MATHEMATICAL ASPECTS OF CONSCIOUSNESS THEORY

Matti Pitkänen

Rinnekatu 2-4 A 8, Karkkila, 03620, Finland

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0.1 PREFACE

Brief summary of TGD

Towards the end of the year 2023 I became convinced that it would be appropriate to prepare collections about books related to TGD and its applications. The finiteness of human lifetime was my first motivation. My second motivation was the deep conviction that TGD will mean a revolution of the scientific world view and I must do my best to make it easier.

The first collection would relate to the TGD proper and its applications to physics. Second collection would relate to TGD inspired theory of consciousness and the third collection to TGD based quantum biology. The books in these collections would focus on much more precise topics than the earlier books and would be shorter. This would make it much easier for the reader to understand what TGD is, when the time is finally mature for the TGD to be taken seriously. This particular book belongs to a collection of books about TGD proper.

The basic ideas of TGD

TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students in the seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 45 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of the embedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, the mainstream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to the same multiplet of the gauge group implying instability of the proton.

Instead of trying to describe in detail the path, which led to TGD as it is now with all its side tracks, it is better to summarize the recent view which of course need not be final.

TGD can be said to be a fusion of special and general relativities. The Relativity Principle (Poincare Invariance) of Special Relativity is combined with the General Coordinate Invariance and Equivalence Principle of General Relativity. TGD involves 3 views of physics: physics geometry, physics as number theory and physics as topological physics in some sense.

Physics as geometry

"Geometro-" in TGD refers to the idea about the geometrization of physics. The geometrization program of Einstein is extended to gauge fields allowing realization in terms of the geometry of surfaces so that Einsteinian space-time as abstract Riemann geometry is replaced with sub-manifold geometry. The basic motivation is the loss of classical conservation laws in General Relativity Theory (GRT)(see **Fig. 12**). Also the interpretation as a generalization of string models by replacing string with 3-D surface is natural.

- Standard model symmetries uniquely fix the choice of 8-D space in which space-time surfaces live to $H = M^4 \times CP_2$ [L118]. Also the notion of twistor is geometrized in terms of surface geometry and the existence of twistor lift fixes the choice of H completely so that TGD is unique [L41, L51](see **Fig. 13**). The geometrization applies even to the quantum theory itself and the space of space-time surfaces - "world of classical worlds" (WCW) - becomes the basic object endowed with Kähler geometry (see **Fig. 14**). The mere mathematical existence of WCW geometry requires that it has maximal isometries, which together twistor lift and number theoretic vision fixes it uniquely [L119].

- General Coordinate Invariance (GCI) for space-time surfaces has dramatic implications. A given 3-surface fixes the space-time surface almost completely as analog of Bohr orbit (preferred extremal). This implies holography and leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K132, L64].

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields in all scales. It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to the phases of ordinary matter predicted by the number theoretic vision and behaving like dark matter but identifiable as matter explaining the missing baryon problem whereas the galactic dark matter would correspond to the dark energy assignable monopole flux tubes as deformations of cosmic strings. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem and p-adic physics solved this problem in terms of p-adic thermodynamics [K32, K68] [L112].

- One of the most recent discoveries of classical TGD is exact general solution of the field equations. Holography can be realized as a generalized holomorphy realized in terms of what I call Hamilton-Jacobi structure [L115]. Space-time surfaces correspond to holomorphic imbeddings of the space-time surface to H with a generalized complex structure defined by the vanishing of 2 analytic functions of 4 generalized complex coordinates of H. These surfaces are automatically minimal surfaces. This is true for any geneneral coordinate invariant action constructed in terms of the induced geometric structures so that the dynamics is universal. Different actions differ only in the sense that singularities at which the minimal surface property fails depend on the action. This affects the scattering amplitudes, which can be constructed in terms of the data related to the singularities [L122].

- Generalized conformal symmetries define an extension of conformal symmetries and one can assign to them Noether charges. Besides this the so called super-symplectic symmetries associated with $\delta M^4_+ \times CP_2$ define isometries of the "world of classical worlds" (WCW), which by holography is essentially the space of Bohr orbits of 3-surfaces as particles so that quantum TGD is expected to reduce to a generalization of wave mechanics.

Physics as number theory

During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

Adelic physics [L38, L39] fusing real and various p-adic physics is part of the number theoretic vision, which provides a kind of dual description for the description based on space-time geometry and the geometry of "world of classical words". Adelic physics predicts two fractal length scale hierarchies: p-adic length scale hierarchy and the hierarchy of dark length scales labelled by $h_{eff} = nh_0$, where n is the dimension of extension of rational. The interpretation of the latter hierarchy is as phases of ordinary matter behaving like dark matter. Quantum coherence is possible in arbitarily long scales. These two hierarchies are closely related. p-Adic primes correspond to ramified primes for a polynomial, whose roots define the extension of rationals: for a given extension this polynomial is not unique.

$M^8 - H$ duality

The concrete realization of the number theoretic vision is based on $M^8 - H$ duality (see **Fig. 15**). What the precise form is this duality is, has been far from clear but the recent form is the simplest one and corresponds to the original view [L121]. M^8 corresponds to octonions O but with the number theoretic metric defined by $Re(o^2)$ rather than the standard norm and giving Minkowskian signature.

The physics in M^8 can be said to be algebraic whereas in H field equations are partial differential equations. The dark matter hierarchy corresponds to a hierarchy of algebraic extensions of rationals inducing that for adeles and has interpretation as an evolutionary hierarchy (see **Fig. 16**). p-Adic physics is an essential part of number theoretic vision and the space-time surfaces are such that at least their M^8 counterparts exists also in p-adic sense. This requires that the analytic function defining the space-time surfaces are polynomials with rational coefficients.

$M^8 - H$ duality relates two complementary visions about physics (see **Fig. 17**), and can be seen as a generalization of the momentum-position duality of wave mechanics, which fails to generalize to quantum field theories (QFTs). $M^8 - H$ duality applies to particles which are 3-surfaces instead of point-like particles.

p-Adic physics

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Principle with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

Hierarchy of Planck constants labelling phases ordinary matter dark matter behaving like dark matter

One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

TGD as an analog of topological QFT

Consider next the attribute "Topological". In condensed matter physical topological physics has become a standard topic. Typically one has fields having values in compact spaces, which are topologically non-trivial. In the TGD framework space-time topology itself is non-trivial as also the topology of $H = M^4 \times CP_2$. Since induced metric is involved with TGD, it is too much to say that TGD is topological QFT but one can for instance say, that space-time surfaces as preferred extremals define representatives for 4-D homological equivalence classes.

The space-time as 4-surface $X^4 \subset H$ has a non-trivial topology in all scales and this together with the notion of many-sheeted space-time brings in something completely new. Topologically trivial Einsteinian space-time emerges only at the QFT limit in which all information about topology is lost (see **Fig. 18**).

Any GCI action satisfying holography=holomorphy principle has the same universal basic extremals: CP_2 type extremals serving basic building bricks of elementary particles, cosmic strings and their thickenings to flux tubes defining a fractal hierarchy of structure extending from CP_2 scale to cosmic scales, and massless extremals (MEs) define space-time correletes for massless particles. World as a set or particles is replaced with a network having particles as nodes and flux tubes as bonds between them serving as correlates of quantum entanglement.

"Topological" could refer also to p-adic number fields obeying p-adic local topology differing radically from the real topology (see **Fig. 19**).

Zero energy ontology

TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

General coordinate invariance leads to the identification of space-time surfaces are analogous to Bohr orbits inside causal diamond (CD). CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). By the already described hologamphy, 3-dimensional data replaces the boundary conditions at single 3-surface involving also normal derivatives with conditions involving no derivates.

In zero energy ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 20** and **Fig. 21**). Quantum states are modes of WCW spinor fields, essentially wave functions in the space WCW consisting of Bohr orbit-like 4-surfaces.

Quantum jumps occur between these and the basic problem of standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes (see **Fig. 22**). This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete. BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions.

In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary of CD and states at it remain unchanged (Zeno effect).

Equivalence Principle in TGD framework

There have been also longstanding problems related to the relationship between inertial mass and gravitational mass, whose identification has been far from obvious.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of CDs defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle in the form expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

  At quantum level, the Equivalence Principle has a surprisingly strong content. In linear Minkowski coordinates, space-time projection of the M^4 spinor connection representing gravitational gauge potentials the coupling to induced spinor fields vanishes. Also the modified Dirac action for the solutions of the modified Dirac equation seems to vanish identically and in TGD perturbative approach separating interaction terms is not possible.

  The modified Dirac equation however fails at the singularities of the minimal surface representing space-time surface and Dirac action reduces to an integral over singularities for the trace of the second fundamental form slashed between the induced spinor field and its conjugate. Also the M^4 part of the trace is non-vanishing and gives rise to the gravitational coupling. The trace gives both standard model vertices and graviton emission vertices. One could say that at the quantum level gravitational and gauge interactions are eliminated everywhere except at the singularities identifiable as defects of the ordinary smooth structure. The exotic smooth structures [L104], possible only in dimension 4, are ordinary smooth structures apart from these defects serving as vertex representing a creation of a fermion-antifermion pair in the induced gauge potentials. The vertex is universal and essentially the trace of the second fundamental form as an analog of the Higgs field and the gravitational constant is proportional to the square of CP_2 radius.

- There is a delicate difference between inertial and gravitational masses. One can assume that the modes of the imbedding space spinor fields are solutions of massles Dirac equation in either $M^4 \times CP_2$ and therefore eigenstates of inertial momentum or in $CD = cd \times CP_2$: in this case they are only mass eigenstates. The mass spectra are identical for these options. Inertial momenta correspond naturally to the Poincare charges in the space of CDs. For the CD option the spinor modes correspond to mass squared eigenstates for which the mode for H^3 with a given value of light-proper time is a unitary irreducible $SO(1,3)$ representation rather than a representation of translation group. These two eigenmode basis correspond to gravitational basis for spinor modes.

Quantum TGD as a generalization of Einstein's geometrization program

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but it turned that this approach fails due to the extreme non-linearity of the theory.

It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as the space of 3-dimensional surfaces. Later 3-surfaces where replaced with 4-surfaces satifying holography and therefore as analogs of Bohr orbits.

- If one assumes Bohr orbitology, then strong correlations between the 3-surfaces at the ends of CD follow and mean holography. It is natural to identify the quantum states of the Universe (and sub-Univeverses) as modes of a formally classical spinor field in WCW. WCW gamma matrices are expressible in terms of oscillator operators of free second quantized spinor fields of H. The induced spinor fields identified projections of H spinor fields to the space-time surfaces satisfy modified Dirac equation for the modified Dirac equation. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- Quantum TGD can be seen as a theory for free spinor fields in WCW having maximal isometries and the generalization of the Super Virasoro conditions gives rise to the analog massless Dirac equation at the level of WCW.

The world of classical worlds and its symmetries

The notion of "World of Classical Worlds" (WCW) emerged around 1985 but found its basic form around 1990. Holography forced by the realization of General Coordinate Invariance forced/allowed to give up the attempts to make sense of the path integral.

A more concrete way to express this view is that WCW does not consist of 3-surfaces as particle-like entities but almost deterministic Bohr orbits assignable to them as preferred extremals of Kähler action so that quantum TGD becomes wave mechanics in WCW combined with Bohr orbitology. This view has profound implications, which can be formulated in terms of zero energy ontology (ZEO), solving among other things the basic paradox of quantum measurement theory. ZEO forms also the backbone of TGD inspired theory of consciousness and quantum biology.

WCW geometry exists only if it has maximal isometries: this statement is a generalization of the discovery of Freed for loop space geometries [A35]. I have proposed [K60, K35, K128, K98, L119] that WCW could be regarded as a union of generalized symmetric spaces labelled by zero modes which do not contribute to the metric. The induced Kähler field is invariant under symplectic transformations of CP_2 and would therefore define zero mode degrees of freedom if one assumes that WCW metric has symplectic transformations as isometries. In particular, Kähler magnetic fluxes would define zero modes and are quantized closed 2-surfaces. The induced metric appearing in Kähler action is however not zero mode degree of freedom. If the action contains volume term, the assumption about union of symmetric spaces is not well-motivated.

Symplectic transformations are not the only candidates for the isometries of WCW. The basic picture about what these maximal isometries could be, is partially inspired by string models.

- A weaker proposal is that the symplectomorphisms of H define only symplectomorphisms of WCW. Extended conformal symmetries define also a candidate for isometry group. Remarkably, light-like boundary has an infinite-dimensional group of isometries which are in 1-1 correspondence with conformal symmetries of $S^2 \subset S^2 \times R_+ = \delta M^4_+$.

- Extended Kac Moody symmetries induced by isometries of δM^4_+ are also natural candidates for isometries. The motivation for the proposal comes from physical intuition deriving from string models. Note they do not include Poincare symmetries, which act naturally as isometries in the moduli space of causal diamonds (CDs) forming the "spine" of WCW.

- The light-like orbits of partonic 2-surfaces might allow separate symmetry algebras. One must however notice that there is exchange of charges between interior degrees of freedom and partonic 2-surfaces. The essential point is that one can assign to these surface conserved charges when the dual light-like coordinate defines time coordinate. This picture also assumes a slicing of space-time surface by by the partonic orbits for which partonic orbits associated with wormrhole throats and boundaries of the space-time surface would be special. This slicing would correspond to Hamilton-Jacobi structure.

- Fractal hierarchy of symmetry algebras with conformal weights, which are non-negative integer multiples of fundamental conformal weights, is essential and distinguishes TGD from string models. Gauge conditions are true only the isomorphic subalgebra and its commutator with the entire algebra and the maximal gauge symmetry to a dynamical symmetry with generators having conformal weights below maximal value. This view also conforms with p-adic mass calculations.

- The realization of the symmetries for 3-surfaces at the boundaries of CD and for light-like orbits of partonic 2-surfaces is known. The problem is how to extend the symmetries to the interior of the space-time surface. It is natural to expect that the symmetries at partonic orbits and light-cone boundary extend to the same symmetries.

After the developments towards the end of 2023, it seems that the extension of conformal and Kac-Moody symmetries of string models to the TGD framework is understood. What about symplectic symmetries, which were originally proposed as isometries of WCW? In this article this question is discussed in detail and it will be found that these symmetries act naturally on 3-D holographic data and one can identify conserved charges. By holography this is in principle enough and might imply that the actions of holomorphic and symplectic symmetry algebras are dual. Holography=holomorphy hypothesis is discussed also in the case of the modified Dirac equation.

About the construction of scattering amplitudes

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far-reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. After having made several guesses for what the counterpart of S-matrix could be, it became clear that the dream about explicit formulas is unrealistic before one has understood what happens in quantum jump.

- In ZEO [K132, L64] one must distinguish between "small" state function reductions (SSFRs) and "big" SFRs (BSFRs). BSFR is the TGD counterpart of the ordinary SFRs and the arrow of the geometric time changes in it. SSFR follows the counterpart of a unitary time evolution and the arrow of the geometric time is preserved in SSFR. The sequence of SSFRs is the TGD counterpart for the sequence of repeated quantum measurements of the same observables in which nothing happens to the state. In TGD something happens in SSFRs and this gives rise to the flow of consciousness. When the set of the observables measured in SSFR does not commute with the previous set of measured observables, BSFR occurs.

The evolution by SSFRs means that also the causal diamond changes. At quantum level one has a wave function in the finite-dimensional moduli space of CDs which can be said to form a spine of WCW [L117]. CDs form a scale hierarchy. SSFRs are preceded by a dispersion in the moduli space of CDs and SSFR means localization in this space.

- There are several S-matrix like entities. One can assign an analog of the S-matrix to each analog of unitary time evolution preceding a given SSFR. One can also assign an analog S-matrix between the eigenstate basis of the previous set of observables and the eigenstate basis of new observers: this S-matrix characterizes BSFR. One can also assign to zero energy states an S-matrix like entity between the states assignable to the two boundaries of CD. These S-matrix like objects can be interpreted as a complex square root of the density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally.

In standard QFTs Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so-called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. The QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

In the TGD framework this generalization of Feynman diagrams indeed emerges unavoidably.

- The counterparts of elementary particles can be identified as closed monopole flux tubes connecting two parallel Minkowskian space-time sheets and have effective ends which are Euclidean wormhole contacts. The 3-D light-like boundaries of wormhole contacts as orbits of partonic 2-surfaces.

  The intuitive picture is that the 3-D light-like partonic orbits replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. A stronger condition is that fermion number is carried by light-like fermion lines at the partonic orbits, which can be identified as boundaries string world sheets.

- The localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In the TGD framework, the fermionic variant of twistor Grassmann formalism combined with the number theoretic vision [L98, L99] led to a stringy variant of the twistor diagrammatics.

- Fundamental fermions are off-mass-shell in the sense that their momentum components are real algebraic integers in an extension of rationals associated with the space-time surfaces inside CD with a momentum unit determined by the CD size scale. Galois confinement states that the momentum components are integer valued for the physical states.

- The twistorial approach suggests also the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras, which would determine the vertices and scattering amplitudes in terms of poly-local symmetries.

The twistorial approach is however extremely abstract and lacks a concrete physical interpretation. The holography=holomorphy vision led to a breakthough in the construction of the scattering amplitudes by solving the problem of identifying interaction vertices [L122].

1. The basic prediction is that space-time surfaces as analogs of Bohr orbits are holomorphic in a generalized sense and are therefore minimal surfaces. The minimal surface property fails at lower-dimensional singularities and the trace of the second fundamental form (SFF) analogous to acceleration associated with the Bohr orbit of the particle as 3-surface has a delta function like singularity but vanishes elsewhere.

2. The minimal surface property expressess masslessness for both fields and particles as 3-surfaces. At singularities masslessness property fails and singularities can be said to serve as sources which also in QFT define scattering amplitudes.

3. The singularities are analogs of poles and cuts for the 4-D generalization of the ordinary holomorphic functions. Also for the ordinary holomorphic functions the Laplace equation as analog massless field equation and expressing analyticity fails. Complex analysis generalizes to dimension 4.

4. The conditions at the singularity give a generalization of Newton's "F=ma"! I ended up where I started more than 50 years ago!

5. In dimension 4, and only there, there is an infinite number of exotic diff structures [**?**], which differ from ordinary ones at singularities of measure zero analogous to defects. These defects correspond naturally to the singularities of minimal surfaces. One can say that for the exotic diff structure there is no singularity.

6. Group theoretically the trace of the SFF can be regarded as a generalization of the Higgs field, which is non-vanishing only at the vertices and this is enough. Singularities take the role of generalized particle vertices and determine the scattering amplitudes. The second fundamental form contracted with the embedding space gamma matrices and slashed between the second quantized induced spinor field and its conjugate gives the universal vertex involving only fermions (bosons are bound states of fermions in TGD). It contains both gauge and gravitational contributions to the scattering amplitudes and there is a complete symmetry between gravitational and gauge interactions. Gravitational couplings come out correctly as the radius squared of CP_2 as also in the classical picture.

7. The study of the modified Dirac equation leads to the conclusion that vertices as singularities and defects contain the standard electroweak gauge contribution coming from the induced spinor connection and a contribution from the M^4 spinor connection. M^4 part of the generalized Higgs can give rise to a graviton as an $L = 1$ rotational state of the flux tube representing the graviton. It is not clear whether M^4 Kähler gauge potential can give rise to a spin 1 particle. The vielbein part of M^4 spinor connection is pure gauge and could give rise to gravitational topological field theory.

Figures

Basic ideas of TGD inspired quantum biology

The following list gives the basic elements of TGD inspired quantum biology.

- Many-sheeted space-time allows the interpretation of the structures of macroscopic world around us in terms of space-time topology. Magnetic/field body acts as intentional agent using biological body as a sensory receptor and motor instrument and controlling biological body and inheriting its hierarchical fractal structure. Fractal hierarchy of EEGs and its variants can be seen as communication and control tools of magnetic body. Also collective levels of consciousness have a natural interpretation in terms of magnetic body. Magnetic body makes also possible entanglement in macroscopic length scales. The braiding of magnetic flux tubes makes possible topological quantum computations and provides a universal mechanism of memory. One can also undersand the real function of various information molecules and corresponding receptors by interpreting the receptors as addresses in quantum computer memory and information molecules as ends of flux tubes which attach to these receptors to form a connection in quantum web.

  Note that also the notion of electric body makes sense [L113]. Quite generally, long range classical gravitational, electric and magnetic fields give rise to very large values of effective Planckl constants. The Nottale's hypothesis of gravitational Planck constant generalizes to electric interactions.

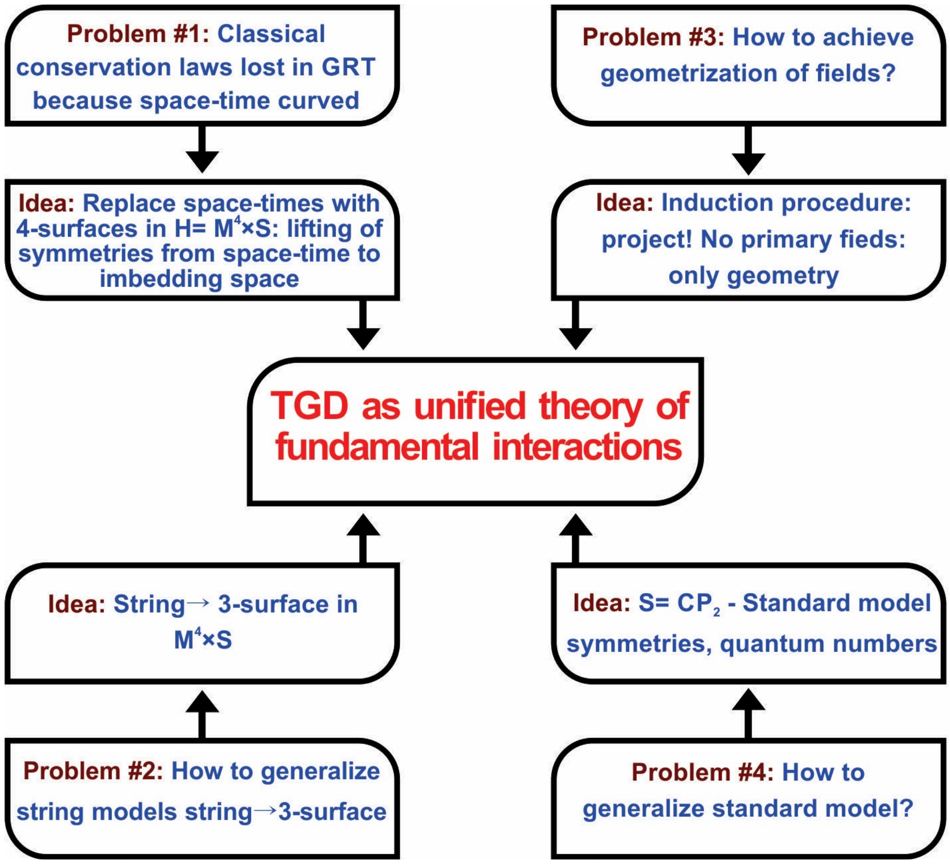


Figure 1: The problems leading to TGD as their solution.

- Magnetic body carrying dark matter and forming an onion-like structure with layers characterized by large values of Planck constant is the key concept of TGD inspired view about Quantum Mind to biology.. Magnetic body is identified as intentional agent using biological body as sensory receptor and motor instrument. EEG and its fractal variants are identified as a communication and control tool of the magnetic body and a fractal hierarchy of analogs of EEG is predicted. Living system is identified as a kind of Indra's net with biomolecules representing the nodes of the net and magnetic flux tubes connections between then.

  The reconnection of magnetic flux tubes and phase transitions changing Planck constant and therefore the lengths of the magnetic flux tubes are identified as basic mechanisms behind DNA replication and analogous processes and also behind the phase transitions associated with the gel phase in cell interior. The braiding of magnetic flux makes possible universal memory representation recording the motions of the basic units connected by flux tubes. Braiding also defines topological quantum computer programs updated continually by the flows of the basic units. The model of DNA as topological quantum computer is discussed as an application. In zero energy ontology the braiding actually generalize to 2-braiding for string world sheets in 4-D space-time and brings in new elements.

- Zero energy ontology (ZEO) makes possible the proposed p-adic description of intentions and cognitions and their transformations to action. Time mirror mechanism based on sending of negative energy signal to geometric past would apply to both long term memory recall, remote metabolism, and realization of intentional acting as an activity beginning in the geometric past in accordance with the findings of Libet. ZEO gives a precise content to the notion of negative energy signal in terms of zero energy state for which the arrow of geometric time is opposite to the standard one.

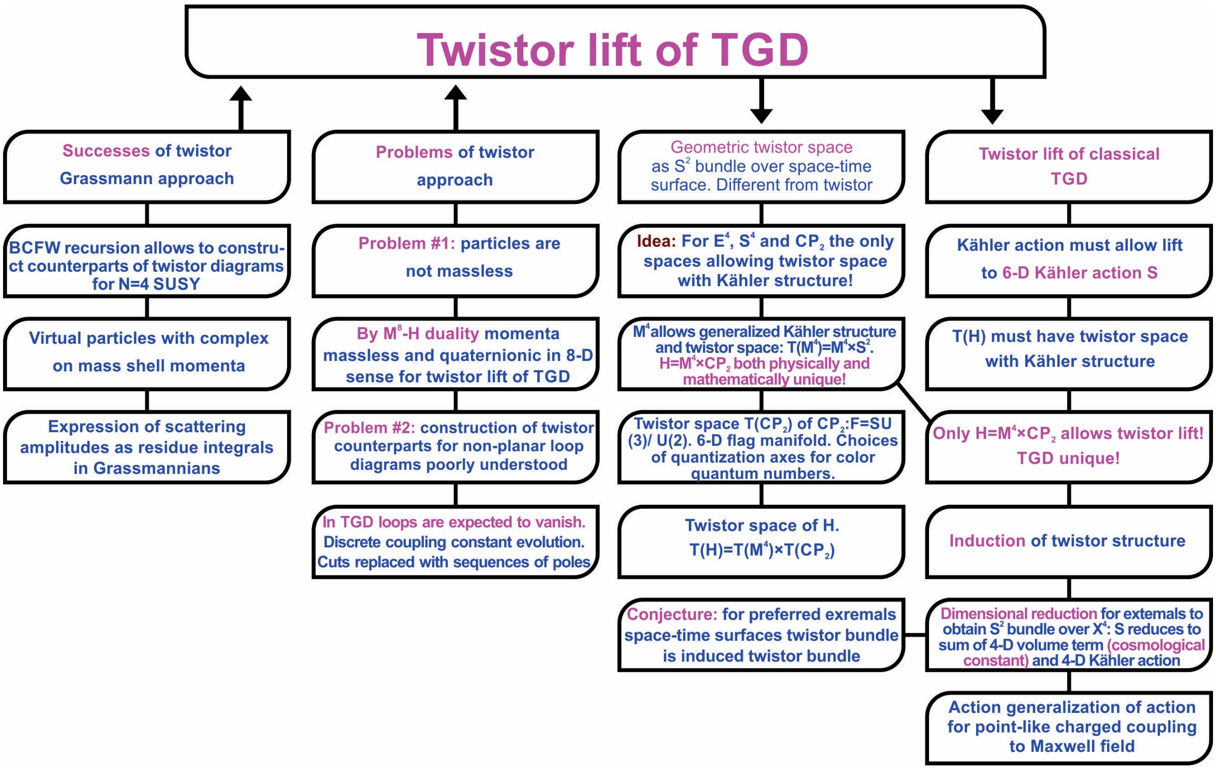


Figure 2: Twistor lift

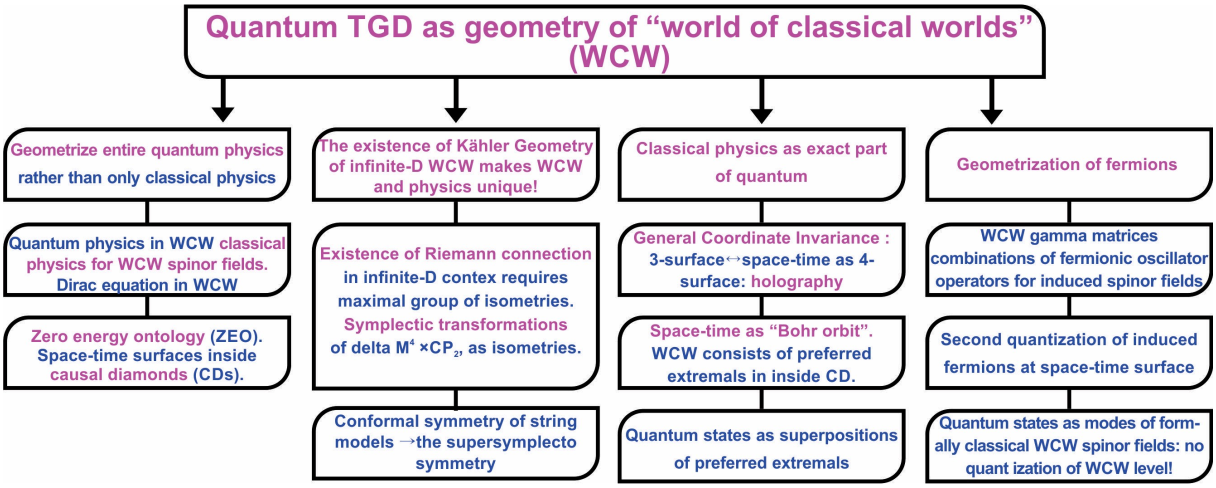


Figure 3: Geometrization of quantum physics in terms of WCW

The associated notion of causal diamond (CD) is essential element and assigns to elementary particles new fundamental time scales which are macroscopic: for electron the time scale is .1 seconds, the fundamental biorhythm. An essentially new element is time-like entanglement which allows to understand among other things the quantum counterparts of Boolean functions in terms of time-like entanglement in fermionic degrees of freedom.

- The assignment of dark matter with a hierarchy of Planck constants gives rise to a hierarchy of macroscopic quantum phases making possible macroscopic and macrotemporal quantum coherence and allowing to understand evolution as a gradual increase of Planck constant. The model for dark nucleons leads to a surprising conclusion: the states of nucleons correspond to DNA, RNA, tRNA, and amino-acids in a natural manner and vertebrate genetic code as correspondence between DNA and amino-acids emerges naturally. This suggests that genetic code is realized at the level of dark hadron physics and living matter in the usual sense provides a secondary representation for it. The hierarchy of Planck constants emerges from basic TGD under rather general assumptions.

- p-Adic physics can be identified as physics of cognition and intentionality. Negentropic entanglement possible for number theoretic entanglement entropy makes sense for rational (and even algebraic) entanglement and leads to the identification of life as something residing in the intersection of real and p-adic worlds. NMP respects negentropic entanglement and the attractive idea is that the experience of understanding and positively colored emotions relate to negentropic entanglement.

- Living matter as conscious hologram is one of the basic ideas of TGD inspired biology and consciousness theory. The basic objection against TGD is that the interference of classical

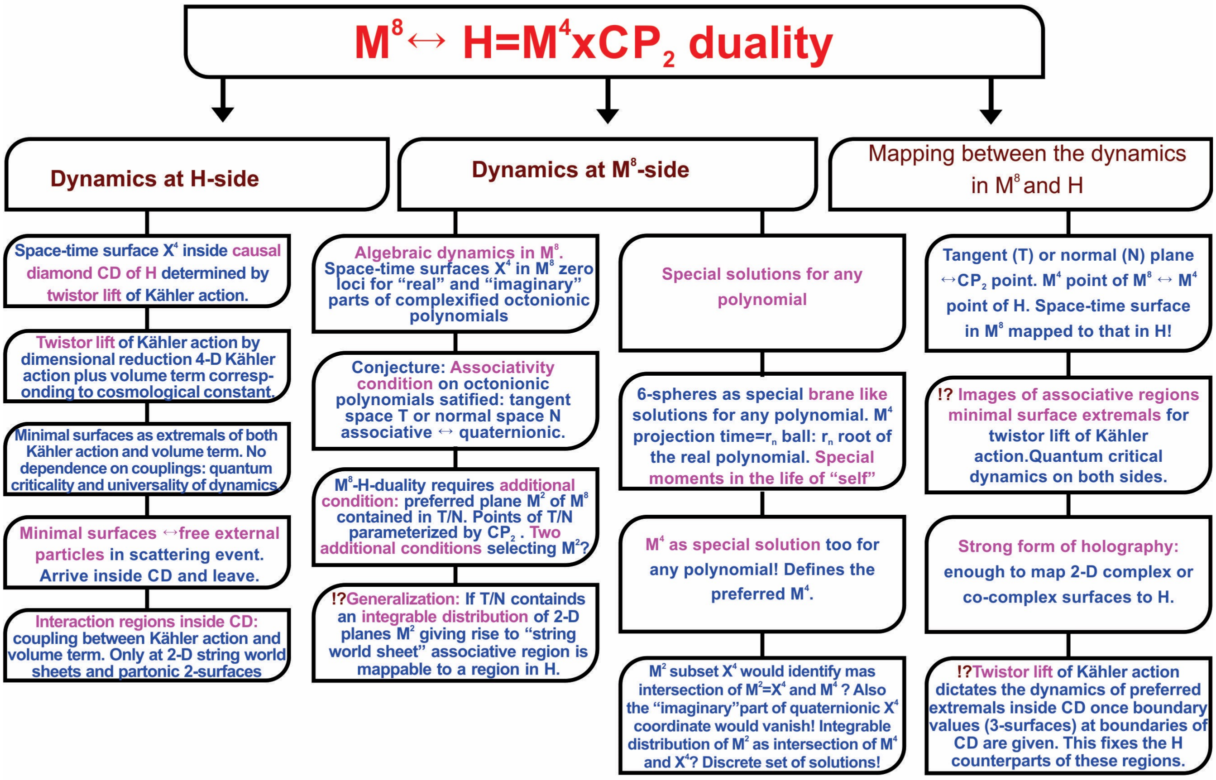


Figure 4: $M^8 - H$ duality

fields is impossible in the standard sense for the reason that that classical fields are not primary dynamical variables in TGD Universe. The resolution is based on the observation that only the interference of the effects caused by these fields can be observed experimentally and that many-sheeted space-time allows to realized the summation of effects in terms of multiple topological condensations of particles to several parallel space-time sheets. One concrete implication is fractality of qualia. Qualia appear in very wide range of scales: our qualia could in fact be those of magnetic body. The proposed mechanism for the generation of qualia realizes the fractality idea.

Various anomalies of living matter have been in vital role in the development of not only TGD view about living matter but also TGD itself.

- TGD approach to living matter was strongly motivated by the findings about the strange behavior of cell membrane and of cellular water, and gel behavior of cytoplasm. Also the findings about effects of ELF em fields on vertebrate brain were decisive and led to the proposal of the hierarchy of Planck constants found later to emerge naturally from the non-determinism of Kähler action. Rather satisfactorily, the other manner to introduce the hierarchy of Planck constants is in terms of gravitational Planck constant: at least in microscopic scales the equivalence of these approaches makes sense and leads to highly non-trivial predictions. The basic testable prediction is that dark photons have cyclotron frequencies inversely proportional to their masses but universal energy spectrum in visible and UV range which corresponds to the transition energies for biomolecules so that they are ideal for biocontrol at the level of both magnetic bodies and at the level of biochemistry.

- Water is in key role in living matter and also in TGD inspired view about living matter. The

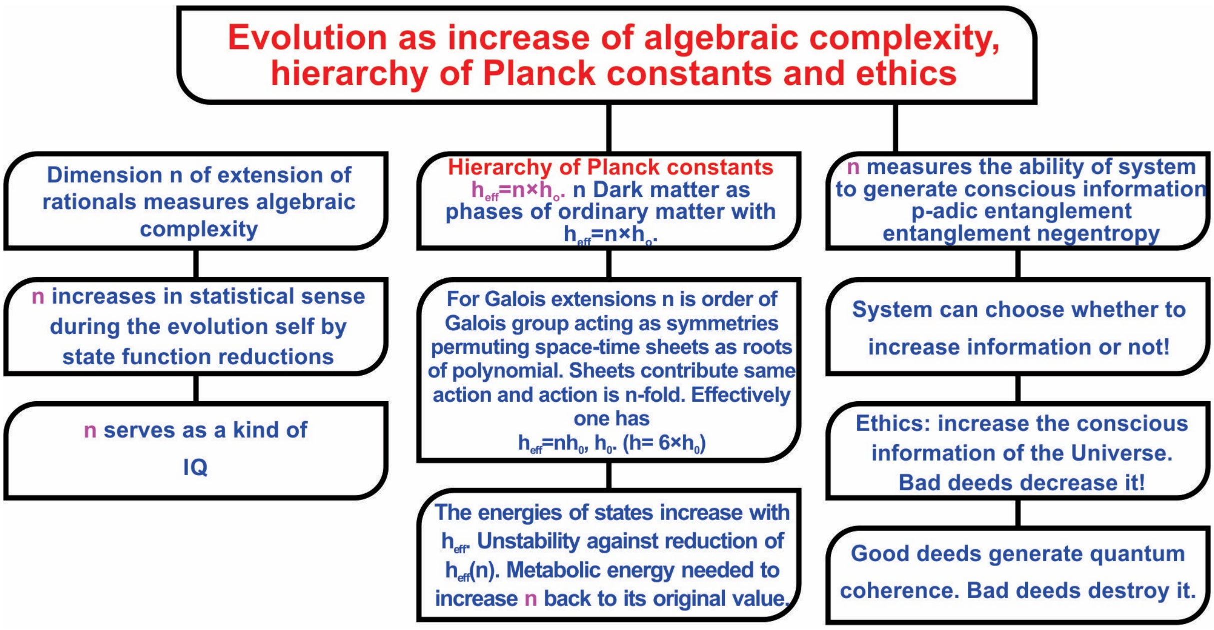


Figure 5: Number theoretic view of evolution

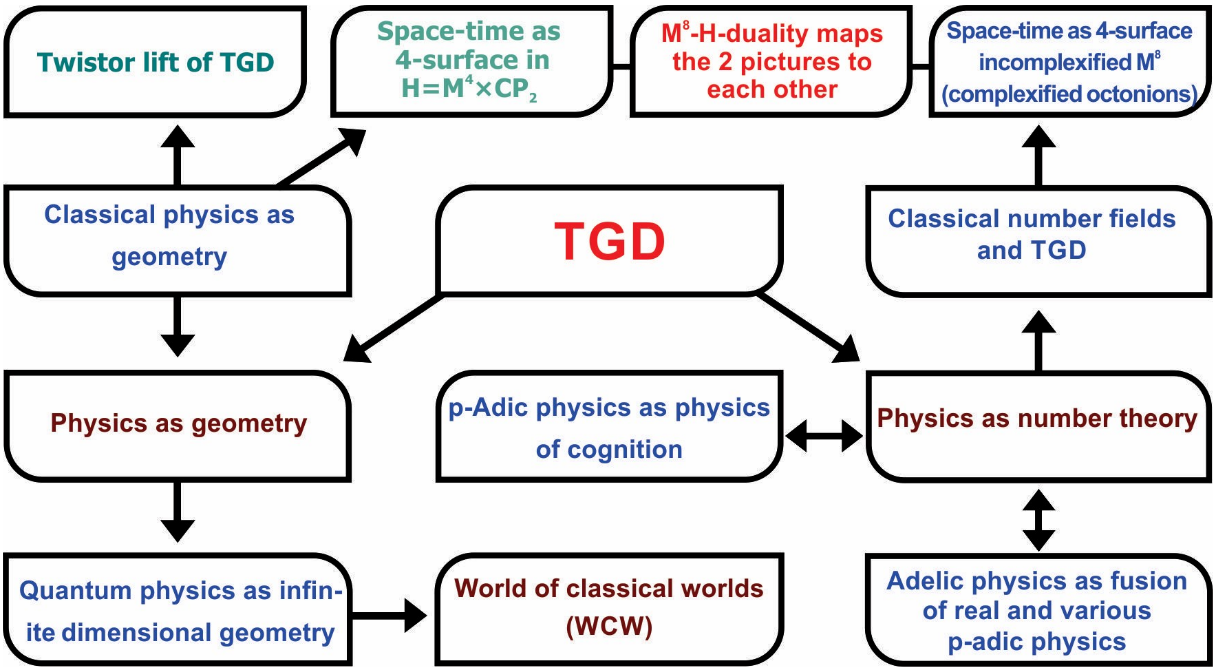


Figure 6: TGD is based on two complementary visions: physics as geometry and physics as number theory.

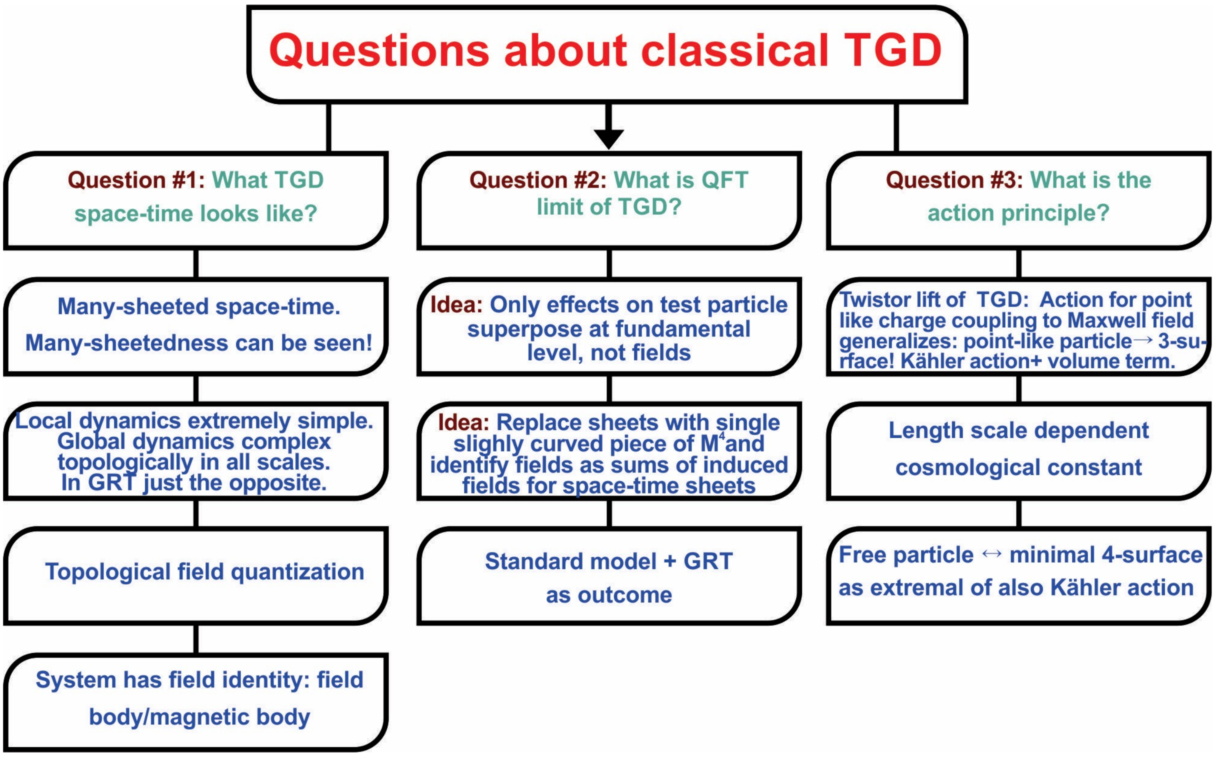


Figure 7: Questions about classical TGD.

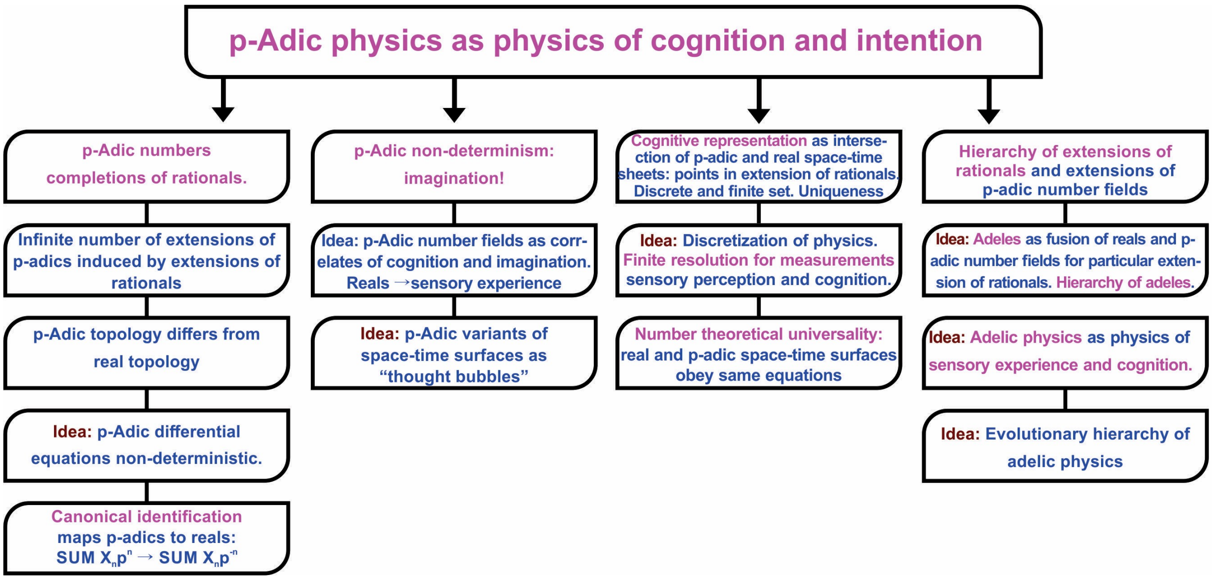


Figure 8: p-Adic physics as physics of cognition and imagination.

anomalies of water lead to a model for dark nuclei as dark proton strings with the surprising prediction that DNA, RNA, aninoacids and even tRNA are in one-one correspondence with the resulting 3-quark states and that vertebrate genetic code emerges naturally. This leads to a vision about water as primordial lifeform still playing a vital role in living organisms. The model of water memory and homeopathy in turn generalizes to a vision about how immune system might have evolved.

- Metabolic energy is necessary for conscious information processing in living matter. This suggests that metabolism should be basically tranfer of negentropic entanglement from nutrients to the organism. ATP could be seen as a molecule of consciousness in this picture and high energy phosphate bond would make possible the transfer of negentropy.

- Pollack effect and its generalizations are in a central role in the TGD inspire quantum biology. In the Pollack effect, the feed of energy allows to increase the value of effective Planck constant so that an ordinary charged particle transforms to its dark variant, being kicked to, say, the gravitational magnetic body of the system itself or some other system such as the Earth or Sun. Charge separation takes place between ordinary biomatter and its magnetic body. Dissipation is extremely small at the magnetic /field body so that Pollack effect makes it possible to realize various biological functions at the magnetic/field body. Photons, in particular solar photons, can provide the energy needed to increase the value of h_{eff} but there are many other possibilities. For instance, the formation of molecular bound states of atoms liberates energy which can be used in the Pollack effect and this process could generate dark matter at the magnetic and more general field bodies.

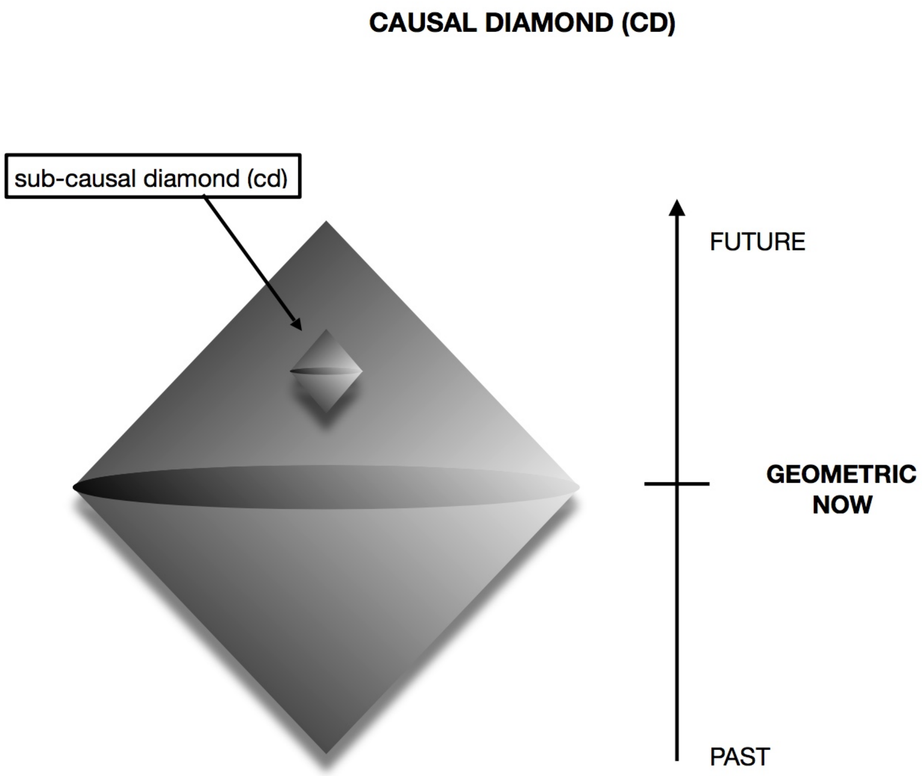


Figure 9: Causal diamond

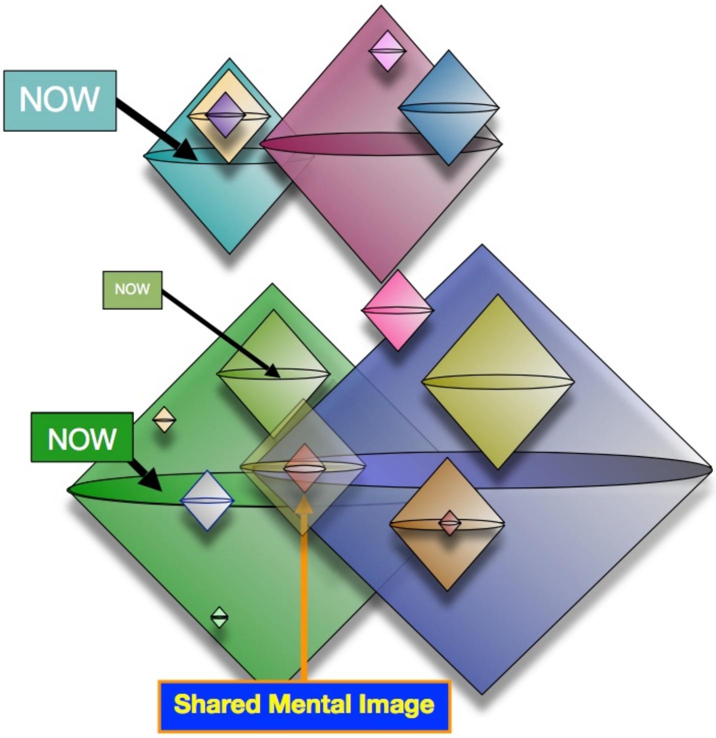


Figure 10: CDs define a fractal “conscious atlas”

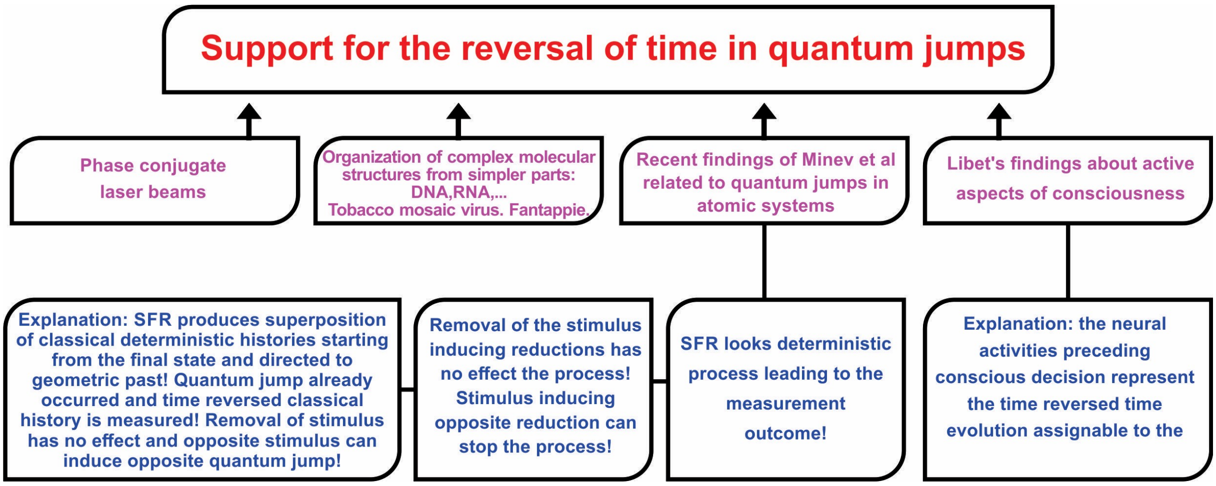


Figure 11: Time reversal occurs in BSFR

Figures

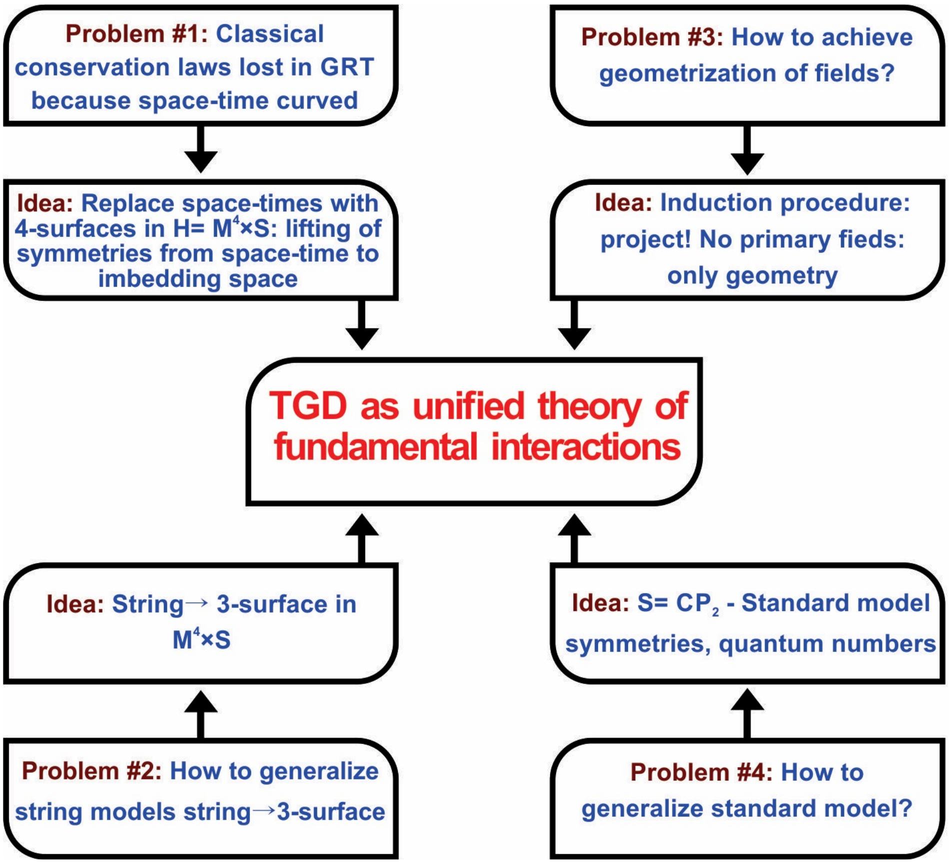
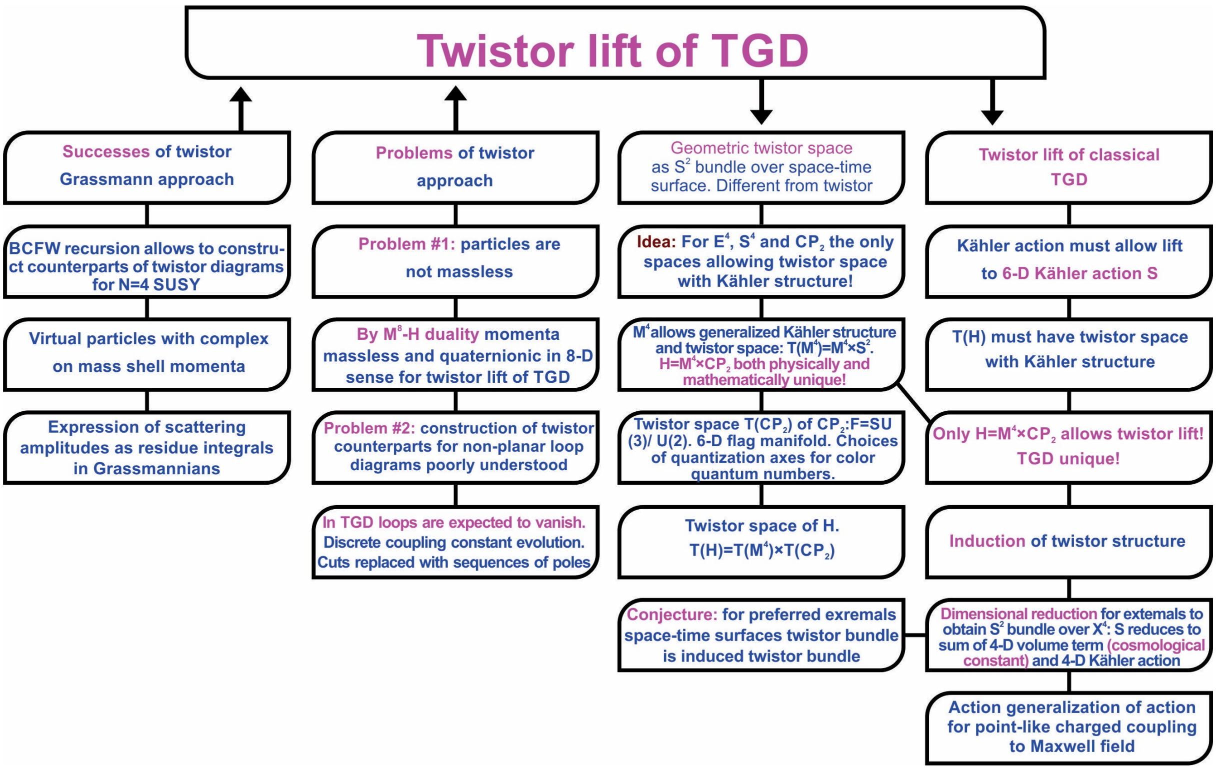


Figure 12: The problems leading to TGD as their solution.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 45 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

Karkkila, April 22, 2024, Finland

Matti Pitkänen



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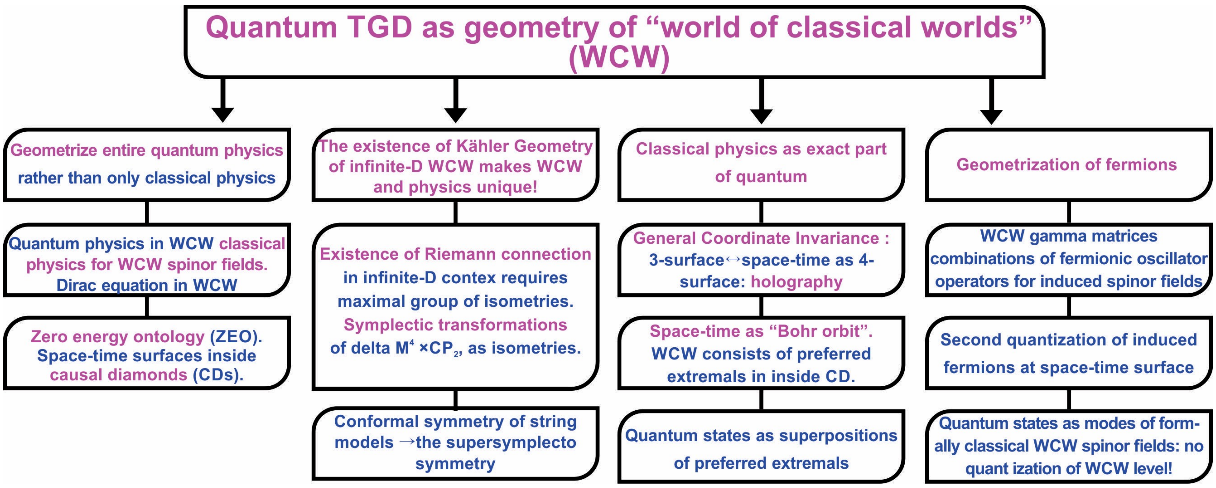


Figure 14: Geometrization of quantum physics in terms of WCW

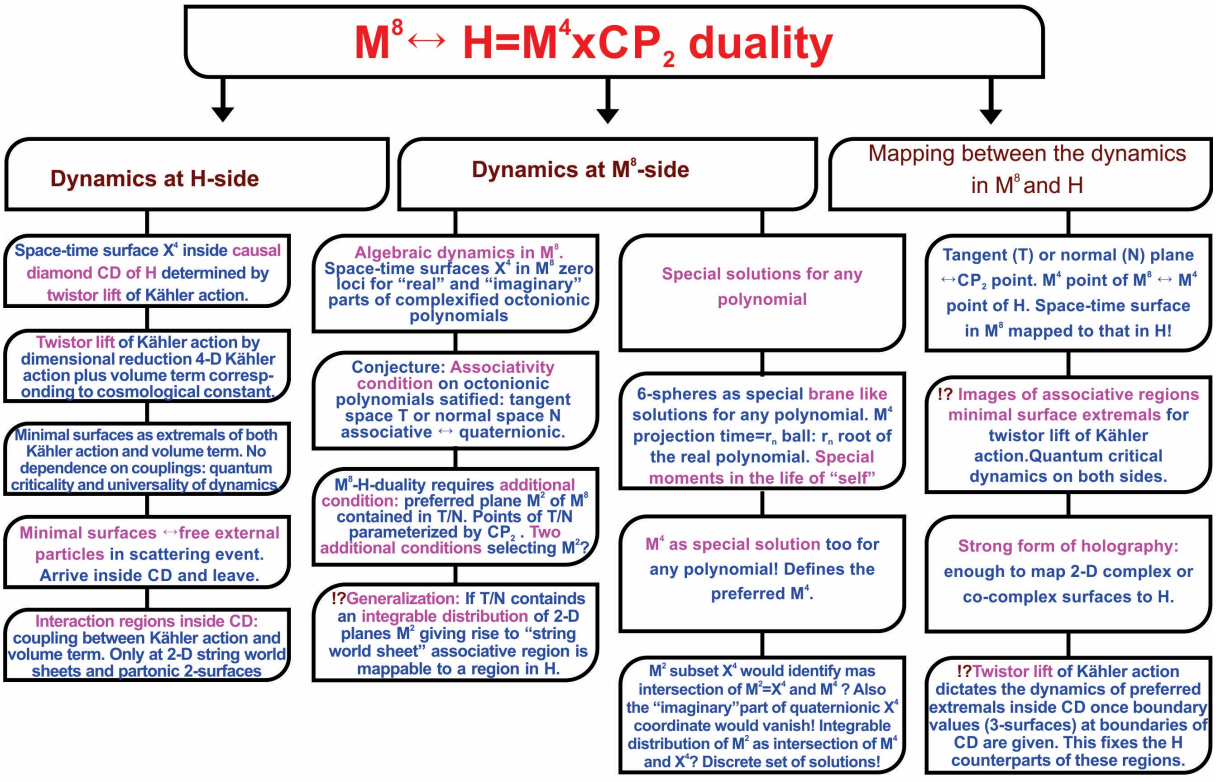


Figure 15: $M^8 - H$ duality

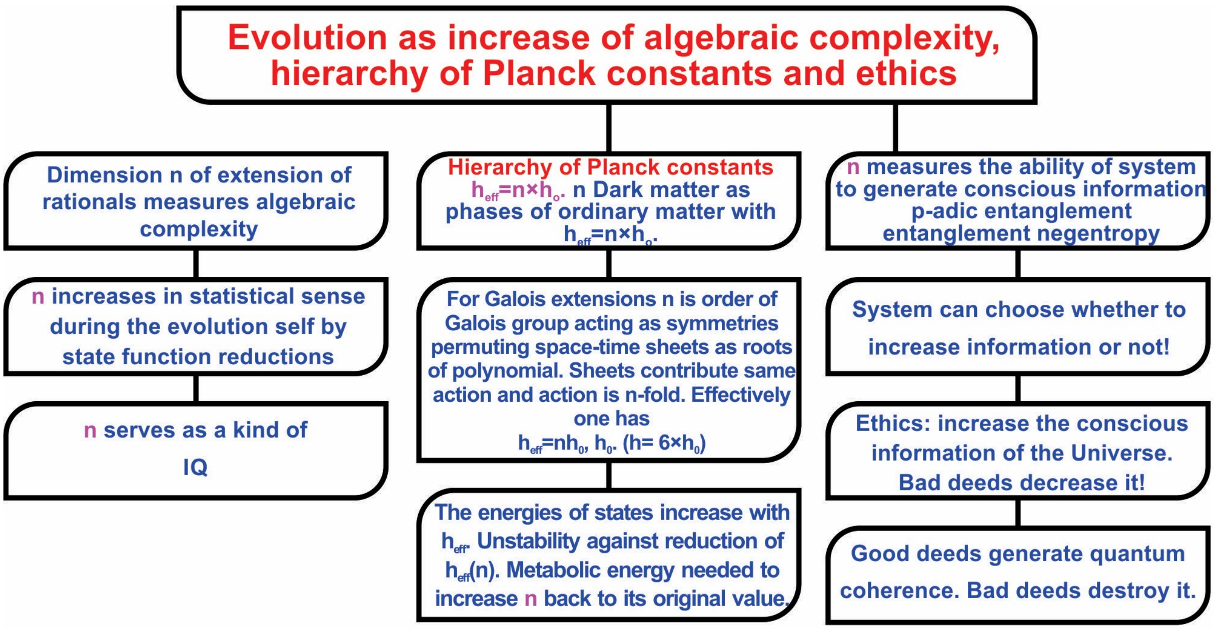


Figure 16: Number theoretic view of evolution

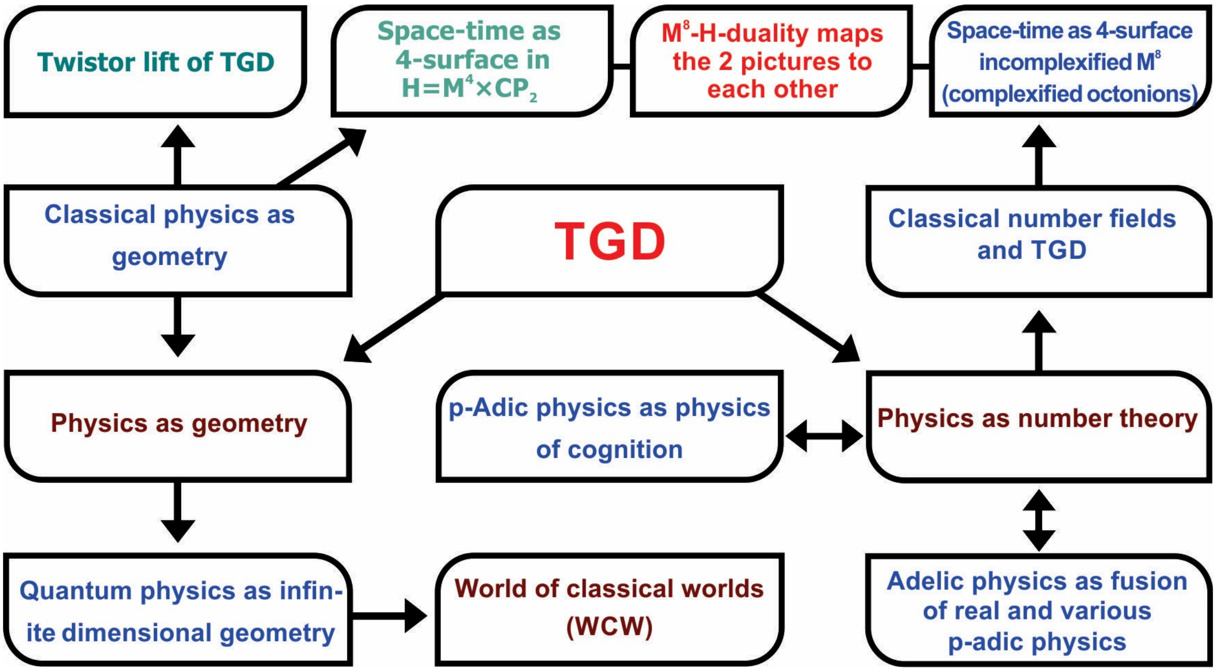


Figure 17: TGD is based on two complementary visions: physics as geometry and physics as number theory.

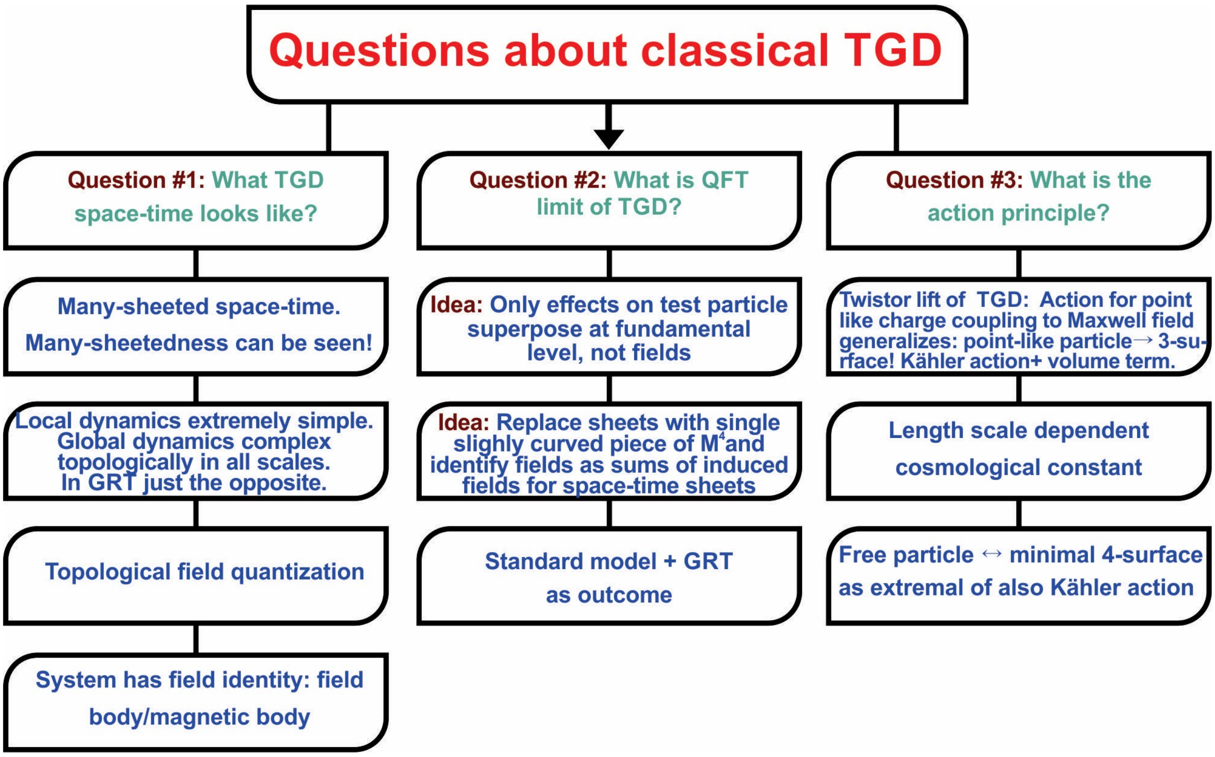


Figure 18: Questions about classical TGD.

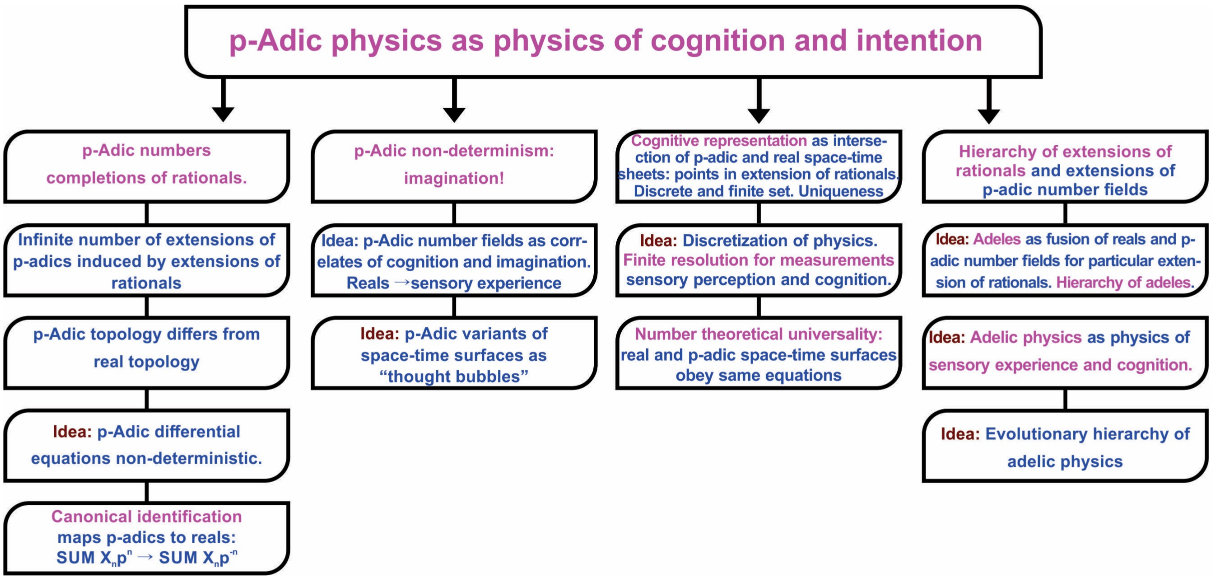


Figure 19: p-Adic physics as physics of cognition and imagination.

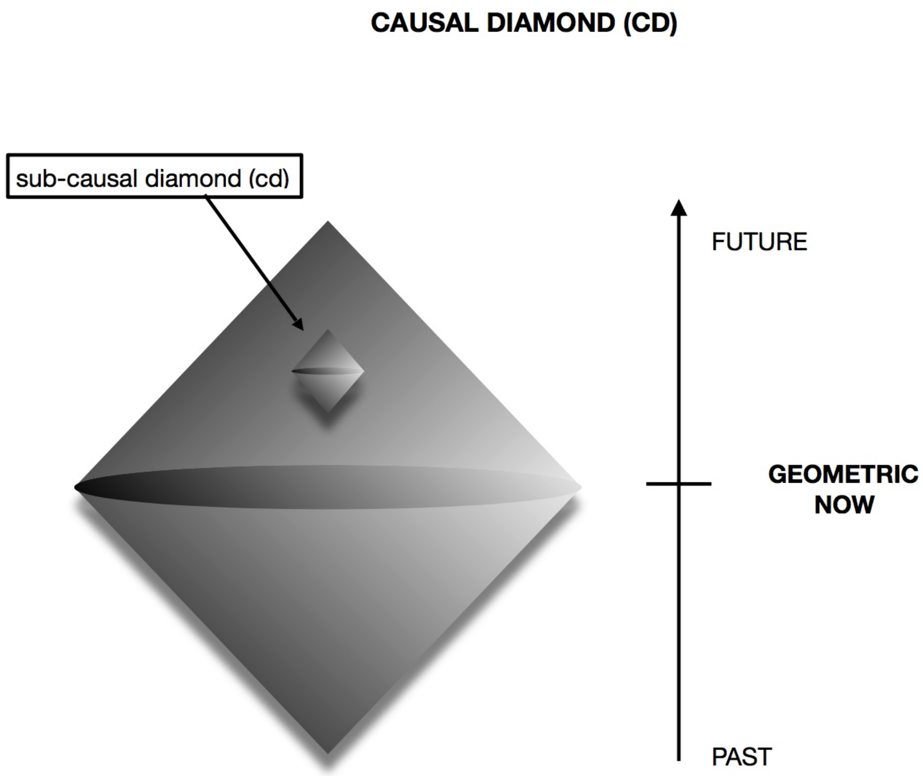
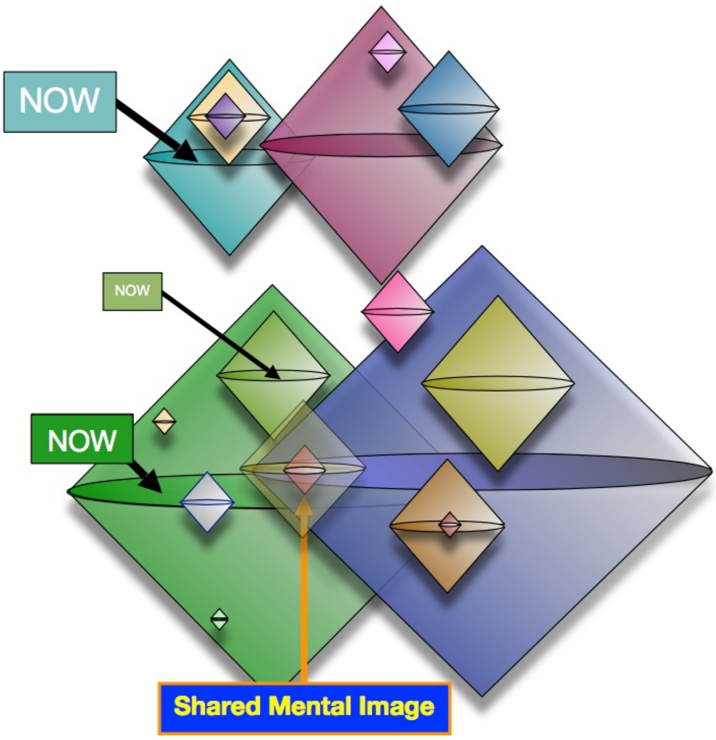


Figure 20: Causal diamond



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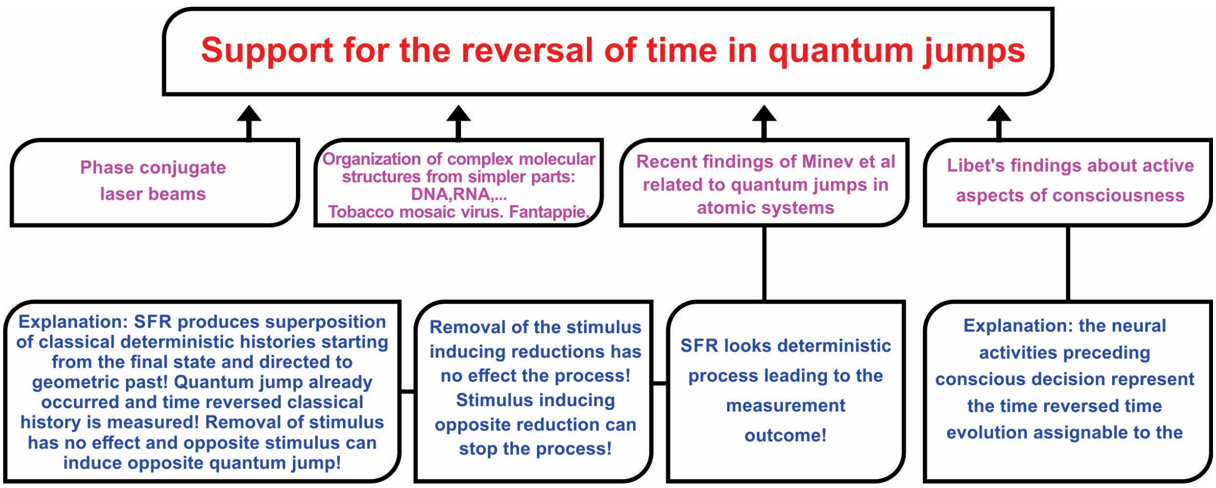

Figure 22: Time reversal occurs in BSFR

ACKNOWLEDGEMENTS

Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family and Kalevi and Ritva Tikkanen and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 45 lonely years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During the last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss my work. Pertti Kärkkäinen is my old physicist friend and has provided continued economic support for a long time. I have also had stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Tommi Ullgren has provided both economic support and encouragement during years. Pekka Rapinoja has offered his help in this respect and I am especially grateful to him for my Python skills.

During the last five years I have had inspiring discussions with many people in Finland interested in TGD. We have had video discussions with Sini Kunnas and had podcast discussions with Marko Manninen related to the TGD based view of physics and consciousness. Marko has also helped in the practical issues related to computers and quite recently he has done a lot of testing of chatGPT helping me to get an overall view of what it is. The discussions in a Zoom group involving Marko Manninen, Tuomas Sorakivi and Rode Majakka have given me the valuable opportunity to clarify my thoughts.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation in CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. I am grateful to Mark McWilliams, Paul Kirsch, Gary Ehlenberg, and Ulla Matfolk and many others for providing links to possibly interesting websites and articles. We have collaborated with Peter Gariaev and Reza Rastmanesh. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the public through the iron wall of academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as an individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without direct support from power holders- even in archives like arXiv.org.

Situation changed as Andrew Adamatsky proposed the writing of a book about TGD when I had already gotten used to the thought that my work would not be published during my lifetime. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loophole. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy.

And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his 73th birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During the period when the right wing held political power this trend was steadily strengthening and the situation is the same as I am writing this. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Karkkila, August 30, 2023, Finland

Matti Pitkänen

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Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K3].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A50]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of

electromagnetic fields are nonvanishing. The correlations functions for weak fields are non-vanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A18] [B22, B16, B17]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B13]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A38, A49, A29, A44].

The identification of the space-time as a sub-manifold [A39, A70] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

[1]There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.1.5 Construction of scattering amplitudes

Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A59, A78, A91]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K76]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their M^8-H duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p=3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M^8_c$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K72].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to "mind stuff", the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \rightarrow f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M_c^8

$Y^4 \subset M_c^8$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M_c^4 point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M_c^4 \subset M_c^8$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} . \tag{1.1.1}$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \rightarrow Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \rightarrow 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M_c^8$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M_c^4 \rightarrow SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.

2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.

4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.

5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same. The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of $M^8 - H$ duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4 \times CP_2$?

1. The selection of $SU(3) \subset G_2$ for ordinary $M^8 - H$ duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3 \subset M^8_c$ and its image $H^3 \subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?

2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3) \subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.

3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of $M^8 - H$ duality as something more fundamental than the ordinary $M^8 - H$ duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3 \times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1) \times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1 \oplus 3 \oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3 \oplus \overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8 \oplus 3 \oplus \overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3) \times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x) \times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E6] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K103].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K91, K92, K89]) support the view that dark matter might be a key player in living matter.

Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K116]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A50]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   $M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L50].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L39]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

1.2 TGD As A Generalization Of Physics To A Theory Consciousness

General Coordinate Invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. The basic idea is that quantum jump can be identified as momentum of consciousness. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [K113, K23, K88, K21, K52, K66, K69, K105, K112].

It is good to list first the basic challenges of TGD inspired theory of consciousness. The challenges can be formulated as questions. Reader can decide how satisfactory the answered proposed by TGD are.

1. What does one mean with quantum jump? Can one overcome the basic problem of the standard quantum measurement theory, that which forcing Bohr to give up totally the idea about objective reality?

2. How do the experienced time and geometric time relate in this framework? How the arrow of subjective time translates to that of geometric time?

3. How to define conscious information? Is it conserved or even increased during time evolution as biological evolution suggests? How does this increase relate to second law implied basically by the randomness of state function reduction?

4. Conscious entities/selves/observers seem to exist. If they are real how do they emerge?

1.2.1 Quantum Jump As A Moment Of Consciousness

The identification of quantum jump between deterministic quantum histories (WCW spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \quad ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is formally analogous to Schrödinger time evolution of infinite duration. The time evolution can however interpreted as a sequence of discrete scalings and Lorentz boosts of causal diamond (CD) and the time corresponds to the change of the proper time distance between between the tips of CD.

In TGD framework S-matrix is generalized to a triplet of U-, M-, and S-matrices. M-matrix is a hermitian square root of density matrix between positive and negative energy states multiplied by universal S-matrix depending on the scale of CD only. The square roots of projection operators form an orthonormal basis. U-matrix and S-matrix are completely universal objects characterizing the dynamics of evolution by self-organization.

The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to S^n, where S is the S-matrix associated with the minimal CD. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S.

U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2}\lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I turns out possible to construct a general representation for the U-matrix reducing its construction to that of S-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be "engineered".

In ZEO U-matrix should correspond relates zero energy states to each other and M matrices defining the rows of U matrix should be assignable to a fixed CD. Zero energy states should have wave function in the moduli space of CDs such that the second boundary of every CD would belong to a boundary of fixed light-cone but second boundary would be free with possible constraint that the distance between the tips of CD is multiple of CP_2 time.

Zero energy states of ZEO correspond in positive energy ontology to physical events and break time reversal invariance. This because either the positive or negative energy part of the state is reduced/equivalently preparated whereas the second end of CD corresponds to a superposition of (negative/positive energy) states with varying particle numbers and single particle quantum numbers just as in ordinary particle physics experiment.

The first state function reduction at given boundary of CD must change the roles of the ends of CDs. This reduction can be followed by a sequence of reductions to the same boundary of CD and not changing the boundary nor the parts of zero energy states associated with it but changing the states at the second end and also quantum distribution of the second boundary in the moduli space of CDs. In standard measurement theory the follow-up reductions would not affect the state at all.

The understanding of how the arrow of time and experience about its flow emerge have been the most difficult problem of TGD inspired theory of consciousness and I have considered several proposals during years having the geometry of future light-cone as the geometric core element.

1. The basic objection is that the arrow of geometric time alternates at embedding space level but we know that arrow of time looks the same in the part of the Universe we live. Possible exceptions however exist, for instance phase conjugate laser beams seem to obey opposite arrow of time. Also biological phenomena might involve non-standard arrow of time at some levels. This led Fantappie [J16] to introduce the notion of syntropy. This suggests that the arrow of time depends on the size scale of CD and of space-time sheet.

2. It took some time to realize that the solution of the problem is trivial in ZEO. In the ordinary quantum measurement theory one must assume that state function reduction can occur repeatedly: the assumption is that nothing happens to the state during repeated reductions. The outcome is Zeno effect: the watched pot does not boil.

   In TGD framework situation is different. Repeated state function reduction leaves the already reduce parts of zero energy state invariant but can change the part of states at the opposite boundary. One must allow a delocalization of the second boundary of CDs and one assumes that the second tip has quantized distance to the fixed one coming as multiple of CP_2 time. Also Lorentz boosts leaving the second CD boundary invariant must be allowed. One must therefore introduce a wave function in the moduli space of CDs with second boundary forming part of fixed light-cone boundary ($\delta M^4_\pm \times CP_2$).

3. The sequence of state function reductions on a fixed boundary of CD leads to the increase of the average temporal distance between the tips of CDs and this gives rise to the experience about flow of time as shifting of contents of perception towards future if the change is what contributes to conscious experience and gives rise to a fixed arrow of time.

4. Contrary to original working hypothesis, state function reduction in the usual sense does not solely determine the ordinary conscious experience. It can however contribute to conscious experience and the act of free will is a good candidate in this respect. TGD view about realization of intentional action assumes that intentional actions involve negative energy signals propagating backwards in geometric time. This would mean that at some level of CD hierarchy the arrow of geometric time indeed changes and the reduction start to occur at opposite boundary of CD at some level of length scale hierarchy.

1.2.2 Negentropy Maximization Principle (NMP)

Information is the basic aspect of consciousness and this motivates the introduction of Negentropy Maximization Principle (NMP) [K72] as the fundamental variational principle of consciousness theory. The amount of negentropy of zero energy state should increase in each quantum jump. The ordinary entanglement entropy is also non-negative so that negentropy could be at best zero. Since p-adic physics is assumed to be a correlate of cognition, it is natural to generalizes Shannon entropy to its number theoretic variant by replacing the probabilities appearing as arguments of logarithms of probabilities with their p-adic norms. This gives negentropy which can be positive so that NMP can generates entanglement.

Consistency with quantum measurement theory allows only negentropic density matrices proportional to unit matrix and negentropy has the largest positive value for the largest power of prime factor of the dimension of density matrix. Entanglement matrix proportional to unitary matrix familiar from quantum computation corresponds to unit density matrix and large $h_{eff} = n \times h$ states are excellent candidates for forming negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book).

The interpretation of negentropic entanglement is as a rule. The instances of the rule correspond to the pairs appearing in the superposition and the large the number of pairs is, the higher the abstraction level of the rule is. NMP is not in conflict with the second law since negentropy in the sense of NMP is not single particle property. Ordinary quantum jumps indeed generate entropy at the level of ensemble as also quantum jumps for states for which the density matrix is direct sum of unit matrices with various dimensions.

NMP forces the negentropic entanglement resources of the Universe to grow and thus implies evolution. I have coined the name "Akashic records" for these resources forming something analogous to library. It has turned out that the only viable option is that negentropic entanglement is experienced directly.

1.2.3 The Notion Of Self

The concept of self seems to be absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness and would be counterpart for observer in quantum measurement theory.

1. The original view was that self corresponds to a subsystem able to remain un-entangled under the sequential informational "time evolutions" U. It is however unclear how it could be possible to avoid generation of entanglement.

2. In ZEO the situation changes. Self corresponds to a sequence of quantum jumps for which the parts of zero energy states at either boundary of CD remaim unchanged. Therefore one can say that self defined in terms of parts of states assignable to this boundary remains unaffected as sub-system and does not generate entanglement. At the other boundary changes occur and give rise to the experience of time flow and arrow of time since the average temporal distance between the tips of CD tends to increase.

   When the reductions begin to occur at the opposite boundary of CD, self "falls asleep": symmetry suggests that new self living in opposite direction of geometric time is generated. Also in biological the change of time direction at some level of hierarchy might take place.

3. It looks natural to assume that the experiences of the self after the last "wake-up" sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its sub-selves as kind of abstracted experience: the experiences of sub-selves S_i are not experienced as such but represent kind of averages $\langle S_{ij}\rangle$ of sub-sub-selves S_{ij}. Entanglement between selves, most naturally realized by the formation of flux tube bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

4. Self corresponds in neuro science to self model defining a model for organism and for the external world. Information or negentropy seems to be necessary for understanding self. Negentropically entangled states - Akashic records - are excellent candidates for selves and would thus correspond to dark matter in TGD sense since the number of states in superposition corresponds to the integer n defining h_{eff}. It is enough that self is potentially conscious: this could mean that it conscious experience about self is generated only in interaction free measurement. Repeated state function reductions to given boundary of CD is second possibility. This would assign irreversibility and definite arrow of time and experience of time flow with self.

5. CDs would serve as embedding space correlates of selves and quantum jumps would be followed by cascades of state function reductions beginning from given CD and proceeding downwards to the smaller scales (smaller CDs). At space-time level space-time sheets in given p-adic length scale would be the natural correlates of selves. One ends also ends up with concrete ideas about how the localization of the contents of sensory experience and cognition to the "upper" (changing) boundary of CD could take place. One cannot exclude the possibility that state function reduction cascades could also take place in parallel branches of the quantum state.

1.2.4 Relationship To Quantum Measurement Theory

TGD based quantum measurement has several new elements. Negentropic entanglement and hierarchy of Planck constants, NMP, the prediction that state function reduction can take place to both boundaries of CD implying that the arrow of geometric time can change (this is expected to occur in microscopic scales whether the arrow of time is not established), and the possibility to understand the flow and arrow of geometric time.

1. The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of WCW spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

2. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It is also consistent with the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

3. Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field, ...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m-M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

   This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the WCW of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. In ZEO state preparation corresponds at some level of the self hierarchy to the a state function reduction to boundary opposite than before. In biology sensory perception and motor action would correspond to state function reduction sequences at opposite boundaries of CDs at some levels of the hierarchy.

Self measurement is governed by Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

1.2.5 Selves Self-Organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [K97]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

ZEO brings in important additional element to the theory of self-organization. The maxima of Kähler function corresponds to the most probable 3-surfaces. Kähler function receives contributions only from the Euclidian regions ("lines" of generalized Feynman diagrams) whereas the contribution to vacuum functional from Minkowskian regions is exponent of imaginary action so that saddle points with stationary phase are in question in these regions. In ZEO 3-surfaces are replaced by pairs of 3-surfaces at opposite boundaries of CD. The maxima actually correspond to temporal patterns of classical fields connecting these 3-surfaces: this means that self-organization is four spatiotemporal rather than spatial patterns - a crucial distinction from the usual view allowing to understand the evolution of behavioral patterns quantally. In biology this allows to understand temporal evolutions of organisms as the most probable self-organization patterns having as correlates the evolutions of the magnetic body of the system.

1.2.6 Classical Non-Determinism Of Kähler Action

A further basic element is non-determinism of Kähler action. This led to the concepts of association sequence and cognitive space-time sheet, which are not wrong notions but replaced by new ones.

1. The huge vacuum degeneracy of the Kähler action suggests strongly that the preferred is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

2. In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration.

Later a more detailed view about non-determinism in the framework of ZEO has emerged and quantum criticality is here the basic notion. The space-time surface connecting two 3-surfaces at the ends of CD is not unique. Conformal transformations which act trivially at the ends of space-time surface generate a continuum of new extremals with the same value of Kähler action and classical conserved quantities. The number n of conformal equivalence classes is finite and defines the value of h_{eff} (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book). There exists a hierarchy of breakdowns of conformal symmetry labelled by n. The fractal hierarchy of CDs gives rise to fractal hierarchy of non-determinisms of this kind.

1.2.7 P-Adic Physics As Physics Of Cognition

A further basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [K108] . The application this notion at the level of the embedding space implies that embedding space has a book like structure with various variants of the embedding space glued together along common rationals (algebraics, see **Fig.** `http://tgdtheory.fi/appfigures/book.jpg` or **Fig. ??** in the appendix of this book). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real embedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic pinary digits a p-valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ pinary digits represent a Boolean logic B^k with k elementary statements (the points of the k-element set in the set theoretic realization) with n taboos which are constrained to be identically true.

1.2.8 P-Adic And Dark Matter Hierarchies And Hierarchy Of Selves

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

1. Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as $\hbar$). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

2. The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [K42].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [K42]. The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [K42]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [K67, K42]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [K42].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [K41, K42]. The larger the value of Planck constant, the longer the life-time of self measured as the increase of the average distance between tips of CDs appearing in the quantum superposition during the period of repeated reductions not affecting the part of the zero energy state at the other boundary of CD- Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like $\hbar$.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self experiencse subselves as separate mental images. Averaging over experiences of sub-selves of sub-self would however occur.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The basic question is what time scale can one assign to the geometric duration of quantum jump measured naturally as the size scale of the space-time region about which quantum jump gives conscious information. This scale is naturally the size scale in which the non-determinism of quantum jump is localized. During years I have made several guesses about this time scales but zero energy ontology and the vision about fractal hierarchy of quantum jumps within quantum jumps leads to a unique identification.

CD as an embedding space correlate of self defines the time scale τ for the space-time region about which the consciousness experience is about. The temporal distances between the tips of CD as come as integer multiples of CP_2 length scales and for prime multiples correspond to what I have christened as secondary p-adic time scales. A reasonable guess is that secondary p-adic time scales are selected during evolution and the primes near powers of two are especially favored. For electron, which corresponds to Mersenne prime $M_{127} = 2^{127} - 1$ this scale corresponds to.1 seconds defining the fundamental time scale of living matter via 10 Hz biorhythm (alpha rhythm). The unexpected prediction is that all elementary particles correspond to time scales possibly relevant to living matter.

Dark matter hierarchy brings additional finesse. For the higher levels of dark matter hierarchy τ is scaled up by $\hbar/\hbar_0$. One could understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ.

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question. The level would determine also the time span of long term memories as discussed in [K42]. The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [K67, K42]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

1.3 Quantum Biology And Quantum Neuroscience In TGD Universe

Quantum biology - rather than only quantum brain - is an essential element of Quantum Mind in TGD Universe. Cells, biomolecules, and even elementary particles are conscious entities and the biological evolution is evolution of consciousness so that it would be very artificial to restrict the discussion to brain, neurons, or microtubules.

1.3.1 Basic Physical Ideas

The following list gives the basic elements of TGD inspire quantum biology.

1. Many-sheeted space-time allows the interpretation of the structures of macroscopic world around us in terms of space-time topology. Magnetic/field body acts as intentional agent using biological body as a sensory receptor and motor instrument and controlling biological body and inheriting its hierarchical fractal structure. Fractal hierarchy of EEGs and its variants can be seen as communication and control tools of magnetic body. Also collective levels of consciousness have a natural interpretation in terms of magnetic body. Magnetic body makes also possible entanglement in macroscopic length scales. The braiding of magnetic flux tubes makes possible topological quantum computations and provides a universal mechanism of memory. One can also undersand the real function of various information molecules and corresponding receptors by interpreting the receptors as addresses in quantum computer memory and information molecules as ends of flux tubes which attach to these receptors to form a connection in quantum web.

2. Magnetic body carrying dark matter and forming an onion-like structure with layers characterized by large values of Planck constant is the key concept of TGD inspired view about Quantum Mind to biology. Magnetic body is identified as intentional agent using biological body as sensory receptor and motor instrument. EEG and its fractal variants are identified as a communication and control tool of the magnetic body and a fractal hierarchy of analogs of EEG is predicted. Living system is identified as a kind of Indra's net with biomolecules representing the nodes of the net and magnetic flux tubes connections between then.

   The reconnection of magnetic flux tubes and phase transitions changing Planck constant and therefore the lengths of the magnetic flux tubes are identified as basic mechanisms behind DNA replication and analogous processes and also behind the phase transitions associated with the gel phase in cell interior. The braiding of magnetic flux makes possible universal memory representation recording the motions of the basic units connected by flux tubes. Braiding also defines topological quantum computer programs updated continually by the flows of the basic units. The model of DNA as topological quantum computer is discussed as an application. In zero energy ontology the braiding actually generalize to 2-braiding for string world sheets in 4-D space-time and brings in new elements.

3. Zero energy ontology (ZEO) makes possible the proposed p-adic description of intentions and cognitions and their transformations to action. Time mirror mechanism (see **Fig.** `http://tgdtheory.fi/appfigures/timemirror.jpg` or **Fig. ??** in the appendix of the book) based on sending of negative energy signal to geometric past would apply to both long term memory recall, remote metabolism, and realization of intentional acting as an activity beginning in the geometric past in accordance with the findings of Libet. ZEO gives a precise content to the notion of negative energy signal in terms of zero energy state for which the arrow of geometric time is opposite to the standard one.

   The associated notion of causal diamond (CD) is essential element and assigns to elementary particles new fundamental time scales which are macroscopic: for electron the time scale is.1 seconds, the fundamental biorhythm. An essentially new element is time-like entanglement which allows to understand among other things the quantum counterparts of Boolean functions in terms of time-like entanglement in fermionic degrees of freedom.

4. The assignment of dark matter with a hierarchy of Planck constants gives rise to a hierarchy of macroscopic quantum phases making possible macroscopic and macrotemporal quantum coherence and allowing to understand evolution as a gradual increase of Planck constant. The model for dark nucleons leads to a surprising conclusion: the states of nucleons correspond to DNA, RNA, tRNA, and amino-acids in a natural manner and vertebrate genetic code as correspondence between DNA and amino-acids emerges naturally. This suggests that genetic code is realized at the level of dark hadron physics and living matter in the usual sense provides a secondary representation for it.

   The hierarchy of Planck constants emerges from basic TGD under rather general assumptions. The key element is the huge vacuum degeneracy which implies that preferred non-vacuum extremals of Kähler action form a 4-D spin glass phase. The basic implications following from the extreme non-linearity of Kähler action is that normal derivatives of embedding space coordinates at 3-D light-like orbits of partonic 2-surfaces and at space-like 3-surfaces at ends of CDs are many-valued functions of canonical momentum densities: this is one of the reasons that forced to develop physics as an infinite-D Kähler geometry vision instead of trying to develop path integral formalism or canonical quantization. A convenient manner to treat the situation is to introduce local many-sheeted covering of embedding space such that the sheets are completely degenerate at partonic 2-surfaces. This leads in natural manner to the hierarchy of Planck constants as effective hierarchy hierarchy and integer multiples of Planck constants emerge naturally.

5. p-Adic physics can be identified as physics of cognition and intentionality. The hierarchy of p-adic length scales predicts a hierarchy of universal metabolic quanta as increments of zero point kinetic energies. Negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) possible for number theoretic entanglement entropy makes sense for rational (and even algebraic) entanglement and leads to the identification of life as something residing in the intersection of real and p-adic worlds. NMP respects negentropic entanglement and the attractive idea is that the experience of understanding and positively colored emotions relate to negentropic entanglement.

6. Living matter as conscious hologram is one of the basic ideas of TGD inspired biology and consciousness theory. The basic objection against TGD is that the interference of classical fields is impossible in the standard sense for the reason that classical fields are not primary dynamical variables in TGD Universe. The resolution is based on the observation that only the interference of the effects caused by these fields can be observed experimentally and that many-sheeted space-time allows to realized the summation of effects in terms of multiple topological condensations of particles to several parallel space-time sheets. One concrete implication is fractality of qualia. Qualia appear in very wide range of scales: our qualia could in fact be those of magnetic body. The proposed mechanism for the generation of qualia realizes the fractality idea.

1.3.2 Brain In TGD Universe

Brain cognizes and one should find physical correlates for cognition. Also the precise role of brain in information processing and its relationship to metabolism should be understood. Here magnetic body brings as a third player to the couple formed by environment and organism.

1. An attractive idea is that the negentropic entanglement can be assigned with magnetic flux tubes somehow and that ATP serves as a correlate for negentropic entanglement. This leads to a rather detailed ideas about the role of phosphate bond and provides interpretation for the fact that the number of valence bonds tend to be maximized in living matter. In a loose sense one could even call ATP a consciousness molecule. The latest view encourages to consider the possibility that negentropic entanglement with what might be called Mother Gaia is what is transferred in metabolism.

2. The view about the function of brain differs from the standard view. The simplest option is that brain is a builder of symbolic representations building percepts and giving them names rather than the seat of primary qualia relevant to our conscious experience. Sensory organs

would carry our primary qualia and brain would build sensory percepts as standardized mental images by using virtual sensory input to the sensory organs. The new view about time is absolutely essential for circumventing the objections against this vision. The prediction is that also neuronal and even cell membranes define sensory maps with primary qualia assignable to the lipids serving as pixels of the sensory screen. These qualia would not however represent our qualia but lower level qualia. At this moment it is not possible to choose between these two options.

3. The role of EEG and its various counterparts at fractally scaled frequency ranges is to make possible communications to the various onion-like layers of the magnetic body and the control by magnetic body. Dark matter at these layers could be seen as the intentional agent and sensory perceiver.

1.3.3 Anomalies

Various anomalies of living matter have been in vital role in the development of not only TGD view about living matter but also TGD itself.

1. TGD approach to living matter was strongly motivated by the findings about strange behavior of cell membrane and of cellular water, and gel behavior of cytoplasm. Also the findings about effects of ELF em fields on vertebrate brain were decisive and led to the proposal of the hierarchy of Planck constants found later to emerge naturally from the non-determinism of Kähler action. Rather satisfactorily, the other manner to introduce the hierarchy of Planck constants is in terms of gravitational Planck constant: at least in microscopic scales the equivalence of these approaches makes sense and leads to highly non-trivial predictions. The basic testable prediction is that dark photons have cyclotron frequencies inversely proportional to their massess but universal energy spectrum in visible and UV range which corresponds to the transition energies for biomolecules so that they are ideal for biocontrol at the level of both magnetic bodies and at the level of biochemistry.

2. Water is in key role in living matter and also in TGD inspired view about living matter. The anomalies of water lead to a model for dark nuclei as dark proton strings with the surprising prediction that DNA, RNA, anino-acids and even tRNA are in one-one correspondence with the resulting 3-quark states and that vertebrate genetic code emerges naturally. This leads to a vision about water as primordial life form still playing a vital role in living organisms. The model of water memory and homeopathy in turn generalizes to a vision about how immune system might have evolved.

3. Metabolic energy is necessary for conscious information processing in living matter. This suggests that metabolism should be basically transfer of negentropic entanglement from nutrients to the organism. ATP could be seen as a molecule of consciousness in this picture and high energy phosphate bond would make possible the transfer of negentropy.

1.4 Bird's Eye of View about the Topics of the Book

The topics of this book are general mathematical ideas, many of them inspired by TGD inspired theory of consciousness.

The topics of this book are general mathematical ideas behind TGD inspired theory of consciousness and quantum TGD itself, many of them inspired by TGD inspired theory of consciousness.

1.4.1 p-Adic physics and consciousness

Physics as a generalized number theory vision received a strong boost from TGD inspired theory of consciousness. There are good reasons for considering p-adic physics as a good candidate for physics of cognition and intention and this leads to a concrete quantum model for how intentions are transformed to actions in quantum jumps transforming p-adic space-time sheets to real ones [K108]

. The logical aspects of cognition would naturally be represented by the discrete projections of p-adic space-time sheets to real embedding space as space-time correlates.

Most points of p-adic space-time sheets are at infinity in real sense so that cognition literally views the real cosmos from outside. The projection to real embedding space is discrete and this could reflect that fact that the part of our cognition which is representable at the level of real physics is bound to be always discrete and actually finite (consider only numerical computations).

1.4.2 Von Neumann algebras and consciousness

The von Neumann algebras known as hyper-finite factors of type II_1 appear naturally in TGD framework: as a matter fact configuration space spinors associated with single point of configuration space ("world of classical worlds") decomposes to a direct integral of these algebras. This alone leads to amazingly strong physical predictions and implies deep connections with conformal field theories, knot-, braid- and quantum groups, and topological quantum computation.

The braids formed by magnetic flux tubes seem to be ideal for the realization of topological quantum computations by coding quantum computations to the braidings of the flux tubes. Living system is populated of molecular structures which form braids so that in TGD Universe bio-systems are basic candidates for topological quantum computers. The possibility to communicate with the geometric past using negative energy bosons suggests that the constraints posed by the non-polynomial computation time on ordinary quantum computations might be circumvented by using time loops, and living matter might utilize this mechanism routinely. The vision about DNA as topological quantum computer is a concrete proposal about how this might be achieved.

1.4.3 Dark matter hierarchy and consciousness

The most recent piece in the big picture is the vision about dark matter as a hierarchy of phases of matter having no local interactions (vertices of Feynman graphs) with other levels. The level of dark matter hierarchy is characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the embedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page. The phase transitions changing the value of Planck constants having interpretation as tunnelling between different pages of the book like structure would induce the phase transitions of gel phases abundant in living matter.

1.4.4 Infinite primes and consciousness

The notion of infinite prime was the first mathematical invention inspired by TGD inspired theory of consciousness. The construction of infinite primes is very much analogous to a repeated second quantization of a super-symmetric arithmetic quantum field theory (with analogs of bound states included). One can also interpret space-time surfaces as representations of infinite primes, integers, and rationals analogous to representations of polynomials as surfaces of their zeros.

Infinite primes form an infinite hierarchy and the realization of this hierarchy at the level of physics would mean that the hierarchy of consciousness is also infinite and that we represent only single level in this hierarchy looking infinitesimal from the point of view of higher levels. The notion of infinite rational predicts also an infinite number of real units with infinitely rich number theoretical anatomy so that single space-time point becomes a Platonia able to represent every quantum state of the entire Universe in its structure. This means kind of Brahman=Atman identity at the level of mathematics.

1.4.5 Categories and consciousness

One can consider the possibility that category theory reflects the basic structures of conscious thought. Although my understanding of category theory is rather meager, I cannot avoid the temptation to discuss the possible applications of category theory to TGD and TGD inspired theory of consciousness. The comparison of the generalized inherent logics associated with categories to the Boolean logic naturally associated with the configuration space spinor fields is also of interest. Category theoretical ideas find a more concrete application in quantum TGD proper in characterization of Feynman diagrams.

1.4.6 Organization of "Mathematical Aspects of Consciousness Theory"

It must be warned that the contents of the book reflect to my views for more than decade ago rather than recent views. Several important new mathematical notions (such as adelic physics, the twistor lift of TGD, and $M^8 - H$ duality) have emerged after that.

The contents of the book are organized in 3 parts.

1. The first 2 chapters of 1st part are devoted to hyper-finite factors of type II_1 and to the physical vision about dark matter provided by the hierarchy of Planck constants. The vision about the mathematical realization of the hierarchy of Planck constants has developed via several side tracks: for instance, I have proposed covering space structure for imbedding space H but in the recent vision it would be for space-time surface itself. Therefore the representation of second chapter might be out-of-date in some aspects.

   The recent views are described in the third chapter about adelic physics. There is also a chapter about $M^8 - H$ duality allowing to see dynamics as purely algebraic (there would be no action principle nor partial differential equations and algebraic equations would determine space-time surfaces at level of M^8).

2. 2nd part is devoted to the idea that a process analogous to topological quantum computation take place at fundamental level in TGD Universe and for the model for how DNA and cell membrane could act as topological quantum computer. The braiding of magnetic flux tubes connecting nucleotides and lipids of the cell membrane is the key element of the model and the model leads to a plethora of ideas about catalyst action and bio-control.

3. The 3rd part of the book contains a chapter about infinite primes and category theoretical ideas. Both are written for more than fifteen years ago and the vision about the role of categories has been updated since then.

1.5 Sources

The eight online books about TGD [K125, K117, K96, K81, K29, K77, K57, K106] and nine online books about TGD inspired theory of consciousness and quantum biology [K113, K23, K88, K21, K52, K66, K69, K105, K112] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.6 The contents of the book

1.6.1 PART I: NEW PHYSICS AND MATHEMATICS INVOLVED WITH TGD

Evolution of Ideas about Hyper-finite Factors in TGD

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors, could provide the mathematics needed to develop a more explicit view about the construction of M-matrix generalizing the notion of S-matrix in zero energy ontology (ZEO). In this chapter I will discuss various aspects of hyper-finite factors and their possible physical interpretation in TGD framework.

1. Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. Therefore also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_{∞} results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. One can start from a local octonionic Clifford algebra in M^8. Associativity (co-associativity) condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the induced gamma matrices associated with the Kähler action span a complex quaternionic (complex co-quaternionic) sub-space at each point of the sub-manifold. This associative (co-associative) sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative (co-associative(algebra and thus to HFF of type II_1.

2. Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a

generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology (ZEO): the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions meaning the analog of state function collapse in zero modes fixing the classical conserved charges equal to the quantal counterparts. Classical charges would be parameters characterizing zero modes.

A concrete construction of M-matrix motivated the recent rather precise view about basic variational principles is proposed. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.

3. Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In ZEO $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$-"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

4. Analogs of quantum matrix groups from finite measurement resolution?

The notion of quantum group replaces ordinary matrices with matrices with non-commutative elements. In TGD framework I have proposed that the notion should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution.

In this article I will consider the notion of quantum matrix inspired by recent view about quantum TGD and it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution. The basic idea is to replace complex matrix elements with operators expressible as products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers.

The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. The weak definition of determinant meaning its development with respect to a fixed row or column does not pose additional conditions. Strong definition of determinant requires its invariance under permutations of rows and columns. The permutation of rows/columns turns out to have interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

The resulting quantum matrices define a more general structure than quantum group but provide a concrete representation and interpretation for quantum group in terms of finite measurement resolution if q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by sign factor invariant under the permutations of both rows and columns. One could also understand the fractal structure of inclusion sequences of hyper-finite factors resulting by recursively replacing operators appearing as matrix elements with quantum matrices.

5. Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

Does TGD Predict Spectrum of Planck Constants?

The quantization of Planck constant has been the basic theme of TGD since 2005. The basic idea was stimulated by the suggestion of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, where the velocity parameter v_0 has the approximate value $v_0 \simeq 2^{-11}$ for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The recent version of the chapter represents the evolution of ideas about quantization of Planck constants from a perspective given by seven years's work with the idea. A very concise summary about the situation is as follows.

1. Basic physical ideas

The basic phenomenological rules are simple.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Effective embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they

have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies.

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. The interpretation of $\hbar_{gr}$ introduced by Nottale in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in astronomical scales. The gravitational Compton length $GM/v_0 = r_S/2v_0$ does not depend on m so that all particles around say Sun say same gravitational Compton length.

   By the independence of gravitational acceleration and gravitational Compton length on particle mass, it is enough to assume that only microscopic particles couple to the dark gravitons propagating along flux tubes mediating gravitational interaction. Therefore $h_{gr} = h_{eff}$ could be true in microscopic scales and would predict that cyclotron energies have no dependence on the mass of the charged particle meaning that the spectrum ordinary photons resulting in the transformation of dark photons to ordinary photons is universal. An attractive identification of these photons would be as bio-photons with energies in visible and UV range and thus inducing molecular transitions making control of biochemistry by dark photons. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation. The energy of the graviton is gigantic unless the emission is assume to take place from a microscopic systems with large but not gigantic h_{gr}.

3. Why Nature would like to have large - maybe even gigantic - value of effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

2. Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of embedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology (ZEO), it became clear that the notion of singular covering space of the embedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. In ZEO 3-surfaces are unions of space-like 3-surface at opposite boundaries of CD. The non-determinism of Kähler action due to the huge vacuum degeneracy would naturally explain the existence of several space-time sheets connecting the two 3-surfaces at the opposite boundaries of CD. Quantum criticality suggests strongly conformal invariance and the identification of n as the number of conformal equivalence classes of these space-time sheets. Also a connection with the notion of negentropic entanglement emerges.

Philosophy of Adelic Physics

The p-adic aspects of Topological Geometrodynamics (TGD) will be discussed. Introduction gives a short summary about classical and quantum TGD. This is needed since the p-adic ideas are inspired by TGD based view about physics.

p-Adic mass calculations relying on p-adic generalization of thermodynamics and super-symplectic and super-conformal symmetries are summarized. Number theoretical existence constrains lead to highly non-trivial and successful physical predictions. The notion of canonical identification mapping p-adic mass squared to real mass squared emerges, and is expected to be a key player of adelic physics allowing to map various invariants from p-adics to reals and vice versa.

A view about p-adicization and adelization of real number based physics is proposed. The proposal is a fusion of real physics and various p-adic physics to single coherent whole achieved by a generalization of number concept by fusing reals and extensions of p-adic numbers induced by given extension of rationals to a larger structure and having the extension of rationals as their intersection.

The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far from obvious and various constraints lead to the idea of number theoretic universality (NTU) and finite measurement resolution realized in terms of number theory. An attractive manner to overcome the problems in case of symmetric spaces relies on the replacement of angle variables and their hyperbolic analogs with their exponentials identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

Also the understanding of the correspondence between real and p-adic physics at various levels - space-time level, embedding space level, and level of "world of classical worlds" (WCW) - is a challenge. The gigantic isometry group of WCW and the maximal isometry group of embedding space give hopes about a resolution of the problems. Strong form of holography (SH) allows a non-local correspondence between real and p-adic space-time surfaces induced by algebraic continuation from common string world sheets and partonic 2-surfaces. Also local correspondence seems intuitively plausible and is based on number theoretic discretization as intersection of real and p-adic surfaces providing automatically finite "cognitive" resolution. he existence p-adic variants of Kähler geometry of WCW is a challenge, and NTU might allow to realize it.

I will also sum up the role of p-adic physics in TGD inspired theory of consciousness. Negentropic entanglement (NE) characterized by number theoretical entanglement negentropy (NEN) plays a key role. Negentropy Maximization Principle (NMP) forces the generation of NE. The interpretation is in terms of evolution as increase of negentropy resources.

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part I

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces.

In the sequel I shall consider the following topics.

1. I will discuss basic notions of algebraic geometry such as algebraic variety, surface, and curve, all rational point of variety central for TGD view about cognitive representation, elliptic curves and surfaces, and rational and potentially rational varieties. Also the notion of Zariski topology and Kodaira dimension are discussed briefly. I am not a mathematician and what hopefully saves me from horrible blunders is physical intuition developed during 4 decades of TGD.

2. It will be shown how M^8-H duality could reduce TGD at fundamental level to octonionic algebraic geometry. Space-time surfaces in M^8 would be algebraic surfaces identified as zero loci for imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial of complexified octonionic variable o_c decomposing as $o_c = q_c^1 + q_c^2 I^4$ and projected to a Minkowskian sub-space M^8 of complexified O. Single real valued polynomial of real variable with algebraic coefficients would determine space-time surface! As proposed already earlier, spacetime surfaces would form commutative and associative algebra with addition, product and functional composition.

   One can interpret the products of polynomials as correlates for free many-particle states with interactions described by added interaction polynomial, which can vanish at boundaries of CDs thanks to the vanishing in Minkowski signature of the complexified norm $q_c\overline{q_c}$ appearing in $RE(P)$ or $IM(P)$ caused by the quaternionic non-commutativity. This leads to the same picture as the view about preferred extremals reducing to minimal surfaces near boundaries of CD. Also zero zero energy ontology (ZEO) could emerge naturally from the failure of number field property for for quaternions at light-cone boundaries.

The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients can give rise to associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

   The hierarchy of notions involved is well-ordering for 1-D structures, commutativity for complex numbers, and associativity for quaternions. This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy Riemann conditions are linear and constant value manifolds are 1-D and thus well-ordered. Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

   In fact, all algebras obtained by Cayley-Dickson construction adding imaginary units to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and M^8-H correspondence could generalize.

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k, $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i. In the generic case this gives 3-D surface.

   In this generic case M^8-H duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H, and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

One can also have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i. Space-time surface would be analogous to a polynomial with a multiple root. The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces except at 3-D boundary surfaces. These regions correspond to interaction regions in which Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes to the level of complex/co-complex surfaces assigned with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces?

   I have proposed commutativity or co-commutatitivity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commutative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part II

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to octonionic algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces.

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4. The super variant of the octonionic geometry relying on octonionic triality makes sense and the geometry of the space-time variety correlates with fermion and antifermion numbers assigned with it. This new view about super-geometry involving also automatic SUSY breaking at the level of space-time geometry.

Also a sketchy proposal for the description of interactions is discussed.

1. The surprise that $RE(P) = 0$ and $IM(P) = 0$ conditions have as singular solutions light-cone interior and its complement and 6-spheres $S^6(t_n)$ with radii t_n given by the roots of the real

$P(t)$, whose octonionic extension defines the space-time variety X^4. The intersections $X^2 = X^4 \cap S^6(t_n)$ are tentatively identified as partonic 2-varieties defining topological interaction vertices.

The idea about the reduction of zero energy states to discrete cognitive representations suggests that interaction vertices at partonic varieties X^2 are associated with the discrete set of intersection points of the sparticle lines at light-like orbits of partonic 2-surfaces belonging to extension of rationals.

2. CDs and therefore also ZEO emerge naturally. For CDs with different origins the products of polynomials fail to commute and associate unless the CDs have tips along real (time) axis. The first option is that all CDs under observation satisfy this condition. Second option allows general CDs.

   The proposal is that the product $\prod P_i$ of polynomials associated with CDs with tips along real axis the condition $IM(\prod P_i) = 0$ reduces to $IM(P_i) = 0$ and criticality conditions guaranteeing associativity and provides a description of the external particles. Inside these CDs $RE(\prod P_i) = 0$ does not reduce to $RE(\prod P_i) = 0$, which automatically gives rise to geometric interactions. For general CDs the situation is more complex.

3. The possibility of super octonionic geometry raises the hope that the twistorial construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY generalizes to TGD in rather straightforward manner to a purely geometric construction. Functional integral over WCW would reduce to summations over polynomials with coefficients in extension of rationals and criticality conditions on the coefficients could make the summation well-defined by bringing in finite measurement resolution.

Scattering diagrams would be determined by points of space-time variety, which are in extension of rationals. In adelic physics the interpretation is as cognitive representations.

1. Cognitive representations are identified as sets of rational points for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8- and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces. For the earlier work related to adelic TGD and cognitive representations see [**?**]

2. Some aspects related to homology charge (Kähler magnetic charge) and genus-generation correspondence are discussed. Both topological quantum numbers are central in the proposed model of elementary particles and it is interesting to see whether the picture is internally consistent and how algebraic variety property affects the situation. Also possible problems related to $h_{eff}/h = n$ hierarchy []adelicphysics realized in terms of n-fold coverings of space-time surfaces are discussed from this perspective.

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III

Cognitive representations are the basic topic of the third chapter related to $M^8 - H$ duality. Cognitive representations are identified as sets of points in extension of rationals for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8- and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces.

The notion is applied in various cases and the connection with $M^8 - H$ duality is rather loose.

1. Extensions of rationals are essentially coders of information. There the possible analogy of extensions of rationals with genes deserves discussion. Extensions, which are not extensions of extensions would be analogous to genes. The notion of conserved gene as number theoretical analogy for Galois extensions as the Galois group of extension which is normal subgroup of Galois extension.

2. The possible physical meaning of the notion of perfectoid introduced by Peter Scholze is discussed in the framework of p-adic physics. Extensions of p-adic numbers involving roots of the prime defining the extension are involved and are not considered previously in TGD framework. There there possible physical meaning deserves discussion.

3. The construction of cognitive representation reduces to a well-known mathematical problem of finding the points of space-time surface with embedding space coordinates in given extension of rationals. The work of Kim and Coates represents new ideas in this respect and there is a natural connection with TGD.

4. One expects that large cognitive representations are winners in the number theoretical fight for survival. Strong form of holography suggests that it is enough to consider cognitive representations restricted to string world sheets and partonic 2-surfaces. If the 2-surface possesses large group of symmetries acting in extension of rationals, one can have large cognitive representations as orbit of point in extension. Examples of highly symmetric 2-D surfaces are geodesic spheres assignable to partonic 2-surfaces and cosmic strings and elliptic curves assignable with string world sheets and cosmic strings.

5. Rationals and their extensions give rise to a unique discretizations of space-time surface (for instance) - cognitive representation - having interpretation in terms of finite measurement resolution. There are however many open questions. Should one allow only octonionic polynomials defined as algebraic continuations of real polynomials or should one allow also analytic functions and regard polynomials as approximations. Zeta functions are especially interesting analytic functions and Defekind zetas characterize extensions of rationals and one can pose physically motivated questions about them.

Could quantum randomness have something to do with classical chaos?

Tim Palmer has proposed that classical chaos and quantum randomness might be related. It came as a surprise to me that these to notions could a have deep relationship in TGD framework.

1. Strong form of Palmer's idea stating that quantum randomness reduces to classical chaos certainly fails but one can consider weaker forms of the idea. Even these variants fail in Copenhagen interpretation since strictly speaking there is no classical reality, only wave function coding for the knowledge about the system. Bohr orbits should be more than approximation and in TGD framework space-time surface as preferred extremal of action is analogous to Bohr orbit and classical physics defined by Bohr orbits is an exact part of quantum theory.

2. In the zero energy ontology (ZEO) of TGD the idea works in weaker form and has very strong implications for the more detailed understanding of ZEO and $M^8 - M^4 \times CP_2$ duality. Ordinary ("big") state functions (BSFRs) meaning the death of the system in a universal sense and re-incarnation with opposite arrow of time would involve quantum criticality accompanied by classical chaos assignable to the correspondence between geometric time and subjective time identified as sequence of "small" state function reductions (SSFRs) as analogs of weak measurements. The findings of Minev et al give strong support for this view and Libet's findings about active aspects of consciousness can be understood if the act of free will corresponds to BSFR.

M^8 picture identifies 4-D space-time surfaces X^4 as roots for "imaginary" or "real" part of octonionic polynomial P_2P_1 obtained as a continuation of real polynomial $P_2(L-r)P_1(r)$, whose arguments have origin at the the tips of B and A and roots a the light-cone boundaries associated with tips. Causal diamond (CD) is identified intersection of future and past directed light-cones light-cones A and B. In the sequences of SSFRs $P_2(L-r)$ assigned to B varies and $P_1(r)$ assigned to A is unaffected. L defines the size of CD as distance $\tau = 2L$ between its tips.

Besides 4-D space-time surfaces there are also brane-like 6-surfaces corresponding to roots $r_{i,k}$ of $P_i(r)$ and defining "special moments in the life of self" having $t_i = r_{i,k}$ ball as M^4_+ projection. The number of roots and their density increases rapidly in the sequence of SSFRs. The condition that the largest root belongs to CD gives a lower bound to it size L as largest root. Note that L increases.

Concerning the approach to chaos, one can consider three options.

Option I: The sequence of steps consisting of unitary evolutions followed by SSFR corresponds to a functional factorization at the level of polynomials as sequence $P_2 = Q_1 \circ Q_2 \circ ...Q_n$. If the size of CD is assumed to increase, also the tip of active boundary of CD must shift so that the argument of P_2 $r - L$ is replaced in each iteration step to with updated argument with larger value of L.

Option II: A completely unexpected connection with the iteration of analytic functions and Julia sets, which are fractals assigned also with chaos interpreted as complexity emerges. In a reasonable approximation quantum time evolution by SSFRs could be induced by an iteration of a polynomial or even an analytic function: $P_2 = P_2 \rightarrow P_2^{\circ 2} \rightarrow ....$ For $P_2(0) = 0$ the roots of the iterate consists of inverse images of roots of P_2 by $P_2^{\circ -k}$ for $k = 0, ..., N-1$.

Suppose that M^8 and X^4 are complexified and thus also $t = r$ and "real" X^4 is the projection of X_c^4 to real M^8. Complexify also the coefficients of polynomials P. If so, the Mandelbrot and Julia sets (`http://tinyurl.com/cplj9pe` and `http://tinyurl.com/cvmr83g`) characterizing fractals would have a physical interpretation in ZEO.

One approaches chaos in the sense that the $N-1$:th inverse images of the roots of P_2 belonging to filled Julia set approach to points of Julia set of P_2 as the number N of iterations increases. Minimal L would increase with N if CD is assumed to contain all roots. The density of the roots in Julia set increases near L since the size of CD is bounded by the size Julia set. One could perhaps say that near the $t = L$ in the middle of CD the life of self when the size of CD has become almost stationary, is the most intensive.

Option III: A conservative option is to consider also real polynomials $P_2(r)$ with real argument r. Only non-negative real roots r_n are of interest whereas in the general case one considers all values of r. For a large N the new roots with possibly one exception would approach to the real Julia set obtained as a real projection of Julia set for complex iteration.

How the size L of CD is determined and when can BSFR occur?

Option I: If L is minimal and thus given by the largest (non-exceptional) root of iterate of P_2 in Julia set, it is bound to increase in the iteration (this option is perhaps too deterministic). L should smaller than the sizes of Julia sets of both$A$ and B since the iteration gives no roots outside Julia sets.

Could BSFR become probable when L as the largest allowed root for iterate P_2 is larger than the size of Julia set of A? There would be no more new "special moments in the life of self" and this would make death (in universal sense) and re-incarnation with opposite arrow of time probable. The size of CD could decrease dramatically in the first iteration for P_1 if it is determined as the largest allowed root of P_1: the re-incarnated self would have childhood.

Option II: The size of CD could be determined in SSFR statistically as an allowed root of P_2. Since the density of roots increases, one would have a lot of choices and quantum criticality and fluctuations of the order of clock time $\tau = 2L$: the order of subjective time would not anymore correspond to that for clock time. BSFR would occur for the same reason as for the first option.

1.6.2 PART II: QUANTUM COMPUTATION IN TGD UNIVERSE

Topological Quantum Computation in TGD Universe

Topological quantum computation (TQC) is one of the most promising approaches to quantum computation. The coding of logical qubits to the entanglement of topological quantum numbers promises to solve the de-coherence problem whereas the S-matrices of topological field theories (modular functors) providing unitary representations for braids would give a realization of quantum computer programs with gates represented as simple braiding operations. Because of their effective 2-dimensionality anyon systems are the best candidates for realizing the representations of braid groups.

TGD allows several new insights related to quantum computation. TGD predicts new information measures as number theoretical negative valued entanglement entropies defined for systems having extended rational entanglement and characterizes bound state entanglement as bound state entanglement. Hierarchy of Planck constants labelling phases of dark matter makes possible macroscopic quantum coherence. Negentropy Maximization Principle and p-adic length scale hierarchy of space-time sheets encourage to believe that Universe itself might do its best to resolve the de-coherence problem. The new view about quantum jump suggests strongly the notion of quantum parallel dissipation so that thermalization in shorter length scales would guarantee coherence in longer length scales. The possibility of negative energies and communications to geometric future in turn might even trivialize the problems caused by long computation times: computation could be iterated again and again by turning the computer on in the geometric past and TGD inspired theory of consciousness predicts that something like this occurs routinely in living matter.

Kähler action defines the basic variational principle of classical TGD and predicts extremely complex but non-chaotic magnetic flux tube structures, which can get knotted and linked. The dimension of CP_2 projection for these structures is $D = 3$. These structures are the corner stone of TGD inspired theory of living matter and provide the braid structures needed by TQC.

Anyons are the key actors of TQC and TGD leads to detailed model of anyons as systems consisting of track of a periodically moving charged particle realized as a flux tube containing the particle inside it. This track would be a space-time correlate for the outcome of dissipative processes producing the asymptotic self-organization pattern. These tracks in general carry vacuum Kähler charge which is topologized when the CP_2 projection of space-time sheet is $D = 3$. This explains charge fractionization predicted to occur also for other charged particles. When a system approaches chaos periodic orbits become slightly aperiodic and the correlate is flux tube which rotates N times before closing. This gives rise to Z_N valued topological quantum number crucial for TQC using anyons ($N = 4$ holds true in this case). Non-Abelian anyons are needed by TQC, and the existence of long range classical electro-weak fields predicted by TGD is an essential prerequisite of non-Abelianity.

Negative energies and zero energy states are of crucial importance of TQC in TGD. The possibility of phase conjugation for fermions would resolve the puzzle of matter-antimatter asymmetry in an elegant manner. Anti-fermions would be present but have negative energies. Quite generally, it is possible to interpret scattering as a creation of pair of positive and negative energy states, the latter representing the final state. One can characterize precisely the deviations of this Eastern world view with respect to the Western world view assuming an objective reality with a positive definite energy and understand why the Western illusion apparently works. In the case of TQC the initial *resp.* final state of braided anyon system would correspond to positive *resp.* negative energy state.

The light-like boundaries of magnetic flux tubes are ideal for TQC. The point is that 3-dimensional light-like quantum states can be interpreted as representations for the time evolution of a two-dimensional system and thus represented self-reflective states being "about something". The light-likeness (no geometric time flow) is a space-time correlate for the ceasing of subjective time flow during macro-temporal quantum coherence. The S-matrices of TQC can be coded to these light-like states such that each elementary braid operation corresponds to positive energy anyons near the boundary of the magnetic flux tube A and negative energy anyons with opposite topological charges residing near the boundary of flux tube B and connected by braided threads representing the quantum gate. Light-like boundaries also force Chern-Simons action as the only possible general coordinate invariant action since the vanishing of the metric determinant does not allow any other candidate. Chern-Simons action indeed defines the modular functor for braid coding for a TQC program.

The comparison of the concrete model for TQC in terms of magnetic flux tubes with the structure of DNA gives tantalizing hints that DNA double strand is a topological quantum computer. Strand *resp.* conjugate strand would carry positive *resp.* negative energy anyon systems. The knotting and linking of DNA double strand would code for 2-gates realized as a unique maximally entangling Yang-Baxter matrix R for 2-state system. The pairs A-T, T-A, C-G, G-C in active state would code for the four braid operations of 3-braid group in 1-qubit Temperley Lieb representation associated with quantum group $SL(2)_q$. On basis of this picture one can identify N-O hydrogen bonds between DNA strands as structural correlates of 3-braids responsible for the nontrivial 1-gates whereas N-N hydrogen bonds would be correlates for the return gates acting as identity gates. Depending on whether the nucleotide is active or not it codes for nontrivial 1-gate or for identity gate so that DNA strand can program itself or be programmed dynamically.

The more recent work has demonstrated the the particular physical realization discussed in this chapter is only one possibily, and that braiding naturally generalizes to 2-braiding in TGD framework with braiding defined for string world sheets in 4-D space-time. Zero energy ontology allows also to understand why TQC programs - naturally identiable as biological programs - are selected as those associated with the maxima of Kähler function, which are now space-time surfaces rather than 3-surfaces.

DNA as Topological Quantum Computer

The chapter represents a vision about how DNA might act as a topological quantum computer). TQC means that the braidings of braid strands define TQC programs and M-matrix (generalization of S-matrix in zero energy ontology) defining the entanglement between states assignable to the end points of strands define the TQC usually coded as unitary time evolution for Schrödinger equation.

Before a representation of the model of TQC general vision about what happens in quantum jump, which at least in formal sense can be regarded as quantum computation (TQC), is represented. Included is also a section about possible modification of thermodynamics required by the possibility of negentropic entanglement. The modification corresponds simply to the replacement $S \to S - N$ for the entropy in standard thermodynamics. The implications of this replacement are however highly non-trivial. The "pessimistic" generalization of the second law allows to understand the thermodynamical aspect of TQC. One can understand why living matter is so effective entropy producer as compared to inanimate matter and also the characteristic decomposition of living systems to highly negentropic and entropic parts as a consequence of generalized second law. ADP-ATP process of metabolism provides a concrete application for the generalized thermodynamics and allows to see this process as a transfer of negentropic entanglement. Also DNA double strand for which sugar-phosphate backbone consists of XMPs, X= A,T,C,G containing negentropy carrying phosphate bonds can be seen as analogous to conscious brain with DNA strands representing right and left hemispheres.

One can end up to the model of TQC in the following manner.

1. Darwinian selection for which the standard theory of self-organization provides a model, should apply also to TQC programs. Tqc programs should correspond to asymptotic self-organization patterns selected by dissipation in the presence of metabolic energy feed. The spatial and temporal pattern of the metabolic energy feed characterizes the TQC program - or equivalently - sub-program call.

2. Since braiding characterizes the TQC program, the self-organization pattern should correspond to a hydrodynamical flow or a pattern of magnetic field inducing the braiding. Braid strands must correspond to magnetic flux tubes of the magnetic body of DNA. If each nucleotide is transversal magnetic dipole it gives rise to transversal flux tubes, which can also connect to the genome of another cell. As a matter fact, the flux tubes would correspond to what I call wormhole magnetic fields having pairs of space-time sheets carrying opposite magnetic fluxes.

3. The output of TQC sub-program is probability distribution for the outcomes of state function reduction so that the sub-program must be repeated very many times. It is represented as four-dimensional patterns for various rates (chemical rates, nerve pulse patterns, EEG power distributions,...) having also identification as temporal densities of zero energy states in various scales. By the fractality of TGD Universe there is a hierarchy of TQCs corresponding to p-adic and dark matter hierarchies. Programs (space-time sheets defining coherence regions) call programs in shorter scale. If the self-organizing system has a periodic behavior each TQC module defines a large number of almost copies of itself asymptotically. Generalized EEG could naturally define this periodic pattern and each period of EEG would correspond to an initiation and halting of TQC. This brings in mind the periodically occurring sol-gel phase transition inside cell near the cell membrane. There is also a connection with hologram idea: EEG rhythm corresponds to reference wave and nerve pulse patters to the wave carrying the information and interfering with the reference wave.

4. Fluid flow must induce the braiding which requires that the ends of braid strands must be anchored to the fluid flow. Recalling that lipid mono-layers of the cell membrane are liquid

crystals and lipids of interior mono-layer have hydrophilic ends pointing towards cell interior, it is easy to guess that DNA nucleotides are connected to lipids by magnetic flux tubes and hydrophilic lipid ends are stuck to the flow.

5. The topology of the braid traversing cell membrane cannot be affected by the hydrodynamical flow. Hence braid strands must be split during TQC. This also induces the desired magnetic isolation from the environment. Halting of TQC reconnects them and make possible the communication of the outcome of TQC.

There are several problems related to the details of the realization.

1. How nucleotides A,T,C,G are coded to the strand color and what this color corresponds to physically? There are two options which could be characterized as fermionic and bosonic.

   (a) Magnetic flux tubes having quark and anti-quark at their ends with u,d and u_c, d_c coding for A,G and T,C. CP conjugation would correspond to conjugation for DNA nucleotides.

   (b) Wormhole magnetic flux tubes having wormhole contact and its CP conjugate at its ends with wormhole contact carrying quark and anti-quark at its throats. The latter are predicted to appear in all length scales in TGD Universe.

2. How to split the braid strands in a controlled manner? High T_c super conductivity suggests a possible mechanism: braid strand can be split only if the supra current flowing through it vanishes. A suitable voltage pulse induces the supra-current and its negative cancels it. The conformation of the lipid could control whether it it can follow the flow or not. The absence of both genuine magnetic monopoles and boundaries however demands that the monopole flux tubes must be closed. One manner to achieve this is to assume that the magnetic flux returns back along second space-time sheet.

   A more realistic variant of this model is based on pairs of flux tubes going through the membrane and carrying opposite currents and parallel (opposite) magnetic fields. Reconnection for the members of the pair occurring the cell membrane effectively cuts both. This conforms with the identification of Cooper pairs as $S = 0$ or $S = 1$ states of electrons at the two flux tubes. The reconnection occurs naturally at the limit when the velocity of electrons and thus current goes to zero.

3. How magnetic flux tubes can be cut without breaking the conservation of the magnetic flux? The notion of wormhole magnetic field could save the situation now: after the splitting the flux returns back along the second space-time sheet of wormhole magnetic field. An alternative solution is based on reconnection of flux tubes. Since only flux tubes of same color can reconnect this process can induce transfer of color: "color inheritance": when applied at the level of amino-acids this leads to a successful model of protein folding. Reconnection makes possible breaking of flux tube connection for both the ordinary magnetic flux tubes and wormhole magnetic flux tubes.

4. How magnetic flux tubes are realized? The interpretation of flux tubes as correlates of directed attention at molecular level leads to concrete picture. Hydrogen bonds are by their asymmetry natural correlates for a directed attention at molecular level. Also flux tubes between acceptors of hydrogen bonds must be allowed and acceptors can be seen as the subjects of directed attention and donors as objects. Examples of acceptors are aromatic rings of nucleotides, $O =$ atoms of phosphates, etc.. A connection with metabolism is obtained if it is assumed that various phosphates XMP, XDP, XTP, $X = A, T, G, C$ act as fundamental acceptors and plugs in the connection lines. The basic metabolic process $ATP \rightarrow ADP + P_i$ allows an interpretation as a reconnection splitting flux tube connection, and the basic function of phosphorylating enzymes would be to build flux tube connections as also of breathing and photosynthesis.

The rest of the article represents a more concrete vision about how DNA might act as a topological quantum computer (TQC). The topics discussed are following.

1. How the basic gates are realized concretely? Gates can be identified as basic braid operations so that the question reduces to how braidings of magnetic flux tubes represent gates and what kind of particles represent the quantum states. The identification of the particles is in terms of quarks: TGD indeed predicts a hierarchy of scaled variants of hadron physics.

2. How the braiding is realized? What do braid strands identified as magnetic flux tubes look like? How the braiding operation is induced? The tentative answer is that color magnetic flux tubes connecting DNA nucleotides to the lipids of nuclear and cell membrane define braid strands and that braiding operations are induced by hydrodynamic flow around membrane generating 2-D flow of liquid crystal defined by the lipids. Also nerve pulse propagation can induced this kind of 2-D flow.

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The model is certainly very speculative and heavily relies on the new physics predicted by TGD. One can also imagine alternative scenarios. The model makes however strong predictions and is therefore testable.

1. The model makes several testable predictions about DNA itself. In particular, matter-antimatter asymmetry and slightly broken isospin symmetry have counterparts at DNA level induced from the breaking of these symmetries for quarks and antiquarks associated with the flux tubes. DNA cell membrane system is not the only possible system that could perform TQC like activities and store memories in braidings: flux tubes could connect biomolecules and the braiding could provide an almost definition for what it is to be living. Even water memory might reduce to braidings.

2. The model leads also to an improved understanding of other roles of the magnetic flux tubes containing dark matter. Phase transitions changing the value of Planck constant for the magnetic flux tubes could be key element of bio-catalysis and electromagnetic long distance communications in living matter. For instance, one ends up to what might be called code for protein folding and bio-catalysis. There is also a fascinating connection with Peter Gariaev's work suggesting that the phase transitions changing Planck constant have been observed and wormhole magnetic flux tubes containing dark matter have been photographed in his experiments.

3. In the proposed vision genes define the hardware and TQC programs the software responsible for what becomes cultural evolution at the higher levels of evolutionary hierarchy. This vision explains also the mystery of introns. The quite recent findings challenging genetic determinism expressed using the term "genetic dark matter" provide support for an existence of new information carrying level at the level of genome identifiable in terms of TQC programs.

It must be emphasized that this model of DNA as TQC is only one option among many. There is large flexibility concerning the identification of fermions involved. For instance A,T,C,G could be represented also in terms of 4 states assignable to two spin half fermions at parallel flux tubes. This would give rise to high T_c superconductor with both $S = 0$ ($S = 1$) Cooper pairs assignet to flux tubes with opposite (parallel) magnetic fields. The spin-spin interaction energy for the Cooper pair would be negative and proportional to h_{eff} and same for all fermion pairs if $h_{eff} = h_{gr}$ hypothesis holds true at microscopic level.

Quantum Gravitation and Topological Quantum Computation

In this article the connection of quantum gravitation, as it is understood in the TGD framework, with topological quantum computation (TQC) is considered. I sketched the first TGD based vision about DNA as a TQCer for about 13 years ago. In particular, a model of the system consisting of DNA and nuclear/cell membrane system acting as a TQCer was discussed.

TGD has evolved a lot after this and there are several motivations for seeing what comes out from combining the recent view about quantum TGD and TGD inspired quantum biology with this model.

1. There is a rather detailed view about the role of dark matter as phases of ordinary matter with the effective Planck constant $h_{eff} = nh_0$. Large values of h_{eff} allow to overcome the problems due to the loss of quantum coherence.

   This leads to the notion of the dark DNA (DDNA), whose codons are realized as dark proton triplets and proposed to accompany the ordinary DNA. Also dark photon triplets are predicted and one ends up to a model of communications and control based on dark cyclotron resonance in which codons serve as addresses and modulation of the signal frequency scale codes the signal to a sequence of pulses. Nerve pulses could be one application.

2. Quite recently, also the understanding of the possible role of quantum gravitation in biochemistry, metabolism, bio-catalysis, and in the function of DNA has considerably increased. The gravitational variants of hydrogen bonds and valence bonds between metal ions having very large value of $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is the gravitational Planck constant originally introduced by Nottale, are in a key role in the model and explain metabolic energy quantum as gravitational energy liberated when dark protons "drops" from a very long gravitational flux tube in the transition $h_{gr} \to h$. Also electronic metabolic energy quantum is predicted and there is empirical support for this.

3. A further motivation comes from the number theoretic vision of quantum TGD. Galois groups as symmetry groups represent new physics and the natural questions are whether Galois groups could give rise to number theoretic variants of anyons and what could the TGD counterparts of the condensed matter (effective) Majorana electrons proposed by Kitaev as anyon like states?

   The answer is that quantum superpositions of symmetric hydrogen bonded structures of form X..H-H+X-H...X are excellent candidates for the seats of dark ($h_{eff} > nh_0 > h$) bi-localized electrons defining TGD analogs of condensed matter Majorana electrons.

   The Galois groups permute the roots of a polynomial, which determines a space-time region by $M^8 - H$ duality. The roots correspond to mass squared values, in general algebraic numbers, and thus to mass hyperboloids in $M^4_c \subset M^8_c$. The H images correspond to 3-hyperboloids with a constant value $a = a_n$ of light-cone proper time. Therefore the Galois group can permute points with time-like separation. Note however that the real or rational parts of two values of a can be same.

   This looks very strange at first but actually confirms with the fact that time-like braidings defining TQC correspond in TGD time-like braidings (involving also reconnections) of string like objects defining string world sheets, which are not now time evolutions of space-like entities as physical state but correspond to time-like entities defining boundary data necessary for fixing holography completely. Their presence is forced by the small failure of the determinism of the action principle involved and is completely analogous to the non-determinism for soap films with frames serving as seats for the failure of determinism.

4. Braidings appear therefore at the level of fundamental TGD and correspond to string world sheets. They are possible only in 4-D space-time but not in string models.

   Also TQC-like processes appear automatically at the level of fundamental physics. In particular, the number theoretical state function reduction cascade for the Galois group following the time evolution induced by braiding can be regarded as a generalization of a decomposition of integers to primes: now primes are replaced by simple groups defining primes for finite groups. Nature is doing number theory!

5. Also zero energy ontology (ZEO) brings in new elements. The change of the arrow of time in "big" state function reductions (BSFRs) implies that dissipation with a reversed arrow of time provides an automatic error correction procedure. Also TQC in which the arrow of time varies for sub-modules, can be considered.

The Possible Role of Spin Glass Phase and P-Adic Thermodynamics in Topological Quantum Computation: the TGD View

Topological quantum computation (TQC) or more generally, a TQC-like process (to be referred as TQC), is one possible application of TGD. The latest article summarizes the recent number theoretic view about TQC in TGD inspired biology. There are several new physics elements involved. Mention only the notion of many-sheeted space-time involving the notions of electric and magnetic body; the new view about quantum theory relying on the M^8-H duality relating number theoretic and geometric views about physics and predicting the hierarchy of effective Planck constants assignable to a hierarchy of extensions of rationals; cognitive representations as unique discretization of space-time surface realizing generalized quantum computationalism; and zero energy ontology (ZEO) suggesting a new vision about quantum error correction. Quantum gravitation plays a key role in the proposal.

The engineering aspects of TQC were not discussed. The question that inspired this article was whether classical computation which relies strongly on non-equilibrium thermodynamics, could provide guidelines to end up with a more detailed view.

This led to a proposal in which p-adic thermodynamics assigned with the TGD based description of spin glasses would play a key role. TQC would involve quantum annealing in the spin glass energy landscape for the fermion states associated with flux tube structures. Anyons would be replaced with representations of the Galois group.

Physical states are however Galois singlets and many fermion states would involve entanglement between irreps of (relative) Galois group associated with spin *resp.* momentum degrees of freedom and give rise to a superposition of Galois singlets. The state function reduction ending TQC would project a tensor product of a given irrep from this superposition.

The entanglement between representations should be engineered in such a manner that the desired outcome of TQC would have the largest entanglement probability. p-Adic thermodynamics could give the entanglement probabilities. A connection with the travelling salesman problem emerges besides the connection with the factorization of the Galois group to prime factors appearing as relative Galois groups, which are simple (prime).

1.6.3 PART III: CATEGORIES, NUMBER THEORY AND CONSCIOUSNESS

Category Theory, Quantum TGD, and TGD Inspired Theory of Consciousness

Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. Category theory might provide the desired systematic approach to fuse together the bundles of general ideas related to the construction of quantum TGD proper. Category theory might also have natural applications in the general theory of consciousness and the theory of cognitive representations.

1. The ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences could be expressed elegantly using the language of the category theory. Quantum classical correspondence might allow a mathematical formulation in terms of structure respecting functors mapping the categories associated with the three kinds of existences to each other. Basic vision is following.

   (a) Self hierarchy would have a functorial map to the hierarchy of space-time sheets and also WCW spinor fields reflect it. Thus the self referentiality of conscious experience would have a functorial formulation (it is possible to be conscious about what one *was* conscious).

(b) The inherent logic for category defined by Heyting algebra must be modified in TGD context. Set theoretic inclusion would be replaced with the topological condensation, which can occur simultaneously to several space-time sheets.

(c) The category of light cones with inclusion as an arrow defining time ordering appears naturally in the construction of the WCW geometry and realizes the cosmologies within cosmologies scenario.

(d) In zero energy ontology (ZEO), which emerged many years after writing the first version of this chapter, causal diamonds (CDs) defined in terms of intersection of future and past directed light-cones form a category with arrow identified as inclusion.

(e) The preferred extremals would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of embedding space respects associativity (co-associativity). The duality would allow to construct new preferred extremals of Kähler action.

2. Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience.

3. Categories possess inherent generalized logic based on set theoretic inclusion which in TGD framework is naturally replaced with topological condensation: the outcome is quantum variants for the notions of sieve, topos, and logic. This suggests the possibility of geometrizing the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through three-valued logic. Also the right-wrong logic of moral rules and beautiful-ugly logic of aesthetics seem to be too naive and might be replaced with a more general quantum logic.

Infinite Primes and Consciousness

Infinite primes are besides p-adicization and the representation of space-time surface as an associative (co-associative) sub-manifold of hyper-octonionic space, basic pillars of the vision about TGD as a generalized number theory and will be discussed in the third part of the multi-chapter devoted to the attempt to articulate this vision as clearly as possible. Infinite primes generate wild philosophical speculations involved and the fate of speculations is usually sad. There are also amazing analogies with basic quantum physics, which make me to take infinite primes seriously.

1. Why infinite primes are unavoidable?

Suppose that 3-surfaces could be characterized by p-adic primes characterizing their effective p-adic topology. p-Adic unitarity implies that each quantum jump involves unitarity evolution U followed by a quantum jump. Simple arguments show that the p-adic prime characterizing the 3-surface representing the entire universe increases in a statistical sense. This leads to a peculiar paradox: if the number of quantum jumps already occurred is infinite, this prime is most naturally infinite. On the other hand, if one assumes that only finite number of quantum jumps have occurred, one encounters the problem of understanding why the initial quantum history was what it was. Furthermore, since the size of the 3-surface representing the entire Universe is infinite, p-adic length scale hypothesis suggest also that the p-adic prime associated with the entire universe is infinite.

These arguments motivate the attempt to construct a theory of infinite primes and to extend quantum TGD so that also infinite primes are possible. Rather surprisingly, one can construct what might be called generating infinite primes by repeating a procedure analogous to a quantization of a super symmetric quantum field theory. At given level of hierarchy one can identify the decomposition of space-time surface to p-adic regions with the corresponding decomposition of the infinite prime to primes at a lower level of infinity: at the basic level are finite primes for which one cannot find any formula.

2. Two views about the role of infinite primes and physics in TGD Universe

Two different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

1. The first view is based on the idea that infinite primes characterize quantum states of the entire Universe. 8-D hyper-octonions make this correspondence very concrete since 8-D hyper-octonions have interpretation as 8-momenta. By quantum-classical correspondence also the decomposition of space-time surfaces to p-adic space-time sheets should be coded by infinite hyper-octonionic primes. Infinite primes could even have a representation as hyper-quaternionic 4-surfaces of 8-D hyper-octonionic embedding space.

2. The second view is based on the idea that infinitely structured space-time points define space-time correlates of mathematical cognition. The mathematical analog of Brahman=Atman identity would however suggest that both views deserve to be taken seriously.

3. Infinite primes and infinite hierarchy of second quantizations

The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. Later it became clear that the process generalizes so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means an enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes. This hierarchy can be identified with the corresponding hierarchy of space-time sheets of the many-sheeted space-time.

One can even try to understand the quantum numbers of physical particles in terms of infinite primes. In particular, the hyper-quaternionic primes correspond four-momenta and mass squared is prime valued for them. The properties of 8-D hyper-octonionic primes motivate the attempt to identify the quantum numbers associated with CP_2 degrees of freedom in terms of these primes. The representations of color group $SU(3)$ are indeed labelled by two integers and the states inside given representation by color hyper-charge and iso-spin.

4. Infinite primes as a bridge between quantum and classical?

An important stimulus came from the observation stimulated by algebraic number theory. Infinite primes can be mapped to polynomial primes and this observation allows to identify completely generally the spectrum of infinite primes whereas hitherto it was possible to construct explicitly only what might be called generating infinite primes.

This in turn led to the idea that it might be possible represent infinite primes (integers) geometrically as surfaces defined by the polynomials associated with infinite primes (integers).

Obviously, infinite primes would serve as a bridge between Fock-space descriptions and geometric descriptions of physics: quantum and classical. Geometric objects could be seen as concrete representations of infinite numbers providing amplification of infinitesimals to macroscopic deformations of space-time surface. We see the infinitesimals as concrete geometric shapes!

5. Various equivalent characterizations of space-times as surfaces

One can imagine several number-theoretic characterizations of the space-time surface.

1. The approach based on octonions and quaternions suggests that space-time surfaces might correspond to associative or hyper-quaternionic surfaces of hyper-octonionic embedding space.

2. Space-time surfaces could be seen as absolute minima of the Kähler action. The challenge is to prove that this characterization is equivalent with the number theoretical dynamics,

6. The representation of infinite complex-octonionic primes as 4-surfaces

The difficulties caused by the Euclidian metric signature of the number theoretical norm forced to give up the idea that space-time surfaces could be regarded as quaternionic sub-manifolds of octonionic space, and to introduce complexified octonions and quaternions resulting by extending quaternionic and octonionic algebra by adding imaginary units multiplied with $\sqrt{-1}$. This spoils the number field property but the notion of prime is not lost. The sub-space of hyper-quaternions *resp.* -octonions is obtained from the algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$. The transition is the number theoretical counterpart for the transition from Riemannian to pseudo-Riemannin geometry performed already in Special Relativity.

The notions of hyper-quaternionic and octonionic manifold make sense but it is implausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Indeed, space-time surfaces can be assumed to be hyper-quaternionic or co-hyper-quaternionic 4-surfaces of 8-dimensional Minkowski space M^8 identifiable as the hyper-octonionic space HO. Since the hyper-quaternionic sub-spaces of HO with a locally fixed complex structure (preferred imaginary unit contained by tangent space at each point of HO) are labelled by CP_2, each (co)-hyper-quaternionic four-surface of HO defines a 4-surface of $M^4 \times CP_2$. One can say that the number-theoretic analog of spontaneous compactification occurs.

Any hyper-octonion analytic function $OH \rightarrow OH$ defines a function $g : OH \rightarrow SU(3)$ acting as the group of octonion automorphisms leaving a preferred imaginary unit invariant, and g in turn defines a foliation of OH and $H = M^4 \times CP_2$ by space-time surfaces. The selection can be local which means that G_2 appears as a local gauge group.

Since the notion of prime makes sense for the complexified octonions, it makes sense also for the hyper-octonions. It is possible to assign to infinite prime of this kind a hyper-octonion analytic polynomial $P : OH \rightarrow OH$ and hence also a foliation of OH and $H = M^4 \times CP_2$ by 4-surfaces. Therefore space-time surface can be seen as a geometric counterpart of a Fock state. The assignment is not unique but determined only up to an element of the local octonionic automorphism group G_2 acting in HO and fixing the local choices of the preferred imaginary unit of the hyper-octonionic tangent plane. In fact, a map $HO \rightarrow S^6$ characterizes the choice since $SO(6)$ acts effectively as a local gauge group.

The construction generalizes to all levels of the hierarchy of infinite primes and produces also representations for integers and rationals associated with hyper-octonionic numbers as space-time surfaces. A close relationship with algebraic geometry results and the polynomials define a natural hierarchical structure in the space of 3-surfaces. By the effective 2-dimensionality naturally associated with infinite primes represented by real polynomials 4-surfaces are determined by data given at partonic 2-surfaces defined by the intersections of 3-D and 7-D light-like causal determinants. In particular, the notions of genus and degree serve as classifiers of the algebraic geometry of the 4-surfaces. The great dream is to prove that this construction yields the solutions to the absolute minimization of Kähler action.

7. Generalization of ordinary number fields: infinite primes and cognition

The introduction of infinite primes, integers, and rationals leads also to a generalization of real numbers since an infinite algebra of real units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity pose technical problems. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum is replaced by multiplication meaning that the set of real units becomes infinitely degenerate.

Infinite primes form an infinite hierarchy so that the points of space-time and embedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point is even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units. For real physics this structure is completely invisible and is relevant only for the physics of cognition. One can say that Universe is an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

Part I

NEW PHYSICS AND MATHEMATICS INVOLVED WITH TGD

Chapter 2

Evolution of Ideas about Hyper-finite Factors in TGD

2.1 Introduction

This chapter has emerged from a splitting of a chapter devote to the possible role of von Neumann algebras known as hyper-finite factors in quantum TGD. Second chapter emerging from the splitting is a representation of basic notions to chapter "Was von Neumann right after all?" [K127] representing only very briefly ideas about application to quantum TGD only briefly.

In the sequel the ideas about TGD applications are reviewed more or less chronologically. A summary about evolution of ideas is in question, not a coherent final structure, and as always the first speculations - in this case roughly for a decade ago - might look rather weird. The vision has however gradually become more realistic looking as deeper physical understanding of factors has evolved slowly.

The mathematics involved is extremely difficult for a physicist like me, and to really learn it at the level of proofs one should reincarnate as a mathematician. Therefore the only practical approach relies on the use of physical intuition to see whether HFFs might the correct structure and what HFFs do mean. What is needed is a concretization of the extremely abstract mathematics involved: mathematics represents only the bones to which physics should add flesh.

2.1.1 Hyper-Finite Factors In Quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated

with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K104] . Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in M^8. Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the Kähler-Dirac gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality [K128, K34] this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II_1.

2.1.2 Hyper-Finite Factors And M-Matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

2.1.3 Connes Tensor Product As A Realization Of Finite Measurement Resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.

2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.

3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$ "averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

2.1.4 Concrete Realization Of The Inclusion Hierarchies

A concrete construction of M-matrix motivated by the recent rather precise view about basic variational principles of TGD allows to identify rather concretely the inclusions of HFFs in TGD framework and relate them to the hierarchies of broken conformal symmetries accompanying quantum criticalities.

1. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.

2. The formulation of scattering amplitudes in terms of Yangian of the super-symplectic algebra leads to a rather detailed view about scattering amplitudes [K116]. In this formulation scattering amplitudes are representations for sequences of algebraic operations connecting collections of elements of Yangian and sequences produce the same result. A huge generalization of the duality symmetry of the hadronic string models is in question.

3. The reduction of the hierarchy of Planck constants $h_{eff}/h = n$ to a hierarchy of quantum criticalities accompanied by a hierarchy of sub-algebras of super-symplectic algebra acting as conformal gauge symmetries leads to the identification of inclusions of HFFs as inclusions of WCW Clifford algebras characterizing by $n(i)$ and $n(i+1) = m(i) \times n(i)$ so that hierarchies of von Neuman algebras, of Planck constants, and of quantum criticalities would be very closely related. In the transition $n(i) \to n(i+1) = m(i) \times n(i)$ the measurement accuracy indeed increases since some conformal gauge degrees of freedom are transformed to physical ones. An open question is whether one could interpret $m(i)$ as the integer characterizing

inclusion: the problem is that also $m(i) = 2$ with $\mathcal{M} : \mathcal{N} = 4$ seems to be allowed whereas Jones inclusions allow only $m \geq 3$.

Even more, number theoretic universality and strong form of holography leads to a detailed vision about the construction of scattering amplitudes suggesting that the hierarchy of algebraic extensions of rationals relates to the above mentioned hierarchies.

2.1.5 Analogs of quantum matrix groups from finite measurement resolution?

The notion of quantum group replaces ordinary matrices with matrices with non-commutative elements. In TGD framework I have proposed that the notion should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution.

In this article I will consider the notion of quantum matrix inspired by recent view about quantum TGD and it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution. The basic idea is to replace complex matrix elements with operators expressible as products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers.

The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. The weak definition of determinant meaning its development with respect to a fixed row or column does not pose additional conditions. Strong definition of determinant requires its invariance under permutations of rows and columns. The permutation of rows/columns turns out to have interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

The resulting quantum matrices define a more general structure than quantum group but provide a concrete representation and interpretation for quantum group in terms of finite measurement resolution if q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by sign factor invariant under the permutations of both rows and columns. One could also understand the fractal structure of inclusion sequences of hyper-finite factors resulting by recursively replacing operators appearing as matrix elements with quantum matrices.

2.1.6 Quantum Spinors And Fuzzy Quantum Mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

This chapter represents a summary about the development of the ideas with last sections representing the recent latest about the realization and role of HFFs in TGD. I have saved the reader from those speculations that have turned out to reflect my own ignorance or are inconsistent with what I regarded established parts of quantum TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

2.2 A Vision About The Role Of HFFs In TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFs of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by ZEO and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

2.2.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A16] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.

2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0,1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.

3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.

4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X,μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.

2. A positive linear functional is weight with $\omega(1)$ finite.

3. A state is a weight with $\omega(1) = 1$.

4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.

5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0, 1, 2, ...$ For factors of type II_1 the values span the range $[0,1]$ and for factors of type II_∞ n the range $[0,\infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

2. Let

$$L^{\infty}(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \quad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{2.2.1}$$

   where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has ${\mathcal{M}_*}^* = \mathcal{M}$.

3. The conjugation $x \rightarrow x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \rightarrow L^2(\mathcal{M})$ via $x \rightarrow x\Omega$. The map $S_0; x\Omega \rightarrow x^*\Omega$ is however non-isometric.

4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A30, A60] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \rightarrow \sigma_t^{\omega} = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a way to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1 h_1(g_2), h_1 h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1 g_1(a_2), g_1 g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K127] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A1] and those of factors of type III by Alain Connes [A20] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the embedding.

The basic facts proved by Jones are following [A1] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a) \ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ , \\ &b) \ \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \end{aligned} \tag{2.2.2}$$

   the numbers at right hand side are known as Beraha numbers [A46] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B35] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed. The Dynkin graphs of Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. $E_6, E_7, and E_8$ correspond to symmetry groups of tetrahedron, octahedron/cube, and icosahedron/dodecahedron. The group for octahedron/cube is missing: what could this mean?

   For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of $SU(2)$ and the interpretation proposed in [A89] is following-

   The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: A_∞ corresponding to $SU(2)$ itself, $A_{-\infty,\infty}$ corresponding to circle group $U(1)$, and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection.

   One can construct also inclusions for which the diagrams corresponding to finite subgroups $G \subset SU(2)$ are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with $n = 6, 7, 8$ for tetrahedron, cube, dodecahedron. These extensions correspond to ADE type Kac-Moody algebras.

   The extension is constructed by constructing first factor R as infinite tensor power of $M_2(C)$ (complexified quaternions). Sub-factor R_0 consists elements of of R of form $Id \otimes x$. $SU(2)$ preserves R_0 and for any subgroup G of $SU(2)$ one can identify the inclusion $N \subset M$ in terms of $N = R_0^G$ and $M = R^G$, where $N = R_0^G$ and $M = R^G$ consists of fixed points of R_0 and R under the action of G. The principal graph for $N \subset M$ is the extended Coxeter-Dynk graph for the subgroup G.

   Physicist might try to interpret this by saying that one considers only sub-algebras R_0^G and R^G of observables invariant under G and obtains extended Dynkin diagram of G defining an

ADE type Kac-Moody algebra. Could the condition that Kac-Moody algebra elements with non-vanishing conformal weight annihilate the physical states state that the state is invariant under R_0 defining measurement resolution. Besides this the states are also invariant under finite group G? Could R_0^G and R^G correspond just to states which are also invariant under finite group G.

Connes tensor product

The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} \quad = \quad \mathcal{M}/\mathcal{N} \otimes \mathcal{N} \ . \tag{2.2.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M}\otimes\mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n \times n$ matrices acts on V from right, V can be regarded as a space formed by $m \times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n \times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m \times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.

2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn} \otimes e^{nr}$$

in the tensor product $V \otimes W$.

3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger \otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m = r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.

4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A75, A30, A60] . There are good arguments showing that in HFFs of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A60] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E \in \mathcal{M}(\mathcal{O})$ can be written as WW^* with $W \in \mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W = 1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.

2. Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.

3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ \ , \ \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ \ .$$

An infinite hierarchy of inclusions of HFFs of type III_1s is induced by set theoretic inclusions.

2.2.2 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. *Conceptual problems*

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?

2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and ZEO, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.

3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?

4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at embedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?

2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?

3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the Kähler-Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the Kähler-Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?

2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

ZEO and factors

The first question concerns the identification of the Hilbert space associated with the factors in ZEO. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.

2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

   It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in ZEO.

1. In ZEO M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFs is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.

2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \to 0$ limit. In quantum TGD the exponent of Kähler-Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Kähler-Dirac action can therefore be regarded as a "square root" of Kähler action.

3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in ZEO based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the Kähler-Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFs involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of Kähler-Dirac action [K128] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.

2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.

3. Quantum criticality means that Kähler-Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two super-conformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.

4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.

5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce

modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L_0^*$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K47] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

2.2.3 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

A proposal for M-matrix

The proposed general picture reduces the core of U-matrix to the construction of S-matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized

eigenvalue $p^k\gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-light geodesics of embedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.

4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K116].

   The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface the vertices would be represented by partonic 2-surfaces. In [K116] the interpretation of scattering ampiltudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are "free". At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

   The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K15] is a remnant of an "idea that came too early". The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond topartial wave in the space S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures **??**, **??** (`http://tgdtheory.fi/appfigures/elparticletgd.jpg` `http://tgdtheory.fi/appfigures/tgdgrpahs.jpg`) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K116] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

Quantum TGD as square root of thermodynamics

ZEO (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A75] [K127]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A75], [K127] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the Kähler-Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8 - M^4 \times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff} = n \times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes.

Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the embedding space [K47]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff} = n \times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. This implies that classical symplectic Noether charges vanish for this sub-algebra. The quantal conformal charges associated with induced spinor fields annihilate the physical states. Therefore it seems that the measured quantities are the symplectic charges and there is not need to introduce any measurement interaction term and the formalism simplifies dramatically.

The resolution increases with $h_{eff}/h = n$. Also the number of of strings connecting partonic 2-surfaces (in practice elementary particles and their dark counterparts plus bound states generated by connecting dark strings) characterizes physically the finite measurement resolution. Their presence is also visible in the geometry of the space-time surfaces through the conditions that induced W fields vanish at them (well-definedness of em charge), and by the condition that the canonical momentum currents for Kähler action define an integrable distribution of planes parallel to the string world sheet. In spirit with holography, preferred extremal is constructed by fixing string world sheets and partonic 2-surfaces and possibly also their light-like orbits (should one fix wormhole contacts is not quite clear). If the analog of AdS/CFT correspondence holds true, the value of Kähler function is expressible as the energy of string defined by area in the effective metric defined by the anti-commutators of K-D gamma matrices.

Super-symplectic algebra, whose charges are represented by Noether charges associated with strings connecting partonic 2-surfaces extends to a Yangian algebra with multi-stringy generators [K116]. The better the measurement resolution, the larger the maximal number of strings associated with the multilocal generator.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

The inclusions of super-symplectic Yangians form a hierarchy and would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra. As h_{eff} increases, infinite number of these gauge degrees of freedom become dynamical and measurement resolution is increased. This picture is definitely in conflict with the original view but the reduction of criticality in the increase of h_{eff} forces it.

Summary

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

2.2.4 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in ZEO

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.

2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.

3. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

4. ZEO gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish ZEO allows a symmetry breaking respecting a chosen Cartan algebra.

5. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFs as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states

modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.

2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.

3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \tag{2.2.4}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{2.2.5}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{2.2.6}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times Tr_N(SS^{\dagger}) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \ . \tag{2.2.7}$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of embedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of embedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

1. In ZEO the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.

2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.

3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.

4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A7] .

5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A7] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.

6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{2.2.8}$$

for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.

2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.

3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \quad (2.2.9)$$

for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the Kähler-Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain Kähler-Dirac gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A53] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor

takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.

2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.

3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

2.2.5 Questions about quantum measurement theory in Zero Energy Ontology

The following summary about quantum measurement theory in ZEO is somewhat out-of-date and somewhat sketchy. For more detailed view see [K72, K124, K9].

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \to P_-) = Tr[P_+M^\dagger P_-M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

Quantum measurements in ZEO

ZEO based quantum measurement theory leads directly to a theory of conscious entities. The basic idea is that state function reduction localizes the second boundary of CD so that it becomes a piece of light-cone boundary (more precisely $\delta M^4_\pm \times CP_2$).

Repeated reductions are possible as in standard quantum measurement theory and leave the passive boundary of CD. Repeated reduction begins with U process generating a superposition of CDs with the active boundary of CD being de-localized in the moduli space of CDs, and is followed by a localization in this moduli space so that single CD is the outcome. This process tends to increase the distance between the ends of the CD and has interpretation as a space-time correlate for the flow of subjective time.

Self as a conscious entity corresponds to this sequence of repeated reductions on passive boundary of CD. The first reduction at opposite boundary means death of self and its re-incarnation at the opposite boundary of CD. Also the increase of Planck constant and generation of negentropic entanglement is expected to be associated with this state function reduction.

Weak form of NMP is the most plausible variational principle to characterize the state function reduction. It does not require maximal negentropy gain for state function reductions but allows it. In other words, the outcome of reduction is n-dimensional eigen space of density matrix space but this space need not have maximum possible dimension and even 1-D ray is possible in which case the entanglement negentropy vanishes for the final state and system becomes isolated from the rest of the world. Weak form of NMP brings in free will and can allow also larger negentropy gain than the strong form if n is a product of primes. The beauty of this option is that one can understand how the generalization of p-adic length scale hypothesis emerges.

Hyper-finite factors of type II_1 and quantum measurement theory with a finite measurement resolution

The realization that the von Neumann algebra known as hyper-finite factor of type II_1 is tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

Hyper-finite factor of type II_1 has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the embedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the embedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in ZEO states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.

2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.

3. This leads also to the notion of quantum group. For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.

4. For HFFs the dimension of infinite-dimensional state space is finite and 1 by convention. For included HFF $\mathcal{N} \subset \mathcal{M}$ the dimension of the tensor factor space containing only the degrees of freedom which are above measurement resolution is given by the index of inclusion $d = \mathcal{M} : \mathcal{N}$. One can say that the dimension associated with degrees of freedom below measurement resolution is $D = 1/d$. This number is never large than 1 for the inclusions and contains a set of discrete values $d = 4cos^2(2\pi/n)$, $n \geq 3$, plus the continuum above it. The fractal generalization of the formula for entanglement entropy gives $S = -log(1/D) = -log(d) \leq 0$ so that one can say that the entanglement negentropy assignable to the projection operators to the sub-factor is positive except for $n = 3$ for which it vanishes. The non-measured degrees of freedom carry information rather than entropy.

5. Clearly both HFFs of type I and II allow entanglement negentropy and allow to assign it with finite measurement resolution. In the case of factors its is not clear whether the weak form of NMP allows makes sense.

As already explained, the topology of the many-sheeted space-time encourages the generalization of the notion of quantum entanglement in such a way that unentangled systems can possess entangled sub-systems. One can say that the entanglement between sub-selves is not visible in the resolution characterizing selves. This makes possible sharing and fusion of mental images central for TGD inspired theory of consciousness. These concepts find a deeper justification from the quantum measurement theory for hyper-finite factors of type II_1 for which the finite measurement resolution is basic notion.

Hierarchies of conformal symmetry breakings, Planck constants, and inclusions of HFFs

The basic almost prediction of TGD is a fractal hierarchy of breakings of symplectic symmetry as a gauge symmetry.

It is good to briefly summarize the basic facts about the symplectic algebra assigned with $\delta M^4_\pm \times CP_2$ first.

1. Symplectic algebra has the structure of Virasoro algebra with respect to the light-like radial coordinate r_M of the light-cone boundary taking the role of complex coordinate for ordinary conformal symmetry. The Hamiltonians generating symplectic symmetries can be chosen to be proportional to functions $f_n(r_M)$. What is the natural choice for $f_n(r_M)$ is not quite clear. Ordinary conformal invariance would suggests $f_n(r_M) = r_M^n$. A more adventurous possibility is that the algebra is generated by Hamiltonians with $f_n(r_M) = r^{-s}$, where s is a root of Riemann Zeta so that one has either $s = 1/2 + iy$ (roots at critical line) or $s = -2n$, $n > 0$ (roots at negative real axis).

2. The set of conformal weights would be linear space spanned by combinations of all roots with integer coefficients $s = n - iy$, $s = \sum n_i y_i$, $n > -n_0$, where $-n_0 \geq 0$ is negative conformal weight. Mass squared is proportional to the total conformal weight and must be real demanding $y = \sum y_i = 0$ for physical states: I call this conformal confinement analogous to color confinement. One could even consider introducing the analog of binding energy as "binding conformal weight".

   Mass squared must be also non-negative (no tachyons) giving $n_0 \geq 0$. The generating conformal weights however have negative real part -1/2 and are thus tachyonic. Rather remarkably, p-adic mass calculations force to assume negative half-integer valued ground state conformal weight. This plus the fact that the zeros of Riemann Zeta has been indeed assigned with critical systems forces to take the Riemannian variant of conformal weight spectrum with seriousness. The algebra allows also now infinite hierarchy of conformal sub-algebras with weights coming as n-ples of the conformal weights of the entire algebra.

3. The outcome would be an infinite number of hierarchies of symplectic conformal symmetry breakings. Only the generators of the sub-algebra of the symplectic algebra with radial conformal weight proportional to n would act as gauge symmetries at given level of the hierarchy. In the hierarchy n_i divides n_{i+1}. In the symmetry breaking $n_i \to n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Gauge degrees of freedom would transform to physical degrees of freedom.

4. What about the conformal Kac-Moody algebras associated with spinor modes. It seems that in this case one can assume that the conformal gauge symmetry is exact just as in string models.

The natural interpretation of the conformal hierarchies $n_i \to n_{i+1}$ would be in terms of increasing measurement resolution.

1. Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and correspond to generators with conformal weight proportional to n_i. Conformal hierarchies and associated hierarchies of Planck constants and n-fold coverings of space-time surface connecting the 3-surfaces at the ends of causal diamond would give a concrete realization of the inclusion hierarchies for hyper-finite factors of type II_1 [K127].

   n_i could correspond to the integer labelling Jones inclusions and associating with them the quantum group phase factor $U_n = exp(i2\pi/n)$, $n \geq 3$ and the index of inclusion given by $|M : N| = 4cos^2(2\pi/n)$ defining the fractal dimension assignable to the degrees of freedom above the measurement resolution. The sub-algebra with weights coming as n-multiples of the basic conformal weights would act as gauge symmetries realizing the idea that these degrees of freedom are below measurement resolution.

2. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view

of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

This interpretation might make sense for the symplectic algebra of $\delta M^4_\pm \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

3. If Kähler action has vanishing total variation under deformations defined by the broken conformal symmetries, the corresponding conformal charges are conserved. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N}=4$ symmetric gauge theories. The deformations defined by symplectic transformations acting gauge symmetries the second variation vanishes and there is not contribution to WCW Kähler metric.

4. One can interpret the situation also in terms of consciousness theory. The larger the value of h_{eff}, the lower the criticality, the more sensitive the measurement instrument since new degrees of freedom become physical, the better the resolution. In p-adic context large n means better resolution in angle degrees of freedom by introducing the phase $exp(i2\pi/n)$ to the algebraic extension and better cognitive resolution. Also the emergence of negentropic entanglement characterized by $n\times n$ unitary matrix with density matrix proportional to unit matrix means higher level conceptualization with more abstract concepts.

The extension of the super-conformal algebra to a larger Yangian algebra is highly suggestive and gives and additional aspect to the notion of measurement resolution.

1. Yangian would be generated from the algebra of super-conformal charges assigned with the points pairs belonging to two partonic 2-surfaces as stringy Noether charges assignable to strings connecting them. For super-conformal algebra associated with pair of partonic surface only single string associated with the partonic 2-surface. This measurement resolution is the almost the poorest possible (no strings at all would be no measurement resolution at all!).

2. Situation improves if one has a collection of strings connecting set of points of partonic 2-surface to other partonic 2-surface(s). This requires generalization of the super-conformal algebra in order to get the appropriate mathematics. Tensor powers of single string super-conformal charges spaces are obviously involved and the extended super-conformal generators must be multi-local and carry multi-stringy information about physics.

3. The generalization at the first step is simple and based on the idea that co-product is the "time inverse" of product assigning to single generator sum of tensor products of generators giving via commutator rise to the generator. The outcome would be expressible using the structure constants of the super-conformal algebra schematically a $Q^1_A = f_A^{BC} Q_B \otimes Q_C$. Here Q_B and Q_C are super-conformal charges associated with separate strings so that 2-local generators are obtained. One can iterate this construction and get a hierarchy of n-local generators involving products of n stringy super-conformal charges. The larger the value of n, the better the resolution, the more information is coded to the fermionic state about the partonic 2-surface and 3-surface. This affects the space-time surface and hence WCW metric but not the 3-surface so that the interpretation in terms of improved measurement resolution makes sense. This super-symplectic Yangian would be behind the quantum groups and Jones inclusions in TGD Universe.

4. n gives also the number of space-time sheets in the singular covering. One possible interpretation is in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

It is not an accident that quantum phases are assignable to Yangian algebras, to quantum groups, and to inclusions of HFFs. The new deep notion added to this existing complex of high level mathematical concepts are hierarchy of Planck constants, dark matter hierarchy, hierarchy of criticalities, and negentropic entanglement representing physical notions. All these aspects represent new physics.

2.2.6 Planar Algebras And Generalized Feynman Diagrams

Planar algebras [A10] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [A31] . In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [K27] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

1. One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.

2. One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.

3. One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.

4. It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

1. Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.

2. Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.

3. The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.

4. The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

5. One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

1. The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.

2. The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

   [Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].

3. Planar diagrams contain lines, which begin and return to the same disk boundary. Also “vacuum bubbles” are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.

4. Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).

5. There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of WCW (“world of classical worlds”) whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

2.2.7 Miscellaneous

The following considerations are somewhat out-of-date: hence the title “Miscellaneous”.

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A89] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A23] .

Fusion rules are indeed something more intricate that the naïve product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A28] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naïve tensor product with something more intricate. The naïvest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naïve an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K33] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A36] .

1. The light-like 3-surfaces X_l^3 defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K5] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K42] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A36] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say 3-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

2.3 Fresh View About Hyper-Finite Factors In TGD Framework

In the following I will discuss the basic ideas about the role of hyper-finite factors in TGD with the background given by a work of more than half decade. First I summarize the input ideas which I combine with the TGD inspired intuitive wisdom about HFFs of type II_1 and their inclusions allowing to represent finite measurement resolution and leading to notion of quantum spaces with algebraic number valued dimension defined by the index of the inclusion.

Also an argument suggesting that the inclusions define “skewed” inclusions of lattices to larger lattices giving rise to quasicrystals is proposed. The core of the argument is that the included HFF of type II_1 algebra is a projection of the including algebra to a subspace with dimension $D \leq 1$. The projection operator defines the analog of a projection of a bigger lattice to the included lattice. Also the fact that the dimension of the tensor product is product of dimensions of factors just like the number of elements in finite group is product of numbers of elements of coset space and subgroup, supports this interpretation.

One also ends up with a detailed identification of the hyper-finite factors in orbital degrees of freedom in terms of symplectic group associated with $\delta M^4_\pm \times CP_2$ and the group algebras of their discrete subgroups define what could be called “orbital degrees of freedom” for WCW spinor fields. By very general argument this group algebra is HFF of type II, maybe even II_1.

2.3.1 Crystals, Quasicrystals, Non-Commutativity And Inclusions Of Hyperfinite Factors Of Type II_1

I list first the basic ideas about non-commutative geometries and give simple argument suggesting that inclusions of HFFs correspond to "skewed" inclusions of lattices as quasicrystals.

1. Quasicrystals (see `http://tinyurl.com/67kz3qo`) (say Penrose tilings) [A13] can be regarded as subsets of real crystals and one can speak about "skewed" inclusion of real lattice to larger lattice as quasicrystal. What this means that included lattice is obtained by projecting the larger lattice to some lower-dimensional subspace of lattice.

2. The argument of Connes concerning definition of non-commutative geometry can be found in the book of Michel Lapidus at page 200. Quantum space is identified as a space of equivalence classes. One assigns to pairs of elements inside equivalence class matrix elements having the element pair as indices (one assumes numerable equivalence class). One considers irreducible representations of the algebra defined by the matrices and identifies the equivalent irreducible representations. If I have understood correctly, the equivalence classes of irreps define a discrete point set representing the equivalence class and it can often happen that there is just single point as one might expect. This I do not quite understand since it requires that all irreps are equivalent.

3. It seems that in the case of linear spaces - von Neumann algebras and accompanying Hilbert spaces - one obtains a connection with the inclusions of HFFs and corresponding quantum factor spaces that should exist as analogs of quantum plane. One replaces matrices with elements labelled by element pairs with linear operators in HFF of type II_1. Index pairs correspond to pairs in linear basis for the HFF or corresponding Hilbert space.

4. Discrete infinite enumerable basis for these operators as a linear space generates a lattice in summation. Inclusion $N \subset M$ defines inclusion of the lattice/crystal for N to the corresponding lattice of M. Physical intuition suggests that if this inclusion is "skewed" one obtains quasicrystal. The fact the index of the inclusion is algebraic number suggests that the coset space M/N is indeed analogous to quasicrystal.

   More precisely, the index of inclusion is defined for hyper-finite factors of type II_1 using the fact that quantum trace of unit matrix equals to unity $Tr(Id(M)) = 1$, and from the tensor product composition $M = (M/N) \times N$ given $Tr(Id(M)) = 1 = Ind(M/N)Tr(P(M \to N))$, where $P(M \to N$ is projection operator from M to N. Clearly, $Ind(M/N) = 1/Tr(P(M \to N))$ defines index as a dimension of quantum space M/N.

   For Jones inclusions characterized by quantum phases $q = exp(i2\pi/n)$, $n = 3, 4, ...$ the values of index are given by $Ind(M/N) = 4cos^2(\pi/n)$, $n = 3, 4, ...$ There is also another range inclusions $Ind(M/N) \geq 4$: note that $Tr(P(M \to N))$ defining the dimension of N as included sub-space is never larger than one for HFFs of type II_1. The projection operator $P(M \to N)$ is obviously the counterpart of the projector projecting lattice to some lower-dimensional sub-space of the lattice.

5. Jones inclusions are between linear spaces but there is a strong analogy with non-linear coset spaces since for the tensor product the dimension is product of dimensions and for discrete coset spaces G/H one has also the product formula $n(G) = n(H) \times n(G/H)$ for the numbers of elements. Noticing that space of quantum amplitudes in discrete space has dimension equal to the number of elements of the space, one could say that Jones inclusion represents quantized variant for classical inclusion raised from the level of discrete space to the level of space of quantum states with the number of elements of set replaced by dimension. In fact, group algebras of infinite and enumerable groups defined HFFs of type II under rather general conditions (see below).

   Could one generalize Jones inclusions so that they would apply to non-linear coset spaces analogs of the linear spaces involved ? For instance, could one think of infinite-dimensional groups G and H for which Lie-algebras defining their tangent spaces can be regarded as HFFs of type II_1? The dimension of the tangent space is dimension of the non-linear manifold: could this mean that the non-linear infinite-dimensional inclusions reduce to tangent space level and thus to the inclusions for Lie-algebras regarded hyper-finite factors of type II_1 or more generally, type II? This would would rise to quantum spaces which have finite but algebraic valued quantum dimension and in TGD framework take into account the finite measurement resolution.

6. To concretize this analogy one can check what is the number of points map from 5-D space containing aperiodic lattice as a projection to a 2-D irrational plane containing only origin as common point with the 5-D lattice. It is easy to get convinced that the projection is 1-to-1 so that the number of points projected to a given point is 1. By the analogy with Jones inclusions this would mean that the included space has same von Neumann dimension 1 - just like the including one. In this case quantum phase equals $q = exp(i2\pi/n)$, $n = 3$ - the lowest possible value of n. Could one imagine the analogs of $n > 3$ inclusions for which the number of points projected to a given point would be larger than 1? In 1-D case the rational lines $y = (k/l)x$ define 1-D rational analogs of quasi crystals. The points $(x, y) = (m, n)$, $m\ mod\ l = 0$ are projected to the same point. The number of points is now infinite and the ratio of points of 2-D lattice and 1-D crystal like structure equals to l and serves as the analog for the quantum dimension $d_q = 4cos^2(\pi/n)$.

To sum up, this this is just physicist's intuition: it could be wrong or something totally trivial from the point of view of mathematician. The main message is that the inclusions of HFFs might define also inclusions of lattices as quasicrystals.

2.3.2 HFFs And Their Inclusions In TGD Framework

In TGD framework the inclusions of HFFs have interpretation in terms of finite measurement resolution. If the inclusions define quasicrystals then finite measurement resolution would lead to quasicrystals.

1. The automorphic action of N in $M \supset N$ and in associated Hilbert space H_M where N acts generates physical operators and accompanying stas (operator rays and rays) not distinguishable from the original one. States in finite measurement resolution correspond to N-rays rather than complex rays. It might be natural to restrict to unitary elements of N.

   This leads to the need to construct the counterpart of coset space M/N and corresponding linear space H_M/H_N. Physical intuition tells that the indices of inclusions defining the "dimension" of M/N are algebraic numbers given by Jones index formula.

2. Here the above argument would assign to the inclusions also inclusions of lattices as quasicrystals.

Degrees of freedom for WCW spinor field

Consider first the identification of various kinds of degrees of freedom in TGD Universe.

1. Very roughly, WCW ("world of classical worlds") spinor is a state generated by fermionic creation operators from vacuum at given 3-surface. WCW spinor field assigns this kind of spinor to each 3-surface. WCW spinor fields decompose to tensor product of spin part (Fock state) and orbital part ("wave" in WCW) just as ordinary spinor fields.

2. The conjecture motivated by super-symmetry has been that both WCW spinors and their orbital parts (analogs of scalar field) define HFFs of type II_1 in quantum fluctuating degrees of freedom.

3. Besides these there are zero modes, which by definition do not contribute to WCW Kähler metric.

   (a) If the zero zero modes are symplectic invariants, they appear only in conformal factor of WCW metric. Symplectically invariant zero modes represent purely classical degrees of freedom - direction of a pointer of measurement apparatus in quantum measurement - and in given experimental arrangement they entangle with quantum fluctuating degrees of freedom in one-one manner so that state function reduction assigns to the outcome of state function reduction position of pointer. I forget symplectically invariant zero modes and other analogous variables in the following and concentrate to the degrees of freedom contributing WCW line-element.

(b) There are also zero modes which are not symplectic invariants and are analogous to degrees of freedom generated by the generators of Kac-Moody algebra having vanishing conformal weight. They represent "center of mass degrees of freedom" and this part of symmetric algebra creates the representations representing the ground states of Kac-Moody representations. Restriction to these degrees of freedom gives QFT limit in string theory. In the following I will speak about "cm degrees of freedom".

The general vision about symplectic degrees of freedom (the analog of "orbital degrees of freedom" for ordinary spinor field) is following.

1. WCW (assignable to given CD) is a union over the sub-WCWs labeled by zero modes and each sub-WCW representing quantum fluctuating degrees of freedom and "cm degrees of freedom" is infinite-D symmetric space. If symplectic group assignable to $\delta M^4_+ \times CP_2$ acts as as isometries of WCW then "orbital degrees of freedom" are parametrized by the symplectic group or its coset space (note that light-cone boundary is 3-D but radial dimension is light-like so that symplectic - or rather contact structure - exists).

   Let S^2 be $r_M = constant$ sphere at light-cone boundary (r_M is the radial light-like coordinate fixed apart from Lorentz transformation). The full symplectic group would act as isometries of WCW but does not - nor cannot do so - act as symmetries of Kähler action except in the huge vacuum sector of the theory correspond to vacuum extremals.

2. WCW Hamiltonians can be deduced as "fluxes" of the Hamiltonians of $\delta M^4_+ \times CP_2$ taken over partonic 2-surfaces. These Hamiltonians expressed as products of Hamiltonians of S^2 and CP_2 multiplied by powers r_M^n. Note that r_M plays the role of the complex coordinate z for Kac-Moody algebras and the group G defining KM is replaced with symplectic group of $S^2 \times CP_2$. Hamiltonians can be assumed to have well-defined spin (SO(3)) and color (SU(3)) quantum numbers.

3. The generators with vanishing radial conformal weight ($n = 0$) correspond to the symplectic group of $S^2 \times CP_2$. They are not symplectic invariants but are zero modes. They would correspond to "cm degrees of freedom" characterizing the ground states of representations of the full symplectic group.

Discretization at the level of WCW

The general vision about finite measurement resolution implies discretization at the level of WCW.

1. Finite measurement resolution at the level of WCW means discretization. Therefore the symplectic groups of $\delta M^4_+ \times CP_2$ *resp.* $S^2 \times CP_2$ are replaced by an enumerable discrete subgroup. WCW is discretized in both quantum fluctuating degrees of freedom and "center of mass" degrees of freedom.

2. The elements of the group algebras of these discrete groups define the "orbitals parts" of WCW spinor fields in discretization. I will later develop an argument stating that they are HFFs of type II - maybe even II_1. Note that also function spaces associated with the coset spaces of these discrete subgroups could be considered.

3. Discretization applies also in the spin degrees of freedom. Since fermionic Fock basis generates quantum counterpart of Boolean algebra the interpretation in terms of the physical correlates of Boolean cognition is motivated (fermion number 1/0 and various spins in decomposition to a tensor product of lower-dimensional spinors represent bits). Note that in ZEO fermion number conservation does not pose problems and zero states actually define what might be regarded as quantum counterparts of Boolean rules $A \rightarrow B$.

4. Note that 3-surfaces correspond by the strong form of GCI/holography to collections of partonic 2-surfaces and string world sheets of space-time surface intersecting at discrete set of points carrying fermionic quantum numbers. WCW spinors are constructed from second quantized induced spinor fields and fermionic Fock algebra generates HFF of type II_1.

Does WCW spinor field decompose to a tensor product of two HFFs of type II_1?

The group algebras associated with infinite discrete subgroups of the symplectic group define the discretized analogs of waves in WCW having quantum fluctuating part and cm part. The proposal is that these group algebras are HFFs of type II_1. The spinorial degrees of freedom correspond to fermionic Fock space and this is known to be HFF. Therefore WCW spinor fields would defined tensor product of HFFs of type II_1. The interpretation would be in terms of supersymmetry at the level of WCW. Super-conformal symmetry is indeed the basic symmetry of TGD so that this result is a physical "must". The argument goes as follows.

1. In non-zero modes WCW is symplectic group of $\delta M^4_+ \times CP_2$ (call this group just $Sympl$) reduces to the analog of Kac-Moody group associated with $S^2 \times CP_2$, where S^2 is $r_M =$ constant sphere of light-cone boundary and z is replaced with radial coordinate. The Hamiltonians, which do not depend on r_M would correspond to zero modes and one could not assign metric to them although symplectic structure is possible. In "cm degrees of freedom" one has symplectic group associated with $S^2 \times CP_2$.

2. Finite measurement resolution, which seems to be coded already in the structure of the preferred extremals and of the solutions of the Kähler-Dirac equation, suggests strongly that this symplectic group is replaced by its discrete subgroup or symmetric coset space. What this group is, depends on measurement resolution defined by the cutoffs inherent to the solutions. These subgroups and coset spaces would define the analogs of Platonic solids in WCW!

3. Why the discrete infinite subgroups of $Sympl$ would lead naturally to HFFs of type II? There is a very general result stating that group algebra of an enumerable discrete group, which has infinite conjugacy classes, and is amenable so that its regular representation in group algebra decomposes to all unitary irreducibles is HFF of type II. See for examples about HFFs of type II listed in Wikipedia article (see `http://tinyurl.com/y8445w8q`) [A6].

4. Suppose that the group algebras associated the discrete subgroups $Sympl$ are indeed HFFs of type II or even type II_1. Their inclusions would define finite measurement resolution the orbital degrees of freedom for WCW spinor fields. Included algebra would create rays of state space not distinguishable experimentally. The inclusion would be characterized by the inclusion of the lattice defined by the generators of included algebra by linearity. One would have inclusion of this lattice to a lattice associated with a larger discrete group. Inclusions of lattices are however known to give rise to quasicrystals (Penrose tilings are basic example), which define basic non-commutative structures. This is indeed what one expects since the dimension of the coset space defined by inclusion is algebraic number rather than integer.

5. Also in fermionic degrees of freedom finite measurement resolution would be realized in terms of inclusions of HFFs- now certainly of type II_1. Therefore one could obtain hierarchies of lattices included as quasicrystals.

What about zero modes which are symplectic invariants and define classical variables? They are certainly discretized too. One might hope that one-one correlation between zero modes (classical variables) and quantum fluctuating degrees of freedom suggested by quantum measurement theory allows to effectively eliminate them. Besides zero modes there are also modular degrees of freedom associated with partonic 2-surfaces defining together with their 4-D tangent space data basis objects by strong form of holography. Also these degrees of freedom are automatically discretized. But could one consider finite measurement resolution also in these degrees of freedom. If the symplectic group of $S^2 \times CP_2$ defines zero modes then one could apply similar argument also in these degrees of freedom to discrete subgroups of $S^2 \times CP_2$.

2.3.3 Little Appendix: Comparison Of WCW Spinor Fields With Ordinary Second Quantized Spinor Fields

In TGD one identifies states of Hilbert space as WCW spinor fields. The analogy with ordinary spinor field helps to understand what they are. I try to explain by comparison with QFT.

Ordinary second quantized spinor fields

Consider first ordinary fermionic QFT in fixed space-time. Ordinary spinor is attached to an space-time point and there is $2^{D/2}$ dimensional space of spin degrees of freedom. Spinor field attaches spinor to every point of space-time in a continuous/smooth manner. Spinor fields satisfying Dirac equation define in Euclidian metric a Hilbert space with a unitary inner product. In Minkowskian case this does not work and one must introduce second quantization and Fock space to get a unitary inner product. This brings in what is essentially a basic realization of HFF of type II_1 as allowed operators acting in this Fock space. It is operator algebra rather than state space which is HFF of type II_1 but they are of course closely related.

Classical WCW spinor fields as quantum states

What happens TGD where one has quantum superpositions of 4-surface/3-surfaces by GCI/partonic 2-surfaces with 4-D tangent space data by strong form of GCI.

1. First guess: space-time point is replaced with 3-surface. Point like particle becomes 3-surface representing particle. WCW spinors are fermionic Fock states at this surface. WCW spinor fields are Fock state as a functional of 3-surface. Inner product decomposes to Fock space inner product plus functional integral over 3-surfaces (no path integral!). One could speak of quantum multiverse. Not single space-time but quantum superposition of them. This quantum multiverse character is something new as compared to QFT.

2. Second guess: forced by ZEO, by geometrization of Feynman diagrams, etc.

   (a) 3-surfaces are actually not connected 3-surfaces. They are collections of components at both ends of CD and connected to single connected structure by 4-surface. Components of 3-surface are like incoming and outgoing particles in connected Feynman diagrams. Lines are identified as regions of Euclidian signature or equivalently as the 3-D light-like boundaries between Minkowskian and Euclidian signature of the induced metric.

   (b) Spinors(!!) are defined now by the fermionic Fock space of second quantized induced spinor fields at these 3-surfaced and by holography at 4-surface. This fermionic Fock space is assigned to all multicomponent 3-surfaces defined in this manner and WCW spinor fields are defined as in the first guess. This brings integration over WCW to the inner product.

3. Third, even more improved guess: motivated by the solution ansatz for preferred extremals and for Kähler-Dirac equation [K128] giving a connection with string models.

   The general solution ansatz restricts all spinor components but right-handed neutrino to string world sheets and partonic 2-surfaces: this means effective 2-dimensionality. String world sheets and partonic 2-surfaces intersect at the common ends of light-like and space-like braids at ends of CD and at along wormhole throat orbits so that effectively discretization occurs. This fermionic Fock space replaces the Fock space of ordinary second quantization.

2.4 The idea of Connes about inherent time evolution of certain algebraic structures from TGD point of view

Jonathan Disckau asked me about what I think about the proposal of Connes represented in the summary of progress of noncommutative geometry in "Noncommutative Geometry Year 2000" [A21] (see `https://arxiv.org/abs/math/0011193`) that certain mathematical structures have inherent time evolution coded into their structure.

I have written years ago about Connes's proposal. At that time I was trying to figure out how to understand the construction of scattering amplitudes in the TGD framework and the proposal of Connes looked attractive. Later I had to give up this idea. However, the basic idea is beautiful. One should only replace the notion of time evolution from a one-parameter group of automorphisms to something more interesting. Also time evolution as increasing algebraic complexity is a more attractive interpretation.

The inclusion hierarchies of hyperfinite factors (HFFs) - closely related to the work of Connes - are a key element of TGD and crucial for understanding evolutionary hierarchies in TGD. Is it possible that mathematical structure evolves in time in some sense? The TGD based answer is that quantum jump as a fundamental evolutionary step - moment of subjective time evolution - is a necessary new element. The sequence of moments of consciousness as quantum jumps would have an interpretation as hopping around in the space of mathematical structures leading to increasingly complex structures.

The generalization of the idea of Connes is discussed in this framework. In particular, the inclusion hierarchies of hyper-finite factors, the extension hierarchies of rationals, and fractal inclusion hierarchies of subalgebras of supersymplectic algebra isomorphic with the entire algebra are proposed to be more or less one and the same thing in TGD framework.

The time evolution operator of Connes could corresponds to super-symplectic algebra (SSA) to the time evolution generated by $exp(iL_0\tau)$ so that the operator Δ of Connes would be identified as $\Delta = exp(L_0)$. This identification allows number theoretical universality if τ is quantized. Furthermore, one ends up with a model for the subjective time evolution by small state function reductions (SSFRs) for SSA with SSA$_n$ gauge conditions: the unitary time evolution for given SSFR would be generated by a linear combination of Virasoro generators not annihilating the states. This model would generalize the model for harmonic oscillator in external force allowing exact S-matrix.

2.4.1 Connes proposal and TGD

In this section I develop in more detail the analog of Connes proposal in TGD framework.

What does Connes suggest?

One must first make clear what the automorphism of HFFs discovered by Connes is.

1. Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. I have described the theory earlier [K76, K48].

