly identical values of p with the same value of k.

14.7 Fermion Masses

In the earlier model the coefficient of $M^2 = kL_0$ had to be assumed to be different for various particle states. $k = 1$ was assumed for bosons and leptons and $k = 2/3$ for quarks. The fact that $k = 1$ holds true for all particles in the model including also super-symplectic invariance forces to modify the earlier construction of quark states. This turns out to be possible without affecting the earlier p-adic mass calculations whose outcome depend in an essential manner on the ground state conformal weights h_{gr} of the fermions (h_{gr} can be negative). The structure of lepton and quark states in color degrees of freedom was discussed in [K60] .

14.7.1 Charged Lepton Mass Ratios

The overall mass scale for lepton and quark masses is determined by the condensation level given by prime $p \simeq 2^k$, k prime by length scale hypothesis. For charged leptons k must correspond to $k = 127$ for electron, $k = 113$ for muon and $k = 107$ for τ. For muon $p = 2^{113} - 1 - 4 * 378$ is assumed (smallest prime below 2^{113} allowing $\sqrt{2}$ but not $\sqrt{3}$). So called Gaussian primes are to complex integers what primes are for the ordinary integers and the Gaussian counterparts of the Mersenne primes are Gaussian primes of form $(1 \pm i)^k - 1$. Rather interestingly, $k = 113$ corresponds to a Gaussian Mersenne so that all charged leptons correspond to generalized Mersenne primes.

For $k = 1$ the leptonic mass squared is integer valued in units of m_0^2 only for the states satisfying

$$p \bmod 3 \neq 2 \ .$$

Only these representations can give rise to massless states. Neutrinos correspond to (p,p) representations with $p \geq 1$ whereas charged leptons correspond to $(p,p+3)$ representations. The earlier mass calculations demonstrate that leptonic masses can be understood if the ground state conformal weight is $h_{gr} = -1$ for charged leptons and $h_{gr} = -2$ for neutrinos.

The contribution of color partial wave to conformal weight is $h_c = (p^2+2p)/3$, $p \geq 1$, for neutrinos and $p = 1$ gives $h_c = 1$ (octet). For charged leptons $h_c = (p^2+5p+6)/3$ gives $h_c = 2$ for $p = 0$ (decouplet). In both cases super-symplectic operator O must have a net conformal weight $h_{sc} = -3$ to produce a correct conformal weight for the ground state. p-adic considerations suggests the use of operators O with super-symplectic conformal weight $z = -1/2 - i\sum n_k y_k$, where $s_k = 1/2 + iy_k$ corresponds to zero of Riemann ζ. If the operators in question are color Hamiltonians in octet representation net super-symplectic conformal weight $h_{sc} = -3$ results. The tensor product of two octets with conjugate super-symplectic conformal weights contains both octet and decouplet so that singlets are obtained. What strengthens the hopes that the construction is not ad hoc is that the same operator appears in the construction of quark states too.

Using CP_2 mass scale m_0^2 [K60] as a p-adic unit, the mass formulas for the charged leptons read as

$$\begin{aligned}
M^2(L) &= A(\nu)\frac{m_0^2}{p(L)} \ , \\
A(e) &= 5 + X(p(e)) \ , \\
A(\mu) &= 14 + X(p(\mu)) \ , \\
A(\tau) &= 65 + X(p(\tau)) \ .
\end{aligned} \tag{14.7.1}$$

$X(\cdot)$ corresponds to the yet unknown second order corrections to the mass squared.

Table 14.4 gives the basic parameters as determined from the mass of electron for some values of Y_e. The mass of top quark favors as maximal value of CP_2 mass which corresponds to $Y_e = 0$.

Y_e	0	.5	.7798
$(m_0/m_{Pl}) \times 10^3$	.2437	.2323	.2266
$K \times 10^{-7}$	2.5262	2.7788	2.9202
$(L_R/\sqrt{G}) \times 10^{-4}$	3.1580	3.3122	3.3954

Table 14.4: Table gives the values of CP_2 mass m_0 using Planck mass $m_{Pl} = 1/\sqrt{G}$ as unit, the ratio $K = R^2/G$ and CP_2 geodesic length $L = 2\pi R$ for $Y_e \in \{0, 0.5, 0.7798\}$.

Table 14.5 lists the lower and upper bounds for the charged lepton mass ratios obtained by taking second order contribution to zero or allowing it to have maximum possible value. The values of lepton masses are $m_e = .510999$ MeV, $m_\mu = 105.76583$ MeV, $m_\tau = 1775$ MeV.

For the maximal value of CP_2 mass the predictions for the mass ratio are systematically too large by a few per cent. From the formulas above it is clear that the second order corrections to mass squared can be such that correct masses result.

τ mass is least sensitive to $X(p(e)) \equiv Y_e$ and the maximum value of $Y_e \equiv Y_{e,max}$ consistent with τ mass corresponds to $Y_{e,max} = .7357$ and $Y_\tau = 1$. This means that the CP_2 mass is at least a fraction .9337 of its maximal value. If Y_L is same for all charged leptons and has the maximal value $Y_{e,max} = .7357$, the predictions for the mass ratios are

$$\begin{aligned}
\frac{m(\mu)_+}{m(\mu)} &= \sqrt{\frac{15}{5}2^7\frac{m_e}{(\mu)}} \simeq 1.0722 \ , \\
\frac{m(\mu)_-}{m(\mu)} &= \sqrt{\frac{14}{6}2^7\frac{m_e}{m_(\mu)}} \simeq 0.9456 \ , \\
\frac{m(\tau)_+}{m(\tau)} &= \sqrt{\frac{66}{5}2^{10}\frac{m_e}{m(\tau)}} \simeq 1.0710 \ , \\
\frac{m(\tau)_-}{m(\tau)} &= \sqrt{\frac{65}{6}2^{10}\frac{m_e}{m(\tau)}} \simeq .9703 \ .
\end{aligned} \tag{14.7.2}$$

Table 14.5: Lower and upper bounds for the charged lepton mass ratios obtained by taking second order contribution to zero or allowing it to have maximum possible value.

$$\begin{aligned}
\frac{m(\mu)_{pr}}{m(\mu)} &= \sqrt{\frac{14+Y_{e,max}}{5+Y_{e,max}}} \times 2^7\frac{m_e}{m(\mu)} \simeq .9922 \ , \\
\frac{m(\tau)_{pr}}{m(\tau)} &= \sqrt{\frac{65+Y_{e,max}}{5+Y_{e,max}}} \times 2^{10}\frac{m_e}{m(\tau)} \simeq .9980 \ .
\end{aligned} \tag{14.7.3}$$

The error is .8 per cent *resp.* .2 per cent for muon *resp.* τ.

The argument leading to estimate for the modular contribution to the mass squared [K60] leaves two options for the coefficient of the modular contribution for $g = 2$ fermions: the value of coefficient is either $X = g$ for $g \leq 1$, $X = 3g - 3$ for $g \geq 2$ or $X = g$ always. For $g = 2$ the predictions are $X = 2$ and $X = 3$ in the two cases. The option $X = 3$ allows slightly larger maximal value of Y_e equal to $Y_{e,max}^{1)} = Y_{e,max} + (5 + Y_{e,max})/66$.

14.7.2 Neutrino Masses

The estimation of neutrino masses is difficult at this stage since the prediction of the primary condensation level is not yet possible and neutrino mixing cannot yet be predicted from the basic principles. The cosmological bounds for neutrino masses however help to put upper bounds on the masses. If one takes seriously the LSND data on neutrino mass measurement of [C67, C22] and the explanation of the atmospheric ν-deficit in terms of $\nu_\mu - \nu_\tau$ mixing [C40, C33] one can deduce that the most plausible condensation level of μ and τ neutrinos is $k = 167$ or $k = 13^2 = 169$ allowed by the more general form of the p-adic length scale hypothesis suggested by the blackhole-elementary particle analogy. One can also deduce information about the mixing matrix associated with the neutrinos so that mass predictions become rather precise. In particular, the mass splitting of μ and τ neutrinos is predicted correctly if one assumes that the mixing matrix is a rational unitary matrix.

Super Virasoro contribution

Using $m_0^2/3$ as a p-adic unit, the expression for the Super Virasoro contribution to the mass squared of neutrinos is given by the formula

$$\begin{aligned}
M^2(SV) &= (s + (3Yp)_R/3)\frac{m_0^2}{p} \ , \\
s &= 4 \text{ or } 5 \ , \\
Y &= \frac{23}{2} + Y_1 \ ,
\end{aligned} \tag{14.7.4}$$

where m_0^2 is universal mass scale. One can consider two possible identifications of neutrinos corresponding to $s(\nu) = 4$ with $\Delta = 2$ and $s(\nu) = 5$ with $\Delta = 1$. The requirement that CKM matrix is sensible forces the asymmetric scenario in which quarks and, by symmetry, also leptons correspond to lowest possible excitation so that one must have $s(\nu) = 4$. Y_1 represents second order contribution to the neutrino mass coming from renormalization effects coming from self energy diagrams involving intermediate gauge bosons. Physical intuition suggest that this contribution is very small so that the precise measurement of the neutrino masses should give an excellent test for the theory.

With the above described assumptions and for $s = 4$, one has the following mass formula for neutrinos

$$\begin{aligned} M^2(\nu) &= A(\nu)\frac{m_0^2}{p(\nu))} \ , \\ A(\nu_e) &= 4 + \frac{(3Y(p(\nu_e)))_R}{3} \ , \\ A(\nu_\mu) &= 13 + \frac{(3Y(p(\nu_\mu)))_R}{3} \ , \\ A(\nu_\tau) &= 64 + \frac{(3Y(p(\nu_\tau)))_R}{3} \ , \\ 3Y &\simeq \frac{1}{2} \ . \end{aligned} \tag{14.7.5}$$

The predictions must be consistent with the recent upper bounds [C24] of order 10 eV, 270 keV and 0.3 MeV for ν_e, ν_μ and ν_τ respectively. The recently reported results of LSND measurement [C22] for $\nu_e- > \nu_\mu$ mixing gives string limits for $\Delta m^2(\nu_e,\nu_\mu)$ and the parameter $sin^2(2\theta)$ characterizing the mixing: the limits are given in the figure 30 of [C22]. The results suggests that the masses of both electron and muon neutrinos are below 5 eV and that mass squared difference $\Delta m^2 = m^2(\nu_\mu) - m^2(\nu_e)$ is between $.25 - 25\ eV^2$. The simplest possibility is that ν_μ and ν_e have common condensation level (in analogy with d and s quarks). There are three candidates for the primary condensation level: namely $k = 163, 167$ and $k = 169$. The p-adic prime associated with the primary condensation level is assumed to be the nearest prime below 2^k allowing p-adic $\sqrt{2}$ but not $\sqrt{3}$ and satisfying $p\ mod\ 4 = 3$. The **Table 14.6** gives the values of various parameters and unmixed neutrino masses in various cases of interest.

k	p	$(3Y)_R/3$	$m(\nu_e)/eV$	$m(\nu_\mu)/eV$	$m(\nu_\tau)/eV$
163	$2^{163} - 4*144 - 1$	1.36	1.78	3.16	6.98
167	$2^{167} - 4*144 - 1$	.34	.45	.79	1.75
169	$2^{169} - 4*210 - 1$	.17	.22	.40	.87

Table 14.6: The values of various parameters and unmixed neutrino masses in various cases of interest.

Could neutrino topologically condense also in other p-adic length scales than $k = 169$?

One must keep mind open for the possibility that there are several p-adic length scales at which neutrinos can condense topologically. Biological length scales are especially interesting in this respect. In fact, all intermediate p-adic length scales $k = 151, 157, 163, 167$ could correspond to metastable neutrino states. The point is that these p-adic lengths scales are number theoretically completely exceptional in the sense that there exist Gaussian Mersenne $2^k \pm i$ (prime in the ring of complex integers) for all these values of k. Since charged leptons, atomic nuclei ($k = 113$) , hadrons and intermediate gauge bosons correspond to ordinary or Gaussian Mersennes, it would not be surprising if the biologically important Gaussian Mersennes would correspond to length scales giving rise to metastable neutrino states. Of course, one can keep mind open for the possibility that $k = 167$ rather than $k = 13^2 = 169$ is the length scale defining the stable neutrino physics.

Neutrino mixing

Consider next the neutrino mixing. A quite general form of the neutrino mixing matrix D given by **Table 14.7** will be considered.

	ν_e	ν_μ	ν_τ
ν_e	c_1	s_1c_3	s_1s_3
ν_μ	$-s_1c_2$	$c_1c_2c_3 - s_2s_3exp(i\delta)$	$c_1c_2s_3 + s_2c_3exp(i\delta)$
ν_τ	$-s_1s_2$	$c_1s_2c_3 + c_2s_3exp(i\delta)$	$c_1s_2s_3 - c_2c_3exp(i\delta)$

Table 14.7: General form of neutrino mixing matrix.

Physical intuition suggests that the angle δ related to CP breaking is small and will be assumed to be vanishing. Topological mixing is active only in modular degrees of freedom and one obtains for the first order terms of mixed masses the expressions

$$\begin{aligned} s(\nu_e) &= 4+9|U_{12}|^2+60|U_{13}|^2 = 4+n_1 \ , \\ s(\nu_\mu) &= 4+9|U_{22}|^2+60|U_{23}|^2 = 4+n_2 \ , \\ s(\nu_\tau) &= 4+9|U_{32}|^2+60|U_{33}|^2 = 4+n_3 \ . \end{aligned} \tag{14.7.6}$$

The requirement that resulting masses are not ultra heavy implies that $s(\nu)$ must be small integers. The condition $n_1+n_2+n_3 = 69$ follows from unitarity. The simplest possibility is that the mixing matrix is a rational unitary matrix. The same ansatz was used successfully to deduce information about the mixing matrices of quarks. If neutrinos are condensed on the same condensation level, rationality implies that $\nu_\mu - \nu_\tau$ mass squared difference must come from the first order contribution to the mass squared and is therefore quantized and bounded from below.

The first piece of information is the atmospheric ν_μ/ν_e ratio, which is roughly by a factor 2 smaller than predicted by standard model [C40]. A possible explanation is the CKM mixing of muon neutrino with τ-neutrino, whereas the mixing with electron neutrino is excluded as an explanation. The latest results from Kamiokande [C40] are in accordance with the mixing $m^2(\nu_\tau) - m^2(\nu_\mu) \simeq 1.6\cdot 10^{-2}\ eV^2$ and mixing angle $sin^2(2\theta) = 1.0$: also the zenith angle dependence of the ratio is in accordance with the mixing interpretation. If mixing matrix is assumed to be rational then only $k = 169$ condensation level is allowed for ν_μ and ν_τ. For this level $\nu_\mu - \nu_\tau$ mass squared difference turns out to be $\Delta m^2 \simeq 10^{-2}\ eV^2$ for $\Delta s \equiv s(\nu_\tau) - s(\nu_\mu) = 1$, which is the only acceptable possibility and predicts $\nu_\mu - \nu_\tau$ mass squared difference correctly within experimental uncertainties! The fact that the predictions for mass squared differences are practically exact, provides a precision test for the rationality assumption.

What is measured in LSND experiment is the probability $P(t,E)$ that ν_μ transforms to ν_e in time t after its production in muon decay as a function of energy E of ν_μ. In the limit that ν_τ and ν_μ masses are identical, the expression of $P(t,E)$ is given by

$$\begin{aligned} P(t,E) &= sin^2(2\theta)sin^2(\frac{\Delta E t}{2}) \ , \\ sin^2(2\theta) &= 4c_1^2s_1^2c_2^2 \ , \end{aligned} \tag{14.7.7}$$

where ΔE is energy difference of ν_μ and ν_e neutrinos and t denotes time. LSND experiment gives stringent conditions on the value of $\sin^2(2\theta)$ as the figure 30 of [C22] shows. In particular, it seems that $sin^2(2\theta)$ must be considerably below 10^{-1} and this implies that s_1^2 must be small enough.

The study of the mass formulas shows that the only possibility to satisfy the constraints for the mass squared and $sin^2(2\theta)$ given by LSND experiment is to assume that the mixing of the electron neutrino with the tau neutrino is much larger than its mixing with the muon neutrino. This means that s_3 is quite near to unity. At the limit $s_3 = 1$ one obtains the following (nonrational) solution of the mass squared conditions for $n_3 = n_2 + 1$ (forced by the atmospheric neutrino data)

$$\begin{aligned} s_1^2 &= \frac{69 - 2n_2 - 1}{60} \ , \\ c_2^2 &= \frac{n_2 - 9}{2n_2 - 17} \ , \\ sin^2(2\theta) &= \frac{4(n_2 - 9)}{51}\frac{(34 - n_2)(n_2 - 4)}{30^2} \ , \\ s(\nu_\mu) - s(\nu_e) &= 3n_2 - 68 \ . \end{aligned} \tag{14.7.8}$$

The study of the LSND data shows that there is only one acceptable solution to the conditions obtained by assuming maximal mass squared difference for ν_e and ν_μ

$$\begin{aligned} n_1 &= 2 \ \ n_2 = 33 \ \ n_3 = 34 \ , \\ s_1^2 &= \frac{1}{30} \ \ c_2^2 = \frac{24}{49} \ , \\ sin^2(2\theta) &= \frac{24}{49}\frac{2}{15}\frac{29}{30} \simeq .0631 \ , \\ s(\nu_\mu) - s(\nu_e)) &= 31 \leftrightarrow .32 \ eV^2 \ . \end{aligned} \tag{14.7.9}$$

That c_2^2 is near 1/2 is not surprise taking into account the almost mass degeneracy of ν_{mu} and ν_τ. From the figure 30 of [C22] it is clear that this solution belongs to 90 per cent likelihood region of LSND experiment but $sin^2(2\theta)$ is about two times larger than the value allowed by Bugey reactor experiment. The study of various constraints given in [C22] shows that the solution is consistent with bounds from all other experiments. If one assumes that $k > 169$ for ν_e $\nu_\mu - \nu_e$ mass difference increases, implying slightly poorer consistency with LSND data.

There are reasons to hope that the actual rational solution can be regarded as a small deformation of this solution obtained by assuming that c_3 is non-vanishing. $s_1^2 = \frac{69-2n_2-1}{60-51c_3^2}$ increases in the deformation by $O(c_3^2)$ term but if c_3 is positive the value of $c_2^2 \simeq \frac{24-102c_1^0c_2^0s_2^0c_3}{49} \sim \frac{24-61c_3}{49}$ decreases by $O(c_3)$ term so that it should be possible to reduce the value of $sin^2(2\theta)$. Consistency with Bugey reactor experiment requires $.030 \leq sin^2(2\theta) < .033$. $sin^2(2\theta) = .032$ is achieved for $s_1^2 \simeq .035$,$s_2^2 \simeq .51$ and $c_3^2 \simeq .068$. The construction of U and D matrices for quarks shows that very stringent number theoretic conditions are obtained and as in case of quarks it might be necessary to allow complex CP breaking phase in the mixing matrix. One might even hope that the solution to the conditions is unique.

For the minimal rational mixing one has $s(\nu_e) = 5$, $s(\nu_\mu) = 36$ and $s(\nu_\tau) = 37$ if unmixed ν_e corresponds to $s = 4$. For $s = 5$ first order contributions are shifted by one unit. The masses ($s = 4$ case) and mass squared differences are given by **Table 14.8**.

k	$m(\nu_e)$	$m(\nu_\mu)$	$m(\nu_\tau)$	$\Delta m^2(\nu_\mu - \nu_e)$	$\Delta m^2(\nu_\tau - \nu_\mu)$
169	.27 eV	.66 eV	.67 eV	.32 eV^2	.01 eV^2

Table 14.8: Mass squared differences for neutrinos.

Predictions for neutrino masses and mass squared splittings for $k = 169$ case.

Evidence for the dynamical mass scale of neutrinos

In recent years (I am writing this towards the end of year 2004 and much later than previous lines) a great progress has been made in the understanding of neutrino masses and neutrino mixing. The pleasant news from TGD perspective is that there is a strong evidence that neutrino masses depend on environment [C55]. In TGD framework this translates to the statement that neutrinos can suffer topological condensation in several p-adic length scales. Not only in the p-adic length scales suggested by the number theoretical considerations but also in longer length scales, as will be found.

The experiments giving information about mass squared differences can be divided into three categories [C55].

1. There along baseline experiments, which include solar neutrino experiments [C29, C44, C54] and [C61] as well as earlier studies of solar neutrinos. These experiments see evidence for the neutrino mixing and involve significant propagation through dense matter. For the solar neutrinos and KamLAND the mass splittings are estimated to be of order $O(8 \times 10^{-5})$ eV^2 or more cautiously 8×10^{-5} eV$^2 < \delta m^2 < 2 \times 10^{-3}$ eV^2. For K2K and atmospheric neutrinos the mass splittings are of order $O(2 \times 10^{-3}) eV^2$ or more cautiously $\delta m^2 > 10^{-3}$eV^2. Thus the scale of mass splitting seems to be smaller for neutrinos in matter than in air, which would suggest that neutrinos able to propagate through a dense matter travel at space-time sheets corresponding to a larger p-adic length scale than in air.
2. There are null short baseline experiments including CHOOZ, Bugey, and Palo Verde reactor experiments, and the higher energy CDHS, JARME, CHORUS, and NOMAD experiments, which involve muonic neutrinos (for references see [C55]. No evidence for neutrino oscillations have been seen in these experiments.
3. The results of LSND experiment [C22] are consistent with oscillations with a mass splitting greater than $3 \times 10^{-2} eV^2$. LSND has been generally been interpreted as necessitating a mixing with sterile neutrino. If neutrino mass scale is dynamical, situation however changes.

If one assumes that the p-adic length scale for the space-time sheets at which neutrinos can propagate is different for matter and air, the situation changes. According to [C55] a mass 3×10^{-2} eV in air could explain the atmospheric results whereas mass of of order .1 eV and $.07 eV^2 < \delta m^2 < .26 eV^2$ would explain the LSND result. These limits are of the same order as the order of magnitude predicted by $k = 169$ topological condensation.

Assuming that the scale of the mass splitting is proportional to the p-adic mass scale squared, one can consider candidates for the topological condensation levels involved.

1. Suppose that $k = 169 = 13^2$ is indeed the condensation level for LSND neutrinos. $k = 173$ would predict $m_{\nu_e} \sim 7 \times 10^{-2}$ eV and $\delta m^2 \sim .02$ eV^2. This could correspond to the masses of neutrinos propagating through air. For $k = 179$ one has $m_{\nu_e} \sim .8 \times 10^{-2}$ eV and $\delta m^2 \sim 3 \times 10^{-4}$ eV^2 which could be associated with solar neutrinos and KamLAND neutrinos.
2. The primes $k = 157, 163, 167$ associated with Gaussian Mersennes would give $\delta m^2(157) = 2^6 \delta m^2(163) = 2^{10} \delta m^2(167) = 2^{12} \delta m^2(169)$ and mass scales $m(157) \sim 22.8$ eV, $m(163) \sim 3.6$ eV, $m(167) \sim .54$ eV. These mass scales are unrealistic or propagating neutrinos. The interpretation consistent with TGD inspired model of condensed matter in which neutrinos screen the classical Z^0 force generated by nucleons would be that condensed matter neutrinos are confined inside these space-time sheets whereas the neutrinos able to propagate through condensed matter travel along $k > 167$ space-time sheets.

The results of MiniBooNE group as a support for the energy dependence of p-adic mass scale of neutrino

The basic prediction of TGD is that neutrino mass scale can depend on neutrino energy and the experimental determinations of neutrino mixing parameters support this prediction. The newest results (11 April 2007) about neutrino oscillations come from MiniBooNE group which has published its first findings [C19] concerning neutrino oscillations in the mass range studied in LSND experiments [C18].

1. The motivation for MiniBooNE

Neutrino oscillations are not well-understood. Three experiments LSND, atmospheric neutrinos, and solar neutrinos show oscillations but in widely different mass regions (1 eV^2 , 3×10^{-3} eV^2, and 8×10^{-5} eV^2).

In TGD framework the explanation would be that neutrinos can appear in several p-adically scaled up variants with different mass scales and therefore different scales for the differences Δm^2 for neutrino masses so that one should not try to try to explain the results of these experiments using single neutrino mass scale. In single-sheeted space-time it is very difficult to imagine that neutrino mass scale would depend on neutrino energy since neutrinos interact so extremely weakly with matter. The best known attempt to assign single mass to all neutrinos has been based on the use of so called sterile neutrinos which do not have electro-weak couplings. This approach is an ad hoc trick and rather ugly mathematically and excluded by the results of MiniBooNE experiments.

2. The result of MiniBooNE experiment

The purpose of the MiniBooNE experiment was to check whether LSND result $\Delta m^2 = 1eV^2$ is genuine. The group used muon neutrino beam and looked whether the transformations of muonic neutrinos to electron neutrinos occur in the mass squared region $\Delta m^2 \simeq 1$ eV^2. No such transitions were found but there was evidence for transformations at low neutrino energies.

What looks first as an over-diplomatic formulation of the result was *MiniBooNE researchers showed conclusively that the LSND results could not be due to simple neutrino oscillation, a phenomenon in which one type of neutrino transforms into another type and back again.* rather than direct refutation of LSND results.

3. LSND and MiniBooNE are consistent in TGD Universe

The habitant of the many-sheeted space-time would not regard the previous statement as a mere diplomatic use of language. It is quite possible that neutrinos studied in MiniBooNE have suffered topological condensation at different space-time sheet than those in LSND if they are in different energy range (the preferred rest system fixed by the space-time sheet of the laboratory or Earth). To see whether this is the case let us look more carefully the experimental arrangements.

1. In LSND experiment 800 MeV proton beam entering in water target and the muon neutrinos resulted in the decay of produced pions. Muonic neutrinos had energies in 60-200 MeV range [C18].
2. In MiniBooNE experiment [C19] 8 GeV muon beam entered Beryllium target and muon neutrinos resulted in the decay of resulting pions and kaons. The resulting muonic neutrinos had energies the range 300-1500 GeV to be compared with 60-200 MeV.

Let us try to make this more explicit.

1. Neutrino energy ranges are quite different so that the experiments need not be directly comparable. The mixing obeys the analog of Schrödinger equation for free particle with energy replaced with $\Delta m^2/E$, where E is neutrino energy. The mixing probability as a function of distance L from the source of muon neutrinos is in 2-component model given by

$$P = sin^2(\theta)sin^2(1.27\Delta m^2 L/E) \ .$$

The characteristic length scale for mixing is $L = E/\Delta m^2$. If L is sufficiently small, the mixing is fifty-fifty already before the muon neutrinos enter the system, where the measurement is carried out and no mixing is detected. If L is considerably longer than the size of the measuring system, no mixing is observed either. Therefore the result can be understood if Δm^2 is much larger or much smaller than E/L, where L is the size of the measuring system and E is the typical neutrino energy.

2. MiniBooNE experiment found evidence for the appearance of electron neutrinos at low neutrino energies (below 500 MeV) which means direct support for the LSND findings and for the dependence of neutron mass scale on its energy relative to the rest system defined by the space-time sheet of laboratory.
3. Uncertainty Principle inspires the guess $L_p \propto 1/E$ implying $m_p \propto E$. Here E is the energy of the neutrino with respect to the rest system defined by the space-time sheet of the laboratory. Solar neutrinos indeed have the lowest energy (below 20 MeV) and the lowest value of Δm^2. However, atmospheric neutrinos have energies starting from few hundreds of MeV and $\Delta; m^2$ is by a factor of order 10 higher. This suggests that the growth of Δm^2 with E^2 is slower than linear. It is perhaps not the energy alone which matters but the space-time sheet at which neutrinos topologically condense. For instance, MiniBooNE neutrinos above 500 MeV would topologically condense at space-time sheets for which the p-adic mass scale is higher than in LSND experiments and one would have $\Delta m^2 >> 1$ eV^2 implying maximal mixing in length scale much shorter than the size of experimental apparatus.
4. One could also argue that topological condensation occurs in condensed matter and that no topological condensation occurs for high enough neutrino energies so that neutrinos remain massless. One can even consider the possibility that the p-adic length scale L_p is proportional to E/m_0^2, where m_0 is proportional to the mass scale associated with non-relativistic neutrinos.

The p-adic mass scale would obey $m_p \propto m_0^2/E$ so that the characteristic mixing length would be by a factor of order 100 longer in MiniBooNE experiment than in LSND.

Comments

Some comments on the proposed scenario are in order: some of the are written much later than the previous text.

1. Mass predictions are consistent with the bound $\Delta m(\nu_\mu, \nu_e) < 2\ eV^2$ coming from the requirement that neutrino mixing does not spoil the so called r-process producing heavy elements in Super Novae [C58].
2. TGD neutrinos cannot solve the dark matter problem: the total neutrino mass required by the cold+hot dark matter models would be about 5 eV. In [K32] a model of galaxies based on string like objects of galaxy size and providing a more exotic source of dark matter, is discussed.
3. One could also consider the explanation of LSND data in terms of the interaction of ν_μ and nucleon via the exchange of $g = 1$ W boson. The fraction of the reactions $\bar{\nu}_\mu + p \rightarrow e^+ + n$ is at low neutrino energies $P \sim \frac{m_W^4(g=0)}{m_W^4(g=1)} sin^2(\theta_c)$, where θ_c denotes Cabibbo angle. Even if the condensation level of $W(g = 1)$ is $k = 89$, the ratio is by a factor of order .05 too small to explain the average $\nu_\mu \rightarrow \nu_e$ transformation probability $P \simeq .003$ extracted from LSND data.
4. The predicted masses exclude MSW and vacuum oscillation solutions to the solar neutrino problem unless one assumes that several condensation levels and thus mass scales are possible for neutrinos. This is indeed suggested by the previous considerations.

14.7.3 Quark Masses

The prediction or quark masses is more difficult due the facts that the deduction of even the p-adic length scale determining the masses of these quarks is a non-trivial task, and the original identification was indeed wrong. Second difficulty is related to the topological mixing of quarks. The new scenario leads to a unique identification of masses with top quark mass as an empirical input and the thermodynamical model of topological mixing as a new theoretical input. Also CKM matrix is predicted highly uniquely.

Basic mass formulas

By the earlier mass calculations and construction of CKM matrix the ground state conformal weights of U and D type quarks must be $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$. The formulas for the eigenvalues of CP_2 spinor Laplacian imply that if m_0^2 is used as a unit, color conformal weight $h_c \equiv m_{CP_2}^2$ is integer for $p\ mod\ = \pm 1$ for U type quark belonging to $(p+1, p)$ type representation and obeying $h_c(U) = (p^2 + 3p + 2)/3$ and for $p\ mod\ 3 = 1$ for D type quark belonging $(p, p+2)$ type representation and obeying $h_c(D) = (p^2 + 4p + 4)/3$. Only these states can be massless since color Hamiltonians have integer valued conformal weights.

In the recent case the minimal $p = 1$ states correspond to $h_c(U) = 2$ and $h_c(D) = 3$. $h_{gr}(U) = -1$ and $h_{gr}(D) = 0$ reproduce the previous results for quark masses required by the construction of CKM matrix. This requires super-symplectic operators O with a net conformal weight $h_{sc} = -3$ just as in the leptonic case. The facts that the values of p are minimal for spinor harmonics and the super-symplectic operator is same for both quarks and leptons suggest that the construction is not had hoc. The real justification would come from the demonstration that $h_{sc} = -3$ defines null state for SCV: this would also explain why h_{sc} would be same for all fermions.

Consider now the mass squared values for quarks. For $h(D) = 0$ and $h(U) = -1$ and using $m_0^2/3$ as a unit the expression for the thermal contribution to the mass squared of quark is given by the formula

$$
\begin{aligned}
M^2 &= (s+X)\frac{m_0^2}{p} \ , \\
s(U) &= 5 \ , \ s(D) = 8 \ , \\
X &\equiv \frac{(3Yp)_R}{3} \ ,
\end{aligned}
\tag{14.7.10}
$$

where the second order contribution Y corresponds to renormalization effects coming and depending on the isospin of the quark. When m_0^2 is used as a unit X is replaced by $X = (Y_p)_R$.

With the above described assumptions one has the following mass formula for quarks

$$
\begin{array}{l}
M^2(q) = A(q)\frac{m_0^2}{p(q)} \ , \\
\\
\begin{array}{lll}
A(u) = 5 + X_U(p(u) \ , & A(c) = 14 + X_U(p(c)) \ , & A(t) = 65 + X_U(p(t)) \ , \\
A(d) = 8 + X_D(p(d)) \ , & A(s) = 17 + X_D(p(s)) \ , & A(b) = 68 + X_D(p(b)) \ .
\end{array}
\end{array}
\tag{14.7.11}
$$

p-Adic length scale hypothesis allows to identify the p-adic primes labelling quarks whereas topological mixing of U and D quarks allows to deduce topological mixing matrices U and D and CKM matrix V and precise values of the masses apart from effects like color magnetic spin orbit splitting, color Coulomb energy, etc..

Integers n_{q_i} satisfying $\sum_i n(U_i) = \sum_i n(D_i) = 69$ characterize the masses of the quarks and also the topological mixing to high degree. The reason that modular contributions remain integers is that in the p-adic context non-trivial rationals would give CP_2 mass scale for the real counterpart of the mass squared. In the absence of mixing the values of integers are $n_d = n_u = 0$, $n_s = n_c = 9$, $n_b = n_t = 60$.

The fact that CKM matrix V expressible as a product $V = U^\dagger D$ of topological mixing matrices is near to a direct sum of 2×2 unit matrix and 1×1 unit matrix motivates the approximation $n_b \simeq n_t$. The large masses of top quark and of $t\bar{t}$ meson encourage to consider a scenario in which $n_t = n_b = n \leq 60$ holds true.

The model for topological mixing matrices and CKM matrix predicts U and D matrices highly uniquely and allows to understand quark and hadron masses in surprisingly detailed level.

1. $n_d = n_u = 60$ is not allowed by number theoretical conditions for U and D matrices and by the basic facts about CKM matrix but $n_t = n_b = 59$ allows almost maximal masses for b and t. This is not yet a complete hit. The unitarity of the mixing matrices and the construction of CKM matrix to be discussed in the next section forces the assignments

$$
(n_d, n_s, n_b) = (5,5,59) \ , \quad (n_u, n_c, n_t) = (5,6,58) \ . \tag{14.7.12}
$$

fixing completely the quark masses apart possible Higgs contribution [K70] . Note that top quark mass is still rather near to its maximal value.