First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \quad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{2.4.1}$$

   where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has ${\mathcal{M}_*}^* = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.

4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.

5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ \ , J\mathcal{M}J = \mathcal{M}' \ \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A30, A60] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.

3. $\omega \rightarrow \sigma_t^{\omega} = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

The definition of Δ^{it} reduces in eigenstate basis of Δ to the definition of complex function d^{it}. Note that is positive so that the logarithm of d is real.

In TGD framework number theoretic universality poses additional conditions. In diagonal basis $e^{log(d)it}$ must exist. A simply manner to solve the conditions is $e = exp(m/r)$ existing p-adically for an extension of rational allowing r:th root of e. This requires also quantization of as a root of unity so that the exponent reduces to a root of unity.

2. Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.

2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists of real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.

3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

3. Objections against the idea of Connes

One can represent objections against this idea.

1. Ordinary time evolution in wave mechanics is a unitary automorphism, so that in this framework they would not have physical meaning but act as gauge transformations. If outer automorphisms define time evolutions, they must act as gauge transformations. One would have an analog of gauge field theory in HFF. This would be of course highly interesting: when I gave up the idea of Connes, I did not consider this possibility. Super-symplectic algebras having fractal structure are however extremely natural candidate for defining HFF and there is infinite number of gauge conditions.

2. An automorphism is indeed in question so that the algebraic system would not be actually affected. Therefore one cannot say that HFF has inherent time evolution and time. However, one can represent in HFF dynamical systems obeying this inherent time evolution. This possibility is highly interesting as a kind of universal gauge theory.

   On the other hand, outer automorphisms affect the trace of the projector defining the identity matrix for a given factor. Does the scaling factor Λ represent some kind of renormalization operation? Could it relate to the action of scalings in the TGD framework where scalings replace time translations at the fundamental level? What the number theoretic vision of TGD could mean? Could this quantize the continuous spectrum of the scalings Λ for HFFs so that they belong to the extension? Could one have a spectrum of Λ for each extension of rationals? Are different extensions related by inclusions of HFFs?

3. The notion of time evolution itself is an essentially Newtonian concept: selecting a preferred time coordinate breaks Lorentz invariance. In TGD however time coordinate is replace by scaling parameter and the situation changes.

4. The proposal of Connes is not general enough if evolution is interpreted as an increase of complexity.

For these reasons I gave up the automorphism proposed by Connes as a candidate for defining time evolution giving rise to scattering amplitudes in TGD framework.

Two views about TGD

The two dual views about what TGD is described briefly in [L81].

1. Physics as geometry of the world of "world of classical worlds" (WCW) identified as the space of space-time surfaces in $M^4 \times CP_2$ [K98]. Twistor lift of TGD [L41] implies that the space-time surfaces are minimal surfaces which can be also regarded as extermals of the Kähler action. This implies holography required by the general coordinate invariance in TGD framework.

2. TGD as generalized number theory forcing to generalize physics to adelic physics [L39] fusing real physics as correlate of sensory experience and various p-adic physics as correlates of cognition. Now space-times are naturally co-associative surfaces in complexified M^8 (complexified octonions) defined as "roots" of octonionic polynomials determined by polynomials with rational coefficients [L73, L74, L90]. Now holography extends dramatically: finite number of rational numbers/roots of rational polynomial/points of space-time region dictate it.

$M^8 - H$ duality relates these two views and is actually a generalization of Fourier transform and realizes generalization of momentum-position duality.

The notion of time evolution in TGD

Concerning various time evolutions in TGD, the general situation is now rather well understood.

There are two quantal time evolutions: geometric one assignable to single CD and and subjective time evolution which reflects the generalization of point-like particle to a 3-surface and the introduction of CD as 4-D perceptive field of particle in ZEO [L64].

1. Geometric time evolution corresponds to the standard scattering amplitudes for which I have a general formula now in terms of zero energy ontology (ZEO) [L84, L73, L74, L90]. The analog of S-matrix corresponds to entanglement coefficients between members of zero energy state at opposite boundaries of causal diamond (CD).

2. Subjective time evolution of conscious entity corresponds to a sequence of "small" state function reductions (SSFRs) as moments of consciousness: each SSFR is preceded by an analog of unitary time evolution, call it U. SSFRs are the TGD counterparts of "weak" measurements.

   $U(t)$ is generated by the scaling generator L_0 scaling light-like radial coordinate of light-cone boundary and is a generalization of corresponding operator in superconformal and string theories and defined for super-symplectic algebras acting as isometries of the world of classical worlds (WCW) [L90]. $U(t)$ is not the exponential of energy as a generator of time translation as in QFTs but an exponential of the mass squared operator and corresponds to the scaling of radial light-like coordinate r of the light-like boundary of CD: r is analogous to the complex coordinate z in conformal field theories.

   Also "big" SFRs (BSFRs) are possible and correspond to "ordinary" SFRs and in TGD framework mean death of self in the universal sense and followed by reincarnation as time reversed subjective time evolution [L54].

3. There is also classical time evolution at the level of space-time surfaces. Here the assumption that X^4 belongs to $H = M^4 \times CP_2$ defines Minkowski coordinates of M^4 as almost unique space-time coordinates of X^4 is the M^4 projection of X^4 is 4-D. This generalizes also to the case of M^8. Symmetries make it possible to identify an essentially a unique time coordinate.

   This means enormous simplification. General coordinate invariance is a marvellous symmetry but it leads to the problem of specifying space-time coordinates that is finding preferred coordinates. This seems impossible since 3-metric is dynamical. M^4 provides a fixed reference system and the problem disappears. M^4 is dynamical by its Minkowskian signature and one can speak about classical signals.

4. There is also classical time evolution for the induced spinor fields. At the level of H the spinor field is a superposition of modes of the massless Dirac operator (massless in 8-D sense). This spinor field is free and second quantized. Second quantization of induced spinor trivializes and this is absolutely crucial for obtaining scattering amplitudes for fermions and avoiding the usual problems for quantization of fermions in curved background.

   The induced spinor field is a restriction of this spinor field to the space-time surface and satisfies modified Dirac equation automatically. There is no need for second quantization at the level of space-time surface and propagators etc.... are directly calculable. This is an enormous simplification.

There are therefore as many as 4 time evolutions and subjective time evolution by BSFRs and possibly also by SSFRs is a natural candidate for time evolution as genuine evolution as emergence of more complex algebraic structures.

Could the inherent time evolution of HFF have a physical meaning in TGD after all?

The idea about inherent time evolution defined by HFF itself as one parameter group of outer automorphisms is very attractive by its universality: physics would become part of mathematics.

1. Thermodynamic interpretation, with inverse temperature identified as an analog of time coordinate, comes first in mind but need not be the correct interpretation.

2. Outer automorphisms should act at a very fundamental level analogous to the state space of topological field theories. Fundamental group is after all in question! The assignment of the S-matrix of particle physics to the outer automorphism does not look reasonable since the time evolution would be with respect to the linear Minkowski coordinate, which is not Lorentz invariant.

For these reasons I gave up the idea of Connes when considering it for the first time. However, TGD inspired theory of consciousness as a generalization of quantum measurement theory has evolved since then and the situation is different now.

The sequence of SSFRs defines subjective time evolution having no counterpart in QFTs. Each SSFR is preceded by a unitary time evolution, which however corresponds to the scaling of the light-like radial coordinate of the light-cone boundary [L90] rather than time translation. Hamiltonian is replaced with the scaling generator L_0 acting as Lorentz invariant mass squared operator so that Lorentz invariance is not lost.

Could the time evolution assignable to L_0 correspond to the outer automorphism of Connes when one poses an infinite number of gauge conditions making inner automorphisms gauge transformations? The connection of Connes proposal with conformal field theories and with TGD is indeed suggestive.

1. Conformally invariant systems obey infinite number of gauge conditions stating that the conformal generators L_n, $n > 0$, annihilate physical states and carry vanishing Noether charges.

   These gauge conditions bring in mind the condition that infinitesimal inner automorphisms do not change the system physically. Does this mean that Connes outer automorphism generates the time evolution and inner automorphisms act as gauge symmetries? One would have an analog of gauge field theory in HFF.

2. In TGD framework one has an infinite hierarchy of systems satisfying conditions analogous to the conformal gauge conditions. The generators of the super-symplectic algebra (SCA) acting as isometries of the "world of classical worlds" (WCW) are labelled by non-negative conformal weight n and it has infinite hierarchy of algebras SCA$_k$ isomorphic to it with conformal weights given by k-multiple of those of the entire algebra, $k = 1, 2, ...$

   Gauge conditions state for SCA_k that the generators of SCA_k and its commutator with SCA annihilate physical states. The interpretation is in terms of a hierarchy of improving measurement resolutions with degrees of freedom below measurement resolution acting like gauge transformations.

   The Connes automorphism would "see" only the time evolution in the degrees of freedom above measurement resolution and as k increases, their number would increase.

In the case of hyperfinite factors of type II_1 (HFFs) the fundamental group of corresponding factor II_∞ consists of all reals: I hope I am right here.

1. The hyperfinite factors of type II_1 and corresponding factors II_∞ are natural in the TGD context. Therefore the spectrum would consist of reals unless one poses additional conditions.

2. Could the automorphisms correspond to the scalings of the lightcone proper time, which replace time translations as fundamental dynamics. Also in string models scalings take the role of time translations.

3. In zero energy ontology (ZEO) the scalings would act in the moduli space of causal diamonds which is finite-dimensional. This moduli space defines the backbone of the "world of classical worlds". WCW itself consists of a union of sub-WCs as bundle structures over CDs [?]. The fiber consists of space-time surfaces inside a given CD analogous to Bohr orbits and satisfying holography reducing to generalized holomorphy. The scalings as automorphisms scale the causal diamonds. The space of CDs is a finite-dimensional coset space and has also other symmetry transformations.

4. The number theoretic vision suggests a quantization of the spectrum of Λ so that for a given extension of rationals the spectrum would belong to the extension. HFFs would be labelled at least partially by the extensions of rationals. The recent view of $M^8 - H$ duality [L121] is dramatically simpler than the earlier view [L73, L74, **?**] and predicts that the space-time regions are determined by a pair of analytic functions with rational coefficients forced by number theoretical universality meaning that the space-time surfaces have interpretation also as p-adic surfaces.

   The simplest analytic functions are polynomials with integer coefficients and if one requires that the coefficients are smaller than the degree of the polynomial, the number of polynomials is finite for a given degree. This would give very precise meaning for the concept of number theoretic evolution.

   There would be an evolutionary hierarchy of pairs of polynomials characterized by increasing complexity and one can assign to these polynomials extension of rationals characterized by ramified primes depending on the polynomials. The ramified primes would have interpretation as p-adic primes characterizing the space-time region considered. Extensions of rationals and ramified primes could also characterize HFFs. This is a rather dramatic conjecture at the level of pure mathematics.

5. Scalings define renormalization group in standard physics. Now they scale the size of the CD. Could the scalings as automorphisms of HFFs correspond to discrete renormalization operations?

Three views about finite measurement resolution

Evolution could be seen physically as improving finite measurement resolution: this applies to both sensory experience and cognition. There are 3 views about finite measurement resolution (FMR) in TGD.

1. Hyper finite factors (HFFs) and FMR

HFFs are an essential part of Connes's work and I encountered them about 15 years ago or so [K127, K48].

The inclusions of hyper-finite factors HFFs provide one of the three - as it seems equivalent - ways to describe finite measurement resolution (FMR) in TGD framework: the included factor defines an analog for gauge degrees of freedom which correspond to those below measurement resolution.

2. Cognitive representations and FMR

Another description for FMR in the framework of adelic physics would be in terms of cognitive representations [L57]. First some background about $M^8 - H$ duality.

1. There are number theoretic and geometric views about dynamics. In algebraic dynamics at the level of M^8, the space-time surfaces are roots of polynomials. There are no partial differential equations like in the geometric dynamics at the level of H.

2. The algebraic "dynamics" of space-time surfaces in M^8 is dictated by co-associativity, which means that the normal space of the space-time surface is associative and thus quaternionic. That normal space rather than tangent space must be associative became clear last year [L73, L74].

3. $M^8 - H$ duality maps these algebraic surfaces in M^8 to $H = M^4 \times CP_2$ and the one obtains the usual dynamics based on variational principle giving minimal surfaces which are non-linear analogs for the solutions of massless field equations. Instead of polynomials the natural functions at the level of H are periodic functions used in Fourier analysis [L90].

At level of complexified M^8 cognitive representation would consist of points of co-associative space-time surface X^4 in complexified M^8 (complexified octonions), whose coordinates belong to extension of rationals and therefore make sense also p-adically for extension of p-adic numbers induced by extension of rationals. $M^8 - H$ duality maps the cognitive representations to H.

Cognitive representations form a hierarchy: the larger the extension of rationals, the larger the number of points in the extension and in the unique discretization of space-time surface. Therefore also the measurement resolution improves.

The surprise was that the cognitive representations which are typically finite, are for the "roots" of octonionic polynomials infinite [L73, L74]. Also in this case the density of points of cognitive representation increases as the dimension of extensions increases.

The understanding of the physical interpretation of $M^8 - H$ duality increased dramatically during the last half year.

1. X^4 in M^8 is highly analogous to momentum space (4-D analog of Fermi ball one might say) and H to position space. Physical states correspond to discrete sets of points - 4-momenta - in X^4. This is just the description used in particle physics for physical states. Time and space in this description are replaced by energy and 4-momentum. At the level of H one space-time and classical fields and one talks about frequencies and wavelengths instead of momenta.

2. $M^8 - H$ duality is a generalization of Fourier transform. Hitherto I have assumed that the space-time surface in M^8 is mapped to H. The momentum space interpretation at the level of M^8 however requires that the image must be a superposition of translates of the image in plane wave with some momentum: only the translates inside some bigger CD are allowed - this means infrared cutoff.

   The total momentum as sum of momenta for two half-cones of CD in M^8 is indeed well-defined. One has a generalization of a plane wave over translational degrees of freedom of CD and restricted to a bigger CD.

   At the limit of infinitely large size for bigger CD, the result is non-vanishing only when the sum of the momenta for two half-cones of CD vanishes: this corresponds to conservation of 4-momentum as a consequence of Poincare invariance rather than assumption as in the earlier approach [L90].

   This generalizes the position-momentum duality of wave mechanics lost in quantum field theory. Point-like particle becomes a quantum superposition of space-time surfaces inside the causal diamond (CD). Plane wave is a plane wave for the superposition of space-time surfaces inside CD having the cm coordinates of CD as argument.

3. Inclusion hierarchy of supersymplectic algebras and FMR

The third inclusion hierarchy allowing to describe finite measurement resolution is defined by supersymplectic algebras acting as the isometries of the "world of classical worlds" (WCW) consisting of space-time surfaces are preferred extremals ("roots" of polynomials in M^8 and minimal surfaces satisfying infinite-D set of additional "gauge conditions" in H).

At a given level of hierarchy generators with conformal weight larger than n act like gauge generators as also their commutators with generators with conformal weight smaller than n correspond to vanishing Noether charges. This defines "gauge conditions".

To sum up, there are therefore 3 hierarchies allowing to describe finite measurement resolution and they must be essentially equivalent in TGD framework.

Three evolutionary hierarchies

There are three evolutionary hierarchies: hierarchies of extensions of extensions of... ofrationals...; inclusions of inclusions of .... of HFFs, and inclusions of isomorphic super symplectic algebras.

1. Extensions of rationals

The extensions of rationals become algebraically increasingly complex as their dimension increases. The co-associative space-time surfaces in M^8 are "roots" of real polynomials with rational coefficients to guarantee number theoretical universality and this means space-time surfaces are characterized by extension of rationals.

Each extension of rationals defines extensions for p-adic number fields and entire adele. The interpretation is as a cognitive leap: the system's intelligence/algebraic complexity increases when the extension is extended further.

The extensions of extensions of .... define hierarchies with Galois groups in certain sense products of extensions involved. Exceptional extensions are those which do not allow this decomposition. In this case Galois group is a simple group. Simple groups are primes of finite groups and correspond to elementary particles of cognition. Kind of fundamental, non-decomposable ideas. Mystic might speak of pure states of consciousnesswith no thoughts.

In the evolution by quantum jumps the dimension of extension increases in statistical sense and evolution is unavoidable. This evolution is due to subjective time evolution by quantum jumps, something which is in spirit with Connes proposal but replaces time evolution by a sequence of evolutionary leaps.

2. Inclusions of HFFs as a hierarchy

HFFs are fractals. They have infinite inclusion hierarchies in which sub-HFF isomorphicto entire HFFs is included to HFF.

Also the hierarchies of inclusions define evolutionary hierarchies: HFF which is isomorphic with original becomes larger and in some sense more complex than the included factor. Also now one has sequences of inclusions of inclusions of.... These sequences would correspond to sequences for extensions of extensions... of rationals. Note that the inclusion hierarchy would be the basic object: not only single HFF in the hierarchy.

3. Inclusions of supersymplectic algebras as an evolutionary hierarchy

The third hierarchy is defined by the fractal hierarchy of sub-algebras of supersymplectic algebra isomorphic to the algebra itself. At a given level of hierarchy generators with conformal weight larger than n correspond to gauge degrees of freedom. As n increases the number of physical degrees of freedom above measurement resolution increases which means evolution. This hierarchy should correspond rather concretely to that for the extensions of rationals. These hierarchies would be essentially one and the same thing in the TGD Universe.

TGD based model for subjective time development

The understanding of subjective time development as sequences of SSFRs preceded by unitary "time" evolution has improved quite considerably recently [L90]. The idea is that the subjective time development as a sequence of scalings at the light-cone boundary generated by the vibrational part $\hat{L}_0$ of the scaling generator $L_0 = p^2 - \hat{L}_0$ (L_0 annihilates the physical states). Also p-adic mass calculations use $\hat{L}_0$.

For more than 10 years ago [K76, K48], I considered the possibility that Connes time evolution operator that he assigned with thermo-dynamical time could have a significant role in the definition of S-matrix in standard sense but had to give up the idea.

It however seems that for super-symplectic algebra $\hat{L}_0$ generates an outer automorphism since the algebra has only generators with conformal with $n > 0$ and its extension to included also generators with $n \leq 0$ is required to introduce L_0: since L_0 contains annihilation operators, it indeed generates outer automorphism in SCA. The two views could be equivalent! Whereas Connes considered thermo-dynamical time evolution, in TGD framework the time evolution would be subjective time evolution by SSFRs.

1. The guess would be that the exponential of the scaling operator L_0 gives the time evolution. The problem is that L_0 annihilates the physical states. The solution of the problem would be the same as in p-adic thermodynamics. L_0 decomposes as $L_0 = p^2 - \hat{L}_0$ and the vibrational part $\hat{L}_0$ this gives mass spectrum as eigenvalues of p^2. The thermo-dynamical state in p-adic thermodynamics is $p^{\hat{L}_0\beta}$. This operator exists p-adically in the p-adic number field defined by prime p.

2. Could unitary subjective time development involve the operator $exp(i2\pi L_0\tau)$ $\tau = log(T/T_0)$? This requires $T/T_0 = exp(n/m)$ guaranteeing that exponential is a root of unity for an eigenstate of L_0. The scalings are discretized and scalings come as powers of $e^{1/m}$. This is possible using extensions of rationals generated by a root of e. The unique feature of p-adics is that e^p is ordinary p-adic number. This alone would give periodic time evolution for eigenstates of L_0 with integer eigenvalues n.

SSA and SSA_n

Supersymplectic algebra SSA has fractal hierarchies of subalgebras SSA_n. The integers in a given hierarchy are of forn $n_1, n_1n_2, n_1n_2n_3, ...$ and correspond naturally to hierarchies of inclusions of HFFs. Conformal weights are positive: $n > 0$. For ordinary conformal algebras also negative weights are allowed. Yangians have only non-negative weights. This is of utmost importance.

SSA_n with generators have radial light-like conformal weights coming as multiples of n. SSA_n annihilates physical states and $[SSA_n, SSA]$ does the same. Hence the generators with conformal weight larger than n annihilate the physical states.

What about generators with conformal weights smaller than n? At least a subset of them need not annihilate the physical states. Since L_n are superpositions of creation operators, the idea that analogs of coherent states could be in question.

It would be nice to have a situation in which L_n, $n < m$ commute. $[L_k, L_l] = 0$ effectively for $k + l \geq m$.

The simplest way to obtain a set of effectively commuting operators is to take the generators L_k, $[m/2] < k < m$, where $[m/2]$ is nearest integer larger than $m/2$.

This raises interesting questions.

1. Could the Virasoro generators $O(\{c_k\}) = \sum_{k\in[m/2],m]} c_kL_k$ as linear combinations of creation operators generate a set of coherent states as eigenstates of their Hermitian conjugates.

2. Some facts about coherent states are in order.

   (a) When one adds to quantum harmonic oscillator Hamiltonian oscillator a time dependent perturbation which lasts for a finite the vacuum state evolves to an oscillator vacuum whose position is displacemented. The displacement is complex and is a Fourier component of the external force $f(t)$ corresponding to the harmonic oscillator frequency ω. Time evolution picks up only this component.

   (b) Coherent state property means that the state is eigenstate of the annihilation creation operator with eivengeu $\alpha = -ig(\omega)$ where $g(omega) = \int f(u)exp(-i\omega u)du$ is Fourier transform of $f(t)$.

   (c) Coherent states are not orthogonal and form an overcomplete set. The overlaps of coherent states are proportional to a Gaussian depending on the complex parameters characterizing them. One can however develop any state in terms of coherent states as a unique expansion since one can represent unitary in terms of coherent states.

   (d) Coherent state obtained from the vacuum state by time evolution in presence of $f(t)$ by a unitary displacement operator $D(\alpha) = exp(\alpha a^\dagger - \overline{\alpha}a)$. (`https://en.wikipedia.org/wiki/Displacement_operator`).

   The displacement operator is a unitary operator and in the general case the displacement is complex. The product of two displacement operators would be apart from a phase factor a displacement operator associated with the sum of displacements.

   (e) Harmonic oscillator coherent states are indeed maximally classical since wave packets have minimal width in both q and p space. Furthermore, the classical expectation values for q and p obey classical equations of motion.

These observations raise interesting questions about how the evolution by SSFRs could be modelled.

1. Instead of harmonic oscillator in q-space, one would have time evolution in the space of scalings of causal diamond parameterized by the scaling parameter $\tau = log(T/T_0)$, where T can be identified as the radial light-like coordinate of light-cone boundary.

   The analogs of harmonic oscillator states would be defined in this space and would be essentially wave packets with ground state minimizing the width of the wave packet.

2. The role of harmonic oscillator Hamiltonian in absence of external force would be taken by the generator $\hat{L}_0$ ($L_0 = p^2 - \hat{L}_0$ acts trivially) and gives rise to mass squared quantization. The situation would be highly analogous to that in p-adic thermodynamics. The role of ω

would be taken by the minimal conformal weight h_{min} such that the eigenvalues of L_0 are its multiples. It seems that this weight must be equal to $h_{min} = 1$.

The commutations of $\hbar L_0$ with L_k, $k > 0$ would be as for L_0 so what the replacement should not affect the situation.

3. The scaling parameter τ is analogous to the spatial coordinate q for the harmonic oscillator. Can one identify the analog of the external force $f(t)$ acting during unitary evolution between two SSFRs? Or is it enough to use only the analog of $g(\omega \to h_{min} = 1)$ - that is the coefficients C_k.

   To identify $f(t)$, one needs a time coordinate t. This was already identified as τ. This one would have $q = t$, which looks strange. The space in which time evolution is the space of scalings and the time evolutions are scalings and thus time evolution means translation in this space. The analog for this would be Hamiltonian $H = i\hbar d/dq$.

   Number theoretical universality allows only the values of $\tau = r/s$ whose exponents give roots of unity. Also $exp(n\tau)$ makes sense p-adically for these values. This would mean that the Fourier transform defining g would become discrete and be sum over the values $f(\tau = r/s)$.

4. What happens if one replaces $\hat{L}_0$ with L_0. In this case one would have the replacement of ω with $h_{vac} = 0$. Also the analog of Fourier transform with zero frequency makes sense. $\hat{L_0} = p^2 - L_0$ is the most natural choice for the Hamiltonian defining the time evolution operator but is trivial. Could $\Delta^{i\tau}$ describe the inherent time evolution. It would be outer automorphism since it is not defined solely in terms of SCA. So: could one have $\Delta = exp(\hat{L_0})$ so that $\Delta^{i\tau}$ coincide with $exp(i\hat{L_0}\tau)$? This would mean the identification

   $$\Delta = exp(\hat{L_0}) \ ,$$

   which is a positive definite operator. The exponents coming from $exp(iL_0\tau)$ can be number theoretically universal if $\tau = log(T/T_0)$ is a rational number implying $T/T_0 = exp(r/s)$, which is possible number theoretically) and the extension of rationals contains some roots of e.

5. Could one have $\Delta = L_0$? Also now that positivity condition would be satisfied if SSA conformal weights satisfy $n > 0$.

   The problem with this operation is that it is not number theoretically universal since the exponents $exp(ilog(n)\tau)$ do not exist p-adically without introducing infinite-D extension of p-adic number making $log(n)$ well-defined.

   What is however intriguing is that the "time" evolution operator $\Delta^{i\tau}$ in the eigenstate basis would have trace equal to $Tr(\Delta^{i\tau}) \sum d(n)n^{i\tau}$, where $d(n)$ is the degeneracy of the state. This is a typical zeta function: for Riemann Zeta one has $d(n) = 1$.

   For $\Delta = exp(L_0)$ option $Tr(\Delta^{i\tau}) = \sum d(n)exp(in\tau)$ exists for $\tau = r/s$ if r:th root of e belongs to the extension of p-adics.

To sum up, one would have Gaussian wave packet as harmonic oscillator vacuum in the space of scaled variants of CD. The unitary time evolution associated with SSFR would displace the peak of the wave packet to a larger scalings. The Gaussian wave function in the space of scaled CDs has been proposed earlier.

Could this time evolution make sense and be even realistic?

1. The analogs of harmonic oscillator states are defined in the space of scalings as Gaussians and states obtained from them using oscillator operators. There would be a wave function in the moduli space of CDs analogous to a state of harmonic oscillator.

2. SSFR following the time evolutions would project to an eigenstate of harmonic oscillator having in general displaced argument. The unitary displacement operator D should commute with the operators having the members of zero energy states at the passive boundary of CD as eigenstates. This poses strong conditions. At least number theoretic measurements could satisfy these conditions.

3. SSFRs are identified as weak measurements as near as possible to classical measurements. Time evolution by the displacement would be indeed highly analogous to classical time evolution for theeharmonic oscillator.

4. The unitary displacement operator corresponds to the arbitrary external force on the harmonic oscillator and it seems that it would be selected in SSFR for the unitary evolution after SSFR. This means fixing the coefficients C_k in the operator $\sum C_k L_k$.

   What is the subjective "time" evolution operator when in the case of SSA_n?

1. The scaling analog of the unitary displacement operator D as $D = \sum exp(\sum C_k L_k - \overline{C}_k L_{-k})$ is highly suggestive and would take the oscillator vacuum to a coherent state. Coefficients C_k would be proportional to τ. There would be a large number of choices for the unitary displacement operator. One can also consider complex values of τ since one has complexified M^8.

2. There should be a normalization for the coefficients: without this it is not possible to talk about a special value of τ does not make sense. For instance, the sum of their moduli squared could be equal to 1. This would give interpretation as a quantum state in the degrees of freedom considered. The width of the Gaussian would increase slowly during the unitary time evolution and be proportional to $log(T/T_0)$.

   The width of the Gaussian would increase slowly as a function of T during the unitary time evolution and be proportional to $log(T/T_0)$. The condition that c_k are proportional the same complex number times τ is too strong.

3. The arbitrariness in the choice of C_k would bring in a kind of non-determinism as a selection of this superposition. The ability to engineer physical systems is in conflict with the determinism of classical physics and also difficult to understand in standard quantum physics. Could one interpret this choice as an analog for engineering a Hamiltonian as in say quantum computation or build-up of an electric circuit for some purpose? Could goal directed action correspond to this choice?

   If so engineerable degrees of freedom would correspond to intermediate degrees of freedom associated with L_k, $[m/2] \leq k \leq m$. They would be totally absent for $k = 1$ and this would correspond to a situation analogous to the standard physics without any intentional action.

2.5 MIP*= RE: What could this mean physically?

I received a very interesting link to a popular article (`https://cutt.ly/sfd5UQF`) explaining a recently discovered deep result in mathematics having implications also in physics. The article [A61] (`https://cutt.ly/rffiYdc`) by Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen has a rather concise title "**MIP*=RE**". In the following I try to express the impressions of a (non-mainstream) physicist about the result.

The following is the result expressed using the concepts of computer science about which I know very little at the hard technical level. The results are however told to state something highly non-trivial about physics.

1. RE (recursively enumerable languages) denotes all problems solvable by computer. P denotes the problems solvable in a polynomial time. NP does not refer to a non-polynomial time but to "non-deterministic polynomial acceptable problems" - I hope this helps the reader- I am a little bit confused! It is not known whether P = NP is true.

2. IP problems (P is now for "prover" that can be solved by a collaboration of an interrogator and prover who tries to convince the interrogator that her proof is convincing with high enough probability. MIP involves multiple l provers treated as criminals trying to prove that they are innocent and being not allowed to communicate. MIP* is the class of solvable problems in which the provers are allowed to entangle.

The finding, which is characterized as shocking, is that *all* problems solvable by a Turing computer belong to this class: MIP*=RE. All problems solvable by computer would reduce to problems in the class MIP*! Quantum computation would indeed add something genuinely new to the classical computation.

Quantum entanglement would play an essential role in quantum computation. Also the implications for physics are highly non-trivial.

1. Connes embedding problem asking whether all infinite-D matrices can always be approximated by finite-D matrices has a negative solution.Therefore MIP*= RE does not hold true for hyperfinite factors of type II_1 (HFFs) central in quantum TGD. Also the Tirelson problem finds a solution. The measurements of commuting observers performed by two observers are equivalent to the measurements of tensor products of observables only in finite-D case and for HFFs. That quantum entanglement would not have any role in HFFs is in conflict with intuition.

2. In the TGD framework finite measurement resolution is realized in terms of HFFs at Hilbert space level and in terms of cognitive representations at space-time level defined purely number-theoretically. This leads to a hierarchy of adeles defined by extensions of rationals and the Hilbert spaces must have algebraic extensions of rationals as a coefficient field. Therefore one cannot in general case find a unitary transformation mapping the entangled situation to an unentangled one, and quantum entanglement plays a key role. It seems that computationalism formulated in terms of recursive functions of natural numbers must be formulated for the hierarchy of extensions of rationals in terms of algebraic integers.

3. In TGD inspired theory of consciousness entanglement between observers could be seen as a kind of telepathy helping to perform conscious quantum computations. Zero energy ontology also suggests a modification of the views about quantum computation. TGD can be formulated also for real and p-adic continua identified as correlates of sensory experience and cognition, and it seems that computational paradigm need not apply in the continuum cases.

2.5.1 Two physically interesting applications

There are two physically interesting applications of the theorem interesting also from the TGD point of view and force to make explicit the assumptions involved.

About the quantum physical interpretation of MP*

To proceed one must clarify the quantum physical interpretation of MIP*.

> Quantum measurement requires entanglement of the observer O with the measured system M. What is basically measured is the density matrix of M (or equivalently that of O). State function reduction gives as an outcome a state, which corresponds to an eigenvalue of the density matrix. Note that this state can be an entangled state if the density matrix has degenerate eigenvalues. Quantum measurement can be regarded as a question to the measured system: "What are the values of given commuting observables?". The final state of quantum measurement provides an eigenstate of the observables as the answer to this question. M would be in the role of the prover and O_i would serve as interrogators.
>
> In the first case multiple interrogators measurements would entangle M with unentangled states of the tensor product $H_1 \otimes H_2$ for O followed by a state function reduction splitting the state of M to un-entangled state in the tensor product $M_1 \otimes M_2$.
>
> In the second case the entire M would be interrogated using entanglement of M with entangled states of $H_1 \otimes H_2$ using measurements of several commuting observables. The theorem would state that interrogation in this manner is more efficient in infinite-D case unless HFFs are involved.

3. This interpretation differs from the interpretation in terms of computational problem solving in which one would have several provers and one interrogator. Could these interpretations be dual as the complete symmetry of the quantum measurement with respect to O and M suggests? In the case of multiple provers (analogous to accused criminals) it is advantageous to isolate them. In the case of multiple interrogators the best result is obtained if the interrogator does not effectively split itself into several ones.

Connes embedding problem and the notion of finite measurement/cognitive resolution

Alain Connes formulated what has become known as Connes embedding problem. The question is whether infinite matrices forming factor of type II_1 can be *always* approximated by finite-D matrices that is imbedded in a *hyperfinite* factor of type II_1 (HFF). Factors of type II and their HFFs are special classes of von Neumann algebras possibly relevant for quantum theory.

This result means that if one has measured of a complete set of for a product of commuting observables acting in the full space, one can find in the finite-dimensional case a unitary transformation transforming the observables to tensor products of observables associated with the factors of a tensor product. In the infinite-D case this is not true.

What seems to put alarms ringing is that in TGD there are excellent arguments suggesting that the state space has HFFs as building bricks. Does the result mean that entanglement cannot help in quantum computation in TGD Universe? I do not want to live in this kind of Universe!

Tsirelson problem

Tsirelson problem (see this) is another problem mentioned in the popular article as a physically interesting application. The problem relates to the mathematical description of quantum measurement.

Three systems are considered. There are two systems O_1 and O_2 representing observers and the third representing the measured system M. The measurement reducing the entanglement between M and O_1 or O_2 can regarded as producing correspondence between state of M and O_1 or O_2, and one can think that O_1 or O_2 measures only obserservables in its own state space as a kind of image of M. There are two ways to see the situation. The provers correspond now to the observers and the two situations correspond to provers without and with entanglement.

Consider first a situation in which one has single Hilbert space H and single observer O. This situation is analogous to IP.

1. The state of the system is described statistically by a density matrix - not necessarily pure state -, whose diagonal elements have interpretation as reduction probabilities of states in this bases. The measurement situation fixes the state basis used. Assume an ensemble of identical copies of the system in this state. Assume that one has a complete set of commuting observables.

2. By measuring all observables for the members of the ensemble one obtains the probabilities as diagonal elements of the density matrix. If the observable is the density matrix having no- degenerate eigenvalues, the situation is simplified dramatically. It is enough to use the density matrix as an observable. TGD based quantum measurement theory assumes that the density matrix describing the entanglement between two subsystems is in a universal observable measure in state function reductions reducing their entanglement.

3. Can one deduce also the state of M as a superposition of states in the basic chosen by the observer? This basis need not be the same as the basis defined by - say density matrix if the system has interacted with some system and this ineracton has led to an eigenstate of the density matrix. Assume that one can prepare the latter basis by a physical process such as this kind of interaction.

   The coefficients of the state form a set of N^2 complex numbers defining a unitary $N \times N$ matrix. Unitarity conditions give N conditions telling that the complex rows and unit vectors: these numbers are given by the measurement of all observables. There are also $N(N-1)$ conditions telling that the rows are orthogonal. Together these $N+N(N-1) = N^2$ numbers fix the elements of the unitary matrix and therefore the complex coefficients of the state basis of the system can be deduced from a complete set of measurements for all elements of the basis.

Consider now the analog of the MIS* involving more than one observer. For simplicity consider two observers.

1. Assume that the state space H of M decomposes to a tensor product $H = H_1 \otimes H_2$ of state spaces H_1 and H_2 such that O_1 measures observables X_1 in H_1 and O_2 measuresobservables X_2 in H_2. The observables X_1 and X_2 commute since they act in different tensor factors. The absence of interaction between the factors corresponds to the inability of the provers to communicate. As in the previous case, one can deduce the probabilities for the various outcomes of the joint measurements interpreted as measurements of a complete set of observables $X_1 \otimes X_2$.

2. One can also think that the two systems form a single system O so that O_1 and O_2 can entangle. This corresponds to a situation in which entanglement between the provers is allowed. Now X_1 and X_2 are not in general independent but also now they must commute. One can deduce the probabilities for various outcomes as eigenstates of observables $X_1 X_2$ and deduce the diagonal elements of the density matrix as probabilities.

Are these ways to see the situation equivalent? Tsirelson demonstrated that this is the case for finite-dimensional Hilbert spaces, which can indeed be decomposed to a tensor product of factors associated with O_1 and O_2. This means that one finds a unitary transformation transforming the entangled situation to an unentangled one and to tensor product observables.

For the infinite-dimensional case the situation remained open. According to the article, the new result implies that this is not the case. For hyperfinite factors the situation can be approximated with a finite-D Hilbert space so that the situations are equivalent in arbitrary precise approximation.

2.5.2 The connection with TGD

The result looks at first a bad news from the TGD point of view, where HFFs are highly suggestive. One must be however very careful with the basic definitions.

Measurement resolution

Measurement resolution is the basic notion.

1. There are intuitive physicist's arguments demonstrating that in TGD the operator algebras involved with TGD are HFFs provides a description of finite measurement resolution. The inclusion of HFFs defines the notion of resolution: included factor represents the degrees of freedom not seen in the resolution used [K127, K48] (`http://tgdtheoryd.fi/pfpool/vNeumann.pdf`) and `http://tgdtheoryd.fi/pfpool/vNeumannnew.pdf`).

   Hyperfinite factors involve new structures like quantum groups and quantum algebras reflecting the presence of additional symmetries: actually the "world of classical worlds" (WCW) as the space of space-time surfaces as maximal group of isometries and this group has a fractal hierarchy of isomorphic groups imply inclusion hierarchies of HFFs. By the analogs of gauge conditions this infinite-D group reduces to a hierarchy of effectively finite-D groups. For quantum groups the infinite number of irreps of the corresponding compact group effectively reduces to a finite number of them, which conforms with the notion of hyper-finiteness.

   It looks that the reduction of the most general quantum theory to TGD-like theory relying on HFFs is not possible. This would not be surprising taking into account gigantic symmetries responsible for the cancellation of infinities in TGD framework and the very existence of WCW geometry.

2. Second TGD based approach to finite resolution is purely number theoretic [L38] and involves adelic physics as a fusion of the real physics with various p-adic physics as correlates of

cognition. Cognitive representations are purely number theoretic and unique discretizations of space-time surfaces defined by a given extension of rationals forming an evolutionary hierarchy: the coordinates for the points of space-time as a 4-surface of the embedding space $H = M^4 \times CP_2$ or of its dual M^8 are in the extension of rationals defining the adele. In the case of M^8 the preferred coordinates are unique apart from time translation. These two views would define descriptions of the finite resolution at the level of space-time and Hilbert space. In particular, the hierarchies of extensions of rationals should define hierarchies of inclusions of HFFs.

For hyperfinite factors the analog of MIP*=RE cannot hold true. Doesn't the TGD Universe allow a solution of all the problems solvable by Turing Computer? There is a loophole in this argument.

1. The point is that for the hierarchy of extensions of rationals also Hilbert spaces have as a coefficient field the extension of rationals! Unitary transformations are restricted to matrices with elements in the extension. In general it is not possible to realize the unitary transformation mapping the entangled situation to an un-entangled one! The weakening of the theorem would hold true for the hierarchy of adeles and entanglement would give something genuinely new for quantum computation!

2. A second deep implication is that the density matrix characterizing the entanglement between two systems cannot in general be diagonalized such that all diagonal elements identifiable as probabilities would be in the extension considered. One would have stable or partially stable entanglement (could the projection make sense for the states or subspaces with entanglement probability in the extension). For these bound states the binding mechanism is purely number theoretical. For a given extension of p-adic numbers one can assign to algebraic entanglement also information measure as a generalization of Shannon entropy as a p-adic entanglement entropy (real valued). This entropy can be negative and the possible interpretation is that the entanglement carries conscious information.

What about transcendental extensions?

During the writing of this article an interesting question popped up.

1. Also transcendental extensions of rationals are possible, and one can consider the generalization of the computationalism by also allowing functions in transcendental extensions. Could the hierarchy of algebraic extensions could continue with transcendental extensions? Could one even play with the idea that the discovery of transcendentals meant a quantum leap leading to an extension involving for instance e and π as basic transcendentals? Could one generalize the notion of polynomial root to a root of a function allowing Taylor expansion $f(x) = \sum q_n x^n$ with rational coefficients so that the roots of $f(x) = 0$ could be used define transcendental extensions of rationals?

2. Powers of e or its root define and infinite-D extensions having the special property that they are finite-D for p-adic number fields because e^p is ordinary p-adic number. In the p-adic context e can be defined as a root of the equation $x^p - \sum p^n/n! = 0$ making sense also for rationals. The numbers $log(p_i)$ such that p_i appears a factor for integers smaller than p define infinite-D extension of both rationals and p-adic numbers. They are obtained as roots of $e^x - p_i = 0$.

3. The numbers $(2n+1)\pi$ $(2n\pi)$ can be defined as roots of $sin(x) = 0$ $(cos(x) = 0$. The extension by π is infinite-dimensional and the conditions defining it would serve as consistency conditions when the extension contains roots of unity and effectively replaces them.

4. What about other transcendentals appearing in mathematical physics? The values $\zeta(n)$ of Riemann Zeta appearing in scattering amplitudes are for even values of n given by $\zeta(2n) = (-1)^{n+1} B_{2n} (2\pi)^{2n}/2(2n+1)!$. This follows from the functional identity for Riemann zeta and from the expression $\zeta(-n) = (-1)^n B_{n+1}/(n+1)$ ($(B(-1/2) = -1/2)$ (`https:`

//cutt.ly/dfgtgmw). The Bernoulli numbers B_n are rational and vanish for odd values of n. An open question is whether also the odd values are proportional to π^n with a rational coefficient or whether they represent "new" transcendentals.

What about the situation for the continuum version of TGD?

At least the cognitively finitely representable physics would have the HFF property with coefficient field of Hilbert spaces replaced by an extension of rationals. Number theoretical universality would suggest that HFF property characterizes also the physics of continuum TGD. This leads to a series of questions.

1. Does the new theorem imply that in the continuum version of TGD all quantum computations allowed by the Turing paradigm for real coefficients field for quantum states are not possible: $MIP* \subset RE$? The hierarchy of extensions of rationals allows utilization of entanglement, and one can even wonder whether one could have $MIP* = RE$ at the limit of algebraic numbers.

2. Could the number theoretic vision force change also the view about quantum computation? What does RE actually mean in this framework? Can one really assume complex entanglement coefficients in computation. Does the computational paradigm makes sense at all in the continuum picture?

   Are both real and p-adic continuum theories unreachable by computation giving rise to cognitive representations in the algebraic intersubsection of the sensory and cognitive worlds? I have indeed identified real continuum physics as a correlate for sensory experience and various p-adic physics as correlates of cognition in TGD: They would represent the computionally unreachable parts of existence.

   Continuum physics involves transcendentals and in mathematics this brings in analytic formulas and partial differential equations. At least at the level of mathematical consciousness the emergence of the notion of continuum means a gigantic step. Also this suggests that transcendentality is something very real and that computation cannot catch all of it.

3. Adelic theorem allows to express the norm of a rational number as a product of inverses of its p-adic norms. Very probably this representation holds true also for the analogs of rationals formed from algebraic integeres. Reals can be approximated by rationals. Could extensions of all p-adic numbers fields restricted to the extension of rationals say about real physics only what can be expressed using language?

Also fermions are highly interesting in the recent context. In TGD spinor structure can be seen as a square root of Kähler geometry, in particular for the "world of classical worlds" (WCW). Fermions are identified as correlates of Boolean cognition. The continuum case for fermions does not follow as a naïve limit of algebraic picture.

1. The quantization of the induced spinors in TGD looks different in discrete and continuum cases. Discrete case is very simple since equal-time anticommutators give discrete Kronecker deltas. In the continuum case one has delta functions possibly causing infinite vacuum energy like divergences in conserved Noether charges (Dirac sea).

2. In [L78] (https://cutt.ly/zfftoK6) I have proposed how these problems could be avoided by avoiding anticommutators giving delta-function. The proposed solution is based on zero energy ontology and TGD based view about space-time. One also obtains a long-sought-for concrete realization for the idea that second quantized induce spinor fields are obtained as restrictions of second quantized free spinor fields in $H = M^4 \times CP_2$ to space-time surface. The fermionic variant of $M^8 - H$-duality [L79] provides further insights and gives a very concrete picture about the dynamics of fermions in TGD.

These considerations relate in an interesting manner to consciousness. Quantum entanglement makes in the TGD framework possible telepathic sharing of mental images represented by sub-selves of self. For the series of discretizations of physics by HFFs and cognitive representations associated with extensions of rationals, the result indeed means something new.

What does one mean with quantum computation in TGD Universe?

The TGD approach raises some questions about computation.

1. The ordinary computational paradigm is formulated for Turing machines manipulating natural numbers by recursive algorithms. Programs would essentially represent a recursive function $n \to f(n)$. What happens to this paradigm when extensions of rationals define cognitive representations as unique space-time discretizations with algebraic numbers as the limit giving rise to a dense in the set of reals.

   The usual picture would be that since reals can be approximated by rationals, the situation is not changed. TGD however suggests that one should replace at least the quantum version of the Turing paradigm by considering functions mapping algebraic integers (algebraic rational) to algebraic integers.

   Quite concretely, one can manipulate algebraic numbers without approximation as a rational and only at the end perform this approximation and computations would construct recursive functions in this manner. This would raise entanglement to an active role even if one has HFFs and even if classical computations could still look very much like ordinary computation using integers.

   This suggests that computationalism usually formulated in terms of recursive functions of natural or rational numbers could be replaced with a hierarchy of computationalisms for the hierarchy of extensions of rationals. One would have recursively definable functions defined and having values in the extensions of rationals. These functions would be analogs of analytic functions (or polynomials) with the complex variable replaced with an integer or a rational of the extension. This poses very powerful constraints and there are good reasons to expect an increase of computational effectiveness. One can hope that at the limit of algebraic numbers of these functions allow arbitrarily precise approximations to real functions. If the real world phenomena can be indeed approximated by cognitive representations in the TGD sense, one can imagine a highly interesting approach to AI.

2. ZEO brings in also time reversal occurring in "big" (ordinary) quantum jumps and this modifies the views about quantum computation. In ZEO based conscious quantum computation halting means "death" and "reincarnation" of conscious entity, self? How the processes involving series of haltings in this sense differs from ordinary quantum computation: could one shorten the computation time by going forth and back in time.

There are many interesting questions to be considered. $M^8 - H$ duality gives justifications for the vision about algebraic physics. TGD leads also to the notion of infinite prime and I have considered the possibility that infinite primes could give a precise meaning for the dimension of infinite-D Hilbert space. Could the number-theoretic view about infinite be considerably richer than the idea about infinity as limit would suggest [K107].

The construction of infinite primes is analogous to a repeated second quantization of arithmetic supersymmetric quantum field theory allowing also bound states at each level and a concrete correspondence with the hierarchy of space-time sheets is suggestive. For the infinite primes at the lowest level of the hierarchy single particle states correspond to rationals and bound states to polynomials and therefore to the sets of their roots. This strongly suggests a connection with M^8 picture.

Could the number field of computable reals (p-adics) be enough for physics?

For some reason I have managed to not encounter the notion of computable number (see `https://cutt.ly/pTeSSfR`) as opposed to that of non-computable number (see `https://cutt.ly/gTeD9vF`). The reason is perhaps that I have been too lazy to take computationalism seriously enough.

Computable real number is a number, which can be produced to an arbitrary accuracy by a Turing computer, which by definition has a finite number of internal states, has input which is natural number and produces output which is natural numbers. Turing computer computes values of a function from natural numbers to itself by applying a recursive algorithm.

The following three formal definitions of the notion are equivalent.

1. The real number a is computable, if it can be expressed in terms of a computable function $n \rightarrow f(n)$ from natural numbers to natural numbers characterized by the property

$$\frac{f(n)-1)}{n} \leq a \leq \ (\frac{f(n)+1)}{n} .$$

For rational $a = q$, $f(n) = nq$ satisfies the conditions. Note that this definition does not work for p-adic numbers since they are not well-ordered.

2. The number a is computable if for an arbitrarily small rational number ϵ there exists a computable function producing a rational number r satisfying $|r - x \leq \epsilon$. This definition works also for p-adic numbers since it involves only the p-adic norm which has values which are powers of p and is therefore real valued.

3. a is computable if there exists a computable sequence of rational numbers r_i converging to a such that $|a - r_i| \leq 2^{-i}$ holds true. This definition works also for 2-adic numbers and its variant obtained by replacing 2 with the p-adic prime p makes sense for p-adic numbers.

The set R_c of computable real numbers and the p-adic counterparts $Q_{p,c}$ of R_c, have highly interesting properties.

1. R_c is enumerable and therefore can be mapped to a subset of rationals: even the ordering can be preserved. Also $Q_{p,c}$ is enumerable but now one cannot speak of ordering. As a consequence, most real (p-adic) numbers are non-computable. Note that the pinary expansion of a rational is periodic after some pinary digit. For a p-adic transcendental this is not the case.

2. Algebraic numbers are computable so that one can regard R_c as a kind of completion of algebraic numbers obtained by adding computable reals. For instance, π and e are computable. 2π can be computed by replacing the unit circle with a regular polygon with n sides and estimating the length as nL_n. L_n the length of the side. e can be computed from the standard formula. Interestingly, e^p is an ordinary p-adic number. An interesting question is whether there are other similar numbers. Certainly many algebraic numbers correspond to ordinary p-adic numbers.

3. R_c $(Q_{p,c})$ is a number field since the arithmetic binary operations $+, -\times, /$ are computable. Also differential and integral calculus can be constructed. The calculation of a derivative as a limit can be carried out by restricting the consideration to computable reals and there is always a computable real between two computable reals. Also Riemann sum can be evaluated as a limit involving only computable reals.

4. An interesting distinction between real and p-adic numbers is that in the sum of real numbers the sum of arbitrarily high digits can affect even all lower digits so that it requires computational work to predict the outcome. For p-adic numbers memory digits affect only the higher digits. This is why p-adic numbers are tailor made for computational purposes.Canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ used in p-adic mass calculations to map p-adic mass squared to its real counterpart [K68] maps p-adics to reals in a continuous manner. For integers this corresponds is 2-to-1 due to the fact that the p-adic numbers $-1 = (p-1)/(1-p)$ and $1/p$ are mapped to p.

5. For computable numbers, one cannot define the relation $=$. One can only define equality in some resolution ϵ. The category theoretical view about equality is also effective and conforms with the physical view.

   Also the relations $\leq$ and $\geq$ fail to have computable counterparts since only the absolute value $|x - y|$ can appear in the definition and one loses the information about the well-ordered nature of reals. For p-adic numbers there is no well-ordering so that nothing is lost. A restriction to non-equal pairs however makes order relation computable. For p-adic numbers the same is true.

6. Computable number is obviously definable but there are also definanable numbers, which are not computable. Examples are Gödel numbers in a given coding scheme for statements, which are true but not provable. More generally, the Gödel numbers coding for undecidable problems such as the halting problem are uncomputable natural numbers in a given coding scheme. Chaitin's constant, which gives the probability that random Turing computation halts, is a non-computable but definable real number.

7. Computable numbers are arithmetic numbers, which are numbers definable in terms of first order logic using Peano's axioms. First order logic does not allow statements about statements and one has an entire hierarchy of statements about... about statements. The hierarchy of infinite primes defines an analogous hierarchy in the TGD framework and is formally similar to a hierarchy of second quantizations [K107].

2.6 Analogs Of Quantum Matrix Groups From Finite Measurement Resolution?

The notion of quantum group [?]eplaces ordinary matrices with matrices with non-commutative elements. This notion is physically very interesting, and in TGD framework I have proposed that it should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution [?] These ideas have developed slowly through various side tracks.

In the sequel I will consider the notion of quantum matrix inspired by the recent view about quantum TGD relying on the notion of finite measurement resolution and show that under some additional conditions it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution.

1. The basic idea is to replace complex matrix elements with operators, which are products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers. Modulus and phase would be non-commuting and have commutation relation analogous to that between momentum and plane-wave in accordance with the idea about quantization of complex numbers.

2. The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. Strong/weak permutation symmetry of determinant requires its invariance apart from sign change under permutations of rows and/or columns. Weak permutation symmetry means development of determinant with respect to a fixed row or column and does not pose additional conditions. For weak permutation symmetry the permutation of rows/columns would however have a natural interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements and here quantum group structure emerges.

3. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

Quantum matrices define a more general structure than quantum group but provide a concrete representation for them in terms of finite measurement resolution, in particular when q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by a sign factor invariant under the permutations of both rows and columns. One can also understand the recursive fractal structure of inclusion sequences of hyper-finite factors resulting by replacing operators appearing as matrix elements with quantum matrices and a concrete connection with quantum groups emerges.

In Zero Energy Ontology (ZEO) M-matrix serving as the basic building brick of unitary U-matrix and identified as a hermitian square root of density matrix provides a possible application for this vision. Especially fascinating is the possibility of hierarchies of measurement resolutions represented as inclusion sequences realized as recursive construction of M-matrices. Quantization would emerge already at the level of complex numbers appearing as M-matrix elements.

This approach might allow to unify various ideas behind TGD. For instance, Yangian algebras emerging naturally in twistor approach are examples of quantum algebras. The hierarchy of Planck constants should have close relationship with inclusions and fractal hierarchy of sub-algebras of super-symplectic and other conformal algebras.

2.6.1 Well-definedness Of The Eigenvalue Problem As A Constraint To Quantum Matrices

Intuition suggests that the presence of degrees of freedom below measurement resolution implies that one must use density matrix description obtained by taking trace over the unobserved degrees of freedom. One could argue that in state function reduction with finite measurement resolution the outcome is not a pure state, or not even negentropically entangled state (possible in TGD framework) but a state described by a density matrix. The challenge is to describe the situation mathematically in an elegant manner.

1. There is present an infinite number of degrees of freedom below measurement resolution with which measured degrees of freedom entangle so that their presence affects the situation. One has a system with finite number degrees of freedom such as two-state system described by a quantum spinor. In this case observables as hermitian operators described by 2×2 matrices would be replaced by quantum matrices with elements, which in general do not commute.

   An attractive generalization of complex numbers appearing as elements of matrices is obtained by replacing them with products $H_{ij} = h_{ij}u_{ij}$ of hermitian operators h_{ij} with non-negative spectrum (modulus of complex number) and unitary operators u_{ij} (phase of complex number) suggests itself. The commutativity of h_{ij} and u_{ij} would look nice but is not necessary and is in conflict with the idea that modulus and phase of an amplitudes do not commute in quantum mechanics.

   Very probably this generalization is trivial for mathematician. One could indeed interpret the generalization in terms of a tensor product of finite-dimensional matrices with possibly infinite-dimensional space of operators of Hilbert space. For the physicist the situation might be different as the following proposal for what hermitian quantum matrices could be suggests.

2. The modulus of complex number is replaced with a hermitian operator having non-negative eigenvalues. The representation as $h = AA^\dagger + A^\dagger A$ is would guarantee this. The phase of complex number would be replaced by a unitary operator U possibly allowing the representation $U = exp(iT)$, T hermitian. The commutativity condition

   $$[h_{ij}, u_{ij}] = 0 \tag{2.6.1}$$

   for a given matrix element is also suggestive but as already noticed, Uncertainty Principle suggests that modulus and phase do not commute as operators. The commutator of modulus and phase would naturally be equal to that between momentum operator and plane wave:

   $$[h_{ij}, u_{ij}] = i\hbar \times u_{ij} \ , \tag{2.6.2}$$

   Here $\hbar = h/2\pi$ can be chosen to be unity in standard quantum theory. In TGD it can be generalized to a hermitian operator H_{eff}/h with an integer valued spectrum of eigenvalues given by $h_{eff}/h = n$ so that ordinary and dark matter sectors would be unified to single structure mathematically.

3. The notions of eigenvalues and eigenvectors for a hermitian operator should generalize. Now hermitian operator H would be a matrix with formally the same structure as $N\times N$ hermitian matrix in commutative number field - say complex numbers - possibly satisfying additional conditions.

Hermitian matrix can be written as

$$H_{ij} = h_{ij}u_{ij} \quad \text{for i>j} \quad H_{ij} = u_{ij}h_{ij} \quad \text{for i<j} \ , \quad H_{ii} = h_i \ . \tag{2.6.3}$$

Hermiticity conditions $H_{ij} = H_{ji}^{\dagger}$ give

$$h_{ij} = h_{ji} \ , \quad u_{ij} = u_{ji}^{\dagger} \ . \tag{2.6.4}$$

Here it has been assumed that one has quantum SU(2). For quantum U(2) one would have $U_{11} = U_{22}^{\dagger} = h_a u_a$ with u_a commuting with other operators. The form of the conditions is same as for ordinary hermitian matrices and it is not necessary to assume commutativity $[h_{ij}, u_{ij}] = 0$. Generalization of Pauli spin matrices provides a simple illustration.

4. The well-definedness of eigenvalue problem gives a strong constraint on the notion of hermitian quantum matrix. Eigenvalues of hermitian operator are determined by the vanishing of determinant $det(H - \lambda I)$. Its expression involves sub-determinants and one must decide whether to demand that the definition of determinant is independent of which column or row one chooses to develop the determinant.

   For ordinary matrix the determinant is expressible as sum of symmetric functions:

$$det(H - \lambda I) = \sum \lambda^n S_n(H) \ . \tag{2.6.5}$$

   Elementary symmetric functions S_n - n-functions in following - have the property that they are sums of contributions from to n-element paths along the matrix with the property that path contains no vertical or horizontal steps. One has a discrete analog of path integral in which time increases in each step by unit. The analogy with fermionic path integral is also obvious. In the non-commutative case non-commutativity poses problems since different orderings of rows (or columns) along the same n-path give different results.

   (a) For the first option one gives up the condition that determinant can be developed with respect to any row or column and defines determinant by developing it with respect to say first row or first column. If one developing with respect to the column (row) the permutations of rows (columns) do not affect the value of determinant or sub-determinants but permutations of columns (rows) do so unless one poses additional conditions stating that the permutations do not affect given contribution to the determinant or sub-determinant. It turns out that this option must be applied in the case of ordinary quantum group. For quantum phase $q = \pm 1$ the determinant is invariant under permutations of both rows and columns.

   (b) Second manner to get rid of difficulty would be that n-path does not depend on the ordering of the rows (columns) differ only by the usual sign factor. For 2×2 case this would give

$$ad - bc = da - cb \ , \quad \text{(Option 2)} \tag{2.6.6}$$

   These conditions state the invariance of the n-path under permutation group S_n permuting rows or columns.

   (c) For the third option the elements along n-paths commute: paths could be said to be "classical". The invariance of N-path in this sense guarantees the invariance of all n-paths. In 2-D case this gives

$$[a, d] = 0 \ , \quad [b, c] = 0 \ . \quad \text{(Option 3)} \tag{2.6.7}$$

5. One should have a well-defined eigenvalue problem. If the n-functions commute, one can diagonalize the corresponding operators simultaneously and the eigenvalues problem reduces to possibly infinite number of ordinary eigenvalue problems corresponding to restrictions to given set of eigenvalues associated with $N-1$ symmetric functions. This gives an additional constraint on quantum matrices.

   In 2-dimensional case one would have the condition

$$[ad - bc, a + d] = 0 \quad . \tag{2.6.8}$$

   Depending on how strong S_2 invariance one requires, one obtains 0, 1, 2 nontrivial conditions for 2×2 quantum matrices and 1 condition from the commutativity of n-functions besides hermiticity conditions.

   For $N \times N$-matrices one would have $N! - 1$ non-trivial conditions from the strong form of permutation invariance guaranteeing the permutation symmetry of n-functions and $N(N-1)/2$ conditions from the commutativity of n-functions.

6. The eigenvectors of the density matrix are obtained in the usual manner for each eigenvalue contributing to quantum eigenvalue. Also the diagonalization can be carried out by a unitary transformation for each eigenvalue separately. Hence the standard approach seems to generalize almost trivially.

   What makes the proposal non-trivial and possibly physically interesting is that the hermitian operators are not assumed to be just tensor products of $N \times N$ hermitian matrices with hermitian operators in Hilbert space.

The notion of unitary quantum matrix should also make sense. The naïve guess is that the exponentiation of a linear combination of ordinary hermitian matrices with coefficients, which are hermitian matrices gives quantum unitary matrices. In the case of U(1) the replacement of exponentiation parameter t in $exp(itX)$ with a hermitian operator gives standard expression for the exponent and it is trivial to see that unitary conditions are satisfied also in this case. Also in the case of SU(2) it is easy to verify that the guess is correct. One must also check that one indeed obtains a group: it could also happen that only semi-group is obtained.

In any case, one could speak of quantum matrix groups with coordinates replaced by hermitian matrices. These quantum matrix group need not be identical with quantum groups in the standard sense of the word. Maybe this could provide one possible meaning for quantization in the case of groups and perhaps also in the case of coset spaces G/H.

2.6.2 The Relationship To Quantum Groups And Quantum Lie Algebras

It is interesting to find out whether quantum matrices give rise to quantum groups under suitable additional conditions. The child's guess for these conditions is that the permutation of rows and columns correspond to braiding for the hermitian moduli h_{ij} defined by unitary operators U_{ij}.

Quantum groups and quantum matrices

The conditions for hermiticity and unitary do not involve quantum parameter q, which suggests that the naïve generalization of the notion of unitary matrix gives unitary group obtained by replacing complex number field with operator algebra gives group with coordinates defined by hermitian operators rather than standard quantum group. This turns out to be the case and it seems that quantum matrices provide a concrete representation for quantum group. The notion of braiding as that for operators h_{ij} can be said to emerge from the notion of quantum matrix.

1. Exponential of quantum hermitian matrix is excellent candidate for quantum unitary matrix. One should check the exponentiation indeed gives rise to a quantum unitary matrix. For $q = \pm 1$ this seems obvious but one should check this separately for other roots of unity. Instead of considering the general case, we consider explicit ansatz for unitary U(2) quantum

matrix as $U = [a, b; -b^\dagger, a^\dagger]$. The conditions for unitary quantum group in the proposed sense would state the orthonormality and unit norm property of rows/columns.

The explicit form of the conditions reads as

$$\begin{array}{ll} ab - ba = 0 \ , & ab^\dagger = b^\dagger a \ , \\ aa^\dagger + bb^\dagger = 1 \ , & a^\dagger a + b^\dagger b = 1 \ . \end{array} \tag{2.6.9}$$

The orthogonality conditions are unique and reduce to the vanishing of commutators.

Normalization conditions involve a choice of ordering. One possible manner to avoid the problem is to assume that both orderings give same unit length for row or column (as done above). If only the other option is assumed then only third or fourth equations is needed. The invariance of determinant under permutation of rows would imply $[a, a^\dagger] = [b, b^\dagger] = 0$ and the ordering problem would disappear.

2. One can look what conditions the explicit representation $U_{ij} = h_{ij}u_{ij}$ or equivalently $[h_a u_a, h_b u_b; -u_b^\dagger h_b, u_a^\dagger h_a]$ gives. The intuitive expectation is that U(2) matrix decomposes to a product of commutating SU(2) matrix and U(1) matrices. This implies that u_a commutes with the other matrices involved. One obtains the conditions

$$h_a h_b = h_b(u_b h_a u_b^\dagger) \ , \quad h_b h_a = (u_b h_a u_b^\dagger) h_b \ . \tag{2.6.10}$$

These conditions state that the permutation of h_a and h_b analogous to braiding operation is a unitary operation.

For the purposes of comparison consider now the corresponding conditions for $SU(2)_q$ matrix.

1. The $SU(2)_q$ matrix $[a, b; b^\dagger, a^\dagger]$ with *real* value of q (see `http://tinyurl.com/yb8tycag`) satisfies the conditions

$$\begin{array}{lll} ba = qab \ , & b^\dagger a = qab^\dagger, & bb^\dagger = b^\dagger b \ , \\ a^\dagger a + q^2 b^\dagger b = 1 \ , & aa^\dagger + bb^\dagger = 1 \ . & \end{array} \tag{2.6.11}$$

This gives $[a^\dagger, a] = (1 - q^2)b^\dagger b$. The above conditions would correspond to $q = \pm 1$ but with complex numbers replaced with operator algebra. q-commutativity obviously replaces ordinary commutativity in the conditions and one can speak of q-orthonormality.

For complex values of q - in particular roots of unity - the condition $a^\dagger a + q^2 b^\dagger b = 1$ is in general not self-consistent since hermitian conjugation transforms q^2 to its complex conjugate. Hence this condition must be dropped for complex roots of unity.

2. Only for $q = \pm 1$ corresponding to Bose-Einstein and Fermi-Dirac statistics the conditions are consistent with the invariance of n-functions (determinant) under permutations of both rows and columns. Indeed, if 2×2 q-determinant is developed with respect to column, the permutation of rows does not affect its value. This is trivially true also in $N \times N$ dimensional case since the permutation of rows does not affect the n-paths at all.

If the symmetry under permutations is weakened, nothing prevents from posing quantum orthogonality conditions also now and the decomposition to a product of positive and hermitian matrices give a concrete meaning to the notion of quantum group.

Do various $n-$functions commute with each other for $SU(2)_q$? The only commutator of this kind is that for the trace and determinant and should vanish:

$$\left[b + b^\dagger, aa^\dagger + bb^\dagger\right] = 0 \quad . \tag{2.6.12}$$

Since $a^\dagger a$ and $aa^\dagger$ are linear combinations of $b^\dagger b = b^\dagger b$, they vanish. Hence it seems that TGD based view about quantum groups is consistent with the standard view.

3. One can look these conditions in TGD framework by restricting the consideration to the case of SU(2) ($u_a = 1$) and using the ansatz $U = [h_a, h_b u_b; -u_b^\dagger h_b, h_a]$. Orthogonality conditions read as

$$h_a h_b = q h_b (u_b h_a u_b^\dagger) \quad , \quad h_b h_a = q(u_b h_a u_b^\dagger) h_b \quad .$$

If q is root of unity, these conditions state that the permutation of h_a and h_b analogous to a unitary braiding operation apart from a multiplication with quantum phase q. For $q = \pm 1$ the sign-factor is that in standard statistics. Braiding picture could help guess the commutators of h_{ij} in the case of $N \times N$ quantum matrices. The permutations of rows and columns would have interpretation as braidings and one could say that braided commutators of matrix elements vanish.

The conditions from the normalization give

$$h_a^2 + h_b^2 = 1 \quad , \quad h_a^2 + q^2 (u_b^\dagger h_b^2 u_b) = 1 \quad . \tag{2.6.13}$$

For complex q the latter condition does not make sense since $h_a^2 - 1$ and $u_b^\dagger h_b^2 u_b$ are hermitian matrices with real eigenvalues. Also for real values of $q \neq \pm 1$ one obtains contradicion since the spectra of unitarily related hermitian operators would differ by scaling factor q^2. Hence one must give up the condition involving q^2 unless one has $q = \pm 1$. Note that the term proportional to q^2 does not allow interpretation in terms of braiding.

4. Roots of unity are natural number theoretically as values of q but number theoretical universality allows the generic value of q would be a complex number existing simultaneously in all p-adic number properly extended. This would suggest the spectrum of q to come as

$$q(m, n) = e^{1/m} exp(\frac{i2\pi}{n}) \quad . \tag{2.6.14}$$

The motivation comes from the fact that e^p is ordinary p-adic number for all p-adic number fields so e and also any root of e defines a finite-dimensional extension of p-adic numbers [K126] [L9]. The roots of unity would be associated to the discretization of the ordinary angles in case of compact matrix groups. Roots of e would be associated with the discretization of hyperbolic angles needed in the case of non-compact matrix groups such as SL(2,C).

Also now unification of various values of q to single single operator Q, which is product of *commuting* hermitian and unitary operators and commuting with the hermitian operator H representing the spectrum of Planck constant would code the spectrum. Skeptic can of course wonder, whether the modulus and phase of Q can be assumed to commute. The relationship between integers associated with H and Q is interesting.

Quantum Lie algebras and quantum matrices

What about quantum Lie algebras? There are many notions of quantum Lie algebra and quantum group. General formulas for the commutation relations are well-known for Drinfeld-Jimbo type quantum groups (see `http://tinyurl.com/yb8tycag`). The simplest guess is that one just poses the defining conditions for quantum group, replaces complex numbers as coefficient module with operator algebra, and poses the above described conditions making possible to speak about eigenvalues and eigen vectors. One might however hope that this representation allows to realize the non-commutativity of matrix elements of quantum Lie algebra in a concrete manner.

1. For SU(2) the commutation relations for the elements X_+, X_-, h read as

$$[h, X_\pm] = \pm X_\pm \quad , \quad [X_+, X_-] = h \quad . \tag{2.6.15}$$

   Here one can use the 2×2 matrix representations for the ladder operators $X^\pm$ and diagonal angular momentum generator h.

2. For $\mathrm{SU}(2)_q$ one has

$$[h, X_\pm] = \pm X_\pm \quad , \quad [X_+, X_-] = \frac{q^h - q^{-h}}{q - q^{-1}} \quad . \tag{2.6.16}$$

3. Using the ansatz for the generators but allowing hermitian operator coefficients in non-diagonal generators $X_\pm$, one obtains the condition

   For $\mathrm{SU}(2)_q$ one would have

$$[X_+, X_-] = h_+^2 = h_-^2 = \frac{q^h - q^{-h}}{q - q^{-1}} \quad . \tag{2.6.17}$$

   Clearly, the proposal might make possible to have concrete representations for the quantum Lie algebras making the decomposition to measurable and directly non-measurable degrees of freedom explicit.

The conclusion is that finite measurement resolution does not lead automatically to standard quantum groups although the proposed realization is consistent with them. Also the quantum phases $q = \pm 1$ $n = 1, 2$ are realized and correspond to strong permutation symmetry and Bose-Einstein and Fermi statistics.

2.6.3 About Possible Applications

The realization for the notion of finite measurement resolution is certainly the basic application but one can imagine also other applications where hermitian and unitary matrices appear.

Density matrix description of degrees of freedom below measurement resolution

Density matrix ρ obtained by tracing over non-observable degrees of freedom is a fundamental example about a hermitian matrix satisfying the additional condition $Tr(\rho) = 1$.

1. A state function reduction with a finite measurement resolution would lead to a non-pure state. This state would be describable using $N \times N$-dimensional quantum hermitian quantum density matrix satisfying the condition $Tr(\rho) = 1$ (or more generally $Tr_q(\rho) = 1$), and satisfying the additional conditions allowing to reduce its diagonalization to that for a collection of ordinary density matrices so that the eigenvalues of ordinary density matrix would be replaced by N quantum eigenvalues defined by infinite-dimensional diagonalized density matrices.

2. One would have N quantum eigenvalues - quantum probabilities - each decomposing to possibly infinite set of ordinary probabilities assignable to the degrees of freedom below measurement resolution and defining density matrix for non-pure states resulting in state function reduction.

Some questions

Some further questions pop up naturally.

1. One might hope that the quantum counterparts of hermitian operators are in some sense universal, at least in TGD framework (by quantum criticality). Could the condition that the commutator of hermitian generators is proportional to $i\hbar$ times hermitian generator pose additional constraints? In 2-D case this condition is satisfied for quantum SU(2) generators and very probably the same is true also in the general case. The possible problems result from the non-commutativity but $(XY)^\dagger = Y^\dagger X^\dagger$ identity takes care that there are no problems.

2. One can also raise physics related questions. What one can say about most general quantum Hamiltonians and their energy spectra, say quantum hydrogen atom? What about quantum angular momentum? If the proposed construction is only a concretization of abstract quantum group construction, then nothing new is expected at the level of representations of quantum groups.

3. Could the spectrum of h_{eff} define a quantum h as a hermitian positive definite operator? Could this allow a description for the presence of dark matter, which is not directly observable? Same question applies to the quantum parameter q.

4. M-matrices are basic building bricks of scattering amplitudes in ZEO. M-matrix is produce of hermitian "complex" square root H of density matrix satisfying $H^2 = \rho$ and unitary S-matrix S. It has been proposed that these matrices commute. The previous consideration relying on basic quantum thinking suggests that they relate like translation generator in radial direction and phase defined by angle and thus satisfy $[H, S] = i(H_{eff}/h) \times S$. This would give enormously powerful additional condition to S-matrix. One can also ask whether M-matrices in presence of degrees of freedom below measurement resolution is quantum version of M-matrix in the proposed sense.

5. Fractality is of of the key notions of TGD and characterizes also hyperfinite factors. I have proposed some realizations of fractality such as infinite primes and finite-dimensional Hilbert spaces taking the role of natural numbers and ordinary sum and product replaced with direct sum and tensor product. One could also imagine a fractal hierarchy of quantum matrices obtained by replacing the operators appearing as matrix elements of quantum matrix element by quantum matrices. This hierarchy could relate to the sequence of inclusions of HFFs.

2.7 Jones Inclusions And Cognitive Consciousness

WCW spinors have a natural interpretation in terms of a quantum version of Boolean algebra. Beliefs of various kinds are the basic element of cognition and obviously involve a representation of the external world or part of it as states of the system defining the believer. Jones inclusions mediating unitary mappings between the spaces of WCWs spinors of two systems are excellent candidates for these maps, and it is interesting to find what one kind of model for beliefs this picture leads to.

The resulting quantum model for beliefs provides a cognitive interpretation for quantum groups and predicts a universal spectrum for the probabilities that a given belief is true. This spectrum depends only on the integer n characterizing the quantum phase $q = exp(i2\pi/n)$ characterizing the Jones inclusion. For $n \neq \infty$ the logic is inherently fuzzy so that absolute knowledge is impossible. $q = 1$ gives ordinary quantum logic with qbits having precise truth values after state function reduction.

2.7.1 Does One Have A Hierarchy Of U- And M-Matrices?

U-matrix describes scattering of zero energy states and since zero energy states can be illustrated in terms of Feynman diagrams one can say that scattering of Feynman diagrams is in question. The initial and final states of the scattering are superpositions of Feynman diagrams characterizing the corresponding M-matrices which contain also the positive square root of density matrix as a factor.

The hypothesis that U-matrix is the tensor product of S-matrix part of M-matrix and its Hermitian conjugate would make U-matrix an object deducible by physical measurements. One cannot of course exclude that something totally new emerges. For instance, the description of quantum jumps creating zero energy state from vacuum might require that U-matrix does not reduce in this manner. One can assign to the U-matrix a square like structure with S-matrix and its Hermitian conjugate assigned with the opposite sides of a square.

One can imagine of constructing higher level physical states as composites of zero energy states by replacing the S-matrix with M-matrix in the square like structure. These states would provide a physical representation of U-matrix. One could define U-matrix for these states in a similar manner. This kind of hierarchy could be continued indefinitely and the hierarchy of higher level U and M-matrices would be labeled by a hierarchy of n-cubes, $n = 1, 2,...$ TGD inspired theory of consciousness suggests that this hierarchy can be interpreted as a hierarchy of abstractions represented in terms of physical states. This hierarchy brings strongly in mind also the hierarchies of n-algebras and n-groups and this forces to consider the possibility that something genuinely new emerges at each step of the hierarchy. A connection with the hierarchies of infinite primes [K107] and Jones inclusions are suggestive.

2.7.2 Feynman Diagrams As Higher Level Particles And Their Scattering As Dynamics Of Self Consciousness

The hierarchy of inclusions of hyper-finite factors of II_1 as counterpart for many-sheeted space-time lead inevitably to the idea that this hierarchy corresponds to a hierarchy of generalized Feynman diagrams for which Feynman diagrams at a given level become particles at the next level. Accepting this idea, one is led to ask what kind of quantum states these Feynman diagrams correspond, how one could describe interactions of these higher level particles, what is the interpretation for these higher level states, and whether they can be detected.

Jones inclusions as analogs of space-time surfaces

The idea about space-time as a 4-surface replicates itself at the level of operator algebra and state space in the sense that Jones inclusion can be seen as a representation of the operator algebra $\mathcal{N}$ as infinite-dimensional linear sub-space (surface) of the operator algebra $\mathcal{M}$. This encourages to think that generalized Feynman diagrams could correspond to image surfaces in II_1 factor having identification as kind of quantum space-time surfaces.

Suppose that the modular S-matrices are representable as the inner automorphisms $\Delta(\mathcal{M}_k^{it}$ assigned to the external lines of Feynman diagrams. This would mean that $\mathcal{N} \subset \mathcal{M}_k$ moves inside $calM_k$ along a geodesic line determined by the inner automorphism. At the vertex the factors $calM_k$ to fuse along $\mathcal{N}$ to form a Connes tensor product. Hence the copies of $\mathcal{N}$ move inside $\mathcal{M}_k$ like incoming 3-surfaces in H and fuse together at the vertex. Since all $\mathcal{M}_k$ are isomorphic to a universal factor $\mathcal{M}$, many-sheeted space-time would have a kind of quantum image inside II_1 factor consisting of pieces which are $d = \mathcal{M} : \mathcal{N}/2$-dimensional quantum spaces according to the identification of the quantum space as subspace of quantum group to be discussed later. In the case of partonic Clifford algebras the dimension would be indeed $d \leq 2$.

The hierarchy of Jones inclusions defines a hierarchy of S-matrices

It is possible to assign to a given Jones inclusion $\mathcal{N} \subset \mathcal{M}$ an entire hierarchy of Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2...$, $\mathcal{M}_0 = N$, $\mathcal{M}_1 = M$. A possible interpretation for these inclusions would be as a sequence of topological condensations.

This sequence also defines a hierarchy of Feynman diagrams inside Feynman diagrams. The factor $\mathcal{M}$ containing the Feynman diagram having as its lines the unitary orbits of $\mathcal{N}$ under $\Delta_{\mathcal{M}}$

becomes a parton in $\mathcal{M}_1$ and its unitary orbits under $\Delta_{\mathcal{M}_1}$ define lines of Feynman diagrams in M_1. The concrete representation for M-matrix or projection of it to some subspace as entanglement coefficients of partons at the ends of a braid assignable to the space-like 3-surface representing a vertex of a higher level Feynman diagram. In this manner quantum dynamics would be coded and simulated by quantum states.

The outcome can be said to be a hierarchy of Feynman diagrams within Feynman diagrams, a fractal structure for which many particle scattering events at a given level become particles at the next level. The particles at the next level represent dynamics at the lower level: they have the property of "being about" representing perhaps the most crucial element of conscious experience. Since net conserved quantum numbers can vanish for a system in TGD Universe, this kind of hierarchy indeed allows a realization as zero energy states. Crossing symmetry can be understood in terms of this picture and has been applied to construct a model for M-matrix at high energy limit [K33] .

One might perhaps say that quantum space-time corresponds to a double inclusion and that further inclusions bring in N-parameter families of space-time surfaces.

Higher level Feynman diagrams

The lines of Feynman diagram in $\mathcal{M}_{n+1}$ are geodesic lines representing orbits of $\mathcal{M}_n$ and this kind of lines meet at vertex and scatter. The evolution along lines is determined by $\Delta_{\mathcal{M}_{n+1}}$. These lines contain within themselves $\mathcal{M}_n$ Feynman diagrams with similar structure and the hierarchy continues down to the lowest level at which ordinary elementary particles are encountered.

For instance, the generalized Feynman diagrams at the second level are ribbon diagrams obtained by thickening the ordinary diagrams in the new time direction. The interpretation as ribbon diagrams crucial for topological quantum computation and suggested to be realizable in terms of zero energy states in [K5] is natural. At each level a new time parameter is introduced so that the dimension of the diagram can be arbitrarily high. The dynamics is not that of ordinary surfaces but the dynamics induced by the $\Delta_{\mathcal{M}_n}$.

Quantum states defined by higher level Feynman diagrams

The intuitive picture is that higher level quantum states corresponds to the self reflective aspect of existence and must provide representations for the quantum dynamics of lower levels in their own structure. This dynamics is characterized by M-matrix whose elements have representation in terms of Feynman diagrams.

1. These states correspond to zero energy states in which initial states have "positive energies" and final states have "negative energies". The net conserved quantum numbers of initial and final state partons compensate each other. Gravitational energies, and more generally gravitational quantum numbers defined as absolute values of the net quantum numbers of initial and final states do not vanish. One can say that thoughts have gravitational mass but no inertial mass.

2. States in sub-spaces of positive and negative energy states are entangled with entanglement coefficients given by M-matrix at the level below.

To make this more concrete, consider first the simplest non-trivial case. In this case the particles can be characterized as ordinary Feynman diagrams, or more precisely as scattering events so that the state is characterized by $\hat{S} = P_{in}SP_{out}$, where S is S-matrix and P_{in} *resp.* P_{out} is the projection to a subspace of initial *resp.* final states. An entangled state with the projection of S-matrix giving the entanglement coefficients is in question.

The larger the domains of projectors P_{in} and P_{out}, the higher the representative capacity of the state. The norm of the non-normalized state $\hat{S}$ is $Tr(\hat{S}\hat{S}^\dagger) \leq 1$ for II_1 factors, and at the limit $\hat{S} = S$ the norm equals to 1. Hence, by II_1 property, the state always entangles infinite number of states, and can in principle code the entire S-matrix to entanglement coefficients.

The states in which positive and negative energy states are entangled by a projection of S-matrix might define only a particular instance of states for which conserved quantum numbers vanish. The model for the interaction of Feynman diagrams discussed below applies also to these more general states.

The interaction of $\mathcal{M}_n$ Feynman diagrams at the second level of hierarchy

What constraints can one pose to the higher level reactions? How Feynman diagrams interact? Consider first the scattering at the second level of hierarchy ($\mathcal{M}_1$), the first level $\mathcal{M}_0$ being assigned to the interactions of the ordinary matter.

1. Conservation laws pose constraints on the scattering at level $\mathcal{M}_1$. The Feynman diagrams can transform to new Feynman diagrams only in such a way that the net quantum numbers are conserved separately for the initial positive energy states and final negative energy states of the diagram. The simplest assumption is that positive energy matter and negative energy matter know nothing about each other and effectively live in separate worlds. The scattering matrix form Feynman diagram like states would thus be simply the tensor product $S \otimes S^{\dagger}$, where S is the S-matrix characterizing the lowest level interactions and identifiable as unitary factor of M-matrix for zero energy states. Reductionism would be realized in the sense that, apart from the new elements brought in by $\Delta_{\mathcal{M}_n}$ defining single particle free dynamics, the lowest level would determine in principle everything occurring at the higher level providing representations about representations about... for what occurs at the basic level. The lowest level would represent the physical world and higher levels the theory about it.

2. The description of hadronic reactions in terms of partons serves as a guide line when one tries to understand higher level Feynman diagrams. The fusion of hadronic space-time sheets corresponds to the vertices $\mathcal{M}_1$. In the vertex the analog of parton plasma is formed by a process known as parton fragmentation. This means that the partonic Feynman diagrams belonging to disjoint copies of $\mathcal{M}_0$ find themselves inside the same copy of $\mathcal{M}_0$. The standard description would apply to the scattering of the initial *resp.* final state partons.

3. After the scattering of partons hadronization takes place. The analog of hadronization in the recent case is the organization of the initial and final state partons to groups I_i and F_i such that the net conserved quantum numbers are same for I_i and F_i. These conditions can be satisfied if the interactions in the plasma phase occur only between particles belonging to the clusters labeled by the index i. Otherwise only single particle states in $\mathcal{M}_1$ would be produced in the reactions in the generic case. The cluster decomposition of S-matrix to a direct sum of terms corresponding to partitions of the initial state particles to clusters which do not interact with each other obviously corresponds to the "hadronization". Therefore no new dynamics need to be introduced.

4. One cannot avoid the question whether the parton picture about hadrons indeed corresponds to a higher level physics of this kind. This would require that hadronic space-time sheets carry the net quantum numbers of hadrons. The net quantum numbers associated with the initial state partons would be naturally identical with the net quantum numbers of hadron. Partons and they negative energy conjugates would provide in this picture a representation of hadron about hadron. This kind of interpretation of partons would make understandable why they cannot be observed directly. A possible objection is that the net gravitational mass of hadron would be three times the gravitational mass deduced from the inertial mass of hadron if partons feed their gravitational fluxes to the space-time sheet carrying Earth's gravitational field.

5. This picture could also relate to the suggested duality between string and parton pictures [K109] . In parton picture hadron is formed from partons represented by space-like 2-surfaces X_i^2 connected by join along boundaries bonds. In string picture partonic 2-surfaces are replaced with string orbits. If one puts positive and negative energy particles at the ends of string diagram one indeed obtains a higher level representation of hadron. If these pictures are dual then also in parton picture positive and negative energies should compensate each other. Interestingly, light-like 3-D causal determinants identified as orbits of partons could be interpreted as orbits of light like string word sheets with "time" coordinate varying in space-like direction.

Scattering of Feynman diagrams at the higher levels of hierarchy

This picture generalizes to the description of higher level Feynman diagrams.

1. Assume that higher level vertices have recursive structure allowing to reduce the Feynman diagrams to ordinary Feynman diagrams by a procedure consisting of finite steps.

2. The lines of diagrams are classified as incoming or outgoing lines according to whether the time orientation of the line is positive or negative. The time orientation is associated with the time parameter t_n characterizing the automorphism $\Delta_{\mathcal{M}_\backslash}^{it_n}$. The incoming and outgoing net quantum numbers compensate each other. These quantum numbers are basically the quantum numbers of the state at the lowest level of the hierarchy.

3. In the vertices the $\mathcal{M}_{n+1}$ particles fuse and $\mathcal{M}_n$ particles form the analog of quark gluon plasma. The initial and final state particles of $\mathcal{M}_n$ Feynman diagram scatter independently and the S-matrix S_{n+1} describing the process is tensor product $S_n \otimes S_n^\dagger$. By the clustering property of S-matrix, this scattering occurs only for groups formed by partons formed by the incoming and outgoing particles $\mathcal{M}_n$ particles and each outgoing $\mathcal{M}_{n+1}$ line contains and irreducible $\mathcal{M}_n$ diagram. By continuing the recursion one finally ends down with ordinary Feynman diagrams.

2.7.3 Logic, Beliefs, And Spinor Fields In The World Of Classical Worlds

Beliefs can be characterized as Boolean value maps $\beta_i(p)$ telling whether i believes in proposition p or not. Additional structure is brought in by introducing the map $\lambda_i(p)$ telling whether p is true or not in the environment of i. The task is to find quantum counterpart for this model.

The spectrum of probabilities for outcomes in state function reduction with finite measurement resolution is universal

Consider quantum two-spinor as a model of a system with finite measurement resolution implying that state function reduction do not anymore lead to a spin state with a precise value but that one can only predict the probability distribution for the outcome of measurement. These probabilities can be also interpreted as truth values of a belief in finite cognitive resolution.

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1 = (z^1\overline{z^1} + \overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2} + \overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.

2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as

$$X_1 = (1/2 + n_1 q^{n_2})r \ \ , \ \ X_2 = (1/2 + n_2 q^{n_1})r \ \ .$$

   The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \to \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \to \infty$.

3. The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2+N_1 n, 1/2+N_2 n)/[1+(N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. This also conforms with the frequency interpretation for probabilities. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \to \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.

4. A possible interpretation for the fuzziness is in terms of finite measurement resolution. The quantized probabilities could be assigned with diagonalized density matrix regarded as matrix with elements which are commuting hermitian operators. The generalized eigenvalues would be eigenvalues spectra. States would not be pure expect at the limits$N_1 \gg N_2$ and $N_2 \gg N_1$. The non-purity of the state could be understood in terms of entanglement with the degrees of freedom below measurement resolution describable in terms of inclusion of von Neumann algebras. One could perhaps say that in finite measurement resolution the outcome of state function reduction is always non-pure state characterized by a universal density matrix obtained by tracing over non-visible degrees of freedom.

WCW spinors as logic statements

In TGD framework the infinite-dimensional WCW (CH) spinor fields defined in CH, the "world of classical worlds", describe quantum states of the Universe [K128] . WCW spinor field can be regarded as a state in infinite-dimensional Fock space and are labeled by a collection of various two valued indices like spin and weak isospin. The interpretation is as a collection of truth values of logic statements one for each fermionic oscillator operator in the state. For instance, spin up and down would correspond to two possible truth values of a proposition characterized by other quantum numbers of the mode.

The hierarchy of space-time sheet could define a physical correlate for the hierarchy of higher order logics (statements about statements about...). The space-time sheet containing N fermions topologically condensed at a larger space-time sheet behaves as a fermion or boson depending on whether N is odd or even. This hierarchy has also a number theoretic counterpart: the construction of infinite primes [K107] corresponds to a repeated second quantization of a super-symmetric quantum field theory.

Quantal description of beliefs

The question is whether TGD inspired theory of consciousness allows a fundamental description of beliefs.

1. Beliefs define a model about some subsystem of universe constructed by the believer. This model can be understood as some kind of representation of real word in the state space representing the beliefs.

2. One can wonder what is the difference between real and p-adic variants of WCW spinor fields and whether they could represent reality and beliefs about reality. WCW spinors (as opposed to spinor fields) are constructible in terms of fermionic oscillator operators and seem to be universal in the sense that one cannot speak about p-adic and real WCW spinors as different objects. Real/p-adic spinor fields however have real/p-adic space-time sheets as arguments. This would suggest that there is no fundamental difference between the logic statements represented by p-adic and real WCW spinors.

3. This vision is realized if the intersection of reality and various p-adicities corresponds to an algebraically universal set of consisting of partonic 2-surfaces and string world sheets for which defining parameters are WCW coordinates in an algebraic extension of rationals defining that for p-adic number fields. Induced spinor fields would be localized at string world sheets and their intersections with partonic 2-surfaces and would be number theoretically universal. If second quantized induced spinor fields are correlates of Boolean cognition, which is behind the entire mathematics, their number theoretical universality is indeed a highly natural condition. Also fermionic anticommutation relations are number theoretically universal. By conformal invariance the conformal moduli of string world sheets and partonic 2-surface would be the natural WCW coordinates for the 2-surfaces in question and I proposed their p-adicization already in p-adic mass calculations for two decades ago.

   This picture would provide an elegant realization for the p-adicization. There would be ne need to map real space-time surfaces directly to p-adic ones and vice versa and one would avoid problems related to general coordinate invariance (GCI) completely. Strong form of holography would assign to partonic surfaces the real and p-adic variants. Already p-adic mass calculations support the presence of cognition in all length scales.

These observations suggest a more concrete view about how beliefs emerge physically.

The idea that p-adic WCW spinor fields could serve as representations of beliefs and real WCW spinor fields as representations of reality looks very nice and conforms with the adelic vision that space-time is adele - a book-like structure contains space-time sheets in various number fields as pages glued together along back for which the parameters characterizing space-time surface are numbers in an algebraic extension of rationals. Real space-time surfaces would be correlates for sensory experience and p-adic space-time sheets for cognition.

2.7.4 Jones Inclusions For Hyperfinite Factors Of Type II_1 As A Model For Symbolic And Cognitive Representations

Consider next a more detailed model for how cognitive representations and beliefs are realized at quantum level. This model generalizes trivially to symbolic representations.

The Clifford algebra of gamma matrices associated with WCW spinor fields corresponds to a von Neumann algebra known as hyper-finite factor of type II_1. The mathematics of these algebras is extremely beautiful and reproduces basic mathematical structures of modern physics (conformal field theories, quantum groups, knot and braid groups,....) from the mere assumption that the world of classical worlds possesses infinite-dimensional Kähler geometry and allows spinor structure.

The almost defining feature is that the infinite-dimensional unit matrix of the Clifford algebra in question has by definition unit trace. Type II_1 factors allow also what are known as Jones inclusions of Clifford algebras $\mathcal{N} \subset \mathcal{M}$. What is special to II_1 factors is that the induced unitary mappings between spinor spaces are genuine inclusions rather than 1-1 maps.

The S-matrix associated with the real-to-p-adic quantum transition inducing belief from reality would naturally define Jones inclusion of CH Clifford algebra $\mathcal{N}$ associated with the real space-time sheet to the Clifford algebra $\mathcal{M}$ associated with the p-adic space-time sheet. The moduli squared of S-matrix elements would define probabilities for pairs or real and belief states.

In Jones inclusion $\mathcal{N} \subset \mathcal{M}$ the factor $\mathcal{N}$ is included in factor $\mathcal{M}$ such that $\mathcal{M}$ can be expressed as $\mathcal{N}$-module over quantum space $\mathcal{M}/\mathcal{N}$ which has fractal dimension given by Jones index $\mathcal{M} : \mathcal{N} = 4cos^2(\pi/n) \leq 4$, $n = 3, 4, ...$ varying in the range $[1, 4]$. The interpretation is as the fractal dimension corresponding to a dimension of Clifford algebra acting in $d = \sqrt{\mathcal{M} : \mathcal{N}}$-dimensional spinor space: d varies in the range $[1, 2]$. The interpretation in terms of a quantal variant of logic is natural.

Probabilistic beliefs

For $\mathcal{M} : \mathcal{N} = 4$ $(n = \infty)$ the dimension of spinor space is $d = 2$ and one can speak about ordinary 2-component spinors with $\mathcal{N}$-valued coefficients representing generalizations of qubits. Hence the inclusion of a given $\mathcal{N}$-spinor as M-spinor can be regarded as a belief on the proposition and for the decomposition to a spinor in N-module $\mathcal{M}/\mathcal{N}$ involves for each index a choice $\mathcal{M}/\mathcal{N}$ spinor component selecting super-position of up and down spins. Hence one has a superposition of truth values in general and one can speak only about probabilistic beliefs. It is not clear whether one can choose the basis in such a way that $\mathcal{M}/\mathcal{N}$ spinor corresponds always to truth value 1. Since WCW spinor field is in question and even if this choice might be possible for a single 3-surface, it need not be possible for deformations of it so that at quantum level one can only speak about probabilistic beliefs.

Fractal probabilistic beliefs

For $d < 2$ the spinor space associated with $\mathcal{M}/\mathcal{N}$ can be regarded as quantum plane having complex quantum dimension d with two non-commuting complex coordinates z^1 and z^2 satisfying $z^1z^2 = qz^2z^1$ and $\overline{z^1z^2} = \overline{q}\overline{z^2z^1}$. These relations are consistent with hermiticity of the real and imaginary parts of z^1 and z^2 which define ordinary quantum planes. Hermiticity also implies that one can identify the complex conjugates of z^i as Hermitian conjugates.

The further commutation relations $[z^1, \overline{z^2}] = [z^2, \overline{z^1}] = 0$ and $[z^1, \overline{z^1}] = [z^2, \overline{z^2}] = r$ give a closed algebra satisfying Jacobi identities. One could argue that $r \geq 0$ should be a function $r(n)$ of the quantum phase $q = exp(i2\pi/n)$ vanishing at the limit $n \to \infty$ to guarantee that the algebra becomes commutative at this limit and truth values can be chosen to be non-fuzzy. $r = sin(\pi/n)$ would be the simplest choice. As will be found, the choice of $r(n)$ does not however affect at all the spectrum for the probabilities of the truth values. $n = \infty$ case corresponding to non-fuzzy quantum logic is also possible and must be treated separately: it corresponds to Kac Moody algebra instead of quantum groups.

The non-commutativity of complex spinor components means that z^1 and z^2 are not independent coordinates: this explains the reduction of the number of the effective number of truth values to $d < 2$. The maximal reduction occurs to $d = 1$ for $n = 3$ so that there is effectively only single truth value and one could perhaps speak about taboo or dogma or complete disappearance of the notions of truth and false (this brings in mind reports about meditative states: in fact $n = 3$ corresponds to a phase in which Planck constant becomes infinite so that the system is maximally quantal).

As non-commuting operators the components of d-spinor are not simultaneously measurable for $d < 2$. It is however possible to measure simultaneously the operators describing the probabilities $z^1\overline{z^1}$ and $z^2\overline{z^2}$ for truth values since these operators commute. An inherently fuzzy Boolean logic would be in question with the additional feature that the spinorial counterparts of statement and its negation cannot be regarded as independent observables although the corresponding probabilities satisfy the defining conditions for commuting observables.

If one can speak of a measurement of probabilities for $d < 2$, it differs from the ordinary quantum measurement in the sense that it cannot involve a state function reduction to a pure qubit meaning irreducible quantal fuzziness. One could speak of fuzzy qbits or fqbits (or quantum qbits) instead of qbits. This picture would provide the long sought interpretation for quantum groups.

The previous picture applies to all representations $M_1 \subset M_2$, where M_1 and M_2 denote either real or p-adic Clifford algebras for some prime p. For instance, real-real Jones inclusion could be interpreted as symbolic representations assignable to a unitary mapping of the states of a subsystem M_1 of the external world to the state space M_2 of another real subsystem. $p_1 \to p_2$ unitary inclusions would in turn map cognitive representations to cognitive representations. There is a strong temptation to assume that these Jones inclusions define unitary maps realizing universe as a universal quantum computer mimicking itself at all levels utilizing cognitive and symbolic representations. Subsystem-system inclusion would naturally define one example of Jones inclusion.

The spectrum of probabilities of truth values is universal

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1 = (z^1\overline{z^1} + \overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2} + \overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.

2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as

   $$X_1 = (1/2 + n_1 q^{n_2})r \quad , \quad X_2 = (1/2 + n_2 q^{n_1})r \quad .$$

   The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \to \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \to \infty$.

3. The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2+N_1 n, 1/2+N_2 n)/[1+(N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. This

also conforms with the frequency interpretation for probabilities. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \to \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.

4. A possible interpretation for the fuzziness is in terms of finite measurement resolution. The quantized probabilities could be assigned with diagonalized density matrix regarded as matrix with elements which are commuting hermitian operators. The generalized eigenvalues would be eigenvalues spectra. States would not be pure expect at the limits$N_1 \gg N_2$ and $N_2 \gg N_1$. The non-purity of the state could be understood in terms of entanglement with the degrees of freedom below measurement resolution describable in terms of inclusion of von Neumann algebras. One could perhaps say that in finite measurement resolution the outcome of state function reduction is always non-pure state characterized by a universal density matrix obtained by tracing over non-visible degrees of freedom.

How to define variants of belief quantum mechanically?

Probabilities of true and false for Jones inclusion characterize the plausibility of the belief and one can ask whether this description is enough to characterize states such as knowledge, misbelief, doubt, delusion, and ignorance. The truth value of $\beta_i(p)$ is determined by the measurement of probability assignable to Jones inclusion on the p-adic side. The truth value of $\lambda_i(p)$ is determined by a similar measurement on the real side. β and λ appear completely symmetrically and one can consider all kinds of triplets $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ assuming that there exist unitary S-matrix like maps mediating a sequence $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ of Jones inclusions. Interestingly, the hierarchies of Jones inclusions are a key concept in the theory of hyper-finite factors of type II_1 and pair of inclusions plays a fundamental role.

Let us restrict the consideration to the situation when $\mathcal{M}_1$ corresponds to a real subsystem of the external world, $\mathcal{M}_2$ its real representation by a real subsystem, and $\mathcal{M}_3$ to p-adic cognitive representation of M_3. Assume that both real and p-adic sides involve a preferred state basis for qubits representing truth and false.

Assume first that both $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\mathcal{M}_2 \subset \mathcal{M}_3$ correspond to $d = 2$ case for which ordinary quantum measurement or truth value is possible giving outcome true or false. Assume further that the truth values have been measured in both M_2 and M_3.

1. Knowledge corresponds to the proposition $\beta_i(p) \wedge \lambda_i(p)$.

2. Misbelief to the proposition $\beta_i(p) \wedge \neq \lambda_i(p)$.
   Knowledge and misbelief would involve both the measurement of real and p-adic probabilities
   .

3. Assume next that one has $d < 2$ form $\mathcal{M}_2 \subset \mathcal{M}_3$. Doubt can be regarded neither belief or disbelief: $\beta_i(p) \wedge \neq \beta_i(\neq p)$: belief is inherently fuzzy although proposition can be non-fuzzy.

   Assume next that truth values in $\mathcal{M}_1 \subset \mathcal{M}_2$ inclusion corresponds to $d < 2$ so that the basic propositions are inherently fuzzy.

4. Delusion is a belief which cannot be justified: $\beta_i(p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. This case is possible if $d = 2$ holds true for $\mathcal{M}_2 \subset \mathcal{M}_3$. Note that also misbelief that cannot be shown wrong is possible.
   In this case truth values cannot be quantum measured for $\mathcal{M}_1 \subset \mathcal{M}_2$ but can be measured for $\mathcal{M}_2 \subset \mathcal{M}_3$. Hence the states are products of pure $\mathcal{M}_3$ states with fuzzy $\mathcal{M}_2$ states.

5. Ignorance corresponds to the proposition $\beta_i(p) \wedge \neq \beta_i(\neq p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. Both real representational states and belief states are inherently fuzzy.

Quite generally, only for $d_1 = d_2 = 2$ ideal knowledge and ideal misbelief are possible. Fuzzy beliefs and logics approach to ordinary one at the limit $n \to \infty$, which according to the proposal of [K103] corresponds to the ordinary value of Planck constant. For other cases these notions are only approximate and quantal approach allows to characterize the goodness of the approximation.

A new kind of inherent quantum uncertainty of knowledge is in question and one could speak about a Uncertainty Principle for cognition and symbolic representations. Also the unification of symbolic and various kinds of cognitive representations deserves to be mentioned.

2.7.5 Intentional Comparison Of Beliefs By Topological Quantum Computation?

Intentional comparison would mean that for a given initial state also the final state of the quantum jump is fixed. This requires the ability to engineer S-matrix so that it leads from a given state to single state only. Any S-matrix representing permutation of the initial states fulfills these conditions. This condition is perhaps unnecessarily strong.

Quantum computation is basically the engineering of S-matrix so that it represents a superposition of parallel computations. In TGD framework topological quantum computation based on the braiding of magnetic flux tubes would be represented as an evolution characterized by braid [K5] . The dynamical evolution would be associated with light-like boundaries of braids. This evolution has dual interpretations either as a limit of time evolution of quantum state (program running) or a quantum state satisfying conformal invariance constraints (program code).

The dual interpretation would mean that conformally invariant states are equivalent with engineered time evolutions and topological computation realized as braiding connecting the quantum states to be compared (beliefs represented as many-fermion states at the boundaries of magnetic flux tubes) could give rise to conscious computational comparison of beliefs. The complexity of braiding would give a measure for how much the states to be compared differ.

Note that quantum computation is defined by a unitary map which could also be interpreted as symbolic representation of states of system M_1 as states of system M_2 mediated by the braid of join along boundaries bonds connecting the two space-time sheets in question and having light-like boundaries. These considerations suggest that the idea about S-matrix of the Universe should be generalized so that the dynamics of the Universe is dynamics of mimicry described by an infinite collection of fermionic S-matrices representable in terms of Jones inclusions.

2.7.6 The Stability Of Fuzzy Qbits And Quantum Computation

The stability of fqbits against state function reduction might have deep implications for quantum computation since quantum spinors would be stable against state function reduction induced by the perturbations inducing de-coherence in the normal situation. If this is really true, and if the only dangerous perturbations are those inducing the phase transition to qbits, the implications for quantum computation could be dramatic. Of course, the rigidity of qbits could be just another way to say that topological quantum computations are stable against thermal perturbations not destroying anyons [K5] .

The stability of fqbits could also be another manner to state the stability of rational, or more generally algebraic, bound state entanglement against state function reduction, which is one of the basic hypothesis of TGD inspired theory of consciousness [K70] . For sequences of Jones inclusions or equivalently, for multiple Connes tensor products, one would obtain tensor products of quantum spinors making possible arbitrary complex configurations of fqbits. Anyonic braids in topological quantum computation would have interpretation as representations for this kind of tensor products.

2.7.7 Fuzzy Quantum Logic And Possible Anomalies In The Experimental Data For The EPR-Bohm Experiment

The experimental data for EPR-Bohm experiment [J13] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [J4] . The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The anomaly

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i,j), where i,j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha,\beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha-\beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha-\beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [J13] are not consistent with this prediction [J5] and this is identified as the anomaly.
2. The QM prediction $E(\alpha,\beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha-\beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [J5] .

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly b) but not anomaly a). A "mundane" explanation for anomaly a) is proposed.

Predictions of fuzzy quantum logic for the probabilities and correlations

1. The description of fuzzy quantum logic in terms statistical ensemble

The fuzzy quantum logic implies that the predictions $P_{i,j}$ for the probabilities should be replaced with ensemble averages over the ensembles defined by fuzzy quantum logic. In practice this means that following replacements should be carried out:

$$\begin{aligned} P_{i,j} &\rightarrow P^2 P_{i,j} + (1-P)^2 P_{i+1,j+1} \\ &+ P(1-P)\left[P_{i,j+1} + P_{i+1,j}\right] \ . \end{aligned} \tag{2.7.1}$$

Here P is one of the state dependent universal probabilities/fuzzy truth values for some value of n characterizing the measurement situation. The concrete predictions would be following

$$\begin{aligned} P_{0,0} = P_{1,1} &\rightarrow A\frac{cos^2(\alpha-\beta)}{2} + B\frac{sin^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{cos^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ P_{0,1} = P_{1,0} &\rightarrow A\frac{sin^2(\alpha-\beta)}{2} + B\frac{cos^2(\alpha-\beta)}{2} \\ &= (A-B)\frac{sin^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ A &= P^2 + (1-P)^2 \ , \ B = 2P(1-P) \ . \end{aligned} \tag{2.7.2}$$

The prediction is that the graphs of probabilities as a function as function of the angle $\alpha-\beta$ are scaled by a factor $1-4P(1-P)$ and shifted upwards by $P(1-P)$. The value of P, and one might hope even the value of n labeling Jones inclusion and the integer m labeling the quantum state might be deducible from the experimental data as the upward shift. The basic prediction is that the maxima of curves measuring probabilities $P_(i,j)$ have minimum at $B/2 = P(1-P)$ and maximum is scaled down to $(A-B)/2 = 1/2 - 2P(1-P)$.

If the P is same for all pairs i,j, the correlation $E = \sum_i (P_{ii} - P_{i,i+1})$ transforms as

$$E(\alpha,\beta) \rightarrow \left[1-4P(1-P)\right] E(\alpha,\beta) \ . \tag{2.7.3}$$

Only the normalization of $E(\alpha,\beta)$ as a function of $\alpha-\beta$ reducing the magnitude of E occurs. In particular the maximum/minimum of E are scaled down from $E=\pm 1$ to $E=\pm(1-4P(1-P))$.

From the figure 1b) of [J5] the scaling down indeed occurs for magnitudes of E with same amount for minimum and maximum. Writing $P=1-\epsilon$ one has $A-B\simeq 1-4\epsilon$ and $B\simeq 2\epsilon$ so that the maximum is in the first approximation predicted to be at $1-4\epsilon$. The graph would give $1-P\simeq\epsilon\simeq .025$. Thus the model explains the reduction of the magnitude for the maximum and minimum of E which was not however considered to be an anomaly in [J4, J5] .

A further prediction is that the identities $P(i,i)+P(i+1,i)=1/2$ should still hold true since one has $P_{i,i}+P_{i,i+1}=(A-B)/2+B=1$. This is implied also by probability conservation. The four curves corresponding to these identities do not however co-incide as the figure 6 of [J5] demonstrates. This is regarded as the basic anomaly in [J4, J5] . From the same figure it is also clear that below $\alpha-\beta<10$ degrees $P_{++}=P_{--}$ $\Delta P_{+-}=-\Delta P_{-+}$ holds true in a reasonable approximation. After that one has also non-vanishing ΔP_{ii} satisfying $\Delta P_{++}=-\Delta P_{--}$. This kind of splittings guarantee the identity $\sum_{ij}P_{ij}=1$. These splittings are not visible in E.

Since probability conservation requires $P_{ii}+P_{ii+1}=1$, a mundane explanation for the discrepancy could be that the failure of the conditions $P_{i,i}+P_{ii+1}=1$ means that the measurement efficiency is too low for P_{+-} and yields too low values of $P_{+-}+P_{--}$ and $P_{+-}+P_{++}$. The constraint $\sum_{ij}P_{ij}=1$ would then yield too high value for P_{-+}. Similar reduction of measurement efficiency for P_{++} could explain the splitting for $\alpha-\beta>10$ degrees.

Clearly asymmetry with respect to exchange of photons or of detectors is in question.

1. The asymmetry of two photon state with respect to the exchange of photons could be considered as a source of asymmetry. This would mean that the photons are not maximally entangled. This could be seen as an alternative "mundane" explanation.

2. The assumption that the parameter P is different for the detectors does not change the situation as is easy to check.

3. One manner to achieve splittings which resemble observed splittings is to assume that the value of the probability parameter P depends on the *polarization pair*: $P=P(i,j)$ so that one has $(P(-,+),P(+,-))=(P+\Delta,P-\Delta)$ and $(P(-,-),P(+,+))=(P+\Delta_1,P-\Delta_1)$. $\Delta\simeq .025$ and $\Delta_1\simeq\Delta/2$ could produce the observed splittings qualitatively. One would however always have $P(i,i)+P(i,i+1)\geq 1/2$. Only if the procedure extracting the correlations uses the constraint $\sum_{i,j}P_{ij}=1$ effectively inducing a constant shift of P_{ij} downwards an asymmetry of observed kind can result. A further objection is that there are no special reason for the values of $P(i,j)$ to satisfy the constraints.

2. Is it possible to say anything about the value of n in the case of EPR-Bohm experiment?

To explain the reduction of the maximum magnitudes of the correlation E from 1 to $\sim .9$ in the experiment discussed above one should have $p_1\simeq .9$. It is interesting to look whether this allows to deduce any information about the valued of n. At the limit of large values of N_in one would have $(N_1-N_2)/(N_1+N_2)\simeq .4$ so that one cannot say anything about n in this case. $(N_1,N_2)=(3,1)$ satisfies the condition exactly. For $n=3$, the smallest possible value of n, this would give $p_1\simeq .88$ and for $n=4$ $p_1=.41$. With high enough precision it might be possible to select between $n=3$ and $n=4$ options if small values of N_i are accepted.

2.7.8 Category Theoretic Formulation For Quantum Measurement Theory With Finite Measurement Resolution?

I have been trying to understand whether category theory might provide some deeper understanding about quantum TGD, not just as a powerful organizer of fuzzy thoughts but also as a tool providing genuine physical insights. Marni Dee Sheppeard (or Kea in her blog Arcadian Functor at `http://tinyurl.com/yb3lsbjq`) is also interested in categories but in much more technical sense. Her dream is to find a category theoretical formulation of M-theory as something, which is not the 11-D something making me rather unhappy as a physicist with second foot still deep in the muds of low energy phenomenology.

Locales, frames, Sierpinski topologies and Sierpinski space

The ideas below popped up when Kea mentioned in M-theory lesson 51 the notions of locale and frame [A5] . In Wikipedia I learned that complete Heyting algebras, which are fundamental to category theory, are objects of three categories with differing arrows. CHey, Loc and its opposite category Frm (arrows reversed). Complete Heyting algebras are partially ordered sets which are complete lattices. Besides the basic logical operations there is also algebra multiplication (I have considered the possible role of categories and Heyting algebras in TGD in [K28]). From Wikipedia I also learned that locales and the dual notion of frames form the foundation of pointless topology [A11] . These topologies are important in topos theory which does not assume axiom of choice.

The so called particular point topology [A9] assumes a selection of single point but I have the physicist's feeling that it is otherwise rather near to pointless topology. Sierpinski topology [A15] is this kind of topology. Sierpinski topology is defined in a simple manner: the set is open only if it contains a given preferred point p. The dual of this topology defined in the obvious sense exists also. Sierpinski space consisting of just two points 0 and 1 is the universal building block of these topologies in the sense that a map of an arbitrary space to Sierpinski space provides it with Sierpinski topology as the induced topology. In category theoretical terms Sierpinski space is the initial object in the category of frames and terminal object in the dual category of locales. This category theoretic reductionism looks highly attractive.

Particular point topologies, their generalization, and number theoretical braids

Pointless, or rather particular point topologies might be very interesting from physicist's point of view. After all, every classical physical measurement has a finite space-time resolution. In TGD framework discretization by number theoretic braids replaces partonic 2-surface with a discrete set consisting of algebraic points in some extension of rationals: this brings in mind something which might be called a topology with a set of particular algebraic points. Could this preferred set belongs to any open set in the particular point topology appropriate in this situation?

Perhaps the physical variant for the axiom of choice could be restricted so that only sets of algebraic points in some extension of rationals can be chosen freely and the choices is defined by the intersection of p-adic and real partonic 2-surfaces and in the framework of TGD inspired theory of consciousness would thus involve the interaction of cognition with the material world. The extension would depend on the position of the physical system in the algebraic evolutionary hierarchy defining also a cognitive hierarchy. Certainly this would fit very nicely to the formulation of quantum TGD unifying real and p-adic physics by gluing real and p-adic number fields to single super-structure via common algebraic points.

Analogs of particular point topologies at the level of state space: finite measurement resolution

There is also a finite measurement resolution in Hilbert space sense not taken into account in the standard quantum measurement theory based on factors of type I. In TGD framework one indeed introduces quantum measurement theory with a finite measurement resolution so that complex rays become included hyper-finite factors of type II_1 (HFFs).

1. Could topology with particular algebraic points have a generalization allowing a category theoretic formulation of the quantum measurement theory without states identified as complex rays?

2. How to achieve this? In the transition of ordinary Boolean logic to quantum logic in the old fashioned sense (von Neuman again!) the set of subsets is replaced with the set of subspaces of Hilbert space. Perhaps this transition has a counterpart as a transition from Sierpinski topology to a structure in which sub-spaces of Hilbert space are quantum subspaces with complex rays replaced with the orbits of subalgebra defining the measurement resolution. Sierpinski space {0,1} would in this generalization be replaced with the quantum counterpart of the space of 2-spinors. Perhaps one should also introduce q-category theory with Heyting algebra being replaced with q-quantum logic.

Fuzzy quantum logic as counterpart for Sierpinksi space

The program formulated above might indeed make sense. The lucky association induced by Kea's blog was to the ideas about fuzzy quantum logic realized in terms of quantum 2-spinor that I had developed a couple of years ago. Fuzzy quantum logic would reflect the finite measurement resolution. I just list the pieces of the argument.

Spinors and qbits: Spinors define a quantal variant of Boolean statements, qbits. One can however go further and define the notion of quantum qbit, qqbit. I indeed did this for couple of years ago (the last section of this chapter).

Q-spinors and qqbits: For q-spinors the two components a and b are not commuting numbers but non-Hermitian operators: $ab = qba$, q a root of unity. This means that one cannot measure both a and b simultaneously, only either of them. $aa^\dagger$ and $bb^\dagger$ however commute so that probabilities for bits 1 and 0 can be measured simultaneously. State function reduction is not possible to a state in which a or b gives zero. The interpretation is that one has q-logic is inherently fuzzy: there are no absolute truths or falsehoods. One can actually predict the spectrum of eigenvalues of probabilities for say 1. Obviously quantum spinors would be state space counterparts of Sierpinski space and for $q \neq 1$ the choice of preferred spinor component is very natural. Perhaps this fuzzy quantum logic replaces the logic defined by the Heyting algebra.

Q-locale: Could one think of generalizing the notion of locale to quantum locale by using the idea that sets are replaced by sub-spaces of Hilbert space in the conventional quantum logic. Q-openness would be defined by identifying quantum spinors as the initial object, q-Sierpinski space. a (resp. b for the dual category) would define q-open set in this space. Q-open sets for other quantum spaces would be defined as inverse images of a (resp. b) for morphisms to this space. Only for q=1 one could have the q-counterpart of rather uninteresting topology in which all sets are open and every map is continuous.

Q-locale and HFFs: The q-Sierpinski character of q-spinors would conform with the very special role of Clifford algebra in the theory of HFFs, in particular, the special role of Jones inclusions to which one can assign spinor representations of $SU(2)$. The Clifford algebra and spinors of the world of classical worlds identifiable as Fock space of quark and lepton spinors is the fundamental example in which 2-spinors and corresponding Clifford algebra serves as basic building brick although tensor powers of any matrix algebra provides a representation of HFF.

Q-measurement theory: Finite measurement resolution (q-quantum measurement theory) means that complex rays are replaced by sub-algebra rays. This would force the Jones inclusions associated with $SU(2)$ spinor representation and would be characterized by quantum phase q and bring in the q-topology and q-spinors. Fuzzyness of qqbits of course correlates with the finite measurement resolution.

Q-n-logos: For other q-representations of SU(2) and for representations of compact groups (Appendix) one would obtain something which might have something to do with quantum n-logos, quantum generalization of n-valued logic. All of these would be however less fundamental and induced by q-morphisms to the fundamental representation in terms of spinors of the world of classical worlds. What would be however very nice that if these q-morphisms are constructible explicitly it would become possible to build up q-representations of various groups using the fundamental physical realization - and as I have conjectured [K96] - McKay correspondence and huge variety of its generalizations would emerge in this manner.

The analogs of Sierpinski spaces: The discrete subgroups of $SU(2)$, and quite generally, the groups Z_n associated with Jones inclusions and leaving the choice of quantization axes invariant, bring in mind the n-point analogs of Sierpinski space with unit element defining the particular point. Note however that $n \geq 3$ holds true always so that one does not obtain Sierpinski space itself. If all these n preferred points belong to any open set it would not be possible to decompose this preferred set to two subsets belonging to disjoint open sets. Recall that the generalized embedding space related to the quantization of Planck constant is obtained by gluing together coverings $M^4 \times CP_2 \to M^4 \times CP_2/G_a \times G_b$ along their common points of base spaces. The topology in question would mean that if some point in the covering belongs to an open set, all of them do so. The interpretation would be that the points of fiber form a single inseparable quantal unit.

Number theoretical braids identified as as subsets of the intersection of real and p-adic variants of algebraic partonic 2-surface define a second candidate for the generalized Sierpinski space with a set of preferred points.

Chapter 3

Does TGD Predict Spectrum of Planck Constants?

3.1 Introduction

The quantization of Planck constant has been the basic them of TGD since 2005 and the perspective in the earlier version of this chapter reflected the situation for about year and one half after the basic idea stimulated by the finding of Nottale [E6] that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$ for the inner planets. The general form of $\hbar_{gr}$ is dictated by Equivalence Principle. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales.

The second crucial empirical input were the anomalies associated with living matter. Mention only the effects of ELF radiation at EEG frequencies on vertebrate brain and anomalous behavior of the ionic currents through cell membrane. If the value of Planck constant is large, the energy of EEG photons is above thermal energy and one can understand the effects on both physiology and behavior. If ionic currents through cell membrane have large Planck constant the scale of quantum coherence is large and one can understand the observed low dissipation in terms of quantum coherence. This approach led to the formula $h_{eff} = n \times h$. Rather recently (2014) it became clear that for microscopic systems the identification $h_{eff} = h_{gr}$ makes sense and predicts universal energy spectrum for cyclotron energies of dark photons identifiable as energy spectrum of bio-photons in TGD inspired quantum biology.

3.1.1 Evolution Of Mathematical Ideas

The original formulation for the hierarchy of Planck constants was in terms of $h_{eff}/h = n$-fold singular coverings of the embedding space $H = M^4 \times CP_2$. Later it turned out that there is no need to postulate these covering spaces although they are a nice auxiliary tool allowing to understand why the phase of matter with different values of n behave like dark matter relative to each other: they are simply at different pages of the book-like structure formed by the covering spaces.

Few years ago it became clear that the hierarchy of Planck constants could be only effective but have the same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $h_{eff} = n \times h$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. This reduces the understanding of the effective hierarchy of Planck constants to quantum variant of multi-furcations for the dynamics of preferred extremals of Kähler action. The number of branches of multi-furcation defines the integer n in $\hbar_{eff} = n\hbar$.

One of the latest steps in the progress was the realization that the hierarchy of Planck constants can be understood in terms of quantum criticality of TGD Universe postulated from the beginning as a way to obtain a unique theory. In accordance with what is known about 2-D critical systems, quantum criticality should correspond to a generalization of conformal invariance. TGD indeed predicts several analogs of super-conformal algebras: so called super-symplectic algebra acting in $\delta M^4_\pm \times CP_2$ should act as isometries of WCW and its generators are labeled by conformal weights. Light-cone boundary $\delta M^4_\pm$ has an extension of conformal symmetries as conformal symmetries and an algebra isomorphic to the ordinary conformal algebra acts as its isometries. The light-like orbits of partonic 2-surfaces allow similar algebra of conformal symmetries and string world sheets and partonic 2-surfaces allow conformal symmetries.

The proposal is that super-symplectic algebra (at least it) defines a hierarchy of broken super-conformal gauge symmetries in the sense that the sub-algebra for which the conformal weights are n-ples of those for the entire algebra acts as gauge conformal symmetries. $n = h_{eff}/h$ giving a connection to the hierarchy of Planck constants would hold true. These sub-algebras are isomorphic to the full algebra and thus form a fractal hierarchy. One has infinite number of hierarchies of broken conformal symmetries defined by the sequences $n(i+1) = m_i \times n(i)$. In the phase transition increasing n conformal gauge symmetry is reduced and some gauge degrees of freedom transform to physical ones and criticality is reduced so that the transition takes place spontaneously. TGD Universe is like a ball at the top of hill at the top of hill at....

This view has far reaching implication for the understanding of living matter and leads to deep connections between different key ideas of TGD. The hierarchy has also a purely number theoretical interpretation in terms of hierarchy of algebraic extensions of rationals appearing naturally in the adelic formulation of quantum TGD. $n = h_{eff}/h$ would naturally correspond to an integer, which is product of so called ramified primes (rational primes for which the decomposition to primes of extension contains higher powers of these primes).

In this framework it becomes obvious that - instead of coverings of embedding space postulated in the original formulation - one has space-time surfaces representable as singular n-fold coverings. The non-determinism of Kähler action - key element of criticality - would be the basic reason for the appearance of singular coverings: two 3-surfaces at the opposite boundaries of CD are connected by n-sheeted space-time surfaces for which the sheets co-incide at the boundaries. Criticality must be accompanied by 4-D variant of conformal gauge invariance already described so that these space-time surfaces are replaced by conformal gauge equivalence classes.

These coverings are highly analogous to the covering space associated with the analytic function $w(z) = z^{1/n}$. If one uses w as a variable, the ordinary conformal symmetries generated by functions of z indeed correspond to the algebra generated by w^n and the sheets of covering correspond to conformal gauge equivalence classes not transformed to each other by conformal transformations.

3.1.2 The Evolution Of Physical Ideas

The evolution of physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The basic idea was that ordinary matter condenses around dark matter which is a phase of matter characterized by non-standard value of Planck constant.

2. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K87] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E6] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with minimum size of order Schwarstchild radius r_S of order scaled up Planck length: $r_S \sim \sqrt{\hbar G}$. Black hole entropy being inversely proportional to $\hbar$ is predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

3. Darkness is a relative concept and due to the fact that particles at different pages of book cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to different page of book are however possible and change Planck constant so that particle exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. This allows to conclude that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K119] , [I69] .

4. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially shocking outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L1, K119] , [L1] .

3.1.3 Basic Physical Picture As It Is Now

The basic phenomenological rules are simple and remained roughly the same during years.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K120].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K87] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. It was [E6] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

3. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

4. The interpretation of the hierarchy of Planck constants as labels for quantum critical systems is especially powerful in TGD inspired quantum biology and consciousness theory. The increase of Planck constant by integer factor occurs spontaneously and means an increase of complexity and sensory and cognitive resolution - in other words evolution. Living matter is however fighting to stay at the existing level of criticality. The reason is that the changes involves state function reduction at the opposite boundary of CD and means death of self followed by re-incarnation.

   Negentropy Maximization Principle [K72] saves the system from this fate if it is able to generate negentropic entanglement by some other means. Metabolic energy suggested already earlier to be a carrier of negentropic entanglement makes this possible. Also other metabolites can carry negentropy. Therefore living systems are eating each other to satisfy the demands of NMP! Why this non-sensical looking Karma's cycle? The sub-systems of self defining sub-selves (mental images) are dying and re-incarnating and generating negentropy: self is a gardener and sub-selves are the fruit trees and the longer self lives, the more fruits are produced. Hence this process, which Buddhist would call attachment to ego is the ways to generate what I have called "Akashic records". Everything has its purpose.

In this chapter I try to summarize the evolution of the ideas related to Planck constant. I have worked hardly to achieve internal consistency but the old theory layers are there and might cause confusion.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

3.2 Experimental Input

In this section basic experimental inputs suggesting a hierarchy of Planck constants and the identification of dark matter as phases with non-standard value of Planck constant are discussed.

3.2.1 Hints For The Existence Of Large $\hbar$ Phases

Quantum classical correspondence suggests the identification of space-time sheets identifiable as quantum coherence regions. Since they can have arbitrarily large sizes, phases with arbitrarily large quantum coherence lengths and arbitrarily long de-coherence times seem to be possible in TGD Universe. In standard physics context this seems highly implausible. If Planck constant can have arbitrarily large values, the situation changes since Compton lengths and other quantum scales are proportional to $\hbar$. Dark matter is excellent candidate for large $\hbar$ phases.

The expression for $\hbar_{gr}$ in the model explaining the Bohr orbits for planets is of form $\hbar_{gr} = GM_1M_2/v_0$ [K103]. This suggests that the interaction is associated with some kind of interface between the systems, perhaps join along boundaries bonds/flux tubes connecting the space-time sheets associated with systems possessing gravitational masses M_1 and M_2. Also a large space-time sheet carrying the mutual classical gravitational field could be in question. This argument generalizes to the case $\hbar/\hbar_0 = Q_1Q_2\alpha/v_0$ in case of generic phase transition to a strongly interacting phase with α describing gauge coupling strength.

There exist indeed some experimental indications for the existence of phases with a large $\hbar$.

1. With inspiration coming from the finding of Nottale [E6] I have proposed an explanation of dark matter as a macroscopic quantum phase with a large value of $\hbar$ [K103]. Any interaction, if sufficiently strong, can lead to this kind of phase. The increase of $\hbar$ would make the fine structure constant α in question small and guarantee the convergence of perturbation series.

2. Living matter could represent a basic example of large $\hbar$ phase [K41, K12]. Even ordinary condensed matter could be "partially dark" in many-sheeted space-time [K43]. In fact, the realization of hierarchy of Planck constants leads to a considerably weaker notion of darkness stating that only the interaction vertices involving particles with different values of Planck constant are impossible and that the notion of darkness is relative notion. For instance, classical interactions and photon exchanges involving a phase transition changing the value of $\hbar$ of photon are possible in this framework.

3. There is claim about a detection in RHIC (Relativistic Heavy Ion Collider in Brookhaven) of states behaving in some respects like mini black holes [C6]. These states could have explanation as color flux tubes at Hagedorn temperature forming a highly tangled state and identifiable as stringy black holes of strong gravitation. The strings would carry a quantum coherent color glass condensate, and would be characterized by a large value of $\hbar$ naturally resulting in confinement phase with a large value of α_s [K104] . The progress in hadronic mass calculations led to a concrete model of color glass condensate of single hadron as many-particle state of super-symplectic gluons [K79, K73] - something completely new from the point of QCD - responsible for non-perturbative aspects of hadron physics. In RHIC events these color glass condensate would fuse to single large condensate. This condensate would be present also in ordinary black-holes and the blackness of black-hole would be darkness.

4. I have also discussed a model for cold fusion based on the assumption that nucleons can be in large $\hbar$ phase. In this case the relevant strong interaction strength is $Q_1Q_2\alpha_{em}$ for two nucleon clusters inside nucleus which can increase $\hbar$ so large that the Compton length of protons becomes of order atomic size and nuclear protons form a macroscopic quantum phase [K43, K41].

3.2.2 Quantum Coherent Dark Matter And $\hbar$

The argument based on gigantic value of $\hbar_{gr}$ explaining darkness of dark mater is attractive but one should be very cautious.

Consider first ordinary QED$e = \sqrt{\alpha 4\pi\hbar}$ appears in vertices so that perturbation expansion in powers of $\sqrt{\hbar}$ basically. This would suggest that large $\hbar$ leads to large effects. All predictions are however in powers of alpha and large $\hbar$ means small higher order corrections. What happens can be understood on basis of dimensional analysis. For instance, cross sections are proportional to $(\hbar/m)^2$, where m is the relevant mass and the remaining factor depends on $\alpha = e^2/(4\pi\hbar)$ only. In the more general case tree amplitudes with n vertices are proportional to e^n and thus to $\hbar^{n/2}$ and loop corrections give only powers of α which get smaller when $\hbar$ increases. This must relate to the powers of $1/\hbar$ from the integration measure associated with the momentum loop integrals affected by the change of α.

Consider now the effects of the scaling of $\hbar$. The scaling of Compton lengths and other quantum kinematical parameters is the most obvious effect. An obvious effect is due to the change of $\hbar$ in the commutation relations and in the change of unit of various quantum numbers. In particular, the right hand side of oscillator operator commutation and anti-commutation relations is scaled. A further effect is due to the scaling of the eigenvalues of the Kähler-Dirac operator $\hbar\Gamma^\alpha D_\alpha$.

The exponent $exp(K)$ of Kähler function K defining perturbation series in WCW degrees of freedom is proportional to $1/g_K^2$ and does not depend on $\hbar$ at all if there is only single Planck constant. The propagator is proportional to g_K^2. This can be achieved also in QED by absorbing e from vertices to e^2 in photon propagator. Hence it would seem that the dependence on α_K (and $\hbar$) must come from vertices which indeed involve Jones inclusions of the II_1 factors of the incoming and outgoing lines.

This however suggests that the dependence of the scattering amplitudes on $\hbar$ is purely kinematical so that all higher radiative corrections would be absent. This seems to leave only one option: the scale factors of covariant CD and CP_2 metrics can vary and might have discrete spectrum of values.

1. The invariance of Kähler action with respect to overall scaling of metric however allows to keep CP_2 metric fixed and consider only a spectrum for the scale factors of M^4 metric.

2. The first guess motivated by Schrödinger equation is that the scaling factor of covariant CD metric corresponds the ratio $r^2 = (\hbar/\hbar_0)^2$. This would mean that the value of Kähler action depends on r^2. The scaling of M^4 coordinate by r the metric reduces to the standard form but if causal diamonds with quantized temporal distance between their tips are the basic building blocks of WCW geometry as zero energy ontology requires, this scaling of $\hbar$ scales the size of CD by r so that genuine effect results since M^4 scalings are not symmetries of Kähler action.

3. In this picture r would code for radiative corrections to Kähler function and thus space-time physics. Even in the case that the radiative corrections to WCW functional integral vanish, as suggested by quantum criticality, they would be actually taken into account.

This kind of dynamics is not consistent with the original view about embedding space and forces to generalize the notion of embedding spaces since it is clear that particles with different Planck constants cannot appear in the same vertex of Feynman diagram. Somehow different values of Planck constant must be analogous to different pages of book having almost copies of embedding space as pages. A possible resolution of the problem cames from the realization that the fundamental structure might be the inclusion hierarchy of number theoretical Clifford algebras from which entire TGD could emerge including generalization of the embedding space concept.

3.2.3 The Phase Transition Changing The Value Of Planck Constant As A Transition To Non-Perturbative Phase

A phase transition increasing $\hbar$ as a transition guaranteeing the convergence of perturbation theory

The general vision is that a phase transition increasing $\hbar$ occurs when perturbation theory ceases to converge. Very roughly, this would occur when the parameter $x = Q_1Q_2\alpha$ becomes larger than one. The net quantum numbers for "spontaneously magnetized" regions provide new natural units for quantum numbers. The assumption that standard quantization rules prevail poses very strong restrictions on allowed physical states and selects a subspace of the original configuration space. One can of course, consider the possibility of giving up these rules at least partially in which case a spectrum of fractionally charged anyon like states would result with confinement guaranteed by the fractionization of charges.

The necessity of large $\hbar$ phases has been actually highly suggestive since the first days of quantum mechanics. The classical looking behavior of macroscopic quantum systems remains still a poorly understood problem and large $\hbar$ phases provide a natural solution of the problem.

In TGD framework quantum coherence regions correspond to space-time sheets. Since their sizes are arbitrarily large the conclusion is that macroscopic and macro-temporal quantum coherence are possible in all scales. Standard quantum theory definitely fails to predict this and the conclusion is that large $\hbar$ phases for which quantum length and time scales are proportional to $\hbar$ and long are needed.

Somewhat paradoxically, large $\hbar$ phases explain the effective classical behavior in long length and time scales. Quantum perturbation theory is an expansion in terms of gauge coupling strengths inversely proportional to $\hbar$ and thus at the limit of large $\hbar$ classical approximation becomes exact. Also the Coulomb contribution to the binding energies of atoms vanishes at this limit. The fact that we experience world as a classical only tells that large $\hbar$ phase is essential for our sensory perception. Of course, this is not the whole story and the full explanation requires a detailed anatomy of quantum jump.

The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar_{gr} = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength $\alpha Q_i Q_j$, where α is the appropriate coupling strength and $Q_i Q_j$ represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician. A primitive formulation for this criterion is the condition $\alpha Q_i Q_j \geq 1$.

The first working hypothesis was the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule turned out to be quite too specific. The mathematically plausible formulation predicts that in principle any rational value for $r = \hbar(M^4)/\hbar(CP_2)$ is possible but there are certain number theoretically preferred values of r such as those coming powers of 2.

3.3 A Generalization of the Notion of Embedding Space as a Realization of the Hierarchy of Planck Constants

In the following the basic ideas concerning the realization of the hierarchy of Planck constants are summarized and after that a summary about generalization of the embedding space is given. In [K87] the important delicacies associated with the Kähler structure of generalized embedding space are discussed. The background for the recent vision is quite different from that for half decade ago. Zero energy ontology and the notion of causal diamond, number theoretic compactification leading to the precise identification of number theoretic braids, the realization of number theoretic universality, and the understanding of the quantum dynamics at the level of Kähler-Dirac action fix to a high degree the vision about generalized embedding space.

3.3.1 Basic Ideas

The first key idea in the geometric realization of the hierarchy of Planck constants emerges from the study of Schrödinger equation and states that Planck constant appears a scaling factor of M^4 metric. Second key idea is the connection with Jones inclusions inspiring an explicit formula for Planck constants. For a long time this idea remained heuristic must-be-true feeling but the recent view about quantum TGD provide a justification for it.

Scaling of Planck constant and scalings of CD and CP_2 metrics

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term has no dependence on it. This makes the scaling of $\hbar$ a non-trivial transformation. If the contravariant metric scales as $r = \hbar/\hbar_0$ the effect of scaling of Planck constant is realized at the level of embedding space geometry provided it is such that it is possible to compare the regions of generalized embedding space having different value of Planck constant.

In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

1. The minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined in such a way that spinor connection corresponds to the quantity $\hbar eQA$, where denotes A gauge potential, and there is no natural manner to separate $\hbar e$ from it.

2. The contravariant CD metric scales like $\hbar^2$. In the case of Dirac operator in $M^4 \times CP_2$ one can assign separate Planck constants to Poincare and color algebras and the scalings of CD

and CP_2 metrics induce scalings of corresponding values of $\hbar^2$. As far as Kähler action is considered, CP_2 metric could be always thought of being scaled to its standard form.

3. Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^i k_i$ rather than four-momentum squared $p^2 = p^i p_i$ in CD degrees of freedom and its analog in CP_2 degrees of freedom. The values of k^2 are proportional to $1/r^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$: analogous conclusion applies in CP_2 degrees of freedom. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

This consideration generalizes to the case of the induced gamma matrices and induced metric in X^4, Kähler-Dirac operator, and Kähler action which carry dynamical information about the ratio $r = \hbar_{eff}/\hbar_0$.

Kähler function codes for a perturbative expansion in powers of $\hbar(CD)/\hbar(CP_2)$

Suppose that one accepts that the spectrum of CD *resp.* CP_2 Planck constants is accompanied by a hierarchy of overall scalings of covariant CD (causal diamond) metric by $(\hbar(M^4)/\hbar_0)^2$ and CP_2 metric by $(\hbar(CP_2)/\hbar_0)^2$ followed by overall scaling by $r^2 = (\hbar_0/\hbar(CP_2))^2$ so that CP_2 metric suffers no scaling and difficulties with isometric gluing procedure of sectors are avoided.

The first implication of this picture is that the Kähler-Dirac operator determined by the induced metric and spinor structure depends on r in a highly nonlinear manner but there is no dependence on the overall scaling of the H metric. This in turn implies that the fermionic oscillator algebra used to define WCW spinor structure and metric depends on the value of r. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite series in powers of r.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over WCW defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant WCW metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of r. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semi-classical" approximation, which might be actually exact, automatically would give a full expansion in powers of r. This is in a sharp contrast to the usual quantization approach.

Jones inclusions and hierarchy of Planck constants

From the beginning it was clear that Jones inclusions of hyper-finite factors of type II_1 are somehow related to the hierarchy of Planck constants. The basic motivation for this belief has been that WCW Clifford algebra provides a canonical example of hyper-finite factor of type II_1 and that Jones inclusion of these Clifford algebras is excellent candidate for a first principle description of finite measurement resolution.

Consider the inclusion $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 [K127]. A deep result is that one can express $\mathcal{M}$ as $\mathcal{N} : \mathcal{M}$-dimensional module over N with fractal dimension $\mathcal{N} : \mathcal{M} = B_n$. $\sqrt{b_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{b_n} = 2cos(\pi/n)$ varying thus in the range $[1, 2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra. The interpretation is that finite measurement resolution introduces correlations between components of quantum spinor implying effective reduction of the dimension of quantum spinors providing a description of the factor space $\mathcal{N}/\mathcal{M}$.

This would suggest that somehow the hierarchy of Planck constants must represent finite measurement resolution and since phase factors coming as roots of unity are naturally associated with Jones inclusions the natural guess was that angular resolution and coupling constant evolution associated with it is in question. This picture would suggest that the realization of the hierarchy of Planck constant in terms of a book like structure of generalized embedding space provides also a geometric realization for a hierarchy of Jones inclusions.

The notion of number theoretic braid and realization that the modified Dirac operator has only finite number of generalized eigenmodes -thanks to the vacuum degeneracy of Kähler action-finally led to the understanding how the notion of finite measurement resolution is coded to the Kähler action and the realized in practice by second quantization of induced spinor fields and how these spinor fields endowed with q-anti-commutation relations give rise to a representations of finite-quantum dimensional factor spaces $\mathcal{N}/\mathcal{M}$ associated with the hierarchy of Jones inclusions having generalized embedding space as space-time correlate. This means enormous simplification since infinite-dimensional spinor fields in infinite-dimensional world of classical worlds are replaced with finite-quantum-dimensional spinor fields in discrete points sets provided by number theoretic braids.

The study of a concrete model for Jones inclusions in terms of finite subgroups G of $SU(2)$ defining sub-algebras of infinite-dimensional Clifford algebra as fixed point sub-algebras leads to what looks like a correct track concerning the understanding of quantization of Planck constants.

The ADE diagrams of A_n and D_{2n} characterize cyclic and dihedral groups whereas those of E_6 and E_8 characterize tetrahedral and icosahedral groups. This approach leads to the hypothesis that the scaling factor of Planck constant assignable to Poincare (color) algebra corresponds to the order of the maximal cyclic subgroup of $G_b \subset SU(2)$ ($G_a \subset SL(2,C)$) acting as symmetry of space-time sheet in CP_2 (CD) degrees of freedom. It predicts arbitrarily large CD and CP_2 Planck constants in the case of A_n and D_{2n} under rather general assumptions.

There are two ways for how G_a and G_b can act as symmetries corresponding to G_i coverings and factors spaces. These coverings and factor spaces are singular and associated with spaces $\hat{CD}\backslash M^2$ and $CP_2\backslash S^2_I$, where S^2_I is homologically trivial geodesic sphere of CP_2. The physical interpretation is that M^2 and S^2_I fix preferred quantization axes for energy and angular moment and color quantum numbers so that also a connection with quantum measurement theory emerges.

3.3.2 The Vision

A brief summary of the basic vision behind the generalization of the embedding space concept needed to realize the hierarchy of Planck constants is in order before going to the detailed representation.

1. The hierarchy of Planck constants cannot be realized without generalizing the notions of embedding space and space-time because particles with different values of Planck constant cannot appear in the same interaction vertex. Some kind of book like structure for the generalized embedding space forced also by p-adicization but in different sense is suggestive. Both M^4 and CP_2 factors would have the book like structure so that a Cartesian product of books would be in question.

2. The study of Schrödinger equation suggests that Planck constant corresponds to a scaling factor of CD metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in CD factor is possible so that the interpretation for scaled up value of $\hbar$ is as scaling of the size of causal diamond CD.

3. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states, and the infinite-D spaces of light-like 3-surfaces inside scaled variants of CD define the fundamental building brick of WCW (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces the effect of scaling on classical and quantum dynamics is non-trivial and a coupling constant evolution results and the coding of radiative corrections to the geometry of space-time sheets becomes possible. The basic geometry of CD suggests that the allowed sizes of CD come in the basic sector $\hbar = \hbar_0$ as powers of two. This predicts p-adic length scale hypothesis and lead to number theoretically universal discretized p-adic coupling constant evolution. Since the scaling is accompanied by a formation of singular coverings and factor spaces, different scales are distinguished at the level of topology. p-Adic length scale hierarchy affords similar characterization of length scales in terms of effective topology.

4. The idea that TGD Universe is quantum critical in some sense is one of the key postulates of quantum TGD. The basic ensuing prediction is that Kähler coupling strength is analogous to critical temperature. Quantum criticality in principle fixes the p-adic evolution of various coupling constants also the value of gravitational constant. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized embedding space. Quantum criticality of TGD Universe means that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the very definition of number theoretic braids to these critical sub-manifolds. At the boundaries of CD associated with positive and negative energy parts of zero energy state in a given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes to regions corresponding to different values of Planck constant much like matter decomposes to several phases at criticality.

The connection with Jones inclusions was originally a purely heuristic guess, and it took half decade to really understand why and how they are involved. The notion of measurement resolution is the key concept.

1. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and the this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized embedding space could indeed serve as correlates for Jones inclusions.

2. The dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field automatically implying cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups associated with Jones inclusions. The interpretation of the Clifford algebra spanned by the fermionic oscillator operators is as a realization for the concept of the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution for angle measurement in the sense that the action of this algebra on zero energy state has no detectable physical effects. $\mathcal{M}$ takes the role of complex numbers in quantum theory and makes physics non-commutative. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

3. G invariance for the elements of the included algebra can be interpreted in terms of finite measurement resolution in the sense that action by G invariant Clifford algebra element has no detectable effects. Quantum groups realize this view about measurement resolution for angle measurement. The G-invariance of the physical states created by fermionic oscillator operators which by definition are not G invariant guarantees that quantum states as a whole have non-fractional quantum numbers so that the leakage between different pages is possible in principle. This hypothesis is consistent with the TGD inspired model of quantum Hall effect [K87].

4. Concerning the formula for Planck constant in terms of the integers n_a and n_b characterizing orders of the maximal cyclic subgroups of groups G_a and G_b defining coverings and factor spaces associated with CD and CP_2 the basic constraint is that the overall scaling of H metric has no effect on physics. What matters is the ratio of Planck constants $r = \hbar(M^4)/\hbar(CP_2)$ appearing as a scaling factor of M^4 metric. This leaves two options if one requires that the Planck constant defines a homomorphism. The model for dark gravitons suggests a unique choice between these two options but one must keep still mind open for the alternative.

5. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 and CP_2 in some sense. The alternative interpretation assigning the inclusions to the two different geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom and is therefore not convincing.

6. The natural question is why the hierarchy of Planck constants is needed. Is it really necessary? Number theoretic Universality suggests that this is the case. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in the p-adic context. All that one can achieve naturally is the notion of phase defined as a root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases purely geometrically and quantum phases q assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on the hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

3.3.3 Hierarchy Of Planck Constants And The Generalization Of The Notion Of Embedding Space

In the following the recent view about structure of embedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E6] that the orbits of the 4 inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K103] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.

2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K104]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or over-critical mass density and the "pressure" associated with these cosmologies is negative.

3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the embedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.

4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact.

The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K119].

5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K87]. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E6] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr} G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L1, K119], [L1].

The most general option for the generalized embedding space

Simple physical arguments pose constraints on the choice of the most general form of the embedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the Kähler-Dirac gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

(b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the embedding space to another one.

1. How the gluing of copies of embedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.

2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.

3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to n_an_b sheets and multiplies YM action action by n_an_b which is equivalent with the $\hbar = n_an_b\hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_an_b\hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

   This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	n_an_b	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_an_b}$

Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} was proposed to define favored as values of n_a in living matter [K42].

The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1 + i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG [K42]. This hypothesis - to be referred to as Mersenne hypothesis - replaces the rather ad hoc proposal $r = \hbar/\hbar_0 = 2^{11k}$ for the preferred values of Planck constant.

How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the embedding space sector with given values of Planck constants are proportional to the corresponding Planck. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

3.4 Updated View About The Hierarchy Of Planck Constants

During last years the work with TGD proper has transformed from the discovery of brave visions to the work of clock smith. The challenge is to fill in the details, to define various notions more precisely, and to eliminate the numerous inconsistencies.

Few years has passed from the latest formulation for the hierarchy of Planck constant. The original hypothesis was that the hierarchy is real. In this formulation the embedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. It was no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

It seems that the time is ripe for checking whether some polishing of this formulation might be needed. In particular, the work with TGD inspired quantum biology suggests a close connection between the hierarchy of Planck constants and negentropic entanglement. Also the connection with anyons and charge fractionalization (see `http://tinyurl.com/y89xp4bu`) has remained somewhat fuzzy [K87]. In particular, it seems that the formulation based on multi-furcations of space-time surfaces to N branches is not general enough: the N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural: as a matter fact, this interpretation was the original one and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

3.4.1 Basic Physical Ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K120].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a

new interpretation for FQHE (see `http://tinyurl.com/y89xp4bu`) (fractional quantum Hall effect) [K87] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale (see `http://tinyurl.com/ya6f3s4l`) [E6] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

   It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the Kähler-Dirac equation.

4. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

3.4.2 Space-Time Correlates For The Hierarchy Of Planck Constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of embedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the embedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents

$\partial L_K/\partial(\partial_\alpha h^k)$ defining the Kähler-Dirac gamma matrices [K128] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of embedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book). What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches coincide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations (see `http://tinyurl.com/2swb2p`) represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

3.4.3 The Relationship To The Original View About The Hierarchy Of Planck Constants

Originally the hierarchy of Planck constant was assumed to correspond to a book like structure having as pages the n-fold coverings of the embedding space for various values of n serving therefore as a page number. The pages are glued together along a 4-D "back" at which the branches of n-furcations are degenerate. This leads to a very elegant picture about how the particles belonging to the different pages of the book interact. All vertices are local and involve only particles with the same value of Planck constant: this is enough for darkness in the sense of particle physics. The interactions between particles belonging to different pages involve exchange of a particle travelling from page to another through the back of the book and thus experiencing a phase transition changing the value of Planck constant.

Is this picture consistent with the picture based on n-furcations? This seems to be the case. The conservation of energy in n-furcation in which several sheets are realized simultaneously is consistent with the conservation of classical conserved quantities only if the space-time sheet before n-furcation involves n identical copies of the original space-time sheet or if the Planck constant is $h_{eff} = nh$. This kind of degenerate many-sheetedness is encountered also in the case of branes. The first option means an n-fold covering of embedding space and h_{eff} is indeed effective Planck constant. Second option means a genuine quantization of Planck constant due to the fact the value of Kähler coupling strength $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is scaled down by $1/n$ factor. The scaling of Planck constant consistent with classical field equations since they involve α_K as an overall multiplicative factor only.

3.4.4 Basic Phenomenological Rules Of Thumb In The New Framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.

2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.

3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.

4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elementary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

   This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar/m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \to k\hbar$ in the formula $E_n = (n+1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.

2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible

coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) automatically.

2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.

3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

3.4.5 Charge Fractionalization And Anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization (see `http://tinyurl.com/26tmhoe`) the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

   Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionization and fractionization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N + 1$: th branch corresponds to the original one. This suggests that angular momentum fractionization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in embedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionization however leads to problems with fractionization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

3.4.6 Negentropic Entanglement Between Branches Of Multi-Furcations

The application of negentropic entanglement and effective hierarchy of Planck constants to photosynthesis and metabolism (see `http://tinyurl.com/yd7j9f5j`) [K65] suggests that these two notions might be closely related. Negentropic entanglement is possible for rational (and even algebraic) entanglement probabilities. If one allows number theoretic variant of Shannon entropy (see `http://tinyurl.com/y6v73ryc`) based on the p-adic norm for the probability appearing as argument of logarithm [K72], it is quite possible to have negative entanglement entropy and the interpretation is as genuine information carried by entanglement. The superposition of state pairs $a_i \otimes b_i$ in entangled state would represent instances of a rule. In the case of Schrödinger cat the rule states that it is better to not open the bottle: understanding the rule consciously however requires that cat is somewhat dead! Entanglement provides information about the relationship between two systems. Shannon entropy represents lack of information about single particle state.

Negentropic entanglement would replace metabolic energy as the basic quantity making life possible. Metabolic energy could generate negentropic entanglement by exciting biomolecules to negentropically entangled states. ATP providing the energy for generating the metabolic entanglement could also itself carry negentropic entanglement, and transfer it to the target by the emission of large $\hbar$ photons.

How the large $\hbar$ photons could carry negentropic entanglement? There are several options to consider and at this stage it is not possible to pinpoint anyone of them as the only possible one. Several of them could also be realized.

1. In zero energy ontology large $\hbar$ photons could carry the negentropic entanglement as entanglement between positive and negative energy parts of the photon state.

2. The negentropic entanglement of large $\hbar$ photon could be also associated with its positive or energy part or both. Large $\hbar_{eff} = n\hbar$ photon with n-fold energy $E = n \times hf$ is n-sheeted structure consisting of n-photons with energy $E = hf$ de-localized in the discrete space formed by the N space-time sheets. The n single photon states can entangle and since the branches effectively form a discrete space, rational and algebraic entanglement is very natural. There are many options for how this could happen. For instance, for N-fold branching the superposition of all $N!/(N-n)!n!$ states obtained by selecting n branches are possible and the resulting state is entangled state. If this interpretation is correct, the vacuum degeneracy and multi-furcations implied by it would the quintessence of life.

3. A further very attractive possibility discovered quite recently is that large $h_{eff} = nh$ is closely related to the negentropic entanglement between the states of *two* n-furcated - that is dark - space-time sheets. In the most recent formulation negentropic entanglement corresponds to a state characterized by $n \times n$ identity matrix resulting from the measurement of density matrix. The number theoretic entanglement negentropy is positive for primes dividing p and there is unique prime for which it is maximal.

The identification of negentropic entanglement as entanglement between branches of a multi-furcation is not the only possible option.

1. One proposal is that non-localized single particle excitations of cyclotron condensate at magnetic flux tubes give rise to negentropic entanglement relevant to living matter. Dark photons could transfer the negentropic entanglement possibly assignable to electron pairs of ATP molecule.

   The negentropic entanglement associated with cyclotron condensate could be associated with the branches of the large $\hbar$ variant of the condensate. In this case single particle excitation would not be sum of single particle excitations at various positions of 3-space but at various sheet of covering representing points of WCW . If each of the n branches carries $1/n$: th part of electron one would have an anyonic state in WCW .

2. One can also make a really crazy question. Could it be that ATP and various bio-molecules form n-particle states at the n-sheet of n-furcation and that the bio-chemistry involves simultaneous reactions of large numbers of biomolecules at these sheets? If so, the chemical reactions would take place as large number of copies.

Note that in this picture the breaking of time reversal symmetry [K9] in the presence of metabolic energy feed would be accompanied by evolution involving repeated multi-furcations leading to increased complexity. TGD based view about the arrow of time implies that for a given CD this evolution has definite direction of time. At the level of ensemble it implies second law but at the level of individual system means increasing complexity.

3.4.7 Dark Variants Of Nuclear And Atomic Physics

During years I have in rather speculative spirit considered the possibility of dark variants of nuclear and atomic - and perhaps even molecular physics. Also the notion of dark cyclotron state is central in the quantum model of living matter. One such notion is the idea that dark nucleons could realize vertebrate genetic code [K123].

Before the real understanding what charge fractionization means it was possible to imagine several variants of say dark atoms depending on whether both nuclei and electrons are dark or whether only electrons are dark and genuinely fractionally charged. The recent picture however fixes these notions completely. Basic building bricks are just ordinary nuclei and atoms and they form n-particle states associated with n-branches of N-furcation with $n = 1, ..., N$. The fractionization for a single particle state de-localized completely to the discrete space of N branches as the analog of plane wave means that single branch carriers charge $1/N$.

The new element is the possibility of n-particle states populating n branches of the N-furcation: note that there is superposition over the states corresponding to different selections of these n branches. $N-k$ and k-nuclei/atoms are in sense conjugates of each other and they can fuse to form N-nuclei/N-atoms which in fermionic case are analogous to Fermi sea with all states filled.

Bio-molecules seem to obey symbolic dynamics which does not depend much on the chemical properties: this has motivated various linguistic metaphors applied in bio-chemistry to describe the interactions between DNA and related molecules. This motivated the wild speculation was that N-atoms and even N-molecules could make possible the emergence of symbolic representations with $n \leq N$ serving as a name of atom/molecule and that k- and $N-k$ atom/molecule would be analogous to opposite sexes in that there would be strong tendency for them to fuse together to form N-atom/-molecule. For instance, in bio-catalysis k- and $N-k$-atoms/molecules would be paired. The recent picture about n and $N-k$ atoms seems to be consistent with these speculations which I had already given up as too crazy. It is difficult to avoid even the speculation that bio-chemistry could replace chemical reactions with their n-multiples. Synchronized quantum jumps would allow to avoid the disastrous effects of state function reductions on quantum coherence. The second manner to say the same thing is that the effective value of Planck constant is large.

3.4.8 What About The Relationship Of Gravitational Planck Constant To Ordinary Planck Constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g^{\alpha\beta}_{eff}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the Kähler-Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g^{kl}_{eff} = K^2 m^{kl}$.

   One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g^{\alpha\beta}_{eff} p_\alpha p_\beta = g^{\alpha\beta}_{eff} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g^{kl}_{eff} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{3.4.1}$$

   Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g^{kl}_{eff} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2 \hbar^2}{K^2 L^2} \quad .$$

   $\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0} \frac{1}{L} \quad . \tag{3.4.2}$$

   For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of the 4 inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g^{\alpha\beta}_{eff} L_\alpha L_\beta = g^{kl}_{eff} L_k L_l = l(l+1)\hbar^2 \quad . \tag{3.4.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \ . \tag{3.4.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

One might counter argue that if gravitational 4- momentum square is proportional to inertial 4-momentum squared, then Equivalence Principle implies that h_{gr} can have only single value. In ZEO however all wormhole throats - also virtual - are massless and the argument fails. The varying h_{gr} can be assigned only with longitudinal and transversal momentum squared separately but not to the ratio of gravitational and inertial 4-momenta squared which both vanish.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m^{kl}_{eff} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m^{kl}_{eff}.

3.4.9 Hierarchy Of Planck Constants And Non-Determinism Of Kähler Action

Originally the hierarchy of Planck constant was inspired by empirical inputs from neuroscience, biology, and from Nottale's observations. That it is possible to understand the hierarchy in terms of non-determinism of Kähler action - the fundamental difference between TGD and quantum field theories and string models - is a victory for TGD approach (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`, or **Fig. ??** in the appendix of this book).

Recall that non-determinism means that all space-time surfaces with CP_2 projection, which is Lagrangian sub-manifold (at most 2-D) of CP_2, carries a vanishing induced Kähler form and is vacuum extremal. The first guess would be that there is a finite number n of space-time sheets connecting given pair of 3-surfaces at the ends of space-time surface at the light-like boundaries of causal diamond (CD). Planck constant would be given as $h_{eff} = n \times h$ in accordance with the earlier interpretation. The degenerate extremals would have same Kähler action and conserved quantities as assumed also in the earlier approach. That the degenerate extremals co-incide at the ends of space-time surface was motivated by mathemtical aesthetics in the earlier approach but finds an interpretation in terms of non-uniqueness of the preferred extremals.

It is essential that these n degrees of freedom are regarded as genuine physical degrees of freedom, which are however discrete. Negentropic entanglement and dark matter would be associated with them naturally. The effective description would be in terms of n-fold singular covering of embedding space becoming singular at the ends of the space-time surface.

I have also assigned hierarchy of Planck constants with the quantum criticality. Quantum criticality means the existence of an entire continuous family of deformations of space-time sheet with same Kähler action and conserved quantities. The deformations would by definition vanish at the ends of space-time surface. The critical deformations would act as gauge transformations identifiable as conformal symmetries indeed expected to be presents since WCW isometries form a conformal algebra and there is also Kac-Moody type algebra present. The proposal has been that the sub-algebras of conformal algebra for which conformal weights are integer multiples of integer $n = 1, 2, ..$ defined a hierarchy of gauge algebras so that the dynamical algebra reduces to n-dimensional one.

These two identifications seem to be mutually inconsistent. The resolution of the conflict comes from the gauge invariance. For a given Kähler action and conserved quantities there would be n conformal equivalence classes of these 4-surfaces rather than n surfaces, and one would have n-fold degeneracy but for conformal equivalence classes of 4-surfaces rather than 4-surfaces. In Minkowskian regions the degenerate preferred extremals are sheets (graphs of a map from M^4 to CP_2).

3.5 Vision About Dark Matter As Phases With Non-Standard Value Of Planck Constant

3.5.1 Dark Rules

It is useful to summarize the basic phenomenological view about dark matter.

The notion of relative darkness

The essential difference between TGD and more conventional models of dark matter is that darkness is only relative concept.

1. Generalized embedding space forms a book like structure and particles at different pages of the book are dark relative to each other since they cannot appear in the same vertex identified as the partonic 2-surface along which light-like 3-surfaces representing the lines of generalized Feynman diagram meet.

2. Particles at different space-time sheets act via classical gauge field and gravitational field and can also exchange gauge bosons and gravitons (as also fermions) provided these particles can leak from page to another. This means that dark matter can be even photographed [I69]. This interpretation is crucial for the model of living matter based on the assumption that dark matter at magnetic body controls matter visible to us. Dark matter can also suffer a phase transition to visible matter by leaking between the pages of the Big Book.

3. The notion of standard value $\hbar_0$ of $\hbar$ is not a relative concept in the sense that it corresponds to rational $r = 1$. In particular, the situation in which both CD and CP_2 correspond to trivial coverings and factor spaces would naturally correspond to standard physics.

Is dark matter anyonic?

In [K87] a detailed model for the Kähler structure of the generalized embedding space is constructed. What makes this model non-trivial is the possibility that CP_2 Kähler form can have gauge parts which would be excluded in full embedding space but are allowed because of singular covering/factor-space property. The model leads to the conclusion that dark matter is anyonic if the partonic 2-surface, which can have macroscopic or even astrophysical size, encloses the tip of CD within it. Therefore the partonic 2-surface is homologically non-trivial when the tip is regarded as a puncture. Fractional charges for anyonic elementary particles imply confinement to the partonic 2-surface and the particles can escape the two surface only via reactions transforming them to ordinary particles. This would mean that the leakage between different pages of the big book is a rare phenomenon. This could partially explain why dark matter is so difficult to observe.

Field body as carrier of dark matter

The notion of "field body" implied by topological field quantization is essential. There would be em, Z^0, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

p-Adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

3.5.2 Phase Transitions Changing Planck Constant

The general picture is that p-adic length scale hierarchy corresponds to p-adic coupling constant evolution and hierarchy of Planck constants to the coupling constant evolution related to phase resolution. Both evolutions imply a book like structure of the generalized embedding space.

Transition to large $\hbar$ phase and failure of perturbation theory

One of the first ideas was that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces the value of gauge coupling strength $\alpha \propto 1/\hbar$ so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

A justification for this picture would be that in non-perturbative phase large quantum fluctuations are present (as functional integral formalism suggests). At space-time level this could mean that space-time sheet is near to a non-deterministic vacuum extremal -at least if homologically trivial geodesic sphere defines the number theoretic braids. At certain critical value of coupling constant strength one expects that the transition amplitude for phase transition becomes very large. The resulting phase would be of course different from the original since typically charge fractionization would occur.

One should understand why the failure of the perturbation theory (expected to occur for $\alpha Q_1Q_2 > 1$) induces the reduction of Clifford algebra, scaling down of CP_2 metric, and whether the G-symmetry is exact or only approximate. A partial understanding already exists. The discrete G symmetry and the reduction of the dimension of Clifford algebra would have interpretation in terms of a loss of degrees of freedom as a strongly bound state is formed. The multiple covering of $M^4_\pm$ accompanying strong binding can be understood as an automatic consequence of G-invariance. A concrete realization for the binding could be charge fractionization which would not allow the particles bound on large light-like 3-surface to escape without transformation to ordinary particles.

Two examples perhaps provide more concrete view about this idea.

1. The proposed scenario can reproduce the huge value of the gravitational Planck constant. One should however develop a convincing argument why non-perturbative phase for the gravitating dark matter leads to a formation of $G_a\times$ covering of $CD\backslash M^2 \times CP_2\backslash S^2_I$ with the huge value of $\hbar_{eff} = n_a/n_b \simeq GM_1M_2/v_0$. The basic argument is that the dimensionless parameter $\alpha_{gr} = GM_1M_2/4\pi\hbar$ should be so small that perturbation theory works. This gives $\hbar_{gr} \geq GM_1M_2/4\pi$ so that order of magnitude is predicted correctly.

2. Color confinement represents the simplest example of a transition to a non-perturbative phase. In this case A_2 and $n = 3$ would be the natural option. The value of Planck constant would be 3 times higher than its value in perturbative QCD. Hadronic space-time sheets

would be 3-fold coverings of $M^4_\pm$ and baryonic quarks of different color would reside on 3 separate sheets of the covering. This would resolve the color statistics paradox suggested by the fact that induced spinor fields do not possess color as spin like quantum number and by the facts that for orbifolds different quarks cannot move in independent CP_2 partial waves assignable to CP_2 cm degrees of freedom as in perturbative phase.

The mechanism of phase transition and selection rules

The mechanism of phase transition is at classical level similar to that for ordinary phase transitions. The partonic 2-surface decomposes to regions corresponding to difference values of $\hbar$ at quantum criticality in such a way that regions in which induced Kähler form is non-vanishing are contained within single page of embedding space. It might be necessary to assume that only a region corresponding to single value of $\hbar$ is possible for partonic 2-surfaces and $\delta CD \times CP_2$ so that quantum criticality would be associated with the intermediate state described by the light-like 3-surface. One could also see the phase transition as a leakage of X^2 from given page to another: this is like going through a closed door through a narrow slit between door and floor. By quantum criticality the points of number theoretic braid are already in the slit.

As in the case of ordinary phase transitions the allowed phase transitions must be consistent with the symmetries involved. This means that if the state is invariant under the maximal cyclic subgroups G_a and G_b then also the final state must satisfy this condition. This gives constraints to the orders of maximal cyclic subgroups Z_a and Z_b for initial and final state: $n(Z_{a_i})$ *resp.* $n(Z_{b_i}))$ must divide $n(Z_{a_f})$ *resp.* $n(Z_{b_f}$ or vice versa in the case that factors of Z_i do not leave invariant the states. If this is the case similar condition must hold true for appropriate subgroups. In particular, powers of prime Z_{p^n}, $n = 1, 2, ...$ define hierarchies of allowed phase transitions.

3.5.3 Coupling Constant Evolution And Hierarchy Of Planck Constants

If the overall vision is correct, quantum TGD would be characterized by two kinds of couplings constant evolutions. p-Adic coupling constant evolution would correspond to length scale resolution and the evolution with respect to Planck constant to phase resolution. Both evolution would have number theoretic interpretation.

Evolution with respect to phase resolution

The coupling constant evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i2\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only iterated square root operation are number theoretically very special since they correspond to algebraic extensions of p-adic numbers obtained by an iterated square root operation, which should emerge first. Therefore systems involving these values of q should be especially abundant in Nature. That arbitrarily high square roots are involved as becomes clear by studying the case $n = 2^k$: $cos(\pi/2^k) = \sqrt{[1 + cos(\pi/2^{k-1})]/2}$.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K80].

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constant assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

The Dynkin diagrams of exceptional Lie groups E_6 and E_8 are exceptional as subgroups of rotation group in the sense that they cannot be reduced to symmetry transformations of plane. They correspond to the symmetry group $S_4 \times Z_2$ of tetrahedron and $A_5 \times Z_2$ of dodecahedron or its dual polytope icosahedron (A_5 is 60-element subgroup of S_5 consisting of even permutations). Maximal cyclic subgroups are Z_4 and Z_5andthus their orders correspond to Fermat polygons. Interestingly, $n = 5$ corresponds to minimum value of n making possible topological quantum computation using braids and also to Golden Mean.

Is there a correlation between the values of p-adic prime and Planck constant?

The obvious question is whether there is a correlation between p-adic length scale and the value of Planck constant. One-to-one correspondence is certainly excluded but loose correlation seems to exist.

1. In [L50] the information about the number theoretic anatomy of Kähler coupling strength is combined with input from p-adic mass calculations predicting α_K to be the value of fine structure constant at the p-adic length scale associated with electron. One can also develop an explicit expression for gravitational constant assuming its renormalization group invariance on basis of dimensional considerations and this model leads to a model for the fraction of volume of the wormhole contact (piece of CP_2 type extremal) from the volume of CP_2 characterizing gauge boson and for similar volume fraction for the piece of the CP_2 type vacuum extremal associated with fermion.

2. The requirement that gravitational constant is renormalization group invariant implies that the volume fraction depends logarithmically on p-adic length scale and Planck constant (characterizing quantum scale). The requirement that this fraction in the range $(0, 1)$ poses a correlation between the rational characterizing Planck constant and p-adic length scale. In particular, for space-time sheets mediating gravitational interaction Planck constant must be larger than $\hbar_0$ above length scale which is about .1 Angstrom. Also an upper bound for $\hbar$ for given p-adic length scale results but is very large. This means that quantum gravitational effects should become important above atomic length scale [L50].

3.6 Some Applications

Below some applications of the hierarchy of Planck constants as a model of dark matter are briefly discussed. The range of applications varying from elementary particle physics to cosmology and I hope that this will convince the reader that the idea has strong physical motivations.

3.6.1 A Simple Model Of Fractional Quantum Hall Effect

The generalization of the embedding space suggests that it could possible to understand fractional quantum Hall effect [D2] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{3.6.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15...,\ 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9...,\ 1/5, 2/9, 3/13...,\ 2/7, 3/11...,\ 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [D2].

The model of Laughlin [D22] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [D13]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of embedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

1. The easiest manner to understand the observed fractions is by assuming that both CD and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [D2, D8]. The fractionized charge is $e/4$ in the latter case [D8, D5]. Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [D13]. $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e \int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather ad hoc.

Therefore the model can be taken as a warm-up exercise only. In [K87], where the delicacies of Kähler structure of generalized embedding space are discussed, also a more detailed of QHE is discussed.

3.6.2 Gravitational Bohr Orbitology

The basic question concerns justification for gravitational Bohr orbitology [K103]. The basic vision is that visible matter identified as matter with $\hbar = \hbar_0$ ($n_a = n_b = 1$) concentrates around dark matter at Bohr orbits for dark matter particles. The question is what these Bohr orbits really mean. Should one in improved approximation relate Bohr orbits to 3-D wave functions for dark matter as ordinary Bohr rules would suggest or do the Bohr orbits have some deeper meaning different from that in wave mechanics. Anyonic variants of partonic 2-surfaces with astrophysical size are a natural guess for the generalization of Bohr orbits.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K103] .

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation -or at least Bohr rules with appropriate interpretation - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

Prediction for the parameter v_0

One of the key questions relate to the value of the parameter v_0. Before the introduction of the hierarchy of Planck constants I proposed that the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n-fold much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. The planetary mass ratios can be produced with an accuracy better than 10 per cent assuming ruler and compass phases.

Further predictions

The study of inclinations (tilt angles with respect to the Earth's orbital plane) leads to a concrete model for the quantum evolution of the planetary system. Only a stepwise breaking of the rotational symmetry and angular momentum Bohr rules plus Newton's equation (or geodesic equation) are needed, and gravitational Schrödinger equation holds true only inside flux quanta for the dark matter.

1. During pre-planetary period dark matter formed a quantum coherent state on the (Z^0) magnetic flux quanta (spherical cells or flux tubes). This made the flux quantum effectively a single rigid body with rotational degrees of freedom corresponding to a sphere or circle (full SO(3) or SO(2) symmetry).

2. In the case of spherical shells associated with inner planets the $SO(3) \rightarrow SO(2)$ symmetry breaking led to the generation of a flux tube with the inclination determined by m and j and a further symmetry breaking, kind of an astral traffic jam inside the flux tube, generated a planet moving inside flux tube. The semiclassical interpretation of the angular momentum

algebra predicts the inclinations of the inner planets. The predicted (real) inclinations are 6 (7) resp. 2.6 (3.4) degrees for Mercury resp. Venus). The predicted (real) inclination of the Earth's spin axis is 24 (23.5) degrees.

3. The $v_0 \rightarrow v_0/5$ transition allowing to understand the radii of the outer planets in the model of Da Rocha and Nottale can be understood as resulting from the splitting of (Z^0) magnetic flux tube to five flux tubes representing Earth and outer planets except Pluto, whose orbital parameters indeed differ dramatically from those of other planets. The flux tube has a shape of a disk with a hole glued to the Earth's spherical flux shell.

   It is important to notice that effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question. This allows to consider also alternative explanations. Perhaps external gravitational perturbations have kicked dark matter from the orbit or Earth to $n = 5k$, $k = 2, 3, ..., 7$ orbits: the fact that the tilt angles for Earth and all outer planets except Pluto are nearly the same, supports this explanation. Or perhaps there exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter and these orbits satisfy $n\ mod\ 5 = 0$ for some reason.

4. A remnant of the dark matter is still in a macroscopic quantum state at the flux quanta. It couples to photons as a quantum coherent state but the coupling is extremely small due to the gigantic value of $\hbar_{gr}$ scaling alpha by $\hbar/\hbar_{gr}$: hence the darkness.

   The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with $n = 1$ orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Comparison with Bohr quantization of planetary orbits

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}}n_{F_{b,2}}/n_{F_{b,1}}n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.

2. Nottale [E6] has suggested that also the harmonics and sub-harmonics of $\hbar_{gr}$ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary [K103]). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in solar system [K103] $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}] * X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet)n_{F,b}(Earth)/n_{F,a}(Earth)n_{F,b}(planet)$. The deviations are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k \prod_k F_{n_k}$, where $F_i = 2^{2^i} + 1$, $i = 0, 1, 2, 3, 4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254} \times 5 \times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [K42]. For $v_0 = 4.6 \times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational

planet	*Me*	*V*	*E*	*M*	*J*
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9\times 5\times 17$	$2^8\times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
planet	*S*	*U*	*N*	*P*	
y	$2^{14}\times 3\times 5\times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^{4}\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 3.1: Table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1+\epsilon)$ of the total mass for Earth gives

$$1+\epsilon = \frac{17}{16} \qquad (3.6.2)$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1-\epsilon) \simeq 4.6\times 10^{-4}$ equals with $v_0(eff) = 1/(2^7\times F_2) = 4.5956\times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v\cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This would suggest that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of CD seems to resolve these problems.

1. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

2. One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of CD, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

Quantum Hall effect and dark anyonic systems in astrophysical scales

Bohr orbitology could be understood if dark matter concentrates on 2-dimensional partonic surfaces usually assigned with elementary particles and having size of order CP_2 radius. The interpretation is in terms of wormhole throats assignable to topologically condensed CP_2 type extremals (fermions) and pairs of them assignable to wormhole contacts (gauge bosons). Wormhole throat defines the light-like 3-surface at which the signature of metric of space-time surface changes from Minkowskian to Euclidian.

Large value of Planck constant would allow partons with astrophysical size. Since anyonic systems are 2-dimensional, the natural idea is that dark matter corresponds to systems carrying large fermion number residing at partonic 2-surfaces of astrophysical size and that visible matter condenses around these. Not only black holes but also ordinary stars, planetary systems, and planets could correspond at the level of dark matter to atom like structures consisting of anyonic 2-surfaces which can have complex topology (flux tubes associated with planetary orbits connected by radial flux tubes to the central spherical anyonic surface). Charge and spin fractionization are key features of anyonic systems and Jones inclusions inspiring the generalization of embedding space indeed involve quantum groups central in the modelling of anyonic systems. Hence one has could hopes that a coherent theoretical picture could emerge along these lines.

This seems to be the case. Anyons and charge and spin fractionization are discussed in detail [K87] and leads to a precise identification of the delicacies involved with the Kähler gauge potential of CP_2 Kähler form in the sectors of the generalized embedding space corresponding to various pages of boook like structures assignable to CD and CP_2. The basic outcome is that anyons correspond geometrically to partonic 2-surfaces at the light-like boundaries of CD containing the tip of CD inside them. This is what gives rise to charge fractionization and also to confinement like effects since elementary particles in anyonic states cannot as such leak to the other pages of the generalized embedding space. G_a and G_b invariance of the states imply that fractionization occurs only at single particle level and total charge is integer valued.

This picture is much more flexible that based on G_a symmetries of CD orbifold since partonic 2-surfaces do not possess any orbifold symmetries in CD sector anymore. In this framework various astrophysical structures such as spokes and circles would be parts of anyonic 2-surfaces with complex topology representing quantum geometrically quantum coherence in the scale of say solar system. Planets would have formed by the condensation of ordinary matter in the vicinity of the anyonic matter. This would predict stars, planetary system, and even planets to have onion-like structure consisting of shells at the level of dark matter. Similar conclusion is suggested also by purely classical model for the final state of star predicting that matter is strongly concentrated at the surface of the star [K122].

Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic.

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

3.6.3 Accelerating Periods Of Cosmic Expansion As PhaseTransitions Increasing The Value Of Planck Constant

There are several pieces of evidence for accelerated expansion, which need not mean cosmological constant, although this is the interpretation adopted in [E4, E2]. Quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the value of the gravitational Planck constant. This assumption provides explanation for the apparent cosmological constant. Also planets are predicted to expand in this manner. This provides a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering the entire surface of Earth but with radius which was one half of the recent one.

The four pieces of evidence for accelerated expansion

1. Supernovas of type Ia

Supernovas of type Ia define standard candles since their luminosity varies in an oscillatory manner and the period is proportional to the luminosity. The period gives luminosity and from this the distance can be deduced by using Hubble's law: $d = cz/H_0$, H_0 Hubble's constant. The observation was that the farther the supernova was the more dimmer it was as it should have been. In other words, Hubble's constant increased with distance and the cosmic expansion was accelerating rather than decelerating as predicted by the standard matter dominated and radiation dominated cosmologies.

2. Mass density is critical and 3-space is flat

It is known that the contribution of ordinary and dark matter explaining the constant velocity of distance stars rotating around galaxy is about 25 per cent from the critical density. Could it be that total mass density is critical?

From the anisotropy of cosmic microwave background one can deduce that this is the case. What criticality means geometrically is that 3-space defined as surface with constant value of cosmic time is flat. This reflects in the spectrum of microwave radiation. The spots representing small anisotropies in the microwave background temperature is 1 degree and this correspond to flat 3-space. If one had dark matter instead of dark energy the size of spot would be.5 degrees!

Thus in a cosmology based on general relativity cosmological constant remains the only viable option. The situation is different in TGD based quantum cosmology based on sub-manifold gravity and hierarchy of gravitational Planck constants.

3. The energy density of vacuum is constant in the size scale of big voids

It was observed that the density of dark energy would be constant in the scale of 10^8 light years. This length scale corresponds to the size of big voids containing galaxies at their boundaries.

4. Integrated Sachs-Wolf effect

Also so called integrated Integrated Sachs-Wolf effect supports accelerated expansion. Very slow variations of mass density are considered. These correspond to gravitational potentials. Cosmic expansion tends to flatten them but mass accretion to form structures compensates this effect so that gravitational potentials are unaffected and there is no effect of CMB. Situation changes if dark matter is replaced with dark energy the accelerated expansion flattening the gravitational potentials wins the tendency of mass accretion to make them deeper. Hence if photon passes by an over-dense region, it receives a little energy. Similarly, photon loses energy when passign by an under-dense region. This effect has been observed.

Accelerated expansion in classical TGD

The minimum TGD based explanation for accelerated expansion involves only the fact that the embeddings of critical cosmologies correspond to accelerated expansion. A more detailed model allows to understand why the critical cosmology appears during some periods.

The first observation is that critical cosmologies (flat 3-space) imbeddable to 8-D embedding space H correspond to negative pressure cosmologies and thus to accelerating expansion. The negativity of the counterpart of pressure in Einstein tensor is due to the fact that space-time sheet is forced to be a 4-D surface in 8-D embedding space. This condition is analogous to a force forcing a particle at the surface of 2-sphere and gives rise to what could be called constraint force. Gravitation in TGD is sub-manifold gravitation whereas in GRT it is manifold gravitation. This would be minimum interpretation involving no assumptions about what mechanism gives rise to the critical periods.

Accelerated expansion and hierarchy of Planck constants

One can go one step further and introduce the hierarchy of Planck constants. The basic difference between TGD and GRT based cosmologies is that TGD cosmology is quantum cosmology. Smooth cosmic expansion is replaced by an expansion occurring in discrete jerks corresponding to the increase of gravitational Planck constant. At space-time level this means the replacement of 8-D embedding space H with a book like structure containing almost-copies of H with various values of Planck constant as pages glued together along critical manifold through which space-time sheet can leak between sectors with different values of $\hbar$. This process is the geometric correlate for the phase transition changing the value of Planck constant.

During these phase transition periods critical cosmology applies and predicts automatically accelerated expansion. Neither genuine negative pressure due to "quintessence" nor cosmological constant is needed. Note that quantum criticality replaces inflationary cosmology and predicts a unique cosmology apart from single parameter. Criticality also explains the fluctuations in microwave temperature as long range fluctuations characterizing criticality.

Accelerated expansion and flatness of 3-cosmology

Observations 1) and 2) about super-novae and critical cosmology (flat 3-space) are consistent with this cosmology. In TGD dark energy must be replaced with dark matter because the mass density is critical during the phase transition. This does not lead to wrong sized spots since it is the increase of Planck constant which induces the accelerated expansion understandable also as a constraint force due to embedding to H.

The size of large voids is the characteristic scale

The TGD based model in its simplest form model assigns the critical periods of expansion to large voids of size 10^8 ly. Also larger and smaller regions can express similar periods and dark space-time sheets are expected to obey same universal "cosmology" apart from a parameter characterizing the duration of the phase transition. Observation 3) that just this length scale defines the scale below which dark energy density is constant is consistent with TGD based model.

The basic prediction is jerk-wise cosmic expansion with jerks analogous to quantum transitions between states of atom increasing the size of atom. The discovery of large voids with size of order 10^8 ly but age much longer than the age of galactic large voids conforms with this prediction. One the other hand, it is known that the size of galactic clusters has not remained constant in very long time scale so that jerk-wise expansion indeed seems to occur.

Do cosmic strings with negative gravitational mass cause the phase transition inducing accelerated expansion

Quantum classical correspondence is the basic principle of quantum TGD and suggest that the effective antigravity manifested by accelerated expansion might have some kind of concrete space-time correlate. A possible correlate is super heavy cosmic string like objects at the center of large voids which have negative gravitational mass under very general assumptions. The repulsive gravitational force created by these objects would drive galaxies to the boundaries of large voids. At some state the pressure of galaxies would become too strong and induce a quantum phase transition forcing the increase of gravitational Planck constant and expansion of the void taking place much faster than the outward drift of the galaxies. This process would repeat itself. In the average sense the cosmic expansion would not be accelerating.

3.6.4 Phase Transition Changing Planck Constant And Expanding Earth Theory

TGD predicts that cosmic expansion at the level of individual astrophysical systems does not take place continuously as in classical gravitation but through discrete quantum phase transitions increasing gravitational Planck constant and thus various quantum length and time scales. The reason would be that stationary quantum states for dark matter in astrophysical length scales cannot expand. One would have the analog of atomic physics in cosmic scales. Increases of $\hbar$ by a power of two are favored in these transitions but also other scalings are possible.

This has quite far reaching implications.

1. These periods have a highly unique description in terms of a critical cosmology for the expanding space-time sheet. The expansion is accelerating. The accelerating cosmic expansion can be assigned to this kind of phase transition in some length scale (TGD Universe is fractal). There is no need to introduce cosmological constant and dark energy would be actually dark matter.

2. The recently observed void which has same size of about 10^8 light years as large voids having galaxies near their boundaries but having an age which is much higher than that of the large voids, would represent one example of jerk-wise expansion.

3. This picture applies also to solar system and planets might be perhaps seen as having once been parts of a more or less connected system, the primordial Sun. The Bohr orbits for inner and outer planets correspond to gravitational Planck constant which is 5 times larger for outer planets. This suggests that the space-time sheet of outer planets has suffered a phase transition increasing the size scale by a factor of 5. Earth can be regarded either as n=1 orbit for Planck constant associated with outer planets or n= 5 orbit for inner planetary system. This might have something to do with the very special position of Earth in planetary system. One could even consider the possibility that both orbits are present as dark matter structures. The phase transition would also explain why n=1 and n=2 Bohr orbits are absent and one only n=3, 4, and 5 are present.

4. Also planets should have experienced this kind of phase transitions increasing the radius: the increase by a factor two would be the simplest situation.

The obvious question - that I did not ask - is whether this kind of phase transition might have occurred for Earth and led from a completely granite covered Earth - Pangeia without seas - to the recent Earth. Neither it did not occur to me to check whether there is any support for a rapid expansion of Earth during some period of its history.

Situation changed when my son visited me last Saturday and told me about a Youtube video [F8] by Neal Adams, an American comic book and commercial artist who has also produced animations for geologists. We looked the amazing video a couple of times and I looked it again yesterday. The video is very impressive artwork but in the lack of references skeptic probably cannot avoid the feeling that Neal Adams might use his highly developed animation skills to cheat you. I found also a polemic article [F1] of Adams but again the references were lacking. Perhaps the reason of polemic tone was that the concrete animation models make the expanding Earth hypothesis very convincing but geologists refuse to consider seriously arguments by a layman without a formal academic background.

The claims of Adams

The basic claims of Adams were following.

1. The radius of Earth has increased during last 185 million years (dinosaurs [I12] appeared for about 230 million years ago) by about factor 2. If this is assumed all continents have formed at that time a single super-continent, Pangeia, filling the entire Earth surface rather than only 1/4 of it since the total area would have grown by a factor of 4. The basic argument was that it is very difficult to imagine Earth with 1/4 of surface containing granite and 3/4 covered by basalt. If the initial situation was covering by mere granite -as would look natural- it is very difficult for a believer in thermodynamics to imagine how the granite would have gathered to a single connected continent.

2. Adams claims that Earth has grown by keeping its density constant, rather than expanded, so that the mass of Earth has grown linearly with radius. Gravitational acceleration would have thus doubled and could provide a partial explanation for the disappearance of dinosaurs: it is difficult to cope in evolving environment when you get slower all the time.

3. Most of the sea floor is very young and the areas covered by the youngest basalt are the largest ones. This Adams interprets this by saying that the expansion of Earth is accelerating. The alternative interpretation is that the flow rate of the magma slows down as it recedes from the ridge where it erupts. The upper bound of 185 million years for the age of sea floor requires that the expansion period - if it is already over - lasted about 185 million years after which the flow increasing the area of the sea floor transformed to a convective flow with subduction so that the area is not increasing anymore.

4. The fact that the continents fit together - not only at the Atlantic side - but also at the Pacific side gives strong support for the idea that the entire planet was once covered by the super-continent. After the emergence of subduction theory this evidence as been dismissed.

5. I am not sure whether Adams mentions the following objections [F2]. Subduction only occurs on the other side of the subduction zone so that the other side should show evidence of being much older in the case that oceanic subduction zones are in question. This is definitely not the case. This is explained in plate tectonics as a change of the subduction direction. My explanation would be that by the symmetry of the situation both oceanic plates bend down so that this would represent new type of boundary not assumed in the tectonic plate theory.

6. As a master visualizer Adams notices that Africa and South-America do not actually fit together in absence of expansion unless one assumes that these continents have suffered a deformation. Continents are not easily deformable stuff. The assumption of expansion implies a perfect fit of *all* continents without deformation.

Knowing that the devil is in the details, I must admit that these arguments look rather convincing to me and what I learned from Wikipedia articles supports this picture.

The critic of Adams of the subduction mechanism

The prevailing tectonic plate theory [F5] has been compared to the Copernican revolution in geology. The theory explains the young age of the seafloor in terms of the decomposition of the litosphere to tectonic plates and the convective flow of magma to which oceanic tectonic plates participate. The magma emerges from the crests of the mid ocean ridges representing a boundary of two plates and leads to the expansion of sea floor. The variations of the polarity of Earth's magnetic field coded in sea floor provide a strong support for the hypothesis that magma emerges from the crests.

The flow back to would take place at so called oceanic trenches [F3] near continents which represent the deepest parts of ocean. This process is known as subduction. In subduction oceanic tectonic plate bends and penetrates below the continental tectonic plate, the material in the oceanic plate gets denser and sinks into the magma. In this manner the oceanic tectonic plate suffers a metamorphosis returning back to the magma: everything which comes from Earth's interior returns back. Subduction mechanism explains elegantly formation of mountains [F4] (orogeny), earth quake zones, and associated zones of volcanic activity [F6] .

Adams is very polemic about the notion of subduction, in particular about the assumption that it generates steady convective cycle. The basic objections of Adams against subduction are following.

1. There are not enough subduction zones to allow a steady situation. According to Adams, the situation resembles that for a flow in a tube which becomes narrower. In a steady situation the flow should accelerate as it approaches subduction zones rather than slow down. Subduction zones should be surrounded by large areas of sea floor with constant age. Just the opposite is suggested by the fact that the youngest portion of sea-floor near the ridges is largest. The presence of zones at which both ocean plates bend down could improve the situation. Also jamming of the flow could occur so that the thickness of oceanic plate increases with the distance from the eruption ridge. Jamming could increase also the density of the oceanic plate and thus the effectiveness of subduction.

2. There is no clear evidence that subduction has occurred at other planets. The usual defense is that the presence of sea is essential for the subduction mechanism.

3. One can also wonder what is the mechanism that led to the formation of single super continent Pangeia covering 1/4 of Earth's surface. How probable the gathering of all separate continents to form single cluster is? The later events would suggest that just the opposite should have occurred from the beginning.

Expanding Earth theories are not new

After I had decided to check the claims of Adams, the first thing that I learned is that Expanding Earth theory [F2], whose existence Adams actually mentions, is by no means new. There are actually many of them.

The general reason why these theories were rejected by the main stream community was the absence of a convincing physical mechanism of expansion or of growth in which the density of Earth remains constant.

1. 1888 Yarkovski postulated some sort of aether absorbed by Earth and transforming to chemical elements (TGD version of aether could be dark matter). 1909 Mantovani postulated thermal expansion but no growth of the Earth's mass.

2. Paul Dirac's idea about changing Planck constant led Pascual Jordan in 1964 to a modification of general relativity predicting slow expansion of planets. The recent measurement of the gravitational constant imply that the upper bound for the relative change of gravitational constant is 10 time too small to produce large enough rate of expansion. Also many other theories have been proposed but they are in general conflict with modern physics.

3. The most modern version of Expanding Earth theory is by Australian geologist Samuel W. Carey. He calculated that in Cambrian period (about 500 million years ago) all continents were stuck together and covered the entire Earth. Deep seas began to evolve then.

Summary of TGD based theory of Expanding Earth

TGD based model differs from the tectonic plate model but allows subduction which cannot imply considerable back-flow of magma. Let us sum up the basic assumptions and implications.

1. The expansion is or was due to a quantum phase transition increasing the value of gravitational Planck constant and forced by the cosmic expansion in the average sense.

2. Tectonic plates do not participate to the expansion and therefore new plate must be formed and the flow of magma from the crests of mid ocean ridges is needed. The decomposition of a single plate covering the entire planet to plates to create the mid ocean ridges is necessary for the generation of new tectonic plate. The decomposition into tectonic plates is thus prediction rather than assumption.

3. The expansion forced the decomposition of Pangeia super-continent covering entire Earth for about 530 million years ago to split into tectonic plates which began to recede as new non-expanding tectonic plate was generated at the ridges creating expanding sea floor. The initiation of the phase transition generated formation of deep seas.

4. The eruption of plasma from the crests of ocean ridges generated oceanic tectonic plates which did not participate to the expansion by density reduction but by growing in size. This led to a reduction of density in the interior of the Earth roughly by a factor 1/8. From the upper bound for the age of the seafloor one can conclude that the period lasted for about 185 million years after which it transformed to convective flow in which the material returned back to the Earth interior. Subduction at continent-ocean floor boundaries and downwards double bending of tectonic plates at the boundaries between two ocean floors were the mechanisms. Thus tectonic plate theory would be more or less the correct description for the recent situation.

5. One can consider the possibility that the subducted tectonic plate does not transform to magma but is fused to the tectonic layer below continent so that it grows to an iceberg like structure. This need not lead to a loss of the successful predictions of plate tectonics explaining the generation of mountains, earthquake zones, zones of volcanic activity, etc...

6. From the video of Adams it becomes clear that the tectonic flow is East-West asymmetric in the sense that the western side is more irregular at large distances from the ocean ridge at the western side. If the magma rotates with slightly lower velocity than the surface of Earth (like liquid in a rotating vessel), the erupting magma would rotate slightly slower than the tectonic plate and asymmetry would be generated.

7. If the planet has not experienced a phase transition increasing the value of Planck constant, there is no need for the decomposition to tectonic plates and one can understand why there is no clear evidence for tectonic plates and subduction in other planets. The conductive flow of magma could occur below this plate and remain invisible.

The biological implications might provide a possibility to test the hypothesis.

1. Great steps of progress in biological evolution are associated with catastrophic geological events generating new evolutionary pressures forcing new solutions to cope in the new situation. Cambrian explosion indeed occurred about 530 years ago (the book "Wonderful Life" of Stephen Gould [I83] explains this revolution in detail) and led to the emergence of multicellular creatures, and generated huge number of new life forms living in seas. Later most of them suffered extinction: large number of phylae and groups emerged which are not present nowadays.

   Thus Cambrian explosion is completely exceptional as compared to all other dramatic events in the evolution in the sense that it created something totally new rather than only making more complex something which already existed. Gould also emphasizes the failure to identify any great change in the environment as a fundamental puzzle of Cambrian explosion. Cambrian explosion is also regarded in many quantum theories of consciousness (including TGD) as a revolution in the evolution of consciousness: for instance, micro-tubuli emerged at this time. The periods of expansion might be necessary for the emergence of multicellular life forms on planets and the fact that they unavoidably occur sooner or later suggests that also life develops unavoidably.

2. TGD predicts a decrease of the surface gravity by a factor 1/4 during this period. The reduction of the surface gravity would have naturally led to the emergence of dinosaurs 230 million years ago as a response coming 45 million years after the accelerated expansion ceased. Other reasons led then to the decline and eventual catastrophic disappearance of the dinosaurs. The reduction of gravity might have had some gradually increasing effects on the shape of organisms also at microscopic level and manifest itself in the evolution of genome during expansion period.

3. A possibly testable prediction following from angular momentum conservation ($\omega R^2 = constant$) is that the duration of day has increased gradually and was four times shorter during the Cambrian era. For instance, genetically coded bio-clocks of simple organisms during the expansion period could have followed the increase of the length of day with certain lag or failed to follow it completely. The simplest known circadian clock is that of the prokaryotic cyanobacteria. Recent research has demonstrated that the circadian clock of Synechococcus elongatus can be reconstituted in vitro with just the three proteins of their central oscillator. This clock has been shown to sustain a 22 hour rhythm over several days upon the addition of ATP: the rhythm is indeed faster than the circadian rhythm. For humans the average innate circadian rhythm is however 24 hours 11 minutes and thus conforms with the fact that human genome has evolved much later than the expansion ceased.

4. Scientists have found a fossil of a sea scorpion with size of 2.5 meters [I56], which has lived for about 10 million years for 400 million years ago in Germany. The gigantic size would conform nicely with the much smaller value of surface gravity at that time. The finding would conform nicely with the much smaller value of surface gravity at that time. Also the emergence of trees could be understood in terms of a gradual growth of the maximum plant size as the surface gravity was reduced. The fact that the oldest known tree fossil is 385 million years old [I71] conforms with this picture.

Did intra-terrestrial life burst to the surface of Earth during Cambrian expansion?

The possibility of intra-terrestrial life [?] is one of the craziest TGD inspired ideas about the evolution of life and it is quite possible that in its strongest form the hypothesis is unrealistic. One can however try to find what one obtains from the combination of the IT hypothesis with the idea of pre-Cambrian granite Earth. Could the harsh pre-Cambrian conditions have allowed only intra-terrestrial multicellular life? Could the Cambrian explosion correspond to the moment of birth for this life in the very concrete sense that the magma flow brought it into the day-light?

1. Gould emphasizes the mysterious fact that very many life forms of Cambrian explosion looked like final products of a long evolutionary process. Could the eruption of magma from the Earth interior have induced a burst of intra-terrestrial life forms to the Earth's surface? This might make sense: the life forms living at the bottom of sea do not need direct solar light so that they could have had intra-terrestrial origin. It is quite possible that Earth's mantle contained low temperature water pockets, where the complex life forms might have evolved in an environment shielded from meteoric bombardment and UV radiation.

2. Sea water is salty. It is often claimed that the average salt concentration inside cell is that of the primordial sea: I do not know whether this claim can be really justified. If the claim is true, the cellular salt concentration should reflect the salt concentration of the water inside the pockets. The water inside water pockets could have been salty due to the diffusion of the salt from ground but need not have been same as that for the ocean water (higher than for cell interior and for obvious reasons). Indeed, the water in the underground reservoirs in arid regions such as Sahara is salty, which is the reason for why agriculture is absent in these regions. Note also that the cells of marine invertebrates are osmoconformers able to cope with the changing salinity of the environment so that the Cambrian revolutionaries could have survived the change in the salt concentration of environment.

3. What applies to Earth should apply also to other similar planets and Mars [E3] is very similar to Earth. The radius is .533 times that for Earth so that after quantum leap doubling the radius and thus Schumann frequency scale (7.8 Hz would be the lowest Schumann frequency)

would be essentially same as for Earth now. Mass is.131 times that for Earth so that surface gravity would be.532 of that for Earth now and would be reduced to.131 meaning quite big dinosaurs! have learned that Mars probably contains large water reservoirs in it's interior and that there is an un-identified source of methane gas usually assigned with the presence of life. Could it be that Mother Mars is pregnant and just waiting for the great quantum leap when it starts to expand and gives rise to a birth of multicellular life forms. Or expressing freely how Bible describes the moment of birth: in the beginning there was only darkness and water and then God saidLet the light come!

To sum up, TGD would provide only the long sought mechanism of expansion and a possible connection with the biological evolution. It would be indeed fascinating if Planck constant changing quantum phase transitions in planetary scale would have profoundly affected the biosphere.

3.6.5 Allais Effect As Evidence For Large Values Of Gravitational Planck Constant?

Allais effect [E1, E9] is a fascinating gravitational anomaly associated with solar eclipses. It was discovered originally by M. Allais, a Nobelist in the field of economy, and has been reproduced in several experiments but not as a rule. The experimental arrangement uses so called paraconical pendulum, which differs from the Foucault pendulum in that the oscillation plane of the pendulum can rotate in certain limits so that the motion occurs effectively at the surface of sphere.

Experimental findings

Consider first a brief summary of the findings of Allais and others [E9].

a) In the ideal situation (that is in the absence of any other forces than gravitation of Earth) paraconical pendulum should behave like a Foucault pendulum. The oscillation plane of the paraconical pendulum however begins to rotate.

b) Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

c) Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5\times 10^{-4}$ [E1, E8] which happens to correspond to the constant $v_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

d) There is also quite recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [E10].

TGD based models for Allais effect

I have already earlier proposed an explanation of the effect in terms of classical Z^0 force [K14]. If the Z^0 charge to mass ratio of pendulum varies and if Earth and Moon are Z^0 conductors, the resulting model is quite flexible and one might hope it could explain the high variation of the experimental results.

The rapid variation of the effect during the eclipse is however a problem for this approach and suggests that gravitational screening or some more general interference effect might be present. Gravitational screening alone cannot however explain Allais effect.

A model based on the idea that gravitational interaction is mediated by topological light rays (MEs) and that gravitons correspond to a gigantic value of the gravitational Planck constant however explains the Allais effect as an interference effect made possible by macroscopic quantum coherence in astrophysical length scales. Equivalence Principle fixes the model to a high degree and one ends up with an explicit formula for the anomalous gravitational acceleration and the general order of magnitude and the large variation of the frequency change as being due to the variation of the distance ratio $r_{S,P}/r_{M,P}$ (S, M, and P refer to Sun, Moon, and pendulum respectively). One can say that the pendulum acts as an interferometer.

3.6.6 Applications To Elementary Particle Physics, Nuclear Physics, And Condensed Matter Physics

The hierarchy of Planck constants could have profound implications for even elementary particle physics since the strong constraints on the existence of new light particles coming from the decay widths of intermediate gauge bosons can be circumvented because direct decays to dark matter are not possible. On the other hand, if light scaled versions of elementary particles exist they must be dark since otherwise their existence would be visible in these decay widths. The constraints on the existence of dark nuclei and dark condensed matter are much milder. Cold fusion and some other anomalies of nuclear and condensed matter physics - in particular the anomalies of water-might have elegant explanation in terms of dark nuclei.

Leptohadron hypothesis

TGD suggests strongly the existence of lepto-hadron [K120]. Lepto-hadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of orto-positronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that lepto-pions (or electro-pions) decay to lepto-nucleon pair $e^+_{ex}e^-_{ex}$ first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged [C5]. Towards the end of 2008 CDF anomaly [C3] gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly [K120] led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

Cold fusion, plasma electrolysis, and burning salt water

The article of Kanarev and Mizuno [D14] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

In nuclear string model nucleon are connected by color bonds representing the color magnetic body of nucleus and having length considerably longer than nuclear size. One can consider also dark nuclei for which the scale of nucleus is of atomic size [L1], [L1]. In this framework can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the catode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

1. The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [E5]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.

2. The so called $H_{1.5}O$ anomaly of water [D15, D12, D20, D10] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.

3. The mysterious behavior burning salt water [D1] can be also understood in the same framework.

4. The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. This kind of transmutations have been reported also in living matter long time ago [C2, C7]. Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

3.6.7 Applications To Biology And Neuroscience

The notion of field or magnetic body regarded as carrier of dark matter with large Planck constant and quantum controller of ordinary matter is the basic idea in the TGD inspired model of living matter.

Do molecular symmetries in living matter relate to non-standard values of Planck constant?

Water is exceptional element and the possibility that G_a as symmetry of singular factor space of CD in water and living matter is intriguing.

1. There is evidence for an icosahedral clustering in [D25] [D16]. Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by CD points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of CD points by CP_2 could be involved.

2. It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.

3. What makes realization of G_a as a symmetry of singular factor space of CD is that the biomolecules most relevant for the functioning of brain (DNA nucleotides, amino-acids acting as neurotransmitters, molecules having hallucinogenic effects) contain aromatic 5- and 6-cycles.

These observations led to an identification of the formula for Planck constant (two alternatives were allowed by the condition that Planck constant is algebraic homomorphism) which was not consistent with the model for dark gravitons. If one accepts the proposed formula of Planck constant, the dark space-time sheets with large Planck constant correspond to factor spaces of both $\hat{CD}\backslash M^2$ and of $CP_2\backslash S^2_I$. This identification is of course possible and it remains to be seen whether it leads to any problems. For gravitational space-time sheets only coverings of both CD and CP_2 make sense and the covering group G_a has very large order and does not correspond to geometric symmetries analogous to those of molecules.

High T_c super-conductivity in living matter

The model for high T_c super-conductivity realized as quantum critical phenomenon predicts the basic scales of cell membrane [K24] from energy minimization and p-adic length scale hypothesis. This leads to the vision that cell membrane and possibly also its scaled up dark fractal variants define Josephson junctions generating Josephson radiation communicating information about the nearby environment to the magnetic body.

Any model of high T_c superconductivity should explain various strange features of high T_c superconductors. One should understand the high value of T_c, the ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap temperature $T_{c_1} > T_c$ and scaling law for resistance for $T_c \leq T < T_{c_1}$, the role of fluctuating charged stripes which are anti-ferromagnetic defects of a Mott insulator, the existence of a critical doping, etc... [D11, D7].

There are reasons to believe that high T_c super-conductors correspond to quantum criticality in which at least two (cusp catastrophe as in van der Waals model), or possibly three or even more phases, are competing. A possible analogy is provided by the triple critical point for water vapor, liquid phase and ice coexist. Instead of long range thermal fluctuations long range quantum fluctuations manifesting themselves as fluctuating stripes are present [D11].

The TGD based model for high T_c super-conductivity [K24] relies on the notions of quantum criticality, general ideas of catastrophe theory, dynamical Planck constant, and many-sheeted space-time. The 4-dimensional spin glass character of space-time dynamics deriving from the vacuum degeneracy of the Kähler action defining the basic variational principle would realize space-time correlates for quantum fluctuations.

1. Two kinds of super-conductivities and ordinary non-super-conducting phase would be competing at quantum criticality at T_c and above it only one super-conducting phase and ordinary conducting phase located at stripes representing ferromagnetic defects making possible formation of $S = 1$ Cooper pairs.

2. The first super-conductivity would be based on exotic Cooper pairs of large $\hbar$ dark electrons with $\hbar = 2^{11}\hbar_0$ and able to have spin $S = 1$, angular momentum $L = 2$, and total angular momentum $J = 2$. Second type of super-conductivity would be based on BCS type Cooper pairs having vanishing spin and bound by phonon interaction. Also they have large $\hbar$ so that gap energy and critical temperature are scaled up in the same proportion. The exotic Cooper pairs are possible below the pseudo gap temperature $T_{c_1} > T_c$ but are unstable against decay to BCS type Cooper pairs which above T_c are unstable against a further decay to conduction electrons flowing along stripes. This would reduce the exotic super-conductivity to finite conductivity obeying the observed scaling law for resistance.

3. The mere assumption that electrons of exotic Cooper pairs feed their electric flux to larger space-time sheet via *two* elementary particle sized wormhole contacts rather than only *one* wormhole contacts implies that the throats of wormhole contacts defining analogs of Higgs field must carry quantum numbers of quark and anti-quark. This inspires the idea that cylindrical space-time sheets, the radius of which turns out to be about about 5 nm, representing zoomed up dark electrons of Cooper pair with Planck constant $\hbar = 2^{11}\hbar_0$ are colored and bound by a scaled up variant of color force to form a color confined state. Formation of Cooper pairs would have nothing to do with direct interactions between electrons. Thus high T_c super-conductivity could be seen as a first indication for the presence of scaled up variant of QCD in mesoscopic length scales.

This picture leads to a concrete model for high T_c superconductors as quantum critical superconductors [K24]. p-Adic length scale hypothesis stating that preferred p-adic primes $p \simeq 2^k$, k integer, with primes (in particular Mersenne primes) preferred, makes the model quantitative.

1. An unexpected prediction is that coherence length ξ is actually $\hbar_{eff}/\hbar_0 = 2^{11}$ times longer than the coherence length 5-10 Angstroms deduced theoretically from gap energy using conventional theory and varies in the range $1 - 5$ μm, the cell nucleus length scale. Hence type

I super-conductor would be in question with stripes as defects of anti-ferromagnetic Mott insulator serving as duals for the magnetic defects of type I super-conductor in nearly critical magnetic field.

2. At quantitative level the model reproduces correctly the four poorly understood photon absorption lines and allows to understand the critical doping ratio from basic principles.

3. The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same. One of the characteristic absorption lines has energy of.05 eV which corresponds to the Josephson energy for neuronal membrane for activation potential $V = 50$ mV. Hence the idea that axons are high T_c superconductors is highly suggestive. Dark matter hierarchy coming in powers $\hbar/\hbar_0 = 2^{k11}$ suggests hierarchy of Josephson junctions needed in TGD based model of EEG [K42].

Magnetic body as a sensory perceiver and intentional agent

The hypothesis that dark magnetic body serves as an intentional agent using biological body as a motor instrument and sensory receptor is consistent with Libet's findings about strange time delays of consciousness. Magnetic body would carry cyclotron Bose-Einstein condensates of various ions. Magnetic body must be able to perform motor control and receive sensory input from biological body.

Cell membrane would be a natural sensor providing information about cell interior and exterior to the magnetic body and dark photons at appropriate frequency range would naturally communicate this information. The strange quantitative co-incidences with the physics of cell membrane and high T_c super-conductivity support the idea that Josephson radiation generated by Josephson currents of dark electrons through cell membrane is responsible for this communication [K42].

Also fractally scaled up versions of cell membrane at higher levels of dark matter hierarchy (in particular those corresponding to powers $n = 2^{k11}$) are possible and the model for EEG indeed relies on this hypothesis. The thickness for the fractal counterpart of cell membrane thickness would be 2^{44} fold and of order of depth of ionosphere! Although this looks weird it is completely consistent with the notion of magnetic body as an intentional agent.

Motor control would be most naturally performed via genome: this is achieved if flux sheets traverse through DNA strands. Flux quantization for large values of Planck constant requires rather large widths for the flux sheets. If flux sheet contains sequences of genomes like the page of book contains lines of text, a coherent gene expression becomes possible at level of organs and even populations and one can speak about super- and hyper-genomes. Introns might relate to the collective gene expression possibly realized electromagnetically rather than only chemically [K24, K25].

Dark cyclotron radiation with photon energy above thermal energy could be used for co-ordination purposes at least. The predicted hierarchy of copies of standard model physics leads to ask whether also dark copies of electro-weak gauge bosons and gluons could be important in living matter. As already mentioned, dark W bosons could make possible charge entanglement and non-local quantum bio-control by inducing voltage differences and thus ionic currents in living matter.

The identification of plasmoids as rotating magnetic flux structures carrying dark ions and electrons as primitive life forms is natural in this framework. There exists experimental support for this identification [I77] but the main objection is the high temperature involved: this objection could be circumvented if large $\hbar$ phase is involved. A model for the pre-biotic evolution relying also on this idea is discussed in [**?**].

At the level of biology there are now several concrete applications leading to a rich spectrum of predictions. Magnetic flux quanta would carry charged particles with large Planck constant.

1. The shortening of the flux tubes connecting biomolecules in a phase transition reducing Planck constant could be a basic mechanism of bio-catalysis and explain the mysterious ability of biomolecules to find each other. Similar process in time direction could explain basic aspects of symbolic memories as scaled down representations of actual events.

2. The strange behavior of cell membrane suggests that a dominating portion of important biological ions are actually dark ions at magnetic flux tubes so that ionic pumps and channels are needed only for visible ions. This leads to a model of nerve pulse explaining its unexpected thermodynamical properties with basic properties of Josephson currents making it un-necessary to use pumps to bring ions back after the pulse. The model predicts automatically EEG as Josephson radiation and explains the synchrony of both kHz radiation and of EEG.

3. The DC currents of Becker could be accompanied by Josephson currents running along flux tubes making possible dissipation free energy transfer and quantum control over long distances and meridians of chinese medicine could correspond to these flux tubes.

4. The model of DNA as topological quantum computer assumes that nucleotides and lipids are connected by ordinary or "wormhole" magnetic flux tubes acting as strands of braid and carrying dark matter with large Planck constant. The model leads to a new vision about TGD in which the assignment of nucleotides to quarks allows to understood basic regularities of DNA not understood from biochemistry.

5. Each physical system corresponds to an onion-like hierarchy of field bodies characterized by p-adic primes and value of Planck constant. The highest value of Planck constant in this hierarchy provides kind of intelligence quotient characterizing the evolutionary level of the system since the time scale of planned action and memory correspond to the temporal distance between tips of corresponding causal diamond (CD). Also the spatial size of the system correlates with the Planck constant. This suggests that great evolutionary leaps correspond to the increase of Planck constant for the highest level of hierarchy of personal magnetic bodies. For instance, neurons would have much more evolved magnetic bodies than ordinary cells.

6. At the level of DNA this vision leads to an idea about hierarchy of genomes. Magnetic flux sheets traversing DNA strands provide a natural mechanism for magnetic body to control the behavior of biological body by controlling gene expression. The quantization of magnetic flux states that magnetic flux is proportional to $\hbar$ and thus means that the larger the value of $\hbar$ is the larger the width of the flux sheet is. For larger values of $\hbar$ single genome is not enough to satisfy this condition. This leads to the idea that the genomes of organs, organism, and even population, can organize like lines of text at the magnetic flux sheets and form in this manner a hierarchy of genomes responsible for a coherent gene expression at level of cell, organ, organism and population and perhaps even entire biosphere. This would also provide a mechanism by which collective consciousness would use its biological body - biosphere.

DNA as topological quantum computer

I ended up with the recent model of TQC in bottom-up manner and this representation is followed also in the text. The model which looks the most plausible one relies on two specific ideas.

1. Sharing of labor means conjugate DNA would do TQC and DNA would "print" the outcome of TQC in terms of mRNA yielding amino-acids in the case of exons. RNA could result also in the case of introns but not always. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of TQC also electromagnetically in terms of standardized field patterns as Gariaev's findings suggest [I63]. Also speech would be a form of gene expression. The quantum states braid (in zero energy ontology) would entangle with characteristic gene expressions. This argument turned out to be based on a slightly wrong belief about DNA: later I learned that both strand and its conjugate are transcribed but in different directions. The symmetry breaking in the case of transcription is only local which is also visible in DNA replication as symmetry breaking between leading and lagging strand. Thus the idea about *entire* leading strand devoted to printing and second strand to TQC must be weakened appropriately.

2. The manipulation of braid strands transversal to DNA must take place at 2-D surface. Here dancing metaphor for topological quantum computation [C4] generalizes. The ends of the

space-like braid are like dancers whose feet are connected by thin threads to a wall so that the dancing pattern entangles the threads. Dancing pattern defines both the time-like braid, the running of classical TQC program and its representation as a dynamical pattern. The space-like braid defined by the entangled threads represents memory storage so that TQC program is automatically written to memory as the braiding of the threads during the TQC. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic reticulum included are good candidates for dancing halls. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

One ends up to the model also in top-down manner.

1. Darwinian selection for which standard theory of self-organization [B3] provides a model, should apply also to TQC programs. TQC programs should correspond to asymptotic self-organization patterns selected by dissipation in the presence of metabolic energy feed. The spatial and temporal pattern of the metabolic energy feed characterizes the TQC program - or equivalently - sub-program call.

2. Since braiding characterizes the TQC program, the self-organization pattern should correspond to a hydrodynamical flow or a pattern of magnetic field inducing the braiding. Braid strands must correspond to magnetic flux tubes of the magnetic body of DNA. If each nucleotide is transversal magnetic dipole it gives rise to transversal flux tubes, which can also connect to the genome of another cell.

3. The output of TQC sub-program is probability distribution for the outcomes of state function reduction so that the sub-program must be repeated very many times. It is represented as four-dimensional patterns for various rates (chemical rates, nerve pulse patterns, EEG power distributions, ...) having also identification as temporal densities of zero energy states in various scales. By the fractality of TGD Universe there is a hierarchy of TQC's corresponding to p-adic and dark matter hierarchies. Programs (space-time sheets defining coherence regions) call programs in shorter scale. If the self-organizing system has a periodic behavior each TQC module defines a large number of almost copies of itself asymptotically. Generalized EEG could naturally define this periodic pattern and each period of EEG would correspond to an initiation and halting of TQC. This brings in mind the periodically occurring sol-gel phase transition inside cell near the cell membrane.

4. Fluid flow must induce the braiding which requires that the ends of braid strands must be anchored to the fluid flow. Recalling that lipid mono-layers of the cell membrane are liquid crystals and lipids of interior mono-layer have hydrophilic ends pointing towards cell interior, it is easy to guess that DNA nucleotides are connected to lipids by magnetic flux tubes and hydrophilic lipid ends are stuck to the flow.

5. The topology of the braid traversing cell membrane cannot affected by the hydrodynamical flow. Hence braid strands must be split during TQC. This also induces the desired magnetic isolation from the environment. Halting of TQC reconnects them and make possible the communication of the outcome of TQC.

6. There are several problems related to the details of the realization. How nucleotides A, T, C, G are coded to strand color and what this color corresponds to? The prediction that wormhole contacts carrying quark and anti-quark at their ends appear in all length scales in TGD Universe resolves the problem. How to split the braid strands in a controlled manner? High T_c super conductivity provides a partial understanding of the situation: braid strand can be split only if the supra current flowing through it vanishes. From the proportionality of Josephson current to the quantity $sin(\int 2eVdt)$ it follows that a suitable voltage pulse V induces DC supra-current and its negative cancels it. The conformation of the lipid controls whether it it can follow the flow or not. How magnetic flux tubes can be cut without breaking the conservation of the magnetic flux? The notion of wormhole magnetic field saves the situation now: after the splitting the flux returns back along the second space-time sheet of wormhole magnetic field.

To sum up, it seems that essentially all new physics involved with TGD based view about quantum biology enter to the model in crucial manner.

Quantum model of nerve pulse and EEG

In this article a unified model of nerve pulse and EEG is discussed.

1. In TGD Universe the function of EEG and its variants is to make possible communications from the cell membrane to the magnetic body and the control of the biological body by the magnetic body via magnetic flux sheets traversing DNA by inducing gene expression. This leads to the notions of super- and hyper-genome predicting coherent gene expression at level of organs and population.

2. The assignment the predicted ranged classical weak and color gauge fields to dark matter hierarchy was a crucial step in the evolution of the model, and led among other things to a model of high T_c superconductivity predicting the basic scales of cell, and also to a generalization of EXG to a hierarchy of ZXGs, WXGs, and GXGs corresponding to Z^0, W bosons and gluons.

3. Dark matter hierarchy and the associated hierarchy of Planck constants plays a key role in the model. For instance, in the case of EEG Planck constant must be so large that the energies of dark EEG photons are above thermal energy at physiological temperatures. The assumption that a considerable fraction of the ionic currents through the cell membrane are dark currents flowing along the magnetic flux tubes explains the strange findings about ionic currents through cell membrane. Concerning the model of nerve pulse generation, the newest input comes from the model of DNA as a topological quantum computer and experimental findings challenging Hodgkin-Huxley model as even approximate description of the situation.

4. The identification of the cell interior as gel phase containing most of water as structured water around cytoskeleton - rather than water containing bio-molecules as solutes as assumed in Hodkin-Huxley model - allows to understand many of the anomalous behaviors associated with the cell membrane and also the different densities of ions in the interior and exterior of cell at qualitative level. The proposal of Pollack that basic biological functions involve phase transitions of gel phase generalizes in TGD framework to a proposal that these phase transitions are induced by quantum phase transitions changing the value of Planck constant. In particular, gel-sol phase transition for the peripheral cytoskeleton induced by the primary wave would accompany nerve pulse propagation. This view about nerve pulse is not consistent with Hodkin-Huxley model.

The model leads to the following picture about nerve pulse and EEG.

1. The system would consist of two superconductors- microtubule space-time sheet and the space-time sheet in cell exterior- connected by Josephson junctions represented by magnetic flux tubes defining also braiding in the model of TQC. The phase difference between two super-conductors would obey Sine-Gordon equation allowing both standing and propagating solitonic solutions. A sequence of rotating gravitational penduli coupled to each other would be the mechanical analog for the system. Soliton sequences having as a mechanical analog penduli rotating with constant velocity but with a constant phase difference between them would generate moving kHz synchronous oscillation. Periodic boundary conditions at the ends of the axon rather than chemistry determine the propagation velocities of kHz waves and kHz synchrony is an automatic consequence since the times taken by the pulses to travel along the axon are multiples of same time unit. Also moving oscillations in EEG range can be considered and would require larger value of Planck constant in accordance with vision about evolution as gradual increase of Planck constant.

2. During nerve pulse one pendulum would be kicked so that it would start to oscillate instead of rotating and this oscillation pattern would move with the velocity of kHz soliton sequence. The velocity of kHz wave and nerve pulse is fixed by periodic boundary conditions at the ends of the axon implying that the time spent by the nerve pulse in traveling along axon is

always a multiple of the same unit: this implies kHz synchrony. The model predicts the value of Planck constant for the magnetic flux tubes associated with Josephson junctions and the predicted force caused by the ionic Josephson currents is of correct order of magnitude for reasonable values of the densities of ions. The model predicts kHz em radiation as Josephson radiation generated by moving soliton sequences. EEG would also correspond to Josephson radiation: it could be generated either by moving or standing soliton sequences (latter are naturally assignable to neuronal cell bodies for which $\hbar$ should be correspondingly larger): synchrony is predicted also now.

3.7 Appendix

3.7.1 About Inclusions Of Hyper-Finite Factors Of Type Ii$_1$

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A66]. It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [A66] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.

2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A66]. For any amenable group G the inclusion is also unique apart from outer automorphism [A41].

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II$_1$ factor [A66]. The construction of Jones leads to a standard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_{\mathcal{N}} \mathcal{M} .... \otimes_{\mathcal{N}} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A66].

1. Jones inclusions

For hyper-finite factors of type II$_1$ Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n) | n \geq 3\} \cap [4, \infty)$ [A66]. They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i-j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A66]. It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r<4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis that only the groups $G_a \times G_b \subset SU(2)\times SU(2) \subset SL(2,C)\times SU(3)$ define orbifold coverings of $H_\pm = CD \times CP_2 \rightarrow H_\pm/G_a \times G_b$.

2. Wasserman's inclusion

Wasserman's construction of $r=4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r=4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1\otimes \mathcal{M})^G \subset (M_2(C)\times \mathcal{M})^G$. According to [A66] Jones inclusions are irreducible also for $r=4$. The definition of Wasserman inclusion for $r=4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2,C)\otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r=4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d=4$ for finite G.

For $r<4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r<4$ inclusion $\mathcal{N}=\mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r=4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \rightarrow H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n\geq 4$) are allowed for $r=4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r<4$, remains open.

3.7.2 Generalization From $Su(2)$ To Arbitrary Compact Group

The inclusions with index $\mathcal{M}:\mathcal{N}<4$ have one-dimensional relative commutant $\mathcal{N}'\cup\mathcal{M}$. The most obvious conjecture that $\mathcal{M}:\mathcal{N}\geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A48] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2=(q-1)e_i+q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q=1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q=exp(i2\pi/l)$, $l=4,5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k\geq 2$. For $SU(2)$ also the value $l=3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} \quad = \quad \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} \quad . \tag{3.7.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l-1$ holds true. Only Young diagrams satisfying the condition $l - k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the embedding space in such a way that only cyclic group Z_n appears in the covering of $M^4 \to M^4/G_a$ or $CP_2 \to CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the embedding space. In the case of $SU(2)$ the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the embedding space level so that Z_3 statistics would be in question.

Chapter 4

Philosophy of Adelic Physics

4.1 Introduction

I have developed during last 39 years a proposal for unifying fundamental interactions which I call "Topological Geometrodynamics" (TGD). During last twenty years TGD has expanded to a theory of consciousness and quantum biology and also p-adic and adelic physics have emerged as one thread in the number theoretical vision about TGD.

Since Quantum TGD and physical arguments have served as basic guidelines in the development of p-adic ideas, the best way to introduce the subject of p-adic physics, is by describing first TGD briefly.

In this article I will consider the p-adic aspects of TGD - the first thread of the number theoretic vision - as I see them at this moment.

1. I will describe p-adic mass calculations based on p-adic generalization of thermodynamics and super-conformal invariance [K68, K32] with number theoretical existence constrains leading to highly non-trivial and successful physical predictions. Here the notion of canonical identification mapping p-adic mass squared to real mass squared emerges and is expected to be key player of adelic physics and allow to map various invariants from p-adics to reals and vice versa.

2. I will propose the formulation of p-adicization of real physics and adelization meaning the fusion of real physics and various p-adic physics to single coherent whole by a generalization of number concept fusing reals and p-adics to larger structure having algebraic extension of rationals as a kind of intersection.

   The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far form obvious, and various constraints lead to the idea of NTU and finite measurement resolution realized in terms of number theory. Maybe the only way to overcome the problems relies on the idea that various angles and their hyperbolic analogs are replaced with their exponentials and identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

   Another challenge is the correspondence between real and p-adic physics at various levels: space-time level, embedding space level, and WCW level. Here the enormous symmetries of WCW and those of embedding space are in crucial role. Strong form of holography (SH) allows a correspondence between real and p-adic space-time surfaces induced by algebraic continuation from string world sheets and partonic 2-surface, which can be said to be common to real and p-adic space-time surfaces.

3. In the last section I will describe the role of p-adic physics in TGD inspired theory of consciousness. The key notion is Negentropic entanglement (NE) characterized in terms of number theoretic entanglement negentropy (NEN). Negentropy Maximization Principle (NMP) would force the growth of NE. The interpretation would be in terms of evolution as increase

of negentropy resources - Akashic records as one might poetically say. The newest finding is that NMP in statistical sense follows from the mere fact that the dimension of extension of rationals defining adeles increases unavoidably in statistical sense - separate NMP would not be necessary.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); World of Classical Worlds (WCW); Strong Form of GCI (SGCI); Strong Form of Holography (SH); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Quantum Criticality (QC); Hyper-finite Factor of Type II_1 (HFF); Number Theoretical Universality (NTU); Canonical Identification (CI); Negentropy Maximization Principle (NMP); Negentropic entanglement (NE); Number Theoretical Entanglement Negentropy (NEN); are the most often occurring acronyms.

4.2 TGD briefly

This section gives a brief summary of classical and quantum TGD, which to my opinion is necessary for understanding the number theoretic vision.

4.2.1 Space-time as 4-surface

TGD forces a new view about space-time as 4-surface of 8-D imbedding space. This view is extremely simple locally but by its many-sheetedness topologically much more complex than GRT space-time.

Energy problem of GRT as starting point

The physical motivation for TGD was what I have christened the energy problem of General Relativity [K130, K26].

1. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The presence of matter curves empty Minkowski space M^4 so that its isometries realized as transformations leaving the distances between points and thus shapes of 4-D objects invariant are lost. Noether's theorem states that symmetries and conservation laws correspond to each other. Hence conservation laws are lost and conserved quantities are ill-defined. Usually this is not seen a practical problem since gravitation is so weak interaction.

2. The proposed way out of the problem is based on the assumption that space-times are imbeddable as 4-surfaces to some 8-dimensional space $H = M^4 \times S$ by replacing the points of 4-D empty Minkowski space with 4-D very small internal space S. The space S is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [K130]. Isometries of space-time are replaced with those of imbedding space. Noether's theorem predicts the classical conserved charges for given general coordinate invariant (GCI) action principle.

Also now the curvature of space-time codes for gravitation. Equivalence Principle (EP) and General Coordinate Invariance (GCI) of GRT augmented with Relativity Principle (RT) of SRT remain the basic principles. Now however the number of solutions to field equations - preferred extremals (PEs) - is dramatically smaller than in Einstein's theory [K10, K19].

1. An unexpected bonus was geometrization classical fields of standard model for $S = CP_2$. Also the space-time counterparts for field quanta emerge naturally but this requires a profound generalization of the notion of space-time: the topological inhomogenities of space-time surface are identified as particles. This means a further huge reduction for dynamical field like variables at the level of single space-time sheet. By general coordinate invariance (GCI) only four imbedding space coordinates appear as variables analogous to classical fields: in a typical GUT their number is hundreds.

2. CP_2 also codes for the standard model quantum numbers in its geometry in the sense that electromagnetic charge and weak isospin emerge from CP_2 geometry: the corresponding symmetries are not isometries so that electroweak symmetry breaking is coded already at this level. Color quantum numbers correspond to the isometries of CP_2 defining an unbroken symmetry: this also conforms with empirical facts. The color of TGD however differs from that in standard model in several aspects and LHC has began to exhibit these differences via the unexpected behavior of what was believed to be quark gluon plasma [K74]. The conservation of baryon and lepton numbers follows as a prediction. Leptons and quarks correspond to opposite chiralities for imbedding space spinors.

3. What remains to be explained in standard model is family replication phenomenon for leptons and quarks. Both quarks and leptons appear as three families identical apart from having different masses. The conjecture was is that fermion families correspond to different topologies for 2-D surfaces characterized by genus telling the number g (genus) of handles attached to sphere to obtain the surface: sphere, torus, ... The 2-surfaces are identified as "partonic 2-surfaces" whose orbits are light-like 3-surface at which the signature of the induced metric of space-time surface transforms from Minkowskian to Euclidian. The partonic orbits replace the lines of Feynman diagrams in TGD Universe in accordance with the replacement of point-like particle with 2-surface.

   Only the three lowest genera are observed experimentally. A possible explanation is in terms of conformal symmetries: the genera $g \leq 2$ allow always Z_2 as a subgroup of conformal symmetries (hyper-ellipticity) whereas higher genera in general do not. The handles of partonic 2-surfaces could form analogs of unbound many-particle states for $g > 2$ with a continuous spectrum of mass squared but for $g = 2$ form a bound state by hyper-ellipticity [K32].

4. Later further arguments in favor of$H = M^4 \times CP_2$ have emerged. One of them relates to twistorialization and twistor lift of TGD [K116, K49, K13]. 4-D Minkowski space is unique space-time with Minkowskian signature of metric in the sense that it allows twistor structure. This is a problem in attempts to introduce twistors to General Relativity Theory (GRT) and a serious obstacle in the quantization based on twistor Grassmann approach, which has demonstrate its enormous power in the quantization of gauge theories. In TGD framework one can ask whether one could lift also the twistor structure to the level of H. M^4 has twistor structure and so does also CP_2: which is the only Euclidian 4-manifold allowing twistor space, which is also a Kähler manifold! This led to the notion of twistor lift of TGD inducing rather recent breakthrough in the understanding of TGD.

TGD can be also seen as a generalization of hadronic string model - not yet superstring model since this model became fashionable two years after the thesis about TGD [K3]. Later it has become clear that string like objects, which look like strings but are actually 3-D are basic stuff of TGD Universe and appear in all scales [K36, K10]. Also strictly 2-D string world sheets popped up in the formulation of quantum TGD (analogy with branes) [**?**]o that one can say that string model in 4-D space-time is part of TGD.

Concluding, TGD generalizes standard model symmetries and provides an incredibly simple proposal for a dynamics: only 4 classical field variables and in fermionic sector only quark and lepton like spinor fields. The basic objection against TGD looks rather obvious in the light of afterwisdom. One loses linear superposition of fields, which holds in good approximation in ordinary field theories, which are almost linear. The solution of the problem to be discussed later relies on the notion many-sheeted space-time [K26].

Many-sheeted space-time

The replacement of the abstract manifold geometry of general relativity with the geometry of 4-surfaces brings in the shape of surface as seen from the perspective of 8-D space-time as additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any variational principle satisfying GCI led soon to the realization that the topological structure of space-time in this framework is much more richer than in GRT.

1. Space-time decomposes into space-time sheets of finite size. This led to the identification of physical objects that we perceive around us as space-time sheets. The original identification of space-time sheet was as a surface of in H with outer boundary. For instance, the outer boundary of the table would be where that particular space-time sheet ends (what "ends" means is not however quite obvious!). We would directly see the complex topology of many-sheeted space-time! Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

   It turned that boundaries are probably excluded by boundary conditions. Rather, two sheets with boundaries must be glued along their boundaries together to get double covering. Sphere can be seen as simplest example of this kind of covering: northern and southern hemispheres are glued along equator together.

2. The original vision was that elementary particles are topological inhomogenities glued to these space-time sheets using topological sum contacts. This means drilling a hole to both sheets and connecting with a very short cylinder. 2-dimensional illustration should give the idea. In this conceptual framework material structures and shapes would not be due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in GRT.

   This view has gradually evolved to much more detailed picture. Elementary particles have wormhole contacts as basic building bricks. Wormhole contact is very small region with *Euclidian (!)* signature of the induced metric connecting two Minkowskian space-time sheets with light-like boundaries carrying spinor fields and there particle quantum numbers. Wormhole contact carries magnetic monopole flux through it and there must be second wormhole contact in order to have closed lines of magnetic flux. Particle world lines are replaced with 3-D light-like surfaces - orbits of partonic 2-surfaces - at which the signature of the induced metric changes.

   One might describe particle as a pair of magnetic monopoles with opposite charges. With some natural assumptions the explanation for the family replication phenomenon in terms of the genus g of the partonic 2-surface is not affected. Bosons emerge as fermion anti-fermion pairs with fermion and anti-fermion at the opposite throats of the wormhole contact. In principle family replication phenomenon should have bosonic analog. This picture assigns to particles strings connecting the two wormhole throats at each space-time sheet so that string model mathematics becomes part of TGD.

The notion of classical field differs in TGD framework in many respects from that in Maxwellian theory.

1. In TGD framework fields do not obey linear superposition and all classical fields are expressible in terms of four imbedding space coordinates in given region of space-time surface. Superposition for classical fields is replaced with *superposition of their effects* [K111, K130] - in full accordance with operationalism. Particle can topologically condense simultaneously to several space-time sheets by generating topological sum contacts (not stable like the wormhole contacts carrying magnetic monopole flux and defining building bricks of particles). Particle "experiences" the superposition of the effects of the classical fields at various space-time sheets rather than the superposition of the fields.

   It is also natural to expect that at macroscopic length scales the physics of classical fields (to be distinguished from that for field quanta) can be explained using only four primary field like variables. Electromagnetic gauge potential has only four components and classical electromagnetc fields give and excellent description of physics. This relates directly to electroweak symmetry breaking in color confinement which in standard model imply the effective absence of weak and color gauge fields in macroscopic scales. TGD however predicts that copies of hadronic physics and electroweak physics could exist in arbitrary long scales [K73] and there are indications that just this makes living matter so different as compared to inanimate matter.

2. The notion of induced gauge field means that one induces electroweak gauge potentials defining so called spinor connection at space-time surface (induction of bundle structure). Induction boils down locally to a projection of the imbedding space vectors representing the spinor connection. The classical fields at the imbedding space level are non-dynamical and fixed and extremely simple: one can say that one has generalization of constant electric field and magnetic fields in CP_2. The dynamics of the 3-surface however implies that induced fields can form arbitrarily complex field patterns. This is essentially dynamics of shadows.

   Induced gauge fields are not equivalent with ordinary free gauge fields. For instance, the attempt to represent constant magnetic or electric field as a space-time time surface has a limited success. Only a finite portion of space-time carrying this field allows realization as 4-surface. I call this topological field quantization. The magnetization of electric and magnetic fluxes is the outcome. Also gravitational field patterns allowing imbedding are very restricted: one implication is that topological with over-critical mass density are not globally imbeddable. This would explain why the mass density in cosmology can be at most critical. This solves one of the mysteries of GRT based cosmology [K104].

   Quite generally, the field patterns are extremely restricted: not only due to imbeddability constraint but also due to the fact that by SH only very restricted set of space-time surfaces can appear solutions of field equations: I speak of preferred extremals (PEs) [K10, K19, K26]. One might speak about archetypes at the level of physics: they are in quite strict sense analogies of Bohr orbits in atomic physics: this is implies by the realization of GCI. This kind of simplicity does not conform with what we observed. The way out is many-sheeted space-time. Although fields do not superpose, particles experience the superposition of effects from the archetypal field configurations (superposition is replaced with set theoretic union).

3. The important implication is that one can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K82]. One can speak about field body or magnetic body of the system.

   Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C1] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K74]. The magnetic flux tubes of magnetic body carry monopole fluxes existing without generating currents. In cosmology the flux tubes assignable to the remnants of cosmic strings make possible long range magnetic fields in all scales impossible in standard cosmology. Also super-conductivity is proposed to rely on dark $h_{eff} = n \times h$ Cooper pairs at pairs of flux tubes carrying monopole flux.

GRT and gauge theory limit of TGD is obtained as an approximation.

1. GRT/gauge theory type description is an approximation obtained by lumping together the space-time sheets to single region of M^4, with gravitational fields and gauge potentials as sums of corresponding induced field quantities at space-time surface geometrized in terms of geometry of H. Gravitational field corresponds to the deviation of the induced metric from Minkowski metric using M^4 coordinates for space-time surface so that the description applies only in long length scale limit.

   Space-time surface has both Minkowskian and Euclidian regions. Euclidian regions are identified in terms of what I call generalized scattering/twistor diagrams. The 3-D boundaries between Euclidian and Minkowskian regions have degenerate induced 4-metric and I call them light-like orbits of partonic 2-surfaces or light-like wormhole throats analogous to blackhole horizons. The interiors of blackholes are replaced with the Euclidian regions and every physical system is characterized by this kind of region.

   Lumping of sheets together implies that global conservation laws cannot hold exactly true for the resulting GRT type space-time. Equivalence Principle (EP) as Einstein's equations stating conservation laws locally follows as a local remnant of Poincare invariance.

2. Euclidian regions are identified as slightly deformed pieces of CP_2 connecting two Minkowskian space-time regions. Partonic 2-surfaces defining their boundaries are connected to each other by magnetic flux tubes carrying monopole flux.

   Wormhole contacts connect two Minkowskian space-time sheets already at elementary particle level, and appear in pairs by the conservation of the monopole flux. Flux tube can be visualized as a highly flattened square traversing along and between the space-time sheets involved. Flux tubes are accompanied by fermionic strings carrying fermion number. Fermionic strings give rise to string world sheets carrying vanishing induced em charged weak fields (otherwise em charge would not be well-defined for spinor modes). String theory in space-time surface becomes part of TGD. Fermions at the ends of strings can get entangled and entanglement can carry information.

3. Strong form of GCI (SGCI) states that light-like orbits of partonic 2-surfaces on one hand and space-like 3-surfaces at the ends of causal diamonds on the other hand provide equivalent descriptions of physics. The outcome is that partonic 2-surfaces and string world sheets at the ends of CD can be regarded as basic dynamical objects.

   Strong form of holography (SH) states the correspondence between quantum description based on these 2-surfaces and 4-D classical space-time description, and generalizes AdS/CFT correspondence. One has huge super-symplectic symmetry algebra acting as isometries of WCW with conformal structure [K35, K98, K128], conformal algebra of light-cone boundary extending the ordinary conformal algebra, and ordinary Kac-Moody and conformal symmetries of string world sheets. This explains why 10-D space-time can be replaced with ordinary space-time and 4-D Minkowski space can be replaced with partonic 2-surfaces and string world sheets. This holography looks very much like the one we are accustomed with!

4.2.2 Zero energy ontology (ZEO)

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology (ZEO) [K76] physical states decompose to pairs of positive and negative energy states such that the net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events.

ZEO and positive energy ontology

ZEO is consistent with the crossing symmetry of QFTs meaning that the final states of the quantum scattering event can be described formally as negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem, which emerges in any theory assuming symmetries giving rise to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state of say cosmology. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in GRT based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

At the level of principle the implications are quite dramatic. In quantum jump as recreation replacing the quantum Universe with a new one it is possible to create entire sub-universes from vacuum without breaking the fundamental conservation laws. From the point of view of consciousness theory the important implication is that "free will" is consistent with the laws of physics. This makes obsolete the basic arguments in favor of materialistic and deterministic world view.

Zero energy states are located inside causal diamond (CD)

By quantum classical correspondence zero energy states must have space-time and imbedding space correlates.

1. Positive and negative energy parts of zero energy state reside at future and past light-like boundaries of causal diamond (CD) identified as intersection of future and past directed light-cones and visualizable as double cone. The analog of CD in cosmology is big bang followed by big crunch. Penrose diagrams provide an excellent 2-D visualization of the notion. CDs form a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs could also intersect.

   The interpretation of CD in TGD inspired theory of consciousness is as an imbedding space correlate for perceptive field of conscious entity: the contents of conscious experience is about the region defined by CD. At the level of space-time sheets the experience come from space-time sheets in the interior of CD. Whether the sheets can be assumed to continue outside CD is still unclear.

   Quantum measurement theory must be modified in ZEO since state function reduction can happen at both boundaries of CD and the reduced states at opposite boundaries are related by time reversal. One can also have quantum superposition of CDs changing between reductions at active boundary followed by localization in the moduli space of CDs with the tip of passive boundary fixed. Quantum measurement theory generalizes to a theory of consciousness with continuous entity identified as a sequence of state function reductions at active (changing) boundary of CD [K9].

2. By number theoretical universality (NTU) the temporal distances between the tips of the intersecting light-cones are assumed to come as integer multiples $T = m \times T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. p-Adic length scale hypothesis [K78, K126] motivates the stonger hypothesis that the distances tend to come as as octaves of T_0: $T = 2^n T_0$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K12] . This means a direct coupling between microscopic and macroscopic scales.

4.2.3 Quantum physics as physics of classical spinor fields in WCW

The notions of Kähler geometry of "World of Classical Worlds" (WCW) and WCW spinor structure are inspired by the vision about the geometrization of the entire quantum theory.

Motivations for WCW

The notion of "World of Classical Worlds" (WCW) [K60, K35, K98] was forced by the failure of both path integral approach and canonical quantization in TGD framework. The idea is that the Kähler function defining WCW Kähler geometry is determined by the real part of an action S determining space-time dynamics and receiving contributions from both Minkowskian and Euclidian regions of space-time surface X^4 (note that $\sqrt{g_4}$ is proportional to imaginary unit in Minkoskian regions).

1. If S is space-time volume both canonical quantization and path integral would make sense at least formally since in principle one could solve the time derivatives of four imbedding space coordinates as functions of canonical momentum densities (general coordinate invariance allows to eliminate four coordinates). The calculation of path integral is however more or less hopeless challenge in practice.

2. A mere space-time volume as action is however not physically attractive. This was thought to leave under consideration only Kähler action S_K - Maxwell action for the induced Kähler form expressible in terms of gauge potential defined by the induced Kähler gauge potential of CP_2. This action has however a huge vacuum degeneracy. Any space-time surface with at most 2-D CP_2 projection, which is Lagrangian sub-manifold of CP_2, is vacuum extremal. Symplectic transformations acting like U(1) gauge transformations generate new vacuum

extremals. They however fail to act as symmetries of non-vacuum extremals so that gauge invariance is not in question: the deviation of the induced metric from flat metric is the reason for the failure. This degeneracy is assumed to give rise to what might be called 4-D spin glass degeneracy meaning that the landscape for the maxima of Kähler function is fractal.

3. Canonical quantization fails because by the extreme non-linearity of the action principle making it is impossible to solve time derivatives explicitly in terms of canonical momentum densities. The problem is especially acute for the canonical imbedding of empty Minkowski space to $M^4 \times CP_2$. The action is vanishing up to fourth order in imbedding space coordinates so that canonical momentum densities vanish identically and there is no hope of defining propagator in path integral approach. The mechanical analog would be criticality around which the potential reduces to $V \propto x^4$. Quantum criticality is indeed a basic aspect of TGD Universe.

The hope held for a long time was that WCW geometry allowing to get rid of path integral would solve the problems. One could however worry about vacuum degeneracy implying that WCW metric becomes extremely degenerate for vacuum extremals and also holography becomes extremely non-unique for them. Also the expected feailure of perturbative approach around M^4 is troublesome.

WCW and twistor lift of TGD

During last year this picture has indeed changed thanks to what might be called twistor lift of TGD [K116, K49, K13] inspired by twistor Grassmann approach to supersymmetric gauge theories [B18]. Remarkably, twistor lift would provide automatically the fundamental couplings of standard model and GRT and also the scale assigned to GUTs as CP_2 radius. PEs would be both extremals of Kähler action and minimal surfaces.

1. The basic observation is E^4, and its Euclidian compactification S^4 and CP_2 are completely unique in that they allow twistor space with Kähler structure [A50]. This was discovered by Hitchin at roughly the same time as I discovered TGD! This generalizes to M^4 having a generalization of ordinary Kähler structure to what I have called Hamilton-Jacobi structure by decomposition $M^4 = M^2 \times E^2$, where M^2 allows hypercomplex structure [K116, K49]. One can consider also integral distributions of tangent decompositions $M^4 = M^2(x) \times E^2(x)$, depending on position. The twistor space has a double fibration by S^2 with base spaces identifiable as M^4 and conformal compactification of M^4 for which metric is defined only up to conformal scaling. The first fibration $M^4 \times S^2$ with a well-defined metric would correspond to the classical TGD.

2. Both Newton's constant G and cosmological constant Λ emerge from twistor lift in M^4 factor. The radius of S^2 is identified in terms of Planck length $l_P = \sqrt{G}$. For CP_2 factor, the radius corresponds to the radius of CP_2 geodesic sphere. 4-D Kähler action can be lifted to 6-D Kähler action only for $M^4 \times CP_2$ so that TGD would be completely unique both mathematically and physically. The twistor space of CP_2 is flag-manifold $SU(3)/U(1) \times U(1)$ having interpretation as the space for the choices of quantization axis of color isospin and hypercharge. This choice could correspond to a selection of Eguchi-Hanson complex coordinates for CP_2 by fixing their phase angles in which isospin and hypercharge rotations induce shifts.

3. The physically motivated conjecture is that the PEs can be lifted to their 6-D twistor bundles with S^2 serving as a fiber, that one induce the twistor structure and the outcome is equal to the twistor structure of space-time surface, and that this condition is at least part of the PE property. This would correspond to the solution of massless wave equations in terms of twistors in the original twistor approach of Penrose [B49]. The analog of spontaneous compactification would lead to 4-D action equal to Kähler action plus volume term. One could of course postulate this action directly without mentioning twistors at all.

   The coefficient of the volume term would correspond to dark energy density characterized by cosmological constant Λ being extremely small in cosmological scales. It removes vacuum degeneracy although the situation remains highly non-perturbative. This can be combined with the earlier conjecture that cosmological constant Λ behaves as $\Lambda \propto 1/p$ under p-adic coupling constant evolution so that Λ would be large in primordial cosmology.

4. The generic extremals of space-time action would depend on coupling parameters, which does not fit with the number theoretic vision inspiring speculations that space-time surface can be seen as quaternionic sub-manifolds of 8-D octonionic space-time [K109], satisfying quaternion analyticity [K49], or a 4-D generalization of holomorphy. By SH the extremals are however "preferred". What could this imply?

   Intriguingly, all known non-vacuum extremals and also CP_2 type vacuum extremals having null-geodesic as M^4 projection are extremals of both Kähler action and volume term separately! The dynamics for volume term and Kähler action effectively decouple and coupling constants do not appear at all in field equations. The twistor lift would only select minimal surface amongst vacuum extremals, modify the Kähler function of WCW identifiable as exponent for the real part of action, and provide a profound mathematical and physical motivation for cosmological constant Λ remaining mysterious GRT framework. One could even hope that preferred extremals are nothing but minimal surface extremals of Kähler action with the vanishing conditions for some sub-algebra of super-symplectic algebra satisfied automatically!

   The analog of decoupling of Kähler action and volume term should take place also for induced spinors. This is expected if mere analyticity properties make spinor modes solutions of modified Dirac equations. This is true in 2-D case Hamilton-Jacobi structure should guarantee this in 4-D case [K128, K49].

   PEs depend on coupling parameters only via boundary conditions stating the vanishing of Noether charges for a sub-algebra of super-symplectic algebra and its commutator with entire algebra. Also the conservation conditions at 3-D light-like surfaces at which the signature of metric changes imply dependence on coupling parameters. These conditions allow the transfer of classical charges between Minkowskian and Euclidian regions necessary to understand momentum exchange between particles and environment classically only if Kähler couplings strength is complex - otherwise there is no exchange of conserved quantities since their real *resp.* imaginary at the two sides [K46]. Interestingly, also in twistor Grassmann approach the massless poles in propagators are complex.

   This picture conforms with the conjecture that discrete p-adic evolution of the Kähler coupling strength in subset of primes near prime powers of two corresponds to complex zeros of zeta [K46]. This conforms also with the conjectured discreteness of p-adic coupling constant evolution by phase transitions changing the values of coupling parameters. One implication is that all loop corrections in functional integral vanish.

5. In path integral approach quantum TGD would be extremely non-perturbative around extremals for which Kähler action vanishes. Same is true also in WCW approach. The cure would be provided by the hierarchy of Planck constants $h_{eff}/h = n$, which effectively scales Λ down to Λ/n. n would be the number sheets of the M^4 covering defined by the space-time surface: the action of Galois group for the number theoretic discretization of space-time surface could give rise to this covering. The finiteness of the volume term in turn forces ZEO: the volume of space-time surface is indeed finite due to the finite size of CD.

Consider now the delicacies of this picture.

1. Should assign also to M^4 the analog of symplectic structure giving an additional contribution to the induced Kähler form? The symmetry between M^4 and CP_2 suggests this, and this term could be highly relevant for the understanding of the observed CP breaking and matter antimatter asymmetry [L43]. Poincare invariance is not lost since the needed moduli space for M^4 Kähler forms would be the moduli space of CDs forced by ZEO in any case, and M^4 Kähler form would serve as the correlate for fixing rest system and spin quantization axis in quantum measurement.

2. Also induced spinor fields are present. The well-definedness of electro-magnetic charge for the spinor modes forces in the generic case the localization of the modes of induced spinor fields at string world sheets (and possibly to partonic 2-surfaces) at which the induced charged weak gauge fields and possibly also neutral Z^0 gauge field vanish. The analogy with branes and super-symmetry force to consider two options.

   Option I: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K101].

   This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

   Option II: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes! In this case 4-D spinor modes do not define space-time super-symmetries.

3. Why the string world sheets coding for effective action should carry vanishing weak gauge fields? If M^4 has the analog of Kähler structure [L43], one can speak about Lagrangian sub-manifolds in the sense that the sum of the symplectic forms of M^4 and CP_2 projected to Lagrangian sub-manifold vanishes. Could the induced spinor fields for effective action be localized to generalized Lagrangian sub-manifolds? This would allow both string world sheets and 4-D space-time surfaces but SH would select 2-D Lagrangian manifolds. At the level of effective action the theory would be incredibly simple.

   Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would coupleonly to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [K20] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of weak group to electromagnetic gauge group.

   The projections of canonical currents of Kähler action to string world sheets would vanish, and the projections of the 4-D modified gamma matrices would define just the induced 2-D metric. If the induced metric of space-time surface reduces to an orthogonal direct sum of string world sheet metric and metric acting in normal space, the flow defined by 4-D canonical momentum currents is parallel to string world sheet. These conditions could define the "boundary" conditions at string world sheets for SH.

This admittedly speculative picture has revolutionized the understanding of both classical and quantum TGD during last year. [K49, K13, K26]. In particular, the construction of single-sheeted PEs as minimal surfaces allows a kind of lego like engineering of more complex PEs [L12]. The minimal surface equations generalize Laplace equation of Newton's gravitational theory to non-linear massless d'Alembert equation with gravitational self-coupling. One obtains the analog of Schwartschild solution and radiative solutions describing also gravitational radiation [K26]. An open question is whether classical theory makes sense if also the analog of Kähler form in M^4 is allowed.

Identification of WCW

The notion of WCW [K60, K35, K98] was inspired by the super-space approach of Wheeler in which 3-geometries are the basic geometric entities.

1. In TGD framework 3-surfaces take this role. Einstein's program for geometrizing classical physics is generalized to a geometrization of entire quantum physics. Hermitian conjugation corresponds to complex conjugation in infinite-dimensional context so that WCW must have Kähler geometry. The geometrization of fermionic statistics/oscillator operators is in terms of gamma matrices of WCW expressible as linear combinations of oscillator operators for second quantized induced spinor field. Formally purely classical spinor modes of WCW represent many fermion states as functionals of 3-surface. One can also interpret gamma matrices as generators of super-conformal symmetries in accordance with the fact that also SUSY involves Clifford algebra.

   In ZEO the entanglement coefficients between positive and negative energy parts of zero energy states determine the S-matrix so that S-matrix would be coded by the modes of WCW spinor fields. Twistor approach to TGD [K49] suggests that the S-matrix reduces completely to the symmetries defined by the multi-local (locus corresponds to partonic 2-surface) generators of the Yangian associated with the super-symplectic algebra.

2. ZEO forces to identify 3-surfaces as pairs of 3-surfaces with members at the opposite boundaries of CD. SH reduces them to a collection of partonic 2-surfaces at boundaries of CD plus number theoretic discretization in space-time interior. Basic geometric objects are pairs of initial and final states (coordinates for both in mechanical analogy) rather than initial states with initial value conditions (coordinates and velocities in mechanical analogy) and initial value problem transforms to boundary value problem. Processes rather than states become the basic elements of ontology: this has far reaching consequences in biology and neuroscience.

3. The realization of GCI requires that the definition of WCW Kähler function assigns to a "physically" 3-surface a unique 4-surface for 4-D general coordinate transformations to act: "physically" could mean "apart from transformations acting as gauge transformations" not affecting the action and conserved classical charges. The outcome is holography.

4. Strong form of holography (SH) would emerge as follows. The condition that light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions are basic geometric entities equivalent with pairs of space-like 3-surfaces at the ends of given causal diamond CD implies SH: partonic 2-surfaces and their 4-D tangent space data should code the physics. One could also speak about almost/effective 2-dimensionality. Tangent space data could in turn be coded by string world sheets. Number theoretical discretitization of space-time interior with preferred coordinates in the extension of rationals could give meaning for "almost".

5. Kähler metric is expressible both in terms of second derivatives of Kähler function K [K60] and as anticommutators of WCW gamma matrices expressible as linear combinations of fermionic oscillator operators. This suggests a close relationship between space-time dynamics and spinor dynamics.

   Super-symplectic symmetry between the action defining space-time surfaces (Kähler action plus volume term) and modified Dirac action would realize this relationship. This is achieved if the modified gamma matrices are defined by the canonical momentum currents of 2-D action associated with string world sheets. These currents are parallel to the string world sheets. This implies the analog of AdS/CFT correspondence requiring only that induced spinor modes at string world sheets determine them in space-time interior (this is like analytic continuation). The localization of spinor modes at string world sheets is *not* required as I believed first.

The geometry of loop spaces developed by Freed [A35] serves as a model in the construction of WCW Kähler geometry [K98].

1. The existence of loop space Riemann connection requires maximal isometry group identifiable as Kac-Moody group so that Killing vector fields span the entire tangent space of the loop space.

2. In TGD framework the properties of Kähler action lead to the idea that WCW is union of homogenous or even symmetric spaces of symplectic algebra acting at the boundary of $\delta CD \subset \delta CD_+ \cup \delta CD_-$, $\delta CD_\pm \subset \delta M^4_\pm \times CP_2$. ZEO requires that the conserved quantum numbers for physical states are opposite for the positive and negative energy parts of the states at the two opposite boundary parts of CD. The symmetric spaces G/H in the union are labelled by zero modes, which do not appear in the line element as differentials but only as parameters of the metric. Conserved Noether charges of isometries and symplectic invariants of examples of zero modes as also the super-symplectic Noether charges invariant under complex conjugation of WCW coordinates.

3. Homogenous spaces of the symplectic group G are obtained by dividing by a subgroup H. An especially attractive option is suggested by the fractal structure of the symplectic algebra containing an infinite hierarchy of sub-algebras G_n for which conformal weights are $n > 0$-multiples of those for G. For this option $H = G_n$ is isomorphic to G and one could have infinite hierarchies of inclusions analogous to the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs). PE property requires almost 2-dimensionality and elimination of huge number of degrees of freedom. The natural condition is that the Noether charges of G_n vanish at the ends of CD. A stronger condition is that also the Noether charges for $[G, G_n]$ vanish. This implies effective normal algebra property and G/G_n acts effectively like group.

   The inclusion of HFFs would define measurement resolution with included factor acting like gauge algebra. Measurement resolution would be naturally determined by the number theoretic discretization of the space-time surface so that physics as geometry and number theory visions would meet each other.

4. This inclusion hierarchy can be identified in terms of quantum criticality (QC). The transitions $n \rightarrow kn$ increasing the value of $n > 0$ reduce QC since pure gauge symmetries are reduced, and new physical super-symplectic degrees of freedom emerge. QC also requires that Kähler couplings strength analogous to temperature is analogous to critical temperature so that the quantum theory is uniquely defined if their is only one critical temperature. Spectrum for α_K seems more plausible and the possibility that Kähler coupling strength depends on the level of the number theoretical hierarchy defined by the allowed extensions of rationals can be considered [K46].

WCW spinor structure

The basic idea is geometrization of quantum states by identifying them as modes of WCW spinor fields [K128, K98]. This requires definition of WCW spinors and WCW spinor structure, WCW gamma matrices and Dirac operator, etc..

The starting point is the definition of WCW gamma matrices using a representation analogous to the usual vielbein representation as linear combinations of flat space gamma matrices. The conceptual leap is the observation that there is no need to assume that the counterparts of flat space gamma matrices have vectorial quantum numbers. Instead, they are identified as fermionic oscillator operators for second quantized free induced spinor fields at space-time surface.

This allows geometrization of the fermionic statistics since WVW spinors for a given 3-surface are analogous to fermionic Fock states. One can also say that spinor structure follows as a square root of metric and also that the spinor basis defines a geometric correlate of Boolean mind [K31]. The dependence of WCW spinor field on 3-surface represents the bosonic degrees of freedom not reducible to many-fermion states. For instance, most of hadron mass would be associated with these degrees of freedom.

Quantum TGD involves Dirac equations at space-time level, imbedding space level, and level of WCW. The dynamics of the induced spinor fields is related by super-symmetry to the action defining space-time surfaces as preferred extremals. [K128, K98].

1. The gamma matrices in the equation - modified gamma matrices - are determined by contractions of the canonical momentum currents of Kähler action with the imbedding space

gamma matrices. The localization at string world sheets for which only induced neutral weak fields or only em field are non-vanishing is accompanied by the integrability condition that various conserved currents run along string world sheets: one can speak of sub-flow. I

2. Modified Dirac equation can be solved exactly just like in the case of string models using holomorphy and the properties of complexified modified gamma matrices. This is expected to be true also in 4-D case by Hamilton-Jacobi structure. If the dynamics of avoidance is realized the modified Dirac equation would be essentially free Dirac equation and holomorphy would allow to solve it.

At the level of WCW one obtains also the analog of massless Dirac equation as the analog of super Virasoro conditions of Super Virasoro algebra.

1. The fermionic counterparts of super-conformal gauge conditions assignable with sub-algebra G_n of supersymplectic conformal symmetry associated with the both light-cone boundary (light-like radial coordinate), with conformal symmetries of light-cone boundary, and with string world sheets.

2. The ground states of supersymplectic representations satisfy massless imbedding space Dirac equation in imbedding space so that Dirac equations in WCW, in imbedding space, and at string world sheets are involved. In twistorialization also massless M^8 Dirac equation emerges in the tangent space M^8 of imbedding space assignable to the partonic 2-surfaces and generalizes the 4-D light-likeness with its 8-D counterpart applying to states with M^4 mass. Here octonionic representation of imbedding space gamma matrices emerges naturally and allows to speak about 8-D analogs of Pauli's sigma matrices [K116].

4.2.4 Quantum criticality, measurement resolution, and hierarchy of Planck constants

The notions of quantum criticality (QC), finite measurement resolution, and hierarchy of Planck constants proposed to give rise to dark matter as phases of ordinary matter are central for TGD [**?**, K127, K47].

These notions relate closely to the strong form of holography (SH) implied by strong form of general coordinate invariance (SGCI). In adelic physics all this would relate closely to the hierarchy of extensions of rationals serving as a correlate for number theoretical evolution.

Finite measurement resolution and fractal inclusion hierarchy of super-symplectic algebras

The fractal hierarchy of isomorphic sub-algebras of supersymplectic algebra - call it g - defines an excellent candidate for the realization of finite the measurement resolution. Similar hierarchies can be assigned also for the extended super-conformal algebra assignable with light-like boundaries of CD and with Kac-Moody and conformal algebras assignable to string world sheets.

An interesting possibility is that the the conformal weights assignable to infinitesimal scaling operator of the light-like radial coordinate of light-cone boundary correspond to zeros of Riemann zeta [K126] [L9]. A kind of dual spectrum would correspond to conformal weights that correspond to logarithms for powers of primes. One can identify the conformal weight as negative of the pole of fermionic zeta $z_F = \zeta(s)/\zeta(2s)$ natural in TGD framework. The real part of conformal weight for the generators is $h_R = -1/4$ for "non-trivial" poles and positive integer $h = n > 0$ for "trivial" poles. There is also a pole for $h = -1$. Hence one obtains tachyonic ground states, which must be assumed also in p-adic mass calculations [K68].

Also the generators of Yangian algebra [K116] integrating the algebras assignable to various partonic 2-surfaces to a multi-local algebra are labelled by a non-negative integer n analogous to conformal weight and telling the number of partonic 2-surfaces involved with the action of the generator. Also this algebra has similar fractal hierarchy of sub-algebras so that the considerations that follow might apply also to it. Now that number of partonic 2-surface would play the role of measurement resolution.

As noticed, there are also other algebras, which allow conformal hierarchy if one can restrict the conformal weights to be non-negative. The first of them generates generalized conformal transformations of light-cone boundary depending on light-like radial coordinate as parameter: also now radial conformal weights for generators can have zeros of zeta as spectrum. As a special case one obtains infinite-dimensional group of isometries of light-cone boundary. Second one corresponds to ordinary conformal and Kac-Moody symmetries for induced spinor fields acting on string world sheets. Also here similar hierarchy of sub-algebras can be considered. In the following argument one restricts to super-symplectic algebra assumed to act as isometries of WCW.

Consider now how the finite measurement resolution could be realized as an infinite hierarchy of super-symplectic gauge symmetry breakings. The physical picture relies on quantum criticality of TGD Universe. The levels of the hierarchy labelled by positive integer n and a ball at the top of ball at... serves as a convenient metaphor.

1. The sub-algebra g_n for which conformal weights of generators (whose commutators give the sub-algebra) are positive integer multiples for those of the entire algebra g defines the algebra acting as pure gauge algebra defining a sub-group of symplectic group. The action of g_n as gauge algebra would mean that it affects on degrees of freedom below the measurement resolution. One can assign to this algebra a coset space G/G_n of the entire symplectic group G and of subgroup G_n. This coset space would describe the dynamical degrees of freedom. If the subgroup were a normal subgroup, the coset space would be a group. This is not the case now since the commutator $[g, g_n]$ of the entire algebra with the sub-algebra does not belong to g_n.

   However, if one poses stronger - physically very attractive - gauge conditions stating that not only g_n but also the commutator algebra $[g, g_n]$ annihilates the physical states and that corresponding classical Noether charges vanish, one obtains effectively a normal subgroup and one has good hopes that coset space acts effectively as group, which is finite-dimensional as far as conformal weights are considered.

2. $n > 0$ is essential for obtaining effective normal algebra property. Without this assumption the commutator $[g, g_n]$ would be entire g. If the spectrum of supersymplectic conformal weights is integer valued it is not obvious why one should pose the restriction $n \geq 1$.

3. In this framework pure conformal invariance could reduce to a finite-dimensional gauge symmetry. A possible interpretation would be in terms of Mc-Kay correspondence [A69] assigning to the inclusions of HFFs labelled by integer $n \geq 3$ a hierarchy of simply laced Lie-groups. Since the included algebra would naturally correspond to degrees of freedom not visible in the resolution used, the interpretation as a dynamical gauge group is suggestive. The dynamical gauge group could correspond to n-dimensional Cartan algebra acting in conformal degrees of freedom identifiable as a simply laced Lie group. This would assign a infinite hierarchy of dynamical gauge symmetries to the broken conformal gauge invariance acting as symmetries of dark matter. This still leaves infinite number of degrees of freedom assignable to the imbedding space Hamiltonians and spectrum generated by zeros of zeta but this might have interpretation in terms of gauging so that additional vanishing conditions for Noether charges are suggestive.

Dark matter as large phases with large gravitational Planck constant $h_{eff} = h_{gr}$

D. Da Rocha and Laurent Nottale [E6] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K103, K83].

1. The proposal is that a Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems and that only the generalizations of Bohr orbits are involved. The space-time sheets in question would carry dark matter.

2. Nottale's hypothesis would predict a gigantic value of $\hbar_{gr}$. Equivalence Principle and the independence of gravitational Compton length $\Lambda_{gr} = \hbar_{gr}/m = GM/v_0 = 2r_S/v_0$ (typically astrophysical scale) on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that $\hbar_{gr}$ would be much smaller. Large $\hbar_{gr}$ could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets, which is quantum coherent in the required time scale [K103].

   One could criticize the hypothesis since it treats the masses M and m asymmetrically: this is only apparently true [**?**].

3. It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta). The cross section of the flux tube corresponds to a sphere $S^2_i \subset CP_2$, $i = I, II$ [K13]. S^2_I is homologically non-trivial carrying Kähler magnetic monopole flux. S^2_{II} is homologically trivial carrying vanishing Kähler magnetic flux but non-vanishing electro-weak flux [K13].

   The flux tubes of type I have both Kähler magnetic energy and dark energy due to the volume action. Flux tubes of type II would have only the volume energy. Both flux tubes could be remnants of cosmic string phase of primordial cosmology. The energy of these flux quanta would be correlated for galactic dark matter and volume action and also magnetic tension would give rise to negative "pressure" forcing accelerated cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside flux tubes identifiable also as dark energy.

4. Both theoretical consistency and certain experimental findings from astrophysics [E7, E11] and biology [K30, K16] suggest the identification $h_{eff} = n \times h = h_{gr}$. The large value of h_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description) [K98]. The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also modified Dirac action is present, part of the interior degrees of freedom associated with the fermionic part of conformal algebra become physical.

   Fermionic oscillator operators could generate super-symmetries and sparticles could correspond to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to an ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) or to a bunch of n low energy gravitons.

Hierarchies of quantum criticalities, Planck constants, and dark matters

Quantum criticality is one of the corner stone assumptions of TGD. In the original approach the value of Kähler coupling strength α_K together with CP_2 radius R fixed quantum TGD and is analogous to critical temperature. Twistor lift [K13] brings in additional coupling constant Λ obeying p-adic coupling constant evolution and Planck length l_G, which like CP_2 radius would not obey coupling constant evolution (as also G). The values of these parameters should be fixed by quantum criticality. What else does quantum criticality mean is however far from obvious, and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K60, K128, K98].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could quantum criticality having classical or perhaps even thermodynamical criticality as its correlate be always accompanied by the generation of dark matter? If this were the case, the recipe would be stupifyingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale [K86, **?**]. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

   The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of the particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

5. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1Z_2e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1Z_2e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

One example of criticality is super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect [D27] and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity [**?**].

But how does quantum criticality relate to number theory and adelic physics? $h_{eff}/h = n$ has been identified as the number of sheets of space-time surface identified as a covering space of some kind. Number theoretic discretization defining the "spine" for a monadic space-time surface [L19] defines also a covering space with Galois group for an extension of rationals acting as covering group. Could n be identifiable as the order for a sub-group of Galois group? If this is the case, the proposed rule for h_{eff} changing phase transitions stating that the reduction of n occurs to its factor would translate to spontaneous symmetry breaking for Galois group and spontaneous - symmetry breakings indeed accompany phase transitions.

TGD variant of AdS/CFT duality

AdS/CFT duality [B34] has provided a powerful approach in the attempts to understand the non-perturbative aspects of super-string theories. The duality states that conformal field theory in

n-dimensional Minkowski space M^n identifiable as a boundary of $n+1$-dimensional space AdS_{n+1} is dual to a string theory in $AdS_{n+1} \times S^{9-n}$.

As a mathematical discovery AdS/CFT duality is extremely interesting but it seems that it need not have much to do with physics as such. From TGD point of view the reason is obvious: the notion of conformal invariance is quite too limited. In TGD framework conformal invariance is extended to a super-symplectic symmetry in $\delta M^4_\pm \times CP_2$, whose Lie-algebra has the structure of conformal algebra. Also ordinary super-conformal symmetries associated with string world sheets are present as well as generalization of 2-D conformal symmetries to their analogs at light-cone boundary and light-like orbits of partonic 2-surfaces. In this framework AdS/CFT duality is expected to be modified.

The matrix elements $G_{K\overline{L}}$ of Kähler metric of WCW can be expressed in two manners. As contractions of the derivatives $\partial_K \partial_{\overline{L}} K$ of the Kähler function of WCW with isometry generators or as anticommutators $\{\Gamma_K, \Gamma_{\overline{L}}\}$ of WCW gamma matrices identified as supersymplectic Noether super charges assignable to fermionic strings connecting partonic 2-surfaces. Kähler function is identified as real part of the action: if coupling parameters are real it reduces to the action for the Euclidian space-time regions with 4-D CP_2 projection and otherwise contains contributions from both Minkowskian and Euclidian regions. The action defines the modified gamma matrices appearing in modified Dirac action as contractions of canonical momentum currents with imbedding space gamma matrices.

This observation suggests that there is a super-symmetry between action and modified Dirac action. The problem is that induced spinor fields naive of SH and also well-definedness of em charge demand the localization of induced spinor modes at 2-D string world sheets. This simply cannot be true. On the other hand, SH only requires that the data about induced spinor fields and space-time surface at the string world sheets is enough to construct the modes in space-time interior.

This leaves two options if one assumes that SH is exact (recall however that the number theoretic interpretation for the hierarchy of Planck constants suggests that the number-theoretic spin of monadic space-time surface represents additional discrete data needed besides that assignable to string world sheets to describe dark matter). As found in the section 4.2.3, there are two options.

Option I: The analog of brane hierarchy is realized at the level of fundamental action. There is a separate fundamental 2-D action assignable with string world sheets - area and topological magnetic flux term - as also world line action assignable to the boundaries of string world sheets. By previous argument string tension should be determined by the value of the cosmological constant Λ obeying -adic coupling constant evolution rather than by G: otherwise there is no hope about gravitationally bound states above Planck scale. String tension would appear as an additional fundamental coupling parameter (perhaps fixed by quantum criticality). This option does not quite conform with the spirit of SH.

Option II: 4-D space-time action and corresponding modified Dirac action defining fundamental actions are expressible as effective actions assignable to string world sheets and their boundaries. String world sheet effective action action could be expressible as string area for the effective metric defined by the anti-commutators of modified gamma matrices at string world sheet. If the sum of the induced Kähler forms of M^4 and CP_2 vanishes at string world sheets the effective metric would be the induced 2-D metric: this together with the observed CP breaking could provide a justification for the introduction of the analog of Kähler form in M^4. String tension would be dynamical rather than determined by l_P and depend on Λ, l_P, R and α_K. This representation of Kähler action would be one aspect of the analog of AdS/CFT duality in TGD framework.

Both options would allow to understand how strings connecting partonic 2-surfaces give rise to the formation of gravitationally bound states. Bound states of macroscopic size are possible only if one allows hierarchy of Planck constants and this is required also by the (extremely) small value of Λ (in cosmic scales).

Consider the concerete realizations for this vision.

1. SGCI requires effective 2-dimensionality. In given UV and IR resolutions partonic 2-surfaces and string world sheets are assignable to a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). Λ would closely relate to the size scale of CD. String world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These

braids carrying fermionic quantum numbers intersect partonic 2-surfaces at discrete points.

2. This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces, whose area by quantum classical correspondence depends on the quantum numbers of the external particles.

String tension of gravitational flux tubes

For Planckian cosmic strings only quantum gravitational bound states of length of order Planck length are possible. There must be a mechanism reducing the string tension. The *effective* string tension assignable to magnetic flux tubes must be inversely proportional to $1/h_{eff}^2$, $h_{eff} = n \times h = h_{gr} = 2\pi GMm/v_0$ in order to obtain gravitationally bound states in macroscopic length scales identified as structures for which partonic 2-surfaces are connected by flux tubes accompanied by fermionic strings.

The reason is that the size scale of (quantum) gravitationally bound states of masses M and m is given by grvitational Compton length $\Lambda_{gr} = GM/v_0$ [K103, K86, ?] assignable to the gravitational flux tubes connecting the masses M and m. If the string tension is of order Λ_{gr}^2 this is achieved since the typical length of string would be Λ_{gr}. Gravitational string tension must be therefore of order $T_{gr} \sim 1/\Lambda_{gr}^2$. How could this be achieved? One can imagine several options and here only the option based on the assumptions

1. Twistor lift makes sense.
2. Fundamental action is 4-D for both space-time and fermionic degrees of freedom and 2-D string world sheet action is an effective action realizing SH. Note effective action makes also possible braid statistics, which does not make sense at fundamental level.
3. Also M^4 carries the analog of Kähler form and the sum of induced Kähler forms from M^4 and CP_2 vanishes at string world sheets and also weak gauge fields vanishes at string world sheets leaving only em field.

is considered since it avoids all the objections that I have been able to invent.

For the twistor lift of TGD [K13] predicting cosmological constant Λ depending on p-adic length scale $\Lambda \propto 1/p$ the gravitational strings would be naturally homologically trivial cosmic strings. These vacuum extremals of Kähler action transform to minimal surface extremals with string tension given by $\rho_{vac}S$, where ρ_{vac} the density of dark energy assignable to the volume term of the action and S the transverse area of the flux tube. One should have $\rho_{vac}S = 8\pi\Lambda S/G = 1/\Lambda_{gr}^2$ so that one would have

$$8\pi\Lambda S = \frac{G}{\Lambda_{gr}^2} \quad .$$

Λ for flux tubes (characterizing the size of CDs containing them) would depend on the gravitational coupling Mm.

4.2.5 Number theoretical vision

Physics as infinite-D spinor geometry of WCW and physics as generalized number theory are the two basic vision about TGD. The number theoretical vision involves three threads [K108, K109, K107].

1. The first thread [K108] involves the notion of number theoretical universality NTU: quantum TGD should make sense in both real and p-adic number fields (and their algebraic extensions induced by extensions of rationals). p-Adic number fields are needed to understand the space-time correlates of cognition and intentionality [K78, K50, K80].

   p-Adic mass calcuations lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [K78, K50]. One of the first applications was the calculation of elementary particle masses [K68]. The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra are involved. Not only the fundamental mass scales would reduce to number theory but also particle masses are predicted correctly under rather mild assumptions and are exponentially sensitive to the p-adic length scale predicted by p-adic length scale hypothesis. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [K68, K32].

2. Second thread [K109] is inspired by the dimensions $D = 1, 2, 4, 8$ of the basic objects of TGD and assumes that classical number fields are in a crucial role in TGD. 8-D imbedding space would have octonionic structure and space-time surfaces would have associative (quaternionic) tangent space or normal space. String world sheets could correspond to commutative surfaces. Also the notion of M^8-H-duality is part of this thread and states that quaternionic 4-surfaces of M^8 containing preferred M^2 in its tangent space can be mapped to PEs in H by assigning to the tangent space CP_2 point parametrizing it. M^2 could be replaced by integrable distribution of $M^2(x)$. If PEs are also quaternionic one has also $H-H$ duality allowing to iterate the map so that PEs form a category. Also quaternion analyticity of PEs is a highly attractive hypothesis [K116]. For instance, it might be possible to interpret string world sheets and partonic 2-surfaces appearing in strong form of holography (SH) as co-dimension 2 surfaces analogous to poles of analytic function in complex plane. Light-like 3-surfaces might be seen as analogs of cuts. The coding of analytic function by its singularities could be seen as analog of SH.

3. The third thread [K107] corresponds to infinite primes and leads to several speculations. The construction of infinite primes is structurally analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory with free particle states characterized by primes. The many-sheeted structure of TGD space-time could reflect directly the structure of infinite prime coding it. Space-time point would become infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography. Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

4.3 p-Adic mass calculations and p-adic thermodynamics

p-Adic mass calculations carried for the first time around 1995 were the stimulus eventually leading to the number theoretical vision as a kind dual for the geometric vision about TGD. In this secton I will roughly describe the calculations [K32, K68] and the questions and challenges raised by them.

4.3.1 p-Adic numbers

Like real numbers, p-adic numbers (`http://tinyurl.com/hmgqtoh`) can be regarded as completions of the rational numbers to a larger number field [K50]. Each prime p defines a p-adic number field allowing the counterparts of the usual arithmetic operations.

1. The basic difference between real and p-adic numbers is that p-adic topology is ultra-metric. Ultrametricity means that the distance function $d(x,y)$ (the counterpart of $|x-y|$ in the real context) satisfies the inequality

$$d(x,z) \leq Max\{d(x,y), d(y,z)\} \quad ,$$

(Max(a, b) denotes maximum of a and b) rather than the usual triangle inequality

$$d(x,z) \leq d(x,y) + d(y,z) \quad .$$

2. The topology defined by p-adic numbers is compact-open. Hence the generalization of manifold obtained by gluing together n-balls fails because smallest open n-balls are just points and one has totally disconnected topology.

3. p-Adic numbers are not well-ordered like real numbers. Therefore one cannot assign orientation to the p-adic number line. This in turn leads to difficulties with attempts to define definite integrals and the notion of differential form although indefinite integral is well-defined. These difficulties serve as important guidelines in the attempts to understand what p-adic physics is and also how to fuse real and various p-adic physics to a larger structure.

4. p-Adic numbers allow an expansion in powers of p analogous to the decimal expansion

$$x = \sum_{n \geq 0} x_n p^n \ ,$$

   and the number of terms in the expansion can be infinite so that p-adic number need not be finite as a real number. The norm of the p-adic number (counterpart of $|x|$ for real numbers) is defined as

$$N_p(x) = \sum_{n \geq 0} x_n p^n = p^{-n_0} \ ,$$

   and depends only very weakly on p-adic number. The ultra-metric distance function can be defined as $d_p(x,y) = N_p(x-y)$.

5. p-Adic numbers allow a generalization of the differential calculus. The basic rules of the p-adic differential calculus are the same as those of the ordinary differential calculus. There is however one important new element: the set of the functions having vanishing p-adic derivative consists of so called pseudo constants, which are analogs of real valued piecewise constant functions. In the real case only constant functions have vanishing derivative. This implies that p-adic differential equations are non-deterministic. This non-determinism is identified as a counterpart of the non-determinism of cognition and imagination [K80].

4.3.2 Model of elementary particle

p-Adic mass calculations [K32, K68] rely heavily on a topological model for elementary particle and it is appropriate to describe it before going to the summary of calculations.

Family replication phenomenon topologically

One of the basic ideas of TGD approach to particle physics has been genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies.

With the advent of zero energy ontology (ZEO) this picture has changed somewhat.

1. The wormhole throats identified as light-like 3-surfaces at with the induced metric of the space-time surface changes its signature from Minkowskian to Euclidian correspond to the light-like orbits of partonic 2-surfaces. One cannot of course exclude the possibility that also boundary components allow to satisfy boundary conditions without assuming vacuum extremal property of nearby space-time surface.

   The intersections of the wormhole throats with the light-like boundaries of causal diamonds (CDs) identified as intersections of future and past directed light cones ($CD \times CP_2$ is actually in question but I will speak about CDs) define special partonic 2-surfaces and the conformal moduli of these partonic 2-surfaces appear in the elementary particle vacuum functionals [K32] naturally. A modification of the original simple picture came from the proposed identification of physical particles as bound states of two wormhole contacts connected by tubes carrying monopole fluxes.

2. For generalized scattering diagrams stringy trouser vertices are replaced with vertices at which the ends of light-like wormhole throats meet. This vertex is the analog of 3-vertex for Feynman diagrams in particle physics lengths scales and for the biological replication (DNA and even cell) in macroscopic length scales.

In this picture the interpretation of the analog of trouser vertex is in terms of propagation of same particle along two different paths. This interpretation is mathematically natural since vertices correspond to 2-manifolds rather than singular 2-manifolds, which are just splitting to two disjoint components. Second complication comes from the weak form of electric-magnetic duality forcing to identify physical particles as weak strings with magnetic monopoles at their ends and one should understand also the possible complications caused by this generalization.

These modifications force to consider several options concerning the identification of light fermions and bosons and one can end up with a unique identification only by making some assumptions. Masslessness of all wormhole throats - also those appearing in internal lines - and dynamical $SU(3)$ symmetry for particle generations are attractive general enough assumptions of this kind. Bosons and their possible spartners would correspond to wormhole contacts with fermion and anti-fermion at the throats of the contact. The expectation was tht free fermions and their possible spartners correspond to CP_2 type vacuum extremals with single wormhole throat. It however turned however that dynamical $SU(3)$ symmetry forces to identify massive (and possibly topologically condensed) fermions as pairs of (g, g) type wormhole contacts. The existence of higher boson families would mean breaking of quark and lepton universality and there are indications for this kind of anomaly [K73] .

The notion of elementary particle vacuum functional

Obviously one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals (EPVFs), is made. The basic assumptions underlying the construction are the following ones [K32].

1. EPVFs depend on the geometric properties of the two-surface X^2 representing elementary particle.
2. EPVFs possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting PE as a generalized Bohr orbit [K60, K10, K19]) and determined in general coordinate invariant manner.
3. ZEO allows to select uniquely the partonic 2-surface as the intersection of the wormhole throat at which the signature of the induced 4-metric changes with either the upper or lower boundary of $CD \times CP_2$. This is essential since otherwise one one could not specify the vacuum functional uniquely.
4. Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.
5. Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.
6. EPVFs are stable against the decay $g \rightarrow g_1 + g_2$ and one particle decay $g \rightarrow g - 1$. This process corresponds to genuine particle decay only for stringy diagrams. For generalized scattering diagrams the interpretation is in terms of propagation along two different paths simultaneously.

In [K32] the construction of EPVFs is described in detail. This requires some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces and the concept of hyper-ellipticity. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are needed. Also possible explanations for the experimental absence of the higher fermion families are considered. Concerning p-adic mass calculations, the key question is how to construct p-adic variants of EPVFs.

4.3.3 p-Adic mass calculations

p-Adic thermodynamics

Consider first the basic ideas of p-adic thermodynamics.

1. p-Adic valued mass squared is identified as as thermal mass in p-adic thermodynamics. Boltzmann weights $exp(-E/T)$ do not make sense if one just replaces exponent function with the p-adic variant of its Taylor series. The reason is that $exp(x)$ has p-adic norm equal to 1 for all acceptable values of the argument x (having p-adic norm smaller than one) so that partition function does not have the usual exponential convergence property. Nothing however prevents from consider Boltzmann weights as powers p^n making sense for integer values of n. Here the p-adic norm approaches zero for $n \to +\infty$: thus the correspondences $e^{-E/T} \leftrightarrow p^{E/T_p}$.

   The values of E/T_p must be quantized to integers. This is guaranteed if E is integer valued in suitable unit of energy and $1/T_p$ has integer valued spectrum using same unit for T_p. Super-conformal invariance guarantees integer valued spectrum of E, which in the recent case corresponds to mass squared. These number theoretical conditions are very powerful and lead to the quantization of also thermal mass squared for given p-adic prime p.

2. The p-adic mass squared is mapped to real number by canonical identification $I : \sum x_n p^n \to \sum x_n p^{-n}$ or its variant for rationals. Canonical identification is continuous and maps powers of p^n to their inverses. One modification of canonical identification maps rationals m/n in their representation in which m and n have no common divisors to $I(m)/I(n)$. The predictions of calculations depend in some cases on which variant one uses but rational option looks the most reasonable choice.

3. p-Adic length scale hypothesis states that preferred p-adic primes correspond to powers of 2: $p \simeq 2^k$, but smaller than 2^k. The values of k form with $p = 2^k - 1$ is prime - Mersenne prime - are especially favored. The nearer the prime p to 2^k, the more favored p is physically. One justification for the hypothesis is that preferred primes have been selected by an evolutionary process.

4. It turns out that p-adic temperature is $T_p = 1$ for fermions. For gauge bosons $T_p \leq 1/2$ seems to be necessary assumption for gauge bosons implying that the contribution to mass squared is very small so that super-symplectic contribution assignable to the wormhole magnetic flux tube dominates for weak bosons. For canonical identification $m/n \to I(m)/I(n)$ second order contribution to fermionic mass squared is very small.

5. The large values of p-adic prime p guarantee that the p-adic thermodynamics convervges extremely rapidly. For $m/n \to I(m)/I(n)$ already the second order contribution is extremely small since the expansion for the real mass squared is in terms of $1/p$ and for electron with $p = M_{127}$ one has $p \sim 10^{38}$. Hence the calculations are essentially exact and errors are those of the model. It is quite possible that calculations could be done exactly using exact expressions for the super-symplectic partition functions generalized to p-adic context. The success of the p-adic mass calculations is especially remarkable because p-adic length scale hypothesis $p \simeq 2^k$ predicts exponential sensitivity of the particle mass scale on k.

Symmetries

The number theoretical existence of p-adic thermodynamics requires powerful symmetries to guarantee integer valued spectrum for the thermalized contribution to the mass squared.

1. Super-conformal symmetry with integer valued conformal weights for Virasoro scaling generator L_0 is essential because it predicts in string models that mass squared is apart from ground state contribution integer valued in suitable units. In TGD framework fermionic string world sheets are characterized by super-conformal symmetry. This gives the p-adic thermodynamics assumed in the calculations. One could however assign Super Virasoro algebra also to super-symplectic algebra having its analog as sub-algebra with positive integer conformal weights. Same applies to the extended conformal algebra of light-cone boundary.

2. TGD however predicts also generalization of conformal symmetry associated with light-cone boundary involving ordinary complex conformal weights and the conformal weight associated with the light-like radial coordinate. For the latter conformal weights for the generators of supersymmetry might be given by $h = -s_n/2$. s_n zero of zeta or pole $h = -s = -1$ of zeta.

   Also super-symplectic symmetries would have similar radial spectrum of conformal weights. Conformal confinement requiring that the conformal weights of states are real implies that the spectrum of conformal weights for physical states consists of non-negative integers as for ordinary superconformal invariance.

   It is not clear whether thermalization occurs in these degrees of freedom except perhaps for trivial conformal weights. These degrees of freedom need not therefore contribute to thermal masses of leptons and quarks but would give dominating contribution to hadron masses and weak boson masses. The negative conformal weights predicted by $h = -s/2$ hypothesis predicts that ground state weight is negative for super-symplectic representations and must be compensated for massless states.

   The assumption that ground state conformal weight is negative and thus tachyonic is essential in case of p-adic mass calculations [K68], and only for massless particles (graviton, photon, gluons) it vanishes or is of order $O(1/p)$. This could be achieved if the ground state of super-symplectic representation has $h = 0$.

3. Modular invariance [K32] assignable to partonic 2-surfaces is a further assumption similar to that made made in string models. This invariance means that for a given genus the dynamical degrees of freedom of the partonic 2-surface correspond to finite-dimensional space of Teichmueller parameters. For genus $g = 0$ this space is trivial.

   Also modular invariance for string world sheets can be considered. By SH the information needed in mass calculations should be assignable to partonic 2-surfaces: the assumption is that one can assign this information to single partonic 2-surface. Stringy contribution would be seen only in scattering amplitudes.

   This might be true only effectively: the recent view about elementary particles is that they are pairs of wormhole contacts connected by flux tubes defining a closed monopole flux and wormhole throats of contact have same genus for light states. Furthermore the quantum numbers of particle are associated with single throat for fermions and with opposite throats of single contact for bosons. The second wormhole contact would carry neutralizing weak charges to realize the finite range of weak interactions as “weak confinement”.

   The number of genera is infinite and one must understand why only three quark and lepton generations are observed. An attractive explanation is in terms of symmetry. For the three lowest genera the partonic 2-surfaces are always hyper-elliptic and have thus global conformal Z_2 symmetry. For higher genera this is not true always and EPVFs constructed from the assumption of modular invariance vanish for the hyper-elliptic surfaces. This suggests that the higher genera are very massive or can be interpreted as many-particle states of handles, which are not bound states but have continuous mass squared.

Contributions to mass squared

There are several contributions to the p-adic thermal mass squared come from the degrees of freedom, which are thermalized.

Super-conformal degrees of freedom associated with string world sheets are certainly thermalized. p-Adic mass calculations strongly suggest that the number of super-conformal tensor factors is $N = 5$ but also $N = 4$ and $N = 6$ can be considered marginally.

I have considered several identifications of tensor factors and not found a compelling alternative. If one assumes that super-symplectic degrees of freedom do not contribute to the thermal mass, string world sheets should explain masses of elementary fermions. Here charged lepton masses are the test bench. One other hand, if super-symplectic degrees of freedom contribute one obtains additional tensor factor assignable to $h = -s/2$, s trivial zero of zeta). Only one tensor factor emerges since Hamiltonians correspond to the products of functions of the coordinates of light-cone boundary and CP_2).

1. $SU(2)_L \times U(1)$ gives 2 tensor factors. $SU(3)$ gives 1 tensor factor. The two transversal degrees of freedom for string world sheet suggest 2 degrees of freedom corresponding to Abelian group E^2. Rotations however transforms these degrees to each other so that 1 tensor factor should emerge. This gives 4 tensor factors. Could it correspond to the degrees of freedom parallel to string at its end assignable to wormhole throat? Could normal vibrations of partonic 2-surface? This would $N = 5$ tensor factors. Another possibility is that the fifth tensor factor comes from super-symplectic Super-Virasoro algebra defined by trivial conformal weights.

2. Super-symplectic contributions need not be present for ordinary elementary fermions. For weak bosons they could give string tension assignable to the magnetic flux tube connecting the wormhole contacts. It is not clear whether this contribution is thermalized. This contribution might be present only for the phases with $h_{eff} = n \times h$. This contribution would dominate in hadron masses.

3. Color degrees of freedom contribute to the ground state mass squared since ground state corresponds to an imbedding space spinor mode massless in 8-D sense. The mass squared contribution corresponds to an eigenvalue of CP_2 spinor d'Alembertian. Its eigenvalues correspond to color multiplets and only the covariantly constant right handed neutrino is color singlet. For the other modes the color representation is non-trivial and depends on weak quantum numbers of the fermion. The construction of the massless state from a tachyonic ground state with conformal weight $h_{vac} = -3$ must involve colored super-Kac Moody generators compensating for the anomalous color charge so that one obtains color single for leptons and color triplet for quarks as massless state.

4. Modular degrees of freedom give a contribution depending on the genus g of the partonic 2-surface. This contribution is estimated by considering p-adic variants of elementary particle vacuum functionals Ω_{vac} [K68] expressible as products of theta functions with the structure of partition function. Theta functions are expressible as sums of exponent functions $exp(X)$ with X defined as a contraction of the matrix Ω_{ij} defined by Teichmueller parameters between integer valued vectors.

   In ZEO the interpretation of Ω_{vac} is as a complex square root of partition functional (quantum theory as complex square root of thermodynamics in ZEO). The integral of $|\Omega|^2$ over allowed moduli has interpretation as partition function. The exponential $exp(Re(X)) = p^{Re(X)/log(p)}$ has interpretation as an exponential of "Hamiltonian" defined by the vacuum conformal weight defined by moduli. $T = log(p)$ is identified as p-adic temperature as in ordinary p-adic thermodynamics.

   NTU requires that the integration over the moduli parameters reduces to a sum over number theoretically universal moduli parameters. The exponents $exp(X)$ must exist p-adically. PE property alone could guarantee this. The exponentials appearing in theta functions should reduce to products $p^k p^{iy} = exp(k/log(p))p^{iy}$ with k is integer and p^{iy} a root of unity. The vacuum expectation value of $Re(X)$ contributing to the mass squared is obtained from the standard formula as logarithmic temperature derivative of the "integral" $\int |\Omega_{vac}|^2$. The formula is same as for the Super-Virasoro contributions apart from the integration reducing to a sum.

   The considerations of the section 4.4.2 [L9] suggest that for given p-adic prime p the exponent $k + iy$ corresponds to a linear combinations of poles of fermionic zeta $z_F(s) = \zeta(s)/\zeta(2s)$ in the class $C(p)$ with non-negative integer coefficients. This class corresponds essentially to the conformal weights of a fractal sub-algebra of super-symplectic algebra. It could give rise also to the complex values of action so that Riemann zeta would define the core of TGD.

   The general dependence of the contribution of genus g to mass squared on g follows from the functional form of EPVF as a product theta functions serving as building brick partition functions apart from overall multiplicative constant and gives a nice agreement with the observed charged lepton mass ratios. The basic feature of the formula is exponential dependence on g.

5. The super-symplectic stringy contribution assignable to the magnetic flux tube dominates for weak bosons and is analogous to the stringy contribution to the hadron masses.

p-Adic mass calculations leave open several questions. What is the precise origin of preferred p-adic primes and of p-adic length scale hypothesis? How to understand the preferred number $N = 5$ of Super-Kac-Moody tensor factors? How to calculate the contribution of super-symplectic degrees of freedom - are they thermalized? Why only 3 lowest genera are light and what are the masses of the predicted bosonic higher genera implying breaking of fermion universality.

4.3.4 p-Adic length scale hypothesis

p-Adic length scale hypothesis [K2, K78] has served as a basic hypothesis of p-adic TGD for several years. This hypothesis states that the scales $L_p = \sqrt{p}l$, $l = 1.376 \cdot 10^4\sqrt{G}$ are fundamental length scale at p-adic condensate level p. The original interpretation of the hypothesis was following:

1. Above the length scale L_p p-adicity sets on and effective course grained space-time or imbedding space topology is p-adic rather than ordinary real topology. Imbedding space topology seems to be more appropriate identification.
2. The length scale L_p serves as a p-adic length scale cutoff for the quantum field theory description of particles. This means that space-time begins to look like Minkowski space so that the QFT $M^4 \to CP_2$ becomes a realistic approximation. Below this length scale string like objects and other particle like 3-surfaces are important.
3. It is un-natural to assume that just single p-adic field would be chosen from the infinite number of possibilities. Rather, there is an infinite number of cutoff length scales. To each prime p there corresponds a cutoff length scale L_p above which p-adic quantum field theory $M^4 \to CP_2$ makes sense and one has a hierarchy of p-adic QFTs. These different p-adic field theories correspond to different hierarchically levels possibly present in the topological condensate. Hierarchical ordering $< p_1 < p_2 < ...$ means that only the surface $p_1 < p_2$ can condense on the surface p_2. The condensed surface can in practice be regarded as a point like particle at level p_2 described by the p-adic conformal field theory below length scale L_{p_2}.

The recent view inspired by adelic physics is that preferred p-adic primes correspond to so called ramified primes for the algebraic extension of rationals defining the adele [K126]. Weak form of Negentropy Maximization Principle (WNMP) [K72] in turn allows to conclude that the length scales corresponding to powers of primes are preferred. Therefore p-adic length scale hypothesis generalizes. There is evidence for 3-adic time scales in biology [I72, I73] and 3-adic time scales can be also assigned with Pythagorean scale in geometric theory of harmony [K93] [L6].

4.3.5 Mersenne primes and Gaussian Mersennes are special

Mersenne primes and their complex counterparts Gaussian Mersennes pop up in p-adic mass calculations and both elementary particle physics, biology [K90], and astrophysics and cosmology [K71] provide support for them.

Mersenne primes

One can also consider the milder requirement that the exponent $\lambda = 2^{\epsilon L_0}$ represents trivial scaling represented by unit in good approximation for some p-adic topology. Not surprisingly, this is the case for $L_0 = mp^k$ since by Fermat's theorem $a^p \ mod \ p = 1$ for any integer a, in particular $a = 2$. This is also the case for $L_0 = mk$ such that $2^k \ mod \ p = 1$ for p prime. This occurs if $2^k - 1$ is Mersenne prime: in this case one has $2^{L_0} = 1$ modulo p so that the sizes of the fractal sub-algebras are exponentially larger than the sizes of $L_0 \propto p^n$ algebras. Note that all scalings a^{L_0} are near to unity for $L_0 = p^n$ whereas now only $a = 2$ gives scalings near unity for Mersenne primes. Perhaps this extended fractality provides the fundamental explanation for the special importance of Mersenne primes.

In this case integrated scalings 2^{L_0} leave the states almost invariant so that even a stronger form of the breaking of the exact conformal invariance would be in question in the super-symplectic case. The representation would be defined by the generators for which conformal weights are odd multiples of n ($M_n = 2^n - 1$) and L_{-kn}, $k > 0$ would generate zero norm states only in order $O(1/M_n)$.

Especially interesting is the hierarchy of primes defined by the so called Combinatorial Hierarchy resulting from TGD based model for abstraction process. The primes are given by $2, 3, 7 = 2^3 - 1, 127 = 2^7 - 1, 2^{127} - 1, ..$: $L_0 = n \times 127$ would correspond to M_{127}-adicity crucial for the memetic code.

Gaussian Mersennes are also special

If one allows also Gaussian primes then the notion of Mersenne prime generalizes: Gaussian Mersennes are of form $(1 \pm i)^n - 1$. In this case one could replace the scaling operations by scaling combined with a twist of $\pi/4$ around some symmetry axis: $1 + i = \sqrt{2}exp(i\pi/4)$ and generalized p-adic fractality would mean that for certain values of n the exponentiated operation consisting of n basic operations would be very near to unity.

1. The integers k associated with the lowest Gaussian Mersennes are following: $2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113$. $k = 113$ corresponds to the p-adic length scale associated with the atomic nucleus and muon. Thus all known charged leptons, rather than only e and τ, as well as nuclear physics length scale, correspond to Mersenne primes in the generalized sense.

2. The primes $k = 151, 157, 163, 167$ define perhaps the most fundamental biological length scales: $k = 151$ corresponds to the thickness of the cell membrane of about ten nanometers and $k = 167$ to cell size about 2.56 μm. This observation also suggests that cellular organisms have evolved to their present form through four basic evolutionary stages. This also encourages to think that $\sqrt{2}exp(i\pi/4)$ operation giving rise to logarithmic spirals abundant in living matter is fundamental dynamical symmetry in bio-matter.

   Logarithmic spiral provides the simplest model for biological growth as a repetition of the basic operation $\sqrt{2}exp(i\pi/4)$. The naive interpretation would be that growth processes consist of $k = 151, 157, 163, 167$ steps involving scaling by $\sqrt{2}$. This however requires the strange looking assumption that growth starts from a structure of size of order CP_2 length. Perhaps this exotic growth process is associated with pair of MEs or magnetic flux tubes of opposite time orientation and energy emergenging CP_2 sized region in a mini big bang type process and that the resulting structure serves as a template for the biological growth.

3. $k = 239, 241, 283, 353, 367, 379, 457$ associated with the next Gaussian Mersennes define astronomical length scales. $k = 239$ and $k = 241$ correspond to the p-adic time scales .55 ms and 1.1 ms: basic time scales associated with nerve pulse transmission are in question. $k = 283$ corresponds to the time scale of 38.6 min. An interesting question is whether this period could define a fundamental biological rhythm. The length scale $L(353)$ corresponds to about 2.6×10^6 light years, roughly the size scale of galaxies. The length scale $L(367) \simeq \times 3.3 \times 10^8$ light years is of same order of magnitude as the size scale of the large voids containing galaxies on their boundaries (note the analogy with cells). $T(379) \simeq 2.1 \times 10^{10}$ years corresponds to the lower bound for the order of the age of the Universe. $T(457) \sim 10^{22}$ years defines a completely superastronomical time and length scale.

4.3.6 Questions

The proposed picture leaves open several questions.

1. Could the descriptions by both real and p-adic thermodynamics be possible? Could they be equivalent (possibly in finite measurement resolution) as is suggested by NTU? The consistency of these descriptions would imply temperature quantization and p-adic length scale hypothesis not possible in purely real context.

2. What could the extension of conformal symmetry to supersymplectic symmetry mean? One possible view is that super-symplectic symmetries correspond to dark degrees of freedom and that only the super-symplectic ground states with negative conformal weights affect the p-adic thermodynamics, which applies only to fermionic degrees of freedom at string world sheets. Super-symplectic degrees of freedom would give the dominant contribution to hadron masses and could contribute also to weak gauge boson masses. $N = 5$ for the needed number

of tensor factors is however a strong constraint and perhaps most naturally obtained when also the super-symplectic Virasoro associated with the trivial zeros of zeta is thermalized.

3. What happens in dark sectors. Preferred extremal property is proposed to mean that the states are annihilated by super-symplectic sub-algebra isomorphic to the original algebra and its commutator with the entire algebra. The conjecture is that this gives rise to Kac-Moody algebras as dynamical symmetries - maybe ADE type algebras, whose Dynkin diagrams characterize the inclusion of HFFs. Does this give an additional tensor factor to super-Virasoro algebra?

4. Superconformal symmetry true in the sense that Super Virasoro conditions hold true. Partition function however depends on mass squared only rather than the entire scaling generator L_0 as thought erratically in the first formulation of p-adic calculation. This does not mean breaking of conformal invariance. Super Virasoro conditions hold true although partition function is for the vibrational part of L_0 determining the mass squared spectrum.

4.4 p-Adicization and adelic physics

This section is devoted to the challenges related to p-adicization and adelization of physics in which the correspondence between real and p-adic numbers via canonical identification serves as the basic building brick. Also the problems associated with p-adic variants of integral, Fourier analysis, Hilbert space, and Riemann geometry should be solved in a way respecting fundamental symmetries and their p-adic variants must be met. The notion of number theoretical universality (NTU) plays a key role here. One should also answer to questions about the origin of preferred primes and p-adic length scale hypothesis.

4.4.1 Challenges

The basic challenges encountered are construction of the p-adic variants of real number based physics, understanding their relationship to real physics, and the fusion of various physics to single coherent whole.

The p-adicization of real physics is not just a straightforward formal generalization of scattering amplitudes of existing theories but requires a deeper understanding of the physics involved. The interpretation of p-adic physics as correlate for cognition and imagination is an important guideline and will be discussed l in more detail in separate section.

Definite integral and Fourier analysis are basic elements of standard physics and their generalization to the p-adic context defines a highly non-trivial challenge. Also the p-adic variants of Riemann geometry and Hilbert space are suggestive. There are however problems.

1. There are problems associated with p-adic definite integral. Riemann sum does not make sense since it approaches zero if the p-adic norm of discretization unit approaches zero. The problems are basically due to the absence of well-orderedness essential for the definition of definite integral and differential forms and their integrals.

   Residue integration might make sense in finite angle resolution. For algebraic extension containing $e^{i\pi/n}$ the number theoretically universal approximation $i\pi = n(e^{i\pi/n} - 1)$ could be used. In twistor approach integrations reduce to multiple residue integrations and since twistor approach generalizes in TGD framework, this approach to integration is very attractive.

   Positivity is a central notion in twistor Grassmannian approach [B21]. Since canonical identification maps p-adic numbers to non-negative real numbers, there is a strong temptation to think that positivity relates to NTU [L10].

2. There are problems with Fourier analysis. The naive generalization of trigonometric functions by replacing e^{ix} with its p-adic counterpart is not physical. Same applies to e^x. Algebraic extensions are needed to get roots of unity ad e as counterparts of the phases and discretization is necessary and has interpretation in terms of finite resolution for angle/phase and its hyperbolic counterpart.

3. The notion of Hilbert space is problematic. The naive generalization of Hilbert space norm square $|x|^2 = \sum x_n \overline{x}_n$ for state $(x_1, x_2, ...)$ can vanish p-adically. Also here NTU could help. State would contain as coefficients only roots of e and unity and only the overall factor could be p-adic number. Coefficients could be restricted to the algebraic numbers generating the algebraic extension of rational numbers and would not contain powers of p or even ordinary p-adic numbers expect in the overall normalization factor.

Second challenge relates to the relationship between real and p-adic physics. Canonical identification (CI) $\sum x_n p^n \to \sum x_n p^{-n}$ or some of its variants should play an important role. CI is expected to map the invariants appearing in scattering amplitudes to their real counterparts.

1. Real and p-adic variants of space-time surfaces should exist and relate to each other somehow. Is this relationship local and involve CI at space-time level or imbedding space level? Or is it only a global and non-local assignment of preferred real extremals to their p-adic counterparts? Or is between these extreme options and involves algebraic discretization of the space-time surface weakening the strong form of SH as already proposed? How do real and p-adic imbedding spaces relate to each other and can this relationship induce local correspondence between preferred extremals (PEs) [K10, K19, K13]?

2. NTU in some sense is a highly suggestive approach to these questions and would suggest that canonical identification applies to isometry invariants whereas angles and hyperbolic angles, or rather the corresponding "phases" belonging to an extension of p-adics containing roots of e and roots of unity are mapped to themselves. Note that the roots of e define extensions of rationals, which induce finite dimensional algebraic extensions of p-adic numbers. This would make possible to define imbedding space in accordance with NTU. Also the Hilbert space could be defined by requiring that its points correspond to number theoretically universal angles expressible in terms of roots of unity.

3. What about real and p-adic variants of WCW? Are they needed at all? Or could their existence be used as a powerful constraint on real physics? The representability of WCW as a union of infinite-dimensional symmetric spaces labelled by zero modes suggests that the same description applies at the level of WCW and imbedding space.

   One cannot circumvent the question about how to generalize functional integral from real WCW to p-adic WCWs. In particular, what is the p-adic variant of the action defining the dynamics of space-time surfaces. In the case of exponent of action the p-adic variant could be defined by assuming algebraic universality: again the roots of e and of unity would be in central role. Also the Kähler structure of WCW implying that Gaussian and metric determinants cancel each other in functional integral, would be absolutely crucial.

   One must remember that the exponents of action for scattering amplitudes for the stationary phase extremal cancel from the path integral representation of scattering amplitudes. Also now this mechanism would allow to get rid of the poorly defined exponent for single minimum. If there is sum over scattering amplitudes assignable to different maxima, normalization sould give ratios of these exponents for different extrema/maxima and only these ratios should belong to the extension of rationals.

   The zero modes of WCW metric are invariants of supersymplectic group so that canonical identification could relate their real and p-adic variants. Zero modes could break NTU and would be behind p-adic thermodynamics and dependence of mass scale on p-adic prime.

The third challenge relates to the fusion of p-adic physics and real physics to a larger structure. Here a generalization of number concept obtained by glueing reals and various p-adics together along an extension of rational numbers inducing the extensions of p-adic numbers is highly suggestive. Adeles associated with the extension of rationals are highly attractive and closely related notion. Real and various p-adic physics would be correlates for sensory and cognitive aspects of the same universal physics rather than separate physics in this framework. One important implication of this view is that real entropy and p-adic negentropies characterize the same entanglement with coefficients in an extension of rationals.

NTU for hyperbolic and ordinary phases is definetely the central idea. How the invariance of angles under conformal transformations does relate to this? Could one perhaps define a discretized version of conformal symmetry preserving the phases defined by the angles between vectors assignable with the tangent spaces of discretized geometric structures and thus respecting NTU? Of should one apply conformal symmetry at Lie algebra level only?

4.4.2 NTU and the correspondence between real and p-adic physics

p-Adic real correspondence is certainly the basic problem of p-adicization and adelization. One can make several general questions about p-adic real correspondence and canonical identification inspired by p-adic mass calculations.

How generally p-adic real correspondence does apply? Could canonical identification for group invariants combined with direct identification of ordinary and hyperbolic phases identified as roots of unity and e apply at WCW and imbedding space level having maximally symmetric geometries? Could this make sense even at space-time level as a correspondence induced from imbedding space level [L19]? Does canonical identification apply locally for the discretizations of space-time surface or only globally for the parameters characterizing PEs (string world sheets and partonic 2-surfaces by SH), which are general coordinate invariant and Poincare invariant quantities?

The following vision seems to be the most feasible one found hitherto.

1. Preservation of symmetries and continuity compete. Lorenz transformations do not commute with canonical identification. This suggests that canonical identification applies only to Lorentz invariants formed from quantum numbers. This is enough in the case of scattering amplitudes. Canonical identification applies only to isometry invariants at the level of WCW and the phases/exponents of ordinary/hyperbolic angles correspond to numbers in the algebraic extension common to extensions of rationals and various p-adics.

2. Canonical identification applies at the level of momentum space and maps p-adic Lorentz invariants of scattering amplitudes to their real counterparts. Phases of angles and their hyperbolic counterparts should correspond to parameters defining extension and should be mapped as such to their p-adic counterparts.

3. The constraints coming from GCI and symmetries do not allow local correspondence but allow to consider its discretized version at space-time leve induced by the correspondence at the level of imbedding space.

   This requires the restriction of isometries and other symmetries to algebraic subgroups defined by the extension of rationals. This would imply reduction of symmetry due to finite cognitive/measurement resolution and should be acceptable. If one wants to realize the ideas about imagination, discretization must be applied also for the space-time interior meaning partial breaking of SH and giving rise to dark matter degrees freedom in TGD sense. SH could apply in real sector for realizable imaginations only. Note that the number of algebraic points of space-time surface is expected to be relatively small.

The correspondence must be considered at the level of imbedding space, space-time, and WCW.

1. At the level of imbedding space p-adic–real correspondence is induced by points in extension of rationals and is totally discontinuous. This requires that space-time dimension is smaller than imbedding space dimension.

2. At space-time level the correspondence involves field equations derivable from a local variational principle make sense also p-adically although the action itself is ill-defined as 4-D integral. The notion of p-adic PE makes sense by strong form of holography applied to 2-surfaces in the intersection. p-Adically however only the vanishing of Noether currents for a sub-algebra of the super-symplectic algebra might make sense. This condition is stronger than the vanishing of Noether charges defined by 3-D integrals.

3. Correspondence at the level of WCW can make sense and reduces to that for string world sheets and partonic 2-surfaces by SH. Real and p-adic 4-surfaces would be obtained by algebraic continuation as PEs from 2-surfaces by assuming that the space-time surface contains

subset of points of imbedding space belonging to the extension of rationals [L19]. p-Adic pseudo constants make p-adic continuation easy. Real continuation need not exist always. p-Adic WCW would be considerably larger than real WCW and make possible a predictive quantum theory of imagination and cognition.

What I have called intersection of realities and p-adicities can be identified as the set of 2-surfaces plus algebraic discretization of space-time interior. Also the values of induced spinor fields at the points of discretization must be given. The parameters characterizing the extremals (say coefficients of polynomials) - WCW coordinates - would be in extension of rationals inducing a finite-D extension of p-adic number fields.

The hierarchy of algebraic extensions induces an evolutionary hierarchy of adeles. The interpretation could be as a mathematical correlate for cosmic evolution realized at the level of the core of WCW defined by the intersection? 2-surfaces could be called space-time genes.

4. Also the p-adic variant Kähler action or at least the exponent of Kähler action defining vacuum functional should be obtainable by algebraic continuation. The weakest condition states that the ratios of action exponents for the maxima of Kähler function to the sum of action exponents for maxima belong to the extension. Without this condition the hopes of satisfying NTU seem rather meager.

4.4.3 NTU at space-time level

What about NTU at space-time level? NTU requires a correspondence between real and p-adic numbers and the details of this corresponds have been a long standing problem.

1. The recent view about the correspondence between real PEs to their p-adic counterparts does not demand discrete local correspondence assumed in the earlier proposal [K129]. The most abstract approach would give up the local correspondence at space-time level altogether, and restrict the preferred coordinates of WCW (having maximal group of isometries) to numbers in the extension of rationals considered. WCW would be discretized.

   Intuitively a more realistic view is a correspondence at space-time level in the sense that real and p-adic space-time sheets intersect at points belonging to the extension of rationals and defining "cognitive representations". Only some p-adic space-time surfaces would have real counterpart.

2. The strongest form of NTU would require that the allowed points of imbedding space belonging an extension of rationals are mapped as such to corresponding extensions of p-adic number fields (no canonical identification). At imbedding space level this correspondence would be extremely discontinuous. The "spines" of space-time surfaces would however contain only a subset of points of extension, and a natural resolution length scale could emerge and prevent the fluctuation. This could be also seen as a reason for why space-times surfaces must be 4-D. The fact that the curve $x^n + y^n = z^n$ has no rational points for $n > 2$, raises the hope that the resolution scale could emerge spontaneously.

3. The notion of monadic geometry discussed in detail in [L19] would realize this idea. Define first a number theoretic discretization of imbedding space in terms of points, whose coordinates in group theoretically preferred coordinate system belong to the extension of rationals considered. One can say that these algebraic points are in the intersection of reality and various p-adicities. Overlapping open sets assigned with this discretization define in the real sector a covering by open sets. In p-adic sector compact-open-topology allows to assign with each point 8^{th} Cartesian power of algebraic extension of p-adic numbers. These compact open sets define analogs for the monads of Leibniz and p-adic variants of field equations make sense inside them.

   The monadic manifold structure of H is induced to space-time surfaces containing discrete subset of points in the algebraic discretization with field equations defining a continuation to space-time surface in given number field, and unique only in finite measurement resolution. This approach would resolve the tension between continuity and symmetries in p-adic–real correspondence: isometry groups would be replaced by their sub-groups with parameters in extension of rationals considered and acting in the intersection of reality and p-adicities.

The Galois group of extension acts non-trivially on the "spines" of space-time surfaces. Hence the number theoretical symmetries act as physical symmetries and define the orbit of given space-time surface as a kind of covering space. The coverings assigned to the hierarchy of Planck constants would naturally correspond to Galois coverings and dark matter would represent number theoretical physics.

This would give rise to a kind of algebraic hierarchy of adelic 4-surfaces identifiable as evolutionary hierarchy: the higher the dimension of the extension, the higher the evolutionary level.

4.4.4 NTU and WCW

p-Adic–real correspondence at the level of WCW

It has not been obvious whether one should perform p-adicization and adelization at the level of WCW. Minimalist could argue that scattering amplitudes are all we want and that their p-adicization and adelization by algebraic continuation can be tolerated only if it can give powerful enough constraints on the amplitudes.

1. The anti-commutations for fermionic oscillator operators are number theoretically universal. Supersymmetry suggests that also WCW bosonic degrees of freedom satisfy NTU. This could mean that the coordinates of p-adic WCW consist of super-symplectic invariants mappable by canonical identification to their real counterparts plus phases and their hyperbolic counterparts expressible as genuinely algebraic numbers common to all number fields. This kind of coordinates are naturally assignable to symmetric spaces [L19].

2. Kähler structure should be mapped from p-adic to real sector and vice versa. Vacuum functional identified as exponent of action should be NTU. Algebraic continuation defined by SH involves p-adic pseudo constants. All p-adic continuations by SH should correspond to the same value of exponent of action obtained by algebraic continuation from its real value. The degeneracy associated with p-adic pseudo-constants would be analogous to gauge invariance - imagination in TGD inspired theory of consciousness.

3. Ist it possible have NTU for WCW functional integration? Or is it enough to realize NTU for scattering amplitudes only. What seems clear that functional integral must reduce to a discrete sum. Physical intuition suggests a sum over maxima of Kähler function forming a subset of PEs representing stationary points. One cannot even exclude the possibility that the set of PEs is discrete and that one can sum over all of them.

   Restriction to maximum/stationary phase approximation gives rise to sum over exponents multiplied with Gaussian determinants. The determinant of Kähler metric however cancels the Gaussian determinants, and one obtains only a sum over the exponents of action.

   The breaking of strong NTU could happen: consider only p-adic mass calculations. This breaking is however associated with the parts of quantum states assignable to the boundaries of CD, not with the vacuum functional.

NTU for functional integral

Number theoretical vision relies on NTU. In fermionic sector NTU is necessary: one cannot speak about real and p-adic fermions as separate entities and fermionic anti-commutation relations are indeed number theoretically universal.

What about NTU in case of functional integral? There are two opposite views.

1. One can define p-adic variants of field equations without difficulties if preferred extremals are minimal surface extremals of Kähler action so that coupling constants do not appear in the solutions. If the extremal property is determined solely by the analyticity properties as it is for various conjectures, it makes sense independent of number field. Therefore there would be no need to continue the functional integral to p-adic sectors. This in accordance with the

philosophy that thought cannot be put in scale. This would be also the option favored by pragmatist.

2. Consciousness theorist might argue that also cognition and imagination allow quantum description. The supersymmetry NTU should apply also to functional integral over WCW (more precisely, its sector defined by CD) involved with the definition of scattering amplitudes.

1. Key observations

The general vision involves some crucial observations.

1. Only the expressions for the scatterings amplitudes should should satisfy NTU. This does not require that the functional integral satisfies NTU.

2. Since the Gaussian and metric determinants cancel in WCW Kähler metric the contributions form maxima are proportional to action exponentials $exp(S_k)$ divided by the $\sum_k exp(S_k)$. Loops vanish by quantum criticality.

3. Scattering amplitudes can be defined as sums over the contributions from the maxima, which would have also stationary phase by the double extremal property made possible by the complex value of α_K. These contributions are normalized by the vacuum amplitude.

   It is enough to require NTU for $X_i = exp(S_i)/\sum_k exp(S_k)$. This requires that $S_k - S_l$ has form $q_1 + q_2 i\pi + q_3 log(n)$. The condition brings in mind homology theory without boundary operation defined by the difference $S_k - S_l$. NTU for both S_k and $exp(S_k)$ would only values of general form $S_k = q_1 + q_2 i\pi + q_3 log(n)$ for S_k and this looks quite too strong a condition.

4. If it is possible to express the 4-D exponentials as single 2-D exponential associated with union of string world sheets, vacuum functional disappears completely from consideration! There is only a sum over discretization with the same effective action and one obtains purely combinatorial expression.

2. What does one mean with functional integral?

The definition of functional integral in WCW is one of the key technical problems of quantum TGD [K126]. NTU states that the integral should be defined simultaneously in all number fields in the intersection of real and p-adic worlds defined by string world sheets and partonic 2-surfaces with WCW coordinates in algebraic extension of rationals and allowing by strong holography continuation to 4-D space-time surface. NTU is powerful constraint and could help in this respect.

1. Path integral is not in question. Rather, the functional integral is analogous to Wiener integral and perhaps allows identification as a genuine integral in the real sector. In p-adic sectors algebraic continuation should give the integral and here number theoretical universality gives excellent hopes. The integral would have exactly the same form in real and p-adic sector and expressible solely in terms of algebraic numbers characterizing algebraic extension and finite roots of e and roots of unity $U_n = exp(i2\pi/n)$ in algebraic extension of p-adic numbers.

   Since vacuum functional $exp(S)$ is exponential of complex action S, the natural idea is that only rational powers e^q and roots of unity and phases $exp(i2\pi q)$ are involved and there is no dependence on p-adic prime p! This is true in the integer part of q is smaller than p so that one does not obtain e^{kp}, which is ordinary p-adic number and would spoil the number theoretic universality. This condition is not possible to satisfy for all values of p unless the value of Kähler function is smaller than 2. One might consider the possibility that the allow primes are above some minimum value.

   The minimal solution to NTU conditions is that the ratios of action exponentials for maxima of Kähler function to the sum of these exponentials belong to the extension of rationals considered.

2. What does one mean with functional integral? TGD is expected to be an integrable in some sense. In integrable QFTs functional integral reduces to a sum over stationary points of the action: typically only single point contributes - at least in good approximation.

   For real α_K and Λ vacuum functional decomposes to a product of exponents of real contribution from Euclidian regions ($\sqrt{g_4}$ real) and imaginary contribution Minkowskian regions ($\sqrt{g_4}$ imaginary). There would be no exchange of momentum between Minkowskian and Euclidian regions. For complex values of α_K [K46] situation changes and Kähler function as real part of action receives contributions from both Euclidian and Minkowskian regions. The imaginary part of action has interpretation as analog of Morse function and action as it appears in QFTs. Now saddle points must be considered.

   PEs satisfy extremely strong conditions [K10, K13]. All classical Noether charges for a sub-algebra associated with super-symplectic algebra and isomorphic to the algebra itself vanish at both ends of CD. The conformal weights of this algebra are $n > 0$-ples of those for the entire algebra. What is fascinating that the condition that the preferred extremals are minimal surface extremals of Kähler action could solve these conditions and guarantee also NTU at the level of space-time surfaces. Supersymplectic boundary conditions at the ends of CD would however pose number theoretic conditions on the coupling parameters. In p-adic case the conditions should reduce to purely local conditions since p-adic charges are not well-defined as integrals.

3. In TGD framework one is constructing zero energy states rather calculating the matrix elements of S-matrix in terms of path integral. This gives certain liberties but a natural expectation is that functional integral as a formal tool at least is involved.

   Could one *define* the functional integral as a discrete sum of contributions of standard form for the preferred extremals, which correspond to maxima in Euclidian regions and associated stationary phase points in Minkowskian regions? Could one assume that WCW spinor field is concentrated along single maximum/stationary point.

   Quantum classical correspondence suggests that in Cartan algebra isometry charges are equal to the quantal charges for quantum states expressible in number theoretically universal manner in terms of fermionic oscillator operators or WCW gamma matrices? Even stronger condition would be that classical correlation functions are identical with quantal ones for allowed space-time surfaces in the quantum superposition. Could the reduction to a discrete sum be interpreted in terms of a finite measurement resolution?

4. In QFT Gaussian determinants produce problems because they are often poorly defined. In the recent case they could also spoil the NTU based on the exceptional properties of e. In the recent case however Gaussian determinant and metric determinant for Kähler metric cancel each other and this problem disappears. One could obtain just a sum over products of roots of e and roots of unity. Thus also Kähler structure seems to be crucial for the dream about NTU.

4.4.5 Breaking of NTU at the level of scattering amplitudes

NTU in strong sense could be broken at the level of scattering amplitudes. At space-time level the breaking does not look natural in the recent framework. Consider only p-adic mass calculations predicting that mass scale depends on p-adic prime. Also for the action strong form of NTU might fail for small p-adic primes since the value of the real part of action would be larger than than p. Should one allow this? What does one actually mean with NTU in the case of action?

Canonical identification is an important element of p-adic mass calculations and might also be needed to map p-adic variants of scattering amplitudes to their real counterparts. The breaking of NTU would take place, when the canonical real valued image of the p-adic scattering amplitude differs from the real scattering amplitude. The interpretation would be in terms of finite measurement resolution. By the finite measurement/cognitive resolution characterized by p one cannot detect the difference.

The simplest form of the canonical identification is $x = \sum_n x_n p^n \to \sum x_n p^{-n}$. Product xy and sum $x + y$ do not in general map to product and sum in canonical identification. The interpretation would be in terms of a finite measurement resolution: $(xy)_R = x_R y_R$ and $(x+y)_R = x_R + y_R$ only modulo finite measurement resolution. p-Adic scattering amplitudes are obtained by algebraic continuation from the intersection by replacing algebraic number valued parameters (such as momenta) by general p-adic numbers. The real images of these amplitudes under canonical identification are in general not identical with real scattering amplitudes the interpretation being in terms of a finite measurement resolution.

In p-adic thermodynamics NTU in the strong sense fails since thermal masses depend on p-adic mass scale. NTU can be broken by the fermionic matrix elements in the functional integral so that the real scattering amplitudes differ from the canonical images of the p-adic scattering amplitudes. For instance, the elementary particle vacuum functionals in the space of Teichmueller parameters for the partonic 2-surfaces and string world sheets should break NTU [K32].

4.4.6 NTU and the spectrum of Kähler coupling strength

During years I have made several attempts to understand coupling evolution in TGD framework. The most convincing proposal has emerged rather recently and relates the spectrum of $1/\alpha_K$ to that for the zeros of Riemann zeta [K46] and to the evolution of the electroweak U(1) couplings strength.

1. The first idea dates back to the discovery of WCW Kähler geometry defined by Kähler function defined by Kähler action (this happened around 1990) [K60]. The only free parameter of the theory is Kähler coupling strength α_K analogous to temperature parameter α_K postulated to be is analogous to critical temperature. Whether only single value or entire spectrum of of values α_K is possible, remained an open question.

   About decade ago I realized that Kähler action is *complex* receiving a real contribution from space-time regions of Euclidian signature of metric and imaginary contribution from the Minkoswkian regions. Euclidian region would give Kähler function and Minkowskian regions analog of QFT action of path integral approach defining also Morse function. Zero energy ontology (ZEO) [K130] led to the interpretation of quantum TGD as complex square root of thermodynamics so that the vacuum functional as exponent of Kähler action could be identified as a complex square root of the ordinary partition function. Kähler function would correspond to the real contribution Kähler action from Euclidian space-time regions. This led to ask whether also Kähler coupling strength might be complex: in analogy with the complexification of gauge coupling strength in theories allowing magnetic monopoles. Complex α_K could allow to explain CP breaking. I proposed that instanton term also reducing to Chern-Simons term could be behind CP breaking.

   The problem is that the dynamics in Minkowskian and Euclidian regions decouple completely and if Euclidian regions serve as space-time correlates for physical objects, there would be no exchanges of classical charges between physical objects. Should one conclude that α_K must be complex?

2. p-Adic mass calculations for 2 decades ago [K68] inspired the idea that length scale evolution is discretized so that the real version of p-adic coupling constant would have discrete set of values labelled by p-adic primes. The simple working hypothesis was that Kähler coupling strength is renormalization group (RG) invariant and only the weak and color coupling strengths depend on the p-adic length scale. The alternative ad hoc hypothesis considered was that gravitational constant is RG invariant. I made several number theoretically motivated ad hoc guesses about coupling constant evolution, in particular a guess for the formula for gravitational coupling in terms of Kähler coupling strength, action for CP_2 type vacuum extremal, p-adic length scale as dimensional quantity. Needless to say these attempts were premature and a hoc.

3. The vision about hierarchy of Planck constants $h_{eff} = n \times h$ and the connection $h_{eff} = h_{gr} = GMm/v_0$, where $v_0 < c = 1$ has dimensions of velocity [?] forced to consider very seriously the hypothesis that Kähler coupling strength has a spectrum of values in one-one correspondence with p-adic length scales. A separate coupling constant evolution associated with h_{eff} induced by $\alpha_K \propto 1/\hbar_{eff} \propto 1/n$ looks natural and was motivated by the idea that

Nature is theoretician friendly: when the situation becomes non-perturbative, Mother Nature comes in rescue and an h_{eff} increasing phase transition makes the situation perturbative again.

Quite recently the number theoretic interpretation of coupling constant evolution [K126] [L9] in terms of a hierarchy of algebraic extensions of rational numbers inducing those of p-adic number fields encouraged to think that $1/\alpha_K$ has spectrum labelled by primes and values of h_{eff}. Two coupling constant evolutions suggest themselves: they could be assigned to length scales and angles which are in p-adic sectors necessarily discretized and describable using only algebraic extensions involve roots of unity replacing angles with discrete phases.

4. Few years ago the relationship of TGD and GRT was finally understood [K122]. GRT space-time is obtained as an approximation as the sheets of the many-sheeted space-time of TGD are replaced with single region of space-time. The gravitational and gauge potential of sheets add together so that linear superposition corresponds to set theoretic union geometrically. This forced to consider the possibility that gauge coupling evolution takes place only at the level of the QFT approximation and α_K has only single value. This is nice but if true, one does not have much to say about the evolution of gauge coupling strengths.

5. The analogy of Riemann zeta function with the partition function of complex square root of thermodynamics suggests that the zeros of zeta have interpretation as inverses of complex temperatures $s = 1/\beta$. Also $1/\alpha_K$ is analogous to temperature. This led to a radical idea to be discussed in detail in the sequel.

   Could the spectrum of $1/\alpha_K$ reduce to that for the zeros of Riemann zeta or - more plausibly - to the spectrum of poles of fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ giving for $k = 1/2$ poles as zeros of zeta and as point $s = 2$? ζ_F is motivated by the fact that fermions are the only fundamental particles in TGD and by the fact that poles of the partition function are naturally associated with quantum criticality whereas the vanishing of ζ and varying sign allow no natural physical interpretation.

   The poles of $\zeta_F(s/2)$ define the spectrum of $1/\alpha_K$ and correspond to zeros of $\zeta(s)$ and to the pole of $\zeta(s/2)$ at $s = 2$. The trivial poles for $s = 2n$, $n = 1, 2, ..$ correspond naturally to the values of $1/\alpha_K$ for different values of $h_{eff} = n \times h$ with n even integer. Complex poles would correspond to ordinary QFT coupling constant evolution. The zeros of zeta in increasing order would correspond to p-adic primes in increasing order and UV limit to smallest value of poles at critical line. One can distinguish the pole $s = 2$ as extreme UV limit at which QFT approximation fails totally. CP_2 length scale indeed corresponds to GUT scale.

6. One can test this hypothesis. $1/\alpha_K$ corresponds to the electroweak U(1) coupling strength so that the identification $1/\alpha_K = 1/\alpha_{U(1)}$ makes sense. One also knows a lot about the evolutions of $1/\alpha_{U(1)}$ and of electromagnetic coupling strength $1/\alpha_{em} = 1/[cos^2(\theta_W)\alpha_{U(1)}$. What does this predict?

   It turns out that at p-adic length scale $k = 131$ ($p \simeq 2^k$ by p-adic length scale hypothesis, which now can be understood number theoretically [K126]) fine structure constant is predicted with .7 per cent accuracy if Weinberg angle is assumed to have its value at atomic scale! It is difficult to believe that this could be a mere accident because also the prediction evolution of $\alpha_{U(1)}$ is correct qualitatively. Note however that for $k = 127$ labelling electron one can reproduce fine structure constant with Weinberg angle deviating about 10 per cent from the measured value of Weinberg angle. Both models will be considered.

7. What about the evolution of weak, color and gravitational coupling strengths? Quantum criticality suggests that the evolution of these couplings strengths is universal and independent of the details of the dynamics. Since one must be able to compare various evolutions and combine them together, the only possibility seems to be that the spectra of gauge coupling strengths are given by the poles of $\zeta_F(w)$ but with argument $w = w(s)$ obtained by a global conformal transformation of upper half plane - that is Möbius transformation (see `http://tinyurl.com/gwjs85b`) with real coefficients (element of $GL(2, R)$) so that one as $\zeta_F((as + b)/(cs + d))$. Rather general arguments force it to be and element of $GL(2, Q)$, $GL(2, Z)$ or maybe even $SL(2, Z)$ ($ad - bc = 1$) satisfying additional constraints. Since TGD

predicts several scaled variants of weak and color interactions, these copies could be perhaps parameterized by some elements of $SL(2,Z)$ and by a scaling factor K.

Could one understand the general qualitative features of color and weak coupling contant evolutions from the properties of corresponding Möbius transformation? At the critical line there can be no poles or zeros but could asymptotic freedom be assigned with a pole of $cs+d$ and color confinement with the zero of $as+b$ at real axes? Pole makes sense only if Kähler action for the preferred extremal vanishes. Vanishing can occur and does so for massless extremals characterizing conformally invariant phase. For zero of $as+b$ vacuum function would be equal to one unless Kähler action is allowed to be infinite: does this make sense?. One can however hope that the values of parameters allow to distinguish between weak and color interactions. It is certainly possible to get an idea about the values of the parameters of the transformation and one ends up with a general model predicting the entire electroweak coupling constant evolution successfully.

To sum up, the big idea is the identification of the spectra of coupling constant strengths as poles of $\zeta_F((as+b/)(cs+d))$ identified as a complex square root of partition function with motivation coming from ZEO, quantum criticality, and super-conformal symmetry; the discretization of the RG flow made possible by the p-adic length scale hypothesis $p \simeq k^k$, k prime; and the assignment of complex zeros of ζ with p-adic primes in increasing order. These assumptions reduce the coupling constant evolution to four real rational or integer valued parameters (a,b,c,d). In the sequel this vision is discussed in more detail.

4.4.7 Other applications of NTU

NTU in the strongest form says that all numbers involved at "basic level" (whatever this means!) of adelic TGD are products of roots of unity and of power of a root of e. This is extremely powerful physics inspired conjecture with a wide range of possible mathematical applications.

1. For instance, vacuum functional defined as an exponent of action for preferred externals would be number of this kind. One could define functional integral as adelic operation in all number fields: essentially as sum of exponents of action for stationary preferred extremals since Gaussian and metric determinants potentially spoiling NTU would cancel each other leaving only the exponent.

2. The implications of NTU for the zeros of Riemann zeta [L9] will be discussed in more detail in the Appendix. Suffice it to say that the observations about Fourier transform for the distribution of loci of non-trivial zeros of zeta together with NTU leads to explicit proposal for the algebraic for of zeros of zeta. The testable proposal is that zeros decompose to disjoint classes $C(p)$ labelled by primes p and the condition that p^{iy} is root of unity in given class $C(p)$.

3. NTU generalises to all Lie groups. Exponents $exp(in_iJ_i/n)$ of lie-algebra generators define generalisations of number theoretically universal group elements and generate a discrete subgroup of compact Lie group. Also hyperbolic "phases" based on the roots $e^{m/n}$ are possible and make possible discretized NTU versions of all Lie-groups expected to play a key role in adelization of TGD.

   NTU generalises also to quaternions and octonions and allows to define them as number theoretically universal entities. Note that ordinary p-adic variants of quaternions and octonions do not give rise to a number field: inverse of quaternion can have vanishing p-adic variant of norm squared satisfying $\sum_n x_n^2 = 0$.

   NTU allows to define also the notion of Hilbert space as an adelic notion. The exponents of angles characterising unit vector of Hilbert space would correspond to roots of unity.

4.4.8 Going to the roots of p-adicity

The basic questions raised by the p-adic mass calculations concern the origin of preferred p-adic primes and of p-adic length scale hypothesis. One can also ask whether there might be a natural origin for p-adicity at the level of WCW.

Preferred primes as ramified primes for extensions of rationals?

Preferred primes as ramified primes for extensions of rationals?

The intuitive feeling is that the notion of preferred prime is something extremely deep and to me the deepest thing I know is number theory. Does one end up with preferred primes in number theory? This question brought to my mind the notion of *ramification of primes* (`http://tinyurl.com/hddljlf`) (more precisely, of prime ideals of number field in its extension), which happens only for special primes in a given extension of number field, say rationals. Ramification is completely analogous to the degeneracy of some roots of polynomial and corresponds to criticality if the polynomial corresponds to criticality (catastrophe theory of Thom is one application). Could this be the mechanism assigning preferred prime(s) to a given elementary system, such as elementary particle? I have not considered their role earlier also their hierarchy is highly relevant in the number theoretical vision about TGD.

1. Stating it very roughly (I hope that mathematicians tolerate this sloppy language of physicist): as one goes from number field K, say rationals Q, to its algebraic extension L, the original prime ideals in the so called *integral closure* (`http://tinyurl.com/js6fpvr`) over integers of K decompose to products of prime ideals of L (prime ideal is a more rigorous manner to express primeness). Note that the general ideal is analog of integer.

   Integral closure for integers of number field K is defined as the set of elements of K, which are roots of some monic polynomial with coefficients, which are integers of K having the form $x^n + a_{n-1}x^{n-1} + ... + a_0$. The integral closures of both K and L are considered. For instance, integral closure of algebraic extension of K over K is the extension itself. The integral closure of complex numbers over ordinary integers is the set of algebraic numbers.

   Prime ideals of K can be decomposed to products of prime ideals of L: $P = \prod P_i^{e_i}$, where e_i is the ramification index. If $e_i > 1$ is true for some i, *ramification* occurs. P_i:s in question are like co-inciding roots of polynomial, which for in thermodynamics and Thom's catastrophe theory corresponds to criticality. Ramification could therefore be a natural aspect of quantum criticality and ramified primes P are good candidates for preferred primes for a given extension of rationals. Note that the ramification make sense also for extensions of given extension of rationals.

2. A physical analogy for the decomposition of ideals to ideals of extension is provided by decomposition of hadrons to valence quarks. Elementary particles becomes composite of more elementary particles in the extension. The decomposition to these more elementary primes is of form $P = \prod P_i^{e(i)}$, the physical analog would be the number of elementary particles of type i in the state (`http://tinyurl.com/h9528pl`). Unramified prime P would be analogous a state with e fermions. Maximally ramified prime would be analogous to Bose-Einstein condensate of e bosons. General ramified prime would be analogous to an e-particle state containing both fermions and condensed bosons. This is of course just a formal analogy.

3. There are two further basic notions related to ramification and characterizing it. *Relative discriminant* is the ideal divided by all ramified ideals in K (integer of K having no ramified prime factors) and relative different for P is the ideal of L divided by all ramified P_i:s (product of prime factors of P in L). These ideals represent the analogs of product of preferred primes P of K and primes P_i of L dividing them. These two integers ideals would characterize the ramification.

In TGD framework the extensions of rationals (`http://tinyurl.com/h9528pl`) and p-adic number fields (`http://tinyurl.com/zq22tvb`) are unavoidable and interpreted as an evolutionary hierarchy physically and cosmological evolution would gradually proceed to more and more complex extensions. One can say that string world sheets and partonic 2-surfaces with parameters of defining functions in increasingly complex extensions of prime emerge during evolution. Therefore ramifications and the preferred primes defined by them are unavoidable. For p-adic number fields the number of extensions is much smaller for instance for $p > 2$ there are only 3 quadratic extensions.

How could ramification relate to p-adic and adelic physics and could it explain preferred primes?

1. Ramified p-adic prime $P = P_i^e$ would be replaced with its e:th root P_i in p-adicization. Same would apply to general ramified primes. Each un-ramified prime of K is replaced with $e = K : L$ primes of L and ramified primes P with $\#\{P_i\} < e$ primes of L: the increase of algebraic dimension is smaller. An interesting question relates to p-adic length scale. What happens to p-adic length scales. Is p-adic prime effectively replaced with e:th root of p-adic prime: $L_p \propto p^{1/2}L_1 \to p^{1/2e}L_1$? The only physical option is that the p-adic temperature for P would be scaled down $T_p = 1/n \to 1/ne$ for its e:th root (for fermions serving as fundamental particles in TGD one actually has $T_p = 1$). Could the lower temperature state be more stable and select the preferred primes as maximimally ramified ones? What about general ramified primes?

2. This need not be the whole story. Some algebraic extensions would be more favored than others and p-adic view about realizable imaginations could be involved. p-Adic pseudo constants are expected to allow p-adic continuations of string world sheets and partonic 2-surfaces to 4-D preferred extremals with number theoretic discretization. For real continuations the situation is more difficult. For preferred extensions - and therefore for corresponding ramified primes - the number of real continuations - realizable imaginations - would be especially large.

   The challenge would be to understand why primes near powers of 2 and possibly also of other small primes would be favored. Why for them the number of realizable imaginations would be especially large so that they would be winners in number theoretical fight for survival?

Can one make this picture more concrete? What kind of algebraic extensions could be considered?

1. In p-adic context a proper definition of counterparts of angle variables as phases allowing definition of the analogs of trigonometric functions requires the introduction of algebraic extension giving rise to some roots of unity. Their number depends on the angular resolution. These roots allow to define the counterparts of ordinary trigonometric functions - the naive generalization based on Taylors series is not periodic - and also allows to defined the counterpart of definite integral in these degrees of freedom as discrete Fourier analysis. For the simplest algebraic extensions defined by $x^n - 1$ for which Galois group is abelian are are unramified so that something else is needed. One has decomposition $P = \prod P_i^{e(i)}$, $e(i) = 1$, analogous to n-fermion state so that simplest cyclic extension does not give rise to a ramification and there are no preferred primes.

2. What kind of polynomials could define preferred algebraic extensions of rationals? Irreducible polynomials are certainly an attractive candidate since any polynomial reduces to a product of them. One can say that they define the elementary particles of number theory. Irreducible polynomials have integer coefficients having the property that they do not decompose to products of polynomials with rational coefficients. It would be wrong to say that only these algebraic extensions can appear but there is a temptation to say that one can reduce the study of extensions to their study. One can even consider the possibility that string world sheets associated with products of irreducible polynomials are unstable against decay to those characterize irreducible polynomials.

3. What can one say about irreducible polynomials? Eisenstein criterion (`http://tinyurl.com/47kxjz` states following. If $Q(x) = \sum_{k=0,..,n} a_k x^k$ is n:th order polynomial with integer coefficients and with the property that there exists at least one prime dividing all coefficients a_i except a_n and that p^2 does not divide a_0, then Q is irreducible. Thus one can assign one or more preferred primes to the algebraic extension defined by an irreducible polynomial Q of this kind - in fact any polynomial allowing ramification. There are also other kinds of irreducible polynomials since Eisenstein's condition is only sufficient but not necessary.

   Furthermore, in the algebraic extension defined by Q, the prime ideals P having the above mentioned characteristic property decompose to an n:th power of single prime ideal P_i: $P = P_i^n$. The primes are maximally/completely ramified.

   A good illustration is provided by equations $x^2 + 1 = 0$ allowing roots $x_\pm = \pm i$ and equation $x^2 + 2px + p = 0$ allowing roots $x_\pm = -p \pm \sqrt{p}p - 1$. In the first case the ideals associated with

$\pm i$ are different. In the second case these ideals are one and the same since $x_+ == -x_- + p$: hence one indeed has ramification. Note that the first example represents also an example of irreducible polynomial, which does not satisfy Eisenstein criterion. In more general case the n conditions on defined by symmetric functions of roots imply that the ideals are one and same when Eisenstein conditions are satisfied.

4. What is so nice that one could readily construct polynomials giving rise to given preferred primes. The complex roots of these polymials could correspond to the points of partonic 2-surfaces carrying fermions and defining the ends of boundaries of string world sheet. It must be however emphasized that the form of the polynomial depends on the choices of the complex coordinate. For instance, the shift $x \to x+1$ transforms $(x^n-1)/(x-1)$ to a polynomial satisfying the Eisenstein criterion. One should be able to fix allowed coordinate changes in such a manner that the extension remains irreducible for all allowed coordinate changes.

   Already the integral shift of the complex coordinate affects the situation. It would seem that only the action of the allowed coordinate changes must reduce to the action of Galois group permuting the roots of polynomials. A natural assumption is that the complex coordinate corresponds to a complex coordinate transforming linearly under subgroup of isometries of the imbedding space.

In the general situation one has $P = \prod P_i^{e(i)}$, $e(i) \geq 1$ so that aso now there are prefered primes so that the appearance of preferred primes is completely general phenomenon.

The origin of p-adic length scale hypothesis?

p-Adic length scale hypothesis emerged from p-adic length scale hypothesis. A possible generalization of this hypothesis is that p-adic primes near powers of prime are physically favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [I73] (`http://tinyurl.com/jbh9m27`) and in music both 2-adicity and 3-adicity could be present: this is discussed in TGD inspired theory of music harmony and genetic code [K93]. See also [L20, L15].

One explanation would be that for preferred primes the number of p-adic space-time sheets representable also as real space-time sheets is maximal. Imagined worlds would be maximally realizable. Preferred p-adic primes would correspond to ramified primes for extensions with the property that the number of realizable imaginations is especially large for them. Why primes satisfying p-adic length scale hypothesis or its generalization would appear as ramified primes for extensions, which are winners in number theoretical evolution?

Also the weak form of NMP (WNMP) applying also to the purely number theoretic form of NMP [K72] might come in rescue here.

1. Entanglement negentropy for a NE [K72] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretical entanglement negentropy (NEN). If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured, which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.

2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real PE as p-adic PE (Note that $p = 1$ makes formally sense but for it the topology is discrete).

3. WNMP [K72, K124] suggests a more feasible explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator. WNMP allows "free will" so that all dimensions $n-k$, $k=0,1,...n-1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.

4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n=2^k=M_k+1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n=2^k-r$ near 2^k produce large entanglement negentropy and would be favored by NMP.

5. This argument suggests a generalization of p-adic length scale hypothesis so that $p=2$ can be replaced by any prime.

4.5 p-Adic physics and consciousness

p-Adic physics as physics of cognition and imagination is an important thread in TGD inspired theory of consciousness. In the sequel I describe briefly the basic of TGD inspired theory of consciousness as generalization of quantum measurement theory to ZEO (ZEO), describe the definition of self, consider the question whether NMP is needed as a separate principle or whether it is implied is in statistical sense by the unavoidable statistical increase of $n=h_{eff}/h$ if identified as a factor of the dimension of Galois group extension of rationals defining the adeles, and finally summarize the vision about how p-adic physics serves as a correlate of cognition and imagination.

4.5.1 From quantum measurement theory to a theory of consciousness

The notion of self can be seen as a generalization of the poorly defined definition of the notion of observer in quantum physics. In the following I take the role of skeptic trying to be as critical as possible.

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. The density matrix was assumed to define the universal observable. Note that a density matrix, which is power series of a product of matrices representing commuting observables has in the generic case eigenstates, which are simultaneous eigenstates of all observables. Second aspect of self was assumed to be the integration of subsequent quantum jumps to coherent whole giving rise to the experienced flow of time.

The precise identification of self allowing to understand both of these aspects turned out to be difficult problem. I became aware the solution of the problem in terms of ZEO (ZEO) only rather recently (2014).

1. Self corresponds to a sequence of quantum jumps integrating to single unit as in the original proposal, but these quantum jumps correspond to state function reductions to a fixed boundary of causal diamond CD leaving the corresponding parts of zero energy states invariant - "small" state function reductions. The parts of zero energy states at second boundary of CD change and even the position of the tip of the opposite boundary changes: one actually has wave function over positions of second boundary (CD sizes roughly) and this wave function changes. In positive energy ontology these repeated state function reductions would have no effect on the state (Zeno effect) but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and self: self is generalized Zeno effect.

2. The first quantum jump to the opposite boundary corresponds to the act of "free will" or birth of re-incarnated self. Hence the act of "free will" changes the arrow of psychological time at some level of hierarchy of CDs. The first reduction to the opposite boundary of CD means "death" of self and "re-incarnation" of time-reversed self at opposite boundary at which the the temporal distance between the tips of CD increases in opposite direction. The

sequence of selves and time reversed selves is analogous to a cosmic expansion for CD. The repeated birth and death of mental images could correspond to this sequence at the level of sub-selves.

3. This allows to understand the relationship between subjective and geometric time and how the arrow of and flow of clock time (psychological time) emerge. The average distance between the tips of CD increases on the average as along as state function functions occur repeatedly at the fixed boundary: situation is analogous to that in diffusion. The localization of contents of conscious experience to boundary of CD gives rise to the illusion that universe is 3-dimensional. The possibility of memories made possibly by hierarchy of CDs demonstrates that this is not the case. Self is simply the sequence of state function reductions at same boundary of CD remaining fixed and the lifetime of self is the total growth of the average temporal distance between the tips of CD.

One can identify several rather abstract state function reductions selecting a sector of WCW.

1. There are quantum measurements inducing localization in the moduli space of CDs with passive boundary and states at it fixed. In particular, a localization in the moduli characterizing the Lorentz transform of the upper tip of CD would be measured. The measured moduli characterize also the analog of symplectic form in M^4 strongly suggested by twistor lift of TGD - that is the rest system (time axis) and spin quantization axes. Of course, also other kinds of reductions are possible.

2. Also a localization to an extension of rationals defining the adeles should occur. Could the value of $n = h_{eff}/h$ be observable? The value of n for given space-time surface at the active boundary of CD could be identified as the order of the smallest Galois group containing all Galois groups assignable to 3-surfaces at the boundary. The superposition of space-time surface would not be eigenstate of n at active boundary unless localization occurs. It is not obvious whether this is consistent with a fixe value of n at passive boundary.

   The measured value of n could be larger or smaller than the value of n at the passive boundary of CD but in statistical sense n would increase by the analogy with diffusion on half line defined by non-negative integers. The distance from the origin unavoidably increases in statistical sense. This would imply evolution as increase of maximal value of negentropy and generation of quantum coherence in increasingly longer scales.

3. A further abstract choice corresponds to the the replacement of the roles of active and passive boundary of CD changing the arrow of clock time and correspond to a death of self and re-incarnation as time-reversed self.

Can one assume that these measurements reduce to measurements of a density matrix of either entangled system as assumed in the earlier formulation of NMP, or should one allow both options. This question actually applies to all quantum measurements and leads to a fundamental philosophical questions unavoidable in all consciousness theories.

1. Do all measurements involve entanglement between the moduli or extensions of two CDs reduced in the measurement of the density matrix? Non-diagonal entanglement would allow final states states, which are not eigenstates of moduli or of n: this looks strange. This could also lead to an infinite regress since it seems that one must assume endless hierarchy of entangled CDs so that the reduction sequence would proceed from top to bottom. It looks natural to regard single CD as a sub-Universe.

   For instance, if a selection of quantization axis of color hypercharge and isospin (localization in the twistor space of CP_2) is involved, one would have an outcome corresponding to a quantum superposition of measurements with different color quantization axis!

   Going philosophical, one can also argue, that the measurement of density matrix is only a reaction to environment and does not allow intentional free will.

2. Can one assume that a mere localization in the moduli space or for the extension of rationals (producing an eigenstate of n) takes place for a fixed CD - a kind of self measurement possible

for even unentangled system? If there is entanglement in these degrees of freedom between two systems (say CDs), it would be reduced in these self measurements but the outcome would not be an eigenstate of density matrix. An interpretation as a realization of intention would be approriate.

3. If one allows both options, the interpretation would be that state function reduction as a measurement of density matrix is only a reaction to environment and self-measurement represents a realization of intention.

4. Self measurements would occur at higher level say as a selection of quantization axis, localization in the moduli space of CD, or selection of extension of rationals. A possible general rule is that measurements at space-time level are reactions as measurements of density matrix whereas a selection of a sector of WCW would be an intentional action. This because formally the quantum states at the level of WCW are as modes of classical WCW spinor field single particle states.

5. If the selections of sectors of WCW at active boundary of CD commute with observables, whose eigenstates appear at passive boundary (briefly *passive observables*) meaning that time reversal commutes with them - they can occur repeatedly during the reduction sequence and self as a generalized Zeno effect makes sense.

   If the selections of WCW sectors at active boundary do not commute with passive observables then volition as a choice of sector of WCW must change the arrow of time. Libet's findings show that conscious choice induces neural activity for a fraction of second before the conscious choice. This would imply the correspondences *"big" measurement changing the arrow of time - self-measurement at the level of WCW - intentional action* and *"small" measurement - measurement at space-time level - reaction.*

   Self as a generalized Zeno effect makes sense only if there are active commuting with passive observables. If the passive observables form a maximal set, the new active observables commuting with them must emerge. The increase of the size of extension of rationals might generate them by expanding the state space so that self would survive only as long at it evolves.

   Otherwise there would be only single unitary time evolution followed by a reduction to opposite boundary. This makes sense only if the sequence of "big" reductions for sub-selves can give rise to the time flow experienced by self: the birth and death of mental images would give rise to flow of time of self.

A hierarchical process starting from given CD and proceeding downwards to shorter scales and stopping when the entanglement is stable is highly suggestive and favors self measurements. What stability could mean will be discussed in the next section. CDs would be a correlate for self hierarchy. One can say also something about the anatomy and correlates of self hierarchy.

1. Self experiences its sub-selves as mental images and even we would represent mental images of some higher level collective self. Everything is conscious but consciousness can be lost or at least it is not possible to have memory about it. The flow of consciousness for a given self could be due to the quantum jump sequences performed by its sub-selves giving rise to mental images.

2. By quantum classical correspondence self has also space-time correlates. One can visualize sub-self as a space-time sheet "glued" by topological sum to the space-time sheet of self. Subsystem is not described as a tensor factor as in the standard description of subsystems. Also sub-selves of selves can entangle negentropically and this gives rise to a sharing of mental images about which stereo vision would be basic example. Quite generally, one could speak of stereo consciousness. Also the experiences of sensed presence [J19] could be understood as a sharing of mental images between brain hemispheres, which are not themselves entangled. This is possible also between different brains. In the normal situation brain hemispheres are entangled.

3. At the level of 8-dimensional imbedding space the natural correlate of self would be CD (causal diamond). At the level of space-time the correlate would be space-time sheet or light-like 3-surface. The contents of consciousness of self would be determined by the space-time sheets in the interior of CD. Without further restrictions the experience of self would be essentially four-dimensional. Memories would be like sensory experiences except that they would be about the geometric past and for some reason are not usually colored by sensory qualia. For instance .1 second time scale defining sensory chronon corresponds to the secondary p-adic time scale characterizing the size of electron's CD (Mersenne prime M_{127}), which suggests that Cooper pairs of electrons are essential for the sensory qualia.

4.5.2 NMP and self

The view about Negentropy Maximization Principle (NMP) [K72] has co-evolved with the notion of self and I have considered many variants of NMP.

1. The original formulation of NMP was in positive energy ontology and made same predictions as standard quantum measurement theory. The new element was that the density matrix of sub-system defines the fundamental observable and the system goes to its eigenstate in state function reduction. As found, the localizations at to WCW sectors define what might be called self-measurements and identifiable as active volitions rather than reactions.

2. In p-adic physics one can assign with rational and even algebraic entanglement probabilities number theoretical entanglement negentropy (NEN) satisfying the same basic axioms as the ordinary Shannon entropy but having negative values and therefore having interpretation as information. The definition of p-adic negentropy (real valued) reads as $S_p = -\sum P_k log(|P_k|_p)$, where $|.|_p$ denotes p-adic norm. The news is that $N_p = -S_p$ can be positive and is positive for rational entanglement probabilities. Real entanglement entropy S is always non-negative.

   NMP would force the generation of negentropic entanglement (NE) and stabilize it. NNE resources of the Universe - one might call them Akashic records- would steadily increase.

3. A decisive step of progress was the realization is that NTU forces all states in adelic physics to have entanglement coefficients in some extension of rationals inducing finite-D extension of p-adic numbers. The same entanglement can be characterized by real entropy S and p-adic negentropies N_p, which can be positive. One can define also total p-adic negentropy: $N = \sum_p N_p$ for all p and total negentropy $N_{tot} = N - S$.

   For rational entanglement probabilities it is easy to demonstrate that the generalization of adelic theorem holds true: $N_{tot} = N - S = 0$. NMP based on N_{tot} rather than N would not say anything about rational entanglement. For extensions of rationals it is easy to find that $N - S > 0$ is possible if entanglement probabilities are of form X_i/n with $|X_i|_p = 1$ and n integer [L17]. Should one identify the total negentropy as difference $N_{tot} = N - S$ or as $N_{tot} = N$?

   Irrespective of answer, large p-adic negentropy seems to force large real entropy: this nicely correlates with the paradoxical finding that living systems tend to be entropic although one would expect just the oppositecite [L17]: this relates in very interesting manner to the work of biologists Jeremy England [I80]. The negentropy would be cognitive negentropy and not visible for ordinary physics.

4. The latest step in the evolution of ideas NMP was the question whether NMP follows from number theory alone just as second law follows form probability theory! This irritates theoretician's ego but is victory for theory. The dimension n of extension is positive integer and cannot but grow in statistical sense in evolution! Since one expects that the maximal value of negentropy (define as $N - S$) must increase with n. Negentropy must increase in long run.

Number theoretic entanglement can be stable

Number theoretical Shannon entropy can serve as a measure for genuine information assignable to a pair of entanglement systems [K72]. Entanglement with coefficients in the extension is always negentropic if entanglement negentropy comes from p-adic sectors only. It can be negentropic if negentropy is defined as the difference of p-adic negentropy and real entropy.

The diagonalized density matrix need not belong to the algebraic extension since the probabilities defining its diagonal elements are eigenvalues of the density matrix as roots of N:th order polynomial, which in the generic case requires n-dimensional algebraic extension of rationals. One can argue that since diagonalization is not possible, also state function reduction selecting one of the eigenstates is impossible unless a phase transition increasing the dimension of algebraic extension used occurs simultaneously. This kind of NE could give rise to cognitive entanglement.

There is also a special kind of NE, which can result if one requires that density matrix serves a universal observable in state function reduction. The outcome of reduction must be an eigen space of density matrix, which is projector to this subspace acting as identity matrix inside it. This kind NE allows all unitarily related basis as eigenstate basis (unitary transformations must belong to the algebraic extension). This kind of NE could serve as a correlate for "enlightened" states of consciousness. Schrödingers cat is in this kind of state stably in superposition of dead and alive and state basis obtained by unitary rotation from this basis is equally good. One can say that there are no discriminations in this state, and this is what is claimed about "enlightened" states too.

The vision about number theoretical evolution suggests that NMP forces the generation of NE resources as NE assignable to the "passive" boundary of CD for which no changes occur during sequence of state function reductions defining self. It would define the unchanging self as negentropy resources, which could be regarded as kind of Akashic records. During the next "re-incarnation" after the first reduction to opposite boundary of CD the NE associated with the reduced state would serve as new Akashic records for the time reversed self. If NMP reduces to the statistical increase of $h_{eff}/h = n$ the consciousness information contents of the Universe increases in statistical sense. In the best possible world of SNMP it would increase steadily.

Does NMP reduce to number theory?

The heretic question that emerged quite recently is whether NMP is actually needed at all! Is NMP a separate principle or could NMP reduced to mere number theory [K72]? Consider first the possibility that NMP is not needed at all as a separate principle.

1. The value of $h_{eff}/h = n$ should increase in the evolution by the phase transitions increasing the dimension of the extension of rationals. $h_{eff}/h = n$ has been identified as the number of sheets of some kind of covering space. The Galois group of extension acts on number theoretic discretizations of the monadic surface and the orbit defines a covering space. Suppose n is the number of sheets of this covering and thus the dimension of the Galois group for the extension of rationals or factor of it.

2. It has been already noticed that the "big" state function reductions giving rise to death and reincarnation of self could correspond to a measurement of $n = h_{eff}$ implied by the measurement of the extension of the rationals defining the adeles. The statistical increase of n follows automatically and implies statistical increase of maximal entanglement negentropy. Entanglement negentropy increases in statistical sense.

   The resulting world would not be the best possible one unlike for a strong form of NMP demanding that negentropy does increaes in "big" state function reductions. n also decrease temporarily and they seem to be needed. In TGD inspired model of bio-catalysis the phase transition reducing the value of n for the magnetic flux tubes connecting reacting bio-molecules allows them to find each other in the molecular soup. This would be crucial for understanding processes like DNA replication and transcription.

3. State function reduction corresponding to the measurement of density matrix could occur to an eigenstate/eigenspace of density matrix only if the corresponding eigenvalue and eigenstate/eigenspace is expressible using numbers in the extension of rationals defining the adele considered. In the generic case these numbers belong to N-dimensional extension of the original extension. This can make the entanglement stable with respect to state the measurements of density matrix.

A phase transition to an extension of an extension containing these coefficients would be required to make possible reduction. A step in number theoretic evolution would occur. Also an entanglement of measured state pairs with those of measuring system in containing the extension of extension would make possible the reduction. Negentropy could be reduced but higher-D extension would provide potential for more negentropic entanglement and NMP would hold true in the statistical sense.

4. If one has higher-D eigen space of density matrix, p-adic negentropy is largest for the entire subspace and the sum of real and p-adic negentropies vanishes for all of them. For negentropy identified as total p-adic negentropy SNMP would select the entire sub-space and NMP would indeed say something explicit about negentropy.

Or is NMP needed as a separate principle?

Hitherto I have postulated NMP as a separate principle [K72]. Strong form of NMP (SNMP) states that Negentropy does not decrease in "big" state function reductions corresponding to death and re-incarnations of self.

One can however argue that SNMP is not realistic. SNMP would force the Universe to be the best possible one, and this does not seem to be the case. Also ethically responsible free will would be very restricted since self would be forced always to do the best deed that is increase maximally the negentropy serving as information resources of the Universe. Giving up separate NMP altogether would allow to have also "Good" and "Evil".

This forces to consider what I christened weak form of NMP (WNMP). Instead of maximal dimension corresponding to N-dimensional projector self can choose also lower-dimensional sub-spaces and 1-D sub-space corresponds to the vanishing entanglement and negentropy assumed in standard quantum measurement theory. As a matter fact, this can also lead to larger negentropy gain since negentropy depends strongly on what is the large power of p in the dimension of the resulting eigen sub-space of density matrix. This could apply also to the purely number theoretical reduction of NMP.

WNMP suggests how to understand the notions of Good and Evil. Various choices in the state function reduction would correspond to Boolean algebra, which suggests an interpretation in terms of what might be called emotional intelligence [K124]. Also it turns out that one can understand how p-adic length scale hypothesis - actually its generalization - emerges from WNMP [K126].

1. One can start from ordinary quantum entanglement. It corresponds to a superposition of pairs of states. Second state corresponds to the internal state of the self and second state to a state of external world or biological body of self. In negentropic quantum entanglement each is replaced with a pair of sub-spaces of state spaces of self and external world. The dimension of the sub-space depends on which pair is in question. In state function reduction one of these pairs is selected and deed is done. How to make some of these deeds good and some bad? Recall that WNMP allows only the possibility to generate NNE but does not force it. WNMP would be like God allowing the possibility to do good but not forcing good deeds.

   Self can choose any sub-space of the subspace defined by $k \leq N$-dimensional projector and 1-D subspace corresponds to the standard quantum measurement. For $k=1$ the state function reduction leads to vanishing negentropy, and separation of self and the target of the action. Negentropy does not increase in this action and self is isolated from the target: kind of price for sin.

   For the maximal dimension of this sub-space the negentropy gain is maximal. This deed would be good and by the proposed criterion NE corresponds to conscious experience with positive emotional coloring. Interestingly, there are $2^k - 1$ possible choices, which is almost the dimension of Boolean algebra consisting of k independent bits. The excluded option corresponds to 0-dimensional sub-space - empty set in set theoretic realization of Boolean algebra. This could relate directly to fermionic oscillator operators defining basis of Boolean algebra - here Fock vacuum would be the excluded state. The deed in this sense would be a choice of how loving the attention towards system of external world is.

2. A map of different choices of k-dimensional sub-spaces to k-fermion states is suggestive. The realization of logic in terms of emotions of different degrees of positivity would be mapped to many-fermion states - perhaps zero energy states with vanishing total fermion number. State function reductions to k-dimensional spaces would be mapped to k-fermion states: quantum jumps to quantum states!

   The problem brings in mind quantum classical correspondence in quantum measurement theory. The direction of the pointer of the measurement apparatus (in very metaphorical sense) corresponds to the outcome of state function reduction, which is now 1-D subspace. For ordinary measurement the pointer has k positions. Now it must have $2^k - 1$ positions. To the discrete space of k pointer positions one must assign fermionic Clifford algebra of second quantized fermionic oscillator operators. The hierarchy of Planck constants and dark matter suggests the realization. Replace the pointer with its space-time k-sheeted covering and consider zero energy energy states made of pairs of k-fermion states at the sheets of the n-sheeted covering? Dark matter would be therefore necessary for cognition. The role of fermions would be to "mark" the k space-time sheets in the covering.

The cautious conclusion is that NMP as a separate principle is not necessary and follows in statistical sense from the unavoidable increase of $n = h_{eff}/h$ identified as dimension of extension of rationals define the adeles if this extension or at least the dimension of its Galois group is observable.

4.5.3 p-Adic physics as correlate of cognition and imagination

The items in the following list give motivations for the proposal that p-adic physics could serve as a correlate for cognition and imagination.

1. By the total disconnectedness of the p-adic topology, p-adic world decomposes naturally into blobs, objects. This happens also in sensory perception. The pinary digits of p-adic number can be assigned to a p-tree. Parisi proposed in the model of spin glass [B30] that p-adic numbers could relate to the mathematical description of cognition and also Khrennikov [J6] has developed this idea. In TGD framework that idea is taken to space-time level: p-adic space-time sheets represent thought bubbles and they correlate with the real ones since they form cognitive reprentations of the real world. SH allows a concrete realization of this.

2. p-Adic non-determinism due to p-adic pseudo constants suggests interpretation in terms of imagination. Given 2-surfaces could allow completion to p-adic preferred extremal but not to a real one so that pure "non-realizable" imagination is in question.

3. Number theoretic negentropy has interpretation as negentropy characterizing information content of entanglement. The superposition of state pairs could be interpreted as a quantum representation for a rule or abstracted association containing its instances as state pairs. Number theoretical negentropy characterizes the relationship of two systems and should not be confused with thermodynamical entropy, which characterizes the uncertainty about the state of single system.

The original vision was that p-adic non-determinism could serve as a correlate for cognition, imagination, and intention. The recent view is much more cautious. Imagination need not completely reduce to p-adic non-determinism since it has also real physics correlates - maybe as partial realizations of SH as in nerve pulse pattern, which does not propagate down to muscles.

A possible interpretation for the solutions of the p-adic field equations would be as geometric correlates of cognition, imagination, and perhaps even intentionality. Plans, intentions, expectations, dreams, and possibly also cognition as imagination in general could have p-adic cognitive space-time sheets as their geometric correlates. A deep principle seems to be involved: incompleteness is the characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

The most feasible view is that the intersections of p-adic and real space-time surfaces define cognitive representations of real space-time surfaces (PEs, [K19, K10, K13]). One could also say that real space-time surface represents sensory aspects of conscious experience and p-adic space-time surfaces its cognitive aspects. Both real and p-adics rather than real or p-adics.

The identification of p-adic pseudo constants as correlates of imagination at space-time level is indeed a further natural idea.

1. The construction of PEs by SH from the data at 2-surfaces is like boundary value problem with number theoretic discretization of space-time surface as additional data. PE property in real context implies strong correlations between string world sheets and partonic 2-surfaces by boundary conditions a them. One cannot choose these 2-surfaces completely independently in real context.

2. In p-adic sectors the integration constants are replaced with pseudo-constants depending on finite number of pinary digits of variables depending on coordinates normal to string world sheets and partonic 2-surfaces. The fixing of the discretization of space-time surface would allow to fix the p-adic pseudo-constants. Once the number theoretic discretization of space-time surface is fixed, the p-adic pseudo-constants can be fixed. Pseudo-constant could allow a large number of p-adic configurations involving string world sheets, partonic 2-surfaces, and number theoretic discretization but not allowed in real context.

   Could these p-adic PEs correspond to imaginations, which in general are not realizable? Could the realizable intentional actions belong to the intersection of real and p-adic WCWs? Could one identify non-realistic imaginations as the modes of WCW spinor fields for which 2-surfaces are not extendable to real space-time surfaces and are localized to 2-surfaces? Could they allow only a partial continuation to real space-time surface. Could nerve pulse pattern representing imagined motor action and not proceeding to the level of muscles correspond to a partially real PE?

   Could imagination and problem solving be search for those collections of string world sheets and partonic 2-surfaces, which allow extension to (realization as) real PEs? If so, p-adic physics would be there as an independent aspect of existence and this is just the original idea. Imagination could be realized in state function reduction, which always selects only those 2-surfaces, which allow continuation to real space-time surfaces. The distinction between only imaginable and also realizable would be the extendability by using strong form of holography.

3. An interesting question is why elementary particles are characterized by preferred p-adic primes (primes near powers of 2, in particular Mersenne primes). Could the number of realizable imaginations for these primes be especially large?

I have the feeling that this view allows respectable mathematical realization of imagination in terms of adelic quantum physics. It is remarkable that SH derivable from - you can guess, SGCI (the Big E again!), plays an absolutely central role in it.

4.6 Appendix: Super-symplectic conformal weights and zeros of Riemann zeta

Since fermions are the only fundamental particles in TGD one could argue that the conformal weight of for the generating elements of supersympelectic algebra could be negatives for the poles of fermionic zeta ζ_F. This demands $n > 0$ as does also the fractal hierarchy of supersymplectic symmetry breakings. NTU of Riemann zeta in some sense is strongly suggested if adelic physics is to make sense.

For ordinary conformal algebras there are only finite number of generating elements ($-2 \leq n \leq 2$). If the radial conformal weights for the generators of g consist of poles of ζ_F, the situation changes. ζ_F is suggested by the observation that fermions are the only fundamental particles in TGD.

1. Riemann Zeta $\zeta(s) = \prod_p (1/(1-p^{-s})$ identifiable formally as a partition function $\zeta_B(s)$ of arithmetic boson gas with bosons with energy $log(p)$ and temperature $1/s = 1/(1/2+iy)$ should be replaced with that of arithmetic fermionic gas given in the product representation by $\zeta_F(s) = \prod_p (1+p^{-s})$ so that the identity $\zeta_B(s))/\zeta_F(s) = \zeta_B(2s)$ follows. This gives

$$\frac{\zeta_B(s)}{\zeta_B(2s)} \ .$$

$\zeta_F(s)$ has zeros at zeros s_n of $\zeta(s)$ and at the pole $s = 1/2$ of $zeta(2s)$. $\zeta_F(s)$ has poles at zeros $s_n/2$ of $\zeta(2s)$ and at pole $s = 1$ of $\zeta(s)$.

The spectrum of $1/T$ would be for the generators of algebra $\{(-1/2+iy)/2, n > 0, -1\}$. In p-adic thermodynamics the p-adic temperature is $1/T = 1/n$ and corresponds to "trivial" poles of ζ_F. Complex values of temperature does not make sense in ordinary thermodynamics. In ZEO quantum theory can be regarded as a square root of thermodynamics and complex temperature parameter makes sense.

2. If the spectrum of conformal weights of the generating elements of the algebra corresponds to poles serving as analogs of propagator poles, it consists of the "trivial" conformal $h = n > 0$- the standard spectrum with $h = 0$ assignable to massless particles excluded - and "non-trivial" $h = -1/4 + iy/2$. There is also a pole at $h = -1$.

   Both the non-trivial pole with real part $h_R = -1/4$ and the pole $h = -1$ correspond to tachyons. I have earlier proposed conformal confinement meaning that the total conformal weight for the state is real. If so, one obtains for a conformally confined two-particle states corresponding to conjugate non-trivial zeros in minimal situation $h_R = -1/2$ assignable to N-S representation.

   In p-adic mass calculations ground state conformal weight must be $-5/2$ [K68]. The negative fermion ground state weight could explain why the ground state conformal weight must be tachyonic $-5/2$. With the required 5 tensor factors one would indeed obtain this with minimal conformal confinement. In fact, arbitrarily large tachyonic conformal weight is possible but physical state should always have conformal weights $h > 0$.

3. $h = 0$ is not possible for generators, which reminds of Higgs mechanism for which the naïve ground states corresponds to tachyonic Higgs. $h = 0$ conformally confined massless states are necessarily composites obtained by applying the generators of Kac-Moody algebra or super-symplectic algebra to the ground state. This is the case according to p-adic mass calculations [K68], and would suggest that the negative ground state conformal weight can be associated with super-symplectic algebra and the remaining contribution comes from ordinary super-conformal generators. Hadronic masses, whose origin is poorly understood, could come from super-symplectic degrees of freedom. There is no need for p-adic thermodynamics in super-symplectic degrees of freedom.

4.6.1 A general formula for the zeros of zeta from NTU

Dyson's comment about Fourier transform of Riemann Zeta [A55] (`http://tinyurl.com/hjbfsuv`) is interesting from the point of NTU for Riemann zeta.

1. The numerical calculation of Fourier transform for the imaginary parts iy of zeros $s = 1/2+iy$ of zeta shows that it is concentrated at discrete set of frequencies coming as $log(p^n)$, p prime. This translates to the statement that the zeros of zeta form a 1-dimensional quasicrystal, a discrete structure Fourier spectrum by definition is also discrete (this of course holds for ordinary crystals as a special case). Also the logarithms of powers of primes would form a quasicrystal, which is very interesting from the point of view of p-adic length scale hypothesis. Primes label the "energies" of elementary fermions and bosons in arithmetic number theory, whose repeated second quantization gives rise to the hierarchy of infinite primes [K107]. The energies for general states are logarithms of integers.

2. Powers p^n label the points of quasicrystal defined by points $log(p^n)$ and Riemann zeta has interpretation as partition function for boson case with this spectrum. Could p^n label also the points of the dual lattice defined by iy.

3. The existence of Fourier transform for points $log(p_i^n)$ for any vector y_a in class $C(p)$ of zeros labelled by p requires $p_i^{iy_a}$ to be a root of unity inside $C(p)$. This could define the sense in

which zeros of zeta are universal. This condition also guarantees that the factor $n^{-1/2-iy}$ appearing in zeta at critical line are number theoretically universal ($p^{1/2}$ is problematic for Q_p: the problem might be solved by eliminating from p-adic analog of zeta the factor $1-p^{-s}$.

(a) One obtains for the pair (p_i, s_a) the condition $log(p_i)y_a = q_{ia}2\pi$, where q_{ia} is a rational number. Dividing the conditions for (i,a) and (j,a) gives

$$p_i = p_j^{q_{ia}/q_{ja}}$$

for every zero s_a so that the ratios q_{ia}/q_{ja} do not depend on s_a. From this one easily deduce $p_i^M = p_j^N$, where M and N are integers so that one ends up with a contradiction.

(b) Dividing the conditions for (i,a) and (i,b) one obtains

$$\frac{y_a}{y_b} = \frac{q_{ia}}{q_{ib}}$$

so that the ratios q_{ia}/q_{ib} do not depend on p_i. The ratios of the imaginary parts of zeta would be therefore rational number which is very strong prediction and zeros could be mapped by scaling y_a/y_1 where y_1 is the zero which smallest imaginary part to rationals.

(c) The impossible consistency conditions for (i,a) and (j,a) can be avoided if each prime and its powers correspond to its own subset of zeros and these subsets of zeros are disjoint: one would have infinite union of sub-quasicrystals labelled by primes and each p-adic number field would correspond to its own subset of zeros: this might be seen as an abstract analog for the decomposition of rational to powers of primes. This decomposition would be natural if for ordinary complex numbers the contribution in the complement of this set to the Fourier trasform vanishes. The conditions $(i,a)$and (i,b) require now that the ratios of zeros are rationals only in the subset associated with p_i.

For the general option the Fourier transform can be delta function for $x = log(p^k)$ and the set $\{y_a(p)\}$ contains N_p zeros. The following argument inspires the conjecture that for each p there is an infinite number N_p of zeros $y_a(p)$ in class $C(p)$ satisfying

$$p^{iy_a(p)} = u(p) = e^{\frac{r(p)}{m(p)}i2\pi} \quad ,$$

where $u(p)$ is a root of unity that is $y_a(p) = 2\pi(m(a) + r(p))/log(p)$ and forming a subset of a lattice with a lattice constant $y_0 = 2\pi/log(p)$, which itself need not be a zero.

In terms of stationary phase approximation the zeros $y_a(p)$ associated with p would have constant stationary phase whereas for $y_a(p_i \neq p))$ the phase $p^{iy_a(p_i)}$ would fail to be stationary. The phase e^{ixy} would be non-stationary also for $x \neq log(p^k)$ as function of y.

1. Assume that for $x = qlog(p)$, where q not a rational, the phases e^{ixy} fail to be roots of unity and are random implying the vanishing/smallness of $F(x)$.

2. Assume that for a given p all powers p^{iy} for $y \notin \{y_a(p)\}$ fail to be roots of unity and are also random so that the contribution of the set $y \notin \{y_a(p)\}$ to $F(p)$ vanishes/is small.

3. For $x = log(p^{k/m})$ the Fourier transform should vanish or be small for $m \neq 1$ (rational roots of primes) and give a non-vanishing contribution for $m = 1$. One has

$$\begin{aligned} &F(x = log(p^{k/m}) = \textstyle\sum_{1\leq a\leq N(p)} e^{k\frac{M(a,p)}{mN(p)}i2\pi}u(p) \quad , \\ &u(p) = e^{\frac{r(p)}{m(p)}i2\pi} \quad . \end{aligned}$$

   Obviously one can always choose $N(a,p) = N(p)$.

4. For the simplest option $N(p) = 1$ one would obtain delta function distribution for $x = log(p^k)$. The sum of the phases associated with $y_a(p)$ and $-y_a(p)$ from the half axes of the critical line would give

$$F(x = log(p^n)) \propto X(p^n) \equiv 2cos(n\frac{r(p)}{m(p)}2\pi) \ .$$

The sign of F would vary.

5. For $x = log(p^{k/m})$ the value of Fourier transform is expected to be small by interference effects if $M(a,p)$ is random integer, and negligible as compared with the value at $x = log(p^k)$. This option is highly attractive. For $N(p) > 1$ and $M(a,p)$ a random integer also $F(x = log(p^k)$ is small by interference effects. Hence it seems that this option is the most natural one.

6. The rational $r(p)/m(p)$ would characterize given prime (one can require that $r(p)$ and $m(p)$ have no common divisors). $F(x)$ is non-vanishing for all powers $x = log(p^n)$ for $m(p)$ odd. For $p = 2$, also $m(2) = 2$ allows to have $|X(2^n)| = 2$. An interesting ad hoc ansatz is $m(p) = p$ or $p^{s(p)}$. One has periodicity in n with period $m(p)$ that is logarithmic wave. This periodicity serves as a test and in principle allows to deduce the value of $r(p)/m(p)$ from the Fourier transform.

What could one conclude from the data (`http://tinyurl.com/hjbfsuv`)?

1. The first graph gives $|F(x = log(p^k)|$ and second graph displays a zoomed up part of $|F(x = log(p^k)|$ for small powers of primes in the range $[2, 19]$. For the first graph the eighth peak $(p = 11)$ is the largest one but in the zoomed graphs this is not the case. Hence something is wrong or the graphs correspond to different approximations suggesting that one should not take them too seriously.

   In any case, the modulus is not constant as function of p^k. For small values of p^k the envelope of the curve decreases and seems to approach constant for large values of p^k (one has $x < 15$ $(e^{15} \simeq 3.3 \times 10^6)$.

2. According to the first graph $|F(x)|$ decreases for $x = klog(p) < 8$, is largest for small primes, and remains below a fixed maximum for $8 < x < 15$. According to the second graph the amplitude decreases for powers of a given prime (say $p = 2$). Clearly, the small primes and their powers have much larger $|F(x)|$ than large primes.

There are many possible reasons for this behavior. Most plausible reason is that the sums involved converge slowly and the approximation used is not good. The inclusion of only 10^4 zeros would show the positions of peaks but would not allow reliable estimate for their intensities.

1. The distribution of zeros could be such that for small primes and their powers the number of zeros is large in the set of 10^4 zeros considered. This would be the case if the distribution of zeros $y_a(p)$ is fractal and gets "thinner" with p so that the number of contributing zeros scales down with p as a power of p, say $1/p$, as suggested by the envelope in the first figure.

2. The infinite sum, which should vanish, converges only very slowly to zero. Consider the contribution $\Delta F(p^k, p_1)$ of zeros not belonging to the class $p_1 \neq p$ to $F(x = log(p^k)) = \sum_{p_i} \Delta F(p^k, p_i)$, which includes also $p_i = p$. $\Delta F(p^k, p_i)$, $p \neq p_1$ should vanish in exact calculation.

   (a) By the proposed hypothesis this contribution reads as

   $$\begin{array}{l} \Delta F(p, p_1) = \sum_a cos\left[X(p^k, p_1)(M(a, p_1) + \frac{r(p_1)}{m(p_1)})2\pi)\right] \ . \\ X(p^k, p_1) = \frac{log(p^k)}{log(p_1)} \ . \end{array}$$

   Here a labels the zeros associated with p_1. If p^k is "approximately divisible" by p^1 in other words, $p^k \simeq np_1$, the sum over finite number of terms gives a large contribution since interference effects are small, and a large number of terms are needed to give a nearly vanishing contribution suggested by the non-stationarity of the phase. This happens in several situations.

(b) The number $\pi(x)$ of primes smaller than x goes asymptotically like $\pi(x) \simeq x/log(x)$ and prime density approximately like $1/log(x) - 1/log(x)^2$ so that the problem is worst for the small primes. The problematic situation is encountered most often for powers p^k of small primes p near larger prime and primes p (also large) near a power of small prime (the envelope of $|F(x)|$ seems to become constant above $x \sim 10^3$).

(c) The worst situation is encountered for $p = 2$ and $p_1 = 2^k - 1$ - a Mersenne prime and $p_1 = 2^{2^k} + 1$, $k \leq 4$ - Fermat prime. For $(p, p_1) = (2^k, M_k)$ one encounters $X(2^k, M_k) = (log(2^k)/log(2^k - 1)$ factor very near to unity for large Mersennes primes. For $(p, p_1) = (M_k, 2)$ one encounters $X(M_k, 2) = (log(2^k - 1)/log(2) \simeq k$. Examples of Mersennes and Fermats are $(3,2), (5,2), (7,2), (17,2), (31,2), (127,2), (257,2), ...$ Powers 2^k, $k = 2, 3, 4, 5, 7, 8, ..$ are also problematic.

(d) Also twin primes are problematic since in this case one has factor $X(p = p_1 + 2, p_1) = \frac{log(p_1+2)}{log(p_1)}$. The region of small primes contains many twin prime pairs: (3,5), (5,7), (11,13), (17,19), (29,31),....

These observations suggest that the problems might be understood as resulting from including too small number of zeros.

3. The predicted periodicity of the distribution with respect to the exponent k of p^k is not consistent with the graph for small values of prime unless the periodic $m(p)$ for small primes is large enough. The above mentioned effects can quite well mask the periodicity. If the first graph is taken at face value for small primes, $r(p)/m(p)$ is near zero, and $m(p)$ is so large that the periodicity does not become manifest for small primes. For $p = 2$ this would require $m(2) > 21$ since the largest power $2^n \simeq e^{15}$ corresponds to $n \sim 21$.

To summarize, the prediction is that for zeros of zeta should divide into disjoint classes $\{y_a(p)\}$ labelled by primes such that within the class labelled by p one has $p^{iy_a(p)} = e^{(r(p)/m(p))i2\pi}$ so that has $y_a(p) = [M(a,p) + r(p)/m(p))]2\pi/log(p)$.

4.6.2 More precise view about zeros of Zeta

There is a very interesting blog post by Mumford (`http://tinyurl.com/zemw27o`), which leads to much more precise formulation of the idea and improved view about the Fourier transform hypothesis: the Fourier transform or its generalization must be defined for all zeros, not only the non-trivial ones and trivial zeros give a background term allowing to understand better the properties of the Fourier transform.

Mumford essentially begins from Riemann's "explicit formula" in von Mangoldt's form.

$$\sum_p \sum_{n\geq 1} log(p)\delta_{p^n}(x) = 1 - \sum_k x^{s_k - 1} - \frac{1}{x(x^2-1)} \quad ,$$

where p denotes prime and s_k a non-trivial zero of zeta. The left hand side represents the distribution associated with powers of primes. The right hand side contains sum over cosines

$$\sum_k x^{s_k - 1} = 2\frac{\sum_k cos(log(x)y_k)}{x^{1/2}} \quad ,$$

where y_k ithe imaginary part of non-trivial zero. Apart from the factor $x^{-1/2}$ this is just the Fourier transform over the distribution of zeros.

There is also a slowly varying term $1 - \frac{1}{x(x^2-1)}$, which has interpretation as the analog of the Fourier transform term but sum over trivial zeros of zeta at $s = -2n$, $n > 0$. The entire expression is analogous to a "Fourier transform" over the distribution of all zeros. Quasicrystal is replaced with union on 1-D quasicrystals.

Therefore the distribution for powers of primes is expressible as "Fourier transform" over the distribution of both trivial and non-trivial zeros rather than only non-trivial zeros as suggested by numerical data to which Dyson [A55] referred to (`http://tinyurl.com/hjbfsuv`). Trivial zeros give a slowly varying background term large for small values of argument x (poles at $x = 0$ and

$x = 1$ - note that also $p = 0$ and $p = 1$ appear effectively as primes) so that the peaks of the distribution are higher for small primes.

The question was how can one obtain this kind of delta function distribution concentrated on powers of primes from a sum over terms $cos(log(x)y_k)$ appearing in the Fourier transform of the distribution of zeros.

Consider $x = p^n$. One must get a constructive interference. Stationary phase approximation is in terms of which physicist thinks. The argument was that a destructive interference occurs for given $x = p^n$ for those zeros for which the cosine does not correspond to a real part of root of unity as one sums over such y_k: random phase approximation gives more or less zero. To get something nontrivial y_k must be proportional to $2\pi \times n(y_k)/log(p)$ in class $C(p)$ to which y_k belongs. If the number of these y_k:s in $C(p)$ is infinite, one obtains delta function in good approximation by destructive interference for other values of argument x.

The guess that the number of zeros in $C(p)$ is infinite is encouraged by the behaviors of the densities of primes one hand and zeros of zeta on the other hand. The number of primes smaller than real number x goes like

$$\pi(x) = N(primes < x) \sim \frac{x}{log(x)}$$

in the sense of distribution. The number of zeros along critical line goes like

$$N(zeros < t) = (t/2\pi) \times log(\frac{t}{2\pi})$$

in the same sense. If the real axis and critical line have same metric measure then one can say that the number of zeros in interval T per number of primes in interval T behaves roughly like

$$\frac{N(zeros < T)}{N(primes < T)} = log(\frac{T}{2\pi}) \times \frac{log(T)}{2\pi}$$

so that at the limit of $T \to \infty$ the number of zeros associated with given prime is infinite. This asumption of course makes the argument a poor man's argument only.

4.6.3 Possible relevance for TGD

What this speculative picture from the point of view of TGD?

1. A possible formulation for NTU for the poles of fermionic Riemann zeta $\zeta_F = \zeta(s)/\zeta(2s)$ could be as a condition that is that the exponents $p^{ks_a(p)/2} = p^{k/4}p^{iky_a(p)/2}$ exist in a number theoretically universal manner for the zeros $s_a(p)$ for given p-adic prime p and for some subset of integers k. If the proposed conditions hold true, exponent reduces $p^{k/4}e^{k(r(p/m(p)i2\pi}$ requiring that k is a multiple of 4. The number of the non-trivial generating elements of super-symplectic algebra in the monomial creating physical state would be a multiple of 4. These monomials would have real part of conformal weight -1. Conformal confinement suggests that these monomials are products of pairs of generators for which imaginary parts cancel.

2. Quasi-crystal property might have an application to TGD. The functions of light-like radial coordinate appearing in the generators of supersymplectic algebra could be of form r^s, s zero of zeta or rather, its imaginary part. The eigenstate property with respect to the radial scaling rd/dr is natural by radial conformal invariance.

   The idea that arithmetic QFT assignable to infinite primes is behind the scenes in turn suggests light-like momenta assignable to the radial coordinate have energies with the dual spectrum $log(p^n)$. This is also suggested by the interpretation of ζ as square root of thermodynamical partition function for boson gas with momentum $log(p)$ and analogous interpretation of ζ_F.

   The two spectra would be associated with radial scalings and with light-like translations of light-cone boundary respecting the direction and light-likeness of the light-like radial vector. $log(p^n)$ spectrum would be associated with light-like momenta whereas p-adic mass scales would characterize states with thermal mass. Note that generalization of p-adic length scale hypothesis raises the scales defined by p^n to a special physical position: this might relate to ideal structure of adeles.

3. Finite measurement resolution suggests that the approximations of Fourier transforms over the distribution of zeros taking into account only a finite number of zeros might have a physical meaning. This might provide additional understand about the origins of generalized p-adic length scale hypothesis stating that primes $p \simeq p_1^k$, p_1 small prime - say Mersenne primes - have a special physical role.

Chapter 5

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part I

5.1 Introduction

There are good reasons to hope that TGD is integrable theory in some sense. Classical physics is an exact part of quantum physics in TGD and during years I have ended up with several proposals for the general solution of classical field equations (classical TGD is an exact part of quantum TGD).

5.1.1 Various approaches to classical TGD

World of classical worlds

The first approach is based on the geometry of the "world of classical worlds" (WCW) [K60, K35, K98].

1. The study of classical field equations led rather early to the realization that preferred extremals for the twistor lift of Kähler action with Minkowskian signature of induced metric define a slicing of space-time surfaces defined by 2-D string world sheets and partonic two-surfaces locally orthogonal to them. The interpretation is in terms of position dependent light-like momentum vector and polarization vector defining the local decompositions $M^2(x) \times E^2(x)$ of tangent space integrating to a foliation by partonic 2-surfaces and string world sheets. I christened this structure Hamilton-Jacobi structure. Its Euclidian counterpart is complex structure in Euclidian regions of space-time surface.

2. The formulation of quantum TGD in terms of spinor fields in WCW [K128] leads to the conclusion that WCW must have Kähler geometry [K60, K35] and has it only if it has maximal group of isometries identified as symplectic transformations of $\delta M^4_\pm \times CP_2$, where $\delta M^4_\pm$ denotes light cone boundary two which upper/lower boundary of causal diamond (CD) belongs. Symplectic Lie algebra extends naturally to supersymplectic algebra (SSA).

3. Space-time surfaces would be preferred extremals of twistor lift of Kähler action [L41] and the conditions realizing strong form of holography (SH) would state that sub-algebra of SSA isomorphic with it and its commutator with SSA give rise to vanishing Noether charges and these charges annihilate physical states or create zero norm states from them. One should solve these conditions.

4. The dynamics involves also fermions. Induced spinor fields are located inside space-time surface but for some yet not completely understood reason only the information about spinor at 2-D string world sheets is needed in the construction of scattering amplitudes. This dynamics would be 2-dimensional. The construction of twistor amplitudes even suggests that

it is 1-dimensional in the sense that 1-D light-like curves at light-like partonic orbits defining boundaries of Minkowskian and Euclidian regions determines the scattering amplitudes. String world sheets are however needed only as correlates for entanglement between fermions at different partonic orbits.

The 2-D character of fermionic dynamics conforms with the strong form of holography (SH) but how the string world sheets and partonic 2-surfaces are selected from Hamilton-Jacobi slicing? Electromagnetic neutrality could select string worlds sheets but one can actually always find a gauge in which the induced classical electroweak field at these surfaces is purely electromagnetic.

Twistor lift of TGD

Second approach to preferred extremals is based on TGD version [K116, K49, K13, L41] of twistor Grassmann approach [B18, B51, B23].

1. The twistor lift of TGD leads to a proposal that space-time surfaces can be represented as sections in their 6-D twistor spaces identified as twistor bundles in the product $T(H) = T(M^4) \times T(CP_2)$ of 6-D twistor spaces of M^4 and CP_2. Twistor structure would be induced from $T(H)$. Kähler action can be lifted to the level of twistor spaces only for $M^4 \times CP_2$ since only for these spaces twistor space allows Kähler structure [A50]. Twistors were originally introduced by Penrose with the motivation that one could apply algebraic geometry in Minkowskian signature. The bundle property is extremely powerful and should be consistent with the algebraic geometrization at the level of M^8_c. The challenge is to formulate the twistor lift at the level of M^8.

2. The twistor lift of Kähler action contains also volume term. Field equations have two kinds of solutions. For the solutions of first kind the dynamics of volume term and Käction are coupled and the interpretation is in terms of interaction regions. Solutions of second kind are minimal surfaces and extremals of both Kähler action and volume term, whose dynamics decouple completely and all coupling constants disappear from the dynamics. These extremals are natural candidates for external particles. For these solutions at least the field equations reduce to the existence of Hamilton-Jacobi structure. The completely universal dynamics of these regions suggests interpretation in terms of maximal quantum criticality characterized by the extension of the usual conformal invariance to its quaternionic analog.

3. A connection with zero energy ontology (ZEO) emerges. Causal diamond (CD, intersection of future and past directed light-cones of M^4 with points replaced by CP_2) would naturally determine the interaction region to which external particles enter through its 2 future and past boundaries. But where does ZEO emerge?

$M^8 - H$ duality

The third approach is based on number theoretic vision [K108, K109, K107, K126].

1. $M^8 - H$ duality [K109, K126, K10] means that one can see space-times as 4-surfaces in either M^8 or $H = M^4 \times CP_2$. One could speak "number theoretical compactification" having however nothing to do with stringy version of compactification, which is dynamical. $M^8 - H$ duality suggests that space-time surfaces in $H = M^4 \times CP_2$ are images of space-time surfaces in M^8 or actually of M^8 projections of complexified space-time surfaces in M^8_c identified as space of complexified octonions. These space-time surfaces could contain the integrated distributions of string world sheets and partonic 2-surfaces mentioned in the previous item. Space-time surfaces must have associative tangent or normal space for $M^8 - H$ correspondence to exist.

2. The fascinating possibility mentioned already earlier is that in M^8 these surfaces could correspond to zero loci for real or imaginary parts of real analytic octonionic polynomials $P(o) = RE(P) + IM(P)I_4$, I_4 an octonionic imaginary unit orthogonal to quaternionic

ones. The condition $IM(P) = 0$ ($RE(P) = 0$) would give associative (co-associative) space-time surface. In the simplest case these functions would be polynomials so that one would have algebraic geometry for algebraically 4-D complex surfaces in 8-D complex space.

Remark: The naive guess that space-time surfaces reduce to quaternionic curves in quaternionic plane fails due to the non-commutativity of quaternions meaning that one has $P(o) = P(q_1, q_2, \overline{q}_1, \overline{q}_2)$ rather than $P(o) = P(q_1, q_2)$.

Remark: Why not rational functions expressible as ratios $R = P_1/P_2$ of octonionic polynomials? It has become clear that one can develop physical arguments in favor of this option. The zero loci for $IM(P_i)$ would represent space-time varieties. Zero loci for $RE(P_1/P_2) = 0$ and $RE(P_1/P_2) = \infty$ would represent their interaction presumably realized as wormhole contacts connecting these varieties. In the sequel most considerations are for polynomials: the replacement of polynomials with rational functions does not introduce big differences and its discussed in the section "Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view" of [L34].

3. The objection against this proposal is obvious. $M^8 - H$ correspondence cannot hold true since the dynamics defined by octonionic polynomials in M^8 contains no coupling constants whereas the dynamics of twistor lift of Kähler action depends on coupling constants in the generic space-time region. However, for space-time surfaces representing external particles entering inside CD at its boundaries this is however not the case! They could satisfy $M^8 - H$ correspondence!

   This suggests that inside CDs the space-time surfaces are not associative/co-associative in M^8 so that $M^8 - H$ correspondence cannot map them to H and the twistor lifted Kähler action and SH take care of the dynamics. External particles are associative and quantum critical and $M^8 - H$ correspondence makes sense. The quantum criticality and associativity at the boundaries of CD poses extremely powerful conditions and allows to satisfy infinite number of vanishing conditions for SSA charges.

   It has later turned out [L51] that it might be possible to take the associativity conditions to extreme in the sense that they would hold everywhere apart from a set of discrete points and space-time surface would be minimal surfaces at all points except this finite set of points. There would be transfer of conserved quantities assignable to the volume term and the 4-D Kähler action (coming as dimensionally reduced 6-D Kähler action for the twistor lift of TGD) only at these points and elementary fermions would be naturally assignable to these points.

4. This picture is consistent with the the explicit formulation of the associativity conditions $Re(P) = 0$ and $IM(P) = 0$ for varieties. The calculations shows that associativity can be realized either by posing a condition making them 3-dimensional except, when the situation is critical in the sense that the 4-D variety is analogous to a double root of polynomial: now however the polynomial corresponds to prime polynomial decomposing to product of polynomials in the extension of rationals such that the product contains higher powers of the factors. One has ramification at the level of polynomial primes so that the criticality condition does not bring anything new but need not make the situation associative. At most 3 conditions need to be applied to guarantee associativity and they might leave the space-time surface 4-D.

5. The coordinates of M^4 as octonionic roots $x + iy$ of the 4 real polynomials need not be real. Space-time surface must have M_c^4 projection, which reduces to M^4. There are two options.

   (a) The original proposal was that the *projection* from M_c^8 to real M^4 (for which M^1 coordinate is real and E^3 coordinates are imaginary with respect to i!) defines the real space-time surface mappable by $M^8 - H$ duality to CP_2. One can howeerver critize the allowance of a nonvanishing imaginary part of space-time surface in M_c^4.

   (b) A more stringent condition is that the roots of the 4 vanishing polynomials as coordinates of M_c^4 belong automatically to M^4 so that m^0 would be real root and m^k, $k = 1, ..., 3$ imaginary with respect to $i \to -i$. M_c^8 coordinates would be invariant ("real") under combined conjugation $i \to -i, I_k \to -I_k$. In the following I will speak about this property as *Minkowskian reality*.

This could allow to identify CDs in very elegant way: outside CD these 4 conditions would not hold true. This option looks more attractive than the first one. Why these conditions can be true just inside CD, should be understood.

6. This octonionic view as also lower-dimensional quaternionic counterpart. In this case one considers 2-D commutative/co-commutative surfaces tentatively identifiable as string world sheets and partonic 2-surfaces. Why not all 2-surfaces appearing in the Hamilton-Jacobi slicing are not selected? The above mechanism would work also now. The commutativity conditions reduce in the generic case give 1-D curve as a solution. The interpretation would be as orbit of point like particle at 3-D partonic orbit appearing in the construction of twistorial amplitudes. In critical situation one would obtains string world sheet serving as a correlate for entanglement between point like particles at its ends: one would have quantum critical bound state.

I have considered also other attempts to define what quaternion structure could mean.

1. One could also consider the possibility that the tangent spaces of space-time surfaces in H are associative or co-associative [K126]. This is not necessary although it seems that this might be the case for the known extremals. If this holds true, one can construct further preferred extremals by functional composition by generalization of $M^8 - H$ correspondence to $H - H$ correspondence.

2. I have considered also the possibility of quaternion analyticity in the sense of generalization of Cauchy-Riemann equations, which tell that left- or right quaternionic differentiation makes sense [L25]. It however seems that this approach is not promising. The conditions are quite too restrictive and bring nothing essentially new. Octonion/quaternion analyticity in the above mentioned sense does not require the analogs of Cauchy-Riemann conditions.

5.1.2 Could one identify space-time surfaces as zero loci for octonionic polynomials with real coefficients?

The identification of space-time surfaces as zero loci of real or imaginary part of octonionic polynomial has several extremely nice features.

1. Octonionic polynomial is an algebraic continuation of a real valued polynomial on real line so that the situation is effectively 1-dimensional! Once the degree of polynomial is known, the value of polynomial at finite number of points are needed to determine it and cognitive representation could give this information! This would strengthen the view strong form of holography (SH) - this conforms with the fact that states in conformal field theory are determined by 1-D data.

2. One can add, sum, multiply, and functionally compose these polynomials provided they correspond to the same quaternionic moduli labelled by CP_2 points and share same time-line containing the origin of quaternionic and octonionic coordinates and real octonions (or actually their complexification by commuting imaginary unit). Classical space-time surfaces - classical worlds - would form an associative and commutative algebra. This algebra induces an analog of group algebra since these operations can be lifted to the level of functions defined in this algebra. These functions form a basic building brick of WCW spinor fields defining quantum states.

3. One can interpret the products of polynomials as correlates for free many-particle states with interactions described by added interaction polynomial, which can vanish at boundaries of CDs. This leads to the same picture as the view about preferred extremals reducing to minimal surfaces near boundaries of CD [L12]. Also zero zero energy ontology (ZEO) could be forced by the failure of number field property for quaternions at light-cone boundaries. It indeed turns out that light-cone boundary emerges quite generally as singular zero locus of polynomials $P(o)$ containing no linear part: this is essentially due to the non-commutativity

of the octonionic units. Also the emergence of CDs can be understood. At this surface the region with $RE(P) = 0$ can transform to $IM(P) = 0$ region. In Euclidian signature this singularity corresponds to single point. A natural conjecture is that also the light-like orbits of partonic 2-surfaces correspond to this kind of singularities for non-trivial Hamilton-Jacobi structures.

4. The reduction to algebraic geometry would mean enormous boost to the vision about cognition with cognitive representations identified as generalized rational points common to reals rationals and various p-adic number fields defining the adele for given extension of rationals. Hamilton-Jacobi structure would result automatically from the decomposition of quaternions to real and imaginary parts which would be now complex numbers.

5. Also a connection with infinite primes is suggestive [K109]. The light-like partonic orbits, partonic 2-surfaces at their ends, and points at the corners of string world sheets might be interpreted in terms of singularities of varying rank and the analog of catastrophe theory emerges.

The great challenge is to prove rigorously that these approaches - or at least some of them - are indeed equivalent. Also it remains to be proven that the zero loci of real/imaginary parts of octonionic polynomials with real coefficients are associative or co-associative. I shall restrict the considerations of this article mostly to $M^8 - H$ duality. The strategy is simple: try to remember all previous objections against $M^8 - H$ duality and invent new ones since this is the best way to make real progress.