2. The constraint that valence quark contribution to pion mass does not exceed pion mass implies the constraint $n(d) \leq 6$ and $n(u) \leq 6$ in accordance with the predictions of the model of topological mixing. $u-d$ mass difference does not affect $\pi^+ - \pi^0$ mass difference and the quark contribution to $m(\pi)$ is predicted to be $\sqrt{(n_d + n_u + 13)/24}\times 136.9$ MeV for the maximal value of CP_2 mass (second order p-adic contribution to electron mass squared vanishes).

The p-adic length scales associated with quarks and quark masses

The identification of p-adic length scales associated with the quarks has turned to be a highly nontrivial problem. The reasons are that for light quarks it is difficult to deduce information about quark masses for hadron masses and that the unknown details of the topological mixing (unknown until the advent of the thermodynamical model [K70]) made possible several p-adic length scales for quarks. It has also become clear that the p-adic length scale can be different form free quark and bound quark and that bound quark p-adic scale can depend on hadron.

Two natural constraints have however emerged from the recent work.

1. Quark contribution to the hadron mass cannot be larger than color contribution and for quarks having $k_q \neq 107$ quark contribution to mass is added to color contribution to the mass. For quarks with same value of k conformal weight rather than mass is additive whereas for quarks with different value of k masses are additive. An important implication is that for diagonal mesons $M = q\overline{q}$ having $k(q) \neq 107$ the condition $m(M) \geq \sqrt{2}m_q$ must hold true. This gives strong constraints on quark masses.
2. The realization that scaled up variants of quarks explain elegantly the masses of light hadrons allows to understand large mass splittings of light hadrons without the introduction of strong isospin-isospin interaction.

The new model for quark masses is based on the following identifications of the p-adic length scales.

1. The nuclear p-adic length scale $L_e(k)$, $k = 113$, corresponds to the p-adic length scale determining the masses of u, d, and s quarks. Note that $k = 113$ corresponds to a so called Gaussian Mersenne. The interpretation is that quark massivation occurs at nuclear space-time sheet at which quarks feed their em fluxes. At $k = 107$ space-time sheet, where quarks feed their color gauge fluxes, the quark masses are vanishing in the first p-adic order. This could be due to the fact that the p-adic temperature is $T_p = 1/2$ at this space-time sheet so that the thermal contribution to the mass squared is negligible. This would reflect the fact that color interactions do not involve any counterpart of Higgs mechanism.
   p-Adic mass calculations turn out to work remarkably well for massive quarks. The reason could be that M_{107} hadron physics means that *allb* quarks feed their color gauge fluxes to $k = 107$ space-time sheets so that color contribution to the masses becomes negligible for heavy quarks as compared to Super-Kac Moody and modular contributions corresponding to em gauge flux fed to $k > 107$ space-time sheets in case of heavy quarks. Note that Z^0 gauge flux is fed to space-time sheets at which neutrinos reside and screen the flux and their size corresponds to the neutrino mass scale. This picture might throw some light to the question of whether and how it might be possible to demonstrate the existence of M_{89} hadron physics. One might argue that $k = 107$ is not allowed as a condensation level in accordance with the idea that color and electro-weak gauge fluxes cannot be fed at the space-time space time sheet since the classical color and electro-weak fields are functionally independent. The identification of η' meson as a bound state of scaled up $k = 107$ quarks is not however consistent with this idea unless one assumes that $k = 107$ space-time sheets in question are separate.
2. The requirement that the masses of diagonal pseudo-scalar mesons of type $M = q\overline{q}$ are larger but as near as possible to the quark contribution $\sqrt{2}m_q$ to the valence quark mass, fixes the p-adic primes $p \simeq 2^k$ associated with c, b quarks but not t since toponium does not exist. These values of k are "nominal" since k seems to be dynamical. c quark corresponds to the p-adic length scale $k(c) = 104 = 2^3 \times 13$. b quark corresponds to $k(b) = 103$ for $n(b) = 5$. Direct determination of p-adic scale from top quark mass gives $k(t) = 94 = 2 \times 47$ so that secondary p-adic length scale is in question.
   Top quark mass tends to be slightly too low as compared to the most recent experimental value of $m(t) = 169.1$ GeV with the allowed range being [164.7, 175.5] GeV [C62] . The optimal situation corresponds to $Y_e = 0$ and $Y_t = 1$ and happens to give top mass exactly equal to the most probable experimental value. It must be emphasized that top quark is experimentally in a unique position since toponium does not exist and top quark mass is that of free top.

In the case of light quarks there are good reasons to believe that the p-adic mass scale of quark is different for free quark and bound state quark and that in case of bound quark it can also depend on hadron. This would explain the notions of valence (constituent) quark and current quark mass as masses of bound state quark and free quark and leads also to a TGD counterpart of Gell-Mann-Okubo mass formula [K70] .

1. Constituent quark masses

Constituent quark masses correspond to masses derived assuming that they are bound to hadrons. If the value of k is assumed to depend on hadron one obtains nice mass formula for light hadrons as will be found later. **Table 14.10** summarizes constituent quark masses as predicted by this model.

2. Current quark masses

Current quark masses would correspond to masses of free quarks which tend to be lower than valence quark masses. Hence k could be larger in the case of light quarks. The table of quark masses in Wikipedia [**?**]ives the value ranges for current quark masses depicted in **Table 14.9** together with TGD predictions for the spectrum of current quark masses.

q	d	u	s
$m(q)_{exp}/MeV$	4-8	1.5-4	80-130
$k(q)$	(122,121,120)	(125,124,123,122)	(114,113,112)
$m(q)/MeV$	(4.5,6.6,9.3)	(1.4,2.0,2.9,4.1)	(74,105,149)
q	c	b	t
$m(q)_{exp}/MeV$	1150-1350	4100-4400	1691
$k(q)$	(106,105)	(105,104)	92
$m(q)/MeV$	(1045,1477)	(3823,5407)	167.8×10^3

Table 14.9: The experimental value ranges for current quark masses [**?**]nd TGD predictions for their values assuming $(n_d, n_s, n_b) = (5, 5, 59)$, $(n_u, n_c, n_t) = (5, 6, 58)$, and $Y_e = 0$. For top quark $Y_t = 0$ is assumed. $Y_t = 1$ would give 169.2 GeV.

Some comments are in order.

1. The long p-adic length associated with light quarks seem to be in conflict with the idea that quarks have sizes smaller than hadron size. The paradox disappears when one realized that $k(q)$ characterizes the electromagnetic "field body" of quark having much larger size than hadron.
2. u and d current quarks correspond to a mass scale not much higher than that of electron and the ranges for mass estimates suggest that u could correspond to scales $k(u) \in (125, 124, 123, 122) = (5^3, 4 \times 31, 3 \times 41, 2 \times 61)$, whereas d would correspond to $k(d) \in (122, 121, 120) = (2 \times 61, 11^2, 3 \times 5 \times 8)$.
3. The TGD based model for nuclei based on the notion of nuclear string leads to the conclusion that exotic copies of $k = 113$ quarks having $k = 127$ are present in nuclei and are responsible for the color binding of nuclei [K93, L3] , [L3] .
4. The predicted values for c and b masses are slightly too low for $(k(c), k(b)) = (106, 105) = (2 \times 53, 3 \times 5 \times 7)$. Second order Higgs contribution could increase the c mass into the range given in [C5] but not that of b.
5. The mass of top quark has been slightly below the experimental estimate for long time. The experimental value has been coming down slowly and the most recent value obtained by CDF [C63] is $m_t = 165.1 \pm 3.3 \pm 3.1$ GeV and consistent with the TGD prediction for $Y_e = Y_t = 0$.

One can talk about constituent and current quark masses simultaneously only if they correspond to dual descriptions. $M^8 - H$ duality [K60] has been indeed suggested to relate the old fashioned low energy description of hadrons in terms of $SO(4)$ symmetry (Skyrme model) and higher energy description of hadrons based on QCD. In QCD description the mass of say baryon would be dominated by the mass associated with super-symplectic quanta carrying color. In $SO(4)$ description constituent quarks would carry most of the hadron mass.

Can Higgs field develop a vacuum expectation in fermionic sector at all?

An important conclusion following from the calculation of lepton and quark masses is that if Higgs contribution is present, it can be of second order p-adically and even negligible, perhaps even vanishing. There is indeed an argument forcing to consider this possibility seriously. The recent view about elementary particles is following.

1. Fermions correspond to CP_2 type vacuum extremals topologically condensed at positive/negative energy space-time sheets carrying quantum numbers at light-like wormhole throat. Higgs and gauge bosons correspond to wormhole contacts connecting positive and negative energy space-time sheets and carrying fermion and anti-fermion quantum numbers at the two light-like wormhole throats.

2. If the values of p-adic temperature are $T_p = 1$ and $T_p = 1/n$, $n > 1$f or fermions and bosons the thermodynamical contribution to the gauge boson mass is negligible.
3. Different p-adic temperatures and Kähler coupling strengths for fermions and bosons make sense if bosonic and fermionic partonic 3-surfaces meet only along their ends at the vertices of generalized Feynman diagrams but have no other common points [K29] . This forces to consider the possibility that fermions cannot develop Higgs vacuum expectation value although they can couple to Higgs. This is not in contradiction with the modification of sigma model of hadrons based on the assumption that vacuum expectation of σ field gives a small contribution to hadron mass [K64] since this field can be assigned to some bosonic space-time sheet pair associated with hadron.
4. Perhaps the most elegant interpretation is that ground state conformal is equal to the square of the eigenvalue of the modified Dirac operator. The ground state conformal weight is negative and its deviation from half odd integer value gives contribution to both fermion and boson masses. The Higgs expectation associated with coherent state of Higgs like wormhole contacts is naturally proportional to this parameter since no other parameter with dimensions of mass is present. Higgs vacuum expectation determines gauge boson masses only apparently if this interpretation is correct. The contribution of the ground state conformal weight to fermion mass square is near to zero. This means that λ is very near to negative half odd integer and therefore no significant difference between fermions and gauge bosons is implied.

q	d	u	s	c	b	t
n_q	4	5	6	6	59	58
s_q	12	10	14	11	67	63
$k(q)$	113	113	113	104	103	94
$m(q)/GeV$	.105	.092	.105	2.191	7.647	167.8

Table 14.10: Constituent quark masses predicted for diagonal mesons assuming $(n_d, n_s, n_b) = (5, 5, 59)$ and $(n_u, n_c, n_t) = (5, 6, 58)$, maximal CP_2 mass scale($Y_e = 0$), and vanishing of second order contributions.

14.8 About The Microscopic Description Of Gauge Boson Massivation

The conjectured QFT limit allows to estimate the quantitative predictions of the theory. This is not however enough. One should identify the microscopic TGD counterparts for various aspects of gauge boson massivation. There is also the question about the consistency of the gauge theory limit with the ZEO inspired view about massivation. The basic challenge are obvious: one should translate notions like Higgs vacuum expectation, massivation of gauge bosons, and finite range of weak interactions to the language of wormhole throats, Kähler magnetic flux tubes, and string world sheets. The proposal is that generalization of super-conformal symmetries to their Yangian counterparts is needed to meet this challenge in mathematically satisfactory manner.

14.8.1 Can P-Adic Thermodynamics Explain The Masses Of Intermediate Gauge Bosons?

The requirement that the electron-intermediate gauge boson mass ratios are sensible, serves as a stringent test for the hypothesis that intermediate gauge boson masses result from the p-adic thermodynamics. It seems that the only possible option is that the parameter k has same value for both bosons, leptons, and quarks:

$$k_B = k_L = k_q = 1 \ .$$

In this case all gauge bosons have $D(0) = 1$ and there are good changes to obtain boson masses correctly. $k = 1$ together with $T_p = 1$ implies that the thermal masses of very many boson states are extremely heavy so that the spectrum of the boson exotics is reduced drastically. For $T_p = 1/2$ the thermal contribution to the mass squared is completely negligible.

Contrary to the original optimistic beliefs based on calculational error, it turned out impossible to predict W/e and Z/e mass ratios correctly in the original p-adic thermodynamics scenario. Although the errors are of order 20-30 percent, they seemed to exclude the explanation for the massivation of gauge bosons using p-adic thermodynamics.

1. The thermal mass squared for a boson state with N active sectors (non-vanishing vacuum weight) is determined by the partition function for the tensor product of N NS type Super Virasoro algebras. The degeneracies of the excited states as a function of N and the weight Δ of the operator creating the massless state are given in the table below.
2. Both W and Z must correspond to $N = 2$ active Super Virasoro sectors for which $D(1) = 1$ and $D(2) = 3$ so that (using the formulas of p-adic thermodynamics the thermal mass squared is $m^2 = k_B(p + 5p^2)$ for $T_p = 1$. The second order contribution to the thermal mass squared is extremely small so that Weinberg angle vanishes in the thermal approximation. $k_B = 1$ gives Z/e mass-ratio which is about 22 per cent too high. For $T_p = 1/2$ thermal masses are completely negligible.
3. The thermal prediction for W-boson mass is the same as for Z^0 mass and thus even worse since the two masses are related $M_W^2 = M_Z^2 cos^2(\theta_W)$.

The conclusion is that p-adic thermodynamics does not produce a natural description for the massivation of weak bosons. For $p = M_{89}$ the mass scale is somewhat too small even if the second order contribution is maximal. If it is characterized by small integer, the contribution is extremely small. An explanation for the value of Weinberg angle is also missing. Hence some additional contribution to mass must be present. Higgsy contribution is not natural in TGD framework but stringy contribution looks very natural.

14.8.2 The Counterpart Of Higgs Vacuum Expectation In TGD

The development of the TGD view about Higgs involves several wrong tracks involving a lot of useless calculation. All this could have been avoided with more precise definition of basic notions. The following view has distilled through several failures and might be taken as starting point.

The basic challenge is to translate the QFT description of gauge boson massivation to microscopic description.

1. One can say that gauge bosons "eat" the components of Higgs. In unitary gauge one gauge rotates Higgs field to electromagnetically neutral direction defined by the vacuum expectation value of Higgs. The rotation matrix codes for the degrees of freedom assignable to non-neutral part of Higgs and they are transferred to the longitudinal components of Higgs in gauge transformation. This gives rise to the third polarization direction for gauge boson. Photon remains massless because em charge commutes with Higgs.
2. The generation of vacuum expectation value has two functions: to make weak gauge bosons massive and to define the electromagnetically neutral direction to which Higgs field is rotated in the transition to the unitary gauge. In TGD framework only the latter function remains for Higgs if p-adic thermodynamics takes care of massivation. The notion of induced gauge field together with CP_2 geometry uniquely defines the electromagnetically neutral direction so that vacuum expectation is not needed. Of course, the essential element is gauge invariance of the Higgs gauge boson couplings. In twistor Grassmann approach gauge invariance is replaced with Yangian symmetry, which is excellent candidate also for basic symmetry of TGD.
3. The massivation of gauge bosons (all particles) involves two contributions. The contribution from p-adic thermodynamics in CP_2 scale (wormhole throat) and the stringy contribution in weak scale more generally, in hadronic scale. The latter contribution cannot be calculated yet. The generalization of p-adic thermodynamics to that for Yangian symmetry instead of mere super-conformal symmetry is probably necessary to achieve this and the construction WCW geometry and spinor structure strongly supports the interpretation in terms of Yangian.

One can look at the situation also at quantitative level.

1. W/Z mass ratio is extremely sensitive test for any model for massivation. In the recent case this requires that string tension for weak gauge boson depends on boson and is proportional

to the appropriate gauge coupling strength depending on Weinberg angle. This is natural if the contribution to mass squared can be regarded as perturbative.

2. Higgs mechanism is characterized by the parameter m_0^2 defining the originally tachyonic mass of Higgs, the dimensionless coupling constant λ defining quartic self-interaction of Higgs. Higgs vacuum expectation is given by $\mu^2 = m_0^2/\lambda$, Higgs mass squared by $m_0^2 = \mu^2\lambda$, and weak boson mass squared is proportional $g^2\mu^2$. In TGD framework λ takes the role of g^2 in stringy picture and the string tensions of bosons are proportional to g_w^2, g_Z^2, λ respectively.
3. Whether λ in TGD framework actually corresponds to the quartic self-coupling of Higgs or just to the numerical factor in Higgs string tension, is not clear. The problem of Higgs mechanism is that the mass of observed Higgs is somewhat too low. This requires fine tuning of the parameters of the theory and SUSY, which was hoped to come in rescue, did not solve the problem. TGD approach promises to solve the problem.

14.8.3 Elementary Particles In ZEO

Let us first summarize what kind of picture ZEO suggests about elementary particles.

1. Kähler magnetically charged wormhole throats are the basic building bricks of elementary particles. The lines of generalized Feynman diagrams are identified as the Euclidian regions of space-time surface. The weak form of electric magnetic duality forces magnetic monopoles and gives classical quantization of the Kähler electric charge. Wormhole throat is a carrier of many-fermion state with parallel momenta and the fermionic oscillator algebra gives rise to a badly broken large $\mathcal{N}$ SUSY [**?**].
2. The first guess would be that elementary fermions correspond to wormhole throats with unit fermion number and bosons to wormhole contacts carrying fermion and anti-fermion at opposite throats. The magnetic charges of wormhole throats do not however allow this option. The reason is that the field lines of Kähler magnetic monopole field must close. Both in the case of fermions and bosons one must have a pair of wormhole contacts (see **Fig.** `http://tgdtheory.fi/appfigures/wormholecontact.jpg` or **Fig. ??** in the appendix of this book) connected by flux tubes. The most general option is that net quantum numbers are distributed amongst the four wormhole throats. A simpler option is that quantum numbers are carried by the second wormhole: fermion quantum numbers would be carried by its second throat and bosonic quantum numbers by fermion and anti-fermion at the opposite throats. All elementary particles would therefore be accompanied by parallel flux tubes and string world sheets.
3. A cautious proposal in its original form was that the throats of the other wormhole contact could carry weak isospin represented in terms of neutrinos and neutralizing the weak isospin of the fermion at second end. This would imply weak neutrality and weak confinement above length scales longer than the length of the flux tube. This condition might be un-necessarily strong.

   The realization of the weak neutrality using pair of left handed neutrino and right handed antineutrino or a conjugate of this state is possible if one allows right-handed neutrino to have also unphysical helicity. The weak screening of a fermion at wormhole throat is possible if ν_R is a constant spinor since in this case Dirac equation trivializes and allows both helicities as solutions. The new element from the solution of the Kähler-Dirac equation is that ν_R would be interior mode de-localized either to the other wormhole contact or to the Minkowskian flux tube. The state at the other end of the flux tube is spartner of left-handed neutrino.

   It must be emphasized that weak confinement is just a proposal and looks somewhat complex: Nature is perhaps not so complex at the basic level. To understand this better, one can think about how M_{89} mesons having quark and antiquark at the ends of long flux tube returning back along second space-time sheet could decay to ordinary quark and antiquark.

14.8.4 Virtual And Real Particles And Gauge Conditions In ZEO

ZEO and twistor Grassmann approach force to build a detailed view about real and virtual particles. ZEO suggests also new approaches to gauge conditions in the attempts to build detailed connection between QFT picture and that provided by TGD. The following is the most conservative one. Of course, also this proposal must be taken with extreme cautiousness.

1. In ZEO all wormhole throats - also those associated with virtual particles - can be regarded as massless. In twistor Grassmann approach [K100] this means that the fermionic propagators can be by residue integration transformed to their inverses which correspond to online massless states but having an unphysical polarization so that the internal lines do not vanish identically.
2. This picture inspired by twistorial considerations is consistent with the simplest picture about Kähler-Dirac action. The boundary term for K-D action is $\sqrt{g_4}\overline{\Psi}\Gamma^n_{K-D}\Psi d^3x$ and due to the localization of spinor modes to 2-D surfaces reduces to a term localized at the boundaries of string world sheets. The normal component Γ^n_{K-D} of the Kähler-Dirac gamma matrices defined by the canonical momentum currents of Kähler action should define the inverse of massless fermionic propagator. If the action of this operator on the induced spinor mode at stringy curves satisfies

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi \quad ,$$

this reduction is achieved. One can pose the condition $g_4 = constant$ as a coordinate condition on stringy curves at the boundaries of CD and the condition would correlate the spinor modes at stringy curve with incoming quantum numbers. This is extremely powerful simplification giving hopes about calculable theory. The residue integral for virtual momenta reduces the situation to integral over on mass shell momenta and only non-physical helicities contribute in internal lines. This would generalize twistorial formulas to fermionic context.

One however ends up with an unexpected prediction which has bothered me for a long time. Consider the representation of massless spin 1 gauge bosons as pairs as wormhole throat carrying fermion and antifermion having net quantum numbers of the boson. Neglect the effects of the second wormhole throat. The problem is that for on-mass shell massless fermion and antifermion with physical helicities the boson has spin 0. Helicity 1 state would require that second fermion has unphysical helicity. What does this mean?

1. Are all on mass shell gauge bosons - including photon - massive? Or is on mass shell massless propagation impossible? Massivation is achieved if the fermion and antifermion have different momentum directions: for instance opposite 3-momen but same sign of energy. Higher order contributions in p-adic thermodynamics could make also photon massive. The 4-D world-lines of fermion and antifermion would not be however parallel, which does not conform with the geometric optics based prejudices.
2. Or could on mass shell gauge bosons have opposite four-momenta so that the second gauge boson would have negative energy? In this manner one could have massless on mass shell states. ZEO ontology certainly allows the identification massless gauge bosons as on mass shell states with opposite directions of four-momenta. This would however require the weakening of the hypothesis that all incoming (outgoing) fundamental fermions have positive (negative) energies to the assumption that only the incoming (outgoing) particles have positive (negative) energies. In the case of massless gauge boson the gauge condition $p \cdot \epsilon = 0$ would be satisfied by the momenta of both fermion and antifermion. With opposite 3-momenta (massivation) but same energy the condition $p_{tot} \cdot \epsilon = 0$ is satisfied for three polarization since in cm system p_{tot} has only time component.
3. The problem is present also for internal lines. Since by residue argument only the unphysical fermion helicities contribute in internal lines, both fermion and antifermion must have unphysical helicity. For the same sign of energy the wormhole throat would behave as scalar particle. Therefore it seems that the energies must have different sign or momenta cannot be strictly parallel. This is required also by the possibility of space-like momenta for virtual bosons.

14.8.5 The Role Of String World Sheets And Magnetic Flux Tubes In Massivation

What is the role of string world sheets and flux tubes in the massivation? At the fundamental level one studies correlation functions for particles and finite correlation length means massivation.

1. String world sheets define as essential element in 4-D description. All particles are basically bi-local objects: pairs of string at parallel space-time sheets extremely near to each other and connected by wormhole contacts at ends. String world sheets are expected to represent correlations between wormhole throats.

2. Correlation length for the propagator of the gauge boson characterizes its mass. Correlation length can be estimated by calculating the correlation function. For bosons this reduces to the calculation of fermionic correlations functions assignable to string world sheets connecting the upper and lower boundaries of CD and having four external fermions at the ends of CD. The perturbation theory reduces to functional integral over space-time sheets and deformation of the space-time sheet inducing the deformation of the induced spinor field expressible as convolution of the propagator associated with the Kähler-Dirac operator with vertex factor defined by the deformation multiplying the spinor field. The external vertices are braid ends at partonic 2-surfaces and internal vertices are in the interior of string world sheet. Recall that the conjecture is that the restriction to the wormhole throat orbits implies the reduction to diagrams involving only propagators connecting braid ends. The challenge is to understand how the coherent state assigned to the Euclidian pion field induces the finite correlation length in the case of gauge bosons other than photon.
3. The non-vanishing commutator of the gauge boson charge matrix with the vacuum expectation assigned to the Euclidian pion must play a key role. The study of the Kähler-Dirac operator suggests that the braid strands contain the Abelianized variant of non-integrable phase factor defined as $exp(i\int Adx)$. If A is identified as string world sheet Hodge dual of Kac-Moody charge the opposite edges of string world sheet with geometry of square given contributions which compensate each other by conservation of Kac-Moody charge if A commutes with the operators building the coherent Higgs state. For photon this would be true. For weak gauge bosons this would not be the case and this gives hopes about obtaining destructive interference leading to a finite correlation length.

One can also consider try to build more concrete ways to understand the finite correlation length.

1. Quantum classical correspondence suggests that string with length of order $L \sim \hbar/E$, $E = \sqrt{p^2+m^2}$ serves as a correlate for particle defined by a pair of wormhole contacts. For massive particle wave length satisfies $L \leq \hbar/m$. Here (p,m) must be replaced with (p_L, m_L) if one takes the notion of longitudinal mass seriously. For photon standard option gives $L = \lambda$ or $L = \lambda_L$ and photon can be a bi-local object connecting arbitrarily distant objects. For the second option small longitudinal mass of photon gives an upper bound for the range of the interaction. Also gluon would have longitudinal mass: this makes sense in QCD where the decomposition $M^4 = M^2 \times E^2$ is basic element of the theory.
2. The magnetic flux tube associated with the particle carries magnetic energy. Magnetic energy grows as the length of flux tube increases. If the flux is quantized magnetic field behaves like $1/S$, where S is the area of the cross section of the flux tube, the total magnetic energy behaves like L/S. The dependence of S on L determines how the magnetic energy depends on L. If the magnetic energy increases as function of L the probability of long flux tubes is small and the particle cannot have large size and therefore mediates short range interactions. For $S \propto L^\alpha \sim \lambda^\alpha$, $\alpha > 1$, the magnetic energy behaves like $\lambda^{-\alpha+1}$ and the thickness of the flux tube scales like $\sqrt{\lambda^\alpha}$. In case of photon one might expect this option to be true. Note that for photon string world sheet one can argue that the natural choice of string is as light-like string so that its length vanishes.

What kind of string world sheets are possible? One can imagine two options.

1. All strings could connect only the wormhole contacts defining a particle as a bi-local object so that particle would be literally the geometric correlate for the interaction between two objects. The notion of free particle would be figment of imagination. This would lead to a rather stringy picture about gauge interactions. The gauge interaction between systems S_1 and S_2 would mean the emission of gauge bosons as flux tubes with charge carrying end at S_1 and neutral end. Absorption of the gauge boson would mean that the neutral end of boson and neutral end of charge particle fuse together line the lines of Feynman diagram at 3-vertex.
2. Second option allows also string world sheets connecting wormhole contacts of different particles so that there is no flux tube accompanying the string world sheet. In this case particles would be independent entities interacting via string world sheets. In this case one could consider the possibility that photon corresponds to string world sheet (or actually parallel pair of them) not accompanied by a magnetic flux tube and that this makes the photon massless at least in excellent approximation.

The first option represents the ontological minimum.

Super-conformal symmetry involves two conformal weight like integers and these correspond to the conformal weight assignable to the radial light-like coordinate appearing in the role of complex coordiante in super-symplectic Hamiltonians and to the spinorial conformal weight assignable to the solutions of Kähler Dirac equation localized to string world sheets. These conformal weights are independent quantum numbers unless one can use the light-like radial coordinate as string coordinate, which is certainly not possible always. The latter conformal weight should correspond to the stringy contribution to the masses of elementary particles and hadron like states. In fact, it is difficult to distinguish between elementary particles and hadrons at the fundamental level since both involve the stringy aspect.

The Yangian symmetry variant of conformal symmetry is highly suggestive and brings in poly-locality with respect to partonic 2-surfaces. This integer would count the number of partonic 2-surfaces to which the generator acts and need not correspond to spinorial conformal weight as one might think first. In any case, Yangian variant of p-adic termodynamics provides an attractive approach concerning the mathematical realization of this vision.

14.8.6 Weak Regge Trajectories

The weak form of electric-magnetic duality suggests strongly the existence of weak Regge trajectories.

1. The most general mass squared formula with spin-orbit interaction term $M^2_{L-S}L\cdot S$ reads as

$$M^2 = nM_1^2 + M_0^2 + M^2_{L-S}L\cdot S \ , \ n = 0,2,4 \ \text{or} \ n = 1,3,5,..., \ . \quad (14.8.1)$$

M_1^2 corresponds to string tension and M_0^2 corresponds to the thermodynamical mass squared and possible other contributions. For a given trajectory even (odd) values of n have same parity and can correspond to excitations of same ground state. From ancient books written about hadronic string model one vaguely recalls that one can have several trajectories (satellites) and if one has something called exchange degeneracy, the even and odd trajectories define single line in M^2-J plane. As already noticed TGD variant of Higgs mechanism combines together $n=0$ states and $n=1$ states to form massive gauge bosons so that the trajectories are not independent.

2. For fermions, possible Higgs, and pseudo-scalar Higgs and their super partners also p-adic thermodynamical contributions are present. M_0^2 must be non-vanishing also for gauge bosons and be equal to the mass squared for the $n=L=1$ spin singlet. By applying the formula to $h=\pm 1$ states one obtains

$$M_0^2 = M^2(boson) \ . \quad (14.8.2)$$

The mass squared for transversal polarizations with $(h,n,L)=(\pm 1, n=L=0, S=1)$ should be same as for the longitudinal polarization with $(h=0, n=L=1, S=1, J=0)$ state. This gives

$$M_1^2 + M_0^2 + M^2_{L-S}L\cdot S = M_0^2 \ . \quad (14.8.3)$$

From $L\cdot S = [J(J+1)-L(L+1)-S(S+1)]/2 = -2$ for $J=0, L=S=1$ one has

$$M^2_{L-S} = -\frac{M_1^2}{2} \ . \quad (14.8.4)$$

Only the value of weak string tension M_1^2 remains open.

3. If one applies this formula to arbitrary $n=L$ one obtains total spins $J=L+1$ and $L-1$ from the tensor product. For $J=L-1$ one obtains

$$M^2 = (2n+1)M_1^2 + M_0^2 \ .$$

For $J=L+1$ only M_0^2 contribution remains so that one would have infinite degeneracy of the lightest states. Therefore stringy mass formula must contain a non-linear term making Regge trajectory curved. The simplest possible generalization which does not affect n=0 and n=1 states is of from

$$M^2 = n(n-1)M_2^2 + (n - \frac{L \cdot S}{2})M_1^2 + M_0^2 \ . \quad (14.8.5)$$

The challenge is to understand the ratio of W and Z^0 masses, which is purely group theoretic and provides a strong support for the massivation by Higgs mechanism.

1. The above formula and empirical facts require

$$\frac{M_0^2(W)}{M_0^2(Z)} = \frac{M^2(W)}{M^2(Z)} = cos^2(\theta_W) \ . \quad (14.8.6)$$

in excellent approximation. Since this parameter measures the interaction energy of the fermion and anti-fermion decomposing the gauge boson depending on the net quantum numbers of the pair, it would look very natural that one would have

$$M_0^2(W) = g_W^2 M_{SU(2)}^2 \ , \ M_0^2(Z) = g_Z^2 M_{SU(2)}^2 \ . \quad (14.8.7)$$

Here $M_{SU(2)}^2$ would be the fundamental mass squared parameter for $SU(2)$ gauge bosons. p-Adic thermodynamics of course gives additional contribution which is vanishing or very small for gauge bosons.

2. The required mass ratio would result in an excellent approximation if one assumes that the mass scales associated with $SU(2)$ and $U(1)$ factors suffer a mixing completely analogous to the mixing of $U(1)$ gauge boson and neutral $SU(2)$ gauge boson W_3 leading to γ and Z^0. Also Higgs, which consists of $SU(2)$ triplet and singlet in TGD Universe, would very naturally suffer similar mixing. Hence $M_0(B)$ for gauge boson B would be analogous to the vacuum expectation of corresponding mixed Higgs component. More precisely, one would have

$$\begin{aligned} M_0(W) &= M_{SU(2)} \ , \\ M_0(Z) &= cos(\theta_W)M_{SU(2)} + sin(\theta_W)M_{U(1)} \ , \\ M_0(\gamma) &= -sin(\theta_W)M_{SU(2)} + cos(\theta_W)M_{U(1)} \ . \end{aligned} \quad (14.8.8)$$

The condition that photon mass is very small and corresponds to IR cutoff mass scale gives $M_0(\gamma) = \epsilon cos(\theta_W)M_{SU(2)}$, where ϵ is very small number, and implies

$$\begin{aligned} \frac{M_{U(1)}}{M(W)} &= tan(\theta_W) + \epsilon \ , \\ \frac{M(\gamma)}{M(W)} &= \epsilon \times cos(\theta_W) \ , \\ \frac{M(Z)}{M(W)} &= \frac{1 + \epsilon \times sin(\theta_W)cos(\theta_W)}{cos(\theta_W)} \ . \end{aligned} \quad (14.8.9)$$

There is a small deviation from the prediction of the standard model for W/Z mass ratio but by the smallness of photon mass the deviation is so small that there is no hope of measuring it. One can of course keep mind open for $\epsilon = 0$. The formulas allow also an interpretation in terms of Higgs vacuum expectations as it must. The vacuum expectation would most naturally correspond to interaction energy between the massless fermion and anti-fermion with opposite 3-momenta at the throats of the wormhole contact and the challenge is to show that the proposed formulas characterize this interaction energy. Since CP_2 geometry codes for standard model symmetries and their breaking, it would not be surprising if this would happen. One cannot exclude the possibility that p-adic thermodynamics contributes to $M_0^2(boson)$. For instance, ϵ might characterize the p-adic thermal mass of photon.
If the mixing applies to the entire Regge trajectories, the above formulas would apply also to weak string tensions, and also photons would belong to Regge trajectories containing high spin excitations.

3. What one can one say about the value of the weak string tension M_1^2? The naïve order of magnitude estimate is $M_1^2 \simeq m_W^2 \simeq 10^4$ GeV^2 is by a factor 1/25 smaller than the direct scaling up of the hadronic string tension about 1 GeV^2 scaled up by a factor 2^{18}. The above argument however allows also the identification as the scaled up variant of hadronic string tension in which case the higher states at weak Regge trajectories would not be easy to discover since

the mass scale defined by string tension would be 512 GeV to be compared with the recent beam energy 7 TeV. Weak string tension need of course not be equal to the scaled up hadronic string tension. Weak string tension - unlike its hadronic counterpart- could also depend on the electromagnetic charge and other characteristics of the particle.

14.8.7 Low Mass Exotic Mesonic Structures As Evidence For Dark Scaled Down Variants Of Weak Bosons?

During last years reports about low mass exotic mesonic structures have appeared. It is interesting to combine these bits of data with the recent view about TGD analog of Higgs mechanism and find whether new predictions become possible. The basic idea is to derive understanding of the low mass exotic structures from LHC data by scaling and understanding of LHC data from data about mesonic structures by scaling back.