5.1.3 Topics to be discussed

Key notions and ideas of algebraic geometry

Before going of octonionic algebraic geometry, I will discuss basic notions of algebraic geometry such as algebraic variety (see `http://tinyurl.com/hl6sjmz`), - surface (see `http://tinyurl.com/y8d5wsmj`), and - curve (see `http://tinyurl.com/nt6tkey`), rational point of variety central for TGD view about cognitive representation, elliptic curves (see `http://tinyurl.com/lovksny`) and - surfaces (see `http://tinyurl.com/yc33a6dg`), and rational points (see `http://tinyurl.com/ybbnnysu`) and potentially rational varieties (see `http://tinyurl.com/yablk4xt`). Also the notion of Zariski topology (see `http://tinyurl.com/h5pv4vk`) and Kodaira dimension (see `http://tinyurl.com/yadoj2ut`) are discussed briefly. I am not a mathematician. What hopefully saves me from horrible blunders is physical intuition developed during 4 decades of TGD.

Much of algebraic geometry is counting numbers of say rational points or of varieties satisfying some conditions. One can also count dimensions of moduli spaces. Hence the basic notions and methods of enumerative geometry are discussed. There is also a discussion of Gromow-Witten invariants and Riemann-Roch theorem having Atyiah-Singer index theorem as a generalization. These notions will be applied in the second part of the article [L34].

$M^8 - H$ duality

$M^8 - H$ duality [K10, K109, K126] would reduce classical TGD to the algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces. Space-time surfaces in M^8 would be algebraic varieties identified as zero loci for imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial of complexified octonionic variable o decomposing as $o = q_c^1 + q_c^2 I_4$ and projected to a Minkowskian sub-space M^8 of o. Single real valued polynomial of real variable with algebraic coefficients would determine space-time surface! As proposed already earlier, spacetime surfaces in M^8 would form commutative and associative algebra with addition, product and functional composition.

As already noticed, the associativity conditions do not allow 4-D solutions except for criticality so that $M^8 - H$ correspondence can hold true only in these space-time regions and one has these nice features at the level of M^8. In critical regions $M^8 - H$ correspondence is true and these features have H counterparts

The basic problem is to understand the map mediating $M^8 - H$ duality mapping the point (m, e) of $M^8 = M_0^4 \times E^4$ to a point (m, s) of $M_0^4 \times CP_2$, where M_0^4 point is obtained as a projection to a suitably chosen $M_0^4 \subset M^8$ and CP_2 point parameterizes the tangent space as quaternionic sub-space containing preferred $M_0^2(x) \subset M^4(x)$. This map involves slightly non-local information and could allow to understand why the preferred extremals at the level of H are determined by partial differential equations rather than algebraic equations. Also the generalization to the level of twistor lift is briefly touched.

Challenges of the octonionic algebraic geometry

The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients determine associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

   This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy Riemann conditions are linear. Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

   In fact, all algebras obtained by Cayley-Dickson construction (see `http://tinyurl.com/ybuyla2k`) by adding imaginary unit repeatedly to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and a $M^8 - H$ correspondence could generalize (maybe even TGD!).

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k, $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i. In the generic case this gives 3-D surface.

   In this generic case $M^8 - H$ duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H, and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

   One can have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i. Space-time surface would be analogous to a polynomial with a multiple root. The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory [A37] emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

   One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces except at 3-D boundary surfaces. These regions correspond to interaction regions in which

Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes also to the level of complex/co-complex surfaces associated with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces? I have proposed commutativity or co-commutatitivity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commutative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

The easiest way to kill $M^8 - H$ duality in the form it is represented here is to prove that 4-D zero loci for imaginary/real parts of octonionic polynomials with real coefficients can never be associative/co-associative being always 3-D. I hope that some professional mathematician would bother to check this.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); Strong Form of GCI (SGCI); Quantum Criticality (QC); Strong Form of Holography (SH); World of Classical Worlds (WCW); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Causal Diamond (CD); Number Theoretical Universality (NTU) are the most often occurring acronyms.

5.2 Some basic notions, ideas, results, and conjectures of algebraic geometry

In this section I will summarize very briefly the basic notions of algebraic geometry needed in the sequel.

5.2.1 Algebraic varieties, curves and surfaces

The basic notion of algebraic geometry is algebraic variety.

1. One considers affine space A^n with n coordinates $x^1, ..., x^n$ having values in a number field K usually assumed to be algebraically closed (note that affine space has no preferred origin like linear space). Algebraic variety is defined as a solution of one or more algebraic equations stating the vanishing of polynomials of n variables: $P^i(x^1, ..., x^n) = 0$, $i = 1, ..., r \leq n$. One can restrict the coefficients of polynomials to p-adic number field or or its extension to an extension of rationals. One talks about polynomials on $k \subset K$.

2. The basic condition is that the variety is not a union of disjoint varieties. This for instance happens, when the polynomial $P(x^1, .., x^n)$ defining co-dimension 1 manifold is product of polynomials $P = \prod_r P_r$. Algebraic variety need not be a manifold meaning that it can have singular points. For instance, for co-dimension 1 variety the Jacobian matrix $\partial P/\partial x^i$ of the polynomial can vanish at singularity.

3. One can define projective varieties (see `http://tinyurl.com/ybsqvy3r`) in projective space P^n having coordinatization in terms of $n+1$ homogenous coordinates $(x^1, ..., x^{n+1})$ in K with points differing by an overall scaling identified. Projective variety is defined as zero locus of homogenous polynomials of $n + 1$ coordinates so that solutions remain solutions under the overall scaling of all coordinates. By identifying the points related by scaling one obtains a surface in P^n. Grassmannian of linear space V^n (not affine space!) is a projective spaces defined as space of k-planes of V^n. These spaces are encountered in twistor Grassmannian approach to scattering amplitudes.

For polynomials of single variable one obtains just the roots of $P_n(x) = 0$ in an algebraic extension assignable to the polynomial. For several variables one can in principle proceed step by step by solving variable x^1 as algebraic function of others from $P_1(x^1,...,x^n) = 0$, proceed to solve x^2 from $P_2(x^1(x^2,...),x^2,...) = 0$ as as algebraic function of the remaining variables, and so one. The algebraic functions involved get increasingly complex but in some exceptional situations the solution has parametric representation in terms of *rational* rather than algebraic functions of parameters t^k. For co-dimension $d_c > 1$ case the intersection of surfaces $P^i = 0$ need not be complete and the tangent spaces of the hyper-surfaces $P^i = 0$ need not intersect transversally in the generic case. Therefore $d_c > 1$ case is not gained so much attention as $d_c = 1$ case.

A more advanced treatment relies on ring theory by assigning to polynomials a ring as the ring of polynomials in the space involved divided by the ring of polynomials vanishing at zero loci of polynomials P^i.

1. The notion of ideal is central and determined as a subring invariant under the multiplication by elements of ring. Prime ideal generalizes the notion of prime and one can say that the notion of integer generalizes to that of ideal. One can also define the notion of fractional ideal.

2. Zariski topology (see `http://tinyurl.com/h5pv4vk`) replacing the topology based on real norm is second highly advanced notion. The closed sets in this topology are algebraic varieties of various dimensions. Since the complement of any algebraic variety is open set this topology and open also in the ordinary real topology, this topology is considerable rougher than the ordinary than the ordinary topology.

Some remarks from the point of view of TGD are in order.

1. In the scenario inspired by $M^8 - H$ duality one has co-dimension 4 surfaces in 8-D complex space. Octonionicity of polynomials however implies huge symmetries since the polynomial is determined by single real polynomial of real variable, whose values at finite number of points determined the polynomial.

2. In TGD the extension of rationals can be assumed to contain also powers for some root of e since in p-adic context this gives rise to a finite-dimensional extensions due to the fact that e^p is ordinary p-adic number. Also a restriction to a finite field are possible and restriction of rational coefficients to their modulo p counterparts reduces the polynomial to polynomial in finite field. This reduction is used as a technical tool. In the case of Diophantine equations (see `http://tinyurl.com/nt6tkey` and `http://tinyurl.com/y8hm4zce`) the coefficients are restricted to be integers.

3. In adelic TGD [L39] [L37] the number fields involved are reals and extensions of p-adic numbers. The coefficient field for the coefficients of polynomials would be naturally extension of rationals or extension of p-adics induced by it. The coefficients of polynomials serve as coordinates of adelic WCW. p-Adic numbers are not algebraically closed and one must assume an extension of p-adic numbers from that for the coefficients one to allow maximal number of roots.

   This suggests an evolutionary process [L42] extending the number field for the coefficients of polynomials. Arbitrary root of polynomial for given extension can be realized only if the original extension is extended further. But this allows polynomial coefficients in this new extension: WCW is now larger. Now one has however roots in even larger extension so that the unavoidable outcome is number theoretic evolution as increase of complexity.

4. What is so remarkable is that octonionic polynomials with rational coefficients could be determined by their values at finite set of points for a polynomial of real argument once the order of polynomial is fixed. Real coordinate corresponds to preferred time axis naturally. A cognitive representation consisting of finite number of rational points could fix the entire space-time surface! This would extend ordinary holography to its discrete variant!

5. Algebraic variety is rather simple object as compared to the solutions of partial differential equations encountered in physics - say those for minimal surfaces. Now one must fix boundary

values or initial values at $n-1$-dimensional surface to fix the solution. For integrable theories the situation can change. In TGD SH suggests that the classical solutions are determined by data at 2-surfaces, which together with conformal invariance could reduce the data to one-dimensional data specified by a polynomial. M^8-H correspondence allows to consider this option seriously.

6. M^8-H duality suggests that space-time surfaces are co-dimension $d_c=4$ algebraic curves in M^8. Could space-time surfaces define closed sets for the analog of Zariski topology? Could string world sheets and partonic 2-surfaces do the same inside space-time surfaces? An interesting question is whether this generalizes also to the level of embedding space H and could perhaps define a topology rougher than real topology in better accord with the notion of finite measurement resolution.

5.2.2 About algebraic curves and surfaces

The realization M^8-H correspondence to be considered allows to understand space-time surfaces as 4-D complex algebraic surfaces X^4_c in the space o of complexified octonions projected to real sub-space of O^c with Minkowskian signature. Due to the non-commutativity of quaternions, the reduction of space-time surfaces to curves in quaternionic plane is not possible. Despite this it is instructive to start from the algebraic geometry of curves and surfaces.

Degree and genus of the algebraic curve

Algebraic curve is defined as zero locus of a polynomial $P(x^1,x^2,...,x^n)$ with x^n in some - preferably algebraically closed - number field K and coefficients in some number field $k\subset K$. In adelic physics K corresponds to real or complex numbers and k to the extension of rationals defining adeles. In p-adic sectors k corresponds to tje extension of p-adic numbers induced by k. In general roots belong to an extension of k.

Degree, genus, and Euler characteristic are the basic characterizers of algebraic curve.

1. The degree d of algebraic curve corresponds to the highest power for the variables appearing in the polynomial. One can also define multi-degree in an obvious manner. A useful geometric interpretation for the degree is that line intersects curve (also complex) of degree d in at most d points as is clear from the fact that the equation of curve reduces the equation for curve to an equation for the roots of d:th order polynomial of single variable.

2. Also the genus g of the curve (see `http://tinyurl.com/ybm3wfue`) is important characteristic. One can distinguish between topological genus, geometric genus and arithmetic genus. For curves these notions are equivalent. The connection between genus and degree d of non-singular algebraic curve is very useful:

$$g = \frac{(d-1)(d-2)}{2} \ . \tag{5.2.1}$$

Spherical topology for complex curves corresponds to $n=1$ and $n=2$.

A more general formula reads as:

$$g = \frac{(d-1)(d-2)}{2}+\frac{n_s}{2} \ . \tag{5.2.2}$$

Here n_s is the number of holes of the curve behaving like holes and increasing the genus.

3. Euler characteristic (for Euler characteristic see `http://tinyurl.com/pp52zd4`) is a homological invariant making sense in arbitrary dimension and also for manifolds. Homological definition based on simplicial homology relies on counting of simplexes of various dimension. The definition in terms of dimensions of homology groups H_n is given by

$$\chi = b_0 - b_1 + b_2 ... + (-1)^n b_n \ , \tag{5.2.3}$$

where b_k is the dimension of k:th homology group (see `http://tinyurl.com/j48ojys`).

The following gives the engineering rules for obtaining Euler characteristic of the surface obtained from simpler building blocks. Note that algebraic variety property is not essential here.

1. Euler characteristic is homotopy invariant so that it does not change one adds homologically trivial space such as E^n as a Cartesian factor.

2. χ is additive under disjoint union. Inclusion-exclusion principle states that if M and N intersect, one has $\chi(M \cup N) = \chi(M) + \chi(N) - \chi(M \cap N)$.

3. Euler characteristic for the connected sum $A\#B$ of n-dimensional manifolds obtained by drilling balls B^n from summands, giving opposite orientation to the boundaries of the hole, and connecting with cylinder $D \times S^{n-1}$ is given by $\chi(A) + \chi(B) - \chi(S^{n-1})$. One has $\chi(S^2) = 2$ and $\chi(D^2) = 1$.

4. The Euler characteristic for product $M \times N$ is $\chi(M) \times \chi(N)$.

5. The Euler characteristic for N-fold covering space M_n is $N \times \chi(M)$ with a correction term coming from the singularities of the covering (ramified covering space).

6. For a fibration $M \to B$ with fiber S, which differs from fiber bundle in that the fibers are only homeomorphic, one has $\chi(M) = \chi(B) \times \chi(S)$.

Euler characteristic and the genus of 2-surface (or complex) curve are related by the equation

$$\chi = 2(1-g) \ . \tag{5.2.4}$$

having values $2, 0, -2, ...$. If the 2-surface has n_s holes (punctures), one has

$$\chi = 2(1-g) - n_s \ . \tag{5.2.5}$$

Punctures must be distinguished from singularities at which some sheets of covering meet at single point.

A formal generalization of the definition of genus for varieties in terms of Euler characteristic makes sense.

$$g = -\frac{\chi}{2} + 1 - \frac{n_s}{2} \ . \tag{5.2.6}$$

Disk has genus 1/2 and drilling of n holes increases genus by $n/2$. Pair of holes gives same contribution to g and the cylinder connecting the holes. Note that for complex curves the definition of puncture is obvious. For real curves the puncture would mean missing point of the curve.

The latter definitions of genus can be identified in terms of Euler characteristic also for higher-dimensional varieties. For curves these notions are equivalent if there are no singularities. For algebraic curves g is same for the real and complex variants of the curve in RP_1 and CP_1 respectively.

Elliptic curves and elliptic surfaces

Elliptic curves (see http://tinyurl.com/lovksny) are cubic curves with no singularities (cusps or self-intersections) having representation of form $y^2 - x^3 - ax - b = 0$. These singularities can occur only at special values of parameters $((a = 0, b = 0)$. Since the degree equals to $d = 3$, elliptic curve has genus $g = 1$.

Elliptic curves allow a group of Abelian symmetries generated by a finite number of generators. The emergence of abelian group structure can be intuitively understood as follows.

1. Given line intersects the curve of degree 3 in at most 3 points. Let P and Q be two of these points. Then there can be also a third intersection point R and by the Z^2 symmetry changing the sign of y also the reflection of R - identify it as $-R$ - belongs to the curve. Define the sum of $P + Q$ to be $-R$.

   The actual proof is slightly more complicated since the number of intersection points for the line with curve can be also 2 or 1. By writing explicit expressions for the coordinates x_R and y_R, one can also find that they are indeed rational if the points P and Q are rational. If the elliptic curve as single rational point it has infinite number of them.

2. The generators with finite order give rise to torsion. The rank of generators of infinite order is called rank and conjectured to be arbitrarily large (see http://tinyurl.com/lovksny) . Therefore elliptic curve is an Abelian group and one talks about Abelian variety. If elliptic curve contains a rational point it contains entire lattice of rational points obtained as shifts of this point.

Remark: Complex elliptic curves are 2-surfaces in complex projective plane CP_2 and therefore highly interesting from TGD point of view. $g = 1$ partonic 2-surfaces would in TGD framework correspond to second generation fermions [K32]. Abelian varieties define a generalization of elliptic curves to higher dimensions and simplest space-time surfaces allowing also large cognitive representations could correspond to such.

Elliptic surfaces (see http://tinyurl.com/yc33a6dg) are fibrations with an algebraic curve as base space and elliptic curve as fiber (fibration is more general notion than fiber space since the fibers are only homeomorphic). The singular fibers failing to be elliptic curves have been classified by Kodaira.

5.2.3 The notion of rational point and its generalization

The notion of algebraic integer (see http://tinyurl.com/y8z389a7) makes sense for any number field as a root of a monic polynomial (polynomial with integer coefficients with coefficient of highest power equal to unity). The field of fractions for given number field consists of ratios of algebraic integers. The same is true for the notion of prime. The more precise definition forces to replace integers and primes with ideals.

Rational varieties are expressible as maps defined by rational functions with rational coefficients in some extension of Q and contain infinite number of rational points. If the variety is not rational, one can ask whether it could allow a dense set of rational points with rational number replaced with the ratio of algebraic integers for some extension of Q. This leads to the idea of potentially rational point, and one can classify algebraic varieties according to whether they are potentially rational or not. The variety is potentially rational if it allows a parameteric representation using rational functions. Otherwise the parametric representation involves algebraic functions such as roots of rational functions.

The interpretation in terms of cognition would be that large enough extension makes the situation “cognitively easy” since cognitive representations involving fermions at the rational points and defining discretizations of the algebraic variety could be arbitrary large. The simpler the surface is cognitively, the large the number of rational points or potentially rational points is.

Complexity of algebraic varieties is measured by Kodaira dimension d_K (see http://tinyurl.com/yadoj2ut). The spectrum for this dimension varies in the range $(-\infty, 0, 1, 2, ...d)$, where d is the algebraic dimension of the variety. Maximal value equals to the ordinary topological dimension d and corresponds to maximal complexity: in this case the set of rational points is finite. Minimal Kodaira dimension is $d_K = -\infty$: in this case the set of rational points is infinite. Rational surfaces are maximally simple and this corresponds to the existence of parametric representations using only rational functions.

Rational points for algebraic curves

The sets of rational points for algebraic curves are rather well understood. Mordelli conjecture proved by Falting as a theorem (see `http://tinyurl.com/y9oq37ce`) states that a curve over Q with genus $g = (d-1)(d-2)/2 > 1$ (degree $d > 3$) has only finitely many rational points.

1. Sphere CP_1 in CP_2 has rational points as a dense set. Quite generally rational surfaces, which by definition allow parametric representation using polynomials with rational coefficients (encountered in context of Du Val singularities characterized by the extended Dynkin diagrams for finite subgroups of $SU(2)$) allow dense set of rational points [A57, A68]).

   $g = 0$ does not yet guarantee that there is dense set of rational points. It is possible to have complex conics (quadratic surface) in CP_2 with no rational points. Note however that this depends on the choice of the coordinates: if origin belongs to the surface, there is at least one rational point

2. Elliptic curve $y^2 - x^3 - ax - b$ in CP_2 (see `http://tinyurl.com/lovksny`) has genus $g = 1$ and has a union of lattices of rational points and of finite cyclic groups of them since it has origin as a rational point. This lattice of points are generated by translations. Note that elliptic curve has no singularities that is self intersections or cusps (for $a = 0, b = 0$ origin is a singularity).

   $g = 1$ does not guarantee that there is infinite number of rational points. Fermat's last theorem and CP_2 as example. $x^d + y^d = z^d$ is projectively invariant statement and therefore defines a curve with genus $g = (d-1)(d-2)/2$ in CP_2 (one has $g = 0, 0, 2, 3, 6, 10, ...$). For $d > 2$, in particular $d = 3$, there are no rational points.

3. $g \geq 2$ curves do not allow a dense set of rational points nor even potentially dense set of rational points.

Remark: In TGD framework algebraic varieties could be zero loci of octonionic polynomials and have algebraic dimension 4 so that the classification for algebraic curves does not help. Octonion analyticity must bring in symmetries which simplify the situation.

Enriques-Kodaira classification

The tables of (see `http://tinyurl.com/ydelr4np`) give an overall view about the Enriques-Kodaira classification of algebraic curves, surfaces, and varieties in terms of Kodaira dimension (see `http://tinyurl.com/yadoj2ut`).

1. For instance, general curves ($g \geq 2$) have $d_K = 1$, elliptic curves ($g = 1$) have $d_K = 0$ and projective line ($g = 0$) has $d_K = -\infty$. $CP_1 \subset CP_2$ is a rational curve so that rational points are dense. Elliptic curves allow infinite number or rational points forming an Abelian group if they containing single rational point and are therefore cognitively easy.

2. Algebraic varieties (with real dimension $d_R = 4$ in complex case) with $d_K = 2$ are surfaces of general type, elliptic surfaces (see `http://tinyurl.com/yc33a6dg`) have $d_K = 1$, surfaces with attribute abelian, hyper-elliptic, K3, and Enriques, have $d_K = 0$.

   Remark: All real 2-surfaces are hyper-elliptic for $g \leq 2$, in other words allow Z_2 as global conformal symmetry. Genus-generation correspondence [K32] for fermions allows to assign to the 3 lowest generations of fermions hyper-elliptic partonic 2-surfaces with genus $g = 0, 1, 2$. These surfaces would have $d_K = 0$ and be rather simple as real surfaces in Kodaira classification. Could one regard them as M^4 projection of complex hyper-elliptic surfaces of real dimension $d_R = 4$? $d_K = -\infty$ holds true for rational surfaces and ruled surfaces, which allow straight line through any point are maximally simple. In complex case the line would be CP_1.

3. The Wikipedia article gives also a table about the classification of algebraic 3-folds. Real algebraic 3-surfaces might well occur in TGD framework. The twistor space for space-time surface might allow realization as complex 3-fold and since it has S^2 has fiber, it would naturally correspond to an uni-ruled surface with $d_K = -\infty$. The table shows that one can build higher dimensional algebraic varieties with $d_K < d$ from lower-dimensional ones as fiber-space like structures, which based or fiber having $d_K < d$. 3-D Abelian varieties and Calabi-Yau 3-folds are complex manifolds with $d_K = 0$, which cannot be engineered in this manner.

4. Space-time surfaces would be surfaces of algebraic dimension 4. Wikipedia tables do not give direct information about this case but one can make guesses on basis of the three tables. Octonionic polynomials are analytic continuations of real polynomials of real variable, which must mean a huge simplification, which also favor cognitive representability. The best that one might have infinite sets of rational points. The examples about extremals of Kähler action does not however favor this wish.

Bombieri-Lang conjecture (see `http://tinyurl.com/y887yn5b`) states that, for any variety X of general type over a number field k, the set of k-rational points of X fails to be Zariski dense (see `http://tinyurl.com/jm9fh74`) in X. This means that , the k-rational points are contained in a *finite* union of lower-dimensional sub-varieties of X. In dimension 1, this is exactly Faltings theorem, since a curve is of general type if and only if it has $g \geq 2$. The conjecture of Vojta (see `http://tinyurl.com/y9sttuu4`) states that varieties of general type cannot be potentially dense. As will be found, these conjectures might be highly relevant for TGD.

5.3 About enumerative algebraic geometry

Algebraic geometry is something very different from Riemann geometry, Kähler geometry, or sub-manifold geometry based on local notions. Sub-manifolds are replaced with sub-varieties defined as zero loci for polynomials with coefficients in the field of rationals or extension of rationals. Partial differential equations are replaced with algebraic ones. One can generalize algebraic geometry to any number field.

String theorists have worked with algebraic geometry with motivation coming from various moduli spaces emerging in string theory. The moduli spaces for closed and open strings possibly in presence of branes are involved. Also Calabi-Yau compacticication leads to algebraic geometry, and topological string theories of type A and B involve also moduli spaces and enumerative algebraic geometry.

In TGD the motivation for enumerative algebraic geometry comes from several sources.

1. Twistor lift of TGD lifts space-time surfaces to their 6-D twistor spaces representable as surfaces in the product of 6-D twistor spaces of M^4 and CP_2 and possessing Kähler structure - this makes these spaces completely unique and strongly suggests the role of algebraic geometry, in particular in the generalization of twistor Grassmannian approach [L34].

2. There are three threads in number theoretic vision: p-adic numbers and adelics, classical number fields, and infinite primes. Adelic physics [L39] as physics of sensory experience and cognition unifies real physics and various p-adic physics in the adele characterized by an extension of rationals inducing those of p-adic number fields. This leads to algebraic geometry and counting of points with embedding space coordinates in the extension of rationals and defining a discrete cognitive representation. The core of the scattering amplitude would be defined by this cognitive representation identifiable in terms of points appearing as arguments of n-point function in QFT picture [L32].

3. $M^8 - M^4 \times CP_2$ duality is the analog of the rather adhoc spontaneous compactification in string models but would be non-dynamical and thus allow to avoid landscape catastrophe. Classical physics would reduce to octonionic algebraic geometry at the level of complexified octonions with several special features due to non-commutativity and non-associativity: space-time could be seen as 4-surface in the complexification of of octonions. The commuting imaginary unit would make possible the realization of algebraic extensions of rationals.

The moduli space for the varieties is discrete if the coefficients of the polynomials are in the extension of rationals. If one poses additional conditions such as associativity of 4-surfaces, the moduli space is further reduced by the resulting criticality conditions realizing quantum criticality at the fundamental level raising hopes about extremely simple formulation of scattering amplitudes at the level of M^8 [L34].

Also complex and co-complex sub-manifolds of associative space-time surface are important and would realize strong form of holography (SH). For non-associative regions of space-time surface it might not be possible to define complex and co-complex surfaces in unique manner since the basic $M^2 \subset M^4$ local flag structure is missing. String world sheets and partonic 2-surfaces and their moduli spaces are indeed in key role and the topology of partonic surfaces plays a key role in understanding of family replication phenomenon in TGD [L32].

In this framework one cannot avoid enumerative algebraic geometry.

1. One might want to know the number of points of sub-variety belonging to the number field defining the coefficients of the polynomials. This problem is very relevant in M^8 formulation of TGD, where these points are carriers of sparticles. In TGD based vision about cognition [L39] they define cognitive representations as points of space-time surface, whose M^8 coordinates can be thought of as belonging to both real number field and to extensions of various p-adic number fields induced by the extension of rationals. If these cognitive representations define the vertices of analogs of twistor Grassmann diagrams in which sparticle lines meet, one would have number theoretically universal adelic formulation of scattering amplitudes and a deep connection between fundamental physics and cognition.

2. Second kind of problem involves a set algebraic surfaces represented as zero loci for polynomials - lines and circles in the simplest situations. One must find the number of algebraic surfaces intersecting or touching the surfaces in this set. Here the notion of incidence is central. Point can be incident on line or two lines (being their intersection), line on plane, etc.. This leads to the notion of Grassmannians and flag-manifolds.

Moduli spaces parameterizing sub-varieties of given kind - lines, circles, algebraic curves of given degree, are central for the more advanced formulation of algebraic geometry. These moduli spaces emerge also in the formulation of TGD. The moduli space of conformal equivalence classes of partonic 2-surfaces is one example involved with the explanation of family replication phenomenon [K32]. One can assign moduli spaces also to octonion and quaternion structures in M^8 (or equivalently with the complexification of E^8). One can identify CP_2 as a moduli space of quaternionic sub-spaces of octonions containing preferred complex sub-space.

One cannot avoid these moduli spaces in the formulation of the scattering amplitudes and this leads to $M^8 - H$ duality. The hard core of the calculation should however reduce to the understanding of the algebraic geometry of 4-surfaces in octonionic space. Clearly, M^8 picture seems to provide the simplest formulation of the number theoretic vision.

5.3.1 Some examples about enumerative algebraic geometry

Some examples give an idea about what enumerative algebraic geometry (see `http://tinyurl.com/y7yzt67b`) is.

1. Consider 4 lines in 3-D space. What is the number of lines intersecting these 4 lines [A85] (see `http://tinyurl.com/ycrbr5aj`). One could deduce the number of lines and lines by writing the explicit equations for the lines with each line characterized by 2+3=5 parameters specifying direction t vector and arbitarily chosen point x_0 on the line. 2+3=5 parameters characterize each sought-for line.

   For intersection points x_i of sought for line with i:th one has $x_i = x_0 + k_i t_0$, $i = 1, ..., 4$ for the sought for line with direction t_0. At the intersection points at the 4 lines one has $x_i = x_{0i} + K_i t_i$ with fixed directions t_i. Combining the two equations for each line one has $4 \times 3 = 12$ equations and 3+4+2 parameters for the sought for line plus 4 parameters K_i for the four lines. This gives 13 unknown parameters corresponding to x_0, k_i, K_i. One would have one parameter set of solutions: something goes wrong.

One has however projective invariance: one can shift x_0 along the line by $x_0 \to x_0 - at$, $k_i \to k_i + a$ and using this freedom assume for instance $k_1 = 0$. This reduces the number of parameters to 12 and one has finite number of solutions in the generic case. Actually the number is 2 in the generic case but can be infinite in some special cases. The challenge is to deduce the number of the solutions by geometric arguments.Below Schubert's argument proving that the number of solutions is 2 will be discussed.

The idea of enumerative geometry is to do this using general geometric arguments allowing to deform the problem topologically to a simpler one in which case the number of solutions is obvious which in the most abstract formulation become topological.

2. Apollonius can be seen as founder of enumerative algebraic geometry. Apollonian circles (see `http://tinyurl.com/ycvxe688`) represent second example. One has 3 circles in plane. What is the number of circles tangential to all these 3 circles. Wikipedia link represents the geometric solution of the problem. The number of circles is 8 in the generic case but there are exceptional cases.

3. In Steiner's conic problem (see `http://tinyurl.com/yahshsjo`) one have 5 conical sections (circles, cones, ellipsoids, hyperbole) in plane. How many different conics tangential to the conics there exist? This problem is rather difficult and the thumb rules of enumerative geometry (dimension counting, Bezout's rule, Schubert calculus) fail. This is a problem in projective geometry where one is forced to introduce moduli space for conics tangential to given conic. This space is algebraic sub-variety of all conics in plane which is 5-D projective space. One must be able to deduce the number of points in the intersection of these sub-varieties so that the original problem in 2-D plane is replaced with a problem in moduli space.

5.3.2 About methods of algebraic enumerative geometry

A brief summary about methods of algebraic geometry is in order to give some idea about what is involved (see `http://tinyurl.com/y7yzt67b`).

1. Dimension counting is the simplest method. If two geometric objects of n-D space have dimensions k and l, there intersection is $n-k-l$-dimensional for $n-k-l \geq 0$ or empty in the generic case. For $k+l=n$ one obtains discrete set of intersection points.

2. Bezout's theorem is a more advanced method. Consider for instance, curves in plane defined by the curves polynomials $x = P^m(y)$ and $x = P^n(y)$ of degrees $k = m$ and $k = n$. The number N of intersection points in the generic case is bounded above by $N = m \times n$ (in this case all roots are real). One can understand this by noticing that one has m roots y_k or given x giving rise to a m-branched graph of function $y = f(x)$. The number of intersections for the graphs of the two polynomials is at most $m \times n$. If one has curve in plane represented by polynomial equation $P^{m,n}(x,y) = 0$, one can also estimate immediately the minimal multi-degree (m,n) for this polynomials.

3. Schubert calculus `http://tinyurl.com/y766ddw2`) is a more advanced but not completely rigorous method of enumerative geometry [A85] (see `http://tinyurl.com/ycrbr5aj`).

   Schubert's vision was that the number of intersection points is stable against deformations in the generic case. This is not quite true always but in exceptional cases one can say that two separate solutions degenerate to single one, just like roots of polynomial can do for suitable values of coefficients.

   For instance, Schubert's solution to the already mentioned problem of finding a line intersecting 4 lines in generic position relies on this assumption. The idea is to deform the situation so that one has two intersecting pairs of lines. One solution to the problem is a line going through the intersection points for line pairs. Second solution is obtained as intersection of the planes. It can happen that planes are parallel in which case this does not work.

Schubert calculus it applies to linear sub-varieties but can be generalized also to non-linear varieties. The notion of incidence allowing a general formulation for intersection and tangentiality (touching) is central. This leads to the notions of flag, flag manifold, and Schubert variety as sub-variety of Grassmannian.

Flag is a hierarchy of incident subspaces $A_0 \subset A_1 \subset A_2 ... \subset A_n$ with the property that the dimension $d_i \leq n$ of A_i satisfies $d_i \geq i$. As a special case this notion leads to the notion of Grassmannian $G(k,n)$ consisting of k-planes in n-dimensional space: in this case A_0 corresponds to k-planes and A_2 to space A_n. More general flag manifolds are moduli spaces and sub-varieties of Grassmannian providing a solution to some conditions. Flag varieties as sub-varieties of Grassmannians are Schubert varieties (see `http://tinyurl.com/y7ehcrzg`). They are also examples of singular varieties. More general Grassmannians are obtained as coset spaces of G/P, where G is algebraic group and P is parabolic sub-group of G.

Remark: CP_2 corresponds to the space of complex lines in C^3. CP_2 can be also understood as the space of quaternionic planes in octonionic 8-space containing fixed 2-plane so that also now one has flag. String world sheets inside space-time surfaces define curved flags with 2-D and 4-D tangent spaces defining an integrable distribution of local flags.

4. Cohomology combined with Poincare duality allows a rigorous formulation of Schubert calculus. Schubert's idea about possibility to deform the generic position corresponds to homotopy invariance, when the degeneracies of the solutions are taken into account. Homology and cohomology become basic tools and the so called cup product for cohomology together with Poincare duality and Künneth formula for the cohomology of Cartesian product in terms of cohomologies of factors allows to deduce intersection numbers algebraically. Schubert cells define a basis for the homology of Grassmannian containing only even-dimensional generators.

   Grassmannians play a key role in twistor Grassmannian approach as auxiliary manifolds. In particular, the singularities of the integrand of the scattering amplitude defined as a multiple residue integral over $G(k,n)$ define a hierarchy of Schubert cells. The so called positive Grassmannian [B21] defines a subset of singularities appearing in the scattering amplitudes of $\mathcal{N} = 4$ SUSY. This hierarchy and its CP_2 counterpart are expected also in TGD framework.

   Remark: Schubert's vision might be relevant for the notion of conscious intelligence. Could problem solving involve the transformation of a problem to a simple critical problem, which is easy but for which some solutions can become degenerate? The transformation of general position for 4 lines to a pair of intersecting lines would be example of this. One can wonder whether quantum criticality could help problem solving by finding critical cases.

5. Moduli spaces of curves and varieties provide the most refined methods. Flag manifolds define basic examples of moduli spaces. Quantum cohomology represents even more refined conceptualization: the varieties (branes in M-theory terminology) are said to be connected or intersect if each of them has a common point with the same pseudo-holomorphic variety ("string world sheet"). Pseudo-holomorphy - which could have minimal surface property as counterpart - implies that the connecting 2-surface is not arbitrary.

   Quantum intersection for the "string world sheet" and "brane" is possible also when it is not stable classically (the co-dimension of brane is smaller than 2). Even in the case that it possible classically quantum intersection makes possible kind of "telepathic" quantum contact mediated by the "string world sheet" naturally involved with the description of quantum entanglement in TGD framework.

5.3.3 Gromow-Witten invariants

Gromow-Witten invariants repreent example of so called quantum invariants natural in string models and M-theory. They provide new invariants in algebraic and symplectic geometry.

Formal definition

Consider first the definition of Gromow-Witten (G-W) invariants (see `http://tinyurl.com/y9b5vbcw`). G-W invariants are rational number valued topological invariants useful in algebraic and symplectic geometry. These quantum invariants give information about these geometries not provided by classical invariants. Despite being rational numbers in the general case G-W invariants in some sense give the number of string world sheets connecting given branes.

1. One considers collection of n surfaces ("branes") with even dimensions in some symplectic manifold X of dimension $D = 2k$ (say Kähler manifold) and pseudo-holomorphic curves ("string world sheets") X^2, which have the property that they connect these n surfaces in the sense that they intersect the "branes" in the marked points x_i, $i = 1, .., n$.

   "Connect" does not reduce to intersection in topologically stable sense since connecting is possible also for branes with dimension smaller than $D - 2$. One allows all surfaces that X^2 that intersects the n surfaces at marked points if they are pseudo-holomorphic even if the basic dimension rule is not satisfied. In 4-dimensional case this does not seem to have implications since partonic 2-surfaces satisfy automatically the dimension rule. The n branes intersect or touch in quantum sense: there is no concrete intersection but intersection with the mediation of "string world sheet".

2. Pseudo-holomorphy means that the Jacobian df of the embedding map $f : X^2 \rightarrow X$ commutes with the symplectic structures j *resp.* J of X^2 *resp.* X: i.e. one has $df(jT) = Jdf(T)$ for any tangent vector T at given point of X^2. For $X^2 = X = C$ this gives Cauchy-Riemann conditions.

   In the symplectic case X^2 is characterized topologically by its genus g and homology class A as surface of X. In algebraic geometry context the degree d of the polynomial defining X^2 replaces A. In TGD X^2 corresponds to string world sheet having also boundary. X^2 has also n marked points $x_1, ..., x_n$ corresponding to intersections with the n surfaces.

3. G-W invariant $GW_{g,n}^{X,A}$ gives the number of pseudo-holomorphic 2-surfaces X^2 connecting n given surfaces in X - each at single marked point. In TGD these surfaces would be partonic 2-surfaces and marked points would be carriers of sparticles.

The explicit definition of G-W invariant is rather hard to understand by a layman like me. I however try to express the basic idea on basis of Wikipedia definition (see `http://tinyurl.com/y9b5vbcw`). I apologize for my primitive understanding of higher algebraic geometry. The article of Vakil [L22] (see `http://tinyurl.com/ybobccub`) discusses the notion of G-W invariant in detail.

1. The situation is conformally invariant meaning that one considers only the conformal equivalence classes for the marked pseudo-holomorphic curves X^2 parameterized by the points of so called Deligne-Mumford moduli space $\overline{M}_{g,n}$ of curves of genus g with n marked points (see `http://tinyurl.com/yaq8n6dp`): note that these curves are just abstract objects without no embedding as surface to X assumed. $\overline{M}_{g,n}$ has *complex* dimension

$$d_0 = 3(g - 1) + n \quad .$$

   n corresponds complex dimensions assignable to the marked points and $3(g - 1)$ correspond to the complex moduli in absence of marked points. This space appears in TGD framework in the construction of elementary particle vacuum functionals [K32].

2. Since these curves must be represented as surfaces in X one must introduces the moduli space $\overline{M}_{g,n}(X, A)$ of their maps f to X with given homology equivalence class. The elements in this space are of form $(C, x_1, .., x_n, f)$ where C is one particular representative of A.

3. The complex dimension d of $\overline{M}_{g,n}(X, A)$ can be calculated. One has

$$d = d_0 + c_1^X(A) + (g - 1)k \quad .$$

   Here $c_1^X(A)$ is the first Chern class defining element of second cohomology of X evaluated for A. For Calabi-Yau manifolds one has $c_1 = 0$. The contribution $(g - 1)k$ to the dimension vanishing for torus topology should have some simple explanation.

4. One defines so called evaluation map ev from $\overline{M}_{g,n}(X,A) \to Y$, $Y = \overline{M}_{g,n} \times X^n$ in terms of stabilization $st(C, x_1, ..., x_n) \in \overline{M}_{g,n}(X,A)$ of C (I understand that stabilization means that the automophism group of the stabilized surface defined by f is finite [A81] (see `http://tinyurl.com/y8r44uhl`). I am not quite sure what the finiteness of the automorphism group means. One might however think that conformal transformations must be in question. One has

$$ev(C, x_1, .., x_n, f) = (st(C, x_1, .., x_n), f(x_1), ...f(x_n)) \ .$$

   Evaluation map assigns to the concrete realization of string world sheet with marked points the abstract curve $st(C, x_1, .., x_n)$ and points $(f(x_i), ..., f(x_n)) \in X^n$ possibly interpretable as positions $f(x_i)$ of n particles. One could say that one has many particle system with particles represented by surfaces of X_i of X connected by X^2 - string world sheet - mediating interaction between X_i via the intersection points.

5. Evaluation map takes the fundamental class of $\overline{M}_{g,n}(X,A)$ in $H_d(\overline{M}_{g,n}(X,A))$ to an element of homology group $H_d(Y)$. This homology equivalence class defines G-W invariant, which is rational valued in the general case.

6. One can make this more concrete by considering homology equivalence class β in $\overline{M}_{g,n}$ and homology equivalence classes α_i, $i = 1, ..., n$ represented by the surfaces X_i. The co-dimensions of these $n+1$ homology equivalence classes must sum up to d. The homologies of $\overline{M}_{g,n}$ and $Y = \overline{M}_{g,n} \times X^n$ induce homology of Y by Künneth formula (see `http://tinyurl.com/yd9ttlfr`) implying that Y has class of $H_d(Y)$ given by the product $\beta \cdot \alpha_1 ... \cdot \alpha_n$.

   One can identify the value of $GW_{g,n}^{X,A}$ for a given class $\beta \cdot \alpha_1 ... \cdot \alpha_n$ as the coefficients in its expansion as sum of all elements in $H_d(Y)$. This coefficient is the value of its intersection product of $GW_{g,n}^{X,A}$ with the product $\beta \cdot \alpha 1 ... \cdot \alpha_n$ and gives element of $H_0(Q)$, which is rational number.

7. There are two non-classical features. Classically intersection must be topologically stable. This would require α_i to have codimension 2 but all even co-dimensions are allowed. That the value for the number of connecting string world sheets is rational number does not conform with the classical geometric intuition. The Wikipedia explanation is that the orbifold singularities for the space $\overline{M}_{g,n}(X,A)$ of stable maps are responsible for rational number.

Application to string theory

Topological string theories give a physical realization of this picture. Here the review article *Instantons, Topological Strings, and Enumerative Geometry* of Szabo [A81] (see `http://tinyurl.com/y8r44uhl`) is very helpful.

1. In M-theory framework and for topological string models of type A and B the physical interpretation for the varieties associated with α_i would be as branes of various dimensions needed to satisfy Dirichlet boundary conditions for strings.

2. In topological string theories one considers sigma model with target space X, which can be rather general. The symplectic or complex structure of X is however essential. X is forced to be 3-D (in complex sense) Calabi-Yau manifold by consistency of quantum theory. Interestingly, the super twistor space $CP(3|4)$ is super Calabi-Yau manifold although CP_3 is not and must therefore have trivial first Chern class c_1 appearing in the formula for the dimension d above. I must admit that I do not understand why this is the case.

   Closed topological strings have no marked points and one has $n = 0$. Open topological strings world sheets meet n branes at points x_i, where they satisfy Dirichlet boundary conditions. Branes an be identified as even-dimensional Lagrangian sub-manifolds with vanishing induced symplectic form.

3. For topological closed string theories of type A one considers holomorphically imbedded curves in X characterized by genus g and homology class A: one speaks of world sheet

instantons. $A = \sum n_i S_i$ is sum over the generating classes S_i with integer coefficients. For given g and A one has analog of product of non-interacting systems at temperatures $1/t_i$ assignable to the homology classes S_i with energies identifiable as n_i. One can assign Boltzmann weight labelled by (g, A) as $Q^\beta = \prod_i Q_i^{n_i}$, $Q_i = exp(-t_i)$.

One can construct partition function for the entire system as sum over Boltzmann weights with degeneracy factors telling the number of world sheet instantons with given (g, A). One can calculate free energy as sum $\sum N_{g,\beta} Q^\beta$ over contributions labelled by (g, A). The coefficients $N_{g,\beta}$ count the rational valued degeneracies of the world sheet instantons of given type and reduce to G-W invariants $GW_{g,0}^{X,A}$.

Remark: If one allows powers of a root $e^{-1/n}$, $t = n$, in the extension of rationals or replace e^{-t} with p^n, partition functions make sense also in the p-adic context.

4. For topological open string theories of type A one has also branes. Homology equivalence classes are relative to the brane configuration. The coefficients $N_{g,\beta}$ are given by $GW_{g,n}^{X,A}$ for a given configuration of branes: the above described general formulas correspond to these.

5. For topological string theories of type B, string world sheets reduce to single point and thus correspond to constant solutions to the field equations of sigma model. Quantum intersection reduces to ordinary intersection and one has $x_1 = x_2 ... = x_n$. G-W invariants involve only classical cohomology and give for $n = 2$ the number of common points for two surfaces in X with dimension d_1 and $d_2 = n - d$. The duality between topological string theories of type A and B related to the mirror symmetry supports the idea that one could generalize the calculation of these invariants in theories B to theories A. It is not clear whether this option as any analog in TGD.

The so called Witten conjecture (see `http://tinyurl.com/yccahv3q`) proved by Kontsevich states that the partition function in one formulation of stringy quantum gravity and having as coefficients of free energy G-W invariants of the target space is same as the partition function in second formulation and expressible in terms of so called tau function associated with KdV hierarchy. This leads to non-trivial identities. Witten conjecture actually follows from the invariance of partition function with respect to half Virasoro algebra and Virasoro conjecture (see `http://tinyurl.com/y7xcc9hm`) stating just this generalizes Witten's conjecture.

5.3.4 Riemann-Roch theorem

Riemann-Roch theorem (RR) is also part of enumerative geometry albeit more abstract. Instead of counting of numbers of points, one counts dimensions of various function spaces associated with Riemann surfaces. RR provides information about the dimensions for the spaces of meromorphic functions and 1-forms with prescribed zeros and poles.

Basic notions

Riemann surface is the basic notion. Riemann surface is orientable is characterized by its genus g and number of holes/punctures in it. Riemann surface can also possess marked points, which seem to be equivalent with punctures. The moduli space of these complex curves is parameterized by a moduli space $\overline{M}_{g,n}$ of curves of genus g with n marked points (see `http://tinyurl.com/yaq8n6dp`) (see `http://tinyurl.com/yaq8n6dp`).

Dolbeault cohomology (see `http://tinyurl.com/y7cvs5sx`) generalizes the notion of differential form so that it has has well-defined degrees with respect to complex coordinates and their conjugates: one can write in general complex manifold this kind of form as

$$\omega = \omega_{i_1 i_2 .. i_n, j_1 j_2 ... j_n} dz^{i_1} \wedge dz^{i_2} ... dz^{i_n} d\overline{z}^{j_1} \wedge d\overline{z}^{j_2} ... dz^{j_n} \quad .$$

The ordinary exterior derivative d is replaced with its holomorphic counterpart ∂ and its conjugate. One can construct the counterparts of cohomology groups (Hodge theory) $H^{p,q} = H^{q,p}$. Betti numbers as numbers $h_{i,j}$ defining the dimensions of the cohomology groups forms of degrees i and j with respect to dz^i and $d\overline{z}^j$. One can define the holomorphic Euler's characteristic as $\chi_C = h_{0,0} - h_{01} = 1 - g$ whereas orinary Euler characteristic is $\chi_R = h_{0,0} - (h_{01} + h_{10}) + h_{1,1} = 2(1 - g)$.

One considers meromorphic functions having poles and zeros as the only singularities (points at which the map does not preserve angles): rational functions provide the basic example. Riemann zeta provides example of meromorphic function not reducing to rational function. Holomorphic functions have only zeros and entire functions have neither zeros nor poles. If analytic functions on Riemann surfaces can be interpreted as maps of compact Riemann surface to itself rather than to complex plane, meromorphy reduces to holomorphy since the point ∞ belongs to the Riemann surface.

The elements of free group of divisors are defined as formal sums of integers associated with the points P of Riemann surface. Divisors $D = \sum_P n(P)$, where (P) is integer, are analogous to integer valued "wave functions" on Riemann surface. The number of points with $n(P) \neq 0$ is countable. The degree of divisor is obtained as the ordinary sum $deg(D)$ of the integers defining the divisor.

Although divisors can be seen as purely formal objects, they are in practice associated to both meromorphic functions and 1-forms. The divisor of a meromorphic function is known as principal divisor. Meromorphic functions and 1-forms differing by a multiplication with meromorphic function are regarded as linearly equivalent - in other words, one can add to a given divisor a divisor of a meromorphic function without changing its equivalence class. It can be shown that all divisors associated with meromorphic 1-forms linearly equivalent and one can talk about canonical divisor. Note that $deg(D)$ is linear invariant since the degree of globally meromorphic function is zero.

The motivation for the divisors is following. Consider the space of meromorphic functions h with the property that the degrees of poles associated with the poles of these functions are not higher than given integers $n(P)$. In other words, one has $\langle h(P)\rangle + D(P) \geq 0$ for all points P ($\langle h\rangle$ is the divisor of h). For $D(P) > 0$ the pole has degree not higher than $D(P)$. For non-positive $D(P)$ the function has zero of order $D(P)$ at least.

Formulation of RR theorem

With these prerequisites it is possibly to formulate RR (for Wikipedia article see `http://tinyurl.com/mdmbcx6`). The Wikipedia article is somewhat confusing and a more precise description of RR can be found in the article "Riemann-Roch theorem" by Vera Talovikova [A90] (see `http://tinyurl.com/ktww7ks`).

Let $l(D)$ be the dimension of the space of meromorphic functions with principal divisor D or 1-forms linearly equivalent with canonical divisor K. Then the equality

$$l(D) - l(K - D) = deg(D) - g + 1 \tag{5.3.1}$$

is true for both meromorphic functions and canonical divisors. For $D = K$ one obtains using $l(0) = 1$

$$l(K) = deg(K) - g + 2 \tag{5.3.2}$$

giving the dimension of the space of canonical divisors. $l(K) > 0$ in general is not in conflict with the fact that canonical divisors are linearly equivalent. $deg(K) = 2g-2$ in the above formula gives $l(K) = g$.

$l(K) = 0$ for $g = 0$ case looks strange: one should actually make notational distinction between dimensions of spaces of meromorphic functions and one-forms (this is done in the article of Talivakova). The explanation is that $l(K)$ here is not the dimension of the space of canonical 1-forms but that of the holomorphic functions with the divisor of K. The canonical form K for the sphere has second order pole at ∞ so that one cannot have meromorphic forms holomorphic outside P.

Riemann's inequality

$$l(D) \geq deg(D) - g + 1 \tag{5.3.3}$$

follows from $l(K - D) \geq 0$, which can be seen as a correction term to the formula

$$l(D) = deg(D) - g + 1 \ . \tag{5.3.4}$$

In what sense this is true, becomes clear from what follows.

The dimension of the space meromorphic functions corresponding to given divisor

The simplest divisor associated with meromorphic function involves only one point. Multiplying a function, which is non-vanishing and finite at P by $(z - z(P))^{-n}$ gives a pole of order n (zero has negative order in this sense). One is interested on the dimension $l(nP)$ of the space nP of meromorphic functions and RR allows to deduce information about $l(nP)$. One is interested on the behavior of $l(nP)$ as function of genus g of Riemann surface (more general situation would allow also punctures). For $n = 0$ one has entire function without poles and zeros. Only constant function is possible: $l(0) = 1$.

First some general observations. K has degree $deg(K) = 2g - 2$, which gives $l(K) = g$. For $n = deg(D) > deg(K) = 2g - 2$ the correction term vanishes since $deg(K - D)$ becomes negative, and one has $l(D) = deg(D) - g + 1$. This gives $l(n = 2g - 1) = g$. Therefore $n \in \{2g - 1, 2g, ...\}$ corresponds to $l(nP) \in \{g, g+1, ...\}$. $n < 2g-2$ corresponds to $l(nP) = 1$. What about the range $n \in \{2, ..., 2g - 2\}$? Note that $2g - 2$ is the negative of the Euler character of Riemann surface.

1. $g = 0$ case. K on sphere. dz canonical 1-form on Riemann sphere covered by two complex coordinate patches. $z \rightarrow w = 1/z$ relates the coordinates. There is second order pole at infinity ($dw = -dz/z^2$). One has therefore $deg(K) = -2$ for sphere in accordance with the general formula $deg(K) = 2g - 2$. The formula $l(nP) = deg(D) + 1$ holds for all n and there is no correction term now. One as $l(nP) = n + 1$.

2. $g = 1$ case.

   One has $deg(K) = 2g - 2 = 0$ for torus reflecting the fact that the canonical form $\omega = dz$ has no poles or zeros (torus is obtained by identifying the cells of a periodic lattice in complex plane). Correction term vanishes since it would have negative degree for all n and one has $l(nP) \in \{1, 1, 2, 3, ...\}$.

3. $g = 2$ case.

   For $n = deg(D) \geq 2 \times 2 - 1 = 3$ gives $l(D) = n - 1$ giving for $n \geq 3$ $l(nP) \in \{2, 3, ...\}$. What about $n = g = 2$? For generic points one has $l(2) = 1$. There are 6 points at which one has $l(D) = 2$ so that there is additional meromorphic function having pole of order 2 at this kind of point. These points are fixed points under Z_2 defining hyper-ellipticity. Note that $g \leq 2$ Riemann surfaces are always hyper-elliptic in the sense that it allows Z_2 as conformal symmetry (see `http://tinyurl.com/y9sdu4o3`).

   One has therefore $l(nP) \in \{1, 1, 1, 2, ..\}$ for a generic point and $l(nP) \in \{1, 1, 2, 2......\}$ for 6 points fixed under Z_2. An interesting question is whether this phenomenon could have physical interpretation in TGD framework.

4. $g > 2$ case.

   For $g > 2$. $l(nP)$ in the range $\{2, 2g - 2\}$ can depend on point and even on the conformal moduli. There are more special points in which correction term differs from that in the generic case. $g = 3$ illustrates the situation. $n \in \{1, 1, 1, 1, 1, 2, ...\}$ is obtained for a generic point. At special points and for $n < 3$ there are also other options for $l(nP)$. Also the dependence of $l(nP)$ on moduli emerges for $g \geq 3$. The natural guess layman is that these points are fixed points of conformal symmetries. Also now hyper-elliptic surfaces allowing projective Z_2 covering are special. In the general case hyper-ellipticity is not possible.

In TGD framework Weierstrass points(see `http://tinyurl.com/y9wehsml`) are of special interest physically.

1. My layman guess is that special points known as Weierstrass points (see http://tinyurl.com/y9wehsml) correspond to singularities for projective coverings for which conformal symmetries permute the sheets of the covering. Several points coincide for the covering since a sub-group of conformal symmetries would act trivially on the Weierstrass point.

   Note that for $g > 2$ Z_2 covering is not possible except for hyper-elliptic surfaces, and one can wonder whether this relates to the experimental absence fo $g > 2$ fermion families [K32]. Second interesting point is that elementary particles indeed correspond to double sheeted structures from the condition that monopole fluxes flow along closed flux tubes (there are no free magnetic monopoles).

2. There is an obvious analogy with the coverings associated with the cognitive representation defined by the points of space-time surface with coordinates in an extension of rationals [L39, L32] [L37]. Fixed points for a sub-group of Galois group generate singularities at which sheets touch each other. These singular points are physically the most interesting and could carry sparticles. The action of discrete conformal groups restricted to cognitive representation could be represented as the action of Galois group on points of cognitive representation. Cognitive representation would indeed represent!

   Remarkably, if the tangent spaces are not parallel for the touching sheets, these points are mapped to several points in H in $M^8 - H$ correspondence. If this picture is correct, the hyper-elliptic symmetry Z_2 of genera $g \leq 2$ could give rise to this kind of exceptional singularities for $g \geq 2$.

   What is worrying that there are two views about twistorial amplitudes. One view relying on the notion of octonionic super-space M^8 [L32] is analogous to that of SUSYs: sparticles can be seen as completely local composites of fermions. Second view relies on embedding space $M^4 \times CP_2$ [L41] and on the identification sparticles as non-local many-fermion states at partonic 2-surfaces. These two views could be actually equivalent by $M^8 - H$ duality.

3. When these singular points are present at partonic 2-surfaces at boundaries of CD and at vertices, the topology of partonic 2-surface is in well-defined sense between g and $g+1$ external particles: one has criticality. The polynomials representing external particles indeed satisfy criticality conditions guaranteeing associativity or co-associativity (quantum criticality of TGD Universe is the basic postulate of quantum TGD). At partonic orbits the touching pieces of partonic 2-surface could separate (g) or fuse ($g+1$). Could this topological mixing give rise to CKM mixing of fermions [K32, K68, K79]?

RR for algebraic varieties and bundles

RR can be generalized to algebraic varieties (see http://tinyurl.com/y9asz4qg). In complex case the real dimension is four so that this generalization is interesting from TGD point of view and will be considered later. The generalization involves rather advanced mathematics such as the notion of sheaf (see http://tinyurl.com/nudhxo6). Zeros and poles appearing in the divisor are for complex surfaces replaced with 2-D varieties so that the generalization is far from trivial.

The following is brief summary based on Wikipedia article.

1. Genus g is replaced with algebraic genus and $deg(D)$ plus correction term is replaced with the intersection number (see http://tinyurl.com/y7dcffb6) for D and $D-K$, where K is the canonical divisor associated with 2-forms, which is also unique apart from linear equivalence Points of divisor are replaced with 2-varieties.

2. The generalization to complex surfaces (with real dimension equal to 4) reads as

$$\chi(D) = \chi(0) + \frac{1}{2} D \cdot (D-K) \ . \tag{5.3.5}$$

   $\chi(D)$ is holomorphic Euler characteristic associated with the divisor. $\chi(0)$ is defined as $\chi(0) = h_{0,0} - h_{0,1} + h_{0,2}$, where $h_{i,j}$ are Betti numbers for holomorphic forms. '$\cdot$' denotes intersection product in cohomology made possibly by Poincare duality. K is canonical two-form which is a section of determinant bundle having unique divisor (their is linear equivalence due to the possibility to multiply with meromorphic function.

One has $\chi(0) = 1 + p_a$, where p_a is arithmetic genus. Noether's formula gives

$$\chi(0) = \frac{c_1^2 + c_2}{12} = \frac{K \cdot K + e}{12} \quad . \tag{5.3.6}$$

c_1^2 is Chern number and $e = c_2$ is topological Euler characteristic.

Clearly the information given by $\chi(D)$ is about Dolbeault homology. For comparison note that RR for curves states $l(D) - l(K - D) = \chi(D) = \chi(0) + deg(D)$.

RR can be also generalized so that it applies to vector bundles. Ordinary RR can be interpreted as applying to a bundle for which the fiber is point. This requires the notion of the inverse bundle defined as a bundle with the property that its direct sum (Whitney sum) with the bundle itself is trivial bundle. One ends up with various characteristic classes, which represent homologically non-trivial forms in the base spaces characterizing the bundle. For instance, the generalizations of RR give information about the dimensions of the spaces of sections of the vector bundle.

Atyiah-Singer index theorem (see `http://tinyurl.com/k6daqco`) deals with so called elliptic operators in compact manifolds and represents a generalization important in recent theoretical physics, in particular gauge theories and string models. The theorem relates analytical index - typically characterizing the dimension for the spectrum of solutions of elliptic operator to a topological index. Elliptic operator is assigned with small perturbations for a given solution of field equations. Perturbations of a given solution of say Yang-Mills equations is a representative example.

5.4 Does $M^8 - H$ duality allow to use the machinery of algebraic geometry?

The machinery of algebraic geometry is extremely powerful. In particular, the number theoretical universality of algebraic geometry implies that same equations make sense for all number fields: this is just what adelic physics [L39] [L37] demands. Therefore it would be extremely nice if one could somehow use this machinery also in TGD framework as it is used in string models. How this could be achieved? There are several guide lines.

1. Twistor lift of TGD [K116, K49, K13, L41] is now a rather well-established idea although a lot of work remains to be done with the details. Twistors were originally introduced in order to be able to use this machinery and involves complexification of Minkowski space M^4 to M_c^4 as an auxiliary tool. Complexification in sufficiently general sense seems to be a necessary auxiliary tool but it cannot be a trick (like Wick rotation) but something fundamental and here complexification at the level of M^8 is suggestive. In the sequel I will used M^4 for M_c^4 and M^8 for M_c^8 unless it is necessary to emphasize that M_c^8 is in question. The essential point is that the Euclidian metric is complexified and it reduces to a real metric in various sub-spaces defining besides Euclidian space also Minkowski spaces with varying signature when the complex coordinates are real or imaginary.

2. If $M^8 - H$ duality holds true, one can solve field equations in $M^8 = M^4 \times E^8$ by assuming that either the tangent space or normal space of the space-time surface X^4 is associative (quaternionic) at each point and contains preferred M^2 in its tangent space. M^2 could depend on x but $M^2(x)$:s should integrate to a 2-surface. This allows to map space-time surface M^8 to a surface in $M^4 \times CP_2$ since tangent spaces are parameterized by points of CP_2 and CP_2 takes the role of moduli space. The image of tangent space as point of CP_2 is same irrespective of whether one has quaternions or complexified quaternions (H_c).

It came a surprise that associativity/co-associativity is possible only if the space-time surface is critical in the sense that some gradients of 8 complex components of the octonionic polynomial P vanish without posing them as additional conditions reducing thus the dimension of the space-time surface. This occurs when the coefficients of P satisfy additional conditions. One obtains associative/co-associative space-time regions and regions without either property and they correspond nicely to two solution types for the twistor lift of Kähler action.

3. Contrary to the original expectations, $M^4 \subset M^8_c$ must be identified as co-associative (co-quaternionic) subspace so that E^4 is the associative/quaternionic sub-space. This allows to have light-cone boundary as the counterpart of point-like singularity in ordinary algebraic geometry and also allows to understand the emergence of CDs and ZEO.

Remark: A useful convention to be used in the sequel. $RE(o)$ and $IM(o)$ denote the real and imaginary parts of the octionion in the decomposition $o = RE(o) + IM(o)I_4$ and $Re(o)$ and $Im(o)$ its real number valued and purely imaginary parts in the usual decomposition.

The problems related to the signature of M^4 have been a longstanding head-ache of M^8 duality.

1. The intuitive vision has been that the problems can be solved by replacing M^8 with its complexification M^8_c identifiable as complexified octonions o. This requires introduction of imaginary unit i commuting with the octonionic units $E^k \leftrightarrow (1, I_1, ..., I_7)$. The real octonionic components are thus replaced with ordinary complex numbers $z_i = x_i + iy_i$.

2. Importantly, complex conjugation $o \rightarrow \overline{o}$ changes only the sign of I_i but *not!* that of i so that the octonionic inner product $(o_1, o_2) = o_1\overline{o}_2 = o_1^k o_2^l \delta_{k,l}$ becomes complex valued. Norm is equal to $O\overline{O} = \sum_i z_i^2$. Both norm and inner product are in general complex valued and real valued only in sub-spaces in which octonionic coordinates are real or imaginary. Sub-spaces have all possible signatures of metric. These sub-spaces are not closed under multiplication and this has been an obstacle in the earlier attempts based on the notion of octonion analyticity. This argument applies also to quaternions and one obtains signatures $(1,1,1,1)$, $(1,1,1,-1)$, $(1,1,-1,-1)$, and $(1,-1,-1,-1)$. Why just the usual Minkowskian signature $(1,-1,-1,-1)$ is physical, should be understood.

   The convention consistent with that used in TGD corresponds to a negative length squared for space-like vectors and positive for time-like vectors. This gives $m = (o^0, io^1, ..., io^7)$ with real o^k. The projection $M^8_c \rightarrow M^8$ defines the projection of $X^4_c \subset M^8_c$ to $X^4 \subset M^8$ serving as the pre-image of $X^4 \subset M^8$ in $M^8 - H$ correspondence.

3. o is not field anymore as is clear from the fact that $1/o = \overline{o}/o\overline{o}$ is formally infinite in Minkowskian sub-spaces, when octonion defines a light-like vector. o (and H_c) remains however a ring so that sum and products are well-defined but division can lead to problems unless one stays inside 7+7-dimensional light-cone with $Re(o\overline{o}) > 0$ ($Re(q\overline{q}) > 0$).

   Although the number field structure is lost, one can still define polynomials needed to define algebraic varieties by requiring their simultaneous vanishing and rational functions make sense inside the light-cone. Also rational functions can be defined but poles are replaced with light-cones in Minkowskian section. Algebraic geometry would thus be forced by the complexification of octonions. This looks to me highly non-trivial! The extension of zeros and poles to light-cones making propagation possible could be a good reason for why Minkowskian signature is physical. Interestingly, the allowed octonionic momenta are light-like quaternions [L41].

4. An interesting question is whether ZEO and the emergence of CDs relates to the failure of field property. It seems now clear that CDs must be assigned even with elementary particles. I have asked whether they could form an analog for the covering of manifold by coordinate patches (in TGD inspired theory of consciousness CDs would be correlates for perceptive fields for conscious entities assignable to CDs [L42]). These observations encourage to ask whether the tips of CD should correspond to a pair formed by two poles/two zeros or by pole and zero assignable to positive and negative energy states.

It turns out that the space-time surfaces in the interior of CD would naturally correspond to non-associative surfaces and only their 3-D boundaries would have associative 4-D tangent spaces allowing mapping to H by M^8-duality, which is enough by holography.

5. The relationship between light-like 3-surface bounding Minkowskian and Euclidian space-time regions and light-like boundaries of CDs is interesting. Could also the partonic orbits be understood a singularities of octonionic polynomials with $IM(P) = RE(P) = 0$?

5.4.1 What does one really mean with $M^8 - H$ duality?

The original proposal was that M^8 duality should map the associative tangent/normal planes of associative/co-associative space-time surface containing preferred M^2, call it M_0^2, to CP_2: the map read as $(m, e) \in M^4 \times E^4 \to (m, s) \in M^4 \times CP_2$. Eventually it became clear that the choice of M^2 can depend on position with $M^2(x)$ forming an integrable distribution to CP_2: this would define what I have called Hamilton-Jacobi structures [K10]. String like objects have minimal surface as M^4 projection for almost any general coordinate invariant action, and internal consistency requires that $M^2(x)$ integrate to a minimal surface. The details are however not understood well enough.

1. M^4 coordinate would correspond simply to projection to a fixed M_0^4 in the decomposition $M^8 = M_0^4 \times E_0^4$. One can however challenge this interpretation. How M_0^4 is chosen? Is it possible to choose it uniquely? Could also M^4 coordinates represent moduli analogous to CP_2 coordinates? What about ZEO?

   There is an elegant general manner to formulate the choice of M_0^4 at the level of M^8. The complexified quaternionic sub-spaces of M_c^8 (M^8) are parameterized by moduli space defining the quaternionic moduli. The moduli space in question is CP_2. The choice of M_0^4 corresponds to fixing of the quaternionic moduli by fixing a point of CP_2.

   Warning: Note that one should be very careful in distinguishing between quaternionic as sub-spaces of M^8 and as the tangent space M^8 of given point of M^8.

2. One can ask whether there could be a connection with ZEO, where CDs play a key role. Indeed, the complexified Minkowski inner product means that the complexified octonions (quaternions) inside M_c^8 (M_c^4) have inverse only inside 7-D (4-D) complexified light-cone and this would motivate the restriction of space-time surfaces inside future or past light-cone or both but not yet force CD.

   If one allows rational functions and even meromorphic functions of octonionic or quaternionic variable, one could consider the possibility of restricting the space-time surface defined as their zeros to a maximally sized region containing no poles.

3. Consider complexified quaternions H_c. Poles $(q\overline{q})^{-n}$, $n \geq 1$ would correspond M^4 light-cone boundaries since $q\overline{q} = 0$ at them. Also zeros $q\overline{q} = 0$, for $n \geq 1$ correspond to light-like boundaries. Could one have two poles with with time-like distance defining CD or a pair of pole and zero?

   There is also a possible connection with the notion of infinite primes [K107]. The notion of infinite prime leads to the proposal that rationals defined as ratios of infinite integers but having unit real norm (and also p-adic norms) could correspond pairs of positive and negative energy states with identical total quantum numbers and located at opposite boundaries of CD. Infinite rationals can be mapped to rational functions. Could positive energy states correspond to the numerators with zeros at second boundary of CD and negative energy states to denominators with zeros at opposite boundary of CD?

Is the choice of the pair (M_0^2, M_0^4) consistent with the properties of known extremals in H

It should be made clear that the notion of associativity/co-associativity (quaternionicity/co-quaternionicity) of the tangent/normal space need not make sense at the level of H. I shall however study this working hypothesis in the sequel.

The choice of the pair (M_0^2, M_0^4) means choosing preferred co-commutative (commutative) sub-space M_0^2 of M^8 defining a subspace of fixed co-quaternionic (quaternionic) sub-space $M_0^4 \subset M^8$.

Remark: M^4 should indeed be the co-associative/co-quaternionic subspace of M^8 if the argument about emergence of CDs is accepted and if $M^8 - H$ correspondence maps associative to associative and co-associative to co-associative.

M_0^4 in turn contains preferred M_0^2 defining co-commutative (hyper-complex) structure. Both M_0^2 and M_0^4 are needed in order to label the choice by CP_2 point (that is as a point of Grassmannian).

Is the projection to a fixed factor $M_0^4 \subset M_0^4 \times E^4$ as a choice of co-quaternionic moduli consistent with what we know about the extremals of twistor lift of Kähler action in H? How could one fix M_0^4 from the data about the extremal in H? One can make similar equations about the choice of M_0^2 as a fixed co-complex moduli characterized by a unit quaternion. Note that this choice is expected to relate closely to the twistor structure and Kähler structure.

It is best to check the proposal for the known extremals in H [K10]. Consider first CP_2 type extremals for which M^4 projection is a piece of light-like geodesic.

1. The CP_2 projection for the image of $X^4 \subset M^8$ differs from single point only if the tangent space isomorphic to M^4 and parameterized by CP_2 point varies. Consider CP_2 type extremals for the twistor lift of Kähler action [**?**]n H having light-like geodesic as M^4 projection as an example. The light-like geodesic defines a light-like vector in the tangent space of CP_2 type extremal. This light-like vector together with its dual spans fixed M^2, which however does not belong to the tangent space so that associative surface would not be in question.

   What about co-associativity or associativity (the latter is favored by above argument)? This property should hold true for the pre-image of CP_2 type extremal in M^8 but I am not able to say anything about this. It is questionable to require this property at the level H but one can of course look what it would give.

   What about associativity for CP_2 tangent space? The normal space of CP_2 type extremal is 3-D (!) since the only the light-like tangent vector of the geodesic and 2 vectors orthogonal to it are orthogonal to CP_2 tangent vectors. For Euclidian signature this would mean that tangent space is 5-D and cannot be associative but now the tangent space is 4-D. Can one still say that tangent space is associative. The co-associativity of the tangent space makes sense trivially. Can one conclude that CP_2 is co-associative.

   The associativity for CP_2 tangent space might make sense since the tangent space is 4-D. The light-like vector k defines M_0^2. The associativity conditions involving two tangent space vectors of CP_2 and the light-like vector k contracted with the corresponding octonion components. The contributions from the components of k to the associator should cancel each other. Since one can change the relative sign of the components of k, this mechanism does not seem to work for all components. Hence associativity cannot hold true. Neither does M_0^2 belong to the normal space since k and its dual are not orthogonal.

   Could one conclude that CP_2 type extremal is co-associative in accordance with the original belief thanks to the light-like projection to M^4? This does not conform with what the singularity considerations for the octonionic polynomials would suggest. Or is it simply not correct to try to apply associativity at the level of H. Or does $M^8 - H$ correspondence map associative tangent spaces to co-associative ones?

2. The normal space M^4 of CP_2 type extremal have all orientations characterized by its CP_2 projection. The normal space must contain the M_0^2 determined by the tangent of the light-like geodesic and this is indeed the case. Note that CP_2 type extremals cannot have entire CP_2 as CP_2 projection: they necessarily have hole at either end, which would be naturally be at the boundary of CD.

   CP_2 type extremals seem to be consistent with $M^8 - H$ correspondence. It however seems that one cannot fix the choice of M_0^4 uniquely in terms of the properties of the extremal. There is a moduli space for M_0^4:s defined by CP_2 and obviously codes for moduli for quaternion structures in octonionic space. The distributions of $M^2(x)$ (minimal surfaces) would code for quaternion structures (decomposition of octonionic coordinates to quaternionic coordinates in turn decomposing to pairs of complex coordinates).

Consider next the associativity condition for cosmic strings in $X^2 \times Y^2 \subset M^4 \times CP_2$. Now CP_2 projection is 2-D complex surfaces and M^4 projection is minimal surface. Situation is clearly associative. How unique the choice of M_0^4 is now?

1. Now $M^2(x)$ depends on position but $M^2(x)$:s define an integrable distribution defining string orbit X^2 as a minimal surface. M_0^4 must contain all surfaces $M^2(x)$, which would fix M_0^4 to a high degree for complex enough cosmic strings.

2. Each point of X^2 corresponds to the same partonic surface $Y^2 \subset CP_2$ labelling the tangent spaces for its pre-image in M^8. All the tangent surfaces $M^2(x) \times E^2(y)$ for $X^2 \times Y^2 \subset M^8$ share only $M^2(x) \subset M_0^4$. M_0^4 must contain all tangent spaces $M^2(x)$ and the inverse image of $Y^2 \subset CP_2$ must belong to the orthogonal complement E^4 of M_0^4. This is completely analogous with the condition $X^2 = X^2 \times Y^2 \subset M^4 \times CP_2$.

   Consider a decomposition $M^8 = M_0^4 \times E^4$, $M_0^4 = M_0^2 \times E_0^2$. If the inverse image of Y^2 at point x belongs to E^4, the M_0^4 projection belongs to M_0^4 also in M^8. If this does not pose any condition on the tangent spaces assignable to the points of Y^2 defining points of CP_2, there are no problems. What could happen that the tangent spaces assignable to Y^2 could force the projection of the inverse image of Y^2 to intersect M_0^4.

One should also understand massless extremals (MEs). How to choose M_0^4 in this case?

1. MEs are given as zeros of arbitrary functions of CP_2 coordinates and 2 M^4 coordinates u and v representing local light-like direction and polarization direction orthogonal to it. In the simplest situation these directions are constant and define $M_0^4 = M_0^2 \times E_0^2$ decomposition everywhere so that M_0^4 is uniquely defined. Same applies also when the directions are not constant. In the general case light-like direction would define the local tangent plane of string world sheet and local polarization plane. Since the dimension of M^4 projection is 4 there seems to be no problems involved.

2. Tangent plane of X^4 is parameterized by CP_2 coordinates depending on 2 coordinates u and v. The surface $X^4 \subset M^8$ must be graph for a map $M_0^4 \to E^4$ so that a 2-parameter deformation of M_0^4 as tangent plane is in question. The distribution of tangent planes of $X^4 \subset M^8$ is 2-D as is also the CP_2 projection in H.

To sum up, the original vision about the associativity properties of the known extremals at level of H survives. On the other hand, CDs emerge if M^4 corresponds to the co-associative part of O. Does this mean that $M^8 - H$ correspondence maps associative to co-associative by multiplying the quaternionic tangent space in M^8 by I_4 to get that in H and vice versa or that the notions of associative and co-associative do not make sense at the level of H? This does not affect the correspondence since the same CP_2 point parametrizes both associative tangent space and its complement.

Space-time surfaces as co-dimension 4 algebraic varieties defined by the vanishing of real or imaginary part of octonionic polynomial?

If the theory intended to be a theory of everything, the solution ansatz for the field equations defining space-time surfaces should be ambitious enough: nothing less than a general solution of field equations should be in question.

1. One cannot exclude the possibility that all analytic functions of complexified octonionic variable with real Taylor or even Laurent coefficients. These would would a commutative and associative algebra. Space-time surfaces would be identified as their zero loci. This option is however number theoretically attractive and can also leads to problems with adelic physics. Since Taylor series at rational point need not anymore give a rational value.

2. Polynomials of complexified octonion variable o with real coefficients define the simplest option but also rational functions formed as ratios of this kind of polynomials must be considered. Polynomials form a non-associative ring allowing sum, product, and functional decomposition as basic operations. If the coefficients o_n of polynomials are complex numbers $o_n = a_n + ib_n$, a_n, b_n real, where i refers to the commutative imaginary unit complexifying the octonions, the ring is associative. It is essential to allow only powers o^n (or $(o - o_0))^n$ with $o_0 = a_0 + ib_0$, a_0, b_0 real numbers). Physically this means that a preferred time axis is fixed. This time axis could connect the tips of CD in ZEO.

   One can write

$$P(o) = \sum_k p_k o^k \equiv RE(P)(q_1, q_2, \overline{q_1}, \overline{q}_2) + IM(P)(q_1, q_2, \overline{q_1}, \overline{q}_2) \times I_4 \quad , p_k \text{ real} \quad , \tag{5.4.1}$$

   where the notations

$$o = q_1 + q_2 I_4 \quad , \quad q_i = z_i^1 + z_i^2 I_2 \quad , \quad \overline{q}_i = z_i^1 - z_i^2 I_2 \quad , z_i^j = x_i^j + iy_i^j \tag{5.4.2}$$

   Note that the conjugation does *not* change the sign of i. Due to the non-commutativity of octonions P^i as functions of quaternions are in general *not* analytic in the sense that they would be polynomials of q_i with real coefficients! They are however analytic functions of z_i. The real and imaginary parts of x_i^j correspond to Minkowskian and Euclidian signatures.

   In adelic physics coefficients o_n of the octonionic polynomials define WCW coordinates and should be rational numbers or rationals in the extension of rationals defining the adele. The polynomials form an associative algebra since associativity holds for powers o^n multiplied by real number. Thus complex analyticity crucial in algebraic geometry would be a key element of adelic physics.

3. If the preferred extremals correspond to the associative algebra formed by these polynomials, one could construct a completely general solution of the field equations as zero loci of their real or imaginary parts and build up of new solutions using algebra operation sum, product, and functional decomposition. One could identify space-time regions as associative or co-associative algebraic varieties in terms of these polynomials and they would form an algebra.

The motivation for this dream comes from 2-D electrostatics, where conducting surfaces correspond to curves at which the real part u or imaginary part v of analytic function $w = f(z) = u + iv$ vanishes. In electrostatics curves form families with curves orthogonal to each other locally and the map $w = u + iv \to v - iu$ defines a duality in which curves of constant potential and the curves defining their normal vectors are mapped to each other.

1. The generalization to the recent situation would be vanishing of the imaginary part $IM(P)$ or real part $RE(P)$ of the octonionic polynomial, where real and imaginary parts are defined via $o = q_c^1 + q_c^2 I_4$. One can consider also the possibility that imaginary or real part has constant value c are restricted to be rational so that one can regard the constant value set also as zero set for a polynomial with constant shift. Note that the rationals could be also complexified by addition of i. One would have

$$RE(P)(z_i^k) \quad \text{or} \quad IM(P)(z_i^k) = c \quad , \quad c = c_0 \text{ rational} \quad . \tag{5.4.3}$$

c_0 must be real. These two options should correspond to the situations in which tangent space or normal space is associative (associativity/co-associativity). Complexified space-time surfaces X^4_c corresponding to different constant values c of imaginary or real part (with respect to i) would define foliations of M^8_c by locally orthogonal 4-dimensional surfaces in M^8_c such that normal space for surface X^4_c would be tangent space for its co-surface.

CDs and ZEO emerges naturally if the $IM(o)$ corresponds to co-quaternionic part of octonion.

2. It must be noticed that one has moduli space for the quaternionic structures even when M^4_0 is fixed. The simplest choices of complexified quaternionic space $H_c = M^4_{c,0}$ containing preferred complex plane $M^2_{c,0}$ and its orthogonal complement are parameterized by CP_2. More complex choices are characterized by the choice of distribution of $M^2(x)$ integrable to (presumably minimal) 2-surface in M^4. Also the choice of the origin matters as found and one has preferred coordinates. Also the 8-D Lorentz boosts give rise to further quaternionic moduli. The physically interesting question concerns the interpretation of space-time surfaces with different moduli. For instance, under which conditions they can interact?

The proposal has several extremely nice features.

1. Single real valued polynomial of real coordinate extended to octonionic polynomial and fixed by real coefficients in extension of rationals would determine space-time surfaces.

2. The notion of analyticity needed in concrete equations is just the ordinary complex analyticity forced by the octonionic complexification: there is no need for the application to have left- or right quaternion analyticity since quaternionic derivatives are not needed. Algebraically one has the most obvious guess for the counterpart of real analyticity for polynomials generalized to octonionic framework and there is no need for the quaternionic generalization of Cauchy-Riemann equations [A92, A65] [A92, A65] (`http://tinyurl.com/yb8l34b5`) plagued by the problems with the definition of differentiation in non-commutative and non-associative context. There would be no problems with non-associativity and non-commutativity thanks to commutativity of complex coordinates with octonionic units.

3. The vanishing of the real or imaginary part gives rise to 4 conditions for 8 complex coordinates z^k_1 and z^k_2 allowing to solve z^k_2 as algebraic functions $z^k_2 = f^k(z^l_1)$ or vice versa. The conditions would reduce to algebraic geometry in complex co-dimension $d_c = 4$ and all methods and concepts of algebraic geometry can be used! Algebraic geometry would become part of TGD as it is part of M-theory too.

5.4.2 Is the associativity of tangent-/normal spaces really achieved?

The non-trivial challenge is to prove that the tangent/normal spaces are indeed associative for the two options. The surfaces X^4_c are indeed associative/co-associative if one considers the *internal* geometry since points are in M^4_c or its orthogonal complement.

One should however prove that X^4_c are also associative *as sub-manifolds* of O and therefore have quaternionic tangent space or normal space at each point parameterized by a point of CP_2 in the case that tangent space containing position dependent M^2_c, which integrate to what might be called a 2-D complexified string world sheet inside M^4_c.

1. The first thing to notice that associativity and quaternionicity need not be identical concepts. Any surface with complex dimension $d < 4$ in O is associative and any surface with dimension $d > 4$ co-associative. Quaternionic and co-quaternionic surfaces are 4-D by definition. One can of course ask whether one should consider a generalization of brane hierarchy of M-theory also in TGD context and allow associativity in its most general sense. In fact, the study of singularity of o^2 shows that 6 and 5-dimensional surfaces are allowed for which the only interpretation would be as co-associative spaces. This exceptional situation is due to the additional symmetries increasing the dimension of the zero locus.

2. One has clearly quaternionicity at the level of o obtained by putting $Y = 0$ and at the level of the tangent space for the resulting surface. The tangent space should be quaternionic. The Jacobian of the map defined by P is such that it takes fixed quaternionic subspace $H_c \rightarrow M^4_{0,c}$

of O to a quaternionic tangent space of X^4. The Jacobian applied to the vectors of H_c gives the octonionic tangent vectors and they should span a quaternionic sub-space.

3. The notion of quaternionic surface is rigorous. $M^8 - H$ correspondence could be actually interpreted in terms of the construction of quaternionic surface in M^8. One has 4-D integrable distribution of quaternionic planes in O with given quaternion structure labelled by points of CP_2 and has representation at the level of H as space-time surface and should be preferred extremals. These quaternion planes should integrate to a slicing by 4-surfaces and their duals. One obtains this slicing by fixing the values 4 of the suitably defined octonionic coordinates P^i, $i = 1,..,8$, to a real constants depending on the surface of the slicing. This gives a space-time surfaces for which tangent space-spaces or normal spaces are quaternionic.

   The first guess for these coordinates P^i be as real or imaginary parts of real polynomials $P(o)$. But how to prove and understand this?

Could the following argument be more than wishful thinking?

1. In complex case an analytic function $w(z) = u + iv$ of $z = x + iy$ mediates a map between complex planes Z and W. One can interpret the imaginary unit appearing in w locally as a tangent vector along $u = constant$ coordinate line.

2. One can interpret also octonionic polynomials with real coefficients as mediating a map from octonionic plane O to second octonionic plane, call if W. The decomposition $P = P^{1)} + P^{2)} I_4$ would have interpretation in terms of coordinates of W with coordinate lines representing quaternions and co-quaternions.

3. This would suggests that the quaternionic coordinate lines in W can be identified as coordinate curves in O - that space-time surfaces - which are quaternionic/co-quaternionic surfaces for $P^1 = constant/P^2 = constant$ lines. One would have a representation of the same thing in two spaces, and if sameness includes also quaternionicity/co-quaternionicity as attributes, then also associativity and co-associativity should hold true.

The most reasonable approach is based on generality. Associativity/quaternionicity means a slicing of octonion space by orthogonal quaternionic and co-quaternionic 4-D surfaces defined by constant value surfaces of octonionic polynomial with real coefficients. This slicing should make sense also for quaternions: one should have a slicing by complex and co-complex (commutative/co-commutative) surfaces and in TGD string world sheets and partonic 2-surfaces assignable to Hamilton-Jacobi structure would define this kind of slicing. In the case of complex numbers one has a slicing in terms of constant value curves for real and imaginary parts of analytic function and Cauchy-Riemann equations should define the property and co-property. The first guess that the tangent space of the curve is real or imaginary is wrong.

Could associativity and commutativity conditions be seen as a generalization of Cauchy-Rieman conditions?

Quaternionicity in the octonionic case, complexity in quaternionic case, and what-ever-it-is in complex case should be seen as a 3-levelled hierarchy of geometric conditions satisfied by polynomial maps with real coefficients for polynomials in case of octonions and quaternions. Of course, also Taylor and even Laurent series might be considered. The "Whatever it is" cannot be nothing but Cauchy-Riemann conditions defining complex analyticity for complex maps.

The hierarchy looks obvious. In the case of Cauchy-Riemann conditions one has commutative and associative structure and Cauchy-Riemann conditions are linear in the partial derivatives. In the case of commutative sub-manifolds of quaternionic space the conditions are quadratic in the partial derivatives. In the case of associative sub-manifolds of octonionic space the conditions are trilinear in partial derivatives. One would have nothing but a generalization of Cauchy-Riemann equations to multilinear equations in dimensions $D = 2^k$, $k = 1, 2, 3$: k-linearity with $k = 1, 2, 3$!

One can continue the hierarchy of division algebras by assuming only algebra property by using Cayley-Dickson construction (see `http://tinyurl.com/ybuyla2k`) by adding repeatedly a non-commuting imaginary unit to the structure already obtained and thus doubling the dimension of the algebra each time. Polynomials with real coefficients should still define an associative and commutative algebra if the proposal is to make sense. All these algebras are indeed power associative: one has $x^m x^n = x^{m+n}$. For instance, sedenions define 16-D algebra. Could this hierarchy corresponds to a hierarchy of analyticities satisfying generalized Cauchy-Riemann conditions?

Complex curves in real plane cannot have real tangent space

Going from octonions to quaternions to complex numbers, could constant value curves of real and imaginary parts of ordinary analytic function in complex plane make sense? The curves $u = 0$ and $v = 0$ of functions $f(z) = u + iv$, $z = x + iy$ define a slicing of plane by orthogonal curves completely analogous to its octonionic and quaternionic variants. Can one say that the tangent vectors for these curves are real/imaginary? For $u = 0$ these curves have tangent $\partial_x u + i\partial_y u$, which is not real unless one has $f(z) = k(x + iy)$, k real.

Reality condition is clearly too strong. In fact, it is the well-ordering of the points of the 1-dimensional curve, which is the property in question and lost for complex numbers and regained at $u = 0$ and $v = 0$ curves. The reasonable interpretation is in terms of hierarchy of conditions multilinear in the gradients of coordinates proposed above and linear Cauchy-Riemann conditions is the only option in the case of complex plane. What is special in this curves that the tangent vectors define flows which by Cauchy-Riemann conditions are divergenceless and irrotational locally.

Pessimistic would conclude that since the conjecture fails except for linear polynomials in complex case, it fails also in the case of quaternions and octonions. For quaternionic polynomial q^2 the conditions are however satisfied and it turns out that the resulting conditions make sense also in the general case. Optimistic would argue that reality condition is not analogous to commutativity and associativity so that this example tells nothing. Less enthusiastic optimist might admit that the reality condition is a natural generalization to complex case but that the conjecture might be true only for a restricted set of polynomials - in complex case of for $f(z) = kz$, k real. In quaternionic and octonionic case but hopefully for a larger set of polynomials with real coefficients, maybe even all polynomials with real coefficients.

Associativity and commmutativity conditions as a generalization of Cauchy-Rieman conditions?

Quaternionicity in the octonionic case, complexity in quaternionic case, and what-ever-it-is in complex case should be seen as a 3-levelled hierarchy of geometric conditions satisfied by polynomial maps with real coefficients for polynomials in case of octonions and quaternions. Of course, also Taylor and even Laurent series might be considered. The "whatever-it-is" cannot be nothing but Cauchy-Riemann conditions defining complex analyticity for complex maps.

The hierarchy looks obvious. In the case of Cauchy-Riemann conditions one has commutative and associative structure and Cauchy-Riemann conditions are linear in the partial derivatives. In the case of commutative sub-manifolds of quaternionic space the conditions are quadratic in the partial derivatives. In the case of associative sub-manifolds of octonionic space the conditions are trilinear in partial derivatives. One would have nothing but a generalization of Cauchy-Riemann equations to multilinear equations in dimensions $D = 2^k$, $k = 1, 2, 3$: k-linearity with $k = 1, 2, 3$!

One can continue the hierarchy of number fields by assuming only algebra property by adding additional imaginary units as done in Cayley-Hamilton construction (see `http://tinyurl.com/ybuyla2k`) by adding repeatedly a non-commuting imaginary unit to the algebra already obtained and thus doubling the dimension of the algebra each time. Polynomials with real coefficients should still define an associative and commutative algebra if the proposal is to make sense. All these algebras are indeed power associative: one has $x^m x^n = x^{m+n}$. For instance, sedenions define 16-D algebra. Could this hierarchy corresponds to a hierarchy of analyticities satisfying generalized Cauchy-Riemann conditions? Could this hierarchy corresponds to a hierarchy of analyticities satisfying generalized Cauchy-Riemann conditions?

One would have also a nice physical interpretation: in the case of quaternions one would have "quaternionic conformal invariance" as conformal invariances inside string world sheets and partonic 2-surfaces in a nice agreement with basic vision about TGD. At the level of octonions would have "quaternionic conformal invariance" inside space-time surfaces and their duals. What selects the preferred commutative or co-commutative surfaces is of course an interesting problem. Is a gauge choice in question? Are these surfaces selected by some special property such as singular character? Or does one have wave function in the set of these surfaces for a given space-time surface?

Could quaternionic polynomials define complex and co-complex surfaces in H_c?

What about complex and co-complex (commutative/co-commutative) surfaces in the space of quaternions? One would have a slicing of the quaternionic space by pairs of complex and co-complex surfaces and would have natural identification as quaternion/Hamilton-Jacobi structure and relate to the decomposition of space-time to string world sheets and partonic 2-surfaces. Now the condition of associativity would be replaced with commutativity.

1. In the quaternionic case the tangent vectors of the 2-D complex sub-variety would be commuting. Can this be the case for the zero loci real polynomials $P(q)$ with $IM(P) = 0$ or $RE(P) = 0$? In this case the commutativity condition is that the tangent vectors have imaginary parts (as quaternions) proportional to each other but can have different real parts. The vanishing of cross product is the condition now and involves only two vectors whereas associativity condition involves 3 vectors and is more difficult.

2. The tangent vectors of a commutative 2-surface commute: $[t^1, t^2] = 0$. The commutator reduces to the vanishing of the cross product for the imaginary parts:

$$Im(t^1) \times Im(t^2) = 0 \ . \tag{5.4.4}$$

3. Expressing z_1^i as a function of z_2^k and using (z_1^i, z_2^k) as coordinates in quaternionic space, the tangent vectors in quaternionic spaces can be written in terms of partial derivatives $\partial z_1^{1)}/\partial z_2^{k)}$ as

$$t_k^i = (\frac{\partial z_1^{i)}}{\partial z_2^{k)}}, \delta_k^i) \ , \tag{5.4.5}$$

Here the first part corresponds to $RE(t^i)$ as quaternion and second part to $IM(t^i)$ as quaternion.

The condition that the vectors are parallel implies

$$\frac{\partial z_1^{1)}}{\partial z_2^{k)}} = 0 \ . \tag{5.4.6}$$

At the commutative 2-surface X^2 $z_1^{1)}$ is constant and $z_1^{2)}$ is a function of $z_2^{1)}$ and $z_2^{2)}$. One would have a graph of a function $z_1^{2)} = f_2(z_2^{k)})$ at X^2 but not elsewhere. One could regard $z_1^{1)}$ as an extremum of a function $z_1^{1)} = f_1(z_2^{k)})$.

How to interpret this result?

1. In the generic case this condition eliminates 1 dimension so that 2-D surface would reduce to a 1-D curve.

2. If one poses constraints on the coefficients of $P(q)$ analogous to the conditions forcing the potential function for say cusp catastrophe to have degenerate extrema at the boundaries of the catastrophe one can get 2-D solution. For these values of parameters the conditions would be equivalent with $RE(P) = 0$ or $IM(P) = 0$ conditions.

   The vanishing of the gradient of z_1^1 would indeed correspond in the case of cups catastrophe to the condition for the co-incidence of two roots of the behavior variable x as extremum of potential function $V(x, a, b)$ fixing the control variable a as function of b.

   This would pose constraints on the coefficients of P not all polynomials would be allowed. This kind of conditions would realize the idea of quantum criticality of TGD at the level of quaternion polynomials. This option is attractive if realizable also at the level of octonion polynomials. This turns out to be the case.

3. One would thus have two kinds of commutative/co-commutative surfaces. The generic 1-D surfaces and 2-D ones which are commutative/commutative and critical and assignable to string world sheets and partonic 2-surfaces. 1-D surfaces would correspond to fermion lines at the orbits of partonic 2-surfaces appearing in the twistor amplitudes in the interaction regions defined by CDS. 2-D surfaces would correspond to the orbits of fermionic strings connecting point-like fermions at their ends and serving as correlates for bound state entanglement for external fermions arriving into CD. This picture would allow also to understand why just some string world sheets and partonic 2-surfaces are selected.

The simplest manner to kill the proposal is to look for $P = q^2$ and $RE(P(q^2)) = 0$ surface. In this case this condition is indeed satisfied. One has

$$\begin{aligned}
&RE(P) = X^{1)} + X^{2)}I_1 \ , \\
&X^{1)} = (z_1^{1)})^2 - (z_1^{2)})^2 + (z_2^{1)})^2 - (z_2^{2)})^2 \ , \quad X^{2)} = 2z_1^{1)}z_1^{2)}I_1 \ , \\
&IM(P) = Y^{1)} + Y^{2)}I_1 \ , \\
&Y^{1)} = (z_2^{1)} + \overline{z_2^{1)}})z_1^{1)} \ , \qquad\qquad Y^{2)} = (z_2^{2)} + \overline{z_2^{2)}})z_1^{2)}
\end{aligned} \tag{5.4.7}$$

$X^{2)} = 0$ gives $z_1^{1)}z_1^{2)} = 0$ so that one has either $z_1^{1)} = 0$ or $z_1^{2)} = 0$. $X^{1)} = 0$ gives for $z_1^{1)} = 0$ $z_1^{2)} = \pm\sqrt{(z_2^{1)})^2 + (z_2^{2)})^2}$.

The partial derivative $\partial z_1^{1)}/\partial z_2^{k)}$ is from implicit function theorem - following from the vanishing of the differential $d(RE(P))$ along the surface - proportional $\partial X^{1)}/\partial z_2^{k)}$, but vanishes as required.

Clearly, the quaternionic variant of the proposal survives in the simplest case its simplest test. 2-D character of the surface would be due to the criticality of q^2 making it possible to satisfy the conditions without the reduction of dimension.

Explicit form of associativity/quaternionicity conditions

Consider now the explicit conditions for associativity in the octonionic case.

1. One should calculate the octonionic tangent (normal) vectors t^i for $X = 0$ in associative ($Y = 0$ in co-associative case) and show that there associators $Ass(t^i, t^j, t^k)$ vanish for all possible or all possible combinations i, j, k. In other words, one that one has

$$Ass(t^i, t^j, t^k) = 0 \ , \quad i, j, k \in \{1, .., 4\} \ , \quad Ass(a, b, c) \equiv (ab)c - a(bc) \ . \tag{5.4.8}$$

   One can cast the condition to simpler from by expressing t^i as octonionic vectors $t_k^i E^k$:

$$Ass(E^a, E^b, E^b) = \equiv f^{abcd}E_d \ \ , \ \ a,b,c,d \in \{1,..,7\} \ \ ,$$
$$f^{abcd} = \epsilon^{abe}\epsilon_e^{\ cd} - \epsilon^{aed}\epsilon^{bc}_{\ \ e} = 2\epsilon^{abe}\epsilon_e^{\ cd} \ \ . \tag{5.4.9}$$

The permutation symbols for a given triplet i,j,k are structures constants for quaternionic inner product and completely antisymmetric (see `http://tinyurl.com/p42tqsq`).. $\epsilon_{ijk} = 1$ is true for the seven triplets $123, 145, 176, 246, 257, 347, 365$ defining quaternionic sub-spaces with 1-D intersections. The anti-associativity condition $(E_iE_j)E_k = -(E_iE_j)E_k$ holds true so that one has obtains the simpler expression for f^{ijks} having values ± 2.

Using this representation $Ass(t^i, t^j, t^k)$ reduces to 7 conditions for each triplet:

$$t^i_r t^j_s t^k_t f^{rstu} = 0 \ \ , \ \ i,j,k \in \{1,..,4\} \ \ , \ \ r,s,t,u \in \{1,..,7\} \ \ . \tag{5.4.10}$$

2. If the vanishing condition $X = 0$ or $Y = 0$ is crucial for associativity then every polynomial is its own case to be studied separately and a general principle behind associativity should be identified: the proposal is as a non-linear generalization of Cauchy-Riemann conditions. As the following little calculation shows, the vanishing condition indeed appears as one calculates partial derivatives $\partial z_1^{k)}/\partial z_2^{l)}$ in the expression for the tangent vectors of the surface deduced from the vanishing gradient of X or Y.

3. I have proposed the octonionic polynomial ansatz already earlier but failed to prove that it gives associative tangent or normal spaces. Besides the intuitive geometric argument I failed to notice that the complex 8-D tangent vectors in coordinates $z_1^{k)}$ or $z_2^{k)}$ for complexified space-time surface and coordinates $(z_1^{k)}, z_2^{k)})$ for o have components

$$\frac{\partial o^i}{\partial z^1_k} \leftrightarrow (\delta^i_k, \frac{\partial z_2^{i)}}{\partial z_1^{k)}})$$

or

$$(\frac{\partial o^i}{\partial z^2_k}) \leftrightarrow (\frac{\partial z_1^{i)}}{\partial z_2^{k)}}, \delta^i_k) \ \ . \tag{5.4.11}$$

These vectors correspond to complexified octonions O_i given by

$$\delta^i_k E^k + \frac{\partial z_2^{i)}}{\partial z_1^{k)}} E^k E_4 \ \ , \tag{5.4.12}$$

where the unit octonions are given by $(E_0, E_1, E_2, E_3) = (1, I_1, I_2, I_3)$ and $(E_5, E_5, E_7, E_8) = (1, I_1, I_2, I_3)E_4$. The vanishing of the associators stating that one has

4. One can calculate the partial derivatives $\frac{\partial z_i^k}{\partial z_j^k}$ explicitly without solving the equations or the complex valued quaternionic components of $RE(P) \equiv X = 0$ or $IM(P) \equiv Y = 0$ (note that X and Y have for complex components labelled by X^i and Y^i respectively.

$$Y^i(z_1^{k)},z_2^{l)}) = c \in R \ , \quad i=1,...,4 \ , \quad \text{associativity} \ ,$$

or

$$X^i(z_1^{k)},z_2^{l)}) = c \in R \ , \quad i=1,...,4 \ , \quad \text{co-associativity} \ . \tag{5.4.13}$$

explicitly and check whether associativity holds true. The derivatives can be deduced from the constancy of Y or X.

5. For instance, if one has $z_2^{k)}$ as function of $z_1^{k)}$, one obtains in the associative case

$$RE(Y)^i{}_k + IM(Y)^i{}_k \frac{\partial z_2^{r)}}{\partial z_1^{k)}} = 0$$

$$RE(Y)^i{}_k \equiv \frac{\partial Y^i}{\partial z_1^{k)}} \ , \qquad IM(Y)^i{}_k \equiv \frac{\partial Y^i}{\partial z_2^{k)}} \ . \tag{5.4.14}$$

In co-associative case one must consider normal vectors expressible in terms of Y^i so that X is replaced with Y in these equations.

This allows to solve the partial derivatives needed in associator conditions

$$\frac{\partial z_2^{i)}}{\partial z_1^{k)}} = \left[Im(Y)^{-1}\right]^i{}_r Re(Y)^r{}_k \ . \tag{5.4.15}$$

6. The vanishing conditions for the associators are however multilinear and one can multiply each factor by the matrix $IM(P)$ without affecting the condition so that $IM(P)^{-1}$ disappears and one obtains the conditions for vectors

$$T_r^i T_s^j T_t^k f^{rstu} = 0 \ , \qquad i,j,k \in \{1,..,4\} \ , \quad r,s,t,u \in \{1,..,7\} \ ,$$

$$T^i = IM(Y)^i{}_k E^k - RE(Y)^i{}_k E^k E_4 \ . \tag{5.4.16}$$

This form of conditions is computationally much more convenient.

How to solve these equations?

1. The antisymmetry of f^{rstu} with respect to the first two indices r,s leads one to ask whether one could have

$$T_r^i T_s^j T_t^k = 0 \tag{5.4.17}$$

for the 7 quaternionic triplets. This is guaranteed if one has either $RE(Y)^i{}_k = \partial Y^i/\partial z_1^k = 0$ (coquaternionic part of T^i) or $IM(Y)^i{}_k = \partial Y^i/\partial z_2^k = 0$ (co-quaternionic part of T^i) for *one* member in each triplet.

The study of the structure constants listed above shows that indices 1,2,3 are contained in all 7 triplets. Same holds for the indices appearing in any quaternionic triplet. Hence it is enough to require that three gradients $RE(Y)^i k = 0$ or $IM(Y)^i{}_k = 0$ $k \in \{1, 2, 3\}$ vanish. This condition is obviously too strong since already single vanishing condition reduces the dimension of space-time variety to 3 in the generic case and it becomes trivially associative.

Octonionic automorphism group G_2 gives additional basis with their own quaternion triplets and the general condition would be that 3 partial derivatives vanish for a triplet obtained from the basic triplet $\{1, 2, 3\}$ by G_2 transformation. It is not quite clear to me whether the G_2 transformation can depend on position on space-time surface.

2. As noticed, the vanishing of all triplets is an un-necessarily strong condition. Already the vanishing of single gradient $RE(Y)^i{}_k$ or $IM(Y)^i{}_k$ reduces the dimension of the surface from 4 to 3 in the generic case. If one accepts that the dimension of associative surface is lower than 4 then single criticality condition is enough to obtain 3-D surface.

   In the generic case associativity holds true only at the 4-D tangent spaces of 3-surfaces at the ends of CD (at light-like partonic orbits it holds true trivially in 4-D) and that the twistor lift of Kähler action determines the space-time surfaces in their interior.

   In this case one can map only the boundaries of space-time surface by $M^8 - H$ duality to H. The criticality at these 3-surfaces dictates the boundary conditions and provides a solution to infinite number of conditions stating the vanishing of SSA Noether charges at space-like boundaries. These space-time regions would correspond to the regions of space-time surfaces inside CDs identifiable as interaction regions, where Kähler action and volume term couple and dynamics depends on coupling constants.

   The mappability of M^8 dynamics to H dynamics in all space-time regions does not look feasible: the dynamics of octonionic polynomials involves no coupling constants whereas twistor lift of Kähler action involves couplings parameters. The dynamics would be non-associative in the geometric sense in the interior of CDs. Notice that also conformal field theories involve slight breaking of associativity and that octonions break associativity only slightly ($a(bc) = -(ab)c$ for octonionic imaginary units). I have discussed the breaking of associativity from TGD viewpoint in [K61] .

3. Twistor lift of Kähler action allows also space-time regions, which are minimal surfaces [L12] and for which the coupling between Kähler action and volume term vanishes. Preferred extremal property reduces to the existence of Hamilton-Jacobi structure as image of the quaternionic structure at the level of M^8. The dynamics is universal as also critical dynamics and independent of coupling constants so that $M^8 - H$ duality makes sense for it. External particles arriving into CD via its boundaries would correspond to critical 4-surfaces: I have discussed their interpretation from the perspective of physics and biology in [L14].

4. One should be able to produce associativity without the reduction of dimension. One can indeed hope of obtaining 4-D associative surfaces by posing conditions on the coefficients of the polynomial P by requiring that one $RE(Y)^i_k$ or $IM(Y)^i_k$, $i = i_1$ -call it just X_1 - should vanish so that Y^i would be critical as function of z_1^k or z_2^k.

   At $X_1 = 0$ would have degenerate zero at the 4-surface. The decomposition of X_1 to a product of monomial factors with root in extension of rationals would have one or more factors appearing at least twice. The associative 4-surfaces would be ramified. Also the physically interesting p-adic primes are conjectured to be ramified in the sense that their decomposition to primes of extension of rationals contains powers of primes of extension. The ramification of the monomial factors is nothing but ramification for polynomials primes in field of rationals in terms of polynomial primes in its extension.

   This could lead to vanishing of say one triplet while keeping $D = 4$. This need not however give rise to associativity in which case also second $RE(Y)^i_i$ or $IM(Y)^i_k$, $i = i_2$, call it X_2, should vanish. The maximal number of X_i would be $n_{max} = 3$. The natural condition consistent with quantum criticality of TGD Universe would be that the variety is associative but maximally quantum critical and has therefore dimension $D = 3$ or $D = 4$. Stronger condition allows only $D = 4$.

These $n \leq 3$ additional conditions make the space-time surface analogous to a catastrophe with $n \leq 3$ behavior variables in Thom's classification of 7 elementary catastrophes with less than 11 control variables [A37]. Thom's theory does not apply now since it has only one potential function $V(x)$ (now $n \leq 3$ corresponding to the critical coordinates Y^i!) as a function of behaviour variables and control variables). Also the number of non-vanishing coefficients in the polynomial having values in an extension of rationals and acting as control variables is unlimited. In quaternionic case the number of potential functions is indeed 1 but the number of control variables unlimited.

5. One should be able to understand the $D = 3$ associative objects - say light-like 3-surfaces or 3-surfaces at the boundaries of CD - as 3-surfaces along which 4-D associative (co-associative) and non-associative (non-co-associative) surfaces are glued together.

   Consider a product P of polynomials allowing 3-D surfaces as necessarily associative zero loci to which a small interaction polynomial vanishing at the boundaries of CD (proportional to o^n, $n > 1$) is added. Could P allow 4-D surface as a zero locus of real or imaginary part and containing the light-like 3-surfaces thanks to the presence of additional parameters coming from the interaction polynomial. Can one say that this small interaction polynomial would generate 4-D space-time in some sense? 4-D associative space-time regions would naturally emerge from the increasing algebraic complexity both via the increase of the degree of the polynomial and the increase of the dimension of the extension of rationals making it easier to satisfy the criticality conditions!

   There are two regions to be considered: the interior and exterior of CD. Could associativity/co-associativity be possible outside CD but not inside CD so that one would indeed have free external particles entering to the non-associative interaction region. Why associativity conditions would be more difficult to satisfy inside CD? Certainly the space-likeness of M^4 points with respect to the preferred origin of M^8 in this region should be crucial since Minkowski norm appears in the expressions of $RE(P)$ and $IM(P)$.

Do the calculations for the associative case generalize to the co-associative case?

1. Suppose that one has possibly associative surface having $RE(P) = 0$. One would have $IM(P) = 0$ for dual space-time surface defining locally normal space of $RE(P) = 0$ surface. This would transform the co-associativity conditions to associativity conditions and the preceding arguments should go through essentially as such.

   Associative and co-associative surfaces would meet at singularity $RE(P) = IM(P) = 0$, which need not be point in Minkowskian signature (see $P = o^2$ example in the Appendix) and can be even 4-D! This raises the possibility that the associative and co-associative surfaces defined by $RE(P) = 0$ and $IM(P) = 0$ meet along 3-D light-like orbits partonic surfaces or 3-D ends of space-time surfaces at the ends of CD.

2. If $D = 3$ for associative surfaces are allowed besides $D = 4$ as boundaries of 4-surfaces, one can ask why not allow $D = 5$ for co-associative surfaces. It seems that they do not have a reasonable interpretation as a surface at which co-associative and non-co-associative 4-D space-time regions would meet. Or could they in some sense be geometric "co-boundaries" of 4-surfaces like branes in M-theory serve as co-boundaries of strings? Could this mean that 4-D space-time-surface is boundary of 5-D co-associative surface defining a TGD variant of brane with strings world sheets replaced with 4-D space-time surfaces?

   What came as a surprise that $P = o^2$ allows 5-D and 6-D surfaces as zero loci of $RE(P)$ or $IM(P)$ as shown in Appendix. The vanishing of the entire o^2 gives 4-D interior or exterior of CD forced also by associativity/co-associativity and thus maximal quantum criticality. This is very probably due to the special properties of o^2 as polynomial: in the generic case the zero loci should be 4-D.

This discussion can be repeated for complex/co-complex surfaces inside space-time surfaces associated with fermionic dynamics.

1. Associativity condition does not force string world sheets and partonic 2-surfaces but they could naturally correspond to commutative or co-commutative varieties inside associative/co-associative varieties.

   In the generic case commutativity/co-commutativity allows only 1-D curves - naturally light-like fermionic world lines at the boundaries of partonic orbits and representing interacting point-like fermions inside CDs and used in the construction of twistor amplitudes [K49, L41]. There is coupling between Kähler part and volume parts of modified Dirac action inside CDs so that coupling constants are visible in the spinor dynamics and in dynamics of string world sheet.

2. At criticality one obtains 2-D commutative/co-commutative surfaces necessarily associated with external particles quantum critical in 4-D sense and allowing quaternionic structure. String world sheets would serve as correlates for bound state entanglement between fermions at their ends. Criticality condition would select string world sheets and partonic 2-surfacs from the slicing of space-time surface provided by quaternionic structure (having Hamilton-Jacobi structure as H-counterpart).

If associativity holds true and fixed M_c^2 is contained in the tangent space of space-time surface, one can map the M^4 projection of the space-time surface to a surface in $M^4 \times CP_2$ so that the quaternionic tangent space at given point is mapped to CP_2 point. One obtains 4-D surface in $H = M^4 \times CP_2$.

1. The condition that fixed M_c^2 belongs to the tangent space of X_c^4 is true in the sense that the coordinates $z_2^{k)}$ do not depend on $z_1^{1)}$ and $z_1^{2)}$ defining the coordinates of M_c^2. It is not clear whether this condition can be satisfied in the general case: octonionic polynomials are expected to imply this dependence un-avoidably.

   The more general condition allows M_c^2 to depend on position but assumes that M_c^2:s associated with different points integrate to a family 2-D surfaces defining a family of complexified string world sheets. In the similar manner the orthogonal complements E_c^2 of M_c^2 would integrate to a family of partonic 2-surfaces. At each point these two tangent spaces and their real projections would define a decomposition analogous to that define by light-like momentum vector and polarization vector orthogonal to it. This decomposition would define decomposition of quaternionic sub-spaces to complexified complex subspace and its co-complex normal space. The decomposition would correspond to Hamilton-Jacobi structure proposed to be central aspect of extremals [K10].

2. What is nice that this decomposition of M_c^4 (M^4) would (and of course should!) follow automatically from the octonionic decomposition. This decomposition is lower-dimensional analog to that of the complexified octonionic space induced by level sets of real octonionic polymials but at lower level and extremely natural due to the inclusion hierarchy of classical number fields. Also M_c^2 could have similar decomposition orthogonal complex curves by the value sets of polynomials. The hierarchy of grids means the realization of the coordinate grid consisting of quaternionic, complex, and real curves for complexified coordinates o^k and their quaternionic and complex variants and is accompanied by corresponding real grids obtained by projecting to M^4 and mapping to CP_2.

   Thus these decompositions would be obtained from the octonionic polynomial decomposing it to real quaternionic and imaginary quaternionic parts first to get a grid of space-time surfaces as constant value sets of either real or imaginary part, doing the same for the non-constant quaternionic part of the octonionic polynomial to get similar grid of complexified 2-surfaces, and repeating this for the complexified complex octonionic part.

Unfortunately, I do not have computer power to check the associativity directly by symbolic calculation. I hope that the reader could perform the numerical calculations in non-trivial cases to this!

General view about solutions to $RE(P) = 0$ and $IM(P) = 0$ conditions

The first challenge is to understand at general level the nature of $RE(P) = 0$ and $IM(P) = 0$ conditions. Appendix shows explicitly for $P(o) = o^2$ that Minkowski signature gives rise to unexpected phenomena. In the following these phenomena are shown to be completely general but not quite what one obtains for $P(o) = o^2$ having double root at origin.

1. Consider first the octonionic polynomials $P(o)$ satisfying $P(0) = 0$ restricted to the light-like boundary δM^8_+ assignable to 8-D CD, where the octonionic norm of o vanishes.

   (a) $P(o)$ reduces along each light-ray of δM^8_+ to the same real valued polynomial $P(t)$ of a real variable t apart from a multiplicative unit $E = (1 + in)/2$ satisfying $E^2 = E$. Here n is purely octonion-imaginary unit vector defining the direction of the light-ray.
   $IM(P) = 0$ corresponds to quaterniocity. If the E^4 ($M^8 = M^4 \times E^4$) projection is vanishing, there is no additional condition. 4-D light-cones $M^4_\pm$ are obtained as solutions of $IM(P) = 0$. Note that $M^4_\pm$ can correspond to any quaternionic subspace.
   If the light-like ray has a non-vanishing projection to E^4, one must have $P(t) = 0$. The solutions form a collection of 6-spheres labelled by the roots t_n of $P(t) = 0$. 6-spheres are not associative.

   (b) $RE(PE) = 0$ corresponding to co-quaternionicity leads to $P(t) = 0$ always and gives a collection of 6-spheres.

2. Suppose now that $P(t)$ is shifted to $P_1(t) = P(t) - c$, c a real number. Also now $M^4_\pm$ is obtained as solutions to $IM(P) = 0$. For $RE(P) = 0$ one obtains two conditions $P(t) = 0$ and $P(t - c) = 0$. The common roots define a subset of 6-spheres which for special values of c is not empty.

The above discussion was limited to δM^8_+ and light-likeness of its points played a central role. What about the interior of 8-D CD?

1. The natural expectation is that in the interior of CD one obtains a 4-D variety X^4. For $IM(P) = 0$ the outcome would be union of X^4 with M^4_+ and the set of 6-spheres for $IM(P) = 0$. 4-D variety would intersect M^4_+ in a discrete set of points and the 6-spheres along 2-D varieties X^2. The higher the degree of P, the larger the number of 6-spheres and these 2-varieties.

2. For $RE(P) = 0$ X^4 would intersect the union of 6-spheres along 2-D varieties. What comes in mind that these 2-varieties correspond in H to partonic 2-surfaces defining light-like 3-surfaces at which the induced metric is degenerate.

3. One can consider also the situation in the complement of 8-D CD which corresponds to the complement of 4-D CD. One expects that $RE(P) = 0$ condition is replaced with $IM(P) = 0$ condition in the complement and $RE(P) = IM(P) = 0$ holds true at the boundary of 4-D CD.

6-spheres and 4-D empty light-cones are special solutions of the conditions and clearly analogs of branes. Should one make the (reluctant-to-me) conclusion that they might be relevant for TGD at the level of M^8.

1. Could M^4_+ (or CDs as 4-D objects) and 6-spheres integrate the space-time varieties inside different 4-D CDs to single connected structure with space-time varieties glued to the 6-spheres along 2-surfaces X^2 perhaps identifiable as pre-images of partonic 2-surfaces and maybe string world sheets? Could the interactions between space-time varieties X^4_i assignable with different CDs be describable by regarding 6-spheres as bridges between X^4_i having only a discrete set of common points. Could one say that X^2_i interact via the 6-sphere somehow. Note however that 6-spheres are not dynamical.

2. One can also have Poincare transforms of 8-D CDs. Could the description of their interactions involve 4-D intersections of corresponding 6-spheres?

3. 6-spheres in $IM(P) = 0$ case do not have image under $M^8 - H$ correspondence. This does not seem to be possible for $RE(P) = 0$ either: it is not possible to map the 2-D normal space to a unique CP_2 point since there is 2-D continuum of quaternionic sub-spaces containing it.

5.4.3 $M^8 - H$ duality: objections and challenges

In the following I try to recall all objections against the reduction of classical physics to octonionic algebraic geometry and against the notion of $M^8 - H$ duality and also invent some new counter arguments and challenges.

Can on really assume distribution of $M^2(x)$?

Hamilton-Jacobi structure means that $M^2(x)$ depends on position and $M^2(x)$ should define an integrable distribution integrating to a 2-D surface. For cosmic string extremals this surface would be minimal surface so that the term "string world sheet" is appropriate. There are objections.

1. It seems that the coefficients of octonionic polynomials cannot contain information about string world sheet, and the only possible choice seems to be that string world sheets and partonic 2-surfaces parallel to it assigned with integrable distribution of orthogonal complements $E^2(x)$ should be coded by quaternionic moduli. It should be possible to define quaternionic coordinates q_i decomposing to pairs of complex coordinates to each choice of $M^2(x) \times E^2(x)$ decomposition of given M_0^4. Octonionic coordinates would be given by $o = q_1 + q_2 I_4$ where q_i are associated with the same quaternionic moduli. The choice of Hamilton-Jacobi structure would not be ad hoc procedure anymore but part of the definition of solutions of field equations at the level of M^8.

2. It would be very nice if the quaternionic structure could be induced from a fixed structure defined for M_c^8 once the choice of curvilinear M^4 coordinates is made. Since Hamiltoni-Jacobi structure [K10] involves a choice of generalized Kähler form for M^4 and since quaternionic structure means that there is full S^2 of Kähler structures determined by quaternionic imaginary units (ordinary Kähler form for sub-space $E^8 \subset M_c^8$) the natural proposal is that Hamilton-Jacobi structures is determined by a particular local choice of the Kähler form for M^4 involving fixing of quaternionic imaginary unit fixing $M^2(x) \subset M_0^4$ identifiable as point of S^2. This might relate closely also to the fixing of twistor structure, which indeed involves also self-dual Kähler form and a similar choice.

3. One can argue that it is not completely clear whether massless extremals (MEs) [K10] allow a general Hamilton-Jacobi structure. It is certainly true that if the light-like direction and orthogonal polarization direction are constant, MEs exist. It is clear that if the form of field equations is preserved and thus reduces to contractions of various tensors with second fundamental form one obtains only contractions of light-like vector with itself or polarization vector and these contractions vanish. For spatially varying directions one could argue that light-like direction codes for a direction of light-like momentum and that problems with local conservation laws expressed by field equations might emerge.

Can one assign to the tangent plane of $X^4 \subset M^8$ a unique CP_2 point when M^2 depends on position

One should show that the choice $s(x) \in CP_2$ for a given distribution of $M^2(x) \subset M^4(x)$ is unique in order to realize the $M^8 - H$ correspondence as a map $M^8 \to H$. It would be even better if one had an analytic formula for $s(x)$ using tangent space-data for $X^4 \subset H$.

1. If $M^2(x) = M_0^2$ holds true but the tangent space $M^4(x)$ depends on position, the assignment of CP_2 point $s(x)$ to the tangent space of $X^4 \subset M^8$ is trivial. When $M^4(x)$ is not constant, the situation is not so easy.

2. The space $M^2(x) \subset M^4(x)$ satisfies also the constraint $M^2(x) \subset M_0^4$ since quaternionic moduli are fixed. To avoid confusion notice that $M^4(x)$ denotes tangent space of X^4 and is different from M_0^4 fixing the quaternionic moduli.

3. $M^2(x)$ determines the local complex subspace and its completion to quaternionic tangent space $M^4(x)$ determines a point $s(x)$ of CP_2. The idea is that M_0^2 defines a standard reference and that one should be able to map $M^2(x)$ to M_0^2 by G_2 automorphism mapping also the $s(x)$ to a unique point $s_0(x) \in CP_2$ defining the CP_2 point assignable to the point of $X^4 \subset M^8$.

4. One can assign to the point x quaternionic unit vector $n(x)$ determining $M^2(x)$ as the direction of the preferred imaginary unit. The G_2 transformation must rotate $n(x)$ to n_0 defining M_0^2 and acts on s. G_2 transformation is not unique since u_1gu_2 has the same effect for $u_i \subset U(2)$ leaving invariant the point of CP_2 for initial and final situation. Hence the equivalence classes of transformations should correspond to a point of 6-dimensional double coset space $U(2)\backslash G_2/U(2)$. Intuitively it seems obvious that the $s_0(x)$ is unique but proof is required.

What about the inverse of $M^8 - H$ duality?

$M^8 - H$ duality should have inverse in the critical regions of $X^4 \subset M^8$, where associativity conditions are satisfied. How could one construct the inverse of $M^8 - H$ duality in these regions? One should map space-time points $(m, s) \in M^4 \times CP_2$ to points $(m, e) = (m, f(m, s)) \in M^8$. $M_0^4 \supset M_0^2$ parameterized by CP_2 point can be chosen arbitrarily and one can require that it corresponds to some space-time point $(m_0, s_0) \in H$. CP_2 point $s(x)$ characterizes the quaternionic tangent space containing $M^2(x)$ and can choose M_0^2 to be $M^2(x_0)$ for conveniently chosen x_0. Coordinates x can be used also for $X^4 \subset M^8$.

One obtains set of points $(m, e) = (m(x), f(m(x), s(x)) \in M^8$ and a distribution of 4-D spaces of labelled by $s(x)$. This requires that the 4-D tangent space spanned by the gradients of $m(x)$ and $f(m(x), s(x))$ and characterized by $s_1 \subset CP_2$ for given $M^2(x)$ by using the above procedure mapping the situation to that for M_0^2 is same as the tangent space determined by $s(x)$: $s(x) = s_1(x)$. Also the associativity conditions should hold true. One should have a formula for s_1 as function of tangent vectors of space-time surface in M^8. The ansatz based on algebraic geometry in M_c^8 should be equivalent with this ansatz. The problem is that the ansatz leads to algebraic functions which cannot be found explicitly. It might be that in practice the correspondence is easy only in the direction $M^8 \to H$.

What one can say about twistor lift of $M^8 - H$ duality?

One can argue that the twistor spaces CP_1 associated with M^4 and E^4 are in no way visible in the dynamics of octonion polynomials and in $M^8 - H$ duality. Hence one could argue that they are not needed for any reasonable purpose. I cannot decide whether this is indeed the case. There I will consider the existence of twistor lift of the M^8 and also the twistor lift $M^8 - H$ duality in the space-time regions, where the tangent spaces satisfy the conditions for the existence of the duality as a map $(m, e) \in M^8 \to (m, s) \in M^4 \times CP_2$ must be considered. This involves some non-trivial delicacies.

1. The twistor bundles of M_c^4 and E_c^4 would be simply $M_c^4 \times CP_1$ and $E_c^4 \times CP_1$. $CP_1 = S^2$ parameterizes direction vectors in 3-D Euclidian space having interpretation as unit quaternions so that this interpretation might make sense. The definition of twistor structure means a selection of a preferred quaternion unit and its representation as Kähler form so that these twistor bundles would have thus Kähler structure. Twistor lift replaces complex quaternionic surfaces with their twistor spaces with induced twistor structure.

2. In M^8 the radii of the spheres CP_1 associated with M^4 and E^4 would be most naturally identical whereas in $M^4 \times CP_2$ they can be different since CP_2 is moduli space. Is the value of the CP_2 radius visible at all in the classical dynamics in the critical associative/co-associative space-time regions, where one has minimal surfaces. Criticality would suggest that besides coupling constants also parameters with dimension of length should disappear from the field equations. At least for the known extremals such as massless extremals, CP_2 type extremals, and cosmic strings CP_2 radius plays no role in the equations. CP_2 radius comes however into play only in interaction regions defined by CDs since $M^8 - H$ duality works only at the

3-D ends of space-time surface and at the partonic orbits. Therefore the different radii for the CP_1 associated with CP_2 and E^4 cause no obvious problems.

Consider now the idea about twistor space as real part of octonionic twistor space regarded as quaternion-complex space.

1. One can regard $CP_1 = S^2$ as the space of unit quaternions and it is natural to replace it with the 6-sphere S^6 of octonionic imaginary units at the level of complexified octonions. The sphere of complexified (by i) unit octonions is non-compact space since the norm is complex valued and this generalization looks neither attractive nor necessary since the projection to real numbers would eliminate the complex part.

   The equations determining the twistor bundle of space-time surface can be indeed formulated as vanishing of the quaternionic imaginary part of S^6 coordinates, and one obtains a reduction to quaternionic sphere S^2 at space-time level.

   If S^2 is identified as sub-manifold $S^2 \subset S^6$, it can be chosen in very many ways (this is of course not necessary). The choices are parameterized by $SO(7)/SO(3) \times SO(4)$ having dimension $D = 12$. This choice has no physical content visible at the level of H. Note that the Kähler structure determining Hamilton-Jaboci structure is fixed by the choice of preferred direction $(M^2(x))$. If all these moduli are allowed, it seems that one has something resembling multiverse, the description at the level of M^8 is deeper one and one must ask whether the space-time surfaces with different twistorial, octonionic, and quaternionic moduli can interact.

2. The resulting octonionic analog of twistor space should be mapped by $M^8 - H$ corresponds to twistor space of space-time surface $T(M^4) \times T(CP_2)$. The radii of twistor spheres of $T(M^4)$ and $T(CP_2)$ are different and this should be also understood. It would seem that the radius of $T(M^4)$ at $H = M^4 \times CP_2$ side should correspond to that of $T(M^4)$ at M^8 side and thus to that of S^6 as its geodesic sphere: Planck length is the natural proposal inspired by the physical interpretation of the twistor lift. The radius of $T(CP_2)$ twistor sphere should correspond to that of CP_2 and is about 2^{12} Planck lengths.

   Therefore the scale of CP_2 would emerge as a scale of moduli space and does not seem to be present at the level of M^8 as a separate scale. M^8 level would correspond to what might be called Planckian realm analogous to that associated with strings before dynamical compactification which is now replaced with number theoretic compactification. The key question is what determines the ratio of the radii of CP_2 scale to Planck for which favored value is 2^{12} [K13]. Could quantum criticality determine this ratio?

5.5 Appendix: o^2 as a simple test case

Octonionic polynomial o^2 serves as a simple testing case. o^2 is not irreducible so that its properties might not be generic and it might be better to study polynomial of form $P(o) = o + po^2$ instead.

Before continuing, some conventions are needed.

1. The convention is that in $M^8 = M^1 \times E^7$ E^7 corresponds to purely imaginary complexified octonions in both octonionic sense and in the sense that they are proportional to i. M^1 corresponds to octonions real in both senses. This corresponds to the signature $(1, -1, -1, -1, ...)$ for M^8 metric obtained as restriction of complexified metric. For $M^4 = M^1 \times E^3$ analogous conventions hold true.

2. Conjugation $o = o_0 + o_k I_k \rightarrow \overline{o} \equiv o_0 - I_k o_k$ does not change the sign of i. Quaternions can be decomposed to real and imaginary parts and some notation is needed. The notation $q = Re(q) + Im(q)$ seems to be the least clumsy one: here $Im(q)$ is 3-vector.

The explicit expression in terms of quaternionic decomposition $o = q_1 + q_2 I_4$ reads as

$$P(o) = o^2 = q_1^2 - q_2\overline{q}_2 + (q_1 q_2 + q_2\overline{q}_1)I_4 \quad . \tag{5.5.1}$$

o corresponds to complexified octonion and there are two options concerning the interpretation of M^4 and E^4. M^4 could correspond to quaternionic or co-quaternionic sub-space. I have assumed the first interpretation hitherto but actually the identification is not obvious. This two cases are different and must be treated both.

With these notations quaternionic inner product reads as

$$\begin{aligned}
&q_1q_2 = Re(q_1q_2) + Im(q_1q_2) \ , \\
&Re(q_1q_2) = Re(q_1)Re(q_2) - Im(q_1)\cdot Im(q_2) \ , \\
&Im(q_1q_2) = Re(q_1)Im(q_2) + Re(q_2)Im(q_1) + Im(q_1)\times Im(q_2) \ .
\end{aligned} \tag{5.5.2}$$

Here $a\cdot b$ denotes the inner product of 3-vectors and $a\times b$ their cross product.

Note that one has real and imaginary parts of octonions as two quaternions and real and imaginary parts of quaternions. To avoid confusion, I will use RE and IM to denote the decomposition of octonions to quaterions and Re and Im for the decomposition of quaternions to real and imaginary parts.

One can express the $RE(o^2)$ as

$$\begin{aligned}
&RE(o^2) \equiv X \equiv q_1^2 - q_2\overline{q}_2 \ , \\
&Re(X) = Re(q_1)^2 - Im(q_1)\cdot Im(q_2) - (Re(q_2)^2 + Im(q_2)\cdot Im(q_2)) \ , \\
&Im(X) = Im(q_1^2) = 2Re(q_1)Im(q_1) \ .
\end{aligned} \tag{5.5.3}$$

For $IM(o^2)$ one has

$$\begin{aligned}
&IM(o^2) \equiv Y = q_1q_2 + q_2\overline{q}_1 \\
&Re(Y) = 2Re(q_1)Re(q_2) \ , \\
&Im(Y) = Re(q_1)Im(q_2) - Re(q_2)Im(q_1) + Im(q_1)\times Im(q_2) \ .
\end{aligned} \tag{5.5.4}$$

The essential point is that only $RE(o^2)$ contains the complexified Euclidian norm $q_2\overline{q_2}$ which becomes Minkowskian of Euclidian norm depending on whether one identifies M^4 as associative or co-associative surface in o_c^8.

5.5.1 Option I: M^4 is quaternionic

Consider first the condition $RE(o^2) = 0$. The condition decomposes to two conditions stating the vanishing of quaternionic real and imaginary parts:

$$\begin{aligned}
&Re(X) = Re(q_1)^2 - Im(q_1)\cdot Im(q_2) - (Re(q_2)^2 + Im(q_2)\cdot Im(q_2)) \equiv N_{M^4}(q_1) - N_{E^4}(q_2) = 0 \ , \\
&Im(X) = Im(q_1^2) = 2Re(q_1)Im(q_1) = 0 \ .
\end{aligned} \tag{5.5.5}$$

$Im(X) = 0$ is satisfied for $Re(q_1) = 0$ or $Im(q_1) = 0$ so that one has two options. This gives 1-D line in time direction of 3-D hyperplane as a solution for M^4 factor.

$Re(X) = 0$ states $N_{M^4}(q_1) = N_{E^4}(q_2)$. q_2 coordinate itself is free. $N_{E^4}(q_2)$ is negative so that q_1 must be space-like with respect to the N_{M^4} so that only the solution $Re(q_1) = 0$ is possible. Therefore one has $Re(q_1) = 0$ and $N_{M^4}(q_1) = N_{E^4}(q_2)$.

One can assign to each E^4 point a section of hyperboloid with $t = 0$ hyper-plane giving a sphere and the surface is 6-dimensional sphere bundle like variety! This is completely unexpected result and presumably is due to the additional accidental symmetries due to the octonionicity. Also the fact that o^2 is not irreducible polynomial is a probably reason since for o the surface is 4-D. The addition of linear term is expected to remove the degeneracy.

Consider next the case $IM(o^2) = 0$. The conditions read now as

$$
\begin{aligned}
Re(Y) &= 2Re(q_1)Re(q_2) = 0 \ , \\
Im(Y) &= Re(q_1)Im(q_2) - Re(q_2)Im(q_1) + Im(q_1)\times Im(q_2) = 0 \ .
\end{aligned}
\tag{5.5.6}
$$

Since cross product is orthogonal to the factors $Im(Y) = 0$ condition requires that $Im(q_1)$ and $Im(q_2)$ are parallel vectors: $Im(q_1) = \lambda Im(q_2)$ and one has the condition $Re(q_1) = \lambda Re(q_2)$ implying $q_1 = \Lambda q_2$. Therefore to each point of E^4 is associated a line of M^4. The surface is 5-dimensional.

It is interesting to look what the situation is if both conditions are true so that one would have a singularity. In this case $Re(q_1) = 0$ and $Re(q_1) = \lambda Re(q_2)$ imply $\lambda = 0$ so that $q_1 = 0$ is obtained and the solution reduces to 4-D E^4, which would be co-associative.

5.5.2 Option II: M^4 is co-quaternionic

This case is obtained by the inspection of the previous calculation by looking what changes the identification of M^4 as co-quaternionic factor means. Now q_1 is Euclidian and q_2 Minkowskian coordinate and $q_2\overline{q}_2$ gives Minkowskian rather than Euclidian norm.

Consider first $RE(o^2) = 0$ case.

$$
\begin{aligned}
Re(X) &= Re(q_1)^2 - Im(q_1)\cdot Im(q_2) - (Re(q_2)^2 + Im(q_2)\cdot Im(q_2)) \equiv N_{M^4}(q_1) - N_{M^4}(q_2) = 0 \ , \\
Im(X) &= Im(q_1^2) = 2Re(q_1)Im(q_1) = 0 \ .
\end{aligned}
\tag{5.5.7}
$$

$N_{M^4}(q_1) - N_{M^4}(q_2) = 0$ condition holds true now besides the condition $Re(q_1) = 0$ or $Im(q_1) = 0$ so that one has also now two options.

1. For $Re(q_1) = 0$ $N_{M^4}(q_1)$ is non-positive and this must be the case for $N_{M^4}(q_2))$ so that the *exterior* of the light-cone is selected. In this case the points of M^4 with fixed N_{M^4} give rise to a 2-D intersection with $Re(q_1) = 0$ hyper-plane that is sphere so that one has 6-D surface, kind of sphere bundle.

2. For $Im(q_1) = 0$ Minkowski norm is positive and so must be corresponding norm in E^4 so that in E^4 surface has future ligt-cone as projection. This surface is 4-D. The emergence of future light-cone might provide justification for the emergence of CDs and zero energy ontology.

For $IM(o^2)$ the discussion is same as in quaternionic case since norm does not appear in the equations.

At singularity both $RE(o^2)$ and $IM(o^2) = 0$ vanish. The condition $q_1 = \Lambda q_2$ reduces to $\Lambda = 0$ so that $q_1 = 0$ is only allowed. This leaves only light-cone boundary under consideration.

The appearance of surfaces with dimension higher than 4 raises the question whether something is wrong. One could of course argue that associativity allows also lower than 4-D surfaces as associative surfaces and higher than 4-D surfaces as co-associative surfaces. At H-level one can say that one has 4-D surfaces. A good guess is that this behavior disappears when the linear term is absent and origin ceases to be a singularity.

Chapter 6

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part II

6.1 Introduction

There are good reasons to hope that TGD is integrable theory in some sense. Classical physics is an exact part of quantum physics in TGD and during years I have ended up with several proposals for the general solution of classical field equations (classical TGD is an exact part of quantum TGD).

6.1.1 Could one identify space-time surfaces as zero loci for octonionic polynomials with real coefficients?

The identification of space-time surfaces as zero loci of real or imaginary part of octonionic polynomial has several extremely nice features.

1. Octonionic polynomial is an algebraic continuation of a real valued polynomial on real line so that the situation is effectively 1-dimensional! Once the degree of polynomial is known, the value of polynomial at finite number of points are needed to determine it and cognitive representation could give this information! This would strengthen the view strong form of holography (SH) - this conforms with the fact that states in conformal field theory are determined by 1-D data.

   Remark: Why not rational functions expressible as ratios $R = P_1/P_2$ of octonionic polynomials? It has become clear that one can develop physical arguments in favor of this option. The zero loci for $IM(P_i)$ would represent space-time varieties. Zero loci for $RE(P_1/P_2) = 0$ and $RE(P_1/P_2) = \infty$ would represent their interaction presumably realized as wormhole contacts connecting these varieties. In the sequel most considerations are for polynomials: the replacement of polynomials with rational functions does not introduce big differences and its discussed in the section "Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view".

2. One can add, sum, multiply, and functionally compose these polynomials provided they correspond to the same quaternionic moduli labelled by CP_2 points and share same time-line containing the origin of quaternionic and octonionic coordinates and real octonions (or actually their complexification by commuting imaginary unit). Classical space-time surfaces - classical worlds - would form an associative and commutative algebra. This algebra induces an analog of group algebra since these operations can be lifted to the level of functions defined in this algebra. These functions form a basic building brick of WCW spinor fields defining quantum states.

3. One can interpret the products of polynomials as correlates for free many-particle states with interactions described by added interaction polynomial, which can vanish at boundaries of CDs. This leads to the same picture as the view about preferred extremals reducing to minimal surfaces near boundaries of CD [L12]. Also zero zero energy ontology (ZEO) could be forced by the failure of number field property for quaternions at light-cone boundaries. It indeed turns out that light-cone boundary emerges quite generally as singular zero locus of polynomials $P(o)$ containing no linear part: this is essentially due to the non-commutativity of the octonionic units. Also the emergence of CDs can be understood. At this surface the region with $RE(P) = 0$ can transform to $IM(P) = 0$ region. In Euclidian signature this singularity corresponds to single point. A natural conjecture is that also the light-like orbits of partonic 2-surfaces correspond to this kind of singularities for non-trivial Hamilton-Jacobi structures.

4. The reduction to algebraic geometry would mean enormous boost to the vision about cognition with cognitive representations identified as generalized rational points common to reals rationals and various p-adic number fields defining the adele for given extension of rationals. Hamilton-Jacobi structure would result automatically from the decomposition of quaternions to real and imaginary parts which would be now complex numbers.

5. Also a connection with infinite primes is suggestive [K109]. The light-like partonic orbits, partonic 2-surfaces at their ends, and points at the corners of string world sheets might be interpreted in terms of singularities of varying rank and the analog of catastrophe theory emerges.

The great challenge is to prove rigorously that these approaches - or at least some of them - are indeed equivalent. Also it remains to be proven that the zero loci of real/imaginary parts of octonionic polynomials with real coefficients are associative or co-associative. I shall restrict the considerations of this article mostly to $M^8 - H$ duality. The strategy is simple: try to remember all previous objections against $M^8 - H$ duality and invent new ones since this is the best way to make real progress.

6.1.2 Topics to be discussed

Challenges of the octonionic algebraic geometry

TGD leads to several proposals for the exact solution of field equations defining space-time surfaces as preferred extremals of twistor lift of Kähler action. So called $M^8 - H$ duality is one of these approaches. The beauty of $M^8 - H$ duality is that it could reduce classical TGD to octonionic algebraic geometry and would immediately provide deep insights to cognitive representation identified as sets of rational points of these surfaces. The construction and interpretation of the octonionic geometry involves several challenges.

1. The fundamental challenge is to prove that the octonionic polynomials with real coefficients determine associative (co-associative) surfaces as the zero loci of their real part $RE(P)$ (imaginary parts $IM(P)$). $RE(P)$ and $IM(P)$ are defined in quaternionic sense. Contrary to the first naive working hypothesis, the identification $M^4 \subset O$ as as a co-associative region turns out to be the correct choice making light-cone boundary a counterpart of point-like singularity essential for the emergence of causal diamonds (CDs).

   This suggests a generalization of Cauchy-Riemann conditions for complex analytic functions to quaternions and octonions. Cauchy Riemann conditions are linear. Quaternionic polynomials with real coefficients define maps for which the 2-D spaces corresponding to vanishing of real/imaginary parts of the polynomial are complex/co-complex or equivalently commutative/co-commutative. Commutativity is expressed by conditions bilinear in partial derivatives. Octonionic polynomials with real coefficients define maps for which 4-D surfaces for which real/imaginary part are quaternionic/co-quaternionic, or equivalently associative/co-associative. The conditions are now 3-linear.

   In fact, all algebras obtained by Cayley-Dickson construction (see `http://tinyurl.com/ybuyla2k`) by adding imaginary unit repeatedly to octonionic algebra are power associative so that polynomials with real coefficients define an associative and commutative algebra. Hence octonion analyticity and a $M^8 - H$ correspondence could generalize (maybe even TGD!).

2. It turns out that in the generic case associative surfaces are 3-D and are obtained by requiring that one of the coordinates $RE(Y)^i$ or $IM(Y)^i$ in the decomposition $Y^i = RE(Y)^i + IM(Y)^i I_4$ of the gradient of $RE(P) = Y = 0$ with respect to the complex coordinates z_i^k, $k = 1, 2$, of O vanishes that is critical as function of quaternionic components z_1^k or z_2^k associated with q_1 and q_2 in the decomposition $o = q_1 + q_2 I_4$, call this component X_i. In the generic case this gives 3-D surface.

   In this generic case $M^8 - H$ duality can map only the 3-surfaces at the boundaries of CD and light-like partonic orbits to H, and only determines the boundary conditions of the dynamics in H determined by the twistor lift of Kähler action. $M^8 - H$ duality would allow to solve the gauge conditions for SSA (vanishing of infinite number of Noether charges) explicitly.

   One can also have criticality. 4-dimensionality can be achieved by posing conditions on the coefficients of the octonionic polynomial P so that the criticality conditions do not reduce the dimension: X_i would have possibly degenerate zero at space-time variety. This can allow 4-D associativity with at most 3 critical components X_i. Space-time surface would be analogous to a polynomial with a multiple root.

   Various components of octonion polynomial P of degree n are polynomials of same degree. Could criticality reduces to the degeneracy of roots for some component polynomials? Could P as a polynomial of real variable have degenerate roots?

   The criticality of X_i conforms with the general vision about quantum criticality of TGD Universe and provides polynomials with universal dynamics of criticality. A generalization of Thom's catastrophe theory [A37] emerges. Criticality should be equivalent to the universal dynamics determined by the twistor lift of Kähler action in H in regions, where Kähler action and volume term decouple and dynamics does not depend on coupling constants.

   One obtains two types of space-time surfaces. Critical and associative (co-associative) surfaces can be mapped by $M^8 - H$ duality to preferred critical extremals for the twistor lift of Kähler action obeying universal dynamics with no dependence on coupling constants and due to the decoupling of Kähler action and volume term: these represent external particles. $M^8 - H$ duality does not apply to non-associative (non-co-associative) space-time surfaces except at 3-D boundary surfaces. These regions correspond to interaction regions in which Kähler action and volume term couple and coupling constants make themselves visible in the dynamics. $M^8 - H$ duality determines boundary conditions.

3. This picture generalizes also to the level of complex/co-complex surfaces associated with fermionic dynamics. Why in some cases 1-D light-like curves at partonic orbits seem to be enough to represent fermions? Why fermionic strings serve as correlates of entanglement for bound states? What selects string world sheets and partonic 2-surfaces from the slicing of space-time surfaces? I have proposed commutativity or co-commutatitivity of string worlds sheets/partonic 2-surfaces in quaternionic sense as number theoretic explanation (tangent space as a sub-space of quaternionic space is commutative/co-commutative at each point). Why not all string world sheets/partonic 2-surfaces in the slicing are not commutative/co-commutative? The answer to these questions is criticality again: in the generic case commutative varieties are 1-D curves. In critical case one has 2-D string worlds sheets and partonic 2-surfaces.

4. The super variant of the octonionic geometry relying on octonionic triality makes sense and the geometry of the space-time variety correlates with fermion and antifermion numbers assigned with it. This new view about super-geometry involving also automatic SUSY breaking at the level of space-time geometry.

Description of interactions

Also a sketchy proposal for the description of interactions is discussed.

1. $IM(P_1P_2) = 0$ is satisfied for $IM(P_1) = 0$ and $IM(P_2) = 0$ since $IM(o_1o_2)$ is linear in $IM(o_i)$ and one obtains union of space-time varieties. $RE(P_1P_2) = 0$ cannot be satisfied in this way since $RE(o_1o_2)$ is not linear in $RE(o_i)$ so that the two varieties interact and this interaction could give rise to a wormhole contact connecting the two space-time varieties.

2. The surprise that $RE(P) = 0$ and $IM(P) = 0$ conditions have as singular solutions light-cone interior and its complement and 6-spheres $S^6(t_n)$ with radii t_n given by the roots of the real $P(t)$, whose octonionic extension defines the space-time variety X^4. The intersections $X^2 = X^4 \cap S^6(t_n)$ are tentatively identified as partonic 2-varieties defining topological interaction vertices. S^6 and therefore also X^2 are doubly critical, S^6 is also singular surface.

   The idea about the reduction of zero energy states to discrete cognitive representations suggests that interaction vertices at partonic varieties X^2 are associated with the discrete set of intersection points of the sparticle lines at light-like orbits of partonic 2-surfaces belonging to extension of rationals.

3. CDs and therefore also ZEO emerge naturally. For CDs with different origins the products of polynomials fail to commute and associate unless the CDs have tips along real (time) axis. The first option is that all CDs under observation satisfy this condition. Second option allows general CDs.

   The proposal is that the product $\prod P_i$ of polynomials associated with CDs with tips along real axis the condition $IM(\prod P_i) = 0$ reduces to $IM(P_i) = 0$ and criticality conditions guaranteeing associativity and provides a description of the external particles. Inside these CDs $RE(\prod P_i) = 0$ does not reduce to $RE(\prod P_i) = 0$, which automatically gives rise to geometric interactions. For general CDs the situation is more complex.

4. The possibility of super-octonionic geometry raises the hope that the twistorial construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY generalizes to TGD in rather straightforward way to a purely geometric construction. Functional integral over WCW would reduce to summations over polynomials with coefficients in extension of rationals and criticality conditions on the coefficients could make the summation well-defined by bringing in finite measurement resolution.

   If scattering diagrams are associated with discrete cognitive representations, one obtains a generalization of twistor formalism involving polygons. Super-octonions as counterparts of super gauge potentials are well-defined if octonionic 8-momenta are quaternionic. Indeed, Grassmannians have quaternionic counterparts but not octonionic ones. There are good hopes that the twistor Grassmann approach to $\mathcal{N} = 4$ SUSY generalizes. The core part in the calculation of the scattering diagram would reduce to the construction of octonionic 4-varieties and identifying the points belonging to the appropriate extension of rationals.

   Twistor Grassmannian construction of scattering amplitudes at the level of M^8 looks feasible. The amplitudes decompose to M^4 and CP_2 parts with similar structure with E^4 spin (electroweak isospin) replacing ordinary spin. The residue integrals over Grassmannians emerging from the conservation of M^4 and E^4 4-momenta would have same form and guarantee Yangian supersymmetry in both sectors. The counterpart for the product of delta functions associated with the "negative helicities" (weak isospins with negative sign) would be expressible as a delta function in the complement of $SU(3)$ Cartan algebra $U(1) \times U(1)$ by using exponential map.

About the analogs of Gromow-Witten invariants and branes in TGD

Gromov-Witten (G-W) invariants belong to the realm of quantum enumerative geometry briefly discussed in [L33]. They count numbers of points in the intersection of varieties ("branes") with quantum intersection identified as the existence of "string world sheet(s)" intersecting the branes. Also octonionic geometry gives rise to brane like objects. G-W invariants are rational numbers but it is proposed that they could be integers in TGD framework.

Riemann-Roch theorem (RR) and its generalization Atyiah-Singer index theorem (AS) relate dimensions of various kinds of moduli spaces to topological invariants. The possible generalizations of RR and AS to octonionic framework and the implications of $M^8 - H$ duality for the possible generalizations are discussed. The adelic hierarchy of extensions of rationals and criticality conditions make the moduli spaces discrete so that one expects kind of particle in box type quantization selecting discrete points of moduli spaces about the dimension.

The discussion of RR as also the notion of infinite primes and infinite rationals as counterparts of zero energy states suggests that rational functions $R = P_1/P_2$ could be more appropriate than mere polynomials. The construction of space-time varieties would not be modified in essential way: one would have zero loci of $IM(P_i)$ identifiable as space-time sheets and zero- and ∞-loci of $RE(P_1/P_2)$ naturally identifiable as wormhole contacts connecting the space-time sheets.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); Strong Form of GCI (SGCI); Quantum Criticality (QC); Strong Form of Holography (SH); World of Classical Worlds (WCW); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Causal Diamond (CD); Number Theoretical Universality (NTU) are the most often occurring acronyms.

6.2 Some challenges of octonionic algebraic geometry

Space-time surfaces in $H = M^4 \times CP_2$ identified as preferred extremals of twistor lift of Kähler action leads to rather detailed view about space-time surfaces as counterparts of particles. Does this picture follow from $X^4 \subset M^8$ picture and does this description bring in something genuinely new?

6.2.1 Could free many-particle states as zero loci for real or imaginary parts for products of octonionic polynomials

In algebraic geometry zeros for the products of polynomials give rise to disjoint varieties, which are disjoint unions of surfaces assignable to the individual surfaces and possibly having lower-dimensional intersections. For instance, for complex curves these intersections consist of points. For complex surfaces they are complex curves.

In the case of octonionic polynomial $P = RE(P) + IM(P)I_4$ (Re and Im are defined in quaternionic sense) one considers zeros of quaternionic polynomial $RE(P)$ or $IM(P)$.

1. Product polynomial $P = P_1P_2$ decomposes to

$$P = RE(P_1)RE(P_2) - IM(P_1)IM(P_2) + (RE(P_1)IM(P_1) + IM(P_1)RE(P_2)I_4 \ .$$

   One can require vanishing of $RE(P)$ or $IM(P)$.

   (a) $IM(P)$ vanishes for

$$(RE(P_1) = 0, RE(P_2) = 0)$$

   or

$$I(m(P_1) = 0, IM(P_2) = 0) \ .$$

   (b) $RE(P)$ vanishes for

$$(RE(P_1) = 0, IM(P_2) = 0)$$

   or

$$IM(P_1) = 0, RE(P_2) = 0) \ .$$

One could reduce the condition $RE(P) = 0$ to $IM(P) = 0$ by replacing $P = P_1 + P_2 I_4$ with $P_2 - P_1 I_4$. If this condition is satisfied for the factors, it is satisfied also for the product. The set of surfaces is a commutative and associative algebra for the condition $IM(P) = 0$. Note that the quaternionic moduli must be same for the members of product. If one has quantum superposition of quaternionic moduli, the many-particle state involves a superposition of products with same moduli.

As found, the condition $IM(P) = 0$ can transform to $RE(P) = 0$ at singularities having $RE(P) = 0, IM(P) = 0$.

2. The commutativity of the product means that the products are analogous to many-boson states. P^n would define an algebraic analog of Bose-Einstein condensate. Does this surface correspond to a state consisting of n identical particles or is this artefact of representation? As a limiting case of product of different polynomials it might have interpretation as genuine n-boson states.

3. The product of two polynomials defines a union of disjoint surfaces having discrete intersection in Euclidian signature. In Minkowskian signature the vanishing of $q\overline{q}$ (conjugation does not affect the sign of i and changes only the sign of I_k!) can give rise to 3-D light-cone. The non-commutativity of quaternions indeed can give rise to combinations of type $q\overline{q}$ in $RE(P)$ and $IM(P)$.

What about interactions?

1. Could one introduce interaction by simply adding a polynomial P_{int} to the product? This polynomial should be small outside interaction region. CD would would define naturally interaction regions and the interaction terms should vanish at the boundaries of CD. This might be possible in Minkowskian signature, where $f(q^2)$ multiplying the interaction term might vanish at the boundary of CD: in Euclidian sector $q\overline{q} = 0$ would imply $q = 0$ but in Minkowskian sector it would give light-cone as solution. One should arrange $IM(P_{int})$ to be proportional to $q\overline{q}$ vanishing at the boundary of CD. Minkowskian signature could be crucial for the possibility to "turning interactions on".

2. If the imaginary part of the interaction term is proportional $f_1(q^2)f_2((q - T)^2)$ (T is real and corresponds to the temporal distance between the tips of CD) with $f_i(0) = 0$, one could obtain asymptotic states reducing to disjoint unions of zero loci of P^i at the boundaries of CD. If the order of of the perturbation terms is higher than the total order of polynomials P^i, one would obtain new roots and particle emission. Non-perturbative situation would correspond to a dramatic modification of the space-time surface as a zero locus of $IM(P)$. This picture would be M^8 counterpart for the reduction of preferred extremals to minimal surfaces analogous to geodesic lines near the boundaries of CD: preferred extremals reduce to extremals of both Kähler action and volume term in these regions [L12].

The singularities of scattering amplitudes at algebraic varieties of Grassmann manifolds are central in the twistor Grassmann program [B18, B51, B23]. Since twistor lift of TGD seems to be the correct manner to formulate classical TGD in H, one can wonder about the connection between space-time surfaces in M^8_c and scattering amplitudes. Witten's formulation of twistor amplitudes in terms of algebraic curves in CP_3 suggests a formulation of scattering amplitudes in terms of the 4-D algebraic varieties in M^8_c as of course, also TGD itself [K49, L41]! Could the huge multi-local Yangian symmetries of twistor Grassmann amplitudes reduce to octonion analyticity.

6.2.2 Two alternative interpretations for the restriction to M^4 subspace of M^8_c

One must complexify M^8 so that one has complexified octonions M^8_c. This means the addition of imaginary unit i commuting with octonionic imaginary units. The vanishing of real or imaginary part of octonionic polynomial in quaternionic sense ($o = q_1 + Jq_2$) defines the space-time surface. Octonionic polynomial itself is obtained from a real polynomial by algebraic continuation so that in information theoretic sense space-time is 1-D. The roots of this real polynomial fix the polynomial and therefore also space-time surface uniquely. 1-D line degenerates to a discrete set of points of an extension in information theoretic sense. In p-adic case one can allow p-adic pseudo constants and this gives a model for imagination.

The octonionic roots $x + iy$ of the real polynomial need not however be real. There are two options.

1. The original proposal in [L32, L34] was that the *projection* from M^8_c to real M^4 (for which M^1 coordinate is real and E^3 coordinates are imaginary with respect to i!) defines the real space-time surface mappable by $M^8 - H$ duality to CP_2.

2. An alternative option is that only the roots of the 4 vanishing polynomials as coordinates of M^4_c belong to M^4 so that m^0 would be real root and m^k, $k = 1, ..., 3$ imaginary with respect to $i \to -i$. M^8_c coordinates would be invariant ("real") under combined conjugation $i \to -i, I_k \to -I_k$. In the following I will speak about this property as *Minkowskian reality*. This could make sense.

   What is remarkable that this could allow to identify CDs in very elegant manner: outside CD these 4 conditions would not hold true. This option looks more attractive than the first one. Why these conditions can be true just inside CD, should be understood.

Consider now this in detail.

1. One can think of starting from one of the 4 vanishing conditions for the components of octonionic polynomial guaranteeing associativity. Assuming real roots and continuing one by one through all 4 conditions to obtain 4-D Minkowskian real regions. The time coordinate of M^4 coordinates is real and others purely imaginary with respect to $i \to -i$. If this region does not connect 3-D surface at the boundaries of real CD, one must make a new trial.

   Cusp catastrophe determined as the zero locus of third order polynomial provides an example. There are regions with single real root, regions with two real roots (complex roots become real and identical) defining V-shaped boundary of cusp and regions with 3 real roots (the interior of the cusp).

2. The restriction of the octonionic polynomial to time axis m^0 identifiable as octonionic real axes is a real polynomial with algebraic coefficients. In this case the root and its conjugate with respect to i would define the same surface. One could say that the Galois group of the real polynomial characterizes the space-time surface although at points other than those at real axis (time axis) the Galois group can be different.

   One could consider the local Galois group of the fourth quaternionic valued polynomial, say the part of quaternionic polynomial corresponding to real unit 1 when other components are required to vanish and give rise to coordinates in $M^8 \subset M^8_c$ - Minkowskian reality. The extension and its Galois group would depend on the point of space-time surface.

   An interesting question is how strong conditions Minkowskian reality poses on the extension. Minkowskian reality seems to imply that E^3 roots are purely real so that for an octonionic polynomial obtained as a continuation of a *real* polynomial one expects that both root and complex conjugate should be allow and that Galois group should contain Z_2 reflection $i \to -i$. Space-time surface would be at least 2-sheeted. Also the model for elementary particles forces this conclusion on physical grounds. Real as opposite to imagined would mean Minkowskian reality in mathematical sense. In the case of polynomials this description would make sense in p-adic case by allowing the coefficients of the polynomial be pseudo constants.

3. What data one could use to fix the space-time surface? Can one start directly from the real polynomial and regard its coefficients as WCW coordinates? This would be easy and elegant. Space-time surface could be determined as Minkowskian real roots of the octonionic polynomial. The condition that the space-time surface has ends at boundaries of given CD and the roots are not Minkowskian real outside it would pose conditions on the polynomial. If the coefficients of the polynomial are p-adic pseudo constants, this condition might be easy to satisfy.

The situation depends also on the coordinates used. For linear coordinates such as Minkowski coordinates Minkowskian reality looks natural. One can however consider also angle like coordinates representable only in terms of complex phases p-adically and coming as roots of unity and requiring complex extension: at H-side they are very natural. For instance, for CP_2 all coordinates would be naturally represented in this manner. For future light-cone one would have hyperbolic angle and 2 ordinary angles plus light-cone proper time which would be real and positive coordinate.

This picture conforms with the proposed picture. The point is that the time coordinate m^k can be real in the sense that they are linear combinations of complex roots, say powers for the roots of unity. $E_c^4 \subset M_c^8$ could be complex and contain also complex roots since $M^8 - H$ duality does not depend on whether tangent space is complex or not. Therefore would could have complex extensions.

6.2.3 Questions related to ZEO and CDs

Octonionic polynomials provide a promising approach to the understanding of ZEO and CDs. Light-like boundary of CD as also light-cone emerge naturally as zeros of octonionic polynomials. This does not yet give CDs and ZEO: one should have intersection of future and past directed light-cones. The intuitive picture is that one has a hierarchy of CDs and that also the space-time surfaces inside different CDs an interact.

Some general observations about CDs

It is good to list some basic features of CDS, which appear as both 4-D and 8-D variants.

1. There are both 4-D and 8-D CDs defined as intersections of future and past directed light-cones with tips at say origin 0 at real point T at quaternionic or octonionic time axis. CDs can be contained inside each other. CDs form a fractal hierarchy with CDs within CDs: one can add smaller CDs with given CD in all possible ways and repeat the process for the sub-CDs. One can also allow overlapping CDs and one can ask whether CDs define the analog of covering of O so that one would have something analogous to a manifold.

2. The boundaries of two CDs (both 4-D and 8-D) can intersect along light-like ray. For 4-D CD the image of this ray in H is light-like ray in M^4 at boundary of CD. For 8-D CD the image is in general curved line and the question is whether the light-like curves representing fermion orbits at the orbits of partonic 2-surfaces could be images of these lines.

3. The 3-surfaces at the boundaries of the two 4-D CDs are expected to have a discrete intersection since 4 + 4 conditions must be satisfied (say $RE(P_i^k)) = 0$ for $i = 1, 2,\ k = 1, 4$. Along line octonionic coordinate reduces effectively to real coordinate since one has $E^2 = E$ for $E = (1 + in)/2$, n octonionic unit. The origins of CDs are shifted by a light-like vector kE so that the light-like coordinates differ by a shift: $t_2 = t_1 - k$. Therefore one has common zero for real polynomials $RE(P_1^k(t))$ and $RE(P_2^k(t - k))$.

   Are these intersection points somehow special physically? Could they correspond to the ends of fermionic lines? Could it happen that the intersection is 1-D in some special cases? The example of o^2 suggest that this might be the case. Does 1-D intersection of 3-surfaces at boundaries of 8-D CDs make possible interaction between space-time surfaces assignable to separate CDs as suggested by the proposed TGD based twistorial construction of scattering amplitudes?

4. Both tips of CD define naturally an origin of quaternionic coordinates for $D = 4$ and the origin of octonionic coordinates for $D = 8$. Real analyticity requires that the octonionic polynomials have real coefficients. This forces the origin of octonionic coordinates to be along the real line (time axis) connecting the tips of CD. Only the translations in this specified direction are symmetries preserving the commutativity and associativity of the polynomial algebra.

5. One expects that also Lorentz boosts of 4-D CDs are relevant. Lorentz boosts leave second boundary of CD invariant and Lorentz transforms the other one. Same applies to 8-D CDs. Lorentz boosts define non-equivalent octonionic and quaternionic structures and it seems that one assume moduli spaces of them.

One can of course ask whether the still somewhat ad hoc notion of CD general enough. Should one generalize it to the analog of the polygonal diagram with light-like geodesic lines as its edges appearing in the twistor Grassmannian approach to scattering diagrams? Octonionic approach gives naturally the light-like boundaries assignable to CDs but leaves open the question whether more complex structures with light-like boundaries are possible. How do the space-time surfaces associated with different quaternionic structures of M^8 and with different positions of tips of CD interact?

The emergence of causal diamonds (CDs)

CDs are a key notion of zero energy ontology (ZEO). They should emerge from the number-theoretic dynamics somehow. How? In the following this question is approached from two different directions.

1. One can ask whether the emergence of CDs could be understood in terms of singularities of octonion polynomials located at the light-like boundaries of CDs. In Minkowskian case the complex norm $q\overline{q}_i$ is present in P. Could this allow to blow up the singular point to a 3-D boundary of light-cone and allow to understand the emergence of causal diamonds (CDs) crucial in ZEO. This question will be considered below.

2. These arguments were developed before the realization that the Minkowskian reality condition discussed in the previous section is natural for the space-time surfaces as roots of the 4 polynomials defining real or imaginary part of octonionic polynomial in quaternionic sense and giving M^4 point as a solution. Minkowskian reality can hold only in some regions of M^4 and an attractive conjecture is that it fails outside CD. CD would be a prediction of number theoretical dynamics and have counterpart also at the level of H.

Consider now the second approach in more detail. The study of the special properties for zero loci of general polynomial $P(o)$ at light-rays of O indeed demonstrated that both 8-D land 4-D light-cones and their complements emerge naturally, and that the M^4 projections of these light-cones and even of their boundaries are 4-D future - or past directed light-cones. What one should understand is how CDs as their intersections, and therefore ZEO, emerge.

1. One manner to obtain CDs naturally is that the polynomials are sums $P(t) = \sum_k P_k(o)$ of products of form $P_k(o) = P_{1,k}(o)P_{2,k}(o-T)$, where T is real octonion defining the time coordinate. Single product of this kind gives two disjoint 4-varieties inside future and past directed light-cones $M^4_+(0)$ and $M^4_-(T)$ for either $RE(P) = 0$ (or $IM(P) = 0$) condition. The complements of these cones correspond to $IM(P) = 0$ (or $RE(P) = 0$) condition.

2. If one has nontrivial sum over the products, one obtains a connected 4-variety due the interaction terms. One has also as special solutions $M^4_\pm$ and the 6-spheres associated with the zeros $P(t)$ or equivalently $P_1(t_1) \equiv P(t)$, $t_1 = T - t$ vanishing at the upper tip of CD. The causal diamond $M^4_+(0) \cap M^4_-(T)$ belongs to the intersection.

   Remark: Also the union $M^4_-(0) \cup M^4_+(T)$ past and future directed light-cones belongs to the intersection but the latter is not considered in the proposed physical interpretation.

3. The time values defined by the roots t_n of $P(t)$ define a sequence of 6-spheres intersecting 4-D CD along 3-balls at times t_n. These time slices of CD must be physically somehow special. Space-time variety intersects 6-spheres along 2-varieties X^2_n at times t_n. The varieties X^2_n are perhaps identifiable as 2-D interaction vertices, pre-images of corresponding vertices in H at which the light-like orbits of partonic 2-surfaces arriving from the opposite boundaries of CD meet.

   The expectation is that in H one as generalized Feynman diagram with interaction vertices at times t_n. The higher the evolutionary level in algebraic sense is, the higher the degree of the polynomial $P(t)$, the number of t_n, and more complex the algebraic numbers t_n. $P(t)$ would be coded by the values of interaction times t_n. If their number is measurable, it would provide important information about the extension of rationals defining the evolutionary level. One can also hope of measuring t_n with some accuracy! Octonionic dynamics would solve the roots of a polynomial! This would give a direct connection with adelic physics [L39] [L38].

Remark: Could corresponding construction for higher algebras obtained by Cayley-Dickson construction solve the "roots" of polynomials with larger number of variables? Or could Cartesian product of octonionic spaces perhaps needed to describe interactions of CDs with arbitrary positions of tips lead to this?

4. Above I have considered only the interiors of light-cones. Also their complements are possible. The natural possibility is that varieties with $RE(P) = 0$ and $IM(P) = 0$ are glued at the boundary of CD, where $RE(P) = IM(P) = 0$ is satisfied. The complement should contain the external (free) particles, and the natural expectation is that in this region the associativity/co-associativity conditions can be satisfied.

5. The 4-varieties representing external particles would be glued at boundaries of CD to the interacting non-associative solution in the complement of CD. The interaction terms should be non-vanishing only inside CD so that in the exterior one would have just product $P(o) = P_{1,k_0}(o)P_{2,k_0}(o-T)$ giving rise to a disjoint union of associative varieties representing external particles. In the interior one could have interaction terms proportional to say $t^2(T-t)^2$ vanishing at the boundaries of CD in accordance with the idea that the interactions are switched one slowly. These terms would spoil the associativity.

Remark: One can also consider sums of the products $\prod_k P_k(o - T_k)$ of n polynomials and this gives a sequence CDs intersecting at their tips. It seems that something else is required to make the picture physical.

6.2.4 About singularities of octonionic algebraic varieties

In Minkowskian signature the notion of singularity for octonionic polynomials involves new aspects as the study of o^2 singular at origin shows (see Appendix). The region in which $RE(o^2) = 0, IM(o^2) = 0$ holds true is 4-D rather than a discrete set of points as one would naïvely expect.

1. At singularity the local dimension of the algebraic variety is reduced. For instance, double cone of 3-space has origin as singular point where it becomes 0-dimensional. A more general example is local pinch in which cylinder becomes infinitely thin at some point. This kind of pinching could occur for fibrations as the fiber contracts to a lower-dimensional space along a sub-variety of the base space.

   A very simple analogy for this kind of singularity is the singularity of $P(x,y) = y^2 - x = 0$ at origin: now the sheets $y = \pm\sqrt{x}$ co-incide at origin. The algebraic functions $y \mp \sqrt{x}$ defining the factorization of $P(x,y)$ co-incide at origin. Quite generally, two or more factors in the factorization of polynomial using algebraic functions co-incide at the singularity. This is completely analogous to the degeneracy or roots of polynomials of single variable.

   The signature of the singularity of algebraic variety determined by the conditions $P^i(z^j) = 0$ is the reduction of the maximal rank r for the matrix formed by the partial derivatives $P^i_j \equiv \partial IM(P)^i/\partial z^j$ ("RE" could replace "IM"). Rank corresponds to the largest dimension of the minor of P^i_j with non-vanishing determinant. Determinant vanishes when two rows of the minor are proportional to each other meaning that two tangent vectors become linearly dependent. When the rank is reduced by Δr, one has $r = r_{max} - \Delta r$ and the local dimension is locally reduced by Δr. One has hierarchy of singularities within singularities.

   The conditions that all independent minors of the P^i_j have reduced rank gives additional constraints and define a sub-variety of the algebraic variety. Note that the dimension of the singularity corresponds to $d_s = \Delta r$ in the sense that the dimension of tangent space at singularity is effectively d_s.

2. In the recent case there are 4 polynomials and 4 complex variables so that $IM(P)^i_j$ is 4×4-matrix. Its rank r can have values in $r = 1,2,3,2,4$. One can use Thom's catastrophe theory as a guideline. Catastrophe decomposes to pieces of various dimensions characterized by the reduction of the rank of the matrix defined by the second derivatives $V_{ij} = \partial_i\partial_j V$ of the potential function defining the catastrophe. For instance, for cusp catastrophe with $V(x,a,b) = x^4 + ax^2 + bx$ one has V-shaped region in (a,b) plane with maximal reduction

of rank to $r = 0$ ($\partial_x^2 V = 0$) at the tip $(a, b) = 0$ at reduction to $r = 1$ at the sides of V, where two roots of $\partial_x V = 4x^3 + 2ax + b = 0$ co-incide requiring that the discriminant of this equation vanishes.

3. In the recent case $IM(P)$ takes the role of complex quaternion valued potential function and the 4 coordinates $z_1^{k)}$ that of behavior variable x for cusp and $z_2^{k)}$ that of control parameters (a, b). The reduction of the rank of $n \times n$ matrix by Δr means that there are r linearly independent rows in the matrix. These give Δr additional conditions besides $IM(P) = 0$ so that the sub-variety along which the singularity takes places as dimension r. One can say that the r-dimensional tangent spaces integrate to the singular variety of dimension r.

   The analogy with branes would be realized as a hierarchical structure of singularities of the spacetime surfaces. This hierarchy of singularities would realize space-time correlates for quantum criticality, which is basic principle of quantum TGD. For instance, the reduction by 3-units would correspond to strings - say at the ends of CD and along the partonic orbits (fermion lines), and maximal reduction might correspond to discrete points - say the ends of fermion lines at partonic 2-surfaces. Also isolated intersection points can be regarded as singularities and are stably present but it does not make sense to add fermions to these points so that cognitive representations are not possible.

4. Note that also the associativity - and commutativity conditions already discuss involved the gradients of $IM(P)^i$ and $RE(P)^i$, which would suggests that these regions can be interpreted as singularities for which the dimension is not lowered by on unit since the vanishing conditions hold true identically by criticality.

There are two cases to be considered. The usual Euclidian case in which pinch reducing the dimension and the Minkowskian case in which metric dimension is reduced locally.

Consider first the Euclidian case.

1. In Euclidian case it is difficult to tell whether all values of Δr are possible since octonion analyticity poses strong conditions on the singularities. The pinch could correspond to the singularity of the covering associated with the space-time surface defined by Galois group for the covering associated with $h_{eff}/h = n$ identifiable as the dimension of the extension [L29]. Therefore there would be very close connection between the extensions of rationals defining the Galois group and the extension of polynomial ring of 8 complex variables z_i^k, $i = 1, 2$, $k = 1, .., 4$ by algebraic functions. At the pinch, which would be algebraic point, the Galois group would have subgroup leaving the coordinates of the point invariant and some sheets of the covering defining roots would co-incide.

2. A very simple analogy for this kind of singularity is the singularity of $P(x, y) = y^2 - x = 0$ at origin: now the sheets $y = \pm\sqrt{x}$ co-incide at origin. The algebraic functions $y \mp \sqrt{x}$ defining the factorization of $P(x, y)$ co-incide at origin. Quite generally, two or more factors in the factorization of polynomial using algebraic functions co-incide at the singularity. This is completely analogous to the degeneracy or roots of polynomials of single variable.

3. Quaternion structure predicts the slicing of M^4 by string world sheets inducing that of space-time surfaces. One must ask whether singular space-time sheets emerge already for the slicing of M^4 by string world sheets. String world sheets could be considered as candidates for $\Delta r = 2$ singularities of this kind. The physical intuition strongly suggests that there indeed physically preferred string world sheets and identification as $\Delta r = 2$ singularities of Euclidian type is attractive. Partonic 2-surfaces are also candidates in this respect. Could some sheets of the $h_{eff}/h = n$ covering co-incide at string world sheets?

Consider next the Minkowskian case. At the level of H the rank of the induced metric is reduced. This reduction need not be same as that for the matrix P_j^i and it is of course not obvious that the partonic orbit allows description as a singularity of algebraic variety.

1. Could the matrix P_j^i take a role analogous to the dual of induced metric and one might hope that the change of the sign for P_j^i for a fixed polynomial at singular surface could be

analogous to the change of the sign of $\sqrt{g_4}$ so that the idea about algebraization of this singularity at level of M^8 might make sense. The information about metric could come from the fact that $IM(P)$ depends on complex valued quaternion norm reducing to Minkowskian metric in Minkowskian sub-space.

2. The condition for the reduction of rank from its maximal value of $r = 4$ to $r = 3$ occurs if one has $det(P) = 0$, which defines co-dimension 1 surface as a sub-variety of space-time surface. The interpretation as co-incidence of two roots should make sense if $IM(P) = 0$. Root pairs would now correspond now to the points at different sides of the singular 3-surface.

   Minkowskian singularity cannot be identified as the 3-D space-like boundary of many-sheeted space-time surface located at the boundary of CD (induced metric is space-like).

   Could this sub-variety be identified as partonic orbit, the common boundary of the Euclidian and Minkowskian regions? This would require that associative region transforms to co-associative one here. $IM(P) = 0$ condition can transform to $RE(P) = 0$ condition if one has $P = 0$ at this surface. Minkowskian variant of point singularity (P^i_j vanishes) would explode it to a light-like partonic orbit.

   What does this imply about the rank of singularity? The condition $IM(P) = RE(P) = 0$ does not reduce the rank if P is linear polynomial and one could consider a hierarchy of reductions of rank. Since $q\overline{q}$ vanishes in Minkowskian sub-space at light-cone boundary rather than at point $q = 0$ only, there are reasons to expect that it appears in P and reduces the rank by $\Delta r = 4$ (see Appendix for the discussion of o^2 case). The rank of the induced 4-metric is however reduced only by $\Delta r = 1$ at partonic orbit. If the complexified complex norm $z\overline{z}$, $z = z_1 + z_2 I_2$ can take the role of $q\overline{q}$, one has $\Delta r = 2$.

3. The reduction of rank to $r = 2$ would give rise to 2-surfaces, which are at the boundaries of 3-D singularities. If partonic orbits correspond to $\Delta r = 1$ singularities one could identify them as partonic 2-surfaces at the ends partonic orbits.

   Could the singularity at partonic 2-surface correspond to the reduction of the rank of the induced metric by 2 units? This is impossible in strict sense since there is only one light-like direction in signature $(1, -1, -1, -1)$. Partonic 2-surface singularity would however correspond to a corner for both Euclidian and Minkowskian regions at which the metrically 2-D but topologically 3-D partonic orbit meets the the space-like 3-surface along the light-like boundary of CD. Also the radial direction for space-like 3-surface could become light-like at partonic 2-surface if the CP_2 coordinates have vanishing gradient with respect to the light-like radial coordinate r_M at the partonic 2-surface. In this sense the rank could be reduced by 2 units. The situation is analogous to that for fold singularity $y^2 - x = 0$.

   String world sheets cannot be subsets of $r = 3$ singularities, which suggests different interpretation for partonic 2-surfaces and string world sheets.

What could this different interpretation be?

1. Perhaps the most convincing interpretation of string world sheets/partonic 2-surfaces has been already discussed (this interpretation would generalize to associative space-time surfaces). They could be commutative/co-commutative (here permutation might be allowed!) sub-manifolds of associative regions of the space-time surface allowing quaternionic tangent spaces so that the notions of commutative and co-commutative make sense. The criticality conditions are satisfied without the reduction of dimension from $d = 2$ to $d = 1$. In non-associative regions string world sheets would reduce to 1-D curves. This would happen at the boundaries of partonic orbits and 3-surfaces at the ends of space-time surface and only the ends of strings at partonic orbits carrying fermion number would be needed to determine twistorial scattering amplitudes [K49, L41].

2. I have also considered an interpretation in terms of singularities of space-time surfaces represented as a sections of their own twistor bundle. Self-intersections of the space-time surface would correspond to 2-D surfaces in this case [L29] and perhaps identifiable as string world sheets. The interpretation mentioned above would be in terms of Euclidian singularities. If this is true, the question is only about whether these two interpretations are consistent with each other.

If I were forced to draw conclusion on basis of these notices, it would be that only $r = 4$ Minkowskian singularities could be interesting and at them $RE(P) = 0$ regions could be transformed to $IM(P) = 0$ regions. Furthermore, the reduction of rank for the induced metric cannot be equal to the reduction of the rank for P^i_j.

6.2.5 The decomposition of space-time surface to Euclidian and Minkowskian regions in octonionic description

The unavoidable outcome of H picture is the decomposition of space-time surface to regions with Minkowskian or Euclidian signature of the induced metric. These regions are bounded by 3-D regions at which the signature of the induced metric is $(0, -1, -1, -1)$ due to the vanishing of the determinant of the induced metric. The boundary is naturally the light-like orbit of partonic 2-surface although one can consider also the possibility that these regions have boundaries intersecting along light-like curves defining boundaries of string world sheets. A more detailed view inspired by the study of extremals is following.

1. Let us assume that the above picture about decomposition of space-time surfaces in H to two kinds regions takes place. The regions where the dynamicis universal minimal surface dynamics have associative pre-image in M^8. The regions where Kähler action and volume term couple the associative pre-image in M^8 exists only at the 3-D boundary regions and M^8 dynamics determines the boundary conditions for H dynamics, which by hologaphy is enough.

2. In the space-time regions having associative pre-image in M^8 one has a fibration of X^4 with with partonic surface as a local base and string world sheet as local fiber. In the interior of space-time region there are no singularities but at the boundary 2-D string world sheets becomes metrically 1-D as 1-D string boundary reduces metrically to 0-D structure analogous to a point. This reduction of dimension would be metric, but not topological.

   The singularity for plane curve $P(x, y) = y^2 - x^3 = 0$ at origin illustrates the difference between Minkowskian and Euclidian singularity. One has $(\partial_x P, \partial_y P) = (-3x^2, 2y)$ vanishing at origin so that $\Delta r = 1$ singularity is in question and the dimension of singular manifold is indeed $r = 0$. From $y = \pm x^{3/2}$, $x \geq 0$. The induced metric $g_{xx} = 1 + (dy/dx)^2 = 1 + (9/4)x$, $x \geq 0$ is however non-singular at origin.

3. If the Euclidian region with pre-image corresponds to a deformation of wormhole contact, the identification as image of a co-associative space-time region in M^8 is natural so that normal space is associative and contains also the preferred $M^2(x)$. In Minkowskian regions the identification as image of associative space-time region in M^8 is natural.

What can one say about the relationship of the M^8 counterparts of neighboring Minkowskian and Euclidian regions?

1. Do these regions intersect along light-like 3-surfaces, 1-D light-like curve (orbit of fermion) or is the intersection disrete set of points possibly assignable to the partonic 2-surface at the boundaries of CD? The M^4 projections of the inverse image of the light-like partonic orbit should co-incide but E^4 projections need not do so. They could be however mappable to the same partonic two surface in $M^8 - H$ correspondence or the images could have at least have light-like curve as common.

2. Is seems impossible for the space-time surfaces determined as zeros of octonionic polynomials to have boundaries. Rather, it seems that the boundary must be between Minkowskian and Euclidian regions of the space-time surface determined by the same octonionic polynomial. At the boundary also associate region would transform to co-associative region suggesting that $IM(P) = RE(P) = 0$ holds allowing to change the condition from $IM(P) = 0$ to $RE(P) = 0$.

Consider now in more detail whether this view can be realized.

1. In $H = M^4 \times CP_2$ the boundary between the Minkowskian and Euclidian space-time regions - light-like partonic 3-surface - is a singularity possible only in Minkowskian signature. Space-time surface X^4 at the boundary is effectively 3-D since one has $\sqrt{g_4} = 0$ meaning that tangent space is effectively 3-D. The 3-D boundary itself is metrically 2-D and this gives rise to the extended conformal invariance defining crucial distinction between TGD and super string models.

2. The singularities of $P(o)$ for o identified as linear coordinate of M^8_c were already considered. The singularities correspond to the boundaries of light-cone and the emergence of CDs can be understood. Could also the light-like orbits of partonic 2-surfaces be understood in the same manner? Does the pre-image of this singularity in M^8 emerge as a singularity of an algebraic variety determined by the vanishing of $IM(P)$ for the octonionic polynomial?

   What is common is that the rank of the induced metric by one unit also now. Now one has however also $det(g_4) = 0$. The singularities correspond to curved light-like 3-surfaces inside space-time surfaces rather than light-like surfaces in M^8: induced metric matters rather than M^4 metric.

3. Could also these regions correspond to singularities of octonionic polynomials at which $P(o) = 0$ is satisfied and associative region transforms to a co-associative region? For $M^2(x) = M^2_0$ this is impossible. Partonic 2-surfaces are planes E^2 now. One should have closed partonic 2-surfaces.

   Could the allowance of quaternionic structures with slicing of X^4 by string world sheets and partonic 2-surfaces help? If one has slicing of string world sheets by dual light-like curves corresponding to light-like coordinates u and v, this slicing gives also rise to a slicing of light-like 3-surfaces and dual light-like coordinate. The pair (u, v) in fact defines the analog of z and $\overline{z}$ in hypercomplex case. Could the singularity of $P(o)$ using the quaternionic coordinates defined by (u, v) and coordinates of partonic 2-surface allow to identify light-like partonic orbits with $det(g_4) = 0$ as a generalization of light-cone boundaries in M^4?

   The decomposition $M^4_0 = M^2_x \times E^2(x)$ associated with quaternionic structure is independent of E^4. In the other hand, tangent space of space-time surface at point decomposes $M^2(x) \times E^2_T(x)$, where $E^2_T(x)$ is in general different from $E^2(x)$. Is this enough to obtain partonic 2-surfaces as singularities with $RE(P) = IM(P) = 0$?

The question whether the boundaries between Minkowskian and Euclidian can correspond to singular regions at which $P(o)$ vanishes and the surface $RE(P) = 0$ transforms to $IM(P) = 0$ surface remains open. What remains poorly understood is the role of the induced metric. My hope is that with a further work the picture could be made more detailed.

6.2.6 About rational points of space-time surface

What one can say about rational points of space-time surface?

1. An important special case corresponds to a generalization of so called rational surfaces for which a parametric representation in terms of 4 complex coordinates t^k exists such that o^k_1 are *rational* functions of t^k. The singularities for 2-complex dimensional surfaces in C^3 or equivalently CP_3 are classified by Du Val [A57, A68] (see `http://tinyurl.com/ydz93hle`).

2. In [L29] [L24] I considered possible singularities of the twistor bundle. These would correspond typically 2-D self-intersections of the embedding of space-time surfaces as 4-D base space of 6-D twistor bundle with sphere as a fiber. They could relate to string world sheets and partonic 2-surfaces and - as already found - are different from singularities at the level of M^8_c. The singularities of string world sheets and partonic 2-surfaces as hyper-complex and co-complex surfaces consist of points and could relate to the singularities at octonionic level.

As already mentioned, Bombieri-Lang conjecture (see `http://tinyurl.com/y887yn5b`) states that, for any variety X of general type over a number field k, the set of k-rational points of X is not Zariski dense (see `http://tinyurl.com/jm9fh74`) in X. Even more, the k-rational points are contained in a *finite* union of lower-dimensional sub-varieties of X.

This conjecture is highly interesting from TGD point of view if one believes in M^8-H duality. Space-time surfaces $X^4 \subset M^8_c$ can be seen as $M^8 = M^4 \times E^4$ projections of zero loci for real or imaginary parts of octonionic polynomials in o. In complex sense they reduce to $M^4 \times E^4$ projections of algebraic co-dimension 4 surfaces in C^8. If Bombieri-Lang conjectures makes sense in this context, it would state that for a space-time surface $X^4 \subset M^8$ of general type the rational points are contained in a *finite* union of lower-dimensional sub-varieties. Also the conjecture of Vojta (see `http://tinyurl.com/y9sttuu4`) stating that varieties of general type cannot be potentially dense is known to be true for curves and support this general vision.

Could the finite union of sub-varieties correspond to string world sheets, partonic 2-surfaces, and their light-like orbits define singularities? But why just singular sub-varieties would be cognitively simple and have small Kodaira dimension d_K allowing large number of rational points? In the case of partonic orbits one might understand this as a reduction of metric dimension. The orbit is effectively 2-dimensional partonic surface metrically and for the genera $g=0,1$ rational points are dense. For string world sheets with handle number smaller than 2 the situation is same.

The proposed realizations of associativity and commutativity provide additional support for this picture. Criticality guaranteeing associativity/commutativity would select preferred space-time surfaces as also string world sheets and partonic 2-surfaces.

Concluding, the general wisdom of algebraic geometry conforms with SH and with the vision about the localization of cognitive representations at 2-surfaces. There are of many possible options for detailed interpretation and certainly the above sketch cannot be correct at the level of details.

6.2.7 About $h_{eff}/h = n$ as the number of sheets of Galois covering

The following considerations were motivated by the observation of a very stupid mistake that I have made repeatedly in some articles about TGD. Planck constant $h_{eff}/h = n$ corresponds naturally to the number of sheets of the covering space defined by the space-time surface.

I have however claimed that one has $n = ord(G)$, where $ord(G)$ is the order of the Galois group G associated with the extension of rationals assignable to the sector of "world of classical worlds" (WCW) and the dynamics of the space-time surface (what this means will be considered below).

This claim of course cannot be true since the generic point of extension G has some subgroup H leaving it invariant and one has $n = ord(G)/ord(H)$ dividing $ord(G)$. Equality holds true only for Abelian extensions with cyclic G. For singular points isotropy group is $H_1 \sup H$ so that $ord(H_1)/ord(H)$ sheets of the covering touch each other. I do not know how I have ended up to a conclusion, which is so obviously wrong, and how I have managed for so long to not notice my blunder.

This observation forced me to consider more precisely what the idea about Galois group acting as a number theoretic symmetry group really means at space-time level and it turned out that M^8-H correspondence [L32] (see `http://tinyurl.com/yd43o2n2`) gives a precise meaning for this idea.

Consider first the action of Galois group (see `http://tinyurl.com/y8grabt2` and `http://tinyurl.com/ydze5psx`).

1. The action of Galois group leaves invariant the number theoretic norm characterizing the extension. The generic orbit of Galois group can be regarded as a discrete coset space G/H, $H \subset G$. The action of Galois group is transitive for irreducible polynomials so that any two points at the orbit are G-related. For the singular points the isotropy group is larger than for generic points and the orbit is G/H_1, $H_1 \sup H$ so that the number of points of the orbit divides n. Since rationals remain invariant under G, the orbit of any rational point contains only single point. The orbit of a point in the complement of rationals under G is analogous to an orbit of a point of sphere under discrete subgroup of $SO(3)$.

   $n = ord(G)/ord(H)$ divides the order $ord(G)$ of Galois group G. The largest possible Galois group for n-D algebraic extension is permutation group S_n. A theorem of Frobenius states that this can be achieved for $n=p$, p prime if there is only single pair of complex roots (see `http://tinyurl.com/y8grabt2`). Prime-dimensional extensions with $h_{eff}/h = p$ would have maximal number theoretical symmetries and could be very special physically: p-adic physics again!

2. The action of G on a point of space-time surface with embedding space coordinates in n-D extension of rationals gives rise to an orbit containing n points except when the isotropy group leaving the point is larger than for a generic point. One therefore obtains singular covering with the sheets of the covering touching each other at singular points. Rational points are maximally singular points at which all sheets of the covering touch each other.

3. At QFT limit of TGD the n dynamically identical sheets of covering are effectively replaced with single one and this effectively replaces h with $h_{eff} = n \times h$ in the exponent of action (Planck constant is still the familiar h at the fundamental level). n is naturally the dimension of the extension and thus satisfies $n \leq ord(G)$. $n = ord(G)$ is satisfied only if G is cyclic group.

The challenge is to define what space-time surface as Galois covering does really mean!

1. The surface considered can be partonic 2-surface, string world sheet, space-like 3-surface at the boundary of CD, light-like orbit of partonic 2-surface, or space-time surface. What one actually has is only the data given by these discrete points having embedding space coordinates in a given extension of rationals. One considers an extension of rationals determined by irreducible polynomial P but in p-adic context also roots of P determine finite-D extensions since e^p is ordinary p-adic number.

2. Somehow this data should give rise to possibly unique continuous surface. At the level of $H = M^4 \times CP_2$ this is impossible unless the dynamics satisfies besides the action principle also a huge number of additional conditions reducing the initial value data ans/or boundary data to a condition that the surface contains a discrete set of algebraic points.

   This condition is horribly strong, much more stringent than holography and even strong holography (SH) implied by the general coordinate invariance (GCI) in TGD framework. However, preferred extremal property at level of $M^4 \times CP_2$ following basically from GCI in TGD context might be equivalent with the reduction of boundary data to discrete data if $M^8 - H$ correspondence [L32] (see `http://tinyurl.com/yd43o2n2`) is accepted. These data would be analogous to discrete data characterizing computer program so that an analog of computationalism would emerge [L26] (see `http://tinyurl.com/y75246rk`).

One can argue that somehow the action of discrete Galois group must have a lift to a continuous flow.

1. The linear superposition of the extension in the field of rationals does not extend uniquely to a linear superposition in the field reals since the expression of real number as sum of units of extension with real coefficients is highly non-unique. Therefore the naïve extension of the extension of Galois group to all points of space-time surface fails.

2. The old idea already due to Riemann is that Galois group is represented as the first homotopy group of the space. The space with homotopy group π_1 has coverings for which points remain invariant under subgroup H of the homotopy group. For the universal covering the number of sheets equals to the order of π_1. For the other coverings there is subgroup $H \subset \pi_1$ leaving the points invariant. For instance, for homotopy group $\pi_1(S^1) = Z$ the subgroup is nZ and one has $Z/nZ = Z_p$ as the group of n-sheeted covering. For physical reasons its seems reasonable to restrict to finite-D Galois extensions and thus to finite homotopy groups.

   $\pi_1 - G$ correspondence would allow to lift the action of Galois group to a flow determined only up to homotopy so that this condition is far from being sufficient.

3. A stronger condition would be that π_1 and therefore also G can be realized as a discrete subgroup of the isometry group of $H = M^4 \times CP_2$ or of M^8 ($M^8 - H$ correspondence) and can be lifted to continuous flow. Also this condition looks too weak to realize the required miracle. This lift is however strongly suggested by Langlands correspondence [K63, K64] (see `http://tinyurl.com/y9x5vkeo`).

The physically natural condition is that the preferred extremal property fixes the surface or at least space-time surface from a very small amount of data. The discrete set of algebraic points in given extension should serve as an analog of boundary data or initial value data.

1. M^8-H correspondence [L32] (see `http://tinyurl.com/yd43o2n2`) could indeed realize this idea. At the level of M^8 space-time surfaces would be algebraic varieties whereas at the level of H they would be preferred extremals of an action principle which is sum of Kähler action and minimal surface term.

   They would thus satisfy partial differential equations implied by the variational principle and infinite number of gauge conditions stating that classical Noether charges vanish for a subgroup of symplectic group of $\delta M^4_\pm \times CP_2$. For twistor lift the condition that the induced twistor structure for the 6-D surface represented as a surface in the 12-D Cartesian product of twistor spaces of M^4 and CP_2 reduces to twistor space of the space-time surface and is thus S^2 bundle over 4-D space-time surface.

   The direct map $M^8 \to H$ is possible in the associative space-time regions of $X^4 \subset M^8$ with quaternionic tangent or normal space. These regions correspond to external particles arriving into causal diamond (CD). As surfaces in H they are minimal surfaces and also extremals of Kähler action and do not depend at all on coupling parameters (universality of quantum criticality realized as associativity). In non-associative regions identified as interaction regions inside CDs the dynamics depends on coupling parameters and the direct map $M^8 \to CP_2$ is not possible but preferred extremal property would fix the image in the interior of CD from the boundary data at the boundaries of CD.

2. At the level of M^8 the situation is very simple since space-time surfaces would correspond to zero loci for $RE(P)$ or $IM(P)$ (RE and IM are defined in quaternionic sense) of an octonionic polynomial P obtained from a real polynomial with coefficients having values in the field of rationals or in an extension of rationals. The extension of rationals would correspond to the extension defined by the roots of the polynomial P.

   If the coefficients are not rational but belong to an extension of rationals with Galois group G_0, the Galois group of the extension defined by the polynomial has G_0 as normal subgroup and one can argue that the relative Galois group $G_{rel} = G/G_0$ takes the role of Galois group.

   It seems that M^8-H correspondence could allow to realize the lift of discrete data to obtain continuous space-time surfaces. The data fixing the real polynomial P and therefore also its octonionic variant are indeed discrete and correspond essentially to the roots of P.

3. One of the elegant features of this picture is that the at the level of M^8 there are highly unique linear coordinates of M^8 consistent with the octonionic structure so that the notion of a M^8 point belonging to extension of rationals does not lead to conflict with GCI. Linear coordinate changes of M^8 coordinates not respecting the property of being a number in extension of rationals would define moduli space so that GCI would be achieved.

Does this option imply the lift of G to π_1 or to even a discrete subgroup of isometries is not clear. Galois group should have a representation as a discrete subgroup of isometry group in order to realize the latter condition and Langlands correspondence supports this as already noticed. Note that only a rather restricted set of Galois groups can be lifted to subgroups of $SU(2)$ appearing in McKay correspondence and hierarchy of inclusions of hyper-finite factors of type II_1 labelled by these subgroups forming so called ADE hierarchy in 1-1 correspondence with ADE type Lie groups [K127, K48] (see `http://tinyurl.com/ybavqvvr`). One must notice that there are additional complexities due to the possibility of quaternionic structure which bring in the Galois group SO(3) of quaternions.

Remark: After writing this article a considerable progress in understanding of $h_{eff}/h = n$ as number of sheets of Galois covering emerged. By M^8-duality space-time surface can be seen as zero locus for real or imaginary part (regarding octonions as sums of quaternionic real and imaginary parts) allows a nice understanding of space-time surface as an $h_{eff}/h = n$-fold Galois covering. M^8 is complexified by adding an imaginary unit i commuting with octonionic imaginary units. Also space-time surface is complexified to 8-D surface in complexified M^8. One can say that ordinary space-time surface is the "real part" of this complexified space-time surface just like x is the real part of a complex number $x + iy$. Space-time surface can be also seen as a root of n:th order polynomial with n complex branches and the projections of complex roots to "real part" of M^8 define space-time surface as an n-fold covering space in which Galois group acts.

6.2.8 Connection with infinite primes

The idea about space-time surfaces as zero loci of polynomials emerged for the first time as I tried to understand the physical interpretation of infinite primes [K107], which were motivated by TGD inspired theory of consciousness. Infinite primes form an infinite hierarchy. At the lowest level the basic entity is the product $X = \prod_p p$ of all finite primes. The physical interpretation could be as an analog of fermionic sea with fermion states labelled by finite primes p.

1. The simplest infinite primes are of form $P = X \pm 1$ as is easy to see. One can construct more complex infinite primes as infinite integers of form $nX/r + mr$. Here r is square free integer, n is integer having no common factors with r, and m can have only factors possessed also by r.

   The interpretation is that r defines fermionic state obtained by kicking from Dirac sea the fermions labelled by the prime factors of r. The integers n and m define bosonic excitations in which k:th power of p corresponds to k bosons in state labelled by p. One can also construct more complex infinite primes as polynomials of X and having no rational factors. In fact, X becomes coordinate variable in the correspondence with polynomials.

2. This process can be repeated at the next level. Now one introduces product $Y = \prod_P P$ of all primes at the previous level and repeats the same construction. These infinite correspond to polynomials of Y with coefficients given by rational functions of X. Primality means irreducibility in the field of rational functions so that solving Y in terms of X would give algebraic function.

3. At the lowest level are ordinary primes. At the next level the infinite primes are indeed infinite in real sense but have p-adic norms equal to unity. They can be mapped to polynomials $P(x_1)$ with rational coefficients and the simplest polynomials are monomials with rational root. Higher polynomials are irreducible polynomials with algebraic roots. At the third level of hierarchy one has polynomials $P(x_2|x_1)$ of two variables. They are polynomials of x_1 with coefficients with are rational functions of x_1. This hierarchy can be continued.

   One can define also infinite integers as products of infinite primes at various levels of hierarchy and even infinite rationals.

4. This hierarchy can be interpreted in terms of a repeated quantization of an arithmetic supersymmetric quantum field theory with elementary particles labelled by primes at given level of hierarchy. Physical picture suggests that the hierarchy of second quantizations is realized also in Nature and corresponds to the hierarchy of space-time sheets.

5. One could consider a mapping $P(x_n|x_{n-1}|..|x_1)$ by a diagonal projection $x_i = x$ to polynomials of single variable x. One could replace x with complexified octonic coordinate o_c. Could this correspondence give rise to octonionic polynomials and could the connection with second quantization give classical space-time correlates of real quantum states assignable to infinite primes and integers? Even quantum states defining counterparts of infinite rationals could be considered. One could require that the real norm of these infinite rationals equals to one. They would define infinite number of real units with arbitrarily complex number theoretical anatomy. The extension of real numbers by these units would mean huge extension of the notion of real number and one could say that each real point corresponds to platonic defined by these units closed under multiplication.

   In ZEO zero energy states formed by pairs of positive and negative energy could correspond to these states physically. The condition that the ratio is unit would have also a physical interpretation in terms of particle content.

6. As already noticed, the notions of analyticity, quaternionicity, and octonionicity could be seen as a manifestation of polynomials in algebras defined by adding repeatedly a new non-commuting imaginary unit to already existing algebra. The dimension of the algebra is doubled in each step so that dimension comes as a power of 2. The algebra of polynomials with real coefficients is commutative and associative. This encourages the crazy idea that the spaces are indeed realized and the generalization of $M^8 - H$ duality holds true at each

level. At level k the counterpart for CP_2 (for $k = 3$) would be as moduli space for sub-spaces of dimension 2^{k-1} for which tangent space reduces to the algebra at level $k-1$. For $k = 2$ CP_1 is the moduli space and could correspond to twistor sphere. Essentially Grassmannian $Gl(2^k, 2^{k-1})$ would be in question. This brings in mind twistor Grassmann approach involving hierarchy of Grassmannians too, which however allows all dimensions. What is interesting that the spinor bundle for space of even dimension d has fiber with dimension $2^{d/2}$.

The number of arguments for the hierarchy of polynomials assignable to the hierarchy of infinite primes increases by one at each step. Hence these two hierarchies are different.

The vanishing of the octonionic polynomials indeed allow a decomposition to products of prime polynomials with roots which in general are algebraic numbers and an exciting possibility is that the prime polynomials have interpretation as counterparts of elementary particles in very general sense.

Infinite primes can be mapped to polynomials and the most natural counterpart for the infinite rational would be as a complexified octonionic rational function $P_1(t)/P_2(t-T)$, where T is real octonion, with coefficients in extension of rationals. This would naturally give the geometry CD. The assignment of opposite boundaries of CD to $P_1(t)$ and $P_2(t-T)$ is suggestive and identification of zero loci of $IM(P_1)$ and $IM(P_2)$ as incoming and outgoing particles would be natural. The zero and ∞ loci for $RE(P_1/P_2)$ would define interaction between these space-time varieties and should give rise to wormhole contacts connecting them. Note that the linearity of $IM(o_1o_2)$ in $IM(o_i)$ and non-linearity of $RE(o_1o_2)$ in $RE(o_i)$ would be a key element behind this identification. This idea will be discussed in more detail in the section "Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view".

6.3 Super variant of octonionic algebraic geometry and space-time surfaces as correlates for fermionic states

Could the octonionic level provide an elegant description of fermions in terms of super variant of octonionic algebraic geometry? Could one even construct scattering amplitudes at the level of M^8 using the variant of the twistor approach discussed in [K49, L41]?

The idea about super-geometry is of course very different from the idea that fermionic statistics is realized in terms of the spinor structure of "world of classical worlds" (WCW) but $M^8 - H$ duality could however map these ideas and also number theoretic and geometric vision to each other. The angel of geometry and the devil of algebra could be dual to each other.

In the following I start from the notion of emergence generalized to the vision that entire physics emerges from the notion of number. This naturally leads to an identification of super-variants of various number fields, in particular of complexified octonions. After that super variants of $RE(P) = 0$ and $IM(P) = 0$ conditions are discussed, and the surprising finding is that the conditions might allow only single fermion states localized at strings. This would allow only single particle in the super-multiplet and would mean breaking of SUSY. This picture would be consistent with the earlier H picture about construction of scattering amplitudes [K49, L41]. Finally the problems related to the detailed physical interpretation are discussed.

6.3.1 About emergence

The notion of emergence is fashionable in the recent day physics, in particular, he belief is that 3-space emerges in some manner. In the sequel I consider briefly the standard view about emergence idea from TGD point of view, then suggest that the emergence in the deepest sense requires emergence of physics from the notion of number and that complexified octonions [L32] [L33, L34, L22, L31] are the most plausible candidate in this respect. After that I will show that number theory generalizes to super-number theory: super-number fields make sense and one can define the notion of super-prime. Every new step of progress creates worry about consistency with the earlier work, now the work done during last months with physics as octonionic algebraic geometry and also this aspect is discussed.

1. The notion of holography is behind the emergence of 3-space and implies that the notion of 2-space is taken as input. This could be justified by conformal invariance.

2. The key idea is that 3-space emerges somehow from entanglement. There is something that must entangle and this something must be labelled by points of space: one must introduce a discretised space. Then one must do some handwaving to make it 3-D - perhaps by arguing that holography based on 2-D holograms is unique by conformal invariance. The next handwave would replace this as a 3-D continuous space at infrared limit.

3. How to get space-time and how to get general coordinate invariance? How to get the symmetries of standard model and special relativity? Somehow all this must be smuggled into the theory when the audience is cheated to direct its attention elsewhere. This Münchausen trick requires a professional magician!

4. One attempt could take as starting point what I call strong form of holography (SH) in which 2-D data determine 4-D physics. Just like 2-D real analytic function determines analytic function of two complex variables in spacetime of 2 complex dimensions by analytic continuation (this hints strongly to quaternions). This is possible if conformal invariance is generalized to that for light-like 3-surfaces such as light-cone boundary. But the emergence magician should do the same without these.

   In TGD one could make this even simpler. Octonionic polynomials and rational functions are obtained from real polynomials of real variable by octonion-analytic continuation. And since polynomials and rational functions P_1/P_2 are in question their values at finite number of discrete points determined them if the orders of P_1 and P_2 are known!

   If one accepts adelic hierarchy based on extensions of rationals the coefficients of polynomials are in extensions of rationals and the situation simplifies further. The criticality conditions guaranteeing associativity for external particles is one more simplification: everything b becomes discrete. The physics at fundamental level could be incredibly simple: discrete number of points determines space-time surfaces as zero loci for $RE(P)$ or $IM(P)$ (octonions are decomposed to two quaternions gives $RE(o)$ and $IM(o)$).

   How this is mapped to physics leading to standard model emerging from the formulation in $M \times CP_2$ This map exists - I call it M^8-H duality - and takes space-time varieties in Minkowskian sector of complexified octonions to a space-time surface in $M^4 \times CP_2$ coding for standard model quantum numbers and classical fields.

How to get all this without bringing in octonionic embedding space: this is the challenge for the emergence-magician! I am afraid this this trick is impossible. I will however propose a deeper for what emergence is. It would not be emergence of space-time and all physics from entanglement but from the notion of number, which is at the base of all mathematics. This view led to a discovery of the notion of super-number field, a completely new mathematical concept, which should show how deep the idea is.

6.3.2 Does physics emerge from the notion of number field?

Concerning emergence one can start from a totally different point of view. Even if one gets rid of space as something fundamental from Hilbert sapce and entanglement, one has not reached the most fundamental level. Structures like Hilbert space, manifold, etc. are not fundamental mathematical structures: they require the notion of number field. Number field is the fundamental notion.

Could entire physics emerge from the notion of number field alone: space-time, fermions, standard model interactions, gravitation? There are good hopes about this in TGD framework if one accepts M^8-H duality and physics as octonionic algebraic geometry! One could however argue that fermions do not follow from the notion of number field alone. The real surprise was that formalizing this more precisely led to a realization that the very notion of number field generalizes to what one could call super-number field!

Emergence of physics from complexified octonionic algebraic geometry

Consider first the situation for number fields postponing the addition of attribute "super" later.

1. Number field endowed with basic arithmetic operations $+$, $-$, $\cdot$, $/$ is the basic notion for anyone wanting to make theoretical physics. There is a rich repertoire of number fields. Finite fields, rationals and their extensions, real numbers, complex numbers, quaternions, and octonions. There also p-adic numbers and their extensions induced by extensions of rationals and fusing into adele forming basic structure of adelic physics. Even the complex, quaternionic, and octonionic rationals and their extensions make sense. p-Adic variants of say octonions must be however restricted to have coefficients belonging to an extension of rationals unless one is willing to give up field property (the p-adic analog of norm squared can vanish in higher p-adic dimensions so that inverse need not exist).

   There are also function fields consisting of functions with local arithmetic operations. Analytic functions of complex variable provides the basic example. If function vanishes at some point its inverse element diverges at the same point. Function fields are derived objects rather than fundamental.

2. Octonions are the largest classical number field and are therefore the natural choice if one wants to reduce physics to the notion of number. Since one wants also algebraic extensions of rationals, it is natural to introduce the notion of complexified octonion by introducing an additional imaginary unit - call it i, commuting with the 7 octonionic imaginary units I_k. One obtains complexified octonions.

   That this is not a global number field anymore turns out to be a blessing physically. Complexified octonion z_kE^k has $z_k = z_k + iy_k$. The complex valued norm of octonion is given by $z_0^2 + ...z_7^2$ (there is no conjugation involved. The norm vanishes at the complex surface $z_0^2 + ...z_7^2 = 0$ defining a 7-D surface in 7-D O_c (the dimension is defined in complex sense). At this surface - complexified light-cone boundary - number field theory property fails but is preserved elsewhere since one can construct the inverse of octonion.

   At the real section M^8 (8-D Minkowski space with one real (imaginary) coordinate and 7 imaginary (real) coordinates the vanishing takes place also. This surface corresponds to the 7-D light-cone boundary of 8-D Minkowskian light-cone. This suggests that light-like propagation is basically due to the complexification of octonions implying local failure of the number field property. Same happens also in other real sections with $0 < n < 8$ real coordinates and $0 < m = 8 - n < 8$ imaginary coordinates and one obtains variant of light-cone with different signatures. Euclidian signature corresponding to $m = 0$ or $m = 8$ is an exception: light-cone boundary reduces to single point in this case and one has genuine number field - no propagation is possible in Euclidian signature.

   Similar argument applies in the case of complexified quaternions Q_c and complexified complex numbers $z_1 + z_2I \in C_c$, where I is octonionic imaginary unit. For Q_c one obtains ordinary 3-D light-cone boundary in real section and 1-D light-cone boundary in the case of C_c. It seems that physics demands complexification! The restriction to real sector follows from the requirement that norm squared reduces to a real number. All real sectors are possible and I have already considered the question whether this should be taken as a prediction of TGD and whether it is testable.

Super-octonionic algebraic geometry

There is also a natural generalization of octonionic TGD to super-octonionic TGD based on octonionic triality. $SO(1,7)$ allows besides 8-D vector representations also spinor representations 8_c and $\overline{8}_c$. This suggests that super variant of number field of octonions might make sense. One would have $o = o_8 + o_{c,8} + \overline{0}_{c,8}$.

1. Should one combine o_8, $o_{c,8}$ and $\overline{o}_{c,8}$ to a coordinate triplet $(o_8, o_{c,8}, \overline{o}_{c,8})$ as done in supersymmetric theories to construct super-fields? The introduction of super-fields as primary dynamical variables is a good idea now since the very idea is to reduce physics to algebraic geometry at the level of M^8. Polynomials of super-octonions defining space-time varieties as zero loci for their real or imaginary part in quaternionic sense could however take the role of super fields. Space-time surface would correspond to zero loci for $RE(P)$ or $IM(P)$.

2. The idea about super-octonions should be consistent with the idea that we live in a complexified number field. How to define the notion of super-octonion? The tensor product $8 \otimes 8_c$ contains 8_c and $8 \otimes 8_{\overline{c}}$ contains $8_{\overline{c}}$ and one can use Glebsch-Gordan coefficients to contract o and θ_c and o and $\overline{\theta}_{c,n}$. The tensor product of 8_c and $8_{\overline{c}}$ defined using structure constants defining octonion product gives 8. Therefore one must have

$$o_s = o + \Psi_c \times \theta_{\overline{c}} + \Psi_{\overline{c}} \times \theta_c \ , \tag{6.3.1}$$

where the products are octonion products. Super parts of super-coordinates would not be just Grassmann numbers but octonionic products of Grassmann numbers with octonionic spinors in 8_c and $\overline{8}_c$. This would bring in the octonionic analogs of spinor fields into the octonionic geometry.

This seems to be consistent with super field theories since octonionic polynomials and even rational functions would give the analogs of super-fields. What TGD would provide would be an algebraic geometrization of super-fields.

3. What is the meaning of the conditions $RE(P) = 0$ and $IM(P) = 0$ for super-octonions? Does this condition hold true for all $d_G = 2^{16}$ super components of $P(o_s)$ or is it enough to pose the condition only for the octonionic part of $P(o)$? In the latter case Ψ_c and $\Psi_{\overline{c}}$ would be free and this does not seem sensical and does not conform with octonionic super-symmetry. Therefore the first option will be studied in the sequel.

If super-octonions for a super variant of number field so that also inverse of super-octonion is well-defined, then even rational functions of complexified super-octonions makes sense and poles have interpretation in terms of 8-D light-fronts (partonic orbits at level of H). The notion must make sense also for other classical number fields, finite fields, rationals and their extensions, and p-adic numbers and their extensions. Does this structure form a generalization of number field to a super counter part of number field? The easiest manner to kill the idea is to check what happens in the case of reals.

1. The super-real would be of form $s = x + y\theta$, $\theta^2 = 0$. Sum and product are obviously well-defined. The inverse is also well-defined and given by $1/s = (x - y\theta))/x^2$. Note that for complex number $x + iy$ the inverse would be $\overline{z}/z\overline{z} = (x - yi)/(x^2 + y^2)$. The formula for super-inverse follows from the same formula as the inverse of complex number by defining conjugate of super-real s as $\overline{s} = x - y\theta$ and the norm squared of s as $|s|^2 = s\overline{s} = x^2$.

One can identify super-integers as $N = m + n\theta$. One can also identify super-real units as number of unit norm. Any number $1_n = 1 + n\theta$ has unit norm and the norms form an Abelian group under multiplication: $1_m 1_n = 1_{m+n}$. Similar non-uniqueness of units occurs also for algebraic extensions of rationals.

2. Could one have super variant of number theory? Can one identify super-primes? Super-norm satisfies the usual defining property $|xy| = |x||y|$. Super-prime is defined only apart from the multiplicative factor 1_m giving not contribution to the norm. This is not a problem but a more rigorous formulation leads to the replacement of primes with prime ideals labelled by primes already in the ordinary number theory.

If the norm of super-prime is ordinary prime it cannot decompose to a product of super-primes. Not all super-primes having given ordinary prime as norm are however independent. If super-primes $p + n\theta$ and $p + m\theta$ differ by a multiplication with unit $1_r = 1 + r\theta$, one has $n - m = pr$. Hence there are only p super-primes with norm p and they can be taken $p_s = p + k\theta$, $k \in \{0, p - 1\}$. A structure analogous to a cyclic group Z_p emerges.

Note that also θ is somewhat analogous to prime although its norm is vanishing.

3. Just for fun, one an ask what is the super counterpart of Riemann Zeta. Riemann zeta can be regarded as an analog of thermodynamical partition function reducing to a product for partition functions for bosonic systems labelled by primes p. The contribution from prime p

is factor $1/(1-p^{-s})$. p^{-s} is analogous to Boltzmann weight $N(E)exp(-E/T)$, where $N(E)$ is number of states with energy E. The degeneracy of states labelled by prime p is for ordinary primes $N(p) = 1$. For super-primes the degeneracy is $N(p) = p$ and the weight becomes $1/(1-N(p)p^{-s}) = 1/(1-p^{-s+1})$. Super Riemann zeta is therefore $zeta(s-1)$ having critical line at $s = 3/2$ rather than at $s = 1/2$ and trivial zeros at real points $s = -1, -3, -5$, rather than at $s = -2, -4, -6, ...$

There are good reasons to expect that the above arguments work also for algebraic extensions of super-rationals and in fact for all number fields, even for super-variants of complex numbers, quaternions and octonions. This because the conditions for invertibility reduce to that for real numbers. One would have a generalization of number theory to super-number theory! Net search gives no references to anything like this. Perhaps the generalization has not been noticed because the physical motivation has been lacking. M^8-H duality would imply that entire physics, including fermion statistics, standard model interactions and gravitation reduces to the notion of number in accordance with number theoretical view about emergence.

Is it possible to satisfy super-variants of $IM(P) = 0$ and $RE(P) = 0$ conditions?

Instead of super-fields one would have a super variant of octonionic algebraic geometry.

1. Super variants of the polynomials and even rational functions make sense and reduce to a sum of octonionic polynomials $P_{kl}\theta_1^k\theta_2^l$, where the integers k and l would be tentatively identified as fermion numbers and θ_k is a shorthand for a monomial of k different thetas. The coefficients in $P_{kl} = P_{kl,n}o^n$ would be given by $P_{kl,n} = P_{n+k+l}B(n+k+l, k+l)$, where $B(r, s) = r!/(r-s)!s!$ is binomial coefficient. The space-time surfaces associated with P_{kl} would be different and they need not be simultaneously critical, which could give rise to a breaking of supersymmetry.

   One would clearly have an upper bound for k and l for given CD. Therefore these many-fermion states must correspond to fundamental particles rather than many-fermion Fock states. One would obtain bosons with non-vanishing fermion numbers if the proposed identification is correct. Octonionic algebraic geometry for single CD would describe only fundamental particles or states with bounded fermion numbers. Fundamental particles would be indeed fundamental also geometrically.

2. One can also now define space-time varieties as zero loci via the conditions $RE(P_s)(o_s) = 0$ or $IM(P_s)(o_s) = 0$. One obtains a collection of 4-surfaces as zero loci of P_{kl}. One would have a correlation with between fermion content and algebraic geometry of the space-time surface unlike in the ordinary super-space approach, where the notion of the geometry remains rather formal and there is no natural coupling between fermionic content and classical geometry. At the level of H this comes from quantum classical correspondence (QCC) stating that the classical Noether charges are equal to eigenvalues of fermionic Noether charges.

In the definition of the first variant of super-octonions I followed the standard idea about what super-coordinates assuming that the super-part of super-octonion is just an anti-commuting Grassmann number without any structure: I just replaced o with $o + \theta_k E^k + \overline{\theta}_k E^k$ regarding θ_k as anticommiting coordinates. Now θ_k receives octonionic coefficient: $\theta_k \rightarrow o_k\theta_k$. θ_k is now analogous to unit vector.

For the super-number field inspired formulation the situation is different since one assigns independent octonionic coordinates to anticommuting degrees of freedom. One has linear space with partially anti-commutative basis. O_c is effectively replaced with O_c^3 so that one has 8+8+8=24-dimensional Cartesian product (it is amusing that the magic dimension 24 for physical polarizations of bosonic string models emerges).

What is the number of equations in the new picture? For N super-coordinates one has 2^N separate monomials analogous to many-fermion states. Now one has $N = 8 + 8 = 16$ and this gives 2^{16} monomials! In the general case $RE = 0$ or $IM = 0$ gives 4 equations for each of the $d_G = 2^{16}$ monomials: the number of equations $RE = 0$ or $IM = 0$ is 4×2^{16} and exceeds the number $d_O = 24$ of octonion valued coordinates. In the original interpretation these equations were regarded as independent and gave different space-time variety for each many-fermion state.

In the new framework these equations cannot be treated independently. One has 24 octonionic coordinates and 2^{16} equations. In the generic case there are no solutions. This is actually what one hopes since otherwise one would have a state involving superposition of many-fermion states with several fermion numbers.

The freedom to pose constraints on the coefficients of Grassmann parameters however allows to reduce degrees of freedom. All coefficients must be however expressible as products of $3\times 8 = 24$ components of super-octonion.

1. One can have solutions for which both 8_c part and $\overline{8}_c$ parts vanish. This gives the familiar 4 equations for 8 variables and 4-surfaces.

2. Consider first options, which fail. If 8_c- or $\overline{8}_c$ part vanishes one has $d_G = 2^8$ and $4\times d_G = 4\times 64$ equations for $dO_=8+8 = 16$ variables having no solutions in the generic case. The restriction of 8_c to its 4-D quaternionic sub-space would give $d_O = 4$ and $4d_G = 4\times 2^4 = 64$ conditions and 16 variables. The reduction to complex sub-space $z_1 + z_2 I$ of super-octonions would give $d_O = 2^2$ and $4\times 2^2 = 16$ conditions for $8 + 2 = 10$ variables.

3. The restriction to 1-D sub-space of super-octonions would give $4\times 2^1 = 8$ conditions and $8 + 1 = 9$ variables. Could the solution be interpreted as 1-D fermionic string assignable to the space-like boundary of space-time surface at the boundary of CD? Skeptic inside me asks whether this could mean the analog of $\mathcal{N} = 1$ SUSY, which is not consistent with H picture.

   Second possibility is restriction to light-like subspace for which powers of light-like octonion reduce effectively to powers of real coordinate. Fermions would be along light-lines in M^8 and along light-like curves in H. The powers of super-octonion have super-part, which belongs to the 1-D super-space in question: only single fermion state is present besides scalar state.

4. There are probably other solutions to the conditions but the presence of fermions certainly forces a localization of fermionic states to lower-dimensional varieties. This is what happens also in H picture. During years the localization of fermion to string worlds sheets and their boundaries has popped up again and again from various arguments. Could one hope that super-number theory provides the eventual argument.

   But how could one understand string world sheets in this framework? If they do not carry fermions at H-level, do they appear naturally as 2-D structures in the ordinary sense?

To sum up, although many details must be checked and up-dated, super-number theory provides and extremely attractive approach promising ultimate emergence as a reduction of physics to the notion of number. When physical theory leads to a discovery of new mathematics, one must take it seriously.

6.3.3 About physical interpretation

Super-octonionic algebraic geometry should be consistent with the H picture in which baryon and lepton numbers as well as other standard model quantum numbers can be understood. There are still many details, which are not properly understood.

The interpretation of theta parameters

The interpretation of theta parameters is not completely straightforward.

1. The first interpretation is that θ_c and $\theta_{\overline{c}}$ correspond to objects with opposite fermion numbers. If this is not the case, one could perhaps define the conjugate of super-coordinate as octonionic conjugate $\overline{o}_s = \overline{o} + \overline{\theta}_1 + \overline{\theta}_2$. This looks ugly but cannot be excluded.

   There is also the question about spinor property. Octonionic spinors are 2-spinors with octonion valued components. Could one say that the coefficients of octonion units have been replaced with Grassmann numbers and the entire 2-component spinor is represented as a pair of θ_c and $\theta_{\overline{c}}$? The two components of spinor in massless theories indeed correspond to massless particle and its antiparticle.

2. One should obtain particles and antiparticles naturally as also separately conserved baryon and lepton numbers (I have also considered the identification of hadrons in terms of anyonic bound states of leptons with fractional charges).

   Quarks and leptons have different coupling to the induced Kähler form at the level of H. It seems impossible to understand this at the level of M^8, where the dynamics is purely algebraic and contains no gauge couplings.

   The difference between quarks and leptons is that they allow color partial waves with triality $t = \pm 1$ and triality $t = 0$. Color partial waves correspond to wave functions in the moduli space CP_2 for $M_0^4 \supset M_0^2$. Could the distinction between quarks and leptons emerge at the level of this moduli space rather than at the fundamental octonionic level? There would be no need for gauge couplings to distinguish between quarks and leptons at the level of M^8. All couplings would follow from the criticality conditions guaranteeing 4-D associativity for external particles (on mass shell states would be critical).

   If so, one would have only the super octonions and θ_c and $\theta_{\overline{c}}$ would correspond to fermions and antifermions with no differentiation to quarks or leptons. Fermion number conservation would be coded by the Grassmann algebra. Quantum classical correspondence (QCC) however suggests that it should be possible to distinguish between quarks and leptons already at M^8 level. Is it really enough that the distinction comes at the level of moduli space for CDs?

One can imagine also other options but they have their problems. Therefore this option will be considered in the sequel.

Questions about quantum numbers

The first questions relate to fermionic statistics.

1. Do super-octonions really realize fermionic statistics and how? The polynomials of super-octonions can have only finite degree in θ and θ_c. One an say that only finite number of fermions are possible at given space-time point. As found, the conditions $IM(P) = 0$ and $RE(P) = 0$ might allow only single fermion strings as solutions perhaps assignable to partonic 2-surfaces.

   Can one allow for given CD arbitrary number of this kind of points as the idea that identical fermions can reside at different points suggests? Or is the number of fermions finite for given CD or correspond to the highest degree monomial of θ and θ_c in P?

   Finite fermion number of CD looks somewhat disappointing at first. The states with high fermion numbers would be described in terms of Cartesian products just like in condensed matter physics. Note however that space-time varieties with different octonionic time axes must be in any case described in this manner. It seems possible to describe the interactions using super-space delta functions stating that the interaction occur only in the intersection points of the space-time surfaces. The delta function would have also super-part as in SUSYs.

2. As found, the theta degree effectively reduces to $d = 1$ for the pointlike solutions, which by above argument are the only possible solutions besides purely bosonic solutions. Only single fermion would be allowed at given point. I have already earlier considered the question whether the partonic 2-surfaces can carry also many-fermion states or not [K49, L41], and adopted the working hypothesis that fermion numbers are not larger than 1 for given wormhole throat, possibly for purely dynamical reasons. This picture however looks too limited. The many fermion states might not however propagate as ordinary particles (the proposal has been that their propagator pole corresponds to higher power of p^2).

The M^8 description of particle quantum numbers should be consistent with H description.

1. Can octonionic super geometry code for the quantum numbers of the particle states? It seems that super-octonionic polynomials multiplied by octonionic multi-spinors inside single CD can code only for the electroweak quantum numbers of fundamental particles besides their fermion and anti-fermion numbers. What about color?

As already suggested, color corresponds to partial waves in CP_2 serving as moduli space for $M_0^4 \supset M_0^2$. Also four-momentum and angular momentum are naturally assigned with the translational degrees for the tip of CD assignable with the fundamental particle.

2. Quarks and leptons have different trialities at H level. How can one understand this at M^8 level. Could the color triality of fermion be determined by the color representation assignable to the color decomposition of octonion as $8 = 1 + 1 + 3 + \overline{3}$. This decomposition occurs for all 3 terms in the super-octonion. Could the octet in question correspond to the term $D(8 \otimes 8_c; 8_c)_k^{mn} o_{c,m} \theta_{c,n} E^k$ and analogous $\theta_{\overline{c}}$ term in super octonion. Only this kind of term survives from the entire super-octonion polynomial at fermionic string for the solutions found.

3. There is however a problem: $8 = 1 + 1 + 3 + \overline{3}$ decomposition is not consistent with the idea that θ_c and $\theta_{\overline{c}}$ have definite fermion numbers. Quarks appear only as 3, not $\overline{3}$. Why $\overline{3}$ from θ term and 3 from $\theta_{\overline{c}}$ term should drop out as allowed single fermion state?

There are also other questions.

1. What about twistors in this framework? $M^4 \times CP_1$ as twistor space with CP_1 coding for the choice of $M_0^2 \subset M_0^4$ allows projection to the usual twistor space CP_3. Twistor wave functions describing spin elegantly would correspond to wave functions in the twistor space and one expects that the notion of super-twistor is well-defined also now. The 6-D twistor space $SU(3)/U(2) \times U(1)$ of CP_2 would code besides the choice of $M_0^4 \supset M_0^2$ also quantization axis for color hypercharge and isospin.

2. The intersection of space-time surfaces with S^6 giving analogs of partonic 2-surfaces might make possible for two sparticle lines to fuse to form a third one at these surfaces. This would define sparticle 3-vertex in very much the same manner as in twistor Grassmann approach to $\mathcal{N} = 4$ SUSY.

   H-picture however supports the alternative option that sparticles just scatter but there is no contact interaction defining analog of 3-vertex. If the lines can carry only single fermion, the H picture about twistor diagrams [K49, L41] would be realized also at the level of M^8! This means breaking of SUSY since only single fermion states from the octonionic SUSY multiplet are realized. This would provide and easy - perhaps too easy - explanation for the failure to find SUSY at LHC.

3. What about the sphere S^6 serving as the moduli space for the choices of M_+^8? Should one have wave functions in S^6 or can one restrict the consideration to single M_+^8? As found, one obtains S^6 also as the zero locus of $Im(P) = 0$ for some radii identifiable as values t_n of time coordinates given as roots of $P(t)$: as matter of fact, $S^6(t_n)$ is a solution of both $RE(P) = 0$ and $IM(P) = 0$. Can one identify the intersections $X^4 \cap S^6$ are 2-D as partonic 2-surfaces serving as topological vertices?

6.4 Could scattering amplitudes be computed in the octonionic framework?

Octonionic algebraic geometry might provide incredibly simple framework for constructing scattering amplitudes since now variational principle is involved and WCW reduces to a discrete set of points in extension of rationals.

6.4.1 Could scattering amplitudes be computed at the level of M^8?

It would be extremely nice if the scattering amplitudes could be computed at the octonionic level by using a generalization of twistor approach in ZEO finding a nice justification at the level of M^8. Something rather similar to $\mathcal{N} = 4$ twistor Grassmann approach suggests itself.

1. In ZEO picture one would consider the situation in which the passive boundary of CD and members of state pairs at it appearing in zero energy state remain fixed during the sequence of state function reductions inducing stepwise drift of the active boundary of CD and change of states at it by unitary U-matrix at each step following by a localization in the moduli space for the positions of the active boundary.

2. At the active boundary one would obtain quantum superposition of states corresponding to different octonionic geometries for the outgoing particles. Instead of functional integral one would have sum over discrete points of WCW. WCW coordinates would be the coefficients of polynomial P in the extension of rationals. This would give undefined result without additional constraints since rationals are a dense set of reals.

   Criticality however serves as a constraint on the coefficients of the polynomials and is expected to realize finite measurement resolution, and hopefully give a well defined finite result in the summation. Criticality for the outgoing states would realize purely number theoretically the cutoff due to finite measurement resolution and would be absolutely essential for the finiteness and well-definedness of the theory.

6.4.2 Interaction vertices for space-time surfaces with the same CD

Consider interaction vertices for space-time surfaces associated with given CD. At the level of H the fundamental interactions vertices are partonic 2-surfaces at which 3 light-like partonic orbits meet. The incoming light-like sparticle lines scatter at this surface and they are not assumed to meet at single vertex. This assumption is motivated because it allows to avoid infinities but one must be ready to challenge it. It is essential that wormhole throats appear in pairs assignable to wormhole contacts and also contacts form pairs by the conservation of Kähler magnetic flux.

What could be the counterpart of this picture at level of M^8?

1. The simplest interaction could be associated with the common stable intersection points of the space-time regions. By dimensional consideration these intersections are stable and form a discrete set. This would however allow only 2-vertices involved in processes like mixing of states. In the generic case the intersection would consist of discrete points.

2. A stronger condition would be that these points belong to the extension of rationals defining adeles or is extension defined by the polynomial P. This would conform with the idea that scattering amplitudes involve only data associated with the points in the extension. The interaction points could be ramified points at which the action of a subgroup H of Galois group G would leave sheets of the Galois covering invariant so that some number of sheets would touch each other. I have discussed this proposal in [L29]. These points could be seen as analogs of interaction points in QFT description in terms of n-point functions and the sum over polynomials would give rise to the analog over integral over different n-point configurations.

3. A possible interpretation is that if the subgroup $H \subset G$ has k-elements the vertex represents meeting of k sparticle lines and thus k-vertex would be in question. This picture is not what the H view about twistor diagrams [L41] suggests: in these diagrams sparticle lines at the light-like orbits of partonic 2-surfaces do not meet at single point but only scatter at partonic 2-surface, where three light-like orbits of partonic 2-surfaces meet.

4. An alternative interpretation is that k-vertex describes the decay of particle to k fractional particles at partonic 2-surfaces and has nothing do with the usual interaction vertex.

This proposal need not describe usual particle scattering. Could the intersection of space-time varieties defined as zero loci for $RE(P_i)$ and $IM(P_i)$ with the special solutions $S^6(t_n)$ and $CD = M^4_+ \cap M^4_-$ define the loci of interaction? It is difficult to believe that these special solutions could be only a beauty spot of the theory. $X^2 = X^4 \cap S^6(t_n)$ is 2-D and $X^0 = X^4 \cap CD$ consists of discrete points.

Consider now the possible role of the singular ($RE(P) = IM(P) = 0$) maximally critical surface $S^6(t_n)$ in the scattering.

1. As already found, the 6-D spheres S^6 with radii t_n given by the zeros of $P(t)$ are universal and have interpretation as $t = t_n$ snapshots of 7-D spherical light front projection to $t = t_n$ 3-balls as cross sections of 4-D CD. Could the 2-D intersection $X^2 = X^4 \cap S^6(t_n)$ play a fundamental role in the description of interaction vertices?

2. Suppose that 3-vertices realize the dynamical realization of octonionic SUSY predicting large number of sparticles. Could one understand in this framework the 3-vertex for the orbits X_i^3 of partonic 2-surfaces meeting each other along their 2-D end defining partonic 2-surface and undersand how 3 fermions lines meet at single point in this picture?

3. Assume that 3 partonic orbits X_i^3, $i = 1, 2, 3$ meet at $X^2 = X^4 \cap S^6(t_n)$. That this occurs could be part of boundary conditions, which should follow from interaction consistency. If fermions just *go through* the X_i^2 in time direction they cannot meet at single point in the generic case. If the sparticle lines however can *move along* X^2 - maybe due the fact that an intersection $X^2 = X^4 \cap S^6(t_n)$ is in question - they intersect in the generic case and fuse to a third fermion line. Note that this portion of fermion line would be space-like whereas outside X^2 the line would be light-like. This can be used as an objection against the idea.

The picture allowing 3-vertices would be different from H picture in which fermion lines only scatter and only 2+2 fermion vertex assignable to topological 3-vertex is fundamental.

1. One would have 2 wormhole contacts carrying fermion and third one carrying fermion anti-fermion pair at its opposite throats and analogous to boson. Of course, one can reproduce the earlier picture by giving up the condition about supersymmetric fermionic 3-vertex. On the other hand, the idea that interactions occur only at discrete points in extension of rationals is extremely attractive.

2. The surprising outcome from the construction of solutions of super-variants of $RE(P) = 0$ and $IM(P) = 0$ conditions was that if the superpart of super-octonion is non-vanishing, the variety can be only 1-D string like entity carrying one-fermion state. This does allow strings with higher fermion number so that the 3-vertex would not be possible! This suggests that fermionic lines appear as sub-varieties of space-time variety.

   If so the original picture [L41] applying at the level of H applies also at the level of M^8. SUSY is broken dynamically allowing only single fermion states localized at strings and scattering of these occurs by classical interactions at the partonic 2-surfaces defining the topological vertices.

3. The only manner to have a point/line containing sparticle with higher fermion number would be as a singularity along which several branches of super-variety degenerate to single point/line: each variety would carry one fermion line. Unbroken octonionic SUSY would characterize singularities of the space-time varieties, which would be unstable so that SUSY would break. Singularities are indeed critical and thus unstable and also tend to possess enhanced symmetries.

What could be the interpretation of $X^0 = X^4 \cap CD$? For instance, could it be that these points code for 4-momenta classically so that quantum classical correspondence (QCC) would be realized also at the level of M^8 although there are no Noether charges now. But what about angular momenta? Could twistorialization realized in terms of the quaternionic structure of M_0^4 help here. What is the role of the intersections of 6-D twistor bundle of X^4 with 6-D twistor bundle of M_0^4 consisting of discrete points?

The interaction vertex would involve delta function telling that the interacting space-time varieties or their regions touch at same point of M^8. Delta function in theta parameter degrees of freedom and Grassmann integral over them would be also involved and guarantee fermion number conservation. Vertex factor should be determined by arguments used in Grassmannian twistor approach. I have developed a proposal in [L41] but this proposal allows only fermion number ± 1 at fermion lines. Now all members of the multiplet would be allowed.

6.4.3 How could the space-time varieties associated with different CDs interact?

The interaction of space-time surfaces inside given CD is well-defined in the octonionic algebraic geometry. The situation is not so clear for different CDs for which the choice of the origin of octonionic coordinates is in general different and polynomial bases for different CDs do not commute nor associate.

The intuitive expectation is that 4-D/8-D CDs can be located everywhere in M^4/M^8. The polynomials with different origins neither commute nor are associative. Their sum is a polynomial whose coefficients are not real. How could one avoid losing the extremely beautiful associative and commutative algebra of polynomials?

1. Should one assume that the physics observable by single conscious observer corresponds to single CD defining the perceptive field of this observer [L42].

2. Or should one give up associativity and allow products (but not sums since one should give up the assumption that the coefficients of polynomials are real) of polynomials associated with different CDs as an analog for the formation of free many-particle states.

Consider first what happens for the single particle solutions defined as solutions of either $RE(P_i) = 0$ or $IM(P_i) = 0$.

1. The polynomials associated with different 8-D CDs do not commute nor associate. Should one allow their products so that one would still *effectively* have a Cartesian product of commutative and associative algebras? This would realize non-commutative and non-associative physics emerging in conformal field theories also at the level of space-time geometry.

2. If the CDs differ by a *real* (time) translation $o_2 = o_1 + T$ one still obtains $IM(P_1) = 0$ and $IM(P_2) = 0$ as solutions to $IM(P_1P_2) = 0$. This applies also to states with more particles. The identification would be in terms of external particles. For $RE(P_1P_2) = 0$ this is not the case. If the interior of CD corresponds to $RE(P_1P_2) = 0$, the dynamics in the interior is not only non-trivial but also non-commutative and non-associative. Non-trivial interaction would be obtained even without interaction terms in the polynomial vanishing at the boundaries of CD!

   Could one consider allowing only CDs with tips at the same real axis but having all sizes scales? This hierarchy of CD would characterize a particular hierarchy of conscious observers - selves having sub-selves (sub-CDs) [L42]. The allowance of only these CD would be analogous to a fixing of quantization axes.

3. What happens if one allows CDs differing by arbitrary octonion translation? Consider external particles. For P_1 and P_2 RE and IM are defined for different decompositions $o_i = RE(o_i) + n_i Im(o_i)$, where n_i, $i = 1, 2$ is a unit octonion.

   What decomposition should one use for P_1P_2? The decomposition for P_1 or P_2 or some other decomposition? One can express $P_2(o_2)$ using o_1 as coordinate but the coefficients multiplying powers of o_1 from *right* would not be real numbers anymore implying $IM(P_2)_1 \neq IM(P_2)_2$. $IM(P_2)_1 = 0$ makes sense but the presence of particle 1 would have affected particle 2 or vice versa.

   Could one argue that the coordinate systems satisfying the condition that some external particles described by P_i have real coefficients and perhaps serving in the role of observers are preferred? Or could one imagine that o_{12} is a kind of center of mass coordinate? In this case the 4-varieties associated with both particles would be affected. What is clear that the choice of the octonionic coordinate origin would affect the space-time varieties of external particles even if they could remain associative/critical.

4. Are there preferred coordinates in which criticality is preserved? For instance, can one achiever criticality for P_2 on coordinates of o_1 if P_1 is critical. Could one see this as a kind of number theoretic observer effect at the level of space-time geometry?

Remark: $P_i(o)$ would reduce to a real polynomial at light-like rays with origin for o_i irrespective of the octonionic coordinate used so that the spheres S^6_i with origin at the origin of o_i as solutions of $P_i(o)=0$ would not be lost.

If one does not give up associativity and commutativity for polynomials, how can one describe the interactions between space-time surfaces inside different CDs at the level of M^8? The following proposal is the simplest one that one can imagine by assuming that interactions take place at discrete points of space-time surfaces with coordinates belonging an extension of rationals.

1. The most straightforward manner would be to introduce Cartesian powers of O and CD:s inside these powers to describe the interaction between CDs with different origin. This would be analogous to what one does in condensed matter physics. What seems clear is that M^8-H correspondence should map all the factors of $(M^8)^n$ to the same $M^4\times CP_2$ by a kind of diagonal projection.

   In topological 3-parton vertex X^2 three light-like partonic orbits along X^4 would meet. X^2 would be the contact of X^4 with S^6 associated with second 8-D CD. Together with SH this gives hopes about an elegant description of interactions in terms of connected space-time varieties.

2. The intersection $X^4_i\cap X^4_j$ consists of discrete set of points. This would suggest that the interaction means transfer of fermion between X^4_1 and X^4_2. The intersection of $X=S^6_1(t_m)\cap S^6_2(t_n)$ is 4-D and space-like. The intersection $X^4_i\cap X$ consists of discrete points could these discrete points allow to construct interaction vertices.

To make this more concrete, assume that the external particles outside the interaction CD (CD$_{int}$) defining the interaction region correspond to associative (or co-associative) space-time varieties with different CDs.

Remark: CDs are now 8-dimensional.

1. One can assign the external particles to the Cartesian factors of $(M^8)^n$ giving $(P_1,...,P_n)$ just like one does in condensed matter physics for particles in 3-space E^3. Inside CD$_{int}$ the Cartesian factors would fuse to single factor and instead of Cartesian product one would have the octonionic product $P=\prod P_i$ plus the condition $RE(P)=0$ (or $IM(P)=0$: one should avoid too strong assumptions at this stage) would give to the space-time surface defining the interaction region.

2. $RE(P)=0$ and $IM(P)=0$ conditions make sense even, when the polynomials do not have origin at common real axis and give rise to 4 conditions for 8 polynomials of 8 complexified octonion components P^i. It is not possible to reduce the situation at the light-like boundaries of 8-D light-cone to a vanishing of polynomial $P(t)$ of real coordinate t anymore, and one loses the the surfaces S^6_i as special solutions and therefore also the partonic 2-surfaces $X^2_i=X^4\cap S^6_i$. Should one assign all X^2_i with the intersections of external particles with the two boundaries $\delta_\pm$ CD of CD defining the interaction region. They would intersect $\delta_\pm$CD at highly unique discrete points defining the sparticle interaction vertices. By 7-dimensionality of $\delta_\pm$CD the intersection points would be at the boundaries of 4-D CD and presumably at light-like partonic orbits at which the induced metric is singular at H side at least just as required by H picture.

   The most general external single-sparticle state would be defined by a product P of mutually commuting and associating polynomials with tips of CD along common real axis and satisfying $IM(P_i)=0$ or $RE(P_i)=0$. This could give both free and bound states of constituents.

3. Different orders and associations for $P=\prod P_i$ give rise to different interaction regions. This requires a sum over the scattering amplitudes $\sum_p T(\prod_i P_{p(i)})$ associated with the permutations p: $(1,...,n)\to(p(1),..,p(n))$ and $T=\sum_p U(p)T(P_{p(1)}...P_{p(n)})$ $(T(AB)+T(BA)$ in the simplest case) with suitable phase factors $U(p)$. Note that one does *not* have a sum over the polynomials $P_{p(1)}...P_{p(n)}$ but over the scattering amplitudes associated with them.

4. Depending on the monomial of theta parameters in super-octonion part of P_i, one has plus or minus signs under the exchange of P_i and P_j. One can also have braid group as a lift of the permutation group. In this case given contribution to the scattering amplitude has a phase factor depending on the permutation (say $T = T(AB) + exp(i\theta)T(BA)$.

   One must also form the sum $T = \sum_{Ass} U(Ass)T(Ass(P))$ over all associations for a given permutation with phase factors $U(Ass)$. Here $T = T((AB)C) + UT(A(BC))$, U phase factor, is the simplest case. One has "association statistics" as the analog of braid statistics. Permutations and associations have now a concrete geometric meaning at the level of space-time geometry - also at the level of H.

5. The geometric realization of permutations and associations could relate to the basic problem encountered in the twistorial construction of the scattering amplitudes. One has essentially sum over the cyclic permutations of the external particles but does not know how to construct the amplitudes for general permutations, which correspond to non-planar Feynman diagrams. The geometric realization of the permutations and associations would solve this problem in TGD framework.

6.4.4 Twistor Grassmannians and algebraic geometry

Twistor Grassmannians provide an application of algebraic geometry involving the above described notions [B21] (see `http://tinyurl.com/yd9tf2ya`). This approach allows extremely elegant expressions for planar amplitudes of $\mathcal{N} = 4$ SYM theory in terms of amplitudes formulated in Grassmannians $G(k, n)$.

It seems that this approach generalizes to TGD in such a way that CP_2 degrees of freedom give rise to additional factors in the amplitudes having form very similar to the M^4 part of amplitudes and involving also $G(k, n)$ with ordinary twistor space CP_3 being replaced with the flag manifold $SU(3)/U(1) \times U(1)$: k would now correspond to the number sparticles with negative weak isospin. Therefore the understanding of the algebraic geometry of twistor amplitudes could be helpful also in TGD framework.

Twistor Grassmannian approach very concisely

I try to compress my non-professional understanding of twistor Grassmann approach to some key points.

1. Twistor Grassmannian approach constructs the scattering amplitudes by fusing 3-vertices (+,-,-) (one positive helicity) and (-,+,+) (one negative helicity) to a more complex diagrams. All particles are on mass shell and massless but complex. If only real massless momenta are allowed the scattering amplitudes would allow only collinear gluons. Incoming particles have real momenta.

   Remark: Remarkably, $M^4 \times CP_2$ twistor lift of TGD predicts also complex Noether charges, in particular momenta, already at classical level. Quantal Noether charges should be hermitian operators with real eigenvalues, which suggests that total Noether charges are real. For conformal weights this condition corresponds to conformal confinement. Also $M^8 - H$ duality requires a complexification of octonions by adding commuting imaginary unit and allows to circumvent problems related to the Minkowski signature since the metric tensor can be regarded as Euclidian metric tensor defining complex value norm as bilinear $m^k m_{kl} m^l$ in complexified M^8 so that real metric is obtained only in sub-spaces with real or purely imaginary coordinates. The additional imaginary unit allows also to define what complex algebraic numbers mean.

   The unique property of 3-vertex is that the twistorial formulation for the conservation of four-momentum implies that in the vertex one has either $\lambda_1 \propto \lambda_2 \propto \lambda_3$ or $\overline{\lambda}_1 \propto \overline{\lambda}_2 \propto \overline{\lambda}_3$. These cases correspond to the 2 3-vertices distinguished notationally by the color of the vertex taken to be white or black [B21].

   Remark: One must allow octonionic super-space in M^8 formulation so that octonionic SUSY broken by CP_2 geometry reducing to the quaternionicity of 8-momenta in given scattering diagram is obtained.

2. The conservation condition for the total four-momentum is quadratic in twistor variables for incoming particles. One can linearize this condition by introducing auxiliary Grassmannian $G(k,n)$ over which the tree amplitude can be expressed as a residue integral. The number theoretical beauty of the multiple residue integral is that it can make sense also p-adically unlike ordinary integral.

   The outcome of residue integral is a sum of residues at discrete set of points. One can construct general planar diagrams containing loops from tree diagrams with loops by BCFW recursion. I have considered the possibility that BCFW recursion is trivial in TGD since coupling constants should be invariant under the addition of loops: the proposed scattering diagrammatics however assumed that scattering vertices reduce to scattering vertices for 2 fermions. The justification for renormalization group invariance would be number theoretical: there is no guarantee that infinite sum of diagrams gives simple function defined in all number fields with parameters in extension of rationals (say rational function).

3. The general form of the Grassmannian integrand in $G(k,n)$ can be deduced and follows from Yangian invariance meaning that one has conformal symmetries and their duals which expand to full infinite-dimensional Yangian symmetry. The denominator of the integrand of planar tree diagram is the product of determinants of $k \times k$ minors for the $k \times n$ matrix providing representation of a point of $G(k,n)$ unique apart from $SL(k,k)$ transformations. Only minors consisting of k consecutive columns are assumed in the product. The residue integral is determined by the poles of the denominator. There are also dynamical singularities allowing the amplitude to be non-vanishing only for some special configurations of the external momenta.

4. On mass-shell diagrams obtained by fusing 3-vertices are highly redundant. One can describe the general diagram by using a disk such that its boundary contains the external particles with positive or negative helicity. The diagram has certain number n_F of faces. There are moves, which do not affect the amplitude and it is possible to reduce the number of faces to minimal one: this gives what is called reduced diagram. Reduced diagrams with n_F faces define a unique $n_F - 1$-dimensional sub-manifold of $G(k,n)$ over which the residue integral can be defined. Since the dimension of $G(k,n)$ is finite, also n_F is finite so that the number of diagrams is finite.

5. On mass shell diagrams can be labelled by the permutations of the external lines. This gives a connection with 1+1-dimensional QFTs and with braid group. In 1+1-D integral QFTs however scattering matrix induces only particle exchanges.

   The permutation has simple geometric description: one starts from the boundary point of the diagram and moves always from left or right depending on the color of the point from which one started. One arrives some other point at the boundary and the final points are different for different starting points so that the process assigns a unique perturbation for a given diagram. Diagrams which are obtained by moves from each other define the same permutation. BFCW bridge which is a way to obtain new Yangian invariant corresponds to a permutation of consecutive external particles in the diagram.

6. The poles of the denominator determine the value of the multiple residue integrals. If one allowed all minors, one would have extremely complex structure of singularities. The allowance only cyclically taken minors simplifies the situation dramatically. Singularities correspond to n subgroups of more than 2 collinear k-vectors implying vanishing of some of the minors.

7. Algebraic geometry comes in rescue in the understanding of singularities. Since residue integral is in question, the choice is rather free and only the homology equivalence class of the cell decomposition matters. The poles for a hierarchy with poles inside poles since given singularity contains sub-singularities. This hierarchy gives rise to a what is known as cell composition - stratification - of Grassmannian consisting of varieties with various dimensions. These sub-varieties define representatives for the homology group of Grassmannian. Schubert cells already mentioned define this kind of stratification.

   Remark: The stratification has very strong analogy of the decomposition of catastrophe in Thom's catastrophe theory to pieces of various dimensions. The smaller the dimension, the higher the criticality involved. A connection with quantum criticality of TGD is therefore highly suggestive.

Cyclicity implies a reduction of the stratification to that for positive Grassmannians for which the points are representable as $k \times n$ matrices with non-negative $k \times k$ determinants. This simplifies the situation even further.

Yangian symmetries have a geometric interpretation as symmetries of the stratification: level 1 Yangian symmetries are diffeomorphisms preserving the cell decomposition.

Problems of twistor approach

Twistor approach is extremely beautiful and elegant but has some problems.

1. The notion of twistor structure is problematic in curved space-times. In TGD framework the twistor structures of M^4 and CP_2 (E^4) induce twistor structure of space-time surface and the problem disappears just like the problems related to classical conservation laws are circumvented. Complexification of octonions allows to solve the problems related to the metric signature in twistorialization.

2. The description of massive particles is a problem. In TGD framework M^8 approach allows to replace massive particles with particles with octonionic momenta light-like in 8-D sense belonging to quaternionic subspace for a given diagram. The situation reduces to that for ordinary twistors in this quaternionic sub-space but since quaternionic sub-space can vary, additional degrees of freedom bringing in CP_2 emerge and manifest themselves as transversal 8-D mass giving real mass in 4-D sense.

3. Non-planar diagrams are also a problem. In TGD framework a natural guess is that they correspond to various permutations of free particle octonionic polynomials. Their product defines interaction region in the interior of CD to which free particles satisfying associativity conditions (quantum criticality) arrive. If the origins of polynomials are not along same time axis, the polynomials do not commute nor associate. One must sum over their permutations and for each permutation over its associations.

6.4.5 About the concrete construction of twistor amplitudes

At H-side the ground states of super-conformal representations are given by the anti-symmetrized products of the modes of H-spinor fields labelled by four-momentum, color quantum numbers, and electroweak (ew) quantum numbers. At partonic 2-surface one has finite number of many fermion states. Single fermion states are assigned with H-spinor basis and the fermion states form a representation of a finite-D Clifford algebra.

M^8 picture should reproduce the physical equivalent of H picture: in particular, one should understand four-momentum, color quantum numbers, ew quantum numbers, and B and L. $M^8 - H$ correspondence requires that the super-twistorial description of scattering amplitudes in M^8 is equivalent with that in H.

The M^8 picture is roughly following.

1. The ground states of super-conformal representations expressible in terms of spinor modes of H correspond at level of M^8 wave functions in super variant of the product $T(M^4) \times T(CP_2)$ of twistor spaces of M^4 and CP_2. This twistor space emerges naturally in $M^8 - H$ correspondence from the quaternionicity condition for 8-momenta.

2. Bosonic M^8 degrees of freedom translate to wave functions in the product $T(M^4) \times T(CP_2)$ labelled by four-momentum and color. Super parts of the M^4 and CP_2 twistors code for spin and ew degrees of freedom and fermion numbers. Only a finite number of spin-ew spin states is possible for a given fundamental particle since one has finite-D Grassmann algebra.

3. Contrary to the earlier expectations [L41], the view about scattering diagrams is very similar to that in $\mathcal{N} = 4$ SUSY. The analog of 3-gluon vertex is fundamental and emerges naturally from number theoretic vision in which scattering diagrams defines a cognitive representation and vertices of the diagram correspond to fusion of sparticle lines.

Identification of H quantum numbers in terms of M^8 quantum numbers

The first challenge is to understand how $M^8 - H$ correspondence maps M^8 quantum numbers to H quantum numbers. At the level of M^8 one does not have action principle and conservation laws must follow from the properties of wave functions in various moduli spaces assignable to 4-D and 8-D CDs that is quaternion and octonion structures. The symmetries of the moduli spaces would dictate the properties of wave functions.

There are three types of symmetries and quantum numbers.

1. WCW quantum numbers

At level of H the quantum numbers in WCW"vibrational"degrees of freedom are associated with the representations of super-symplectic group acting as isometries of WCW. Super-symplectic generators correspond to Hamiltonians labelled by color and angular momentum quantum numbers for $SU(3) \times SO(3)$. In $M^4_\pm$ there are also super-symplectic conformal weights assignable to the radial light-coordinate in $\delta M^4_\pm$. These conformal weights could be complex and might relate closely to the zeros of Riemann zeta [K46]. Physical states should however have integer valued conformal weights (conformal confinement).

At the level of M^8 WCW "vibrational"degrees of freedom are discrete and correspond to the degree of the octonionic polynomial P and its coefficients in the extension of rationals considered. WCW integration reduces to a discrete sum, which should be well-defined by the criticality conditions on the coefficients of the polynomials. $M^8 - H$ correspondence guarantees that 4-varieties in M^8 are mappable to space-time surfaces in H. Therefore also quantum numbers should be mappable to each other.

There are also spinorial degrees of freedom associated with WCW spinors with spin-like quantum numbers assignable to fermionic oscillator operators labelled by spin, ew quantum numbers, fermion numbers, and by super-symplectic conformal weights.

2. Quantum numbers assignable to isometries of H.

These quantum numbers are special assignable to the ground states of the representations of Kac-Moody algebras associated with light-like partonic orbits.

1. The isometry group of H consists of Poincare group and color group for CP_2. M^8 isometries correspond to $8-D$ Poincare group. Only G_2 respects given octonion structure and 8-D Lorentz transformations transform to each other different octonion structures. Quantum numbers consist of 8-momentum and analogs of spin and ew spin. $M^8 - H$ correspondence is non-trivial since one must map light-like quaternionic 8-momenta to 4-momenta and color quantum numbers.

2. There are quantum numbers assignable to cm spinor degrees of freedom. They correspond for both M^8 and H to 8-D spinors and give rise to spin and ew quantum numbers. For these quantum numbers $M^8 - H$ correspondence is trivial. At the level of H baryon and lepton numbers are assignable to the conserved chiralities of H-spinors.

Quantum classical correspondence (QCC) is a key piece of TGD.

1. At the level of H QCC states that the eigenvalues of the fermionic Noether charges are equal to the classical bosonic Noether charges in Cartan algebra implies that fermionic quantum number as also ew quantum numbers and spin have correlates at the level of space-time geometry.

2. A the level of M^8 QCC is very concrete. Both bosonic and superpart of octonions have the decomposition $1+\overline{1}+3+\overline{3}$ under color rotations. Each monomial of theta parameters characterizes one particular many-fermion state containing leptons/antileptons and quarks/antiquarks. Leptons/antileptons are assignable to complexified octonionic units $(1 \pm iI_1)/\sqrt{2}$ defining preferred octonion plane M_2 and quarks/antiquarks are assignable to triplet and antitriplet, which also involve complexified octonion units. One obtains breaking of SUSY in the sense that space-time varieties assignable to different theta monomials are different (one can argue that the sum $8_s + \overline{8}_s$ can be regarded as real).

Purely leptonic and antileptonic varieties correspond to 1 and $\overline{1}$ and quark and antiquark varieties to 3 and $\overline{3}$ and the monomial transforms as a tensor product of thetas. The monomial has well defined quark and lepton numbers and the interpretation is that it characterizes fundamental sparticle. At the level of H this kind of correspondence follows form QCC.

3. Also super-momentum leads to a characterization of spin and fermion numbers of the state since delta function expressing conservation of super-momentum codes the supersymmetry for scattering amplitudes and gives rise to vertices conserving fermion numbers. Does this mean QCC in the sense that the super parts of super-momentum and super twistor should be associated with space-time varieties with same fermion and spin content?

How the light-like quaternionic 8-momenta are mapped to H quantum numbers?

The key challenge is to understand how the light-like quaternionic 8-momenta are mapped to massive M^4 momenta and color quantum numbers.

1. One has wave function in the space of CP_2 quaternionic four-momenta. M_0^4 momentum can be identified as M_0^2 projection and in general massive unless M_0^2 and M_0^4 are chosen so that the light-like M^8 momentum belongs to M_0^2. The situation is analogous to that in the partonic description of hadron scattering.

   The space of quaternionic sub-spaces $M_0^4 \supset M_0^2$ with this property is parameterized by CP_2, and one obtains color partial waves. The inclusion of the choice of quantization axis extends this space to $T(CP_2) = SU(3)/U(1)\times U(1)$. Without quaternionicity/associativity condition the space of momenta would correspond to M^8.

   The wave functions in the moduli space for the position of the tip of CD and for the choice $M_0^2 \supset M_0^4$ specifying M_0^4 twistor structure and choice of quantization axis of spin correspond to wave functions in the twistor space CP_3 of $M_\pm^4$ coding for momentum and spin.

   Remark: The inclusion of M^4 spin quantization axis characterized by the choice of M_0^2 extends M_0^4 to geometric twistor space $T(M^4) = M_0^4 \times S^2 \supset M_0^2$ having bundle projection to CP_3. Twistorialization means essentially the inclusion of the choice of various quantization axis as degrees of freedom. This space is for symmetry group G the space G/H, where H is the Cartan sub-group of G. This description might make sense also at the level of super-symplectic and super-Kac-Moody symmetries.

2. Ordinary octonionic degrees of freedom for super-octonions in M^8 must be mapped to $M^4 \times CP_2$ cm degrees of freedom. Super octonionic parts should correspond to fermionic and spin and electroweak degrees of freedom. The space of super-twistorial states should same as the space of the super-symplectic grounds states describable in terms H-spinor modes.

3. One has wave function in the moduli space of CDs. The states in M^8 are labelled by quaternionic super-momenta. Bosonic part must correspond to four-momentum and color and super-part to spin and ew quantum numbers of CP_2. This part of the moduli space wave function is characterized by the spin and ew spin quantum numbers of the fundamental particle. Wave functions in the super counterpart of $T(M^4) \times T(CP_2)$ allow to characterize these degrees of freedom without the introduction of spinors and should correspond to the ground states of super-conformal representations in H.

It seems that H-description is an abstract description at the level moduli spaces and M^8 description for single space-time variety represents reduction to the primary level, where number theory dictates the dynamics.

Octonionic twistors and super-twistors

How to define octonionic twistors? Or is it enough to identify quaternionic/associative twistors as sub-spaces of octonionic twistors?

1. Ordinary twistors and super-twistors

Consider first how ordinary twistors and their super counterparts could be defined, and how they could allow an elegant description of spin and ew quantum numbers as quantum numbers analogous to angular momenta.

1. Ordinary twistors are defined as pairs of 2-spinors giving rise to a representation of four-momentum. The spinors are complex spinors transforming as a doublet representation of SL(2,C) and its conjugate.

   The 2-spinors are related by incidence relation, a linear condition in which M^4 coordinates represented as 2×2 matrix appears linearly [L41]. The expression of four-momentum is bilinear in the spinors and invariant under complex scalings of the 2-spinors compensating each other so that instead of 8-D space one has actually 6-D space, which reduces to CP_3 to which the geometric twistor space $M^4 \times S^2$ has a projection.

2. For light-like four-momenta p the determinant of the matrix having the two 2-spinors as rows and representing p as a point of M^4 vanishes. Wave functions in CP_3 allow to describe spin in terms of bosonic wave function. What is so beautiful is that this puts particles with different spin in a democratic position.

   Super-twistors allow to integrate the states constructible as many-fermion states of $\mathcal{N}$ elementary fermions in the same representations involving several spins. The many-fermion states - sparticles - are in 1-1-correspondence with Grassmann algebra basis.

3. The description of massless particles in terms of M^4 (super-)twistors is elegant but one encounters problems in the case of massive particles [K116, K49, L41].

2. Octonionic twistors at the level of M^8?

How to define octonionic twistors at the level of M^8?

1. At the level of M^8 one has light-like 8-momenta. The M^4 momentum identified as M_0^4 projection can there be massive. This solves the basic problem of the standard twistor approach.

2. The additional assumption is that the 8-momenta in given vertex of scattering diagram belong to the same quaternionic sub-space $M_0^4 \subset M^8$ satisfying $M_0^4 \supset M_0^2$. This effectively transforms momentum space $M^4 \times E^4$ to $M^4 \times CP_2$. A stronger condition is that all momenta in a given diagram belong to the same sub-space $M_0^4 \supset M_0^2$.

   Remark: Quaternionicity implies that the 8-momentum is time-like or light-like if one requires that quaternionicity for an arbitrary choice of the octonionic structure (the action of 8-D Poincare group gives rise transforms octonionic structures to each other).

3. Complex 2-spinors are replaced with complexified octonionic spinors which must be consistent quaternionicity condition for 8-momenta. A good guess is that the spinors belong to a quaternionic sub-space of octonions too. This is expected to transform them effectively to quaternionic spinors. Without effective quaternionicity the number of 2-spinor components would be 8 rather than 4 times larger than for ordinary 2-spinors.

   Remark: One has complexified octonions (i commutes with the octonionic imaginary units E_k).

4. Octonionic/quaternionic twistors should be pairs of octonionic/quaternionic 2-spinors determined only modulo octonionic/quaternionic scaling. If quaternionicity holds true, the number of 2-spinor components is 4 times larger than usually. Does this mean that one has basically quaternionic twistors plus moduli space CP_2 for $M_0^4 \supset M_0^2$. One should be able to express octonionic twistors as bi-linears formed from 2 octonionic/quaternionic 2-spinors. Octonionic option should give the octonionic counterpart OP_3 of Grassmannian CP_3, which does not however exist.

   Remark: Octonions allow only projective plane OP_2 as the octonionic counterpart of CP_2 (see `http://tinyurl.com/ybwaeu2s`) but do not allow higher-D projective spaces nor Grassmannians (see `http://tinyurl.com/ybm8ubef`, whereas reals, complex numbers, and quaternions do so. The non-existence of Grassmannians for rings obtained by Cayley-Dickson construction could mean that M^8-H correspondence and TGD do not generalize beyond octonions.

Does the restriction to quaternionic 8-momenta the Grassmannians to be quaternionic (sub-spaces of octonions). This would give quaternionic counterpart HP_3 of CP_3. Quaternions indeed allow projective spaces and Grassmannians and (see `http://tinyurl.com/y9htjstc` and `http://tinyurl.com/y87gpq8l`).

Remark: One can wonder whether non-commutativity forces to distinguish between left- and right Grassmannians (points as lines $\{c(q_1,..,q_n)|c\in H\}$ or as lines as lines $\{(q_1,..,q_n)c|c\in H\}$.

5. Concerning the generalization to octonionic case, it is crucial to realize that the 2×2-matrix representing four-momentum as a pair 2-spinor can be regarded as an element in the sub-space of complexified quaternions. The representation of four-momentum would be as sum of $p_8 = p_1^k\sigma_k + I_4 p_2^k\sigma_k$, where I_4 octonionic imaginary unit orthogonal to σ_k representing quaternionic units.

   No! The twistorial representation of the 4-momentum is already quaternionic! Choosing the decomposition of M^8 to quaternionic sub-space and its complement suitably, one has $IM(p_8)=0$ for quaternionic 8-momenta and one obtains standard representation of 4-momentum in this sub-space! The only new element is that one has now moduli specifying the quaternionic sub-space. If the sub-space contains a fixed M_0^2 one obtains just CP_2 and ordinary twistor codes for the choices of M_0^2. If the choice of color quantization axes matters as it indeed does, one has twistor space $SU(3)/U(1)\times U(1)$ instead of CP_2. This would suggest that ordinary representation of scattering amplitudes reduces apart from the presence of CP_2 twistor to the usual representation.

   One can hope for a reduction to ordinary twistors and projective spaces, moduli space CP_2 for quaternion structures, and moduli space for the choices of real axis of octonion structures. One can even consider the possibility [L41] of using standard M_0^2 with the property that M^8 momentum reduces to M_0^2 momentum and coding the information about real M_0^2 to moduli. This could reduce the twistor space to $RP(3)$ associated with M_0^2 is considered and solve the problems related to the signature of M^4. Note however that the complexification of octonions in any case allow to regard the metric as Euclidian albeit complexified so that these problems should disappear.

3. Octonionic super-twistors at the level of M^8?

Should one generalize the notion of super-twistor to octonionic context or can one do by using only the moduli space and the fact that octonionic geometry codes for various components of octonion as analog of super-field? It seems that super-twistors are needed.

1. It seems that super-twistors are needed. Octonionic super-momentum would appear in the super variant of momentum conserving delta function resulting in the integration over translational moduli. In twistor Grassmann approach this delta function is super-twistorialized and this leads to the amazingly simple expressions for the scattering amplitudes.

2. At the level of M^8 one should generalize ordinary momentum to super-momentum and perform super-twistorialization. Different monomials of theta parameters emerging from super part of momentum conserving delta function (for $\mathcal{N}=1$ one has $\delta(\theta-\theta_0)=exp(i\theta-\theta_0)/i)$ correspond to different spin states of the super multiplet and anti-commutativity guarantees correct statistics. At the level of H the finite-D Clifford algebra of 8-spinors at fixed point of H gives states obtained as monomials or polynomials for the components of super-momentum in M^8.

3. Octonionic super-momentum satisfying quaternionicity condition can be defined as a combination of ordinary octonionic 8-momentum and super-parts transforming like 8_s and $\overline{8}_s$. One can express the octonionic super-momentum as a bilinear of the super-spinors defining quaternionic super-twistor. Quaternionicity is assumed at least for the octonionic super-momenta

in the same vertex. Hence the M^4 part of the super-twistorialization reduces to that in SUSYs and one obtains standard formulas. The new elements is the super-twistorialization of $T(CP_2)$.

Remark: Octonionic SUSY involving $8+8_s+\overline{8}_s$ would be an analog of $\mathcal{N}=8$ SUSY associated with maximal supergravity (see `http://tinyurl.com/nv3aajy`) and in M^4 degrees of freedom twistorialization should be straightforward.

The octonionic super-momentum belongs to a quaternionic sub-space labelled by CP_2 point and corresponds to a particular sub-space M_0^2 in which it is light-like (has no other octonionic components). M_0^2 is characterized by point of S^2 point of twistor space $M^4\times S^2$ having bundle projection to CP_3.

4. That the twistor space $T(CP_2) = SU(3)/U(1)\times U(1)$ coding for the color quantization axes rather than only CP_2 emerges must relate to the presence of electroweak quantum numbers related to the super part of octonionic momentum. Why the rotations of $SU(2)\times U(1) \subset SU(3)$ have indeed interpretation also as tangent space-rotations interpreted as electroweak rotations. The transformations having an effect on the choice of quantization axies are parameterized by S^2 relating naturally to the choice of $SO(4)$ quantization axis in E^4 and coded by the geometric twistor space $T(E^4) = E^4\times S^2$.

5. Since the super-structure is very closely related to the construction of the exterior algebra in the tangent space, super-twistorialization of $T(CP_2)$ should be possible. Octonionic triality could be also in a key role and octonionic structure in the tangent space of $SU(3)$ is highly suggestive. $SU(3)$ triality could relate to the octonionic triality.

   $SU(3)/U(1)\times U(1)$ is analogous with the ordinary twistor space CP_3 obtained from C^4 as a projective space. Now however $U(1)\times U(1)$ instead of group of complex scalings would define the equivalence classes. Generalization of projective space would be in question. The super-part of twistor would be obtained as $U(1)\times U(1)$ equivalence class and gauge choice should be possible to get manifestly 6-D representation. One can ask whether the CP_2 counterparts of higher- D Grassmannians appear at the level of generalized twistor diagrams: could the spaces $SU(n)/G$, H Cartan group correspond to these spaces?

4. How the wave functions in super-counterpart of $T(CP_2)$ correspond to quantum states in CP_2 degrees of freedom?

In CP_2 spinor partial waves have vanishing triality $t=0$ for leptonic chirality and $t=\pm 1$ for quarks and antiquarks. One can say that the triality $t\neq 1$ states are possible thanks to the anomalous hypercharge equal to fractional electromagnetic charge $Y_A = Q_{em}$ of quarks: this gives also correlation between color quantum numbers and electroweak quantum numbers which is wrong for spinor partial waves. The super-symplectic and super Kac-Moody algebras however bring in vibrationals degrees of freedom and one obtains correct quantum number assignments [K68].

This mechanism should have a counterpart at the level of the super variant of the twistor space $T(CP_2) = SU(3)/U(1)\times U(1)$. The group algebra of $SU(3)$ gives the scalar wave functions for all irreps of $SU(3)$ as matrix elements. Allowing only matrix elements that are left- or right invariant under $U(1)\times U(1)$ one obtains all irreps realized in $T(CP_2)$ as scalar wave functions. These representations have $t=0$. The situation would be analogous for scalar functions in CP_2. One must however obtain also electroweak quantum numbers and $t\neq 0$ colored states. Here the octonionic algebraic geometry and superpart of the $T(CP_2)$ should come in rescue. The electroweak degrees of freedom in CP_2 should correspond to the super-parts of twistors.

The $SU(3)$ triplets assignable to the triplets 3 and $\overline{3}$ of space-time surfaces would make possible also the $t=\pm 1$ states. Color would be associated with the octonionic geometry. The simplest possibility would be that one has just tensor products of the triplets with $SU(3)/U(1)\times U(1)$ partial waves. In the case of CP_2 there is however a correlation between color partial waves and electroweak quantum numbers and the same is expected also now between super-part of the twistor and geometric color wave function: minimum correlation is via $Y_A = Q_{em}$. The minimal option is that the number theoretic color for the octonionic variety modifies the transformation properties of $T(CP_2)$ wave function only by a phase factor due to $Y_A = Q_{em}$ as in the case of CP_2.

The most elegant outcome would be that super-twistorial state basis in $T(M^4) timesT(CP_2)$ is equivalent with the state basis defined by super-symplectic and super Kac-Moody representations in H.

About the analogs of twistor diagrams

There seems to be a strong analogy with the construction of twistor amplitudes in $\mathcal{N} = 4$ SUSY [B18, B51, B23] and one can hope of obtaining a purely geometric analog of SUSY with dynamics of fields replaced by the dynamics of algebraic super-octonionic surfaces.

1. Number theoretical vision leads to the proposal that the scattering amplitudes involve only data at discrete points of the space-time variety belonging to extension of rationals defining cognitive representation. The identification of these points has been already considered in the case of partonic orbits entering to the partonic 2-vertex and for the regions of space-time surfaces intersecting at discrete set of points. Scattering diagrams should therefore correspond to polygons with vertices of polygons defining cognitive representation and lines assignable to the external fundamental particles with given quark and lepton numbers having correlates at the level of space-time geometry. This occurs also in twistor Grassmannian approach [B18, B51, B23].

   Since polynomials determine space-time surfaces, this data is enough to determine the space-time variety completely. Indeed, the zeros of $P(t)$ determining the space-time variety give also rise to a set of spheres $S^6(t_n)$ and partonic 2-surfaces $X^2(t_n) = X^4 \cap S^6(t_n)$, where t_n is root of $P(t)$. The discretization need not mean a loss of information. The scattering amplitudes would be expressible as an analog of n-point function with points having coordinates in the extension of rationals.

2. (Super) octonion as"field"in X^4 is dynamically analogous to (super) gauge potentials and super-octonion to its super variant. (Super) gauge potentials are replaced with M^8 (super-) octonion coordinate and gauge interactions are geometrized. Here I encounter a problem with terminology. Neither sparticle nor sboson sounds good. Hence I will talk about sparticles.

3. The amplitude for a given space-time variety contains no information M^8-momentum. M^8-momentum emerges as a label for a wave function in the moduli space of 4-D and 8-D CDs involving both translational and orientational degrees of freedom. For fixed time axis the orientational degrees of freedom reduce to rotational degrees of freedom identifiable in terms of the twistor sphere S^2. The delta functions expressing conservation of 8-D quaternionic super-momentum in M^8 coming from the integration over the moduli space of 8-D translations.

   As found, quaternionicity of 8-momenta implies that standard M^4 twistor description of momenta applies but one obtains CP_2 twistors as additional contribution. This is of course what one would intuitively expect.

   8-D momentum conservation in turn translates to the conservation of momentum and color quantum numbers in the manner described. The amplitudes in momentum and color degrees of freedom reduce to kinematics as in SUSYs. It is however not clear whether one should also perform number theoretical discretization of various moduli spaces.

   In any case, it seems that all the details of the scattering amplitudes related to moduli spaces reduce to symmetries and the core of calculations reduces to the construction of space-time varieties as zero loci of octonionic polynomials and identification of the points of the 4-varieties in extension of rationals. Classical theory would indeed be an exact part of the quantum theory.

4. Quaternionic 8-D light-likeness reduces the situation to the level of ordinary complex and thus even positive (real) Grassmannians. This is crucial from the p-adic point of view. CP_2 twistors characterizes the moduli related to the choice of quaternionic sub-space, where 8-momentum reduces to ordinary 4-momentum. M^4 parts of the scattering amplitudes in twistor Grassmann approach should be essentially the same as in $\mathcal{N} = 4$ SUSY apart from the replacement of super degrees of freedom with super-octonionic ones. The challenge is

to generalize the formalism so that it applies also to CP_2 twistors. The challenge would be to generalize the formalism so that it applies also to CP_2 twistors. The M^4 and CP_2 degrees of freedom are expected to factorize in twistorial amplitudes. A good guess is that the scattering amplitudes are obtained as residue integrals in the analogs of Grassmannians associated with $T(CP_2)$. Could one have Grassmannians also now?

Consider the formula of tree amplitude for n gluons with k negative helicities conjectured Arkani-Hamed *et al* in the twistor Grassmannian approach [B23]. The amplitude follows from the twistorial representation for momentum conservation and is equal to an $k \times n$-fold multiple residue integral over the complex variables $C_{\alpha a}$ defining coordinates for Grassmannian $Gl(n,k)$ and reduces to a sum over residues. The integrand is the inverse for the product of all $k \times k$ minors of the matrix $C_{\alpha a}$ in cyclic order and the resides corresponds to zeros for one or more minors. This part does not depend on twistor variables. The dependence on n twistor variables comes from the product $\prod_{\alpha=1}^{k} \delta(C_{\alpha a} W^a)$ of k delta functions related to momentum conservation. W^a denotes super-twistors in the 8-D representation, which is linear. One has projective invariance and therefore a reduction to $T(M^4) = CP_3 = SU(4)/SU(3) \times U(1)$.

Could this formula generalize almost as such to $T(CP_2)$ and come from the conservation of E^4 momentum? One has n sparticles to which super-twistors in $T(CP_2)$ are assigned. The first guess is that the sign of helicity are replaced by the sign of electroweak isospin - essentially E^4 spin at the level of M^8. For electromagnetic charge identified as the analog of helicity one would have problems in the case of neutrinos. $T(M^4) = CP_3 = SU(4)/SU(3) \times U(1)$ is replaced with $T(CP_2) = SU(3)/U(1) \times U(1)$. $T(CP_2)$ does not have a representation as a projective space but there is a close analogy since the group of complex scalings is replaced with $U(1) \times U(1)$. The (apparent) linearity is lost but one represent the points of $T(CP_2)$ as exponentials of $su(3)$ Lie-algebra elements with vanishing $u(1) \times u(1)$ part. The resulting 3 complex coordinates are analogous to two complex CP_2 coordinates. The basic difference between M^4 and CP_2 degrees of freedom would come from the exponential representation of twistors.

5. By Yangian invariance one should obtain very similar formulas for the amplitudes except that one has instead of $\mathcal{N} = 4$ SUSY $\mathcal{N} = 8$ octonionic SUSY analogous to $\mathcal{N} = 8$ SUGRA.

Trying to understand the fundamental 3-vertex

Due to its unique twistorial properties as far as realization of four-momentum conservation is considered 3-vertex is fundamental in the construction of scattering diagrams in twistor Grassmannian approach to $\mathcal{N} = 4$ SYM [B21] (see `http://tinyurl.com/yd9tf2ya`). Twistor Grassmann approach suggests that 3-vertex with complexified light-like 8-momenta represents the basic building brick representing from which more complex diagrams can be constructed using the BCFW recursion formula [B21]. In TGD 3-vertex generalized to 8-D light-like quaternionic momenta should be highly analogous to the 4-D 3-vertex and in a well-defined sense reduce to it if all momenta of the diagram belong to the same quaternionic sub-space M_0^4. It is however not completely clear how 3-vertex emerges in TGD framework.

1. A possible identification of the 3-vertex at the level of M^8 would be as a vertex at which 3 sparticle lines with light-like complexified quaternionic 8-momenta meet. This vertex would be associated with the partonic vertex $X^2(t_n) = X^4 \cap S^6(t_n)$. Incoming sparticle lines at the light-like partonic orbits identified as boundaries of string world sheets (for entangled states at least) would be light-like.

   Does the fusion of two sparticle lines to third one require that either or both fusing lines become space-like - say pieces of geodesic line inside the Euclidian space-time region- bounded by the partonic orbit? The identification of the lines of twistor diagrams as carriers of light-like complexified quaternionic momenta in 8-D sense does not encourage this interpretation (also classical momenta are complex). Should one pose the fusion of the light-like lines as a boundary condition? Or should one give up the idea that sparticle lines make sense inside interaction region?

2. As found, one can challenge the assumption about the existence of string world sheets as commutative regions in the non-associative interaction region. Could one have just fermion lines as light-like curves at partonic orbits inside CD? Or cannot one have even them?

   Even if the polynomial $\prod_i P_i$ defining the interaction region is product of polynomials with origins of octonionic coordinates not along the same real line, the 7-D light-cones of M^8 associated with the particles still make sense in the sense that $P_i(o_i) = 0$ reduces at it to $P_i(t_i) = 0$, t_i real number, giving spheres $S^6(t_i(n))$ and partonic 2-surfaces and vertices $X_2(t_i(n))$. The light-like curves as geodesics the boundary of 7-D light-cones mapped to light-like curves along partonic orbits in H would not be lost inside interaction regions.

3. At the level of H this relates to a long standing interpretational problem related to the notion of induced spinor fields. SH suggests strongly the localization of the induced spinor fields at string world sheets and even at sparton lines in absence of entanglement. Super-conformal symmetry however requires that induced spinor fields are 4-D and thus seems to favor delocalization. The information theoretic interpretation is that the induced spinor fields at string world sheets or even at sparton lines contain all information needed to construct the scattering amplitudes. One can also say that string world sheets and sparton lines correspond to a description in terms of an effective action.

Could the M^8 view about twistorial scattering amplitudes be consistent with the earlier H picture?

The proposed M^8 picture involving super coordinates of M^8 and super-twistors does not conform with the earlier proposal for the construction of scattering amplitudes at the level of H [L41]. In H picture the introduction of super-space does not look natural, and one can say that fundamental fermions are the only fundamental particles [K49, L41]. The H view about super-symmetry is as broken supersymmetry in which many fermion states at partonic 2-surfaces give rise to supermultiplets such that fermions are at different points. Fermion 4-vertex would be the fundamental vertex and involve classical scattering without fusion of fermion lines. Only a redistribution of fermion and anti-fermion lines among the orbits of partonic 2-surfaces would take place in scattering and one would have kind of OZI rule.

Could this H view conform with the recent M^8 view much closer to the SUSY picture. The intuitive idea without a rigorous justification has been that the fermion lines at partonic 2-surfaces correspond to singularities of many-sheeted space-time surface at which some sheets co-incide. M^8 sparticle consists effectively of n fermions at the same point in M^8. Could it be mapped by M^8-H duality to n fermions at distinct locations of partonic 2-surface in H?

$M^8 - H$ correspondence maps the points of $M^4 \subset M^4 \times E^4$ to points of $M^4 \subset M^4 \times CP_2$. The tangent plane of space-time surface containing a preferred M^2 is mapped to a point of CP_2. If the effective n-fermion state M^8 is at point at which n sheets of space-time surface co-incide and if the tangent spaces of different sheets are not identical, which is quite possible and even plausible, the point is indeed mapped to n points of H with same M^4 coordinates but different CP_2 coordinates and sparticle would be mapped to a genuine many-fermion state. But what happens to scalar sparticle. Should one regard it as a pure gauge degree of freedom in accordance with the chiral symmetry at the level of M^8 and H?

6.5 From amplituhedron to associahedron

Lubos has a nice blog posting (see `http://tinyurl.com/y7ywhxew`) explaining the proposal represented in the newest article by Nima Arkani-Hamed, Yuntao Bai, Song He, Gongwang Yan [**?**]see `http://tinyurl.com/ya8zstll`). Amplituhedron is generalized to a purely combinatorial notion of associahedron and shown to make sense also in string theory context (particular bracketing). The hope is that the generalization of amplituhedron to associahedron allows to compute also the contributions of non-planar diagrams to the scattering amplitudes - at least in $\mathcal{N} = 4$ SYM. Also the proposal is made that color corresponds to something less trivial than Chan-Paton factors.

The remaining problem is that 4-D conformal invariance requires massless particles and TGD allows to overcome this problem by using a generalization of the notion of twistor: masslessness is realized in 8-D sense and particles massless in 8-D sense can be massive in 4-D sense.

In TGD non-associativity at the level of arguments of scattering amplitude corresponds to that for octonions: one can assign to space-time surfaces octonionic polynomials and induce arithmetic operations for space-time surface from those for polymials (or even rational or analytic functions). I have already earlier [L32] demonstrated that associahedron and construction of scattering amplitudes by summing over different permutations and associations of external particles (space-time surfaces). Therefore the notion of associahedron makes sense also in TGD framework and summation reduces to "integration" over the faces of associahedron. TGD thus provides a concrete interpretation for the associations and permutations at the level of space-time geometry.

In TGD framework the description of color and four-momentum is unified at the level and the notion of twistor generalizes: one has twistors in 8-D space-time instead of twistors in 4-D space-time so Chan-Paton factors are replaced with something non-trivial.

6.5.1 Associahedrons and scattering amplitudes

The following describes briefly the basic idea between associahedrons.

Permutations and associations

One starts from a non-commutative and non-associative algebra with product (in TGD framework this algebra is formed by octonionic polynomials with real coefficients defining space-time surfaces as the zero loci of their real or imaginary parts in quaternionic sense. One can indeed multiply space-time surface by multiplying corresponding polynomials! Also sum is possible. If one allows rational functions also division becomes possible.

All permutations of the product of n elements are in principle different. This is due to non-commutativity. All associations for a given ordering obtained by scattering bracket pairs in the product are also different in general. In the simplest case one has either $a(bc)$ or $(ab)c$ and these 2 give different outcomes. These primitive associations are building bricks of general associations: for instance, abc does not have well-defined meaning in non-associative case.

If the product contains n factors, one can proceed recursively to build all associations allowed by it. Decompose the n factors to groups of m and $n-m$ factors. Continue by decomposing these two groups to two groups and repeat until you have have groups consisting of 1 or two elements. You get a large number of associations and you can write a computer code computing recursively the number $N(n)$ of associations for n letters.

Two examples help to understand. For $n = 3$ letters one obviously has $N(n = 3) = 2$. For $n = 4$ one has $N(4) = 5$: decompose first $abcd$ to $(abc)d$, $a(bcd)$ and $(ab)(cd)$ and then the two 3 letter groups to two groups: this gives $N(4) = 2 + 2 + 1 = 5$associations and associahedron in 3-D space has therefore 5 faces.

Geometric representation of association as face of associahedron

Associations of n letters can be represented geometrically as so called Stasheff polytope (see `http://tinyurl.com/q9ga785`). The idea is that each association of n letters corresponds to a face of polytope in $n-2$-dimensional space with faces represented by the associations.

Associahedron is constructed by using the condition that adjacent faces (now 2-D polygons) intersecting along common face (now 1-D edges). The number of edges of the face codes for the structure particular association. Neighboring faces are obtained by doing minimal change which means replacement of some $(ab)c$ with $a(bc)$ appearing in the association as a building bricks or vice versa. This means that the changes are carried out at the root level.

How does this relate to particle physics?

In scattering amplitude letters correspond to external particles. Scattering amplitude must be invariant under permutations and associations of the external particles. In particular, this means that one sums over all associations by assigning an amplitude to each association. Geometrically this means that one "integrates" over the boundary of associahedron by assigning to each face an amplitude. This leads to the notion of associahedron generalizing that of amplituhedron.

Personally I find it difficult to believe that the mere combinatorial structure leading to associahedron would fix the theory completely. It is however clear that it poses very strong conditions on the structure of scattering amplitudes. Especially so if the scattering amplitudes are defined in terms of "volumes" of the polyhedrons involved so that the scattering amplitude has singularities at the faces of associahedron.

An important constraint on the scattering amplitudes is the realization of the Yangian generalization of conformal symmetries of Minkowski space. The representation of the scattering amplitudes utilizing moduli spaces (projective spaces of various dimensions) and associahedron indeed allows Yangian symmetries as diffeomorphisms of associahedron respecting the positivity constraint. The hope is that the generalization of amplituhedron to associahedron allows to generalize the construction of scattering amplitudes to include also the contribution of non-planar diagrams of at $\mathcal{N} = 4$ SYM in QFT framework.

6.5.2 Associations and permutations in TGD framework

Also in the number theoretical vision about quantum TGD one encounters associativity constraings leading to the notion of associahedron. This is closely related to the generalization of twistor approach to TGD forcing to introduce 8-D analogs of twistors [L32] (see `http://tinyurl.com/yd43o2n2`).

Non-associativity is induced by octonic non-associativity

As found in [L32], non-associativity at the level of space-time geometry and at the level of scattering amplitudes is induced from octonionic non-associativity in M^8.

1. By $M^8 - H$ duality ($H = M^4 \times CP_2$) the scattering are assignable to complexified 4-surfaces in complexified M^8. Complexified M^8 is obtained by adding imaginary unit i commutating with octonionic units I_k, $k = 1, , .., 7$. Real space-time surfaces are obtained as restrictions to a Minkowskian subspace complexified M^8 in which the complexified metric reduces to real valued 8-D Minkowski metric. This allows to define notions like Kähler structure in Minkowskian signature and the notion of Wick rotations ceases to be ad hoc concept. Without complexification one does not obtain algebraic geometry allowing to reduces the dynamics defined by partial differential equations for preferred extremals in H to purely algebraic conditions in M^8. This means huge simplications but the simplicity is lost at the QFT-GRT limit when many-sheeted space-time is replaced with slightly curved piece of M^4.

2. The real 4-surface is determined by a vanishing condition for the real or imaginary part of octonionic polynomial with $RE(P)$ and $IM(P)$ defined by the composition of octonion to two quaternions: $o = RE(o) + I_4 IM(o)$, where I_4 is octonionic unit orthogonal to a quaternionic sub-space and $RE(o)$ and $IM(o)$ are quaternions. The coefficients of the polynomials are assumed to be real. The products of octonionic polynomials are also octonionic polynomials (this holds for also for general power series with real coefficients (no dependence on I_k. The product is not however neither commutative nor associative without additional conditions. Permutations and their associations define different space-time surfaces. The exchange of particles changes space-time surface. Even associations do it. Both non-commutativity and non-associativity have a geometric meaning at the level of space-time geometry!

3. For space-time surfaces representing external particles associativity is assumed to hold true: this in fact guarantees $M^8 - H$ correspondence for them! For interaction regions associativity does not hold true but the field equations and preferred extremal property allow to construct the counterpart of space-time surface in H from the boundary data at the boundaries of CD fixing the ends of space-time surface.

   Associativity poses quantization conditions on the coefficients of the polynomial determining it. The conditions are interpreted in terms of quantum criticality. In the interaction region identified naturally as causal diamond (CD), associativity does not hold true. For instance, if external particles as space-time surfaces correspond to vanishing of $RE(P_i)$ for polynomials representing particles labelled by i, the interaction region (CD) could correspond to the vanishing of $IM(P_i)$ and associativity would fail. At the level of H associativity and criticality corresponds to minimal surface property so that quantum criticality corresponds to universal free particle dynamics having no dependence on coupling constants.

4. Scattering amplitudes must be commutative and associative with respect to their arguments which are now external particles represented by polynomials P_i This requires that scattering amplitude is sum over amplitudes assignable to 4-surfaces obtained by allowing all permutations and all associations of a given permutation. Associations can be described combinatorially by the associahedron!

   Remark:. In quantum theory associative statistics allowing associations to be represented by phase factors can be considered (this would be associative analog of Fermi statistics). Even a generalization of braid statistics can be considered.

Yangian variants of various symmetries are a central piece also in TGD although supersymmetries are realized in different manner and generalized to super-conformal symmetries: these include generalization of super-conformal symmetries by replacing 2-D surfaces with light-like 3-surfaces, supersymplectic symmetries and dynamical Kac-Moody symmetries serving as remnants of these symmetries after supersymplectic gauge conditions characterizing preferred extremals are applied, and Kac-Moody symmetries associated with the isometries of H . The representation of Yangian symmetries as diffeomorphisms of the associahedron respecting positivity constraint encourages to think that associahedron is a useful auxiliary tool also in TGD.

Is color something more than Chan-Paton factors?

Nima *et al* talk also about color structure of the scattering amplitudes usually regarded as trivial. It is claimed that this is actually not the case and that there is non-trivial dynamics involved. This is indeed the case in TGD framework. Also color quantum numbers are twistorialized in terms of the twistor space of CP_2, and one performs a twistorialization at the level of M^8 and $M^4 \times CP_2$. At the level of M^8 momenta and color quantum numbers correspond to associative 8-momenta. Massless particles are now massless in 8-D sense but can be massive in 4-D sense. This solves one of the basic difficulty of the ordinary twistor approach. A further bonus is that the choice of the embedding space H becomes unique: only the twistor spaces of S^4 (and generalized twistor space of M^4 and CP_2 have Kähler structure playing a crucial role in the twistorialization of TGD. To sum up, all roads lead to Rome. Everyone is well-come to Rome!

6.5.3 Questions inspired by quantum associations

Associations have (or seem to have) different meaning depending on whether one is talking about cognition or mathematics. In mathematics the associations correspond to different bracketings of mathematical expressions involving symbols denoting mathematical objects and operations between them. The meaning of the expression - in the case that it has meaning - depends on the bracketing of the expression. For instance, one has $a(b+c) \neq (ab)+c$, that is $ab+ac \neq ab+c$). Note that one can change the order of bracket and operation but not that of bracket and object.

For ordinary product and sum of real numbers one has associativity: $a(bc) = (ab)c$ and $a + (b+c) = (a+b)+c$. Most algebraic operations such as group product are associative. Associativity of product holds true for reals, complex numbers, and quaternions but not for octonions and this would be fundamental in both classical and quantum TGD.

The building of different associations means different groupings of n objects. This can be done recursively. Divide first the objects to two groups, divide these tow groups to two groups each, and continue until you jave division of 3 objects to two groups - that is abc divided into $(ab)c$ or $a(bc)$. Numbers 3 and 2 are clearly the magic numbers.

This inspire several speculative quetions related to the twistorial construction of scattering amplitudes as associative singlets, the general structure of quantum entanglement, quantum measurement cascade as formation of association, the associative structure of many-sheeted space-time as a kind of linguistic structure, spin glass as a strongly associative system, and even the tendency of social structures to form associations leading from a fully democratic paradise to cliques of cliques of ... .

1. In standard twistor approach 3-gluon amplitude is the fundamental building brick of twistor amplitudes constructed from on-shell-amplitudes with complex momenta recursively. Also in TGD proposal this holds true. This would naturally follow from the fact that associations

can be reduced recursively to those of 3 objects. 2- and 3-vertex would correspond to a fundamental associations. The association defined 2-particle pairing (both associated particles having either positive or negative helicities for twistor amplitudes) and 3-vertex would have universal structure although the states would be in general decompose to associations.

2. Consider first the space-time picture about scattering [L32]. CD defines interaction region for scattering amplitudes. External particles entering or leaving CD correspond to associative space-time surfaces in the sense that the tangent space or normal space for these space-time surfaces is associative. This gives rise to M^8-H correspondence.

   These surfaces correspond to zero loci for the imaginary parts (in quaternionic sense) for octonionic polynomial with coefficients, which are real in octonionic sense. The product of $\prod_i P_i)$ of polynomials with same octonion structure satisfying $IM(P_i)=0$ has also vanishing imaginary part and space-time surface corresponds to a disjoint union of surfaces associated with factors so that these states can be said to be non-interacting.

   Neither the choice of quaternion structure nor the choice of the direction of time axis assignable to the octonionic real unit need be same for external particles: if it is the particles correspond to same external particle. This requires that one treats the space of external particles (4-surfaces) as a Cartesian product of of single particle 4-surfaces as in ordinary scattering theory.

   Space-time surfaces inside CD are non-associative in the sense that the neither normal nor tangent space is associative: $M^8-M^4\times CP_2$ correspondence fails and space-time surfaces inside CD must be constructed by applying boundary conditions defining preferred extremals. Now the real part of $RE(\prod_i P_i)$ in quaternionic sense vanishes: there is genuine interaction even when the incoming particles correspond to the same octonion structure since one does not have union of surfaces with vanishing $RE(P_i)$. This follows from s rather trivial observation holding true already for complex numbers: imaginary part of zw vanishes if it vanishes for z and w but this does not hold true for the real part. If octonionic structures are different, the interaction is present irrespective of whether one assumes $RE(\prod_i P_i)=0$ or $IM(\prod_i P_i)=0$. $RE(\prod_i P_i)=0$ is favoured since for $IM(\prod_i P_i)=0$ one would obtain solutions for which $IM(P_i)=0$ would vanish for the i:th particle: the scattering dynamics would select i:th particle as non-interacting one.

3. The proposal is that the entire scattering amplitude defined by the zero energy state - is associative, perhaps in the projective sense meaning that the amplitudes related to different associations relate by a phase factor (recall that complexified octonions are considered), which could be even octonionic. This would be achieved by summing over all possible associations.

4. Quantum classical correspondence (QCC) suggests that in ZEO the zero energy states - that is scattering amplitudes determined by the classically non-associative dynamics inside CD - form a representation for the non-associative product of space-time surfaces defined by the condition $RE(\prod_i P_i)=0$. Could the scattering amplitude be constructed from products of octonion valued single particle amplitudes. This kind of condition would pose strong constraints on the theory. Could the scattering amplitudes associated with different associations be octonionic - may be differing by octonion-valued phase factors - and could only their sum be real in octonionic sense (recall that complexified octonions involving imaginary unit i commuting with the octonionic imaginary units are considered)?

One can look the situation also from the point of view of positive and negative energy states defining zero energy states as they pairs.

1. The formation of association as subset is like formation of bound state of bound states of ... . Could each external line of zero energy state have the structure of association? Could also the internal entanglement associated with a given external line be characterized in terms of association.

   Could the so called monogamy theorem stating that only two-particle entanglement can be maximal correspond to the decomposing of $n=3$ association to one- and two-particle associations? If quantum entanglement is behind associations in cognitive sense, the cognitive meaning of association could reduce to its mathematical meaning.

An interesting question relates to the notion of identical particle: are the many-particle states of identical particles invariant under associations or do they transform by phase factor under association. Does a generalization of braid statistics make sense?

2. In ZEO based quantum measurement theory the cascade of quantum measurements proceeds from long to short scales and at each step decomposes a given system to two subsystems. The cascade stops when the reduction of entanglement is impossible: this is the case if the entanglement probabilities belong to an extension of extension of rationals characterizing the extension in question. This cascade is nothing but a formation of an association! Since only the state at the second boundary of CD changes, the natural interpretation is that state function reduction mean a selection of association in 3-D sense.

3. The division of n objects to groups has also social meaning: all social groups tend to divide into cliques spoiling the dream about full democracy. Only a group with 2 members - Romeo and Julia or Adam and Eve - can be a full democracy in practice. Already in a group of 3 members 2 members tend to form a clique leaving the third member outside. Jules and Catherine, Jim and Catherine, or maybe Jules and Jim! Only a paradise allows a full democracy in which non-associativity holds true. In ZEO it would be realized only at the quantum critical external lines of scattering diagram and quantum criticality means instability. Quantum superposition of all associations could realize this democracy in 4-D sense.

A further perspective is provided by many-sheeted space-time providing classical correlate for quantum dynamics.

1. Many-sheeted space-time means that physical states have a hierarchical structure - just like associations do. Could the formation of association (AB) correspond basically to a formation of flux tube bond between A and B to give AB and serve as space-time correlate for (negentropic) entanglement. Could ((AB)C) would correspond to (AB) and (C) "topologically condensed" to a larger surface. If so, the hierarchical structure of many-sheeted space-time would represent associations and also the basic structures of language.

2. Spin glass (see `http://tinyurl.com/y9yyq8ga`) is a system characterized by so called frustrations. Spin glass as a thermodynamical system has a very large number of minima of free energy and one has fractal energy landscape with valleys inside valleys. Typically there is a competition between different pairings (associations) of the basic building bricks of the system.

   Could spin glass be describable in terms of associations? The modelling of spin glass leads to the introduction of ultrametric topology characterizing the natural distance function for the free energy landscape. Interestingly, p-adic topologies are ultrametric. In TGD framework I have considered the possibility that space-time is like 4-D spin glass: this idea was originally inspired by the huge vacuum degeneracy of Kähler action. The twistor lift of TGD breaks this degeneracy but 4-D spin glass idea could still be relevant.

6.6 Gromov-Witten invariants, Riemann-Roch theorem, and Atyiah-Singer index theorem from TGD point of view

Gromov-Witten (G-W) invariants, Riemann-Roch theorem (RR), and Atyiah-Singer index theorem (AS) are applied in advanced algebraic geometry, and it is interesting to see whether they could have counterparts in TGD framework. The basic difference between TGD and conventional algebraic geometry is due to the adelic hierarchy demanding that the coefficients of polynomials involved are in given extension of rationals. Continuous moduli spaces are replaced with discrete ones by number theoretical quantization due to the criticality guaranteeing associativity of tangent or normal space. $M^8 - H$ duality brings in powerful consistency conditions: counting of allowed combinations of coefficients of polynomials on M^8 side and counting of dimensions on H side using

AS should give same results. $M^8 - H$ duality might be in fact analogous to the mirror symmetry of M-theory.

6.6.1 About the analogs of Gromow-Witten invariants and branes in TGD

Gromow-Witten invariants, whose definition was discussed in [L33], play a central role in superstring theories and M-theory and are closely related to branes. For instance, partition functions can be expressed in terms of these invariants giving additional invariants of symplectic and algebraic geometries. Hence it is interesting to look whether they could be important also in TGD framework.

1. As such the definition of G-W invariants discussed in [L33] do not make sense in TGD framework. For instance, space-time surface is not a closed symplectic manifold whereas M^8 and H are analogs of symplectic spaces. Minkowskian regions of space-time surface have Hamilton-Jacobi structure at the level of both M^8 and H and this might replace the symplectic structure. Space-time surfaces are not closed manifolds.

   Physical intuition however suggests that the generalization exists. The fact that Minkowskian metric and Euclidian metric for complexified octonions are obtained in various sectors for which complex valued length squared is real suggests that signature is not a problem. Kähler form for complexified z gives as special case analog of Kähler form for E^4 and M^4.

2. The quantum intersection defines a description of interactions in terms of string world sheets. If I have understood G-W invariant correctly, one could have for $D > 4$-dimensional symplectic spaces besides partonic $2k-2$-D surfaces also surfaces with smaller but even dimension identifiable as branes of various dimensions. Branes would correspond to a generalization of relative cohomology. In TGD framework one has $2k = 4$ and the partonic 2-surfaces have dimension 2 so that classical intersections consisting of discrete points are possible and stable for string world sheets and partonic 2-surfaces. This is a unique feature of 4-D space-time.

   One might think a generalization of G-W invariant allowing to see string world sheets as connecting the spaced-like 3-surfaces at the boundaries of CDs and light-like orbits of partonic 2-surfaces. The intersection is not discrete now and marked points would naturally correspond to the ends points of strings at partonic 2-surfaces associated with the boundaries of CD and with the vertices of topological scattering diagrams.

3. The idea about 2-D string world sheet as interaction region could generalize in TGD to space-time surface inside CD defining 4-D interaction region. In [L34] one indeed ends up with amazingly similar description of interactions for n external particles entering CD and represented as zero loci for quaternion valued "real" part $RE(P)$ or "imaginary" part $IM(P)$ for the complexified octonionic polynomial.

   Associativity forces quantum criticality posing conditions on the coefficients of the polynomials. Polynomials with the origin of octonion coordinate along the same real axis commute and associate. Since the origins are different for external particles in the general case, the polynomials representing particles neither commute nor associate inside the interaction region defined by CD but one can also now define zero loci for both $RE(\prod P_i)$ and $IM(\prod P_i)$ giving $P_i = 0$ for some i. Now different permutations and different associations give rise to different interaction regions and amplitude must be sum over all these.

   3-vertices would correspond to conditions $P_i = 0$ for 3 indices i simultaneously. The strongest condition is that 3 partonic 2-surfaces X_i^2 co-incide: this condition does not satisfy classical dimension rule and should be posed as essentially 4-D boundary condition. Two partonic 2-surfaces $X_i^2(t_i(n))$ intersect at discrete set of points: could one assume that the sparticle lines intersect and there fusion is forced by boundary condition? Or could one imagine that partonic 2-surfaces turns back in time and second partonic 2-surface intersects it at the turning point?

4. In 4-D context string world sheets are associated with magnetic flux tubes connecting partonic orbits and together with strings serve as correlates for negentropic entanglement assignable

to the p-adic sectors of the adele considered, to attention in consciousness theory, and to remote mental interactions in general and occurring routinely between magnetic body and biological body also in ordinary biology. This raises the question whether "quantum touch" generalizes from 2-D string world sheets to 4-D space-time surface (magnetic flux tubes) connecting 3-surfaces at the orbits and partonic orbits.

5. The above formulation applies to closed symplectic manifolds X. One can however generalize the formulation to algebraic geometry. Now the algebraic curve X^2 is characterized by genus g and order of polynomial n defining it. This formulation looks very natural in M^8 picture.

An interesting question is whether the notion of brane makes sense in TGD framework.

1. In TGD branes inside space-time variety are replaced by partonic 2-surfaces and possibly by their light-like orbits at which the induced metric changes signature. These surfaces are metrically 2-D. String world sheets inside space-time surfaces have discrete intersection with the partonic 2-surfaces. The intersection of strings as space-like *resp.* light-like boundaries of string world sheet with partonic orbit sheet *resp.* space-like 3-D ends of space-time surface at boundaries of CD is also discrete classically.

2. An interesting question concerns the role of 6-spheres $S^6(t_n)$ appearing as special solutions to the octonionic zero locus conditions solving both $RE(P_n) = 0$ and $IM(P_n) = 0$ requiring $P_n(o) = 0$. This can be true at 7-D light cone $o = et$, e light-like vector and t a real parameter. The roots t_n of $P(t) = 0$ give 6-spheres $S^6(t_n)$ with radius t_n as solutions to the singularity condition. As found, one can assign to each factor P_i in the product of polynomials defining many-particle state in interaction region its own partonic 2-surfaces $X^2(t_n)$ related to the solution of $P_i(t) = 0$

   Could one interpret 6-spheres as brane like objects, which can be connected by 2-D "free" string world sheets as 2-varieties in M^8 and having discrete intersection with them implied by the classical dimension condition for the intersection. Free string world sheets would be something new and could be seen as trivially associative surfaces whereas 6-spheres would represent trivially co-associative surfaces in M^8.

   The 2-D intersections of $S^6(t_n)$ with space-time surfaces define partonic 2-surfaces X^2 appearing at then ends of space-time and as vertices of topological diagrams. Light-like sparticle lines along parton orbits would fuse at the partonic 2-surfaces and give rise to the analog of 3-vertex in $\mathcal{N} = 4$ SUSY.

Some further TGD inspired remarks are in order.

1. Virasoro conjecture generalizing Witten conjecture involves half Virasoro algebra. Super-Virasoro algebra algebra and its super-symplectic counterpart (SSA) play a key role in the formulation of TGD at level of H. Also these algebras are half algebras. The analogs of super-conformal conformal gauge conditions state that sub-algebra of SSA with conformal weights coming as n-ples of those for entire algebra and its commutator with entire SSA give rise to vanishing Noether charges and annihilate physical states.

   These conditions are conjecture to fix the preferred extremals and serve as boundary conditions allowing the formulation of $M^8 - H$ correspondence inside space-time regions (interaction regions), where the associativity conditions fail to be true and direct $M^8 - H$ correspondence does not make sense. Non-trivial solutions to these conditions are possible only if one assumes half super-conformal and half super-symplectic algebras. Otherwise the generators of the entire SSA annihilate the physical states and all SSA Noether charges vanish. The invariance of partition function for string world sheets in this sense could be interpreted in terms of emergent dynamical symmetries.

2. Just for fun one can consider the conjecture that the reduction of quantum intersections to classical intersections mediated by string world sheets implies that the numbers of string world sheets as given by the analog of G-W invariants are integers.

6.6.2 Does Riemann-Roch theorem have applications to TGD?

Riemann-Roch theorem (RR) (see http://tinyurl.com/mdmbcx6) is a central piece of algebraic geometry. Atyiah-Singer index theorem is one of its generalizations relating the solution spectrum of partial differential equations and topological data. For instance, characteristic classes classifying bundles associated with Yang-Mills theories (see http://tinyurl.com/y9xvkhyy) have applications in gauge theories and string models.

The advent of octonionic approach to the dynamics of space-time surfaces inspired by M^8-H duality [L32] [L33, L34] gives hopes that dynamics at the level of complexified octonionic M^8 could reduce to algebraic equations plus criticality conditions guaranteeing associativity for space-time surfaces representing external particles, in interaction region commutativity and associativity would be broken. The complexification of octonionic M^8 replacing norm in flat space metric with its complexification would unify various signatures for flat space metric and allow to overcome the problems due to Minkowskian signature. Wick rotation would not be a mere calculational trick.

For these reasons time might be ripe for applications of possibly existing generalization of RR to TGD framework. In the following I summarize my admittedly unprofessional understanding of RR discussing the generalization of RR for complex algebraic surfaces having real dimension 4: this is obviously interesting from TGD point of view.

I will also consider the possible interpretation of RR in TGD framework. One interesting idea is possible identification of light-like 3-surfaces and curves (string boundaries) as generalized poles and zeros with topological (but not metric) dimension one unit higher than in Euclidian signature.

Could a generalization of Riemann-Roch theorem be useful in TGD framework?

The generalization of RR for algebraic varieties, in particular for complex surfaces (real dimension equal to 4) exists. In M^8 picture the complexified metric Minkowskian signature need not cause any problems since the situation can be reduced to Euclidian sector. Clearly, this picture would provide a realization of Wick rotation as more than a trick to calculate scattering amplitudes.

Consider first the motivations for the desire of having analog of Riemann-Roch theorem (RR) at the level of space-time surfaces in M^8.

1. It would be very nice if partonic 2-surfaces would have interpretation as analogs of zeros or poles of a meromorphic function. RR applies to the divisors characterizing meromorphic functions and 2-forms, and one could hope of obtaining information about the dimensions of these function spaces giving rise to octonionic space-time varieties. Note however that the reduction to real polynomials or even rational functions might be already enough to give the needed information. Rational functions are required by the simplest generalization whereas the earlier approach assumed only polynomials. This generalization does not however change the construction of space-time varieties as zero loci of polynomials in an essential manner as will be found.

2. One would like to count the degeneracies for the intersections of 2-surfaces of space-time surface and here RR might help since its generalization to complex surfaces involves intersection form as was found in the brief summary of RR for complex surfaces with real dimension 4 (see Eq. 5.3.5).

   In particular, one would like to know about the intersections of partonic 2-surfaces and string world sheets defining the points at which fermions reside. The intersection form reduces the problem via Poincare duality to 2-cohomology of space-time surfaces. More generally, it is known that the intersection form for 2-surfaces tells a lot about the topology of 4-D manifolds (see http://tinyurl.com/y8tmqtef). This conforms with SH. Gromow-Witten invariants [L22] (see http://tinyurl.com/ybobccub) are more advanced rational valued invariants but might reduce to integer valued in variants in TGD framework [L34].

There are also other challenges to which RR might relate.

1. One would like to know whether the intersection points for string world sheets and partonic 2-surfaces can belong in an extension of rationals used for adele. If the points belong to

cognitive representations and subgroup of Galois group acts trivially then the number of points is reduces as the points at its orbit fuse together. The sheets of the Galois covering would intersect at point. The images of the fused points in H could be disjoint points since tangent spaces need not be parallel.

2. One would also like to have idea about what makes partonic 2-surfaces and string world sheets so special. In 2-D space-time one would have points instead of 2-surfaces. The obvious idea is that at the level of M^8 these 2-surfaces are in some sense analogous to poles and zeros of meromorphic functions. At the level of H the non-local character of $M^8 - H$ would imply that preferred extremals are solutions of an action principle giving partial differential equations.

What could be the analogs of zeros and poles of meromorphic function?

The basic challenge is to define what notions like pole, zero, meromorphic function, and divisor could mean in TGD context. The most natural approach based on a simple observation that rational functions need not define map of space-time surface to itself. Even though rational function can have pole inside CD, the point ∞ need not belong to the space-time variety defined the rational functions. Hence one can try the modification of the original hypothesis by replacing the octonionic polynomials with rational functions. One cannot exclude the possibility that although the interior of CD contains only finite points, the external particles outside CD could extend to infinity.

1. For octonionic analytic polynomials the notion of zero is well-defined. The notion of pole is well-defined only if one allows rational functions $R = P_1(o)/P_2(o)$ so that poles would correspond to zeros for the denominator of rational function. 0 and ∞ are both unaffected by multiplication and ∞ also by addition so that they are algebraically special. There are several variants of this picture. The most general option is that for a given variety zeros of both P_i are allowed.

2. The zeros of $IM(P_1) = 0$ and $IM(P_2) = 0$ would give solutions as unions of surfaces associated with P_i. This is because $IM(o_1o_2) = IM(o_1)RE(o_2) + IM(o_2)RE(o_1)$. There is no need to emphasize how important this property of IM for product is. One might say that one has two surfaces which behave like free non-interacting particles.

3. These surfaces should however interact somehow. The intuitive expectation is that the two solutions are glued by wormhole contacts connecting partonic 2-surfaces corresponding to $IM(P_1) = 0$ and $IM(P_2) = 0 = \infty$. For $RE(P_i) = 0$ and $RE(P_i) = \infty$ the solutions do not reduce to separate solutions $RE(P_1) = 0$ and $RE(P_2) = 0$. The reason is that the real part of o_1o_2 satisfies $Re(o_1o_2) = Re(o_1)Re(o_2) - Im(o_1)Im(o_2)$. There is a genuine interaction, which should generate the wormhole contact. Only at points for which $P_1 = 0$ and $P_2 = 0$ holds true, $RE(P_1) = 0$ and $RE(P_2) = 0$ are satisfied simultaneously. This happens in the discrete intersection of partonic 2-surfaces.

4. Elementary particles correspond even for $h_{eff} = h$ to two-sheeted structures with partonic surfaces defining wormhole throats. The model for elementary particles requires that particles are minimally 2-sheeted structures since otherwise the conservation of monopole Kähler magnetic flux cannot be satisfied: the flux is transferred between space-time sheets through wormhole contacts with Euclidian signature of induced metric and one obtains closed flux loop. Euclidian wormhole contact would connect the two Minkowskian sheets. Could the Minkowskian sheets corresponds to zeros $IM(P_i)$ for P_1 and P_2 and could wormhole contacts emerge as zeros of $RE(P_1/P_2)$?

One can however wonder whether this picture could allow more detailed specification. The simplest possibility would be following. The basic condition is that CD emerges automatically from this picture.

1. The simplest possibility is that one has $P_1(o)$ and $P_2(T - o)$ with the origin of octions at the "lower" tip of CD. One would have $P_1(0) = 0$ and $P_2(0) = 0$. $P_1(o)$ would give rise to the "lower" boundary of CD and $P_2(T - o)$ to the "upper" boundary of CD.

ZEO combined with the ideas inspired by infinite rationals as counterparts of space-time surfaces connecting 3-surfaces at opposite boundaries of CD [K107] would suggest that the opposite boundaries of CD could correspond zeros and poles respectively and the ratio $P_1(o)/P_2(T-o)$ and to zeros of P_1 resp. P_2 assignable to different boundaries of CD. Both light-like parton orbits and string world sheets would interpolate between the two boundaries of CD at which partonic 2-surface would correspond to zeros and poles.

The notion divisor would be a straightforward generalization of this notion in the case of complex plane. What would matter would be the rational function $P_1(t)/P_2(T-t)$ extended from the real (time) axis of octonions to the entire space of complexified octonions. Positive degree of divisor would multiply $P_1(t)$ with $(t-t_1)^m$ inducing a new zero at or increasing the order of existing zero at t_1. Negative orders n would multiply the denominator by $(t-t_1)^n$.

2. One can also consider the possibility that both boundaries of CD emerge for both P_1 and P_2 and without assigning either boundary of CD with P_i. In this case P_i would be sum over terms $P_{ik} = P_{ia_k}(o)P_{ib_k}(T-o)$ of this kind of products satisfying $P_{ia_k}(0) = 0$ and $P_{ib_k}(0) = 0$.

One can imagine also an alternative approach in which 0 and ∞ correspond to opposite tips of CD and have geometric meaning. Now zeros and poles would correspond to 2-surfaces, which need not be partonic. Note that in the case of Riemann surfaces ∞ can represent any point. This approach does not however look attractive.

Could one generalize RR to octonionic algebraic varieties?

RR is associated with complex structure, which in TGD framework seems to make sense independent of signature thanks to complexification of octonions. Divisors are the key notion and characterize what might be called local winding numbers. De-Rham cohomology is replaced with much richer Dolbeault cohomology (see `http://tinyurl.com/y7cvs5sx`) since the notion of continuity is replaced with that of meromorphy. Symplectic approach about which G-W invariants for symplectic manifolds provide an example define a different approach and now one has ordinary cohomology.

An interesting question is whether M^8-H-duality corresponds to the mirror symmetry of string models (see `http://tinyurl.com/yc2m2e5m`) relating complex structures and symplectic structures. If this were the case, M^8 would correspond to complex structure and H to symplectic structure.

RR for curves gives information about dimensions for the spaces of meromorphic functions having poles with order not higher than specified by divisor. This kind of interpretation would be very attractive now since the poles and zeros represented as partonic 2-surfaces would have direct physical interpretation in terms of external particles and interaction vertices. RR for curves involves poles with orders not higher than specified by the divisor and gives a formula for the dimension of the space of meromorphic functions fora given divisor. As a special case give the dimension $l(nD)$ for a given divisor.

Could something similar be true in TGD framework?

1. Arithmetic genus makes sense for polynomials $P(t)$ since t can be naturally complexified giving a complex curve with well-defined arithmetic genus. What could correspond to the intersection form for 2-surfaces representing D and $K-D$? The most straightforward possibility is that partonic 2-surfaces correspond to poles and zeros.

Divisor $-D$ would correspond to the inverse of P_2/P_1 representing it. $D-K$ would also a well-defined meaning provided the canonical divisor associated with holomorphic 2-form has well-defined meaning in the Dolbeault cohomology of the space-time surface with complex structure. RR would give direct information about the space of space-time varieties defined by $RE(P) = 0$ or $IM(P) = 0$ condition.

One could hope of obtaining information about intersection form for string world sheets and partonic 2-surfaces. Whether the divisor $D-K$ has anything to do string world sheets, is of course far from clear.

2. Complexification means that field property fails in the sense that complexified Euclidian norm vanishes and the inverse of complexified octonion/quaternion/complex number is infinite formally. For Euclidian sector with real coordinates this does not happen but does take place when some coordinates are real and some imaginary so that signature is effectively Minkowskian signature.

   At 7-D light-cone of M^8 the condition $P(o) = 0$ reduces to a condition for real polynomial $P(t) = 0$ giving roots t_n. Partonic 2-varieties are intersections of 4-D space-time varieties with 6-spheres with radii t_n. There are good reasons to expect that the 3-D light-like orbits of partonic 3-surfaces are intersections of space-time variety with 7-D light-cone boundary and their H counterparts are obtained as images under $M^8 - H$ duality.

   For light-like complefixied octonionic points the inverse of octonion does not exist since the complexified norm vanishes. Could the light-like 3-surfaces as partonic orbits correspond to images under $M^8 - H$ duality for zeros and/or poles as 3-D light-like surfaces? Could also the light-like boundaries of strings correspond to this kind of generalized poles or zeros? This could give a dynamical realization for the notions of zero and pole and increase the topological dimension of pole and zero for both 2-varieties and 4-varieties by one unit. The metric dimension would be unaffected and this implies huge extension of conformal symmetries central in TGD since the light-like coordinate appears as additional parameter in the infinitesimal generators of symmetries.

Could one formulate the counterpart of RR at the level of H? The interpretation of $M^8 - H$ duality as analog of mirror symmetry (see `http://tinyurl.com/yc2m2e5m`) suggests this. In this case the first guess for the identification of the counterpart of canonical divisor could be as Kähler form of CP_2. This description would provide symplectic dual for the description based on divisors at the level of M^8. G-W invariants and their possible generalization are natural candidates in this respect.

6.6.3 Could the TGD variant of Atyiah-Singer index theorem be useful in TGD?

Atyiah-Singer index theorem (AS) is one of the generalizations of RR and has shown its power in gauge field theories and string models as a method to deduce the dimensions of various moduli spaces for the solutions of field equations. A natural question is whether AS could be useful in TGD and whether the predictions of AS at H side could be consistent with $M^8 - H$ duality suggesting very simple counting for the numbers of solutions at M^8 side as coefficient combinations of polynomials in given extension of rationals satisfying criticality conditions. One can also ask whether the hierarchy of degrees n for octonion polynomials could correspond to the fractal hierarchy of generalized conformal sub-algebras with conformal weights coming as n-multiples for those for the entire algebras.

Atyiah-Singer index theorem (AS) and other generalizations of RR involve extremely abstract concepts. The best manner to get some idea about AS is to learn the motivations for it. The article `http://tinyurl.com/yc49lljp` gives a very nice general view about the motivations of Atyiah-Singer index theorem and also avoids killing the reader with details.

Solving problems of algebraic geometry is very demanding. The spectrum of solutions can be discrete (say number of points of space-time surface having linear M^8 coordinates in an extension of rationals) or continuous such as the space of roots for n:th order polynomials with real coefficients.

An even more difficult challenge is solving of partial differential equations in some space, call it X, of say Yang-Mills gauge field coupled to matter fields. In this case the set of solutions is typically continuous moduli space.

One can however pose easier questions. What is the number of solutions in counting problem? What is the dimension of the moduli space of solutions? Atiyiah-Singer index theorem relates this number - analytic index - to topological index expressible in terms of topological invariants assignable to complexified tangent bundle of X and to the bundle structure - call it field bundle - accompanying the fields for which field equations are formulated.

AS very briefly

Consider first the assumptions of AS.

1. The idea is to study perturbations of a given solution and linearize the equations in some manifold X often assumed to be compact. This leads to a linear partial differential equations defined by linear operator P. One can deduce the dimension of the solution space of P. This number defines the dimension of the tangent space of solution space of full partial differential equations, call it moduli space.

2. The idea is to assign to the partial differential operator P its symbol $\sigma(P)$ obtained by replacing derivatives with what might be called momentum components. The reversal of this operaion is familiar from elementary wave mechanics: $p_i \rightarrow id/dx^i$. This operation can be formulated in terms of co-tangent bundle. The resulting object is purely algebraic. If this matrix is reversible for all momentum values and points of X, one says that the operator is elliptic.

   Note that for field equations in Minkowski space M^4 the invertibility constraint is not satisfied and this produces problems. For instance, for massive M^4 d'Alembertian for scalar field the symbol is four-momentum squared, which vanishes, when on-mass shell condition is satisfied. Wick rotation is somewhat questionable manner to escape this problem. One replaces Minkowski space with its Euclidian counterpart or by 4-sphere. If all goes well the dimension of the solution space does not depend on the signature of the metric.

3. In the general case one studies linear equation of form $DP = f$, where f is homogenuity term representing external perturbation. f can also vanish. Quite generally, one can write the dimension of the solution space as

$$Ind_{anal}(P) = dim(ker(P)) - dim(coker(P)) \quad . \tag{6.6.1}$$

   $ker(P)$ denotes the solution space for $DP = 0$ without taking into account the possible restrictions coming from the fact that f can involve part f_0 satisfying $Df_0 = 0$ (for instance, f_0 corresponds to resonance frequency of oscillator system) nor boundary conditions guaranteing hermiticity. Indeed, the hermitian conjugate $D^\dagger$ of D is not automatically identical with D. $D^\dagger$ is defined in terms of the inner product for small perturbations as

$$\langle D^\dagger P_1^* | DP_2\rangle = \langle P_1 | DP_2\rangle \quad . \tag{6.6.2}$$

   The inner product involves integration over X and partial integrations transfer the action of partial derivatives from P_2 to P_1^*. This however gives boundary terms given by surface integral and hermiticity requires that they vanish. This poses additional conditions on P and contributes to $dim(coker(P))$.

The challenge is to calculate $Ind_{anal}(P)$ and here AS is of enormous help. AS relates analytical index $Ind_{anal}(P)$ for P to topological index $Ind_{top}(\sigma(P))$ for its symbol $\sigma(P)$.

1. $Ind_{top}(\sigma(P))$ involves only data associated with the topology X and with the bundles associated with field variables. In the case of Yang-Mills fields coupled to matter the bundle is the bundle associated with the matter fields with a connection determined by Yang-Mills gauge potentials. So called Todd class $Td(X)$ brings in information about the topology of complexified tangent bundle.

2. $Ind_{top}(\sigma(P))$ is not at all easy to define but is rather easily calculable as integrals of various invariants assignable to the bundle structure involved. Say instanton density for YM fields and various topological invariants expressing the topological invariants associated with the metric of the space. What is so nice and so non-trivial is that the dimension of the moduli space for non-linear partial differential equations is determined by topological invariants. Much of the dynamics reduces to topology.

The expression for $Ind_{top}(\sigma(P))$ involves besides σ_P topological data related to the field bundle and to the complexified tangent bundle. The expression Ind_{top} as a function of the symbol $\sigma(P)$ is given by

$$Ind_{top}(\sigma(P)) = (-1)^n \langle ch(\sigma(P)) \cdot Td(T_C(X), [X]) \rangle \quad . \tag{6.6.3}$$

The expression involves various topological data.

1. Dimension of X.

2. The quantity $\langle x.y \rangle$ involving cup product $x.y$ of cohomology classes, which contains a contribution in the highest homology group $H^n(X)$ of X corresponding to the dimension of X and is contracted with this fundamental class $[X]$. $\langle x.y \rangle$ denotes matrix trace for the operator $ch(\sigma(P))$ formed as polynomial of $\sigma(P)$. $[X]$ denotes so called fundamental class fr X belonging to H^n and defines the orientation of X.

3. Chern character $ch_E(t)$ (see `http://tinyurl.com/ybavu66h`). I must admit that I ended up to a garden of branching paths while trying to understand the definition of ch_E is. In any case, $ch_E(t)$ characterizes complex vector bundle E expressible in terms of Chern classes (see `http://tinyurl.com/y8jlaznc`) of E. E is the bundle assignable to field variables, say Yang Mills fields and various matter fields.

   Both direct sums and tensor products of fiber spaces of bundles are possible and the nice feature of Chern class is that it is additive under tensor product and multiplicative under direct sum. The fiber space of the entire bundle is now direct sum of the tangent space of X and field space, which suggests that $Ind(top)$ is actually the analog of Chern character for the entire bundle.

   $t = \sigma P$ has interpretation as an argument appearing in the definition of Chern class generalized to Chern character. $t = \sigma(P)$ would naturally correspond to a matrix valued argument of the polynomial defining Chern class as cohomology element. $ch(\sigma(P))$ is a polynomial of the linear operator defined by symbol $\sigma(P)$. ch_E for given complex vector bundle is a polynomial, whose coefficients are relatively easily calculable as topological invariants assignable to bundle E. E must be the field bundle now.

4. Todd class $Td(T_C(X))$ for the complexified tangent bundle (see `http://tinyurl.com/yckv4w84`) appears also in the expression. Note that also now the complexification occurs. The cup product gives element in $H^n(X)$, which is contracted with fundamental class $[X]$ and integrated over X.

AS and TGD

The dynamics of TGD involves two levels: the level of complexified M^8 (or equivalently E^8) and the level of H related to $M^8 - H$ correspondence.

1. At the level of M^8 one has algebraic equations rather than partial differential equations and the situation is extremely simple as compared to the situation for a general action principle. At the level of H one has action principle and partial differential equations plus infinite number of gauge conditions selecting preferred extremals and making dynamics for partial differential equations dual to the dynamics determined by purely number theoretic conditions.

   The space-time varieties representing external particles outside CDs in M^8 satisfy associativity conditions for tangent space or normal space and reducing to criticality conditions for the real coefficients of the polynomials defining the space-time variety. In the interior of CDs associativity conditions are not satisfied but the boundary conditions fix the values of the coefficients to be those determined by criticality conditions guaranteing associativity outside the CD.

   In the interiors space-time surfaces of CDs M^8-duality does not apply but associativity of tangent spaces or normal spaces at the boundary of CD fixes boundary values and minimal surface dynamics and strong form of holography (SH) fixes the space-time surfaces in the interior of CD.

2. For the H-images of space-time varieties in H under M^8-H duality the dynamics is universal coupling constant independent critical dynamics of minimal surfaces reducing to holomorphy in appropriate sense. For minimal surfaces the 4-D Kähler current density vanishes so that the solutions are 4-D analogs of geodesic lines outside CD. Inside CD interactions are coupled on and this current is non-vanishing. Infinite number of gauge conditions for various half conformal algebras in generalized sense code at H side for the number theoretical critical conditions at M^8 side. The sub-algebra with conformal weights coming as n-ples of the entire algebra and its commutator with entire algebra gives rise to vanishing classical Noether charges. An attractive assumption is that the value of n at H side corresponds to the order n of the polynomials at M^8 side.

3. The coefficients of polynomials $P(o)$ determining space-time varieties are real numbers (also complexified reals can be considered without losing associativity) restricted to be numbers in extension of rationals. This makes it possible to speak about p-adic variants of the space-time surfaces at the level of M^8 at least.

Could Atyiah-Singer theorem have relevance for TGD?

1. For real polynomials it is easy to calculate the dimension of the moduli space by counting the number of independent real (in octonionic sense) coefficients of the polynomials of real variable (one cannot exclude that the coefficients are in complex extension of rationals). Criticality conditions reduce this number and the condition that coefficients are in extension of rationals reduces it further. One has quite nice overall view about the number of solutions and one can see them as subset of continuous moduli space. If M^8-H duality really works then this gives also the number of preferred extremals at H side.

2. This picture is not quite complete. It assumes fixing of 8-D CD in M^8 as well as fixing of the decomposition $M^2 \subset M^4 \subset M^4 \times E^4$. This brings in moduli space for different choices of octonion structures (8-D Lorentz group is involved). Also moduli spaces for partonic 2-surfaces are involved. Number theoretical universality seems to require that also these moduli spaces have only points with coordinates in extension of rationals involved.

3. In principle one can try to formulate the counterpart of AS at H side for the linearization of minimal surface equations, which are nothing but the counterpart of massless field equations in a fixed background metric. Note that additional conditions come from the requirement that the term from Kähler action reduces to minimal surface term.

   Discrete sets of solutions for the extensions of rationals should correspond to each other at the two sides. One can also ask whether the dimensions for the effective continuous moduli spaces labelled by n characterizing the sub-algebras of various conformal algebras isomorphic to the entire algebra and those for the polynomials of order n satisfying criticality conditions. One would have a number theoretic analog for a particle in box leading to the quantization of momenta.

All this is of course very speculative and motivated only by the general physical vision. If the speculations were true, they would mean huge amount of new mathematics.

6.7 Intersection form for 4-manifolds, knots and 2-knots, smooth exotics, and TGD

Gary Ehlenberger sent a highly interesting commentary related to smooth structures in R^4 discussed in the article of Gompf [A80] (`https://cutt.ly/eMracmf`) and more generally to exotics smoothness discussed from the point of view of mathematical physics in the book of Asselman-Maluga and Brans [A87] (`https://cutt.ly/DMuOdYr`). I am grateful for these links for Gary.

6.7.1 Basic ideas

The role of intersection forms in TGD

The intersection form of 4-manifold (https://cutt.ly/jMriNdI) characterizing partially its 2-homology is a central notion in the study of the smooth structures. I am not a topologist but have two good reasons to get interested on intersection forms.

1. In the TGD framework [L96], the intersection form describes the intersections of string world sheets and partonic 2-surfaces and therefore is of direct physical interest [K62, L34].

2. Knots have an important role in TGD. The 1-homology of the knot complement characterizes the knot. Time evolution defines a knot cobordism as a 2-surface consisting of knotted string world sheets and partonic 2-surfaces. A natural guess is that the 2-homology for the 4-D complement of this cobordism characterizes the knot cobordism. Also 2-knots are possible in 4-D space-time and a natural guess is that knot cobordism defines a 2-knot.

   The intersection form for the complement for cobordism as a way to classify these two-knots is therefore highly interesting in the TGD framework. One can also ask what the counterpart for the opening of a 1-knot by repeatedly modifying the knot diagram could mean in the case of 2-knots and what its physical meaning could be in the TGD Universe. Could this opening or more general knot-cobordism of 2-knot take place in zero energy ontology (ZEO) [L64, L93, L100] as a sequence of discrete quantum jumps leading from the initial 2-knot to the final one.

Why exotic smooth structures are not possible in TGD?

The existence of exotic 4-manifolds [A80, A87, A42] could be an anomaly in the TGD framework. In the articles [A80, A42] the term anomaly is indeed used. Could these anomalies cancel in the TGD framework?

The first naive guess was that the exotic smooth structures are not possible in TGD but it turned out that this is not trivially true. The reason is that the smooth structure of the space-time surface is not induced from that of H unlike topology. One could induce smooth structure by assuming it given for the space-time surface so that exotics would be possible. This would however bring an ad hoc element to TGD. This raises the question of how it is induced.

1. This led to the idea of a holography of smoothness, which means that the smooth structure at the boundary of the manifold determines the smooth structure in the interior. Suppose that the holography of smoothness holds true. In ZEO, space-time surfaces indeed have 3-D ends with a unique smooth structure at the light-like boundaries of the causal diamond $CD = cd \times CP_2 \subset H = M^4 \times CP_2$, where cd is defined in terms of the intersection of future and past directed light-cones of M^4. One could say that the absence of exotics implies that $D = 4$ is the maximal dimension of space-time.

2. The differentiable structure for $X^4 \subset M^8$, obtained by the smooth holography, could be induced to $X^4 \subset H$ by $M^8 - H$-duality. Second possibility is based on the map of mass shell hyperboloids to light-cone proper time $a = constant$ hyperboloids of H belonging to the space-time surfaces and to a holography applied to these.

3. There is however an objection against holography of smoothness (https://cutt.ly/3MewYOt). In the last section of the article, I develop a counter argument against the objection. It states that the exotic smooth structures reduce to the ordinary one in a complement of a set consisting of arbitrarily small balls so that local defects are the condensed matter analogy for an exotic smooth structure.

6.7.2 Intersection form in the case of 4-surfaces

Intersection form (https://cutt.ly/jMriNdI) for homologically trivial 2-surfaces of the space-time surface and 2-homology for the complement of these surfaces can be physically important in tGD framework.

Intersection forms in 2-D case

It is good to explain the notion of intersection form by starting from 1-homology. The intersection form for 1-homology is encountered for a cylinder with ends fixed. In this case, one has relative homology and homologically trivial curves are curves connecting the ends of string and characterized by a winding number.

In the case of torus obtained by identifying the ends of cylinder, one obtains two winding numbers (m,n) corresponding to to homologically non-trivial circles at torus. The intersection number for curves (m,n) and (p,q) at torus is $N = mq - np$ and for curves at cylinder one as $(m,n) = (1,n)$ giving $N = n - q$.

The antisymmetric intersection form is defined as 2×2 matrix defining intersections for the basis of the homology with $(m,n) = (1,0)$ and $(n,m) = (0,1)$ and is given by $(0,1;-1,0)$.

Intersection for 4-surfaces in TGD context

In TGD, the intersection form for a 4-surface identified as space-time surface could have a rather concrete physical interpretation and the stringy part of TGD physics would actually realize it concretely.

1. $M^8 - H$ duality requires that the 4-surface in M^8 has quaternionic/associative normal space: this distribution of normal spaces is integrable and integrates to the 4-surface in M^8.

   The normal must also contain a commutative (complex) sub-space at each point. Only this allows us to parametrize normal spaces by points of CP_2 and map them to space-time surfaces in $H = M^4 \times CP_2$. The integral distribution of these commutative sub-spaces defines a 2-D surface. Physically, these surfaces would correspond to string world sheets and partonic 2-surfaces.

2. String world sheets and partonic 2-surfaces, regarded as objects in relative homology (modulo ends of the space-time surfaces at the boundaries of causal diamond (CD)), can intersect as 2-D objects inside the space-time surface and the intersection form characterizes them.

   There is an analogy with the cylinder: time-like direction corresponds to the cylinder axis and a homologically non-trivial 2-surface of CP_2 corresponds to the circle at the cylinder.

3. If the second homology of the space-time surface is trivial, the naive expectation is that the intersections of string world sheets are not stable under large enough deformations of the string world sheets. Same applies to intersecting plane curves. At the cylinder, the situation is different since the relative first homology is non-trivial and spanned by two generators: the circle and a line connecting the ends of the cylinder.

   The intersection form is however non-trivial as in the case of the cylinder for 2-surfaces having 2-D homologically non-trivial CP_2 projection. They would represent M^4 deformations of 2-D homologically trivial surfaces of CP_2 just like a helical orbit along a cylinder surface. A 2-D generalization of CP_2 type extremal would have a light-like curve or light-like geodesic as M^4 projection and could define light-partonic orbit.

4. The intersection of string world sheet and partonic 2-surface can be stable however. Partonic 2-surface is a boundary of a wormhole contact connecting two space-time sheets.

   Consider a string arriving along space-time sheet A, going through the wormhole contact, and continuing along sheet B. The string has an intersection point with both wormhole throats. This intersection is stable against deformations. The orbit of this string intersects the light-like orbit of the partonic 2-surface along the light-like curve.

   One has a non-trivial intersection form with the number of intersections with partonic 2-surfaces equal to 1. In analogy with cylinder, also the intersections of 2-surfaces with 2-D homologically trivial CP_2 projection are unavoidable and reflect the non-trivial intersection form of CP_2.

6.7.3 About ordinary knots

Ordinary knots and 3-topologies are related and the natural expectation is that also 2-knots and 4-topologies are related.

About knot invariants

Consider first knot invariants (`https://cutt.ly/DMrgs14`)at the general level.

1. One important knot invariant of ordinary knots is the 1-homology of the complement and the associated first homotopy group whose abelianization gives the homology group.

2. The complement of the knot can be given a metric of a hyperbolic 3-manifold, which corresponds to a unit cell for a tessellation of the mass shell. $M^8 - H$ duality suggests that the intersection X^3 of 4-surface of M^8 with mass shell $H^3_m \subset M^4 \subset M^8$ is a hyperbolic manifold and identical with the hyperbolic manifold associated with the complement of a knot of H^3_a realized as light-cone proper time $a = constant$ hyperboloid of $M^4 \subset H$ and closed knotted and linked strings as ends of string world sheets at H^3_a.

   The evolution of the strings defined by the string world sheets would define a 1-knot cobordism. The 2-homology of the knot complement should characterize the topological evolution of the 1-homology of the knot.

Opening of knots and links by knot cobordisms

The procedure leading to the trivialization of knot or link can be used to define knot invariants and the procedure itself characterizes knot.

1. Ordinary knot is described by a knot diagram obtained as a projection of the knot to the plane. It contains intersections of lines and the intersection contains information telling which line is above and which line is below.

2. The opening of the knot or link to give a trivial knot or link, which is used in the construction of knot invariants, is a sequence of violent operations. In the basic step strings portions go through each other and therefore suffer a reconnection. This operation can therefore change the 1-homology of the 3-D knot complement.

   Knot or link can be modified by forcing two intersecting strands of the plane projection to go through each other. Locally the basic operation for two links is the same as for the pieces of knot. The transformation of the knot or link to a trivial knot or link corresponds to some sequence of these operations and can be used to define a knot invariants. This operation is not unique since there are moves which do not affect the knot.

   The basic opening operation can be also seen as a time evolution, knot cobordism, in which the first portion, call it A, remains unchanged and the second portion, call it B, draws a 2-D surface in E^3. A intersects the 2-D orbit at a single point.

3. The 2-homology for the string world sheets and partonic 2-surfaces as 2-surfaces in space-time serves as an invariant of knot cobordism and represents the topological dynamics of ordinary 1-knots of 3-surface and links formed by strings or flux tubes in 3-surface as cobordism defining the time evolution of a knot to another knot.

   In particular, the intersection form for the 2-homology of the complement of the cobordism defines an invariant of cobordism. This intersection form must be distinguished from the intersection form for the second homology of the space-time surface rather than the 2-knot complement.

4. One can also consider more general sequences of basic operations transforming two knots or links to each other as knot-/link cobordisms, which involve self intersections of the knots. Does this mean that the intersection form characterizes the knot cobordism. Could a string diagram involving reconnections describe the cobordism process.

Stringy description of knot cobordisms

$M^8 - H$ duality [L73, L74, L111, L107] requires string word sheets and partonic 2-surfaces. This implies that TGD physics represents the 2-homology of both space-time surfaces and the homology of the complement of the knotted links defined by them.

Although the "non-homological" intersections of string world sheets can be eliminated by a suitable deformation of the string world sheet, they should have a physical meaning. This comes from the observation that they affect nontrivially the 1-homology of the knot complement as 3-D time=constant slice.

The first thing that I am able to imagine is that strings reconnect. This is nothing but the trouser vertex for strings so that intersection form would define topological string dynamics in some sense. These reconnections play a key role in TGD, also in TGD inspired quantum biology.

The dynamics of partonic 2-surfaces and string world sheets could relate to knot cobordisms, possibly leading to the opening of ordinary knot,

6.7.4 What about 2-knots and their cobordisms?

2-D closed surfaces in 4-D space give rise to 2-knots. What is the physical meaning of 2-knots of string world sheets? What could 2-knots for orbits of linear molecules or associated magnetic flux tubes mean physically and from the point of view of quantum information theory? One can try to understand 2-knots by generalizing the ideas related to the ordinary knots.

1. Intuitively it seems that the cobordism of a 1-knot defines a 2-knot. It is not clear to me whether all 2-knots for space-time surfaces connecting the boundaries of CD can be regarded as this kind of cobordisms of 1-knots.

2. The 2-homology of the complement of 2-knot should define a 2-knot invariant. In particular, the intersection form should define a 2-knot invariant.

3. The opening of 1-knot by repeating the above described basic operation is central in the construction of knot invariants and the sequence of the operations can be said to be knot invariant modulo moves leaving the knot unaffected.

   The opening or a more general cobordism of a 2-knot could be seen as a time evolution with respect to a time parameter t_5 parametrizing the isotopy of space-time surface. The local cobordism can keep the first portion of 2-knot, call it A, unchanged and deform another portion, call it B, so that a 3-D orbit at the space-time surface is obtained. For each value of t_5, the portions A and B of 2-knot have in the generic case only points as intersections.

   This would suggest that an intersection point of A and B is generated in the operation and moves during the t_5 time evolution along A along 1-D curve during the process. This process would be the basic operation used repeatedly to open 2-knot or to transform it to another 2-knot.

4. In quantum TGD, a sequence of quantum jumps, quantum cobordism, would have the same effect as t_5 time evolution. This brings in mind DNA transcription and replication as a process proceeding along a DNA strand parallel to the monopole flux tube as a sequence of SFRs involving direct contact between DNA strand and enzymes catalyzing the process and also of corresponding flux tubes. An interesting possibility is that these quantum cobordisms appear routinely in biochemistry of the fundamental linear bio-molecules such as DNA, RNA, tRNA, and amino-acids [K52, K4, K123, K1, K133, L6] [L52].

   The quantum cobordism of 2-knot is possible only in ZEO, where the quantum state as a time= constant snapshot is replaced with a superposition of space-time surfaces.

6.7.5 Could the existence of exotic smooth structures pose problems for TGD?

The article of Gabor Etesi [A42] (`https://cutt.ly/2Md7JWP`) gives a good idea about the physical significance of the existence of exotic smooth structures and how they destroy the cosmic censorship hypothesis (CCH of GRT stating that spacetimes of GRT are globally hyperbolic so that there are no time-like loops.

Smooth anomaly

No compact smoothable topological 4-manifold is known, which would allow only a single smooth structure. Even worse, the number of exotics is infinite in every known case! In the case of non-compact smoothable manifolds, which are physically of special interest, there is no obstruction against smoothness and they typically carry an uncountable family of exotic smooth structures.

One can argue that this is a catastrophe for classical general relativity since smoothness is an essential prerequisite for tensory analysis and partial differential equations. This also destroys hopes that the path integral formulation of quantum gravitation, involving path integral over all possible space-time geometries, could make sense. The term anomaly is certainly well-deserved.

Note however that for 3-geometries appearing as basic objects in Wheeler's superspace approach, the situation is different since for $D < 3$ there is only a single smooth structure. If one has holography, meaning that 3-geometry dictates 4-geometry, it might be possible to avoid the catastrophe.

The failure of the CCH is the basic message of Etesi's article. Any exotic R^4 fails to be globally hyperbolic and Etesi shows that it is possible to construct exact vacuum solutions representing curved space-times which violate the CCH. In other words, GRT is plagued by causal anomalies.

Etesi constructs a vacuum solution of Einstein's equations with a vanishing cosmological constant which is non-flat and could be interpreted as a pure gravitational radiation. This also represents one particular aspect of the energy problem of GRT: solutions with gravitational radiation should not be vacua.

1. Etesi takes any exotic R^4 which has the topology of $S^3 \times R$ and has an exotic smooth structure, which is not a Cartesian product. Etesi maps maps R^4 to CP_2, which is obtained from C^2 by gluing CP_1 to it as a maximal ball B^3_r for which the radial Eguchi-Hanson coordinate approaches infinity: $r \to \infty$. The exotic smooth structure is induced by this map. The image of the exotic atlas defines atlas. The metric is that of CP_2 but $SU(3)$ does not act as smooth isometries anymore.
2. After this Etesi performs Wick rotation to Minkowskian signature and obtains a vacuum solution of Einstein's equations for any exotic smooth structure of R^4.

In TGD, the question of exotic smoothness is encountered both at the level of embedding space and associated fixed spaces and at the level of space-time surfaces and their 6-D twistor space analogies. Could TGD solve the smooth anomaly?

Can embedding space and related spaces have exotic smooth structure?

One can first worry about the exotic smooth structures possibly associated with the M^4, CP_2, $H = M^4 \times CP_2$, causal diamond CD= $cd \times CP_2$, where cd is the intersection of the future and past directed light-cones of M^4, and with M^8. One can also worry about the twistor spaces CP_3 *resp.* $SU(3)/U(1) \times U(1)$ associated with M^4 *resp.* CP_2.

The key assumption of TGD is that all these structures have maximal isometry groups so that they relate very closely to Lie groups, whose unique smooth structures are expected to determine their smooth structures.

1. The first sigh of relief is that all Lie groups have the standard smooth structure. In particular, exotic R^4 does not allow translations and Lorentz transformations as isometries. I dare to conclude that also the symmetric spaces like CP_2 and hyperbolic spaces such as $H^n = SO(1,n)/SO(n)$ are non-exotic since they provide a representation of a Lie group as isometries and the smoothness of the Lie group is inherited. This would mean that the charts for the coset space G/H would be obtained from the charts for G by an identification of the points of charts related by action of subgroup H.

   Note that the mass shell H^3, as any 3-surface, has a unique smooth structure by its dimension.
2. Second sigh of relief is that twistor spaces CP_3 and $SU(3)/U(1) \times U(1)$ have by their isometries and their coset space structure a standard smooth structure.

In accordance with the vision that the dynamics of fields is geometrized to that of surfaces, the space-time surface is replaced by the analog of twistor space represented by a 6-surface with a structure of S^2 bundle with space-time surface X^4 as a base-space in the 12-D product of twistor spaces of M^4 and CP_2 and by its dimension $D = 6$ can have only the standard smooth structure unless it somehow decomposes to $(S^3 \times R) \times R^2$. Holography of smoothness would prevent this since it has boundaries because X^4 as base space has boundaries at the boundaries of CD.

If exotic smoothness is allowed at the space-time level in the proposed sense ordinary smooth structure could be possible at the level of twistor space in the complement of a Cartesian product of the fiber space S^2 with a discrete set of points associated with partonic 2-surfaces.

3. *cd* is an intersection of future and past directed light-cones of M^4. Future/past directed light-cone could be seen as a subset of M^4 and implies standard smooth structure is possible. Coordinate atlas of M^4 is restricted to *cd* and one can use Minkowski coordinates also inside the *cd*. *cd* could be also seen as a pile of light-cone boundaries $S^2 \times R_+$ and by its dimension $S^2 \times R$ allows only one smooth structure.

4. M^8 is a subspace of complexified octonions and has the structure of 8-D translation group, which implies standard smooth structure.

The conclusion is that continuous symmetries of the geometry dictate standard smoothness at the level of embedding space and related structures.

Could TGD eliminate the smoothness anomaly or provide a physical interpretation for it?

The question of exotic smoothness is encountered both at the level of embedding space and associated fixed spaces and at the level of space-time surfaces and their 6-D twistor space analogies.

What does the induction of a differentiable structure really mean? Here my naive expectations turn out to be wrong. If a sub-manifold $S \subset H$ can be regarded as an embedding of smooth manifold N to $S \subset H$, the embedding $N \rightarrow S \subset H$ induces a smooth structure in S (`https://cutt.ly/tMtvG79`). The problem is that the smooth structure would not be induced from H but from N and for a given 4-D manifold embedded to H one could also have exotic smooth structures. This induction of smooth structure is of course physically adhoc.

It is not possible to induce the smooth structure from H to sub-manifold. The atlas defining the smooth structure in H cannot define the charts for a sub-manifold (surface). For standard R^4 one has only one atlas.

1. Could holography of smoothness make sense in the general case?

The first trial to get rid of exotics [A87] was based on the holography of smoothness and did not involve TGD. Could a smooth structure at the boundary of a 4-manifold could dictate that of the manifold uniquely. Could one speak of holography for smoothness? Manifolds with boundaries would have the standard smooth structure.

1. The obvious objection is that the coordinate atlas for 3-D boundary cannot determine 4-D atlas in any way because the boundary cannot have information of the topology of the interior.

2. The holography for smoothness is also argued to fail (`https://cutt.ly/3MewYOt`). Assume a 4-manifold W with 2 different smooth structures. Remove a ball B^4 belonging to an open set U and construct a smooth structure at its boundary S^3. Assume that this smooth structure can be continued to W. If the continuation is unique, the restrictions of the 2 smooth structures in the complement of B^4 would be equivalent but it is argued that they are not.

3. The first layman objection is that the two smooth structures of W are equivalent in the complement $W - B^3$ of an arbitrary small ball $B^3 \subset W$ but not in the entire W. This would be analogous to coordinate singularity. For instance, a single coordinate chart is enough for a sphere in the complement of an arbitrarily small disk.

An exotic smooth structure would be like a local defect in condensed matter physics. In fact it turned out that this intuitive idea is correct: it can be shown that the exotic smooth structures are equivalent with standard smooth structure in a complement of a set having co-dimension zero (`https://cutt.ly/7MbGqx2`). This does not save the holography of smoothness in the general case but gives valuable hints for how exotic smoothness might be realized in TGD framework.

2. Could holography of smoothness make sense in the TGD framework?

Could $M^8 - H$ duality and holography make holography of smoothness possible in the TGD framework?

1. In the TGD framework space-time is 4-surface rather than abstract 4-manifold. 4-D general coordinate invariance, assuming that 3-surfaces as generalization of point-like particles are the basic objects, suggests a fully deterministic holography. A small failure of determinism is however possible and expected, and means that space-time surfaces analogous to Bohr orbits become fundamental objects. Could one avoid the smooth anomaly in this framework?

   The 8-D embedding space topology induces 4-D topology. My first naive intuition was that the 4-D smooth structure, which I believed to be somehow inducible from that of $H = M^4 \times CP_2$, cannot be exotic so that in TGD physics the exotics could not be realized. But can one really exclude the possibility that the induced smooth structure could be exotic as a 4-D smooth structure?

2. In the TGD framework and at the level of $H = M^4 \times cP_2$, one can argue that the holography implied by the general coordinate invariance somehow determines the smooth structure in the interior of space-time surface from the coordinate atlas at the boundary. One would have a holography of smoothness. It is however not obvious why this unique structure should be the standard one.

3. One has also holography in M^8 and this induces holography in H by $M^8 - H$ duality. The 3-surfaces X^3 inducing the holography in M^8 are parts of mass shells, which are hyperbolic spaces $H^3 \subset M^4 \subset M^8$. 3-surfaces X^3 could be even hyperbolic 3-manifolds as unit cells of tessellations of H^3. These hyperbolic manifolds have unique smooth structures as manifolds with dimension $D < 4$.

   The hypothesis is that one can assign to these 3-surfaces a 4-surface by a number theoretic dynamics requiring that the normal space is associative, that is quaternionic [L73, L74]. The additional condition is that the normal space contains commutative subspace makes it possible to parametrize normal spaces by points of CP_2. $M^8 - H$ duality would map a given normal space to a point of CP_2. $M^8 - H$ duality makes sense also for the twistor lift.

4. A more general statement would be as follows. A set of 3-surfaces as sub-manifolds of mass shells H^3_m determined by the roots of polynomial P having interpretation as mass square values defining the 4-surface in M^8 take the role of the boundaries. Mass-shells H^3_m or partonic 2-surfaces associated with them having particle interpretation could correspond to discontinuities of derivatives and even correspond to failure of manifold property analogous to that occurring for Feybman diagrams so that the holography of smoothness would decompose to a piece-wise holography.

   The regions of $X^4 \subset M^8$ connecting two sub-sequent mass shells would have a unique smooth structure induced by the hyperbolic manifolds H^3 at the ends.

It is important to notice that the holography of smoothness does not force the smooth 4-D structure to be the standard one.

3. Could the exotic smooth structures have a physical interpretation in the TGD framework?

In the TGD framework, exotic smooth structures could also have a physical interpretation. As noticed, the failure of the standard smooth structure can be thought to occur at a point set of dimension zero and correspond to a set of point defects in condensed matter physics. This could have a deep physical meaning.

1. The space-time surfaces in $H = M^4 \times CP_2$ are images of 4-D surfaces of M^8 by M^8-H-duality. The proposal is that they reduce to minimal surfaces analogous to soap films spanned by frames. Regions of both Minkowskian and Euclidean signature are predicted and the latter correspond to wormhole contacts represented by CP_2 type extremals. The boundary between the Minkowskian and Euclidean region is a light-like 3-surface representing the orbit of partonic 2-surface identified as wormhole throat carrying fermionic lines as boundaries of string world sheets connecting orbits of partonic 2-surfaces.

2. These fermionic lines are counterparts of the lines of ordinary Feynman graphs, and have ends at the partonic 2-surfaces located at the light-like boundaries of CD and in the interior of the space-time surface. The partonic surfaces, actually a pair of them as opposite throats of wormhole contact, in the interior define topological vertices, at which light-like partonic orbits meet along their ends.

3. These points should be somehow special. Number theoretically they should correspond points with coordinates in an extension of rationals for a polynomial P defining 4-surface in H and space-time surface in H by M^8-H duality. What comes first in mind is that the throats touch each other at these points so that the distance between Minkowskian space-time sheets vanishes. This is analogous to singularities of Fermi surface encountered in topological condensed matter physics: the energy bands touch each other. In TGD, the partonic 2-surfaces at the mass shells of M^4 defined by the roots of P are indeed analogs of Fermi surfaces at the level of $M^4 \subset M^8$, having interpretation as analog of momentum space.

   Could these points correspond to the defects of the standard smooth structure in X^4? Note that the branching at the partonic 2-surface defining a topological vertex implies the local failure of the manifold property. Note that the vertices of an ordinary Feynman diagram imply that it is not a smooth 1-manifold.

4. Could the interpretation be that the 4-manifold obtained by removing the partonic 2-surface has exotic smooth structure with the defect of ordinary smooth structure assignable to the partonic 2-surface at its end. The situation would be rather similar to that for the representation of exotic R^4 as a surface in CP_2 with the sphere at infinity removed [A42].

5. The failure of the cosmic censorship would make possible a pair creation. As explained, the fermionic lines can indeed turn backwards in time by going through the wormhole throat and turn backwards in time. The above picture suggests that this turning occurs only at the singularities at which the partonic throats touch each other. The QFT analog would be as a local vertex for pair creation.

6. If all fermions at a given boundary of CD have the same sign of energy, fermions which have returned back to the boundary of CD, should correspond to antifermions without a change in the sign of energy. This would make pair creation without fermionic 4-vertices possible.

   If only the total energy has a fixed sign at a given boundary of CD, the returned fermion could have a negative energy and correspond to an annihilation operator. This view is nearer to the QFT picture and the idea that physical states are Galois confined states of virtual fundamental fermions with momentum components, which are algebraic integers. One can also ask whether the reversal of the arrow of time for the fermionic lines could give rise to gravitational quantum computation as proposed in [A87].

4. A more detailed model for the exotic smooth structure associated with a topological 3-vertex

One can ask what happens to the 4-surface near the topological 3-particle vertex and what is the geometric interpretation of the point defect. The first is whether the description of the situation is possible both in M^8 and H. Here one must consider momentum conservation.

1. By Uncertainty Principle and momentum conservation at the level of M^8, the incoming real momenta of the particle reaction are integers in the scale defined by CD. In the standard QFT picture, the momenta at the vertex of physical particles are at different mass shells.

In M^8 picture, the mass squared values of virtual fermions are in general algebraic and also complex roots of a polynomial defining the 3-D mass shells H^3_m of $M^4 \subset M^8$, determining 4-surface by associative holography.

In the standard wave mechanical picture assumed also in TGD, a given topological vertex, describable in terms of partonic 2-surfaces, would correspond to a multi-local vertex in M^8 in accordance with the representation of a local n-vertex in M^4 as convolution of n-local vertices in momentum space realizing momentum conservation.

2. $M^8 - H$ duality maps M^4 momenta by inversion to positions in $M^4 \subset H$. This encourages the question whether the topological vertex could be described also in M^8 as a partonic surface at single algebraic mass shell in M^8, mapped by $M^8 - H$ duality to a single $a = constant$ hyperboloid in $M^4 \subset H$.

   The virtual momenta at the level of M^8 are algebraic, in general complex, integers. The algebraic mass squared values at the mass shell of M^8 would be the same for all particles of the vertex. This kind of correspondence does not make sense if $M^8 - H$ duality applies to the full algebraic momenta. The assumption has been that it applies to the rational parts of the momenta.

3. The rational parts of the algebraic integer valued 4-momenta of virtual fermions are in general not at the same mass shell. Could this make possible a description in terms of partonic 2-surfaces at fixed mass *resp.* $a = consant$ shell at the level of M^8 *resp.* H?

   The classical space-time surface in H, partonic 2-surfaces and fermion lines at them are characterized by classical momenta by Noether's theorem. Quantum classical correspondence, realized in ZEO as Bohr orbitology, suggests that the classical 4-momenta assignable to these objects correspond to the rational parts of the momenta at M^8 mass shell. Could the rational projections of M^8 momenta at H^3_n correspond to different mass squared values at given H^3?

4. Note that this additional symmetry for complexified momentum space and position space descriptions would be analogous to the duality of twistor amplitudes position space and the space of area momenta.

How to describe the topological vertex in H? The goal is to understand how exotic smooth structure and its point defects could emerge from this picture. The physical picture applied hitherto is as follows.

1. 3 partonic orbits meet at a vertex described by a partonic 2-surface. Assume that they are located to single $a = constant$ $H^3 \subset M^4 \subset H$.

2. The partonic wormhole throats appear as pairs at the opposite Minkowskian space-time sheets. There are three pairs corresponding to 3 external particle lines and one line which must be a bosonic line describing fermion-antifermion bound state disappears: this corresponds to a boson absorption (or emission).

   The opposite throats carry opposite magnetic monopole charges. The only possibility, not noticed before, is that the opposite wormhole throats for the partoni orbit, which ends at the vertex, must coincide at the vertex. The minimal option is that the exotic smooth structure is associated with this partonic orbit turning back in time. The two partonic orbits, which bind 4-D Euclidean regions as wormhole throats, would fuse to a larger 4-D surface with an exotic smooth structure.

   Fermion-antifermion annihilation occurs at a point at which fermion and antifermion lines meet. The first guess is that this point corresponds to the defect of the smooth structure.

3. There is an analogy with the construction of Etesi [A42]in which a homologically non-trivial ball CP_1 glued to the C^2 at infinity to construct an exotic smooth structure. One dimension disappears for the glued 3-surface at infinity.

   In the partonic vertex, one has actually two homologically non-trivial 2-surfaces with opposite homology charges as boundaries between wormhole contact and Minkowskian regions and they fuse together in the partonic vertex. Also now, one dimension disappears as the partonic 2-surfaces become identical so that 3-D wormhole contact contracts to single 2-D partonic 2-surface.

4. The defect for the smooth structure associated with the fusion of the pair of wormhole orbits should correspond to a point at which fermion and antifermion lines meet.

   This suggests that the throats do not fuse instantaneously but gradually. The fusion would start from a single touching point identifiable asd the fermion-antifermion vertex, serving as a seed of a phase transition, and would proceed to the entire wormhole contact so that it reduces to a partonic 2-surface.

   One can argue that one has a problem if this surface is homologically non-trivial. Could the process make the closed partonic 2-surface homologically trivial. A simplified example is the fusion of two circles with opposite winding numbers ± 1 on a cylinder. The outcome is two homologically non-trivial circles of opposite orientations on top of each other. The phase transition starting from a point would correspond to a touching of the circles.

A couple of further comments are in order.

1. The connection of the pair of wormhole throats to the associative holography is an interesting question. The 4-D tangent planes of $X^4 \subset M^8$ mass shell correspond to points of CP_2. They would be different at the two parallel sheets.

   At the mass shell H^3_m the branches would coincide. The presence of two tangent planes could give rise to two different holographic orbits, which coincide at the initial mass shell and gradually diverge from each other just as in the above model for the fusion of partonic 2-surfaces. The failure of the strict determinism for the associative holography at the partonic 2-surface would make in TGD the analogy of fermion-antifermion annihilation vertex possible.

2. There is also an analogy with the cusp catastrophe in which the projection of the cusp catastrophe as a 2-surface in 3-D space with behavior variable x and two control parameters (a, b) has a boundary at which two real roots of a polynomial of degree 3 coincide. The projection to the (a, b) plane gives a sharp shape, whose boundary is a V-shaped curve in which the sides of V become parallel at the vertex. The vertex corresponds to maximal criticality. The particle vertex would be a critical phenomenon in accordance with the interpretation as a phase transition.

6.7.6 Is a master formula for the scattering amplitudes possible?

Marko Manninen asked whether TGD can in some sense be reduced to a single equation or principle is very interesting. My basic answer is that one could reduce TGD to a handful of basic principles but formula analogous to $F = ma$ is not possible. However, at the level of classical physics, one could perhaps say that general coordinate invariance $\rightarrow$ holography $\leftarrow$ 4-D generalization of holomorphy [**?**]educe the representations of preferred extremals as analogs of Bohr orbits for particles as 3-surfaces to a representation analogous to that of a holomorphic function.

Can one hope something analogous to happen at the level of scattering amplitudes? Is some kind of a master formula possible? I have considered many options, even replacing the S-matrix with the Kähler metric in the fermionic degrees of freedom [L85]. The motivation was that the rows of the matrix defining Kähler metric define unit vectors allowing interpretation in terms of probability conservation. However, it seems that the concept of zero energy state alone makes the definition unambiguous and unitarity is possible without additional assumptions.

1. In standard quantum field theory, correlation functions for quantum fields give rise to scattering amplitudes. In TGD, the fields are replaced by the spinor fields of the "world of classical worlds" (WCW) which can regarded as superpositions of pairs of multi-fermion states restricted at the 3-D surfaces at the ends of the 4-D Bohr orbits at the boundaries of CD.

   These 3-surfaces are extremely strongly but not completely correlated by holography implied by 4-D general coordinate invariance. The modes of WCW spinor fields at the 3-D surfaces correspond to irreducible unitary representations of various symmetries, which include supersymplectic symmetries of WCW and Kac-Moody type symmetries [K35, K98] [L96, L111, L118]. Hence the inner product is unitary.

2. Whatever the detailed form of the 3-D parts of the modes of WCW spinor fields at the boundaries of CD is, they can be constructed from ordinary many fermion states. These many-fermion state correspond in the number theoretic vision of TGD to Galois singlets realizing Galois confinement [L118, L114, L116]. They are states constructed at the level of M^8 from fermion with momenta whose components are possibly complex algebraic integers in the algebraic extension of rationals defining the 4-D region of M^8 mapped to H by M^8-H duality. Complex momentum means that the corresponding state decomposes to plane waves with a continuum of momenta. The presence of Euclidian wormhole contact makes already the classical momenta complex.

   Galois confined states have momenta, whose components are integers in the momentum scale defined by the causal diamond (CD). Galois confinement defines a universal mechanism for the formation of bound states. The induced spinor fields are second quantized free spinor fields in H and their Dirac propagators are therefore fixed. This means an enormou calculational simplification.

3. The inner products of these WCW spinor fields restricted to 3-surfaces determine the scattering amplitudes. They are non-trivial since the modes of WCW spinor fields are located at opposite boundaries of CD. These inner products define the zero energy state identifiable as such as scattering amplitudes. This is the case also in wave mechanics and quantum TGD is indeed wave mechanics for particles identified as 3-surfaces.

4. There is also a functional integral of these amplitudes over the WCW, i.e. over the 4-D Bohr orbits. This defines a unitary inner product. The functional integral replaces the path integral of field theory and is mathematically well-defined since the Kähler function, appearing in the exponent defining vacuum functional, is a non-local function of the 3-surface so that standard local divergences due to the point-like nature of particles disappear. Also the standard problems due to the presence of a Hessian coming from a Gaussian determinant is canceled by the square foot of the determinant of the Kähler metric appearing in the integration measure [K60, K98].

5. The restriction of the second quantized spinor fields to 4-surfaces and zero-energy ontology are absolutely essential. Induction turns free fermion fields into interacting ones. The spinor fields of H are free and define a trivial field theory in H. The restriction to space-time surfaces changes the situation. Non-trivial scattering amplitudes are obtained since the fermionic propagators restricted to the space-time surface are not anymore free propagators in H. Therefore the restriction of WCW spinors to the boundaries of CD makes the fermions interact in exactly the same way as it makes the induced spinor connection and the metric dynamical.

There are a lot of details involved that I don't understand, but it would seem that a simple "master formula" is possible. Nothing essentially new seems to be needed. There is however one more important "but".

Are pair production and boson emission possible?

The question that I have pondered a lot is whether the pair production and emission of bosons are possible in the TGD Universe. In this process the fermion number is conserved, but fermion and antifermion numbers are not conserved separately. In free field theories they are, and in the interacting quantum field theories, the introduction of boson fermion interaction vertices is necessary. This brings infinities into the theory.

1. In TGD, the second quantized fermions in H are free and the boson fields are not included as primary fields but are bound states of fermions and antifermions. Is it possible to produce pairs at all and therefore also bosons? For example, is the emission of a photon from an electron possible? If a photon is a fermion-antifermion pair, then the fermion and antifermion numbers cannot be preserved separately. How to achieve this?

2. If fundamental fermions correspond to light-like curves at light-like orbit of partonic 2-surfaces, pair creation requires that that fermion trajectory turns in time direction. At this point velocity is infinite and this looks like a causal anomaly. There are two options: the fermion changes the sign of its energy or transforms to antiferion with the same sign of energy.

   Different signs of energy is not possible since the annihilation operator creating the fermion with opposite energy would annihilate either the final state or some fermion in the final state so that both fermion and antifermion numbers of the final state would be the same as those of the initial state.

   On the other hand, it can be said that positive energy antifermions propagate backwards in time because in the free fermion field since the terms proportional to fermion creation operators and antifermion annihilation operators appear in the expression of the field as sum of spinor modes.

   Therefore a fermion-antifermion pair with positive energies can be created and corresponds to a pair of creation operators. It could also correspond to a boson emission and to a field theory vertex, in which the fermion, antifermion and boson occur. In TGD, however, the boson fields are not included as primary fields. Is such a "vertex without a vertex" possible at all?

3. Can one find an interpretation for this creation of a pair that is in harmony with the standard view. Space-time surfaces are associated with induced classical gauge potentials. In standard field theory, they couple to fermion-antifermion pairs, and pairs can be created in classical fields. The modified Dirac equation [K128] and the Dirac equation in H also have such a coupling. Now the modified Dirac equation holds true at the fermion lines at the light-like orbits of the partonic 2-surface. Does the creation of pairs happen in this way? It might do so: also in the path integral formalism of field theories, bosons basically correspond to classical fields and the vertex is just this except that in TGD fermions are restricted to 1-D lines.

Fundamental fermion pair creation vertices as local defects of the standard smooth structure of the space-time surface?

Here comes the possible connection with a very general mathematical problem of general relativity that I have already discussed.

1. Causal anomalies as time loops that break causality are more the rule than an exception in general relativity the essence of the causal anomaly is the reversal of the arrow of time. Causal anomalies correspond to exotic diffeo-structures that are possible only in dimension $D = 4$! Their number is infinite.

2. Quite generally, the exotic smooth structures reduce to defects of the usual differentiable structure and have measure zero. Assume that they are point like defects. Exotic differentiable structures are also possible in TGD, and the proposal is that the associated defects correspond to a creation of fermion-fermion pairs for emission of fermion pairs of of gauge bosons and Higgs particle identified in TGD as bound states of fermion-antifermion pairs. This picture generalizes also to the case of gravitons, which would involve a pair of vertices of this kind. The presence of 2 vertices might relate to the weakness of the gravitational interaction.

   The reversal of the fermion line in time direction would correspond to a creation of a fermion-antifermion pair: fermion and antiferion would have the same sign of energy. This would be a causal anomaly in the sense that the time direction of the fermion line is reversed so that it becomes an antifermion.

   I have proposed that this causal anomaly is identifiable as an anomaly of differentiable structure so that emission of bosons and fermion pairs would only be possible in dimension 4: the space-time dimension would be unique!

3. But why would a point-like local defect of the differentiable structure correspond to a fermion pair creation vertex. In TGD, the point-like fermions correspond to 1-D light-like curves at the light-like orbit of the partonic 2-surface.

   In the pair creation vertex in presence of classical induced gauge potentials, one would have a V-shaped world line of fermion turning backwards in time meaning that antifermion is transformed to fermion. The antifermion and fermion numbers are not separately conserved although the total fermion number is. If one assumes that the modified Dirac equation holds true along the entire fermion worldline, there would be no pair creation.

   If it holds true only outside the V-shaped vertex the modified Dirac action for the V-shaped fermion libe can be transformed to a difference of antifermion number equal to the discontinuity of the antifermion part of the fermion current identified as an operator at the vertex. This would give rise to a non-trivial vertex and the modified gamma matrices would code information about classical bosonic action.

4. The 1-D curve formed by fermion and antifermion trajectories with opposite time direction turns backwards in time at the vertex. At the vertex, the curve is not differentiable and this is what the local defect of the standard smooth differentiable structure would mean physically!

Master formula for the scattering amplitudes: finally?

Most pieces that have been identified over the years in order to develop a master formula for the scattering amplitudes are as such more or less correct but always partially misunderstood. Maybe the time is finally ripe for the fusion of these pieces to a single coherent whole. I will try to list the pieces into a story in the following.

1. The vacuum functional, which is the exponential Kähler function defined by the classical bosonic action defining the preferred extremal a an analog of Bohr orbit, is the starting point. Physically, the Kähler function corresponds to the bosonic action (e.g. EYM) in field theories.

   Because holography is almost unique, it replaces the path integral by a sum over 4-D Bohr trajectories as a functional integral over 3-surfaces plus discrete sum.

2. However, the fermionic part of the action is missing. I have proposed a long time ago a super symmetrization of the WCW Kähler function by adding to it what I call modified Dirac action. It relies on modified gamma matrices modified gamma matrices Γ^α, which are contractions $\Gamma_k T^{\alpha k}$ of H gamma matrices Γ_k with the canonical momentum currents $T^{\alpha k} = \partial L/\partial_{\partial_\alpha h^k}$ defined by the Lagrangian L. Modified Dirac action is therefore determined by the bosonic action from the requirement of supersymmetry. This supersymmetry is however quite different from the SUSY associated with the standard model and it assigns to fermonic Noether currents their super counterparts.

   Bosonic field equations for the space-time surface actually follow as hermiticity conditions for the modified Dirac equation. These equations also guarantee the conservation of fermion number(s). The overall super symmetrized action that defines super symmetrized Kähler function in WCW would be unambiguous. One would get exactly the same master formula as in quantum field theories, but without the path integral.

3. The overall super symmetrized action is sum of contributions assignable to the space-time surface itself, its 3-D light-like parton orbits as boundaries between Minkowskian regions and Euclidian wormhole contact, 2-D string world sheets and their 1-D boundaries as orbits of point-like fermions. These 1-D boundaries are the most important and analogous to the lines of ordinary Feynman diagrams. One obtains a dimensional hierarchy.

4. One can assign to these objects of varying dimension actions defined in terms of the induced geometry and spinor structure. The supersymmetric actions for the preferred extremals analogous to Bohr orbit in turn give contributions to the super symmetrized Kähler function as an analogue of the YM action so that, apart from the reduction of path integral to a sum over 4-D Bohr orbits, there is a very close analogy with the standard quantum field theory.

However, some problems are encountered.

1. It seems natural to assume that a modified Dirac equation holds true. I have presented an argument for how it indeed emerges from the induction for the second quantized spinor field in H restricted to the space-time surface assuming modified Dirac action.

   The problem is, however, that the fermionic action, which should define vertex for fermion pair creation, disappears completely if Dirac's equation holds everywhere! One would not obtain interaction vertices in which pairs of fermions arise from classical induced fields. Something goes wrong. In this vertex total fermion number is conserved but fermion and antifermion numbers are changed since antifermion transforms to fermion at the V-shaped vertex: this condition should be essential.

2. If one gives up the modified Dirac equation, the fermionic action does not disappear. In this case, one should construct a Dirac propagator for the modified Dirac operator. This is an impossible task in practice.

   Moreover, the construction of the propagator is not even necessary and in conflict with the fact that the induced spinor fields are second quantized spinors of H restricted to the space-time surface and the propagators are therefore well-defined and calculable and define the propagation at the space-time surface.

3. Should we conclude that the modified Dirac equation cannot hold everywhere? What these, presumably lower-dimensional regions of space-time surface, are and could they give the interaction vertices as topological vertices?

The key question is how to understand geometrically the emission of fermion pairs and bosons as their bound states?

1. I have previously derived a topological description for reaction vertices. The fundamental 1 $\rightarrow$ 2 vertex (for example e$\rightarrow$e+ gamma) generalizes the basic vertex of Feynman diagrams, where a fermion emits a boson or a boson decays into a pair of fermions. Three lines meet at the ends.

   In TGD, this vertex can topologically correspond to the decomposition of a 3-surface into two 3-surfaces, to the decomposition of a partonic 2-surface into two, to the decomposition of a string into two, and finally, to the turning of the fermion line backwards from time. One can say that the n-surfaces are glued together along their $n-1$-dimensional ends, just like the 1-surfaces are glued at the vertex in the Feynman diagram.

2. In the previous section, I already discussed how to identify vertex for fermion-antifermion pair creation as a V-shaped turning point of a 1-D fermion line. The fermion line turns back in time and fermion becomes an antifermion. In TGD, the quantized boson field at the vertex is replaced by a classical boson field. This description is basically the same as in the ordinary path integral where the gauge potentials are classical.

   The problem was that if the modified Dirac equation holds everywhere, there are no pair creation vertices. The solution of the problem is that the modified Dirac equation at the V-shaped vertex cannot hold true.

   What this means physically is that fermion and antifermion numbers are not separately conserved in the vertex. The modified Dirac action for the fermion line can be transformed to the change of antifermion number as operator (or fermion number at the vertex) expressible as the change of the antifermion part of the fermion number. This is expressible as the discontinuity of a corresponding part of the conserved current at the vertex. This picture conforms with the appearance of gauge currents in gauge theory vertices. Notice that modified gamma matrices determined by the bosonic action appear in the current.

3. This argument was limited to 1-D objects but can be generalized to higher-dimensional defects by assuming that the modified Dirac equation holds true everywhere except at defects represented as vertices, which become surfaces. The modified Dirac action reduces to an integral of the discontinuity of say antifermion current at the vertex, i.e. the change of the antifermion charge as an operator.

What remains more precisely understood and generalized, is the connection with the irreducible exotic smooth structures possible only in 4-D space-time.

1. TGD strongly suggests that 0-dimensional vertices generalize to topological vertices representable as surfaces of dimension $n = 0, 1, 2, 3$ assignable to objects carrying induced spinor field. In the $1 \rightarrow 2$ vertex, the orbit of an $n < 4$- dimensional surface would turn back in the direction of time and would define a V-shaped structure in time direction. These would be the various topological vertices that I have previously arrived at, but guided by a physical intuition. Also now the vertex would boild down to the discontinuity of say antifermion current instead of the current itself at the vertex.

2. It is known that exotic smooth structures reduce to standard ones except in a set of defects having measure zero. Also non-point-like defects might be possible in contrast to what I assumed at first. If the defects are surfaces, their dimension is less than 4. If not, then only the direction of fermion lines could change.

   If the generalization is possible, also 1-D, 2-D, and 3-D defects, defining an entire hierarchy of particles of different dimensions, is possible. As a matter of fact, a longstanding issue has been whether this prediction should be taken seriously. Note that in topological condensed matter physics, defects with various dimensions are commonplace. One talks about bulk states, boundary states, edge states and point-like singularities. In this would predict hierarchy of fermionic object of various dimensions.

To summarize, exotic smooth structures would give vertices without vertices assuming only free fermions fields and no primary boson fields! And this is possible only in space-time dimension 4!

6.8 A possible connection with family replication phenomenon?

In TGD framework the genus g of the partonic 2-surfaces is proposed to label fermion families [K32, K68, K73]. One can characterize by genus g the topology of light-like partonic orbits and identify the three fermion generators as 2-surfaces with genus $g = 0, 1, 2$ with the special property that they are always hyper-elliptic. Quantum mechanically also topological mixing giving rise to CKM mixing is possible. The view is that given connected 3-surface can contain several light-like 3-surface with different genera. For instance, hadrons would be such surfaces.

There are however questions to be answered.

1. The genera $g = 0, 1, 2$ assigned with the free fermion families correspond to Riemann surfaces, which are always hyper-elliptic allowing therefore Z_2 as a global conformal symmetry. These complex curves correspond to degrees $n = 2, 3, 4$ for the corresponding polynomials. For $n \leq 4$ can write explicit solutions for the roots of the polynomials. Could there be a deep connection between particle physics and mathematical cognition?

2. The homology and genus for 2-surfaces of CP_2 correlate with each other [A76]: is this consistent with the proposed topologicization of color hypercharge implying color confinement?

3. $h_{eff}/h = n$ hypothesis means that dark variant of particle particle characterized by genus g is n-fold covering of this surface. In the general case the genus of covering is different. Is this consistent with the genus-generation correspondence?

4. The degree of complex curve correlates with the genus of the curve. Is generation-genus correspondence consistent with the assumption that partonic 2-surfaces have algebraic curve as CP_2 projection (this need not be the case)?

6.8.1 How the homology charge and genus correlate?

Complex surfaces in CP_2 are highly interesting from TGD point of view.

1. The model for elementary particles assumes that the partonic 2-surfaces carrying fermion number are homologically non-trivial, in other words they carry Kähler magnetic monopole flux having values $q = \pm 1$ and $q = \pm 2$. The idea is that color hyper charge $Y = \{\pm 2/3, \pm 1/3\}$ is proportional to n for quarks and color confinement topologizes to the vanishing of total homology charge [K73].

2. The explanation of the family replication phenomenon [K32] in terms of genus-generation correspondence states that the three quarks and lepton generations correspond to the three lowest genera $g = 0, 1, 2$ for partonic 2-surfaces. Only these genera are always hyper-elliptic allowing thus a global Z_2 conformal symmetry. The physical vision is that for higher genera the handles behave like free particles. Is this proposal consistent with the proposal for the topologization of color confinement?

There is a result [A76] (page 124) stating that if the homology charge q is divisible by 2 then one must have $g \geq q^2/4 - 1$. If q is divisible by h, which is odd power of prime, one has $g \geq (q^2/4 - 1) - (q^2/4h^2)$. For $q = 2$ the theorem allows $g \geq 0$ so that all genera with color hyper charge $Y = \pm 2/3$ are realized.

The theorem says however nothing about $q = 0, 1$. These charges can be assigned to the two different geodesic spheres of CP_2 with $g = 0$ remaining invariant under SO(3) and U(2) subgroups of SU(3) respectively. Is $g > 0$ possible for $q = 1$ as the universality of topological color confinement would require? For $q = 3$ one would have $g \geq 1$. For $q = 4$ $h = 2$ divides q and one has $g \geq 2$. It would seem $g \geq 5$. The conditions become more restrictive for higher q, which suggests that for $q = 0, 1$ one has $g \geq 0$ so that the topologization of color hypercharge would make sense.

6.8.2 Euler characteristic and genus for the covering of partonic 2-surface

Hierarchy of Planck constants $h_{eff}/h = n$ means a hierarchy of space-time surfaces identifiable as n-fold coverings. The proposal is that the number of sheets in absence of singularities is maximal possible and equals to the dimension of the extension dividing the order of its Galois group.

The Euler characteristic of n-fold covering in absence of singular points is $\chi_n = n\chi$. If there are singular (ramified) points these give a correction term given by Riemann-Hurwitz formula (see `http://tinyurl.com/y7n2acub`.)

In absence of singularities one has from $\chi = -2(g - 1)$ and $\chi_n = n\chi$

$$g_n = n(g - 1) + 1 \ . \tag{6.8.1}$$

For $n = 1$ this indeed gives $g_1 = g$ independent of g. One can also combine this with the formula $g = (d - 1)(d - 2)/2$ holding for non-singular algebraic curves of degree d.

Singularities are unavoidable at algebraic points of cognitive representations at which some subgroup of Galois group leaves the point invariant (say rational point in ordinary sense). One can consider the possibility that fermions are located at the singular points at which several sheets of covering touch each other. This would give a correction factor to the formula. If the projection map from the covering to based is of form $\Pi(z) = z^n$ at the singular point P, one says that singularity has ramimifaction index $e_P = n$ and the algebraic genus would increase to

$$g_n = n(g - 1) + 1 + \frac{1}{2}\sum_P (e_P - 1) \ . \tag{6.8.2}$$

Indeed, singularities mean that sheets touch each other at singular points and this increases connectivity.

Under what conditions the genus of dark partonic surface with $n > 1$ can be same as that of the ordinary partonic surface representing visible matter? For the genera $g = 0$ and $g = 1$ this is possible so that these genera would be in an exceptional role also from the point of view of dark matter.

1. For $g = 1$ one has $g_n = g = 1$ independent of n in absence of singular point. Torus topology (assignable to muon and (c,s) quarks) is exceptional. In presence of singularities the genus would increase by the $\sum_P (e_P - 1)/2$ independent of the value of n. The lattice of points for elliptic surfaces would suggest existence of infinite number of singular points if the abelian group operations preserve the singular character of the points so that the genus would become infinite.

2. For $g = 0$ one would have $g_n = -n + 1$ in absence of singularities. Only $n = 1$ - ordinary matter - is possible without singularities. Dark matter is however possible if singularities are allowed. For sphere one would obtain $g_n = -n + 1 + \sum_P (e_P - 1)/2 \geq 0$. The condition $n \leq \sum_P (e_P - 1)/2 + 1$ must therefore hold true for $g \geq 0$.

   The condition $g_n = -n + 1 + \sum_P (e_P - 1)/2 = g = 0$ gives $\sum_P (e_P - 1) = 2(n - 1)$. For spherical topology it is possible to have dense set of rational points so that it is possible create cognitive representations with arbitrary number of points which can be also singular. One might argue that this kind of situation corresponds to a non-perturbative phase.

3. For $g = 2$ one would have $g_n = n + 1 + \sum_P (e_P - 1)/2$ and genus would grow with n even in absence of singularities and would be very large for large values of h_{eff}. $g_n = 2$ is obtained with $n = 1$ (ordinary matter) and no singular points not even allowed for $n = 1$. $g_n = g = 2$ is not possible for $n > 1$.

   Note that dark $g \geq 2$ fermions cannot correspond to lower generation fermions with singular points of covering. More generally, one could say that $g \geq 2$ fermions can exists only with standard value of Planck constant unless they are singular coverings of $g < 2$ fermions.

What is clear that the model of dark matter predicts breaking of universality. This breaking is not seen in the standard model couplings but makes it visible in amore delicate manner and might allow to understand why the masses of fermions increase with generation index.

6.8.3 All genera are not representable as non-singular algebraic curves

Suppose for a moment that partonic 2-surfaces correspond to rational maps of algebraic curves in CP_2 to M^4 that is deformations of these curves in M^4 direction. This assumption is of course questionable but deserves to be sttudied.

The formula (for algebraic curve see http://tinyurl.com/nt6tkey)

$$g = \frac{(d-1)(d-2)}{2} + \frac{\sum \delta_s}{2} \quad ,$$

where $\delta_s > 0$ characterizes the singularity, does not allow all genera for algebraic curves for $\sum \delta_s = 0$: one has $g = 0, 0, 1, 3, 6, 10, ..$ for $d = 1, 2, ...$

For instance, $g = 2$, which would correspond in TGD to third quark or lepton generation is not possible without singularities for $d = 3$ curve having $g = 1$ without singularities!

This raises questions. Could the third fermion generation actually correspond to $g = 3$? Or does it correspond to $g = 2$ 2-surface of CP_2, which is more general surface than algebraic curve meaning that it is not representable as complex surface? Or could third generation fermions correspond to $g = 0$ or $g = 1$ curves with singular point of covering by Galois group so that several sheets touch each other?

To sum up, if the results for algebraic varieties generalize to TGD framework, they suggest notable differences between different fermion families. Universality of standard model interactions says that the only differences between fermion families are due to the differ masses. It is not clear whether the different masses could be due to the differences at number theoretical level and dark matter sectors.

1. All genera can appear as as ordinary matter ($d = 1$). Dark variants of $g = 1$ states have $g_d = 1$ automatically in absence of singular points. Dark variants of $g = 0$ states must have singular point in order to give $g_n = 0$. Dark variants of $g = 2$ states with $g_d = 2$ are obtained from $g = 1$ states with singularities. The special role of the two lowest is analogous to their special role for algebraic curves.

2. If one assumes that partonic 2-surfacs correspond to algebraic curves, one obtains again that $g = 2$ surfaces must correspond to singular $g = 0$ and $g = 1$ which could be dark in TGD sense.

6.9 Summary and future prospects

In the following I give a brief summary about what has been done. I concentrate on $M^8 - H$ duality since the most significant results are achieved here.

It is fair to say that the new view answers the following a long list of open questions.

1. When $M^8 - H$ correspondence is true (to be honest, this question emerged during this work!)? What are the explicit formulas expressing associativity of the tangent space or normal space of the 4-surface?

   The key element is the formulation in terms of complexified M^8 - M^8_c - identified in terms of octonions and restriction $M^8_c \to M^8$. One loses the number field property but for polynomials ring property is enough. The level surfaces for real and imaginary parts of octonionic polynomials with real coefficients define 4-D surfaces in the generic case.

   Associativity condition is an additional condition reducing the dimension of the space-time surface unless some components of $RE(P)$ or $IM(P)$ are critical meaning that also their gradients vanish. This conforms with the quantum criticality of TGD and provides a concrete first principle realization for it.

   An important property of $IM(P_1P_2)$ is its linearity with respect to $IM(P_i)$ implying that this condition gives the surfaces $IM(P_i) = 0$ as solutions. This generalizes by induction to $IM(P_1P_2...P_n)$. For $RE(P_1P_2) = 0$ linearity does not hold true and there is a genuine interaction. A physically attractive idea idea is that $RE(P_1P_2) = 0$ holds true inside CDs and for wormhole contacts between space-time sheets with Minkoskian signature. One can generalizes this also to $IM(P_1/P_2)$ and $RE(P_1/P_2)$ if rational functions are allowed. Note however that the origins of octonionic coordinates in P_i must be on the octonionic real line.

2. How this picture corresponds to twistor lift? The twistor lift of Kähler action (dimensionally reduced Kähler action in twistor space of space-time surface) one obtains two kinds of space-time regions. The regions, which are minimal surfaces and obey dynamics having no dependence on coupling constants, correspond naturally to the critical regions in M^8 and H.

   There are also regions in which one does not have extremal property for both Kähler action and volume term and the dynamics depends on coupling constant at the level of H. These regions are associative only at their 3-D ends at boundaries of CD and at partonic orbits, and the associativity conditions at these 3-surfaces force the initial values to satisfy the conditions guaranteeing preferred extremal property. The non-associative space-time regions are assigned with the interiors of CDs. . The particle orbit like space-time surfaces entering to CD are critical and correspond to external particles.

   It has later turned out [L51] that it might be possible to take the associativity conditions to extreme in the sense that they would hold everywhere apart from a set of discrete points and space-time surface would be minimal surfaces at all points except this finite set of points. There would be transfer of conserved quantities assignable to the volume term and the 4-D Kähler action (coming as dimensionally reduced 6-D Kähler action for the twistor lift of TGD) only at these points and elementary fermions would be naturally assignable to these points.

3. The surprise was that $M^4 \subset M^8$ is naturally co-associative. If associativity holds true also at the level of H, $M^4 \subset H$ must be associative. This is possible if $M^8 - H$ duality maps tangent space in M^8 to normal space in H and vice versa.

4. The connection to the realization of the preferred extremal property in terms of gauge conditions of subalgebra of SSA is highly suggestive. Octonionic polynomials critical at the boundaries of space-time surfaces would determine by $M^8 - H$ correspondence the solution to the gauge conditions and thus initial values and by holography the space-time surfaces in H.

5. A beautiful connection between algebraic geometry and particle physics emerges. Free many-particle states as disjoint critical 4-surfaces can be described by products of corresponding polynomials satisfying criticality conditions. These particles enter into CD , and the non-associative and non-critical portions of the space-time surface inside CD describe the interactions. One can define the notion of interaction polynomial as a term added to the product of polynomials. It can vanish at the boundary of CD and forces the 4-surface to be connected inside CD. It also spoils associativity: interactions are switched on. For bound states the coefficients of interaction polynomial are such that one obtains a bound state as associative space-time surface.

6. This picture generalizes to the level of quaternions. One can speak about 2-surfaces of space-time surface with commutative or co-commutative tangent space. Also these 2-surfaces would be critical. In the generic case commutativity/co-commutativity allows only 1-D curves.

   At partonic orbits defining boundaries between Minkowskian and Euclidian space-time regions inside CD the string world sheets degenerate to the 1-D orbits of point like particles at their boundaries. This conforms with the twistorial description of scattering amplitudes in terms of point like fermions.

   For critical space-time surfaces representing incoming states string world sheets are possible as commutative/co-commutative surfaces (as also partonic 2-surfaces) and serve as correlates for (long range) entaglement) assignable also to macroscopically quantum coherent system ($h_{eff}/h = n$ hierarchy implied by adelic physics).

7. The octonionic polynomials with real coefficients form a commutative and associative algebra allowing besides algebraic operations function composition. Space-time surfaces therefore form an algebra and WCW has algebra structure. This could be true for the entire hierarchy of Cayley-Dickson algebras, and one would have a highly non-trivial generalization of the conformal invariance and Cauchy-Riemann conditions to their n-linear counterparts at the n:th level of hierarchy with $n = 1, 2, 3, ..$ for complex numbers, quaternions, octonions,... One can even wonder whether TGD generalizes to this entire hierarchy!

8. In the original version of this article I did not realize that there are two options for realizing the idea that the M_c^4 projection of space-time surface in M_c^8 must belong to M^4.

   (a) I proposed that the *projection* from M_c^8 to real M^4 (for which M^1 coordinate is real and E^3 coordinates are imaginary with respect to i!) defines the real space-time surface mappable by $M^8 - H$ duality to CP_2 [L32].

   (b) An alternative option, which I have not considered in the original versions of [L32, L34] is that only the roots of the 4 vanishing polynomials as coordinates of M_c^4 belong to M^4 so that m^0 would be real root and m^k, $k = 1, ..., 3$ imaginary with respect to $i \to -i$. M_c^8 coordinates would be invariant ("real") under combined conjugation $i \to -i, I_k \to -I_k$. In the following I will speak about this property as *Minkowskian reality.* This could make sense. Outside CD these conditions would not hold true. This option looks more attractive than the first one. Why these condition can be true just inside CD, should be understood.

9. The use of polynomials or rational functions could be also an approximation. Analytic functions of real variable extended to octonionic functions would define the most general space-time surfaces but the limitations of cognition would force to use polynomial approximation. The degree n of the polynomial determining also $h_{eff} = nh_0$ would determine the quality of the approximation and at the same time the "IQ" of the system.

All big pieces of quantum TGD are now tightly interlinked.

1. The notion of causal diamond (CD) and therefore also ZEO can be now regarded as a consequence of the number theoretic vision and $M^8 - H$ correspondence, which is also understood physically.

2. The hierarchy of algebraic extensions of rationals defining evolutionary hierarchy corresponds to the hierarchy of octonionic polynomials.

3. Associative varieties for which the dynamics is critical are mapped to minimal surfaces with universal dynamics without any dependence on coupling constants as predicted by twistor lift of TGD. The 3-D associative boundaries of non-associative 4-varieties are mapped to initial values of space-time surfaces inside CDs for which there is coupling between Kähler action and volume term.

4. Free many particle states as algebraic 4-varieties correspond to product polynomials in the complement of CD and are associative. Inside CD the addition of interaction terms vanishing at its boundaries spoils associativity and makes these varieties connected.

5. The super variant of the octonionic algebraic geometry makes sense, and one obtains a beautiful correlation between the fermion content of the state and corresponding space-time variety. This suggests that twistorial construction indeed generalizes. Criticality for the external particles giving rise to additional constraints on the coefficients of polynomials could make possible to have well-define summation over corresponding varieties.

What mathematical challenges one must meet?

1. One should prove more rigorously that criticality is possible without the reduction of dimension of the space-time surface.

2. One must demonstrate that SSA conditions can be true for the images of the associative regions (with 3-D or 4-D). This would obviously pose strong conditions on the values of coupling constants at the level of H.

Concerning the description of interactions there are several challenges.

1. Do associative space-time regions have minimal surface extremals as images in H and indeed obeying universal critical dynamics? As found, the study of the known extremals supports this view.

2. Could one construct the scattering amplitudes at the level of M^8? Here the possible problems are caused by the exponents of action (Kähler action and volume term) at H side. Twistorial construction [L41] however leads to a proposal that the exponents actually cancel. This happens if the scattering amplitude can be thought as an analog of Gaussian path integral around single extremum of action and conforms with the integrability of the theory. In fact, nothing prevents from defining zero energy states in this manner! If this holds true then it might be possible to construct scattering amplitudes at the level of M^8.

3. What about coupling constants? Coupling constants make themselves visible at H side both via the vanishing conditions for Noether charges in sub-algebra of SSA and via the values of the non-vanishing Noether charges. $M^8 - H$ correspondence determining the 3-D boundaries of interaction regions within CDs suggests that these couplings must emerge from the level M^8 via the criticality conditions posing conditions on the coefficients of the octonionic polynomials coding for interactions.

   Could all coupling constant emerge from the criticality conditions at the level of M^8? The ratio of R^2/l_P^2 of CP_2 scale and Planck length appears at H level. Also this parameter should emerge from $M^8 - H$ correspondence and thus from criticality at M^8 level. Physics would reduce to a generalization of the catastrophe theory of Rene Thom!

4. The description of interactions at the space-time surface associated with single CD should be M^8 counterpart of the H picture in which 3 light-like partonic orbits meet at common end topological vertex - defined by a partonic 2-surface and fermions scatter without touching. Now one has octonionic sparticle lines and interaction vertex becomes possible. This conforms with the idea that interactions take place at discrete points belonging to the extension of rationals. The partonic 2-surfaces defining topological vertices would naturally correspond to the intersections $X^2 = X^4 \cap S^6(t_n)$. If sparticle lines are allowed to move along this space-like 2-surface (the line becomes space-like) they can intersect and give rise to a fusion vertex producing the third fermionic line.

The partonic 2-surfaces defining topological vertices would naturally correspond to the intersections $X^2 = X^4 \cap S^6(t_n)$, which satisfy $RE(P) = IM(P) = 0$ and are singular and doubly critical. If sparticle lines are allowed to move along this space-like 2-surface (the line becomes space-like) they can intersect and give rise to a fusion vertex producing the third fermionic line.

5. Real analyticity requires that the octonionic polynomials have real coefficients. This forces the origin of octonionic coordinates to be at real line (time axis) in the octonionic sense, and guarantees the associativity and commutativity of the polynomials. Arbitrary CDs cannot be located along this line. Can one assume that all CDs involved with *observable* processes satisfy this condition?

   If not, how do the 4-varieties associated with octonionic polynomials with different origins interact? How could one avoid losing the extremely beautiful associative and commutative algebra? It seems that one cannot form their products and sums and must form the Cartesian product of M^8:s with different tips for CDS and formulate the interaction in this framework. In the case of space-time surfaces associated with different CDs the discrete intersections of space-time surfaces would define the interaction vertices.

6. Super-octonionic geometry suggests that the twistorial construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY generalizes to TGD in rather straightforward manner to a purely geometric construction. Functional integral over WCW would reduce to summations over polynomials with coefficients in an appropriate extension of rationals and criticality conditions on the coefficients could make the summation well-defined by bringing in finite measurement resolution.

   If scattering diagrams are associated with discrete cognitive representations, one obtains a generalization of super-twistor formalism involving polygons. Super-octonions as counterparts of super gauge potentials are well-defined if octonionic 8-momenta are quaternionic: indeed, Grassmannians have quaternionic counterparts but not octonionic ones. There are good hopes that the twistor Grassmann approach to $\mathcal{N} = 4$ SUSY generalizes. The core part in the calculation of the scattering diagram would reduce to the construction of octonionic 4-varieties and identifying the points belonging to the extension of rationals considered. The rest would be dictated by symmetries and integrations over various moduli spaces, which should be number theoretically universal so that residue calculus strongly suggests itself.

7. What is the connection with super conformal variant of Yangian symmetry, whose generalization in TGD framework is highly suggestive? Twistorial construction of scattering amplitudes at the level of M^8 looks highly promising idea and could also realize Yangian supersymmetry. The conjecture is that the twistorial amplitudes decompose to M^4 and CP_2 parts with similar structure with E^4 spin (electroweak isospin) replacing ordinary spin and that the integrands in Grassmannians emerging from the conservation of M^4 and E^4 4-momenta are identical in the two cases and thus guarantee Yangian supersymmetry in both sectors. The only difference would be due to the product of delta functions associated with the "negative helicities" (weak isospins with negative sign) expressible as a delta function in the complement of $SU(3)$ Cartan algebra $U(1) \times U(1)$ by using exponential map.

It is appropriate to close with a question about fundamentals.

1. The basic structure at M^8 side consists of complexified octonions. The metric tensor for the complexified inner product for complexified octonions (no complex conjugation with respect to i for the vectors in the inner product) can be taken to have any signature $(\epsilon_1, ..., \epsilon_8)$, $\epsilon_i = \pm 1$. By allowing some coordinates to be real and some coordinates imaginary one obtains effectively any signature from say purely Euclidian signature. What matters is that the restriction of complexified metric to the allowed sub-space is real. These sub-spaces are linear Lagrangian manifolds for Kähler form representing the commuting imaginary unit i. There is analogy with wave mechanics. Why M^8 -actually M^4 - should be so special real section? Why not some other signature?

2. The first observation is that the CP_2 point labelling tangent space is independent of the signature so that the problem reduces to the question why M^4 rather than some other signature $(\epsilon_1, .., \epsilon_4)$. The intersection of real subspaces with different signatures and same origin $(t,r) = 0$ is the common sub-space with the same signature. For instance, for $(1, -1, -1, -1)$ and $(-1, -1, -1, -1)$ this subspace is 3-D $t = 0$ plane sharing with CD the lower tips of CD. For $(-1, 1, 1, 1)$ and $(1, 1, 1, 1)$ the situation is same. For $(1, -1, -1, -1)$ and $(1, 1, -1, -1)$ $z = 0$ holds in the intersection having as common with the lower boundary of CD the boundary of 3-D light-cone. One obtains in a similar manner boundaries of 2-D and 1-D light-cones as intersections.

3. What about CDs in various signatures? For a fully Euclidian signature the counterparts for the interiors of CDs reduce to 4-D intervals $t \in [0, T]$ and their exteriors and thus the space-time varieties representing incoming particles reduce to pairs of points $(t, r) = (0, 0)$ and $(t, r) = (T, 0)$: it does not make sense to speak about external particles. For other signatures the external particles correspond to 4-D surfaces and dynamics makes sense. The CDs associated with the real sectors intersect at boundaries of lower dimensional CDs: these lower-dimensional boundaries are analogous to subspaces of Big Bang (BB) and Big Crunch (BC).

4. I have not found any good argument for selecting $M^4 = M^{1,3}$ as a unique signature. Should one allow also other real sections? Could the quantum numbers be transferred between sectors of different signature at BB and BC? The counterpart of Lorentz group acting as a symmetry group depends on signature and would change in the transfer. Conservation laws should be satisfied in this kind of process if it is possible. For instance, in the leakage from $M^4 = M^{1,3}$ to Mi, j, say $M^{2,2}$, the intersection would be $M^{1,2}$. Momentum components for which signature changes, should vanish if this is true. Angular momentum quantization axis normal to the plane is defined by two axis with the same signature. If the signatures of these axes are preserved, angular momentum projection in this direction should be conserved. The amplitude for the transfer would involve integral over either boundary component of the lower-dimensional CD.