1. The article *Search for low-mass exotic mesonic structures: II. attempts to understand the experimental results* by Taticheff and Tomasi-Gustafsson (see `http://tinyurl.com/ybq323yy`) [C65] mentions evidence for exotic mesonic structures. The motivation came from the observation of a narrow range of dimuon masses in $\Sigma^+ \rightarrow pP^0$, $P^0 \rightarrow \mu^-\mu^+$ in the decays of P^0 with mass of $214.3 \pm .5$ MeV: muon mass is 105.7 MeV giving $2m_\mu = 211.4$ MeV. Mesonlike exotic states with masses $M = 62, 80, 100, 181, 198, 215, 227.5$, and 235 MeV are reported. This fine structure of states with mass difference 20-40 MeV between nearby states is reported for also for some baryons.
2. The preprint *Observation of the E(38) boson* by Kh.U. Abraamyan *et al* (see `http://tinyurl.com/y7zer8dw`) [C11, C12, C27] reports the observation of what they call E(38) boson decaying to gamma pair observed in d(2.0 GeV/n)+C,d(3.0 GeV/n)+Cu and p(4.6 GeV)+C reactions in experiments carried in JINR Nuclotron.

If these results can be replicated they mean a revolution in nuclear and hadron physics. What strongly suggests itself is a fine structure for ordinary hadron states in much smaller energy scale than characterizing hadronic states. Unfortunately the main stream, in particular the theoreticians interested in beyond standard model physics, regard the physics of strong interactions and weak interactions as closed chapters of physics, and are not interested on results obtained in nuclear collisions.

In TGD framework situation is different. The basic characteristic of TGD Universe is fractality. This predicts new physics in all scales although standard model symmetries are fundamental unlike in GUTs and are reduced to number theory. p-Adic length scale hypothesis characterizes the fractality.

1. In TGD Universe p-adic length scale hypothesis predicts the possibility of scaled versions of both strong and weak interactions. The basic objection against new light bosons is that the decay widths of weak bosons do not allow them. A possible manner to circumvent the objection is that the new light states correspond to dark matter in the sense that the value of Planck constant is not the standard one but its integer multiple [K42].
   The assumption that only particles with the same value of Planck constant can appear in the vertex, would explain why weak bosons do not decay directly to light dark particles. One must however allow the transformation of gauge bosons to their dark counterparts. The 2-particle vertex is characterized by a coupling having dimensions of mass squared in the case of bosons, and p-adic length scale hypothesis suggests that the primary p-adic mass scale characterizes the parameter (the secondary p-adic mass scale is lower by factor $1/\sqrt{p}$ and would give extremely small transformation rate).
2. Ordinary strong interactions correspond to Mersenne prime M_n, $n = 2^{107} - 1$, in the sense that hadronic space-time sheets correspond to this p-adic prime. Light quarks correspond to space-time sheets identifiable as color magnetic flux tubes, which are much larger than hadron itself. M_{89} hadron physics has hadronic mass scale 512 times higher than ordinary hadron physics and should be observed at LHC. There exist some pieces of evidence for the mesons of this hadron physics but masked by the Higgsteria. The expectation is that Minkowskian M_{89} pion has mass around 140 GeV assigned to CDF bump (see `http://tinyurl.com/yc98cau6`) [C16].
3. In the leptonic sector there is evidence for lepto-hadron physics for all charged leptons labelled by Mersenne primes M_{127}, $M_{G,113}$ (Gaussian Mersenne), and M_{107} [K104]. One can

ask whether the above mentioned resonance P^0 decaying to $\mu^-\mu^+$ pair motivating the work described in [C65] could correspond to pion of muon-hadron physics consisting of a pair of color octet excitations of muon. Its production would presumably take place via production of virtual gluon pair decaying to a pair of color octet muons.

Returning to the observations of [C65]: the reported meson-like exotic states seem to be arranged along Regge trajectories but with string tension lower than that for the ordinary Regge trajectories with string tension $T = .9$ GeV^2. String tension increases slowly with mass of meson like state and has three values $T/GeV^2 \in \{1/390, 1/149.7, 1/32.5\}$ in the piecewise linear fit discussed in the article. The TGD inspired proposal is that IR Regge trajectories assignable to the color magnetic flux tubes accompanying quarks are in question. For instance, in hadrons u and d quarks - understood as constituent quarks - would have $k = 113$ quarks and string tension would be by naïve scaling by a factor $2^{107-113} = 1/64$ lower: as a matter of fact, the largest value of the string tension is twice this value. For current quark with mass scale around 5 MeV the string tension would be by a factor of order $2^{107-121} = 2^{-16}$ lower.

Clearly, a lot of new physics is predicted and it begins to look that fractality - one of the key predictions of TGD - might be realized both in the sense of hierarchy of Planck constants (scaled variants with same mass) and p-adic length scale hypothesis (scaled variants with varying masses). Both hierarchies would represent dark matter if one assumes that the values of Planck constant and p-adic length scale are same in given vertex. The testing of predictions is not however expected to be easy since one must understand how ordinary matter transforms to dark matter and vice versa. Consider only the fact, that only recently the exotic meson like states have been observed and modern nuclear physics regarded often as more or less trivial low energy phenomenology was born born about 80 years ago when Chadwick discovered neutron.

14.8.8 Cautious Conclusions

The discussion of TGD counterpart of Higgs mechanism gives support for the following general picture.

1. p-Adic thermodynamics for wormhole contacts contributes to the masses of all particles including photon and gluons: in these cases the contributions are however small. For fermions they dominate. For weak bosons the contribution from string tension of string connecting wormhole contacts as the correct group theoretical prediction for the W/Z mass ratio demonstrates. The mere spin 1 character for gauge bosons implies that they are massive in 4-D sense unless massless fermion and anti-fermion have opposite signs of energy. Higgs provides the longitudinal components of weak bosons by gauge invariance and CP_2 geometry defines unitary gauge so that Higgs vacuum expectation value is not needed. The non-existence of covariantly constant CP_2 vector field does not mean absence of Higgs like particle as believed first but only the impossibility of Higgs vacuum expectation value.
   The usual space-time SUSY associated with embedding space in TGD framework is not needed, and there are strong arguments suggesting that it is not present [**?**] For space-time regarded as 4-surfaces one obtains 2-D super-conformal invariance for fermions localized at 2-surfaces and for right-handed neutrino it extends to 4-D superconformal symmetry generalizing ordinary SUSY to infinite-D symmetry.
2. The basic predictions to LHC are following. M_{89} hadron physics, whose pion was first proposed to be identifiable as Higgs like particle, will be discovered. The findings from RHIC and LHC concerning collisions of heavy ions and protons and heavy ions already provide support for the existence of string like objects identifiable as mesons of M_{89} physics. Fermi satellite has produced evidence for a particle with mass around 140 GeV and this particle could correspond to the pion of M_{89} physics. This particle should be observed also at LHC and CDF reported already earlier evidence for it. There has been also indications for other mesons of M_{89} physics from LHC discussed in [K64].
3. Fermion and boson massivation by Higgs mechanism could emerge unavoidably as a theoretical artefact if one requires the existence of QFT limit leading unavoidably to a description in terms of Higgs mechanism. In the real microscopic theory p-adic thermodynamics for wormhole contacts and strings connecting them would describe fermion massivation, and might describe even boson massivation in terms of long parts of flux tubes. Situation remains open in this

respect. Therefore the observation of decays of Higgs at expected rate to fermion pairs cannot kill TGD based vision.

The new view about Higgs combined with the stringy vision about twistor Grassmannian [K100] allows to see several conjectures related to ZEO in new light and also throw away some conjectures such as the idea about restriction of virtual momenta to plane $M^2 \subset M^4$.

1. The basic conjecture related to the perturbation theory is that wormhole throats are massless on mass shell states in embedding space sense: this would hold true also for virtual particles and brings in mind what happens in twistor program. The recent progress [K113] in the construction of n-point functions leads to explicit general formulas for them expressing them in terms of a functional integral over four-surfaces. The deformation of the space-time surface fixes the deformation of basis for induced spinor fields and one obtains a perturbation theory in which correlation functions for embedding space coordinates and fermionic propagator defined by the inverse of the Kähler-Dirac operator appear as building bricks and the electroweak gauge coupling of the Kähler-Dirac operator define the basic vertex. This operator is indeed 2-D for all other fermions than right-handed neutrino.
2. The functional integral gives some expressions for amplitudes which resemble twistor amplitudes in the sense that the vertices define polygons and external fermions are massless although gauge bosons as their bound states are massive. This suggests a stringy generalization of twistor Grassmannian approach [K100]. The residue integral would replace 4-D integrations of virtual fermion momenta to integrals over massless momenta. The outcome would be non-vanishing for non-physical helicities of virtual fermion. Also the problem due to the fact that fermionic Super Virasoro generator carries fermion number in TGD framework disappears.
3. There are two conformal weights involved. The conformal weight associated with the light-like radial coordinate of $\delta M^4_\pm$ and the spinorial conformal weight associated with the fermionic string connecting wormhole throats and throats of wormhole contact. Are these conformal weights independent or not? For instance, could one use radial light-like coordinate as string coordinate in the generic situation so that the conformal weights would not define independent quantum numbers? This does not look feasible. The Yangian variant of conformal algebra [A29] [B25, B17, B18] involves two integers. Second integer would naturally be the number of partonic 2-surfaces acted by the generator characterizing the poly-locality of Yangian generators, and it is not clear whether it has anything to do with the spinorial conformal weight. One can of course consider also three integers! This would be in accordance with the idea that the basic objects are 3-dimensional.

   If the conjecture that Yangian invariance realized in terms of Grassmannians makes sense, it could allow to deduce the outcome of the functional integral over four-surfaces and one could hope that TGD can be transformed to a calculable theory. Also p-adic mass calculations should be formulated using p-adic thermodynamics assuming Yangian invariance and enlargened conformal algebra.

14.9 Calculation Of Hadron Masses And Topological Mixing Of Quarks

The calculation of quark masses is not enough since one must also understand CKM mixing of quarks in order to calculate hadron massess. A model for CKM matrix and hadron masses is constructed in [K70] and here only a brief summary about basic ideas involved is given.

14.9.1 Topological Mixing Of Quarks

In TGD framework CKM mixing is induced by topological mixing of quarks (that is 2-dimensional topologies characterized by genus). The strongest number theoretical constraint on mixing matrices would be that they are rational. Perhaps a more natural constraint is that they are expressible in terms of roots of unity for some finite dimensional algebraic extension of rationals and therefore also p-adic numbers.

Number theoretical constraints on topological mixing can be realized by assuming that topological mixing leads to a thermodynamical equilibrium subject to constraints from the integer valued modular contributions remaing integer valued in the mixing. This gives an upper bound of 1200 for the number of different U and D matrices and the input from top quark mass and $\pi^+ - \pi^0$ mass difference implies that physical U and D matrices can be constructed as small perturbations of matrices expressible as direct sum of essentially unique 2×2 and 1×1 matrices. The maximally entropic solutions can be found numerically by using the fact that only the probabilities p_{11} and p_{21} can be varied freely. The solutions are unique in the accuracy used, which suggests that the system allows only single thermodynamical phase.

The matrices U and D associated with the probability matrices can be deduced straightforwardly in the standard gauge. The U and D matrices derived from the probabilities determined by the entropy maximization turn out to be unitary for most values of integers n_1 and n_2 characterizing the lowest order contribution to quark mass. This is a highly non-trivial result and means that mass and probability constraints together with entropy maximization define a sub-manifold of $SU(3)$ regarded as a sub-manifold in 9-D complex space. The choice $(n(u), n(c)) = (4, n)$, $n < 9$, does not allow unitary U whereas $(n(u), n(c)) = (5, 6)$ does. This choice is still consistent with top quark mass and together with $n(d) = n(s) = 5$ it leads to a rather reasonable CKM matrix with a value of CP breaking invariant within experimental limits. The elements V_{i3} and V_{3i}, $i = 1, 2$ are however roughly twice larger than their experimental values deduced assuming standard model. V_{31} is too large by a factor 1.6. The possibility of scaled up variants of light quarks could lead to too small experimental estimates for these matrix elements. The whole parameter space has not been scanned so that better candidates for CKM matrices might well exist.

14.9.2 Higgsy Contribution To Fermion Masses Is Negligible

There are good reasons to believe that Higgs expectation for the fermionic space-time sheets is vanishing although fermions couple to Higgs. Thus p-adic thermodynamics would explain fermion masses completely. This together with the fact that the prediction of the model for the top quark mass is consistent with the most recent limits on it, fixes the CP_2 mass scale with a high accuracy to the maximal one obtained if second order contribution to electron's p-adic mass squared vanishes. This is very strong constraint on the model.

14.9.3 The P-Adic Length Scale Of Quark Is Dynamical

The assumption about the presence of scaled up variants of light quarks in light hadrons leads to a surprisingly successful model for pseudo scalar meson masses using only quark masses and the assumption mass squared is additive for quarks with same p-adic length scale and mass for quarks labelled by different primes p. This conforms with the idea that pseudo scalar mesons are Goldstone bosons in the sense that color Coulombic and magnetic contributions to the mass cancel each other. Also the mass differences between hadrons containing different numbers of strange and heavy quarks can be understood if s, b and c quarks appear as several scaled up versions.

This hypothesis yields surprisingly good fit for meson masses but for some mesons the predicted mass is slightly too high. The reduction of CP_2 mass scale to cure the situation is not possible since top quark mass would become too low. In case of diagonal mesons for which quarks correspond to same p-adic prime, quark contribution to mass squared can be reduced by ordinary color interactions and in the case of non-diagonal mesons one can require that quark contribution is not larger than meson mass.

It should be however made clear that the notion of quark mass is problematic. One can speak about current quark masses and constituent quark masses. For u and d quarks constituent quark masses have scale 10^2 GeV are much higher than current quark masses having scale 10 GeV. For current quarks the dominating contribution to hadron mass would come from super-symplectic bosons at quantum level and at more phenomenological level from hadronic string tension. The open question is which option to choose or whether one should regard the two descriptions as duals of each other based on $M^8 - H$ duality. M^8 description would be natural at low energies since $SO(4)$ takes the role of color group. One could also say that current quarks are created in de-confinement phase transition which involves change of the p-adic length scale characterizing the quark. Somewhat counter intuitively but in accordance with Uncertainty Principle this length scale would increase but one could assign it the color magnetic field body of the quark.

14.9.4 Super-Symplectic Bosons At Hadronic Space-Time Sheet Can Explain The Constant Contribution To Baryonic Masses

Current quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD which could be characterized in terms of constituent quark masses in M^8 picture and in terms of current quark masses and string tension or super-symplectic bosons in $M^4 \times CP_2$ picture.

Super-symplectic gluons provide an attractive description of this contribution. They need not exclude more phenomenological description in terms of string tension. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent. Super-symplectic gluons also provide a possible solution to the spin puzzle of proton.

Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight.

In the case of mesons pion could contain super-symplectic boson of first generation preventing the large negative contribution of the color magnetic spin-spin interaction to make pion a tachyon. For heavier bosons super-symplectic boson need not to be assumed. The preferred role of pion would relate to the fact that its mass scale is below QCD Λ.

14.9.5 Description Of Color Magnetic Spin-Spin Splitting In Terms Of Conformal Weight

What remains to be understood are the contributions of color Coulombic and magnetic interactions to the mass squared. There are contributions coming from both ordinary gluons and super-symplectic gluons and the latter is expected to dominate by the large value of color coupling strength.

Conformal weight replaces energy as the basic variable but group theoretical structure of color magnetic contribution to the conformal weight associated with hadronic space-time sheet ($k = 107$) is same as in case of energy. The predictions for the masses of mesons are not so good than for baryons, and one might criticize the application of the format of perturbative QCD in an essentially non-perturbative situation.

The comparison of the super-symplectic conformal weights associated with spin 0 and spin 1 states and spin 1/2 and spin 3/2 states shows that the different masses of these states could be understood in terms of the super-symplectic particle contents of the state correlating with the total quark spin. The resulting model allows excellent predictions also for the meson masses and implies that only pion and kaon can be regarded as Goldstone boson like states. The model based on spin-spin splittings is consistent with the model.

To sum up, the model provides an excellent understanding of baryon and meson masses. This success is highly non-trivial since the fit involves only the integers characterizing the p-adic length scales of quarks and the integers characterizing color magnetic spin-spin splitting plus p-adic thermodynamics and topological mixing for super-symplectic gluons. The next challenge would be to predict the correlation of hadron spin with super-symplectic particle content in case of long-lived hadrons.

Chapter 15

New Physics Predicted by TGD

15.1 Introduction

TGD predicts a lot of new physics and it is quite possible that this new physics becomes visible at LHC. Although calculational formalism is still lacking, p-adic length scale hypothesis allows to make precise quantitative predictions for particle masses by using simple scaling arguments. Actually there is already now evidence for effects providing further support for TGD based view about QCD and first rumors about super-symmetric particles have appeared.

Before detailed discussion it is good to summarize what elements of quantum TGD are responsible for new physics.

1. The new view about particles relies on their identification as partonic 2-surfaces (plus 4-D tangent space data to be precise). This effective metric 2-dimensionality implies generalization of the notion of Feynman diagram and holography in strong sense. One implication is the notion of field identity or field body making sense also for elementary particles and the Lamb shift anomaly of muonic hydrogen could be explained in terms of field bodies of quarks.
2. The topological explanation for family replication phenomenon implies genus generation correspondence and predicts in principle infinite number of fermion families. One can however develop a rather general argument based on the notion of conformal symmetry known as hyper-ellipticity stating that only the genera $g = 0, 1, 2$ are light [?] What "light" means is however an open question. If light means something below CP_2 mass there is no hope of observing new fermion families at LHC. If it means weak mass scale situation changes.
   For bosons the implications of family replication phenomenon can be understood from the fact that they can be regarded as pairs of fermion and anti-fermion assignable to the opposite wormhole throats of wormhole throat. This means that bosons formally belong to octet and singlet representations of dynamical SU(3) for which 3 fermion families define 3-D representation. Singlet would correspond to ordinary gauge bosons. Also interacting fermions suffer topological condensation and correspond to wormhole contact. One can either assume that the resulting wormhole throat has the topology of sphere or that the genus is same for both throats.
3. The view about space-time supersymmetry differs from the standard view in many respects. First of all, the super symmetries are not associated with Majorana spinors. Super generators correspond to the fermionic oscillator operators assignable to leptonic and quark-like induced spinors and there is in principle infinite number of them so that formally one would have $\mathcal{N} = \infty$ SUSY. I have discussed the required modification of the formalism of SUSY theories in [?]nd it turns out that effectively one obtains just $\mathcal{N} = 1$ SUSY required by experimental constraints. The reason is that the fermion states with higher fermion number define only short range interactions analogous to van der Waals forces. Right handed neutrino generates this super-symmetry broken by the mixing of the M^4 chiralities implied by the mixing of M^4 and CP_2 gamma matrices for induced gamma matrices. The simplest assumption is that particles and their superpartners obey the same mass formula but that the p-adic length scale can be different for them.
4. The new view about particle massivation based on p-adic thermodynamics raises the question

about the role of Higgs field. The vacuum expectation value (VEV) of Higgs is not feasible in TGD since CP_2 does not allow covariantly constant holomorphic vector fields. The original too strong conclusion from this was that TGD does not allow Higgs. Higgs VEV is not needed for the selection of preferred electromagnetic direction in electro-weak gauge algebra (unitary gauge) since CP_2 geometry does that. p-Adic thermodynamics explains fermion masses bout the masses of weak bosons cannot be understood on basis of p-adic thermodynamics alone giving extremely small second order contribution only and failing to explain W/Z mass ratio. Weak boson mass can be associated to the string tension of the strings connecting the throats of two wormhole contacts associated with elementary particle (two of them are needed since the monopole magnetic flux must have closed field lines).

At M^4 QFT limit Higgs VEV is the only possible description of massivation. Dimensional gradient coupling to Higgs field developing VEV explains fermion masses at this limit. The dimensional coupling is same for all fermions so that one avoids the loss of "naturalness" due to the huge variation of Higgs-fermion couplings in the usual description.

The stringy contribution to elementary particle mass cannot be calculated from the first principles. A generalization of p-adic thermodynamics based on the generalization of super-conformal algebra is highly suggestive. There would be two conformal weights corresponding the the conformal weight assignable to the radial light-like coordinate of light-cone boundary and to the stringy coordinate and third integer characterizing the poly-locality of the generator of Yangian associated with this algebra (n-local generator acts on n partonic 2-surfaces simultaneously).

5. One of the basic distinctions between TGD and standard model is the new view about color.

   (a) The first implication is separate conservation of quark and lepton quantum numbers implying the stability of proton against the decay via the channels predicted by GUTs. This does not mean that proton would be absolutely stable. p-Adic and dark length scale hierarchies indeed predict the existence of scale variants of quarks and leptons and proton could decay to hadons of some zoomed up copy of hadrons physics. These decays should be slow and presumably they would involve phase transition changing the value of Planck constant characterizing proton. It might be that the simultaneous increase of Planck constant for all quarks occurs with very low rate.

   (b) Also color excitations of leptons and quarks are in principle possible. Detailed calculations would be required to see whether their mass scale is given by CP_2 mass scale. The so called lepto-hadron physics proposed to explain certain anomalies associated with both electron, muon, and τ lepton could be understood in terms of color octet excitations of leptons [**?**]

6. Fractal hierarchies of weak and hadronic physics labelled by p-adic primes and by the levels of dark matter hierarchy are highly suggestive. Ordinary hadron physics corresponds to $M_{107} = 2^{107} - 1$ One especially interesting candidate would be scaled up hadronic physics which would correspond to $M_{89} = 2^{89} - 1$ defining the p-adic prime of weak bosons. The corresponding string tension is about 512 GeV and it might be possible to see the first signatures of this physics at LHC. Nuclear string model in turn predicts that nuclei correspond to nuclear strings of nucleons connected by colored flux tubes having light quarks at their ends. The interpretation might be in terms of M_{127} hadron physics. In biologically most interesting length scale range 10 nm-2.5 μm contains four electron Compton lengths $L_e(k) = \sqrt{5}L)k)$ associated with Gaussian Mersennes and the conjecture is that these and other Gaussian Mersennes are associated with zoomed up variants of hadron physics relevant for living matter. Cosmic rays might also reveal copies of hadron physics corresponding to M_{61} and M_{31}

   The well-definedness of em charge for the modes of induced spinor fields localizes them at 2-D surfaces with vanishing W fields and also Z^0 field above weak scale. This allows to avoid undesirable parity breaking effects.

7. Weak form of electric magnetic duality implies that the fermions and anti-fermions associated with both leptons and bosons are Kähler magnetic monopoles accompanied by monopoles of opposite magnetic charge and with opposite weak isospin. For quarks Kähler magnetic charge need not cancel and cancellation might occur only in hadronic length scale. The magnetic flux tubes behave like string like objects and if the string tension is determined by weak length scale, these string aspects should become visible at LHC. If the string tension is 512 GeV the

situation becomes less promising.

In this chapter the predicted new elementary particle physics and possible indications for it are discussed. Second chapter is devoted to new hadron physics and scaled up variants of hardon physics in both quark and lepton sector.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L22].

15.2 Scaled Variants Of Quarks And Leptons

15.2.1 Fractally Scaled Up Versions Of Quarks

The strange anomalies of neutrino oscillations [C55] suggesting that neutrino mass scale depends on environment can be understood if neutrinos can suffer topological condensation in several p-adic length scales [K60] . The obvious question whether this could occur also in the case of quarks led to a very fruitful developments leading to the understanding of hadronic mass spectrum in terms of scaled up variants of quarks. Also the mass distribution of top quark candidate exhibits structure which could be interpreted in terms of heavy variants of light quarks. The ALEPH anomaly [C8], which I first erratically explained in terms of a light top quark has a nice explanation in terms of b quark condensed at $k = 97$ level and having mass ~ 55 GeV. These points are discussed in detail in [K70] .

The emergence of ALEPH results [C8] meant a an important twist in the development of ideas related to the identification of top quark. In the LEP 1.5 run with $E_{cm} = 130 - 140\ GeV$, ALEPH found 14 e^+e^- annihilation events, which pass their 4-jet criteria whereas 7.1 events are expected from standard model physics. Pairs of dijets with vanishing mass difference are in question and dijets could result from the decay of a new particle with mass about 55 GeV.

The data do not allow to conclude whether the new particle candidate is a fermion or boson. Top quark pairs produced in e^+e^- annihilation could produce 4-jets via gluon emission but this mechanism does not lead to an enhancement of 4-jet fraction. No $b\bar{b}b\bar{b}$ jets have been observed and only one event containing b has been identified so that the interpretation in terms of top quark is not possible unless there exists some new decay channel, which dominates in decays and leads to hadronic jets not initiated by b quarks. For option 2), which seems to be the only sensible option, this kind of decay channels are absent.

Super symmetrized standard model suggests the interpretation in terms of super partners of quarks or/and gauge bosons [C48] . It seems now safe to conclude that TGD does not predict sparticles. If the exotic particles are gluons their presence does not affect Z^0 and W decay widths. If the condensation level of gluons is $k = 97$ and mixing is absent the gluon masses are given by $m_g(0) = 0$, $m_g(1) = 19.2\ GeV$ and $m_g(2) = 49.5\ GeV$ for option 1) and assuming $k = 97$ and hadronic mass renormalization. It is however very difficult to understand how a pair of $g = 2$ gluons could be created in e^+e^- annihilation. Moreover, for option 2), which seems to be the only sensible option, the gluon masses are $m_g(0) = 0$, $m_g(1) = m_g(2) = 30.6\ GeV$ for $k = 97$. In this case also other values of k are possible since strong decays of quarks are not possible.

The strong variations in the order of magnitude of mass squared differences between neutrino families [C55] can be understood if they can suffer a topological condensation in several p-adic length scales. One can ask whether also t and b quark could do the same. In absence of mixing effects the masses of $k = 97$ t and b quarks would be given by $m_t \simeq 48.7$ GeV and $m_b \simeq 52.3$ GeV taking into account the hadronic mass renormalization. Topological mixing reduces the masses somewhat. The fact that b quarks are not observed in the final state leaves only $b(97)$ as a realistic option. Since Z^0 boson mass is ~ 94 GeV, $b(97)$ does not appreciably affect Z^0 boson decay width. The observed anomalies concentrate at cm energy about 105 GeV. This energy is 15 percent smaller than the total mass of top pair. The discrepancy could be understood as resulting from the binding energy of the $b(97)\bar{b}(97)$ bound states. Binding energy should be a fraction of order $\alpha_s \simeq .1$ of the total energy and about ten per cent so that consistency is achieved.

15.2.2 Toponium at 30.4 GeV?

Prof. Matt Strassler tells about a gem found from old data files of ALEPH experiment (see `http://tinyurl.com/ze6l5wr`) by Arno Heisner [C7](see `http://tinyurl.com/hy8ugf4`). The 3-sigma bump appears at 30.40 GeV and could be a statistical fluctuation and probably is so. It has been found to decay to muon pairs and b-quark pairs. The particle that Strassler christens V (V for vector) would have spin 1.

Years ago [K64] I have commented a candidate for scaled down top quark reported by Aleph: this had mass around 55 GeV and the proposal was that it corresponds to p-adically scaled up b quark with estimated mass of 52.3 GeV.

Could TGD allow to identify V as a scaled up variant of some spin 1 meson?

1. p-Adic length scale hypothesis states that particle mass scales correspond to certain primes $p \simeq 2^k$, $k > 0$ integer. Prime values of k are of special interest. Ordinary hadronic space-time sheets would correspond to hadronic space-time sheets labelled by Mersenne prime $p = M_{107} = 2^{107} - 1$ and quarks would be labelled by corresponding integers k.
2. For low mass mesons the contribution from color magnetic flux tubes to mass dominates whereas for higher mass mesons consisting of heavy quarks heavy quark contribution is dominant. This suggests that the large mass of V must result by an upwards scaling of some light quark mass or downwards scaling of top quark mass by a power of square root of 2.
3. The mass of b quark is around 4.2-4.6 GeV and Upsilon meson has mass about 9.5 GeV so that at most about 1.4 GeV from total mass would correspond to the non-perturbative color contribution partially from the magnetic body. Top quark mass is about 172.4 GeV and p-adic mass calculations suggest $k = 94$ (M_{89}) for top. If the masses for heavy quark mesons are additive as the example of Upsilon suggests, the non-existing top pair vector meson (toponium) (see `http://tinyurl.com/nfzhnej`) would have mass about $m(toponium) = 2 \times 172.4$ GeV $= 344.8$ GeV.
4. Could the observed bump correspond to p-adically scaled down version of toponium with $k = 94 + 7 = 101$, which is prime? The mass of toponium would be 30.47 GeV, which is consistent with the mass of the bump. If this picture is correct, V would be premature toponium able to exist for prime $k = 101$. Its decays to b quark pair are consistent with this.
5. Tommaso Dorigo (see `http://tinyurl.com/zhgyecd`) argues that the signal is spurious since the produced muons tend to be parallel to b quarks in cm system of Z^0. Matt Strassler identifies the production mechanism as a direct decay of Z^0 and in this case Tommaso would be right: the direct 3-particle decay of $Z^0 \rightarrow b + \overline{b} + V$ would produce different angular distribution for V. One cannot of course exclude the possibility that the interpretation of Tommaso is that muon pairs are from decays of V in its own rest frame in which case they certainly cannot be parallel to b quarks. So elementary mistake from a professional particle physicist looks rather implausible. The challenge of the experiments was indeed to distinguish the muon pairs from muons resulting from b quarks decaying semileptonically and being highly parallel to b quarks. A further objection of Tommaso is that the gluons should have roughly opposite momenta and fusion seems highly implausible classically since the gluons tend to be emitted in opposite directions. Quantally the argument does not look so lethal if one thinks in terms of plane waves rather than wave packets. Also fermion exchange is involved so that the fusion is not local process.
6. How the bump appearing in $Z^0 \rightarrow b + \overline{b} + V$ would be produced if toponium is in question? The mechanism would be essentially the same as in the production of Ψ/J meson by a $c + \overline{c}$ pair. The lowest order diagram would correspond to gluon fusion. Both b and $\overline{b}$ emit gluon and these could annihilate to a top pair and these would form the bound state. Do virtual t and $\overline{t}$ have ordinary masses 172 GeV or scaled down masses of about 15 GeV? The checking which option is correct would require numerical calculation and a model for the fusion of the pair to toponium.
   That the momenta of muons are parallel to those of b and $\overline{b}$ might be understood. One can approximate gluons with energy about 15 GeV as a brehmstrahlung almost parallel/antiparallel to the direction of b /$\overline{b}$ both having energy about 45 GeV in the cm system of Z^0. In cm they would combine to V with helicity in direction of axis nearly parallel to the direction defined by the opposite momenta of b and $\overline{b}$. The V with spin 1 would decay to a muon pair with helicities

in the direction of this axis, and since relativistic muons are in question, the momenta would by helicity conservation tend to be in the direction of this axis as observed.

Are there other indications for scaled variants of quarks?

1. Tony Smith [C64] has talked about indications for several mass peaks for top quark. I have discussed this in [K70] in terms of p-adic length scale hypothesis. There is evidence for a sharp peak in the mass distribution of the top quark in 140-150 GeV range). There is also a peak slightly below 120 GeV, which could correspond to a p-adically scaled down variant t quark with $k = 93$ having mass 121.6 GeV for $(Y_e = 0, Y_t = 1)$. There is also a small peak also around 265 GeV which could relate to $m(t(95)) = 243.2$ GeV. Therefore top could appear at least at p-adic scales $k = 93, 94, 95$. This argument does not explain the peak in 140-150 GeV range rather near to top quark mass.
2. What about Aleph anomaly? The value of $k(b)$ in $p_b \simeq 2^{k_b}$ uncertain. $k(b) = 103$ is one possible value. In [K64]. I have considered the explanation of Aleph anomaly in terms of $k = 96$ variant of b quark. The mass scaling would be by factor of $2^{7/2}$, which would assign to mass $m_b = 4.6$ GeV mass of about 52 GeV to be compared with 55 GeV.

To sum up, the objections of Tommasso Dorigo might well kill the toponium proposal and the bump is probably a statistical fluctuation. It is however amazing that its mass comes out correctly from p-adic length scale hypothesis which does not allow fitting.

Aleph anomaly just refuses to disappear

I learned about evidence for a bump around 28 GeV (see `https://arxiv.org/abs/1808.01890`). The title of the preprint is "*Search for resonances in the mass spectrum of muon pairs produced in association with b quark jets in proton-proton collisions at $\sqrt{s}$= 8 and 13 TeV*". An excess of events above the background near a dimuon mass of 28 GeV is observed in the 8 TeV data, corresponding to local significances of 4.2 and 2.9 standard deviations for the first and second event categories, respectively. At 13 TeV data the excess is milder. This induced two dejavu experiences.

1. First dejavu

Last year (2018) came a report from Aleph titled "Observation of an excess at 30 GeV in the opposite sign di-muon spectra of $Z \rightarrow b\overline{b} + X$ events recorded by the ALEPH experiment at LEP" (see `https://arxiv.org/pdf/1610.06536.pdf`). The article represents re-analysis of data from 1991-1992. The energy brings strongly in mind 28 GeV bump.

TGD - or more precisely p-adic fractality - suggests the existence of p-adically scaled variants of quarks and leptons with masses coming as powers of 2 (or perhaps even $\sqrt{2}$. They would be like octaves of a fundamental tone represented by the particle. Neutrino physics is plagued by anomalies and octaves of neutrino could resolve these problems.

Could one understand 30 GeV bump - possibly same as 28 GeV bump in TGD framework? b quark has mass 4.12 GeV or 4.65 GeV depending on the scheme used to estimate it. b quark could correspond to p-adic length scale $L(k)$ for $k = 103$ but the identification of the p-adic scale is not quite clear. p-Adically scaling b-quark mass taken to be 4.12 GeV by factor 4 gives about 16.5 GeV ($k = 103 - 4 = 99$), which is one half of 32 GeV: could this correspond to the proposed 30 GeV resonance or even 28 GeV resonance? One must remember that these estimates are rough since already QCD estimates for b quark mass vary about 10 per cent.

28 GeV bump could correspond to p-adically scaled variant of b with $k = 99$. b quark would indeed appear as octaves. But how to understand the discrepancy: could one imagine that there are actually two mesons involved and analogous to pion and rho meson?

2. Second dejavu

Concerning quarks, I remember an old anomaly reported by Aleph at 56 GeV. This anomaly is mentioned in a preprint published last year (see `https://arxiv.org/pdf/hep-ph/9608264.pdf`) and there is reference to old paper: ALEPH Collaboration, D. Buskulic *et al.*, CERN preprint PPE/96–052.. What was observed was 4-jet events consisting of dijets with invariant mass around 55 GeV. What makes this interesting is that the mass of 28 GeV particle candidate would be one half of the mass of a particle with mass of mass of 56 GeV particle, quite near to 55 GeV.