   Could the leakage between signatures be detected as disappearance of matter for CDs in elementary particle scales or lab scales?

5. One can also raise a question about the role of WCW geometry as a continuous infinite-D geometry: could the discretization by cognitive representations making WCW effectively discrete mean its loss? It seems that this cannot be the case. At least in the real sector continuum must be present and the discretization reflects only the discreteness of cognitive representations. In principle continuous WCW could make sense also in p-adic sectors of the adele.

   The identification of space-time surfaces as zero loci of polynomials generalizes to rational functions and even transcendental functions although the existence of the p-adic counterparts of these functions requires additional conditions. Could one interpret the representation in terms of polynomials and possibly rational functions as an approximation? Could the hierarchy of approximations obtained in this manner give rise to a hierarchy of hyper-finite factors of type II_1 defining a hierarchy of measurement resolutions [K127]?

Chapter 7

Does $M^8 - H$ duality reduce classical TGD to octonionic algebraic geometry?: Part III

7.1 Introduction

In the third chapter about $M^8 - H$ duality the question whether the space-time surfaces in M^8 allow a global slicing by string world sheets X^2 defined by an integrable distribution of local tangent spaces $M^2(x) \subset M^4$ and their orthogonal duals or whether there is only a discrete set of surfaces X^2 is discussed. Discrete set is obtained by requiring that space-time surface or its normal space contains string world sheet as a complex (commutative) sub-manifold. By the strong form of holography (SH) this is enough to deduce the image of $X^4 \subset M^8$ in H from the boundary data consisting of the H-images of X^2 and metrically 2-D light-like partonic orbits X^3_L of topological dimension $D = 3$.

Also the relation of $M^8 - H$ duality to p-adic length scale hypothesis and dark matter hierarchy are discussed and it is shown that the notion of p-adic length scale emerging from p-adic mass calculations emerges also geometrically.

The fermionic aspects of $M^8 - H$ duality are discussed: the basic purely number theoretic elements are the octonionic realization of M^8 spinors and the replacement of Dirac equation as a partial differential equation with an algebraic equation for octonionic spinors. Dirac equation for octonionic spinors is analogous to the algebraic momentum space variant of the ordinary Dirac equation. This provides also considerable understanding about the bosonic aspects of $M^8 - H$ duality. In particular, the pre-images of $X^3_L \subset X^4 \subset H$ in M^8 correspond to mass shells for massless octonionic spinor modes realized as light-like 3-surfaces in M^8. One can say that M^8 picture realizes the momentum space dual of the modified Dirac equation in $X^4 \subset H$. Twistor Grassmannian picture supports the view that spinor modes also in H are localized to $X^3_L \subset X^4$, and obey the modified Dirac equation associated with Chern-Simons term.

Cognitive representations is the third basic topic of the chapter. Cognitive representations are identified as sets of points in an extension of rationals for algebraic varieties with "active" points containing fermion. The representations are discussed at both M^8- and H level. General conjectures from algebraic geometry support the vision that these sets are concentrated at lower-dimensional algebraic varieties such as string world sheets and partonic 2-surfaces and their 3-D orbits identifiable also as singularities of these surfaces. For the earlier work related to adelic TGD and cognitive representations see [L39, L24, L29].

The notion is applied in various cases and the connection with $M^8 - H$ duality is rather loose.

1. Extensions of rationals are essentially coders of information. There the possible analogy of extensions of rationals with genes deserves discussion. Extensions, which are not extensions of extensions would be analogous to genes. The notion of conserved gene as number theoretical

analogy for Galois extensions as the Galois group of extension which is normal subgroup of Galois extension.

2. The work of Peter Scholze [A71] based on the notion of perfectoid has raised a lot of interest in the community of algebraic geometers. One application of the notion relates to the attempt to generalize algebraic geometry by replacing polynomials with analytic functions satisfying suitable restrictions. Also in TGD this kind of generalization might be needed at the level of $M^4 \times CP_2$ whereas at the level of M^8 algebraic geometry might be enough. The notion of perfectoid as an extension of p-adic numbers Q_p allowing all p:th roots of p-adic prime p is central and provides a powerful technical tool when combined with its dual, which is function field with characteristic p.

   Could perfectoids have a role in TGD? The infinite-dimensionality of perfectoid is in conflict with the vision about finiteness of cognition. For other p-adic number fields Q_q, $q \neq p$ the extension containing p:th roots of p would be however finite-dimensional even in the case of perfectoid. Furthermore, one has an entire hierarchy of almost-perfectoids allowing powers of p^m:th roots of p-adic numbers. The larger the value of m, the larger the number of points in the extension of rationals used, and the larger the number of points in cognitive representations consisting of points with coordinates in the extension of rationals. The emergence of almost-perfectoids could be seen in the adelic physics framework as an outcome of evolution forcing the emergence of increasingly complex extensions of rationals [L30].

3. The construction of cognitive representation represents a well-known mathematical problem of finding the points of space-time surface with embedding space coordinates in given extension of rationals. Number theorist Minhyong Kim [A56, A67] has speculated about very interesting general connection between number theory and physics. The reading of a popular article about Kim's work revealed that number theoretic vision about physics provided by TGD has led to a very similar ideas and suggests a concrete realization of Kim's ideas [L63]. In the following I briefly summarize what I call identification problem. The identification of points of algebraic surface with coordinates, which are rational or in extension of rationals, is in question. In TGD framework the embedding space coordinates for points of space-time surface belonging to the extension of rationals defining the adelic physics in question are common to reals and all extensions of p-adics induced by the extension. These points define what I call cognitive representation, whose construction means solving of the identification problem.

   Cognitive representation defines discretized coordinates for a point of "world of classical worlds" (WCW) taking the role of the space of spaces in Kim's approach. The symmetries of this space are proposed by Kim to help to solve the identification problem. The maximal isometries of WCW necessary for the existence of its Kähler geometry provide symmetries identifiable as symplectic symmetries. The discrete subgroup respecting extension of rationals acts as symmetries of cognitive representations of space-time surfaces in WCW, and one can identify symplectic invariants characterizing the space-time surfaces at the orbits of the symplectic group.

4. One expects that large cognitive representations are winners in the number theoretical fight for survival. Strong form of holography suggests that it is enough to consider cognitive representations restricted to string world sheets and partonic 2-surfaces. If the 2-surface possesses large group of symmetries acting in extension of rationals, one can have large cognitive representations as orbit of point in extension. Examples of highly symmetric 2-D surfaces are geodesic spheres assignable to partonic 2-surfaces and cosmic strings and elliptic curves assignable with string world sheets and cosmic strings [L72].

5. Rationals and their extensions give rise to a unique discretizations of space-time surface (for instance) - cognitive representation - having interpretation in terms of finite measurement resolution. There are howevever many open questions. Should one allow only octonionic polynomials defined as algebraic continuations of real polynomials or should one allow also analytic functions and regard polynomials as approximations. Zeta functions are especially interesting analytic functions and Dekekind zetas characterize extensions of rationals and one can pose physically motivated questions about them [L53].

7.2 About $M^8 - H$-duality, p-adic length scale hypothesis and dark matter hierarchy

$M^8 - H$ duality, p-adic length scale hypothesis and dark matter hierarchy as phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ are basic assumptions of TGD, which all reduce to number theoretic vision. In the sequel $M^8 - H$ duality, p-adic length scale hypothesis and dark matter hierarchy are discussed from number theoretic perspective.

Several new results emerge. Strong form of holography (SH) allows to weaken strong form of $M^8 - H$ duality mapping space-time surfaces $X^4 \subset M^8$ to $H = M^4 \times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained.

M^8 duality allows to relate p-adic length scales L_p to differences for the roots of the polynomial defining the extension defining "special moments in the life of self" assignable causal diamond (CD) central in zero energy ontology (ZEO). Hence p-adic length scale hypothesis emerges both from p-adic mass calculations and $M^8 - H$ duality. It is proposed that the size scale of CD correspond to the largest dark scale nL_p for the extension and that the sub-extensions of extensions could define hierarchy of sub-CDs. Skyrmions are an important notion if nuclear and hadron physics, $M^8 - H$ dyality suggests an interpretation of skyrmion number as winding number as that for a map defined by complex polynomial.

7.2.1 Some background

A summary of the basic notions and ideas involved is in order.

p-Adic length scale hypothesis

In p-adic mass calculations [K68] real mass squared is obtained by so called canonical identification from p-adic valued mass squared identified as analog of thermodynamical mass squared using p-adic generelization of thermodynamics assuming super-conformal invariance and Kac-Moody algebras assignable to isometries ad holonomies of $H = M^4 \times CP_2$. This implies that the mass squared is essentially the expectation value of sum of scaling generators associated with various tensor factors of the representations for the direct sum of super-conformal algebras and if the number of factors is 5 one obtains rather predictive scenario since the p-adic temperature T_p must be inverse integer in order that the analogs of Boltzmann factors identified essentially as p^{L_0/T_p}.

The p-adic mass squared is of form $Xp + O(p^2)$ and mapped to $X/p + O(1/p^2)$. For the p-adic primes assignable to elementary particles ($M_{127} = 2^{127} - 1$ for electron) the higher order corrections are in general extremely small unless the coefficient of second order contribution is larger integer of order p so that calculations are practically exact.

Elementary particles seem to correspond to p-adic primes near powers 2^k. Corresponding p-adic length - and time scales would come as half-octaves of basic scale if all integers k are allowed. For odd values of k one would have octaves as analog for period doubling. In chaotic systems also the generalization of period doubling in which prime $p = 2$ is replaced by some other small prime appear and there is indeed evidence for powers of $p = 3$ (period tripling as approach to chaos). Many elementary particles and also hadron physics and electroweak physics seem to correspond to Mersenne primes and Gaussian Mersennes which are maximally near to powers of 2.

For given prime p also higher powers of p define p-adic length scales: for instance, for electron the secondary p-adic time scale is .1 seconds characterizing fundamental bio-rhythm. Quite generally, elementary particles would be accompanied by macroscopic length and time scales perhaps assignable to their magnetic bodies or causal diamonds (CDs) accompanying them.

This inspired p-adic length scale hypothesis stating the size scales of space-time surface correspond to primes near half-octaves of 2. The predictions of p-adic are exponentially sensitive to the value of k and their success gives strong support for p-adic length scale hypothesis. This hypothesis applied not only to elementary particle physics but also to biology and even astrophysics and cosmology. TGD Universe could be p-adic fractal.

Dark matter as phases of ordinary matter with $h_{eff} = nh_0$

The identification of dark matter as phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ is second key hypothesis of TGD. To be precise, these phases behave like dark matter and galactic dark matter could correspond to dark energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes.

There are good arguments in favor of the identification $h = 6h_0$ [L16, L47]. "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining space-time surface as a covering. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

$M^8 - H$ duality

$M^8 - H$ duality ($H = M^4 \times CP_2$) [L60] has taken a central role in TGD framework. $M^8 - H$ duality allows to identify space-time regions as "roots" of octonionic polynomials P in complexified M^8 - M^8_c - or as minimal surfaces in $H = M^4 \times CP_2$ having 2-D singularities.

Remark:O_c,H_c,C_c,R_c will be used in the sequel for complexifications of octonions, quaternions, etc.. number fields using commuting imaginary unit i appearing naturally via the roots of real polynomials.

The precise form of $M^8 - H$ duality has however remained unclear. Two assumptions are involved.

1. Associativity stating that the tangent or normal space of at the point of the space-time space-time surface M^8 is associative - that is quaternionic. There are good reasons to believe that this is true for the polynomial ansatz everywhere but there is no rigorous proof.

2. The tangent space of the point of space-time surface at points mappable from M^8 to H must contain fixed $M^2 \subset M^4 \subset M^8$ or an integrable distribution of $M^2(x)$ so that the 2-surface of M^4 determined by it belongs to space-time surface.

The strongest, global form of $M^8 - H$ duality states that $M^2(x)$ is contained to tangent spaces of X^4 at all points x. Strong form of holography (SH) states allows also the option for which this holds true only for 2-D surfaces - string world sheets and partonic 2-surfaces - therefore mappable to H and that SH allows to determined $X^4 \subset H$ from this data. In the following a realization of this weaker form of $M^8 - H$ duality is found. Note however that one cannot exclude the possibility that also associativity is true only at these surfaces for the polynomial ansatz.

Number theoretic origin of p-adic primes and dark matter

There are several questions to be answered. How to fuse real number based physics with various p-adic physics? How p-adic length scale hypothesis and dark matter hypothesis emerge from TGD?

The properties of p-adic number fields and the strange failure of complete non-determinism for p-adic differential equations led to the proposal that p-adic physics might serve as a correlate for cognition, imagination, and intention. This led to a development of number theoretic vision which I call adelic physics. A given adele corresponds to a fusion of reals and extensions of various p-adic number fields induced by a given extension of rationals.

The notion of space-time generalizes to a book like structure having real space-time surfaces and their p-adic counterparts as pages. The common points of pages defining is back correspond to points with coordinates in the extension of rationals considered. This discretization of space-time surface is in general finite and unique and is identified as what I call cognitive representation. The Galois group of extension becomes symmetry group in cognitive degrees of freedom. The ramified primes of extension are exceptionally interesting and are identified as preferred p-adic primes for the extension considered.

The basic challenge is to identify dark scale. There are some reasons to expect correlation between p-adic and dark scales which would mean that the dark scale would depend on ramified primes, which characterize roots of the polynomial defining the extensions and are thus not defined completely by extension alone. Same extension can be defined by many polynomials. The naïve guess is that the scale is proportional to the dimension n of extension serving as a measure for algebraic complexity (there are also other measures). p-Adic length scales L_p would be proportional nL_p, p ramified prime of extension? The motivation would be that quantum scales are typically proportional to Planck constant. It turns out that the identification of CD scale as dark scale is rather natural.

7.2.2 New results about $M^8 - H$ duality

In the sequel some new results about M^8-H duality are deduced. Strong form of holography (SH) allows to weaken the assumptions making possible $M^8 - H$ duality. It would be enough to map only certain complex 2-D sub-manifolds of quaternionic space-time surface in M^8 to H: SH would allow to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds would be determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and they form a discrete set.

Strong form of holography (SH)

Ordinary 3-D holography is forced by general coordinate invariance (GCI) and loosely states that the data at 3-D surfaces allows to determined space-time surface $X^4 \subset H$. In ZEO 3-surfaces correspond to pairs of 3-surfaces with members at the opposite light-like boundaries of causal diamond (CD) and are analogous to initial and final states of deterministic time evolution as Bohr orbit.

This poses additional strong conditions on the space-time surface.

1. The conjecture is that these conditions state the vanishing of super-symplectic Noether charges for a sub-algebra of super-symplectic algebra SC_n with radial conformal weights coming as n-multiples of those for the entire algebra SC and its commutator $[SC_n, SC]$ with the entire algebra: these conditions generalize super conformal conditions and one obtains a hierarchy of realizations.

   This hierarchy of minimal surfaces would naturally corresponds to the hierarchy of extensions of rationals with n identifiable as dimension of the extension giving rise to effective Planck constant. At the level of Hilbert spaces the inclusion hierarchies for extensions could also correspond to the inclusion hierarchies of hyper-finite factors of type I_1 [K127] so that M^8-H duality would imply beautiful connections between key ideas of TGD.

2. Second conjecture is that the preferred extremals (PEs) are extremals of both the volume term and Kähler action term of the action resulting by dimensional reduction making possible the induction of twistor structure from the product of twistor spaces of M^4 and CP_2 to 6-D S^2 bundle over X^4 defining the analog of twistor space. These twistor spaces must have Kähler structure since action for 6-D surfaces is Kähler action - it exists only in these two cases [A50] so that TGD is unique.

Strong form of holography (SH) is a strengthening of 3-D holography. Strong form of GCI requires that one can use either the data associated either with

- light-like 3-surfaces defining partonic orbits as surfaces at which signature of the induced metric changes from Euclidian to Minkowskian or
- the space-like 3-surfaces at the ends of CD to determine space-time surface as PE (in case that it exists).

This suggests that the data at the intersections of these 2-surfaces defined by partonic 2-surfaces might be enough for holography. A slightly weaker form of SH is that also string world sheets intersecting partonic orbits along their 1-D boundaries is needed and this form seems more realistic.

SH allows to weaken strong form of M^8-H duality mapping space-time surfaces $X^4 \subset M^8$ to $H = M^4 \times CP_2$ that it allows to map only certain complex 2-D sub-manifolds of quaternionic space-time surface to H: SH allows to determine $X^4 \subset H$ from this 2-D data. Complex sub-manifolds are determined by conditions completely analogous to those determined space-time surface as quaternionic sub-manifold and only discrete set of them is obtained.

Space-time as algebraic surface in M_c^8 regarded complexified octonions

The octonionic polynomial giving rise to space-time surface as its "root" is obtained from ordinary real polynomial P with rational coefficients by algebraic continuation. The conjecture is that the identification in terms of roots of polynomials of even real analytic functions guarantees associativity and one can formulate this as rather convincing argument [?] Space-time surface X_c^4 is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. These options correspond to complexified-quaternionic tangent- or normal spaces. For $P(x) = x^n + ..$ ordinary roots are algebraic integers. The real 4-D space-time surface is projection of this surface from M_c^8 to M^8. One could drop the subscripts "$_c$" but in the sequel they will be kept.

M_c^4 appears as a special solution for any polynomial P. M_c^4 seems to be like a universal reference solution with which to compare other solutions.

One obtains also brane-like 6-surfaces as 6-spheres as universal solutions. They have M^4 projection, which is a piece of hyper-surface for which Minkowski time as time coordinate of CD corresponds to a root $t = r_n$ of P. For monic polynomials these time values are algebraic integers and Galois group permutes them.

One cannot exclude rational functions or even real analytic functions in the sense that Taylor coefficients are octonionically real (proportional to octonionic real unit). Number theoretical vision - adelic physics [?, ?] suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

How do the solutions assignable to the opposite boundaries of CD relate to each other?

CD has two boundaries. The polynomials associated with them could be different in the general formulation discussed in [L77, L82] but they could be also same. How are the solutions associated with opposite boundaries of CD glued together in a continuous manner?

1. The polynomials assignable to the opposite boundaries of CD are allowed to be polynomials of o *resp.* $(o - T)$: here T is the distance between the tips of CD.

2. CD brings in mind the realization of conformal invariance at sphere: the two hemispheres correspond to powers of z and $1/z$: the condition $z = \overline{1/z}$ at unit circle is essential and there is no real conjugation. How the sphere is replaced with 8-D CD which is also complexified. The absence of conjugation looks natural also now: could CD contain a 3-surface analogous to the unit circle of sphere at which the analog of $z = \overline{1/z}$ holds true? If so, one has $P(o, z) = P(1/o, z)$ and the solutions representing roots fo $P(o, z)$ and $P(1/o, z)$ can be glued together.

   Note that $1/o$ can be expressed as $\overline{o}/o\overline{o}$ when the Minkowskian norm squared $\overline{o}o$ is non-vanishing and one has polynomial equation also now. This condition is true outside the boundary of 8-D light-cone, in particular near the upper boundary of CD.

   The counter part for the length squared of octonion in Minkowskian signature is light-one proper time coordinate $a^2 = t^2 - r^2$ for M_+^8. Replacing o which scaled dimensionless variable $o_1 = o/(T/2)$ the gluing take place along $a = T/2$ hyperboloid.

One has algebraic holomorphy with respect to o but also anti-holomorphy is possible. What could these two options correspond to? Could the space-time surfaces assignable to self and its time-reversal relate by octonionic conjugation $o \to \overline{o}$ relating two Fock vacuums annihilated by fermionic annihilation *resp.* creation operators?

In [L77, L82] the possibility that the sequence of SSFRs or BSFRs could involve iteration of the polynomial defining space-time surface - actually different polynomials were allowed for two boundaries. There are 3 options: each SSFR would involve the replacement $Q = P \circ .. \circ P \to P \circ Q$, the replacement occurs only when new "special moments in the life of self" defined by the roots of P as $t = r_n$ balls of cd, or the replacement can occur in BSFR when the metabolic resources do not allow to continue the iteration (the increase of h_{eff} during iteration increases the needed metabolic feed).

The iteration is compatible with the proposed picture. The assumption $P(0) = 0$ implies that iterates of P contain also the roots of P as roots - they are like conserved genes. Also the 8-D light-cone boundary remains invariant under iteration. Even more general function decompositions $P \rightarrow Q \rightarrow P$ are consistent with the proposed picture.

Brane-like solutions

One obtains also 6-D brane-like solutions to the equations.

1. In general the zero loci for imaginary or real part are 4-D but the 7-D light-cone δM^8_+ of M^8 with tip at the origin of coordinates is an exception [L33, L34, L35]. At δM^8_+ the octonionic coordinate o is light-like and one can write $o = re$, where 8-D time coordinate and radial coordinate are related by $t = r$ and one has $e = (1 + e_r)/\sqrt{2}$ such that one as $e^2 = e$.

   Polynomial $P(o)$ can be written at δM^8_+ as $P(o) = P(r)e$ and its roots correspond to 6-spheres S^6 represented as surfaces $t_M = t = r_N$, $r_M = \sqrt{r_N^2 - r_E^2} \leq r_N$, $r_E \leq r_N$, where the value of Minkowski time $t = r = r_N$ is a root of $P(r)$ and r_M denotes radial Minkowski coordinate. The points with distance r_M from origin of $t = r_N$ ball of M^4 has as fiber 3-sphere with radius $r = \sqrt{r_N^2 - r_E^2}$. At the boundary of S^3 contracts to a point.

2. These 6-spheres are analogous to 6-D branes in that the 4-D solutions would intersect them in the generic case along 2-D surfaces X^2. The boundaries $r_M = r_N$ of balls belong to the boundary of M^4 light-cone. In this case the intersection would be that of 4-D and 3-D surface, and empty in the generic case (it is however quite not clear whether topological notion of "genericity" applies to octonionic polynomials with very special symmetry properties).

3. The 6-spheres $t_M = r_N$ would be very special. At these 6-spheres the 4-D space-time surfaces X^4 as usual roots of $P(o)$ could meet. Brane picture suggests that the 4-D solutions connect the 6-D branes with different values of r_n.

   The basic assumption has been that particle vertices are 2-D partonic 2-surfaces and light-like 3-D surfaces - partonic orbits identified as boundaries between Minkowskian and Euclidian regions of space-time surface in the induced metric (at least at H level) - meet along their 2-D ends X^2 at these partonic 2-surfaces. This would generalize the vertices of ordinary Feynman diagrams. Obviously this would make the definition of the generalized vertices mathematically elegant and simple.

   Note that this does not require that space-time surfaces X^4 meet along 3-D surfaces at S^6. The interpretation of the times t_n as moments of phase transition like phenomena is suggestive. ZEO based theory of consciousness suggests interpretation as moments for state function reductions analogous to weak measurements ad giving rise to the flow of experienced time.

4. One could perhaps interpret the free selection of 2-D partonic surfaces at the 6-D roots as initial data fixing the 4-D roots of polynomials. This would give precise content to strong form of holography (SH), which is one of the central ideas of TGD and strengthens the 3-D holography coded by ZEO alone in the sense that pairs of 3-surfaces at boundaries of CD define unique preferred extremals. The reduction to 2-D holography would be due to preferred extremal property realizing the huge symplectic symmetries and making $M^8 - H$ duality possible as also classical twistor lift.

   I have also considered the possibility that 2-D string world sheets in M^8 could correspond to intersections $X^4 \cap S^6$? This is not possible since time coordinate t_M constant at the roots and varies at string world sheets.

   Note that the compexification of M^8 (or equivalently octonionic E^8) allows to consider also different variants for the signature of the 6-D roots and hyperbolic spaces would appear for $(\epsilon_1, \epsilon_i, .., \epsilon_8)$, $epsilon_i = \pm 1$ signatures. Their physical interpretation - if any - remains open at this moment.

5. The universal 6-D brane-like solutions S_c^6 have also lower-D counterparts. The condition determining X^2 states that the C_c-valued "real" or "imaginary" for the non-vanishing Q_c-valued "real" or "imaginary" for P vanishes. This condition allows universal brane-like solution as a restriction of O_c to M_c^4 (that is CD_c) and corresponds to the complexified time=constant hyperplanes defined by the roots $t = r_n$ of P defining "special moments in the life of self" assignable to CD. The condition for reality in R_c sense in turn gives roots of $t = r_n$ a hyper-surfaces in M_c^2.

Explicit realization of $M^8 - H$ duality

$M^8 - H$ duality allows to map space-time surfaces in M^8 to H so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D singularities in H satisfying an infinite number of additional conditions stating vanishing of Noether charges for super-symplectic algebra actings as isometries for the "world of classical worlds" (WCW). Twistor lift allows variants of this duality. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of extensions of rationals defining an evolutionary hierarchy. This forms the basis for the number theoretical vision about TGD.

$M^8 - H$ duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions.

1. Associativity condition for tangent-/normal space is the first essential condition for the existence of $M^8 - H$ duality and means that tangent - or normal space is quaternionic.

2. The tangent space of space-time surface and thus space-time surface itself must contain a preferred $M_c^2 \subset M_c^4$ or more generally, an integrable distribution of tangent spaces $M_c^2(x)$ and similar distribution of their complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X_c^2 \subset X_c^4$ in R_c sense. $E_c^2(x)$ would correspond to partonic 2-surface.

   One can imagine two realizations for this condition.

   Option I: Global option states that the distributions $M_c^2(x)$ and $E_c^2(x)$ define slicing of X_c^4.

   Option II: Only discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce space-time surfaces in H. This would be the minimal option.

   That the selection between these options is not trivial is suggested by following.

1. For massless extremals (MEs, topological light rays) parameterized by light-like vector vector k defining $M^2 \subset M^2 \times E^2 \subset M^4$ at each point and by space-like polarization vector ϵ depending on single transversal coordinate of E^2 [K10].

2. CP_2 coordinates have an arbitrary dependence on both $u = k \cdot m$ and $w = \epsilon \cdot m$ and can be also multivalued functions of u and w. Single light-like vector k is enough to identify M^2. CP_2 type extremals having metric and Kähler form of CP_2 have light-like geodesic as M^4 projection defining M^2 and its complement E^2 in the normal space.

3. String like objects $X^2 \times Y^2 \subset M^4 \times CP_2$ are minimal surfaces and X^2 defines the distribution of $M^2(x) \subset M^4$. Y^2 ddefines the complement of this distribution.

   Option I is realized in all 3 cases. It is not clear whether M^2 can depend on position in the first 2 cases and also CP_2 point in the third case. It could be that only a discrete set of these string world sheets assignable to wormhole contacts representing massless particles is possible (**Option II**).

   How these conditions would be realized?

1. The basic observation is that X^2c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c valued "real" or "imaginary" part in C_c sense for P vanishes. M_c^2 would be the simplest solution but also more general complex sub-manifolds $X_c^2 \subset M_c^4$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. As found, there are also classes special global solutions for which the choice of M_c^2 is global and does not depend on space-time point. The interpretation would be in terms of modes of classical massless fields characterized by polarization and momentum. If the identification of M_c^2 is correct, these surfaces are however unstable against perturbations generating discrete string world sheets and orbits of partonic 2-surfaces having interpretation space-time counterparts of quanta. That fields are detected via their quanta was the revolutionary observation that led to quantum theory. Could quantum measurement induce the instability decomposing the field to quanta at the level of space-time topology?

3. One can generalize this condition so that it selects 1-D surface in X_c^2. By assuming that R_c-valued "real" or "imaginary" part of quaternionic part of P at this 2-surface vanishes. one obtains preferred M_c^1 or E_c^1 containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \rightarrow C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

   This option could be made possible by SH. SH states that preferred extremals are determined by data at 2-D surfaces of X^4. Even if the conditions defining X_c^2 have only a discrete set of solutions, SH at the level of H could allow to deduce the preferred extremals from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. Associativity and existence of $M^2(x)$ would be required only at the 2-D surfaces.

4. I have proposed that physical string world sheets and partonic 2-surfaces appear as singularities and correspond to 2-D folds of space-time surfaces at which the dimension of the quaternionic tangent space degenerates from 4 to 2 [L59] [K10]. This interpretation is consistent with a book like structure with 2-pages. Also 1-D real and imaginary manifolds could be interpreted as folds or equivalently books with 2 pages.

   For the singular surfaces the dimension quaternionic tangent or normal space would reduce from 4 to 2 and it is not possible to assign CP_2 point to the tangent space. This does not of course preclude the singular surfaces and they could be analogous to poles of analytic function. Light-like orbits of partonic 2-surfaces would in turn correspond to cuts.

5. What could the normal space singularity mean at the level of H? Second fundamental form defining vector basis in normal space is expected to vanish. This would be the case for minimal surfaces.

   (a) String world sheets with Minkowskian signature (in M^4 actually) are expected to be minimal surfaces. In this case T matters and string world sheets could be mapped to H by $M^8 - H$ duality and SH would work for them.

   (b) The light-like orbits of partonic 2-surfaces with Euclidian signature in H would serve as analogs of cuts. N is expected to matter and partonic 2-surfaces should be minimal surfaces. Their branching of partonic 2-surfaces is thus possible and would make possible (note the analogy with the branching of soap films) for them to appear as 2-D vertices in H.

   The problem is to identify the pre-images of partonic 2-surfaces in M^8. The light-likeness of the orbits of partonic 2-surfaces (induced 4-metric changes its signature and degenerates to 3-D) should be important. Could light-likeness in this sense define the pre-images partonic orbits in M^8?

Remark: It must be emphasized that SH makes possible $M^8 - H$ correspondence assuming that also associativity conditions hold true only at partonic 2-surfaces and string world sheets. Thus one could give up the conjecture that the polynomial ansatz implies that tangent or normal spaces are associative. Proving that this is the case for the tangent/normal spaces of these 2-surfaces should be easier.

Does $M^8 - H$ duality relate hadron physics at high and low energies?

During the writing of this article I realized that $M^8 - H$ duality has very nice interpretation in terms of symmetries. For $H = M^4 \times CP_2$ the isometries correspond to Poincare symmetries and color $SU(3)$ plus electroweak symmetries as holonomies of CP_2. For octonionic M^8 the subgroup $SU(3) \subset G_2$ is the sub-group of octonionic automorphisms leaving fixed octonionic imaginary unit invariant - this is essential for $M^8 - H$ duality. $SU(3)$ is also subgroup of $SO(6) \equiv SU(4)$ acting as rotation on $M^8 = M^2 \times E^6$. The subgroup of the holonomy group of $SO(4)$ for E^4 factor of $M^8 = M^4 \times E^4$ is $SU(2) \times U(1)$ and corresponds to electroweak symmetries. One can say that at the level of M^8 one has symmetry breaking from $SO(6)$ to $SU(3)$ and from $SO(4) = SU(2) \times SO(3)$ to $U(2)$.

This interpretation gives a justification for the earlier proposal that the descriptions provided by the old-fashioned low energy hadron physics assuming $SU(2)_L \times SU(2)_R$ and acting acting as covering group for isometries $SO(4)$ of E^4 and by high energy hadron physics relying on color group $SU(3)$ are dual to each other.

Skyrmions and $M^8 - H$ duality

I received a link (https://tinyurl.com/ycathr3u) to an article telling about research (https://tinyurl.com/yddwhr2o) carried out for skyrmions, which are very general condensed matter quasiparticles. They were found to replicate like DNA and cells. I realized that I have not clarified myself the possibility of skyrmions on TGD world and decided to clarify my thoughts.

1. What skyrmions are?

Consider first what skyrmions are.

1. Skyrmions are topological entities. One has some order parameter having values in some compact space S. This parameter is defined in say 3-ball such that the parameter is constant at the boundary meaning that one has effectively 3-sphere. If the 3rd homotopy group of S characterizing topology equivalence classes of maps from 3-sphere to S is non-trivial, you get soliton-llike entities, stable field configurations not deformable to trivial ones (constant value). Skyrmions can be assigned to space S which is coset space $SU(2)_L \times SU(2)_R / SU(2)_V$, essentially S^3 and are labelled by conserved integer-valued topological quantum number.

2. One can imagine variants of this. For instance, one can replace 3-ball with disk. $SO(3) = S^3$ with 2-sphere S^2. The example considered in the article corresponds to discretized situation in which one has magnetic dipoles/spins at points of say discretized disk such that spins have same direction about boundary circle. The distribution of directions of spin can give rise to skyrmion-like entity. Second option is distribution of molecules which do not have symmetry axis so that as rigid bodies the space of their orientations is discretized version of $SO(3)$. The field would be the orientation of a molecule of lattice and one has also now discrete analogs of skyrmions.

3. More generally, skyrmions emerge naturally in old-fashioned hadron physics, where $SU(2)_L \times SU(2)_R / SU(2)_V$ involves left-handed, right-handed and vectorial subgroups of $SO(4) = SU(2)_L \times SU(2)_R$. The realization would be in terms of 4-component field (π, σ), where π is charged pion with 3 components - axial vector - and σ which is scalar. The additional constraint $\pi \cdot \pi + \sigma^2 = constant$ defines 3-sphere so that one has field with values in S^3. There are models assigning this kind of skyrmion with nucleon, atomic nuclei, and also in the bag model of hadrons bag can be thought of as a hole inside skyrmion. These models seem to have something to do with reality so that a natural question is whether skyrmions might appear in TGD.

2. Skyrmion number as winding number

In TGD framework one can regard space-time as 4-surface in either octonionic M^8_c, c refers here to complexification by an imaginary unit i commuting with octonions, or in $M^4 \times CP_2$. For the solution surfaces M^8 has natural decomposition $M^8 = M^2 \times E^6$ and E^6 has $SO(6)$ as isometry group containing subgroup $SU(3)$ having automorphisms of octonions as subgroup leaving M^2

invariant. $SO(6) = SU(4)$ contains $SU(3)$ as subgroup, which has interpretation as isometries of CP_2 and counterpart of color gauge group. This supports $M^8 - H$ duality, whose most recent form is discussed in [L75].

The map $S^3 \to S^3$ defining skyrmion could be taken as a phenomenological consequence of $M^8 - H$ duality implying the old-fashioned description of hadrons involving broken $SO(4)$ symmetry (PCAC) and unbroken symmetry for diagonal group $SO(3)_V$ (CCV). The analog of $(\pi, sigma)$ field could correspond to a B-E condensate of pions $(\pi, sigma)$.

The obvious question is whether the map $S^3 \to S^3$ defining skyrmion could have a deeper interpretation in TGD framework. I failed to find any elegant formulation. One could however generalize and ask whether skyrmion like entities characterize by winding number are predicted by basic TGD.

1. In the models of nucleon and nuclei the interpretation of conserved topological skyrmion number is as baryon number. This number should correspond to the homotopy class of the map in question, essentially winding number. For polynomials of complex number degree corresponds to winding number. Could the degree $n = h_{eff}/h_0$ of polynomial P having interpretation as effective Planck constant and measure of complexity - kind of number theoretic IQ - be identifiable as skyrmion number? Could it be interpreted as baryon number too?

2. For leptons regarded as local 3 anti-quark composites in TGD based view about SUSY [L65] the same interpretation would make sense. It seems however that the winding number must have both signs. Degree is n is however non-negative.

   Here complexification of M^8 to M^8_c is essential. One an allow both holomorphic and anti-holomorphic continuations of real polynomials P (with rational coefficients) using complexification defined by commutative imaginary unit i in M^8_c so that one has polynomials $P(z)$ *resp.* $P(\overline{z})$ in turn algebraically continued to complexified octonionic polynomials $P(z, o)$ *resp.* $P(\overline{z}, o)$.

   Particles *resp.* antiparticles would correspond to the roots of octonionic polynomial $P(z, o)$ resp. $P(\overline{z}, o)$ meaning space-time geometrization of the particle-antiparticle dichotomy and would be conjugates of each other. This could give a nice physical interpretation to the somewhat mysterious complex roots of P.

3. More detailed formulation

To make this formulation more detailed on must ask how 4-D space-time surfaces correspond to 8-D "roots" for the "imaginary" ("real") part of complexified octonionic polynomial as surfaces in M^8_c.

1. Equations state the simultaneous vanishing of the 4 components of complexified quaternion valued polynomial having degree n and with coefficients depending on the components of O_c, which are regarded as complex numbers $x + iy$, where i commutes with octonionic units. The coefficients of polynomials depend on complex coordinates associated with non-vanishing "real" ("imaginary") part of the O_c valued polynomial.

2. To get perspective, one can compare the situation with that in catastrophe theory in which one considers roots for the gradient of potential function of behavior variables x^i. Potential function is polynomial having control variables as parameters. Now behavior variable correspond "imaginary" ("real") part and control variables to "real" ("imaginary") of octonionic polynomial.

   For a polynomial with real coefficients the solution divides to regions in which some roots are real and some roots are complex. In the case of cusp catastrophe one has cusp region with 3-D region of the parameter defined by behavior variable x and 2 control parameters with 3 real roots, the region in which one has one real root. The boundaries for the projection of 3-sheeted cusp to the plane defined by control variables correspond to degeneration of two complex roots to one real root.

   In the recent case it is not clear whether one cannot require the M^8_c coordinates for space-time surface to be real but to be in $M^8 = M^1 + iE^7$.

3. Allowing complex roots gives 8-D space-time surfaces. How to obtain real 4-D space-time surfaces?

   (a) One could project space-time surfaces to real M^8 to obtain 4-D real space-time surfaces. For M^8 this would mean projection to $M^1 + iE^7$ and in time direction the real part of root is accepted and is same for the root and its conjugate. For E^7 this would mean that imaginary part is accepted and means that conjugate roots correspond to different space-time surfaces and the notion of baryon number is realized at space-time level.

   (b) If one allows only real roots, the complex conjugation proposed to relate fermions and anti-fermions would be lost.

4. One can select for 4 complex M^8_c coordinates X^k of the surface and the remaining 4 coordinates Y^k can be formally solved as roots of n:th degree polynomial with dynamical coefficients depending on X^k and the remaining Y^k. This is expected to give rise to preferred extremals with varying dimension of M^4 and CP_2 projections.

5. It seems that all roots must be complex.

   (a) The holomorphy of the polynomials with respect to the complex M^8_c coordinates implies that the coefficients are complex in the generic point M^8_c. If so, all 4 roots are in general complex but do not appear as conjugate pairs. The naïve guess is that the maximal number of solutions would be n^4 for a given choice of M^8 coordinates solved as roots. An open question is whether one can select subset of roots and what happens at $t = r_n$ surfaces: could different solutions be glued together at them.

   (b) Just for completeness one can consider also the case that the dynamical coefficients are real - this is true in the E^8 sector and whether it has physical meaning is not clear. In this case the roots come as real roots and pairs formed by complex root and its conjugate. The solution surface can be divided into regions depending on the character of 4 roots. The n roots consist of complex root pairs and real roots. The members or complex root pairs are mapped to same point in E^8.

4. Could skyrmions in TGD sense replicate?

What about the observation that condensed matter skyrmions replicate? Could this have analog at fundamental level?

1. The assignment of conserved topological quantum number to the skyrmion is not consistent with replication unless the skyrmion numbers of outgoing states sum up to that of the initial state. If the system is open one can circumvent this objection. The replication would be like replication of DNA in which nucleotides of new DNA strands are brought to the system to form new strands.

2. It would be fascinating if all skyrmions would correspond to space-time surfaces at fundamental M^8 level. If so, skyrmion property also in magnetic sense could be induced by from a deeper geometric skyrmion property of the MB of the system. The openness of the system would be essential to guarantee conservation of baryon number. Here the fact that leptons and baryons have opposite baryon numbers helps in TGD framework. Note also ordinary DNA replication could correspond to replication of MB and thus of skyrmion sequences.

7.2.3 About p-adic length scale hypothesis and dark matter hierarchy

It is good to introduce first some background related to p-adic length scale hypothesis discussed in chapters of [K77] and dark matter hierarchy discussed in chapters [K58, K59], in particular in chatper [?].

General form of p-adic length scale hypothesis

The most general form of p-adic length scale hypothesis does not pose conditions on allowed p-adic primes and emerges from p-adic mass calculations [K32, K68, K79]. It has two forms corresponding to massive particles and massless particles.

1. For massive particles the preferred p-adic mass calculations based on p-adic thermodynamics predicts the p-adic mass squared m^2 to be proportional to p or its power- the real counterpart of m^2 is proportional to $1/p$ or its power. In the simplest case one has

   $$m^2 = \frac{X}{p}\frac{\hbar}{L_0} \ ,$$

   where L_0 is apart from numerical constant the length R of CP_2 geodesic circle. X is a numerical constant not far from unity. $X \geq 1$ is small integer in good approximation. For instance for electron one has $x = 5$.

   By Uncertainty Principle the Compton length of particle is characterizing the size of 3-surfaces assignable to particle are proportional to $\sqrt{p}$:

   $$L_c(m) = \frac{\hbar}{m} = \sqrt{\frac{1}{X}}L_p \ , \quad L_p = \sqrt{p}L_0 = \ .$$

   Here L_p is p-adic length scale and corresponds to minimal mass for given p-adic prime. p-Adic length scale would be would characterize the size of the 3-surface assignable to the particle and would correspond to Compton length.

2. For massless particles mass vanishes and the above picture is not possible unless there is very small mass coming from p-adic thermodynamics and determined by the size scale of CD - this is quite possible. The preferred time/spatial scales p-adic energy- equivalently 3-momentum are proportional to p-adic prime p or its power. The real energy is proportional to $1/p$. At the embedding space level the size of scale causal diamond (CD) [L64] would be proportional to p: $L = T = pL_0$, $L_0 = T_0$ for $c = 1$. The interpretation in terms of Uncertainty Principle is possible.

   There would be therefore two levels: space-time level and embedding space level . At the space-time level the primary p-adic length scale would be proportional to $\sqrt{p}$ whereas the p-adic length scale at embedding space-time would correspond to secondary p-adic length scale proportional to p. The secondary p-adic length scales would assign to elementary new physics in macroscopic scales. For electron the size scale of CD would be about .1 seconds, the time scale associated with the fundamental bio-rhythm of about 10 Hz.

3. A third piece in the picture is adelic physics [L39, L38] inspiring the hypothesis that effective Planck constant h_{eff} given by $h_{eff}/h_0 = n$, $h = 6h_0$, labels the phases of ordinary matter identified as dark matter. n would correspond to the dimension of extension of rationals.

   The connection between preferred primes and the value of $n = h_{eff}/h_0$ is interesting. One proposal is that preferred primes p in p-adic length scale hypothesis determining the mass scale of particle correspond to so called ramified primes, which characterize the extensions. The p-adic variant of the polynomial defining space-time surfaces in M^8 picture would have vanishing discriminant in order $O(p)$. Since discriminant is proportional to the product of differences of different roots of the polynomial, two roots would be very near to each other p-adically. This would be mathematical correlate for criticality in p-adic sense.

   $M^8 - H$ duality [L60, L57] leads to the prediction that the roots r_n of polynomial defining the space-time region in M^8 correspond to preferred time values $t = t_n = \propto r_n$- I have called $t = t_n$ "special moments in the life of self". Since the squares for the differences for the roots are proportional to ramified primes, these time differences would code for ramified primes assignable to the space-time surface. There would be several p-adic time scales involved and they would be coded by $t_{ij} = r_i - r_j$, whose moduli squared are divided by so called ramified primes defining excellent candidates for preferred p-adic primes. p-Adic physics would make itself visible at the level of space-time surface in terms of "special moments in the life of self".

4. p-Adic length scales emerge naturally from M^8-H duality [L60, L57]. Ramified primes would in M^8 picture appear as factors of time differences associated with "special moments in the life of self" associated with CD [L57]. One has $|t_i-t_j| \propto \sqrt{p_{ij}}$, p_{ij} ramified prime. It is essential that square root of ramified prime appears here.

   This suggests strongly that p-adic length scale hypothesis is realized at the level of space-time surface and there are several p-adic length scales present coded to the time differences. Knowing of the polynomial would give information about p-adic physics involved. If dark scales correlate with p-adic length scales as proposed, the definition of dark scale should assume the dependence of ramified primes quite generally rather than as a result of number theoretic survival of fittest as one might also think.

   The factors t_i-t_j are proportional - not only to the typically very large p-adic prime p_{max} charactering the system - but also smaller primes or their powers. Could the scales in question be of form $l_p=\sqrt{X}\sqrt{p_{max}}L_0$ rather than p-adic length scales $L_{p_{ram}}$ defined by various ramified primes. Here X would be integer consisting of small ramified primes.

   p-Adic mass calculations predict in an excellent approximation the mass of the particle is given by $m=(\sqrt{X}/\sqrt{p})m_0$, X small integer and $m_0=1/L_0$. Compton length would be given by $L_c(p)=\sqrt{p}/\sqrt{X})L_0$. The identification $l_p=L_c(p)$ would be attractive but is not possible unless one has $X=1$. In this case one would be considering p-adic length scale L_p. the interpretation in terms of multi-p-adicity seems to be the realistic option.

About more detailed form of p-adic length scale hypothesis

More specific form of p-adic length scale hypothesis poses conditions on physically preferred p-adic primes. There are several guesses for preferred primes. They could be primes near to integer powers 2^k, where k could be positive integer, which could satisfy additional conditions such as being odd, prime or be associated with Mersenne prime or Gaussian Mersenne. One can consider also powers of other small primes such as $p=2,3,5$. p-Adic length scale hypothesis in is basic form would generalize the notion of period doubling. For odd values of k one would indeed obtain period doubling, tripling, etc... suggesting strongly chaos theoretic origin.

1. p-Adic length scale hypothesis in its basic form

Consider first p-adic length scale hypothesis in its basic form.

1. In its basic form states that primes $p\simeq 2^k$ are preferred p-adic primes and correspond by p-adic mass calculations p-adic length scales $L_p\equiv L(k)\propto\sqrt{p}=2^{k/2}$. Mersenne primes and primes associated with Gaussian Mersennes as especially favored primes and charged leptons ($k\in\{127,113,107\}$) and Higgs boson ($k=89$) correspond to them. Also hadron physics ($k=107$) and nuclear physics ($k=113$) correspond to these scales. One can assign also to hadron physics Mersenne prime and the conjecture is that Mersennes and Gaussian Mersennes define scaled variants of hadron physics and electroweak physics. In the length scale between cell membrane thickness fo 10 nm and nuclear size about 2.5 μm there are as many as 4 Gaussian Mersennes corresponding to $k\in\{151,157,163,167\}$.

   Mersenne primes correspond to prime values of k and I have proposed that k is prime for fundamental p-adic length scales quite generally. There are also however also other p-adic length scales - for instance, for quarks k need not be prime - and it has remained unclear what criterion could select the preferred exponents k. One can consider also the option that odd values of k defined fundamental p-adic length scales.

2. What makes p-adic length scale hypothesis powerful is that masses of say scaled up variant of hadron physics can be estimated by simple scaling arguments. It is convenient to use electron's p-adic length scale and calculate other p-adic length scales by scaling $L(k)=2^{(k-127)/2}L(127)$.

Here one must make clear that there has been a confusion in the definitions, which was originally due to a calculational error.

1. I identified the p-adic length scale $L(151)$ mistakenly as $L(151) = 2^{(k-127)/2}L_e(127)$ by using instead of $L(127)$ electron Compton length $L_e \simeq L(127/\sqrt{5}$. The notation for these scales would be therefore $L_e(k)$ identified as $L_e(k) = 2^{(k-127)/2}L_e(127)$ and I have tried to use it systematically but failed to use the wrong notation in informal discussions.

2. This mistake might reflect highly non-trivial physics. It is scaled up variants of L_e which seem to appear in physics. For instance, $L_e(151) \simeq 10$ nm corresponds to basic scale in living matter. Why the biological important scales should correspond to scaled up Compton lengths for electron? Could dark electrons with scaled up Compton scales equal to $L_e(k)$ be important in these scales? And what about the real p-adic length scales relate to these scales by a scaling factor $\sqrt{5} \simeq 2.23$?

2. Possible modifications of the p-adic length scale hypothesis

One can consider also possible modifications of the p-adic length scale hypothesis. In an attempt to understand the scales associated with INW structures in terms of p-adic length scale hypothesis it occurred to me that the scales which do not correspond to Mersenne primes or Gaussian Mersennes might be generated somehow from the these scales.

1. Geometric mean $L = \sqrt{L(k_1)L(k_2)}$ would length scale which would correspond to L_p with $p \simeq 2^{(k_1+k_2)/2}$. This is of the required form only if $k = k_1 + k_2$ is even so that k_1 and k_2 are both even or odd. If one starts from Mersennes and Gaussian Mersennes the condition is satisfied. The value of $k = (k_1 + k_2)/2$ can be also even.

   Remark: The geometric mean $(127 + 107)/2 = 117$ of electronic and hadronic Mersennes corresponding to mass 16 MeV rather near to the mass of so called X boson [L21] (`https://tinyurl.com/ya3yuzeb`).

2. One can also consider the formula $L = (L(k_1)L(k_2)..L(k_n))^{1/n}$ but in this case the scale would correspond to prime $p \simeq 2^{k_1+...k_n)/n}$. Since $(k_1 + ..k_n)/n$ is integer only if $k_1 + ...k_n$ is proportional to n.

What about the allowed values of fundamental integers k? It seems that one must allow all odd integers.

1. If only prime values of k are allowed, one can obtain obtain for twin prime pair $(k-1, k+1)$ even integer k as geometric mean $\sqrt{k}$ if k is square. If prime k is not a member of this kind of pair, it is not possible to get integers $k-1$ and $k+1$. If only prime values of k are fundamental, one could assign to $k = 89$ characterizing Higgs boson weak bosons $k = 90$ possibly characterizing weak bosons. Therefore it seems that one must allow all odd integers with the additional condition already explained.

2. Just for fun one can check whether $k = 161$ forced by the argument related to electroweak scale and h_{eff} corresponds to a geometric mean of two Gaussian Mersennes. One has $k(k_1, k_2) = (k_2 + k_2)/2$ giving the list $k(151, 157) = 154)$, $k(151, 163) = 157$ Gaussian Mersenne itself, $k(151, 167) = 159$, $k(157, 163) = 160$, $k(157, 167) = 162$, $k(163, 167) = 165$. Unfortunately, $k = 161$ does not belong to this set. If one allows all odd values of k as fundamental, the problem disappears.

One can also consider refinements of p-adic length scale hypothesis in its basic form.

1. One can consider also a generalization of p-adic length scale hypothesis to allow length scales coming as powers of small primes. The small primes $p = 2, 3, 5$ assignable to Platonic solids would be especially interesting. $p = 2, 3, 5$ and also Fermat primes and Mersenne primes are maximally near to powers of two and their powers would define secondary and higher p-adic length scales. In this sense the extension would not actually bring anything new.

   There is evidence for the occurrence of long p-adic time scales coming as powers of 3 [I72, I73] (`http://tinyurl.com/ycesc5mq`) and [K80] (`https://tinyurl.com/y8camqlt`. Furthermore, prime 5 and Golden Mean are related closely to DNA helical structure. Portion of DNA with L(151) contains 10 DNA codons and is the minimal length containing an integer number of codons.

2. The presence of length scales associated with 1 nm and 2 nm thick structures encourage to consider the possibility of p-adic primes near integers $2^k3^l5^m$ defining generators of multiplicative ideals of integers. They do not satisfy the maximal nearness criterion anymore but would be near to integers representable as products of powers of primes maximally near to powers of two.

What could be the interpretation of the integer k appearing in $p\simeq 2^k$? Elementary particle quantum numbers would be associated with wormhole contacts with size scale of CP_2 whereas elementary particles correspond to p-adic size scale about Compton length. What could determine the size scale of wormhole contact? I have proposed that to p-adic length scale there is associated a scale characterizing wormhole contact and depending logarithmically on it and corresponds to $L_k=(1/2)log(p)L_0=(k/2)log(2)L_0$. The generalization of this hypothesis to the case of $p\simeq 2^k3^l5^m...$ be straightforward and be $L_{k,l,m}=(1/2)(klog(2)+llog(3)+mlog(5)+..)$.

Dark scales and scales of CDs and their relation to p-adic length scale hierarchy

There are two length scale hierarchies. p-Adic length scale hierarchy assignable to space-time surfaces and the dark hierarchy assignable to CDs. One should find an identification of dark scales and understand their relationship to p-adic length scales.

1. Identification of dark scales

The dimension n of the extension provides the roughest measure for its complexity via the formula $h_{eff}/h_0=n$. The basic - rather ad hoc - assumption has been that n as dimension of extension defines not only h_{eff} but also the size scale of CD via $L=nL_0$.

This assumption need not be true generally and already the attempt to understand gravitational constant [L76] as a prediction of TGD led to the proposal that gravitational Planck constant $h_{gr}=n_{gr}h_0=GMm/v_0$ [E6] could be coded by the data relating to a normal subgroup of Galois group appearing as a factor of n.

The most general option is that dark scale is coded by a data related to extension of its sub-extension and this data involves ramified primes. Ramified primes depend on the polynomial defining the extension and there is large number polynomials defining the same extension. Therefore ramified ramifies code information also about polynomial and dynamics of space-time surface.

First some observations.

1. For Galois extension the order n has a natural decomposition to a product of orders n_i of its normal subgroups serving also as dimensions of corresponding extensions: $n=\prod_i n_i$. This implies a decomposition of the group algebra of Galois group to a tensor product of state spaces with dimensions n_i [L82].

2. Could one actually identify several dark scales as the proposed identifications of gravitational, electromagnetic, etc variants of h_{eff} suggest? The hierarchy of normal subgroups of Galois group of rationals corresponds to sub-groups with orders given by $N(i,1)=n_in_{i-1}...n_{i-1}$ of n define orders for the normal subgroups of Galois group. For extensions of $k-1$:th extension of rationals one has $N(i,k)=n_in_{i-1}...n_{i-k}$. The most general option is that these normal subgroups provide only the data allowing to associate dark scales to each of them. The spectrum of h_{eff} could correspond to the $\{N_{i,k}\}$ or at least the set $\{N_{i,1}\}$.

3. The extensions with prime dimension $n=p$ have no non-trivial normal subgroups and $n=p$ would hold for them. For these extensions the state space of group algebra is prime as Hilbert space and does not decompose to tensor product so that it would represent fundamental system. Could these extensions be of special interest physically? SSFRs would naturally involve state function reduction cascades proceeding downwards along hierarchy of normal subgroups and would represent cognitive measurements [L82].

The original guess was that dark scale $L_D=nL_p$, where n is the order n for the extensions and p is a ramified prime for the extension. A generalized form would allow $L_D=N(i,1)L_{p_k}$ for the sub-extension such that p_k is ramified prime for the sub-extension.

2. Can one identify the size scale of CD as dark scale?

It would be natural if the scale of CD would be determined by the extension of rationals. Or more generally, the scales of CD and hierarchy of sub-CDs associated with the extension would be determined by the inclusion hierarchy of extensions and thus correspond to the hierarchy of normal sub-groups of Galois group.

The simplest option would be $L_{CD}=L_D$ so that the size scales of sub-CD would correspond dark scales for sub-extension given by $L_{CD,i}=N(i,1)L_{p_k}$, p_k ramified prime of sub-extension.

1. The differences $|r_i-r_j|$ would correspond to differences for Minkowski time of CD. CD need not contain all values of hyperplanes $t=r_i$ and the evolution by SSFR would gradually bring in day-light all roots r_n of the polynomial P defining space-time surface as "very special moments in the life of self". If the size scale of CD is so large that also the largest value of $|r_i|$ is inside the upper or lower half of CD, the size scale of CD would correspond roughly to the largest p-adic length scale.

   CD contains sub-CDs and these could correspond to normal subgroups of Galois extension as extension of extension of ....

2. One can ask what happens when all special moments $t=r_n$ have been experienced? Does BSFR meaning death of conscious entity take place or is there some other option? In [L77] I considered a proposal for how chaos could emerge via iterations of P during the sequence of SSFRs.

   One could argue that when CD has reached by SSFRs following unitary evolutions a size for which all roots r_n have become visible, the evolution could continues by the replacement of P with $P\circ P$, and so on. This would give rise to iteration and space-time analog for the approach to chaos.

3. Eventually the evolution by SSFRs must stop. Biological arguments suggests that metabolic limitations cause the death of self since the metabolic energy feed is not enough to preserve the distribution of values of h_{eff} (energies increase with $h_{eff}\propto Nn$, for N:th iteration and h_{eff} is reduced spontaneously) [L83].

7.3 Fermionic variant of M^8-H duality

The topics of this section is M^8-H duality for fermions. Consider first the bosonic counterpart of M^8-H duality.

1. The octonionic polynomial giving rise to space-time surface X^4 as its "root" is obtained from ordinary real polynomial P with rational coefficients by algebraic continuation. The conjecture is that the identification in terms of roots of polynomials of even real analytic functions guarantees associativity and one can formulate this as rather convincing argument [L33, L34, L35]. Space-time surface X^4_c is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. These options correspond to complexified-quaternionic tangent- or normal spaces. For $P(x)=x^n+..$ ordinary roots are algebraic integers. The real 4-D space-time surface is projection of this surface from M^8_c to M^8. One could drop the subscripts "$_c$" but in the sequel they will be kept.

   M^4_c appears as a special solution for any polynomial P. M^4_c seems to be like a universal reference solution with which to compare other solutions.

   One obtains also brane-like 6-surfaces as 6-spheres as universal solutions. They have M^4 projection, which is a piece of hyper-surface for which Minkowski time as time coordinate of CD corresponds to a root $t=r_n$ of P. For monic polynomials these time values are algebraic integers and Galois group permutes them.

2. One cannot exclude rational functions or even real analytic functions in the sense that Taylor coefficients are octonionically real (proportional to octonionic real unit). Number theoretical vision - adelic physics [L39], suggests that polynomial coefficients are rational or perhaps

in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

The generalization of the relationship between reals, extensions of p-adic number fields, and algebraic numbers in their intersection is suggestive. The "world of classical worlds" (WCW) would contain the space-time surfaces defined by polynomials with general real coefficients. Real WCW would be continuous space in real topology. The surfaces defined by rational or perhaps even algebraic coefficients for given extension would represent the intersection of real WCW with the p-adic variants of WCW labelled by the extension.

3. $M^8 - H$ duality requires additional condition realized as condition that also space-time surface itself contains 2-surfaces having commutative (complex) tangent or normal space. These surfaces can be 2-D also in metric sense that is light-like 3-D surfaces. The number of these surfaces is finite in generic case and they do not define a slicing of X^4 as was the first expectation. Strong form of holography (SH) makes it possible to map these surfaces and their tangent/normal spaces to 2-D surfaces $M4 \times CP_2$ and to serve as boundary values for the partial differential equations for variational principle defined by twistor lift. Space-time surfaces in H would be minimal surface apart from singularities.

Concerning M^8-H duality for fermions, there are strong guidelines: also fermionic dynamics should be algebraic and number theoretical.

1. Spinors should be octonionic. I have already earlier considered their possible physical interpretation. [L5].

2. Dirac equation as linear partial differential equation should be replaced with a linear algebraic equation for octonionic spinors which are complexified octonions. The momentum space variant of the ordinary Dirac equation is an algebraic equation and the proposal is obvious: $P\Psi = 0$, where P is the octonionic continuation of the polynomial defining the space-time surface and multiplication is in octonionic sense. The conjugation in O_c is induced by the conjugation of the commuting imaginary unit i. The square of the Dirac operator is real if the space-time surface corresponds to the projection $O_c \to M^8 \to M^4$ with real time coordinate and imaginary spatial coordinates so that the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

   The masslessness condition restricts the solutions to light-like 3-surfaces $m_{kl}P^kP^l = 0$ in Minkowskian sector analogous to mass shells in momentum space - just as in the case of ordinary massless Dirac equation. $P(o)$ rather than octonionic coordinate o would define momentum. These mass shells should be mapped to light-like partonic orbits in H.

3. This picture leads to the earlier phenomenological picture about induced spinors in H. Twistor Grassmann approach suggests the localization of the induced spinor fields at light-like partonic orbits in H. If the induced spinor field allows a continuation from 3-D partonic orbits to the interior of X^4, it would serve as a counterpart of virtual particle in accordance with quantum field theoretical picture.

7.3.1 $M^8 - H$ duality for space-time surfaces

It is good to explain M^8-H duality for space-time surfaces before discussing it in fermionic sector.

Space-time as 4-surface in $M^8_c = O_c$

One can regard real space-time surface $X^4 \subset M^8$ as a M^8−-projection of $X^4_c \subset M^8_c = O_c$. M^4_c is identified as complexified quaternions H_c [L60, L75]. The dynamics is purely algebraic and therefore local an associativity is the basic dynamical principle.

1. The basic condition is associativity of $X^4 \subset M^8$ in the sense that either the tangent space or normal space is associative - that is quaternionic. This would be realized if X^4_c as a root for

the quaternion-valued "real" or "imaginary part" for the O_c algebraic continuation of real analytic function $P(x)$ in octonionic sense. Number theoretical universality requires that the Taylor coefficients are rational numbers and that only polynomials are considered.

The 4-surfaces with associative normal space could correspond to elementary particle like entities with Euclidian signature (CP_2 type extremals) and those with associative tangent space to their interaction regions with Minkowskian signature. These two kinds space-time surfaces could meet along these 6-branes suggesting that interaction vertices are located at these branes.

2. The conditions allow also exceptional solutions for any polynomial for which both "real" and "imaginary" parts of the octonionic polynomial vanish. Brane-like solutions correspond to 6-spheres S^6 having $t = r_n$ 3-ball B^3 of light-cone as M^4 projection: here r_n is a root of the real polynomial with rational coefficients and can be also complex - one reason for complexification by commuting imaginary unit i. For scattering amplitudes the topological vertices as 2-surfaces would be located at the intersections of X_c^4 with 6-brane. Also Minkowski space M^4 is a universal solution appearing for any polynomial and would provide a universal reference space-time surface.

3. Polynomials with rational coefficients define EQs and these extensions form a hierarchy realized at the level of physics as evolutionary hierarchy. Given extension induces extensions of p-adic number fields and adeles and one obtains a hierarchy of adelic physics. The dimension n of extension allows interpretation in terms of effective Planck constant $h_{eff} = n \times h_0$. The phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$ behave like dark matter and galactic dark matter could correspond to classical energy in TGD sense assignable to cosmic strings thickened to magnetic flux tubes. It is not completely clear whether number galactic dark matter must have $h_{eff} > h$. Dark energy in would correspond to the volume part of the energy of the flux tubes.

   There are good arguments in favor of the identification $h = 6h_0$ [L45]. "Effective" means that the actual value of Planck constant is h_0 but in many-sheeted space-time n counts the number of symmetry related space-time sheets defining X^4 as a covering space locally. Each sheet gives identical contribution to action and this implies that effective value of Planck constant is nh_0.

   The ramified primes of extension in turn are identified as preferrred p-adic primes. The moduli for the time differences $|t_r - t_s|$ have identification as p-adic time scales assignable to ramified primes [L75]. For ramified primes the p-adic variants of polynomials have degenerate zeros in $O(p) = 0$ approximation having interpretation in terms of quantum criticality central in TGD inspired biology.

4. During the preparation of this article I made a trivial but overall important observation. Standard Minkowski signature emerges as a prediction if conjugation in O_c corresponds to the conjugation with respect to commuting imaginary unit i rather than octonionic imaginary units as though earlier. If the space-time surface corresponds to the projection $O_c \rightarrow M^8 \rightarrow M^4$ with real time coordinate and imaginary spatial coordinates the metric defined by the octonionic norm is real and has Minkowskian signature. Hence the notion of Minkowski metric reduces to octonionic norm for O_c - a purely number theoretic notion.

Realization of $M^8 - H$ duality

$M^8 - H$ duality allows to $X^4 \subset M^8$ to $X^4 \subset H$ so that one has two equivalent descriptions for the space-time surfaces as algebraic surfaces in M^8 and as minimal surfaces with 2-D preferred 2-surfaces defining holography making possible $M^8 - H$ duality and possibly appearing as singularities in H. The dynamics of minimal surfaces, which are also extremals of Kähler action, reduces for known extremals to purely algebraic conditions analogous to holomorphy conditions in string models and thus involving only gradients of coordinates. This condition should hold generally and should induce the required huge reduction of degrees of freedom proposed to be realized also in terms of the vanishing of super-symplectic Noether charges already mentioned [K98].

Twistor lift allows several variants of this basic duality [L71]. M^8_H duality predicts that space-time surfaces form a hierarchy induced by the hierarchy of EQs defining an evolutionary hierarchy. This forms the basics for the number theoretical vision about TGD.

As already noticed, $X^4 \subset M^8$ would satisfy an infinite number of additional conditions stating vanishing of Noether charges for a sub-algebra $SSA_n \subset SSA$ of super-symplectic algebra SSA actings as isometries of WCW.

$M^8 - H$ duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions [L60].

1. Associativity condition for tangent-/normal spaces is the first essential condition for the existence of $M^8 - H$ duality and means that tangent - or normal space is associative/quaternionic.

2. Each tangent space of X^4 at x must contain a preferred $M^2_c(x) \subset M^4_c$ such that $M^2_c(x)$ define an integrable distribution and therefore complexified string world sheet in M^4_c. This gives similar distribution for their orthogonal complements $E^2c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense. $E^2_c(x)$ would correspond to partonic 2-surface. This condition generalizes for X^4 with quaternionic normal space. A possible interpretation is as a space-time correlate for the selection of quantization axes for energy (rest system) and spin.

   One can imagine two realizations for the additional condition.

Option I: Global option states that the distributions $M^2_c(x)$ and $E^2_c(x)$ define a slicing of X^4_c.

Option II: Only a discrete set of 2-surfaces satisfying the conditions exist, they are mapped to H, and strong form of holography (SH) applied in H allows to deduce $X^4 \subset H$. This would be the minimal option.

It seems that only **Option II** can be realized.

1. The basic observation is that X^2_c can be fixed by posing to the non-vanishing H_c-valued part of octonionic polynomial P condition that the C_c-valued "real" or "imaginary" part in C_c sense for P vanishes. M^2_c would be the simplest solution but also more general complex sub-manifolds $X^2_c \subset M^4_c$ are possible. This condition allows only a discrete set of 2-surfaces as its solutions so that it works only for **Option II**.

   These surfaces would be like the families of curves in complex plane defined by $u = 0$ an $v = 0$ curves of analytic function $f(z) = u + iv$. One should have family of polynomials differing by a constant term, which should be real so that $v = 0$ surfaces would form a discrete set.

2. SH makes possible $M^8 - H$ duality assuming that associativity conditions hold true only at 2-surfaces including partonic 2-surfaces or string world sheets or perhaps both. Thus one can give up the conjecture that the polynomial ansatz implies the additional condition globally.

   SH indeed states that PEs are determined by data at 2-D surfaces of X^4. Even if the conditions defining X^2_c have only a discrete set of solutions, SH at the level of H could allow to deduce the PEs from the data provided by the images of these 2-surfaces under $M^8 - H$ duality. The existence of $M^2(x)$ would be required only at the 2-D surfaces.

3. There is however a delicacy involved: X^2 might be 2-D only metrically but not topologically! The 3-D light-like surfaces X^3_L indeed have metric dimension $D = 2$ since the induced 4-metric degenerates to 2-D metric at them. Therefore their pre-images in M^8 would be natural candidates for the singularities at which the dimension of the quaternionic tangent or normal space reduces to $D = 2$ [L59] [K10]. If this happens, SH would not be quite so strong as expected. The study of fermionic variant of $M^8 - H$-duality supports this conclusion.

One can generalize the condition selecting X^2_c so that it selects 1-D surface inside X^2_c. By assuming that R_c-valued "real" or "imaginary" part of complex part of P sense at this 2-surface vanishes. One obtains preferred M^1_c or E^1_c containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as a complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_c \to C_c \to H_c \to O_c$ realized as surfaces.

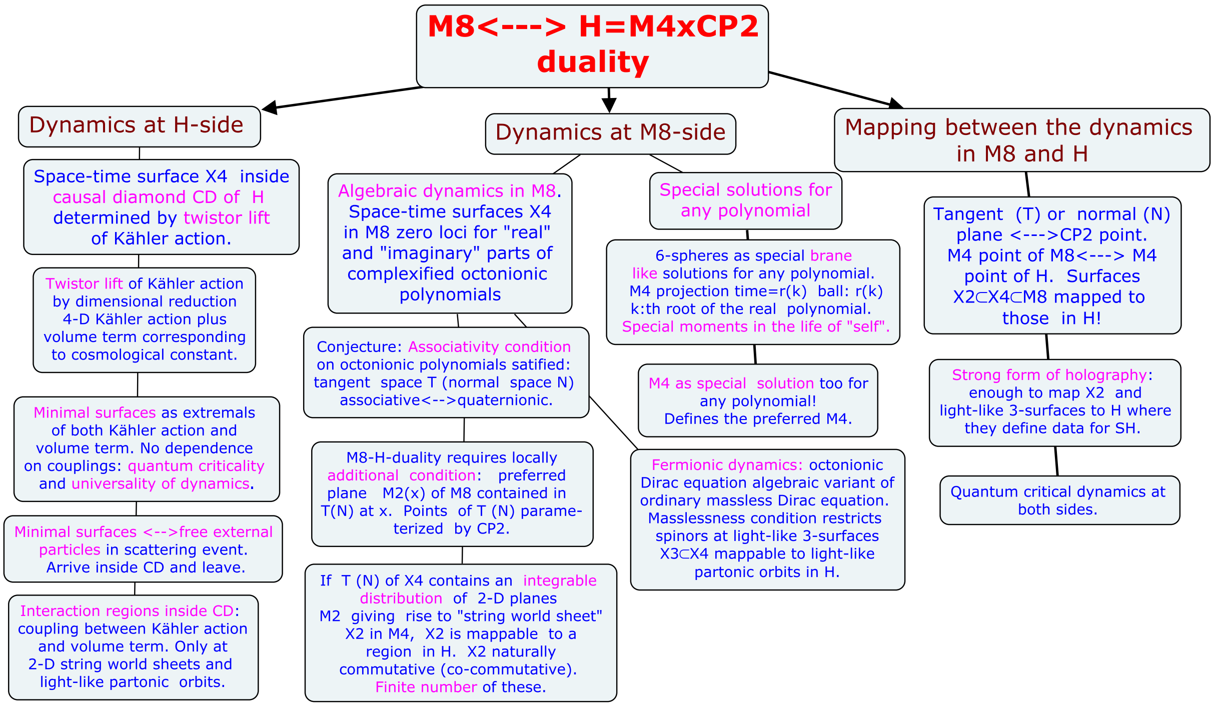

Figure 7.1: $M^8 - H$ duality.

7.3.2 What about $M^8 - H$ duality in the fermionic sector?

During the preparation of this article I become aware of the fact that the realization M^8-H duality in the fermionic sector has remained poorly understood. This led to a considerable integration of the ideas about $M^8 - H$ duality also in the bosonic sector and the existing phenomenological picture follows now from $M^8 - H$ duality. There are powerful mathematical guidelines available.

Octonionic spinors

By supersymmetry, octonionicity should have also fermionic counterpart.

1. The interpretation of M^8_c as complexified octonions suggests that one should use complexified octonionic spinors in M^8_c. This is also suggested by SO(1,7) triality unique for dimension $d = 8$ and stating that the dimensions of vector representation, spinor representation and its conjugate are same and equal to $D = 8$. I have already earlier considered the possibility to interpret M^8 spinors as octonionic [L5]. Both octonionic gamma matrices and spinors have interpretation as octonions and gamma matrices satisfy the usual anti-commutation rules. The product for gamma matrices and gamma matrices and spinors is replaced with non-associative octonionic product.

2. Octonionic spinors allow only one M^8-chirality, which conforms with the assumption of TGD inspired SUSY that only quarks are fundamental fermions and leptons are their local composites [L65].

3. The decomposition of $X^2 \subset X^4 \subset M^8$ corresponding to $R \subset C \subset Q \subset O$ should have analog for the O_c spinors as a tensor product decomposition. The special feature of dimension $D = 8$ is that the dimensions of spinor spaces associated with these factors are indeed 1, 2, 4, and 8 and correspond to dimensions for the surfaces!

   One can define for octonionic spinors associative/co-associative sub-spaces as quaternionic/co-quaternionic spinors by posing chirality conditions. For $X^4 \subset M^8_c$ one could define the analogs of projection operators $P_\pm = (1 \pm \gamma_5)/2$ as projection operators to either factor of the spinor space as tensor product of spinor space associated with the tangent and normal spaces of X^4: the analog of γ_5 would correspond to tangent or normal space depending on whether tangent or normal space is associative. For the spinors with definite chirality there would be no entanglement between the tensor factors. The condition would generalize the chirality condition for massless M^4 spinors to a condition holding for the local M^4 appearing as tangent/normal space of X^4.

4. The chirality condition makes sense also for $X^2 \subset X^4$ identified as complex/co-complex surface of X^4. Now γ_5 is replaced with γ_3 and states that the spinor has well-defined spin in the direction of axis defined by the decomposition of X^2 tangent space to $M^1 \times E^1$ with M^1 defining real octonion axis and selecting rest frame. Interpretation in terms of quantum measurement theory is suggestive.

What about tangent space quantum numbers in M^8 picture. In H-picture they correspond to spin and electroweak quantum numbers. In M^8 picture the geometric tangent space group for a rest system is product $SU(2) \times SU(2)$ with possible modifications due to octonionicity reducing tangent space group to those respecting octonionic automorphisms.

What about the sigma matrices for the octonionic gamma matrices? The surprise is that the commutators of M^4 sigma matries and those of E^4 sigma matrices close to the sama $SO(3)$ algebra allowing interpretation as representation for quaternionic automorphisms. Lorentz boosts are represented trivially, which conforms with the fact that octonion structure fixes unique rest system. Analogous result holds in E^4 degrees of freedom. Besides this one has unit matrix assignable to the generalize spinor structure of CP_2 so that also electroweak $U(1)$ factor is obtained.

One can understand this result by noticing that octonionic spinors correspond to 2 copies of a tensor products of the spinor doublets associated with spin and weak isospin. One has $2 \otimes 2 = 3 \oplus 1$ so that one must have $1 \oplus 3 \oplus 1 \oplus 3$. The octonionic spinors indeed decompose like $1+1+3+\overline{3}$ under $SU(3)$ representing automophisms of the octonions. $SO(3)$ could be interpreted as $SO(3) \subset SU(3)$. $SU(3)$ would be represented as tangent space rotations.

Dirac equation as partial differential equation must be replaced by an algebraic equation

Algebraization of dynamics should be also supersymmetric. The modified Dirac equation in H is linear partial differential equation and should correspond to a linear algebraic equation in M^8.

1. The key observation is that for the ordinary Dirac equation the momentum space variant of Dirac equation for momentum eigenstates is algebraic! Could the interpretation for $M^8 - H$ duality as an analog of momentum-position duality of wave mechanics considered already earlier make sense! This could also have something to do with the dual descriptions of twistorial scattering amplitudes in terms of either twistor and momentum twistors. Already the earlier work excludes the interpretation of the octonionic coordinate o as 8-momentum. Rather, $P(o)$ has this interpretation and o corrresponds to embedding space coordinate.

2. The first guess for the counterpart of the modified Dirac equation at the level of $X^4 \subset M^8$ is $P\Psi = 0$, where Ψ is octonionic spinor and the octonionic polynomial P defining the space-time surface can be seen as a generalization of momentum space Dirac operator with octonion units representing gamma matrices. If associativity/co-associativity holds true, the equation becomes quaternionic/co-quaternionic and reduces to the 4-D analog of massless Dirac equation and of modified Dirac equation in H. Associativity hols true if also Ψ satisfies associativity/co-associativity condition as proposed above.

3. What about the square of the Dirac operator? There are 3 conjugations involved: quaternionic conjugation assumed in the earlier work, conjugation with respect to i, and their combination. The analog of octonionic norm squared defined as the product $o_c o_c^*$ with conjugation with respect to i only, gives Minkowskian metric $m_{kl}o^k\overline{o}^l$ as its real part. The imaginary part of the norm squared is vanishing for the projection $O_c \to M^8 \to M^4$ so that time coordinate is real and spatial coordinates imaginary. Therefore Dirac equation allows solutions only for the M^4 projection X^4 and M^4 ($M8$) signature of the metric can be said to be an outcome of quaternionicity (octonionicity) alone in accordance with the duality between metric and algebraic pictures.

   Both $P^\dagger P$ and PP should annihilate Ψ. $P^\dagger P\Psi = 0$ gives $m_{kl}P^k\overline{P}^l = 0$ as the analog of vanishing mass squared in M^4 signature in both associative and co-associative cases. $PP\Psi = 0$ reduces to $P\Psi = 0$ by masslessness condition. One could perhaps interpret the projection $X_c^4 \to M^8 \to M^4$ in terms of Uncertainty Principle.

   There is a $U(1)$ symmetry involved: instead of the plane M^8 one can choose any plane obtained by a rotation $exp(i\phi)$ from it. Could it realize quark number conservation in M^8 picture?

   For $P = o$ having only $o = 0$ as root $Po = 0$ reduces to $o^\dagger o = 0$ and o takes the role of momentum, which is however vanishing. 6-D brane like solutions S^6 having $t = r_n$ balls $B^3 \subset CD_4$ as M^4 projections one has $P = 0$ so that the Dirac equation trivializes and does not pose conditions on Ψ. o would have interpretation as space-time coordinates and $P(o)$ as position dependent momentum components P^k.

   The variation of P at mass shell of M_c^8 (to be precise) could be interpreted in terms of the width of the wave packet representing particle. Since the light-like curve at partonic 2-surface for fermion at X_L^3 is not a geodesic, mass squared in M^4 sense is not vanishing. Could one understand mass squared and the decay width of the particle geometrically? Note that mass squared is predicted also by p-adic thermodynamics [K68].

4. The masslessness condition restricts the spinors at 3-D light-cone boundary in $P(M^8)$. M^8-H duality [L60] suggests that this boundary is mapped to $X_L^3 \subset H$ defining the light-like orbit of the partonic 2-surface in H. The identification of the images of $P_kP^k = 0$ surfaces as X_L^3 gives a very powerful constraint on SH and M^8-H duality.

5. Also at 2-surfaces $X^2 \subset X^4$ an the variant Dirac equation would hold true and should commute with the corresponding chirality condition. Now $D^\dagger D\Psi = 0$ gives 2-D variant of masslessness condition with 2-momentum components represented by those of P. 2-D masslessness locates the spinor to a 1-D curve X_L^1. Its H-image would naturally contain the boundary of the string word sheet at X_L^3 assumed to carry fermion quantum numbers and also the boundary of string world sheet at the light-like boundary of CD_4. The interior of string world sheet in H would not carry induced spinor field.

6. The general solution for both 4-D and 2-D cases can be written as $\Psi = P\Psi_0$, Ψ_0 a constant spinor - this in a complete analogy with the solution of modified Dirac equation in H. P depends on position: the WKB approximation using plane waves with position dependent momentum seems to be nearer to reality than one might expect.

The phenomenological picture at H-level follows from the M^8-picture

Remarkably, the partly phenomenological picture developed at the level of H is reproduced at the level of M^8. Whether the induced spinor fields in the interior of X^4 are present or not, has been long standing question since they do not seem to have any role in the physical picture. The proposed picture answers this question.

Consider now the explicit realization of M^8-H-duality for fermions.

1. SH and the expected analogy with the bosonic variant of M^8-H duality lead to the first guess. The spinor modes in $X^4 \subset M^8$ restricted to X^2 can be mapped by M^8-H-duality to those at their images $X^2 \subset H$, and define boundary conditions allowing to deduce the

solution of the modified Dirac equation at $X^4 \subset H$. X^2 would correspond to string world sheets having boundaries X^1_L at X^3_L.

The guess is not quite correct. Algebraic Dirac equation requires that the solutions are restricted to the 3-D and 1-D mass shells $P_k P^k = 0$ in M^8. This should remain true also in H and X^3_L and their 1-D intersections X^1_L with string world sheets remain. Fermions would live at boundaries. This is just the picture proposed for the TGD counterparts of the twistor amplitudes and corresponds to that used in twistor Grassmann approach!

For 2-D case constant octonionic spinors Ψ_0 and gamma matrix algebra are equivalent with the ordinary Weyl spinors and gamma matrix algebra and can be mapped as such to H. This gives one additional reason for why SH must be involved.

2. At the level of H the first guess is that the modified Dirac equation $D\Psi = 0$ is true for D based on the modified gamma matrices associated with both volume action and Kähler action. This would select preferred solutions of modified Dirac equation and conform with the vanishing of super-symplectic Noether charges for SSA_n for the spinor modes. The guess is not quite correct. The restriction of the induced spinors to X^3_L requires that Chern-Simons action at X^3_L defines the modified Drac action.

3. The question has been whether the 2-D modified Dirac action emerges as a singular part of 4-D modified Dirac action assignable to singular 2-surface or can one assign an independent 2-D Dirac action assignable to 2-surfaces selected by some other criterion. For singular surfaces $M^8 - H$ duality fails since tangent space would reduce to 2-D space so that only their images can appear in SH at the level of H.

   This supports the view that singular surfaces are actually 3-D mass shells M^8 mapped to X^3_L for which 4-D tangent space is 2-D by the vanishing of $\sqrt{g_4}$ and light-likeness. String world sheets would correspond to non-singular $X^2 \subset M^8$ mapped to H and defining data for SH and their boundaries $X^1_L \subset X^3_L$ and $X^1_L \subset CD_4$ would define fermionic variant of SH.

What about the modified Dirac operator D in H?

1. For X^3_L modified Dirac equation $D\Psi = 0$ based on 4-D action S containing volume and Kähler term is problematic since the induced metric fails to have inverse at X^3_L. The only possible action is Chern-Simons action S_{CS} used in topological quantum field theories and now defined as sum of C-S terms for Kähler actions in M^4 and CP_2 degrees of freedom. The presence of M^4 part of Kähler form of M^8 is forced by the twistor lift, and would give rise to small CP breaking effects explaining matter antimatter asymmetry [L65]. S_{C-S} could emerge as a limit of 4-D action.

   The modified Dirac operator D_{C-S} uses modified gamma matrices identified as contractions $\Gamma^\alpha_{CS} = T^{\alpha k}\gamma_k$, where $T^{\alpha k} = \partial L_{CS}/\partial(\partial_\alpha h^k)$ are canonical momentum currents for S_{C-S} defined by a standard formula.

2. CP_2 part would give conserved Noether currents for color in and M^4 part Poincare quantum numbers: the apparently small CP breaking term would give masses for quarks and leptons! The bosonic Noether current $J_{B,A}$ for Killing vector j^k_A would be proportional to $J^\alpha_{B,A} = T^\alpha_k j_A k$ and given by $J_{B,A} = \epsilon^{\alpha\beta\gamma}[J_{\beta\gamma}A_k + A_\beta J_{\gamma k}]j^k_A$.

   Fermionic Noether current would be $J_{F,A} = \overline{\Psi} J^\alpha \Psi$ 3-D Riemann spaces allow coordinates in which the metric tensor is a direct sum of 1-D and 2-D contributions and are analogous to expectation values of bosonic Noether currents. One can also identify also finite number of Noether super currents by replacing $\overline{\Psi}$ or Ψ by its modes.

3. In the case of X^3_L the 1-D part light-like part would vanish. If also induced Kähler form is non-vanishing only in 2-D degrees of freedom, the Noether charge densities J^t reduce to $J^t = JA_k j^k_A$, $J = \epsilon^{\alpha\beta\gamma}J_{\beta\gamma}$ defining magnetic flux. Modified Dirac operator would reduce to $D = JA_k\gamma^k D_t$ and 3-D solutions would be covariantly constant spinors along the light-like geodesics parameterized by the points 2-D cross section. One could say that the number of solutions is finite and corresponds to covariantly constant modes continued from X^1_L to X^3_L. This picture is just what twistor Grassmannian approach led to [L51].

A comment inspired by the ZEO based quantum measurement theory

I cannot resist the temptation to make a comment relating to quantum measurement theory inspired by zero energy ontology (ZEO) extending to a theory of consciousness [L64, L82, L83].

I have proposed [L75, L77] that the time evolution by "big" state function reductions (BSFRs) could be induced by iteration of real polynomial P - at least in some special cases. The foots of the real polynomial P would define a fractal at the limit of larger number of iterations. The roots of n-fold iterate $\circ^n P$ would contain the inverse images under $\circ^{-n+1}P$ of roots of P and for $P(0) = 0$ the inverse image $\circ^n P$ would consist of inverse images under $\circ^{-k}P$, $k = 0, ..., n-1$, of roots of P.

Also the mass shells for $\circ^n P$ would be unions of inverses images under $\circ^{-k}P$, $k = 0, , ..., n-1$, of roots of P. This gives rather concrete view about evolution of M^4 projections of the partonic orbits. A rough approximate expression for the largest root of real P approximated as $P(x) \simeq a_n x^n + an - 1ix^{n-1}$ for large x is $x_{max} \sim a_n/a_{n-1}$. For $\circ^n P$ one obtains the same estimate. This suggests that the size scales of the partonic orbits are same for the iterates. The mass shells would not differ dramatically: could they have an interpretation in terms of mass splitting?

The evolution by iteration would add new partonic orbits and preserve the existing ones: this brings in mind conservation of genes in biological evolution. This is true also for a more general evolution allowing general functional decomposition $Q \rightarrow Q \circ P$ to occur in BSFR.

What next in TGD?

The construction of scattering amplitudes has been the dream impossible that has driven me for decades. Maybe the understanding of fermionic $M^8 - H$ duality provides the needed additional conceptual tools. The key observation is utterly trivial but far reaching: there are 3 possible conjugations for octonions corresponding to the conjugation of commutative imaginary unit or of octonionic imaginary units or both of them. 1st norm gives a real valued norm squared in Minkowski signature natural at M^8 level! Second one gives a complex valued norm squared in Euclidian signature. 1st and 2nd norms are equivalent for octonions light-like with respect to the first norm. The 3rd conjugation gives a real-valued Euclidian norm natural at the level of Hilbert space.

1. M^8 picture looks simple. Space-time surfaces in M^8 can be constructed from real polynomials with real (rational) coefficients, actually knowledge of their roots is enough. Discrete data - roots of the polynomial!- determine space-time surface as associative or co-associative region! Besides this one must pose additional condition selecting 2-D string world sheets and 3-D light-like surfaces as orbits of partonic 2-surfaces. These would define strong form of holography (SH) allowing to map space-time surfaces in M^8 to $M^4 \times CP_2$.

2. Could SH generalize to the level of scattering amplitudes expressible in terms of n-point functions of CFT?! Could the n points correspond to the roots of the polynomial defining space-time region!

   Algebraic continuation to quaternion valued scattering amplitudes analogous to that giving space-time sheets from the data coded SH should be the key idea. Their moduli squared are real - this led to the emergence of Minkowski metric for complexified octonions/quaternions) would give the real scattering rates: this is enough! This would mean a number theoretic generalization of quantum theory.

3. One can start from complex numbers and string world sheets/partonic 2-surfaces. Conformal field theories (CFTs) in 2-D play fundamental role in the construction of scattering string theories and in modelling 2-D statistical systems. In TGD 2-D surfaces (2-D at least metrically) code for information about space-time surface by strong holography (SH) .

   Are CFTs at partonic 2-surfaces and string world sheets the basic building bricks? Could 2-D conformal invariance dictate the data needed to construct the scattering amplitudes for given space-time region defined by causal diamond (CD) taking the role of sphere S^2 in CFTs. Could the generalization for metrically 2-D light-like 3-surfaces be needed at the level of "world of classical worlds" (WCW) when states are superpositions of space-time surfaces, preferred extremals?

The challenge is to develop a concrete number theoretic hierarchy for scattering amplitudes: $R \rightarrow C \rightarrow H \rightarrow O$ - actually their complexifications.

1. In the case of fermions one can start from 1-D data at light-like boundaries LB of string world sheets at light-like orbits of partonic 2-surfaces. Fermionic propagators assignable to LB would be coded by 2-D Minkowskian QFT in manner analogous to that in twistor Grassmann approach. n-point vertices would be expressible in terms of Euclidian n-point functions for partonic 2-surfaces: the latter element would be new as compared to QFTs since point-like vertex is replaced with partonic 2-surface.

2. The fusion (product?) of these Minkowskian and Euclidian CFT entities corresponding to different realization of complex numbers as sub-field of quaternions would give rise to 4-D quaternionic valued scattering amplitudes for given space-time sheet. Most importantly: there moduli squared are real for both norms.

   It is not quite clear whether one must use the 1st Minkowskian norm requiring "time-like" scattering amplitudes to achieve non-negative probabilities or use the 3rd norm to get the ordinary positive-definite Hilbert space norm. A generalization of quantum theory (CFT) from complex numbers to quaternions (quaternionic "CFT") would be in question.

3. What about several space-time sheets? Could one allow fusion of different quaternionic scattering amplitudes corresponding to different quaternionic sub-spaces of complexified octonions to get octonion-valued non-associative scattering amplitudes. Again scattering rates would be real. This would be a further generalization of quantum theory.

There is also the challenge to relate M^8- and H-pictures at the level of WCW. The formulation of physics in terms of WCW geometry [K98, L70] leads to the hypothesis that WCW Kähler geometry is determined by Kähler function identified as the 4-D action resulting by dimensional reduction of 6-D surfaces in the product of twistor spaces of M^4 and CP_2 to twistor bundles having S^2 as fiber and space-time surface $X^4 \subset H$ as base. The 6-D Kähler action reduces to the sum of 4-D Kähler action and volume term having interpretation in terms of cosmological constant.

The question is whether the Kähler function - an essentially geometric notion - can have a counterpart at the level of M^8.

1. SH suggests that the Kähler function identified in the proposed manner can be expressed by using 2-D data or at least metrically 2-D data (light-like partonic orbits and light-like boundaries of CD). Note that each WCW would correspond to a particular CD.

2. Since 2-D conformal symmetry is involved, one expects also modular invariance meaning that WCW Kähler function is modular invariant, so that they have the same value for $X^4 \subset H$ for which partonic 2-surfaces have induced metric in the same conformal equivalence class.

3. Also the analogs of Kac-Moody type symmetries would be realized as symmetries of Kähler function. The algebra of super-symplectic symmetries of the light-cone boundary can be regarded as an analog of Kac-Moody algebra. Light-cone boundary has topology $S^2 \times R_+$ where R_+ corresponds to radial light-like ray parameterized by radial light-like coordinate r. Super symplectic transformations of $S^2 \times CP_2$ depend on the light-like radial coordinate r, which is analogous to the complex coordinate z for he Kac-Moody algebras.

   The infinitesimal super-symplectic transformations form algebra SSA with generators proportional to powers r^n . The Kac-Moody invariance for physical states generalizes to a hierarchy of similar invariances. There is infinite fractal hierarchy of sub-algebras $SSA_n \subset SSA$ with conformal weights coming as n-multiples of those for SSA. For physical states SSA_n and $[SSA_n, SSA]$ would act as gauge symmetries. They would leave invariant also Kähler function in the sector WCW_n defined by n. This would define a hierarchy of sub- WCWs of the WCW assignable to given CD.

   The sector WCW$_n$ could correspond to extensions of rationals with dimension n, and one would have inclusion hierarchies consisting of sequences of n_i with n_i dividing n_{i+1}. These inclusion hierarchies would naturally correspond to those for hyper-finite factors of type II$_1$ [K127].

7.4 Cognitive representations and algebraic geometry

The general vision about cognition is realized in terms of adelic physics as physics of sensory experience and cognition [L39, L37]. Rational points and their generalization as ratios of algebraic integers for geometric objects would define cognitive representations as points common for real and various p-adic variants of the space-time surface. The finite-dimensionality for induced p-adic extensions allows also extensions of rationals involving root of e and its powers. This picture applies both at space-time level, embedding space level, and at the level of space-time surfaces but basically reduces to embedding space level. Hence counting of the (generalized) rational points for geometric objects would be determination of the cognitive representability.

7.4.1 Cognitive representations as sets of generalized rational points

The set of rational points depends on the coordinates chosen and one can argue that one must allow different cognitive representations and classify them according to their effectiveness.

How uniquely the M^8_c coordinates can be chosen?

1. Polynomial property allows only linear transformations of the complex octonionic coordinates with coefficients which belong to the extension of rationals used. This poses extremely strong restrictions on the allowed representations once the quaternionic moduli defining a foliation of M^4_0 is chosen. One has therefore moduli space of quaternionic structures. One must also fix the time axis in M^4 assignable to real octonions.
2. One can also define several inequivalent octonionic structures and associate a moduli space to these. The moduli space for octonionic structures would correspond to the space of $M^4_0 \subset M^8$s as quaternionic planes containing fixed M^2_0. One can allow even allow Lorentz transforms mixing real and imaginary octonionic coordinates. It seems that these moduli are not relevant at the level of H.

What could the precise definition of rationality?

1. The coordinates of point are rational in the sense defined by the extension of rationals used. Suppose that one considers parametric representations of surfaces as maps from space-time surface to embedding space. Suppose that one uses as space-time coordinates subset of preferred coordinates for embedding space. These coordinate changes cannot be global and one space-time surface decomposes to regions in which different coordinates apply.
2. The coordinate transformations between over-lapping regions are birational in the sense that both the map and its inverse are in terms of rational functions. This makes the notion of rationality global.
3. When cognitively easy rational parametric representations are possible? For algebraic curves with $g \geq 2$ in CP_2 represented as zeros of polynomials this cannot the case since the number of rational points is finite for instance for $g \geq 2$ surfaces. There is simple explanation for this. Solving second complex coordinate in terms of the other one gives it as an algebraic function for $g \geq 2$: this must be the reason for the loss of dense set of rational points. For elliptic surfaces $y^2 - x^3 - ax - b = 0$ y^2 is however polynomial of x and one can find rational parametric representation by taking y^2 as coordinate [L29]. For $g = 0$ one has linear equations and one obtains dense set of rational points. For conic sections one can also have dense set of rational points but not always. Generalizing from this it would seem that the failure to have rational parametric representation is the basic reason for the loss of dense set of rational points.

This picture does not work for general surfaces but generalizes for algebraic varieties defined by several polynomial equations. The co-dimension $d_c = 1$ case is however unique and the most studied one since for several polynomial equations one encounters technical difficulties when the intersection of the surfaces defined by the d_c polynomials need not be complete for $d_c > 1$. In the recent situation one has $d_c = 4$ but octonion analyticity could be powerful enough symmetry to solve the problem of non-complete intersections by eliminating them or providing a physical interpretation for them.

7.4.2 Cognitive representations assuming $M^8 - H$ duality

Many questions should be answered.

1. Can one generalize the results applying to algebraic varieties? Could the general vision about rational and potentially dense set of rational points generalize?. At M^8 side the description of space-time surfaces as algebraic varieties indeed conforms with this picture. Could one understand SH from the fact that real analyticity octonionic polynomials are determined by ordinary polynomial real coordinate completely? In information theoretic sense sense SH reduces to 1-D holography and the polynomial property makes the situation effectively discrete since finite number of points of real axis allows to determine the octonionic polynomial completely! It is a pity that one cannot measure octonionic polynomial directly!

2. Also the notion of Zariski dimension should make sense in TGD at M^8 side. Preferred extremals define the notion of closed set for given CD at M^8 side? It would indeed seem that one define Zariski topology at the level of M^8_c. Zariski topology would require 4-surfaces, string world sheets, or partonic 2-surfaces and even 1-D curves. This picture conforms with the recent view about TGD and resembles the M-theory picture, where one has branes. SH suggests that the analog of Zariski dimension of space-time surface reduces to that for strings world sheets and partonic 2-surfaces and that even these are analogous to 1-D curves by complex analyticity. Integrability of TGD and preferred extremal property would indeed suggest simplicity.

   $M^8 - H$ hypothesis suggests that these conjectures make sense also at H side. String world sheets, partonic 2-surface, space-like 3-surfaces at the ends of space-time surface at boundaries of CD, and light-like 3-surfaces correspond to closed sets also at the level of WCW in the topology most natural for WCW.

3. Also the problems related to Minkowskian signature could be solved. String world sheets are problematic because of the Minkowskian signature. They however have the topology of disk plus handles suggesting immediately a vision about cognitive representations in terms of rational points. One can can complexify string world sheets and it seems possible to apply the results of algebraic geometry holding true in Euclidian signature. This would be analogous to the Wick rotation used in QFTs and also in twistor Grassmann approach.

4. What about algebraic geometrization of the twistor lift? How complex are twistor spaces of M^4, CP_2 and space-time surface? How can one generalize twistor lift to the level of M^8. S^2 bundle structure and the fact that S^2 allows a dense set of rational suggests that the complexity of twistor space is that of space-time surface itself so that the situation actually reduces to the level of space-time surfaces.

Suppose one accepts $M^8 - H$ duality requiring that the tangent space of space-time surface at given point x contains $M^2(x)$ such that $M^2(x)$ define an integrable distribution giving rise to string world sheets and their orthogonal complements give rise to partonic 2-surfaces. This would give rise to a foliation of the space-time surface by string world sheets and partonic 2-surface conjecture on basis of the properties of extremals of Kähler action. As found these foliations could correspond to quaternion structures that is allowed choices of quaterionic coordinates.

Should one define cognitive representations at the level of M^8 or at the level of $M^4 \times CP_2$? Or both? For M^8 option the condition that space-time point belongs to an extension of rationals applies at the level of M^8 coordinates. For $M^4 \times CP_2$ option cognitive representations are at the level of M^4 and CP_2 parameterizing the points of M^4 and their tangent spaces. The formal study of partial differential equations alone does not help much in counting the number of rational points. One can define cognitive representation in very many ways, and some cognitive representation could be preferred only because they are more efficient than others. Hence both cognitive representations seems to be acceptable.

Some cognitive representations are more efficient than others. General coordinate invariance (GCI) at the level of cognition is broken. The precise determination of cognitive efficiency is a challenge in itself. For instance, the use of coordinates for which coordinate lines are orbits of subgroups of the symmetry group should be highly efficient. Only coordinate transformations mediated by bi-rational maps can take polynomial representations to polynomial representations.

It might well be that only a rational (in generalized sense) sub-group G_2 of octonionic automorphisms is allowed. For rational surfaces allowing parametric representation in terms of polynomial functions the rational points form a dense set.

The cognitive resolution for a dense set of rational points is unrealistically high since cognitive representation would contain infinite number of points. Hence one must tighten the notion of cognitive representation. The rational points must contain a fermion. Fermions are indeed are identified as correlates for Boolean cognition [K31]. This would suggests a view in which cognitive representations are realized at the light-like orbits of partonic 2-surfaces at which Minkowskian associative and Euclidian co-associative space-time surfaces meet. The general wisdom is that rational points are localized to lower-dimensional sub-varieties (Bombieri-Lang conjecture): this conforms with the view that fermion lines reside at the orbits of partonic 2-surfaces.

7.4.3 Are the known extremals in H easily cognitively representable?

Suppose that one takes TGD inspired adelic view about cognition seriously. If cognitive representations correspond to rational points for an extension of rationals, then the surfaces allowing large number of this kind of points are easily representable cognitively by adding fermions to these points. One could even speculate that mathematical cognition invents those geometric objects, which are easily cognitively representable and thus have a large number of rational points.

Could the known extremals of twistor lift be cognitively easy?

Also TGD is outcome of mathematical cognition. Could the known extremals of the twistor lift of Kähler action be cognitively easy? This is suggested by the fact that even such a pariah class theoretician as I am have managed to discover then! Positive answer could be seen as support for the proposed description of cognition!

1. If one believes in $M^8 - H$ duality and the proposed identification of associative and co-associative space-time surfaces in terms of algebraic surfaces in octonionic space M^8_c, the generalization of the results of algebraic geometry should give overall view about the cognitive representations at the level of M^8. In particular, surfaces allowing rational parametric representation (polynomials would have rational coefficients) would allow dense set or rational points since the images of rational points are rational. Rationals are understood here as ratios of algebraic integers in extension of rationals.

2. Also for H the existence of parameter representation using preferred H-coordinates and rational functions with rational coefficients implies that rational points are dense. If $M^8 - H$ correspondence maps the parametric representations in terms of rational functions to similar representations, dense set of rational points is preserved in the correspondence. There is however no obvious reason why $M^8 - H$ duality should have this nice property.

   One can even play with the idea that the surfaces, which are cognitively difficult at the M^8 side, might be cognitively easy at H-side or vice versa. Of course, if the explicit representation as algebraic functions makes sense at M^8 side, this side looks cognitively ridiculously easy as compared to H side. The preferred extremal property and SH can however change the situation.

3. At M^8 side and for a given point of M^4 there are several points of E^4 (or vice versa) if the degree of the polynomial is larger than $n = 1$ so that for the image of the surface H there are several CP_2 points for a given point of M^4 (or vice versa) depending on the choice of coordinates. This is what the notion of the many-sheeted space-time predicts.

4. The equations for the surface at H side are obtained by a composite map assigning first to the coordinates of $X^4 \subset M^8$ point of $M^4 \times E^4$, and then assigning to the points of $X^4 \subset M^8$ CP_2 coordinates of the tangent space of the point. At this step the slightly non-local tangent space information is fed in and the surfaces in $M^4 \times CP_2$ cannot be given by zeros of polynomials. The indeed satisfy instead of algebraic equations partial differential equations given by the Kähler action for the twistor lift TGD. Algebraic equations instead of partial differential equations suggests that the M^8 representation is much simpler than H-representation. On the other hand, reduction to algebraic equations at M^8 side could have interpretation in terms of the conjectured complete integrability of TGD [K10, K116].

Testing the idea about self-reference

In any case, it is possible to test the idea about self-reference by looking whether the known extremals in H are cognitively easy and even have a dense set of rational points in natural coordinates. Here I will consider the situation at the level of $M^4 \times CP_2$. It was already found that the known extremals can have inverse images in M^8.

1. Canonically imbedded M^4 with linear coordinates and constant CP_2 coordinates rational is the simple example about preferred extremal and it seems that TGD based cosmology at microscopic relies on these extremals. In this case it is obvious that one has a dense set of rational points at both sides. Could this somehow relate to the fact that physics as physics M^4 was discovered before general relativity?

   Canonically imbedded M^4 corresponds to a first order octonionic polynomial for which imaginary part is put to constant so that tangent space is same everywhere and corresponds to a constant CP_2 coordinate.

2. CP_2 type extremals have 4-D CP_2 projection and light-like geodesic line of M^4 as M^4 projection. One can choose the time parameter as a function of CP_2 coordinates in infinitely many ways. Clearly the rational points are dense in any CP_2 coordinates.

3. Massless extremals (MEs) are given as zeros of arbitrary functions of CP_2 coordinates and 2 M^4 coordinates representing local light-like direction and polarization direction orthogonal to it. In the simplest situation these directions are constant. In the general case light-like direction would define tangent space of string world sheet giving rise also to a distribution of ortogonal polarization planes. This is consistent with the general properties of the M^8 representation and corresponds to the decomposition of quaternionic tangent plane to complex plane and its complement. One can ask whether one should allow only polynomials with rational coefficients as octonionic polynomials.

4. String like objects $X^2 \times Y^2$ with $X^2 \subset M^4$ a minimal surface and Y^2 complex or Lagrangian surface of CP_2 are also basic extremals and their deformations in M^4 directions are expected to give rise to magnetic flux tubes.

   If Y^2 is complex surface with genus $g = 0$ rational points are dense. Also for $g = 1$ one obtains a dense set of rational points in some extension of rationals. For elliptic curves one has lattice of rational points. What happens for Lagrangian surfaces Y^2? In this case one does not have complex curves but real co-dimension 2 surfaces. There is no obvious objection why these surfaces would not be possible.

5. What about string world sheets? If the string world is static $M^2 \subset M^4$ one has a dense set of rational points. One however expects something more complex. If the string world sheet is rational map M^2 to its orthogonal complement E^2 one has rational surface. For rotating strings this does not make sense except for certain period of time. If the choice of the quaternion structure corresponds to a choice of minimal surface in M^4 as integrable distribution for $M^2(x)$, the coordinates associated with the Hamilton-Jacobi structure could make the situation simple.

   If one restricts the consideration the intersections of partonic 2-surfaces and string world sheets at two boundaries of CD the situation simplifies and the question is only about the rationality of the M^4 coordinates at rational points of $Y^2 \subset CP_2$. This would simplify the situation enormously and might even allow to use existing knowledge.

6. The slicing of of space-time surfaces by string world sheets and partonic 2-surfaces required by Hamilton-Jacobi structure could be seen as a fibering analogous to that possessed by elliptic surfaces. This suggest that M^8 counterparts of spacetime surfaces are not of general type in Kodaira classification and that the number of rational points can be large. If the existence of Hamilton-Jacobi structure does not allow handles, this factor would be cognitively simple. This would however suggests that fermion number is not localized at the ends of strings only - as assumed in the construction of scattering amplitudes inspired by twistor Grassmann approach [K49] - but also to the interior of the light-like curves inside string world sheets.

7.4.4 Twistor lift and cognitive representations

What about twistor lift of TGD replacing space-time surfaces with their twistor spaces. Consider first M^8 side.

1. At M^8 side S^2 seems to introduce nothing new. One might expect that the situation does not change at H-side since space-time surfaces are obtained essentially by dimensional reduction and the possible problem relates to the choice of base space as section of is twistor bundle and the embedding of space-time as base space could have singularities at the boundary of Euclidian and Minkowskian space-time regions as discussed in [L29].

   At the side of M^8 the proposed induction of twistor structure is just a projection of the twistor sphere S^6 to its geodesic sphere and one has 4-D moduli space for geodesic spheres $S^2 \subset S^6$. If one interprets the choice of $S^2 \subset S^6$ as as a section in the moduli space, the moduli of S^2 can depend on the point of space-time surface. Note that there are is also a position dependent choice of preferred point of S^2 representing Kähler form, and this choice is good candidate for giving rise to Hamilton-Jacobi structures with position dependent M^2.

2. The notion of Kodaira dimension is defined also for co-dimension 4 algebraic varieties in M^8_c. The cognitively easiest spacetime surfaces would allow rational parametric representation with complex coordinates serving as parameters. If this is not possible, one has algebraic functions, which makes the situation much more complex so that the number of rational points would be small.

3. For some complex enough extensions of rationals the set of rational points can be dense. $g \geq 2$ genera are basic example and one expects also in more general case that polynomials involving powers larger than $n = 4$ make the situation problematic. The condition that real or imaginary part of real analytic octonionic polynomial is in question is a strong symmetry expected to faciliate cognitive representability.

4. The general intuitive wisdom from algebraic geometry is that the rational points are dense only in lower-dimensional sub-varieties (Bombieri-Lang and Vojta conjectures mentioned in the first section). The general vision inspired by SH and the proposal for the construction of twistor amplitudes indeed is that the algebraic points (rational in generalized sense) defining cognitive representations are associated with the intersections of string world sheets and partonic 2-surfaces to which fermions are assigned. This would suggest that partonic 2-surfaces and string world sheets contain the cognitive representation, which under additional conditions can contain very many points.

5. An interesting question concerns the M^8 counterparts of partonic 2-surfaces as space-time regions with Minkowskian and Euclidian signature. The partonic orbits representing the boundaries between these regions should be mapped to each other by $M^8 - H$ duality. This conforms with the fact that induced metric must have degenerate signature $(0, -1, -1, -1)$ at partonic orbits. Can one assume that the topologies of partonic 2-surfaces at two sides are identical? Consider partonic 2-surface of genus g in $M^4 \times CP_2$ - say at the boundary of CD. It should be inverse image of a 2-surface in $M^4 \times E^4$ such that the tangent space of this surface labelled by CP_2 coordinates is mapped to a 2-surface in $M^4 \times CP_2$. If the inverse of $M^8 - H$ correspondence is continuous one expects that g is preserved.

Consider next the H-side. There is a conjecture that for Cartesian product the Kodaira dimension is sum $d_K = \sum_i d_{K,i}$ of the Kodaira dimensions for factors. Suppose that CP_1 fiber as surface in the 12-D twistor bundle $T(M^4) \times T(CP_2)$ has Kodaira dimension $d_K(CP_1) = -\infty$ (it is expected to be rational surface) then the fact that the bundle decomposes to Cartesian product locally and rational points are pairs of rational points in the factors, is indeed consistent with the proposal. S^2 would give dense set of rational points in S^2 and the bundle would have infinite number of rational points.

In TGD context, it is however space-time surface which matters. Space-time surface as section of the bundle would not however have a dense set of points in the general case and the relevant Kodaira dimension be $d_K = d_K(X^4)$. One can of course ask whether the space-time surface as an algebraic section (not many of them) of the twistor bundle could chosen to be cognitively simple.

7.4.5 What does cognitive representability really mean?

The following considerations reflect the ideas inspired by Face Book debate with Santeri Satama (SS) relating to the notion of number and the notion of cognitive representation.

SS wants to accept only those numbers that are constructible, and SS mentioned the notion of demonstrability due to Gödel. According to my impression demonstrability means that number can be constructed by a finite algorithm or at least that the information needed to construct the number can be constructed by a finite algorithm although the construction itself would not be possible as digit sequence in finite time. If the constructibility condition is taken to extreme, one is left only with rationals.

As a physicists, I cannot consider starting to do physics armed only with rationals: for instance, continuous symmetries and the notion of Riemann manifold would be lost. My basic view is that we should identify the limitations of cognitive representability as limitations for what can exist. I talked about cognitive representability of numbers central in the adelic physics approach to TGD. Not all real numbers are cognitively representable and need not be so.

Numbers in the extensions of rationals would be cognitively representable as points with coordinates in an extension of rationals. The coordinates themselves are highly unique in the octonionic approach to TGD and different coordinates choices for complexified octonionic M^8 are related by transformations changing the moduli of the octonion structure. Hence one avoids problems with general coordinate invariance). Not only algebraic extensions of rationals are allowed. Neper number e is an exceptional transcendental in that e^p is p-adic number and finite-D extensions of p-adic numbers by powers for root of e are possible.

My own basic interest is to find a deeper intuitive justification for why algebraic numbers shoud be cognitively representable. The naïve view about cognitive representability is that the number can be produced in a finite number of steps using an algorithm. This would leave only rationals under consideration and would mean intellectual time travel to ancient Greece.

Situation changes if one requires that only the information about the construction of number can be produced in a finite number of steps using an algorithm. This would replace construction with the recipe for construction and lead to a higher abstraction level. The concrete construction itself need not be possible in a finite time as bit sequence but could be possible physically ($\sqrt{2}$ as a diagonal of unit square, one can of course wonder where to buy ideal unit squares). Both number theory and geometry would be needed.

Stern-Brocot tree associated with partial fractions indeed allows to identify rationals as finite paths connecting the root of S-B tree to the rational in question. Algebraic numbers can be identified as infinite periodic paths so that finite amount of information specifies the path. Transcendental numbers would correspond to infinite non-periodic paths. A very close analogy with chaos theory suggests itself.

Demonstrability viz. cognitive representability

SS talked about demonstrable numbers. According to Gödel demonstrable number would be representable by a formula G, which is provable in some axiom system. I understand this that G would give a recipe for constructing that number. In computer programs this can even mean infinite loop, which is easy to write but impossible to realize in practice. Here comes the possibility that demonstrability does not mean constructibility in finite number of steps but only a finite recipe for this.

The requirement that all numbers are demonstrable looks strange to me. I would talk about cognitive representability and reals and p-adic number fields emerge unavoidably as prerequisites for this notion: cognitive representation must be about something in order to be a representation.

About precise construction of reals or something bigger - such as surreals - containing them, there are many views and I am not mathematician enough to take strong stance here. Note however that if one accepts surreals as being demonstrable (I do not really understand what this could mean) one also accept reals as such. These delicacies are not very interesting for the formulation of physics as it is now.

The algorithm defining G defines a proof. But what does proof mean? Proof in mathematical sense would reduce in TGD framework be a purely cognitive act and assignable to the p-adic sectors of adele. Mathematicians however tend to forget that for physicist the demonstration is also experimental. Physicist does not believe unless he sees: sensory perception is needed. Experimental proofs are what physicists want. The existence of $\sqrt{2}$ as a diagonal of unit square is experimentally demonstrable in the sense of being cognitively representable but not deducible from the axioms for rational numbers. As a physicist I cannot but accept both sensory and cognitive aspects of existence.

Instead of demonstrable numbers I prefer to talk about cognitively representable numbers.

1. All numbers are cognizable (p-adic) or sensorily perceivable (real). These must form continua if one wants to avoid problems in the construction of physical theories, where continuous symmetries are in a key role.

   Some numbers but not all are also *cognitively representable* that is being in the intersection reals and p-adics - that is in extension of rationals if one allows extensions of p-adics induced by extensions of rationals. This generalizes to intersection of space-time surfaces with real/p-adic coordinates, which are highly unique linear coordinates at octonionic level so that objections relating to a loss of general coordinate invariance are circumvented. General coordinate transformations reduce to automorphisms of octonions.

   The relationship to the axiom of choice is interesting. Should axiom of choice be restricted to the points of complexified octonions with coordinates in extensions of rationals? Only points in the extensions could be selected and this selection process would be physical in the sense that fermions providing realization of quantum Boolean algebra would reside at these points [K31]. In preferred octonionic coordinates the M^8 coordinates of these points would be in given extension of rationals. At the limit of algebraic numbers these points would form a dense set of reals.

   Remark: The spinor structure of "world of classical worlds" (WCW) gives rise to WCW spinors as fermionic Fock states at given 3-surface. In ZEO many-fermion states have interpretation in terms of superpositions of pairs of Boolean statements $A \to B$ with A and B represented as many-fermion states at the ends of space-time surface located at the opposite light-like boundaries of causal diamond (CD). One could say that quantum Boolean logic emerges as square root of Kähler geometry of WCW.

   At partonic 2-surfaces these special points correspond to points at which fermions can be localized so that the representation is physical. Universe itself would come in rescue to make representability possible. One would not anymore try to construct mathematics and physics as distinct independent disciplines.

   Even observer as conscious entity is necessarily brought into both mathematics and physics. TGD Universe as a spinor field in WCW is re-created state function reduction by reduction and evolves: evolution for given CD corresponds to the increase of the size of extension of rationals in statistical sense. Hence also mathematics with fixed axioms is replaced with a q dynamical structure adding to itself new axioms discovery by discovery [L38, L39].

2. Rationals as cognitively representable numbers conforms with naïve intuition. One can however criticize the assumption that also algebraic numbers are such. Consider $\sqrt{2}$: one can simply define it as length of diagonal of unit square and this gives a meter stick of length $\sqrt{2}$: one can represent any algebraic number of form $m + n\sqrt{2}$ by using meter stricks with length of 1 and $\sqrt{2}$. Cognitive representation is also sensory representation and would bring in additional manner to represent numbers.

   Note that algebraic numbers in n-dimensional extension are points of n-dimensional space and their cognitive representations as points on real axis obtained by using the meter sticks assignable to the algebraic numbers defining base vectors. This should generalize to the roots arbitrary polynomials with rational or even algebraic coefficients. Essentially projection form n-D extension to 1-D real line is in question. This kind of projection might be important in number theoretical dynamics. For instance, quasi-periodic quasi-crystals are obtained from higher-D periodic crystals as projections.

   n-D algebraic extensions of p-adics induced by those of rationals might also related to our ability to imagine higher-dimensional spaces.

3. In TGD Universe cognitive representability would emerge from fundamental physics. Extensions of rationals define a hierarchy of adeles and octonionic surfaces are defined as zero loci

for real or imaginary parts (in quaternionic sense) of polynomials of real argument with coefficients in extension continued to octonionic polynomials [L32]. The zeros of real polynomial have a direct physical interpretation and would represent algebraic numbers physically. They would give the temporal positions of partonic 2-surfaces representing particles at light-like boundary of CD.

4. Note that all calculations with algebraic numbers can be done without using approximations for the genuinely algebraic numbers defining the basis for the extension. This actually simplifies enormously the calculation and one avoids accumulating errors. Only at the end one represents the algebraic units concretely and is forced to use rational approximation unless one uses above kind of cognitive representation.

For these reasons I do not feel any need to get rid of algebraics or even transcendentals. Sensory aspects of experience require reals and cognitive aspects of experience require p-adic numbers fields and one ends up with adelic physics. Cognitive representations are in the intersection of reality and various p-adicities, something expressible as formulas and concrete physical realizations or at least finite recipes for them.

What the cognitive representability of algebraic numbers could mean?

Algebraic numbers should be in some sense simple in order to be cognitively representable.

1. For rationals representation as partial fractions produces the rational number by using a finite number of steps. One starts from the top of Stern-Brocot (S-B) tree (see `http://tinyurl.com/yb6ldekq`) and moves to right or left at each step and ends up to the rational number appearing only once in S-B tree.
2. Algebraic numbers cannot be produced in a finite number of steps. During the discussion I however realized that one can produce the information needed to construct the algebraic number in a finite number of steps. One steps to a new level of abstraction by replacing the object with the information allowing to construct the object using infinite number of steps but repeating the same sub-algorithm with finite number of steps: infinite loop would be in question.

   Similar abstraction takes place as one makes a step from the level of space-time surface to the level of WCW. Space-time surface with a continuum of points is represented by a finite number of WCW coordinates, in the octonionic representation of space-time surface by the coefficients of polynomial of finite degree belonging to an extension of rationals [L32]. Criticality conditions pose additional conditions on the coefficients. Finite number of algebraic points at space-time surface determines the entire space-time surface under these conditions! Simple names for complex things replacing the complex things is the essence of cognition!
3. The interpretation for expansions of numbers in given base suggests an analog with complexity theory and symbolic dynamics associated with division. For cognitively representable numbers the information about this dynamics should be coded by an algorithm with finite steps. Periodic orbit or fixed point orbit would be the dynamical analog for simplicity. Non-periodic orbit would correspond to complexity and possibly also chaos.

These ideas led to two approaches in attempt to understand the cognitive representability of algebraic numbers.

1. Generalized rationals in extensions of rationals as periodic orbits for the dynamics of division

The first approach allows to represent ratios of algebraic integers for given extension using periodic expansion in the base so that a finite amount of information is needed to code the number if one accepts the numbers defining the basis of the algebraic extension as given.

1. Rationals allow periodic expansion with respect to any base. For p-adic numbers the base is naturally prime. Therefore the information about rational is finite. One can see the expansion as a periodic orbit in dynamics determining the expansion by division m/n in given base. Periodicity follows from the fact that the output of the division algorithm for a given digit has only a finite number of outcomes so that the process begins to repeat itself sooner or later.

2. This generalizes to generalized rationals in given extension of rationals defined as ratios of algebraic integers. One can reduce the division to the construction of the expansion of ordinary rational identified as number theoretic norm $|N|$ of the denominator in the extension of rationals considered.

   The norm $|N|$ of N is the determinant $|N| = det(N)$ for the linear map of extension induced by multiplication with N. $det(N)$ is ordinary (possibly p-adic) integer. This is achieved by multiplying $1/N$ by $n-1$ conjugates of N both in numerator and denominator so that one obtains product of $n-1$ conjugates in the numerator and $det(N)$ in the denominator. The computation of $1/N$ as series in the base used reduces to that in the case of rationals.

3. One has now periodic orbits in n-dimensional space defined by algebraic extensions which for ordinary rationals reduced to periodic orbits in 1-D space. This supports the interpretation of numbers as orbits of number theoretic dynamics determining the next digit of the generalized rational for given base. This picture also suggests that transcendentals correspond to non-periodic orbits. Some transcendentals could still allow a finite algorithm: in this case the dynamics would be still deterministic. Some transcendentals would be chaotic.

4. Given expansion of algebraic number is same for all extensions of rationals containing the extension in question and the ultimate extension corresponds to algebraic numbers.

The problem of this approach is that the algebraic numbers defining the extension do not have representation and must be accepted as irreducibles.

2. Algebraic numbers as infinite periodic orbits in the dynamics of partial fractions

Second approach is based on partial fractions and Stern-Brocot tree (see `http://tinyurl.com/yb6ldekq`, see also `http://tinyurl.com/yc6hhboo`) and indeed allows to see information about algebraic numbers as constructible by using an algorithm with finite number of steps, which is allowed if one accepts abstraction as basic aspect of cognition. I had managed to not become aware of this possibility and am grateful for SS for mentioning the representation of algebraics in terms of S-B tree.

1. The definition S-B tree is simple: if m/n and m'/n' are any neighboring rationals at given level in the tree, one adds $(m+m')/(n+n')$ between them and obtains in this manner the next level in the tree. By starting from $(0/1)$ and $(1/0)$ as representations of zero and ∞ one obtains $(0/1)(1/1)(1/0)$ as the next level. One can continue in this manner ad infinitum. The nodes of S-B tree represent rational points and it can be shown that given rational appears only once in the tree.

   Given rational can be represented as a finite path beginning from 1/1 at the top of tree consisting of left moves L and right moves R and ending to the rational which appears only once in S-B tree. Rational can be thus constructured by a sequences $R^{a_0}L^{a_1}L^{a_2}$.... characterized by the sequence $a_0; a_1, a_2$.... For instance, $4/11 = 0+1/(2+x)$, $x = 1/(1+1/3)$ corresponds to $R^0L^2R^1L^{3-1}$ labelled by $0; 2, 1, 3$.

2. Algebraic numbers correspond to infinite but periodic paths in S-B tree in the sense that some sequence of L:s and R:s characterized by sequences of non-negative integers starts to repeat itself. Periodicity means that the information needed to construct the number is finite.

   The actual construction as a digit sequence representing algebraic number requires infinite amount of time. In TGD framework octonionic physics would come in rescue and construct algebraic numbers as roots of polynomials having concrete interpretations as coordinate values assignable to fermions at partonic 2-surfaces.

3. Transcendentals would correspond to non-periodic infinite sequences of L:s and R:s. This does not exclude the possibility that these sequences are expressible in terms of some rule involving finite number of steps so that the amount of information would be also now finite. Information about number would be replaced by information about rule.

   This picture conforms with the ideas about transition to chaos. Rationals have finite paths. A possible dynamical analog is particle coming at rest due to the dissipation. Algebraic numbers would correspond to periodic orbits possible in presence of dissipation if there is external feed of energy. They would correspond to dynamical self-organization patterns.

Remark: If one interprets the situation in terms of conservative dynamics, rationals would correspond to potential minima and algebraic numbers closed orbits around them.

The assignment of period doubling and p-pling to this dynamics as the dimension of extension increases is an attractive idea. One would expect that the complexity of periodic orbits increases as the degree of the defining irreducible polynomial increases. Algebraic numbers as maximal extension of rationals possibly also containing extension containing all rational roots of e and transcendentals would correspond to chaos.

Transcendentals would correspond to non-periodic orbits. These orbits need not be always chaotic in the sense of being non-predictable. For instance, Neper number e can be said to be p-adically algebraic number (e^p is p-adic integer albeit infinite as real integer). Does the sequence of L:s and R:s allow a formula for the powers of L and R in this case?

4. TGD should be an integrable theory. This suggests that scattering amplitudes involve only cognitive representations as number theoretic vision indeed strongly suggests [L32]. Cognitively representable numbers would correspond to the integrable sub-dynamics [L44]. Also in chaotic systems both periodic and chaotic orbits are present. Complexity theory for characterization of real numbers exists. The basic idea is that complexity is measured by the length of the shortest program needed to code the bit sequences coding for the number.

Surreals and ZEO

The following comment is not directly related to cognitive representability but since it emerged during discussion, I will include it. SS favors surreals (see `http://tinyurl.com/86jatas`) as ultimate number field containing reals as sub-field. I must admit that my knowledge and understanding of surreals is rather fragmentary.

I am agnostic in these issues and see no conflict between TGD view about numbers and surreals. Personally I however like very much infinite primes, integers, and rationals over surreals since they allow infinite numbers to have number theoretical anatomy [K107]. A further reason is that the construction of infinite primes resembles structurally repeated second quantization of the arithmetic number field theory and could have direct space-time correlate at the level of many-sheeted space-time. One ends up also to a generalization of real number. Infinity can be seen as something related to real norm: everything is finite with respect to various p-adic norms.

Infinite rationals with unit real norm and various p-adic norms bring in infinitely complex number theoretic anatomy, which could be even able to represent even the huge WCW and the space of WCW spinor fields. One could speak of number theoretical holography or algebraic Brahman=Atman principle. One would have just complexified octonions with infinitely richly structure points.

Surreals are represented in terms of pairs of sets. One starts the recursive construction from empty set identified as 0. The definition says that the pairs (.|.) of sets defining surreals x and y satisfy $x \leq y$ if the left hand part of x as set is to left from the pair defining y and the right hand part of y is to the right from the pair defining x. This does not imply that one has always $x < y$, $y < x$ or $x = y$ as for reals.

What is interesting that the pair of sets defining surreal x is analogous to a pair of states at boundaries of CD defining zero energy state. Is there a connection with zero energy ontology (ZEO)? One could perhaps say at the level of CD - forgetting everything related to zero energy states - following. The number represented by CD_1 - say represented as the distance between its tip - is smaller than than the number represented by CD_2, if CD_1 is inside CD_2. This conforms with the left and righ rule if left and right correspond to the opposite boundaries of CD. A more detailed definition would presumably say that CD_1 can be moved so that it is inside CD_2.

What makes this also interesting is that CD is the geometric correlate for self, conscious entity, also mathematical mental image about number.

7.5 Galois groups and genes

In an article discussing a TGD inspired model for possible variations of G_{eff} [L48], I ended up with an old idea that subgroups of Galois group could be analogous to conserved genes in that they could be conserved in number theoretic evolution. In small variations such as above variation

Galois subgroups as genes would change only a little bit. For instance, the dimension of Galois subgroup would change.

The analogy between subgoups of Galois groups and genes goes also in other direction. I have proposed long time ago that genes (or maybe even DNA codons) could be labelled by $h_{eff}/h = n$. This would mean that genes (or even codons) are labelled by a Galois group of Galois extension (see `http://tinyurl.com/zu5ey96`) of rationals with dimension n defining the number of sheets of space-time surface as covering space. This could give a concrete dynamical and geometric meaning for the notion of gene and it might be possible some day to understand why given gene correlates with particular function. This is of course one of the big problems of biology.

7.5.1 Could DNA sequence define an inclusion hierarchy of Galois extensions?

One should have some kind of procedure giving rise to hierarchies of Galois groups assignable to genes. One would also like to assign to letter, codon and gene and extension of rationals and its Galois group. The natural starting point would be a sequence of so called intermediate Galois extensions E^H leading from rationals or some extension K of rationals to the final extension E. Galois extension has the property that if a polynomial with coefficients in K has single root in E, also other roots are in E meaning that the polynomial with coefficients K factorizes into a product of linear polynomials. For Galois extensions the defining polynomials are irreducible so that they do not reduce to a product of polynomials.

Any sub-group $H \subset Gal(E/K))$ leaves the intermediate extension E^H invariant in element-wise manner as a sub-field of E (see `http://tinyurl.com/y958drcy`). Any subgroup $H \subset Gal(E/K))$ defines an intermediate extension E^H and subgroup $H_1 \subset H_2 \subset ...$ define a hierarchy of extensions $E^{H_1} > E^{H_2} > E^{H_3}...$ with decreasing dimension. The subgroups H are normal - in other words $Gal(E)$ leaves them invariant and $Gal(E)/H$ is group. The order $|H|$ is the dimension of E as an extension of E^H. This is a highly non-trivial piece of information. The dimension of E factorizes to a product $\prod_i |H_i|$ of dimensions for a sequence of groups H_i.

Could a sequence of DNA letters/codons somehow define a sequence of extensions? Could one assign to a given letter/codon a definite group H_i so that a sequence of letters/codons would correspond a product of some kind for these groups or should one be satisfied only with the assignment of a standard kind of extension to a letter/codon?

Irreducible polynomials define Galois extensions and one should understand what happens to an irreducible polynomial of an extension E^H in a further extension to E. The degree of E^H increases by a factor, which is dimension of E/E^H and also the dimension of H. Is there a standard manner to construct irreducible extensions of this kind?

1. What comes into mathematically uneducated mind of physicist is the functional decomposition $P^{m+n}(x) = P^m(P^n(x))$ of polynomials assignable to sub-units (letters/codons/genes) with coefficients in K for a algebraic counterpart for the product of sub-units. $P^m(P^n(x))$ would be a polynomial of degree $n+m$ in K and polynomial of degree m in E^H and one could assign to a given gene a fixed polynomial obtained as an iterated function composition. Intuitively it seems clear that in the generic case $P^m(P^n(x))$ does not decompose to a product of lower order polynomials. One could use also polynomials assignable to codons or letters as basic units. Also polynomials of genes could be fused in the same manner.

2. If this indeed gives a Galois extension, the dimension m of the intermediate extension should be same as the order of its Galois group. Composition would be non-commutative but associative as the physical picture demands. The longer the gene, the higher the algebraic complexity would be. Could functional decomposition define the rule for who extensions and Galois groups correspond to genes? Very naïvely, functional decomposition in mathematical sense would correspond to composition of functions in biological sense.

3. This picture would conform with $M^8 - M^4 \times CP_2$ correspondence [L32] in which the construction of space-time surface at level of M^8 reduces to the construction of zero loci of polynomials of octonions, with rational coefficients. DNA letters, codons, and genes would correspond to polynomials of this kind.

7.5.2 Could one say anything about the Galois groups of DNA letters?

A fascinating possibility is that this picture could allow to say something non-trivial about the Galois groups of DNA letters.

1. Since $n = h_{eff}/h$ serves as a kind of quantum IQ, and since molecular structures consisting of large number of particles are very complex, one could argue that n for DNA or its dark variant realized as dark proton sequences can be rather large and depend on the evolutionary level of organism and even the type of cell (neuron viz. soma cell). On the other, hand one could argue that in some sense DNA, which is often thought as information processor, could be analogous to an integrable quantum field theory and be solvable in some sense. Notice also that one can start from a background defined by given extension K of rationals and consider polynomials with coefficients in K. Under some conditions situation could be like that for rationals.

2. The simplest guess would be that the 4 DNA letters correspond to 4 non-trivial finite groups with smaller possible orders: the cyclic groups Z_2, Z_3 with orders 2 and 3 plus 2 finite groups of order 4 (see the table of finite groups in `http://tinyurl.com/j8d5uyh`). The groups of order 4 are cyclic group $Z_4 = Z_2 \times Z_2$ and Klein group $Z_2 \oplus Z_2$ acting as a symmetry group of rectangle that is not square - its elements have square equal to unit element. All these 4 groups are Abelian. Polynomial equations of degree not larger than 4 can be solved exactly in the sense that one can write their roots in terms of radicals.

3. Could there exist some kind of connection between the number 4 of DNA letters and 4 polynomials of degree less than 5 for whose roots one an write closed expressions in terms of radicals as Galois found? Could it be that the polynomials obtained by a a repeated functional composition of the polynomials of DNA letters have also this solvability property?

   This could be the case! Galois theory states that the roots of polynomial are solvable by radicals if and only if the Galois group is solvable meaning that it can be constructed from abelian groups using Abelian extensions (see `https://cutt.ly/4RuXmGo`).

   Solvability translates to a statement that the group allows so called sub-normal series $1 < G_0 < G_1 ... < G_k$ such that G_{j-1} is normal subgroup of G_j and G_j/G_{j-1} is an abelian group. An equivalent condition is that the derived series $G \triangleright G^{(1)} \triangleright G^{(2)} \triangleright ...$ in which $j+1$:th group is commutator group of G_j ends to trivial group. If one constructs the iterated polynomials by using only the 4 polynomials with Abelian Galois groups, the intuition of physicist suggests that the solvability condition is guaranteed! Wikipedia article also informs that for finite groups solvable group is a group whose composition series has only factors which are cyclic groups of prime order.

   Abelian groups are trivially solvable, nilpotent groups are solvable, p-groups (having order, which is power prime) are solvable and all finite p-groups are nilpotent. Every group with order less than 60 elements is solvable. Fourth order polynomials can have at most S_4 with 24 elements as Galois groups and are thus solvable. Fifth order polynomials can have the smallest non-solvable group, which is alternating group A_5 with 60 elements as Galois group and in this case are not solvable. S_n is not solvable for $n > 4$ and by the finding that S_n as Galois group is favored by its special properties (see `https://arxiv.org/pdf/1511.06446.pdf`).

   A_5 acts as the group icosahedral orientation preserving isometries (rotations). Icosahedron and tetrahedron glued to it along one triangular face play a key role in TGD inspired model of bio-harmony and of genetic code [L6, L49]. The gluing of tetrahedron increases the number of codons from 60 to 64. The gluing of tetrahedron to icosahedron also reduces the order of isometry group to the rotations leaving the common face fixed and makes it solvable: could this explain why the ugly looking gluing of tetrahedron to icosahedron is needed? Could the smallest solvable groups and smallest non-solvable group be crucial for understanding the number theory of the genetic code.

An interesting question inspired by $M^8 - H$-duality [L32] is whether the solvability could be posed on octonionic polynomials as a condition guaranteeing that TGD is integrable theory in number theoretical sense or perhaps following from the conditions posed on the octonionic polynomials. Space-time surfaces in M^8 would correspond to zero loci of real/imaginary parts (in quaternionic sense) for octonionic polynomials obtained from rational polynomials by analytic continuation. Could solvability relate to the condition guaranteeing M^8 duality boiling down to the condition that the tangent spaces of space-time surface are labelled by points of CP_2. This requires that tangent or normal space is associative (quaternionic) and that it contains fixed complex subspace of octonions or perhaps more generally, there exists an integrable distribution of complex subspaces of octonions defining an analog of string world sheet.

What could the interpretation for the events in which the dimension of the extension of rationals increases? Galois extension is extensions of an extension with relative Galois group $Gal(rel) = Gal(new)/Gal(old)$. Here $Gal(old)$ is a normal subgroup of $Gal(new)$. A highly attractive possibility is that evolutionary sequences quite generally (not only in biology) correspond to this kind of sequences of Galois extensions. The relative Galois groups in the sequence would be analogous to conserved genes, and genes could indeed correspond to Galois groups [K37] [L32]. To my best understanding this corresponds to a situation in which the new polynomial P_{m+n} defining the new extension is a polynomial P_m having as argument the old polynomial $P_n(x)$: $P_{m+n}(x) = P_m(P_n(x))$.

What about the interpretation at the level of conscious experience? A possible interpretation is that the quantum jump leading to an extension of an extension corresponds to an emergence of a reflective level of consciousness giving rise to a conscious experience about experience. The abstraction level of the system becomes higher as is natural since number theoretic evolution as an increase of algebraic complexity is in question.

This picture could have a counterpart also in terms of the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs). The included factor M and including factor N would correspond to extensions of rationals labelled by Galois groups $Gal(M)$ and $Gal(N)$ having $Gal(M) \subset Gal(M)$ as normal subgroup so that the factor group $Gal(N)/Gal(M)$ would be the relative Galois group for the larger extension as extension of the smaller extension. I have indeed proposed [L50] that the inclusions for which included and including factor consist of operators which are invariant under discrete subgroup of $SU(2)$ generalizes so that all Galois groups are possible. One would have Galois confinement analogous to color confinement: the operators generating physical states could have Galois quantum numbers but the physical states would be Galois singlets.

7.6 Could the precursors of perfectoids emerge in TGD?

In algebraic-geometry community the work of Peter Scholze [A71] (see http://tinyurl.com/y7h2sms7) introducing the notion of perfectoid related to p-adic geometry has raised a lot of interest. There are two excellent popular articles about perfectoids: the first article in AMS (see http://tinyurl.com/ydx38vk4) and second one in Quanta Magazine (see http://tinyurl.com/yc2mxxqh). I had heard already earlier about the work of Scholze but was too lazy to even attempt to understand what is buried under the horrible technicalities of modern mathematical prose. Rachel Francon re-directed my attention to the work of Scholze (see http://tinyurl.com/yb46oza6). The work of Scholze is interesting also from TGD point of view since the construction of p-adic geometry is a highly non-trivial challenge in TGD.

1. One should define first the notion of continuous manifold but compact-open characteristic of p-adic topology makes the definition of open set essential for the definition of topology problematic. Even single point is open so that hopes about p-adic manifold seem to decay to dust. One should pose restrictions on the allowed open sets and p-adic balls with radii coming as powers of p are the natural candidates. p-Adic balls are either disjoint or nested: note that also this is in conflict with intuitive picture about covering of manifold with open sets. All this strangeness originates in the special features of p-adic distance function known as ultra-metricity. Note however that for extensions of p-adic numbers one can say that the Cartesian products of p-adic 1-balls at different genuinely algebraic points of extension along particular axis of extension are disjoint.

2. At level of M^8 the p-adic variants of algebraic varieties defined as zero loci of polynomials do not seem to be a problem. Equations are algebraic conditions and do not involve derivatives like partial differential equations naturally encountered if Taylor series instead of polynomials are allowed. Analytic functions might be encountered at level of $H = M^4 \times CP_2$ and here p-adic geometry might well be needed.

The idea is to define the generalization of p-adic algebraic geometry in terms of p-adic function fields using definitions very similar to those used in algebraic geometry. For instance, generalization of variety corresponds to zero locus for an ideal of p-adic valued function field. p-Adic ball of say unit radius is taken as the basic structure taking the role of open ball in the topology of ordinary manifolds. This kind of analytic geometry allowing all power series with suitable restrictions to function field rather than allowing only polynomials is something different from algebraic geometry making sense for p-adic numbers and even for finite fields.

3. One would like to generalize the notion of analytic geometry even to the case of number fields with characteristic p (p-multiple of element vanishes), in particular for finite fields F_p and for function fields $F_p[t]$. Here one encounters difficulties. For instance, the factorial $1/n!$ appearing as normalization factor of forms diverges if p divides it. Also the failure of Frobenius homomorphism to be automorphism for $F_p[t]$ causes difficulties in the understanding of Galois groups.

The work of Scholze has led to a breakthrough in unifying the existing ideas in the new framework provided by the notion of perfectoid. The work is highly technical and involves infinite-D extension of ordinary p-adic numbers adding all powers of all roots p^{1/p^m}, $m = 1, 2....$ Formally, an extension by powers of p^{1/p^∞} is in question.

This looks strange at first but it guarantees that all p-adic numbers in the extension have p:th roots, one might say that one forms a p-fold covering/wrapping of extension somewhat analogous to complex numbers. This number field is called perfectoid since it is perfect meaning that Frobenius homomorphism $a \rightarrow a^p$ is automorphism by construction. $Frob$ is injection always and by requiring that p:th roots exist always, it becomes also a surjection.

This number field has same Galois groups for all of its extensions as the function field $G[t]$ associated with the union of function fields $G = F_p[t^{1/p^m}]$. Automorphism property of $Frob$ saves from the difficulties with the factorization of polynomials and p-adic arithmetics involving remainders is replaced with purely local modulo p arithmetics.

7.6.1 About motivations of Scholze

Scholze has several motivations for this work. Since I am not a mathematician, I am unable to really understand all of this at deep level but feel that my duty as user of this mathematics is at least to try!

1. Diophantine equations is a study of polynomial equations in several variables, say $x^2 + 2xy + y = 0$. The solutions are required to be integer valued: in the example considered $x = y = 0$ and $x = -y = -1$ is such a solution. For integers the study of the solution is very difficult and one approach is to study these equations modulo p that is reduced the equations to finite field G_p for any p. The equations simplify enormously since ane has $a^p = a$ in F_p. This identity in fact defines so called Frobenius homomorphism acting as automorphism for finite fields. This holds true also for more complex fields with characteristic p say the ring $F_p[t]$ of power series of t with coefficients in F_p.

   The powers of variables, say x, appearing in the equation is reduced to at most x^{p-1}. One can study the solutions also in p-adic number fields. The idea is to find first whether finite field solution, that is solution modulo p, does exist. If this is the case, one can calculate higher powers in p. If the series contains finite number of terms, one has solution also in the sense of ordinary integers.

2. One of the related challenges is the generalization of the notion of variety to a geometry defined in arbitrary number field. One would like to have the notion of geometry also for finite fields, and for their generalizations such as $F_p[t]$ characterized by characteristic p ($px = 0$ holds true for any element of the field). For fields of characteristic 1 - extensions of rationals, real, and p-adic number fields) $xp = 0$ not hold true for any $x \neq 0$. Any field containing rationals as sub-field, being thus local field, is said to have characteristic equal to 1. For local fields the challenge is relatively easy.

3. The situation becomes more difficult if one wants a generalization of differential geometry. In differential geometry differential forms are in a key role. One wants to define the notion of differential form in fields of characteristic p and construct a generalization of cohomology

theory. This would generalize the notion of topology to p-adic context and even for finite fields of finite character. A lot of work has been indeed done and Grothendieck has been the leading pioneer.

The analogs of cohomology groups have values in the field of p-adic numbers instead of ordinary integers and provide representations for Galois groups for the extensions of rationals inducing extensions of p-adic numbers and finite fields.

In ordinary homology theory non-contractible sub-manifolds of various dimensions correspond to direct summands Z (group of integers) for homology groups and by Poincare duality those for cohomology groups. For Galois groups Z is replaced with Z_N. N depends on extension to which Galois group is associated and if N is divisible by p one encounters technical problems.

There are many characteristic p- and p-adic cohomologies such as etale cohomology, chrystalline cohomology, algebraic de-Rham cohomology. Also Hodge theory for complex differential forms generalizes. These cohomologies should be related by homomorphism and category theoretic thinking the proof of the homomorphism requires the construction of appropriate functor between them.

The integrals of forms over sub-varieties define the elements of cohomology groups in ordinary cohomology and should have p-adic counterparts. Since p-adic numbers are not well-ordered, definite integral has no straightforward generalization to p-adic context. One might however be able to define integrals analogous to those associated with differential forms and depending only on the topology of sub-manifold over which they are taken. These integrals would be analogous to multiple residue integrals, which are the crux of the twistor approach to scattering amplitudes in super-symmetric gauge theories. One technical difficulty is that for a field of finite characteristic the derivative of X^p is pX^{p-1} and vanishes. This does not allow to define what integral $\int X^{p-1}dX$ could mean. Also $1/n!$ appears as natural normalization factor of forms but if p divides it, it becomes infinite.

7.6.2 Attempt to understand the notion of perfectoid

Consider now the basic ideas behind the notion of perfectoid.

1. For finite finite fields F_p Frobenius homomorphism $a \to a^p$ is automorphism since one has $a^p = a$ in modulo p arithmetics. A field with this property is called perfect and all local fields are perfect. Perfectness means that an algebraic number in any extension L of perfect field K is a root of a separable minimal polynomial. Separability means that the number of roots in the algebraic closure of K of the polynomial is maximal and the roots are distinct.

2. All fields containing rationals as sub-fields are perfect. For fields of characteristic p $Frob$ need not be a surjection so that perfectness is lost. For instance, for $F_p[t]$ $Frob$ is trivially injection but surjective property is lost: $t^{1/p}$ is not integer power of t.

   One can however extend the field to make it perfect. The trick is simple: add to $F_p[t]$ all fractional powers t^{1/p^n} so that all p:th roots exist and $Frob$ becomes and automorphism. The automorphism property of $Frob$ allows to get rid of technical problems related to a factorization of polynomials. The resulting extension is infinite-dimensional but satisfies the perfectness property allowing to understand Galois groups, which play key role in various cohomology theories in characteristic p.

3. Let $K = Q_p[p^{1/p^\infty}]$ denote the infinite-dimensional extension of p-adic number field Q_p by adding all powers of p^m:th roots for all all $m = 1, 2, ...$ This is not the most general option: K could be also only a ring. The outcome is perfect field although it does not of course have Frobenius automorphism since characteristic equals to 1.

   One can divide K by p to get K/p as the analog of finite field F_p as its infinite-dimensional extension. K/p allows all p:th roots by construction and $Frob$ is automorphism so that K/p is perfect by construction.

   The structure obtained in this manner is closely related to a perfect field with characteristic p having same Galois groups for all its extensions. This object is computationally much more attractive and allows to prove theorems in p-adic geometry. This motivates the term perfectoid.

4. One can assign to K another object, which is also perfectoid but has characteristic p. The correspondence is as follows.
   (a) Let F_p be finite field. F_p is perfect since it allows trivially all p:th roots by $a^p = a$. The ring $F_p[t]$ is however not prefect since t^{1/p^m} is not integer power of t. One must modify $F_p[t]$ to obtain a perfect field. Let $G_m = F_p[t^{1/p^m}]$ be the ring of formal series in powers of t^{1/p^m} defining also function field. These serious are called t-adic and one can define t-adic norm.
   (b) Define t-adic function field K_b called the **tilt** of K as

   $$K_b = \cup_{m=1,...}(K/p)[t^{1/pm}][t] \ .$$

   One has all possible power series with coefficients in K/p involving all roots t^{1/p^m}, $m = 1, 2, ...$, besides powers of positive integer powers of t. This function field has characteristic p and all roots exist by construction and $Frob$ is automorphism. K_b/t is perfect meaning that the minimal polynomials for the for given analog of algebraic number in any of its extensions allows separable polynomial with maximal number of roots in its closure.

   This sounds rather complicated! In any case, K_b/t has same number theoretical structure as $Q_p[p^{1/p^\infty}]/p$ meaning that Galois groups for all of its extensions are canonically isomorphic to those for extensions of K. Arithmetics modulo p is much simpler than p-adic arithmetic since products are purely local and there is no need to take care about remainders in arithmetic operations, this object is much easier to handle.

   Note that also p-adic number fields fields Q_p as also $F_p = Q_p/p$ are perfect but the analog of $K_b = F_b[t]$ fails to be perfect.

7.6.3 Second attempt to understand the notions of perfectoid and its tilt

This subsection is written roughly year after the first version of the text. I hope that it reflects a genuine increase in my understanding.

1. Scholze introduces first the notion of perfectoid. This requires some background notions. The characteristic p for field is defined as the integer p (prime) for which $px = 0$ for all elements x. Frobenius homomorphism (Frob familiarly) is defined as $Frob : x \rightarrow x^p$. For a field of characteristic p $Frob$ is an algebra homomorphism mapping product to product and sum to sum: this is very nice and relatively easy to show even by a layman like me.
2. Perfectoid is a field having either characteristic $p = 0$ (reals, p-adics for instance) or for which $Frob$ is a surjection meaning that $Frob$ maps at least one number to a given number x.
3. For finite fields $Frob$ is identity: $x^p = x$ as proved already by Fermat. For reals and p-adic number fields with characteristic p=0 it maps all elements to unit element and is not a surjection. Field is perfect if it has either $p = 0$ (reals, p-adics) or if Frobenius is surjection. Finite fields are obviously perfectoids too.

Scholze introduces besides perfectoids K also what he calls tilt K_b of the perfectoid. K_b is infinite-D extension of p-adic numbers by iterated p:th roots p-adic numbers: the units of the extension correspond to the roots p^{1/p^k}. They are something between p-adic number fields and reals and leads to theorems giving totally new insights to arithmetic geometry. Unfortunately, my technical skills in mathematics are hopelessly limited to say anything about these theorems.

1. As we learned during the first student year of mathematics, real numbers can be defined as Cauchy sequences of rationals converging to a real number, which can be also algebraic number or transcendental. The elements in the tilt K_b would be this kind of sequences.
2. Scholze starts from (say) p-adic numbers and considers infinite sequence of iterates of $1/p$:th roots. At given step $x \rightarrow x^{1/p}$. This gives the sequence $(x, x^{1/p}, x^{1/p^2}, x^{1/p^3}, ...)$ identified as an element of the tilt K_b. At the limit one obtains $1/p^\infty$ root of x.

   Remark: For finite fields each step is trivial ($x^p = x$) so that nothing interesting results: one has $(x, x, x, x, ...)$

(a) For p-adic number fields the situation is non-trivial. $x^{1/p}$ exists as p-adic number for all p-adic numbers with unit norm having $x = x_0 + x_1 p + ...$ In the lowest order $x \simeq x_0$ the root is just x since x is effectively an element of finite field in this approximation. One can develop the $x^{1/p}$ to a power series in p and continue the iteration. The sequence obtained defines an element of tilt K_b of field K, now p-adic numbers.

(b) If the p-adic number x has norm p^n, $n \neq 0$ and is therefore not p-adic unit, the root operation makes sense only if one performs an extension of p-adic numbers containing all the roots p^{1/p^k} . These roots define one particular kind of extension of p-adic numbers and the extension is infinite-dimensional since all roots are needed. One can approximate K_b by taking only finite number iterated roots.

3. The tilt is said to be fractal: this is easy to understand from the presence of the iterated p:th root. Each step in the sequence is like zooming. One might say that p-adic scale becomes p:th root of itself. In TGD the p-adic length scale L_p is proportional to $p^{1/2}$: does the scaling mean that the p-adic length scale would defined hierarchy of scales proportional to $p^{1/2kp}$: root of itself and approach the CP_2 scale since the root of p approaches unity. Tilts as extensions by iterated roots would improve the length scale resolution.

One day later after writing this I got the feeling that I might have vaguely understood one more important thing about the tilt of p-adic number field: changing of the characteristic 0 of p-adic number field to characteristics $p > 0$ of the corresponding finite field for its tilt. What could this mean?

1. Characteristic p (p is the prime labelling p-adic number field) means $px = 0$. This property makes the mathematics of finite fields extremely simple: in the summation one need not take care of the residue as in the case of reals and p-adics. The tilt of the p-adic number field would have the same property! In the infinite sequence of the p-adic numbers coming as iterated p:th roots of the starting point p-adic number one can sum each p-adic number separately. This is really cute if true!

2. It seems that one can formulate the arithmetics problem in the tilt where it becomes in principle as simple as in finite field with only p elements! Does the existence of solution in this case imply its existence in the case of p-adic numbers? But doesn't the situation remain the same concerning the existence of the solution in the case of rational numbers? The infinite series defining p-adic number must correspond a sequence in which binary digits repeat with some period to give a rational number: rational solution is like a periodic solution of a dynamical system whereas non-rational solution is like chaotic orbit having no periodicity? In the tilt one can also have solutions in which some iterated root of p appears: these cannot belong to rationals but to their extension by an iterated root of p.

The results of Scholze could be highly relevant for the number theoretic view about TGD in which octonionic generalization of arithmetic geometry plays a key role since the points of space-time surface with coordinates in extension of rationals defining adele and also what I call cognitive representations determining the entire space-time surface if $M^8 - H$ duality holds true (space-time surfaces would be analogous to roots of polynomials). Unfortunately, my technical skills in mathematics needed are hopelessly limited.

TGD inspires the question is whether this kind of extensions could be interesting physically. At the limit of infinite dimension one would get an ideal situation not realizable physically if one believes that finite-dimensionality is basic property of extensions of p-adic numbers appearing in number theoretical quantum physics (they would related to cognitive representations in TGD). Adelic physics [L39] involves all finite-D extensions of rationals and the extensions of p-adic number fields induced by them and thus also cutoffs of extensions of type K_b- which I have called precursors of K_b.

How this relates to Witt vectors?

Witt vectors provide an alternative representation of p-adic arithmetics of p-adic integers in which the sum and product are reduced to purely local digit-wise operations for each power of p for the components of Witt vector so that one need not worry about carry pinary digit.

1. The idea is to consider the sequence consisting pinary cutoffs to p-adic number $xmodp^n$ and identify p-adic integer as this kind of sequence as n approaches infinity. This is natural approach when one identifies finite measurement resolution or cognitive resolution as a cutoff in some power of p^n. One simply forms the numbers $X_n = x \ mod \ p^{n+1}$: for numbers $1, ..., p-1$ they are called Teichmueller representatives and only they are needed to construct the sequences for general x. One codes this sequence of pinary cutoffs to Witt vector.
2. The non-trivial observation made by studying sums of p-adic numbers is that the sequence $X_0, X_1, X_2, ...$ of approximations define a sequence of components of Witt vector as $W_0 = X_0$, $W_1 = X_0^p + pX_1$, $W_2 = X_0^{(}p^2) + pX_1^p + p^2X_2$, ... or more formally $W_n = Sum_{i<n}p^iZX_i^{[}p^{(}n-i)]$.
3. The non-trivial point is that Witt vectors form a commutative ring with local digit-wise multiplication and sum modulo p: there no carry digits. Effectively one obtains infinite Cartesian power of finite field F_p. This means a great simplification in arithmetics. One can do the arithmetics using Witt vectors and deduce the sum and product from their product.
4. Witt vectors are universal. In particular, they generalize to any extension of p-adic numbers. Could Witt vectors bring in something new from physics point of view? Could they allow a formulation for the hierarchy of pinary cutoffs giving some new insights? For instance, neuro-computationalist might ask whether brain could perform p-adic arithmetics using a linear array of modules (neurons or neuron groups) labelled by $n = 1, 2, ...$ calculates sum or product for component W_n of Witt vector? No transfer of carry bits between modules would be needed. There is of course the problem of transforming p-adic integers to Witt vectors and back - it is not easy to imagine a natural realization for a module performing this transformation. Is there any practical formulation for say p-adic differential calculus in terms of Witt vectors?

I would seem that Witt vectors might relate in an interesting manner to the notion of perfectoid. The basic result proved by Petter Scholtze is that the completion $\cup_n Q_p(p^{1/p^n})$ of p-adic numbers by adding p^n:th roots and the completion of Laurent series $F_p((t))$ to $\cup_n F_p((t^{1/p^n}))$ have isomorphic absolute Galois groups and in this sense are one and same thing. On the other hand, p-adic integers can be mapped to a subring of $F_p(t)$ consisting of Taylor series with elements allowing interpretation as Witt vectors.

7.6.4 TGD view about p-adic geometries

As already mentioned, it is possible to define p-adic counterparts of n-forms and also various p-adic cohomologies with coefficient field taken as p-adic numbers and these constructions presumably make sense in TGD framework too. The so called rigid analytic geometry is the standard proposal for what p-adic geometry might be.

The very close correspondence between real space-time surfaces and their p-adic variants plays realized in terms of cognitive representations [L42, L38, L32] plays a key role in TGD framework and distinguishes it from approaches trying to formulate p-adic geometry as a notion independent of real geometry.

Ordinary approaches to p-adic geometry concentrate the attention to single p-adic prime. In the adelic approach of TGD one considers both reals and all p-adic number fields simultaneously.

Also in TGD framework Galois groups take key role in this framework and effectively replace homotopy groups and act on points of cognitive representations consisting of points with coordinates in extension of rationals shared by real and p-adic space-time surfaces. One could say that homotopy groups at level of sensory experience are replaced by Galois at the level of cognition. It also seems that there is very close connection between Galois groups and various symmetry groups. Galois groups would provide representations for discrete subgroups of symmetry groups.

In TGD framework there is strong motivation for formulating the analog of Riemannian geometry of $H = M^4 \times CP_2$ for p-adic variants of H. This would mean p-adic variant of Kähler geometry. The same challenge is encountered even at the level of "World of Classical Worlds" (WCW) having Kähler geometry with maximal isometries. p-Adic Riemann geometry and n-forms make sense locally as tensors but integrals defining distances do not make sense p-adically and it seems that the dream about global geometry in p-adic context is not realizable. This makes sense: p-adic physics is a correlate for cognition and one cannot put thoughts in weigh or measure their length.

Formulation of adelic geometry in terms of cognitive representations

Consider now the key ideas of adelic geometry and of cognitive representations.

1. The king idea is that p-adic geometries in TGD framework consists of p-adic balls of possibly varying radii p^n assignable to points of space-time surface for which the preferred embedding space coordinates are in the extension of rationals. At level of M^8 octonion property fixes preferred coordinates highly uniquely. At level of H preferred coordinates come from symmetries.

   These points define a cognitive representation and inside p-adic points the solution of field equations is p-adic variant of real solution in some sense. At M^8 level the field equations would be algebraic equations and real-p-adic correspondence would be very straightforward. Cognitive representations would make sense at both M^8 level and H level.

   Remark: In ordinary homology theory the decomposition of real manifold to simplexes reduces topology to homology theory. One forgets completely the interiors of simplices. Could the cognitive representations with points labelling the p-adic balls could be seen as analogous to decompositions to simplices. If so, homology would emerge as something number theoretically universal. The larger the extension of rationals, the more precise the resolution of homology would be. Therefore p-adic homology and cohomology as its Poincare dual would reduce to their real counterparts in the cognitive resolution used.

2. M^8-H correspondence would play a key role in mapping the associative regions of space-time varieties in M^8 to those in H. There are two kinds of regions. Associative regions in which polynomials defining the surfaces satisfy criticality conditions and non-associative regions. Associative regions represent external particles arriving in CDs and non-associative regions interaction regions within CDs.

3. In associative regions one has minimal surface dynamics (geodesic motion) at level of H and coupling parameters disappear from the field equations in accordance with quantum criticality. The challenge is to prove that $M^8 - H$ correspondence is consistent with the minimal surface dynamics n H. The dynamics in these regions is determined in M^8 as zero loci of polynomials satisfying quantum criticality conditions guaranteeing associativity and is deterministic also in p-adic sectors since derivatives are not involved and pseudo constants depending on finite number of pinary digits and having vanishing derivative do not appear. $M^8 - H$ correspondence guarantees determinism in p-adic sectors also in H.

4. In non-associative regions M^8-H correspondence does not make sense since the tangent space of space-time variety cannot be labelled by CP_2 point and the real and p-adic H counterparts of these regions would be constructed from boundary data and using field equations of a variational principle (sum of the volume term and Kähler action term), which in non-associative regions gives a dynamics completely analogous to that of charged particle in induced Kähler field. Now however the field characterizes extended particle itself.

   Boundary data would correspond to partonic 2-surfaces and string world sheets and possibly also the 3-surfaces at the ends of space-time surface at boundaries of CD and the light-like orbits of partonic 2-surfaces. At these surfaces the 4-D (!) tangent/normal space of space-time surface would be associative and could be mapped by $M^8 - H$ correspondence from M^8 to H and give rise to boundary conditions.

   Due to the existence of p-adic pseudo-constants the p-adic dynamics determined by the action principle in non-associative regions inside CD would not be deterministic in p-adic sectors. The interpretation would be in terms of freedom of imagination. It could even happen that boundary values are consistent with the existence of space-time surface in p-adic sense but not with the existence of real space-time surfaces. Not all that can be imagined is realizable.

At the level of M^8 this vision seems to have no obvious problems. Inside each ball the same algebraic equations stating vanishing of $IM(P)$ (imaginary part of P in quaternionic sense) hold true. At the level of H one has second order partial differential equations, which also make sense also p-adically. Besides this one has infinite number of boundary conditions stating the vanishing of Noether charges assignable to sub-algebra super-symplectic algebra and its commutator with the entire algebra at the 3-surfaces at the boundaries of CD. Are these two descriptions really equivalent?

During writing I discovered an argument, which skeptic might see as an objection against $M^8 - H$ correspondence.

1. M^8 correspondence maps the space-time varieties in M^8 in non-local manner to those in $H = M^4 \times CP_2$. CP_2 coordinates characterize the tangent space of space-time variety in M^8 and this might produce technical problems. One can map the real variety to H and find the points of the image variety satisfying the condition and demand that they define the "spine" of the p-adic surface in p-adic H.

2. The points in extensions of rationals in H need not be images of those in M^8 but should this be the case? Is this really possible? M^4 point in $M^4 \times E^4$ would be mapped to $M^4 \subset M^4 \times CP_2$: this is trivial. 4-D associative tangent/normal space at m containing preferred M^2 would be characterized by CP_2 coordinates: this is the essence of $M^8 - H$ correspondence. How could one guarantee that the CP_2 coordinates characterizing the tangent space are really in the extension of rationals considered? If not, then the points of cognitive representation in H are not images of points of cognitive representation in M^8. Does this matter?

Are almost-perfectoids evolutionary winners in TGD Universe?

One could take perfectoids and perfectoid spaces as a mere technical tool of highly refiner mathematical cognition. Since cognition is basic aspect of TGD Universe, one could also ask perfectoids or more realistically, almost-perfectoids, could be an outcome of cognitive evolution in TGD Universe?

1. p-Adic algebraic varieties are defined as zero loci of polynomials. In the octonionic M^8 approach identifying space-time varieties as zero loci for RE or IM of octonionic polynomial (RE and IM in quaternionic sense) this allows to define p-adic variants of space-time surfaces as varieties obeying same polynomial equations as their real counterparts provided the coefficients of octonion polynomials obtainable from real polynomials by analytic continuation are in an extension of rationals inducing also extension of p-adic numbers.

   The points with coordinates in the extension of rationals common to real and p-adic variants of M^8 identified as cognitive representations are in key role. One can see p-adic space-time surfaces as collections of "monads" labelled by these points at which Cartesian product of 1-D p-adic balls in each coordinate degree. The radius of the p-adic ball can vary. Inside each ball the same polynomial equations are satisfied so that the monads indeed reflect other monads.

   Kind of algebraic hologram would be in question consisting of the monads. The points in extension allow to define ordinary real distance between monads. Only finite number of monads would be involved since the number of points in extension tends to be finite. As the extension increases, this number increases. Cognitive representations become more complex: evolution as increase of algebraic complexity takes place.

2. Finite-dimensionality for the allowed extensions of p-adic number fields is motivated by the idea about finiteness of cognition. Perfectoids are however infinite-dimensional. Number theoretical universality demands that on only extensions of p-adics induced by those of rationals are allowed and defined extension of the entire adele. Extensions should be therefore be induced by the same extension of rationals for all p-adic number fields.

   Perfectoids correspond to an extension of Q_p apparently depending on p. This dependence is in conflict with number theoretical universality if real. This extension could be induced by corresponding extension of rationals for all p-adic number fields. For p-adic numbers Q_q $q \neq p$ all equation $a^{p^n} = x$ reduces to $a^n = x \ mod \ p$ and this in term to $a^m = x \ mod \ p$, $m = n \ mod \ p$. Finite-dimensional extension is needed to have all roots of required kind! This extension is therefore finite-D for all $q \neq p$ and infinite-D for p.

3. What about infinite-dimensionality of the extension. The real world is rarely perfect and our thoughts about it even less so, and one could argue that we should be happy with almost-perfectoids! "Almost" would mean extension induced by powers of p^{1/p^m} for large enough m, which is however not infinite. A finite-dimensional extension approaching perfectoid asymptotically is quite possible!

4. One could see the almost perfectoid as an outcome of evolution and perfectoid as the asymptotic states. High dimension of extension means that p-adic numbers and extension of rationals have large number of common numbers so that also cognitive representations contain a large number of common points. Maybe the p-adic number fields, which are evolutionary winners, have managed to evolve to especially high-dimensional almost-perfectoids! Note however that also the roots of e can be considered as extensions of rationals since corresponding p-adic extensions are finite-dimensional. Similar evolution can be considered also now.

   To get some perspective mote that for large primes such as $M_{127} = 2^{127} - 1$ characterizing electron the lowest almost perfectoid would give powers of $M_{127}^{1/M_{127}} = (2^{127} - 1)^{1/(2^{127}-1)} \sim 1 + log(2)2^{-120}$! The lattice of points in extension is extremely dense near real unit. The density of of points in cognitive representations near this point would be huge. Note that the length scales comes as negative powers of two, which brings in mind p-adic length scale hypothesis [K77].

Although the octonionic formulation in terms of polynomials (or rational functions identifying space-time varieties as zeros or poles of $RE(P)$ or $IM(P)$ is attractive in its simplicity, one can also consider the possibility of allowing analytic functions of octonion coordinate obtained from real analytic functions. These define complex analytic functions with commutative imaginary unit used to complexify octonions. Could meromorphic functions real analytic at real axis having only zeros and poles be allowed? The condition that all p-adic variants of these functions exist simultaneously is non-trivial. Coefficients must be in the extension of rationals considered and convergence poses restrictions. For instance, e^x converges only for $|x|_p < 1$. These functions might appear at the level of H.

7.7 Secret Link Uncovered Between Pure Math and Physics

I learned about a possible existence of a very interesting link between pure mathematics and physics (see `http://tinyurl.com/y86bckmo`). The article told about ideas of number theorist Minhyong Kim working at the University of Oxford. As I read the popular article, I realized it is something very familiar to me but from totally different view point.

Number theoretician encounters the problem of finding rational points of an algebraic curve defined as real or complex variant in which case the curve is 2-D surface and 1-D in complex sense. The curve is defined as root of polynomials polynomials or several of them. The polynomial have typically rational coefficients but also coefficients in extension of rationals are possible.

For instance, Fermat's theorem is about whether $x^n + y^n = 1$, $n = 1, 2, 3, ...$ has rational solutions for $n \geq 1$. For $n = 1$, and $n = 2$ it has, and these solutions can be found. It is now known that for $n > 2$ no solutions do exist. Quite generally, it is known that the number is finite rather than infinite in the generic case.

A more general problem is that of finding points in some algebraic extension of rationals. Also the coefficients of polynomials can be numbers in the extension of rationals. A less demanding problem is mere counting of rational points or points in the extension of rationals and a lot of progress has been achieved in this problem. One can also dream of classifying the surfaces by the character of the set of the points in extension.

I have consider the identification problem earlier in [L32] and I glue here a piece of text summarizing some basic results. The generic properties of sets of rational points for algebraic curves are rather well understood. Mordelli conjecture proved by Falting as a theorem (see `http://tinyurl.com/y9oq37ce`) states that a curve over Q with genus $g = (d-1)(d-2)/2 > 1$ (degree $d > 3$) has only finitely many rational points.

1. Sphere CP_1 in CP_2 has rational points as a dense set. Quite generally rational surfaces, which by definition allow parametric representation using polynomials with rational coefficients (encountered in context of Du Val singularities characterized by the extended Dynkin diagrams for finite subgroups of $SU(2)$) allow dense set of rational points [A57, A68]).

   $g = 0$ does not yet guarantee that there is dense set of rational points. It is possible to have complex conics (quadratic surface) in CP_2 with no rational points. Note however that this depends on the choice of the coordinates: if origin belongs to the surface, there is at least one rational point

2. Elliptic curve $y^2 - x^3 - ax - b$ in CP_2 (see `http://tinyurl.com/lovksny`) has genus $g = 1$ and has a union of lattices of rational points and of finite cyclic groups of them since it has origin as a rational point. This lattice of points are generated by translations. Note that elliptic curve has no singularities that is self intersections or cusps (for $a = 0, b = 0$ origin is a singularity).

   $g = 1$ does not guarantee that there is infinite number of rational points. Fermat's last theorem and CP_2 as example. $x^d + y^d = z^d$ is projectively invariant statement and therefore defines a curve with genus $g = (d-1)(d-2)/2$ in CP_2 (one has $g = 0, 0, 2, 3, 6, 10, ...$). For $d > 2$, in particular $d = 3$, there are no rational points.

3. $g \geq 2$ curves do not allow a dense set of rational points nor even potentially dense set of rational points.

In my article [L32] providing TGD perspective about the role of algebraic geometry in physics, one can find basic results related to the identification problem including web links and references to literature.

7.7.1 Connection with TGD and physics of cognition

The identification problem is extremely difficult even for mathematicians - to say nothing about humble physicist like me with hopelessly limited mathematical skills. It is however just this problem which I encounter in TGD inspired vision about adelic physics [L38, L39, L32]. Recall that in TGD space-times are 4-surfaces in $H = M^4 \times CP_2$, preferred extremals of the variational principle defining the theory [K98, L51].

1. In this approach p-adic physics for various primes p provide the correlates for cognition: there are several motivations for this vision. Ordinary physics describing sensory experience and the new p-adic physics describing cognition for various primes p are fused to what I called adelic physics. The adelic physics is characterized by extension of rationals inducing extensions of various p-adic number fields. The dimension n of extension characterizes kind of intelligence quotient and evolutionary level since algebraic complexity is the larger, the larger the value of n is. The connection with quantum physics comes from the conjecture that n is essentially effective Planck constant $h_{eff}/h_0 = n$ characterizing a hierarchy of dark matters. The larger the value of n the longer the scale of quantum coherence and the higher the evolutionary level, the more refined the cognition.

2. An essential notion is that of cognitive representation [K80] [L39, L32]. It has several realizations. One of them is the representation as a set of points common to reals and extensions of various p-adic number fields induced by the extension of rationals. These space-time points have points in the extension of rationals considered defining the adele. The coordinates are the embedding space coordinates of a point of the space-time surface. The symmetries of embedding space provide highly unique embedding space coordinates.

3. The gigantic challenge is to find these points common to real number field and extensions of various p-adic number fields appearing in the adele.

4. If this were not enough, one must solve an even tougher problem. In TGD the notion of "world of classical worlds" (WCW) is also a central notion [K98]. It consists of space-time surfaces in embedding space $H = M^4 \times CP_2$, which are so called preferred extremals of the action principle of theory. Quantum physics would reduce to geometrization of WCW and construction of classical spinor fields in WCW and representing basically many-fermion states: only the quantum jump would be genuinely quantal in quantum theory.

   There are good reasons to expect that space-time surfaces are minimal surfaces with 2-D singularities, which are string world sheets - also minimal surfaces [L51, L59]. This gives nice geometrization of gauge theories since minimal surfaces equations are geometric counterparts for massless field equations.

   One must find the algebraic points, the cognitive representation, for all these preferred extremals representing points of WCW (one must have preferred coordinates for H - the symmetries of embedding space crucial for TGD and making it unique, provide the preferred coordinates)!

5. What is so beautiful is that in given cognitive resolution defined by the extension of rationals inducing the discretization of space-time surface, the cognitive representation defines the coordinates of the space-time surfaces as a point of WCW. In finite cognitive and measurement resolution this huge infinite-dimensional space WCW discretizes and the situation can be handled using finite mathematics.

7.7.2 Connection with Kim's work

So: what is then the connection with the work and ideas of Kim. There has been a lot of progress in understanding the problem: here I an only refer to the popular article.

1. One step of progress has been the realization that if one uses the fact that the solutions are common to both reals and various p-adic number fields helps a lot. The reason is that for rational points the rationality implies that the solution of equation representable as infinite power series of p contains only finite number powers of p. If one manages to prove the this happens for even single prime, a rational solution has been found.

   The use of reals and all p-adic numbers fields is nothing but adelic physics. Real surfaces and all its p-adic variants form pages of a book like structure with infinite number of pages. The rational points or points in extension of rationals are the cognitive representation and are points common to all pages in the back of the book.

   This generalizes also to algebraic extensions of rationals. Solving the number theoretic problem is in TGD framework nothing but finding the points of the cognitive representation. The surprise for me was that this viewpoint helps in the problem rather than making it more complex.

   There are however problematic situations in some cases the hypothesis about finite set of algebraic points need not make sense. A good example is Fermat for $x + y = 1$. All rational points and also algebraic points are solutions. For $x^2+y^2 = 1$ the set of Pythagorean triangles characterizing the solutions is infinite. How to cope with these situations in which one has accidental symmetries as one might say?

2. Kim argues that one can make even further progress by considering the situation from even wider perspective by making the problem even bigger. Introduce what the popular article (see `http://tinyurl.com/y86bckmo`) calls the space of spaces. The space of string world sheets is what string models suggests. WCW is what TGD suggests. One can get a wider perspective of the problem of finding algebraic points of a surface by considering the problem in the space of surfaces and at this level it might be possible to gain much more understanding. The notion of WCW would not mean horrible complication of a horribly complex problem but possible manner to understand the problem!

   The popular article mentioned in the beginning mentions so called Selmer varieties as a possible candidate for the space of spaces. From the Wikipedia article (see `http://tinyurl.com/y27so3f2`) telling about Kim one can find a link to an article [A56] related to Selmer varieties. This article goes over my physicist's head but might give for a more mathematically oriented reader some grasp about what is involved. One can find also a list of publications of Kim (see `http://people.maths.ox.ac.uk/kimm/`.

   Kim also suggests that the spaces of gauge field configurations could provide the spaces of spaces. The list contains an article [A67] with title *Arithmetic Gauge Theory: A Brief Introduction* (see `http://tinyurl.com/y66mphkh`) , which might help physicist to understand the ideas. An arithmetic variant of gauge theory could provide the needed space of spaces.

7.7.3 Can one make Kim's idea about the role of symmetries more concrete in TGD framework?

The crux of the Kim's idea is that somehow symmetries of space of spaces could come in rescue in the attempts to understand the rational points of surface. The notion of WCW suggest in TGD framework rather concrete realization of this idea that I have discussed from the point of view of construction of quantum states.

1. A little bit more of zero energy ontology (ZEO) is needed to follow the argument. In ZEO causal diamonds (CDs) are central. CDs are defined as intersections of future and past directed light-cones with points replaced with CP_2 and forming a scale hierarchy are central. Space-time surfaces are preferred extremals with ends at the opposite boundaries of CD indeed looking like diamond. Symplectic group for the boundaries of causal diamond (CD) is the group of isometries of WCW [K98] [L51]. Maximal isometry group is required to guarantee that the WCW Kähler geometry has Riemann connection - this was discovered for loop spaces by Dan Freed [A35]. Its Lie algebra has structure of Kac- Moody algebra with respect to the light-like radial coordinate of the light-like boundary of CD, which is piece of light-cone boundary. This infinite-D group plays central role in quantum TGD: it acts as maximal group of WCW isometries and zero energy states are invariant under its action at opposite boundaries.
2. As one replaces space-time surface with a cognitive representation associated with an extension of rationals, WCW isometries are replaced with their infinite discrete subgroup acting in the number field define by the extension of rationals defining the adele. These discrete isometries do not leave the cognitive representation invariant but replace with it new one having the same number of points and one obtains entire orbit of cognitive representations. This is what the emergence of symmetries in wider conceptual framework would mean.
3. One can in fact construct invariants of the symplectic group. Symplectic transformations leave invariant the Kähler magnetic fluxes associated with geodesic polygons with edges identified as geodesic lines of H. There are also higher-D symplectic invariants. The simplest polygons are geodesic triangles. The symplectic fluxes associated with the geodesic triangles define symplectic invariants characterizing the cognitive representation. For the twistor lift one must allow also M^4 to have analog of Kähler form and it would be responsible for CP violation and matter antimatter asymmetry [L27]. Also this defines symplectic invariants so that one obtains them for both M^4 and CP_2 projections and can characterize the cognitive representations in terms of these invariants. Note that the existence of twistor lift fixes the choice of H uniquely since M^4 and CP_2 are the only 4-D spaces allowing twistor space with Kähler structure [A50] necessary for defining the twistor lift of Kähler action.

   More complex cognitive representations in an extension containing the given extension are obtained by adding points with coordinates in the larger extension and this gives rise to new geodesic triangles and new invariants. A natural restriction could be that the polynomial defining the extension characterizing the preferred extremal via M^8-H duality defines the maximal extension involved.
4. Also in this framework one can have accidental symmetries. For instance, M^4 with CP_2 coordinates taken to be constant is a minimal surface, and all rational and algebraic points for given extension belong to the cognitive representation so that they are infinite. Could this has something to do with the fact that we understand M^4 so well and have even identified space-time with Minkowski space! Linear structure would be cognitively easy for the same reason and this could explain why we must linearize.

   CP_2 type extremals with light-like M^4 geodesic as M^4 projection is second example of accidental symmetries. The number of rational or algebraic points with rational M^4 coordinates at light-like curve is infinite - the situation is very similar to $x+y=1$ for Fermat. Simplest cosmic strings are geodesic sub-manifolds, that is products of plane $M^2 \subset M^4$ and CP_2 geodesic sphere. Also they have exceptional symmetries.

   What is interesting from the point of view of proposed model of cognition is that these cognitively easy objects play a central role in TGD: their deformations represent more complex dynamical situations. For instance, replacing planar string with string world sheet replaces cognitive representation with a discrete or perhaps even finite one in M^4 degrees of freedom.
5. A further TGD based simplification would be M^8-H ($H=M^4\times CP_2$) duality in which space-time surfaces at the level of M^8 are algebraic surfaces, which are mapped to surfaces in H identified as preferred extremals of action principle by the M^8-H duality [L32]. Algebraic surfaces satisfying algebraic equations are very simple as compared to preferred extremals satisfying partial differential equations but "preferred" is what makes possible the duality. This huge simplification of the solution space of field equations guarantees holography necessitated by general coordinate invariance implying that space-time surfaces are analogous to

Bohr orbits. It would also guarantee the huge symmetries of WCW making it possible to have Kähler geometry.

This suggests in TGD framework that one finds the cognitive representation at the level of M^8 using methods of algebraic geometry and maps the points to H by using the $M^8 - H$ duality. TGD and octonionic variant of algebraic geometry would meet each other.

It must be made clear that now solutions are not points but 4-D surfaces and this probably means also that points in extension of rationals are replaced with surfaces with embedding space coordinates defining function in extensions of rational functions rather than rationals. This would bring in algebraic functions. This might provide also a simplification by providing a more general perspective. Also octonionic analyticity is extremely powerful constraint that might help.

7.8 Cognitive representations for partonic 2-surfaces, string world sheets, and string like objects

Cognitive representations are identified as points of space-time surface $X^4 \subset M^4 \times CP_2$ having embedding space coordinates in the extension of of rationals defined by the polynomial defined by the M^8 pre-image of X^4 under $M^8 - H$ correspondence [L33, L34, L67, L60, L58, L53]. Cognitive representations have become key piece in the formulation of scattering amplitudes [L62] . One might argue that number theoretic evolution as increase of the dimension of the extension of rationals favors space-time surfaces with especially large cognitive representations since the larger the number of points in the representation is, the more faithful the representation is.

One can pose several questions if one accepts the idea that space-time surfaces with large cognitive representations are survivors.

1. Preferred p-adic primes are proposed to correspond to the ramified primes of the extension [L69]. The proposal is that the p-adic counterparts of space-time surfaces are identifiable as imaginations whereas real space-time surfaces correspond to realities. p-Adic space-time surfaces would have the embedding space points in extension of rationals as common with real surfaces and large number of these points would make the representation realistic. Note that the number of points in extension does not depend on p-adic prime.

   Could some extensions have an especially high number of points in the cognitive representation so that the corresponding ramified primes could be seen as survivors in number theoretical fight for survival, so to say? Galois group of the extension acts on cognitive representation. Galois extension of an extension has the Galois group of the original extension as normal subgroup so that ormal Galois group is analogous to a conserved gene.

2. Also the type of extremal matters. For instance, for instance canonically imbedded M^4 and CP_2 contain all points of extension. These surfaces correspond to the vanishing of real or imaginary part (in quaternionic sense) for a linear octonionic polynomial $P(o) = o$! As a matter of fact, this is true for all known preferred extremals under rather mild additional conditions. Boundary conditions posed at both ends of CD in ZEO exclude these surfaces and the actual space-time surfaces are expected to be their deformations.

3. Could the surfaces for which the number of points in cognitive representation is high, be the ones most easily discovered by mathematical mind? The experience with TGD supports positive answer: in TGD the known extremals [K10] are examples of such mathematical objects! If so, one should try to identify mathematical objects with high symmetries and look whether they allow TGD realization.

4. One must also specify more precisely what cognitive representation means. Strong form of holography (SH) states that the information gives at 2-D surfaces - string world sheets and partonic 2-surfaces - is enough to determine the space-time surfaces. This suggests that it is enough to consider cognitive representation restricted to these 2-surfaces. What kind of 2-surfaces are the cognitively fittest one? It would not be surprising if surfaces with large symmetries acting in extension were favored and elliptic curves with discrete 2-D translation group indeed turn out to be assigable string world sheets as singularities and string like objects. In the case of partonic 2-surfaces geodesic sphere of CP_2 is similar object.

All known extremals, in particular preferred extremals, are good candidates in this respect because of their high symmetries. By strong form of holography (SH) partonic 2-surfaces and string world sheets are expected to give rise to cognitive representations. Also cosmic strings are expected to carry them. Under what conditions these representations are large?

7.8.1 Partonic 2-surfaces as seats of cognitive representations

One can start from SH and look the situation more concretely. The situation for partonic 2-surfaces has been considered already earlier [L68, L57] but deserves a separate discussion.

1. Octonionic polynomials allow special solutions for which the entire polynomial vanishes. This happens at 6-sphere S^6 at the boundary of 8-D light-cone. S^6 is analogous to brane and has radius $R = r_n$, which is a root of the real polynomial with rational coefficients algebraically continued to the octonionic polynomial.

   S^6 has the ball B^3 of radius r_n of the light-cone M^4_+ with time coordinate $t = r_n$ as analog of base space and sphere S^3 of E^4 with radius $R = \sqrt{r_n^2 - r^2}$, r the radial coordinate of B^3 as an analog of fiber. The analog of the fiber contracts to a point at the boundary of the light-cone. The points with B^3 projection and E^4 coordinates in extension of rationals belong to the cognitive representation. The condition that $R^2 = x_i x^i = r_n^2 - r^2$ is square of a number of extension is rather mild and allows infinite number of solutions.

2. The 4-D space-time surfaces X^4 are obtained as generic solutions of $Im(P(o)) = 0$ or $Re(P(o)) = 0$. Their intersection with S^6 - partonic 2-surface X^2 - is 2-D. The assumption is that the incoming and outgoing 4-D space-time surfaces representing orbits of particles in topological sense are glued together at X^2 and possibly also in their interiors. X^2 serves as an analog of vertex for 3-D particles. This gives rise to topological analogs of Feynman diagrams.

   In the generic case the number of points in cognitive representation restricted to X^2 is finite unless the partonic 2-surface X^2 is special - say correspond to a geodesic spere of S^6.

3. The discrete isometries and conformal symmetries of the cognitive representation restricted to X^2 possibly represented as elements of Galois group might play a role. For $X^2 = S^2$ the finite discrete subgroups of $SO(3)$ giving rise to finite tessellations and appearing in ADE correspondence might be relevant. For genera $g = 01, 2$ conformal symmetry Z_2 is always possible but for higher genera only in the case of hyper-elliptic surfaces- this used to explain why only $g = 0, 1, 2$ correspond to observed particles [K32] whereas higher genera could be regarded as many-particle states of handles having continuous mass spectrum. Torus is an exceptional case and one can ask whether discrete subgroup of its isometries could be realized.

4. In TGD inspired theory of consciousness [L42, L57] the moments $t = r_n$ corresponds to "very special moments in the life of self". They would be also cognitively very special - kind of eureka moments with a very large number of points in cognitive representation. The question is whether these surfaces might be relevant for understanding the nature of mathematical consciousness and how the mathematical notions emerge at space-time level.

7.8.2 Ellipticity

Surfaces with discrete translational symmetries is a natural candidate for a surface with very large cognitive representation. Are their analogs possible? The notions of elliptic function, curve, and surface suggest themselves as a starting point.

1. Elliptic functions (`http://tinyurl.com/gpugcnh`) have 2-D discrete group of translations as symmetries and are therefore doubly periodic and thus identifiable as functions on torus.

   Weierstrass elliptic functions $\mathcal{P}(z; \omega_1, \omega_2)$ (`http://tinyurl.com/ycu8oa4r`) are defined on torus and labelled by the conformal equivalence class $\lambda = \omega_1/\omega_2$ of torus identified as the ratio $\lambda = \omega_1/\omega_2$ of the complex numbers ω_i defining the periodicities of the lattice involved. Functions $\mathcal{P}(z; \omega_1, \omega_2)$ are of special interest as far as elliptic curves are considered and defines an embedding of elliptic curve to CP_2 as will be found.

   If the periods are in extension of rationals then values in the extension appear infinitely many times. Elliptic functions are not polynomials. Although the polynomials giving rise to octonionic polynomials could be replaced by analytic functions it seems that elliptic functions are not the case of primary interest. Note however that the roots r_n could be also complex and could correspond to values of elliptic function forming a lattice.

2. Elliptic curves (`http://tinyurl.com/lovksny`) are defined by the polynomial equation

$$y^2 \ = \ P(x) = x^3 + ax + b \ . \tag{7.8.1}$$

An algebraic curve of genus 1 allowing 2-D discrete translations as symmetries is in question. If a point of elliptic curve has coordinates in extension of rationals then 2-D discrete translation acting in extension give rise to infinite number of points in the cognitive representation. Clearly, the 2-D vectors spanning the lattice defining the group must be in extension of rationals.

One can indeed define commutative sum $P+Q$ for the points of the elliptic curve. The detailed definition of the group law and its geometric illustration can be found in Wikipedia article (`http://tinyurl.com/lovksny`).

1. Consider real case for simplicity so that elliptic curve is planar curve. $y^2 = P(x) = x^3+ax+b$ must be non-negative to guarantee that y is real. $P(x) \geq 0$ defines a curve in upper (x,y) plane extending from some negative value x_{min} corresponding to $y^2 = P(x_{min}) = 0$ to the right. Given value of y can correspond to 3 real roots or 1 real root of $P_y(x) = y^2 - P(x)$. At the two extrema of $P_y(x)$ 2 real roots co-incide. The graph of $y = \pm\sqrt{P(x)}$ is reflection symmetric having two branches beginning from $(x_{min}, y = 0)$.
2. The negative $-P$ is obtained by reflection with respect to x-axis taking y_P to $-y_P$. Neutral element O is identified as point a infinity (assuming compactification of the plane to a sphere) which goes to itself under reflection $y \rightarrow -y$.
3. One assigns to the points P and Q of the elliptic curve a line $y = sx+d$ containing them so that one has $s = (y_p - y_Q)/(x_P - x_Q)$. In the generic case the line intersects the elliptic curve also at third point R since $P_{y=sx+d}(x)$ is third order polynomial having three roots (x_P, x_Q, x_R). It can happen that 2 roots are complex and one has 1 real root. At criticalityfor the transiton from 3 to 1 real roots one has $x_Q = x_R$.

   Geometrically one can distinguish between 4 cases.

   - The roots P, Q, R of $P_{y=sx+d}(x)$ are different and finite: one defines the sum as $P+Q = -R$.
   - $P \neq Q$ and $Q = R$ (roots Q and R are degenerate): $P+Q+Q = O$ giving $R = -P/2$.
   - P and Q are at a line parallel to y-axis and one has $R = O$: $P+Q+O = O$ and $P = -Q$.
   - P is double root of $P_{y=sx+d}(x)$ with tangent parallel to y-axis at the point $(x_{min}, y = 0)$ at which the elliptic curve begins so that one has $R = O$: $P+P+O = O$ gives $P = -P$. This corresponds to torsion.
4. Elliptic surfaces (see `http://tinyurl.com/yc33a6dg`) define a generalization of elliptic curves and are defined for 4-D complex manifolds. Fiber is required to be smooth and has genus 1.

7.8.3 String world sheets and elliptic curves

In twistor lift of TGD space-time surfaces identifiable as minimal surfaces with singularities, which are string world sheets and partonic 2-surfaces. Preferred extremal property means that space-time surfaces are extremals of both Kähler action and volume action except at singularities.

Are string world sheets with very large number of points in cognitive representation possible? One has right to expect that string world sheets allow special kind of symmetries allowing large, even infinite number of points at the limit of large sheet and related by symmetries acting in the extension of rationals. If one of the points is in the extension, also other symmetry related points are in the extension. For a non-compact group, say translation one would have infinite number of points in the representation but the finite size of CD would pose a limitation to the number of points.

String world sheets are good candidates for the realization of elliptic curves.

1. The general conjecture is that preferred extremals allow what I call Hamilton-Jacobi structure for M^4 [K98]. The distribution of tangent spaces having decomposition $M^4(x) = M^2(x) \times E^2(x)$ would be integrable giving rise to a family of string world sheets Y^2 and partonic 2-surfaces X^2 more general than those defined above. X^2 and Y^2 are orthogonal to each other at each point of X^4. One can introduce local light-cone coordinates (u, v) for Y^2 and local E^2 complex coordinate w for X^2.

2. Space-time surface itself would be a deformation of M^4 with Hamilton-Jacobi structure in CP_2 direction. w coordinate as function $w(z)$ of CP_2 complex coordinate z or vice versa would define the string world sheet. This would be a transversal deformation of the basic string world sheet Y^2: stringy dynamics is indeed transversal.

3. The idea about maximal cognitive representation suggests that $w \leftrightarrow z$ correspondence defines elliptic curve. One would have $y^2 = P(x) = x^3 + ax + b$ with either $(y = w, x = z)$ or $(y = z, x = w)$. A natural conjecture is that for the space-time surface corresponding to a given extension K of rationals the coefficients a an b belong to K so that the algebraic complexity of string world sheet would increase in number theoretic evolution [L66]. The orbit of a algebraic point at string world sheet would be lattice made finite by the size of CD. Elliptic curves would define very special deformed string world sheets in space-time.

4. It is interesting to consider the pre-image of given point y ($y = w$ or $y = z$) covering point x. One has $y = \pm\sqrt{u}$, $u = P(x)$ corresponding to group element and its negative: there are two points of covering given value of u. $u = P(x)$ covers 3 values of x. The values of x would belong to 6-fold covering of rationals. The number theoretic interpretation for the effective Planck constant $h_{eff} = nh_0$ states that n is the number of sheets for space-time surface as covering.

   There is evidence that $h_{eff} = h$ corresponds to $n = 6$ [L16]. Could 6-fold covering of rationals be fundamental since it gives very large cognitive representation at the level of string world sheets?

   For extensions K of rationals the x coordinates for the points of cognitive representation would belong to 6-D extension of K.

5. Ellipticity condition would apply on the string world sheets themselves. In the number theoretic vision string world sheets would correspond at M^8 level to singularities at which the quaternionic tangent space degenerates to 2-D complex space. Are these conditions consistent with each other? It would seem that the two conditions would select cognitively very special string world sheets and partonic 2-surfaces defining by strong form of holography (SH) space-time surface as a hologram in SH. Consciousness theorist interested in mathematical cognition might ask whether the notion of elliptic surfaces have been discovered just because it is cognitively very special. In the case of partonic 2-surfaces geodesic sphere of CP_2 is similar object.

7.8.4 String like objects and elliptic curves

String like objects - cosmic strings - and their deformations, are fundamental entities in TGD based cosmology and astrophysics and also in TGD inspired quantum biology. One can assign elliptic curves also to string like objects.

1. Quite generally, the products $X^2 \times Y^2 \subset M^4$ of string world sheets X^2 and complex surfaces Y^2 of CP_2 define extremals that I have called cosmic strings [K10].

2. Elliptic curves allow a standard embedding to CP_2 as complex surfaces constructible in terms of Weierstrass elliptic function $\mathcal{P}(z)$ (`http://tinyurl.com/ycu8oa4r`) satisfying the identity

$$[\mathcal{P}'(z)]^2 = [\mathcal{P}(z)]^3 - g_2\mathcal{P}(z) - g_3 \quad . \tag{7.8.2}$$

   Here g_2 and g_3 are modular invariants. This identity is of the same form as the condition $y^2 = x^3 + ax + b$ with identifications $y = \mathcal{P}'(z), x = \mathcal{P}(z)$ and $(a = -g_2, b = -g_3)$. From the expression

$$y^2 = x(x-1)(x-\lambda) \tag{7.8.3}$$

in terms of the modular invariant $\lambda = \omega_1/\omega_2$ of torus one obtains

$$g_2 = \frac{4^{1/3}}{3}(\lambda^2 - \lambda + 1 \ , \quad g_3 = \frac{1}{27}(\lambda+1)(2\lambda^2 - 5\lambda + 2) \ . \tag{7.8.4}$$

Note that third root of a appears in the formula. The so called modular discriminant

$$\Delta = g_2^3 - 27g_3^2 = \lambda^2(\lambda-1)^2 \ . \tag{7.8.5}$$

vanishes for $\lambda = 0$ and $\lambda = 1$ for which the lattice degenerates.

3. The embedding of the elliptic curve to CP_2 can be expressed in projective coordinates of CP_2 as

$$(z^1, z^2, z^3) = (\xi^1, \xi^2, 1) = (\frac{\mathcal{P}'(w)}{2}, \mathcal{P}(w), 1) \ . \tag{7.8.6}$$

7.9 Are fundamental entities discrete or continuous and what discretization at fundamental level could mean?

There was an interesting FB discussion about discrete and continuum. I decided to write down my thoughts and emphasize those points that I see as important.

7.9.1 Is discretization fundamental or not?

The conversation inspired the question whether discreteness is something fundamental or not. If it is assumed to be fundamental, one encounters problems. The discrete structures are not unique. One has deep problem with the known space-time symmetries. Symmetries are reduced to discrete subgroup or totally lost. A further problem is the fact that in order to do physics, one must bring in topology and length measurements.

In discrete situation topology, in particular space-time dimension, must be put in via homology effectively already meaning use of embedding to Euclidian space. Length measurement remains completely ad hoc. The construction of discrete metric is highly non-unique procedure and the discrete analog of of say Einstein's theory (Regge calculus) is rather clumsy. One feeds in information, which was not there by using hand weaving arguments like infrared limit. It is possible to approximate continuum by discretization but discrete to continuum won't go.

In hype physics these hand weaving arguments are general. For instance, the emergence of 3-space from discrete Hilbert space is one attempt to get continuum. One puts in what is factually a discretization of 3-space and then gets 3-space back at IR limit and shouts "Eureka!".

7.9.2 Can one make discretizations unique?

Then discussion went to numerics. Numerics is for mathematicians same as eating for poets. One cannot avoid it but luckily you can find people doing the necessary programming if you are a professor. Finite discretization is necessary in numerics and is highly unique.

I do not have anything personal against discretization as a numerical tool. Just the opposite, I see finite discretization as absolutely essential element of adelic physics as an attempt to describe also the correlates of cognition in terms of p-adic physics with p-adic space-time sheets as correlates of "thought bubbles" [L39, L38]. Cognition is discrete and finite and uses rational numbers: this is the basic clue.

1. Cognitive representations are discretizations of (for instance) space-time surface. One can say that physics itself builds its cognitive representation in all scales using p-adic space-time sheets. They should be unique once measurement resolution is characterized if one is really talking about fundamental physics.

   The idea abou tp-adic physics as physics of cognition indeed led to powerful calculational recipes. In p-adic thermodynamics the predictions come in power series of p-adic prime p and for the values of p assignable to elementary particles the two lowest terms give practically exact result [K68]. Corrections are of order 10^{-76} for electron characterized by Mersenne prime $M_{127} = 2^{127} - 1 \sim 10^{38}$.

2. Adelic physics [L39] provides the formulation of p-adic physics: it is assumed that cognition is universal. Adele is a book like structure having as pages reals and extensions of various p-adic number fields induced by given extension of rationals. Each extension of rationals defines its own extension of the rational adele by inducing extensions of p-adic number fields. Common points between pages consist of points in extension of rationals. The books associated with the adeles give rise to an infinite library.

   At space-time level the points with coordinates in extension define what I call cognitive representation. In the generic case it is discrete and has finite number of points. The loss of general coordinate invariance is the obvious objection. In TGD however the symmetries of the embedding space fix the coordinates used highly uniquely. $M^8 - H$ duality ($H = M^4 \times CP_2$) and octonionic interpretation implies that M^8 octonionic linear coordinates are highly unique [L32, L60]. Note that M^8 must be complexified. Different coordinatizations correspond to different octonionic structures- to different moduli - related by Poincare transformations of M^8. Only rational time translations as transformations of octonionic real coordinate are allowed as coordinate changes respecting octonionic structure.

3. Discretization by cognitive representation is unique for given extension of rationals defining the measurement resolution. At the limit of algebraic numbers algebraic points form a dense set of real space-time surface and p-adic space-time surfaces so that the measurement resolution is ideal. One avoids the usual infinities of quantum field theories induced by continuous delta functions, which for cognitive representations are replaced with Kronecker deltas. This seems to be the best that one can achieve with algebraic extensions of rationals. Also for transcendental extensions the situation is discrete.

   This leads to a number theoretic vision about second quantization of induced spinor fields central for the construction of gamma matrices defining the spinor structure of "world of classical worlds" (WCW) providing the arena of quantum dynamics in TGD analogous to the super-space of Wheeler [K98]. One ends up to a construction allowing to understand TGD view about SUSY as necessary aspect of second quantization of fermions and leads to the conclusions that in the simplest scenario only quarks are elementary fermions and leptons can be seen as their local composites analogous to super partners.

4. Given polynomial defining space-time surfaces in M^8 defines via its roots extension of rationals. The hierarchy of extensions defines an evolutionary hierarchy. The dimension n of extension defines kind of IQ measuring algebraic complexity and n corresponds also to effective Planck constant labelling phases of dark matter in TGD sense so that a direct connection with physics emerges.

   Embedding space assigns to a discretization a natural metric. Distances between points of metric are geodesic distances computed at the level of embedding space.

5. An unexpected finding was that the equations defining space-time surfaces as roots of real or imaginary parts of octonionic polynomials have also 6-D brane like entities with topology of S^6 as solutions [L57, L67]. These entities intersect space-time surfaces at 3-D sections for which linear M^4 time is constant. 4-D roots can be glued together along these branes. These solutions turn out to have an interpretation in TGD based theory of quantum measurement extending to a theory of consciousness. The interpretation as moments of "small" state function reductions as counterparts of so called weak measurements. They could correspond to special moments in the life of conscious entity.

7.9.3 Can discretization be performed without lattices?

For a systems obeying dynamics defined by partial differential equations, the introduction of lattices seems to be necessary aspect of discretization. The problem is that the replacement of derivatives with discrete approximations however means that there is no hope about exact results. In the general case the discretization for partial differential equations involving derivatives forces to introduce lattice like structures. This is not needed in TGD.

1. At the level of M^8 ordinary polynomials give rise to octonionic polynomials and space-time surfaces are algebraic surfaces for which imaginary or real part of octonionic polynomial in quaternionic sense vanishes. The equations are purely algebraic involving no partial derivatives and there is no need for lattice discretization.

   For surfaces defined by polynomials the roots of polynomial are enough to fix the polynomials and therefore also the space-time surface uniquely: discretization is not an approximation but gives an exact result! This could be called number theoretical holography and generalizes the ordinary holography. Space-time surfaces are coded by the roots of polynomials with rational coefficients.