My proposal for the identification of the 55 GeV bump was as a meson formed from scaled variants b and $\overline{b}$ corresponding to p-adic prime $p \simeq 2^k$, $k = 96$. The above argument suggests

$k = 99 - 2 = 97$. Note that the production of the 28 GeV bump decaying to muon pair is associated with production of b quark and second jet.

3. What the resonance are and how could they be produced?

The troubling question is why the two masses around 28 GeV ad 30 GeV? Even worse: for 30 GeV candidate a dip is reported in at 28 GeV! Could the two candidates correspond to $\pi(28)$ and $\rho(30)$ having slightly different masses by color-magnetc spin-spin splitting?

The production mechanism should explain why the resonance is associated with b-quark and jet and also why two different mass values suggest themselves.

1. If one has 56 GeV pseudo-scalar resonance consisting mostly of $b\bar{b}$ - call it $\pi(56)$, it could couple to Z^0 by standard instanton density coupling, and one could have the decay $Z \rightarrow Z + \pi(56)$. The final state virtual Z would produce the b-tag in its decay.
2. $\pi(56)$ in turn would decay strongly to $\pi(28) + \rho(30)$ with spin 1 and analogous to the rho meson partner of ordinary pion. Masses would be naturally different for π and ρ.

It is easy to check that the observed spin-spin splitting is consistent with the simplest model for the spin-spin splitting obtained by extrapolating the for ordinary $\pi - \rho$ system.

1. At these mass scales the spin-spin splitting proportional to color magnetic moments and thus to inverses of the b quark masses should be small and indeed is.
2. Consider first ordinary $\pi - \rho$ system. The predicted masses due to spin-spin splitting are $m(\pi) = m - \Delta/2$ and $m(\rho) = m + 3\Delta/2)$, where one has $m = (3m(\pi) + m(\rho))/4$ and $\Delta = (m(\rho) - m(\pi))/2$. For $\pi - \rho$ system one has $r_1 = \Delta m/m \simeq .5$.
   $\Delta m/m$ is due to the interaction of color magnetic moments and of form xr, $r\alpha_s^2 m^2(\pi)/m^2(d)$. The small masses of u and d quarks - $m(d) \simeq 4.8$ MeV (Wikipedia value, the estimate vary widely) - implies that $m(\pi)/m(d) \simeq 28.2$ is rather large. The value of α_s is larger than $\alpha_s = .1$ achieved at higher energies, which gives $r_2 = \alpha_s^2 m^2(\pi)/m^2(d) > .28$. One has $r_1/r_2 \simeq .57$.
3. For $\pi(28) - \rho(30)$ system the values of the parameters are $m \simeq 29$ GeV and $\Delta m = 2$ GeV and $r_1 = \Delta m/m \simeq .07$. The mass ratio is roughly $m(\pi)/m(b) = 2$ for heavy mesons for which quark mass dominates in the meson mass. For $\alpha_s = .1$ the order of magnitude for $r_2 = \alpha_s^2 m^2(\pi(28))/m^2(b)$ is $r_2 \simeq .04$ and one has $r_1/r_2 = .57$ to be compared with $r_1/r_2 = .56$ for ordinary $\pi(28) - \rho(30)$ system so that the model looks realistic.
   Interestingly, the same value of α_s works in both cases: does this provide support for the TGD view about renormalization group invariance of coupling strengths [L63, L71]? This invariance is not global but implies discrete coupling constant evolution.

15.2.3 Could Neutrinos Appear In Several P-Adic Mass Scales?

There are some indications that neutrinos can appear in several mass scales from neutrino oscillations [C4]. These oscillations can be classified to vacuum oscillations and to solar neutrino oscillations believed to be due to the so called MSW effect in the dense matter of Sun. There are also indications that the mixing is different for neutrinos and antineutrinos [C23, C3].

In TGD framework p-adic length scale hypothesis might explain these findings. The basic vision is that the p-adic length scale of neutrino can vary so that the mass squared scale comes as octaves. Mixing matrices would be universal. The large discrepancy between LSND and MiniBoone results [C23] contra solar neutrino results could be understood if electron and muon neutrinos have same p-adic mass scale for solar neutrinos but for LSND and MiniBoone the mass scale of either neutrino type is scaled up. The existence of a sterile neutrino [C47] suggested as an explanation of the findings would be replaced by p-adically scaled up variant of ordinary neutrino having standard weak interactions. This scaling up can be different for neutrinos and antineutrinos as suggested by the fact that the anomaly is present only for antineutrinos.

The different values of Δm^2 for neutrinos and antineutrinos in MINOS experiment [C3] can be understood if the p-adic mass scale for neutrinos increases by one unit. The breaking of CP and CPT would be spontaneous and realized as a choice of different p-adic mass scales and could be understood in ZEO. Similar mechanism would break supersymmetry and explain large differences between the mass scales of elementary fermions, which for same p-adic prime would have mass scales differing not too much.

Experimental results

There several different type of experimental approaches to study the oscillations. One can study the deficit of electron type solar electron neutrinos (Kamiokande, Super-Kamiokande); one can measure the deficit of muon to electron flux ratio measuring the rate for the transformation of ν_μ to ν_τ (super-Kamiokande); one can study directly the deficit of ν_e ($\overline{\nu}_e$) neutrinos due to transformation to ν_μ $\nu_{\overline{\mu}}$ coming from nuclear reactor with energies in the same range as for solar neutrinos (KamLAND); and one can also study neutrinos from particle accelerators in much higher energy range such as solar neutrino oscillations (K2K,LSND,Miniboone,Minos).

1. *Solar neutrino experiments and atmospheric neutrino experiments*

The rate of neutrino oscillations is sensitive to the mass squared differences Δm^2_{12}, Δm^2_{12}, Δm^2_{13} and corresponding mixing angles θ_{12}, θ_{13}, θ_{23} between ν_e, ν_μ, and ν_τ (ordered in obvious manner). Solar neutrino experiments allow to determine $sin^2(2\theta_{12})$ and Δm^2_{12}. The experiments involving atmospheric neutrino oscillations allow to determine $sin^2(2\theta_{23})$ and Δm^2_{23}.

The estimates of the mixing parameters obtained from solar neutrino experiments and atmospheric neutrino experiments are $sin^2(2\theta_{13}) = 0.08$, $sin^2(2\theta_{23}) = 0.95$, and $sin^2(2\theta_{12}) = 0.86$. The mixing between ν_e and ν_τ is very small. The mixing between ν_e and ν_μ, and ν_μ and ν_τ tends is rather near to maximal. The estimates for the mass squared differences are $\Delta m^2_{12} = 8 \times 10^{-5}$ eV^2, $\Delta m^2_{23} \simeq \Delta m^2_{13} = 2.4 \times 10^{-3}$ eV^2. The mass squared differences have obviously very different scale but this need not means that the same is true for mass squared values.

2. *The results of LSND and MiniBoone*

LSND experiment measuring the transformation of $\overline{\nu}_\mu$ to $\overline{\nu}_e$ gave a totally different estimate for Δm^2_{12} than solar neutrino experiments MiniBoone [C47]. If one assumes same value of $sin^2(\theta_{12})^2 \simeq .86$ one obtains $\Delta m^2_{23} \sim .1$ eV^2 to be compared with $\Delta m^2_{12} = 8 \times 10^{-5}$ eV^2. This result is known as LSND anomaly and led to the hypothesis that there exists a sterile neutrino having no weak interactions and mixing with the ordinary electron neutrino and inducing a rapid mixing caused by the large value of Δm^2. The purpose of MiniBoone experiment [C23] was to test LSND anomaly.

1. It was found that the two-neutrino fit for the oscillations for $\nu_\mu \to \nu_e$ is not consistent with LSND results. There is an unexplained 3σ electron excess for $E < 475$ MeV. For $E > 475$ MeV the two-neutrino fit is not consistent with LSND fit. The estimate for Δm^2 is in the range $.1 - 1$ eV^2 and differs dramatically from the solar neutrino data.
2. For antineutrinos there is a small 1.3σ electron excess for $E < 475$ MeV. For $E > 475$ MeV the excess is 3 per cent consistent with null. Two-neutrino oscillation fits are consistent with LSND. The best fit gives $(\Delta m^2_{12}, sin^2(2\theta_{12}) = (0.064\ \ eV^2, 0.96)$. The value of Δm^2_{12} is by a factor 800 larger than that estimated from solar neutrino experiments.

All other experiments (see the table of the summary of [C47] about sterile neutrino hypothesis) are consistent with the absence of $\nu_\mu \to n_e$ and $\overline{\nu}_\mu \to \overline{\nu}_e$ mixing and only LSND and MiniBoone report an indication for a signal. If one however takes these findings seriously they suggest that neutrinos and antineutrinos behave differently in the experimental situations considered. Two-neutrino scenarios for the mixing (no sterile neutrinos) are consistent with data for either neutrinos or antineutrinos but not both [C47].

3. *The results of MINOS group*

The MINOS group at Fermi National Accelerator Laboratory has reported evidence that the mass squared differences between neutrinos are not same for neutrinos and antineutrinos [C3]. In this case one measures the disappearance of ν_μ and $\overline{\nu}_\mu$ neutrinos from high energy beam beam in the range .5-1 GeV and the dominating contribution comes from the transformation to τ neutrinos. Δm^2_{23} is reported to be about 40 percent larger for antineutrinos than for neutrinos. There is 5 percent probability that the mass squared differences are same. The best fits for the basic parameters are $(\Delta m^2_{23} = 2.35 \times 10^{-3}, sin^2(2\theta_{23} = 1)$ for neutrinos with error margin for Δm^2 being about 5 per cent and $(\Delta m^2_{23} = 3.36 \times 10^{-3}, sin^2(2\theta_{23}) = .86)$ for antineutrinos with errors margin around 10 per cent. The ratio of mass squared differences is $r \equiv \Delta m^2(\overline{\nu})/\Delta m^2(\nu) = 1.42$. If one assumes $sin^2(2\theta_{23}) = 1$ in both cases the ratio comes as $r = 1.3$.

Explanation of findings in terms of p-adic length scale hypothesis

p-Adic length scale hypothesis predicts that fermions can correspond to several values of p-adic prime meaning that the mass squared comes as octaves (powers of two). The simplest model for the neutrino mixing assumes universal topological mixing matrices and therefore for CKM matrices so that the results should be understood in terms of different p-adic mass scales. Even CP breaking and CPT breaking at fundamental level is un-necessary although it would occur spontaneously in the experimental situation selecting different p-adic mass scales for neutrinos and antineutrinos. The expression for the mixing probability a function of neutrino energy in two-neutrino model for the mixing is of form

$$P(E) = sin^2(2\theta)sin^2(X) \ \ , \ \ X = k \times \Delta m^2 \times \frac{L}{E} \ \ .$$

Here k is a numerical constant, L is the length travelled, and E is neutrino energy.

1. *LSND and MiniBoone results*

LSND and MiniBoone results are inconsistent with solar neutrino data since the value of Δm_{12}^2 is by a factor 800 larger than that estimated from solar neutrino experiments. This could be understood if in solar neutrino experiments ν_μ and ν_w correspond to the same p-adic mass scale $k = k_0$ and have very nearly identical masses so that Δm^2 scale is much smaller than the mass squared scale. If either p-adic scale is changed from k_0 to $k_0 + k$, the mass squared difference increases dramatically. The counterpart of the sterile neutrino would be a p-adically scaled up version of the ordinary neutrino having standard electro-weak interactions. The p-adic mass scale would correspond to the mass scale defined by Δm^2 in LSND and MiniBoone experiments and therefore a mass scale in the range .3-1 eV. The electron Compton scale assignable to eV mass scale could correspond to $k = 167$, which corresponds to cell length scale of 2.5 μm. $k = 167$ defines one of the Gaussian Mersennes $M_{G,k} = (1 + i)^k - 1$. $L_e(k) = \sqrt{5}L(k)$, $k = 151, 157, 163, 167$, varies in the range 10 nm (cell membrane thickness) and 2.5 μm defining the size of cell nucleus. These scales could be fundamental for the understanding of living matter [K38] .

2. *MINOS results*

One must assume also now that the p-adic mass scales for ν_τ and $\overline{\nu}_\tau$ are near to each other in the "normal" experimental situation. Assuming that the mass squared scales of ν_μ or $\overline{\nu}_\mu$ come as 2^{-k} powers of $m_{\nu_\mu}^2 = m_{\nu_\tau}^2 + \Delta m^2$, one obtains

$$m_{\nu_\tau}^2(k_0) - m_{\overline{\nu}_\mu}^2(k_0 + k) = (1 - 2^{-k})m^2_{nu_\tau} - 2^{-k}\Delta m_0^2 \ \ .$$

For $k = 1$ this gives

$$r \ = \ \frac{\Delta m^2(k=2)}{\Delta m^2(k=1)} = \frac{\frac{3}{2} - \frac{2r}{3}}{1-r} \ \ , \ \ r = \frac{\Delta m_0^2}{m_{\nu_\tau}^2} \ \ . \tag{15.2.1}$$

One has $r \geq 3/2$ for $r > 0$ if one has $m_{\nu_\tau} > m_{\nu_\mu}$ for the same p-adic length scale. The experimental ratio $r \simeq 1.3$ could be understood for $r \simeq -.31$. The experimental uncertainties certainly allow the value $r = 1.5$ for $k(\overline{\nu}_\mu) = 1$ and $k(\nu_\mu) = 2$.

This result implies that the mass scale of ν_μ and ν_τ differ by a factor 1/2 in the "normal" situation so that mass squared scale of ν_τ would be of order 5×10^{-3} eV^2. The mass scales for $\overline{\nu}_\tau$ and ν_τ would about .07 eV and .05 eV. In the LSND and MiniBoone experiments the p-adic mass scale of other neutrino would be around .1-1 eV so that different p-adic mass scale large by a factor $2^{k/2}$, $2 \leq 2 \leq 7$ would be in question. The different resuts from various experiments could be perhaps understood in terms of the sensitivity of the p-adic mass scale to the experimental situation. Neutrino energy could serve as a control parameter.

CPT breaking [B3] requires the breaking of Lorentz invariance. ZEO could therefore allow a spontaneous breaking of CP and CPT. This might relate to matter antimatter asymmetry at the level of given CD.

There is some evidence that the mixing matrices for neutrinos and antineutrinos are different in the experimental situations considered [C3, C23]. This would require CPT breaking in the standard QFT framework. In TGD p-adic length scale hypothesis allowing neutrinos to reside in several p-adic mass scales. Hence one could have apparent CPT breaking if the measurement arrangements for neutrinos and antineutrinos select different p-adic length scales for them [K64] .

Is CP and T breaking possible in ZEO?

The CKM matrices for quarks and possibly also leptons break CP and T. Could one understand the breaking of CP and T at fundamental level in TGD framework?

1. In standard QFT framework Chern-Simons term breaks CP and T. Kähler action indeed reduces to Chern-Simons terms for the proposed ansatz for preferred extremals assuming that weak form of electric-magnetic duality holds true.
   In TGD framework one must however distinguish between space-time coordinates and embedding space coordinates. CP breaking occurs at the embedding space level but instanton term and Chern-Simons term are odd under P and T only at the space-time level and thus distinguish between different orientations of space-time surface. Only if one identifies P and T at space-time level with these transformations at embedding space level, one has hope of interpreting CP and T breaking as spontaneous breaking of these symmetries for Kähler action and basically due to the weak form of electric-magnetic duality and vanishing of $j \cdot A$ term for the preferred extremals. This identification is possible for space-time regions allowing representation as graphs of maps $M^4 \to CP_2$.
2. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.
3. The GRT-QFT limit of TGD obtained by lumping together various space-time sheets to a region of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of sheets from Minkowski metric. Gauge potentials for the effective space-time would idenfied as sums of gauge potentials for space-time sheets. At this limit the identification of P and T at space-time level and embedding space level would be natural. Could the resulting effective theory in Minkowski space or GRT space-time break CP and T slightly? If so, CKM matrices for quarks and fermions would emerge as a result of representing different topologies for wormhole throats with different topologies as single point like particle with additional genus quantum number.
4. Could the breaking of CP and T relate to the generation of the arrow of time? The arrow of time relates to the fact that state function reduction can occur at either boundary of CD [K9]. Zero energy states do not change at the boundary at which reduction occurs repeatedly but the change at the other boundary and also the wave function for the position of the second boundary of CD changes in each quantum jump so that the average temporal distance between the tips of CD increases. This gives to the arrow of psychological time, and in TGD inspired theory of consciousness "self" as a counterpart of observed can be identified as sequence of quantum jumps for which the state function reduction occurs at a fixed boundary of CD. The sequence of reductions at fixed boundary breaks T-invariance and has interpretation as irreversibility. The standard view is that the irreversibility has nothing to do with breaking of T-invariance but it might be that in elementary particle scales irreversibility might manifest as small breaking of T-invariance.

Is CPT breaking needed/possible?

Different values of Δm_{ij}^2 for neutrinos and antineutrinos would require in standard QFT framework not only the violation of CP but also CPT [B3] which is the cherished symmetry of quantum field theories. CPT symmetry states that when one reverses time's arrow, reverses the signs of momenta and replaces particles with their antiparticles, the resulting Universe obeys the same laws as the original one. CPT invariance follows from Lorentz invariance, Lorentz invariance of vacuum state, and from the assumption that energy is bounded from below. On the other hand, CPT violation requires the breaking of Lorentz invariance.

In TGD framework this kind of violation does not seem to be necessary at fundamental level since p-adic scale hypothesis allowing neutrinos and also other fermions to have several mass scales coming as half-octaves of a basic mass scale for given quantum numbers. In fact, even in TGD inspired low energy hadron physics quarks appear in several mass scales. One could explain the different choice of the p-adic mass scales as being due to the experimental arrangement which selects different p-adic length scales for neutrinos and antineutrinos so that one could speak about spontaneous breaking of CP and possibly CPT. The CP breaking at the fundamental level which is however expected to be small in the case considered. The basic prediction of TGD and relates to the CP breaking of Chern-Simons action inducing CP breaking in the Kähler-Dirac action defining the fermionic propagator [L8]. For preferred extremals Kähler action would indeed reduce to Chern-Simons terms by weak form of electric-magnetic duality.

In TGD one has breaking of translational invariance and the symmetry group reduces to Lorentz group leaving the tip of CD invariant. Positive and negative energy parts of zero energy states correspond to different Lorentz groups and zero energy states are superpositions of state pairs with differen values of mass squared. Is the breaking of Lorentz invariance in this sense enough for breaking of CPT is not clear.

One can indeed consider the possibility of a spontaneous breaking of CPT symmetry in TGD framework since for a given CD (causal diamond defined as the intersection of future and past directed light-cones whose size scales are assumed to come as octaves) the Lorentz invariance is broken due to the preferred time direction (rest system) defined by the time-like line connecting the tips of CD. Since the world of classical worlds is union of CDs with all boosts included the Lorentz invariance is not violated at the level of WCW. Spontaneous symmetry breaking would be analogous to that for the solutions of field equations possessing the symmetry themselves. The mechanism of breaking would be same as that for supersymmetry. For same p-adic length scale particles and their super-partners would have same masses and only the selection of the p-adic mass scale would induces the mass splitting.

Encountering the puzzle of inert neutrinos once again

Sabine Hossenfelder had an interesting link to Quanta Magazine article "*On a Hunt for a Ghost of a Particle*" telling about the plans of particle physicist Janet Conrad to find the inert neutrino (see `http://tinyurl.com/ybhcjwu6`).

The attribute "sterile" or "inert" (I prefer the latter since it is more respectful!) comes from the assumption this new kind of neutrino does not have even weak interactions and feels only gravitation. There are indications for the existence of inert neutrino from LSND experiments (see `http://tinyurl.com/y7ktyfrs`) and some Mini-Boone experiments(see `http://tinyurl.com/y74hmq7c`). In standard model it would be interpreted as fourth generation neutrino which would suggest also the existence of other fourth generation fermions. For this there is no experimental support.

The problem of inert neutrino is very interesting also from TGD point of view. TGD predicts also right handed neutrino with no electroweak couplings but mixes with left handed neutrino by a new interaction produced by the mixing of M^4 and CP_2 gamma matrices: this is a unique feature of induced spinor structure and serves as a signature of sub-manifold geometry and one signature distinguishing TGD from standard model. Only massive neutrino with both helicities remains and behaves in good approximation as a left handed neutrino.

There are indeed indications in both LSND and MiniBoone experiments for inert neutrino. But only in some of them. And not in the ICECUBE experiment (see `http://tinyurl.com/h79dyj3`) performed at was South Pole. Special circumstances are required. "Special circumstances" need not mean bad experimentation. Why this strange behavior?

1. The evidence for the existence of inert neutrino, call it $\overline{\nu}_I$, came from antineutrino mixing $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ manifesting as mass squared difference between muonic and electronic antineutrinos. This difference was $\Delta m^2(LSND) = 1 - 10\ eV^2$ in the LSND experiment. The other two mass squared differences deduced from solar neutrino mixing and atmospheric neutrino mixing were $\Delta m^2(sol) = 8 \times 10^{-5}\ eV^2$ and $\Delta m^2(atm) = 2.5 \times 10^{-3}\ eV^2$ respectively.

2. The inert neutrino interpretation would be that actually $\overline{\nu}_\mu \rightarrow \overline{\nu}_I$ takes place and the mass squared difference for $\overline{\nu}_\mu and \overline{\nu}_I$ determines the mixing.

1. The explanation based on several p-adic mass scales for neutrinos

The first TGD inspired explanation proposed for a long time ago relies on p-adic length scale hypothesis predicting that neutrinos can exist in several p-adic length scales for which mass squared scale ratios come as powers of 2. Mass squared differences would also differ by a power of two. Indeed, the mass squared differences from solar and atmospheric experiments are in ratio 2^{-5} so that the model looks promising!

Writing $\Delta m^2(LSND) = x\ eV^2$ the condition $m^2(LSND)/m^2(atm) = 2^k$ has 2 possible solutions corresponding to $k = 9$, or $k = 10$ and $x = 2.5$ and $x = 1.25$. The corresponding mass squared differences 2.5 eV^2 and 1.25 eV^2.

The interpretation would be that the three measurement outcomes correspond to 3 neutrinos with nearly identical masses in given p-adic mass scale k but having different p-adc mass scales. The atmospheric and solar p-adic length scales would comes as powers $(L(atm), L(sol)) = (2^{n/2}, 2^{(n+10)/2}) \times L(k(LSND))$, $n = 9$ or $n = 10$. For $n = 10$ the mass squared scales would come as powers of 2^{10}.

How to estimate the value of $k(LSND)$?

1. Empirical data and p-adic mass calculations suggest that neutrino mass is of order .1 eV . The most natural candidates for p-adic mass scales would correspond to $k = 163, 167$ or $k = 169$. The first primes $k = 163, 167$ correspond to Gaussian Mersenne primes $M_{G,n} = (1 + i)^n - 1$ and to p-adic length scales $L(163) = 640$ nm and $L(167) = 2.56$ μm.
2. p-Adic mass calculations [K60] predict that the ratio $x = \Delta m^2/m^2$ for $\mu - e$ system has upper bound $x \sim .4$. This does not take into account the mixing effects but should give upper bound for the mass squared difference affected by the mixing.
3. The condition $\Delta m^2/m^2 = .4 \times x$, where $x \leq 1$ parametrizes the mass difference assuming $\Delta m(LSND)^2 = 2.5\ eV^2$ gives $m^2(LSND) \sim 6.25\ eV^2/x$.
   $x = 1/4$ would give $(k(LSND), k(atm), k(sol)) = (157, 167, 177)$. $k(LSND)$ and $k(atm)$ label two Gaussian Mersenne primes $M_{G,k} = (1+i)^k$ in the series $k = 151, 157, 163, 167$ of Gaussian Mersennes. The scale $L(151) = 10$ nm defines cell membrane thickness. All these scales could be relevant for DNA coiling. $k(sol) = 177$ is not Mersenne prime nor even prime. The correspoding p-adic length scale is 82 μm perhaps assignable to neuron. Note that $k = 179$ is prime.

This explanation looks rather nice because the mass squared difference ratios come as powers of two. What seems clear that the longer the path of neutrino travelled from the source to the detector, the smaller than mass squared: in other words one has $k(LSND) < k(atm) < k(sol)$. This suggest that neutrinos transform to lower mass neutrinos during the travel $k(LSND) \rightarrow k(atm) \rightarrow k(sol)$. The sequence could contains also other p-adic length scales.

What really happens when neutrino characterised by p-adic length scale $L(k_1)$ transforms to a neutrino characterized by p-adic length scale $L(k_2)$.

1. The simplest possibility would be that $k_1 \rightarrow k_2$ corresponds to a 2-particle vertex. The conservation of energy and momentum however prevent this process unless one has $\Delta m^2 = 0$. The emission of weak boson is not kinematically possible since Z^0 boson is so massive. For instance, solar neutrinos have energies in MeV range. The presence of classical Z^0 field could make the transformation possible and TGD indeed predicts classical Z^0 fields with long range. The simplest assumption is that all classical electroweak gauge fields except photon field vanish at string world sheets. This could in fact be guaranteed by gauge choice analogous to the unitary gauge.
2. The twistor lift of TGD however provides an alternative option. Twistor lift predicts that also M^4 has the analog of Kähler structure characterized by the Kähler form $J(M^4)$ which is covariantly constant and self-dual and thus corresponds to parallel electric and magnetic components of equal strength. One expects that this gives rise to both classical and quantum field coupling to fermion number, call this $U(1)$ gauge field U. The presence of $J(M^4)$ induces P, T, and CP breaking and could be responsible for CP breaking in both leptonic and quark sectors and also explain matter antimatter asymmetry [L37, L41] as well as large parity violation in living matter (chiral selection). The coupling constant strength α_1 is rather small due

to the constraints coming from atomic physics (new $U(1)$ boson couples to fermion number and this causes a small scaling of the energy levels). One has $\alpha_1 \sim 10^{-9}$, which is also the number characterizing matter antimatter asymmetry as ratio of the baryon density to CMB photon density.
Already the classical long ranged U field could induce the neutrino transitions. $k_1 \rightarrow k_2$ transition could become allowed by conservation laws also by emission of U boson. The simplest situation corresponds to parallel momenta for neutrinos and U. Conservation laws of energy and momentum give $E_1 = \sqrt{p_1^2 + m_1^2} = E_2 + E(U) = \sqrt{p_2^2 + m_2^2} + E(U)$, $p_1 = p_2 + p(U)$. Masslessness gives $E(U) = p(U)$. This would give in good approximation $p_2/p_1 = m_1^2/m_2^2$ and $E(U) = p_1 - p_2 = p_1(1 - m_1^2/m_2^2)$.
One can ask whether CKM mixing for quarks could involve similar mechanism explaining the CP breaking. Also the transitions changing $h_{eff}/h = n$ could involve U boson emission.

2. *The explanation based on several p-adic mass scales for neutrinos*

Second TGD inspired interpretation would be as a transformation of ordinary neutrino to a dark variant of ordinary neutrino with $h_{eff}/h = n$ occurring only if the situation is quantum critical (what would this mean now?). Dark neutrino would behave like inert neutrino. One cannot exclude this option but it does not give quantitative predictions.

This proposal need not however be in conflict with the first one since the transition $k(LSND) \rightarrow k_1$ could produce dark neutrino with different value of $h_{eff}/h = 2^{\Delta k}$ scaling up the Compton scale by this factor. This transition could be followed by a transition back to a particle with p-adic length scale scaled up by 2^{2k}. I have proposed that p-adic phase transitions occurring at criticality requiring $h_{eff}/h > 1$ are important in biology [K58].

There is evidence for a similar effect in the case of neutron decays. Neutron lifetime is found to be considerably longer than predicted. The TGD explanation [K64] is that part of protons resulting in the beta decays of neutrino transform to dark protons and remain undetected so that lifetime looks longer than it really is [L55] (see `http://tinyurl.com/yc8d7sed`). Note however that also now conservation laws give constraints and the emission of U photon might be involved also in this case. As a matter of fact, one can consider the possibility that the phase transition changing $h_{eff}/h = n$ involve the emission of U photon too. The mere mixing of the ordinary and dark variants of particle would induce mass splitting and U photon would take care of energy momentum conservation.

LSND anomaly is here again!

MinibooNe collaboration published a highly interesting preprint [C20] "Observation of a Significant Excess of Electron-Like Events in the MiniBooNE Short-Baseline Neutrino Experiment" (see `https://arxiv.org/abs/1805.12028`).

The findings give strong support for old and forgotten LSND anomaly - forgotten because it is in so blatant conflict with the standard model wisdom. The significance level of the anomaly is 6.1 sigmas in the new experiment. 5 sigma is regarded as the threshold for a discovery. It is nice to see this fellow again: anomalies are the theoreticians best friends.

To me this seems like a very important event from the point of view of standard model and even theoretical particle physics: this anomaly together with other anomalies raises hopes that the patient could leave the sickbed after illness that has lasted for more than four decades after becoming a victim of the GUT infection.

LSND as also other experiments are consistent with neutrino mixing model. LSND however produces electron excess as compared to other neutrino experiments. Anomaly means that the parameters of the neutrino mixing matrix (masses, mixing angles, phases) are not enough to explain all experiments.

One manner to explain the anomaly would be fourth "inert" neutrino having no couplings to electroweak bosons. TGD predicts both right and left-handed neutrinos and right-handed ones would not couple electroweakly. In massivation they would however combine to single massive neutrino just like in Higgs massivation Higgs gives components for massive gauge bosons and only neutral Higgs having no coupling to photon remains. Therefore this line of thought does not loo promising in TGD framework.

For many years ago I explained the LSND neutrino anomaly in TGD framework as being due to the fact that neutrinos can correspond to several p-adic mass scales. p-Adic mass scale coming as power of $2^{1/2}$ would bring in the needed additional parameter. The new particles could be ordinary neutrinos with different p-adic mass scales. The neutrinos used in experiment would have p-adic length scale depending on their origin. Lab, Earth's atmosphere, Sun, ... It is possible that the neutrinos transform during their travel to less massive neutrinos.

What is intriguing that the p-adic length scale range that can be considered as candidates for neutrino Compton lengths is biologically extremely interesting. This range could correspond to the p-adic length scales $L(k) \sim 2^{(k-151)/2}L(151)$, $k = 151, 157, 163, 167$ varying from cell membrane thickness 10 nm to 2.5 μm. These length scales correspond to Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$. The appearance of four of 4 Gaussian Mersennes in such a short length scale interval is a number theoretic miracle. Could neutrinos or their dark variants with $h_{eff} = n \times h_0$ together with dark variants weak bosons effectively massless below their Compton length have a fundamental role in quantum biology?

Remark: $h = 6 \times h_0$ is the most plausible option at this moment [L31, L60] (see `http://tinyurl.com/ybxlqqsj` and `http://tinyurl.com/yafndef9`).

15.3 Family Replication Phenomenon And Super-Symmetry

15.3.1 Family Replication Phenomenon For Bosons

TGD predicts that also gauge bosons, with gravitons included, should be characterized by family replication phenomenon but not quite in the expected manner. The first expectation was that these gauge bosons would have at least 3 light generations just like quarks and leptons.

Only within last years it has become clear that there is a deep difference between fermions and gauge bosons. Elementary fermions and particles super-conformally related to elementary fermions correspond to single throat of a wormhole contact assignable to a topologically condensed CP_2 type vacuum extremal whereas gauge bosons would correspond to a wormhole throat pair assignable to wormhole contact connecting two space-time sheets. Wormhole throats correspond to light-like partonic 3-surfaces at which the signature of the induced metric changes.

In the case of 3 generations gauge bosons can be arranged to octet and singlet representations of a dynamical SU(3) and octet bosons for which wormhole throats have different genus could be massive and effectively absent from the spectrum.

Exotic gauge boson octet would induce particle reactions in which conserved handle number would be exchanged between incoming particles such that total handle number of boson would be difference of the handle numbers of positive and negative energy throat. These gauge bosons would induce flavor changing but genus conserving neutral current. There is no evidence for this kind of currents at low energies which suggests that octet mesons are heavy. Typical reaction would be $\mu + e \rightarrow e + \mu$ scattering by exchange of $\Delta g = 1$ exotic photon.

15.3.2 Supersymmetry In Crisis

Supersymmetry is very beautiful generalization of the ordinary symmetry concept by generalizing Lie-algebra by allowing grading such that ordinary Lie algebra generators are accompanied by super-generators transforming in some representation of the Lie algebra for which Lie-algebra commutators are replaced with anti-commutators. In the case of Poincare group the super-generators would transform like spinors. Clifford algebras are actually super-algebras. Gamma matrices anti-commute to metric tensor and transform like vectors under the vielbein group (SO(n) in Euclidian signature). In supersymmetric gauge theories one introduced super translations anti-commuting to ordinary translations.

Supersymmetry algebras defined in this manner are characterized by the number of super-generators and in the simplest situation their number is one: one speaks about $\mathcal{N} = 1$ SUSY and minimal super-symmetric extension of standard model (MSSM) in this case. These models are most studied because they are the simplest ones. They have however the strange property that the spinors generating SUSY are Majorana spinors- real in well-defined sense unlike Dirac spinors. This implies that fermion number is conserved only modulo two: this has not been observed experimentally. A second problem is that the proposed mechanisms for the breaking of SUSY do not look feasible.

LHC results suggest MSSM does not become visible at LHC energies. This does not exclude more complex scenarios hiding simplest $\mathcal{N} = 1$ to higher energies but the number of real believers is decreasing. Something is definitely wrong and one must be ready to consider more complex options or totally new view abot SUSY.

What is the analog of SUSY in TGD framework? I must admit that I am still fighting to gain understanding of SUSY in TGD framework [K88]. That I can still imagine several scenarios shows that I have not yet completely understood the problem but I am working hardly to avoid falling to the sin of sloppying myself.

At the basic level one has super-conformal invariance generated in the fermion sector by the super-conformal charges assignable to the strings emanating from partonic 2-surfaces and connecting them to each other. For elementary particles one has 2 wormhole contacts and 4 wormhole throats. If the number of strings is just one, one has symplectic super-conformal symmetry, which is already huge. Several strings must be allowed and this leads to the Yangian variant of super-conformal symmetry, which is multi-local (multi-stringy).

One can also say that fermionic oscillator operators generate infinite-D super-algebra. One can restrict the consideration to lowest conformal weights if spinorial super-conformal invariance acts as gauge symmetry so that one obtains a finite-D algebra with generators labelled by electroweak quantum numbers of quarks and leptons. This super-symmetry is badly broken but contains the algebra generated by right-handed neutrino and its conjugate as sub-algebra.