2. What about the field equations at the level of $H = M^4 \times CP_2$? $M^8 - H$ duality maps these surfaces to preferred extremals as 4-surfaces in H analogous to Bohr orbits. Twistor lift of TGD predicts that they should be minimal surfaces with 2-D singularities being also extremals of 4-D Kähler action. The field equations would reduce locally to purely algebraic conditions. In properly chosen coordinates for H they are expected to be determined in terms of polynomials coding for the same extension of rationals as their M^8 counterparts so that the degree should be same [L60]. This would allow to deduce the partial derivatives of embedding space for the image surfaces without lattice approximation.

3. The simplest assumption is that the polynomials have rational coefficients. Number theoretic universality allows to consider also algebraic coefficients. In both cases also WCW is discretized and given point -space-time surface in QCD has coordinates given by the points of the number theoretically universal cognitive representation of the space-time surface. Even real coefficients are possible. This would allow to obtain WCW as a continuum central for the construction of WCW metric but is not consistent with number theoretical universality.

   Can one have polynomial/functions with rational coefficients and discretization of WCW without lattice but without losing WCW metric? Maybe the same trick that works at space-time level works also in WCW!

   (a) The group WCW isometries is identified as symplectic transformations of $\delta M^4_\pm \times CP_2$ ($\delta M^4_\pm$ denotes light-cone boundary) containing the boundary of causal diamond CD. The Lie algebra *Sympl* of this group is analogous half-Kac Moody algebra having symplectic transformations of $S^2 \times CP_2$ as counterpart of finite-D Lie group has fractal structure containing infinite number of sub-algebras $Sympl_n$ isomorphic to algebra itself: the conformal weights assignable to radial light-like coordinate are n-multiples of those for the entire algebra. Note that conformal weights of *Sympl* are non-negative.

   (b) One formulation for the preferred extremal property is in terms of infinite number of analogs of gauge conditions stating the vanishing of classical and also Noether charges for $Sympl_n$ and $[Sympl_n, Sympl]$. The conditions generalize to the super-counterpart of *Sympl* and apply also to quantum states rather than only space-time surfaces. In fact, while writing this I realized that - contrary to the original claim - also the vanishing of the Noether charges of higher commutators is required so that effectively $Sympl_n$ would define normal subgroup of *Sympl*. These conditions does not follow automatically.

   The Hamiltonians of $Sympl(S^2 \times CP_2)$ are also labelled by the representations of the product of the rotation group $SO(3) \subset SO(3,1)$ of S^2 and color group $SU(3)$ together forming the analog of the Lie group defining Kac-Moody group. This group does not have have the fractal hierarchy of subgroups. The strongest condition is that the algebra corresponding to Hamiltonian isometries does not annihilate the physical states.

   The space of states satisfying the gauge conditions is finite-D and that WCW becomes effectively finite-dimensional. A coset space associated with *Sympl* would be in question and it would have maximal symmetries as also WCW. The geometry of the reduced WCW, WCW_{red} could be deduced from symmetry considerations alone.

(c) Number theoretic discretization would correspond to a selection of points of this subspace with the coordinates in the extension of rationals.The metric of WCW$_{red,n}$ at the points of discretization would be known and no lattice discretization would be needed. The gauge conditions are analogous to massless Dirac equation in WCW and could be solved in the points of discretization without introducing the lattice to approximate derivatives. As a matter fact, Dirac equation can be formulated solely in terms of the generators of $Sympl$.

(d) This effectively restricts WCW to $WCW_{red,n}$ in turn reduced to its discrete subset - since infinite number of WCW coordinates are fixed. If this sub-space can be regarded as realization of infinite number of algebraic conditions by polynomials with rational coefficients one can assign to it extension of rationals defining naturally the discretization of WCW$_{red,n}$. This extension is naturally the same as for space-time surfaces involved so that the degree of polynomials defining WCW$_{red,n}$ would be naturally n and same as that for the polynomial defining the space-time surface. WCW$_{red,n}$ would decompose to union of spaces WCW$_{red,E_n}$ labelled by extensions E_n of rationals with same dimension n.
There is analogy with gauge fixing. WCW$_{red,E_n}$ is a coset space of WCW defined by the gauge conditions. One can represent this coset space as a sub-manifold of WCW by taking one representative point from each coset. This choice is not unique but one can hope finding a gauge choice realized by an infinite number of polynomials of degree n defining same extension of rationals as the polynomial defining the space-time surfaces in question.

(e) WCW spinor fields would be always restricted to finite-D algebraic surface of WCW$_{red,E_n}$ expressible in terms of algebraic equations. Finite measurement resolution indeed strongly suggests that WCW spinor field mode is non-vanishing only in a region parameterized in WCW by finite number of parameters. There is also a second manner to see this. WCW$_{red,E_n}$ could be also seen as $n+4$-dimensional surface in WCW.

(f) One can make this more concrete. Cognitive representation by points of space-time surface with coordinates in the extension - possibly satisfying additional conditions such as belonging to the 2-D vertices at which space-time surfaces representing different roots meet - provides WCW coordinates of given space-time surface. Minimum number of points corresponds to the dimension of extension so that the selection of coordinate can be redundant. As the values of these coordinates vary, one obtains coordinatization for the sector of WCW$_{red,E_n}$. An interesting question is whether one could represent the distances of space-time surfaces in this space in terms of the data provided by the points of discretization.
An interesting question is whether one can represent the distances of space-time surfaces in this space in terms of the data provided by the points of cognitive representation. One can define distance between two disjoint surfaces as the minimum of distance between the points of 2-surfaces. Could something like this work now? The points would be restricted to the cognitive representations. Could one define the distance between two cognitive representations with same number N of points in the following manner.
Consider all bipartitions formed by the cognitive representations obtained by connecting their points together in 1-1 manner. There are N! bipartitions of this kind if the number of points is N. Calculate the sum of the squares of the embedding space distances between paired points. Find the bipartition for which this distance squared is minimum and define the distance between cognitive representations as this distance. This definition works also when the numbers of points are different.

(g) If there quantum states are the basic objects and there is nothing "physical" behind them one can ask how we can imagine mathematical structures which different from basic structure of TGD. Could quantum states of TGD Universe in some sense represent all mathematical objects which are internally consistent. One could indeed say that at the level of WCW all n+4-D manifolds can be represented concretely in terms of WCW spinor fields localized to n-D subspaces of WCW. WCW spinor fields can represent concept of 4-surface of WCW$_{red,n}$ as a quantum superposition of its instance and define at the same time $n+4$-D surfaces [L70] [L59, L63, L62, L70].

7.9.4 Simple extensions of rationals as codons of space-time genetic code

A fascinating idea is that extensions of rationals define the analog of genetic code for space-time surfaces, which would therefore represent number theory and also finite groups.

(a) The extensions of rationals define an infinite hierarchy: the proposal is that the dimension of extensions corresponds to the integer n characterizing subalgebra $Sympl_n$. This would give direct correspondence between the inclusions of HFFs assigned to the hierarchy of algebras $Sympl_n$ and hierarchy of extensions of rationals with dimension n.
Galois group for a extension of extension contains Galois group of extension as normal subgroup and is therefore *not simple*. Extension hierarchies correspond to inclusion hierarchies for normal subgroups. Simple Galois groups are in very special position and associated with what one might call simple extensions serving as fundamental building bricks of inclusion hierarchies. They would be like elementary particles and define fundamental space-time regions. Their Galois groups would act as groups of physical symmetries.

(b) One can therefore talk about elementary space-time surfaces in M^8 and their compositions by function composition of octonionic polynomials. Simple groups would label elementary space-time regions. They have been classified: (see `http://tinyurl.com/y3xh4hrh`). The famous Monster groups are well-known examples about simple finite groups and would have also space-time counterparts. Also the finite subgroups of Lie groups are special and those of $SU(2)$ are associated with Platonic solids and seem to play key role in TGD inspired quantum biology. In particular, vertebrate genetic code can be assigned to icosahedral group.

(c) There is also an analogy with genes. Extensions with simple Galois groups could be seen as codons and sequences of extension obtained by functional composition as analogs of genes. I have even conjectured that the space-time surfaces associated with genes could quite concretely correspond to extensions of extensions of extensions of ...

7.9.5 Are octonionic polynomials enough or are also analytic functions needed?

I already touched the question whether also analytic functions with rational coefficients (number theoretical universality) might be needed.

(a) The roots of analytic functions generate extension of rationals. If the roots involve transcendental numbers they define infinite extensions of rationals. Neper number e is very special in this sense since e^p is ordinary p-adic number for all primes p so that the induced extension is finite-dimensional. One could thus allow it without losing number theoretical universality. The addition of π gives infinite-D extension but one could do by adding only roots of unity to achieve finite-D extensions with finite accuracy of phase measurement. Phases would be number theoretically universal but not angles.

(b) One could of course consider only transcendental functions with rational roots. Trigonometric function $sin(x/2\pi)$ serves as a simple example. One can also argue that since physics involves in an essential manner trigonometric functions via Fourier analysis, the inclusion of analytic functions with algebraic roots must be allowed.

(c) What about analytic functions as limits of polynomials with rational coefficients such that the number of roots becomes infinite at the limit? Also their imaginary and real part can vanish in quaternionic sense and could define space-time surfaces - analogs of transcendentals as space-time surfaces. It is not clear whether these could be allowed or not.

Could one have a universal polynomial like function giving algebraic numbers as the extension of rationals defined by its algebraic roots? Could Riemann zeta (see `http://tinyurl.com/nfbkrsx`) code algebraic numbers as an extension via its roots. I have conjectured that roots of Riemann zeta are algebraic numbers: could they span all algebraic numbers?

It is known that the real or imaginary part of Riemann zeta along $s = 1/2$ critical line can approximate any function to arbitrary accuracy: also this would fit with universality. Could one think that the space-time surface defined as root of octonionic continuation of zeta could be universal entity analogous to a fixed point of iteration in the construction of fractals? This does not look plausible.

4. One can construct iterates of Riemann zeta having at least the same roots as zeta by the rule

$$\begin{array}{l} f_0(s) = \zeta(s) \ , \\ f_n(s) = \zeta(f_{n-1}(s)) - \zeta(0), \ \ \zeta(0) = -1/2 \ . \end{array} \tag{7.9.1}$$

ζ is not a fixed point of this iteration as the fractal universality would suggest. The set of roots however is. Should one be happy with this.

5. Riemann zeta has also counterpart in all extensions of rationals known as Dedekind zeta (see `http://tinyurl.com/y5grktv`) [L36, L69, L61]. Riemann zeta is therefore not unique. One can ask whether Dedekind zetas associated with simple Galois groups are special and whether Dedekind zetas associated with extensions of extensions of .... can be constructed by using the Dedekind zetas of simple extensions. How do the roots of Dedekind zeta depend on the associated extension of rationals? How the roots of Dedekind zeta for extension of extension defined by composite of two polynomials depend on extensions involved? Are the roots union for the roots associated with the composites?

6. What about forming composites of Dedekind zetas? Categorical according to my primitive understanding raises the question whether a composition of extensions could correspond to a composition of functions. Could Dedekind zeta for a composite of extensions be obtained from a composite of Dedekind zetas for extensions? Requiring that roots of extension E_1 are preserved would give formula

$$\zeta_{D,E_1E_2} = \zeta_{D,E_1} \circ \zeta_{D,E_2} - \zeta_{D,E_1}(0) \ . \tag{7.9.2}$$

The zeta function would be obtained by an iteration of simple zeta functions labelled by simple extensions. The inverse image for the set of roots of ζ_{D,E_1} under ζ_{D,E_2} that is the set $\zeta_{D,E_2}^{-1}(roots(\zeta_{D,E_1})$ would define also roots of ζ_{D,E_1E_2}. This looks rather sensible.

But what about iteration of Riemann zeta, which corresponds to trivial extension? Riemann ζ is not invariant under iteration although its roots are. Should one accept this and say that it is the set of roots which defines the invariant. Could one say that the iterates of various Dedekind zetas define entities which are somehow universal.

Chapter 8

Could quantum randomness have something to do with classical chaos?

8.1 Introduction

There was an interesting guest post by Tim Palmer in the blog of Sabine Hosssenfelder (`http://tinyurl.com/yx7htn3u`).

8.1.1 Palmer's idea

Consider first what was said in the post "Undecidability, Uncomputability and the Unity of Physics. Part 1" by Tim Palmer.

1. I understood (perhaps mis-) that the idea is to reduce quantum randomness to classical chaos. If this is taken to mean that quantum theory reduces to chaos theory, I will not follow. The precise rules of quantum measurement having interpretation as measurements performed for the observables - typically generators of symmetries - are very restrictive and it is extremely difficult to image that classical physics could explain them. Quantum theory is much more than probability theory. Probabilities are essentially moduli squared for probability amplitudes and this gives rise to interference and entanglement. Therefore the idea of reducing state function reduction (SFR) and quantum randomness to classical chaos does not look promising. One could however consider the possibility classical chaos is in some sense as a correlate for quantum randomness or associated with state function reductions.
2. The difficulty to combine general relativity (GRT) to quantum gravity was mentioned. The difficulty is basically due to the loss of Poincare symmetries in curved space-time. Already string models solve the problem by assuming that strings live in M^{10} or its spontaneous compactification. Strings are however 2-D, not 4-D, and this leads to a catastrophe. In TGD $H = M^4 \times CP_2$ allows to have Poincare invariance and conservation laws are not lost. In QFT picture this means that the existence of energy guarantees existence of Hamiltonian defining time evolution operator and S-matrix.
3. It was noticed that chaos in quantum theory cannot be assigned to Schrödinger equation. This is true and applies quite generally to unitary time evolution generated by unitary S-matrix acting linearly. It as also noticed that in statistical mechanism Liouville operator defines a linear equation for phase space probability distribution analogous to Schrödinger equation. Liouville equation allows the classical system to be non-linear and chaotic. Could Schrödinger equation in some sense replace Liouville equation in in quantum theory since phase space ceases to make sense by Uncertainty Principle.

   Could Schrödinger equation allow in some sense non-linear chaotic classical systems? In Copenhagen interpretation no classical system exists except at macroscopic limit as an approximation. One has only wave function coding for the knowledge about physical system changing in quantum measurement. There is no classical reality and there are no classical orbits of particle since one gives up the notion of Bohr orbit. Could Bohr orbit be more than approximation?

The author considers also the question about definition of chaos.

1. Chaos is difficult to define in GRT. The replacement time coordinate with its logarithm exponentially growing difference becomes linearly growing and one does not have chaos. By general coordinate invariance this definition of chaos does not therefore make sense.
2. Strange attractors are typical asymptotic situations in chaotic systems and can make sense also in general relativity (GRT). They represent lower dimensional manifolds to which the dynamics of the system is restricted asymptotically. It is not possible to predict to which strange attractor the chaotic dynamical system ends up. This definition of chaos makes sense also in GRT.

Remark: One must remember that the notion of chaos is often used in misleading sense. The increase of complexity looks like chaos for external observer but need not have anything to do with genuine chaos.

8.1.2 Could TGD allow realization of Palmer's idea in some form?

It came as a surprise to me that these to notions could a have deep relationship in TGD framework.

1. Strong form of Palmer's idea stating that quantum randomness reduces to classical chaos certainly fails but one can consider weaker forms of the idea. Even these variants fail in Copenhagen interpretation since strictly speaking there is no classical reality, only wave function coding for the knowledge about the system. Bohr orbits should be more than approximation and in TGD framework space-time surface as preferred extremal of action is analogous to Bohr orbit and classical physics defined by Bohr orbits is an exact part of quantum theory.
2. In the zero energy ontology (ZEO) of TGD the idea works in weaker form and has very strong implications for the more detailed understanding of ZEO and $M^8-M^4\times CP_2$ duality. Ordinary ("big") state functions (BSFRs) meaning the death of the system in a universal sense and re-incarnation with opposite arrow of time would involve quantum criticality accompanied by classical chaos assignable to the correspondence between geometric time and subjective time identified as sequence of "small" state function reductions (SSFRs) as analogs of weak measurements. The findings of Minev *et al* [L54] give strong support for this view [L54] and Libet's findings about active aspects of consciousness [J7] can be understood if the act of free will corresponds to BSFR.

M^8 picture identifies 4-D space-time surfaces X^4 as roots for "imaginary" or "real" part of octonionic polynomial P_2P_1 obtained as a continuation of real polynomial $P_2(L-r)P_1(r)$, whose arguments have origin at the the tips of B and A and roots a the light-cone boundaries associated with tips. Causal diamond (CD) is identified intersection of future and past directed light-cones light-cones A and B. In the sequences of SSFRs $P_2(L-r)$ assigned to B varies and $P_1(r)$ assigned to A is unaffected. L defines the size of CD as distance $\tau=2L$ between its tips.

Besides 4-S space-time surfaces there are also brane-like 6-surfaces corresponding to roots $r_{i,k}$ of $P_i(r)$ and defining "special moments in the life of self" having $t_i=r_{i,k}$ ball as M^4_+ projection. The number of roots and their density increases rapidly in the sequence of SSFRs. The condition that the largest root belongs to CD gives a lower bound to it size L as largest root. Note that L increases.

Concerning the approach to chaos, one can consider three options.

Option I: The sequence of steps consisting of unitary evolutions followed by SSFR corresponds to a functional factorization at the level of polynomials as sequence $P_2=Q_1\circ Q_2\circ ...Q_n$. The size L of CD increases if it corresponds to the largest root, also the tip of active boundary of CD must shift so that the argument of P_2 $L-r$ is replaced in each iteration step to with updated argument with larger value of L identifiable as the largest root of P_2.

Option II: A completely unexpected connection with the iteration of analytic functions and Julia sets, which are fractals assigned also with chaos interpreted as complexity emerges. In a reasonable approximation quantum time evolution by SSFRs could be induced by an iteration of a polynomial or even an analytic function: $P_2 = P_2 \to P_2^{\circ 2} \to ....$ For $P_2(0) = 0$ the roots of the iterate consists of inverse images of roots of P_2 by $P_2^{\circ -k}$ for $k = 0, ..., N-1$.

Suppose that M^8 and X^4 are complexified and thus also $t = r$ and "real" X^4 is the projection of X_c^4 to real M^8. Complexify also the coefficients of polynomials P. If so, the Mandelbrot and Julia sets (`http://tinyurl.com/cplj9pe` and `http://tinyurl.com/cvmr83g`) characterizing fractals would have a physical interpretation in ZEO.

Chaos is approached in the sense that the inverse images of the roots of P_2 assumed to belong to filled Julia set approaching to points of Julia set of P_2 as the number N of iterations increases in statistical sense. The size L as largest root of $P_2^{\circ N}$ would increase with N if CD is assumed to contain all roots. The density of the roots in Julia set increases near L since the size of CD is bounded by the size Julia set. One could perhaps say that near the $t = L$ in the middle of CD the life of self when the size of CD has become almost stationary, is the most intensive.

Option III: A conservative option is to consider only real polynomials $P_2(r)$ with real argument r. Only non-negative real roots r_n are of interest whereas in the general case one considers all values of r. For a large N the inverse iterates of the roots of P_2 would approach to the real Julia set obtained as a real projection of Julia set for complex iteration.

How the size L of CD is determined and when can BSFR occur?

Option I: If L is minimal and thus given by the largest root of $P_2^{\circ N}$ in Julia set, it is bound to increase in the iteration (this option is perhaps too deterministic). Should L be smaller than the sizes of Julia sets of both$A$ and B if the iteration gives no roots outside Julia set.

Could BSFR become probable when L as the largest allowed root for $P_2^{\circ N}$ is larger than the size of Julia set of A? There would be no more new "special moments in the life of self" and this would make death and re-incarnation with opposite arrow of time probable. The size of CD could decrease dramatically in the first iteration for P_1 if it is determined as the largest allowed root of P_1: the re-incarnated self would have childhood.

Option II: The size of CD could be determined in SSFR statistically as an allowed root of P_2. Since the density of roots increases, one would have a lot of choices and quantum criticality and fluctuations of the order of clock time $\tau = 2L$: the order of subjective time would not anymore correspond to that for clock time. BSFR would occur for the same reason as for the first option.

The fact that fractals quite generally assignable to iteration (`http://tinyurl.com/ctmcdx5`) appear everywhere gives direct support for the ZEO based view about consciousness and self-organization and would give a completely new meaning for "self" in "self-organization" [L61]. Fractals, quantum measurement theory, theory of self-organization, and theory of consciousness would be closely related.

8.2 Could classical chaos and state function reduction relate to each other in TGD Universe?

In the sequel the idea about connection between chaos in some sense and state function reductions as they are understood in ZEO is discussed.

8.2.1 Classical physics is an exact part of quantum physics in TGD

Concerning the relation between classical and quantum the situation changes in TGD framework. Classical physics becomes an exact part of quantum theory. In zero energy ontology (ZEO) quantum states are superpositions of space-time surfaces preferred extremals of basic variational principle connecting 3-surfaces at opposite boundaries of causal diamond (CD). This solves the well-known basic problem of quantum measurement theory. Unitary time evolution operator or its generalization are totally different things from classical time evolution defined by highly non-linear field equations. There is nothing preventing quantum counterpart of chaos - it need not be classical chaos at space-time level but could correspond to some other form of chaos. Ordinary state function reduction in ZEO involves naturally quantum criticality involving long range quantum fluctuations assignable to chaotic systems so that the correlation between classical chaos defined in proper manner and state function reduction might make sense.

8.2.2 TGD space-time and $M^8 - H$ duality

$M^8 - H$ duality combined with zero energy ontology (ZEO) is central for the TGD inspired proposal for the connection between chaos and quantum.

Basic vision

Consider first what TGD space-time is.

1. In TGD framework space-times can be regarded 4-surfaces in $H = M^4 \times CP_2$ or in complexifiation of octonionic M^8. Linear Minkowski coordinates or Robertson-Walker coordinates for light-cone (used in TGD based cosmology) provide highly unique coordinate choice and this problem disappears. Exponential divergence in M^4 coordinates could be used as a symptom for a chaotic behavior.
2. The solutions of field equations are preferred extremals satisfying extremely powerful additional conditions giving rise to a huge generalization of the ordinary 2-D conformal symmetry to 4-D context. In fact, twistor twist of TGD predicts that one has minimal surfaces, which are also extremals of 4-D Kähler action apart from 2-D singularities identifiable as string world sheets and partonic 2-surfaces having a number theoretical interpretation. The huge symmetries act as maximal isometry group of "world of classical worlds" (WCW) consisting of preferred extremals connecting pair of 3-surfaces, whose members are located at boundaries of causal diamond (CD). These symmetries strongly suggest that TGD represents completely integrable system and thus non-chaotic and diametrical opposite of a chaotic system. Therefore the chaos - if present - would be something different.

$M^8 - H$ duality suggests an analogous picture at the level of M^8. $M^8 - H$ duality in itse most restrictive form states that space-time surfaces are characterized by "roots" of rational polynomials extended to complexified octonionic ones by replacing the real coordinate by octonionic coordinate o [L33, L34, L35].

1. One can define the imaginary and real parts $IM(P)$ and $RE(P)$ of $P(o)$ in octonionic sense by using the decomposition of octonions $o = q_1 + I_4 q_2$ to two quaternions so that $IM(P)$ and $RE(P)$ are quaternion valued. For 4-D space-time surfaces one has either $IM(P) = 0$ or $RE(P) = 0$ in the generic case. The curve defined by the vanishing of imaginary or real part of complex function serves as the analog.
2. If the condition $P(0) = 0$ is satisfied, the boundary of δM^8_+ of M^8 light-cone is special. By the light-likeness of δM^8_+ points the polynomial $P(o)$ at δM^8_+ reduces to ordinary real polynomial $P(r)$ of the radial M^4 coordinate r identifiable as linear M^4 time coordinate t: $r = t$.

   Octonionic roots $P(o) = 0$ at M^8 light-cone reduce to roots $t = r_n$ of the real polynomial $P(r)$ and give rise to 6-D exceptional solutions with $IM(P) = RE(P) = 0$ vanish. The solutions are located to δM^8_+ and have topology of 6-sphere S^6 having 3-balls B^3 with $t = r_n$ as of M^4_+ projections. The "fiber" at point of B^3 with radial M^4 coordinate $r_M \leq r_n$ is 3-sphere $S^3 \subset E^4 \subset M^8 = M^4 \times E^4$ contracting to point at the δM^4_+.

   These 6-D objects are analogous to 5-branes in string theory and define "special moments in the life of self". At these surfaces the 4-D "roots" for $IM(P)$ or $RE(P)$ intersect and intersection is 2-D partonic surface having interpretation as a generalization of vertex for particles generalized to 3-D surfaces (instead of strings). In string theory string world sheets have boundaries at branes. Strings are replaced with space-time surfaces and branes with "special moments in the life of self".

   Quite generally, one can consider gluing 4-D "roots" for different polynomials P_1 and P_2 at surface $t = r_n$ when r_n is common root. For instance, P and its iterates $P^{\circ N}$ having r_n and the lower inverse iterates as common roots can be glued in this manner.
3. It is possible complexify M^8 and thus also r. Complexification is natural since the roots of P are in general complex. Also 4- space-time surface is complexified to 8-D surface and real space-time surface can be identified as its real projection.

To sum up, space-time surfaces would be coded a polynomial with rational or at most algebraic coefficients. Essentially the discrete data provided by the roots r_n of P would dictate the space-time surface so that one would have extremely powerful form of holography.

One can consider generalizations of the simplest picture.

1. One can also consider a generalization of polynomials to general analytic functions F of octonions obtained as octonionic continuation of a real function with rational Taylor coefficients: the identification of space-time surfaces as "roots" of $IM(F)$ or $RE(F)$ makes sense.
2. What is intriguing that for space-time surfaces for which $IM(F_1) = 0$ and $IM(F_2) = 0$, one has $IM(F_1F_2) = RE(F_1)IM(F_2) + IM(F_1)RE(F_2) = 0$. One can multiply space-time surfaces by multiplying the polynomials. Multiplication is possible also when one has $RE(F_1) = 0$ and $IM(F_2) = 0$ or $RE(F_2) = 0$ or $IM(F_1) = 0$ since one has $RE(F_1F_2) = RE(F_1)RE(F_2) - IM(F_1)IM(F_2) = 0$.

   For $IM(F) = 0$ type space-time surfaces one can even define polynomials analytic functions of the space-time surface with rational Taylor coefficients. One could speak of functions having space-time surface as argument, space-time surface itself would behave like number.
3. One can also form functional composites $P \circ Q$ (also for analytic functions with complex coefficients). Since $P \circ Q$ at $IM(Q) = 0$ surface is quaternionic, its image by P is quaterionic and satisfies $IM(P \circ Q) = 0$ so that one obtains a new solution. One can iterate space-time surfaces defined by $Im(P) = 0$ condition by iterating these polynomials to give $P, P^{circ2}, ..., P^{\circ N}...$ From $IM(P) = 0$ solutions one obtains a solutions with $RE(Q) = 0$ by multiplying the M^8 coordinates with I_4 appearing in $o = q_1 + I_4q_2$.

   The $Im(P) = 0$ solutions can be iterated to give $P \to P \circ P \to ..$, which suggests that the sequence of SSFRs could at least approximately correspond to the dynamics of iterations and generalizations of Mandelbrot and Julia sets and other complex fractals and also their space-time counterparts. Chaos (or rather, complexity theory) including also these fractals could be naturally part of TGD!

Building many-particle states at the level of M^8

The polynomials defining surfaces in M^8 are defined in preferred M^8 coordinates with preferred selection of M^8 time axis M^1 as real octonionic axis and one octonionic imaginary axes characterizing subspace $M^2 \subset M^8$. $M^4 \subset M^8$ is quaternionic subspace containing M^2. Different choices of $M^4 \sup M^2$ are labelled by points of CP_2 and $M^8 - H$ duality maps these choices to points of CP_2.

The origin of $M8$ coordinates coordinates must be at M^1 so that the 8-D Poincare symmetry reduces to time translations and rotations of around spatial coordinate axis M^2 respecting the rationality of polynomial coefficients or in more general case the extension of rationals associated with the coefficients. This corresponds to a selection of quantization axis for energy and angular momentum and could have a deeper meaning in quantum measurement theory.

The Lorentz transformations of M^4 change the direction of time axis and also M^2 in the general case and generate new octonionic structure and quaternonic structure. One should understand how space-time regions as roots of octonionic polynomials with different rest frames relate to each other.

The intuitive picture is that each particle as a region determined by octonionic polynomial corresponds to its own CD and rest frame determined by its 4-momentum in fixed coordinate frame for M^4. Also quantization axis of spin fixed. One can assign CD for to interacting many particle system with common rest frame. One can speak of external (incoming and outgoing) free particles with their own CDs characterizing their rest systems. The challenges is to related the polynomials P_n associated with the external particles to the polynomial characterizing the interacting system.

1. Assume that the polynomial defining the CD is product P_1P_2 of polynomials P_1 and P_2 assignable to its active and passive boundaries with origins of octonionic coordinates at the tips $t = 0$ and $t = \tau$ of CD. If the space-time surface reduces to the root of P_1 at passive boundary and root of $IM(P_2)$ at active boundary, one could say that the 3-surfaces at these boundaries correspond to P_1 and P_2 asymptotically. If these conditions are true everywhere, one has two un-correlated space-time surfaces, which does not make sense. $IM(P_1)RE(P_2) + RE(P_1)IM(P_2) = 0$ indeed allows more general solutions than $IM(P_1) = 0$ and $IM(P_2) = 0$ everywhere. The fact that the boundaries correspond to special 6-D brane like solutions in M^8 suggests that it is possible to pose the boundary condition $IM(P_1) = 0$ *resp.* $IM(P_2) = 0$ at the boundaries.

2. The formation of products is possible also at the boundaries so that one can assume that P_i at the boundary of many-particle CD is with product $P_i = \prod_k P_{ik}$. The boundary conditions would read read $P_{ik} = 0$ at active *resp.* passive boundary of many-particle CD respectively. The interpretation would be that P_{ik} corresponds to an external particle which is in interacting state at active boundary. In the interior of many-particle CD only the condition $Im(P_1P_2) = 0$ would hold true so that interactions of particles would have algebraic description.

3. One should also understand how the external particles characterized by CDs with different rest frame are glued to the boundary of many-particle CD. Assume that M^4 is same for all these particles so that CP_2 coordinates are same. The boundaries of 4-D CDs are 3-D light-cones with different origins so that their M^4 intersection is 2-D defining a 2-D surface at the boundary of CD. The interpretation in terms of partonic 2-surface suggests itself. The partonic 2-surfaces of free particle and its interacting variant would be same at the intersection.

   The gluing should correspond to a root $t = r_n$ of polynomial defining a "special moment in the life of self". The roots of P_1 and its Lorentz boots as values of coordinates at light-radial geodesic are related by Lorentz boost and are not same in general. One could require that the root r_n and its Lorentz boost belong to the 2-D interaction of two light-cones and thus define two points of partonic 2-surface. These points would not be identical and the interaction would be non-local in the scale of partonic 2-surface. It seems that the condition that root r_n and Lorentz boost $L(rm)$ co-incide would pose too strong constraints on external momenta.

8.2.3 In what sense chaos/complexity could emerge in TGD Universe?

Consider now in what sense chaos (or complexity, one must be precise here) could emerge in TGD framework?

1. Chaos (or complexity) could be an approximate property emerging in number theoretical discretization for cognitive representations labelled by extensions of rationals as the dimension of extension and therefore algebraic complexity increases ad the number of points in cognitive representation as points of M^8 with coordinates in the extension of rationals increases. The minimal number of points corresponds to the degree of the polynomial determining the extension. At the limit of maximal complexity the extension would consists of algebraic numbers and the cognitive representation would be dense subset of space-time surface. It is not clear whether the roots r_n are also dense along time axis.

2. Also transcendental extensions of rationals can be considered. Typically they are infinite-D in both real and p-adic sectors. Exponential function is however number-theoretically completely unique. Neper number e and its roots define infinite-D extensions of rationals but - rather remarkably - finite-dimensional extensions of p-adic numbers since e^p is ordinary p-adic number. Extension of rationals would become infinite-D but the induced extensions of rationals would remain finite-D in accordance with the idea that cognition is always finite-D.

   Could one allow e and its roots and thus exponential functions besides polynomials? Could exponential divergence be the hallmark of chaos or perhaps the first step in the transition to transcendental chaos (or rather, complexity)? Could chaos (complexity) in real sense be possible for extensions of rationals generated by a root of e? One can however argue that the finite dimension of induced p-adic extensions means that cognitive chaos is not yet present.

   For general transcendentals the dimensions of p-adic extensions are infinite and one would have also cognitive chaos (infinite complexity). Could the transition to chaos means the emergence of analytic functions with rational coefficients having also roots, which are transcendentals. Chaos would mean that one can only approximated f analytic function as a polynomial giving approximation for the roots. By $M^8 - H$ duality these roots would correspond to values of M^4 time inside light-cone, preferred moments of time [L57]. These would become transcendental and in general p-adic extension would become infinite-D.

3. An interesting analogy with real numbers emerges. Real numbers have expansion in powers of any integer, in particular any prime p. The sequence defined by the coefficients of the expansion are analogous to an orbit of a discrete dynamical system. For transcendentals the expansion is unpredictable and analogous to a chaotic orbit.

For rationals this expansion is periodic so that one has analog of a periodic orbit. This applies also to expansion of rationals formed from the integers in finite-D extensions of rationals. One must of course accept that the algebraic numbers defining the roots do not allow periodic expansion but one can do all calculations in extension and perform approximation only at end of computation. Therefore the extensions of rationals represent also islands of order in the ocean of trancendental chaos. Could one see he gradual increase of the dimension of extension of rationals as a transition to chaos: of course, chaos would be wrong term since increase in algebraic complexity, which corresponds to evolution in TGD Universe is in question. Cognition becomes more and more refined.

4. As found, space-time surfaces behave like numbers and one can have functions having space-time surface as argument. Could the picture about emergence of chaos for reals be translated to the level of space-time surfaces identified as "roots" of octonion analytic function in M^8? The polynomial space-time surfaces would represent islands of order in chaos defined by general analytic functions with rational Taylor coefficients.

Can one imagine a connection between quantum randomness and chaos?

To my view, the reduction of quantum randomness to classical chaos is definitely excluded. Quantum classical correspondence allows however to consider a looser connection between quantum randomness and chaos.

1. The following considerations lead to a formulation of a more precise view about the sequence of steps consisting of a unitary evolution followed by SSFR as a a model of self. M^8-H duality involving representation of space-time surface in terms of a polymial with rational coefficients leads to an approximate model of the quantal time evolution by SSFRs as quantum counterpart for an iteration of a polynomial map, and gives a direct connection with chaos as algebraic complexity in the sense of generalization of Mandelbrot and Julia sets (`http://tinyurl.com/cplj9pe` and `http://tinyurl.com/cvmr83g`).
   The identification of time evolution as iteration $P \rightarrow P^{\circ 2} \rightarrow ...$ is very probably only an approximation. More general picture would assume that the corresponds to a functional factorization of P as $P = P_1 \circ P_2 \circ ... \circ P_n$. Even this assumption can be only approximate.
2. The fixed points of iteration would correspond to asymptotics for the evolution of space-time surface defined by iteration and approach of CD to a fixed point CD. This conforms with the idea that fixed points of iteration as representations of fractals, criticality and chaos. Chaos understood as genuine chaos could correspond to a fluctuation of the arrow of time in the sequence of SSFRs as a fixed point of iteration is reached.

It must be of course made clear that the view about M^8-H duality already considered and the view about the emergence of fractals to be discussed are only one of the many options that one can imagine and involve many poorly understood aspecs. Only time will tell whether the proposals work and how they must be improved.

Chaos and time

TGD Universe has gigantic symmeries [K35, K98] and looks like a completely integrable system and the idea about genuine chaos at space-time level does not look attractive. M^8-H duality suggests that chaos - actually complexity - in the sense of Mandelbrot fractals looks more promising idea. ZEO int turn suggests that chaos could be associated with the relationship between geometric and subjective time in the sense that the orderings of the two times would not be strictly identical.

1. Often the chaos is taken to mean increase of complexity (Mandelbrot and Julia sets), which actually means a diametric opposite of chaos. In TGD framework a more promising connection is between finite measurement resolution and complexity as that for extension of rationals. For trivial extensions of rationals the points of cognitive representation have rational M^8 (and becase also H-) coordinates. All other points fail to have a cognitive representation. For extensions of rationals the number of points in cognitive representations increases: the increase of cognitive complexity has actually nothing to do with emergence of a genuine chaos. Here one must be however very cautious and one must consider ZEO view about state function reduction in detail to see what happens.

2. $M^8 - H$ duality allows to consider a concrete example. The roots r_n of real rational polynomials P or even analytic functions correspond "special moments in the life of self". Could the increase of complexity be understood in terms of what happens for the roots. The number of these moments equals to the degree n of P and cognitive representation more and more complex since the dimension of extension equals to n: this could occur in BSFRs at least. The clock defined by the moments roots $t = r_k$ could become more precise. It will be found that in presence of quantum criticality the emerging complexity could also correspond to a genuine chaos.

3. One can define clock time as a temporal distance τ between tips of CD after "small" state function reduction (SSFR), which corresponds to weak measurement in standard picture. Passive boundary and the states at the passive boundary of CD remain unchanged (generalized Zeno effect) and the states at active boundary is change. Also the distance between tips of CD changes but increases in statistical sense.

   The statistical nature of the change implies that the ordering for subjective time as sequence of SSFRs is not quite the same as that for τ (one could of course assume that only increase of the CD size is possible in BSFR but this would be an ad hoc assumption). This corresponds to a kind of quantum randomness due to the state function reductions. If the number of roots is large and the average time chronon is small, the changes of time order could occur often. Could this have interpretation as a genuine chaos in short time scales due to SSFRs? This need not correspond to a genuine chaos at the level of space-surfaces as preferred extremals. Chaos as algebraic complexity could however increases and would be consistent with complete integrability: this happends in n increases in BSFRs.

Chaos in death according to ZEO

The assignment of a genuine chaos to death looks natural from what we know about biological death. Could this assignment make sense in ZEO where BSFR corresponds in a well-defined sense to death?

1. Recall that BSFR corresponds to ordinary state function reduction in which the arrow of geometric time identifiable as distance between the tips of CD changes: self dies and re-incarnates with an opposite arrow of time. The active boundary of CD becomes passive. The passive boundary becomes active and the size of CD starts to statistically increase in opposite time direction in SSFRs. The former passive boundary CD can remain at the critical moment but could also shift towards the former active boundary - the re-incarnated self would have small CD and could have "childhood."

   The continual increase of CD looks strange. Also our mental images would increase in size and unless one makes special assumptions (say that the average change of the size of CD is proportional to its size (scaling)) one ends up with difficulties. Time evolution as stepwise scaling would be indeed natural.

2. Under what conditions does BSFR - death and reincarnation - occur? A quantum criticality implying instability against BSFR should be involved. The size scales of CD as temporal distances τ between its tips would have critical values τ_{cr} at which death of self in this universal sense could take place. τ_{cr} could be integer multiple of CP_2 length scale with allowed integers being primes of preferred primes allowed by p-adic length scale hypothesis. Criticality indeed involves long range fluctuations assigned with chaotic behavior: the simplest example is the transition to chaos in convection as energy feed to the system increases.

3. A concrete model for SSFRs [L64] suggests that one can assign to CD temperature T satisfying $T \propto 1/\tau$ so that the evolution of self would correspond to T as analog of cosmic temperature. Death could correspond to a critical temperature T_{cr} (τ_{cr}) and would be unavoidable. The quantum criticality assignable to death could correspond to the emergence of a genuine temporal chaos. The time order would become more and more ill-defined, and time τ would go forth and back so that eventually one would $\tau = \tau_{crit}$ as size of CD and death would occur. This however requires that the number of roots r_n increases so that also their density increases. This requires that the degree of the polynomial P defining the extension increases. Can this be consistent with the assumption that passive boundary does not change?

Remark: Why I take this seriously is that I have had near death experience being in clinically unconscious but actually conscious state and I experienced quite literally the flow of time forth and back and was fighting to preserve the usual arrow of time.

4. This picture applies to all BSFRs and SSFRs and therefore to ordinary state functions reductions in all scales: the findings of Minev *et al* [L54] can be understood if the arrow of time indeed changes [L54]. There would be a connection between state function reductions and chaos understood as genuine chaos. The idea that this chaos corresponds to a strange attractor at space-time level is not plausible. Rather it could be analogous to chaos in the sense of an attractor of iteration of complex function by functional decomposition. Fixed point is also a fractal and corresponds to criticality.

What gives rise to the lethal quantum criticality, BSFR, and death?

What could give rise to quantum criticality leading to death and reincarnation of self as BSFR?

1. If P remains the same during SSFRs, one could think that once the CD size is so large that all "special moments in the life of self" have been experienced as time values $\tau = r_n$, the system is ready to die. But how could this give rise to quantum criticality?
2. Assume that CD is defined as the intersection of future and past light-cones and the polynomial P corresponds to a product $P_1(r)P_2(L-r)$ of polynomials associated with these two light-cones such that P_i vanishes at the tip of its light-cone corresponding to $r = 0$ *resp.* $L - r = 0$. P_1 associated with the passive boundary of CD would not change in SSFRs but P_2 associated with the active boundary would change. Most importantly its degree would increase and the number of roots and their density would increase too. Eventually the density of active roots would become so high that death as BSFR is bound to occur as event $\tau = \tau_{cr}$.

   Remark: One can consider two options: real M^8 and real r or complexified M^8 and complex r.
3. As already noticed, if the space-time surface reduces to the root of P_1 at passive boundary and root of P_2 at active boundary, one could say that the 3-surfaces at these boundaries correspond to P_1 and P_2 asymptotically. The fact that the boundaries correspond to special 6-D brane like solutions in M^8 sugests that the boundary conditions are possible.
4. The statistically increasing extension of rationals would correspond to "personal" evolution for the changing part of self during life cycle. Note that $n = h_{eff}/h_0$ corresponds to the scale of quantum coherence thus increasing. This extension would define the evolutionary level of the unchanging part ("soul") during the next re-incarnation.

Could polynomial iteration approximate quantum time evolution by SSFRs in statistical sense?

I have considered rather concrete models for the counterpart of S-matrix for given space-time surface [L50, L51, L65] but the deeper understanding of the sequence of SSFRs is still lacking although quite concrete proposals already exists.

Number theoretical vision suggests that also the time evolution by SSFRs should reduce to number theory being induced by some natural number theoretical dynamics.

1. The most general option is that in each SSFR a superposition over extensions defined by various polynomials with varying rational coefficients is generated. The idea about the correspondence of the sequence of SSFRs with a functional decomposition of polynomials is however attractive.
2. The sequence of unitary evolutions brings strongly in mind the iteration $U \to U^2 \to U^3$.... One can however consider also the possibly $U \to U_1U \to U_2U_1U$.... The obvious guess for the iteration of U is that it is induced by a functional iteration of polynomial P_2 assigned to the active boundary of CD $P_2 \to P_2 \circ P_2 \to$ .... The more general option would not be iteration anymore but a composition of form $P_2 \to P_3 \circ P_2 \to$ .....

   The boundary conditions at the boundary of CD and at gluing points - possibly $t = r_n$ surfaces to which 6-branes are assignable as special solutions and identified as "special moments in the life of self" could make the superpositions of functional composites more probable contributions in the superposition. The polynomial $P \circ Q$ has same roots as Q (for $P(0) = Q(0) = 0$) and this favors conservative state function reductions preserving the state already achieved.

Iteration would be even more conservative option. If the solutions assignable to P and Q are to be glued together along brane with $t = r_n$ they must share r_n as root. This would favor iterations if one has superposition over different rational coefficient values for P and Q with fixed degree.

Remark: Also critical points of Q as zeros of derivative are preserved in $Q \to P \to Q$ as one finds by applying chain rule. For iteration both the new critical points/roots of $P \circ P^{\circ k}$ are inverse images of critical points/roots of $P^{\circ k}$. Only roots are of significance in the picture considered.

3. Superpositions of different iterates generated in the unitary time evolution preceding SSFR are required by the model of temporal chaos. SSFR selects extension of rationals and thus fixed iteration. In statistical sense the degree of iteration is bound to increase so that in statistical sense quantum iteration reduces to classical one. At the limit of fixed point of iteration the number of critical points $t = p_n$ and roots $t = r_n$ of the iterate increases as also their density along time axis and temporal chaos emerges leading to fluctuation of CD size τ.

4. Iteration of the real polynomial P satisfying $P(0) = 0$ would mean that one would have a series extensions obtained as powers of generating extension: E, $E \circ E$, $E \circ E \circ E$,... conserving the roots of E provided the polynomials involved vanish at origin: $P(0) = 0$. The proposal has been that biological evolution corresponds to a more general series of extensions $E_1, E_2 \circ E_1, E_3 \circ E_2 \circ E_1, ...$ Also now Galois groups in the series of them would be conserved. I have proposed that Galois groups are analogs of conserved genes [L32, L35].

The proposed picture is only one possibility to interpret evolution of self as iteration leading to chaos in the proposed sense.

1. One could argue that the polynomial $P_{nk} = P_n \circ .... \circ P_n$ associated with the active boundary remains the same during SSFRs as long as possible. This because the increase of degree from nk to $n(k+1)$ in $P_{nk} \to P_{nk} \circ P_n$ increases h_{eff} by factor $(k+1)/k$ so that the metabolic feed needed to preserve the value of h_{eff} increases.

   Rather, when all roots of the polynomials P assignable to the active boundary of CD are revealed in the gradual increase of CD preserving P_{nk}, the transition $P_{nk} \to P_{nk} \circ P_n$ could occur provided the metabolic resources allow this. Otherwise BSFR occurs and self dies and re-incarnates. The idea that BSFR occurs when metabolic resources are not available is discussed in [L83].

2. Could $P_{nk} \to P_{nk} \circ P_n$ occur only in BSFRs so that the degree n of P would be preserved during single life cycle of self - that n can increase only in BSFRs was indeed the original guess.

8.2.4 Basic facts about iteration of real polynomials

The iteration of real polynomials and also more general functions can be understood graphically. Assign to a x point $y = f(x)$ of the graph and reflect through the line $y = x$ and project to the graph to obtain the image point $x_1 = f(x)$. Fixed points $x = f(x)$ correspond to the intersections of the line $y = x$ and graph $y = f(x)$. The magnitude $|df/dx|$ at the intersection point determines whether it is attractor ($|df/dx| < 1$ or repellor ($|df/dx| \geq 1$) in which case large jumps in the value of x can occur, as one can easily check. Quite generally iteration in the part of the graph below (above) $y = x$ decreases (increases) x. Real polynomial $c - x^2$ provides a simple example.

Feigenbaum discovered by iterating logistic map numerically (`http://tinyurl.com/u3zwmar`) that the approach to chaos - not only for logistic map but - for real functions $f(x)$ with one quadratic maximum and depending on a varied parameter a is universal. Period-doubling bifurcations occur at parameter values satisfying at the limit $n \to \infty$

$$\frac{a_{N-1} - a_{N-2}}{a_N - a_{N-1}} \to 4.669201609... \ .$$

Second universality relates to the widths of tines - distances between the branches of bifurcation - appearing in the sequence of bifurcations. The ratio between width of the tine to widths of its sub-tines approaches at the limit $N \to \infty$ to constant given by

$$\alpha = 2.502907875095892822283902873218... \ .$$

.

In TGD framework conservative option would correspond to real M^4 so that the coordinates t and r would be real and the polynomials P_1 an P_2 would have real coefficients. The time evolution by iterations of P_2 would reduce to an iteration of a real polynomial P_2.

The number of real roots is in general smaller than the degree n of the polynomial. Only non-negative roots can be considered since one as $r \geq 0$ and $r = 0$ is a root. This condition could generalize to complex polynomials of complexified r as a condition $Re(r_c) \geq 0$ guaranteeing that roots are in the upper half plane for the variable $z = ir_c$.

The real polynomial $P(x)$ of degree n one has either positive or negative values between neighboring roots and at least one extremum between them. The n roots of $P_n(x)$ gives rise to Nn roots in N:th iteration and only non-negative ones are allowed. Since the roots are below the axis $y = s$, the root is obtained from the inverse of the roots by reflecting with respect to $y = x$ and projecting to the graph. The inverse of this operation increases the root. One has special case of complex iteration.

8.2.5 What about TGD analogs of Mandelbrot -, Julia-, and Fatou sets?

What about the interpretation of Mandelbrot -, Julia-, and Fatou sets (http://tinyurl.com/cplj9pe and http://tinyurl.com/cvmr83g) in the proposed picture? Could the iteration of P_2 define analogs of Mandelbrot and Julia fractals? This would give the long-sought-for connection between quantum physics and Mandelbrot and Julias sets, which are simply too beautiful objects to lack a physical application. Period-doubling bifurcations (http://tinyurl.com/t2swmdg) are involved with the iteration of real functions and relate closely to the complex fractals when the polynomials considered have real coefficients.

1. In the simplest situation both Mandelbrot and Julia sets are fractals associated with the iteration of complex polynomial $P_c(z) = z^2 + c$ where z and c are complex numbers (note that in TGD would have $c = 0$ in this case). One can consider also more general polynomials and even rational functions, in particular polynomial $f = P_2$ defined earlier, and replace $z = 0$ with any critical point satisfying $df/dz = 0$. Even meromorphic transcendental functions can be considered: what is required that the image contains the domain.

2. Mandelbrot set M is defined as the region of the plane spanned by the values of c for which the iteration starting from the critical point z_{cr} does not lead to infinity. Physically the restriction to Mandelbrot set looks natural.

3. For rational functions Julia set J_c (http://tinyurl.com/cplj9pe corresponds to a fixed value of c, and is defined as points z for which are unstable in the sense that for an arbitrary small perturbation of z iteration can lead to infinity. Inside J_c the iteration is repelling: $|f(w) - f(z)| > |w - z|$ for all w in neighbourhood of z within J_c. One can say that the behavior is chaotic within J_c and regular in its complement - Fatou set. Julia set can contain also cycles and iteration in J_c leads to these cycles. These cycles are analogs of the limit cycles appearing in the iteration of real-valued function discovered by Feigenbaum (http://tinyurl.com/u3zwmar).

   For polynomials Julia set can be identified as the boundary of the filled Julia set consisting of points for which iterates remain bounded. Also the inverse iterates in this set remain bounded. The filled J_c - denote it by $J_{c,in}$ - can be regarded as a set of points, which are inverse images of fixed points of the polynomial. All points except at most two points of J_c can be regarded as points in the limiting set for the union $\cup_n f^{-n}(z)$ of the inverse images for the points z in filled Julia set. Julia set and its complement Fatou set are invariant under both P and P^{-1} and therefore also under their functional powers. Julia set is the set of pre-images for practically any point of J_c: this can be used for computational purposes. If I have understood correctly there can be single exceptional point for which this is not the case.

J_c can be regarded as a fractal curve. For parameter values inside M J_c is connected, which seems counter intuitive. For c outside the M, Julia set is a discrete Cantor space, Fatou dust.

What is remarkable from TGD point of view is that the new roots obtained in N:th step of iteration are $N-1$:th inverse images of the roots of P. Since polynomial iteration takes sufficiently distant points to ∞, its inverse does the opposite so that the roots of $P^{\circ N}$ are bounded: this strongly suggests that the roots of $P^{\circ N}$ are in J_c if those of P_2 are. One can say that the situation becomes chaotic at the large N limit since the number of roots increases without bound.

4. Fatou set F_c can be identified as the complement of Julia set. Fatou set fills the complex plane densely and has disjoint components, which contain at least one point with $df/dz = 0$ unless Fatou set contains $z = \infty$. Note however that critical point is ot fixed point as in gradient dynamics. This allows to deduce the number or at least upper bound for the number of components of Fatou set, which equals to the degree n of polynomial in the generic case. All components have entire J_c accumulation points. Since the points of J_c are infinitely near to more than 2 disjoint sets for Fatou set with more than 2 components, J_c cannot be a smooth curve in this case being thus fractal. However, the Julia set of $P = z^2 + c$ is also fractal although Fatou set has only two components corresponding to the critical point $z = 0$ and $z = \infty$.

A couple of examples are in order: for $P(z) = z^2$ Julia set is unit circle S^1 and Fatou set has interior and exterior of S^1 as its components. The cycles in Julia set correspond to roots of unity and the orbits of other points form dense sets of unit circle. For $P(z) = z^2 - 2$ Julia set is the interval $(-2, 2)$ having fixed points as its ends. Fatou set has only one component as the complement of Julia set. For $P(z) = z^2 + c$, c complex Julia set is in general fractal. Hence the roots of the polynomial need not belong to Julia set.

Emergence of Mandelbrot and Julia sets from ZEO assuming $M^8 - H$ duality

Consider now the application to TGD assuming $M^8 - H$ duality [L33, L34, L35, L60] .

1. In TGD framework complex numbers $x + iy$ emerge in the complexification of M^8 and i commutes with octonionic units. If space-time surfaces are identified as real projection of their complexified variants obtained as roots of polynomials one can consider also polynomials with complexified coefficients c. Note that c would be complex rational but one can also consider complex algebraic numbers. The most general situation corresponds to analytic functions with complex rational Taylor coefficients. Complex argument with complex coefficients is possible space-time surface is identified by projection the complex space-time surface to real part of complexified M^8 [L33, L34, L34].
2. The complexified light-like coordinate r at the active boundary CD defines the analog of z plane in which iterates of P_2 act. r corresponds directly to the complexified linear time coordinate t of M^8 (time-axis connects tips of CD) and the roots r_n of P_2 correspond to the "special moments in the life of self" as time values $t = r_n$. Assume that $P_2(0)$ vanishes so that r_n are also roots of iterates.
3. Julia set J_c bounds filled Julia set $J_{c,in}$ of the complexified r-plane, whose interior points remain inside $J_{c,in}$ in the iterations by fixed P_2. Julia set J_c is connected but the Fatou set as its complement has several components labelled by the $n-1$ points p_k satisfying $dP_2(z)/dz = 0$ and by $z = \infty$ so that Fatou set has n components. The inverse iterates of roots need not belong to Fatou sets not containing ∞ or to the filled Julia set.
4. There are several Mandelbrot sets and the extrema of P_2 satisfying $dP_2/dr = 0$ label them. The extrema of P_2 are also extrema of its iterates. There are $n-1$ extrema p_k. In the real case they can be classified as either attractors or repellors but in complex situation they correspond to saddle points. Denote by $M(p_n)$ the region of parameter space of polynomial coefficients c for which the iteration starting starting at $p(n)$ does not lead outside it.

   In the real case the iteration of P_2 leads to the attractors $t = p_k$. In complex case the situation is not so simple and the basic of attraction is replaced with the Fatou set $F_c(p_k)$.

   Since c parameterizes points in the space of polynomials characterizing space-time surfaces in TGD, Mandelbrot set can be defined as a sub-space of "world of classical worlds" (WCW).

Inside $M(p_n)$ the iteration maps r_n to a point $M_{in}(r_n)$. Note that also new roots emerge in each iteration and the Mandelbrot set for the iterates contains more components.

Remark: In TGD only the roots of P_2 are interesting. The roots of iterates are inverse iterates of roots of P_2.

Could one understand the size of CD and its evolution during the iteration of P_2?

1. Consider first the situation for real time $t = r$ and real polynomials. Since the boundary of CD contains only the roots $t = r_n$, the simplest guess is that the size of CD corresponds to the largest root of $P_2^{\circ N}$. The size of CD would increase in the iterations. The inverse images of the roots approach to Julia set so that the real counterpart of Julia set is important for understanding the asymptotic situation. Mandelbrot set defines the coefficient values for which iteration does not lead to infinity.
2. The situation is essentially the same for complexified time. The size of CD would correspond to the modulus for the largest of the iterate root and increases during iteration. The size of CD approaches to that for a point in Julia set.

Could the iteration lead to a stationary size of CD?

One can represent an objection to the idea that quantum iteration of P_2 could be more than an approximation.

1. Suppose that the size of CD is determined by the maximum for the iterates of the roots of P_2. Suppose that the parameters c are fixed and belong to Mandelbrot set $M(p_k)$. For given c there is therefore an upper for $\tau = 2r$ given by $r = r_{max}(c, p_k)$ for the Fatou set $F_c(p_k)$. One gets stuck to fixed τ since maximal root cannot become larger than $r_{max}(c)$ in the iteration. Note that in this situation the number of roots of $P_2^{\circ k}$ increases and if they corresponds to "special moments in the life of self", this could lead to quantum criticality and occurrence of BSFR.
2. Fluctuations of τ in the sequences of SSFRs is possible if superpositions of iterates are allowed. This could cause BSFR would occur and eventually second BSFR would eventually lead to the original situation. If P_2 is not modified, the iteration continues and one is still at criticality. BSFR soon occurs and same repeats itself.

Is this situation acceptable? Maybe - I have considered the possibility that the size of CD remains below some upper bound [L64, L55]. The selves such as our mental images could continue to live in the geometric past and memories would be communications with them. Or should one get rid of this situation? How?

1. Assume that SSFR creates a superposition of iterates with varying values of parameters c belonging to the Mandelbrot set $M(P_2)$. The value of $r_{max}(c, p_k)$ depends on c and it is possible to increase the value of τ in statistical sense if SSFR selects the values of c suitably. The value of L would be however given by maximal root and would remain below the maximum r_{max} of $r_{max}(c, p_k)$ in $M(P_2)$ if c belongs to $M(P_2)$. $\tau = 2L$ would remain below the maximum for the size of $J_c(P_2)$ in $M(P_2)$. One would get stuck if this size is finite, which is the case if $r_{max}(c, p_k)$ is bounded as function of c and p_k?

   Is $r_{max}(c, p_k)$ bounded? The polynomials with given degree of can have arbitrarily large roots and critical points in the same extension of rationals. Therefore it might be possible to avoid getting stuck if there is no restriction on the size of the roots of P_2 in the superposition over different values of c.

When death occurs and can self have a childhood?

I hope that talking about death and reincarnation does not irritate the reader too much. I use these terms as precisely defined technical terms applying universally. There are two extreme options for what happens to the former passive boundary in BSFR. The real situation could be between these two.

1. The first shift after reincarnation is to geometric past so that CD size increases.

2. The first shift is towards the former active boundary so that the size of CD decreases at least to the size of CD when the iteration of P_2 began. The reincarnated self would have "childhood" and would start from scratch so to say.

Consider P_1P_2 option. Suppose that time evolution is induced by iteration of either polynomial and maximal root defines the size of the size of CD. What happens to P_1?

1. Could the new functional iteration start from where it stopped in previous re-incarnation: if P_1 is n:th functional power of Q ($P_1 = Q^{\circ n}$), the first step would corresponds to $P_1 \rightarrow Q \circ P_1$. This conservative option does not quite correspond to the idea that one starts from scratch.
2. If P_1 can change, could one require that P_1 is replaced with a polynomial, which is minimal in the sense that it is not functional power of form $P_{1,new} = Q_{new}^{\circ n}$. Or could one even require that it is functional prime having prime valued degree: $n = p$. This would mean starting from scratch except that the algebraic extension of P_2 is fixed.

Probably these options represent only extreme situations. The most general option is that BSFR generates a state, which corresponds to a superposition of extensions of rationals characterized by polynomials P_2P_1, P_2 fixed, and from these one is selected.

Suppose that L as the size of CD is minimal and thus given by the largest root of $P_2^{\circ N}$ in the filled Julia set, it is bound to increase in the iteration (this option is perhaps too deterministic). Under what conditions can BSFR occur? Can the re-incarnated self have childhood?

1. One can argue that L should be smaller than the sizes of Julia sets of both A and B since the iteration gives no roots outside Julia set. This would require iteration to stop when the largest root of P_2^{circN} exceeds the size of the Julia set of A. When applied to B this condition would prevent BSFRs in the opposite time direction would prevent the growth of CD and it would become stationary. This condition looks too deterministic.
2. This picture suggests that the unitary evolution preceding SSFR creates a superposition of iterates $P_2^{\circ N}$ and that the size of CD as outcome of SSFR is determined statistically as a maximal root for $P_2^{\circ N}$ selected in the iteration. N could also decrease. Since the density of roots increases, one would have a lot of choices for the maximal root and quantum criticality and fluctuations of the order of clock time $\tau = 2L$: the order of subjective time would not anymore correspond to that for clock time.
3. Could BSFR become only probable as L as the largest root for the iterate $P_2^{\circ N}$ has exceeded the size of Julia set of A? A quantum analogy with super-cooling comes in mind. The size of CD boundary at side A would contain more volume than needed to store the information provided by the roots r_n and bring no new "special moments in the life of self" at A side. At B side the density of these moments would eventually become large enough so that the reduction of the size of CD destroying part of these moments would mean only a loss of precision. Could this make death and re-incarnation with an opposite arrow of time probable?

   If $P_2^{\circ N}$ is achieved during the life cycle, the reduction in the size of CD in BSFR would reduce N to $N_1 < N$. For $P_1 = Q_1^M$ similar reduction of M to $M_1 < M$ would take place. If one returns to the situation when the iterated started, all new "special moments" are lost. Nothing would have been learned but one could start from scratch and live a childhood, as one might say.

   In the proposed picture - one of many - the opposite boundaries of CD would correspond to both short and long range quantum fluctuations. Could this observation be raised to a guiding principle: could one even say that the opposite boundaries of CD give holistic and reductionistic representations.
4. Do the roots of $P_2^{\circ N}$ belonging to filled Julia set approach the Julia set as N increases? Or are they located randomly inside Julia set? Indeed, the inverse iterate of a root of P_2 is larger than the root as one finds graphically. The $P_2^{\circ N}$ does the same for the roots $P_2^{\circ N}$. If this argument is correct, the density of the roots is largest near Julia set and near the maximum $L - t = L - r$ near the corner of CD.
5. The proposed picture is interesting from the point of view of consciousness theory. Action would be near the corner of CD in the sense that conscious experience would gain most of its content in Minkowskian sense here whereas larger smaller values of $L - r$.

This does not mean paradox since the size of CD inreases and special moments already experienced are shifted to the future direction and would define the unchanging part - "soul" - of the next re-incarnation. This could be seen as wisdom gained in the previous life [L64].

6. Suppose that the approach to chaos in the iteration of P_2 indeed leads to death and re-incarnation. Can one avoid this or at least increase the span of life cycle? Could one start a new life by replacing P_2 with some polynomial Q_2 in the iteration so that the new iterates would be of form $Q_2^{N_2} \circ P_2^{\circ N_1}$. If the replacement is done sufficiently early, the development of chaos might be delayed since reaching the boundary of Julia set of Q would require quite a many iterations if its largest root is larger than that for P_2. This is also true if the degree of Q_2 is small enough.

Unexpected observations about memories

Some comments about memories in the model of self based on iteration.

1. The conscious activity is at the corner of CD in middle of CD if the new roots define "special moments in the life of self" as conscious experiences. The roots r_n of $P_2^{\circ N}$ defining already experienced special moments shift to Minkowskian geometric future as CD increases in size. Subjective memories are in Minkowskian future and become in re-incarnation stable memories about previous life!

   Subjective memories from recent and previous life could be obtained by communications with geometric future and past involving time reflection of the signal so that the constraints due to the finite light velocity can be overcome.

   One can ask whether self can have "remember" or "anticipate" also external world. If this is possible then the "memories" are indeed from geometric past and "anticipations" from geometric future.

2. The view about subjective memories raises interesting speculations (to be made with tongue in cheek). Consider an unlucky theoretician who believes that he has discovered wonderful theory and has used his lifetime to develop it. Unfortunately, colleagues have not shown a slightest to his theory. Although personal fame might not matter for him, he might be interested in knowing during his lifetime whether his life work will ever gain recognition. Is this possible in TGD Universe?

   Suppose that dreams involve sub-selves representing signals to Minkowskian future and their time reflection inside CD (re-incarnation). If sub-selves near the boundary of CD are able to send time signals to geometric future they might get information about the external world, maybe even about what colleagues think about the theory of unlucky theoretician. Dreams might allow to receive this information indirectly. Dreams might even involve meetings with colleagues of geometricfuture and if their behavior is very respectful, unlucky theoretician might wonder whether his work might have been recognized or is this only wishful thinking!

3. Usually it is thought the recollection of past is not good idea. One can however argue that it communication not only with subjective past but also with objective future (the world external to personal CD). This would give information about the external world of geometric future and also increase the span the time scale of conscious experience and of temporal quantum coherence. This might helpful or a theoretician not interested in fashionable thinking only.

8.3 Can one define the analogs of Mandelbrot and Julia sets in TGD framework?

The stimulus to this contribution came from the question related to possible higher-dimensional analogs of Mandelbrot and Julia sets (see this). The notion complex analyticity plays a key role in the definition of these notions and it is not all clear whether one can define these analogs.

I have already earlier considered the iteration of polynomials in the TGD framework [?] suggesting the TGD counterparts of these notions. These considerations however rely on a view of $M^8 - H$ duality which is replaced with dramatically simpler variant and utilizing the hologra-phy=holomorphy principle [L121] so that it is time to update these ideas.

This principle states that space-time surfaces are analogous to Bohr orbits for particles which are 3-D surfaces rather than point-like particles. Holography is realized in terms of space-time surfaces which can be regarded as complex surfaces in $H = M^4 \times CP_2$ in the generalized sense. This means that one can give H 4 generalized complex coordinates and 3 such generalized complex coordinates can be used for the 4-surface. These surfaces are always minimal surfaces irrespective of the action defining them as its externals and the action makes itself visible only at the singularities of the space-time surface.

8.3.1 Ordinary Mandelbrot and Julia sets

Consider first the ordinary Mandelbrot and Julia sets.

1. The simplest example of the situation is the map $f: z \to z^2+c$. One can consider the iteration of f by starting from a selected point z and look for various values of complex parameter c whether the iteration converges or diverges to infinity. The interface between the sets of the complex c-plane is 1-D Mandelbrot set and is a fractal. One can generalize the iteration to an arbitrary rational function f, in particular polynomials.
2. For polynomials of degree n also consider $n-1$ parameters c_i, $i = 1, ..., n$, to obtain $n-1$ complex-dimensional analog of Mandelbrot set as boundaries of between regions where the iteration lead or does not lead to infinity. For $n = 2$ one obtains a 4-D set.
3. One can also fix the parameter c and consider the iteration of f. Now the complex z-plane decomposes to two a finite region with a finite number of components and its complement, Fatou set. The iteration does not lead out from the finite region but diverges in the complement. The 1-D fractal boundary between these regions is the Julia set.

8.3.2 Holography= holomorphy principle

The generalization to the TGD framework relies heavily on holography=holomorphy principle.

1. In the recent formulation of TGD, holography required by the realization of General Coordinate Invariance is realized in terms of two functions f_1, f_2 of 4 analogs of generalized complex coordinates, one of them corresponds to the light-like (hypercomplex) M^4 coordinate for a surface $X^2 \subset M^4$ and the 3 complex coordinates to those of Y^2 orthogonal to X^2 and the two complex coordinates of CP_2.

   Space-time surfaces are defined by requiring the vanishing of these two functions: $(f_1, f_2) = (0, 0)$. They are minimal surfaces irrespective of the action as long it is general coordinate invariant and constructible in terms of the induced geometry.
2. In the number theoretic vision of TGD, $M^8 - H$-duality [L121] maps the space-time as a holomorphic surface $X^4 \subset H$ is mapped to an associative 4-surface $Y^4 \subset M^8$. The condition for holography in M^8 is that the normal space of Y^4 is quaternionic.

   In the number theoretic vision, the functions f_i are naturally rational functions or polynomials of the 4 generalized complex coordinates. I have proposed that the coefficients of polynomials are rationals or even integers, which in the most stringent approach are smaller than the degree of the polynomial. In the most general situation one could have analytic functions with rational Taylor coefficients.

   The polynomials $f_i = P_i$ form a hierarchy with respect to the degree of P_i, and the iteration defined is analogous to that appearing in the 2-D situation. The iteration of P_i gives a hierarchy of algebraic extensions, which are central in the TGD view of evolution as an increase of algebraic complexity. The iteratikon would also give a hierarchy of increasingly complex space-time surface and the approach to chaos at the level of space-time would correspond to approach of Mandelbrot or Julia set.
3. In the TGD context, there are 4-complex coordinates instead of 1 complex coordinate z. The iteration occurs in H and the vanishing conditions for the iterates define a sequence of 4-surfaces. The initial surface is defined by the conditions $(f_1, f_2) = 0$. This set is analogous to the set $f(z) = 0$ for ordinary Julia sets.

   One could consider the iteration as $(f_1, f_2) \to (f_1 \circ f_1, f_2 \circ f_2)$ continued indefinitely. One could also iterate only f_1 or f_2. Each step defines by the vanishing conditions a 4-D surface, which would be analogous to the image of the $z = 0$ in the 2-D iteration. The iterates form a sequence of 4-surfaces of H analogous to a sequence of iterates of z in the complex plane.

The sequence of 4-surfaces also defines a sequence of points in the "world of classical worlds" (WCW) analogous to the sequence of points $z, f(z), ...$ This conforms with the idea that 3-surface is a generalization of point-like particles, which by holography can be replaced by a Bohr orbit-like 4-surface.

4. Also in this case, one can see whether the iteration converges to a finite result or not. In the zero energy ontology (ZEO), convergence could mean that the iterates of X^4 stay within a causal diamond CD having a finite volume.

8.3.3 The counterparts of Mandelbrot and Julia sets at the level of WCW

What the WCW analogy of the Mandelbrot and Julia sets could look like?

1. Consider first the Mandelbrot set. One could start from a set of roots of $(f_1, f_2) = (c_1, c_2)$ equivalent with the roots of $(f_1 - c_1, f_2 - c_2) = (0, 0)$. Here c_1 and c_2 define complex parameters analogous to the parameter c of the Mandelbrot sent. One can iterate the two functions for all pairs (c_1, c_2). One can look whether the iteration converges or not and identify the Mandelbrot set as the critical set of parameters (c_1, c_2). The naive expectation is that this set is 3-D dimensional fractal.

2. The definition of Julia set requires a complex plane as possible initial points of the iteration. Now the iteration of $(f_1, f_2) = 0$ fixes the starting point (not necessarily uniquely since 3-D surface does not fix the Bohr orbit uniquely: this is the basic motivation for ZEO). The analogy with the initial point of iteration suggests that we can assume $(f_1, f_2) = (c_1, c_2)$ but this leads to the analog of the Mandelbrot set. The notions coincide at the level of WCW.

3. Mandelbrot and Julia sets and their generalizations are critical in a well-defined sense. Whether iteration could be relevant for quantum dynamics is of course an open question. Certainly it could correspond to number theoretic evolution in which the dimension of the algebraic extension rapidly increases. For instance, one could one consider a WCW spinor field as a wave function in the set of converging iterates. Quantum criticality would correspond to WCW spinor fields restricted to the Mandelbrot or Julia sets.

Could the 3-D analogs of Mandelbrot and Julia sets correspond to the light-like partonic orbits defining boundaries between Euclidean and Minkowskian regions of the space-time surface and space-time boundaries? Can the extremely complex fractal structure as sub-manifold be consistent with the differentiability essential for the induced geometry? Could light-likeness help here.

8.3.4 Do the analogs of Mandelbrot and Julia sets exist at the level of space-time?

Could one identify the 3-D analogs of Mandelbrot and Julia sets for a given space-time surface? There are two approaches.

1. The parameter space (c_1, c_2) for a given initial point h of H for iterations of $f_1 - c_1, f_2 - c_2)$ defines a 4-D complex subspace of WCW. Could one identify this subset as a space-time surface and interpret the coordinates of H as parameters? If so, there would be a duality, which would represent the complement of the Fatou set (the thick Julia set) defined as a subset of WCW as a space-time surface!

2. One could also consider fixed points of iteration for which iteration defines a holomorphic map of space-time surface to itself. One can consider generalized holomorphic transformations of H leaving X^4 invariant locally. If they are 1-1 maps they have interpretation as general coordinate transformations. Otherwise they have a non-trivial physical effect so that the analog of the Julia set has a physical meaning. For these transformations one can indeed find the 3-D analog of Julia set as a subset of the space-time surface. This set could define singular surface or boundary of the space-time surface.

8.3.5 Could Mandelbrot and Julia sets have 2-D analogs in TGD?

What about the 2-D analogs of the ordinary Julia sets? Could one identify the counterparts of the 2-D complex plane (coordinate z) and parameter space (coordinate c).

1. Hamilton-Jacobi structure defines what the generalized complex structure is [L115] and defines a slicing of M^4 in terms of integrable distributions of string world sheets and partonic 2-surfaces transversal or even orthogonal to each other. Partonic 2-surface could play the role of complex plane and string world sheet the role of the parameter space or vice versa.

   Partonic 2-surfaces *resp.* and string world sheet having complex *resp.* hyper-complex structures would therefore be in a key role. M^8-H duality maps these surfaces to complex *resp.* co-complex surfaces of octonions having Minkowskian norm defined as number theoretically as $Re(o^2)$.

2. In the case of Julia sets, one could consider generalized holomorphic transformations of H mapping X^4 to itself as a 4-surface but not reducing to 1-1 maps. If f_2 (f_1) acts trivially at the partonic 2-surface Y^2 (string world sheet X^2), the iteration reduces to that for f_1 (f_2). Within string world sheets and partonic 2-surfaces the iteration defines Julia set and its hyperbolic analog in the standard way. One can argue that string world sheets and partonic 2-surfaces should correspond to singularities in some sense. Singularity could mean this fixed point property.

   The natural proposal is that the light-like 3-surfaces defining boundaries between Euclidean and Minkowskian regions of the space-time surface define light-like orbits of the partonic 2-surface. And string world sheets are minimal surfaces having light-like 1-D boundaries at the partonic 2-surface having physical interpretation as world-lines of fermions.

   One could also iterate only f_1 or f_2 allow the parameter c of the initial value of f_1 to vary. This would give the analog of Mandelbrot set as a set of 2-D surfaces of H and it might have dual representation as a 2-surface.

3. The 2-D analog of the Mandelbrot set could correspond to a set of 2-surfaces obtained by fixing a point of the string world sheet X^2. Also now one could consider holomorphic maps leaving the space-time surface locally but not acting 1-1 way. The points of Y^2 would define the values of the complex parameter c remaining invariant under these maps. The convergence of the iteration of f_1 in the same sense as for the Mandelbrot fractal would define the Mandelbrot set as a critical set. For the dual of the Mandelbrot set X^2 and Y^2 would change their roles.

Part II

TOPOLOGICAL QUANTUM COMPUTATION IN TGD UNIVERSE

Chapter 9

Topological Quantum Computation in TGD Universe

9.1 Introduction

Quantum computation is perhaps one of the most rapidly evolving branches of theoretical physics. TGD inspired theory of consciousness has led to new insights about quantum computation and in this chapter I want to discuss these ideas in a more organized way.

There are three mathematically equivalent approaches to quantum computation [B4] : quantum Turing machines, quantum circuits, and topological quantum computation (TQC). In fact, the realization that TGD Universe seems to be ideal place to perform TQC [B25]. [C4] served as the stimulus for writing this chapter.

Quite generally, quantum computation allows to solve problems which are NP hard, that is the time required to solve the problem increases exponentially with the number of variables using classical computer but only polynomially using quantum computer. The topological realization of the computer program using so called braids resulting when threads are weaved to 2-dimensional patterns is very robust so that de-coherence, which is the basic nuisance of quantum computation, ceases to be a problem. More precisely, the error probability is proportional to $exp(-\alpha l)$, where l is the length scale characterizing the distance between strands of the braid [B25].

9.1.1 Evolution Of Basic Ideas Of Quantum Computation

The notion of quantum computation goes back to Feynman [B48] who demonstrated that some computational tasks boil down to problems of solving quantum evolution of some physical system, say electrons scattering from each other. Many of these computations are NP hard, which means that the number of computational steps required grows exponentially with the number of variables involved so that they become quickly unsolvable using ordinary computers. A quicker way to do the computation is to make a physical experiment. A further bonus is that if you can solve one NP hard problem, you can solve many equivalent NP hard problems. What is new that quantum computation is not deterministic so that computation must be carried out several times and probability distribution for the outcomes allows to deduce the answer. Often however the situation is such that it is easy to check whether the outcome provides the sought for solution.

Years later David Deutch [B11] transformed Feynman's ideas into a detailed theory of quantum computation demonstrating how to encode quantum computation in a quantum system and researchers started to develop applications. One of the key factors in the computer security is cryptography which relies on the fact that the factorization of large integers to primes is a NP hard problem. Peter Shor [B47] discovered an algorithm, which allows to carry out the factorization in time, which is exponentially shorter than by using ordinary computers. A second example is problem of searching a particular from a set of N items, which requires time proportional to N classically but quantally only a time proportional to $\sqrt{N}$.

The key notion is quantum entanglement which allows to store information in the relationship between systems, qubits in the simplest situation. This means that information storage capacity increases exponentially as a function of number of qubits rather than only linearly. This explains why NP hard problems which require time increasing exponentially with the number of variables can be solved using quantum computers. It also means exponentially larger information storage capacity than possible classically.

Recall that there are three equivalent approaches to quantum computation: quantum Turing machine, quantum circuits, and topology based unitary modular functor approach. In quantum circuit approach the unitary time evolution defining the quantum computation is assumed to be decomposable to a product of more elementary operations defined by unitary operators associated with quantum gates. The number of different gates needed is surprisingly small: only 1-gates generating unitary transformations of single qubit, and a 2-gate representing a transformation which together with 1-gates is able to generate entanglement are needed to generate a dense subgroup of unitary group $U(2^n)$ in the case of n-qubit system. 2-gate could be conditional NOT (CNOT). The first 1-gate can induce a phase factor to the qubit 0 and do nothing for qubit 1. Second 1-gate could form orthogonal square roots of bits 1 and 0 as superposition of 1 and 0 with identical probabilities.

The formal definition of the quantum computation using quantum circuit is as a computation of the value of a Boolean function of n Boolean arguments, for instance the k:th bit of the largest prime factor of a given integer. The unitary operator U is constructed as a product of operators associated with the basic gates. It is said that the function coding the problem belongs to the class BQP (function is computable with a bounded error in polynomial time) if there exists a classical polynomial-time (in string length) algorithm for specifying the quantum circuit. The first qubit of the outgoing n-qubit is measured and the probability that the value is 0 determines the value of the bit to be calculated. For instance, for $p(0) \geq 2/3$ the bit is 0 and for $p(0) \geq 1/3$ the bit is 1. The evaluation of the outcome is probabilistic and requires a repeat the computation sufficiently many times.

The basic problem of quantum computation is the extremely fragility of the physical qubit (say spin). The fragility can be avoided by mapping q-bits to logical qubits realized as highly entangled states of many qubits and quantum error-correcting codes and fault tolerant methods [B36, B46, B12] rely on this.

The space W of the logical qubits is known as a code space. The sub-space W of physical states of space $Y = V \otimes V..... \otimes V$ is called k-code if the effect of any k-local operator (affecting only k tensor factors of Y linearly but leaving the remaining factors invariant) followed by an orthogonal projection to W is multiplication by scalar. This means that k-local operator modify the states only in directions orthogonal to W.

These spaces indeed exist and it can be shown that the quantum information coded in W is not affected by the errors operating in fewer than $k/2$ of the n particles. Note that $k = 3$ is enough to guarantee stability with respect to 1-local errors. In this way it is possible to correct the errors by repeated quantum measurements and by a suitable choice of the sub-space eliminate the errors due to the local changes of qubits by just performing a projection of the state back to the subspace (quantum measurement).

If the error magnitude is below so called accuracy threshold, arbitrary long quantum computations are reliable. The estimates for this constant vary between 10^{-5} and 10^{-3}. This is beyond current technologies. Error correction is based on the representation of qubit as a logical qubit defined as a state in a linear sub-space of the tensor product of several qubits.

Topological quantum computation [B25] provides an alternative approach to minimize the errors caused by de-coherence. Conceptually the modular functor approach [B25, B29] is considerably more abstract than quantum circuit approach. Unitary modular functor is the S-matrix of a topological quantum field theory. It defines a unitary evolution realizing the quantum computation in macroscopic topological ground states degrees of freedom. The nice feature of this approach is that the notion of physical qubit becomes redundant and the code space defined by the logical qubits can be represented in terms topological and thus non-local degrees of freedom which are stable against local perturbations as required.

9.1.2 Quantum Computation And TGD

Concerning quantum computation [B4] in general, TGD TGD inspired theory of consciousness provides several new insights.

Quantum jump as elementary particle of consciousness and cognition

Quantum jump is interpreted as a fundamental cognitive process leading from creative confusion via analysis to an experience of understanding, and involves TGD counterpart of the unitary process followed by state function reduction and state preparation. One can say that quantum jump is the elementary particle of consciousness and that selves consists of sequences of quantum jump just like hadrons, nuclei, atoms, molecules,... consist basically of elementary particles. Self loses its consciousness when it generates bound state entanglement with environment. The conscious experience of self is in a well-defined sense a statistical average over the quantum jump during which self exists. During macro-temporal quantum coherence during macro-temporal quantum coherence a sequence of quantum jumps integrates effectively to a single moment of consciousness and effectively defines single unitary time evolution followed by state function reduction and preparation. This means a fractal hierarchy of consciousness very closely related to the corresponding hierarchy for bound states of elementary particles and structure formed from them.

Negentropy Maximization Principle guarantees maximal entanglement

Negentropy Maximization Principle is the basic dynamical principle constraining what happens in state reduction and self measurement steps of state preparation. Each self measurement involves a decomposition of system into two parts. The decomposition is dictated by the requirement that the reduction of entanglement entropy in self measurement is maximal. Self measurement can lead to either unentangled state or to entangled state with density matrix which is proportional to unit matrix (density matrix is the observable measured). In the latter case maximally entangled state typically involved with quantum computers results as an outcome. Hence Nature itself would favor maximally entangling 2-gates. Note however that self measurement occurs only if it increases the entanglement negentropy.

Number theoretical information measures and extended rational entanglement as bound state entanglement

The emerging number theoretical notion of information allows to interpret the entanglement for which entanglement probabilities are rational (or belong to an extension of rational numbers defining a finite extension of p-adic numbers) as bound state entanglement with positive information content. Macro-temporal quantum coherence corresponds to a formation of bound entanglement stable against state function reduction and preparation processes.

Spin glass degeneracy, which is the basic characteristic of the variational principle defining space-time dynamics, implies a huge number of vacuum degrees of freedom, and is the key mechanism behind macro-temporal quantum coherence. Spin glass degrees of freedom are also ideal candidates qubit degrees of freedom. As a matter fact, p-adic length scale hierarchy suggests that qubit represents only the lowest level in the hierarchy of qupits defining p-dimensional state spaces, p prime.

Time mirror mechanism and negative energies

The new view about time, in particular the possibility of communications with and control of geometric past, suggests the possibility of circumventing the restrictions posed by time for quantum computation. Iteration based on initiation of quantum computation again and again in geometric past would make possible practically instantaneous information processing.

Space-time sheets with negative time orientation carry negative energies. Also the possibility of phase conjugation of fermions is strongly suggestive. It is also possible that anti-fermions possess negative energies in phases corresponding to macroscopic length scales. This would explain matter-antimatter asymmetry in elegant way. Zero energy states would be ideal for quantum computation purposes and could be even created intentionally by first generating a p-adic surface representing the state and then transforming it to a real surface.

The most predictive and elegant cosmology assumes that the net quantum numbers of the Universe vanish so that quantum jumps would occur between different kinds of vacua. Crossing symmetry makes this option almost consistent with the idea about objective reality with definite conserved total quantum numbers but requires that quantum states of 3-dimensional quantum theory represent S-matrices of 2-dimensional quantum field theory. These quantum states are thus about something. The boundaries of space-time surface are most naturally light-like 3-surfaces space-time surface and are limiting cases of space-like 3-surface and time evolution of 2-surface. Hence they would act naturally as space-time correlates for the reflective level of consciousness.

9.1.3 TGD And The New Physics Associated With TQC

TGD predicts the new physics making possible to realized braids as entangled flux tubes and also provides a detailed model explaing basic facts about anyons.

Topologically quantized magnetic flux tube structures as braids

Quantum classical correspondence suggests that the absolute minimization of Kähler action, which might make sense for Eucdliain regions, could correspond to a space-time representation of second law and that the 4-surfaces approach asymptotically space-time representations of systems which do not dissipate anymore. The correlate for the absence of dissipation is the vanishing of Lorentz 4-force associated with the induced Kähler field. This condition can be regarded as a generalization of Beltrami condition for magnetic fields and leads to very explicit general solutions of field equations [K19].

The outcome is a general classification of solutions based on the dimension of CP_2 projection. The most unstable phase corresponds to $D = 2$-dimensional projection and is analogous to a ferromagnetic phase. $D = 4$ projection corresponds to chaotic de-magnetized phase and $D = 3$ is the extremely complex but ordered phase at the boundary between chaos and order. This phase was identified as the phase responsible for the main characteristics of living systems [K85, K84]. It is also ideal for quantum computations since magnetic field lines form extremely complex linked and knotted structures.

The flux tube structures representing topologically quantized fields, which have $D = 3$ -dimensional CP_2 projection, are knotted, linked and braided, and carry an infinite number of conserved topological charges labelled by representations of color group. They seem to be tailor-made for defining the braid structure needed by TQC. The boundaries of the magnetic flux tubes correspond to light-like 3-surfaces with respect to the induced metric (being thus metrically 2-dimensional and allowing conformal invariance) and can be interpreted either as 3-surfaces or time-evolutions of 2-dimensional systems so that S-matrix of 2-D system can be coded into the quantum state of conformally invariant 3-D system.

Anyons in TGD

TGD suggests a many-sheeted model for anyons used in the modelling of quantum Hall effect [D26, D17, D21]. Quantum-classical correspondence requires that dissipation has space-time correlates. Hence a periodic motion should create a permanent track in space-time. This kind of track would be naturally magnetic flux tube like structure surrounding the Bohr orbit of the charged particle in the magnetic field. Anyon would be electron plus its track.

The magnetic field inside magnetic flux tubes impels the anyons to the surface of the magnetic flux tube and a highly conductive state results. The partial fusion of the flux tubes along their boundaries makes possible de-localization of valence anyons localized at the boundaries of flux tubes and implies a dramatic increase of longitudinal conductivity. When magnetic field is gradually increased the radii of flux tubes and the increase of the net flux brings in new flux tubes. The competition of these effects leads to the emergence of quantum Hall plateaus and sudden increase of the longitudinal conductivity σ_{xx}.

The simplest model explains only the filling fractions $\nu = 1/m$, m odd. The filling fractions $\nu = N/m$, m odd, require a more complex model. The transition to chaos means that periodic orbits become gradually more and more non-periodic: closed orbits fail to close after the first turn and do so only after N 2π rotations. Tracks would become N-branched surfaces. In N-branched space-time the single-valued analytic two particle wave functions $(\xi_k - \xi_l)^m$ of Laughlin [D21] correspond to multiple valued wave functions $(z_k - z_l)^{m/N}$ at its M^4_+ projection and give rise to a filling fraction $\nu = N/m$. The filling fraction $\nu = N/m$, m even, requires composite fermions [D28]. Anyon tracks can indeed contain up to $2N$ electrons if both directions of spin are allowed so that a rich spectroscopy is predicted: in particular anyonic super-conductivity becomes possible by 2-fermion composites. The branching gives rise to Z_N-valued topological charge.

One might think that fractional charges could be only apparent and result from the multi-branched character as charges associated with a single branch. This does not seem to be the case. Rather, the fractional charges result from the additional contribution of the vacuum Kähler charge of the anyonic flux tube to the charge of anyon. For $D = 3$ Kähler charge is topologized in the sense that the charge density is proportional to the Chern-Simons term. Also anyon spin could become genuinely fractional due to the vacuum contribution of the Kähler field to the spin. Besides electronic anyons also anyons associated with various ions are predicted and certain strange experimental findings about fractional Larmor frequencies of proton in water environment [D23] , [J22] have an elegant explanation in terms of protonic anyons with $\nu = 3/5$. In this case however the magnetic field was weaker than the Earth's magnetic field so that the belief that anyons are possible only in systems carrying very strong magnetic fields would be wrong.

In TGD framework anyons as punctures of plane would be replaced by wormhole like tubes connecting different points of the boundary of the magnetic flux tube and are predicted to always appear as pairs as they indeed do. Detailed arguments demonstrate that TGD anyons are for $N = 4$ ($\nu = 4/m$) ideal for realizing the scenario of [B25] for TQC.

The TGD inspired model of non-Abelian anyons is consistent with the model of anyons based on spontaneous symmetry breaking of a gauge symmetry G to a discrete sub-group H dynamically [A72]. The breaking of electro-weak gauge symmetry for classical electro-weak gauge fields occurs at the space-time sheets associated with the magnetic flux tubes defining the strands of braid. Symmetry breaking implies that elements of holonomy group span H. This group is also a discrete subgroup of color group acting as isotropy group of the many-branched surface describing anyon track inside the magnetic flux tube. Thus the elements of the holonomy group are mapped to a elements of discrete subgroup of the isometry group leading from branch to another one but leaving many-branched surface invariant.

Witten-Chern-Simons action and light-like 3-surfaces

The magnetic field inside magnetic flux tube expels anyons at the boundary of the flux tube. In quantum TGD framework light-like 3-surfaces of space-time surface and future light cone are in key role since they define causal determinants for Kähler action. They also provide a universal way to satisfy boundary conditions. Hence also the boundaries of magnetic flux tube structures could be light like surfaces with respect to the induced metric of space-time sheet and would be somewhat like black hole horizons. By their metric 2-dimensionality they allow conformal invariance and due the vanishing of the metric determinant the only coordinate invariant action is Chern-Simons action associated Kähler gauge potential or with the induced electro-weak gauge potentials.

The quantum states associated with the light-like boundaries would be naturally "self-reflective" states in the sense that they correspond to S-matrix elements of the Witten-Chern-Simons topological field theory. Modular functors could results as restriction of the S-matrix to ground state degrees of freedom and Chern-Simons topological quantum field theory is a promising candidate for defining the modular functors [A36, A62].

Braid group B_n is isomorphic to the first homotopy group of the configuration space $C_n(R^2)$ of n particles. $C_n(R^2)$ is $((R^2)_n - D)/S_n$, where D is the singularity represented by the configurations in which the positions of 2 or more particles. and be regarded also as the configuration associated with plane with $n+1$ punctures with $n+1$:th particle regarded as inert. The infinite order of the braid group is solely due to the 2-dimensionality. Hence the dimension $D = 4$ for space-time is unique also in the sense that it makes possible TQC.

9.1.4 TGD And TQC

Many-sheeted space-time concept, the possibility of negative energies, and Negentropy Maximization Principle inspire rather concrete ideas about TQC. NMP gives good hopes that the laws of Nature could take care of building fine-tuned entanglement generating 2-gates whereas 1-gates could be reduced to 2-gates for logical qubits realized using physical qubits realized as Z^4 charges and not existing as free qubits.

Only 2-gates are needed

The entanglement of qubits is algebraic which corresponds in TGD Universe to bound state entanglement. Negentropy Maximization Principle implies that maximal entanglement results automatically in quantum jump. This might saves from the fine-tuning of the 2-gates. In particular, the maximally entangling Yang-Baxter R-matrix is consistent with NMP.

TGD suggests a rather detailed physical realization of the model of [B25] for anyonic quantum computation. The findings about strong correlation between quantum entanglement and topological entanglement are apparently contradicted by the Temperley-Lie representations for braid groups using only single qubit. The resolution of the paradox is based on the observation that in TGD framework batches containing anyon Cooper pair (AA) and single anyon (instead of two anyons as in the model of [B25]) allow to represent single qubit as a logical qubit, and that mixing gate and phase gate can be represented as swap operations s_1 and s_2. Hence also 1-gates are induced by the purely topological 2-gate action, and since NMP maximizes quantum entanglement, Nature itself would take care of the fine-tuning also in this case. The quantum group representation based on $q = exp(i2\pi/5)$ is the simplest representation satisfying various constraints and is also physically very attractive. [B25, B29].

TGD makes possible zero energy TQC

TGD allows also negative energies: besides phase conjugate photons also phase conjugate fermions and anti-fermions are possible, and matter-antimatter asymmetry might be only apparent and due to the ground state for which fermion energies are positive and anti-fermion energies negative.

This would make in principle possible zero energy topological quantum computations. The least one could hope wold be the performance of TQC in doubles of positive and negative energy computations making possible error detection by comparison. The TGD based model for anyon computation however leads to expect that negative energies play much more important role.

The idea is that the quantum states of light-like 3-surfaces represent 2-dimensional time evolutions (in particular modular functors) and that braid operations correspond to zero energy states with initial state represented by positive energy anyons and final state represented by negative energy anyons. The simplest way to realize braid operations is by putting positive *resp.* negative energy anyons near the boundary of tube T_1 *resp.* T_2. Opposite topological charges are at the ends of the magnetic threads connecting the positive energy anyons at T_1 with the negative energy anyons at T_2. The braiding for the threads would code the quantum gates physically.

Before continuing a humble confession is in order: I am not a professional in the area of quantum information science. Despite this, my hope is that the speculations below might serve as an inspiration for real professionals in the field and help them to realize that TGD Universe provides an ideal arena for quantum information processing, and that the new view about time, space-time, and information suggests a generalization of the existing paradigm to a much more powerful one.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

9.2 Existing View About Topological Quantum Computation

In the sequel the evolution of ideas related to topological quantum computation, dance metaphor, and the idea about realizing the computation using a system exhibiting so called non-Abelian Quantum Hall effect, are discussed.

9.2.1 Evolution Of Ideas About TQC

The history of the TQC paradigm is as old as that of QC and involves the contribution of several Fields Medalists. At 1987 to-be Fields Medalist Vaughan Jones [A89] demonstrated that the von Neumann algebras encountered in quantum theory are related to the theory of knots and allow to distinguish between very complex knots. Vaughan also demonstrated that a given knot can be characterized in terms an array of bits. The knot is oriented by assigning an arrow to each of its points and projected to a plane. The bit sequence is determined by a sequence of bits defined by the self-intersections of the knot's projection to plane. The value of the bit in a given intersection changes when the orientation of either line changes or when the line on top of another is moved under it. Since the logic operations performed by the gates of computer can be coded to matrices consisting of 0s and 1s, this means that tying a know can encode the logic operations necessary for computation.

String theorist Edward Witten [A36], also a Fields Medalist, connected the work of Jones to quantum physics by showing that performing measurements to a system described by a 3-dimensional topological quantum field theory defined by non-Abelian Chern-Simons action is equivalent with performing the computation that a particular braid encodes. The braids are determined by linked word lines of the particles of the topological quantum field theory. What makes braids and quantum computation so special is that the coding of the braiding pattern to a bit sequence gives rise to a code, which corresponds to a code solving NP hard problem using classical computer.

1989 computer scientist Alexei Kitaev [B7] demonstrated that Witten's topological quantum field theory could form a basis for a computer. Then Fields Medalist Michael Freedman entered the scene and in collaboration with Kitaev, Michael Larson and Zhenghan Wang developed a vision of how to build a topological quantum computer [B25, B29] using system exhibiting so called non-Abelian quantum Hall effect [D18].

The key notion is Z_4 valued topological charge which has values 1 and 3 for anyons and 0 and 2 for their Cooper pairs. For a system of $2n$ non-Abelian anyon pairs created from vacuum there are n-1 anyon qubits analogous to spin. The notion of physical qubit is not needed at all and logical qubit is coded to the topological charge of the anyon Cooper pair. The basic idea is to utilize entanglement between Z_4 valued topological charges to achieve quantum information storage stable against de-coherence. The swap of neighboring strands of the braid is the topological correlate of a 2-gate which as such does not generate entanglement but can give rise to a transformation such as CNOT. When combined with 1-gates taking square root of qubit and relative phase, this 2-gate is able to generate $U(2^n)$.

The swap can be represented as the so called braid Yang-Baxter R-matrix characterizing also the deviation of quantum groups from ordinary groups [B38]. Quite generally, all unitary Yang-Baxter R-matrices are entangling when combined with square root gate except for special values of parameters characterizing them and thus there is a rich repertoire of topologically realized quantum gates. Temperley-Lieb representation provides a 1-qubit representation for swaps in 3-braid system [B38, B29]. The measurement of qubit reduces to the measurement of the topological charge of the anyon Cooper pair: in the case that it vanishes (qubit 0) the anyon Cooper pair can annihilate and this serves as the physical signature.

9.2.2 Topological Quantum Computation As Quantum Dance

Although topological quantum computation involves very abstract and technical mathematical thinking, it is possible to illustrate how it occurs by a very elegant metaphor. With tongue in cheek one could say that topological quantum computation occurs like a dance. Dancers form couples and in this dancing floor the partners can be also of same sex. Dancers can change their partners. If the partners are of the same sex, they define bit 1 and if they are of opposite sex they define bit 0.

To simplify things one can arrange dancers into a row or several rows such that neighboring partners along the row form a couple. The simplest situation corresponds to a single row of dancers able to make twists of 180 degrees permuting the dancers and able to change the partner to a new one any time. Dance corresponds to a pattern of tracks of dancers at the floor. This pattern can be lifted to a three-dimensional pattern introducing time as a third dimension. When one looks the tracks of a row of dancers in this 2+1-dimensional space-time, one finds that the tracks of the dancers form a complex weaved pattern known as braiding. The braid codes for the computation. The braiding consists of primitive swap operations in which two neighboring word lines twist around each other.

The values of the bits giving the result of the final state of the calculation can be detected since there is something very special which partners with opposite sex can do and do it sooner or later. Just by looking which pairs do it allows to deduce the values of the bits. The alert reader has of course guessed already now that the physical characterization for the sex is as a Z^4 valued topological charge, which is of opposite sign for the different sexes forming Cooper pairs, and that the thing that partners of opposite sex can do is to annihilate! All that is needed to look for those pairs which annihilate after the dance evening to detect the 0s in the row of bits. The coding of the sex to the sign of the topological charge implies also robustness.

It is however essential that the value of topological charge for a given particle in the final state is not completely definite (this is completely general feature of all quantum computations). One can tell only with certain probability that given couple in the final state is male-female or male-male or female-female and the probabilities in question code for the braid pattern in turn coding for quantum logic circuit. Hence one must consider an ensemble of braid calculations to deduce these probabilities.

The basic computational operation permuting the neighboring topological charges is topological so that the program represented by the braiding pattern is very stable against perturbations. The values of the topological charges are also stable. Hence the topological quantum computation is a very robust process and immune to quantum de-coherence even in the standard physics context.

9.2.3 Braids And Gates

In order to understand better how braids define gates one must introduce some mathematical notions related to the braids.

Braid groups

Artin introduced the braid groups bearing his name as groups generated by the elements, which correspond to the cross section between neighboring strands of the braid. The definition of these groups is discussed in detail in [B38]. For a braid having $n+1$ strands the Artin group B_{n1} has n generators s_i. The generators satisfy certain relations. Depending on whether the line coming from left is above the line coming from right one has s_i or s_i^{-1}. The elements s_i and s_j commute for $i<j$ and $i>j+1$: $s_is_j=s_js_i$, which only says that two swaps which do not have common lines commute. For $i=j$ and $i=j+1$ commutativity is not assumed and this correspond to the situation in which the swaps act on common lines.

As already mentioned, Artin's braid group B_n is isomorphic with the homotopy group $\pi_1((R^2)^n/S_{n+1})$ of plane with $n+1$ punctures. B_n is infinite-dimensional because the conditions $s_i^2=1$ added to the defining relations in the case of permutation group S_n are not included. The infinite-dimensionality of homotopy groups reflects the very special topological role of 2-dimensional spaces.

One must consider also variants of braid groups encountered when all particles in question are not identical particles. The reason is that braid operation must be replaced by a 2π rotation of particle A around B when the particles are not identical.

1. Consider first the situation in which all particles are non-identical. The first homotopy group of $R^2)^n-D$, where D represents points configurations for which two or more points are identical is identical with the colored braid group B_n^c defined by $n+1$ punctures in plane such that $n+1$: th is passive (punctures are usually imagined to be located on line). Since particles are not identical the braid operation must be replaced by monodromy in which i: th particle makes 2π rotation around j: th particle. This group has generators

$$\gamma_{ij}=s_i...s_{j-2}s_{j-1}^2s_{j-2}...s_i^{-1} \quad ,i<j \quad , \tag{9.2.1}$$

and can be regarded as a subgroup of the braid group.

2. When several representatives of a given particle species are present the so called partially colored braid group B_n^{pc} is believed to describe the situation. For pairs of identical particles the generators are braid generators and for non-identical particles monodromies appear as generators. It will be found later that in case of anyon bound states, the ordinary braid group

with the assumption that braid operation can lead to a temporary decay and recombination of anyons to a bound state, might be a more appropriate model for what happens in braiding.

3. When all particles are identical, one has the braid group B_n, which corresponds to the fundamental group of $C_n(R^2) = ((R^2)^n - D)/S_n$. Division by S_n expresses the identicality of particles.

Extended Artin's group

Artin's group can be extended by introducing any group G and forming its tensor power $G^{\otimes^n} = G \otimes ... \otimes G$ by assigning to every strand of the braid group G. The extended group is formed from elements of $g_1 \otimes g_2 .... \otimes g_n$ and s_i by posing additional relations $g_i s_j = s_j g_i$ for $i < j$ and $i > j+1$. The interpretation of these relations is completely analogous to the corresponding one for the Artin's group.

If G allows representation in some space V one can look for the representations of the extended Artin's group in the space $V^{\otimes^n}$. In particular, unitary representations are possible. The space in question can also represent physical states of for instance anyonic system and the element g_i associated with the lines of the braid can represent the unitary operators characterizing the time development of the strand between up to the moment when it experiences a swap operation represented by s_i after this operation g_i becomes $s_i g_i s_i^{-1}$.

Braids, Yang-Baxter relations, and quantum groups

Artin's braid groups can be related directly to the so called quantum groups and Yang-Baxter relations. Yang-Baxter relations follow from the relation $s_1 s_2 s_1 = s_2 s_1 s_2$ by noticing that these operations permute the lines 123 of the braid to the order 321. By assigning to a swap operation permuting i:th and j:th line group element R_{ij} when i:th line goes over the j:th line, and noticing that $R_{ij}i$ acts in the tensor product $V_i \otimes V_j$, one can write the relation for braids in a form

$$R_{32}R_{13}R_{12} = R_{12}R_{13}R_{23} \quad .$$

Braid Yang-Baxter relations are equivalent with the so called algebraic Yang-Baxter relations encountered in quantum group theory. Algebraic R can be written as $R_a = RS$, where S is the matrix representing swap operation as a mere permutation. For a suitable choice R_a provides the fundamental representations for the elements of the quantum group $SL(n)_q$ when V is n-dimensional.

The equations represent n^6 equations for n^4 unknowns and are highly over-determined so that solving the equations is a difficult challenge. Equations have symmetries which are obvious on basis of the topological interpretation. Scaling and automorphism induced by linear transformations of V act as symmetries, and the exchange of tensor factors in $V \otimes V$ and transposition are symmetries as also shift of all indices by a constant amount (using modulo N arithmetics).

Unitary R-matrices

Quite a lot is known about the general solutions of the Yang-Baxter equations and for $n = 2$ the general unitary solutions of equations is known [B32]. All of these solutions are entangling and define thus universal 1-gates except for certain parameter values.

The first solution is

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \cdot & \cdot & 1 \\ \cdot & 1 & -1 & \cdot \\ \cdot & 1 & 1 & \cdot \\ -1 & \cdot & \cdot & 1 \end{pmatrix} \tag{9.2.2}$$

and contains no free parameters (dots denote zeros). This R-matrix is strongly entangling. Note that the condition $R^8 = 1$ is satisfied. The defining relations for Artin's braid group allow also more general solutions obtained by multiplying R with an arbitrary phase factor. This would mean that $R^8 = 1$ constraint is not satisfied anymore. One can argue that over-all phase does not matter: on the other hand, the over all phase is visible in knot invariants defined by the trace of R.

The second and third solution come as families labelled four phases a, b, c and d:

$$R'(a,b,c,d) = \frac{1}{\sqrt{2}} \begin{pmatrix} a & \cdot & \cdot & \cdot \\ \cdot & & b & \cdot \\ \cdot & c & \cdot & \cdot \\ \cdot & \cdot & \cdot & d \end{pmatrix}$$

$$R''(a,b,c,d) = \frac{1}{\sqrt{2}} \begin{pmatrix} \cdot & \cdot & \cdot & a \\ \cdot & b & \cdot & \cdot \\ \cdot & \cdot & c & \cdot \\ d & \cdot & \cdot & \cdot \end{pmatrix} \tag{9.2.3}$$

These matrices are not as such entangling. The products $U_1 \otimes U_2 R V_1 \otimes V_2$, where U_i and V_i are 2×2 unitary matrices, are however entangling matrices and thus act as universal gates for $ad - bc \neq 0$ guaranteeing that the state $a|11\rangle + b|10\rangle + |01\rangle + |00\rangle$ is entangled.

It deserves to be noticed that the swap matrix

$$S = R'(1,1,1,1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & & 1 & \cdot \\ \cdot & 1 & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \tag{9.2.4}$$

permuting the qubits does not define universal gate. This is understandable since in this representation of braid group reduces it to permutation group and situation becomes completely classical.

One can write all solutions R of braid Yang-Baxter equation in the form $R = R_a$, where R_a is the solution of so called algebraic Yang-Baxter equation. The interpretation is that the swap matrix S represents the completely classical part of the swap operation since it acts as a mere permutation whereas R_a represents genuine quantum effects related to the swap operation.

In the article of Kauffman [B38] its is demonstrated explicitly how to construct CNOT gate as a product MRN, where M and N are products of single particle gates. This article contains also a beautiful discussion about how the traces of the unitary matrices defined by the braids define knot invariants. For instance, the matrix R satisfies $R^8 = 1$ so that the invariants constructed using R as 2-gate cannot distinguish between knots containing n and $n + 8k$ sub-sequent swaps. Note however that the multiplication of R with a phase factor allows to get rid of the 8-periodicity.

Knots, links, braids, and quantum 2-gates

In [B38] basic facts about knots, links, and their relation to braids are discussed. Knot diagrams are introduced, the so called Reidermeister moves and homeomorphisms of plane as isotopies of knots and links are discussed. Also the notion of braid closure producing knots or links is introduced together with the theorem of Markov stating that any knot and link corresponds to some (not unique) braid. Markov moves as braid deformations leaving corresponding knots and links invariant are discussed and it the immediate implication is that traces of the braid matrices define knot invariants. In particular, the traces of the unitary matrices defined by R-matrix define invariants having same value for the knots and links resulting in the braid closure.

In [B38] the state preparation and quantum measurement allowing to deduce the absolute value of the trace of the unitary matrix associated with the braid defining the quantum computer is discussed as an example how quantum computations could occur in practice. The braid in question is product of the braid defining the invariant and trivial braid with same number n of strands. The incoming state is maximally entangled state formed $\sum_n |n\rangle \otimes |n\rangle$, where n runs over all possible bit sequences defined by the tensor product of n qubits. Quantum measurement performs a projection to this state and from the measurements it is possible to deduce the absolute value of the trace defining the knot invariant.

9.2.4 About Quantum Hall Effect And Theories Of Quantum Hall Effect

Using the dance metaphor for TQC, the system must be such that it is possible to distinguish between the different sexes of dancers. The proposal of [B25] is that the system exhibiting so called non-Abelian Quantum Hall effect [D17, D19] could make possible realization of the topological computation.

The most elegant models of quantum Hall effect are in terms of anyons regarded as singularities due to the symmetry breaking of gauge group G down to a finite sub-group H, which can be also non-Abelian. Concerning the description of the dynamics of topological degrees of freedom topological quantum field theories based on Chern-Simons action are the most promising approach.

Quantum Hall effect

Quantum Hall effect [D26, D17] occurs in 2-dimensional systems, typically a slab carrying a longitudinal voltage V causing longitudinal current j. A magnetic field orthogonal to the slab generates a transversal current component j_T by Lorentz force. j_T is proportional to the voltage V along the slab and the dimensionless coefficient is known as transversal conductivity. Classically the coefficients is proportional ne/B, where n is 2-dimensional electron density and should have a continuous spectrum. The finding that came as surprise was that the change of the coefficient as a function of parameters like magnetic field strength and temperature occurred as discrete steps of same size. In integer quantum Hall effect the coefficient is quantized to $2\nu\alpha$, $\alpha = e^2/4\pi$, such that ν is integer.

Later came the finding that also smaller steps corresponding to the filling fraction $\nu = 1/3$ of the basic step were present and could be understood if the charge of electron would have been replaced with $\nu = 1/3$ of its ordinary value. Later also QH effect with wide large range of filling fractions of form $\nu = k/m$ was observed.

The model explaining the QH effect is based on pseudo particles known as anyons [A72], [D17]. According to the general argument of [D26] anyons have fractional charge νe. Also the TGD based model for fractionization to be discussed later suggests that the anyon charge should be νe quite generally. The braid statistics of anyon is believed to be fractional so that anyons are neither bosons nor fermions. Non-fractional statistics is absolutely essential for the vacuum degeneracy used to represent logical qubits.

In the case of Abelian anyons the gauge potential corresponds to the vector potential of the divergence free velocity field or equivalently of incompressible anyon current. For non-Abelian anyons the field theory defined by Chern-Simons action is free field theory and in well-defined sense trivial although it defines knot invariants. For non-Abelian anyons situation would be different. They would carry non-Abelian gauge charges possibly related to a symmetry breaking to a discrete subgroup H of gauge group [A72] each of them defining an incompressible hydrodynamical flow. Non-Abelian QH effect has not yet been convincingly demonstrated experimentally. According to [B25] the anyons associated with the filling fraction $\nu = 5/2$ are a good candidate for non-Abelian anyons and in this case the charge of electron is reduced to $Q = 1/4$ rather than being $Q = \nu e$.

Non-Abelian anyons [D17, D18] are always created in pairs since they carry a conserved topological charge. In the model of [B25] this charge should have values in 4-element group Z_4 so that it is conserved only modulo 4 so that charges +2 and -2 are equivalent as are also charges 3 and -1. The state of n anyon pairs created from vacuum can be show to possess 2^{n-1}-dimensional vacuum degeneracy [D19]: later a TGD based argument for why this is the case is constructed. When two anyons fuse the 2^{n-1}-dimensional state space decomposes to 2^{n-2}-dimensional tensor factors corresponding to anyon Cooper pairs with topological charges 2 and 0. The topological "spin" is ideal for representing logical qubits. Since free topological charges are not possible the notion of physical qubit does not make sense (note the analogy with quarks). The measurement of topological qubit reduces to a measurement of whether anyon Cooper pair has vanishing topological charge or not.

Quantum Hall effect as a spontaneous symmetry breaking down to a discrete subgroup of the gauge group

The system exhibiting quantum Hall effect is effectively 2-dimensional. Fractional statistics suggests that topological defects, anyons, allowing a description in terms of the representations of the homotopy group of $((R^2)^n - D)/S_n$. The gauge theory description would be in terms of spontaneous symmetry breaking of the gauge group G to a finite subgroup H by a Higgs mechanism [A72] , [D17]. This would make all gauge degrees of freedom massive and leave only topological degrees of freedom. What is unexpected that also non-Abelian topological degrees of freedom are in principle possible. Quantum Hall effect is Abelian or non-Abelian depending on whether the group H has this property.

In the symmetry breaking $G \rightarrow H$ the non-Abelian gauge fluxes defined as non-integrable phase factors $Pexp(i \oint A_\mu dx^\mu)$ around large circles (surrounding singularities (so that field approaches a pure gauge configuration) are elements of the first homotopy group of G/H, which is H in the case that H is discrete group and G is simple. An idealized manner to model the situation [D17] is to assume that the connection is pure gauge and defined by an H-valued function which is many-valued such that the values for different branches are related by a gauge transformation in H. In the general case a gauge transformation of a non-trivial gauge field by a multi-valued element of the gauge group would give rise to a similar situation.

One can characterize a given topological singularity magnetically by an element in conjugacy class C of H representing the transformation of H induced by a 2π rotation around singularity. The elements of C define states in given magnetic representation. Electrically the particles are characterized by an irreducible representations of the subgroup of $H_C \subset H$ which commutes with an arbitrarily chosen element of the conjugacy class C.

The action of $h(B)$ resulting on particle A when it makes a closed turn around B reduces in magnetic degrees of freedom to translation in conjugacy class combined with the action of element of H_C in electric degrees of freedom. Closed paths correspond to elements of the braid group $B_n(X^2)$ identifiable as the mapping class group of the punctured 2-surface X^2 and this means that symmetry breaking $G \rightarrow H$ defines a representation of the braid group. The construction of these representations is discussed in [D17] and leads naturally via the group algebra of H to the so called quantum double $D(H)$ of H, which is a quasi-triangular Hopf algebra allowing non-trivial representations of braid group.

Anyons could be singularities of gauge fields, perhaps even non-Abelian gauge fields, and the latter ones could be modelled by these representations. In particular, braid operations could be represented using anyons.

Witten-Chern-Simons action and topological quantum field theories

The Wess-Zumino-Witten action used to model 2-dimensional critical systems consists of a 2-dimensional conformally invariant term for the chiral field having values in group G combined with 2+1-dimensional term defined as the integral of Chern-Simons 3-form over a 3-space containing 2-D space as its boundary. This term is purely topological and identifiable as winding number for the map from 3-dimensional space to G. The coefficient of this term is integer k in suitable normalization. k gives the value of central extension of the Kac-Moody algebra defined by the theory.

One can couple the chiral field $g(x)$ to gauge potential defined for some subgroup of G_1 of G. If the G_1 coincides with G, the chiral field can be gauged away by a suitable gauge transformation and the theory becomes purely topological Witten-Chern-Simons theory. Pure gauge field configuration represented either as flat gauge fields with non-trivial holonomy over homotopically non-trivial paths or as multi-valued gauge group elements however remain and the remaining degrees of freedom correspond to the topological degrees of freedom.

Witten-Chern-Simons theories are labelled by a positive integer k giving the value of central extension of the Kac-Moody algebra defined by the theory. The connection with Wess-Zumino-Witten theory come from the fact that the highest weight states associated with the representations of the Kac-Moody algebra of WZW theory are in one-one correspondence with the representations R_i possible for Wilson loops in the topological quantum field theory.

In the Abelian case case 2+1-dimensional Chern-Simons action density is essentially the inner product $A\wedge dA$ of the vector potential and magnetic field known as helicity density and the theory in question is a free field theory. In the non-Abelian case the action is defined by the 3-form

$$\frac{k}{4\pi}Tr\left(A\wedge(dA+\frac{2}{3}A\wedge A)\right)$$

and contains also interaction term so that the field theory defined by the exponential of the interaction term is non-trivial.

In topological quantum field theory the usual n-point correlation functions defined by the functional integral are replaced by the functional averages for $Diff^3$ invariant quantities defined in terms of non-integrable phase factors defined by ordered exponentials over closed loops. One can consider arbitrary number of loops which can be knotted, linked, and braided. These quantities define both knot and 3-manifold invariants (the functional integral for zero link in particular). The perturbative calculation of the quantum averages leads directly to the Gaussian linking numbers and infinite number of perturbative link and not invariants.

The experience gained from topological quantum field theories defined by Chern-Simons action has led to a very elegant and surprisingly simple category theoretical approach to the topological quantum field theory [A27, A83] allowing to assign invariants to knots, links, braids, and tangles and also to 3-manifolds for which braids as morphisms are replaced with cobordisms. The so called modular Hopf algebras, in particular quantum groups $Sl(2)_q$ with q a root of unity, are in key role in this approach. Also the connection between links and 3-manifolds can be understood since closed, oriented, 3-manifolds can be constructed from each other by surgery based on links.

Witten's article [A36] "Quantum Field Theory and the Jones Polynomial" is full of ingenious constructions, and for a physicist it is the easiest and certainly highly enjoyable manner to learn about knots and 3-manifolds. For these reasons a little bit more detailed sum up is perhaps in order.

1. Witten discusses first the quantization of Chern-Simons action at the weak coupling limit $k\rightarrow\infty$. First it is shown how the functional integration around flat connections defines a topological invariant for 3-manifolds in the case of a trivial Wilson loop. Next a canonical quantization is performed in the case $X^3=\Sigma^2\times R^1$: in the Coulomb gauge $A_3=0$ the action reduces to a sum of $n=dim(G)$ Abelian Chern-Simons actions with a non-linear constraint expressing the vanishing of the gauge field. The configuration space consists thus of flat non-Abelian connections, which are characterized by their holonomy groups and allows Kähler manifold structure.
2. Perhaps the most elegant quantal element of the approach is the decomposition of the 3-manifold to two pieces glued together along 2-manifold implying the decomposition of the functional integral to a product of functional integrals over the pieces. This together with the basic properties of Hilbert of complex numbers (to which the partition functions defined by the functional integrals over the two pieces belong) allows almost a miracle like deduction of the basic results about the behavior of 3-manifold and link invariants under a connected sum, and leads to the crucial skein relations allowing to calculate the invariants by decomposing the link step by step to a union of unknotted, unlinked Wilson loops, which can be calculated exactly for $SU(N)$. The decomposition by skein relations gives rise to a partition function like representation of invariants and allows to understand the connection between knot theory and statistical [A93]. A direct relationship with conformal field theories and Wess-Zumino-Witten model emerges via Wilson loops associated with the highest weight representations for Kac Moody algebras.
3. A similar decomposition procedure applies also to the calculation of 3-manifold invariants using link surgery to transform 3-manifolds to each other, with 3-manifold invariants being defined as Wilson loops associated with the homology generators of these (solid) tori using representations R_i appearing as highest weight representations of the loop algebra of torus. Surgery operations are represented as mapping class group operations acting in the Hilbert space defined by the invariants for representations R_i for the original 3-manifold. The outcome is explicit formulas for the invariants of trivial knots and 3-manifold invariant of S^3 for $G=SU(N)$, in terms of which more complex invariants are expressible.
4. For $SU(N)$ the invariants are expressible as functions of the phase $q=exp(i2\pi/(k+N))$ associated with quantum groups. Note that for $SU(2)$ and $k=3$, the invariants are expressible

in terms of Golden Ratio. The central charge $k = 3$ is in a special position since it gives rise to $k + 1 = 4$-vertex representing naturally 2-gate physically. Witten-Chern-Simons theories define universal unitary modular functors characterizing quantum computations [B29].

Chern-Simons action for anyons

In the case of quantum Hall effect the Chern-Simons action has been deduced from a model of electrons as a 2-dimensional incompressible fluid [D21]. Incompressibility requires that the electron current has a vanishing divergence, which makes it analogous to a magnetic field. The expressibility of the current as a curl of a vector potential b, and a detailed study of the interaction Lagrangian leads to the identification of an Abelian Chern-Simons for b as a low energy effective action. This action is Abelian, whereas the anyonic realization of quantum computation would suggest a non-Abelian Chern-Simons action.

Non-Abelian Chern-Simons action could result in the symmetry breaking of a non-Abelian gauge group G, most naturally electro-weak gauge group, to a non-Abelian discrete subgroup H [A72] so that states would be labelled by representations of H and anyons would be characterized magnetically H-valued non-Abelian magnetic fluxes each of them defining its own incompressible hydro-dynamical flow. As will be found, TGD predicts a non-Abelian Chern-Simons term associated with electroweak long range classical fields.

9.2.5 Topological Quantum Computation Using Braids And Anyons

By the general mathematical results braids are able to code all quantum logic operations [B5]. In particular, braids allow to realize any quantum circuit consisting of single particle gates acting on qubits and two particle gates acting on pairs of qubits. The coding of braid requires a classical computation which can be done in polynomial time. The coding requires that each dancer is able to remember its dancing history by coding it into its own state.

The general ideas are following.

1. The ground states of anyonic system characterize the logical qubits, One assumes non-Abelian anyons with Z_4 -valued topological charge so that a system of n anyon pairs created from vacuum allows 2^{n-1}-fold anyon degeneracy [D19]. The system is decomposed into blocks containing one anyonic Cooper pair with $Q_T \in \{2, 0\}$ and two anyons with such topological charges that the net topological charge vanishes. One can say that the states $(0, 1 - 1)$ and $(0, -1, +1))$ represent logical qubit 0 whereas the states $(2, -1, -1)$ and $(2, +1, +1)$ represent logical qubit 1. This would suggest 2^2-fold degeneracy but actually the degeneracy is 2-fold.

   Free physical qubits are not possible and at least four particles are indeed necessarily in order to represent logical qubit. The reason is that the conservation of Z^4 charge would not allow mixing of qubits 1 and 0, in particular the Hadamard 1-gate generating square root of qubit would break the conservation of topological charge. The square root of qubit can be generated only if 2 units of topological charge is transferred between anyon and anyon Cooper pair. Thus qubits can be represented as entangled states of anyon Cooper pair and anyon and the fourth anyon is needed to achieve vanishing total topological charge in the batch.

2. In the initial state of the system the anyonic Cooper pairs have $Q_T = 0$ and the two anyons have opposite topological charges inside each block. The initial state codes no information unlike in ordinary computation but the information is represented by the braid. Of course, also more general configurations are possible. Anyons are assumed to evolve like free particles except during swap operations and their time evolution is described by single particle Hamiltonians.

   Free particle approximation fails when the anyons are too near to each other as during braid operations. The space of logical qubits is realized as k-code defined by the 2^{n-1} ground states, which are stable against local single particle perturbations for $k = 3$ Witten-Chern-Simons action. In the more general case the stability against n-particle perturbations with $n < [k/2]$ is achieved but the gates would become $[k/2]$-particle gates (for $k = 5$ this would give 6-particle vertices).

3. Anyonic system provides a unitary modular functor as the S-matrix associated with the anyon system whose time evolution is fixed by the pre-existing braid structure. What this means that

the S-matrices associated with the braids can be multiplied and thus a unitary representation for the group formed by braids results. The vacuum degeneracy of anyon system makes this representation non-trivial. By the NP complexity of braids it is possible to code any quantum logic operation by a particular braid [B41]. There exists a powerful approximation theorem allowing to achieve this coding classically in polynomial time [B5]. From the properties of the R-matrices inducing gate operations it is indeed clear that two gates can be realized. The Hadamard 1-gate could be realized as 2-gate in the system formed by anyon Cooper pair and anyon.

4. In [B25] the time evolution is regarded as a discrete sequence of modifications of single anyon Hamiltonians induced by swaps [B42]. If the modifications define a closed loop in the space of Hamiltonians the resulting unitary operators define a representation of braid group in a dense discrete sub-group of $U(2^n)$. The swap operation is 2-local operation acting like a 2-gate and induces quantum logical operation modifying also single particle Hamiltonians. What is important that this modification maps the space of the ground states to a new one and only if the modifications correspond to a closed loop the final state is in the same code space as the initial state. What time evolution does is to affect the topological charges of anyon Cooper pairs representing qubits inside the 4-anyon batches defined by the braids.

   In quantum field theory the analog but not equivalent of this description would be following. Quite generally, a given particle in the final state has suffered a unitary transformation, which is an ordered product consisting of two kinds of unitary operators. Unitary single particle operators $U_n = Pexp(i\int_{t_n}^{t_{n+1}} H_0 dt)$ are analogs of operators describing single qubit gate and play the role of anyon propagators during no-swap periods. Two-particle unitary operators $U_{swap} = Pexp(i\int H_{swap} dt)$ are analogous to four-particle interactions and describe the effect of braid operations inducing entanglement of states having opposite values of topological charge but conserving the net topological charge of the anyon pair. This entanglement is completely analogous to spin entanglement. In particular, the braid operation mixes different states of the anyon. The unitary time development operator generating entangled state of anyons and defined by the braid structure represents the operation performed by the quantum circuit and the quantum measurement in the final state selects a particular final state.

5. Formally the computation halts with a measurement of the topological charge of the left-most anyon Cooper pair when the outcome is just single bit. If decay occurs with sufficiently high probability it is concluded that the value of the computed bit is 0, otherwise 1.

9.3 General Implications Of TGD For Quantum Computation

TGD based view about time and space-time could have rather dramatic implications for quantum computation in general and these implications deserve to be discussed briefly.

9.3.1 Time Need Not Be A Problem For Quantum Computations In TGD Universe

Communication with and control of the geometric past is the basic mechanism of intentional action, sensory perception, and long term memory in TGD inspired theory of consciousness. The possibility to send negative energy signals to the geometric past allows also instantaneous computations with respect to subjective time defined by a sequence of quantum jumps. The outcome of computation back to the past where it defines initial values of the next round of iteration. Time would cease to be a limiting factor to computation.

9.3.2 New View About Information

The notion of information is very problematic even in the classical physics and in quantum realm this concept becomes even more enigmatic. TGD inspired theory consciousness has inspired number theoretic ideas about quantum information which are still developing. The standard definition of entanglement entropy relies on the Shannon's formula: $S = -\sum_k p_k log(p_k)$. This entropy is always non-negative and tells that the best one can achieve is entanglement with zero entropy.

The generalization of the notion of entanglement entropy to the p-adic context however led to realization that entanglement for which entanglement probabilities are rational or in an extension of rational numbers defining a finite extension of p-adics allows a hierarchy of entanglement entropies S_p labelled by primes. These entropies are defined as $S_p = -\sum_k p_k log(|p_k|_p)$, where $|p_k|_p$ denotes the p-adic norm of probability. S_p can be negative and in this case defines a genuine information measure. For given entanglement probabilities S_p has a minimum for some value p_0 of prime p, and S_{p_0} could be taken as a measure for the information carried by the entanglement in question whereas entanglement in real and p-adic continua would be entropic. The entanglement with negative entanglement entropy is identified as bound state entanglement.

Since quantum computers by definition apply states for which entanglement coefficients belong to a finite algebraic extension of rational numbers, the resulting states, if ideal, should be bound states. Also finite-dimensional extensions of p-adic numbers by transcendentals are possible. For instance, the extension by the $p-1$ first powers of e (e^p is ordinary p-adic number in R_p). As an extension of rationals this extension would be discrete but infinite-dimensional. Macro-temporal quantum coherence can be identified as being due to bound state formation in appropriate degrees of freedom and implying that state preparation and state function reduction effectively ceases to occur in these degrees of freedom.

Macro-temporal quantum coherence effectively binds a sequence of quantum jumps to single quantum jump so that the effective duration of unitary evolution is stretched from about 10^4 Planck times to arbitrary long time span. Also quantum computations can be regarded as this kind of extended moments of consciousness.

9.3.3 Number Theoretic Vision About Quantum Jump As A Building Block Of Conscious Experience

The generalization of number concept resulting when reals and various p-adic number fields are fused to a book like structure obtaining by gluing them along rational numbers common to all these number fields leads to an extremely general view about what happens in quantum jump identified as basic building block of conscious experience. First of all, the unitary process U generates a formal superposition of states belonging to different number fields including their extensions. Negentropy Maximization Principle [K72] constrains the dynamics of state preparation and state function reduction following U so that the final state contains only rational or extended rational entanglement with positive information content. At the level of conscious experience this process can be interpreted as a cognitive process or analysis leading to a state containing only bound state entanglement serving as a correlate for the experience of understanding. Thus quantum information science and quantum theory of consciousness seem to meet each other.

In the standard approach to quantum computing entanglement is not bound state entanglement. If bound state entanglement is really the entanglement which is possible for quantum computer, the entanglement of qubits might not serve as a universal entanglement currency. That is, the reduction of the general two-particle entanglement to entanglement between N qubits might not be possible in TGD framework.

The conclusion that only bound state entanglement is possible in quantum computation in human time scales is however based on the somewhat questionable heuristic assumption that subjective time has the same universal rate, that is the average increment Δt of the geometric time in single quantum jump does not depend on the space-time sheet, and is of order CP_2 time about 10^4 Planck times. The conclusion could be circumvented if one assumes that Δt depends on the space-time sheet involved: for instance, instead of CP_2 time Δt could be of order p-adic time scale T_p for a space-time sheet labelled by p-adic prime p and increase like $\sqrt{p}$. In this case the unitary operator defining quantum computation would be simply that defining the unitary process U.

9.3.4 Dissipative Quantum Parallelism?

The new view about quantum jump implies that state function reduction and preparation process decomposes into a hierarchy of these processes occurring in various scales: dissipation would occur in quantum parallel manner with each p-adic scale defining one level in the hierarchy. At space-time level this would correspond to almost independent quantum dynamics at parallel space-time sheets labelled by p-adic primes. In particular, dissipative processes can occur in short scales while the dynamics in longer scales is non-dissipative. This would explain why the description of hadrons as dissipative systems consisting of quarks and gluons in short scales is consistent with the description of hadrons as genuine quantum systems in long scales. Dissipative quantum parallelism would also mean that thermodynamics at shorter length scales would stabilize the dynamics at longer length scales and in this manner favor scaled up quantum coherence.

NMR systems [B4] might represent an example about dissipative quantum parallelism. Room temperature NMR (nuclear magnetic resonance) systems use highly redundant replicas of qubits which have very long coherence times. Quantum gates using radio frequency pulses to modify the spin evolution have been implemented, and even effective Hamiltonians have been synthesized. Quantum computations and dynamics of other quantum systems have been simulated and quantum error protocols have been realized. These successes are unexpected since the energy scale of cyclotron states is much below the thermal energy. This has raised fundamental questions about the power of quantum information processing in highly mixed states, and it might be that dissipative quantum parallelism is needed to explain the successes.

Magnetized systems could realize quite concretely the renormalization group philosophy in the sense that the magnetic fields due to the magnetization at the atomic space-time sheets could define a return flux along larger space-time sheets as magnetic flux quanta (by topological flux quantization) defining effective block spins serving as thermally stabilized qubits for a long length scale quantum parallel dynamics. For an external magnetic field $B \sim 10$ Tesla the magnetic length is $L \sim 10$ nm and corresponds to the p-adic length scale $L(k = 151)$. The induced magnetization is $M \sim n\mu^2 B/T$, where n is the density of nuclei and $\mu = ge/2m_p$ is the magnetic moment of nucleus. For solid matter density the magnetization is by a factor ~ 10 weaker than the Earth's magnetic field and corresponds to a magnetic length $L \sim 15$ μm: the p-adic length scale is around $L(171)$. For 10^{22} spins per block spin used for NMR simulations the size of block spin should be $\sim$ 1mm solid matter density so that single block spin would contain roughly 10^6 magnetization flux quanta containing 10^{16} spins each. The magnetization flux quanta serving as logical qubits could allow to circumvent the standard physics upper bound for scaling up of about 10 logical qubits [B4].

9.3.5 Negative Energies And Quantum Computation

In TGD universe space-times are 4-surfaces so that negative energies are possible due to the fact that the sign of energy depends on time orientation (energy momentum tensor is replaced by a collection of conserved momentum currents). This has several implications. Negative energy photons having phase conjugate photons as physical correlates of photons play a key role in TGD inspired theory of consciousness and living matter and there are also indications that magnetic flux tubes structures with negative energies are important.

Negative energies makes possible communications to the geometric past, and time mirror mechanism (see **Fig. ??** in the appendix of this book) involving generation of negative energy photons is the key mechanism of intentional action and plays central role in the model for the functioning of bio-systems. In principle this could allow to circumvent the problems due to the time required by computation by initiating computation in the geometric past and iterating this process. The most elegant and predictive cosmology is that for which the net conserved quantities of the universe vanish due the natural boundary condition that nothing flows into the future light cone through its boundaries representing the moment of big bang.

Also topological quantum field theories describe systems for which conserved quantities associated with the isometries of space-time, such as energy and momentum, vanish. Hence the natural question is whether negative energies making possible zero energy states might also make possible also zero energy quantum computations.

Crossing symmetry and Eastern and Western views about what happens in scattering

The hypothesis that all physical states have vanishing net quantum numbers (Eastern view) forces to interpret the scattering events of particle physics as quantum jumps between different vacua. This interpretation is in a satisfactory consistency with the assumption about existence of objective reality characterized by a positive energy (Western view) if crossing symmetry holds so that WCW spinor fields can be regarded as S-matrix elements between initial state defined by positive energy particles and negative energy state defined by negative energy particles. As a matter fact, the proposal for the S-matrix of TGD at elementary particle level relies on this idea: the amplitudes for the transition from vacuum to states having vanishing net quantum numbers with positive and negative energy states interpreted as incoming and outgoing states are assumed to be interpretable as S-matrix elements.

More generally, one could require that scattering between any pair of states with zero net energies and representing S-matrix allows interpretation as a scattering between positive energy states. This requirement is satisfied if their exists an entire self-reflective hierarchy of S-matrices in the sense that the S-matrix between states representing S-matrices S_1 and S_2 would be the tensor product $S_1 \otimes S_2$. At the observational level the experience the usual sequence of observations $|m_1\rangle \rightarrow |m_2\rangle ..... \rightarrow |m_n\rangle ...$ based on belief about objective reality with non-vanishing conserved net quantum numbers would correspond to a sequence $(|m_1 \rightarrow m_2\rangle \rightarrow |m_2 \rightarrow m_3\rangle ...$ between "self-reflective" zero energy states. These sequences are expected to be of special importance since the contribution of the unit matrix to S-matrix $S = 1 + iT$ gives dominating contribution unless interactions are strong. This sequence would result in the approximation that $S_2 = 1 + iT_2$ in $S = S_1 \otimes S_2$ is diagonal. The fact that the scattering for macroscopic systems tends to be in forward direction would help to create the materialistic illusion about unique objective reality.

It should be possible to test whether the Eastern or Western view is correct by looking what happens strong interacting systems where the western view should fail. The Eastern view is consistent with the basic vision about quantum jumps between quantum histories having as a counterpart the change of the geometric past at space-time level.

Negative energy anti-fermions and matter-antimatter asymmetry

The assumption that space-time is 4-surface means that the sign of energy depends on time orientation so that negative energies are possible. Phase conjugate photons [D24] are excellent candidates for negative energy photons propagating into geometric past.

Also the phase conjugate fermions make in principle sense and one can indeed perform Dirac quantization in four ways such that a) both fermions and anti-fermions have positive/negative energies, b) fermions (anti-fermions) have positive energies and anti-fermions (fermions) have negative energies. The corresponding ground state correspond to Dirac seas obtained by applying the product of a) all fermionic and anti-fermionic annihilation (creation) operators to vacuum, b) all fermionic creation (annihilation) operators and anti-fermionic annihilation (creation) operators to vacuum. The ground states of a) have infinite vacuum energy which is either negative or positive whereas the ground states of b) have vanishing vacuum energy. The case b) with positive fermionic and negative anti-fermionic energies could correspond to long length scales in which are matter-antisymmetric due to the effective absence of anti-fermions ("effective" meaning that no-one has tried to detect the negative energy anti-fermions). The case a) with positive energies could naturally correspond to the phase studied in elementary particle physics.

If gravitational and inertial masses have same magnitude and same sign, consistency with empirical facts requires that positive and negative energy matter must have been separated in cosmological length scales. Gravitational repulsion might be the mechanism causing this. Applying naïvely Newton's equations to a system of two bodies with energies $E_1 > 0$ and $-E_2 < 0$ and assuming only gravitational force, one finds that the sign of force for the motion in relative coordinates is determined by the sign of the reduced mass $-E_1E_2/(E_1 - E_2)$, which is negative for $E_1 > |E_2|$: positive masses would act repulsively on smaller negative masses. For $E_1 = -E_2$ the motion in the relative coordinate becomes free motion and both systems experience same acceleration which for E_1 corresponds to a repulsive force. The reader has probably already asked whether the observed acceleration of the cosmological expansion interpreted in terms of cosmological constant due to vacuum energy could actually correspond to a repulsive force between positive and negative energy matter.

It is possible to create pairs of positive energy fermions and negative energy fermions from vacuum. For instance, annihilation of photons and phase conjugate photons could create electron and negative energy positron pairs with a vanishing net energy. Magnetic flux tubes having positive and negative energies carrying fermions and negative energy positrons pairs of photons and their phase conjugates via fermion anti-fermion annihilation. The obvious idea is to perform zero energy topological quantum computations by using anyons of positive energy and anti-anyons of negative energy plus their Cooper pairs. This idea will be discussed later in more detail.

9.4 TGD Based New Physics Related To Topological Quantum Computation

For a long the belief was that absolute minimum property defines the basic dynamical principle of space-time physics. This might make sense in space-time regions of Euclidian signature, where Kähler action is non-negative but not in Minkowskian regions, where the contribution to the exponent defining vacuum functional is imaginary. The reduction of the theory to the level of Kähler-Dirac action [K128] made it however clear that the preferred extremals defining the analogs of Bohr orbits must be critical in the sense of having an infinite number of deformations for which the second variation of Kähler action vanishes. The criticality of Kähler action would thus the basic dynamical principle of space-time dynamics. Purely number theoretic conditions in turn suggest the conclusion that space-time surfaces must be hyper-quaternionic in the sense that the Kähler-Dirac gamma matrices span hyper-quaternionic (asociative) or co-hyper-quaternionic (co-associative) plane at each point of the spacce-time surface. "Co-" means that the orthogonal complement of this plane is hyper-quaternionic (associative). Whether criticality and associativity (co-associativity) are consistent is not clear.

For a long time it remained an open question whether the known solutions of field equations are building blocks of preferred extremals of Kähler action or represent only the simplest extremals one can imagine and perhaps devoid of any real significance. Quantum-classical correspondence meant a great progress in the understanding the solution spectrum of field equations. Among other things, this principle requires that the dissipative quantum dynamics leading to non-dissipating asymptotic self-organization patterns should have the vanishing of the Lorentz 4-force as space-time correlate. The absence of dissipation in the sense of vanishing of Lorentz 4-force is a natural correlate for the absence of dissipation in quantum computations.

The vanishing of Lorentz 4-force generalizes the so called Beltrami conditions [B8, B14] stating the vanishing of Lorentz force for purely magnetic field configurations and these conditions reduce in many cases to topological conditions. The study of classical field equations predicts three phases corresponding to non-vacuum solutions of field equations possessing vanishing Lorentz force. The dimension D of CP_2 projection of the space-time sheet serves as classifier of the phases.

1. $D = 2$ phase is analogous to ferro-magnetic phase possible in low temperatures and relatively simple, $D = 4$ phase is in turn analogous to a chaotic de-magnetized high temperature phase.
2. $D = 3$ phase represents spin glass phase, kind of boundary region between order and chaos possible in a finite temperature range and is an ideal candidate for the field body serving as a template for living systems. $D = 3$ phase allows infinite number of conserved topological charges having interpretation as invariants describing the linking of the magnetic field lines. This phase is also the phase in which topological quantum computations are possible.

9.4.1 Topologically Quantized Generalized Beltrami Fields And Braiding

From the construction of the solutions of field equations in terms topologically quantized fields it is obvious that TGD Universe is tailor made for TQC.

$D = 3$ phase allows infinite number of topological charges characterizing the linking of magnetic field lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

For $D = 2$ the topological charge densities vanish identically, for $D = 3$ they are in general non-vanishing and conserved, whereas for $D = 4$ they are not conserved. The transition to $D = 4$ phase can thus be used to erase quantum computer programs realized as braids. The 3-dimensional CP_2 projection provides an economical manner to represent the braided world line pattern of dancers and would be the space where the 3-dimensional quantum field theory would be defined.

The topological charge can also vanish for $D = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D = 3$ solutions. Note however that in boundaries can still remain super-conducting and it seems that this occurs in the case of anyons.

Chern-Simons action is known as helicity in electrodynamics [B31]. Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r - r')}{|r - r'|^3} \times B(r') \right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW of 3-surfaces defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges would contribute to WCW metric a part which would define a Kähler magnetic knot invariant. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

The color partial wave degeneracy of topological charges inspires the idea that also anyons could move in color partial waves identifiable in terms of "rigid body rotation" of the magnetic flux tube of anyon in CP_2 degrees of freedom. Their presence could explain non-Abelianity of Chern-Simons action and bring in new kind bits increasing the computational capacity of the topological quantum computer. The idea about the importance of macroscopic color is not new in TGD context. The fact that non-vanishing Kähler field is always accompanied by a classical color field (proportional to it) has motivated the proposal that colored excitations in macroscopic length scales are important in living matter and that colors as visual qualia correspond to increments of color quantum numbers in quantum phase transitions giving rise to visual sensations.

Knot theory, 3-manifold topology, and $D = 3$ solutions of field equations

Topological quantum field theory (TQFT) [A83] demonstrates a deep connection between links and 3-topology, and one might hope that this connection could be re-interpreted in terms of embeddings of 3-manifolds to $H = M^4_+ \times CP_2$ as surfaces having 3-dimensional CP_2 projection, call it X^3 in the sequel. $D = 3$ suggests itself because in this case Chern-Simons action density for the induced Kähler field is generically non-vanishing and defines an infinite number of classical charges identifiable as Kähler magnetic canonical covariants invariant under $Diff(M^4_+)$. The field topology of Kähler magnetic field should be in a key role in the understanding of these invariants.

1. Could 3-D CP_2 projection of 3-surface provide a representation of 3-topology?

Witten-Chern-Simons theory for a given 3-manifold defines invariants which characterize both the topology of 3-manifold and the link. Why this is the case can be understood from the construction of 3-manifolds by drilling a tubular neighborhood of a link in S^3 and by gluing the tori back to get a new 3-manifolds. The links with some moves defining link equivalences are known to be in one-one correspondence with closed 3-manifolds and the axiomatic formulation of TQFT [A83] as a modular functor clarifies this correspondence. The question is whether the CP_2 projection of the 3-surface could under some assumptions be represented by a link so that one could understand the connection between the links and topology of 3-manifolds.

In order to get some idea about what might happen consider the CP_2 projection X^3 of 3-surface. Assume that X^3 is obtained from S^3 represented as a 3-surface in CP_2 by removing from S^3 a tubular link consisting of linked and knotted solid tori $D^2 \times S^1$. Since the 3-surface is closed, it must have folds at the boundaries being thus representable as a two-valued map $S^3 \to M^4_+$ near the folds. Assume that this is the case everywhere. The two halves of the 3-surface corresponding to the two branches of the map would be glued together along the boundary of the tubular link by identification maps which are in the general case characterized by the mapping class group of 2-torus. The gluing maps are defined inside the overlapping coordinate batches containing the boundary $S^1 \times S^1$ and are maps between the pairs (Ψ_i, Φ_i), $i = 1, 2$ of the angular coordinates parameterizing the tori.

Define longitude as a representative for the $a + nb$ of the homology group of the 2-torus. The integer n defines so called framing and means that the longitude twists n times around torus. As a matter fact, TQFT requires bi-framing: at the level of Chern-Simons perturbation theory bi-framing is necessary in order to define self linking numbers. Define meridian as the generator of the homology group of the complement of solid torus in S^3. It is enough to glue the carved torus back in such a way that meridian is mapped to longitude and longitude to minus meridian. This map corresponds to the $SL(2, C)$ element

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

Also other identification maps defined by $SL(2, Z)$ matrices are possible but one can do using only this. Note that the two component $SL(2, Z)$ spinors defined as superpositions of the generators

(a, b) of the homology group of torus are candidates for the topological correlates of spinors. In the gluing process the tori become knotted and linked when seen in the coordinates of the complement of the solid tori.

This construction would represent the link surgery of 3-manifolds in terms of CP_2 projections of 3-surfaces of H. Unfortunately this representation does not seem to be the only one. One can construct closed three-manifolds also by the so called Heegaard splitting. Remove from S^3 D_g, a solid sphere with g handles having boundary S_g, and glue the resulting surface with its oppositely oriented copy along boundaries. The gluing maps are classified by the mapping class group of S_g. Any closed orientable 3-manifold can be obtained by this kind of procedure for some value of g. Also this construction could be interpreted in terms of a fold at the boundary of the CP_2 projection for a 2-valued graph $S^3 \to M^4_+$. Whether link surgery representation and Heegaard splitting could be transformed to each other by say pinching D_g to separate tori is not clear to me.

When the graph $CP_2 \to M^4_+$ is at most 2-valued, the intricacies due to the embedding of the 3-manifold are at minimum, and the link associated with the projection should give information about 3-topology and perhaps even characterize it. Also the classical topological charges associated with Kähler Chern-Simons action could give this kind of information.

2. Knotting and linking for 3-surfaces

The intricacies related to embedding become important in small co-dimensions and it is of considerable interest to find what can happen in the case of 3-surfaces. For 1-dimensional links and knots the projection to a plane, the shadow of the knot, characterizes the link/knot and allows to deduce link and knot invariants purely combinatorially by gradually removing the intersection points and writing a contribution to the link invariant determined by the orientations of intersecting strands and by which of them is above the other. Thus also the generalization of knot and link diagrams is of interest.

Linking of m- and n-dimensional sub-manifolds of D-dimensional manifold H_D occurs when the condition $m + n = D - 1$ holds true. The n-dimensional sub-manifold intersects $m + 1$-dimensional surfaces having m-dimensional manifold as its boundary at discrete points, and it is usually not possible to remove these points by deforming the surfaces without intersections in some intermediate stage. The generalization of the link diagram results as a projection $D-1$-dimensional disk D^{D-1} of H_D.

3-surfaces link in dimension $D = 7$ so that the linking of 3-surfaces occurs quite generally in time=constant section of the embedding space. A link diagram would result as a projection to $E^2 \times CP_2$, E^2 a 2-dimensional plane: putting CP_2 coordinates constant gives ordinary link diagram in E^2. For magnetic flux tubes the reduction to 2-dimensional linking by idealizing flux tubes with 1-dimensional strings makes sense.

Knotting occurs in codimension 2 that is for an n-manifold imbedded in $D = n + 2$-dimensional manifold. Knotting can be understood as follows. Knotted surface spans locally $n + 1$-dimensional 2-sided n+1-disk D^{n+1} (disk for ordinary knot). The portion of surface going through D^{n+1} can be idealized with a 1-dimensional thread going through it and by $n+2 = D$ knotting is locally linking of this 1-dimensional thread with n-dimensional manifold. N-dimensional knots define n+1-dimensional knots by so called spinning. Take an n-knot with the topology of sphere S^n such that the knotted part is above $n+1$-plane of $n+2$-dimensional space R^{n+2} $(z \geq 0)$, cut off the part below plane $(z < 0)$, introduce an additional dimension (t) and make a 2π rotation for the resulting knot in $z - t$ plane. The resulting manifold is a knotted S^{n+1}. The counterpart of the knot diagram would be a projection to $n + 1$-dimensional sub-manifold, most naturally disk D^{n+1}, of the embedding space.

3-surfaces could become knotted under some conditions. Vacuum extremals correspond to 4-surfaces $X^4 \subset M^4_+ \times Y^2$ whereas the four-surfaces $X^4 \subset M^4_+ \times S^2$, S^2 homologically non-trivial geodesic sphere, define their own "sub-theory". In both cases 3-surfaces in time=constant section of embedding space can get knotted in the sense that un-knotting requires giving up the defining condition temporarily. The counterpart of the knot diagram is the projection to $E^2 \times X^2$, $X^2 = Y^2$ or S^2, where E^2 is plane of M^4_+. For constant values of CP_2 coordinates ordinary knot diagram would result. Reduction to ordinary knot diagrams would naturally occur for $D = 2$ magnetic flux tubes. The knotting occurs also for 4-surfaces themselves in $M^4_+ \times X^2$: knot diagram is now defined as projection to $E^3 \times X^2$.

3. Could the magnetic field topology of 3-manifold be able to mimic other 3-topologies?

In $D = 3$ case the topological charges associated with Kähler Chern-Simons term characterize the linking of the field lines of the Kähler gauge potential A. What $dA \wedge A \neq 0$ means that field lines are linked and it is not possible to define a coordinate varying along the field lines of A. This is impossible even locally since the $dA \wedge A \neq 0$ condition is equivalent with non existence of a scalar functions k and Φ such that $\nabla\Phi = kA$ guaranteeing that Φ would be the sought for global coordinate.

One can idealize the situation a little bit and think of a field configuration for which magnetic flux is concentrated at one-dimensional closed lines. The vector potential would in this case be simply $A = \nabla(k\Psi + l\Phi)$, where Ψ is an angle coordinate around the singular line and Φ a coordinate along the singular circle. In this idealized situation the failure to have a global coordinate would be due to the singularities of otherwise global coordinates along one-dimensional linked and knotted circles. The reason is that the field lines of A and B rotate helically around the singular circle and the points (x, y, z) with constant values of x, y are on a helix which becomes singular at z-axis. Since the replacement of a field configuration with a non-singular field configuration but having same field line topology does not affect the global field line topology, one might hope of characterizing the field topology by its singularities along linked and knotted circles also in the general case.

Just similar linked and knotted circles are used to construct 3-manifolds in the link surgery which would suggest that the singularities of the field line topology of X^3 code the non-trivial 3-topology resulting when the singularities are removed by link surgery. Physically the longitude defining the framing $a + nb$ would correspond to the field line of A making an $n2\pi$ twist along the singular circle. Meridian would correspond to a circle in the plane of B. The bi-framing necessitated by TQFT would have a physical interpretation in terms of the helical field lines of A and B rotating around the singular circle. At the level of fields the gluing operation would mean a gauge transformation such that the meridians would become the field lines of the gauge transformed A and being non-helical could be continued to the interior of the glued torus without singularities. Simple non-helical magnetic torus would be in question.

This means that the magnetic field patterns of a given 3-manifold could mimic the topologies of other 3-manifolds. The topological mimicry of this kind would be a very robust manner to represent information and might be directly relevant to TQC. For instance, the computation of topological invariants of 3-manifold Y^3 could be coded by the field pattern of X^3 representing the link surgery producing the 3-manifold from S^3, and the physical realization of TQC program could directly utilize the singularities of this field pattern. Topological magnetized flux tubes glued to the back-ground 3-surface along the singular field lines of A could provide the braiding.

This mimicry could also induce transitions to the new topology and relate directly to 3-manifold surgery performed by a physical system. This transition would quite concretely mean gluing of simple $D = 2$ magnetic flux tubes along their boundaries to the larger $D = 3$ space-time sheet from which similar flux tube has been cut away.

4. A connection with anyons?

There is also a possible connection with anyons. Anyons are thought to correspond to singularities of gauge fields resulting in a symmetry breaking of gauge group to a finite subgroup H and are associated with homotopically non-trivial loops of $C_n = ((R^2)^n - D)/S_n$ represented as elements of H. Could the singularities of gauge fields relate to the singularities of the link surgery so that the singularities would be more or less identifiable as anyons? Could N-branched anyons be identified in terms of framings $a + Nb$ associated with the gluing map? $D = 3$ solutions allow the so called contact structure [K19], which means a decomposition of the coordinates of CP_2 projection to a longitudinal coordinate s and a complex coordinate w. Could this decomposition generalize the notion of effective 2-dimensionality crucial for the notion of anyon?

5. What about Witten's quantal link invariants?

Witten's quantal link invariants define natural multiplicative factors of WCW spinor fields identifiable as representations of two 2-dimensional topological evolution. In Witten's approach these invariants are defined as functional averages of non-integrable phase factors associated with a given link in a given 3-manifold. TGD does not allow any natural functional integral over gauge field configurations for a fixed 3-surface unless one is willing to introduce fictive non-Abelian gauge fields. Although this is not a problem as such, the representation of the invariants in terms of inherent properties of the 3-surface or corresponding 4-surfaces would be highly desirable.

Functional integral representation is not the only possibility. Quantum classical correspondence combined with topological field quantization implied by the preferred extremal property generalizing Bohr rules to the field context gives hopes that the 3-surfaces themselves might be able to represent 3-manifold invariants classically. In $D = 3$ case the quantized exponents of Kähler-Chern-Simons action and $SU(2)_L$ Chern-Simons action could define 3-manifold invariants. These invariants would satisfy the obvious multiplicativity conditions and could correspond to the phase factors due to the framing dependence of Witten's invariants identifying the loops of surgery link as Wilson loops. These phase factors are powers of $U = exp(i2\pi c/24)$, where c is the central charge of the Virasoro representation defined by Kac Moody representation. One has $c = k \times dim(g)/(k + c_g/2)$, which gives $U = exp(i2\pi k/8(k + 2))$ for $SU(2)$. The dependence on k differs from what one might naïvely expect. For this reason, and also because the classical Wilson loops do not depend explicitly on k, the value of k appearing in Chern-Simons action should be fixed by the internal consistency and be a constant of Nature according to TGD. The guess is that k possesses the minimal value $k = 3$ allowing a universal modular functor for $SU(2)$ with $q = exp(i2\pi/5)$.

The loops associated with the topological singularities of the Kähler gauge potential (typically the center lines of helical field configurations) would in turn define natural Wilson loops, and since the holonomies around these loops are also topologically quantized, they could define invariants of 3-manifolds obtained by performing surgery around these lines. The behavior of the induced gauge fields should be universal near the singularities in the sense that the holonomies associated with the CP_2 projections of the singularities to CP_2 would be universal. This expectation is encouraged by the notion of quantum criticality in general and in particular, by the interpretation of $D = 3$ phase as a critical system analogous to spin glass.

The exponent of Chern-Simons action can explain only the phase factors due to the framing, which are usually regarded as an unavoidable nuisance. This might be however all that is needed. For the manifolds of type $X^2 \times S^1$ all link invariants are either equal to unity or vanish. Surgery would allow to build 3-manifold invariants from those of $S^2 \times S^1$. For instance, surgery gives the invariant $Z(S^3)$ in terms of $Z(S^2 \times S^1, R_i)$ and mapping class group action coded into the linking of the field lines.

Holonomies can be also seen as multi-valued $SU(2)_L$ gauge transformations and can be mapped to a multi-valued transformations in the $SU(2)$ subgroup of $SU(3)$ acting on 3-surface as a geometric transformations and making it multi-branched. This makes sense if the holonomies define a finite group so that the gauge transformation is finitely many-valued. This description might apply to the 3-manifold resulting in a surgery defined by the Wilson loops identifiable as branched covering of the initial manifold.

The construction makes also sense for the holonomies defined by the classical $SU(3)$ gauge fields defined by the projections of the isometry currents. Furthermore, the fact that any CP_2 Hamiltonian defines a conserved topological charge in $D = 3$ phase should have a deep significance. At the level of WCW geometry the finite-dimensional group defining Kac Moody algebra is replaced with the group of canonical transformations of CP_2. Perhaps one could extend the notion of Wilson loop for the algebra of canonical transformations of CP_2 so that the representations R_i of the gauge group would be replaced by matrix representations of the canonical algebra. That the trace of the identity matrix is infinite in this case need not be a problem since one can simply redefine the trace to have value one.

Braids as topologically quantized magnetic fields

$D = 3$ space-time sheets would define complex braiding structures with flux tubes possessing infinite number of topological charges characterizing the linking of field lines. The world lines of the quantum computing dancers could thus correspond to the flux tubes that can get knotted, linked, and braided. This idea conforms with the earlier idea that the various knotted and linked structures formed by linear bio-molecules define some kind of computer programs.

1. *Boundaries of magnetic flux tubes as light-like 3-surfaces*

Field equations for Kähler action are satisfied identically at boundaries if the boundaries of magnetic flux tubes (and space-time sheet in general) are light-like in the induced metric. In M^4_+ metric the flux tubes could look static structures. Light-likeness allows an interpretation of the boundary state either as a 3-dimensional quantum state or as a time-evolution of a 2-dimensional quantum state. This conforms with the idea that quantum computation is cognitive, self reflective process so that quantum state is about something rather than something. There would be no need to force particles to flow through the braid structure to build up time-like braid whereas for time-like boundaries of magnetic flux tubes a time-like braid results only if the topologically charged particles flow through the flux tubes with the same average velocity so that the length along flux tubes is mapped to time.

Using the terminology of consciousness theory, one could say that during quantum dance the dancers are in trance being entangled to a single macro-temporally coherent state which represents single collective consciousness, and wake up to individual dancers when the dance ends. Quantum classical correspondence suggests that the generation of bound state entanglement between dancers requires tangled flux tubes connecting the space-time sheets of anyons (braid of flux tubes again!): dancers share mental images whereas direct contact between magnetic flux tubes defining the braid is not necessary. The bound state entanglement between sub-systems of unentangled systems is made possible by the many-sheeted space-time. This kind of entanglement could be interpreted as entanglement not visible in scales of larger flux tubes so that the notion is natural in the philosophy based on the idea of length scale resolution.

2. How braids are generated?

The encoding of the program to a braid could be a mechanical process: a bundle of magnetic flux tubes with one end fixed would be gradually weaved to a braid by stretching and performing the needed elementary twists. The time to perform the braiding mechanically requires classical computer program and the time needed to carry out the braiding depends polynomially on the number of strands.

The process could also occur by a quantum jump generating the braided flux tubes in single flash and perhaps even intentionally in living systems (flux tubes with negative topological charge could have negative energy so that it would require no energy to generate the structure from vacuum). The interaction with environment could be used to select the desired braids. Also ensembles of braids might be imagined. Living matter might have discovered this mechanism and used int intentionally.

3. Topological quantization, many-sheetedness, and localization

Localization of modular functors is one of the key problems in topological quantum computation (see the article of Freedman [B42]. For anyonic computation this would mean in the ideal case a decomposition of the system into batches containing 4 anyons each so that these anyon groups interact only during swap operations.

The role of topological quantization would be to select of a portion of the magnetic field defining the braid as a macroscopic structure. Topological field quantization realizes elegantly the requirement that single particle time evolutions between swaps involve no interaction with other anyons.

Also many-sheetedness is important. The (AA) pair and two anyons would correspond braids inside braids and as it turns out this gives more flexibility in construction of quantum computation since the 1-gates associated with logical qubits of 4-batch can belong to different representation of braid group than that associated with braiding of the batches.

9.4.2 Quantum Hall Effect And Fractional Charges In TGD

In fractional QH effect anyons possess fractional electromagnetic charges. Also fractional spin is possible. TGD explains fractional charges as being due to multi-branched character of space-time sheets. Also the Z_n-valued topological charge associated with anyons has natural explanation.

Basic TGD inspired ideas about quantum Hall effect

Quantum Hall effect is observed in low temperature systems when the intensity of a strong magnetic field perpendicular to the current carrying slab is varied adiabatically. Classically quantum Hall effect can be understood as a generation of a transversal electric field, which exactly cancels the magnetic Lorentz force. This gives $E = -j \times B/ne$. The resulting current can be also understood as due to a drift velocity proportional to $E \times B$ generated in electric and magnetic fields orthogonal to each other and allowing to cancel Lorentz force. This picture leads to the classical expression for transversal Hall conductivity as $\sigma_{xy} = ne/B$. σ_{xy} should vary continuously as a function of the magnetic field and 2-dimensional electron density n.

In quantum Hall effect σ_{xy} is piece-wise constant and quantized with relative precision of about 10^{-10}. The second remarkable feature is that the longitudinal conductivity σ_{xx} is very high at plateaus: variations by 13 orders of magnitude are observed. The system is also very sensitive to small perturbations.

Consider now what these qualitative observations might mean in TGD context.

1. Sensitivity to small perturbations means criticality. TGD Universe is quantum critical and quantum criticality reduces to the spin glass degeneracy due to the enormous vacuum degeneracy of the theory. The $D = 2$ and $D = 3$ non-vacuum phases predicted by the generalized Beltrami ansatz are this in-stability might play important role in the effect.

2. The magnetic fields are genuinely classical fields in TGD framework, and for $D = 2$ proportional to induced Kähler magnetic field. The canonical symmetries of CP_2 act like $U(1)$ gauge transformations on the induced gauge field but are not gauge symmetries since canonical transformations change the shape of 3-surface and affect both classical gravitational fields and electro-weak and color gauge fields. Hence different gauges for classical Kähler field represent magnetic fields for which topological field quanta can have widely differing and physically non-equivalent shapes. For instance, tube like quanta act effectively as insulators whereas magnetic walls parallel to the slab act as conducting wires.

   Wall like flux tubes parallel to the slab perhaps formed by a partial fusion of magnetic flux tubes along their boundaries would give rise to high longitudinal conductivity. For disjoint flux tubes the motion would be around the flux tubes and the electrons would get stuck inside these tubes. By quantum criticality and by $D < 4$ property the magnetic flux tube structures are unstable against perturbations, in particular the variation of the magnetic field strength itself. The transitions from a plateau to a new one would correspond to the decay of the magnetic walls back to disjoint flux tubes followed by a generation of walls again so that conductivity is very high outside transition regions. The variation of any parameter, such as temperature, is expected to be able to cause similar effects implying dramatic changes in Hall conductivity.

   The percolation model for the quantum Hall effect represents slab as a landscape with mountains and valleys and the varied external parameter, say B or free electron density, as the sea level. For the critical values of sea level narrow regions carrying so called edge states allow liquid to fill large regions appear and implies increase of conductivity. Obviously percolation model differs from the model based on criticality for which the landscape itself is highly fragile and a small perturbation can develop new valleys and mountains.

3. The effective 2-dimensionality implies that the solutions of Schrödinger equation of electron in external magnetic field are products of any analytic function with a Gaussian representing the ground state of a harmonic oscillator. Analyticity means that the kinetic energy is completely degenerate for these solutions. The Lauhglin ansatz for the state functions of electron in the external magnetic field is many-electron generalization of these solutions: the wave functions consists of products of terms of form $(z_i - z_j)^m$, m odd integer from Fermi statistics.

   The N-particle variant of Laughlin's ansatz allows to deduce that the system is incompressible. The key observation is that the probability density for the many-particle state has an interpretation as a Boltzmann factor for a fictive two-dimensional plasma in electric field created by constant charge density [D26, D21]. The probability density is extremely sensitive to the changes of the positions of electrons giving rise to the constant electron density. The screening of charge in this fictive plasma implies the filling fraction $\nu = 1/m$, m odd integer and requires charge fractionization $e \to e/m$. The explanation of the filling fractions $\nu = N/m$

would require multi-valued wave functions $(z_i - z_j)^{N/m}$. In single-sheeted space-time this leads to problems. TGD suggests that these wave functions are single valued but defined on N-branched surface.

The degeneracy with respect to kinetic energy brings in mind the spin glass degeneracy induced by the vacuum degeneracy of the Kähler action. The Dirac equation for the induced spinors is not ordinary Dirac equation but super-symmetrically related to the field equations associated with Kähler action. Also it allows vacuum degeneracy. One cannot exclude the possibility that also this aspect is involved at deeper level.

4. The fractionization of charge in quantum Hall effect challenges the idea that charged particles of the incompressible liquid are electrons and this leads to the notion of anyon. Quantum-classical correspondence inspires the idea that although dissipation is absent, it has left its signature as a track associated with electron. This track is magnetic flux tube surrounding the classical orbit of electron and electron is confined inside it. This reduces the dissipative effects and explains the increase of conductivity. The rule that there is single electron state per magnetic flux quantum follows if Bohr quantization is applied to the radii of the orbits. The fractional charge of anyon would result from a contribution of classical Kähler charge of anyon flux tube to the charge of the anyon. This charge is topologized in $D = 3$ phase.

Anyons as multi-branched flux tubes representing charged particle plus its track

Electrons (in fact, any charged particles) moving inside magnetic flux tubes move along circular paths classically. The solutions of the field equations with vanishing Lorentz 4-force correspond to asymptotic patterns for which dissipation has already done its job and is absent. Dissipation has however definite effects on the final state of the system, and one can argue that the periodic motion of the charged particle has created what might called its "track". The track would be realized as a circular or helical flux tube rotating around field lines of the magnetic field. The corresponding cyclotron states would be localized inside tracks. Simplest tracks are circular ones and correspond to absence of motion in the direction of the magnetic field. Anyons could be identified as systems formed as particles plus the tracks containing them.

1. Many-branched tracks and approach to chaos

When the system approaches chaos one expects the periodic circular tracks become non-periodic. One however expects that this process occurs in steps so that the tracks are periodic in the sense that they close after N 2π rotations with the value of N increasing gradually. The requirement that Kähler energy stays finite suggests also this. A basic example of this kind of track is obtained when the phase angles Ψ and Φ of complex CP_2 coordinates (ξ^1, ξ^2) have finitely multi-valued dependence on the coordinate ϕ of cylindrical coordinates: $(\Psi, \Phi) = (m_1/N, m_2/N)\phi)$. The space-sheet would be many-branched and it would take N turns of 2π to get back to the point were one started. The phase factors behave as a phase of a spinning particle having effective fractional spin $1/N$. I have proposed this kind of mechanism as an explanation of so called hydrino atoms claimed to have the spectrum of hydrogen atom but with energies scaled up by N^2 [K118] , [D9]. The first guess that N corresponds to m in $\nu = 1/m$ is wrong. Rather, N corresponds to N in $\nu = N/m$ which means many-valued Laughlin wave functions in single branched space-time.

Similar argument applies also in CP_2 degrees of freedom. Only the N-multiples of 2π rotations by CP_2 isometries corresponding to color hyper charge and color iso-spin would affect trivially the point of multi-branched surface. Since the contribution of Kähler charge to electromagnetic charge corresponds also to anomalous hyper-charge of spinor field in question, an additional geometric contribution to the anomalous hypercharge would mean anomalous electromagnetic charge.

It must be emphasized the fractionization of the isometry charges is only effective and results from the interpretation of isometries as space-time transformations rather than transformation rotating entire space-time sheet in embedding space. Also classical charges are effectively fractionized in the sense that single branch gives in a symmetric situation a fraction of $1/n$ of the entire charge. Later it will be found that also a genuine fractionization occurs and is due to the classical topologized Kähler charge of the anyon track.

2. Modelling anyons in terms of gauge group and isometry group

Anyons can be modelled in terms of the gauge symmetry breaking $SU(2)_L \to H$, where H is discrete sub-group. The breaking of gauge symmetry results by the action of multi-valued gauge transformation $g(x)$ such that different branches of the multi-valued map are related by the action of H.

1. The standard description of anyons is based on spontaneous symmetry breaking of a gauge symmetry G to a discrete sub-group H dynamically [A72]. The gauge field has suffered multi-valued gauge transformation such that the elements of H permute the different branches of $g(x)$. The puncture is characterized by the element of the H associated with the loop surrounding puncture. In the idealized situation that gauge field vanishes, the parallel translation of a particle around puncture affects the particle state, itself a representation of G, by the element of the homotopy $\pi_1(G/H) = H$ identifiable as non-Abelian magnetic charge. Thus holonomy group corresponds to homotopy group of G/H which in turn equals to H. This in turn implies that the infinite-dimensional braid group whose elements define holonomies in turn is represented in H.
2. In TGD framework the multi-valuedness of $g(x)$ corresponds to a many-branched character of 4-surface. This in turn induces a branching of both magnetic flux tube and anyon tracks describable in terms of $H \subset SU(2)_L$ acting as an isotropy group for the boundaries of the magnetic flux tubes. H can correspond only to a non-Abelian subgroup $SU(2)_L$ of the electro-weak gauge group for the induced (classical) electro-weak gauge fields since the Chern-Simons action associated with the classical color gauge fields vanishes identically. The electro-weak holonomy group would reduce to a discrete group H around loops defined by anyonic flux tubes surrounding magnetic field lines inside the magnetic flux tubes containing anyons. The reduction to H need to occur only at the boundaries of the space-time sheet where conducting anyons would reside: boundaries indeed correspond to asymptotia in well-defined sense. Electro-weak symmetry group can be regarded as a sub-group of color group of isometries in a well-defined sense so that H can be regarded also as a subgroup of color group acting as isotropies of the multi-branched surface at least in the in regions where gauge field vanishes.
3. For branched surfaces the points obtained by moving around the puncture correspond in a good approximation to some elements of $h \in H$ leading to a new branch but the 2-surface as a whole however remains invariant. The braid group of the punctured 2-surface would be also now represented as transformations of H. The simplest situation is obtained when H is a cyclic group Z_N of the $U(1)$ group of CP_2 geodesic in such a way that 2π rotation around symmetry axis corresponds to the generating element $exp(i2\pi/N)$ of Z_N.
   Dihedral group D_n having order $2n$ and acting as symmetries of n-polygon of the plane is especially interesting candidate for H. For $n = 2$ the group is Abelian group $Z_2 \times Z_2$ whereas for $n > 2$ D_n is a non-Abelian sub-group of the permutation group S_n. The cyclic group Z_4 crucial for TQC is a sub-group of D_4 acting as symmetries of square. D_4 has a 2-dimensional faithful representation. The numbers of elements for the conjugacy classes are 1, 1, 2, 2, 2. The sub-group commuting with a fixed element of a conjugacy class is D_4 for the 1-element conjugacy classes and cyclic group Z_4 for 2-element conjugacy classes. Hence 2-valued magnetic flux would be accompanied by Z_4 valued "electric charge" identifiable as a cyclic group permuting the branches.

3. Can one understand the increase in conductivity and filling fractions at plateaus?

Quantum Hall effect involves the increase of longitudinal conductivity by a factor of order 10^{13} [D26]. The reduction of dissipation could be understood as being caused by the fact that anyonic electrons are closed inside the magnetic flux tubes representing their tracks so that their interactions with matter and thus also dissipation are reduced.

Laughlin's theory [D26, D21] gives almost universal description of many aspects of quantum Hall effect and the question arises whether Laughlin's wave functions are defined on possibly multi-branched space-time sheet X^4 or at projection of X^4 to M^4_+. Since most theoreticians that I know still live in single sheeted space-time, one can start with the most conservative assumption that they are defined at the projection to M^4_+. The wave functions of one-electron state giving rise filling fraction $\nu = 1/m$ are constructed of $(z_i - z_j)^m$, where m is odd by Fermi statistics.

Also rational filling fractions of form $\nu = 1/m = N/n$ have been observed. These could relate to the presence of states whose projections to M^4 are multi-valued and which thus do not have any "classical" counterpart. For N-branched surface the single-valued wave functions $(\xi_i - \xi_j)^n$, n odd by Fermi statistics, correspond to apparently multi-valued wave functions $(z_i - z_j)^{n/N}$ at M^4 projection with fractional relative angular momenta $m = n/N$. The filling fraction would be $\nu = N/n$, n odd. All filling fractions reported in [D26] have n odd with n varying in the range $1-7$. N has the values $1,2,3,4,5,7,9$. Also values $N = 12,13$ for which $n = 5$ are reported [B25].

The filling fractions $\nu = N/n = 5/2, 3/8, 3/10$ reported in [D28] would require even values of n conflicting with Fermi statistics. Obviously Lauhglin's model fails in this case and the question is whether one these fractions could correspond to bosonic anyons, perhaps Cooper pairs of electrons inside track flux tubes. The Z_N valued charge associated with N-branched surfaces indeed allows the maximum $2N$ electrons per anyon. Bosonic anyons are indeed the building block of the TQC model of [B25]. The anyon Cooper pairs could be this kind of states and their BE condensation would make possible genuine super-conductivity rather than only exceptionally high value of conductivity.

One can imagine also more complex multi-electron wave functions than those of Laughlin. The so called conformal blocks representing correlation functions of conformal quantum field theories are natural candidates for the wave functions [D19] and they appear naturally as state functions of in topological quantum field theories. For instance, wave functions which are products of factors $(z^k - z^l)^2$ with the Pfaffian $Pf(A_{kl})$ of the matrix $A_{kl} = 1/(z_k - z_l)$ guaranteeing anti-symmetrization have been used to explain even values of m [D19].

4. N-branched space-time surfaces make possible Z_N valued topological charge

According to [D19] that $2n$ non-Abelian anyon pairs with charge $1/4$ created from vacuum gives rise to a 2^{n-1}-fold degenerate ground state. It is also argued that filling fraction $5/2$ could correspond to this charge [B25]. TGD suggests somewhat different interpretation. 4-fold branching implies automatically the Z_4-valued topological charge crucial for anyonic quantum computation. For 4-branched space-time surface the contribution of a single branch to electron's charge is indeed $1/4$ units but this has nothing to do with the actual charge fractionization. The value of ν is of form $\nu = /m$ and electromagnetic charge equals to $\nu = 4e/m$ in this kind of situation.

If anyons (electron plus flux tube representing its track) have Z_4 charges 1 and 3, their Cooper pairs have charges 0 and 2. The double-fold degeneracy for anyon's topological charge means that it possesses topological spin conserved modulo 4. In presence of $2n$ anyon pairs one would expect 2^n-fold degeneracy. The requirement that the net topological charge vanishes modulo 4 however fixes the topological charge of n: th pair so that 2^{n-1} fold degeneracy results.

A possible interpretation for Z_N-valued topological charge is as fractional angular momenta k/N associated with the phases $exp(ik2\pi/N)$, $k = 0,1,....,n-1$ of particles in multi-branched surfaces. The projections of these wave functions to single-branched space-time would be many-valued. If electro-weak gauge group breaks down to a discrete subgroup H for magnetic flux tubes carrying anyonic "tracks", this symmetry breakdown could induce their multi-branched property in the sense rotation by 2π would correspond to H isometry leading to a different branch.

Topologization of Kähler charge as an explanation for charge fractionization

The argument based on what happens when one adds one anyon to the anyon system by utilizing Faraday's law [D26] leads to the conclusion that anyon charge is fractional and given by νe. The anyonic flux tube along boundary of the flux tube corresponds to the left hand side in the Faraday's equation

$$\oint E \cdot dl = -\frac{d\Phi}{dt} \ .$$

By expressing E in turns of current using transversal conductivity and integrating with respect to time, one obtains

$$Q = \nu e$$

for the charge associated with a single anyon. Hence the addition of the anyon means an addition of a fractional charge νe to the system. This argument should survive as such the 1-branched situation so that at least in this case the fractional charges should be real.

In N-branched case the closed loop $\oint E \cdot dl$ around magnetic flux tube corresponds to N-branched anyon and surrounds the magnetic flux tube N times. This would suggest so that net magnetic flux should be N times the one associated with single but unclosed 2π rotation. Hence the formula would seem to hold true as such also now for the total charge of the anyon and the conclusion is that charge fractionization is real and cannot be an effective effect due to fractionization of charge at single branch of anyon flux tube.

One of the basic differences between TGD and Maxwell's theory is the possibility of vacuum charges and this provides an explanation of the effect is in terms of vacuum Kähler charge. Kähler charge contributes $e/2$ to the charge of electron. Anyon flux tube can generate vacuum Kähler charge changing the net charge of the anyon. If the anyon charge equals to νe the conclusions are following.

1. The vacuum Kähler charge of the anyon track is $q = (\nu - 1)e$.
2. The dimension of the CP_2 projection of the anyon flux tube must be $D = 3$ since only in this case the topologization of anyon charge becomes possible so that the charge density is proportional to the Chern-Simons term $A \wedge dA/4\pi$. Anyon flux tubes cannot be super-conducting in the sense that non-integrable phase factor $exp(\int A \cdot dl)$ would define global order parameter. The boundaries of anyonic flux tubes could however remain potentially super-conducting and anyon Cooper pairs would be expelled there by Meissner effect. This gives super-conductivity in length scale of single flux tube. Conductivity and super-conductivity in long length scales requires that magnetic flux tubes are glued together along their boundaries partially.
3. By Bohr quantization anyon tracks can have $r_n = \sqrt{n} \times r_B$, $n \leq m$, where r_m corresponds to the radius of the magnetic flux tube carrying m flux quanta. Only the tracks with radius r_m contribute to boundary conductivity and super-conductivity giving $\nu = 1/m$ for singly branched surfaces.

   The states with $\nu = N/m$ cannot correspond to non-super-conducting anyonic tracks with radii r_n, $n < m$, n odd, since these cannot contribute to boundary conductivity. The many-branched character however allows an N-fold degeneracy corresponding to the fractional angular momentum states $exp(ik\phi/N)$, $k = 0, ..., N-1$ of electron inside anyon flux tubes of radius r_m. k is obviously a an excellent candidate for the Z_N-valued topological charge crucial for anyonic quantum computation. Z_4 is uniquely selected by the braid matrix R.

   Only part of the anyonic Fermi sea need to be filled so that filling fractions $\nu = k/m$, $k = 1, ..., N$ are possible. Charges νe are possible if each electron inside anyon track contributes $1/m$ units to the fractional vacuum Kähler charge. This is achieved if the radius of the anyonic flux tube grows as $\sqrt{k/m}$ when electrons are added. The anyon tracks containing several electrons give rise to composite fermions with fermion number up to $2N$ if both directions of electron spin are allowed.
4. Charge fractionization requires vacuum Kähler charge has rational values $Q_K = (\nu - 1)e$. The quantization indeed occurs for the helicity defined by Chern-Simons term $A \wedge dA/4\pi$. For compact 3-spaces without boundary the helicity can be interpreted as an integer valued invariant characterizing the linking of two disjoint closed curves defined by the magnetic field lines. This topological charge can be also related to the asymptotic Hopf invariant proposed by Arnold [B50], which in non-compact case has a continuum of values. Vacuum Kähler current is obtained from the topological current $A \wedge dA/4\pi$ by multiplying it with a function of CP_2 coordinates completely fixed by the field equations. There are thus reasons to expect that vacuum Kähler charge and also the topological charges obtained by multiplying Chern Simons current by $SU(3)$ Hamiltonians are quantized for compact 3-surfaces but that the presence of boundaries replaces integers by rationals.

What happens in quantum Hall system when the strength of the external magnetic field is increased?

The proposed mechanism of anyonic conductivity allows to understand what occurs in quantum Hall system when the intensity of the magnetic field is gradually increased.

1. Percolation picture encourages to think that magnetic flux tubes fuse partially along their boundaries in a transition to anyon conductivity so that the anyonic states localized at the boundaries of flux tubes become de-localized much like electrons in metals. Laughlin's states provide an idealized description for these states. Also anyons, whose tracks have Bohr radii r_m smaller than the radius r_B of the magnetic flux tube could be present but they would not participate in this localization. Clearly, the anyons at the boundaries of magnetic flux tubes are highly analogous to valence electrons in atomic physics.
2. As the intensity of the magnetic field B increases, the areas a of the flux tubes decreases as $a \propto 1/B$: this means that the existing contacts between neighboring flux tubes tend to be destroyed so that anyon conductivity is reduced. On the other hand, new magnetic flux tubes must emerge by the constancy of the average magnetic flux implying $dn/da \propto B$ for the average density of flux tubes. This increases the probability that the newly generated flux tubes can partially fuse with the existing flux tubes.
3. If the flux tubes are not completely free to move and change their shape by area preserving transformations, one can imagine that for certain value ranges of B the generation of new magnetic flux tubes is not favored since there is simply no room for the newcomers. The Fermi statistics of the anyonic electrons at the boundaries of flux tubes might relate to this non-hospitable behavior. At certain critical values of the magnetic field the sizes of flux tubes become however so small that the situation changes and the new flux tubes penetrate the system and via the partial fusion with the existing flux tubes increase dramatically the conductivity.

Also protonic anyons are possible

According to the TGD based model, any charged particle can form anyons and the strength of the magnetic field does not seem to be crucial for the occurrence of the effect and it could occur even in the Earth's magnetic field. The change of the cyclotron and Larmor frequencies of the charged particle in an external magnetic field to a value corresponding to the fractional charge provides a clear experimental signature for both the presence of anyons and for their the fractional charge.

Interestingly, water displays a strange scaling of proton's cyclotron frequency in an external magnetic field [D23] , [J22]. In an alternating magnetic field of.1551 Gauss (Eearth's field has a nominal value of.58 Gauss) a strong absorption at frequency $f = 156$ Hz was observed. The frequency was halved when D_2O was used and varied linearly with the field strength. The resonance frequency however deviated from proton's Larmor frequency, which suggests that a protonic anyon is in question. The Larmor frequency would be in this case $f_L = r \times \nu eB/2m_p$, where $r = \mu_p/\mu_B = 2.2792743$ is the ratio of proton's actual magnetic moment to its value for a point like proton. The experimental data gives $\nu = .6003 = 3/5$ with the accuracy of 5×10^{-4} so that 3-branched protonic anyons with $m = 5$ would be responsible for the effect.

If this interpretation is correct, entire p-adic hierarchy of anyonic NMR spectroscopies associated with various atomic nuclei would become possible. Bosonic anyon atoms and Cooper pairs of fermionic anyon atom could also form macroscopic quantum phases making possible super-conductivity very sensitive to the value of the average magnetic field and bio-systems and brain could utilize this feature.

9.4.3 Does The Quantization Of Planck Constant Transform Integer Quantum Hall Effect To Fractional Quantum Hall Effect?

The model for topological quantum computation inspired the idea that Planck constant might be dynamical and quantized. The work of Nottale [E6] gave a strong boost to concrete development of the idea and it took year and half to end up with a proposal about how basic quantum TGD could allow quantization Planck constant associated with M^4 and CP_2 degrees of freedom such that the scaling factor of the metric in M^4 degrees of freedom corresponds to the scaling of $\hbar$ in CP_2 degrees of freedom and vice versa [K47]. The dynamical character of the scaling factors of M^4 and CP_2 metrics makes sense if space-time and embedding space, and in fact the entire quantum TGD, emerge from a local version of an infinite-dimensional Clifford algebra existing only in dimension $D = 8$ [K127].

The predicted scaling factors of Planck constant correspond to the integers n defining the quantum phases $q = exp(i\pi/n)$ characterizing Jones inclusions. A more precise characterization of Jones inclusion is in terms of group $G_b \subset SU(2) \subset SU(3)$ in CP_2 degrees of freedom and $G_a \subset SL(2,C)$ in M^4 degrees of freedom. In quantum group phase space-time surfaces have exact symmetry such that to a given point of M^4 corresponds an entire G_b orbit of CP_2 points and vice versa. Thus space-time sheet becomes $N(G_a)$ fold covering of CP_2 and $N(G_b)$-fold covering of M^4. This allows an elegant topological interpretation for the fractionization of quantum numbers. The integer n corresponds to the order of maximal cyclic subgroup of G.

In the scaling $\hbar_0 \rightarrow n\hbar_0$ of M^4 Planck constant fine structure constant would scale as

$$\alpha = \frac{e^2}{4\pi\hbar c} \rightarrow \frac{\alpha}{n} \ ,$$

and the formula for Hall conductance would transform to

$$\sigma_H \rightarrow \frac{\nu}{n}\alpha \ .$$

Fractional quantum Hall effect would be integer quantum Hall effect but with scaled down α. The apparent fractional filling fraction $\nu = m/n$ would directly code the quantum phase $q = exp(i\pi/n)$ in the case that m obtains all possible values. A complete classification for possible phase transitions yielding fractional quantum Hall effect in terms of finite subgroups $G \subset SU(2) \subset SU(3)$ given by ADE diagrams would emerge (A_n, D_{2n}, E_6 and E_8 are possible). What would be also nice that CP_2 would make itself directly manifest at the level of condensed matter physics.

9.4.4 Why 2+1-Dimensional Conformally Invariant Witten-Chern-Simons Theory Should Work For Anyons?

Wess-Zumino-Witten theories are 2-dimensional conformally invariant quantum field theories with dynamical variables in some group G. The action contains the usual 2-dimensional kinetic term for group variables allowing conformal group action as a dynamical symmetry plus winding number defined associated with the mapping of 3-surface to G which is $Diff^4$ invariant. The coefficient of this term is quantized to integer.

If one couples this theory to a gauge potential, the original chiral field can be transformed away and only a Chern-Simons term defined for the 3-manifold having the 2-dimensional space as boundary remains. Also the coefficient k of Chern-Simons term is quantized to integer. Chern-Simons-Witten action has close connection with Wess-Zumino-Witten theory. In particular, the states of the topological quantum field theory are in one-one correspondence with highest weights of the WZW action.

The appearance of 2+1-dimensional $Diff^3$ invariant action can be understood from the fundamentals of TGD.

1. Light-like 3-surfaces of both future light-cone M^4_+ and of space-time surface X^4 itself are in a key role in the construction of quantum TGD since they define causal determinants for Kähler action.

2. At the space-time level both the boundaries of X^4 and elementary particle horizons surrounding the orbits of wormhole contacts define light-like 3-surfaces. The field equations are satisfied identically at light-like boundaries. Of course, the projections of the the light-like surfaces of X^4 to Minkowski space need not look light-like at all, and even boundaries of magnetic flux tubes could be light-like.

   Light-like 3-surfaces are metrically 2-dimensional and allow a generalized conformal invariance crucial for the construction of quantum TGD. At the level of embedding space conformal super-symplectic invariance results. At the space-time level the outcome is conformal invariance highly analogous to the Kac Moody symmetry of super string models [K35, K109]. In fact, there are good reasons to believe that the three-dimensional Chern-Simons action appears even in the construction of configuration space metric and give an additional contribution to the configuration space metric when the light-like boundaries of 3-surface have 3-dimensional CP_2 projection.

3. By the effective two-dimensionality the Wess-Zumino-Witten action containing Chern-Simons term is an excellent candidate for the quantum description of S-matrix associated with the light-like 3-surfaces since by the vanishing of the metric determinant one cannot define any general coordinate invariant 3-dimensional action other than Chern-Simons action. The boundaries of the braid formed by the magnetic flux tubes having light-like boundaries, perhaps having flux tubes between swapped flux tubes would define the 2+1-dimensional space-time associated with a braid, would define the arena of Witten-Chern-Simons theory describing anyons. This S-matrix can be interpreted also as characterizing either a 3-dimensional quantum state since light-like boundaries are limiting cases of space-like 3-surfaces.

4. Kähler action defines an Abelian Chern-Simons term and the induced electroweak gauge fields define a non-Abelian variant of this term. The Chern-Simons action associated with the classical color degrees of freedom vanishes as is easy to find. The classical color fields are identified as projections of Killing vector fields of color group: $A^c_\alpha = j^A_k \partial_\alpha s^k \tau_A = J_k{}^r \partial_r H^A \partial_\alpha s^k$. The classical color gauge field is proportional to the induced Kähler form: $F^c_{\alpha\beta} = H^A J_{\alpha\beta} \tau_A$. A little calculation shows that the instanton density vanishes by the identity $H_A H^A = 1$ (this identity is forced by the necessary color-singletness of the YM action density and is easy to check in the simpler case of S^2.

5. Since qubit realizes the fundamental representation of the quantum group $SU(2)_q$, $SU(2)$ is in a unique role concerning the construction of modular functors and quantum computation using Chern-Simons action. The quantum group corresponding to $q = exp(i2\pi/r)$, $r = 5$ is realized for the level $k = 3$ Chern-Simons action and satisfies the constraint $r = k + c_g$, where $c_g = 2$ is the so called dual Coxeter number of $SU(2)$ [B25, B29, B41].

   The exponent non-Abelian $SU(2)_L \times U(1)$ Chern-Simons action combined with the corresponding action for Kähler form so that effective reduction to $SU(2)_L$ occurs, could appear as a multiplicative factor of the WCW spinor fields defined in the WCW . Since 3-dimensional quantum state would represent a 2-dimensional time evolution the role of these phase factor would be very analogous to the role of ordinary Chern-Simons action.

9.5 Topological Quantum Computation In TGD Universe

The general philosophy behind TQC inspires the dream that the existence of basic gates, in particular the maximally entangling 2-gate R, is guaranteed by the laws of Nature so that no fine tuning would be needed to build the gates. Negentropy Maximization Principle, originally developed in context of TGD inspired theory of consciousness, is a natural candidate for this kind of Law of Nature.

9.5.1 Concrete Realization Of Quantum Gates

The bold dream is that besides 2-gates also 1-gates are realized by the basic laws of Nature. The topological realization of the 3-braid representation in terms of Temperley-Lie algebra allows the reduction of 1-gates to 2-gates.

NMP and TQC

Quantum jump involves a cascade of self measurements in which the system under consideration can be though of as decomposing to two parts which are either un-entangled or possess rational or extended rational entanglement in the final state. The sub-system is selected by the requirement that entanglement negentropy gain is maximal in the measurement of the density matrix characterizing the entanglement of the sub-system with its complement.

In the case case that the density matrix before the self measurement decomposes into a direct sum of matrices of dimensions N_i, such that $N_i > 1$ holds true for some values of i, say i_0, the final state is a rationally entangled and thus a bound state. i_0 is fixed by the requirement that the number theoretic entropy for the final state maximally negative and equals to $klog(p)$, where p^k is the largest power of prime dividing N_{i_0}. This means that maximally entangled state results and the density matrix is proportional to a unit matrix as it is also for the entanglement produced by R. In case of R the density matrix is $1/2$ times 2-dimensional unit matrix so that bound state entanglement negentropy is 1 bit.

The question is what occurs if the density matrix contains a part for which entanglement probabilities are extended rational but not identical. In this case the entanglement negentropy is positive and one could argue that no self-measurement occurs for this state and it remains entangled. If so then the measurement of the density matrix would occur only when it increases entanglement negentropy. This looks the only sensible option since otherwise only bound state entanglement with identical entanglement probabilities would be possible. This question is relevant also because Temperley-Lieb representation using $(AA)-A-A$ system involves entanglement with entanglement probabilities which are not identical.

In the case that the 2-gate itself is not directly entangling as in case of R' and R'', NMP should select just the quantum history, that single particle gates at it guarantee maximum entanglement negentropy. Thus NMP would come in rescue and give hopes that various gates are realized by Nature.

Non-Abelian anyon systems are modelled in terms of punctures of plane and Chern-Simons action for the incompressible vector potential of hydrodynamical flow. It is interesting to find how these ideas relate to the TGD description.

Non-Abelian anyons reside at boundaries of magnetic flux tubes in TGD

In [B25] anyons are modelled in terms of punctures of plane defined by the slab carrying Hall current. In TGD the punctures correspond naturally to magnetic flux tubes defining the braid. It is now however obvious under what conditions the braid containing the TGD counterpart of (AA)-A-A system can be described as a punctured disk if the flux tubes describing the tracks of valence anyons are very near to the boundaries of the magnetic flux tubes. Rather, the punctured disk is replaced with the closed boundary of the magnetic flux tube or of the structure formed by the partial fusion of several magnetic flux tubes. This microscopic description and is consistent with Laughlin's model only if it is understood as a long length scale description.

Non-Abelian charges require singularities and punctures but a two-surface which is boundary does not allow punctures. The punctures assigned with an anyon pair would become narrow wormhole threads traversing through the interior of the magnetic flux tube and connecting the punctures like wormholes connect two points of an apple. It is also possible that the threads connect the surfaces of two nearby magnetic flux tubes. The wormhole like character conforms with the fact that non-Abelian anyons appear always in pairs.

The case in which which the ends of the wormhole thread belong to different neighboring magnetic flux tubes, call them T_1 and T_2, is especially interesting as far as the model for TQC is considered. The state of $(AA) - A - A$ system before (after) the 3-braid operation would be identifiable as anyons near the surface of T_1 (T_2). If only sufficiently local operations are allowed, the braid group would be same as for anyons inside disk. This means consistency with the anyon model of [B25] for TQC requiring that the dimension for the space of ground states is 2^{n-1} in a system consisting of n anyon pairs.

The possibility of negative energies allows inspires the idea that the anyons at T_2 have negative energies so that the anyon system would have a vanishing net energy. This would conform with the idea that the scattering from initial to final state is equivalent with the creation of zero energy state for which initial (final) state particles have positive (negative) energies, and with the fact that the boundaries of magnetic flux tubes are light-like systems for which 3-D quantum state is representation for a 2-D time evolution.

Since the correlation between anyons at the ends of the wormhole thread is purely topological, the most plausible option is that they behave as free anyons dynamically. Assuming 4-branched anyon surfaces, the charges of anyons would be of form $Q = \nu_A e$, $\nu_A = 4/m$, m odd.

Consider now the representation of 3-braid group. That the mapping class group for the 3-braid system should have a 2-dimensional representation is obvious from the fact that the group has same generators as the mapping class group for torus which is represented by as $SL(2,Z)$ matrices acting on the homology of torus having two generators a, b corresponding to the two non-contractible circles around torus. 3-braid group would be necessarily represented in Temperley-Lieb representation.

The character of the anyon bound state is important for braid representations.

1. If anyons form loosely bound states (AA), the electrons are at different tracks and the charge is additive in the process so that one has $Q_{AA} = 2Q_A = 8/m$, m odd, which is at odds with statistics. It might be that the naïve rule of assigning fractional charge to the state does not hold true for loosely bound bosonic anyons. In this case $(AA) - A$ system with charge states $((1,-1),1)$ and $((1,1),-1)$ would be enough for realizing 1-gates in TQC. The braid operation s_2 of Temperley-Lieb representation represented $(A_1A_2) - A_3 \rightarrow (A_1A_3) - A_2$ would correspond to an exchange of the dance partner by a temporary decay of (A_1A_2) followed by a recombination to a quantum superposition of (A_1A_2) and (A_1A_3) and could be regarded as an ordinary braid operation rather than monodromy. The relative phase 1-gate would correspond to s_1 represented as braid operation for A_1 and A_2 inside (A_1A_2).

2. If anyons form tightly bound states (AA) in the sense that single anyonic flux tube carries two electrons, charge need not be additive so that bound states could have charges $Q = 4/2m_1$ so that the vacuum Kähler charge $Q_K = 4(1/m_1 - 2/m)$ would be created in the process. This would stabilize (AA) state and would mean that the braid operation $(A_1A_2) - A_3 \rightarrow (A_1A_3) - A_2$ cannot occur via a temporary decay to free anyons and it might be necessary to replace 3-braid group by a partially colored 3-braid group for $(AA) - A - A$ system which is sub-group of 3-braid group and has generators s_1^2 (two swaps for $(AA) - A$) and s_2 (swap for $A - A$) instead of s_1 and s_2. Also in this case a microscopic mechanism changing the value of (AA) Z^4 charge is needed and the situation might reduce to the case a) after all.

   The Temperley Lieb representation for this group is obtained by simply taking square of the generator inducing entanglement (s_2 rather than s_1 in the notation used!). The topological charge assignments for $(AA)-A-A$ system are $((1,-1),1,-1)$ and $((1,1),-1,-1)$. s_1^2 would correspond to the group element generating $(AA) - A$ entanglement and s_2 acting on $A - A$ pair would correspond to phase generating group element.

Braid representations and 4-branched anyon surfaces

Some comments about braid representations in relation to Z_N- valued topological charges are in order.

1. Yang-Baxter braid representation using the maximally entangling braid matrix R is especially attractive option. For anyonic computation with Z_4-valued topological charge R is the unique 2-gate conserving the net topological charge (note that the mixing of the $|1,1\rangle$ and $|-1,-1\rangle$ is allowed). On the other, R allows only the conservation of Z_4 value topological charge. This suggests that the entanglement between logical qubits represented by $(AA) - A - A$ batches is is generated by R. The physical implication is that only $\nu = 4/n$ 4-branched anyons could be used for TQC.

2. In TGD framework the entangling braid representation inside batches responsible for 1-gates need not be the same since batches correspond to magnetic flux tubes. In standard physics context it would be harder to defend this kind of assumption. As will be found 3-braid Temperley-Lieb representation is very natural for 1-gates. The implication is that the n-braid system with braids represented as 4-batches would have 2^n-dimensional space of logical qubits in fact identical with the space of realizable qubits.

3. Also n-braid Temperley-Lieb representations are possible and the explicit expressions of the braiding matrices for 6-braid case suggest that Z_4 topological charge is conserved also now [B29]. In this case the dimension of the space of logical qubits is for highly favored value of quantum group parameter $q = exp(i\pi/5)$ given by the Fibonacci number $F(n)$ for n-braid case and behaves as Φ^{4n} asymptotically so that this option would be more effective. From $\Phi^4 = 1 + 3\Phi \simeq 8.03$ one can say that single 4-batch carries 3 bits of information instead of one. This is as it must be since topological charge is not conserved inside batches separately for this option.

4. $(AA) - A$ representation based on Z_4-valued topological charge is unique in that the space of logical qubits would be the space of topologically realizable qubits. Quantum superposition of logical qubits could could be represented $(AA) - A$ entangled state of form $a|2,-1\rangle + b|0,1\rangle$ generated by braid action. Relative phase could be generated by braid operation acting on the entangled state of anyons of (AA) Cooper pair. Since the superposition of logical

cubits corresponds to an entangled state $a|2,-1\rangle + b|0,1\rangle$ for which coefficients are extended rational numbers, the number theoretic realization of the bound state property could pose severe conditions on possible relative phases.

9.5.2 Temperley-Lieb Representations

The articles of Kaufmann [B38] and Freedman [B29, B42] provide enjoyable introduction to braid groups and to Tempeley-Lie representations. In the sequel Temperley-Lieb representations are discussed from TGD view point.

Temperley-Lieb representation for 3-braid group

In [B38] it is explained how the so called Temperley-Lie algebra defined by 2×2-matrices I, U_1, U_2 satisfying the relations $U_1^2 = dU_1$, $U_2^2 = dU_2$, $U_1U_2U_1 = U_2$, $U_2U_1U_2 = U_1$ allows a unitary representation of Artin's braid group by unitary 2×2 matrices. The explicit representations of the matrices U_1 and U_2 (note that U_i/d acts as a projector) given by

$$\begin{aligned} U_1 &= \begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix} , \\ U_2 &= \begin{pmatrix} \frac{1}{d} & \sqrt{1-\frac{1}{d^2}} \\ \sqrt{1-\frac{1}{d^2}} & d-\frac{1}{d} \end{pmatrix} . \end{aligned} \tag{9.5.1}$$

Note that the eigenvalues of U_i are d and 0. The representation of the elements s_1 and s_2 of the 3-braid group is given by

$$\begin{aligned} \Phi(s_1) &= AI + A^{-1}U_1 = \begin{pmatrix} -U^{-3} & 0 \\ 0 & U \end{pmatrix} , \\ \Phi(s_2) &= AI + A^{-1}U_2 = \begin{pmatrix} -\frac{U^3}{d} & \frac{U^{-1}}{\sqrt{1-(1/d)^2}} \\ \frac{U^{-1}}{\sqrt{1-(1/d)^2}} & \frac{U^{-5}}{d} \end{pmatrix} , \\ U &= exp(i\phi) . \end{aligned} \tag{9.5.2}$$

Here the condition $d = -A^2 - A^{-2}$ is satisfied. For $A = exp(i\phi)$, with $|\phi| \leq \pi/6$ or $|\pi - \phi| \leq \pi/6$, the representation is unitary. The constraint comes from the requirement $d > 1$. From the basic representation it follows that the eigenvalues of $\Phi(s_i)$ are $-exp(-3i\phi)$ and $exp(i\phi)$.