The basic question is whether covariantly constant right handed neutrino generators $\mathcal{N} = \in$ SUSY or whether the SUSY is generated as approximate symmetry by adding massless right-handed neutrino to the state thus changing its four-momentum. The problem with the first option is that it the standard norm of the state is naturally proportional to four-momentum and vanishes at the limit of vanishing four-momentum: is it possible to circumvent this problem somehow? In the following I summarize the situation as it seems just now.

1. In TGD framework $\mathcal{N} = 1$ SUSY is excluded since B and L and conserved separately and embedding space spinors are not Majorana spinors. The possible analog of space-time SUSY should be a remnant of a much larger super-conformal symmetry in which the Clifford algebra generated by fermionic oscillator operators giving also rise to the Clifford algebra generated by the gamma matrices of the "world of classical worlds" (WCW) and assignable with string world sheets. This algebra is indeed part of infinite-D super-conformal algebra behind quantum TGD. One can construct explicitly the conserved super conformal charges accompanying ordinary charges and one obtains something analogous to $\mathcal{N} = \infty$ super algebra. This SUSY is however badly broken by electroweak interactions.
2. The localization of induced spinors to string world sheets emerges from the condition that electromagnetic charge is well-defined for the modes of induced spinor fields. There is however an exception: covariantly constant right handed neutrino spinor ν_R: it can be de-localized along entire space-time surface. Right-handed neutrino has no couplings to electroweak fields. It couples however to left handed neutrino by induced gamma matrices except when it is covariantly constant. Note that standard model does not predict ν_R but its existence is necessary if neutrinos develop Dirac mass. ν_R is indeed something which must be considered carefully in any generalization of standard model.

Could covariantly constant right handed neutrinos generate SUSY?

Could covariantly constant right-handed spinors generate exact $\mathcal{N} = 2$ SUSY? There are two spin directions for them meaning the analog $\mathcal{N} = 2$ Poincare SUSY. Could these spin directions correspond to right-handed neutrino and antineutrino. This SUSY would not look like Poincare SUSY for which anti-commutator of super generators would be proportional to four-momentum. The problem is that four-momentum vanishes for covariantly constant spinors! Does this mean that the sparticles generated by covariantly constant ν_R are zero norm states and represent super gauge degrees of freedom? This might well be the case although I have considered also alternative scenarios.

What about non-covariantly constant right-handed neutrinos?

Both embedding space spinor harmonics and the Kähler-Dirac equation have also right-handed neutrino spinor modes not constant in M^4 and localized to the partonic orbits. If these are responsible for SUSY then SUSY is broken.

1. Consider first the situation at space-time level. Both induced gamma matrices and their generalizations to Kähler-Dirac gamma matrices defined as contractions of embedding space gamma matrices with the canonical momentum currents for Kähler action are superpositions of M^4 and CP_2 parts. This gives rise to the mixing of right-handed and left-handed neutrinos. Note that non-covariantly constant right-handed neutrinos must be localized at string world sheets.
   This in turn leads neutrino massivation and SUSY breaking. Given particle would be accompanied by sparticles containing varying number of right-handed neutrinos and antineutrinos localized at partonic 2-surfaces.
2. One an consider also the SUSY breaking at embedding space level. The ground states of the representations of extended conformal algebras are constructed in terms of spinor harmonics of the embedding space and form the addition of right-handed neutrino with non-vanishing four-momentum would make sense. But the non-vanishing four-momentum means that the members of the super-multiplet cannot have same masses. This is one manner to state what SUSY breaking is.

What one can say about the masses of sparticles?

The simplest form of massivation would be that all members of the super-multiplet obey the same mass formula but that the p-adic length scales associated with them are different. This could allow very heavy sparticles. What fixes the p-adic mass scales of sparticles? If this scale is CP_2 mass scale SUSY would be experimentally unreachable. The estimate below does not support this option.

One can consider the possibility that SUSY breaking makes sparticles unstable against phase transition to their dark variants with $h_{eff} = n \times h$. Sparticles could have same mass but be non-observable as dark matter not appearing in same vertices as ordinary matter! Geometrically the addition of right-handed neutrino to the state would induce many-sheeted covering in this case with right handed neutrino perhaps associated with different space-time sheet of the covering.

This idea need not be so outlandish at it looks first.

1. The generation of many-sheeted covering has interpretation in terms of breaking of conformal invariance. The sub-algebra for which conformal weights are n-tuples of integers becomes the algebra of conformal transformations and the remaining conformal generators do note represent gauge degrees of freedom anymore. They could however represent conserved conformal charges still.
2. This generalization of conformal symmetry breaking gives rise to infinite number of fractal hierarchies formed by sub-algebras of conformal algebra and is also something new and a fruit of an attempt to avoid sloppy thinking. The breaking of conformal symmetry is indeed expected in massivation related to the SUSY breaking.

The following poor man's estimate supports the idea about dark sfermions and the view that sfermions cannot be very heavy.

1. Neutrino mixing rate should correspond to the mass scale of neutrinos known to be in eV range for ordinary value of Planck constant. For $h_{eff}/h = n$ it is reduced by factor $1/n$, when mass kept constant. Hence sfermions could be stabilized by making them dark.
2. A very rough order of magnitude estimate for sfermion mass scale is obtained from Uncertainty Principle: particle mass should be higher than its decay rate. Therefore an estimate for the decay rate of sfermion could give a lower bound for its mass scale.
3. Assume the transformation $\nu_R \rightarrow \nu_L$ makes sfermion unstable against the decay to fermion and ordinary neutrino. If so, the decay rate would be dictated by the mixing rate and therefore to neutrino mass scale for the ordinary value of Planck constant. Particles and sparticles would have the same p-adic mass scale. Large h_{eff} could however make sfermion dark, stable, and non-observable.

A rough model for the neutrino mixing in TGD framework

The mixing of neutrinos would be the basic mechanism in the decays of sfermions. The following argument tries to capture what is essential in this process.

1. Conformal invariance requires that the string ends at which fermions are localized at wormhole throats are light-like curves. In fact, light-likeness gives rise to Virasosoro conditions.
2. Mixing is described by a vertex residing at partonic surface at which two partonic orbits join. Localization of fermions to string boundaries reduces the problem to a problem completely analogous to the coupling of point particle coupled to external gauge field. What is new that orbit of the particle has edge at partonic 2-surface. Edge breaks conformal invariance since one cannot say that curve is light-like at the edge. At edge neutrino transforms from right-handed to left handed one.
3. In complete analogy with $\overline{\Psi}\gamma^t A_t\Psi$ vertex for the point-like particle with spin in external field, the amplitude describing $nu_R-\nu_L$ transition involves matrix elements of form $\overline{\nu}_R\Gamma^t(CP_2)Z_t\nu_L$ at the vertex of the CP_2 part of the Kähler-Dirac gamma matrix and classical Z^0 field.
How Γ^t is identified? The Kähler-Dirac gamma matrices associated with the interior need not be well-defined at the light-like surface and light-like curve. One basis of weak form of electric magnetic duality the Kähler-Dirac gamma matrix corresponds to the canonical momentum density associated with the Chern-Simons term for Kähler action. This gamma matrix contains only the CP_2 part.

The following provides as more detailed view.

1. Let us denote by $\Gamma^t_{CP_2}(in/out)$ the CP_2 part of the Kähler-Dirac gamma matrix at string at at partonic 2-surface and by Z^0_t the value of Z^0 gauge potential along boundary of string world sheet. The direction of string line in embedding space changes at the partonic 2-surface. The question is what happens to the Kähler-Dirac action at the vertex.
2. For incoming and outgoing lines the equation
$$D(in/out)\Psi(in/out) = p^k(in,out)\gamma_k\Psi(in/out) \ ,$$
where the Kähler-Dirac operator is $D(in/out) = \Gamma^t(in/out)D_t$, is assumed. ν_R corresponds to "in" and ν_R to "out". It implies that lines corresponds to massless M^4 Dirac propagator and one obtains something resembling ordinary perturbation theory.
It also implies that the residue integration over fermionic internal momenta gives as a residue massless fermion lines with non-physical helicities as one can expect in twistor approach. For physical particles the four-momenta are massless but in complex sense and the imaginary part comes classical from four-momenta assignable to the lines of generalized Feynman diagram possessing Euclidian signature of induced metric so that the square root of the metric determinant differs by imaginary unit from that in Minkowskian regions.
3. In the vertex $D(in/out)$ could act in $\Psi(out/in)$ and the natural idea is that $\nu_R-\nu_L$ mixing is due to this so that it would be described the classical weak current couplings $\overline{\nu}_R\Gamma^t_{CP_2}(out)Z^0_t(in)\nu_L$ and $\overline{\nu}_R\Gamma^t_{CP_2}(out)Z^0_t(in)\nu_L$.

To get some idea about orders of magnitude assume that the CP_2 projection of string boundary is geodesic circle thus describable as $\Phi=\omega t$, where Φ is angle coordinate for the circle and t is Minkowski time coordinate. The contribution of CP_2 to the induced metric g_{tt} is $\Delta g_{tt} = -R^2\omega^2$.

1. In the first approximation string end is a light-like curve in Minkowski space meaning that CP_2 contribution to the induced metric vanishes. Neutrino mixing vanishes at this limit.
2. For a non-vanishing value of ωR the mixing and the order of magnitude for mixing rate and neutrino mass is expected to be $R\sim\omega$ and $m\sim\omega/h$. p-Adic length scale hypothesis and the experimental value of neutrino mass allows to estimate m to correspond to p-adic mass to be of order eV so that the corresponding p-adic prime p could be $p\simeq 2^{167}$. Note that $k=127$ defines largest of the four Gaussian Mersennes $M_{G,k}=(1+i)^k-1$ appearing in the length scale range 10 nm -2.5 μm. Hence the decay rate for ordinary Planck constant would be of order $R\sim 10^{14}$/s but large value of Planck constant could reduced it dramatically. In living matter reductions by a factor 10^{-12} can be considered.

To sum up, the space-time SUSY in TGD sense would differ crucially from SUSY in the standard sense. There would no Majorana spinors and sparticles could correspond to dark phase of matter with non-standard value of Planck constant. The signatures of the standard SUSY do not apply to TGD. Of course, a lot of professional work would be needed to derive the signatures of TGD SUSY.

15.4 New Hadron Physics

15.4.1 Leptohadron Physics

TGD suggest strongly ("predicts" is perhaps too strong expression) the existence of color excited leptons. The mass calculations based on p-adic thermodynamics and p-adic conformal invariance lead to a rather detailed picture about color excited leptons.

1. The simplest color excited neutrinos and charged leptons belong to the color octets ν_8 and L_{10} and $L_{\overline{10}}$ decouplet representations respectively and lepto-hadrons are formed as the color singlet bound states of these and possible other representations. Electro-weak symmetry suggests strongly that the minimal representation content is octet and decouplets for both neutrinos and charged leptons.
2. The basic mass scale for lepto-hadron physics is completely fixed by p-adic length scale hypothesis. The first guess is that color excited leptons have the levels $k = 127, 113, 107, ...$ ($p \simeq 2^k$, k prime or power of prime) associated with charged leptons as primary condensation levels. p-Adic length scale hypothesis allows however also the level $k = 11^2 = 121$ in case of electronic lepto-hadrons. Thus both $k = 127$ and $k = 121$ must be considered as a candidate for the level associated with the observed lepto-hadrons. If also lepto-hadrons correspond non-perturbatively to exotic Super Virasoro representations, lepto-pion mass relates to pion mass by the scaling factor $L(107)/L(k) = k^{(107-k)/2}$. For $k = 121$ one has $m_{\pi_L} \simeq 1.057$ MeV which compares favorably with the mass $m_{\pi_L} \simeq 1.062$ MeV of the lowest observed state: thus $k = 121$ is the best candidate contrary to the earlier beliefs. The mass spectrum of lepto-hadrons is expected to have same general characteristics as hadronic mass spectrum and a satisfactory description should be based on string tension concept. Regge slope is predicted to be of order $\alpha' \simeq 1.02/MeV^2$ for $k = 121$. The masses of ground state lepto-hadrons are calculable once primary condensation levels for colored leptons and the CKM matrix describing the mixing of color excited lepton families is known.

The strongest counter arguments against color excited leptons are the following ones.

1. The decay widths of Z^0 and W boson allow only $N = 3$ light particles with neutrino quantum numbers. The introduction of new light elementary particles seems to make the decay widths of Z^0 and W intolerably large.
2. Lepto-hadrons should have been seen in e^+e^- scattering at energies above few MeV. In particular, lepto-hadronic counterparts of hadron jets should have been observed.

A possible resolution of these problems is provided by the loss of asymptotic freedom in lepto-hadron physics. Lepto-hadron physics would effectively exist in a rather limited energy range about one MeV.

The development of the ideas about dark matter hierarchy [**?**, K93, K39, K37] led however to a much more elegant solution of the problem.

1. TGD predicts an infinite hierarchy of various kinds of dark matters which in particular means a hierarchy of color and electro-weak physics with weak mass scales labelled by appropriate p-adic primes different from M_{89}: the simplest option is that also ordinary photons and gluons are labelled by M_{89}.
2. There are number theoretical selection rules telling which particles can interact with each other. The assignment of a collection of primes to elementary particle as characterizer of p-adic primes characterizing the particles coupling directly to it, is inspired by the notion of infinite primes [K94] , and discussed in [**?**] . Only particles characterized by integers having common prime factors can interact by the exchange of elementary bosons: the p-adic length scale of boson corresponds to a common primes.

3. Also the physics characterized by different values of h_{eff} are dark with respect to each other as far quantum coherent gauge interactions are considered. Laser beams might well correspond to photons characterized by p-adic prime different from M_{89} and de-coherence for the beam would mean decay to ordinary photons. De-coherence interaction involves scaling down of the Compton length characterizing the size of the space-time of particle implying that particles do not anymore overlap so that macroscopic quantum coherence is lost.
4. Those dark physics which are dark relative to each other can interact only via graviton exchange. If lepto-hadrons correspond to a physics for which weak bosons correspond to a p-adic prime different from M_{89}, intermediate gauge bosons cannot have direct decays to colored excitations of leptons irrespective of whether the QCD in question is asymptotically free or not. Neither are there direct interactions between the QED:s and QCD:s in question if M_{89} characterizes also ordinary photons and gluons. These ideas are discussed and applied in detail in [**?**, K93, K39] .

Skeptic reader might stop the reading after these counter arguments unless there were definite experimental evidence supporting the lepto-hadron hypothesis.

1. The production of anomalous e^+e^- pairs in heavy ion collisions (energies just above the Coulomb barrier) suggests the existence of pseudo-scalar particles decaying to e^+e^- pairs. A natural identification is as lepto-pions that is bound states of color octet excitations of e^+ and e^-.
2. The second puzzle, Karmen anomaly, is quite recent [C31] . It has been found that in charge pion decay the distribution for the number of neutrinos accompanying muon in decay $\pi \rightarrow \mu+\nu_\mu$ as a function of time seems to have a small shoulder at $t_0 \sim ms$. A possible explanation is the decay of charged pion to muon plus some new weakly interacting particle with mass of order 30 MeV [C10] : the production and decay of this particle would proceed via mixing with muon neutrino. TGD suggests the identification of this state as color singlet leptobaryon of, say type $L_B = f_{abc}L_8^aL_8^b\bar{L}_8^c$, having electro-weak quantum numbers of neutrino.
3. The third puzzle is the anomalously high decay rate of orto-positronium. [C43] . e^+e^- annihilation to virtual photon followed by the decay to real photon plus virtual lepto-pion followed by the decay of the virtual lepto-pion to real photon pair, $\pi_L\gamma\gamma$ coupling being determined by axial anomaly, provides a possible explanation of the puzzle.
4. There exists also evidence for anomalously large production of low energy e^+e^- pairs [C30, C41, C35, C59] in hadronic collisions, which might be basically due to the production of lepto-hadrons via the decay of virtual photons to colored leptons.

In this chapter a revised form of lepto-hadron hypothesis is described.

1. Sigma model realization of PCAC hypothesis allows to determine the decay widths of lepto-pion and lepto-sigma to photon pairs and e^+e^- pairs. Ortopositronium anomaly determines the value of $f(\pi_L)$ and therefore the value of lepto-pion-lepto-nucleon coupling and the decay rate of lepto-pion to two photons. Various decay widths are in accordance with the experimental data and corrections to electro-weak decay rates of neutron and muon are small.
2. One can consider several alternative interpretations for the resonances.
   Option 1: For the minimal color representation content, three lepto-pions are predicted corresponding to $8, 10, \overline{10}$ representations of the color group. If the lightest lepto-nucleons e_{ex} have masses only slightly larger than electron mass, the anomalous e^+e^- could be actually $e_{ex}^+ + e_{ex}^-$ pairs produced in the decays of lepto-pions. One could identify 1.062, 1.63 and 1.77 MeV states as the three lepto-pions corresponding to $8, 10, \overline{10}$ representations and also understand why the latter two resonances have nearly degenerate masses. Since d and s quarks have same primary condensation level and same weak quantum numbers as colored e and μ, one might argue that also colored e and μ correspond to $k = 121$. From the mass ratio of the colored e and μ, as predicted by TGD, the mass of the muonic lepto-pion should be about 1.8 MeV in the absence of topological mixing. This suggests that 1.83 MeV state corresponds to the lightest $g = 1$ lepto-pion.
   Option 2: If one believes sigma model (in ordinary hadron physics the existence of sigma meson is not established and its width is certainly very large if it exists), then lepto-pions are accompanied by sigma scalars. If lepto-sigmas decay dominantly to e^+e^- pairs (this might be forced by kinematics) then one could adopt the previous sceneario and could identify 1.062

state as lepto-pion and 1.63, 1.77 and 1.83 MeV states as lepto-sigmas rather than lepto-pions. The fact that muonic lepto-pion should have mass about 1.8 MeV in the absence of topological mixing, suggests that the masses of lepto-sigma and lepto-pion should be rather close to each other.

Option 3: One could also interpret the resonances as string model "satellite states" having interpretation as radial excitations of the ground state lepto-pion and lepto-sigma. This identification is not however so plausible as the genuinely TGD based identification and will not be discussed in the sequel.

3. PCAC hypothesis and sigma model leads to a general model for lepto-hadron production in the electromagnetic fields of the colliding nuclei and production rates for lepto-pion and other lepto-hadrons are closely related to the Fourier transform of the instanton density $\bar{E}\cdot\bar{B}$ of the electromagnetic field created by nuclei. The first source of anomalous e^+e^- pairs is the production of $\sigma_L\pi_L$ pairs from vacuum followed by $\sigma_L \to e^+e^-$ decay. If $e^+_{ex}e^-_{ex}$ pairs rather than genuine e^+e^- pairs are in question, the production is production of lepto-pions from vacuum followed by lepto-pion decay to lepto-nucleon pair.

   Option 1: For the production of lepto-nucleon pairs the cross section is only slightly below the experimental upper bound for the production of the anomalous e^+e^- pairs and the decay rate of lepto-pion to lepto-nucleon pair is of correct order of magnitude.

   Option 2: The rough order of magnitude estimate for the production cross section of anomalous e^+e^- pairs via $\sigma_l\pi_l$ pair creation followed by $\sigma_L \to e^+e^-$ decay, is by a factor of order $1/\sum N_c^2$ (N_c is the total number of states for a given colour representation and sum over the representations contributing to the ortopositronium anomaly appears) smaller than the reported cross section in case of 1.8 MeV resonance. The discrepancy could be due to the neglect of the large radiative corrections (the coupling $g(\pi_L\pi_L\sigma_L) = g(\sigma_L\sigma_L\sigma_L)$ is very large) and also due to the uncertainties in the value of the measured cross section.

   Given the unclear status of sigma in hadron physics, one has a temptation to conclude that anomalous e^+e^- pairs actually correspond to lepto-nucleon pairs.

4. The vision about dark matter suggests that direct couplings between leptons and lepto-hadrons are absent in which case no new effects in the direct interactions of ordinary leptons are predicted. If colored leptons couple directly to ordinary leptons, several new physics effects such as resonances in photon-photon scattering at cm energy equal to lepto-pion masses and the production of $e_{ex}\bar{e}_{ex}$ (e_{ex} is leptobaryon with quantum numbers of electron) and $e_{ex}\bar{e}$ pairs in heavy ion collisions, are possible. Lepto-pion exchange would give dominating contribution to $\nu - e$ and $\bar{\nu} - e$ scattering at low energies. Lepto-hadron jets should be observed in e^+e^- annihilation at energies above few MeV:s unless the loss of asymptotic freedom restricts lepto-hadronic physics to a very narrow energy range and perhaps to entirely non-perturbative regime of lepto-hadronic QCD.

During 18 years after the first published version of the model also evidence for colored μ has emerged. Towards the end of 2008 CDF anomaly gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

15.4.2 Evidence For TGD View About QCD Plasma

The emergence of the first interesting findings from LHC by CMS collaboration [C17, C1] provide new insights to the TGD picture about the phase transition from QCD plasma to hadronic phase and inspired also the updating of the model of RHIC events (mainly elimination of some remnants from the time when the ideas about hierarchy of Planck constants had just born).

In some proton-proton collisions more than hundred particles are produced suggesting a single object from which they are produced. Since the density of matter approaches to that observed in heavy ion collisions for five years ago at RHIC, a formation of quark gluon plasma and its subsequent decay is what one would expect. The observations are not however quite what QCD plasma picture would allow to expect. Of course, already the RHIC results disagreed with what QCD expectations. What is so striking is the evolution of long range correlations between particles in events containing more than 90 particles as the transverse momentum of the particles increases in the range 1-3 GeV (see the excellent description of the correlations by Lubos Motl in his blog [C6]).

One studies correlation function for two particles as a function of two variables. The first variable is the difference $\Delta\phi$ for the emission angles and second is essentially the difference for the velocities described relativistically by the difference $\Delta\eta$ for hyperbolic angles. As the transverse momentum p_T increases the correlation function develops structure. Around origin of $\Delta\eta$ axis a widening plateau develops near $\Delta\phi = 0$. Also a wide ridge with almost constant value as function of $\Delta\eta$ develops near $\Delta\phi = \pi$. The interpretation is that particles tend to move collinearly and or in opposite directions. In the latter case their velocity differences are large since they move in opposite directions so that a long ridge develops in $\Delta\eta$ direction in the graph.

Ideal QCD plasma would predict no correlations between particles and therefore no structures like this. The radiation of particles would be like blackbody radiation with no correlations between photons. The description in terms of string like object proposed also by Lubos Motl on basis of analysis of the graph showing the distributions as an explanation of correlations looks attractive. The decay of a string like structure producing particles at its both ends moving nearly parallel to the string to opposite directions could be in question.

Since the densities of particles approach those at RHIC, I would bet that the explanation (whatever it is!) of the hydrodynamical behavior observed at RHIC for some years ago should apply also now. The introduction of string like objects in this model was natural since in TGD framework even ordinary nuclei are string like objects with nucleons connected by color flux tubes [L3] , [L3] : this predicts a lot of new nuclear physics for which there is evidence. The basic idea was that in the high density hadronic color flux tubes associated with the colliding nucleon connect to form long highly entangled hadronic strings containing quark gluon plasma. The decay of these structures would explain the strange correlations. It must be however emphasized that in the recent case the initial state consists of two protons rather than heavy nuclei so that the long hadronic string could form from the QCD like quark gluon plasma at criticality when long range fluctuations emerge.

The main assumptions of the model for the RHIC events and those observed now deserve to be summarized. Consider first the "macroscopic description".

1. A critical system associated with confinement-deconfinement transition of the quark-gluon plasma formed in the collision and inhibiting long range correlations would be in question.
2. The proposed hydrodynamic space-time description was in terms of a scaled variant of what I call critical cosmology defining a universal space-time correlate for criticality: the specific property of this cosmology is that the mass contained by comoving volume approaches to zero at the initial moment so that Big Bang begins as a silent whisper and is not so scaring. Criticality means flat 3-space instead of Lobatchevski space and means breaking of Lorentz invariance to SO(4). Breaking of Lorentz invariance was indeed observed for particle distributions but now I am not so sure whether it has much to do with this.

The microscopic level the description would be like follows.

1. A highly entangled long hadronic string like object (color-magnetic flux tube) would be formed at high density of nucleons via the fusion of ordinary hadronic color-magnetic flux tubes to much longer one and containing quark gluon plasma. In QCD world plasma would not be at flux tube.
2. This geometrically (and perhaps also quantally!) entangled string like object would straighten and split to hadrons in the subsequent "cosmological evolution" and yield large numbers of almost collinear particles. The initial situation should be apart from scaling similar as in cosmology where a highly entangled soup of cosmic strings (magnetic flux tubes) precedes the space-time as we understand it. Maybe ordinary cosmology could provide analogy as galaxies arranged to form linear structures?
3. This structure would have also black hole like aspects but in totally different sense as the 10-D hadronic black-hole proposed by Nastase to describe the findings. Note that M-theorists identify black holes as highly entangled strings: in TGD 1-D strings are replaced by 3-D string like objects.

15.4.3 The Incredibly Shrinking Proton

The discovery by Pohl *et al* (2010) [C42] was that the charge radius of proton deduced from detuerium - the muonic version of hydrogen atom - is .842 fm and about 4 per cent smaller than .875 fm than the charge radius deduced from hydrogen atom [C50, C53] is in complete conflict with the cherished belief that atomic physics belongs to the museum of science (for details see the Wikipedia article `http://tinyurl.com/jkt2mkv`). The title of the article *Quantum electrodynamics-a chink in the armour?* of the article published in Nature [C42] expresses well the possible implications, which might actually go well extend beyond QED.

Quite recently (2016) new more precise data has emerged from Pohl *et al* [C45] (see `http://tinyurl.com/jd2hwuq`). Now the reduction of charge radius of muonic variant of deuterium is measured. The charge radius is reduced from 2.1424 fm to 2.1256 fm and the reduction is .012 fm, which is about .8 per cent (see `http://tinyurl.com/j4z3yp9`). The charge radius of proton deduced from it is reported to be consistent with the charge radius deduced from deuterium. The anomaly seems therefore to be real. Deuterium data provide a further challenge for various models. The finding is a problem of QED or to the standard view about what proton is. Lamb shift [C2] is the effect distinguishing between the states hydrogen atom having otherwise the same energy but different angular momentum. The effect is due to the quantum fluctuations of the electromagnetic field. The energy shift factorizes to a product of two expressions. The first one describes the effect of these zero point fluctuations on the position of electron or muon and the second one characterizes the average of nuclear charge density as "seen" by electron or muon. The latter one should be same as in the case of ordinary hydrogen atom but it is not. Does this mean that the presence of muon reduces the charge radius of proton as determined from muon wave function? This of course looks implausible since the radius of proton is so small. Note that the compression of the muon's wave function has the same effect.

Before continuing it is good to recall that QED and quantum field theories in general have difficulties with the description of bound states: something which has not received too much attention. For instance, van der Waals force at molecular scales is a problem. A possible TGD based explanation and a possible solution of difficulties proposed for two decades ago is that for bound states the two charged particles (say nucleus and electron or two atoms) correspond to two 3-D surfaces glued by flux tubes rather than being idealized to points of Minkowski space. This would make the non-relativistic description based on Schrödinger amplitude natural and replace the description based on Bethe-Salpeter equation having horrible mathematical properties.

In the following two models of the anomaly will be discussed.

1. The basic idea of the original model is that muon has some probability to end up to the magnetic flux tubes assignable to proton. In this state it would not contribute to the ordinary Schrödinger amplitude. The effect of this would be reduction of $|\Psi|^2$ near origin and apparent reduction of the charge radius of proton. The weakness of the model is that it cannot make quantitative prediction for the size of the effect. Even the sign is questionable. Only S-wave binding energy is affected considerably but does the binding energy really increase by the interaction of muon with the quarks at magnetic flux tubes? Is the average of the charge density seen by muon in S wave state larger, in other words does it spend more time near proton or do the quarks spend more time at the flux tubes?
2. Second option is inspired by data about breaking of universality of weak interactions in neutral B decays possibly manifesting itself also in the anomaly in the magnetic moment of muon. Also the different values of the charge radius deduced from hydrogen atom and muonium could reflect the breaking of universality. In the original model the breaking of universality is only effective.
3. TGD indeed predicts a dynamical U(3) gauge symmetry whose 8+1 gauge bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Throats are characterized by genus $g = 0, 1, 2$, so that bosons are superpositions of states labelled by (g_1, g_2). Fermions correspond to single wormhole throat carrying fermion number and behave as U(3) triplet labelled by g.
The charged gauge bosons with different genera for wormhole throats are expected to be very massive. The 3 neutral gauge bosons with same genus at both throats are superpositions of states (g, g) are expected to be lighter. Their charge matrices are orthogonal and necessarily break the universality of electroweak interactions. For the lowest boson family - ordinary gauge

bosons - the charge matrix is proportional to unit matrix. The exchange of second generation bosons Z_1^0 and γ_1 would give rise to Yukawa potential increasing the binding energies of S-wave states. Therefore Lamb shift defined as difference between energies of S and P waves is increased and the charge radius deduced from Lamb shift becomes smaller.

4. The model thus predicts a correct sign for the effect but the size of the effect from naïve estimate assuming only γ_1 contribution and $\alpha_1 = \alpha$ ad $M = 2.9$ TeV is almost by an order of magnitude too small. The values of the gauge couplings α_1 and $\alpha_1 Z, 1$ are free parameters as also the mixing angles between states (g, g). The effect is also proportional to the ratio $(m_\mu/M(boson)^2$. It turns out that the inclusion of Z_1^0 contribution and assumption α_1 and $\alpha_1 Z, 1$ are near color coupling strength α_s gives a correct prediction.

Basic facts and notions

In this section the basic TGD inspired ideas and notions - in particular the notion of field body - are introduced and the general mechanism possibly explaining the reduction of the effective charge radius relying on the leakage of muon wave function to the flux tubes associated with u quarks is introduced. After this the value of leakage probability is estimated from the standard formula for the Lamb shift in the experimental situation considered.

1. Basic notions of TGD which might be relevant for the problem

Can one say anything interesting about the possible mechanism behind the anomaly if one accepts TGD framework? How the presence of muon could reduce the charge radius of proton? Let us first list the basic facts and notions.

1. One can say that the size of muonic hydrogen characterized by Bohr radius is by factor $m_e/m_\mu = 1/211.4 = 4.7\times 10^{-4}$ smaller than for hydrogen atom and equals to 250 fm. Hydrogen atom Bohr radius is .53 Angstroms.
2. Proton contains 2 quarks with charge 2e/3 and one d quark which charge -e/3. These quarks are light. The last determination of u and d quark masses [C36] (see `http://tinyurl.com/zqbj7x4`) gives masses, which are $m_u = 2$ MeV and $m_d = 5$ MeV (I leave out the error bars). The standard view is that the contribution of quarks to proton mass is of same order of magnitude. This would mean that quarks are not too relativistic meaning that one can assign to them a size of order Compton wave length of order $4 \times r_e \simeq 600$ fm in the case of u quark (roughly twice the Bohr radius of muonic hydrogen) and $10 \times r_e \simeq 24$ fm in the case of d quark. These wavelengths are much longer than the proton charge radius and for u quark more than twice longer than the Bohr radius of the muonic hydrogen. That parts of proton would be hundreds of times larger than proton itself sounds a rather weird idea. One could of course argue that the scales in question do not correspond to anything geometric. In TGD framework this is not the way out since quantum classical correspondence requires this geometric correlate.
3. There is also the notion of classical radius of electron and quark. It is given by $r = \alpha\hbar/m$ and is in the case of electron this radius is 2.8 fm whereas proton charge radius is .877 fm and smaller. The dependence on Planck constant is only apparent as it should be since classical radius is in question. For u quark the classical radius is .52 fm and smaller than proton charge radius. The constraint that the classical radii of quarks are smaller than proton charge radius gives a lower bound of quark masses: p-adic scaling of u quark mass by $2^{-1/2}$ would give classical radius .73 fm which still satisfies the bound. TGD framework the proper generalization would be $r = \alpha_K\hbar/m$, where α_K is Kähler coupling strength defining the fundamental coupling constant of the theory and quantized from quantum criticality. Its value is very near or equal to fine structure constant in electron length scale.
4. The intuitive picture is that light-like 3-surfaces assignable to quarks describe random motion of partonic 2-surfaces with light-velocity. This is analogous to zitterbewegung assigned classically to the ordinary Dirac equation. The notion of braid emerges from the localization of the modes of the induced spinor field to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces carrying vanishing W fields and Z^0 field at least above weak scale. It is implied by well-definedness of em charge for the modes of Kähler-Dirac action. The orbits of partonic 2-surface effectively reduces to braids carrying fermionic quantum numbers. These braids in

turn define higher level braids which would move inside a structure characterizing the particle geometrically. Internal consistency suggests that the classical radius $r = \alpha_K \hbar/m$ characterizes the size scale of the zitterbewegung orbits of quarks.

I cannot resist the temptation to emphasize the fact that Bohr orbitology is now reasonably well understood. The solutions of field equations with higher than 3-D CP_2 projection describing radiation fields allow only generalizations of plane waves but not their superpositions in accordance with the fact it is these modes that are observed. For massless extremals with 2-D CP_2 projection superposition is possible only for parallel light-like wave vectors. Furthermore, the restriction of the solutions of the Chern-Simons Dirac equation at light-like 3-surfaces to braid strands gives the analogs of Bohr orbits. Wave functions of -say electron in atom- are wave functions for the position of wormhole throat and thus for braid strands so that Bohr's theory becomes part of quantum theory.

5. In TGD framework quantum classical correspondence requires -or at least strongly suggests- that also the p-adic length scales assignable to u and d quarks have geometrical correlates. That quarks would have sizes much larger than proton itself how sounds rather paradoxical and could be used as an objection against p-adic length scale hypothesis. Topological field quantization however leads to the notion of field body as a structure consisting of flux tube-sandthe identification of this geometric correlate would be in terms of Kähler (or color-, or electro-) magnetic body of proton consisting of color flux tubes beginning from space-time sheets of valence quarks and having length scale of order Compton wavelength much longer than the size of proton itself. Magnetic loops and electric flux tubes would be in question. Also secondary p-adic length scale characterizes field body. For instance, in the case of electron the causal diamond assigned to electron would correspond to the time scale of .1 seconds defining an important bio-rhythm.

2. Could the notion of field body explain the anomaly?

The large Compton radii of quarks and the notion of field body encourage the attempt to imagine a mechanism affecting the charge radius of proton as determined from electron's or muon's wave function.

1. Muon's wave function is compressed to a volume, which is about 8 million times smaller than the corresponding volume in the case of electron. The Compton radius of u quark more that twice larger than the Bohr radius of muonic hydrogen so that muon should interact directly with the field body of u quark. The field body of d quark would have size 24 fm which is about ten times smaller than the Bohr radius so that one can say that the volume in which muons sees the field body of d quark is only one thousandth of the total volume. The main effect would be therefore due to the two u quarks having total charge of 4e/3.

   One can say that muon begins to "see" the field bodies of u quarks and interacts directly with u quarks rather than with proton via its electromagnetic field body. With d quarks it would still interact via protons field body to which d quark should feed its electromagnetic flux. This could be quite enough to explain why the charge radius of proton determined from the expectation value defined by its wave function is smaller for muonium than for hydrogen. One must of course notice that this brings in also direct magnetic interactions with u quarks.
2. What could be the basic mechanism for the reduction of charge radius? Could it be that the muon is caught with some probability into the flux tubes of u quarks and that Schrödinger amplitude for this kind state vanishes near the origin? If so, this portion of state would not contribute to the charge radius and the since the portion ordinary state would smaller, this would imply an effective reduction of the charge radius determined from experimental data using the standard theory since the reduction of the norm of the standard part of the state would be erratically interpreted as a reduction of the charge radius.
3. This effect would be of course present also in the case of electron but in this case the u quarks correspond to a volume which million times smaller than the volume defined by Bohr radius so that electron does not in practice "see" the quark sub-structure of proton. The probability P for getting caught would be in a good approximation proportional to the value of $|\Psi(r_u)|^2$ and in the first approximation one would have

$$\frac{P_e}{P_\mu} \sim (a_\mu/a_e)^3 = (m_e/m_\mu)^3 \sim 10^{-7} \ .$$

from the proportionality $\Psi_i \propto 1/a_i^{3/2}$, i=e,$\mu$.

3. A general formula for Lamb shift in terms of proton charge radius

The charge radius of proton is determined from the Lamb shift between 2S- and 2P states of muonic hydrogen. Without this effect resulting from vacuum polarization of photon Dirac equation for hydrogen would predict identical energies for these states. The calculation reduces to the calculation of vacuum polarization of photon inducing to the Coulomb potential and an additional vacuum polarization term. Besides this effect one must also take into account the finite size of the proton which can be coded in terms of the form factor deducible from scattering data. It is just this correction which makes it possible to determine the charge radius of proton from the Lamb shift.

1. In the article [C9] the basic theoretical results related to the Lamb shift in terms of the vacuum polarization of photon are discussed. Proton's charge density is in this representation is expressed in terms of proton form factor in principle deducible from the scattering data. Two special cases can be distinguished corresponding to the point like proton for which Lamb shift is non-vanishing only for S wave states and non-point like proton for which energy shift is present also for other states. The theoretical expression for the Lamb shift involves very refined calculations. Between 2P and 2S states the expression for the Lamb shift is of form

$$\Delta E(2P_{3/2}^{F=2}2S_{1/2}^{F=1} = a - br_p^2 + cr_p^3 = 209.968(5)5.2248 \times r_p^2 + 0.0347 \times r_p^3 \;\; meV \;\; . \quad (15.4.1)$$

where the charge radius $r_p = .8750$ is expressed in femtometers and energy in meVs.

2. The general expression of Lamb shift is given in terms of the form factor by

$$\begin{aligned} E(2P-2S) &= \int \frac{d^3q}{2\pi)^3} \times (-4\pi\alpha)\frac{F(q^2)}{q^2}\frac{\Pi(q^2)}{q^2} \times X \;\; , \\ X &= \int (|\Psi_{2P}(r)|^2 - |\Psi_{2S}(r)|^2) exp(iq\cdot r) dV \;\; . \end{aligned} \quad (15.4.2)$$

Here Π is is a scalar representing vacuum polarization due to decay of photon to virtual pairs.

The model to be discussed predicts that the effect is due to a leakage from "standard" state to what I call flux tube state. This means a multiplication of $|\Psi_{2P}|^2$ with the normalization factor $1/N$ of the standard state orthogonalized with respect to flux tube state. It is essential that $1/N$ is larger than unity so that the effect is a genuine quantum effect not understandable in terms of classical probability.

The modification of the formula is due to the normalization of the 2P and 2S states. These are in general different. The normalization factor $1/N$ is same for all terms in the expression of Lamb shift for a given state but in general different for 2S and 2P states. Since the lowest order term dominates by a factor of ~ 40 over the second one, one one can conclude that the modification should affect the lowest order term by about 4 per cent. Since the second term is negative and the modification of the first term is interpreted as a modification of the second term when r_p is estimated from the standard formula, the first term must increase by about 4 per cent. This is achieved if this state is orthogonalized with respect to the flux tube state. For states Ψ_0 and Ψ_{tube} with unit norm this means the modification

$$\begin{aligned} \Psi_0 &\rightarrow \frac{1}{1-|C|^2} \times (\Psi_i - C\Psi_{tube}) \;\; , \\ C &= \langle \Psi_{tube}|\Psi_0\rangle \;\; . \end{aligned} \quad (15.4.3)$$

In the lowest order approximation one obtains

$$a - br_p^2 + cr_p^3 \;\; \rightarrow \;\; (1+|C|^2)a - br_p^2 + cr_p^3 \;\; . \quad (15.4.4)$$

Using instead of this expression the standard formula gives a wrong estimate r_p from the condition

$$a - b\hat{r}_p^2 + c\hat{r}_p^3 \rightarrow (1+|C|^2)a - br_p^2 + cr_p^3 \ . \tag{15.4.5}$$

This gives the equivalent conditions

$$\begin{aligned} \hat{r}_p^2 &= r_p^2 - \frac{|C|^2 a}{b} \ , \\ P_{tube} &\equiv |C|^2 \simeq 2\frac{b}{a} \times r_p^2 \times \frac{(r_p - \hat{r_p})}{r_p}) \ . \end{aligned} \tag{15.4.6}$$

The resulting estimate for the leakage probability is $P_{tube} \simeq .0015$. The model should be able to reproduce this probability.

A model for the coupling between standard states and flux tube states

Just for fun one can look whether the idea about confinement of muon to quark flux tube carrying electric flux could make sense.

1. Assume that the quark is accompanied by a flux tube carrying electric flux $\int EdS = -\int \nabla\Phi \cdot dS = q$, where $q = 2e/3 = ke$ is the u quark charge. The potential created by the u quark at the proton end of the flux tube with transversal area $S = \pi R^2$ idealized as effectively 1-D structure is

$$\Phi = -\frac{ke}{\pi R^2}|x| + \Phi_0 \ . \tag{15.4.7}$$

The normalization factor comes from the condition that the total electric flux is q. The value of the additive constant V_0 is fixed by the condition that the potential coincides with Coulomb potential at $r = r_u$, where r_u is u quark Compton length. This gives

$$e\Phi_0 = \frac{e^2}{r_u} + Kr_u \ , \ K = \frac{ke^2}{\pi R^2} \ . \tag{15.4.8}$$

2. Parameter R should be of order of magnitude of charge radius $\alpha_K r_u$ of u quark is free parameter in some limits. $\alpha_K = \alpha$ is expected to hold true in excellent approximation. Therefore a convenient parameterization is

$$R = z\alpha r_u \ . \tag{15.4.9}$$

This gives

$$K = \frac{4k}{\alpha r_u^2} \ , \ e\Phi_0 = 4(\pi\alpha + \frac{k}{\alpha})\frac{1}{r_u} \ . \tag{15.4.10}$$

3. The requirement that electron with four times larger charge radius that u quark can topologically condensed inside the flux tube without a change in the average radius of the flux tube (and thus in a reduction in p-adic length scale increasing its mass by a factor 4!) suggests that $z \geq 4$ holds true at least far away from proton. Near proton the condition that the radius of the flux tube is smaller than electron's charge radius is satisfied for $z = 1$.

1. Reduction of Schrödinger equation at flux tube to Airy equation

The 1-D Schrödinger equation at flux tube has as its solutions Airy functions and the related functions known as "Bairy" functions.

1. What one has is a one-dimensional Schrödinger equation of general form

$$-\frac{\hbar^2}{2m_\mu}\frac{d^2\Psi}{dx^2} + (Kx - e\Phi_0)\Psi = E\Psi \ , \ K = \frac{ke^2}{\pi R^2} \ . \tag{15.4.11}$$

By performing a linear coordinate change

$$u = (\frac{2m_\mu K}{\hbar^2})^{1/3}(x - x_E) \ , \ x_E = \frac{-|E| + e\Phi_0}{K} \ , \tag{15.4.12}$$

one obtains

$$\frac{d^2\Psi}{du^2} - u\Psi = 0 \ . \quad (15.4.13)$$

This differential equation is known as Airy equation (or Stokes equation) and defines special functions $Ai(x)$ known as Airy functions and related functions $Bi(x)$ referred to as "Bairy" functions [B1] . Airy functions characterize the intensity near an optical directional caustic such as that of rainbow.

2. The explicit expressions for $A_i(u)$ and $Bi(u)$ are is given by

$$\begin{aligned} Ai(u) &= \frac{1}{\pi}\int_0^\infty cos(\frac{1}{3}t^3 + ut)dt \ , \\ Bi(u) &= \frac{1}{\pi}\int_0^\infty \left[exp(-\frac{1}{3}t^3) + sin(\frac{1}{3}t^3 + ut)dt\right] \ . \end{aligned} \quad (15.4.14)$$

$Ai(u)$ oscillates rapidly for negative values of u having interpretation in terms of real wave vector and goes exponentially to zero for $u > 0$. $Bi(u)$ oscillates also for negative values of x but increases exponentially for positive values of u. The oscillatory behavior and its character become obvious by noticing that stationary phase approximation is possible for $x < 0$.
The approximate expressions of $Ai(u)$ and $Bi(u)$ for $u > 0$ are given by

$$\begin{aligned} Ai(u) &\sim \frac{1}{2\pi^{1/2}}exp(-\frac{2}{3}u^{3/2})u^{-1/4} \ , \\ Bi(u) &\sim \frac{1}{\pi^{1/2}}exp(\frac{2}{3}u^{3/2})u^{-1/4} \ . \end{aligned} \quad (15.4.15)$$

For $u < 0$ one has

$$\begin{aligned} Ai(u) &\sim \frac{1}{\pi^{1/2}}sin(\frac{2}{3}(-u)^{3/2})(-u)^{-1/4} \ , \\ Bi(u) &\sim \frac{1}{\pi^{1/2}}cos(\frac{2}{3}(-u)^{3/2})(-u)^{-1/4} \ . \end{aligned} \quad (15.4.16)$$

3. $u = 0$ corresponds to the turning point of the classical motion where the kinetic energy changes sign. $x = 0$ and $x = r_u$ correspond to the points

$$\begin{aligned} u_{min} \equiv u(0) &= -(\frac{2m_\mu K}{\hbar^2})^{1/3}x_E \ , \\ u_{max} \equiv u(r_u) &= (\frac{2m_\mu K}{\hbar^2})^{1/3}(r_u - x_E) \ , \\ x_E &= \frac{-|E| + e\Phi_0}{K} \ . \end{aligned} \quad (15.4.17)$$

4. The general solution is

$$\Psi = aAi(u) + bBi(u) \ . \quad (15.4.18)$$

The natural boundary condition is the vanishing of Ψ at the lower end of the flux tube giving

$$\frac{b}{a} = -\frac{Ai(u(0))}{Bi(u(0))} \ . \quad (15.4.19)$$

A non-vanishing value of b implies that the solution increases exponentially for positive values of the argument and the solution can be regarded as being concentrated in an excellent approximation near the upper end of the flux tube.
Second boundary condition is perhaps most naturally the condition that the energy is same for the flux tube amplitude as for the standard solution. Alternative boundary conditions would require the vanishing of the solution at both ends of the flux tube and in this case one obtains very large number of solutions as WKB approximation demonstrates. The normalization of the state so that it has a unit norm fixes the magnitude of the coefficients a and b since one can choose them to be real.

2. Estimate for the probability that muon is caught to the flux tube

The simplest estimate for the muon to be caught to the flux tube state characterized by the same energy as standard state is the overlap integral of the ordinary hydrogen wave function of muon and of the effectively one-dimensional flux tube. What one means with overlap integral is however not quite obvious.

1. The basic condition is that the modified "standard" state is orthogonal to the flux tube state. One can write the expression of a general state as

$$\begin{aligned} \Psi_{nlm} &\rightarrow N \times (\Psi_{nlm} - C(E,nlm)\Phi_{nlm}) \ , \\ \Phi_{nlm} &= Y_{lm}\Psi_E \ , \\ C(E,nlm) &= \langle \Psi_E | \Psi_{nlm} \rangle \ . \end{aligned} \tag{15.4.20}$$

   Here Φ_{nlm} depends a flux tube state in which spherical harmonics is wave function in the space of orientations of the flux tube and Ψ_E is flux tube state with same energy as standard state. Here an inner product between standard states and flux tube states is introduced.

2. Assuming same energy for flux tube state and standard state, the expression for the total total probability for ending up to single flux tube would be determined from the orthogonality condition as

$$P_{nlm} = \frac{|C(E,nlm)|^2}{1-|C(E,lmn)|^2} \ . \tag{15.4.21}$$

   Here E refers to the common energy of flux tube state and standard state. The fact that flux tube states vanish at the lower end of the flux tube implies that they do not contribute to the expression for average charge density. The reduced contribution of the standard part implies that the attempt to interpret the experimental results in "standard model" gives a reduced value of the charge radius. The size of the contribution is given by P_{nlm} whose value should be about 4 per cent.

One can consider two alternative forms for the inner product between standard states and flux tube states. Intuitively it is clear that an overlap between the two wave functions must be in question.

1. The simplest possibility is that one takes only overlap at the upper end of the flux tube which defines 2-D surface. Second possibility is that the overlap is over entire flux tube projection at the space-time sheet of atom.

$$\begin{aligned} \langle \Psi_E | \Psi_{nlm} \rangle &= \int_{end} \overline{\Psi}_r \Psi_{nlm} dS \ \ \text{(Option I)} \ , \\ \langle \Psi_E | \Psi_{nlm} \rangle &= \int_{tube} \overline{\Psi}_r \Psi_{nlm} dV \ \ \text{(Option II)} \ . \end{aligned} \tag{15.4.22}$$

2. For option I the inner product is non-vanishing only if Ψ_E is non-vanishing at the end of the flux tube. This would mean that electron ends up to the flux tube through its end. The inner product is dimensionless without introduction of a dimensional coupling parameter if the inner product for flux tube states is defined by 1-dimensional integral: one might criticize this assumption as illogical. Unitarity might be a problem since the local behaviour of the flux tube wave function at the end of the flux tube could imply that the contribution of the flux tube state in the quantum state dominates and this does not look plausible. One can of course consider the introduction to the inner product a coefficient representing coupling constant but this would mean loss of predictivity. Schrödinger equation at the end of the flux tubes guarantees the conservation of the probability current only if the energy of flux tube state is same as that of standard state or if the flux tube Schrödinger amplitude vanishes at the end of the flux tube.
3. For option II there are no problems with unitary since the overlap probability is always smaller than unity. Option II however involves overlap between standard states and flux tube states even when the wave function at the upper end of the flux tube vanishes. One can however consider the possibility that the possible flux tube states are orthogonalized with respect to standard states with leakage to flux tubes. The interpretation for the overlap integral would be that electron ends up to the flux tube via the formation of wormhole contact.

3. Option I fails

The considerations will be first restricted to the simpler option I. The generalization of the results of calculation to option II is rather straighforward. It turns out that option II gives correct order of magnitude for the reduction of charge radius for reasonable parameter values.

1. In a good approximation one can express the overlap integrals over the flux tube end (option I) as

$$\begin{aligned} C(E,nlm) &= \int_{tube} \overline{\Psi}_E \Psi_{nlm} dS \simeq \pi R^2 \times Y_{lm} \times C(E,nl) \ , \\ C(E,nl) &= \overline{\Psi}_E(r_u) R_{nl}(r_u) \ . \end{aligned} \tag{15.4.23}$$

An explicit expression for the coefficients can be deduced by using expression for Ψ_E as a superposition of Airy and Bairy functions. This gives

$$\begin{aligned} C(E,nl) &= \overline{\Psi}_E(r_u) R_{nl}(r_u) \ , \\ \Psi_E(x) &= a_E Ai(u_E) + bBi(u_E) \ , \ \frac{a_E}{b_E} = -\frac{Bi(u_E(0))}{Ai(u_E(0))} \ , \\ u_E(x) &= (\frac{2m_\mu K}{\hbar^2})^{1/3}(x - x_E) \ , \ x_E = \frac{|E| - e\Phi_0}{K} \ , \\ K &= \frac{ke^2}{\pi R^2} \ , \ R = z\alpha_K r_u \ , \ k = \frac{2}{3} \ . \end{aligned} \tag{15.4.24}$$

The normalization of the coefficients is fixed from the condition that a and b chosen in such a way that Ψ has unit norm. For these boundary conditions Bi is expected to dominate completely in the sum and the solution can be regarded as exponentially decreasing function concentrated around the upper end of the flux tube.

In order to get a quantitative view about the situation one can express the parameters u_{min} and u_{max} in terms of the basic dimensionless parameters of the problem.

1. One obtains

$$\begin{aligned} u_{min} \equiv u(0) &= -2(\frac{k}{z\alpha})^{1/3}\left[1 + \pi\frac{z}{k}\alpha^2(1 - \frac{1}{2}\alpha r)\right] \times r^{1/3} \ , \\ u_{max} \equiv u(r_u) &= u(0) + 2\frac{k}{z\alpha} \times r^{1/3} \ , \\ r &= \frac{m_\mu}{m_u} \ , \ R = z\alpha r_u \ . \end{aligned} \tag{15.4.25}$$

Using the numerical values of the parameters one obtains for $z = 1$ and $\alpha = 1/137$ the values $u_{min} = -33.807$ and $u_{max} = 651.69$. The value of u_{max} is so large that the normalization is in practice fixed by the exponential behavior of Bi for the suggested boundary conditions.

2. The normalization constant is in good approximation defined by the integral of the approximate form of Bi^2 over positive values of u and one has

$$N^2 \simeq \frac{dx}{du} \times \int_{u_{min}}^{u_{max}} Bi(u)^2 du \ , \ \frac{dx}{du} = \frac{1}{2}(\frac{z^2\alpha}{k})^{1/3} \times r^{1/3} r_u \ , \tag{15.4.26}$$

By taking $t = exp(\frac{4}{3}u^{3/2})$ as integration variable one obtains

$$\begin{aligned} \int_{u_{min}}^{u_{max}} Bi(u)^2 du &\simeq \pi^{-1} \int_{u_{min}}^{u_{max}} exp(\frac{4}{3}u^{3/2}) u^{-1/2} du \\ &= (\frac{4}{3})^{2/3} \pi^{-1} \int_{t_{min}}^{t_{max}} \frac{dt}{log(t)^{2/3}} \simeq \frac{1}{\pi} \frac{exp(\frac{4}{3}u_{max}^{3/2})}{u_{max}} \ . \end{aligned} \tag{15.4.27}$$

This gives for the normalization factor the expression

$$N \simeq \frac{1}{2}(\frac{z^2\alpha}{k})^{2/3} r^{1/3} r_u^{1/2} exp(\frac{2}{3}u_{max}^{3/2}) \ . \tag{15.4.28}$$

3. One obtains for the value of Ψ_E at the end of the flux tube the estimate

$$\Psi_E(r_u) = \frac{Bi(u_{max})}{N} \simeq 2\pi^{-1/2} \times (\frac{k}{z^2\alpha})^{2/3} r^{1/3} r_u^{-1/2} \ , \ r = \frac{r_u}{r_\mu} \ . \quad (15.4.29)$$

4. The inner product defined as overlap integral gives for the ground state

$$\begin{aligned} C_{E,00} &= \Psi_E(r_u) \times \Psi_{1,0,0}(r_u) \times \pi R^2 \\ &= 2\pi^{-1/2}(\frac{k}{z^2\alpha})^{2/3} r^{1/3} r_u^{-1/2} \times (\frac{1}{\pi a(\mu)^3})^{1/2} \times exp(-\alpha r) \times \pi z^2 \alpha^2 r_u^2 \\ &= 2\pi^{1/2} k^{2/3} z^{2/3} r^{11/6} \alpha^{17/6} exp(-\alpha r) \ . \end{aligned} \quad (15.4.30)$$

The relative reduction of charge radius equals to $P = C^2_{E,00}$. For $z = 1$ one obtains $P = C^2_{E,00} = 5.5 \times 10^{-6}$, which is by three orders of magnitude smaller than the value needed for $P_{tube} = C^2_{E,20} = .0015$. The obvious explanation for the smallness is the α^2 factor coming from the area of flux tube in the inner product.

4. Option II could work

The failure of the simplest model is essentially due to the inner product. For option II the inner product for the flux tube states involves the integral over the area of flux tube so that the normalization factor for the state is obtained from the previous one by the replacement $N \to N/\sqrt{\pi R^2}$. In the integral over the flux tube the exponent function is is in the first approximation equal to constant since the wave function for ground state is at the end of the flux tube only by a factor .678 smaller than at the origin and the wave function is strongly concentrated near the end of the flux tube. The inner product defined by the overlap integral over the flux tube implies $N \to NS^{1/2}$, $S = \pi R^2 = z^2\alpha^2 r_u^2$. In good approximation the inner product for option II means the replacement

$$\begin{aligned} C_{E,n0} &\to A \times B \times C_{E,n0} \ , \\ A &= \frac{\frac{dx}{du}}{\sqrt{\pi R^2}} = \frac{1}{2\sqrt{\pi}} z^{-1/3} k^{-1/3} \alpha^{-2/3} r^{1/3} \ , \\ B &= \frac{\int Bi(u)du}{\sqrt{Bi(u_{max})}} = u_{max}^{-1/4} = 2^{-1/4} z^{1/2} k^{-1/4} \alpha^{1/4} r^{-1/12} \ . \end{aligned} \quad (15.4.31)$$

Using the expression

$$R_{20}(r_u) = \frac{1}{2\sqrt{2}} \times (\frac{1}{a_\mu})^{3/2} \times (2 - r\alpha) \times exp(-r\alpha) \ , \ r = \frac{r_u}{r_\mu} \quad (15.4.32)$$

one obtains for $C_{E,20}$ the expression

$$C_{E,20} = 2^{-3/4} z^{5/6} k^{1/12} \alpha^{29/12} r^{25/12} \times (2 - r\alpha) \times exp(-r\alpha) \ . \quad (15.4.33)$$

By the earlier general argument one should have $P_{tube} = |C_{E,20}|^2 \simeq .0015$. $P_{tube} = .0015$ is obtained for $z = 1$ and $N = 2$ corresponding to single flux tube per u quark. If the flux tubes are in opposite directions, the leakage into 2P state vanishes. Note that this leakage does not affect the value of the coefficient a in the general formula for the Lamb shift. The radius of the flux tube is by a factor 1/4 smaller than the classical radius of electron and one could argue that this makes it impossible for electron to topologically condense at the flux tube. For $z = 4$ one would have $P_{tube} = .015$ which is 10 times too large a value. Note that the nucleus possess a wave function for the orientation of the flux tube. If this corresponds to S-wave state then only the leakage beween S-wave states and standard states is possible.

Are exotic flux tube bound states possible?

There seems to be no deep reason forbidding the possibility of genuine flux tube states decoupling from the standard states completely. To get some idea about the energy eigenvalues one can apply WKB approximation. This approach should work now: in fact, the study on WKB approximation near turning point by using linearization of the potential leads always to Airy equation so that the linear potential represents an ideal situation for WKB approximation. As noticed these states do not seem to be directly relevant for the recent situation. The fact that these states have larger binding energies than the ordinary states of hydrogen atom might make possible to liberate energy by inducing transitions to these states.

1. Assume that a bound state with a negative energy E is formed inside the flux tube. This means that the condition $p^2 = 2m(E-V) \geq 0$, $V = -e\Phi$, holds true in the region $x \leq x_{max} < r_u$ and $p^2 = 2m(E-V) < 0$ in the region $r_u > x \geq x_{max}$. The expression for x_{max} is

$$x_{max} = \frac{\pi R^2}{k}(-\frac{|E|}{e^2} + \frac{1}{r_u} + \frac{kr_u}{\pi R^2})\hbar \ . \tag{15.4.34}$$

   $x_{max} < r_u$ holds true if one has

$$|E| < \frac{e^2}{r_u} = E_{max} \ . \tag{15.4.35}$$

   The ratio of this energy to the ground state energy of muonic hydrogen is from $E(1) = e^2/2a(\mu)$ and $a = \hbar/\alpha m$ given by

$$\frac{E_{max}}{E(n=1)} = \frac{2m_u}{\alpha m_\mu} \simeq 5.185 \ . \tag{15.4.36}$$

   This encourages to think that the ground state energy could be reduced by the formation of this kind of bound state if it is possible to find a value of n in the allowed range. The physical state would of course contain only a small fraction of this state. In the case of electron the increase of the binding energy is even more dramatic since one has

$$\frac{E_{max}}{E(n=1)} = \frac{2m_u}{\alpha m_e} = \frac{8}{\alpha} \simeq 1096 \ . \tag{15.4.37}$$

   Obviously the formation of this kind of states could provide a new source of energy. There have been claims about anomalous energy production in hydrogen [D9] . I have discussed these claims from TGD viewpoint in [K102]

2. One can apply WKB quantization in the region where the momentum is real to get the condition

$$I = \int_0^{x_{max}} \sqrt{2m(E+e\Phi)}\frac{dx}{\hbar} = n + \frac{1}{2} \ . \tag{15.4.38}$$

   By performing the integral one obtains the quantization condition

$$\begin{aligned} I &= k^{-1}(8\pi\alpha)^{1/2} \times \frac{R^2}{r_u^{3/2} r_\mu} \times A^{3/2} = n + \frac{1}{2} \ , \\ A &= 1 + kx^2 - \frac{|E| r_u}{e^2} \ , \\ x &= \frac{r_u}{R} \ , \ k = \frac{2}{3\pi} \ , \ r_i = \frac{\hbar}{m_i} \ . \end{aligned} \tag{15.4.39}$$

3. Parameter R should be of order of magnitude of charge radius $\alpha_K r_u$ of u quark is free parameter in some limits. $\alpha_K = \alpha$ is expected to hold true in excellent approximation. Therefore a convenient parameterization is

$$R = z\alpha r_u \ . \tag{15.4.40}$$

   This gives for the binding energy the general expression in terms of the ground state binging energy $E(1,\mu)$ of muonic hydrogen as

$$\begin{aligned}
|E| &= C\times E(1,\mu) \ , \\
C &= D\times (1+Kz^{-2}\alpha^{-2}-(\frac{y}{z^2})^{2/3}\times (n+1/2)^{2/3}) \ , \\
D &= 2y\times (\frac{K^2}{8\pi\alpha})^{1/3} \ , \\
y &= \frac{m_u}{m_\mu} \ , \ K=\frac{2}{3\pi} \ .
\end{aligned} \quad (15.4.41)$$

4. There is a finite number of bound states. The above mentioned consistency conditions coming from $0 < x_{max} < r_\mu$ give $0 < C < C_{max} = 5.185$ restricting the allowed value of n to some interval. One obtains the estimates

$$\begin{aligned}
n_{min} &\simeq \frac{z^2}{y}(1+Kz^{-2}\alpha^{-2}-\frac{C_{max}}{D})^{3/2}-\frac{1}{2} \ , \\
n_{max} &= \frac{z^2}{y}(1+Kz^{-2}\alpha^{-2})^{3/2}-\frac{1}{2} \ .
\end{aligned} \quad (15.4.42)$$

Very large value of n is required by the consistency condition. The calculation gives $n_{min} \in \{1.22\times 10^7, 4.59\times 10^6, 1.48\times 10^5\}$ and $n_{max} \in \{1.33\times 10^7, 6.66\times 10^6, 3.34\times 10^6\}$ for $z \in \{1,2,4\}$. This would be a very large number of allowed bound states -about 3.2×10^6 for $z=1$.

The WKB state behaves as a plane wave below x_{max} and sum of exponentially decaying and increasing amplitudes above x_{max}:

$$\begin{aligned}
&\frac{1}{\sqrt{k(x)}}\left[Aexp(i\int_0^x k(y)dy)+Bexp(-i\int_0^x k(y)dy)\right] \ , \\
&\frac{1}{\sqrt{\kappa(x)}}\left[Cexp(-\int_{x_{max}}^x \kappa(y)dy+Dexp(\int_{x_{max}}^x \kappa(y)dy\right] \ , \\
&k(x)=\sqrt{2m(-|E|+e\Phi)} \ , \ \kappa(x)\sqrt{2m(|E|-e\Phi)} \ .
\end{aligned} \quad (15.4.43)$$

At the classical turning point these two amplitudes must be identical.

The next task is to decide about natural boundary conditions. Two types of boundary conditions must be considered. The basic condition is that genuine flux tube states are in question. This requires that the inner product between flux tube states and standard states defined by the integral over flux tube ends vanishes. This is guaranteed if the Schrödinger amplitude for the flux tube state vanishes at the ends of the flux tube so that flux tube behaves like an infinite potential well. The condition $\Psi(0) = 0$ at the lower end of the flux tube would give $A = -B$. Combined with the continuity condition at the turning point these conditions imply that Ψ can be assumed to be real. The $\Psi(r_u) = 0$ gives a condition leading to the quantization of energy.

The wave function over the directions of flux tube with a given value of n is given by the spherical harmonics assigned to the state (n,l,m).

Could second generation of weak bosons explain the reduction of proton charge radius?

The above proposed speculative model is not the only one that one can imagine. The observation could be explained also as breaking of the universality of weak interactions. Also other anomalies challenging the universality exists. The decays of neutral B-meson to lepton pairs should be same apart from corrections coming from different lepton masses by universality but this does not seem to be the case [K64]. There is also anomaly in muon's magnetic moment discussed briefly in [K88]. This leads to ask whether the breaking of universality could be due to the failure of universality of electroweak interactions.

The proposal for the explanation of the muon's anomalous magnetic moment and anomaly in the decays of B-meson is inspired by a recent very special di-electron event and involves higher generations of weak bosons predicted by TGD leading to a breaking of lepton universality. Both Tommaso Dorigo (`http://tinyurl.com/pfw7qqm`) and Lubos Motl (`http://tinyurl.com/hqzat92`) tell about a spectacular 2.9 TeV di-electron event not observed in previous LHC runs. Single event of this kind is of course most probably just a fluctuation but human mind is such that it tries to see something deeper in it - even if practically all trials of this kind are chasing of mirages.

Since the decay is leptonic, the typical question is whether the dreamed for state could be an exotic Z boson. This is also the reaction in TGD framework. The first question to ask is whether weak bosons assignable to Mersenne prime M_{89} have scaled up copies assignable to Gaussian Mersenne M_{79}. The scaling factor for mass would be $2^{(89-79)/2} = 32$. When applied to Z mass equal to about .09 TeV one obtains 2.88 TeV, not far from 2.9 TeV. Eureka!? Looks like a direct scaled up version of Z!? W should have similar variant around 2.6 TeV.

TGD indeed predicts exotic weak bosons and also gluons.

1. TGD based explanation of family replication phenomenon in terms of genus-generation correspondence forces to ask whether gauge bosons identifiable as pairs of fermion and antifermion at opposite throats of wormhole contact could have bosonic counterpart for family replication. Dynamical SU(3) assignable to three lowest fermion generations labelled by the genus of partonic 2-surface (wormhole throat) means that fermions are combinatorially SU(3) triplets. Could 2.9 TeV state - if it would exist - correspond to this kind of state in the tensor product of triplet and antitriplet? The mass of the state should depend besides p-adic mass scale also on the structure of SU(3) state so that the mass would be different. This difference should be very small.
2. Dynamical SU(3) could be broken so that wormhole contacts with different genera for the throats would be more massive than those with the same genera. This would give SU(3) singlet and two neutral states, which are analogs of $\eta\prime$ and η and π^0 in Gell-Mann's quark model. The masses of the analogs of η and π^0 and the analog of $\eta\prime$, which I have identified as standard weak boson would have different masses. But how large is the mass difference?
3. These 3 states are expected top have identical mass for the same p-adic mass scale, if the mass comes mostly from the analog of hadronic string tension assignable to magnetic flux tube. connecting the two wormhole contacts associates with any elementary particle in TGD framework (this is forced by the condition that the flux tube carrying monopole flux is closed and makes a very flattened square shaped structure with the long sides of the square at different space-time sheets). p-Adic thermodynamics would give a very small contribution genus dependent contribution to mass if p-adic temperature is $T = 1/2$ as one must assume for gauge bosons ($T = 1$ for fermions). Hence 2.95 TeV state could indeed correspond to this kind of state.

Could the exchange of massive $M_{G,79}$ photon and Z^0 give rise to additional electromagnetic interaction inducing the breaking of Universality?

1. The additional contribution in the effective Coulomb potential is Yukawa potential. In S-wave state this would give a contribution to the binding energy in a good approximation given by the expectation value of the Yukawa potential, which can be parameterized as

$$V(r) = g^2 \frac{e^{-Mr}}{r} \ , \quad g^2 = 4\pi k\alpha \ . \tag{15.4.44}$$

. The expectation differs from zero significantly only in S-wave state characterized by principal quantum number n. Since the exponent function goes exponentially to zero in the p-adic length scale associated with 2.9 TeV mass, which is roughly by a factor 32 times shorter than intermediate boson mass scale, hydrogen atom wave function is constant in excellent approximation in the effective integration volume. This gives for the energy shift

$$\begin{aligned} \Delta E &= g^2 |\Psi(0|^2 \times I \ , \\ |\Psi(0|^2 &= \frac{2^2}{n^2}\frac{1}{a_0^3} \ , \ a_0 = \frac{1}{m\alpha} \ , \\ I &= \int \frac{e^{-Mr}}{r} r^2 dr d\Omega = \frac{4\pi}{M^2} \ . \end{aligned} \tag{15.4.45}$$

For the energy shift and its ratio to ground state energy

$$E_n = \frac{\alpha^2}{2n^2} \times m \tag{15.4.46}$$

on obtains the expression

$$\begin{aligned} \Delta E_n &= \frac{64\pi^2\alpha}{n^2}\alpha^3(\frac{m}{M})^2 \times m \ , \\ \frac{\Delta E_n}{E_n} &= 2^7\pi^2\alpha^2k^2(\frac{m}{M})^2 \ . \end{aligned} \tag{15.4.47}$$

For $k = 1$ and $M = 2.9$ TeV one has $\Delta E_n/E_n \simeq 8.9 \times 10^{-11}$ for muon.

Consider next Lamb shift.

1. Lamb shift as difference of energies between S and P wave states (see http://tinyurl.com/y99ctyn4) is approximately given by

$$\frac{\Delta_n(Lamb)}{E_n} = \frac{13\alpha^3}{2n} \ . \tag{15.4.48}$$

For $n = 2$ this gives $\Delta_2(Lamb)/E_2 = 4.9 \times 10^{-7}$.

2. Recall that the previous parameterization for the theoretical Lamb shift reads as

$$\Delta E(r_p(th)) = a - br_p^2 + cr_p^3 = 209.968(5)5.2248 \times r_p^2 + 0.0347 \times r_p^3 \ meV \ . \tag{15.4.49}$$

where the charge radius $r_p = .8750$ is expressed in femtometers and energy in meVs.

3. The reduction of r_p by 3.3 per cent allows to estimate the reduction of Lamb shift (attractive additional potential reduces it). The relative change of the Lamb shift is

$$\begin{aligned} x &= \frac{\Delta E(r_p(th)) - \Delta E(r_p(exp))}{\Delta E(r_p(th))} \\ &= \frac{5.2248 \times (r_p^2(th) - r_p^2(exp)) + 0.0347 \times (r_p^3(th) - r_p^3(exp))}{209.968(5)5.2248 \times r_p^2(th) + 0.0347 \times r_p^3(th)} \ . \end{aligned} \tag{15.4.50}$$

The estimate gives $x = 1.2 \times 10^{-3}$.

This value can be compared with the prediction. For $n = 2$ ratio of $\Delta E_n/\Delta E_n(Lamb)/$ is

$$x = \frac{\Delta E_n}{\Delta E_n(Lamb)} = k^2 \times \frac{2^9\pi^2}{13\alpha} \times (\frac{m}{M})^2 \ . \tag{15.4.51}$$

For $M = 2.9$ TeV the numerical estimate gives $x \simeq k^2 \times 10^{-4}$. The value of x deduced from experimental data is $x \simeq 1.2 \times 10^{-3}$. For $k = 3$ a correct order of magnitude is obtained. There are thus good hopes that the model works.

The contribution of Z_1^0 exchange is neglected in the above estimate. Is it present and can it explain the discrepancy?

1. In the case of deuterium the weak isospins of proton and deuterium are opposite so that their contributions to the Z_1^0 vector potential cancel. If Z_1^0 contribution for proton can be neglected, one has $\Delta r_p = \Delta r_d$.

One however has $\Delta r_p \simeq 2.75\Delta r_d$. Hence Z_1^0 contribution to Δr_p should satisfy $\Delta r_p(Z_1^0) \simeq 1.75 \times \Delta r_p(\gamma_1)$. This requires $\alpha_{Z,1} > \alpha_1$, which is true also for the ordinary gauge bosons. The weak isospins of electron and proton are opposite so that the atom is weak isospin singlet in Abelian sense, and one has $I_p^3 I_\mu^3 = -1/4$ and attractive interaction. The condition relating r_p and r_Z suggests

$$\frac{\alpha_{Z,1}}{\alpha_1} \simeq \frac{28}{6} = 4 + \frac{1}{3} \ .$$

In standard model one has $\alpha_Z/\alpha = 1/[sin^2(\theta_W)cos^2(\theta_W)] = 5.6$ for $sin^2(\theta_W) = .23$. One has upper bound $\alpha_{Z,1}/\alpha_1 \geq 4$ saturated for $sin^2(\theta_{W,1}) = 1/2$. Weinberg angle can be expressed as

$$sin^2(\theta_{W,1}) = \frac{1}{2}\left[1 - \sqrt{1 - 4\frac{\alpha_1}{\alpha_{Z,1}}}\right] \ .$$

$\alpha_{Z,1}/\alpha_1 \simeq 28/6$ gives $sin^2(\theta_{W,1}) = \frac{1}{2}[1-\sqrt{1/7}] \simeq .31$.
The contribution to the axial part of the potential depending on spin need not cancel and could give a spin dependent contribution for both proton and deuteron.

2. If the scale of α_1 and $\alpha_{Z,1}$ is that of $\alpha_s \simeq .1$ at TeV energy scale and if the factor 2.75 emerges in the proposed manner, one has $k^2 \simeq 2.75 \times 10 = 27.5$ rather near to the rough estimate $k^2 \simeq 27$ from data for proton. This would give $\alpha_1 \simeq 1/13.7$.
Note however than there are mixing angles involved corresponding to the diagonal hermitian family charge matrix $Q = (a, b, c)$ satisfying $a^2 + b^2 + c^2 = 1$ and the condition $a + b + c = 0$ expressing the orthogonality with the electromagnetic charge matrix $(1, 1, 1)/\sqrt{3}$ expressing electroweak universality for ordinary electroweak bosons. For instance, one could have $(a, b, c) = (0, 1, -1)/\sqrt{2}$ for the second generation and $(a, b, c) = (2, -1, -1)/\sqrt{6}$ for the third generation. In this case the above estimate would would be scaled down: $\alpha_1 \rightarrow 2\alpha_1/3 \simeq 1/20.5$.

To sum up, the proposed model is successful at quantitative level allowing to understand the different changes for charge radius for proton and deuteron and estimate the values of electroweak couplings of the second generation of weak bosons apart from the uncertainty due to the family charge matrix. Muon's magnetic moment anomaly and decays of neutral B allow to test the model and perhaps fix the remaining two mixing angles.

15.4.4 Misbehaving b-quarks and the magnetic body of proton

Science news tells about misbehaving bottom quarks (see `http://tinyurl.com/jpkwey4` and ICHEP conference talk at `http://tinyurl.com/z4lqtvz`). Or perhaps one should talk about misbehaving b-hadrons - hadrons containing b- quarks. The mis-behavior appears in proton-proton collisions at LHC. This is not the only anomaly associated with proton. The spin of proton is still poorly understood and proton charge radius if quite not what it should be. Now we learn that there are more b-containing hadrons (b-hadrons) in the directions deviating considerably from the direction of proton beam: discrepancy factor is of order two.

How this could reflect the structure of proton? Color magnetic flux tubes are the new TGD based element in the model or proton: could they help? I assign to proton color magnetic flux tubes with size scale much larger than proton size - something like electron Compton length: most of the mass of proton is color magnetic energy associated with these tubes and they define the non-perturbative aspect of hadron physics in TGD framework. For instance, constituent quarks would be valence quarks plus their color flux tubes. Current quarks just the quarks whose masses give rather small contribution to proton mass.

What happens when two protons collide? In cm system the dipolar flux tubes get contracted in the direction of motion by Lorentz contraction. Suppose b-hadrons tend to leave proton along the color magnetic flux tubes (also ordinary em flux tubes could be in question). Lorentz contraction of flux tubes means that they tend to leave in directions orthogonal to the collision axis. Could this explain the misbehavior of b-hadrons?

But why only b-hadrons or some fraction of them should behave in this manner? Why not also lighter hadrons containing c and s? Could this relate to the much smaller size of b-quark defined by its Compton length $L = \hbar/m(b)$, $m(b) = 4.2 GeV$, which is much shorter than the Compton length of u-quark (the mass of constituent u quark is something like 300 MeV and the mass of current u quark is few MeVs. Could it be that lighter hadrons do not leave proton along flux tubes? Why? Are these hadrons or corresponding quarks too large to fit (topologically condense) inside protonic flux tube? b-quark is much more massive and has considerably smaller size than say c-quark with mass $m(c) = 1.5$ GeV and could be able to topologically condense inside the protonic flux tube. c quark should be too large, which suggests that the radius of flux tubes is larger than proton Compton length. This picture conforms with the view of perturbative QCD in which the primary processes take place at parton level. The hadronization would occur in longer time scale and generate the magnetic bodies of outgoing hadrons. The alternative idea that also the color magnetic body of hadron should fit inside the protonic color flux tube is not consistent with this view.

15.4.5 Dark Nuclear Strings As Analogs Of DNA-, RNA- And Amino-Acid Sequences And Baryonic Realization Of Genetic Code?

Water memory is one of the ugly words in the vocabulary of a main stream scientist. The work of pioneers is however now carrying fruit. The group led by Jean-Luc Montagnier, who received Nobel prize for discovering HIV virus, has found strong evidence for water memory and detailed information about the mechanism involved [K48, K103] , [I6] . The work leading to the discovery was motivated by the following mysterious finding. When the water solution containing human cells infected by bacteria was filtered in purpose of sterilizing it, it indeed satisfied the criteria for the absence of infected cells immediately after the procedure. When one however adds human cells to the filtrate, infected cells appear within few weeks. If this is really the case and if the filter does what it is believed to do, this raises the question whether there might be a representation of genetic code based on nano-structures able to leak through the filter with pores size below 200 nm.

The question is whether dark nuclear strings might provide a representation of the genetic code. In fact, I posed this question year before the results of the experiment came with motivation coming from attempts to understand water memory. The outcome was a totally unexpected finding: the states of dark nucleons formed from three quarks can be naturally grouped to multiplets in one-one correspondence with 64 DNAs, 64 RNAS, and 20 amino-acids and there is natural mapping of DNA and RNA type states to amino-acid type states such that the numbers of DNAs/RNAs mapped to given amino-acid are same as for the vertebrate genetic code.

The basic idea is simple. Since baryons consist of 3 quarks just as DNA codons consist of three nucleotides, one might ask whether codons could correspond to baryons obtained as open strings with quarks connected by two color flux tubes. This representation would be based on entanglement rather than letter sequences. The question is therefore whether the dark baryons constructed as string of 3 quarks using color flux tubes could realize 64 codons and whether 20 amino-acids could be identified as equivalence classes of some equivalence relation between 64 fundamental codons in a natural manner.

The following model indeed reproduces the genetic code directly from a model of dark neutral baryons as strings of 3 quarks connected by color flux tubes.

1. Dark nuclear baryons are considered as a fundamental realization of DNA codons and constructed as open strings of 3 dark quarks connected by two colored flux tubes, which can be also charged. The baryonic strings cannot combine to form a strictly linear structure since strict rotational invariance would not allow the quark strings to have angular momentum with respect to the quantization axis defined by the nuclear string. The independent rotation of quark strings and breaking of rotational symmetry from SO(3) to SO(2) induced by the direction of the nuclear string is essential for the model.
   (a) Baryonic strings could form a helical nuclear string (stability might require this) locally parallel to DNA, RNA, or amino-acid) helix with rotations acting either along the axis of the DNA or along the local axis of DNA along helix. The rotation of a flux tube portion around an axis parallel to the local axis along DNA helix requires that magnetic flux tube has a kink in this portion. An interesting question is whether this kink has correlate at the level of DNA too. Notice that color bonds appear in two scales corresponding to these two strings. The model of DNA as topological quantum computer [K6] allows a modification in which dark nuclear string of this kind is parallel to DNA and each codon has a flux tube connection to the lipid of cell membrane or possibly to some other bio-molecule.
   (b) The analogs of DNA -, RNA -, and of amino-acid sequences could also correspond to sequences of dark baryons in which baryons would be 3-quark strings in the plane transversal to the dark nuclear string and expected to rotate by stringy boundary conditions. Thus one would have nuclear string consisting of short baryonic strings not connected along their ends. In this case all baryons would be free to rotate.
2. The new element as compared to the standard quark model is that between both dark quarks and dark baryons can be charged carrying charge $0, \pm 1$. This is assumed also in nuclear string model and there is empirical support for the existence of exotic nuclei containing charged color bonds between nuclei.
3. The net charge of the dark baryons in question is assumed to vanish to minimize Coulomb repulsion:

$$\sum_q Q_{em}(q) = - \sum_{flux\ tubes} Q_{em}(flux\ tube) \quad . \tag{15.4.52}$$

This kind of selection is natural taking into account the breaking of isospin symmetry. In the recent case the breaking cannot however be as large as for ordinary baryons (implying large mass difference between Δ and nucleon states).

4. One can classify the states of the open 3-quark string by the total charges and spins associated with 3 quarks and to the two color bonds. Total em charges of quarks vary in the range $Z_B \in \{2, 1, 0, -1\}$ and total color bond charges in the range $Z_b \in \{2, 1, 0, -1, -2\}$. Only neutral states are allowed. Total quark spin projection varies in the range $J_B = 3/2, 1/2, -1/2, -3/2$ and the total flux tube spin projection in the range $J_b = 2, 1, -1, -2$. If one takes for a given total charge assumed to be vanishing one representative from each class (J_B, J_b), one obtains $4 \times 5 = 20$ states which is the number of amino-acids. Thus genetic code might be realized at the level of baryons by mapping the neutral states with a given spin projection to single representative state with the same spin projection. The problem is to find whether one can identify the analogs of DNA, RNA and amino-acids as baryon like states.

States in the quark degrees of freedom

One must construct many-particle states both in quark and flux tube degrees of freedom. These states can be constructed as representations of rotation group SU(2) and strong isospin group SU(2) by using the standard tensor product rule $j_1 \times j_2 = j_1 + j_2 \oplus j_1 + j_2 - 1 \oplus ... \oplus |j_1 - j_2|$ for the representation of SU(2) and Fermi statistics and Bose-Einstein statistics are used to deduce correlations between total spin and total isospin (for instance, $J = I$ rule holds true in quark degrees of freedom). Charge neutrality is assumed and the breaking of rotational symmetry in the direction of nuclear string is assumed.

Consider first the states of dark baryons in quark degrees of freedom.

1. The tensor product $2 \otimes 2 \otimes 2$ is involved in both cases. Without any additional constraints this tensor product decomposes as $(3 \oplus 1) \otimes 2 = 4 \oplus 2 \oplus 2$: 8 states altogether. This is what one should have for DNA and RNA candidates. If one has only identical quarks uuu or ddd, Pauli exclusion rule allows only the 4-D spin 3/2 representation corresponding to completely symmetric representation -just as in standard quark model. These 4 states correspond to a candidate for amino-acids. Thus RNA and DNA should correspond to states of type uud and ddu and amino-acids to states of type uuu or ddd. What this means physically will be considered later.

2. Due to spin-statistics constraint only the representations with $(J, I) = (3/2, 3/2)$ (Δ resonance) and the second $(J, I) = (1/2, 1/2)$ (proton and neutron) are realized as free baryons. Now of course a dark -possibly p-adically scaled up - variant of QCD is considered so that more general baryonic states are possible. By the way, the spin statistics problem which forced to introduce quark color strongly suggests that the construction of the codons as sequences of 3 nucleons - which one might also consider - is not a good idea.

3. Second nucleon like spin doublet - call it 2_{odd} - has wrong parity in the sense that it would require $L = 1$ ground state for two identical quarks (uu or dd pair). Dropping 2_{odd} and using only $4 \oplus 2$ for the rotation group would give degeneracies $(1, 2, 2, 1)$ and 6 states only. All the representations in $4 \oplus 2 \oplus 2_{odd}$ are needed to get 8 states with a given quark charge and one should transform the wrong parity doublet to positive parity doublet somehow. Since open string geometry breaks rotational symmetry to a subgroup SO(2) of rotations acting along the direction of the string and since the boundary conditions on baryonic strings force their ends to rotate with light velocity, the attractive possibility is to add a baryonic stringy excitation with angular momentum projection $L_z = -1$ to the wrong parity doublet so that the parity comes out correctly. $L_z = -1$ orbital angular momentum for the relative motion of uu or dd quark pair in the open 3-quark string would be in question. The degeneracies for spin projection value $J_z = 3/2, ..., -3/2$ are $(1, 2, 3, 2)$. Genetic code means spin projection mapping the states in $4 \oplus 2 \oplus 2_{odd}$ to 4.

States in the flux tube degrees of freedom

Consider next the states in flux tube degrees of freedom.

1. The situation is analogous to a construction of mesons from quarks and antiquarks and one obtains the analogs of π meson (pion) with spin 0 and ρ meson with spin 1 since spin statistics forces $J = I$ condition also now. States of a given charge for a flux tube correspond to the tensor product $2 \otimes 2 = 3 \oplus 1$ for the rotation group.
2. Without any further constraints the tensor product $3 \otimes 3 = 5 \oplus 3 \oplus 1$ for the flux tubes states gives 8+1 states. By dropping the scalar state this gives 8 states required by DNA and RNA analogs. The degeneracies of the states for DNA/RNA type realization with a given spin projection for $5 \oplus 3$ are $(1, 2, 2, 2, 1)$. $8\times$ 8 states result altogether for both *uud* and *udd* for which color bonds have different charges. Also for *ddd* state with quark charge -1 one obtains $5 \oplus 3$ states giving 40 states altogether.
3. If the charges of the color bonds are identical as the are for *uuu* type states serving as candidates for the counterparts of amino-acids bosonic statistics allows only 5 states ($J = 2$ state). Hence 20 counterparts of amino-acids are obtained for *uuu*. Genetic code means the projection of the states of $5 \oplus 3$ to those of 5 with the same spin projection and same total charge.

Analogs of DNA,RNA, amino-acids, and of translation and transcription mechanisms

Consider next the identification of analogs of DNA, RNA and amino-acids and the baryonic realization of the genetic code, translation and transcription.

1. The analogs of DNA and RNA can be identified dark baryons with quark content *uud*, *ddu* with color bonds having different charges. There are 3 color bond pairs corresponding to charge pairs $(q_1, q_2) = (-1, 0), (-1, 1), (0, 1)$ (the order of charges does not matter). The condition that the total charge of dark baryon vanishes allows for *uud* only the bond pair $(-1, 0)$ and for *udd* only the pair $(-1, 1)$. These thus only single neutral dark baryon of type *uud resp. udd*: these would be the analogous of DNA and RNA codons. Amino-acids would correspond to *uuu* states with identical color bonds with charges $(-1, -1)$, $(0, 0)$, or $(1, 1)$. *uuu* with color bond charges (-1,-1) is the only neutral state. Hence only the analogs of DNA, RNA, and amino-acids are obtained, which is rather remarkable result.
2. The basic transcription and translation machinery could be realized as processes in which the analog of DNA can replicate, and can be transcribed to the analog of mRNA in turn translated to the analogs of amino-acids. In terms of flux tube connections the realization of genetic code, transcription, and translation, would mean that only dark baryons with same total quark spin and same total color bond spin can be connected by flux tubes. Charges are of course identical since they vanish.
3. Genetic code maps of $(4\oplus2\oplus2)\otimes(5\oplus3)$ to the states of 4×5. The most natural map takes the states with a given spin to a state with the same spin so that the code is unique. This would give the degeneracies $D(k)$ as products of numbers $D_B \in \{1, 2, 3, 2\}$ and $D_b \in \{1, 2, 2, 2, 1\}$: $D = D_B \times D_b$. Only the observed degeneracies $D = 1, 2, 3, 4, 6$ are predicted. The numbers $N(k)$ of amino-acids coded by D codons would be

$$[N(1), N(2), N(3), N(4), N(6)] = [2, 7, 2, 6, 3] \ .$$

   The correct numbers for vertebrate nuclear code are $(N(1), N(2), N(3), N(4), N(6)) = (2, 9, 1, 5, 3)$. Some kind of symmetry breaking must take place and should relate to the emergence of stopping codons. If one codon in second 3-plet becomes stopping codon, the 3-plet becomes doublet. If 2 codons in 4-plet become stopping codons it also becomes doublet and one obtains the correct result $(2, 9, 1, 5, 3)$!
4. Stopping codons would most naturally correspond to the codons, which involve the $L_z = -1$ relative rotational excitation of *uu* or *dd* type quark pair. For the 3-plet the two candidates for the stopping codon state are $|1/2, -1/2\rangle \otimes \{|2, k\rangle\}$, $k = 2, -2$. The total spins are $J_z = 3/2$ and $J_z = -7/2$. The three candidates for the 4-plet from which two states are thrown out are $|1/2, -3/2\rangle \otimes \{|2, k\rangle, |1, k\rangle\}$, $k = 1, 0, -1$. The total spins are now $J_z = -1/2, -3/2, -5/2$. One guess is that the states with smallest value of J_z are dropped which would mean that $J_z = -7/2$ states in 3-plet and $J_z = -5/2$ states 4-plet become stopping codons.

5. One can ask why just vertebrate code? Why not vertebrate mitochondrial code, which has unbroken $A-G$ and $T-C$ symmetries with respect to the third nucleotide. And is it possible to understand the rarely occurring variants of the genetic code in this framework? One explanation is that the baryonic realization is the fundamental one and biochemical realization has gradually evolved from non-faithful realization to a faithful one as kind of emulation of dark nuclear physics. Also the role of tRNA in the realization of the code is crucial and could explain the fact that the code can be context sensitive for some codons.

Understanding the symmetries of the code

Quantum entanglement between quarks and color flux tubes would be essential for the baryonic realization of the genetic code whereas chemical realization could be said to be classical. Quantal aspect means that one cannot decompose to codon to letters anymore. This raises questions concerning the symmetries of the code.

1. What is the counterpart for the conjugation $ZYZ \rightarrow X_cY_cZ_c$ for the codons?
2. The conjugation of the second nucleotide Y having chemical interpretation in terms of hydrophoby-hydrophily dichotomy in biology. In DNA as TQC model it corresponds to matter-antimatter conjugation for quarks associated with flux tubes connecting DNA nucleotides to the lipids of the cell membrane. What is the interpretation in now?
3. The A-G, T-C symmetries with respect to the third nucleotide Z allow an interpretation as weak isospin symmetry in DNA as TQC model. Can one identify counterpart of this symmetry when the decomposition into individual nucleotides does not make sense?

Natural candidates for the building blocks of the analogs of these symmetries are the change of the sign of the spin direction for quarks and for flux tubes.

1. For quarks the spin projections are always non-vanishing so that the map has no fixed points. For flux tube spin the states of spin $S_z = 0$ are fixed points. The change of the sign of quark spin projection must therefore be present for both $XYZ \rightarrow X_cY_cZ_c$ and $Y \rightarrow Y_c$ but also something else might be needed. Note that without the symmetry breaking $(1,3,3,1) \rightarrow (1,2,3,2)$ the code table would be symmetric in the permutation of 2 first and 2 last columns of the code table induced by both full conjugation and conjugation of Y.
2. The analogs of the approximate $A-G$ and $T-C$ symmetries cannot involve the change of spin direction in neither quark nor flux tube sector. These symmetries act inside the A-G and T-C sub-2-columns of the 4-columns defining the rows of the code table. Hence this symmetry must permute the states of same spin inside 5 and 3 for flux tubes and 4 and 2 for quarks but leave 2_{odd} invariant. This guarantees that for the two non-degenerate codons coding for only single amino-acid and one of the codons inside triplet the action is trivial. Hence the baryonic analog of the approximate $A-G$ and $T-C$ symmetry would be exact symmetry and be due to the basic definition of the genetic code as a mapping states of same flux tube spin and quark spin to single representative state. The existence of full 4-columns coding for the same amino-acid would be due to the fact that states with same quark spin inside $(2,3,2)$ code for the same amino-acid.
3. A detailed comparison of the code table with the code table in spin representation should allow to fix their correspondence uniquely apart from permutations of n-plets and thus also the representation of the conjugations. What is clear that Y conjugation must involve the change of quark spin direction whereas Z conjugation which maps typically 2-plets to each other must involve the permutation of states with same J_z for the flux tubes. It is not quite clear what X conjugation correspond to.

Some comments about the physics behind the code

Consider next some particle physicist's objections against this picture.

1. The realization of the code requires the dark scaled variants of spin 3/2 baryons known as Δ resonance and the analogs (and only the analogs) of spin 1 mesons known as ρ mesons. The lifetime of these states is very short in ordinary hadron physics. Now one has a scaled up variant of hadron physics: possibly in both dark and p-adic senses with latter allowing arbitrarily small overall mass scales. Hence the lifetimes of states can be scaled up.

2. Both the absolute and relative mass differences between Δ and N *resp.* ρ and π are large in ordinary hadron physics and this makes the decays of Δ and ρ possible kinematically. This is due to color magnetic spin-spin splitting proportional to the color coupling strength $\alpha_s \sim .1$, which is large. In the recent case α_s could be considerably smaller - say of the same order of magnitude as fine structure constant 1/137 - so that the mass splittings could be so small as to make decays impossible.
3. Dark hadrons could have lower mass scale than the ordinary ones if scaled up variants of quarks in p-adic sense are in question. Note that the model for cold fusion that inspired the idea about genetic code requires that dark nuclear strings have the same mass scale as ordinary baryons. In any case, the most general option inspired by the vision about hierarchy of conscious entities extended to a hierarchy of life forms is that several dark and p-adic scaled up variants of baryons realizing genetic code are possible.
4. The heaviest objection relates to the addition of $L_z = -1$ excitation to $S_z = |1/2, \pm 1/2\rangle_{odd}$ states which transforms the degeneracies of the quark spin states from $(1,3,3,1)$ to $(1,2,3,2)$. The only reasonable answer is that the breaking of the full rotation symmetry reduces $SO(3)$ to $SO(2)$. Also the fact that the states of massless particles are labeled by the representation of $SO(2)$ might be of some relevance. The deeper level explanation in TGD framework might be as follows. The generalized embedding space is constructed by gluing almost copies of the 8-D embedding space with different Planck constants together along a 4-D subspace like pages of book along a common back. The construction involves symmetry breaking in both rotational and color degrees of freedom to Cartan sub-group and the interpretation is as a geometric representation for the selection of the quantization axis. Quantum TGD is indeed meant to be a geometrization of the entire quantum physics as a physics of the classical spinor fields in the "world of classical worlds" so that also the choice of measurement axis must have a geometric description.

The conclusion is that genetic code can be understand as a map of stringy baryonic states induced by the projection of all states with same spin projection to a representative state with the same spin projection. Genetic code would be realized at the level of dark nuclear physics and biochemical representation would be only one particular higher level representation of the code. A hierarchy of dark baryon realizations corresponding to p-adic and dark matter hierarchies can be considered. Translation and transcription machinery would be realized by flux tubes connecting only states with same quark spin and flux tube spin. Charge neutrality is essential for having only the analogs of DNA, RNA and amino-acids and would guarantee the em stability of the states.

15.5 Cosmic Rays And Mersenne Primes

Sabine Hossenfelder has written two excellent blog postings about cosmic rays. The first one is about the GKZ (see `http://tinyurl.com/ybdflmgl`) cutoff for cosmic ray energies and second one about possible indications for new physics above 100 TeV (see `http://tinyurl.com/ydewc2ug`). This inspired me to read what I have said about cosmic rays and Mersenne primes- this was around 1996 - immediately after performing for the first time p-adic mass calculations. It was unpleasant to find that some pieces of the text contained a stupid mistake related to the notion of cosmic ray energy. I had forgotten to take into account the fact that the cosmic ray energies are in the rest system of Earth- what a shame! The recent version should be free of worst kind of blunders. Before continuing it should be noticed I am now living year 2012 and this section was written for the first time for around 1996 - and as it became clear - contained some blunders due to the confusion with what one means with cosmic ray energy. The recent version should be free of worst kind of blunders.

TGD suggests the existence of a scaled up copy of hadron physics associated with each Mersenne prime $M_n = 2^n - 1$, n prime: M_{107} corresponds to ordinary hadron physics. Also lepto-hadrons are predicted. Also Gaussian Mersennes $(1+i)^k - 1$, could correspond to hadron physics. Four of them ($k = 151, 157, 163, 167$) are in the biologically interesting length scale range between cell membrane thickness and the size of cell nucleus. Also leptonic counterparts of hadron physics assignable to certain Mersennes are predicted and there is evidence for them (see `http://tinyurl.com/ybfkptns`) [K104].

The scaled up variants of hadron physics corresponding to $k < 107$ are of special interest. $k = 89$ defines the interesting Mersenne prime at LHC, and the near future will probably tell whether the 125 GeV signal corresponds to Higgs or a pion of M_{89} physics. Also cosmic ray spectrum could provide support for M_{89} hadrons and quite recent cosmic ray observations [C60] are claimed to provide support for new physics around 100 TeV (see `http://tinyurl.com/y8s8swa5`). M_{89} proton would correspond to.5 TeV mass considerably below 100 TeV but this mass scale could correspond to a mass scale of a scaled up copy of a heavy quark of M_{107} hadron physics: a naïve scaling of top quark mass by factor 512 would give mass about 87 TeV. Also the lighter hadrons of M_{89} hadron physics should contribute to cosmic ray spectrum and there are indeed indications for this.

The mechanisms giving rise to ultra high energy cosmic rays are poorly understood. The standard explanation would be acceleration in huge magnetic fields. TGD suggests a new mechanism based on the decay cascade of cosmic strings. The basis idea is that cosmic string decays *cosmic string* $\rightarrow M_2$ *hadrons* $\rightarrow M_3$ *hadrons* ....$\rightarrow M_{61} \rightarrow M_{89} \rightarrow M_{107}$ *hadrons* could be a new source of cosmic rays. Also variants of this scenario with decay cascade beginning from larger Mersenne prime can be considered. One expects that the decay cascade leads rapidly to extremely energetic ordinary hadrons, which can collide with ordinary hadrons in atmosphere and create hadrons of scaled variants of ordinary hadron physics. These cosmic ray events could serve as a signature for the existence of these scale up variants of hadron physics.

1. Centauro events and the peculiar events associated with $E > 10^5$ GeV radiation from Cygnus X-3. E refers to energy in Earth's rest frame and for a collision with proton the cm energy would be $E_{cm} = \sqrt{2EM} > 10$ TeV in good approximation whereas M_{89} variant of proton would have mass of.5 TeV. These events be understood as being due to the collisions of energetic M_{89} hadrons with ordinary hadrons (nucleons) in the atmosphere.
2. The decay $\pi_n \rightarrow \gamma\gamma$ produces a peak in the spectrum of the cosmic gamma rays at energy $\frac{m(\pi_n)}{2}$. These produce peaks in cosmic gamma ray spectrum at energies which depend on the energy of π_n in the rest system of Earth. If the pion is at rest in the cm system of incoming proton and atmospheric proton one can estimate the energy of the peak if the total energy of the shower can be estimated reliably.
3. The slope in the hadronic cosmic ray spectrum changes at $E = 3 \cdot 10^6$ GeV. This corresponds to the energy $E_{cm} = 2.5$ TeV in the cm system of cosmic ray hadron and atmospheric proton. This is not very far from M_{89} proton mass .5 TeV. The creation of M_{89} hadrons in atmospheric collisions could explain the change of the slope.
4. The ultra-higher energy cosmic ray radiation having energies of order 10^9 GeV in Earth's rest system apparently consisting of protons and nuclei not lighter than Fe might be actually dominated by gamma rays: at these energies γ and p induced showers have same muon content. $E = 10^9$ GeV corresponds to $E_{cm} = \sqrt{2Em_p} = 4 \times 10^4$ GeV. M_{89} nucleon would correspond to mass scale 512 GeV.
5. So called GKZ cutoff should take place for cosmic gamma ray spectrum due to the collisions with the cosmic microwave background. This should occur around $E = 6 \times 10^{10}$ GeV, which corresponds to $E_{cm} = 3.5 \times 10^5$ GeV. Cosmic ray events above this cutoff (see `http://tinyurl.com/y75jho96`) are however claimed. There should be some mechanism allowing for ultra high energy cosmic rays to propagate over much longer distances as allowed by the limits. Cosmic rays should be able to propagate without collisions. Many-sheeted space-time suggests ways for how gamma rays could avoid collisions with microwave background. For instance, gamma rays could be dark in TGD sense and therefore have large value of Planck constant. One can even imagine exotic variants of hadrons, which differ from ordinary hadrons in that they do not have quarks and therefore no interactions with the microwave background.
6. The highest energies of cosmic rays are around $E = 10^{11}$ GeV, which corresponds to $E_{cm} = 4 \times 10^5$ GeV. M_{61} nucleon and pion correspond to the mass scale of 6×10^6 GeV and 8.4×10^5 GeV. These events might correspond to the creation of M_{61} hadrons in atmosphere.

The identification of the hadronic space-time sheet as super-symplectic mini black-hole [K70] suggests the science fictive possibility that part of ultra-high energy cosmic rays could be also protons which have lost their valence quarks. These particles would have essentially same mass as proton and would behave like mini black-holes consisting of dark matter. They could even give a large contribution to the dark matter. Since electro-weak interactions are absent, the scattering from microwave background is absent, and they could propagate over much longer distances than ordinary particles. An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV in the rest system of Earth are super-symplectic mini black-holes associated with M_{107} hadron physics or some other copy of hadron physics.

15.5.1 Mersenne Primes And Mass Scales

p-Adic mass calculations lead to quite detailed predictions for elementary particle masses. In particular, there are reasons to believe that the most important fundamental elementary particle mass scales correspond to Mersenne primes $M_n = 2^n - 1$, $n = 2, 3, 7, 13, 17, 19, ...$

$$\begin{aligned} m_n^2 &= \frac{m_0^2}{M_n} , \\ m_0 &\simeq 1.41 \cdot \frac{10^{-4}}{\sqrt{G}} , \end{aligned} \tag{15.5.1}$$

where $\sqrt{G}$ is Planck length. The lower bound for n can be of course larger than $n = 2$. The known elementary particle mass scales were identified as mass scales associated identified with Mersenne primes $M_{127} \simeq 10^{38}$ (leptons), M_{107} (hadrons) and M_{89} (intermediate gauge bosons). Of course, also other p-adic length scales are possible and it is quite possible that not all Mersenne primes are realized. On the other hand, also Gaussian Mersennes could be important (muon and atomic nuclei corresponds to Gaussian Mersenne $(1 + i)^k - 1$ with $k = 113$).

Theory predicts also some higher mass scales corresponding to the Mersenne primes M_n for $n = 89, 61, 31, 19, 17, 13, 7, 3$ and suggests the existence of a scaled up copy of hadron physics with each of these mass scales. In particular, masses should be related by simple scalings to the masses of the ordinary hadrons.

An attractive first working hypothesis hypothesis is that the color interactions of the particles of level M_n can be described using the ordinary QCD scaled up to the level M_n so that masses and the confinement mass scale Λ is scaled up by the factor $\sqrt{M_n/M_{107}}$.

$$\Lambda_n = \sqrt{\frac{M_n}{M_{107}}} \Lambda . \tag{15.5.2}$$

In particular, the naïve scaling prediction for the masses of the exotic pions associated with M_n is given by

$$m(\pi_n) = \sqrt{\frac{M_n}{M_{107}}} m_\pi . \tag{15.5.3}$$

Here $m_\pi \simeq 135\ MeV$ is the mass of the ordinary pion. This estimte is of course extremely naïve and the recent LHC data suggests that the 125 GeV Higgs candidate could be M_{89} pion. The mass would be two times higher than the naïve estimate gives. p-Adic scalings by small powers of $\sqrt{2}$ must be considered in these estimates.

The interactions between the different level hadrons are mediated by the emission of electro-weak gauge bosons and by gluons with cm energies larger than the energy defined by the confinement scale of level with smaller p. The decay of the exotic hadrons at level M_{n_k} to exotic hadrons at level $M_{n_{k+1}}$ must take place by a transition sequence leading from the effective M_{n_k}-adic space-time topology to effective $M_{n_{k+1}}$-adic topology. All intermediate p-adic topologies might be involved.

15.5.2 Cosmic Strings And Cosmic Rays

Cosmic strings are fundamental objects in quantum TGD and dominated during early cosmology.

Cosmic strings

Cosmic strings (not quite the same thing in TGD as in GUTs) are basic objects in TGD inspired cosmology [K32, K90].

1. In TGD inspired galaxy model galaxies are regarded as mass concentrations around cosmic strings and the energy of the string corresponds to the dark energy whereas the particles condensed at cosmic strings and magnetic flux tubes resulting from them during cosmic expansion correspond to dark matter [K32, K90]. The galactic nuclei, often regarded as candidates for black holes, are the most probable seats for decaying highly entangled cosmic strings.
2. Galaxies are known to organize to form larger linear structures. This can be understood if the highly entangled galactic strings organize around long strings like pearls in necklace. Long strings could correspond to galactic jets and their gravitational field could explain the constant velocity spectrum of distant stars in the galactic halo.
3. In [K32, K90, K89] it is suggested that decaying cosmic strings might provide a common explanation for the energy production of quasars, galactic jets and gamma ray bursters and that the visible matter in galaxies could be regarded as decay products of cosmic strings. The magnetic and Z^0 magnetic flux tubes resulting during the cosmic expansion from cosmic strings allow to assign at least part of gamma ray bursts to neutron stars. Hot spots (with temperature even as high as $T \sim \frac{10^{-3,5}}{\sqrt{G}}$) in the cosmic string emitting ultra high energy cosmic rays might be created under the violent conditions prevailing in the galactic nucleus.

The decay of the cosmic strings provides a possible mechanism for the production of the exotic hadrons and in particular, exotic pions. In [C34] the idea that cosmic strings might produce gamma rays by decaying first into "X" particles with mass of order 10^{15} GeV and then to gamma rays, was proposed. As authors notice this model has some potential difficulties resulting from the direct production of gamma rays in the source region and the presence of intensive electromagnetic fields near the source. These difficulties are overcome if cosmic strings decay first into exotic hadrons of type M_{n_0}, $n_0 \geq 3$ of energy of order $2^{-n_0+2}10^{25}$ GeV, which in turn decay to exotic hadrons corresponding to M_k, $k > n_0$ via ordinary color interaction, and so on so that a sequence of M_k: s starting some value of n_0 in $n = 2, 3, 7, 13, 17, 19, 31, 61, 89, 107$ is obtained. The value of n remains open at this stage and depends on the temperature of the hot spot and much smaller temperatures than the $T \sim m_0$ are possible: favored temperatures are the temperatures $T_n \sim m_n$ at which M_n hadrons become unstable against thermal decay.

Decays of cosmic strings as producer of high energy cosmic gamma rays

In [C57] the gamma ray signatures from ordinary cosmic strings were considered and a dynamical QCD based model for the decay of cosmic string was developed. In this model the final state particles were assumed to be ordinary hadrons and final state interactions were neglected. In the recent case the string decays first to M_{n_0} hadrons and the time scale of for color interaction between M_{n_0} hadrons is extremely short (given by the length scale defined by the inverse of π_{n_0} mass) as compared to the time time scale in case of ordinary hadrons. Therefore the interactions between the final state particles must be taken into account and there are good reasons to expect that thermal equilibrium sets on and much simpler thermodynamic description of the process becomes possible.

A possible description for the decaying part of the highly tangled cosmic string is as a "fireball" containing various M_{n_0} ($n \geq 3$) partons in thermal equilibrium at Hagedorn temperature T_{n_0} of order $T_{n_0} \sim m_{n_0} = 2^{-2+n_0}\frac{10^{-4}}{k\sqrt{G}}$, $k \simeq 1.288$. The experimental discoveries made in RHIC suggest [C56] that high energy nuclear collisions create instead of quark gluon plasma a liquid like phase involving gluonic BE condensate christened as color glass condensate. Also black hole like behavior is suggested by the experiments.

RHIC findings inspire a TGD based model for this phase as a macroscopic quantum phase condensed on a highly tangled color magnetic string at Hagedorn temperature. The model relies also on the notion of dynamical but quantized $\hbar$ [K37] and its recent form to the realization that super-symplectic many-particle states at hadronic space-time sheets give dominating contribution to the baryonic mass and explain hadronic masses with an excellent accuracy.

This phase has no direct gauge interactions with ordinary matter and is identified in TGD framework as a particular instance of dark matter. Quite generally, quantum coherent dark matter would reside at magnetic flux tubes idealizable as string like objects with string tension determined by the p-adic length scale and thus outside the "ordinary" space-time. This suggests that color glass condensate forms when hadronic space-time sheets fuse to single long string like object containing large number of super-symplectic bosons.

Color glass condensate has black-hole like properties by its electro-weak darkness and there are excellent reasons to believe that also ordinary black holes could by their large density correspond to states in which super-symplectic matter would form single connected string like structure (if Planck constant is larger for super-symplectic hadrons, this fusion is even more probable).

This inspires the following mechanism for the decay of exotic boson.

1. The tangled cosmic string begins to cool down and when the temperature becomes smaller than $m(\pi_{n_0})$ mass it has decayed to M_{n_1} matter which in turn continues to decay to M_{n_2} matter. The decay to M_{n_1} matter could occur via a sequence $n_0 \rightarrow n_0 - 1 \rightarrow ...n_1$ of phase transitions corresponding to the intermediate p-adic length scales $p \simeq 2^k$, $n_1 \geq k > n_0$. Of course, all intermediate p-adic length scales are in principle possible so that the process would be practically continuous and analogous to p-adic length scale evolution with $p \simeq 2^k$ representing more stable intermediate states.
2. The first possibility is that virtual hadrons decay to virtual hadrons in the transition $k \rightarrow k-1$. The alternative option is that the density of final state hadrons is so high that they fuse to form a single highly entangled hadronic string at Hagedorn temperature T_{k-1} so that the process would resemble an evaporation of a hadronic black hole staying in quark plasma phase without freezing to hadrons in the intermediate states. This entangled string would contain partons as "color glass condensate".
3. The process continues until all particles have decayed to ordinary hadrons. Part of the M_n low energy thermal pions decay to gamma ray pairs and produce a characteristic peak in cosmic gamma ray spectrum at energies $E_n = \frac{m(\pi_n)}{2}$ (possibly red-shifted by the expansion of the Universe). The decay of the cosmic string generates also ultra high energy hadronic cosmic rays, say protons. Since the creation of ordinary hadron with ultra high energy is certainly a rare process there are good hopes of avoiding the problems related to the direct production of protons by cosmic strings (these protons produce two high flux of low energy gamma rays, when interacting with cosmic microwave background [C34]).

Topologically condensed cosmic strings as analogs super-symplectic black-holes?

Super-symplectic matter has very stringy character. For instance, it obeys stringy mass formula due the additivity and quantization of mass squared as multiples of p-adic mass scale squared [K70]. The ensuing additivity of mass squared defines a universal formula for binding energy having no independence on interaction mechanism. Highly entangled strings carrying super-symplectic dark matter are indeed excellent candidates for TGD variants of black-holes. The space-time sheet containing the highly entangled cosmic string is separated from environment by a wormhole contact with a radius of black-hole horizon. Schwartschild radius has also interpretation as Compton length with Planck constant equal to gravitational Planck constant $\hbar/\hbar_0 = 2GM^2$. In this framework the proposed decay of cosmic strings would represent nothing but the TGD counterpart of Hawking radiation. Presumably the value of p-adic prime in primordial stage was as small as possible, even $p = 2$ can be considered.

Exotic cosmic ray events and exotic hadrons

One signature of the exotic hadrons is related to the interaction of the ultra high energy gamma rays with the atmosphere. What can happen is that gamma rays in the presence of an atmospheric nucleus decay to virtual exotic quark pair associated with M_{n_k}, which in turn produces a cascade of exotic hadrons associated with M_{n_k} through the ordinary scaled up color interaction. These hadrons in turn decay $M_{n_{k+1}}$ type hadrons via mechanisms to be discussed later. At the last step ordinary hadrons are produced. The collision creates in the atmospheric nucleus the analog of quark gluon plasma which forms a second kind of fireball decaying to ordinary hadrons. RHIC experiments have already discovered these fireballs and identified them as color glass condensates [C56]. It must be emphasized that it is far from clear whether QCD really predicts this phase.

These showers differ from ordinary gamma ray showers in several respects.

1. Exotic hadrons can have small momenta and the decay products can have isotropic angular distribution so that the shower created by gamma rays looks like that created by a massive particle.
2. The muon content is expected to be similar to that of a typical hadronic shower generated by proton and larger than the muon content of ordinary gamma ray shower [C51].
3. Due to the kinematics of the reactions of type $\gamma + p \rightarrow H_{M_n} + ... + p$ the only possibility at the available gamma ray energies is that M_{89} hadrons are produced at gamma ray energies above 10 TeV. The masses of these hadrons are predicted to be above 70 GeV and this suggests that these hadrons might be identified incorrectly as heavy nuclei (heavier than ^{56}Fe). These signatures will be discussed in more detail in the sequel in relation to Centauro type events, Cygnus X-3 events and other exotic cosmic ray events. For a good review for these events and models form them see the review article [C25].

Some cosmic ray events [C46, C21] have total laboratory energy as high as 3000 TeV which suggests that the shower contains hadron like particles, which are more penetrating than ordinary hadrons.

1. One might argue that exotic hadrons corresponding M_k, $k > 107$with interact only electro-weakly (color is confined in the length scale associated with M_n) with the atmosphere one might argue that they are more penetrating than the ordinary hadrons.
2. The observed highly penetrating fireballs could also correspond super-symplectic dark matter part of incoming, possibly exotic, hadron fused with that for a hadron of atmosphere. Both hadrons would have lost their valence quarks in the collision just as in the case of Pomeron events. Large fraction of the collision energy would be transformed to super-symplectic quanta in the process and give rise to a large color spin glass condensate. These condensates would have no direct electro-weak interactions with ordinary matter which would explain their long penetration lengths in the atmosphere. Sooner or later the color glass condensate would decay to hadrons by the analog of blackhole evaporation. This process is different from QCD type hadronization process occurring in hadronic collisions and this might allow to understand the anomalously low production of neutral pions.

Exotic mesons can also decay to lepton pairs and neutral exotic pions produce gamma pairs. These gamma pairs in principle provide a signature for the presence of exotic pions in the cosmic ray shower. If M_{89} proton is sufficiently long-lived enough they might be detectable.The properties of Centauro type events however suggest that M_{89} protons are short lived.

Chapter i

Appendix

A-1 Introduction

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of embedding space and related spaces are discussed and the relationship of CP_2 to the standard model is summarized. The basic vision is simple: the geometry of the embedding space $H = M^4 \times CP_2$ geometrizes standard model symmetries and quantum numbers. The assumption that space-time surfaces are basic objects, brings in dynamics as dynamics of 3-D surfaces based on the induced geometry. Second quantization of free spinor fields of H induces quantization at the level of H, which means a dramatic simplification.

The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Also the relationship to string models is discussed briefly.

Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure analogous to adele [L53, L52]. In the recent view of quantum TGD [L130], both notions reduce to physics as number theory vision, which relies on $M^8 - H$ duality [L99, L100] and is complementary to the physics as geometry vision.

Zero energy ontology (ZEO) [L80] [K115] has become a central part of quantum TGD and leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory having quantum biology as an application. Also these aspects of TGD are briefly discussed.

A-2 Embedding space $M^4 \times CP_2$

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that embedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Embedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) [L80, L120] [K115] causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure [A57] so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2.1 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrate that it codes for the symmetries of standard models via its isometries and holonomies.

CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A48] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi+\Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi-\Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , \quad & e^1 &= \tfrac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \tfrac{r\sigma_2}{\sqrt{F}} \ , \quad & e^3 &= \tfrac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{array}{llllll} e^0 & = & \frac{dr}{F} \ , & e^1 & = & \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 & = & \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 & = & \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{array} \quad \text{(A-2.11)}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \quad \text{(A-2.12)}$$

From this expression one finds that at coordinate infinity $r = \infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \quad \text{(A-2.13)}$$

is given by

$$\begin{array}{llllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ . \end{array} \quad \text{(A-2.14)}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{llllll} R_{01} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \quad \text{(A-2.15)}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -is_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \quad \text{(A-2.16)}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \quad \text{(A-2.17)}$$

The condition states that J and g give representations of real unit and imaginary units related by the formula $i^2 = -1$.

Kähler form is expressible locally in terms of Kähler gauge potential

$$J = dB \ , \quad \text{(A-2.18)}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

$dJ = ddB = 0$ gives the topological half of Maxwell equations (vanishing of magnetic charges and Faraday's induction law) and self-duality $*J = J$ reduces the remaining equations to $dJ = 0$. Hence the Kähler form can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling).

The magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2} dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F} sin\Theta d\Theta \wedge d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical (or symplectic or Darboux) coordinates in which the Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= -\frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A39]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential

$exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the embedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A81] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \;\text{ for }\; X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \;\text{ or equivalently }\; (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \;\text{ or equivalently }\; (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2.2 CP_2 geometry and Standard Model symmetries

Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B35] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-2.23}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \otimes \gamma_5$, $1 \otimes \gamma_5$ and $\gamma_5 \otimes 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4)$ having as its covering group $SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \quad \text{(A-2.24)}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{llllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ , \end{array} \quad \text{(A-2.25)}$$

and

$$B = 2re^3 \ , \quad \text{(A-2.26)}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \quad \text{(A-2.27)}$$

where one have defined

$$\begin{array}{lll} I_L^1 & = & \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 & = & \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{array} \quad \text{(A-2.28)}$$

A_{ch} is clearly left handed so that one can perform the identification of the gauge potential as

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \quad \text{(A-2.29)}$$

where $W^\pm$ denotes the charged intermediate vector boson.

The covariantly constant curvature tensor is given by

$$\begin{array}{l} R_{01} = -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} = -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\ R_{03} = 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , \\ R_{12} = 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \quad \text{(A-2.30)}$$

The charged part of the curvature tensor is left handed.

This is to be compared with the Weyl tensor, which defines a representation of quaternionic imaginary units.

$$W_{03} = W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ ,$$
$$W_{01} = W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ ,$$
$$W_{02} = W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ . \quad \text{(A-2.31)}$$

The charged part of the Weyl tensor is right-handed and that the relative sign of the two terms in the curvature tensor and Weyl tensor are opposite.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$X = re^3 \ ,$$
$$Y = \frac{e^3}{r} \ , \quad \text{(A-2.32)}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\bar{\gamma} = aX + bY \ ,$$
$$\bar{Z}^0 = cX + dY \ , \quad \text{(A-2.33)}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \quad \text{(A-2.34)}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \quad \text{(A-2.35)}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \quad \text{(A-2.36)}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$Q_{em} = \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ ,$$
$$I_L^3 = \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \quad \text{(A-2.37)}$$

The fields γ and Z^0 are defined via the relations

$$\gamma = 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ ,$$
$$Z^0 = 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \quad \text{(A-2.38)}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-2.39}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of the Weinberg angle is a dynamical problem. The original approach was based on the assumption that it makes sense to talk about electroweak action defined at fundamental level and introduce a symmetry breaking by adding an additional term proportional to Kähler action. The recent view is that Kähler action plus volume term defines the fundamental action.

The Weinberg angle is completely fixed if one requires that the electroweak action contains no cross term of type γZ^0. This leads to a definite value for the Weinberg angle.

One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle. As a matter fact, color gauge action identifying color gauge field as proportional to $H^A J_{\alpha\beta}$ is proportional to Kähler action. A possible interpretation would be as a sum of electroweak and color gauge interactions.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \tag{A-2.40}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0\wedge e^3 + e^1\wedge e^2) \ , \\ R_{12} &= 2(e^0\wedge e^3 + 2e^1\wedge e^2) \ , \\ J &= 2(e^0\wedge e^3 + e^1\wedge e^2) \ , \end{aligned} \tag{A-2.41}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.42}$$

Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-2.43}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-2.44}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-2.45}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$X = -\frac{K}{2g^2} + \frac{fp}{18} ,$$
$$K = Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] , \quad \text{(A-2.46)}$$

This parameter can be calculated by substituting the values of quark and lepton charges and weak isospins.

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9}\right] , \quad \text{(A-2.47)}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18+n_i^2))} . \quad \text{(A-2.48)}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2}+28)} . \quad \text{(A-2.49)}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is not far from the typical value 9/24 of GUTs at high energies [B10]. The experimental value at the scale length scale of the electron can be deduced from the ratio of W and Z boson masses as $sin^2\theta_W = 1-(m_W/m_Z)^2 \simeq .22290$. This ratio and also the weak boson masses depend on the length scale.

If one interprets the additional term proportional to J as color action, one could perhaps interpret the value of Weinberg angle as expressing a connection between strong and weak coupling constant evolution. The limit $f \rightarrow 0$ should correspond to an infinite value of color coupling strength and at this limit one would have $sin^2\theta_W = \frac{9}{28}$ for $f/g^2 \rightarrow 0$. This does not make sense since the Weinberg angle is in the standard model much smaller in QCD scale Λ corresponding roughly to pion mass scale. The Weinberg angle is in principle predicted by the p-adic coupling constant evolution fixed by the number theoretical vision of TGD.

One could however have a sum of electroweak action, correction terms changing the value of Weinberg angle, and color action and coupling constant evolution could be understood in terms of the coupling parameters involved.

Electroweak symmetry breaking

One of the hardest challenges in the development of the TGD based view of weak symmetry breaking was the fact that classical field equations allow space-time surfaces with finite but arbitrarily large size. For a fixed space-time surface, the induced gauge fields, including classical weak fields, are long ranged. On the other hand, the large mass for weak bosons would require a short correlation length. How can one understand this together with the fact that a photon has a long correlation length?

In zero energy ontology quantum states are superpositions of space-time surfaces as analogs of almost unique Bohr orbits of particles identified as 3-D surfaces. For some reason the superposition should be such that the quantum averages of weak gauge boson fields vanish below the weak scale whereas the quantum average of electromagnetic fields is non-vanishing.

This is indeed the case.

1. The supersymplectic symmetries form isometries of the world of classical worlds (WCW) and they act in CP_2 degrees of freedom as symplectic transformations leaving the CP_2 symplectic form J invariant and therefore also its contribution to the electromagnetic field since this part is the same for all space-time surfaces in the superposition of space-time surfaces as a representation of supersymplectic isometry group (as a special case a representation of color group).
2. In TGD, color and electroweak symmetries acting as holonomies are not independent and for the $SU(2)_L$ part of induced spinor connection the symplectic transformations induces $SU(2)_L \times U(1)_R$ gauge transformation. This suggests that the quantum expectations of the induced weak fields over the space-time surfaces vanish above the quantum coherence scale. The averages of W and of the left handed part of Z^0 should therefore vanish.
3. $\langle Z^0 \rangle$ should vanish. For $U(1)_R$ part of Z^0, the action of gauge transformation is trivial in gauge theory. Now however the space-time surface changes under symplectic transformations and this could make the average of the right-handed part of Z^0 vanishing. The vanishing of the average of the axial part of the Z^0 is suggested by the partially conserved axial current hypothesis.

One can formulate this picture quantitatively.

1. The electromagnetic field [L137] contains, besides the induced Kähler form, also the induced curvature form R_{12}, which couples vectorially. Conserved vector current hypothesis suggests that the average of R_{12} is non-vanishing. One can express the neutral part of the induced gauge field in terms of induced spinor curvature and Kähler form J as

$$\begin{array}{rcl} R_{03} & = & 2(2e^0 \wedge e^3 + e^1 \wedge e^2) = J + 2e^0 \wedge e^3 \ , \\ J & = & 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} & = & 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) = 3J - 2e^0 \wedge e^3 \quad , \end{array} \tag{A-2.50}$$

2. The induced fields γ and Z^0 (photon and Z- boson) can be expressed as

$$\begin{array}{rcl} \gamma & = & 3J - sin^2\theta_W R_{12} \ , \\ Z^0 & = & 2R_{03} = 2(J + 2e^0 \wedge e^3) \qquad \text{(A-2.51)} \\ per. & & \qquad \text{(A-2.52)} \end{array}$$

The condition $\langle Z^0 \rangle = 0$ gives $2\langle e^0 \wedge e^3 \rangle = -2J$ and this in turn gives $\langle R_{12} \rangle = 4J$. The average over γ would be

$$\langle \gamma \rangle = (3 - 4sin^2\theta_W)J \ .$$

For $sin^2\theta_W = 3/4 \ langle\gamma\rangle$ would vanish.

The quantum averages of classical weak fields quite generally vanish. What about correlation functions?

1. One expects that the correlators of classical weak fields as color invariants, and perhaps even symplectic invariants, are non-vanishing below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as hadron.
2. The intuitive picture is that in longer length scales one has disjoint 3-surfaces with a size scale of Compton length. If the states associated with two disjoint 3-surfaces are separately color invariant there are no correlations in color degrees of freedom and correlators reduce to the products of expectations of classical weak fields and vanish. This could also hold when the 3-surfaces are connected by flux tube bonds.
   Below the Compton length weak bosons would thus behave as correlated massless fields. The Compton lengths of weak bosons are proportional to the value of effective Planck constant h_{eff} and in living systems the Compton lengths are proposed to be even of the order of cell size. This would explain the mysterious chiral selection in living systems requiring large parity violation.
3. What about the averages and correlators of color gauge fields? Classical color gauge fields are proportional to the products of Hamiltonians of color isometries induced Kähler form

and the expectations of color Hamiltonians give vanishing average above Compton length and therefore vanishing average. Correlators are non-vanishing below the hadron scale. Gluons do not propagate in long scales for the same reason as weak bosons. This is implied by color confinement, which has also classical description in the sense that 3-surfaces have necessarily a finite size.
A large value of h_{eff} allows colored states even in biological scales below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as dark hadron.

Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.
2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B13] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-2.53}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-2.54}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-2.55}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-3.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has embedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if embedding space at their image points. In the recent case the embedding of space-time surface to embedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the embedding space to the space-time surface (see `http://tgdtheory.fi/appfigures/induct.jpg`).

Induction procedure makes sense also for the spinor fields of embedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with embedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://tgdtheory.fi/appfigures/induct.jpg`.

A-3.2 Induced gauge fields for space-times for which CP$_2$ projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP$_2$ projection, only vacuum extremals and space-time surfaces for which CP$_2$ projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r=\infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP$_2$ projection is $r=\infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \ .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP$_2$ projection color rotations and weak symmetries commute.

A-3.3 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four embedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of embedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-3.4 Embedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.
2. Spinor harmonics of embedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of embedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the embedding space spinor connection carries W gauge potentials one can say that the embedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.
2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.
3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.
4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.
5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-3.5 About induced gauge fields

In the following the induced gauge fields are studied for general space-time surface without assuming the preferred extremal property (Bohr orbit property). Therefore the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F} sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-3.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.2}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k+cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-3.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \tag{A-3.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{A-3.5}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.
3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2} Z^0 \ . \qquad \text{(A-3.6)}
\end{aligned}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \qquad \text{(A-3.7)}
\end{aligned}$$

and is useful in the construction of vacuum embedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned}
\Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\
\Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \qquad \text{(A-3.8)}
\end{aligned}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global embedding for, say a constant magnetic field. Although global embedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \qquad \text{(A-3.9)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-4 The relationship of TGD to QFT and string models

The recent view of the relationship of TGD to QFT and string models has developed slowly during years and it seems that in a certain sense TGD means a return to roots: instead of QFT like description involving path integral one would have wave mechanics for 3-surfaces.

A-4.1 TGD as a generalization of wave mechanism obtained by replacing point-like particles with 3-surfaces

The first vision of TGD was as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

The later work has revealed that TGD could be seen as a generalization of the wave mechanism based on the replacement of a point-like particle with 3-D surface. This is due to holography implied by general coordinate invariance. The definition of the metric of the "world of classical worlds" (WCW) must assign a unique or at least almost unique space-time surface to a given 3-surface. This 4-surface is analogous to Bohr orbit so that also Bohr orbitology becomes an exact part of quantum physics. The failure of strict determinism forces to replace 3-surfaces with 4-surfaces and this leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K52, K31, K85] [L121, L130].

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

A-4.2 Extension of superconformal invariance

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in a completely unique position as far as symmetries are considered.

In fact, this leads to a generalization of the Kac-Moody type symmetries of string models. $\delta M^4_+ \times CP_2$ allows huge supersymplectic symmetries for which the radial light-like coordinate of

δM^4_+ plays the role of complex string coordinate in string models. These symmetries are assumed to act as isometries of WCW.

A-4.3 String-like objects and strings

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of the TGD Universe and the inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string-like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situations in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

A-4.4 TGD view of elementary particles

The TGD based view about elementary particles has two key aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidean signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.
2. At the level of H Fermion number is carried by the modes of the induced spinor field. In space-time regions with Minkowski signature the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle orbit corresponds to a 4-D generalization of a world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidean signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at the same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having a hadronic string as a physical counterpart. Long strings should be distinguished from wormhole contacts which due to their superconformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.
2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.
3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particles along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K52, K85].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

A-5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [K87] [L121, L125, L126] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$ of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A57] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.
2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.
3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.
   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.
   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

A-5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.
2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L130]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.
   The super symplectic algebra A has an infinite hierarchy of sub-algebras [L130] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.
   This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.
2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L130] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by M^8-H duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.
2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L130] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P) = n(braid) = n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.
3. The actions related to different values of $n(P) = n(braid) = n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.
WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.
4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.
A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L128] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i = 2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r = 1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.
A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.
5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p \simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K64, K65]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.
   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L128], and the forward scattering amplitude corresponds to the propagator.
   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.
2. According to [L128], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.
   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L128].
3. p-Adic length scale hypothesis [L131] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.
   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.
   Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

In [L130] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K52, K31]. As isometries they would naturally permute the maxima with each other.

A-6 Number theoretic vision of TGD

Physics as number theory vision is complementary to the physics as geometry vision and has developed gradually since 1993. Langlands program is the counterpart of this vision in mathematics [L129].

The notion of p-adic number fields emerged with the motivation coming from the observation that elementary particle mass scales and mass ratios could be understood in terms of the so-called p-adic length scale hypothesis [K68, K60, K28]. The fusion of the various p-adic physics leads to what I call adelic physics [L53, L52]. Later the hypothesis about hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter emerged [K34, K35, K36, K36].

Eventually this led to that the values of effective Planck constant could be identified as the dimension of an algebraic extension of rationals assignable to polynomials with rational coefficients. This led to the number theoretic vision in which so-called M^8-H duality [L99, L100] plays a key role. M^8 (actually a complexification of real M^8) is analogous to momentum space so that the duality generalizes momentum position duality for point-like particles. M^8 has an interpretation as complexified octonions.

The dynamics of 4-surfaces in M^8 is coded by polynomials with rational coefficients, whose roots define mass shells H^3 of $M^4 \subset M^8$. It has turned out that the polynomials satisfy stringent additional conditions and one can speak of number theoretic holography [L128, L129]. Also the ordinary 3→ 4 holography is needed to assign 4-surfaces with these 3-D mass shells. The number theoretic dynamics is based on the condition that the normal space of the 4-surface in M^8 is associative (quaternionic) and contains a commutative complex sub-space. This makes it possible to assign to this surface space-time surface in $H = M^4 \times CP_2$.

At the level of H the space-time surfaces are by holography preferred extremals and are assumed to be determined by the twistor lift of TGD [K87] giving rise to an action which is sum of the Kähler action and volume term. The preferred extremals would be minimal surfaces analogous to soap films spanned by frames. Outside frames they would be simultaneous extremals of the Kähler action, which requires a generalization of the holomorphy characterizing string world sheets.

In the following only p-adic numbers and hierarchy of Planck constants will be discussed.

A-6.1 p-Adic numbers and TGD

p-Adic number fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A35]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k , \ x(k) = 0, ..., p-1 \ . \tag{A-6.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \quad \text{(A-6.3)}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \quad \text{(A-6.4)}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \quad \text{(A-6.5)}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.
2. Distances of points x and y inside single class are smaller than distances between different classes.
3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B31]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

1. Basic form of the canonical identification

There exists a natural continuous map $I: R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \quad \text{(A-6.6)}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \quad \text{(A-6.7)}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1} \sum_{k=0,..} p^{k} \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \quad \text{(A-6.8)}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

2. The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \quad \text{(A-6.9)}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-6.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

3. Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

4. Generalization of number concept and notion of embedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic embedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real embedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real embedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles".

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used
2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution
3. Canonical identification violates general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic embedding space with chart maps to real embedding space and assuming preferred coordinates made possible by isometries of embedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-6.2 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the embedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of embedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of embedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-6.3 $M^8 - H$ duality as it is towards the end of 2021

The view of $M^8 - H$ duality (see Appendix **??**) has changed considerably towards the end 2021 [L121] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L121] of the $M^8 - H$ duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L121] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L99, L100]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$. This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L73]. This notion does not make sense at the level M^8 anymore.

The modified $M^8 - H$ duality forces us to modify the original interpretation [L121]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L116] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L121]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

A-7 Zero energy ontology (ZEO)

ZEO is implied by the holography forced in the TGD framework by general coordinate invariance.

A-7.1 Basic motivations and ideas of ZEO

The following gives a brief summary of ZEO [L80] [K115].

1. In ZEO quantum states are not 3-dimensional but superpositions of 4-dimensional deterministic time evolutions connecting ordinary initial 3-dimensional states. By holography they are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution. One can say that in the TGD framework general coordinate invariance implies holography and the slight failure of its determinism in turn forces ZEO.
   Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced with a new superposition. Classical determinism of individual time evolution is not violated and this solves the basic paradox of quantum measurement theory. There are two kinds of quantum jumps: ordinary ("big") state function reductions (BSFRs) changing the arrow of time and "small" state function reductions (SSFRs) (weak measurements) preserving it and giving rise to the analog of Zeno effect [L80].
2. To avoid getting totally confused it is good to emphasize some aspects of ZEO.
   (a) ZEO does not mean that physical states in the usual 3-D sense as snapshots of time evolution would have zero energy state pairs defining zero energy states as initial and final states have same conserved quantities such as energy. Conservation implies that one can adopt the conventions that the values of conserved quantities are opposite for these states so that their sum vanishes: one can think that incoming and outgoing particles come from geometric past and future is the picture used in quantum field theories.
   (b) ZEO means two times: subjective time as sequence of quantum jumps and geometric time as space-time coordinate. These times are identifiable but are strongly correlated.
3. In BSFRs the arrow of time is changed and the time evolution in the final state occurs backwards with respect to the time of the external observer. BSFRs can occur in all scales since TGD predicts a hierarchy of effective Planck constants with arbitrarily large values. There is empirical support for BSFRs.
   (a) The findings of Minev et al [L69] in atomic scale can be explained by the same mechanism [L69]. In BSFR a final zero energy state as a superposition of classical deterministic time evolutions emerges and for an observer with a standard arrow of time looks like a superposition of deterministic smooth time evolutions leading to the final state. Interestingly, once this evolution has started, it cannot be stopped unless one changes the stimulus signal inducing the evolution in which case the process does not lead to anywhere: the interpretation would be that BSFR back to the initial state occurs!
   (b) Libets' experiments about active aspects of consciousness [J3] can be understood. Subject person raises his finger and neural activity starts before the conscious decision to do so. In the physicalistic framework it is thought to lead to raising of the finger. The problem with the explanation is that the activity beginning .5 seconds earlier seems to be dissipation with a reversed arrow of time: from chaotic and disordered to ordered at around .15 seconds. ZEO explanation is that macroscopic quantum jump occurred and generated a signal proceeding backwards in time and generated neural activity and dissipated to randomness.

(c) Earthquakes involve a strange anomaly: they are preceded by ELF radiation. One would expect that they generate ELF radiation. The identification as BSFR would explain the anomaly [L72]. In biology the reversal of the arrow of time would occur routinely and be a central element of biological self-organization, in particular self-organized quantum criticality (see [L77, L139].

A-7.2 Some implications of ZEO

ZEO has profound implications for understanding self-organization and self-organized quantum criticality in terms of dissipation with non-standard arrow of time looking like generation of structures [L77, L139]. ZEO could also allow understanding of what planned actions - like realizing the experiment under consideration - could be.

1. Second law in the standard sense does not favor - perhaps even not allow - realization of planned actions. ZEO forces a generalization of thermodynamics: dissipation with a non-standard arrow of time for a subsystem would look like self-organization and planned action and its realization.
   Could most if not all planned action be like this - induced by BSFR in the geometric future and only apparently planned? There would be however the experience of planning and realizing induced by the signals from geometric future by a higher level in the hierarchy of conscious entities predicted by TGD! In long time scales we would be realizing our fates or wishes of higher level conscious entities rather than agents with completely free will.
2. The notion of magnetic body (MB) serving as a boss of ordinary matter would be central. MB carries dark matter as $h_{eff} = nh_0$ phases of ordinary matter with n serving as a measure for algebraic complexity of extension of rationals as its dimension and defining a kind of universal IQ. There is a hierarchy of these phases and MBs labelled by extension of rationals and the value of n.
   MBs would form a hierarchy of bosses - a realization for master slave hierarchy. Ordinary matter would be at the bottom and its coherent behavior would be induced from quantum coherence at higher levels. BSFR for higher level MB would give rise to what looks like planned actions and experienced as planned action at the lower levels of hierarchy. One could speak of planned actions inducing a cascade of planned actions in shorter time scales and eventually proceeding to atomic level.

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. http://tgdtheory.fi/appfigures/reconect2.jpg

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. http://tgdtheory.fi/appfigures/cat.jpg

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred embedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. http://tgdtheory.fi/appfigures/padictoreal.jpg

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. `http://tgdtheory.fi/appfigures/sharing.jpg`

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** `http://tgdtheory.fi/appfigures/timemirror.jpg` or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. `http://tgdtheory.fi/appfigures/timemirror.jpg`

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