Tihs 3-braid representation is a special case of a more general Temperley-Lieb-Jones representation discussed in [B29] using notations $A = \sqrt{-1}exp(-i2\pi/4r)$, $s = A^2$, and $q = A^4$. In this case all eigen-values of all representation matrices are -1 and $q = exp(-i2\pi/r)$. This representation results by multiplying Temperley-Lieb representation above with an over-all phase factor $exp(4i\phi)$ and by the replacement $A = exp(i\phi) \rightarrow \sqrt{-1}A$.

Constraints on the parameters of Temperley-Lieb representation

The basic mathematical requirement is that besides entangling 2-gate there is minimum set of 1-gates generating infinite sub-group of $U(2)$. Further conditions come from the requirement that a braid representation is in question. In the proposal of [B25, B29] the 1-gates are realized using Temperley-Lieb 3-braid representation. It is found that there are strong constraints to the representation and that relative phase gate generating the phase $exp(i\phi) = exp(i2\pi/5)$ is the simplest solution to the constraints.

The motivation comes from the findings made already by Witten in his pioneering work related to the topological quantum field theories and one can find a good representation about what is involve din [A83].

Topological quantum field theories can produce unitary modular functors when the $A = q^{1/4} = exp(i\phi)$ characterizing the quantum group multiplication is a root of unity so that the quantum enveloping algebra $U(Sl(2))_q$ defined as the quantum version of the enveloping algebra

$U(Sl(2))$ is not homomorphic with $U(Sl(2))$ and theory does not trivialize. Besides this, q must satisfy some consistency conditions. First of all, $A^{4n} = 1$ must be satisfied for some value of n so that A is either a primitive l: th, $2l$: th of unity for l odd, or $4l$: th primitive root of unity.

This condition relates directly to the fact that the quantum integers $[n]_q = (A^{2n} - A^{-2n})/(A^2 - A^{-2})$ vanish for $n \geq l$ so that the representations for a highest weight n larger than l are not irreducible. This implies that the theory simplifies dramatically since these representations can be truncated away but can cause also additional difficulties in the definition of link invariants. Indeed, as Witten found in his original construction, the topological field theories are unitary for $U(Sl(2))_q$ only for $A = exp(ik\pi/2l)$, k not dividing $2l$, and $A = exp(i\pi/l)$, l odd (no multiples are allowed) [A83]. $n = 2l = 10$, which is the physically favored choice, corresponds to the relative phase $4\phi = 2\pi/5$.

Golden Mean and quantum computation

Temperley-Lieb representation based on $q = exp(i2\pi/5)$ is highly preferred physically.

1. One might hope that the Yang-Baxter representation based on maximally entangling braid matrix R might work. $R^8 = 1$ constraint is however not consistent with Temperley-Lieb representations. The reason is that $\Phi^8(s_1) = 1$ gives $\phi = \pi/4 > \pi/6$ so that unitarity constraint is not satisfied. $\phi = exp(i2\pi/16)$ corresponding $r = 4$ and to the matrix $\Phi(s_2) = \hat{R} = exp(i2\pi/16) \times R$ allows to satisfy the unitarity constraint. This would look like a very natural looking selection since $\Phi(s_2)$ would act as a Hadamard gate and NMP would imply identical entanglement probabilities if a bound state results in a quantum jump. Unfortunately, s_1 and s_2 do not generate a dense subgroup of $U(2)$ in this case as shown in [B29].

2. $\phi = \pi/10$ corresponding to $r = 5$ and Golden Mean satisfies all constraints coming from quantum computation and knot theory. That is it spans a dense subgroup of $U(2)$, and allows the realization of modular functor defined by Witten-Chern-Simons $SU(2)$ action for $k = 3$, which is physically highly attractive since the condition

$$r = k + c_g(SU(2))$$

connecting r, k and the dual Coxeter number $c_g(SU(N)) = n$ in WCS theories is satisfied for $SU(2)$ in this case for $r = 5$ and $k = 3$.

$SU(2)$ would have interpretation as the left-handed electro-weak gauge group $SU(2)_L$ associated with classical electro-weak gauge fields. The symmetry breaking of $SU(2)_L$ down to a discrete subgroup of $SU(2)_L$ yielding anyons would relate naturally to this. The conservation of the topologized Kähler charge would correlate with the fact that there is no symmetry breaking in the classical $U(1)$ sector. $k = 3$ Chern-Simons theory is also known to share the same universality class as simple 4-body Hamiltonian [B25] (larger values of k would correspond to $k + 1$-body Hamiltonians).

3. Number theoretical vision about intentional systems suggests that the preferred relative phases are algebraic numbers or more generally numbers which belong to a finite-dimensional extension of p-adic numbers. The idea about p-adic cognitive evolution as a gradual generation of increasingly complex algebraic extensions of rationals allows to see the extension containing Golden Mean $\Phi = (1 + \sqrt{5})/2$ as one of the simplest extensions. The relative phase $exp(i4\phi) = exp(i2\pi/5)$ is expressible in an extension containing $\sqrt{\Phi}$ and Φ: one has $cos(4\phi) = (\Phi - 1)/2$ and $sin(4\phi) = \sqrt{5\Phi}/2$.

The general number theoretical ideas about cognition support the view that Golden Mean is in a very special role in the number theoretical world order. This would be due to the fact that $log(\Phi)/\pi$ is a rational number. This hypothesis would explain scaling hierarchies based on powers of Golden Mean. One could argue that the geometry of the braid should reflect directly the value of the $A = exp(i2\phi)$. The angle increment per single DNA nucleotide is $\phi/2 = 2\pi/10$ for DNA double strand (note that q would be $exp(i\pi/10)$, which raises the question whether DNA might be a topological quantum computer.

Bratteli diagram for $n = 5$ case, Fibonacci numbers, and microtubuli

Finite-dimensional von Neumann algebras can be conveniently characterized in terms of Bratteli diagrams [A93]. For instance, the diagram a) of the figure **??** at the end of the chapter represents the inclusion $N \subset M$, where $N = M_2(C) \otimes C$, $M = M_6(C) \otimes M_3(C) \otimes C$. The diagram expresses the embeddings of elements $A \otimes x$ of $M_2(C) \otimes C$ to $M_6(C)$ as a tensor product $A_1 \otimes A_2 \otimes x$

$$
\begin{aligned}
A_1 &= \begin{pmatrix} A & . & . \\ . & A & . \\ . & . & A \end{pmatrix} , \\
A_2 &= \begin{pmatrix} A & . \\ . & x \end{pmatrix} .
\end{aligned}
\tag{9.5.3}
$$

Bratteli diagrams of infinite-dimensional von Neumann algebras are obtained as limiting cases of finite-dimensional ones (see **Fig. 9.1**).

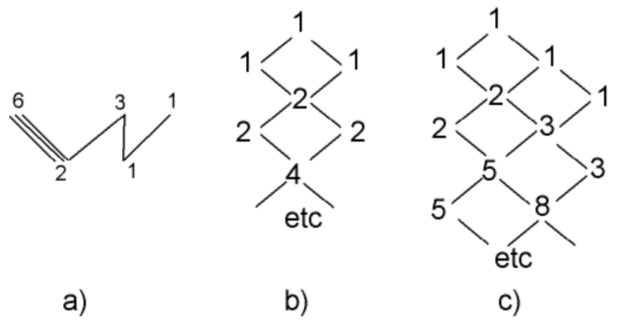

Figure 9.1: a) Illustration of Bratteli diagram. b) and c) give Bratteli diagrams for $n = 4$ and $n = 5$ Temperley Lieb algebras

2. Temperley Lieb algebras approximate II_1 factors

The hierarchy of inclusions of with $|M_{i+1} : M_i| = r$ defines a hierarchy of Temperley-Lieb algebras characterizable using Bratteli diagrams. The diagrams b) and c) of the figure **??** at the end of the chapter characterize the Bratteli diagrams for $n = 4$ and $n = 5$. For $n = 4$ the dimensions of algebras come in powers of 2 in accordance with the fact $r = 2$ is the dimension of the effective tensor factor of II_1.

For $n = 5$ and $B_m = \{1, e_1, ..., e_m\}$ the dimensions of the two tensor factors of the Temperley Lieb-representation are two subsequent Fibonacci numbers F_{m-1}, F_m ($F_{m+1} = F_m + F_{m-1}$, $F_1 = 1, F_2 = 1$) so that the dimension of the tensor product is $dim(B_m) = F_m F_{m-1}$. One has $dim(B_{m+1})/dim(B_m) = F_m/F_{m-2} \rightarrow \Phi^2 = 1 + \Phi$, the dimension of the effective tensor factor for the corresponding hierarchy of II_1 factors. Hence the two dimensional hierarchies "approximate" each other. In fact, this result holds completely generally.

The fact that r is approximated by an integer in braid representations is highly interesting from the point of view of TQC. For 3-braid representation the dimension of Temperley-Lieb representation is 2 for all values of n so that 3-braid representation defines single (topo)logical qubit as $(AA) - A - A$ realization indeed assumes. One could optimistically say that TGD based physics automatically realizes topological qubit in terms of 3-braid representation and the challenge is to understand the details of this realization.

2. Why Golden Mean should be favored?

The following argument suggests a physical reason for why just Golden Mean should be favored in the magnetic flux tube systems.

1. Arnold [B50] has shown that if Lorentz 3-force satisfies the condition $F_B = q(\nabla \times B) \times B = q\nabla\Phi$, then the field lines of the magnetic field lie on $\Phi = constant$ tori. On the other hand, the vanishing of the Lorentz 4-forces for solutions of field equations representing asymptotic self-organized states, which are the "survivors" selected by dissipation, equates magnetic force with the negative of the electric force expressible as qE, $E = -\nabla\Phi + \partial_t A$, which is gradient if the vector potential does not depend on time. Since the vector potential depends on three CP_2 coordinates only for $D = 3$, this seems to be the case.
2. The celebrated Kolmogorov-Arnold-Moser (KAM) theorem is about the stability of systems, whose orbits are on invariant tori characterized by the frequencies associated with the n independent harmonic oscillator like degrees of freedom. The theorem states that the tori for which the frequency ratios are rational are highly unstable against perturbations: this is due to resonance effects. The more "irrational" the frequencies are, the higher the stability of the orbits is, and the most stable situation corresponds to frequencies whose ratio is Golden Mean. In quantum context the frequencies for wave motion on torus would correspond to multiples $\omega_i = n2\pi/L_i$, L_i the circumference of torus. This poor man's argument would suggest that the ratio of the circumferences of the most stable magnetic tori should be given by Golden Mean in the most stable situation: perhaps one might talk about Golden Tori!

3. Golden Mean and microtubuli

What makes this observation so interesting is that Fibonacci numbers appear repeatedly in the geometry of living matter. For instance, micro-tubuli, which are speculated to be systems performing quantum computation, represent in their structure the hierarchy Fibonacci numbers $5, 8, 13$, which brings in mind the tensor product representation $5 \otimes 8$ of B_5 (5 braid strands!) and leads to ask whether this Temperley-Lieb representation could be somehow realized using microtubular geometry.

According to the arguments of [B25] the state of n anyons corresponds to 2^{n-1} topological degrees of freedom and code space corresponds to F_n-dimensional sub-space of this space. The two conformations of tubulin dimer define the standard candidate for qubit, and one could assume that the conformation correlates strongly with the underlying topological qubit. A sequence of 5 *resp.* 8 tubulin dimers would give 2^4 *resp.* 2^7-dimensional space with $F_5 = 5$- *resp.* $F_7 = 13$-dimensional code sub-space so that numbers come out nicely. The changes of tubulin dimer conformations would be induced by the braid groups B_4 and B_7. B_4 would be most naturally realized in terms of a unit of 5-dimers by regarding the 4 first tubulins as braided punctures and 5th tubulin as the passive puncture. B_7 would be realized in a similar manner using a unit of 8 tubulin dimers.

Flux tubes would connect the subsequent dimers along the helical 5-strand *resp.* 8-strand defined by the microtubule. Nearest neighbor swap for the flux tubes would induce the change of the tubulin conformation and induce also entanglement between neighboring conformations. A full 2π helical twist along microtubule would correspond to 13 basic steps and would define a natural TQC program module. In accordance with the interpretation of II_1 factor hierarchy, (magnetic or electric) flux tubes could be assumed to correspond to $r = 2$ II_1 factor and thus carry 2-dimensional representations of $n = 5$ or $n = 4$ 3-braid group. These qubits could be realized as topological qubits using $(AA) - A$ system.

Topological entanglement as space-time correlate of quantum entanglement

Quantum-classical correspondence encourages to think that bound state formation is represented at the space-time level as a formation of join along boundaries bonds connecting the boundaries of 3-space sheets. In particular, the formation of entangled bound states would correspond to a topological entanglement for the flux tubes forming braids. The light-likeness of the boundaries of the bonds gives a further support for this identification. During macro-temporal quantum coherence a sequence of quantum jumps binds effectively to single quantum jump and subjective time effectively ceases to run. The light-likeness for the boundaries of bonds means that geometric time stops and is thus natural space-time correlate for the subjective experience during macro-temporal quantum coherence.

Also the work with TQC lends support for a a deep connection between quantum entanglement and topological entanglement in the sense that the knot invariants constructed using entangling 2-gate R can detect linking. Temperley-Lieb representation for 3-braids however suggests that topological entanglement allows also single qubit representations for with quantum entanglement plays no role. One can however wonder whether the entanglement might enter into the picture in some natural manner in the quantum computation of Temperley-Lieb representation. The idea is simple: perhaps the physics of $(AA) - A - A$ system forces single qubit representation through the simple fact that the state space reduces in 4-batch to single qubit by topological constraints.

For TQC the logical qubits correspond to entangled states of anyon Cooper pair (AA) and second anyon A so that the quantum superposition of qubits corresponds to an entangled state in general. Several arguments suggest that logical qubits would provide Temperley-Lieb representation in a natural manner.

1. The number of braids inside 4-anyon batch (or 3-anyon batch in case that (AA) can decay temporarily during braid operation) 3 so that by the universality this system allows to compute the unitary Temperley-Lieb braid representation. The space of logical qubits equals to the entire state space since the number of qubits represented by topological ground state degeneracy is 1 instead of the expected three since $2n$ anyon system gives rise to 2^{n-1}-fold vacuum degeneracy. The degeneracy is same even when two of the anyons fuse to anyon Cooper pair. Thus it would seem that the 3-braid system in question automatically produces 1-qubit representation of 3-braid group.
2. The braiding matrices $\Phi(s_1)$ and $\Phi(s_2)$ are different and only $\Phi(s_2)$ mixes qubit values. This can be interpreted as the presence of two inherently different braid operations such that only the second braiding operation can generate entanglement of states serving as building blocks of logical qubits. The description of anyons as 2-dimensional wormholes led to precisely this picture. The braid group reduces to braid group for one half of anyons since anyon and its partner at the end of wormhole are head and feet of single dancer, and the anyon pair (AA) forming bound state can change partner during swap operation with anyon A and this generates quantum entanglement. The swap for anyons inside (AA) can generate only relative phase.
3. The vanishing of the topological charge in a pairwise manner is the symmetry which reduces the dimension of the representation space to 2^{n-1} as already found. For $n = 4$ only single topological qubit results. The conservation and vanishing of the net topological charge inside each batch gives a constraint, which is satisfied by the maximally entangling R-matrix R so that it could take care of braiding between different 4-batches and one would have different braid representation for 4-batches and braids consisting of them. Topological quantization justifies this picture physically. Only phase generating *physical* 1-gates are allowed since Hadamard gate would break the conservation of topological charge whereas for *logical* 1-gates entanglement generating 2-gates can generate mixing without the breaking of the conservation of topological charges.

Summary

It deserves to summarize the key elements of the proposed model for which the localization (in the precise sense defined in [B42]) made possible by topological field quantization and Z_4 valued topological charge are absolutely essential prerequisites.

1. $2n$-anyon system has 2^{n-1}-fold ground state degeneracy, which for $n = 2$ leaves only single logical qubit. In standard physics framework $(AA)-A-A$ is minimal option because the total homology charge of the system must vanish. In TGD $(AA) - A$ system is enough to represent 3-braid system if the braid operation between AA and A can be realized as an exchange of the dancing partner. This option makes sense because the anyons with opposite topological charges at the ends of wormhole threads can be negative energy anyons representing the final state of the braid operation. A pair of magnetic flux tubes is needed to realize single anyon-system containing braid.
2. Maximally entangling R-matrix realizes braid interactions between $(AA) - A$ systems realized as 3-braids inside larger braids and the space of logical qubits is equivalent with the space of realizable qubits. The topological charges are conserved separately for each $(AA) - A$ system. Also the more general realization based on n-braid representations of Temperley-Lieb algebra

is formally possible but the different topological realization of braiding operations does not support this possibility.

3. Temperley-Lieb 3-braid representation for $(AA)-A-A$ system allows to realize also 1-gates as braid operations so that topology would allow to avoid the fine-tuning associated with 1-gates. Temperley-Lieb representation for $\phi = exp(i\pi/10)$ satisfies all basic constraints and provides representation of the modular functor expressible using $k = 3$ Witten-Chern-Simons action. Physically 1-gates are realizable using Φ_1 acting as phase gate for anyon pair inside (AA) and $\Phi(s_2)$ entangling (AA) and A by partner exchange. The existence of single qubit braid representations apparently conflicting with the identification of topological entanglement as a correlate of quantum entanglement has an explanation in terms of quantum computation under topological symmetries.

9.5.3 Zero Energy Topological Quantum Computations

As already described, TGD suggests a radical re-interpretation for matter antimatter asymmetry in long length scales. The asymmetry would be due to the fact that ground state for fermion system corresponds to infinite sea of negative energy fermions and positive energy anti-fermions so that fermions would have positive energies and anti-fermions negative energies.

The obvious implication is the possibility to interpret scattering between positive energy states as a creation of a zero energy state with outgoing particles represented as negative energy particles. The fact that the quantum states of 3-dimensional light-like boundaries of 3-surfaces represent evolutions of 2-dimensional quantum systems suggests a realization of topological quantum computations using physical boundary states consisting of positive energy anyons representing the initial state of anyon system and negative energy anyons representing the outcome of the braid operation.

The simplest scenario simply introduces negative energy charge conjugate of the $(AA) - A$ system so that no deviations from the proposed scenario are needed. Both calculation and its conjugate are performed. This picture is the only possible one if one assumes that given space-time sheet contains either positive or negative energy particles but not both and very natural if one assumes ordinary fermionic vacuum. The quantum computing system would could be generated without any energy costs and even intentionally by first generating the p-adic space-time sheets responsible for the magnetic flux tubes and anyons and then transformed to their real counterparts in quantum jump. This double degeneracy is analogous to that associated with DNA double strand and could be used for error correction purposes: if the calculation has been run correctly both anyon Cooper pairs and their charge conjugates should decay with the same probability.

Negative energies could have much deeper role in TQC. This option emerges naturally in the wormhole handle realization of TQC. The TGD realization of 1-gates in 3-braid Temperley-Lieb representation uses anyons of opposite topological charges at the opposite ends of threads connecting magnetic flux tube boundaries. Single 3-braid unit would correspond to positive energy electronic anyons at the first flux tube boundary and negative energy positronic anyons at the second flux tube boundary. The sequences of 1-gates represented as 3-braid operations would be coded by a sequence of 3-braids representing generators of 3-braid group along a pair of magnetic flux tubes. Of course, also n-braid operations could be coded in the similar manner in series. Hence TQC could be realized using only two magnetic flux tubes with n-braids connecting their boundaries in series.

Condensed matter physicist would probably argue that all this could be achieved by using electrons in strand and holes in the conjugate strand instead of negative energy positrons: this would require only established physics. One can however ask whether negative energy positrons could appear routinely in condensed matter physics. For instance, holes might in some circumstances be generated by a creation of an almost zero energy pair such that positron annihilates with a fermion below the Fermi surface. The signature for this would be a photon pair consisting of ordinary and phase conjugate photons.

The proposed interpretation of the S-matrix in the Universe having vanishing net quantum numbers encourages to think that the S-matrices of 2+1-dimensional field theories based on Witten-Chern-Simons action defined in the space of zero (net) energy states could define physical states for quantum TGD. Thus the 2+1-dimensional S-matrix could define quantum states of 4-dimensional theory having interpretation as states representing "self-reflective" level representing in itself the

S-matrix of a lower-dimensional theory. The identification of the quantum state as S-matrix indeed makes sense for light-like surfaces which can be regarded as limiting cases of space-like 3-surfaces defining physical state and time-like surfaces defining a time evolution of the state of 2-dimensional system.

Time evolution would define also an evolution in topological degrees of freedom characterizing ground states. Quantum states associated with light-like (with respect to the induced metric of space-time sheet) 3-dimensional boundaries of say magnetic flux tubes would define quantum computations as modular functors. This conforms with quantum-classical correspondence since braids, the classical states, indeed define quantum computations.

The important implication would be that a configuration which looks static would code for the dynamic braiding. One could understand the quantum computation in this framework as signals propagating through the strands and being affected by the gate. Even at the limit when the signal propagates with light velocity along boundary of braid the situation looks static from outside. Time evolution as a state could be characterized as sequence of many-anyon states such that basic braid operations are realized as zero energy states with initial state realized using positive energy anyons and final state realized using negative energy energy anyons differing by the appropriate gate operation from the positive energy state.

In the case of n-braid system the state representing the S-matrix $S = S^1S^2....S^n$ associated with a concatenation of n elementary braid operations would look like

$$\begin{array}{rcl} |S\rangle & = & P_{k_1}S^1_{k_1k_2}P_{k_2}S^2_{k_2k_3}P_{k_3}S^3_{k_3k_4}... \ , \\ P_k & = & |k,<\rangle|k,>\rangle \ . \end{array} \tag{9.5.4}$$

Here S^k are S-matrices associated with gates representing simple braiding operations s_k for $n+1$ threads connecting the magnetic flux tubes. P_k represents a trivial transition $|k\rangle \rightarrow |k \rightarrow k\rangle$ as zero energy state $|k, > 0\rangle|k,<\rangle$. The states P_k represent matrix elements of the identification map from positive energy Hilbert space to its negative energy dual.

What would happen can be visualized in two alternative ways.

1. For this option the braid maps occur always from flux tube 1 to flux tube 2. A braiding transition from 1 to 2 is represented by S^{k_1}; a trivial transition from 2 to 1 is represented by P_k; a braiding transition from 1 to 2 is represented by S^{k_2}, etc... In this case flux tube 1 contains positive energy anyons and flux tube 2 the negative energy anyons.

2. An alternative representation is the one in which P_k represents transition along the strand so that S^k *resp.* S^{k+1} corresponds to braiding transition from strand 1 to 2 *resp.* 2 to 1. In this case both flux tubes contain both positive and negative energy anyons.

9.6 Quantum computations without definite causal structure: TGD view

I encountered a link to a interesting popular article *"Causal Witness" Provides First Experimental Evidence Of Indefinite Causal Order* (see `http://tinyurl.com/lwaurk3`). The article tells about an article *Experimental verification of an indefinite causal order* by Rubio *et al* [B27](see `http://tinyurl.com/ltamjbv`). In the following are my first impressions.

In TGD Zero Energy Ontology (ZEO) replaces ordinary ontology and the arrow of time is not fixed, and it is interesting to see whether superposition of different causal orders related by time inversion T for causal diamond (CD) and SWITCH could be realized in ZEO. The twistor lift of TGD leads to the proposal that CD is accompanied by a Minkowskian generalization of self-dual Kähler form $J(CD)$. Although the moduli space of CDs allows to avoid breaking of Poincare invariance, self-duality of $J(CD)$ leads to violation of T implying that different causal orders correspond to disjoint sectors of "world of classical worlds". This makes possible also superposition of different causal orders and SWITCH would map these sectors to each other.

9.6.1 Indefinite causal order

The abstract of the article Rubio *et al* might give some idea about what is involved.

Investigating the role of causal order in quantum mechanics has recently revealed that the causal relations of events may not be a priori well-defined in quantum theory. Although this has triggered a growing interest on the theoretical side, creating processes without a causal order is an experimental task. We report the first decisive demonstration of a process with an indefinite causal order. To do this, we quantify how incompatible our setup is with a definite causal order by measuring a "causal witness". This mathematical object incorporates a series of measurements that are designed to yield a certain outcome only if the process under examination is not consistent with any well-defined causal order. In our experiment, we perform a measurement in a superposition of causal orders—without destroying the coherence—to acquire information both inside and outside of a causally non-ordered process. Using this information, we experimentally determine a causal witness, demonstrating by almost 7 SDs that the experimentally implemented process does not have a definite causal order.

Unfortunately, I do not have prerequisites to say anything interesting about the delicacies of the experiment itself. Since causal order is fixed by that associated with space-time in standard physics, the implications of the experiment could be world view changing. The key quantum information theoretic notions are causal order, causal separability, quantum wittness, quantum process called SWITCH changing causal order, and superposition of causal orders.

1. The notion of causal order is discussed in the article "Quantum correlations with no causal order" by Oreshkov *et al* [B44] (see `http://tinyurl.com/l7wb5zh`). One has two events A and B. If they are causally separable, one can tell which causes which. In Minkowski space causally separable events would connected by a time-like curve. If not, one cannot speak about causal order. One can tell whether A precedes B or vice versa. For light-like distances, the situation is not so clear.

   Relaxing the standard assumption about fixed arrow of time one can at quantum level consider also a situation in which one has quantum superposition of different causal orders. One has causal non-separability.

2. The notion of causal wittness [B19] (see `http://tinyurl.com/jwzo3lq`) provides a method allowing to deduce experimentally whether the process is causally separable or not. The notion is similar to that of entanglement wittness (see `http://tinyurl.com/mwjb7um`) allowing to deduce whether the two systems are entangled. Essentially one has observable whose expectation is negative for states with indefinite causal order and positive for those with definite causal order. Causal wittness is not universal but must be constructed for each causally indefinite state separately. The construction of causal wittness expectation value of operator is far from trivial and requires deeper understanding of operator theory. The abstract definition goes as follows:

   Causal wittness represents a set of quantum operations, such as unitaries, channels, state preparations, and measurements, whose expectation value is non-negative as long as all the operations are performed in a definite causal order, i.e., as long as only causally separable resources are used. The observation of a negative expectation value is thus sufficient to conclude that the operations were not performed in a definite order.

   Causal witness can be constructed efficiently and the construction is discussed in [B20] (see `http://tinyurl.com/lxer962`).

3. SWITCH is a further basic notion. One has two events A and B, which can be connected by a time-like curve. One can tell whether A precedes B or vice versa. SWITCH is a quantum operation switching the causal order. The obvious manner to do this would permute A and B and would require "time travel" not allowed in standard physics. Obviously, SWITCH cannot be realized as operation respecting fixed causal order.

4. If superpositions of causal orders are possible, one can have a situation in which causal order is indefinite. Also this is something which does not conform the ordinary view about physics in fixed space-time but is allowed by postulates for quantum computation and SWITCH represents an example of quantum computation impossible with a fixed causal order.

Needless to say, the notions of causal order and superposition of causal orders are revolutionary ideas and the article claims that they have been experimentally verified. Standard physics framework does not allow SWITCH. Therefore there are excellent motivations to find whether these notions and the operation of SWITCH could be understood in TGD framework.

In TGD Zero Energy Ontology (ZEO) replaces ordinary ontology and the arrow of time is not fixed, and it is interesting to see whether superposition of different causal orders related by time inversion T for causal diamond (CD) and SWITCH could be realized in ZEO. The twistor lift of TGD leads to the proposal that CD is accompanied by a Minkowskian generalization of self-dual Kähler form $J(CD)$ [K13, L41]. Although the moduli space of CDs allows to avoid breaking of Poincare invariance, self-duality of $J(CD)$ leads to violation of T implying that different causal orders correspond to disjoint sectors of "world of classical worlds". This makes possible also superposition of different causal orders and SWITCH would map these sectors to each other.

9.6.2 ZEO and discrete symmetries for twistor lift of TGD

Some background about TGD is necessary in order to proceed.

1. Zero Energy Ontology (ZEO) is the cornerstone of TGD and TGD inspired theory of consciousness. Zero energy states appear as two variants and correspond to different WCW spinor fields (WCW for "world of classical worlds"). I have proposed that they correspond also to WCW spinor fields localized to different sectors of WCW but this might be un-necessarily strong assumption.

   Zero energy states for given basis have been subject to a state function reduction at either boundary of CD - passive boundary. Neither the passive boundary nor the members of state pairs at it appearing in the superposition of state pairs are affected in repeated state function reductions. One has what I have called generalized Zeno effect identified as conscious entity - self.

   At the opposite boundary the states evolve: every state function reduction at active boundary is preceded by a unitary time evolution ending to a localization of the active boundary of CD, which can be also seen as a state function reduction [L42]. The temporal distance between the tips of CD increases in this process and gives rise to clock time and experienced flow of time.

   Eventually the first reduction to the opposite boundary occurs and the roles of active and passive boundary of CD are changed. One can say that time reversed zero energy state is obtained and begins to evolve. The first reduction to the opposite boundary would mean death of the conscious entity defined by the sequence of state function reductions at the same boundary and generation of time reversed re-incarnation of self.

2. Violation of T - to be discussed below - would also imply asymmetry between selves and their time reversals. For instance, the average duration for the state reduction sequences keeping boundary fixed could be different and second causal order could dominate giving rise to a dominating arrow of time. Since these reduction sequences are identified as correlates for conscious selves, the time reversed re-incarnations would live much shorter time. Biological systems might be an exception: in TGD inspired theory of consciousness sensory perception and motor action are time reversals of each other.

3. Fermionic oscillator operators associated with induced spinor fields allow to represent WCW gamma matrices as their linear combinations: Fermi statistics is geometrized. Fermionic oscillator operators define also quantum Boolean algebra in the sense that fermion numbers 1/0 correspond to the two Boolean values. One could say that quantum logic is square root of WCW Kähler geometry. This allows to interpretation the S-matrix for fermions as quantum Boolean map between quatum Boolean algebras at opposite boundaries. This is obviously important when one talks about quantum computation.

   In the approach to twistor amplitudes [K49, L41] fermions are localized at the boundaries of string world sheets defining light-like curves at the 3-D light-like orbits of partonic 2-surfaces, at which the signature of the induced metric changes from Minkowskian to Euclidian and has a vanishing determinant so that tangent space is effectively 3-D. The interpretation is in terms of strong form of holography (SH) stating that the data determining both space-time surface as preferred extremal and modes of the induced spinor field in the interior of space-time surface. SH predicts that both the bosonic and fermionic 4-D actions reduce to 2-D effective actions for string world sheets.

The implication is that fermion states at the boundaries of CD are localized at discrete points of partonic 2-surfaces. One has of course amplitude over different locations of fermions at partonic 2-surfaces. The presence of fermionic string world sheets correlates the fermions at different partonic 2-surfaces and serves as correlate for entanglement in fermionic degrees of freedom.

4. The twistor lift of TGD [K49, K13, L41] has led to a rather detailed understanding of discrete symmetries CP, P,T. If M^4 factor of embedding space - or more precisely CD - is endowed with a generalized *self-dual* Kähler form $J(M^4)$ (analogs of magnetic and electric fields of same magnitude and direction), new violations of CP, P and T occurring in long scales and having no counterpart in standard model emerge. The reason is that CP, P and T do not respect the self-duality of M^4 Kähler form. The violations of Poincare invariance are avoided if one assumes moduli space for CDs containing the Lorentz boosts and translations of CD.

   The first guess is that T leaving the center point of CD invariant applied to the CD maps the 3-surfaces at the boundaries of CD to each other. The violation of T however implies that the image of the pair need not allow preferred extremal (space-time surface) connecting its members.

   One can however define the temporal mirror image of pair by mapping only the 3-surface at the passive boundary to the opposite boundary: preferred extremal property would determine the 3-surface at the passive boundary. This could imply that the sub-WCWs formed by pairs and the time reversals are disjoint and form different sectors of WCW as indeed assumed in [L42]. This realization of T allows also the possibility that the dynamics of preferred extremals is not strictly deterministic (true at least for p-adic space-time sheets).

   In absence of T violation T operation would also permute the values fermionic states partonic 2-surfaces at the boundaries of CD but if T is violated, can map only the state at the passive boundary to the opposite boundary and determine the state at original boundary from the hermitian conjugate of S-matrix in opposite time direction. The fermionic state at the opposite boundary would be superposition of states having only same total quantum numbers as the state at the passive boundary. The quantum numbers of individual fermions would not be sharp for non-trivial S-matrix if the zero energy states and their T images correspond to same sector of WCW.

   If T is not violated, zero energy states and their T-images would not correspond to the same sector of WCW. Obviously they would correspond to opposite causal orders, see below) This would force to give up the assumption that states at passive boundary are state function reduced. If T is violated globally, the zero energy states and their T-reversals would correspond to disjoint sectors of WCW, and the sectors would only correspond to different arrows of time. This option gives hopes about WCW localization the outcome of the measurement of causal order.

   The union of sub-WCWs with opposite arrow of time is a space with fixed causal order plus additional binary digit characterizing the causal order. The state function for this binary digit could fix the causal order and quantum computation generating superposition of causal orders should generate entanglement with this bit.

5. What about the situation for a union of CDs? Different CDs should be able to have their own arrow of time. For instance, there are reasons to think that in living matter one can have subsystems with non-standard arrow of time [J16]. Also phase conjugate laser ray could be also example of this. This requires that WCW spinor fields associated with a union of CDs form a tensor product. Causal order need not be same for all CDs but characterizes the sub-WCW associated with CD forming a Cartesian factor of WCW so that WCW spinors for the CDs form tensor product.

9.6.3 Two views about SWITCH and superposition of causal orders

One can imagine two approaches to the identification of SWITCH operation and superposition of causal orders.

Option I: Unitary SWITCH as time reversal

The first option corresponds to corresponds to unitary "time travel" option.

1. As already proposed, SWITCH as a unitary operation could correspond to T. Time reversal operation T applied to the 3-surfaces at the boundaries of CD would naturally change the causal order for the zero energy state. If T is violated the state and its T-image belong to separate sectors of WCW. One could also have a superposition of zero energy states related by T and having different causal orders and localizable to the two sectors of WCW.

2. Is it possible to perform SWITCH as a unitary (as a matter fact, antiunitary) operation? If T maps the disjoint sectors to each other and maps fermionic time evolutions to their time reversals, SWITCH maps the two sectors of WCW to each other.

   Can one realize SWITCH mathematically as a unitary operation between fermionic state spaces. This seems possible: the tensor product of $S \otimes S^{\dagger}$ on tensor product of fermionic Fock spaces would realize this map. Whether SWITCH can be realized physically is of course another question.

Option II: Non-unitary SWITCH as the first state function reduction to the opposite boundary of CD

Could non-unitary SWITCH be realized as the first state function reduction to the opposite boundary - death of self followed by a re-incarnation as time-reversed self? In this case SWITCH is neither unitary nor deterministic. This SWITCH corresponds to non-unitary "time travel" option in the sense that self identified as passive boundary makes a time travel to the opposite boundary of CD by re-incarnating in non-deterministic manner.

What about the superposition of self and time reversed self as a superposition of causal orders? Schrödinger cat would be more than a catchy metaphor: it would indeed be a superposition of cat and re-incarnated cat! Should one take this seriously?

If the CDs with different arrow of time correspond to different sectors of WCW, different causal orders correspond to states localized in these sectors. A superposition of causal orders would correspond to WCW spinor field having component in both these sectors. If the state function reduction to opposite boundary of CD takes place at the level of entire WCW - or more realistically, for a Cartesian factor of WCW, it must be accompanied by a localization to either sector of WCW in order to avoid paradoxes.

I have already earlier work with TGD inspired ideas related to quantum computation. For more than decade ago I developed rather speculative model topological quantum computation in TGD framework [K4, K123]. One speculation about super-effective quantum computations is inspired by the analogy of selves with quantum computations halting, when the self dies and re-incarnates as time reversed self with clock time running in opposite direction at opposite boundary of CD [K9]. This would mean using the non-unitary SWITCH realized as state function reduction in quantum computation.

This allows to imagine a series of quantum computations at opposite boundaries proceeding as a sequence of re-incarnations so that the size of CD and thus clock time would grow in opposite directions during subsequent incarnations [L18]. Although the re-incarnation as time reversed self could have long life-time, it would not be seen by an observer near the former re-incarnation and the re-re-incarnation would appear at clock-time, which need not be much later than the time of previous death. One could imagine that the time-reversed selves could have very long life-time or that the self could die and re-incarnate very many times without any-one noticing it! The large amount of time spent in time reversed mode could explain the miraculous cognitive feats of mathematicians like Ramajunan and also the magic computational abilities of idiot savants able to factorize large integers without any idea about the notion of prime.

9.6.4 Higher level quantum computations and ZEO

From the article of Rubio *et al* one ends up to an article *Quantum computations without definite causal structure* by Chiribella *et al* (see `http://tinyurl.com/lgjkzhx`). The article considers a rather far reaching generalization of quantum computation. Ordinary quantum computation is a time evolution of quantum states followed by a state function reduction. Since the outcome of state function reduction halting the quantum computer program is non-deterministic, the extraction of the result involves statistical averaging over a large enough number of quantum computations to get the outcome, say prime factorization or a period of periodic function.

The notion of classical computation is generalized by Church. The computation need not be a function but can assign function to a function. One can continue this abstraction hierarchy indefinitely and it is realized formally in terms of so called Λ calculus (see `http://tinyurl.com/829fea8`). Could this hierarchy be extended to quantum computations? The quantum computation in question would be kind of super-computation assigning to quantum computation a quantum computation and entire hierarchy of quantum computations.

In TGD this kind of hierarchies emerge naturally. At space-time level there is hierarchy of space-time sheets: space-time sheet is (topologically) condensed to a larger space-time sheet and contains smaller space-time sheets condensed at it. The hierarchy of infinite primes corresponds to an infinite hierarchy of second quantizations and could relate to this hierarchy [K107] (see `http://tinyurl.com/m3tuo9q`. In each scale space-time sheets would be particles consisting of smaller particles consisting of ... Even galaxy could be seen as elementary particle in some scale characterizing the galactic space-time sheet.

The analog of quantum computational hierarchy emerges quite concretely in ZEO. The simplest zero energy states have positive and negative energy states with opposite total quantum numbers at the opposite boundaries of CD (intersection of future and past directed light-cones). One can have CDs within CDs. Furthermore, the positive/negative energy states assignable to the boundaries of CD could be also zero energy states associated with smaller CDs near the boundaries of CD. The simplest zero energy states correspond to quantum evolution representing ordinary quantum computation. Higher level zero energy stats would represent time evolution assigning to a quantum computation represented as zero energy state at the boundary of CD a second quantum computation at opposite boundary of CD.

The fermionic representation of quantum Boolean algebra makes this hierarchy quite concrete. At lowest level unitary evolution connects positive and negative energy fermionic states at opposite boundaries of CD and unitary S-matrix characterizes the computation. Higher level computations connect zero energy states assignable to sub-CDs near the boundaries of CD.

9.6.5 Is conscious experience without definite causal order possible?

The exciting question is what the superposition of causal orders could mean from the point of view of conscious experience. What seems obvious is that in the superposition of selves with opposite arrows of clock-time there should be no experience about the flow of time in definite direction. Dissipation is associated with the thermodynamical arrow of time. Therefore also the sensory experience about dissipation expected to have unpleasant emotional color should be absent. This brings in mind the reports of meditators about experiences of timelessness. These states are also characterized by words like “bliss” and “enlightenment”.

Why I find this aspects so interesting is due to my personal experience for about 32 years ago. I of course know that this kind of personal reminiscences in an article intended to be scientific, is like writing one’s own academic death sentence. But I also know I long ago done this so that I have nothing to lose! The priests of the materialistic church will never bother to take seriously anything that I have written so that it does not really matter! This experience - I dared to talk about enlightenment experience - changed my personal life profoundly, and led to the decision to continue work with TGD instead of doing full-day job to make money and keeping TGD as a kind of hobby. The experience also forced to realize that our normal conscious experience is only a dim shadow of what it can be and stimulated the passion to understand consciousness.

In this experience my body went to a kind of light flowing state: liquid is what comes in mind. All unpleasant sensations in body characterizing the everyday life (at least mine!) suddenly disappeared as this phase transition propagated through my body. As a physicist I characterized this as absence of dissipation, and I talked to myself about a state of whole-body consciousness.

There was also the experience about moving in space in cosmic scales and the experience about the presence of realities very different the familiar one. Somehow I saw these different worlds from above, in bird’s eye of view. I also experienced what I would call time travel and re-incarnation in some other world.

Decades later I would ask whether my sensory consciousness could have been replaced with that only about my magnetic body only. In the beginning of the experience there was indeed a concrete feeling that my body size had increased with some factor. I even had the feeling the factor was about 137 (inverse of the fine structure constant) but this interpretation was probably forced by my attempt to associate the experience with something familiar to physicist! Although I did all the time my best to understand what I was experiencing, I did not direct my attention to my time experience, and cannot say whether I experienced the presence or absence of time or time flow.

Towards the end of the experience I was clinically unconscious for about day or so. I was however conscious. For instance, I experienced quite concretely how the arrow of time flow started to fluctuate forth and back. I somehow knew that permanent change would mean death and I was fighting to preserve the usual arrow of time. My childhood friend, who certainly did not know much about physics, told about about alternation of the arrow of time during a state that was classified by psychiatrists as an acute psychosis.

9.7 Retrocausality and TGD

The comments below were inspired by a popular article "Physicists provide support for retrocausal quantum theory, in which the future influences the past" in Phys.org (see `http://tinyurl.com/yd4rwsg7`) telling about the preprint "Is a time symmetric interpretation of quantum theory possible without retrocausality?" of Leifer and Pusey related to the notion of retrocausality [B43] (see `http://tinyurl.com/yd59jvd5`). Retrocausality means the possibility of causal influences propagating in non-standard time direction. Retrocausality has been also proposed by Cramer as a possible manner to obtain deterministic quantum mechanics and allowing to interpret wave functions as real objects. Bell theorem and Kochen-Specker theorem however pose difficult challenges for this program and the condition that the theory is classical in strong sense (all observables have well-defined values) seems impossible.

The work is interesting from TGD view point for several reasons.

1. TGD leads to a new view about reality solving the basic problem of quantum measurement theory. In ZEO quantum states are replaced by zero energy states which are analogous to pairs of initial and final states in ordinary ontology and can be regarded as superpositions of classical deterministic time evolutions. The sequence of state function reductions means sequence of re-creations of the superpositions of classical realities. The TGD based view about scattering amplitudes has a rather concrete connection with the view of Cramer as I interpret it. There is however no attempt to reduce quantum theory to a purely classical theory. The notion of "world of classical worlds" consisting of classical realities identified as space-time surfaces replaces space-time as a fixed observer independent reality in TGD.

2. Retrocausality is basic aspect of TGD. Zero Energy Ontology (ZEO) predicts that both arrows of time are possible. In this sense TGD is time symmetric. On the other hand, the twistor lift of TGD predicts a violation of time reflection T and this might imply that second arrow of causality dominates in some sense. The ZEO based view about state function reduction essential for TGD inspired theory of consciousness and implying generalized Zeno effect giving rise to conscious entities -"selves" - is also essential. One might say that when conscious entity dies it re-incarnates as time-reversed self.

3. The possibility of superposing states with opposite causal arrows [B27] (see `http://tinyurl.com/ltamjbv`) is a fascinating idea and its plausibility is discussed already earlier in TGD framework [L40] (see `http://tinyurl.com/y9tgfxbf`).

In the sequel I will discuss the articles from TGD point of view criticizing the hidden assumptions about the nature of time leading to the well-known problems of quantum measurement theory and consider also the concrete implications for theories of consciousness. Also the empirical evidence for retrocausality is discussed briefly. Contrary to the article the discussion is non-technical: I do not believe that the introduction of technicalities helps to understand the deep conceptual problems involved and possible solutions to them.

9.7.1 Retrocausality

In this section I will explain my own view about retrocausality but will not introduce the TGD view yet.

Retrocausality: with or without real quantum states

Leifer and Pusey use as a starting point the work of Hue Price [B33] (see `http://tinyurl.com/yaa8wogr`), which claims that if quantum states are real and quantum world is time-symmetric then theory must allow retrocausal influences.

What does one mean when one says that quantum states are real? In standard ontology (PEO) this is usually taken to mean that physics is deterministic and universe corresponds to single solution of field equations. This leads of course to conflict with the facts behind quantum measurement theory. State function reductions are not deterministic. This has led to various interpretations such as Copenhagen interpretation giving up ontology altogether and assuming only epistemology: wave function describes only our knowledge about something, which does not exist. One has paradox.

Retrocausality has been proposed to save the notion of reality as something unique and deterministic. A stronger condition is that all observables have sharp values as in classical mechanics. For Schrödinger amplitudes - which can be seen also as purely classical objects - the observables are defined by expectation values of operators and simultaneous eigenstates of non-commuting observables are not possible and classicality in strong sense fails.

Cramer's transcactional interpretation of quantum theory (see `http://tinyurl.com/zpupb8g`) indeed assumes that both causal arrows are possible.

1. To my best understanding this would mean that there are two time evolutions: the usual one from past to future and the retrocausal one from future to past. At some 3-dimensional hyper-surface of space-time these time evolutions would meet each other and be glued together. At this hyper-surface there would be discontinuities. This picture might lead to the standard statistical predictions of quantum measurement theory such as reduction probabilities if the two states would correspond to eigenstates of corresponding measured observables and transition amplitudes are given by Born rule. Note that Cramer's theory is completely deterministic and there is no room for free will.

2. Realism should be consistent with the experimentally verified Bell theorem supporting non-locality of quantum theory made possible by quantum entanglement. The hope is that retrocausality is consistent with Bell theorem predicting correlations between distant measurements not possible to understand in terms of classical probabilities. The key point is the interference of amplitudes which in quantum theory replaces summation of probabilities.

3. There is also Kochen-Specker theorem (see `http://tinyurl.com/q4vb9j5`) stating that it is not possible to have classical description of all quantum observables when their algebra is non-commutative. If realism is taken to mean that all observables have well-defined values then there is no hidden variable theory allowing realism. Even retrocausality can help only if realism is formulated in less demanding manner.

Leifer and Pusey give up the debatable assumption about reality of quantum states and claim of having proved the same result. The article is rather technical and I have not checked the details. Intuitively the result of course looks rather obvious. Of course, it is possible that theory is not fully time-symmetric and there are still retrocausal influences. For instance, the violation of time reflection symmetry and this could make the ordinary causal influences longer lasting than the retrocausal ones.

Cannot have both time symmetry and no-retrocausality

Leifer and Pusey show that time symmetry and non-allowance of retrocausality leads to a contradiction but they do not assume the reality of quantum states in the sense of PEO. Time symmetry implies that forward and backward processes have same probabilities. Impossibility of retrocausality obviously requires that the probabilities for retrocausal processes vanish.

Intuitively it is clear that time symmetry is more or less equivalent with the possibility of retrocausality meaning possibility of signals propagating in non-standard time direction. There are however many poorly understood issues.

1. What does one mean with signal? How can one conclude that the time evolution of say electromagnetic field corresponds to signal (or influence) in a given time direction? One possibility is that positive frequency photons correspond to signals to future. In quantum field theories (QFTs) positive energy photons correspond to creation operators and negative energy photons to annihilation operators. By $E = hf$ and its generalization positive frequencies correspond to positive energies. This would code for the selection of arrow of time and causality. This selection has nothing to do with thermodynamics and second law. One could call this ontology positive energy ontology (PEO).
2. It is far from obvious whether the time of physicist geometrized by Einstein can be identified with the experienced time is correct. This identification is done also in the work of Leifer and Pusey. The arrow of time is naturally assignable to subjective time but not to geometric time. Certainly these two times correlate strongly. We can experience subjective time directly but also use physical processes such as oscillators serving as clocks measuring geometric time, and this gives correspondence between the sequence of mental images and the position of the pointer of the clock.

   This strong correlation does not justify the identification of the two times. For instance, subjective time seems to have no future, only the moment "Now" is experienced directly, and there are only memories (even sensory ones) about the past. Geometric time has no preferred value and seems to correspond to eternity. There are therefore dramatic differences between the two times but for some reason they are usually identified.
3. Thermodynamics and second law are certainly closely related to subjective time. Thermodynamical predictions in turn rely kinetic theory with various reaction rates deduced from quantum theory. Non-determinism of state function reduction is what gives to the second law basically. Entropy increases due to state function reduction in the direction of time, which corresponds to a direction in which energies (frequencies for photons) are positive. If subjective time and geometric time are not identified this assumption becomes questionable and one can wonder whether the causal arrow is property of Universe or of quantum state only. For instance, could thermodynamical arrow correspond to non-standard arrow of geometric time?

   What time symmetry would mean if one does not identify subjective and geometric time? Is there symmetry with respect to subjective or geometric time or both? Could the causal arrow be a property of quantum state? Is it possible to have both causal arrows without conflict with second law or should one generalize second law so that it applies in reverse direction of geometric time for systems allowing retrocausality?

9.7.2 The notions of reality and retrocausality in TGD context

Consider next what one can say about the notion reality and retrocausality in TGD framework.

About the notion of reality in TGD framework

In TGD framework the question about the relationship between geometric and experienced time leads to a new view about state function reduction solving the basic paradox of standard quantum measurement theory and forces to replace the notion of classical reality with quantum superposition of realities. The new notions are "world of classical worlds" (WCW) and zero energy ontology (ZEO).

1. The key question is whether subjective and geometric time are identical. Could it be that these times are not same? If so, one would have two causalities: causality of state function reductions and causality of field equations. Could state function reductions occur between entire deterministic time evolutions rather than tinkering with single time evolution making it non-deterministic?

   This would force to give up the idea about single 4-D reality and replace it with a space of realities. In quantum theory one would be forced to speak of quantum superposition of these classical realities. Each state function reduction would re-create the superposition of 4-D classical realities identified as deterministic time evolutions. One would have realism in more general sense: quantum states would be quantum superpositions of classical realities giving rise to the "world of classical worlds" (WCW). Quantum jumps would allow continual re-creation of classical realities making possible evolution.

2. What does one mean with WCW? The notion of WCW from the view about TGD as a generalization of quantum field theory and string models. One replaces point-like particles with 3-D surfaces, whose 4-D orbits have interpretation as space-time surfaces. This of course means a considerable generalization of the notion of space-time. Particles can be seen as smaller space-times glued to larger space-times looking like particles in a rougher resolution.

   Given sector of WCW correspond to the space of space-time surfaces (generalizing particle world lines) inside 8-D causal diamond (CD), which is diamond like intersection of future and past directed light-cones with points replaced with CP_2. By holography these space-time surfaces are determined by 3-D surfaces at opposite boundaries of CD. To be precise, instead of ordinary holography one has strong form of holography (SH) meaning that 2-D data determine the classical dynamics and also quantum dynamics to high extent.

   This view is obviously something new. Classically physical states are not characterized by the initial values - of say coordinates and velocities at *time=constant* snapshot - but by their boundary values at the future and past boundaries of CD. Initial value problem has changed to boundary value problem. One gives initial and final values but only for coordinates, one might say. One has ZEO rather than PEO. This is essential from the point of view of retrocausality.

3. By its infinite-dimensionality WCW Kähler geometry is essentially unique and even the choice of the embedding space as $H = M^4 \times CP_2$ is unique by twistorial considerations [L41]. One might say WCW as the space of classical realities is the unique reality, maybe one should call it **THE REALITY**.

4. The wave functions in WCW correspond to WCW spinor fields. WCW spinors correspond to many-fermion states at given space-time surface and spinor fields are these spinors extended to spinor fields in WCW. All fundamental particles reduce to many-fermion states. This picture is a completely straightforward generalization of the notion of wave function in Minkowski space obtained by replacing point like particles with 3-surfaces. "Center of mass" degrees of freedom for 3-surfaces indeed are indeed characterized by $M^4 \times CP_2$-coordinates.

   These WCW spinor fields can be said to be purely classical spinor fields. There is no second quantization at WCW level. Therefore the only genuinely quantal aspect of quantum TGD would be state function reduction, which makes possible conscious entities with free will and leads to the notion of subjective time besides geometric time.

5. Quantum classical correspondence (QCC) is one of the basic principles of TGD and says that classical physics is exact part of quantum physics. The space-time surfaces in the quantum superposition are preferred extremals of certain action principle. "Preferred" means that SH holds true. In standard path integral approach one would have path integral over all possible space-time surfaces, now only over the preferred extremals. For integrable theories about which TGD seems to be an example, these two views are more or less equivalent. QCC states that the classical Cartan algebra Noether charges for preferred extremals in superposition are identical with eigenvalues of corresponding quantal charges. One could consider even the possibility that all classical Noether charges for the preferred extremals in the superposition are same as the expectation values of their quantum counterparts.

6. Cramer's view about state function reduction as gluing together of causal and retrocausal solutions of field equations together at 3-D surface has a highly interesting analogy in TGD. Elementary particle vertices correspond to this kind of gluing of corresponding space-time surfaces together along their ends [K49, L41]. At partonic level one has analogy of three-particle Feynman vertex. The three external lines of vertex correspond to three 3-D lightlike orbits of partonic 2-surfaces defining boundaries between space-time regions with Minkowskian and Euclidian signature of the induced metric. The vertex corresponds to partonic 2-surface at which these orbits are glued together along their ends. There is also gluing of space-time surfaces along their 3-D ends which could be located to boundaries of a sub-CD within larger

CD containing initial and final states of particle reaction at its boundaries. The amplitudes at vertices are obtained using the QFT analog of Born rule.

QCC would require that each space-time surface in the superposition of space-time surfaces in CD satisfies the Cramer type rules for each vertex involving sub-CD. The superposition of space-time surfaces would be superposition of potential state function reductions! The real state function reduction would pick up of them!

To sum up, in TGD there is no attempt to get rid of the non-determinism of state function reduction or force the reality to be classical in the sense of classical mechanics (local realism with well-defined values for all observables). Classical Noether charges are well-defined for all space-time surfaces but it is impossible to localize WCW spinor field to single space-time surface. This is already impossible by the fact that there is always finite measurement resolution: this notion indeed plays key role in TGD framework and involves p-adic length scale hierarchies and hierarchy of Planck constants labelling dark matter as phases of ordinary matter. Cramer's rule however resembles very strongly the TGD view about classical space-time correlates of particle reactions.

To my view the most precious gift of quantum theory based on ZEO is the possibility to understand free will without conflict with the determinism of basic field equations and various various trying to force old-fashioned reality give up this gift.

ZEO based view about time, state function reduction, and consciousness

In ZEO quantum measurement theory extends to a theory of consciousness: observer ceases to be an outsider and becomes part of the physical world also mathematically. The detailed discussion of various issues and of recent situation of TGD inspired theory of consciousness can be found in [L42].

The basic idea is that consciousness (actually not a property of anything) is in the state function reduction, between the two quantum realities rather than being a property of quantum reality. This resolves various problems of monistic and dualistic approaches, and one could say that TGD ontology is tri-partistic: classical existence at the space-time level (space-time surfaces), existence at quantum level (zero energy states), and conscious existence at the level of state function reductions. Adelic physics implies further division of realities to "real" and p-adic sectors serving as correlates for sensory and cognitive aspects of conscious experience.

The theory has developed slowly. ZEO meant breakthrough and led gradually through twists and turns to a notion of self surprisingly similar to the original idea. Negentropy maximization principle (NMP) was for a long time regarded as a separate principle but its statistical form follows automatically from adelic physics [L37, L39]. The understanding of the notion of time has been the main challenge.

The basic notion is that of self.

1. Self corresponds to a sequence of quantum jumps integrating to single unit as in the original proposal, but these quantum jumps correspond to repeated state function reductions leaving both the passive boundary of CD and the corresponding parts of zero energy states (state pairs) invariant. The parts of zero energy states at the active boundary of CD change and even the position of the tip of the opposite boundary changes: one actually has wave function over positions of second boundary (CD sizes roughly) and this wave function changes. In positive energy ontology these repeated state function reductions would have no effect on the state (Zeno effect) but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and self: self is generalized Zeno effect.

2. The first quantum jump to the opposite boundary corresponds to the act of "free will" or birth of re-incarnated self. Hence the act of "free will" changes the arrow of psychological time at some level of hierarchy of CDs. The first reduction to the opposite boundary of CD means "death" of self and "re-incarnation" of time-reversed self at opposite boundary at which the the temporal distance between the tips of CD increases in opposite direction. The sequence of selves and time reversed selves is analogous to a cosmic expansion for CD. The repeated birth and death of mental images could correspond to this sequence at the level of sub-selves.

3. This allows to understand the relationship between subjective and geometric time and how the arrow of and flow of clock time (psychological time) emerge. The average distance between

the tips of CD increases on the average as along as state function functions occur repeatedly at the fixed boundary: situation is analogous to that in diffusion. The localization of contents of conscious experience to boundary of CD gives rise to the illusion that universe is 3-dimensional. The possibility of memories made possible by hierarchy of CDs demonstrates that this is not the case. Self is simply the sequence of state function reductions at the same boundary of CD remaining fixed and the lifetime of self is the total growth of the average temporal distance between the tips of CD.

4. It is important to notice that one has actually self hierarchy as a counterpart for the existence of hierarchy of systems. Sub-selves correspond to mental images of self, which in turn defines mental image of a higher level self. The proposal is that sub-selves of sub-self are experienced as averages. One might say that TGD predicts pan-psychism in well-defined sense.

   The new view about subsystem makes possible sharing of mental images by entanglement although selves are un-entangled at their own level and thus define separate conscious entities. The new view about subsystem follows naturally from the notion of many-sheeted space-time: space-time sheets can be disjoint although smaller space-time sheets glued to them by wormhole contacts with Euclidian signature of induced metric can have magnetic flux tube connections serving as correlates of attention.

It is clear that selves and their time reversals correspond to causality and retrocausality. Self experiences that signals arrive from geometric past always: the roles of past and future are however changed in the re-incarnation of self.

ZEO can be said to be time symmetric. There is however a breaking of time symmetry in the sense that the twistor lift TGD violates T (and also of CP and P) realized as a time reflection with respect to the center of CD [L23, L27]. An interesting question is whether this asymmetry could favor the second causal arrow in some sense. For instance, could the life cycles of self with standard arrow of time be considerably longer than those for time reversed selves? This would be due to T-non-invariance of the probabilities for the first reduction to the opposite boundary of CD. Could the longevity in standard time direction emerge in long length scales? For elementary particles the durations of selves are expected to be short since the usual rules for state function reduction apply to the reductions meaning death of self.

There are processes in which the arrow of time seems to be non-standard and a fascinating question is whether these ghostly time reversed selves could be observed, and whether even communications with them could be possible! Some people believe in communications between deceased and alive and study of the communications with deceased is part of parapsychology: could there be some seed of truth in these beliefs?

Possible implications and experimental support for retrocausality

Retrocausality implies that signals can propagate in both directions of time. The signals could even be time reflected at the boundary of CD, which would mean state function reduction changing the arrow of time in the case of signal (note that there is hierarchy of CDs and selves). These reflections could make possible apparently superluminal communications and communication with future and past.

1. The TGD based model for long term memories and precognition relies on the idea that memories involve time reflection from either boundary of CD [K95, ?, K124, K9].

   The model for motor actions as induced by signals sent to the brain of the geometric past relies on the same idea and explains Libet's strange finding that conscious action is preceded by neuronal activity [J7] used usually to argue that free will is illusion.

   Since the signals propagating to non-standard direction of time has negative energy, one can consider also a model of remote metabolism in which the system needing metabolic energy sends negative energy signal to a system able to provide it, say population reverted laser. This quantum credit card mechanism making possible instantaneous reactions would have obvious evolutionary value and would also favor co-operation.

2. Fantappie [J16] was probably the first theoretical physicist to propose that causal arrow might vary in in living matter and introduced the notion of syntropy, which would correspond entropy growing in nonstandard direction of time. There is quite a number of bio-systems which might

be retrocausal at some level. One example is the self-assembly of bio-macromolecules (say tobacco mosaic virus): retrocausally a decay would be in question. Also phase conjugate laser beams [D4, D6] and phase conjugate sound waves are known to obey second law in wrong time direction. For some reason these empirical facts have not captured the attention of theoreticians. In ZEO these findings find a natural explanation.

3. The possibility of time reflection implies that light velocity ceases to be a barrier for communications. One can even speculate with the possibility that conscious entities in distant galaxies could communicate using this mechanism. The altered states of consciousness caused by various psychedelics involve often the experiences about encountering representatives of other civilizations and one can ask whether these encounters are due to remote sensory experiences based on the above mechanism involving both classical and quantum communication (entanglement) [L7] [K114, K110]. Could it be that some sensory receptors (perhaps all) are connections to magnetic flux tubes which can connect the brain to even remote galaxies? If this were the case, one must ask whether our ideas really originate in our brains.

4. One can even imagine that causal arrow is not definite in the sense that one can have quantum states, which are superpositions of states with opposite causal arrows, and there is even a claim that the existence of these states have been verified experimentally by quantum measuring the arrow of causality - causal witness is the name for this observable: see the article *Experimental verification of an indefinite causal order* by Rubio *et al* [B27](see `http://tinyurl.com/ltamjbv`). The popular article *"Causal Witness" Provides First Experimental Evidence Of Indefinite Causal Order* (see `http://tinyurl.com/lwaurk3`) summarizes the work of Rubio *et al.*

   If the finding is real it is revolutionary: in the standard physics framework it is very difficult to imagine how a superposition of different causal arrows could be possible. In the case of superposition of two causal orders the measurement of causal witness has two outcomes and both are claimed to be possible with certain probabilities.

   Does TGD allow the superposition of causal arrows? One can obviously decompose the sub-WCW associated with given CD to sectors with well-defined causal arrow: they are related by time reflection T and are indeed different by T violation for classical dynamics. The roles of passive and active boundaries would be changed for the T-related sectors [L40] (see `http://tinyurl.com/y9tgfxbf`). Superposition of causal arrows would mean a state having component in both sectors. This makes sense if state function reduction to the opposite boundary is preceded by the measurement of the causal witness. Therefore the formation of this superposition and refusal to measure causal witness would be a recipe for immortality!

   The localizations in the sequence of reductions to the active boundary must occur in complete synchrony for the components in the superposition if they occur at all. A stronger condition is that the two reduction sequences cease so that the time flows stop in both directions: there would be no observables commuting with the observables diagonalized at the passive boundary to be measured anymore [L42]. Does the absence of a well-defined causal arrow alone imply an experience of timelessness or must also the time flow stop? Could the enlightened states reported by meditators and involving experience of timelessness have something in common with this kind of states?

9.8 Still about the notion of causal indefiniteness in TGD framework

The motivation for this comment came from a popular article "*Quantum mischief rewrites the laws of cause and effect*" (`https://cutt.ly/2xEP5Vd`), which tells both about the theoretical work of Lucien Hardy [B39](`https://arxiv.org/pdf/gr-qc/0509120.pdf`) and related experimental work, in particular about the following experimental finding [?] (`https://advances.sciencemag.org/content/3/3/e1602589.short`.

Photon beam goes through a spin splitter to form a superposition of photons going along two paths. At the first path they go through A and then through B having some effect on the photons. At second splitter the order of A and B is changed. After that the beams are superposed and it is found that the photons in a causally indefinite state in the sense that the effects of both AB and BA are superposed. In classical physics this is impossible.

The finding is claimed to demonstrate causal indefiniteness: one does not know whether A causes B or B causes A. Classically - that is in the framework provided by fixed causal order dictated by Minkowski space - this seems to be the case.

Is this interpretation correct? Is one really forced to give up causality in the standard form? The rules of standard quantum theory are consistent with the finding but should one change the views about the notion of space-time?

9.8.1 Background

What happens to causality in quantum gravity?

Lightcone of M^4 characterizes the causal structure of Minkowski space in special relativity and is the basic notion of QFTs. In curved space-time of GRT, the light-cone however depends on the metric of space-time. Causal structure is dynamical. The intuitive view is that in quantum gravity causality becomes somehow fuzzy since there is no unique space-time anymore. What this non-uniqueness means is not clear. For instance, could it correspond to what happens in the path integral over space-times?

The problem is that one cannot compare the causal structure for different space-times because the light-cones characterizing them are in different space-times. If the space-times had common coordinates, the comparison would become possible but one cannot assume this.

Lucien Hardy wanted to understand what happens for causality in quantum gravity [B39]. Hardy proposed a method to test whether events in separate space-time regions are causally related.

1. The method allows to formulate dynamical causality operationally in terms of correlations for measurements performed for regions of space-time. He also introduces the notion of an elementary region from which more complex regions giving rise to causaloids are built. Elementary region corresponds to a space-time region in which some measurement giving a definite result is performed.

   One is interested in the correlations between measurement results associated with disjoint elementary regions and in principle all measurements should reduce to a deduction of such correlations. If two disjoint regions of this kind are causally correlated, the measurement outcomes are correlated. The basic interest is in the conditional probabilities for various outcomes from the measurement of observable F_2 in region R_2 given that the measurement outcome for F_1 in R_1 is known.

   Hardy calls these structures causaloids and proposes that causaloids can be composed to form larger causaloids. No fixed causal structure is assumed and even the notion of time is in principle un-necessary in this formulation for experimental deduction of causal structure. Hardy suggests that the quantization of gravity could be performed using the notion of causaloid.

Quantum switch

Second input comes from quantum computation. Giulio Chiribella and colleagues were interested on what kind of computations are possible [B24] (`https://cutt.ly/wxEJtDF`). Classical computation can be characterized as a recursive function mapping natural numbers to natural numbers. One can build more complex functions from given functions by composition of functions. In classical computation the functions are represented as networks of Boolean gates. In quantum computation the quantum gates are used. Now the situation is more complex, since the outcome of the computation is deduced from the probabilities for various outcomes emerging as the quantum computation halts.

Chiribella and colleagues asked what kind of functions are possible. They ended up with the notion of quantum switch. Beam splitter divides the incoming photon beam to two branches. For the first branch function BA is realizes and for the second branch function AB so that one can speak of two different causal orders. After this the beams superpose. If AB and BA can be realized as causal orders, one could say that the resulting state is causally indefinite. If AB and BA are interpreted as quantum computations wihtout halting one can say that quantum switch realizes a superposition of two computations.

The quantum switch can be realized in the laboratory for the first time by Giulia Rubino *et al* [B28](https://cutt.ly/pxEH5wQ. One can measure the polarization of the outgoing photons from the quantum switch to see whether the photons carry information about both AB and BA. This was found to be the case and the findings have been interpreted by saying that that photon experiences causal indefiniteness.

Several technological applications such as communication over noisy channels (https://cutt.ly/2xEJfVB) and quantum refrigerator based on indefinite causal order (ICO) (https://cutt.ly/cxEJcj9) have been proposed.

Hardy also proposes a Quantum Equivalence Principle [B40] stating that one can find a common reference frame for various deformations of a given space-time metric such that the light-cones of various space-time metrics co-incide in these coordinates at least locally.

9.8.2 TGD view about indefiniteness

In the TGD framework [L81] the basic problem of quantum gravity due to the dynamical nature of causality disappears since the embedding space defines the pre-existing causal structure inducing the causal structure at space-time surfaces.

1. In the TGD framework the space-time of GRT is replaced with a 4-surface in $H = M^4 \times CP_2$. The topology of the space-time surface is non-trivial in all scales which leads to the notion of many-sheeted space-time allowing to reduce matter as shape to the space-time topology. Matter is not something in space-time but topological inhomogenities of space-time surface with size and shape - space-time sheets.

2. The basic conceptual problem in the quantization GRT is due to inability to compare different space-times, in particular their causal structures. This problem disappears in TGD.

   The fixed causal structure of Minkowski space M^4 defines the causal structure with induces causal structures of space-time surfaces in terms of induced metric. One can also quite concretely compare the light-cones of different space-time surfaces determined by the induced metric.

   One can also use M^4 linear coordinates as common coordinates for all space-time surfaces. If the space-time surface does not have 4-D M^4 projection, one can choose a subset of H-coordinates as space-time coordinates. By its maximal symmetries H allows very limited set of preferred coordinates sothat the problems produced by general coordinate invariance are circumvented.

   In TGD framework the surface property of space-time realizes the Quantum Equivalence Principle a at the level of embedding space in the same way as isometries of the space-time as source of conservation laws are lifted to the level of the embedding space.

3. In quantum TGD, also zero energy ontology (ZEO) and causal diamond (CD) are basic notions. CD represents the perceptive field of a conscious entity. The notion of CD resembles the notion of causaloid. One can assign a CD to any quantum system, even elementary particle. CDs form an analog of an atlas consisting of charts and there is a fractal hierarchy with CDs inside CDs, and also overlapping CDs.

4. Zero energy states inside CDs represent particle states as extended objects rather than points. Zero energy states associated with CD correspond to superpositions of space-time surfaces identifiable as preferred extremals of an action principle deriving from the twistor lift of TGD. Minimal surfaces, which are also extermals of so called Kähler action, are in question - minimal surface property geometrizes the notion of massless field and extrelality also for Kähler poses extremely powerful additional conditions guaranteeing Bohr orbit like character of the space-time surface needed to realize general coordinate invariance.

   The important point is that the configurations AB and BA appearing in quantum switch corresponds to a space-time surface represents a branching of 3-surfaces representing photon propagation to two pieces at beam splitter and recombination back to single 3-surface making it possible for the photon wave functions interfere. Causal indefiniteness in the proposed sense does *not* mean that the direction of the causal arrow as an arrow of time is changed and in TGD framework it is not natural to speak about causal indefiniteness.

5. Concerning the understanding of the causality at quantum level in TGD Universe, induction is the key notion. All geometric structures are induced from those of H. This applies to metric, spinor connection, spinor structure, and twistor structure.

   In particular, second quantized spinor field of H is a superposition of modes of the massless Dirac operator of H, which can be solved explicitly and one can calculate Dirac propagator [L84]. The induction of the second quantized spinor field means a restriction to space-time surface and propagators at space-time surface are simply propagators in H. There are no problems with causality since it is induced from H to space-time surfaces.

9.8.3 Also a genuine change of the arrow of time is possible in ZEO

TGD however predicts a different kind of causal anomaly: the arrow of time can change and induce the change of the thermo-dynamical arrow of time [L64].

1. TGD predicts that two kinds of fermionic vacua corresponding intuitively to Dirac seas for which either all negative energy states or positive energy states are filled. They are present also in QFTs but one selects only the second one.

   The fermionic creation/annuhilation operators for the first vacuum act like annihilation/creation operators for the second vacuum. In ZEO these two fermionic vacua are associated with the opposite boundaries of CD.

2. Zero energy states as pairs of states assignable to the boundaries of CD. By conservation laws one can say that the total quantum numbers of CD vanish so that the total quantum numbers for the boundaries of CD are opposite.

3. In ZEO [L64, L91] there are two kinds of state function reductions (SFRs): "big" SFRs (BSFRs) as counterparts of ordinary SFRs and "small" SFRs (SSFRs) as counterparts of "weak" measurements as quantum analogs of classical measurements. In BSFRs the arrow of time changes and therefore cause and effect change their roles.

4. Either boundary of the CD is the passive boundary. Neither the passive boundary nor states at it change during the sequences of SSFRs. One can say that the Zeno effect is realized at the passive boundary.Active boundary recedes from the passive boundary in a sequence of scalings of CD followed by SSFRs preceded by unitary time evolutions. Therefore also the states at the active boundary change.

5. In BSFR the active boundary becomes passive and vice versa. The time reversal occurs for dark matter with $h_{eff} = nh_0$ residing at magnetic bodies, and since MB controls the dynamics of ordinary matter, BSFRs for MB induce effective change of the arrow of time for the ordinary matter in scales much longer than it would occur normally in BSFRs in the scale of microcosmos.

6. The change of the thermo-dynamical arrow of time changes in BSFR implies thermo-dynamical anomalies such as generation of gradients observed in systems with a reversed arrow of time [L61]. For instance, the time reversed system can effectively extract thermal energy from the environment. Actually this would be dissipation with a reversed arrow of time. Time reversal also makes self-organized quantum criticality (QSOC) possible [L123], and homeostasis could be seen as the biological manifestation of QSOC.

9.9 Appendix: A Generalization Of The Notion Of Embedding Space

In the following the recent view about structure of embedding space forced by the quantization of Planck constant is described. This view has developed much before the original version of this chapter was written.

The original idea was that the proposed modification of the embedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the embedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of embedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

9.9.1 Both Covering Spaces And Factor Spaces Are Possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, , where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. Zero energy ontology forces to modify this picture somewhat. In zero energy ontology causal diamonds (CDs) defined as the intersections of future and past directed light-cones are loci for zero energy states containing positive and negative energy parts of state at the two light-cone boundaries. The location of CD in M^4 is arbitrary but p-adic length scale hypothesis suggests that the temporal distances between tips of CD come as powers of 2 using CP_2 size as unit. Thus M^4 is replaces by CD and $\hat{M}^4$ is replaced with $\hat{CD}$ defined in obvious manner.

3. H_4 represents a straight cosmic string inside CD. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD} \times \hat{CP}_2$ implying that surfaces in $CD \times S^2$ and $(M^2 \cap CD) \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

4. The covering spaces in question would correspond to the Cartesian products $\hat{CD}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{CD}$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing $M^2 \cap CD$ and S^2 with its orbit under G_a (say tetrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD}\hat{\times}G_a$ *resp.* $\hat{CP}_2\hat{\times}G_b$.

5. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds $M^2 \cap CD$ or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds to exceptional groups in the ADE correspondence). For instance, in the case of $M^2 \cap CD$ the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6. Also the orbifolds $\hat{CD}/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$ and $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the embedding space to another one.

1. How the gluing of copies of embedding space at $(M^2 \cap CD) \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.
2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in CD degrees of freedom. This is not the case. Light-likeness in $(M^2 \cap CD) \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset (M^2 \cap CD) \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $(M^2 \cap CD) \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of $(M^2 \cap CD)$ as M^2 projection. Hence no sudden change of the size X^2 occurs.
3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

9.9.2 Do Factor Spaces And Coverings Correspond To The Two Kinds Of Jones Inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.
2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.
3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labeled by a subset of discrete subgroups of SU(2).
4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group

elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tetrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tetrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized WCW spinor fields in the WCW labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.

2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.

3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{H}t\hat{imes}(G_a \times G_b)$ option with obvious formulas for hybrid cases.

4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{H}t\hat{imes}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.

5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X) \to r(X)/n$ for factor space and $r(X) \to nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.

6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

9.9.3 A Simple Model Of Fractional Quantum Hall Effect

The generalization of the embedding space suggests that it could possible to understand fractional quantum Hall effect [D2] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned}\sigma &= \nu\times\frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{9.9.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...4/3, 7/5$ $1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [D2].

The model of Laughlin [D21] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [D13]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of embedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are four combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this there are two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. On the following just for fun consideration option I is considered although the conservation of number of states in the phase transition changing $\hbar$ favors option II.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. The appearance of $\nu = 5/2$ has been observed [D8]. The fractionized charge is $e/4$ in this case. Since $n_i > 3$ holds true if coverings are correlates for Jones inclusions, this requires to $n_b = 4$ and $n_a = 10$. n_b predicting a correct fractionization of charge. The alternative option would be $n_b = 2$ that also Z_2 would appear as the fundamental group of the covering space. Filling fraction $1/2$ corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [D13]. $n_b = 2$ is however inconsistent with the observed fractionization of electric charge and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e\int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F\int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6\times10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low

temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the identification of charge unit is rather ad hoc. Therefore this model can be taken only as a warm-up exercise. In [K87] Quantum Hall effect and charge fractionization are discussed in detail and one ends up with a rather detailed view about the delicacies of the Kähler structure of generalized embedding space.

Chapter 10

DNA as Topological Quantum Computer

10.1 Introduction

Large values of Planck constant makes possible all kinds of quantum computations [B4, B48, B11, B47]. What makes topological quantum computation (TQC) [B25, B45, B38, B29, B7] so attractive is that the computational operations are very robust and there are hopes that external perturbations do not spoil the quantum coherence in this case. The basic problem is how to create, detect, and control the dark matter with large $\hbar$. The natural looking strategy would be to assume that living matter, say a system consisting of DNA and cell membranes, performs TQC and to look for consequences.

There are many questions. How the TQC could be performed? Does TQC hypothesis might allow to understand the structure of living cell at a deeper level? What does this hypothesis predict about DNA itself? One of the challenges is to fuse the vision about living system as a conscious hologram with the DNA as TQC vision. The experimental findings of Peter Gariaev [I61, I69] might provide a breakthrough in this respect. In particular, the very simple experiment in which one irradiates DNA sample using ordinary light in UV-IR range and photographs the scattered light seems to allow an interpretation as providing a photograph of magnetic flux tubes containing dark matter. If this is really the case, then the bottle neck problem of how to make dark matter visible and how to manipulate it would have been resolved in principle. The experiment of Gariaev and collaborators [I69] also show that the photographs are obtained only in the presence of DNA sample. This leaves open the question whether the magnetic flux tubes associated with instruments are there in absence of DNA and only made visible by DNA or generated by the presence of DNA.

10.1.1 Basic Ideas Of TQC

The basic idea of topological quantum computation (TQC) is to code TQC programs to braiding patterns (analogous to linking and knotting). A nice metaphor for TQC is as dance. Dancing pattern in time direction defines the TQC program. This kind of patterns are defined by any objects moving around so that the Universe might be performing topological quantum computation like activities in all scales.

One assigns to the strands of the braid elementary particles. The S-matrix coding for TQC is determined by purely topological consideration as a representation for braiding operation. It is essential that the particles are in anyonic phase: this means in TGD framework that the value of Planck constant differs from its standard value. Tqc as any quantum computation halts in state function reduction which corresponds to the measurement of say spins of the particles involved.

As in the case of ordinary computers one can reduce the hardware to basic gates. The basic 2-gate is represented by a purely topological operation in which two neighboring braid strands are twisted by π. 1-particle gate corresponds to a phase multiplication of the quantum state associated with braid strand. This operation is not purely topological and requires large Planck constant to overcome the effects of thermal noise.

In TGD framework TQC differs somewhat from the ordinary one.

1. Zero energy ontology (ZEO) means that physical states decompose into pairs of positive and negative energy states at the "upper" and "lower" light-like boundaries boundaries of $CD \times CP_2$, where CD denotes causal diamond identified as the intersection of the future and past directed light-cones (in the sequel CD is used for $CD \times CP_2$ in order to make notations more elegant). Positive and negative energy states have opposite values of conserved quantum numbers. The interpretation is as an event, say particle scattering, in positive energy ontology. The time like entanglement coefficients define S-matrix, or rather M-matrix, and this matrix can be interpreted as coding for physical laws in the structure of physical state as quantum superposition of statements "A implies B" with A and B represented as positive and negative energy parts of quantum state. The halting of topological quantum computation would select this kind of statement.
2. The new view about quantum state as essentially 4-D notion implies that the outcome of TQC is expressed as a four-dimensional pattern at space-time sheet rather than as time=constant final state. All kinds of patterns would provide a representation of this kind. In particular, holograms formed by large $\hbar$ photons emitted by Josephson currents, including EEG as a special case, would define particular kind of representation of outcome.

10.1.2 Identification Of Hardware Of TQC And TQC Programs

One challenge is to identify the hardware of TQC and realization of TQC programs.

1. Living cell is an excellent candidate in this respect. The lipid layers of the cell membrane is 2-D liquid crystal and the 2-D motion of lipids would define naturally the braiding if the lipids are connected to DNA nucleotides. This motion might be induced by the self organization patterns of metabolically driven liquid flow in the vicinity of lipid layer both in interior and exterior of cell membrane and thus self-organization patters of the water flow would define the TQC programs.
2. This identification of braiding implies that TQC as dancing pattern is coded automatically to memory in the sense that lipids connected to nucleotides are like dancers whose feet are connected to the wall of the dancing hall define automatically space-like braiding as the threads connected to their feet get braided. This braiding would define universal memory realized not only as tissue memory but related also to water memory [?].
3. It is natural to require that the genetic code is somehow represented as property of braids strands. This is achieved if strands are "colored" so that A,T,C,G correspond to four different "colors". This leads to the hypothesis that flux tubes assignable to nucleotides are wormhole magnetic flux tubes such that the ends of the two sheets carry quark and antiquark (*resp.* antiquark and quark) quantum numbers. This gives mapping A,T,C,G to u, u_c, d, d_c. These quarks are not ordinary quarks but their scaled variants predicted by the fractal hierarchy of color and electro-weak physics. Chiral selection in living matter could be explained by the hierarchy of weak physics. The findings of topologist Barbara Shipman about mathematical structure of honeybee dance led her to proposed that the color symmetries of quarks are in some mysterious way involved with honeybee cognition and this model would justify her intuition [A25].
4. One should identify the representation of qubit. Ordinary spin is not optimal since the representation of 1-gates would require a modification of direction of magnetic field in turn requiring modification of direction of flux tubes. A more elegant representation is based on quark color which means effectively 3-valued logic: true, false, and undefined, also used in ordinary computers and is natural in a situation in which information is only partial. In this case 1-gates would correspond to color rotations for space-time sheets requiring no rotation of the magnetic field.

In this framework genes define the hardware of TQC rather than genetic programs. This means that the evolution takes place also at the level of TQC programs meaning that strict genetic determinism fails. There are also good reasons to believe that these TQC programs can be inherited to some degree. This could explain the huge differences between us and our cousins in spite of almost the identical genetic codes and explains also cultural evolution and the observation that our children seem to learn more easily those things that we have already learned [I82]. It must be added that DNA as TQC paradigm seems to generalizedDNA, lipids, proteins, water molecules,... can have flux tubes connecting them together and this is enough to generate braidings and TQC programs. Even water could be performing simple TQC or at least building memory representations based on braiding of flux tubes connecting water molecules.

10.1.3 How Much TQC Resembles Ordinary Computation?

If God made us to his own image one can ask whether we made computers images of ourselves in some respects. Taking this seriously one ends up asking whether facts familiar to us from ordinary computers and world wide web might have counterparts in DNA as TQC paradigm.

1. Can one identify program files as space-like braiding patterns. Can one differentiate between program files and data files?
2. In ordinary computers electromagnetic signalling is in key role. The vision about living matter as conscious holograms suggests that this is the case also now. In particular, the idea that entire biosphere forms a TQC web communicating electromagnetically information and control signals looks natural. Topological light rays (MEs) make possible precisely targeted communications with light velocity without any change in pulse shape. Gariaev's findings [I61] that the irradiation of DNA by laser light induces emission of radio wave photons having biological effects on living matter at distances of tens of kilometers supports this kind of picture. Also the model of EEG in which the magnetic body controls the biological body also from astrophysical distances conforms with this picture.
3. The calling of computer programs by simply clicking the icon or typing the name of program followed by return is an extremely economic ways to initiate complex computer programs. This also means that one can construct arbitrarily complex combinations from given basic modules and call this complex by a single name if the modules are able to call each other. This kind of program call mechanism could be realized at the level of TQC by DNA. Since the intronic portion of genome increases with the evolutionary level and is about 98 per cent for humans, one can ask whether introns would contain representations for names of program modules. If so, introns would express themselves electromagnetically by transcribing the nucleotide to a temporal pattern of electromagnetic radiation activating desired subprogram call, presumably the conjugate of intronic portion as DNA sequence. A hierarchical sequence of subprogram calls proceeding downwards at intronic level and eventually activating the TQC program leading to gene expression is suggestive.

   [I61] [I61] has found that laser radiation scattering from given DNA activates only genomes which contain an address coded as temporal pattern for the direction of polarization plane. If flux tubes are super-conducting and there is strong parity breaking (chiral selection) then Faraday rotation for photons traveling through the wormhole flux tube code nucleotide to an angle characterizing the rotation of polarization plane. User id and password would be kind of immune system against externally induced gene expression.
4. Could nerve pulses establish only the connection between receiver and sender neurons as long magnetic flux tubes? Real communication would take place by electromagnetic signals along the flux tube, using topological light ray (ME) attached to flux tube, and by entanglement. Could neural transmitters specify which parts of genomes are in contact and thus serve as a kind of directory address inside the receiving genome?

10.1.4 Basic Predictions Of DNA As TQC Hypothesis

DNA as TQC hypothesis leads to several testable predictions about DNA itself.

Anomalous em charge

The model for DNA as TQC assigns to flux tubes starting from DNA an anomalous em charge. This means that the total charge of DNA nucleotide using e as unit is $Q = -2 + Q(q)$, where -2 is the charge of phosphate group and $Q(q) = -/+2/3, +/-1/3$ is the electromagnetic charge of quark associated with "upper" sheet of wormhole magnetic flux tube. If the phosphate group is not present one has $Q = Q(q)$. In the presence of phosphate bonds the anomalous charge makes possible the coding of nucleotides to the rotation of angle of polarization plane resulting as photon travels along magnetic flux tube. The anomalous em charge should be visible as an anomalous voltage created by DNA. It would be relatively easy to test this prediction by using various kinds of DNA:s.

Does breaking of matter antimatter and isospin symmetries happen at the level of DNA and mRNA?

The nice feature of the model is that it allows to interpret the slightly broken A-G and T-C symmetries of genetic code with respect to the third nucleotide Z of codon XYZ in terms of the analog of strong isospin symmetry at quark level at wormhole magnetic flux tubes. Also matter-antimatter dichotomy has a chemical analog in the sense that if the letter Y of codon corresponds to quark u, d (antiquark u_c, d_c), the codon codes for hydrophobic (hydrophilic) amino-acid. It is also known that the first letter X of the codon codes for the reaction path leading from a precursor to an amino-acid. These facts play a key role in the model for code of protein folding and catalysis. The basic assumption generalizing base pairing for DNA nucleotides is that wormhole flux tubes can connect an amino-acid inside protein only to molecules (amino-acids, DNA, mRNA, or tRNA) for which Y letter is conjugate to that associated with the amino-acid. This means that the reduction of Planck constant leading to the shortening of the flux tube can bring only these amino-acids together so that only these molecules can find each other in biocatalysis: this would mean kind of code of bio-catalysis.

The fact that matter-antimatter and isospin symmetries are broken in Nature suggests that the same occurs at the level of DNA for quarks and anti-quarks coding for nucleotides. One would expect that genes and other parts of genome differ in the sense that the anomalous em charge, isospin, and net quark number (vanishes for matter antimatter symmetric situation) differ for them. From Wikipedia [I85] one learns that there are rules about distribution of nucleotides which cannot be understood on basis of chemistry. The rules could be understood in terms of new physics. Chargaff's rules state that these symmetries hold true in one per cent approximation at the level of entire chromosomes. Szybalski's rules [I85] state that they fail for genes. There is also a rule stating that in good approximation both strands contain the same portion of DNA transcribed to mRNA. This implies that at mRNA level the sign of matter antimatter asymmetry is always the same: this is analogous to the breaking of matter antimatter asymmetry in cosmology (only matter is observed).

It would be interesting to study systematically the breaking of these symmetries for a sufficiently large sample of genes and also other in parts of genome where a compensating symmetry breaking must occur. That the irradiation of DNA by laser light induces emission of radio wave photons having biological effects on living matter at distances of tens of kilometers supports this kind of picture. Also the model of EEG in which magnetic body controls biological body from astrophysical distances conforms with this picture.

It must be emphasized that this model of DNA as TQC is only one possibility. There is large flexibility concerning the identification of fermions involved. For instance A,T,C,G could be represented also in terms of 4 states assignable to two spin half fermions at parallel flux tubes. This would give rise to high T_c superconductor with both $S = 0$ ($S = 1$) Cooper pairs assignet to flux tubes with opposite (parallel) magnetic fields. The spin-spin interaction energy for the Cooper pair would be negative and proportional to h_{eff} and same for all fermion pairs if $h_{eff} = h_{gr}$ hypothesis holds true at microscopic level.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

10.2 Basic Concepts And Ideas

The following represents a brief overall view about the notions of quantum jump, unitary process described by unitary U-matrix between zero energy states having as its orthogonal rows M-matrices between positive and negative energy parts of zero energy states identifiable as counterpart of ordinary S-matrix and of Negentropy Maximization Principle (NMP) governing the dynamics of state function reduction cascade.

10.2.1 What Happens In Quantum Jump?

Quantum jump involves U process and state function reduction cascade. Negentropy Maximization Principle implies second law for the standard view about state function reduction: second law states that the ensemble entropy increases by the randomness of the outcome of the state function reduction process. When negentropic entanglement possible in what might be called intersection of the real and various p-adic worlds is present the situation is not so clear. Before proceeding to consider the modification of the second law one must define more precisely what U process is.

The simplest view about quantum jump is as a unitary U-process followed by as a cascade of state function reductions proceeding from top to bottom. But what is the top?

1. In positive energy ontology it would be entire Universe. Quantum classical correspondence suggests that one should be able to assign to quantum jump a duration of geometric time. For this proposal this time is most naturally infinite.
2. The vision about fractal hierarchy of selves and quantum jumps together with ZEO suggests a more refined view about quantum jump in which. U-process and subsequent state function reduction cascade could occur independently for disjoint CDs. For a given CD the new sub-CDs (representing mental images of the corresponding self) can be created and old destroyed so that the only constraint would be that only disjoint CDs can perform quantum jumps independently. For this option the duration of geometric time assignable to the quantum jump would naturally correspond to the temporal distance between the tips of CD: p-adic length scale hypothesis and number theoretical vision suggest that this distance comes as an octave of CP_2 time scale (prime or integer multiple is the more general option). For infinitely large CD this would mean infinite duration. This picture is consistent with the TGD view about how the arrow of subjective time induces the arrow of geometric time [K9].

10.2.2 M-Matrix

The unitary U-matrix characterizing the unitary process has as its rows orthogonal M-matrices characterized by in general non-unitarity M-matrices. M-matrix decomposes into a product of positive definite diagonal square roots of density matrix and unitary S-matrix measurement in particle physics experiment. M-matrix represents both the time-like entanglement between positive and negative energy parts of zero energy states with opposite quantum numbers and space-like entanglement for the positive and negative energy states.

Time-like and space-like entanglement in zero energy ontology

M-matrix for each summand is product of Hermitian square root of density matrix and unitary S-matrix multiplied by a square root of probability having interpretation as analog for Boltzmann weight or probability defined by density matrix (note that it is essential to have $Tr(Id) = 1$ for factors of type II_1. If factor of type I_∞ are present situation is more complex. This means that quantum computations are highly universal and M-matrices are characterized by the inclusion $\mathcal{N} \subset \mathcal{M}$ in each summand defining measurement resolution. Hermitian elements of $\mathcal{N}$ act as symmetries of M-matrix. The identification of the reducible entanglement characterized by Boltzmann weight like parameters in terms of thermal equilibrium would allow to interpret quantum theory as square root of thermodynamics.

If the entanglement probabilities defined by S-matrix and assignable to $\mathcal{N}$ rays do not belong to the algebraic extension used then a full state function reduction is prevented by NMP. Ff the generalized Boltzmann weights are also algebraic then also thermal entanglement is irreducible. In p-adic thermodynamics for Virasoro generator L_0 and using some cutoff for conformal weights the Boltzmann weights are rational numbers expressible using powers of p-adic prime p.

Effects of finite temperature

Usually finite temperature is seen as a problem for quantum computation. In TGD framework the effect of finite temperature is to replace zero energy states formed as pairs of positive and negative energy states with a superposition in which energy varies.

One has an ensemble of space-time sheets which should represent nearly replicas of the quantum computation. There are two cases to be considered.

1. If the thermal entanglement is reducible then each space-time sheet gives outcome corresponding to a well defined energy and one must form an average over these outcomes.
2. If thermal entanglement is irreducible each space-time sheet corresponds to a quantum superposition of space-time sheets, and if the outcome is represented classically as rates and temporal field patterns, it should reflect thermal average of the outcomes as such.

If the degrees of freedom assignable to topological quantum computation do not depend on the energy of the state, thermal width does not affect at all the relevant probabilities. The probabilities are actually affected even in the case of TQC since 1-gates are not purely topological and the effects of temperature in spin degrees of freedom are unavoidable. If T grows the probability distribution for the outcomes flattens and it becomes difficult to select the desired outcome as that appearing with the maximal probability.

10.2.3 About NMP And Quantum Jump

NMP is assumed to be the variational principle telling what can happen in quantum jump and says that the information content of conscious experience for the entire system is maximized. In zero energy ontology (ZEO) the definition of NMP is far from trivial and the recent progress - as I believe - in the understanding of structure of quantum jump forces to check carefully the details related to NMP. A very intimate connection between quantum criticality, life as something in the intersection of realities and p-adicities, hierarchy of effective values of Planck constant, negentropic entanglement (NE), and p-adic view about cognition emerges. One ends up also with an argument why p-adic sector is necessary if one wants to speak about conscious information. I will proceed by making questions.

What happens in single state function reduction?

State function reduction is a measurement of density matrix. The condition that a measurement of density matrix takes place implies standard measurement theory on both real and p-adic sectors: system ends to an *eigen-space* of density matrix. This is true in both real and p-adic sectors. NMP is stronger principle at the real side and implies state function reduction to 1-D subspace - its eigenstate.

The resulting N-dimensional space has however rational entanglement probabilities $p = 1/N$ so that one can say that it is the intersection of realities and p-adicities. If the number theoretic variant of entanglement entropy is used as a measure for the amount of entropy carried by entanglement rather than either entangled system, the state carries genuine information and is stable with respect to NMP if the p-adic prime p divides N. NMP allows only single p-adic prime for real $\rightarrow$ p-adic transition: the power of this prime appears is the largest power of prime appearing in the prime decomposition of N. Degeneracy means also criticality so that ordinary quantum measurement theory for the density matrix favors criticality and NMP fixes the p-adic prime uniquely.

If one - contrary to the above conclusion - assumes that NMP holds true in the entire p-adic sector, NMP gives in p-adic sector rise to a *reduction* of the negentropy in state function reduction if the original situation is negentropic and the eigen-spaces of the density matrix are 1-dimensional. This situation is avoided if one assumes that state function reduction cascade in real or genuinely p-adic sector occurs first (without NMP) and gives therefore rise to N-dimensional eigen spaces. The state is negentropic and stable if the p-adic prime p divides N. Negentropy is generated.

The real state can be transformed to a p-adic one in quantum jump (defining cognitive map) if the entanglement coefficients are rational or belong to an algebraic extension of p-adic numbers in the case that algebraic extension of p-adic numbers is allowed (number theoretic evolution gradually generates them). The density matrix can be expressed as sum of projection operators multiplied by probabilities for the projection to the corresponding sub-spaces. After state function reduction cascade the probabilities are rational numbers of form $p = 1/N$.

Number theoretic entanglement entropy also allows to avoid some objections related to fermionic and bosonic statistics. Fermionic and bosonic statistics require complete anti-symmetrization/symmetrization. This implies entanglement which cannot be reduced away. By looking for symmetrized or antisymmetrized 2-particle state consisting of spin 1/2 fermions as the simplest example one finds that the density matrix for either particle is the simply unit 2×2 matrix. This is stable under NMP based on number theoretic negentropy. One expects that the same result holds true in the general case. The interpretation would be that particle symmetrization/antisymmetrization carries negentropy.

The degeneracy of the density matrix is of course not a generic phenomenon and one can argue that it corresponds to some very special kind of physics. The identification of space-time correlates for the hierarchy for the effective values $\hbar_{eff} = n\hbar$ of Planck constant as n-furcations of space-time sheet suggests strongly the identification of this physics in terms of this hierarchy. Hence quantum criticality, the essence of life as something in the rational intersection of realities and p-adicities, the hierarchy of effective values of $\hbar$, negentropic quantum entanglement, and the possibility to make real-p-adic transitions and thus cognition and intentionality would be very intimately related. This is a highly satisfactory outcome, since these ideas have been rather loosely related hitherto.

What happens in quantum jump?

Suppose that everything can be reduced to what happens for a given CD characterized by a scale. There are at least two questions to be answered.

1. There are two processes involved. State function reduction and quantum jump transforming real state to p-adic state (matter to cognition) and vice versa (intention to action). Do these transitions occur independently or not? Does the ordering of the processes matter? It has turned out that the mathematical realization of this picture is very difficult and that these transformations are not even needed in the adelic vision where cognitionandsensory aspects realized as p-adic and real space-time sheets are both present in all scales.

2. State function reduction cascade in turn consists of two different kinds of state function reductions. The M-matrix characterizing the zero energy state is product of square root of density matrix and of unitary S-matrix and the first step means the measurement of the projection operator. It defines a density matrix for both upper and lower boundary of CD and these density matrices are essentially same.

   (a) At the first step a measurement of the density matrix between positive and negative energy parts of the quantum state takes place for CD. One can regard both the lower and upper boundary as an eigenstate of density matrix in absence of NE. The measurement is thus completely symmetric with respect to the boundaries of CDs. At the real sector this leads to a 1-D eigen-space of density matrix if NMP holds true. In the intersection of real and p-adic sectors this need not be the case if the eigenvalues of the density matrix have degeneracy. Zero energy state becomes stable against further state function reductions! The interactions with the external world can of course destroy the stability sooner or later. An interesting question is whether so called higher states of consciousness relate to this kind of states.

   (b) If the first step gave rise to 1-D eigen-space of the density matrix, a state function reduction cascade at either upper of lower boundary of CD proceeding from long to short scales. At given step divides the sub-system into two systems and the sub-system-complement pair which produces maximum negentropy gain is subject to quantum measurement maximizing negentropy gain. The process stops at given subsystem resulting in the process if the resulting eigen-space is 1-D or has NE (p-adic prime p divides the dimension N of eigenspace in the intersection of reality and p-adicity).

10.2.4 Hyper-Finite Factors Of Type Ii_1 And Quantum Measurement Theory With A Finite Measurement Resolution

The realization that the von Neumann algebra known as hyper-finite factor of type II_1 is tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

Hyper-finite factor of type II_1 has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW . Also the local Clifford algebra of the embedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the embedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in zero energy ontology states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.
2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.
3. This leads also to the notion of quantum group. For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.
4. The realization for quantum measurement theory modulo finite measurement resolution is in terms of M-matrices defined in terms of Connes tensor product which essentially means that the included hyper-finite factor N takes the role of complex num bers.

As already explained, the topology of the many-sheeted space-time encourages the generalization of the notion of quantum entanglement in such a way that unentangled systems can possess entangled sub-systems. One can say that the entanglement between sub-selves is not visible in the resolution characterizing selves. This makes possible sharing and fusion of mental images central for TGD inspired theory of consciousness. These concepts find a deeper justification from the quantum measurement theory for hyper-finite factors of type II_1 for which the finite measurement resolution is basic notion.

Also the notions of resolution and monitoring pop up naturally in this framework. p-Adic probabilities relate very naturally to hyper-finite factors of type II_1 and extend the expressive power of the ordinary probability theory. p-Adic thermodynamics with conformal cutoff is very natural for hyper-finite factors of type II_1 and explains p-adic length scale hypothesis $p \simeq 2^k$, k prime characterizing exponentially smaller p-adic length scale.

10.2.5 NMP And Biology

The notion of self is crucial for the understanding of bio-systems and consciousness. It seems that the negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) is the decisive element of life and that one can say that in metaphorical sense life resides in the intersection of real and p-adic worlds.

Generalization of the notion of information

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In

TGD state preparation process is realized as a sequence of self measurements and state preparation for next quantum jump is state reduction for the previous quantum jump. In zero energy ontology one can interpret the state preparation for positive and negative energy parts of the state as reduction and preparation in the sense of standard physics. Each self measurement means a decomposition of the sub-system involved to two unentangled parts unless the system is bound state. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

Bound state entanglement is stable against self measurement simply because energy conservation prevents the decay to a pair of free (uncorrelated) subsystems. The generalized definition of entanglement entropy allows to assign a negative value of entanglement entropy to rational and algebraic entanglement, so that this kind of entanglement would actually carry information, in fact conscious information (experience of understanding). This kind of entanglement cannot be reduced in state function reduction. Macro-temporal quantum coherence could correspond to a generation of either bound state entanglement or negentropic entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images. Generation of negentropic entanglement would involve experience about expansion of consciousness and that of bound states entanglement a loss of consciousness.

The mathematical models for quantum computers typically operate with systems for which entanglement probabilities are identical. Also rational numbers are involved. Does this mean that negentropic entanglement makes possible quantum computation? This does not seem to be the case. State function reduction with random outcomes is a central element of quantum computation which suggests that quantum computation must be based on entropic entanglement with large enough value of $\hbar$ to overcome the restrictions caused by the interactions with the external world. The negentropic entanglement in turn would relate to conscious information processing involving experience of understanding represented by negentropic entanglement. Negentropic entanglement would make possible conscious cellular automaton type information processing much closer to that carried out by ordinary computers and this information processing might be equally important in living systems.

Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

NMP and negentropic entanglement demanding entanglement probabilities which are equal to inverse of integer, is the starting point. Rational and even algebraic entanglement coefficients make sense in the intersection of real and p-adic words, which suggests that in some sense life and conscious intelligence reside in the intersection of the real and p-adic worlds.

What could be this intersection of realities and p-adicities?

1. The facts that fermionic oscillator operators are correlates for Boolean cognition and that induced spinor fields are restricted to string world sheets and partonic 2-surfaces suggests that the intersection consists of these 2-surfaces.
2. Strong form of holography allows a rather elegant adelization of TGD by a construction of space-time surfaces by algebraic continuations of these 2-surfaces defined by parameters in algebraic extension of rationals inducing that for various p-adic number fields to real or p-adic number fields. Scattering amplitudes could be defined also by a similar algebraic contination. By conformal invariance the conformal moduli characterizing the 2-surfaces would defined the parameters.

This suggests a rather concrete view about the fundamental quantum correlates of life and intelligence.

1. For the minimal option life would be effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests. There are good reasons to expect that only the data from the intersection of real and p-adic string world sheets partonic two-surfaces appears in U-matrix so that the data localizable to strings connecting partonic 2-surfaces would dictate the scattering amplitudes.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving [K4]. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua. Life as a critical phenomenon in the number theoretical sense would be one aspect of quantum criticality of TGD Universe besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K97].

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

Later progress in understanding of quantum TGD allows to refine and simplify this view dramatically. The idea about p-adic-to-real transition for space-time sheets as a correlate for the transformation of intention to action has turned out to be un-necessary and also hard to realize mathematically. In adelic vision real and p-adic numbers are aspects of existence in all length scales and mean that cognition is present at all levels rather than emerging. Intentions have interpretation in terms of state function reductions in ZEO and there is no need to identify p-adic space-time sheets as their correlates.

That only algebraic extensions are possible is of course only a working hypothesis. Also finite-dimensional extensions of p-adic numbers involving transcendentals are possible and might in fact be necessary. Consider for instance the extension containing $e, e^2, .., e^{p-1}$ as units (e^p is ordinary p-adic number. Infinite number of analogous finite-dimensional extensions can be constructed by taking a function of integer variable such that $f(p)$ exists both p-adically and as a real transcendental number. The powers of $f(p)^{1/n}$ for a fixed value of n define a finite-dimensional transcendental extension of p-adic numbers if the roots do not exist p-adically.

Numbers like $log(p)$ and π cannot belong to a finite-dimensional extension of p-adic numbers [K50]. One cannot of course take any strong attitude concerning the possibility of infinite-dimensional extensions of p-adic numbers but the working hypothesis has been that they are absent. The phases $exp(i2\pi/n)$ define finite dimensional extensions allowing to replace the notion of angle in finite measurement resolution with the corresponding phase factors in finite measurement. The functions $exp(i2\pi q/n)$, where q is arbitrary p-adic integers define in a natural manner the physical counterparts of plane waves and angular momentum eigenstates not allowing an identification as ordinary p-adic exponential functions. They are clearly strictly periodic functions of q with a finite value set. If n is divisible by a power of p, these functions are continuous since the values of the function for q and $q + kp^n$ are identical for large enough values of n. This condition is essential and means in the case of plane waves that the size scale of a system (say one-dimensional box) is multiple of a power of p.

Evolution and second law

Evolution has many facets in TGD framework.

1. A natural characterization of evolution is in terms of p-adic topology relating naturally to cognition. p-Adic primes near powers of two are favored if CDs have the proposed discrete size spectrum. From the point of view of self this would be essentially cosmic expansion in discrete jumps. CDs and can be characterized by powers of 2 and if partonic 2-surfaces correspond to effective p-adic p-adic topology characterized by a power of two, one obtains

the commeasurability of the secondary p-adic time scale of particle and that of CD in good approximation.

2. The notion of infinite primes motivates the hypothesis that the many-sheeted structure of space-time can be coded by infinite primes [K107]. The number of primes larger than given infinite prime P is infinitely larger than the number of primes than P. The infinite prime P characterizing the entire universe decomposes in a well defined manner to finite primes and p-adic evolution at the level of entire universe is implied by local p-adic evolution at the level of selves. Therefore maximum entanglement negentropy gain for p-adic self increases at least as $log(p)$ with p in the long run. This kind of relationship might hold true for real selves of p-adic physics is physics of cognitive representations of real physics as suggested by the success of p-adic mass calculations. Thus it should be possible to assign definite p-adic prime to each partonic 2-surface.

3. A further aspect of evolution relates to the hierarchy of Planck constants implying that at dark matter levels rational or at least integer multiples of the favored p-adic time scales are realized. The latter option is favored by the idea that the book like structure with pages consisting of many-sheeted coverings of CD and CP_2, and correlates with the emergence of algebraic extensions of p-adic numbers defined by the roots $exp(i2\pi/n)$ of unity. For the latter option evolution by quantum jumps would automatically imply the drifting of the partonic 2-surfaces to the pages of books labelled by increasing values of Planck constant. For more general option one might argue that drifting to pages with small values of Planck constant is also possible. This would give kind of antizooms of long length scale physics to short scales. Both kind of temporal zooms could be crucial for conscious intelligence building scaled models about time evolution in various scales.

4. The generation of negentropic entanglement between different number fields would of course be the fundamental aspect of evolution. It would give rise to increasingly complex and negentropic sensory perceptions and cognitive representations based on conscious rules coded by negentropic entanglement. This would justify the association concept as it used in neuro-science. Negentropic entanglement could be also crucial for the basic mechanism of metabolism and make possible conscious co-operation even in nano-scales.

Just for fun one can play also with numbers.

1. The highest dark matter level associated with self corresponds to its geometric duration which can be arbitrarily long: the typical duration of the memory span gives an idea about the level of dark matter hierarchy involved if one assumes that the time scale.1 seconds assignable to electrons is the fundamental time scale. If the time scale T of human life cycle corresponds to a secondary p-adic time scale then $T = 100$ years gives the rough estimate $r \equiv \hbar/\hbar_0 = 2^{33}$ if this time scale corresponds to that for dark electron. The corresponding primary p-adic time length scale corresponds to $k = 160$ and is 2.2×10^{-7} meters.

2. If human time scale -taken to be $T = 100$ years- corresponds to primary p-adic time scale of electron, one must have roughly $r = 2^{97}$.

I have already discussed the second law in TGD framework and it seems that its applies only when the time scale of perception is longer than the time scale characterizing the level of the p-adic and dark matter hierarchy. Second law as it is usually stated can be seen as an unavoidable implication of the materialistic ontology.

Stable entanglement and quantum metabolism as different sides of the same coin

The notion of binding has two meanings. Binding as a formation of bound state and binding as a fusion of mental images to larger ones essential for the functioning of brain and regarded as one the big problems of consciousness theory.

Only bound state entanglement and negentropic entanglement are stable against the state reduction process. Hence the fusion of the mental images implies the formation of a bound entropic state- in this case the two interpretations of binding are equivalent- or a negentropic state, which need not be bound state.

1. In the case of negentropic entanglement bound state need not be formed and the interesting possibility is that the negentropic entanglement could give rise to stable states without binding energy. This could allow to understand the mysterious high energy phosphate bond to which metabolic energy is assigned in ATP molecule containing three phosphates and liberated as ATP decays to ADP and phosphate molecule. Negentropic entanglement could also explain the stability of DNA and other highly charged biopolymers. In this framework the liberation of metabolic (negentropic) energy would involve dropping of electrons to a larger space-time sheets accompanying the process $ATP \rightarrow ADP + P_i$. A detailed model of this process is discussed in [?].

   In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

2. The formation of bound state entanglement is expected to involve a liberation of the binding energy and this energy might be a usable energy. This process could perhaps be coined as quantum metabolism and one could say that quantum metabolism and formation of bound states are different sides of the same coin. It is known that an intense neural activity, although it is accompanied by an enhanced blood flow to the region surrounding the neural activity, does not involve an enhanced oxidative metabolism [J14] (that is $ATP \rightarrow ADP$ process and its reversal). A possible explanation is that quantum metabolism accompanying the binding is involved. Note that the bound state is sooner or later destroyed by the thermal noise so that this mechanism would in a rather clever manner utilize thermal energy by applying what might be called buy now–pay later principle.

If these interpretations are correct, there would be two modes of metabolism corresponding to two different kinds of fusion of mental images.

10.2.6 Generalization Of Thermodynamics Allowing Negentropic Entanglement And A Model For Conscious Information Processing

The possibility of negetropic entanglement in TGD framework means that the second law of thermodynamics must be modified. The most obvious modification means only the replacement $S \rightarrow S - N$, where S is thermodynamical entropy and N the negentropy associated with negentropic entanglement. Hence the basic formulas of thermodynamics remain formally as such. The generalization leads to a thermodynamical model for how conscious information is generated and how metabolism relates to this. One can also understand why living matter is so effective entropy producer as compared to inanimate matter and the characteristic decomposition of living systems to highly negentropic and entropic parts.

A pessimistic modification of thermodynamics to take into account negentropic entanglement

What does the presence of the negentropic entanglement mean from the point of view of thermodynamics? There are two obvious options to consider. The optimistic option is just the standard thermodynamics saying nothing about negentropy generation. Indeed, number theoretic entanglement negentropy characterizes information carried by entanglement rather than ignorance about individual entangled state and is therefore not identifiable as thermodynamics entropy. The pessimistic option is that the generation of negentropy must be accompanied by a generation of at least the same amount of entropy: the good news is that this entropy can be carried by different system and it is possible to have genuinely negentropic systems.

The following consideration is restricted to the pessimistic option. Some-one might argue that this provides a more realistic view about the world we live in. One must however remember that evolution is an empirical fact as also dark matter about which we know practically nothing. Furthermore, I am unable to image a concrete mechanism guaranteeing that the generation of negentropic entanglement would be accompanied by a generation of ensemble entropy. The two negentropies are indeed different although there exists a connection. A large degeneracy of states to the geometric realization of $h_{eff} = nh$ hierarchy, implies a large thermodynamical entropy. It also makes possible negentropic entanglement with large number theoretical entanglement negentropy. Therefore large entropic resources implied degeneracy of states can be transformed to large negentropic resources by NMP. Dirt can be transformed to jewels (by love for which negentropic entanglement could serve as a quantum correlate!).

Second ZEO based prediction is that the arrow of thermodynamical time defined by the asymmetry between positive and negative energy parts of zero energy states alternates. There is evidence for this kind of alternation and this lead Fantappie to introduce the notion of syntropy long time ago [J16]. This aspect is not considered in the sequel.

1. One must generalize the basic expression for energy differential

$$dE = TdS - dW \rightarrow T(dS - dN) - dW \ . \tag{10.2.1}$$

This means that there are two kinds of energies given out by the system. The useful work dW and negentropic energy TdN. For steam engine only dW is present. For ideal system only negentropic energy would be present.

2. What happens to the second law? The pessimistic guess is that generation of negentropy requires a generation of at least same amount of entropy so that one would have

$$\Delta S - \Delta N \geq 0 \ . \tag{10.2.2}$$

Here S can be interpreted as a sum of two terms. The first part corresponds to the ensemble entropy generated by the randomness of ordinary quantum jumps, and second part to the entropy assignable as maximal entanglement entropy assignable to the decompositions of bound state to two parts. N corresponds to maximal negentropy for the decompositions of negentropic sub-system to pairs. One can criticize these definitions and a possible modification of could be as as the average for the entanglement entropies over this kind of decompositions.

3. Quite generally, Clausius inequality allowing to deduce extremization conditions for various thermodynamical potentials generalizes to

$$T_0(\Delta S - \Delta N) - \Delta E - P_0\Delta V \geq 0 \ . \tag{10.2.3}$$

where T_0 and P_0 and temperature and pressure of heat bath. Living systems would be entropy producers and this seems to conform with what we see around us.

For instance, for a system in constant volume one would have

$$\Delta S - \Delta N - \frac{\Delta E}{T} \geq 0 \ . \tag{10.2.4}$$

so that systems developing negentropy would also generate thermodynamics entropy. For a system in heat bath one has $T = T_0$ and Clausius inequality gives

$$\Delta F = -\Delta W \tag{10.2.5}$$

stating that increase of free energy at constant temperature requires work done on the system ($dW < 0$): otherwise $\Delta F \leq 0$ holds true.

By using the variable $S - N$ instead of S all formulas reduce formally to standard thermodynamics except that S can be negative.

The analog of Carnot cycle as a simple model for information processing in living matter

Carnot engine transforms heat to work. Costa de Beauregard [J9], [J9] has proposed a modification of Carnot engine as a model for information processing. One can consider Carnot engine and its information theoretic analog in this framework.

1. The basic equation for Carnot engine is

$$dW = dQ_{in} - dQ_{out} \geq 0 \ . \tag{10.2.6}$$

   Optimal efficiency corresponds to $dS_{out} = dS_{in}$.

2. The information theoretic analog of Carnot engine proposed by Beauregard does not perform work and one would have

$$dW = 0 \ , \tag{10.2.7}$$

   and

$$dN = dS_{out} - dS_{in} \geq 0 \ . \tag{10.2.8}$$

   The interpretation would be that incoming entropy flow leaves the computer in a state of higher entropy and the difference corresponds to information dN fed to say printer. The increase of entropy would have interpretation in terms of erasing of data from computer memory.

   The problematic aspect of the model is that it requires $T_{in} > T_{out}$ in order to have $dN > 0$. For living systems one has however typically $T_{in} < T_{out}$. Already for $T_{in} = T_{out}$ the situation trivializes since one has

$$dN = 0 \tag{10.2.9}$$

   by $dW = 0$ and $dS = dQ/T$.

3. In the recent case however a more general condition

$$T_{in}d(S_{in} - N_{in}) - T_{out}d(S_{out} - N_{out}) \geq 0 \tag{10.2.10}$$

   holds true and allows to generate conscious information provided it is compensated by thermodynamical entropy. Note that the temperature of the environment can be even lower than the temperatures of the system.

   It is also possible to transform information to work as the expression for the differential $dF = -SdT - TdN - dW$ of the generalized free energy $E = E - TS$ shows. The increase of dW for the work done by the system is compensated by the reduction of information dN so that system loses negentropy in the process keeping dF constant. The loss of negentropy couild be interpreted in terms of a loss of metabolic energy which corresponds to negentropic entanglement for AMP, ADP, and ATP molecules.

Basic biological implications

Some clarifying comments about biological implications are in order.

1. There is no need to restrict the consideration to equilibrium systems. First of all, the environment and living system are in general at different temperatures and temperature difference is typically of wrong sign for the model of Beauregard to work in this context. Beauregard's model is of course a model for computation, not for the generation of negentropic mental images. Maybe cognitive machine might be proper term for what the modified model could describe.

2. Quite generally, self-organization requires a feed of energy to the system so that one has flow equilibrium. In the case of living system this feed of energy is metabolic energy associated with the negentropic entanglement transferred to the system in the ATP-ADP process. Self-organization driven by negentropic entanglement leads to standardized negentropic mental images automatically as asymptotic self-organization patterns in 4-D sense (CDs within CDs within... : CD denotes causal diamond defined as cartesian produc to the intersection of the future and past directed light-cones with CP_2, which is the key notion in zero energy ontology).

3. No explicit assumptions about computational aspects of the process has been made. Just a generation of conscious information identified in terms of negentropic entanglement is assumed. The basic character quantum jump as U-process followed by the cascade of state function reductions represents a fractal hierarchy of what can be seen as quantum computations and are distinguished from classical computations in that the process proceeds from top to bottom rather than being a local process. The result of computation is represented using statistical ensembles defined by sub-CDs at various levels of the hierarchy and is in principle communicable by classical fields (say EEG patterns in the case of brain) to higher levels of self hierarchy which in turn can induces the same distributions so that communication of the objective aspects of the experience with the mediation of "medium" is possible. The presence of the "medium" seems unavoidable. Magnetic body would be this medium in TGD inspired biology.

Living matter involves also another aspect made possible by the generalized second law obtained by the replacement $S \rightarrow S - N$. Subsystem can have also negative net entropy and split to two highly negentropic and entropic pieces. In the extreme situation this is nothing but excretion, which is absolutely essential element of being alive but sometimes forgotten from the lists of properties distinguishing living matter from inanimate matter. It is not at all clear whether this is possible for standard non-equilibrium systems defining information as a reduction of disorder. At all levels of the fractal hierarchy division into negentropic and entropic subsystems is expected.

This picture seems to be in accordance with basic chemistry of energy metabolism.

1. The process creating both negentropy and entropy would be standardized in living matter and mean a generation of high energy phosphate bonds assignable to AMP, ADP, and ATP containing 1, 2, and 3 phosphates respectively besides the sugar residue. Sugar residue is basic nutrient and would provide the stored metabolic energy transformed to the negentropic energy of the high energy phosphate bonds if the proposed view is correct. Also other DNA nucleotides such as G can appear besides A but in metabolism A has a preferred role.

2. The basic metabolic cycle provides ADP with an additional phosphate energizing it to ATP and the reverse process transfers the metabolic energy and also negentropic entanglement to the acceptor molecule. Also ADP can provide metabolic energy by transforming to AMP when ATP is not available in sufficient amounts. That the catabolism of AMP creates urea excreted out of the system fits with the general picture. The catabolism for nutrients would create the entropy compensating for the negentropy of the high energy phosphate bonds.

3. The backbone of DNA is made of sugar and phosphate residues and corresponds to a sequence of XMP, $X = A, T, C, G$ with each XMP presumably containing single high energy phosphate bond serving as a storage or potential source of negentropy. This conforms with the view that DNA carries conscious information.

Negentropic and entropic entanglement are assumed to generate mental images with opposite emotional colors. This connects information processing with emotions. From neuroscience point of view this is not a news: peptides are molecules of emotions on one hand and molecules of information on the other hand [J8]. The well-known specialization of the left and right hand sides of the amygdala to experience positive and negatively colored emotions could be seen as one instance of this connection and representing also an example about fractal negentropic-entropic differentiation.

10.3 How Quantum Computation In TGD Universe Differs From Standard Quantum Computation?

Many problems of quantum computation in standard sense might relate to a wrong view about quantum theory. If TGD Universe is the physical universe, the situation would improve in many respects. There is the new fractal view about quantum jump and observer as "self" ; there is p-adic length scale hierarchy and hierarchy of Planck constants as well as self hierarchy; there is a new view about entanglement and the possibility of irreducible entanglement carrying genuine information and making possible quantum superposition of fractal quantum computations and quantum parallel dissipation; there is zero energy ontology, the notion of M-matrix allowing to understand quantum theory as a square root of thermodynamics, the notion of measurement resolution allowing to identify M-matrix in terms of Connes tensor product; there is also the notion of magnetic body providing one promising realization for braids in TQC, etc... This section gives a short summary of these aspects of TGD.

There is also a second motivation for this section. Quantum TGD and TGD inspired theory of consciousness involve quite a bundle of new ideas and the continual checking of internal consistency by writing it through again and again is of utmost importance. This section can be also seen as this kind of checking. I can only represent apologies to the benevolent reader: this is a work in rapid progress.

10.3.1 General Ideas Related To Topological Quantum Computation

Topological computation relies heavily on the representation of TQC program as a braiding. There are many kinds of braidings. Number theoretic braids are defined by the orbits of minima of vacuum expectation of Higgs at light-like partonic 3-surfaces (and also at space-like 3-surfaces). There are braidings defined by Kähler gauge potential (possibly equivalent with number theoretic ones) and by Kähler magnetic field. Magnetic flux tubes and partonic 2-surfaces interpreted as strands of define braidings whose strands are not infinitely thin. A very concrete and very complex time-like braiding is defined by the motions of people at the surface of globe: perhaps this sometimes purposeless-looking fuss has a deeper purpose: maybe those at the higher levels of dark matter hierarchy are using us to carry out complex topological quantum computations)!

General vision about quantum computation

In TGD Universe the hierarchy of Planck constants gives excellent prerequisites for all kinds of quantum computations. The general vision about quantum computation (TQC) would result as a special case and would look like follows.

1. Time-like entanglement between positive and negative energy parts of zero energy states would define the analogs of qc-programs. Space-like quantum entanglement between ends of strands whose motion defines time-like braids would provide a representation of q-information.
2. Both time- and space-like quantum entanglement would correspond to Connes tensor product expressing the finiteness of the measurement resolution between the states defined at ends of space-like braids whose orbits define time like braiding. The characterization of the measurement resolution would thus define both possible q-data and tq-programs as representations for "laws of physics".
3. The braiding between DNA strands with each nucleotide defining one strand transversal to DNA realized in terms of magnetic flux tubes was my first bet for the representation of space-like braiding in living matter. It turned out that the braiding is more naturally defined by flux tubes connecting nucleotides to the lipids of nuclear-, cell-, and endoplasma membranes. Also braidings between other microtubules and axonal membrane can be considered. The conjectured hierarchy of genomes giving rise to quantum coherent gene expressions in various scales would correspond to computational hierarchy.

About the relation between space-like and time-like number theoretic braidings

The relationship between space- and time-like braidings is interesting and there might be some connections also to 4-D topological gauge theories suggested by geometric Langlands program discussed in the previous posting and also in [K63].

1. The braidings along light-like surfaces modify space-like braiding if the moving ends of the space-like braids at partonic 3-surfaces define time-like braids. From TQC point of view the interpretation would be that TQC program is written to memory represented as the modification of space-like braiding in 1-1 correspondence with the time-like braiding.
2. The orbits of space-like braids define codimension two sub-manifolds of 4-D space-time surface and can become knotted. Presumably time-like braiding gives rise to a non-trivial "2-braid". Could also the "2-braiding" based on this knotting be of importance? Do 2-connections of n-category theorists emerge somehow as auxiliary tools? Could 2-knotting bring additional structure into the topological QFT defined by 1-braidings and Chern-Simons action?
3. The strands of dynamically evolving braids could in principle go through each other so that time evolution can transform braid to a new one also in this manner. This is especially clear from standard representation of knots by their planar projections. The points where intersection occurs correspond to self-intersection points of 2-surface as a sub-manifold of space-time surface. Topological QFT: s are also used to classify intersection numbers of 2-dimensional surfaces understood as homological equivalence classes. Now these intersection points would be associated with "braid cobordism".

Quantum computation as quantum superposition of classical computations?

It is often said that quantum computation is quantum super-position of classical computations. In standard path integral picture this does not make sense since between initial and final states represented by classical fields one has quantum superposition over *all* classical field configurations representing classical computations in very abstract sense. The metaphor is as good as the perturbation theory around the minimum of the classical action is as an approximation.

In TGD framework the classical space-time surface is a preferred extremal of Kähler action so that apart from effects caused by the failure of complete determinism, the metaphor makes sense precisely. Besides this there is of course the computation associated with the spin like degrees of freedom in which one has entanglement and which one cannot describe in this manner.

For TQC a particular classical computation would reduce to the time evolution of braids and would be coded by 2-knot. Classical computation would be coded to the manipulation of the braid. Note that the branching of strands of generalized number theoretical braids has interpretation as classical communication.

The identification of topological quantum states

Quantum states of TQC should correspond to topologically robust degrees of freedom separating neatly from non-topological ones.

1. The generalization of the embedding space inspired by the hierarchy of Planck constants suggests an identification of this kind of states as elements of the group algebra of discrete subgroup of $SO(3)$ associated with the group defining covering of M^4 or CP_2 or both in large $\hbar$ sector. One would have wave functions in the discrete space defined by the homotopy group of the covering transforming according to the representations of the group. This is by definition something robust and separated from non-topological degrees of freedom (standard model quantum numbers). There would be also a direct connection with anyons.
2. An especially interesting group is dodecahedral group corresponding to the minimal quantum phase $q = exp(2\pi/5)$ (Golden Mean) allowing a universal topological quantum computation: this group corresponds to Dynkin diagram for E_8 by the ALE correspondence. Interestingly, neuronal synapses involve clathrin molecules [I10] associated with microtubule ends possessing dodecahedral symmetry.

Some questions

A conjecture inspired by the inclusions of HFFs is that these states can be also regarded as representations of various gauge groups which TGD dynamics is conjectured to be able to mimic so that one might have connection with non-Abelian Chern-Simons theories where topological S-matrix is constructed in terms of path integral over connections: these connections would be only an auxiliary tool in TGD framework.

1. Do these additional degrees of freedom give only rise to topological variants of gauge- and conformal field theories? Note that if the earlier conjecture that entire dynamics of these theories could be mimicked, it would be best to perform TQC at quantum criticality where either M^4 or CP_2 dynamical degrees of freedom or both disappear.
2. Could it be advantageous to perform TQC near quantum criticality? For instance, could one construct magnetic braidings in the visible sector near q-criticality using existing technology and then induce phase transition changing Planck constant by varying some parameter, say temperature.

10.3.2 Fractal Hierarchies

Fractal hierarchies are the essence of TGD. There is hierarchy of space-time sheets labelled by preferred p-adic primes. There is hierarchy of Planck constants reflecting a book like structure of the generalized embedding space and identified in terms of a hierarchy of dark matters. These hierarchies correspond at the level of conscious experience to a hierarchy of conscious entities - selves: self experiences its sub-selves as mental images.

Fractal hierarchies mean completely new element in the model for quantum computation. The decomposition of quantum computation to a fractal hierarchy of quantum computations is one implication of this hierarchy and means that each quantum computation proceeds from longer to shorter time scales $T_n = T_0 2^{-n}$ as a cascade like process such that at each level there is a large number of quantum computations performed with various values of input parameters defined by the output at previous level. Under some additional assumptions to be discussed later this hierarchy involves at a given level a large number of replicas of a given sub-module of TQC so that the output of single fractal sub-module gives automatically probabilities for various outcomes as required.

10.3.3 Irreducible Entanglement And Possibility Of Quantum Parallel Quantum Computation

The basic distinction from standard measurement theory is irreducible entanglement not reduced in quantum jump. There are two kinds of irreducible entanglement: both are negentropic. First kind of irreducible entanglement corresponds to a density matrix, which is proportional to $n \times n$ unit matrix and is naturally associated with the $h_{eff} = nh$ hierarchy. If the entanglement matrix is unitary, density matrix is proportional to unit matrix. One can consider various restrictions on the unitary matrix and these were already discussed. The assumption that the unitary matrix is representable as TQC with basic gate defined by braiding operation is very natural and gives connection between consciousness and quantum computation.

One can imagine also a second candidate for irreducible entanglement. If the density matrix belongs to an algebraic extension of p-adic numbers, one can assign to it number theoretic negentropy. The diagonalized density matrix can however belong to a higher-dimensional algebraic extension than the matrix elements of the entanglement matrix itself. Does this mean that state function reduction can take place only if it is accompanied by an evolutionary step increasing the dimension of algebraic extension involved?

NMP and the possibility of irreducible entanglement

Negentropy Maximimization Principle (NMP) states that entanglement entropy is minimized in quantum jump. For standard Shannon entropy this would lead to a final state which corresponds to a ray of state space. If entanglement probabilities are rational - or even algebraic - one can replace Shannon entropy with its number theoretic counterpart in which p-adic norm of probability replaces the probability in the argument of logarithm: $log(p_n) \rightarrow log(|p_n|_p)$. This entropy can have negative values. It is not quite clear whether prime p should be chosen to maximize the number theoretic negentropy or whether p is the p-adic prime characterizing the light-like partonic 3-surface in question.

Obviously NMP favors generation of irreducible entanglement which however can be reduced in U process. Irreducible entanglement is something completely new and the proposed interpretation is in terms of experience of various kinds of conscious experiences with positive content such as understanding.

Quantum superposition of unitarily evolving quantum states generalizes to a quantum superposition of quantum jump sequences defining dissipative time evolutions. Dissipating quarks inside quantum coherent hadrons would provide a basic example of this kind of situation.

Quantum parallel quantum computations and conscious experience

The combination of quantum parallel quantum jump sequences with the fractal hierarchies of scales implies the possibility of quantum parallel quantum computations. In ordinary quantum computation halting selects single computation but in the recent case arbitrarily large number of computations can be carried out simultaneously at various branches of entangled state. The probability distribution for the outcomes is obtained using only single computation.

One would have quantum superposition of space-time sheets (assignable to the maxima of Kähler function) each representing classically the outcome of a particular computation. Each branch would correspond to its own conscious experience but the entire system would correspond to a self experiencing consciously the outcome of computation as intuitive and holistic understanding, and abstraction. Emotions and emotional intellect could correspond to this kind of non-symbolic representation for the outcome of computation as analogs for collective parameters like temperature and pressure.

Delicacies

There are several delicacies involved.

1. The above argument works for factors of type I. For HFFs of type II_1 the finite measurement resolution characterized in terms of the inclusion $\mathcal{N} \subset \mathcal{M}$ mean is that state function reduction takes place to $\mathcal{N}$-ray. There are good reasons to expect that the notion of number theoretic entanglement negentropy generalizes also to this case. Note that the entanglement associated with $\mathcal{N}$ is below measurement resolution.

2. In TGD inspired theory of consciousness irreducible entanglement makes possible sharing and fusion of mental images. At space-time level the space-time sheets corresponding to selves are disjoint but the space-time sheets topologically condensed at them are joined typically by what I call flux tubes identifiable as braid strands (magnetic flux quanta). In topological computation with finite measurement resolution this kind of entanglement with environment would be below the natural resolution and would not be a problem.

3. State function reduction means quantum jump to an eigen state of density matrix. Suppose that density matrix has rational elements. Number theoretic vision forces to ask whether the quantum jump to eigen state is possible if the eigenvalues of ρ do not belong to the algebraic extension of rationals and p-adic numbers used. If not, then one would have number theoretically irreducible entanglement depending on the algebraic extension used. If the eigenvalues actually define the extension there would be no restrictions: this option is definitely simpler.

4. Fuzzy quantum logic [K127] brings also complications. What happens in the case of quantum spinors that spin ceases to be observable and one cannot reduce the state to spin up or spin down. Rather, one can measure only the eigenvalues for the probability operator for spin up (and thus for spin down) so that one has fuzzy quantum logic characterized by quantum phase. Inclusions of HFFs are characterized by quantum phases and a possible interpretation is that the quantum parallelism related to the finite measurement resolution could give rise to fuzzy qubits. Also the number theoretic quantum parallelism implied by number theoretic NMP could effectively make probabilities as operators. The probabilities for various outcomes would correspond to outcomes of quantum parallel state function reductions.

10.3.4 Possible Problems Related To Quantum Computation

At least following problems are encountered in quantum computation.

1. How to preserve quantum coherence for a long enough time so that unitary evolution can be achieved?
2. The outcome of calculation is always probability distribution: for instance, the output with maximum probability can correspond to the result of computation. The problem is how to replicate the computation to achieve the desired accuracy. Or more precisely, how to produce replicas of the hardware of quantum computer defined in terms of classical physics?
3. How to isolate the quantum computer from the external world during computation and despite this feed in the inputs and extract the outputs?

The notion of coherence region in TGD framework

In standard framework one can speak about coherence in two senses. At the level of Schrödinger amplitudes one speaks about coherence region inside which it makes sense to speak about Schrödinger time evolution. This notion is rather defined.

In TGD framework coherence region is identifiable as a region inside which the Kähler-Dirac equation holds true. Strictly speaking, this region corresponds to a light-like partonic 3-surface whereas 4-D space-time sheet corresponds to coherence region for classical fields. p-Adic length scale hierarchy and hierarchy of Planck constants means that arbitrarily large coherence regions are possible.

The precise definition for the notion of coherence region and the presence of scale hierarchies imply that the coherence in the case of single quantum computation is not a problem in TGD framework. De-coherence time or coherence time correspond to the temporal span of space-time sheet and a hierarchy coming in powers of two for a given value of Planck constant is predicted by basic quantum TGD. p-Adic length scale hypothesis and favored values of Planck constant would naturally reflect this fundamental fractal hierarchy.

De-coherence of density matrix and replicas of TQC

Second phenomenological description boils down to the assumption that non-diagonal elements of the density matrix in some preferred basis (involving spatial localization of particles) approach to zero. The existence of more or less faithful replicas of space-time sheet in given scale allows to identify the counterpart of this notion in TGD context. De-coherence would mean a loss of information in the averaging of M-matrix and density matrix associated with these space-time sheets.

Topological computations are probabilistic. This means that one has a collection of space-time sheets such that each space-time sheet corresponds to more or less the same TQC and therefore the same M-matrix. If M is too random (in the limits allowed by Connes tensor product), the analog of generalized phase information represented by its "phase" - S-matrix - is useless.

In order to avoid de-coherence in this sense, the space-time sheets must be approximate copies of each other. Almost copies are expected to result by dissipation leading to asymptotic self-organization patterns depending only weakly on initial conditions and having also space-time correlates. Obviously, the role of dissipation in eliminating effects of de-coherence in TQC would be something new. The enormous symmetries of M-matrix, the uniqueness of S-matrix for given resolution and parameters characterizing braiding, fractality, and generalized Bohr orbit property of space-time sheets, plus dissipation give good hopes that almost replicas can be obtained.

Isolation and representations of the outcome of TQC

The interaction with environment makes quantum computation difficult. In the case of topological quantum computation this interaction corresponds to the formation of braid strands connecting the computing space-time sheet with space-time sheets in environment. The environment is four-dimensional in TGD framework and an isolation in time direction might be required. The space-time sheets responsible for replicas of TQC should not be connected by light-like braids strands having time-like projections in M^4.

Length scale hierarchy coming in powers of two and finite measurement resolution might help considerably. Finite measurement resolution means that those strands which connect space-time sheets topologically condensed to the space-time sheets in question do not induce entanglement visible at this level and should not affect TQC in the resolution used.

Hence only the elimination of strands responsible for TQC at given level and connecting computing space-time sheet to space-time sheets at same level in environment is necessary and would require magnetic isolation. Note that super-conductivity might provide this kind of isolation. This kind of elimination could involve the same mechanism as the initiation of TQC which cuts the braid strands so the initiation and isolation might be more or less the same thing.

Strands reconnect after the halting of TQC and would make possible the communication of the outcome of computation along strands by using say em currents in turn generating generalized EEG, nerve pulse patterns, gene expression, etc... halting and initiation could be more or less synonymous with isolation and communication of the outcome of TQC.

How to express the outcome of quantum computation?

The outcome of quantum computation is basically a representation of probabilities for the outcome of TQC. There are two representations for the outcome of TQC. Symbolic representation which quite generally is in terms of probability distributions represented in terms "classical space-time" physics. The rates for various processes having basically interpretation as geometro-temporal densities would represent the probabilities just as in the case of particle physics experiment. For TQC in living matter this would correspond to gene expression, neural firing, EEG patterns, ...

A representation as a conscious experience is another (and actually the ultimate) representation of the outcome. It need not have any symbolic counterpart since it is felt. Intuition, emotions and emotional intelligence would naturally relate to this kind of representation made possible by irreducible entanglement. This representation would be based on fuzzy qubits and would mean that the outcome would be true or false only with certain probability. This unreliability would be felt consciously.

The proposed model of TQC combined with basic facts about theta waves [J18, J23] to be discussed in the subsection about the role of supra currents in TQC suggests that EEG rhythm (say theta rhythm) and correlated firing patterns correspond to the isolation at the first half period of TQC and random firing at second half period to the sub-sequent TQC: s at shorter time scales coming as negative powers of 2. The fractal hierarchy of time scales would correspond to a hierarchy of frequency scales for generalized EEG and power spectra at these scales would give information about the outcome of TQC. Synchronization would be obviously an essential element in this picture and could be understood in terms of classical dynamics which defines space-time surface as a generalized Bohr orbit.

Tqc would be analogous to the generation of a dynamical hologram or "conscious hologram" [K22]. EEG rhythm would correspond to reference wave generated by magnetic body as control and coordination signal and the contributions of spikes to EEG generated by neurons would correspond to the incoming wave interfering with the reference wave.

How data is fed into submodules of TQC?

Scale hierarchy obviously gives TQC a fractal modular structure and the question is how data is fed to submodules at shorter length scales. There are certainly interactions between different levels of scale hierarchy. The general ideas about master-slave hierarchy assigned with self-organization support the hypothesis that these interactions are directed from longer to shorter scales and have interpretation as a specialization of input data to TQC sub-modules represented by smaller space-time sheets of hierarchy. The call of submodule would occur when the TQC of the calling module halts and the result of computation is expressed as a 4-D pattern. The lower level module would start only after the halting of TQC (with respect to subjective time at least) and the durations of resulting TQC's would come as $T_n = 2^{-n}T_0$ that geometric series of TQC's would become possible. There would be entire family of TQC's at lower level corresponding to different values of input parameters from calling module.

One of the ideas assigned to hyper-computation [B1] is that one can have infinite series of computations with durations comings as negative powers of 2 (Zeno paradox obviously inspires this idea). In TGD framework there can be however only a finite series of these TQC's since CP_2 time scale poses a lower bound for the duration of TQC. One might of course ask whether the spectrum of Planck constant could help in this respect.

The role of dissipation and energy feed

Dissipation plays key role in the theory of self-organizing systems [B3]. Its role is to serve as a Darwinian selector. Without an external energy feed the outcome is a situation in which all organized motions disappear. In presence of energy feed highly unique self-organization patterns depending only very weakly on the initial conditions emerge.

In the case of TQC one function of dissipation would be to drive the braidings to static standard configurations, and perhaps even effectively eliminate fluctuations in non-topological degrees of freedom. Note that magnetic fields are important for 1-gates. Magnetic flux conservation however saves magnetic fields from dissipation.

External energy feed is needed in order to generate new braidings. For the proposed model of cellular TQC the flow of intracellular water induces the braiding and requires energy feed. Also now dissipation would drive this flow to standard patterns coding for TQC programs. Metabolic energy would be also needed in order to control whether lipids can flow or not by generating cis type unsaturated bonds. Obviously, energy flows defining self organization patterns would define TQC programs.

Is it possible to realize arbitrary TQC?

The 4-D spin glass degeneracy of TGD Universe due to the enormous vacuum degeneracy of Kähler action gives good hopes that the classical dynamics for braidings allows to realize every possible TQC program. As a consequence, space-time sheets decompose to maximal non-deterministic regions representing basic modules of TQC. Similar decomposition takes place at the level of light-like partonic 3-surfaces and means decomposition to 3-D regions inside which conformal invariance eliminates light-like direction as dynamical degree of freedom so that the dynamics is effectively that of 2-dimensional object. Since these 3-D regions behave as independent units as far as longitudinal conformal invariance is considered, one can say that light-like 3-surfaces are 3-dimensional in discretized sense. In fact, for 2-D regions standard conformal invariance implies similar effective reduction to 1-dimensional dynamics realized in terms of a net of strings and means that 2-dimensionality is realized only in discretized sense.

10.3.5 Negentropic Entanglement, NMP, Braiding And TQC

Negentropic entanglement, NMP, braiding and TQC

Negentropic entanglement for which number theoretic entropy characterized by p-adic prime is negative so that entanglement carries information, is in key role in TGD inspired theory of consciousness and quantum biology.

1. The key feature of 2-particle negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) is that density matrix is projector and thus proportional to unit matrix so that the assumption that state function reduction corresponds to the measurement of density matrix does not imply state function reduction to one-dimensional sub-space. This special kind of degenerate density matrix emerges naturally for the hierarchy $h_{eff} = n \times h$ interpreted in terms of a hierarchy of dark matter phases. I have already earlier considered explicit realizations of negentropic entanglement assuming that E is invariant under the group of unitary or orthogonal transformations (also subgroups of unitary group can be considered -say symplectic group). One can however consider much more general options and this leads to a connection with topological quantum computation (TQC).
2. Entanglement matrix E equal to $1/\sqrt{n}$ factor times unitary matrix U (as a special case orthogonal matrix O) defines a density matrix given by $\rho = UU^\dagger/n = Id_n/n$, which is group invariant. One has NE respected by state function reduction if NMP is assumed. This would give huge number of negentropically entangled states providing a representation for

some unitary group or its subgroup (such as symplectic group). In principle any unitary representation of any Lie group would allow representation in terms of NE. In principle any unitary representation of any Lie group would allow a representation in terms of NE.

3. In physics as generalized number theory vision, a natural condition is that the matrix elements of E belong to the algebraic extension of p-adic numbers used so that discreted algebraic subgroups of unitary or orthogonal group are selected. This realizes evolutionary hierarchy as a hierarchy of p-adic number fields and their algebraic extensions, and one can imagine that evolution of cognition proceeds by the generation of negentropically entangled systems with increasing algebraic dimensions and increasing dimension reflecting itself as an increase of the largest prime power dividing n and defining the p-adic prime in question.

4. One fascinating implication is the ability of TGD Universe to emulate itself like Turing machine: unitary S-matrix codes for scattering amplitudes and therefore for physics and negentropically entangled subsystem could represent sub-matrix for S-matrix as rules representing "the laws of physics" in the approximation that the world corresponds to n-dimension Hilbert space. Also the limit $n \rightarrow \infty$ makes sense, especially so in the p-adic context where real infinity can correspond to finite number in the sense of p-adic norm. Here also dimensions n given as products of powers of infinite primes can be formally considered.

One can consider various restrictions on E.

1. In 2-particle case the stronger condition that E is group invariant implies that unitary matrix is identity matrix apart from an overall phase factor: $U = exp(i\phi)Id$. In orthogonal case the phase factor is ± 1. For n-particle NE one can consider group invariant states by using n-dimensional permutation tensor $\epsilon_{i_1,...i_n}$.

2. One can give up the group invariance of E and consider only the weaker condition that permutation is represented as transposition of entanglement matrix: $C_{ij} \rightarrow C_{ij}$. Symmetry/antisymmetry under particle exchange would correspond to $C_{ji} = \epsilon C_{ij}$, $\epsilon = \pm 1$. This would give in orthogonal case $OO^T = O^2 = Id$ and $UU^* = Id$ in unitary case.

   In the unitary case particle exchange could be identified as hermitian conjugation $C_{ij} \rightarrow C^*_{ji}$ and one would have $U^2 = Id$. Euclidian gamma matrices γ_i define unitary and hermitian generators of Clifford algebra having dimension 2^{2m} for $n = 2m$ and $n = 2m+1$. It is relatively easy to verify that the squares of completely anti-symmetrized products of k gamma matrices representing exterior algebra normalized by factor $1/\sqrt{k!}$ are equal to unit matrix. For $k = n$ the anti-symmetrized product gives essentially permutation symbol times the product $\prod_k \gamma_k$. In this manner one can construct entanglement matrices representing negentropic bi-partite entanglement.

3. The possibility of taking tensor products $\epsilon_{ij..k...n}\gamma_i \otimes \gamma_j .. \otimes \gamma_k$ of k gamma matrices means that one can has also co-product of gamma matrices. What is interesting is that quantum groups important in topological quantum computation as well as the Yangian algebra associated with twistor Grassmann approach to scattering amplitudes possess co-algebra structure. TGD leads also to the proposal that this structure plays a central role in the construction of scattering amplitudes. Physically the co-product is time reversal of product representing fusion of particles.

4. One can go even further. In 2-dimensional QFTs braid statistics replaces ordinary statistics. The natural question is what braid statistics could correspond to at the level of NE. Braiding matrix is unitary so that it defines NE. Braiding as a flow replaces the particle exchange and lifts permutation group to braid group serving as its infinite covering.

   The allowed unitary matrices representing braiding in tensor product are constructed using braiding matrix R representing the exchange for two braid strands? The well-known Yang-Baxter equation for R defined in tensor product as an invertible element (`http://tinyurl.com/yax3j6mr`) expresses the associativity of braiding operation. Concretely it states that the two braidings leading from 123 to 321 produce the same result. Entanglement matrices constructed R as basic operation would correspond to unitary matrices providing a representation for braids and each braid would give rise to one particular NE.

   This would give a direct connection with TQC for which the entanglement matrix defines a density matrix proportional to $n \times n$ unit matrix: R defines the basic gate [B38]. Braids would provide a concrete space-time correlate for NE giving rise to "Akashic records". Note that in string theory-GRT framework this old idea of TGD has been recently introduced by Maldacena and Sussking as a proposal that wormholes connecting blackholes provide a description of entanglement.

I have indeed proposed the interpretation of braidings as fundamental memory representations much before the vision about Akashic records. This kind of entanglement matrix need not represent only time-like entanglement but can be also associated also with space-like entanglement. The connection with braiding matrices supports the view that magnetic flux tubes are carriers of negentropically entangled matter and also suggests that this kind of entanglement between -say- DNA and nuclear or cell membrane gives rise to TQC.

Some comments concerning the covering space degrees of freedom associated with $h_{eff} = n \times h$ viz. ordinary degrees of freedom are in order.

1. Negentropic entanglement with n entangled states would correspond naturally to $h_{eff} = n \times h$ and is assigned with "many-particle" states, which can be localized to the sheets of covering but one cannot exclude similar entanglement in other degrees of freedom. Group invariance leaves only group singlets and states which are not singlets are allowed only in special cases. For instance for $SU(2)$ the state $|j,m\rangle = |1,0\rangle$ represented as 2-particle state of 2 spin $1/2$ particles is negentropically entangled whereas the states $|j,m\rangle = |1,\pm 1\rangle$ are pure.

2. Negentropic entanglement associated with $h_{eff} = n \times h$ could factorize as tensor product from other degrees of freedom. Negentropic entanglement would be localised to the covering space degrees of freedom but there would be entropic entanglement in the ordinary degrees of freedom - say spin. The large value of h_{eff} would however scale up the quantum coherence time and length also in the ordinary degrees of freedom. For entanglement matrix this would correspond to a direct sum proportional to unitary matrices so that also density matrix would be a direct sum of matrices $p_n E_n = p_n Id_n/n$, $\sum p_n = 1$ correspond ing to various values of "other quantum numbers", and state function reduction could take place to any subspace in the decomposition. Also more general entanglement matrices for which the dimensions of direct summands vary, are possible.

3. One can argue that NMP in form does not allow halting of quantum computation. This is not true. The computation halts but in different manner since negentropic entanglement tends to be generated even for weak form of NMP. Weak form of NMP allows also ordinary state function reduction. State function reduction is not need if NE can be directly experienced and self represents this mental image as a kind of abstraction or rule with the state pairs in the superposition representing the instances of the rule.

   It might be also possible to deduce the structure of negentropically entangled state by an interaction free quantum measurement replacing the state function reduction with "externalised" state function reduction. One could speak of interaction free TQC. This TQC would be reading of "Akashic records".

4. One could also counter argue that NMP allows the transfer of NE from the system so that TQC halts. NMP allows this if some another system receives at least the negentropy contained by NE. The interpretation would be as the increase of information obtained by a conscious observer about the outcome of halted quantum computation.

   Metabolism could quite concretely correspond the transfer of NE associated with the NE between nutrient molecules and some system. This would satisfy the demands of NMP and make possible for the organism to avoid the first state function reduction to the opposite boundary of CD (death) In [K86] it is suggested that this system can be of astrophysical size, say gravitational Mother Gaia with magnetic flux tubes characterized by gravitational Planck constant $\hbar_{gr} = GMm/v_0 = \hbar_{eff} = n \times \hbar$, where v_0 is a parameter with dimensions of velocity. There is experimental evidence for dark matter shell around Earth [K103] and there are highly interesting connection to the hypothesis identifying bio-photons as decay products of dark photons located at magnetic flux tubes and having $h_{eff} = h_{gr}$.

10.4 DNA As Topological Quantum Computer

Braids [A2] code for topological quantum computation. One can imagine many possible identifications of braids but this is not essential for what follows. What is highly non-trivial is that the motion of the ends of strands defines both time-like and space-like braidings with latter defining in a well-defined sense a written version of the TQC program, kind of log file. The manipulation of braids is a central element of TQC and if DNA really performs TQC, the biological unit modifying braidings should be easy to identify. An obvious signature is the 2-dimensional character of this unit.

10.4.1 Conjugate DNA As Performer Of TQC And Lipids As Quantum Dancers

In this section the considerations are restricted to DNA as TQC. It is however quite possible that also RNA and other biomolecules could be involved with TQC like process.

Sharing of labor

The braid strands must begin from DNA double strands. Precisely which part of DNA does perform TQC? Genes? Introns [I22] ? Or could it be conjugate DNA which performs TQC? The function of conjugate DNA has indeed remained a mystery and sharing of labor suggests itself.

Conjugate DNA would do TQC and DNA would "print" the outcome of TQC in terms of RNA yielding amino-acids in the case of exons. RNA could the outcome in the case of introns. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of TQC also electromagnetically in terms of standardized field patterns. Also speech would be a form of gene expression. The quantum states braid would entangle with characteristic gene expressions. This hypothesis will be taken as starting point in the following considerations.

Cell membranes as modifiers of braidings defining TQC programs?

The manipulation of braid strands transversal to DNA must take place at 2-D surface. The ends of the space-like braid are dancers whose dancing pattern defines the time-like braid, the running of classical TQC program. Space-like braid represents memory storage and TQC program is automatically written to memory during the TQC. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic reticulum included are good candidates for dancing hall. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

1. Consider first the anatomy of membranes. Cell membrane [I4] and membranes of nuclear envelope [I34] consist of 2 lipid [I25] layers whose hydrophobic ends point towards interior. There is no water here nor any direct perturbations from the environment or interior milieu of cell. Nuclear envelope consists of two membranes having between them an empty volume of thickness 20-40 nm. The inner membrane consists of two lipid layers like ordinary cell membrane and outer membrane is connected continuously to endoplasmic reticulum [I14], which forms a highly folded cell membrane. Many biologists believe that cell nucleus is a prokaryote, which began to live in symbiosis with a prokaryote defining the cell membrane.
2. What makes dancing possible is that the phospholipid layers of the cell membrane are liquid crystals [D3]: the lipids can move freely in the horizontal direction but not vertically. "Phospho" could relate closely to the metabolic energy needs of dancers. If these lipids are self-organized around braid strands, their dancing patterns along the membrane surface would be an ideal manner to modify braidings since the lipids would have standard positions in a lattice. This would be like dancing on a chessboard. Note that the internal structure of lipid does not matter in this picture since it is braid color dicated by DNA nucleotide which matters. As a matter fact, living matter is full of self-organizing liquid crystals and one can wonder whether the deeper purpose of their life be running and simultaneous documentation of TQC programs?

3. Ordinary computers have an operating system: a collection of standard programs - the system - and similar situation should prevail now. The "printing" of outputs of TQC would represent example of this kind of standard program. This TQC program should not receive any input from the environment of the nucleus and should therefore correspond to braid strands connecting conjugate strand with strand. Braid strands would go only through the inner nuclear membrane and return back and would not be affected much since the volume between inner and outer nuclear membranes is empty. This assumption looks ad hoc but it will be found that the requirement that these programs are inherited as such in the cell replication necessitates this kind of structure (see the section "Cell replication and TQC").
4. The braid strands starting from the conjugate DNA could traverse several time through the highly folded endoplasmic reticulum but without leaving cell interior and return back to nucleus and modify TQC by intracellular input. Braid strands could also traverse the cell membrane and thus receive information about the exterior of cell. Both of these TQC programs could be present also in prokaryotes [I41] but the braid strands would always return back to the DNA, which can be also in another cell. In multicellulars (eukaryotes [I16]) braid strands could continue to another cell and give rise to "social" TQC programs performed by the multicellular organisms. Note that the topological character of braiding does not require isolation of braiding from environment. It might be however advantageous to have some kind of sensory receptors amplifying sensory input to standardized re-braiding patterns. Various receptors in cell membrane would serve this purpose.
5. Braid strands can end up at the parquet defined by ends of the inner phospholipid layer: their distance of inner and outer parquet is few nanometers. They could also extend further.
   (a) If one is interested in connecting cell nucleus to the membrane of another cell, the simpler option is the formation of hole defined by a protein attached to cell membrane. In this case only the environment of the second cell affects the braiding assignable to the first cell nucleus.
   (b) The bi-layered structure of the cell membrane could be essential for the build-up of more complex TQC programs since the strands arriving at two nearby hydrophobic 2-surfaces could combine to form longer strands. The formation of longer strands could mean the fusion of the two nearby hydrophobic two-surfaces in the region considered.
      i. The original naïve idea was that TQC could begin with the cutting of the strands so that non-trivial braiding could be generated via lipid dance and TQC would halt when strands would recombine and define a modified braiding. There are however strong objections about cutting since boundaries are not favored by boundary connections.
      ii. If there is a U-shaped flux tube from interior to the exterior and returning back, reconnection at cell membrane could create two U-shaped loops inside and outside the cell membrane. The U-shaped loop could also correspond make the turn through wormhole contact so that the effective splitting would create two wormhole contacts. The latter option fixes completely DNA TQC option based on quarks and antiquarks. If Cooper pairs of charged spin 1/2 fermions forming four spin states assumed to correspond A, T, C, G the first option is the natural one.

   This would allow to connect cell nucleus and cell membrane to a larger TQC unit and cells to multicellular TQC units so that the modification of TQC programs by feeding the information from the exteriors of cells - essential for the survival of multicellulars - would become possible.

Gene expression and other basic genetic functions from TQC point of view

It is useful to try to imagine how gene expression might relate to the halting of TQC. There are of course myriads of alternatives for detailed realizations, and one can only play with thoughts to build a reasonable guess about what might happen.

1. Qubits for transcription factors and other regulators

Genetics is consistent with the hypothesis that genes correspond to those TQC moduli whose outputs determine whether genes are expressed or not. The naïve first guess would be that the value of single qubit determines whether the gene *is* expressed or not. Next guess replaces "*is* " with "*can be* ".

Indeed, gene expression involves promoters, enhancers and silencers [I42]. Promoters are portions of the genome near genes and recognized by proteins known as transcription factors [I48]. Transcription factors bind to the promoter and recruit RNA polymerase, an enzyme that synthesizes RNA. In prokaryotes RNA polymerase itself acts as the transcription factor. For eukaryotes situation is more complex: at least seven transcription factors are involved with the recruitment of the RNA polymerase II catalyzing the transcription of the messenger RNA. There are also transcription factors for transcription factors and transcription factor for the transcription factor itself.

The implication is that several qubits must have value "Yes" for the actual expression to occur since several transcription factors are involved with the expression of the gene in general. In the simplest situation this would mean that the computation halts to a measurement of single qubit for subset of genes including at least those coding for transcription factors and other regulators of gene expression.

2. Intron-exon qubit

Genes would have very many final states since each nucleotide is expected to correspond to at least single qubit. Without further measurements that state of nucleotides would remain highly entangled for each gene. Also these other qubits are expected to become increasingly important during evolution.

For instance, eukaryotic gene expression involves a transcription of RNA and splicing out of pieces of RNA which are not translated to amino-acids (introns). Also the notion of gene is known to become increasingly dynamical during the evolution of eukaryotes so that the expressive power of genome increases. A single qubit associated with each codon telling whether it is spliced out or not would allow maximal flexibility. Tqc would define what genes are and the expressive power of genes would be due to the evolution of TQC programs: very much like in the case of ordinary computers. Stopping sign codon and starting codon would automatically tell where the gene begins and ends if the corresponding qubit is "Yes". In this picture the old fashioned static genes of prokaryotes without splicings would correspond to TQC programs for which the portions of genome with a given value of splicing qubit are connected.

3. What about braids between DNA, RNA, tRNA and amino-acids

This simplified picture might have created the impression that amino-acids are quantum outsiders obeying classical bio-chemistry. For instance, transcription factors would in this picture end up to the promoter by a random process and "Print" would only increase the density of the transcription factor. If DNA is able to perform TQC, it would however seem very strange if it would be happy with this rather dull realization of other central functions of the genetic apparatus.

One can indeed consider besides the braids connecting DNA and its conjugate - crucial for the success of replication - also braids connecting DNA to mRNA and other forms of RNA, mRNA to tRNA, and tRNA to amino-acids. These braids would provide the topological realization of the genetic code and would increase dramatically the precision and effectiveness of the transcription and translation if these processes correspond to quantum transitions at the level of dark matter leading more or less deterministically to the desired outcome at the level of visible matter be it formation of DNA doublet strand, of DNA-mRNA association, of mRNA-tRNA association or tRNA-amino-acid association.

For instance, a temporary reduction of the value of Planck constant for these braids would contract these to such a small size that these associations would result with a high probability. The increase of Planck constant for braids could in turn induce the transfer of mRNA from the nucleus, the opening of DNA double strand during transcription and mitosis.

Also DNA-amino-acid braids might be possible in some special cases. The braiding between regions of DNA at which proteins bind could be a completely general phenomenon. In particular, the promoter region of gene could be connected by braids to the transcription factors of the gene and the halting of TQC computation to printing command could induce the reduction of Planck constant for these braids inducing the binding of the transcription factor binds to the promoter region. In a similar manner, the region of DNA at which RNA polymerase binds could be connected by braid strands to the RNA polymerase.

How braid color is represented?

If braid strands carry 4-color (A, T, C, G) then also lipid strands should carry this kind of 4-color. The lipids whose hydrophobic ends can be joined to form longer strand should have same color. This color need not be chemical in TGD Universe.

Only braid strands of the same color can be connected as TQC halts. This poses strong restrictions on the model.

1. Do braid strands appear as patches possessing same color?

Color conservation is achieved if the two lipid layers decompose in a similar manner into regions of fixed color and the 2-D flow is restricted inside this kind of region at both layers. A four-colored map of cell membrane would be in question! Liquid crystal structure [I4] applies only up to length scale of $L(151) = 10$ nm and this suggests that lipid layer decomposes into structural units of size $L(151)$ defining also cell membrane thickness. These regions might correspond to minimal regions of fixed color containing $N \sim 10^2$ lipids.

The controversial notion of lipid raft [I27] was inspired by the immiscibility of ordered and disordered liquid phases in a liquid model of membrane. The organization to connected regions of particular phase could be a phenomenon analogous to a separation of phases in percolation. Many cell functions implicate the existence of lipid rafts. The size of lipid rafts has remained open and could be anywhere between 1 and 1000 nm. Also the time scale for the existence of a lipid raft is unknown. A line tension between different regions is predicted in hydrodynamical model but not observed. If the decomposition into ordered and disordered phases is time independent, ordered phases could correspond to those involved with TQC and possess a fixed color. If disordered phases contain no braid strands the mixing of different colors is avoided. The problem with this option is that it restricts dramatically the possible braidings.

If one takes this option seriously, the challenge is to make patches and patch color (A, T, C, G) visible. Perhaps one could try to mark regions of portions of lipid layer by some marker to find whether the lipid layer decomposes to non-mixing regions.

Quantum criticality suggests that that the patches of lipid layer have a fractal structure corresponding to a hierarchy of TQC program modules. The hydrodynamics would be thus fractal: patches containing patches.... moving with respect to each other would correspond to braids containing braids containing... such that sub-braids behave as braid strands. In principle this is also a testable prediction.

2. Does braid color corresponds to some chemical property?

The conserved braid color is not necessary for the model but would imply genetic coding of the TQC hardware so that sexual reproduction would induce an evolution of TQC hardware. Braid color would also make the coupling of foreign DNA to the TQC performed by the organism difficult and realize an immune system at the level of quantum information processing.

The conservation of braid color poses however considerable problems. The concentration of braid strands of the same color to patches would guarantee the conservation but would restrict the possible braiding dramatically. A more attractive option is that the strands of same color find each other automatically by energy minimization after the halting of TQC. Electromagnetic Coulomb interaction would be the most natural candidate for the interaction in question. Braid color would define a faithful genetic code at the level of nucleotides. It would induce long range correlation between properties of DNA strand and the dynamics of cell immediately after the halting of TQC.

The idea that color could be a chemical property of phospholipids does not seem plausible. The lipid asymmetry of the inner and outer monolayers excludes the assignment of color to the hydrophilic groups PS, PI, PE, PCh. Fatty acids have $N = 14, ..., 24$ carbon atoms and $N = 16$ and 18 are the most common cases so that one could consider the possibility that the 4 most common feet pairs could correspond to the resulting combinations. It is however extremely difficult to understand how long range correlation between DNA nucleotide and fatty acid pair could be created.

3. Does braid color correspond to neutral quark pairs?

It seems that the color should be a property of the braid strand. In TGD inspired model of high T_c super-conductivity [K24] wormhole contacts having u and $\overline{d}$ and d and $\overline{u}$ quarks at the two wormhole throats feed electron's gauge flux to larger space-time sheet. The long range correlation between electrons of Cooper pairs is created by color confinement for an appropriate scaled up variant of chromo-dynamics which are allowed by TGD. Hence the neutral pairs of colored quarks whose members are located the ends of braid strand acting like color flux tube connecting the nucleotide to the lipid could code DNA color to QCD color.

For the pairs $u\overline{d}$ with net em charge the quark and anti-quark have the same sign of em charge and tend to repel each other. Hence the minimization of electro-magnetic Coulomb energy favors the neutral configurations $u\overline{u}$, $d\overline{d}$ and $u\overline{u}$, and $d\overline{d}$ coding for A, T, C, G in some order.

After the halting of TQC only these pairs would form with a high probability. The reconnection of the strands would mean a formation of a short color flux tube between the strands and the annihilation of quark pair to gluon. Note that single braid strand would connect DNA color and its conjugate rather than identical colors so that braid strands connecting two DNA strands (conjugate strands) should always traverse through an even (odd) number of cell membranes. The only plausible looking option is that nucleotides A, T, G, C are mapped to pairs of quark and anti-quarks at the ends of braid strand. Symmetries pose constraints on this coding.

1. By the basic assumptions charge conjugation must correspond to DNA conjugation so that one A and T would be coded to quark pair, say $q\overline{q}$ and its conjugate $\overline{q}q$. Same for C and G.
2. An additional aesthetically appealing working hypothesis is that *both* A and G with the same number of aromatic cycles (three) correspond to $q\overline{q}$ (or its conjugate).

   This would leave four options:

$$\begin{array}{ll}
(A,G)\rightarrow (u\overline{u},d\overline{d}) \ , & (T,C)\rightarrow (\overline{u}u,\overline{d}d) \ , \\
(A,G)\rightarrow (d\overline{d},u\overline{u}) \ , & (T,C)\rightarrow (\overline{d}d),\overline{u}u) \ , \\
(T,C)\rightarrow (u\overline{u},d\overline{d}) \ , & (A,G)\rightarrow (\overline{u}u,\overline{d}d) \ , \\
(T,C)\rightarrow (d\overline{d},u\overline{u}) \ , & (A,G)\rightarrow (\overline{d}d),\overline{u}u) \ .
\end{array} \tag{10.4.1}$$

It is an experimental problem to deduce which of these correspondences - if any - is realized.

Some general predictions

During TQC the lipids of the two lipid layers should define independent units of lipid hydrodynamics whereas after halting of TQC they should behave as single dynamical unit. Later it will be found that these two phases should correspond to high T_c superconductivity for electrons (Cooper pairs would bind the lipid pair to form single unit) and its absence. This prediction is testable.

The differentiation of cells should directly correspond to the formation of a mapping of a particular part of genome to cell membrane. For neurons the gene expression is maximal which conforms with the fact that neurons can have very large size. Axon might be also part of the map. Stem cells represent the opposite extreme and in this case minimum amount of genome should be mapped to cell membrane. The prediction is that the evolution of cell should be reflected in the evolution of the genome-membrane map.

Quantitative test for the proposal

There is a simple quantitative test for the proposal. A hierarchy of TQC programs is predicted, which means that the number of lipids in the nuclear inner membrane should be larger or at least of the same order of magnitude that the number of nucleotides. For definiteness take the radius of the lipid molecule to be about 5 Angstroms (probably somewhat too large) and the radius of the nuclear membrane about 2.5 μm.

For our own species the total length of DNA strand is about one meter and there are 30 nucleotides per 10 nm. This gives 6.3×10^7 nucleotides: the number of intronic nucleotides is only by few per cent smaller. The total number of lipids in the nuclear inner membrane is roughly 10^8. The number of lipids is roughly twice the number nucleotides. The number of lipids in the membrane of a large neuron of radius of order 10^{-4} meters is about 10^{11}. The fact that the cell membrane is highly convoluted increases the number of lipids available. Folding would make possible to combine several modules in sequence by the proposed connections between hydrophobic surfaces.

10.4.2 How Quantum States Are Realized?

Quantum states should be assigned to the ends of the braid strands and therefore to the nucleotides of DNA and conjugate DNA. The states should correspond to many-particle states of anyons and fractional electrons and quarks and anti-quarks are the basic candidates.

Anyons represent quantum states

The multi-sheeted character of space-time surface as a 4-surface in a book like structure having as pages covering spaces of the embedding space (very roughly, see the appendix) would imply additional degrees of freedom corresponding to the group algebra of the group $G \supset Z_n$ defining the covering. Especially interesting groups are tetra-hedral, octahedral, and icosahedral groups whose action does not map any plane to itself. Group algebra would give rise to $n(G)$ quantum states. If electrons are labeled by elements of group algebra this gives $2^{n(G)}$-fold additional degeneracy corresponding to many-electron states at sheets of covering. The vacuum state would be excluded so that $2^{n(G)}-1$ states would result. If only Cooper pairs are allowed one would have $m_n = 2^{n(G)-1}-1$ states.

This picture suggests the fractionization of some fermionic charges such as em charge, spin, and fermion number. This aspect is discussed in detail in the Appendix. Single fermion state would be replaced by a set of states with fractional quantum numbers and one would have an analogy with the full electronic shell of atom in the sense that a state containing maximum number of anyonic fermions with the same spin direction would have the quantum numbers of the ordinary fermion.

One can consider two alternative options.

1. The fractionization of charges inspired the idea that catalytic hot spots correspond to "half" hydrogen bonds containing dark fractionally charged electron meaning that the Fermi sea for electronic anyons is not completely filled [K12]. The formation of hydrogen bond would mean a fusion of "half hydrogen bond" and its conjugate having by definition a compensating fractional charges guaranteeing that the net em charge and electron number of the resulting state are those of the ordinary electron pair and the state is stable as an analog of the full electron shell. Half hydrogen bonds would assign to bio-molecules "names" as sequences of half hydrogen bonds and only molecules whose "names" are conjugates of each other would form stable hydrogen bonded pairs. Therefore symbolic dynamics would enter the biology via bio-catalysis. Concerning quantum computation the problem is that the full shell assigned to hydrogen bond corresponds to only single state and cannot carry information.

2. The assignment of braids and fractionally charged anyonic quarks and anti-quarks would realize very similar symbolic dynamics. One cannot exclude the possibility that leptonic charges fractionize to same values as quark charges.

This suggest the following picture.

1. One could assign the fractional quantum numbers to the quarks and anti-quarks at the ends of the flux tubes defining the braid strands. This hypothesis is consistent with the correspondence between nucleotides and quarks and assigns anyonic quantum states to the ends of the braid. Wormhole magnetic fields would distinguish between matter in vivo and in vitro. This option is certainly favored by Occam's razor in TGD Universe.

2. Hydrogen bonds connect the DNA strands which suggests that fractionally charged quantum states at the ends of braids might be assignable to the ends of hydrogen bonds. The model for plasma electrolysis of Kanarev [L1] leads to a proposal that new physics is involved with hydrogen bonds. The presence of fractionally charged particles at the ends of bond might provide alternative explanation for the electrostatic properties of hydrogen bonds usually explained in terms of a modification electronic charge distribution by donor-acceptor mechanism. There would exists entire hierarchy of hydrogen bonds corresponding to the increasing values of Planck constant. DNA and even hydrogen bonds associated with water might correspond to a larger value of Planck constant for mammals than for bacteria.

3. The model for protein folding code [K7] leads to a cautious conclusion that flux tubes are prerequisites for the formation of hydrogen bonds although not identifiable with them. The

$$\begin{array}{lll} \{M_n\} & \{n(G)\} & \\ & \{n_b\} & \\ \{2,7,127,2^{127}-1,?\} & \{4,8,128,2^{127},?\} & \{2,6,126,2^{126},?\} \\ \{5,31,2^{31}-1\} & \{6,32,2^{31}\} & \{4,30,2^{30}\} \\ \{13,2^{13}-1\} & \{14,2^{13}\} & \{12,2^{12}\} \\ \{17,2^{17}-1\} & \{18,2^{17}\} & \{16,2^{16}\} \\ \{19,2^{19}-1\} & \{20,2^{19}\} & \{18,2^{18}\} \\ \{61,2^{61}-1\} & \{62,2^{61}\} & \{60,2^{60}\} \\ \{89,2^{89}-1\} & \{90,2^{89}\} & \{88,2^{88}\} \\ \{107,2^{107}-1\} & \{108,2^{107}\} & \{106,2^{106}\} \end{array} \tag{10.4.2}$$

Table 10.1: Hierachies of Mersenne primes

model predicts also the existence of long flux tubes between acceptors of hydrogen bonds (such as $O =$, and aromatic rings assignable to DNA nucleotides, amino-acid backbone, phosphates, XYP, $X = A, T, G, C$, $Y = M, D, T$). This hypothesis would allow detailed identification of places to which quantum states are assigned.

Hierarchy of genetic codes defined by Mersenne primes

The model for the hierarchy of genetic codes inspires the question whether the favored values of $n(G) - 1$ correspond to Mersenne primes [A8]. **Table 10.1** lists the lowest hierarchies. Most of them are short.

The number of states assignable to M_n is $M_n = 2^n - 1$ which does not correspond to full n bits: the reason is that one of the states is not physically realizable. 2^{n-1} states have interpretation as maximal number of statements consistent with an atomic statement (single bit fixed) and to $n_b = n - 1$ bits. **Table 10.1** lists the values of n_b for Mersenne primes.

Notice that micro-tubules decompose into 13 parallel helices consisting of 13 tubulin dimers. Could these helices with the conformation of the last tubulin dimer serving as a kind of parity bit realize M_{13} code?

There would be a nice connection with the basic phenomenology of ordinary computers. The value of the integer $n-1$ associated with Mersenne primes would be analogous to the number of bits of the basic information unit of processor. During the evolution of PCs it has evolved from 8 to 32 and is also power of 2.

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10.4.3 The Role Of High T_c Superconductivity In TQC

A simple model for braid strands leads to the understanding of how high T_c super conductivity assigned with cell membrane [K42] could relate to TQC. The most plausible identification of braid strands is as magnetic or wormhole magnetic flux tubes consisting of pairs of flux tubes connected by wormhole contacts whose throats carry fermion and anti-fermion such that their rotational motion at least partially generates the antiparallel magnetic fluxes at the two sheets of flux tube. The latter option is favored by the model of TQC but one must of course keep mind open for variants of the model involving only ordinary flux tubes. Both kinds of flux tubes can carry charged particles such as protons, electrons, and biologically important ions as dark matter with large Planck constant and the model for nerve pulse and EEG indeed relies on this assumption [K94].

Currents at space-like braid strands

If space-like braid strands are identified as idealized structures obtained from 3-D tube like structures by replacing them with 1-D strands, one can regard the braiding as a purely geometrical knotting of braid strands.

The simplest realization of the braid strand as magnetic flux tube would be as a hollow cylindrical surface connecting conjugate DNA nucleotide to cell membrane and going through 5- and/or 6- cycles associated with the sugar backbone of conjugate DNA nucleotides. The free electron pairs associated with the aromatic cycles would carry the current creating the magnetic field needed.

For wormhole magnetic flux one would have pair of this kind of hollow cylinders connected by wormhole contacts and carrying opposite magnetic fluxes. In this case the currents created by wormhole contacts would give rise to the antiparallel magnetic fluxes at the space-time sheets of wormhole contact and could serve as controllers of TQC. I have indeed proposed long time ago that so called wormhole Bose-Einstein condensates might be fundamental for the quantum control in living matter [K131]. In this case the presence of supra currents at either sheet would generate asymmetry between the magnetic fluxes.

There are two extreme options for both kinds of magnetic fields. For B-option magnetic field is parallel to the strand and vector potential rotates around it. For A-option vector potential is parallel to the strand and magnetic field rotates around it. The general case corresponds to the hybrid of these options and involves helical magnetic field, vector potential, and current.

1. For B-option current flowing around the cylindrical tube in the transversal direction would generate the magnetic field. The splitting of the flux tube would require that magnetic flux vanishes requiring that the current should go to zero in the process. This would make possible selection of a part of DNA strand participating to TQC.

2. For A-option the magnetic field lines of the braid would rotate around the cylinder. This kind of field is created by a current in the direction of cylinder. In the beginning of TQC the strand would split and the current of electron pairs would stop flowing and the magnetic field would disappear. Also now the initiation of computation would require stopping of the current and should be made selectively at DNA.

   The control of the TQC should rely on currents of electron pairs (perhaps Cooper pairs) associated with the braid strands. Supra currents would have quantized values and they are therefore very attractive candidates. The (supra) currents could also bind lipids to pairs so that they would define single dynamical unit in 2-D hydrodynamical flow. One can also think that Cooper pairs with electrons assignable to different members of lipid pair bind it to a single dynamical unit.

Do supra currents generate magnetic fields?

Energetic considerations favor the possibility that supra currents create the magnetic fields associated with the braid strands defined by magnetic flux tubes. In the case of wormhole magnetic flux tubes supra currents could generate additional magnetic fields present only at the second sheet of the flux tube.

Supra current would be created by a voltage pulse ΔV, which gives rise to a constant supra current after it has ceased. Supra current would be destroyed by a voltage pulse of opposite sign. Therefore voltage pulses could define an elegant fundamental control mechanism allowing to select the parts of genome participating to TQC. This kind of voltage pulse could be collectively initiated at cell membrane or at DNA. Note that constant voltage gives rise to an oscillating supra current.

Josephson current through the cell membrane would be also responsible for dark Josephson radiation determining that part of EEG which corresponds to the correlate of neuronal activity [K42] . Note that TGD predicts a fractal hierarchy of EEGs and that ordinary EEG is only one level in this hierarchy. The pulse initiating or stopping TQC would correspond in EEG to a phase shift by a constant amount

$$\Delta\Phi = Ze\Delta VT/\hbar \ ,$$

where T is the duration of pulse and ΔV its magnitude.

The contribution of Josephson current to EEG responsible for beta and theta bands interpreted as satellites of alpha band should be absent during TQC and only EEG rhythm would be present. The periods dominated by EEG rhythm should be observed as EEG correlates for problem solving situations (say mouse in a maze) presumably involving TQC. The dominance of slow EEG rhythms during sleep and meditation would have interpretation in terms of TQC.

Topological considerations

The existence of supra current requires that the flow allows for a complex phase $exp(i\Psi)$ such that supra current is proportional to $\nabla\Psi$. This requires integrability in the sense that one can assign to the flow lines of A or B (combination of them in the case of A-B braid) a coordinate variable Ψ varying along the flow lines. In the case of a general vector field X this requires $\nabla\Psi = \Phi X$ giving $\nabla \times X = -\nabla\Phi/\Phi$ as an integrability condition. This condition defines what is known as Beltrami flow [K19].

The perturbation of the flux tube, which spoils integrability in a region covering the entire cross section of flux tube means either the loss of super-conductivity or the disappearance of the net supra current. In the case of the A-braid, the topological mechanism causing this is the increase in the dimension of the CP_2 projection of the flux tube so that it becomes 3-D [K19], where I have also considered the possibility that 3-D character of CP_2 projection is what transforms the living matter to a spin glass type phase in which very complex self-organization patterns emerge. This would conform with the idea that in TQC takes place in this phase.

Fractal memory storage and TQC

If Josephson current through cell membrane ceases during TQC, TQC manifests itself as the presence of only EEG rhythm characterized by an appropriate cyclotron frequency. Synchronous neuron firing might therefore relate to TQC. The original idea that a phase shift of EEG is induced by the voltage initiating TQC - although wrong - was however useful in that it inspired the question whether the initiation of TQC could have something to do with what is known as a place coding by phase shifts performed by hippocampal pyramidal cells [J18, J23]. The playing with this idea provides important insights about the construction of quantum memories and demonstrates the amazing explanatory power of the paradigm once again.

The model also makes explicit important conceptual differences between TQC a la TGD and in the ordinary sense of wordin particular those related to different view about the relation between subjective and geometric time.

1. In TGD TQC corresponds to the unitary process U taking place following by a state function reduction and preparation. It replaces WCW ("world of classical worlds") spinor field with a new one. WCW spinor field represent generalization of time evolution of Schrödinger equation so that a quantum jump occurs between entire time evolutions. Ordinary TQC corresponds to Hamiltonian time development starting at time $t = 0$ and halting at $t = T$ to a state function reduction.

2. In TGD the expression of the result of TQC is essentially 4-D pattern of gene expression (spiking pattern in the recent case). In usual TQC it would be 3-D pattern emerging as the computation halts at time t. Each moment of consciousness can be seen as a process in which a kind of 4-D statue is carved by starting from a rough sketch and proceeding to shorter details and building fractally scaled down variants of the basic pattern. Our life cycle would be a particular example of this process and would be repeated again and again but of course not as an exact copy of the previous one.

1. Empirical findings

The place coding by phase shifts was discovered by O'Reefe and Recce [J18]. In [J23] Y. Yamaguchi describes the vision in which memory formation by so called theta phase coding is essential for the emergence of intelligence. It is known that hippocampal pyramidal cells have "place property" being activated at specific "place field" position defined by an environment consisting of recognizable objects serving as landmarks. The temporal change of the percept is accompanied by a sequence of place unit activities. The theta cells exhibit change in firing phase distributions relative to the theta rhythm and the relative phase with respect to theta phase gradually increases as the rat traverses the place field. In a cell population the temporal sequence is transformed into a phase shift sequence of firing spikes of individual cells within each theta cycle.

Thus a temporal sequence of percepts is transformed into a phase shift sequence of individual spikes of neurons within each theta cycle along linear array of neurons effectively representing time axis. Essentially a time compressed representation of the original events is created bringing in mind temporal hologram. Each event (object or activity in perceptive field) is represented by a firing of one particular neuron at time τ_n measured from the beginning of the theta cycle. τ_n is obtained by scaling down the real time value t_n of the event. Note that there is some upper bound for the total duration of memory if scaling factor is constant.

This scaling down - story telling - seems to be a fundamental aspect of memory. Our memories can even abstract the entire life history to a handful of important events represented as a story lasting only few seconds. This scaling down is thought to be important not only for the representation of the contextual information but also for the memory storage in the hippocampus. Yamaguchi and collaborators have also found that the gradual phase shift occurs at half theta cycle whereas firings at the other half cycle show no correlation [J23] . One should also find an interpretation for this.

2. TGD based interpretation of findings

How this picture relates to TGD based 4-D view about memory in which primary memories are stored in the brain of the geometric past?

1. The simplest option is the initiation of TQC like process in the beginning of each theta cycle of period T and having geometric duration $T/2$. The transition $T \rightarrow T/2$ conforms nicely with the fundamental hierarchy of time scales comings as powers defining the hierarchy of measurement resolutions and associated with inclusions of hyperfinite factors of type II_1 [K127]. That firing is random at second half of cycle could simply mean that no TQC is performed and that the second half is used to code the actual events of "geometric now".
2. In accordance with the vision about the hierarchy of Planck constants defining a hierarchy of time scales of long term memories and of planned action, the scaled down variants of memories would be obtained by down-wards scaling of Planck constant for the dark space-time sheet representing the original memory. In principle a scaling by any factor $1/n$ (actually by any rational) is possible and would imply the scaling down of the geometric time span of TQC and of light-like braids. One would have TQC's inside TQC's and braids within braids (flux quanta within flux quanta). The coding of the memories to braidings would be an automatic process as almost so also the formation of their zoomed down variants.
3. A mapping of the time evolution defining memory to a linear array of neurons would take place. This can be understood if the scaled down variant (scaled down value of $\hbar$) of the space-time sheet representing original memory is parallel to the linear neuron array and contains at scaled down time value t_n a stimulus forcing n^{th} neuron to fire. The 4-D character of the expression of the outcome of TQC allows to achieve this automatically without complex program structure.

To sum up, it seems that the scaling of Planck constant of time like braids provides a further fundamental mechanism not present in standard TQC allowing to build fractally scaled down variants of not only memories but TQC's in general. The ability to simulate in shorter time scale is a certainly very important prerequisite of intelligent and planned behavior. This ability has also a space-like counterpart: it will be found that the scaling of Planck constant associated with space-like braids connecting bio-molecules might play a fundamental role in DNA replication, control of transcription by proteins, and translation of mRNA to proteins. A further suggestive conclusion is that the period T associated with a given EEG rhythm defines a sequence of TQC's having geometric span $T/2$ each: the rest of the period would be used to perceive the environment of the geometric now. The fractal hierarchy of EEGs would mean that there are TQC's within TQC's in a very wide range of time scales.

10.4.4 Codes And TQC

TGD suggests the existence of several (genetic) codes besides 3-codon code [K53, **?**]. The experience from ordinary computers and the fact that genes in general do not correspond to $3n$ nucleotides encourages to take this idea more seriously. The use of different codes would allow to tell what kind of information a given piece of DNA strand represents. DNA strand would be like a drawing of building containing figures (3-code) and various kinds of text (other codes). A simple drawing for the building would become a complex manual containing mostly text as the evolution proceeds: for humans 96 per cent of code would corresponds to introns perhaps obeying some other code.

The hierarchy of genetic codes is obtained by starting from n basic statements and going to the meta level by forming all possible statements about them (higher order logics) and throwing away one which is not physically realizable (it would correspond to empty set in the set theoretic realization). This allows $2^n - 1$ statements and one can select 2^{n-1} of statements consistent with atomic statement (one bit fixed) and say that these are true and give kind of axiomatics about world. The remaining statements are false. DNA would realize only these statements.

The hierarchy of Mersenne primes $M_n = 2^n - 1$ with $M_{n(next)} = M_{M_n}$ starting from $n = 2$ with $M_2 = 3$ gives rise to 1-code with 4 codons, 3-code with 64 codons, and $3 \times 21 = 63$-code with 2^{126} codons [K53] realized as sequences of 63 nucleotides (the length of 63-codon is about $2L(151)$, roughly twice the cell membrane thickness. It is not known whether this Combinatorial Hierarchy continues ad infinitum. Hilbert conjectured that this is the case.

In the model of pre-biotic evolution also 2-codons appear and 3-code is formed as the fusion of 1- and 2-codes. The problem is that 2-code is not predicted by the basic Combinatorial Hierarchy associated with $n = 2$.

There are however also other Mersenne hierarchies and the next hierarchy allows the realization of the 2-code. This Combinatorial Hierarchy begins from Fermat prime $n = 2^k + 1 = 5$ with $M_5 = 2^5 - 1 = 31$ gives rise to a code with 16 codons realized as 2-codons (2 nucleotides). Second level corresponds to Mersenne prime $M_{31} = 2^{31} - 1$ and a code with $2^{30=15\times 2}$ codons realized by sequences of 15 3-codons containing 45 nucleotides. This corresponds to DNA length of 15 nm, or length scale $3L(149)$, where $L(149) = 5$ nm defines the thickness of the lipid layer of cell membrane. $L(151) = 10$ nm corresponds to 3 full 2π twists for DNA double strand. The model for 3-code as fusion of 1- and 2-codes suggests that also this hierarchy - which probably does not continue further - is realized.

There are also further short Combinatorial hierarchies corresponding to Mersenne primes [A8].

1. $n = 13$ defines Mersenne prime M_{13}. The code would have $2^{12=6\times 2}$ codons representable as sequences of 6 nucleotides or 2 3-codons. This code might be associated with microtubuli.

2. The Fermat prime $17 = 2^4 + 1$ defines Mersenne prime M_{17} and the code would have $2^{16=8\times 2}$ codons representable as sequences of 8 nucleotides.

3. $n = 19$ defines Mersenne prime M_{19} and code would have $2^{18=9\times 2}$ codons representable as sequences of 9 nucleotides or three DNA codons.

4. The next Mersennes are M_{31} belonging to $n = 5$ hierarchy, M_{61} with $2^{60=30\times 2}$ codons represented by 30-codons. This corresponds to DNA length $L(151) = 10$ nm (cell membrane thickness). M_{89} (44-codons), M_{107} (53-codons) and M_{127} (belonging to the basic hierarchy) are the next Mersennes. Next Mersenne corresponds to M_{521} (260-codon) and to completely super-astrophysical p-adic length scale and might not be present in the hierarchy.

   This hierarchy is realized at the level of elementary particle physics and might appear also at the level of DNA. The 1-, 2-, 3-, 6-, 8-, and 9-codons would define lowest Combinatorial Hierarchies.

10.5 How To Realize The Basic Gates?

In order to have a more concrete view about realization of TQC, one must understand how quantum computation can be reduced to a construction of braidings from fundamental unitary operations. The article "Braiding Operators are Universal Quantum Gates" by Kaufmann and Lomonaco [B38] contains a very lucid summary of how braids can be used in topological quantum computation.

1. The identification of the braiding operator R - a unitary solution of Yang-Baxter equation - as a universal 2-gate is discussed. In the following I sum up only those points which are most relevant for the recent discussion.

2. One can assign to braids both knots and links and the assignment is not unique without additional conditions. The so called braid closure assigns a unique knot to a given braid by connecting n^{th} incoming strand to n^{th} outgoing strand without generating additional knotting. All braids related by so called Markov moves yield the same knot. The Markov

trace (q-trace actually) of the unitary braiding S-matrix U is a knot invariant characterizing the braid closure.

3. Braid closure can be mimicked by a topological quantum computation for the original n-braid plus trivial n-braid and this leads to a quantum computation of the modulus of the Markov trace of U. The probability for the diagonal transition for one particular element of Bell basis (whose states are maximally entangled) gives the modulus squared of the trace. The closure can be mimicked quantum computationally.

10.5.1 Universality Of TQC

Quantum computer is universal if all unitary transformations of n^{th} tensor power of a finite-dimensional state space V can be realized. Universality is achieved by using only two kinds of gates. The gates of first type are single particle gates realizing arbitrary unitary transformation of $U(2)$ in the case of qubits. Only single 2-particle gate is necessary and universality is guaranteed if the corresponding unitary transformation is entangling for some state pair. The standard choice for the 2-gate is CNOT acting on bit pair (t, c). The value of the control bit c remains of course unchanged and the value of the target bit changes for $c = 1$ and remains unchanged for $c = 0$.

10.5.2 The Fundamental Braiding Operation As A Universal 2-Gate

The realization of CNOT or gate equivalent to it is the key problem in topological quantum computation. For instance, the slow de-coherence of photons makes quantum optics a promising approach but the realization of CNOT requires strongly nonlinear optics. The interaction of control and target photon should be such that for second polarization of the control photon target photon changes its direction but keeps it for the second polarization direction.

For braids CNOT can be expressed in terms of the fundamental braiding operation e_n representing the exchange of the strands n and $n+1$ of the braid represented as a unitary matrix R acting on $V_n \otimes V_{n+1}$.

The basic condition on R is Yang-Baxter equation expressing the defining condition $e_n e_{n+1} e_n = e_{n+1} e_n e_{n+1}$ for braid group generators. The solutions of Yang-Baxter equation for spinors are well-known and CNOT can be expressed in the general case as a transformation of form $A_1 \otimes A_2 R A_3 \otimes A_4$ in which single particle operators A_i act on incoming and outgoing lines. 3-braid is the simplest possible braid able to perform interesting TQC, which suggests that genetic codons are associated with 3-braids.

The dance of lipids on chessboard defined by the lipid layer would reduce R to an exchange of neighboring lipids. For instance, the matrix $R = DS$, $D = diag(1, 1, 1, -1)$ and $S = e_{11} + e_{23} + e_{32} + e_{44}$ the swap matrix permuting the neighboring spins satisfies Yang-Baxter equation and is entangling.

10.5.3 What The Replacement Of Linear Braid With Planar Braid Could-Mean?

Standard braids are essentially linear objects in plane. The possibility to perform the basic braiding operation for the nearest neighbors in two different directions must affect the situation somehow.

1. Classically it would seem that the tensor product defined by a linear array must be replaced by a tensor product defined by the lattice defined by lipids. Braid strands would be labelled by two indices and the relations for braid group would be affected in an obvious manner.

2. The fact that DNA is a linear structure would suggests that the situation is actually effectively one-dimensional, and that the points of the lipid layer inherit the linear ordering of nucleotides of DNA strand. One can however ask whether the genuine 2-dimensionality could provide a mathematical realization for possible long range correlations between distant nucleotides n and $n + N$ for some N. p-Adic effective topology for DNA might become manifest via this kind of correlations and would predict that N is power of some prime p which might depend on organism's evolutionary level.

3. Quantum conformal invariance would suggest effective one-dimensionality in the sense that only the observables associated with a suitably chosen linear braid commute. One might also speak about topological quantum computation in a direction transversal to the braid strands giving a slicing of the cell membrane to parallel braid strands. This might mean an additional computational power.

4. Partonic picture would suggest a generalization of the linear braid to a structure consisting of curves defining the decomposition of membrane surface regions such that conformal invariance applies separately in each region: this would mean breaking of conformal invariance and 2-dimensionality in discrete sense. Each region would define a one parameter set of topological quantum computations. These regions might corresponds to genes. If each lipid defines its own conformal patch one would have a planar braid.

10.5.4 Single Particle Gates

The realization of single particle gates as $U(2)$ transformations leads naturally to the extension of the braid group by assigning to the strands sequences of group elements satisfying the group multiplication rules. The group elements associated with a n^{th} strand commute with the generators of braid group which do not act on n^{th} strand. G would be naturally subgroup of the covering group of rotation group acting in spin degrees of spin 1/2 object. Since $U(1)$ transformations generate only an overall phase to the state, the presence of this factor might not be necessary. A possible candidate for $U(1)$ factor is as a rotation induced by a time-like parallel translation defined by the electromagnetic scalar potential $\Phi = A_t$.

One of the challenges is the realization of single particle gates representing $U(2)$ rotation of the qubit. The first thing to come mind was that $U(2)$ corresponds to $U(2)$ rotation induced by magnetic field and electric fields. A more elegant realization is in terms of $SU(3)$ rotation, where $SU(3)$ is color group associated with strong interactions. This looks rather weird but there is direct evidence for the prediction that color $SU(3)$ is associated with TQC and thus cognition: something that does not come first in mind! I have myself written text about the strange finding of topologist Barbara Shipman suggesting that quarks are in some mysterious manner involved with honeybee dance and proposed an interpretation.

The realization of qubit as ordinary spin

A possible realization for single particle gate $s \subset SU(2)$ would be as $SU(2)$ rotation induced by a magnetic pulse. This transformation is fixed by the rotation axis and rotation angle around this axes. This kind of transformation would result by applying to the strand a magnetic pulse with magnetic field in the direction of rotation axes. The duration of the pulse determines the rotation angle. Pulse could be created by bringing a magnetic flux tube to the system, letting it act for the required time, and moving it away. $U(1)$ phase factor could result from the electromagnetic gauge potential as a non-integrable phase factor $exp(ie \int A_t dt/\hbar)$ coming from the presence of scale potential $\Phi = A_t$ in the Hamiltonian.

Conrete model for realization of 1-gates in terms of ordinary rotations

What could be the simplest realization of the $U(2)$ transformation in the case of cell membrane assuming that it corresponds to ordinary rotation?

1. There should be a dark spin 1/2 particle associated with each lipid, electron or proton most plausibly. TGD based model for high T_c superconductivity [K24] predicts that Cooper pairs correspond to pairs of cylindrical space-time sheets with electrons at the two space-time sheets. The size scale of the entire Cooper corresponds to p-adic length scale $L(151)$ defining the thickness of the cell membrane and cylindrical structure to $L(149)$, the thickness of lipid layer so that electrons are the natural candidates for TQC. The Cooper pair BE condensate would fuse the lipid pairs to form particles of lipid liquid.

2. Starting of TQC requires the splitting of electron Cooper pairs and its halting the formation of Cooper pairs again. The initiation of TQC could involve increase of temperature or an introduction of magnetic field destroying the Cooper pairs. Tqc could be also controlled by

supra currents flowing along cylindrical flux tubes connecting 5- and/or aromatic cycles of conjugate DNA nucleotides to the cell membrane. The cutting of the current flow would make it possible for braid strand to split and TQC to begin.

3. By shifting a magnetic flux tube or sheet parallel to the cell membrane to the position of the portion of membrane participating to TQC is the simplest manner to achieve this. Halting could be achieved by removing the flux tube. The unitary rotation induced by the constant background magnetic field would not represent gate and it should be possible to eliminate its effect from TQC proper.
4. The gate would mean the application of a magnetic pulse much stronger than background magnetic field on the braid strands ending at the lipid layer. The model for the communication of sensory data to the magnetic body requires that magnetic flux tubes go through the cell membrane. This would suggest that the direction of the magnetic flux tube is temporarily altered and that the flux tube then covers part of the lipid for the required period of time.

   The realization of the single particle gates requires electromagnetic interactions. That single particle gates are not purely topological transformations could bring in the problems caused by a de-coherence due to electromagnetic perturbations. The large values of Planck constant playing a key role in the TGD based model of living matter could save the situation. The large value of $\hbar$ would be also required by the anyonic character of the system necessary to obtain R-matrix defining a universal 2-gate.

   The minimum time needed to inducing full 2π rotation around the magnetic axes would be essentially the inverse of cyclotron frequency for the particle in question in the magnetic field considered$T = 1/f_c = 2\pi m/ZeB$. For electrons in the dark magnetic field of $B = .2$ Gauss assigned to living matter in the quantum model of EEG this frequency would be about $f_c = .6$ MHz. For protons one would have $f_c = 300$ Hz. For a magnetic field of Tesla the time scales would be reduced by a factor 2×10^{-5}.

The realization of 1-gate in terms of color rotations

One can criticize the model of 1-gates based on ordinary spin. The introduction of magnetic pulses does not look an attractive idea and seems to require additional structures besides magnetic flux tubes (MEs?). It would be much nicer to assign the magnetic field with the flux tubes defining the braid strands. The rotation of magnetic field would however require changing the direction of braid strands. This does not look natural. Could one do without this rotation by identifying spin like degree of freedom in some other manner? This is indeed possible.

TGD predicts a hierarchy of copies of scaled up variants of both weak and color interactions and these play a key role in TGD inspired model of living matter. Both weak isospin and color isospin could be considered as alternatives for the ordinary spin as a realization of qubit in TGD framework. Below color isospin is discussed but one could consider also a realization in terms of nuclei and their exotic counterparts [L1], [L1] differing only by the replacement of neutral color bond between nuclei of nuclear string with a charged one. Charge entanglement between nuclei would guarantee overall charge conservation.

1. Each space-time sheet of braid strands contains quark and antiquark at its ends. Color isospin and hypercharge label their states. Two of the quarks of the color triplet form doublet with respect to color isospin and the third is singlet and has different hyper charge Y. Hence qubit could be realized in terms of color isospin I_3 instead of ordinary spin but third quark would be inert in the Boolean sense. Qubit could be also replaced with qutrit and isospin singlet could be identified as a statement with ill-defined truth value. Trits are used also in ordinary computers. In TGD framework finite measurement resolution implies fuzzy qubits and the third state might relate to this fuzziness. Also Gödelian interpretation can be consideredthe quark state with vanishing isospin would be associated with counterparts of undecidable propositions to which one cannot assign truth value (consider sensory input which is so ambiguous that one cannot tell what is there or a situation in which one cannot decide whether to do something or not). Note that hyper-charge would induce naturally the U(1) factor affecting the over all phase of qubit but affecting differently to the third quark.
2. Magnetic flux tubes are also color magnetic flux tubes carrying non-vanishing classical color gauge field in the case that they are non-vacuum extremals. The holonomy group of classical

color field is an Abelian subgroup of the $U(1)\times U(1)$ Cartan subgroup of color group. Classical color magnetic field defines the choice of quantization axes for color quantum numbers. For instance, magnetic moment is replaced with color magnetic moment and this replacement is in key role in simple model for color magnetic spin spin splittings between spin 0 and 1 mesons as well as spin 1/2 and 3/2 baryons.

3. There is a symmetry breaking of color symmetry to subgroup $U(1)_{I_3} \times U(1)_Y$ and color singletness is in TGD framework replaced by a weaker condition stating that physical states have vanishing net color quantum numbers. This makes possible the measurement of color quantum numbers in the manner similar to that for spin. For instance, color singlet formed by quark and antiquark with opposite color quantum numbers can in the measurement of color quantum numbers of quark reduce to a state in which quark has definite color quantum numbers. This state is a superposition of states with vanishing Y and I_3 in color singlet and color octet representations. Strong form of color confinement would not allow this kind of measurement.

4. Color rotation in general changes the directions of quantization axis of I_3 and Y and generates a new state basis. Since $U(1) \times U(1)$ leaves the state basis invariant, the space defined by the choices of quantization axes is 6-dimensional flag manifold $F = SU(3)/U(1) \times U(1)$. In contrast to standard model, color rotations in general do not leave classical electromagnetic field invariant since classical em field is a superposition of color invariant induced Kähler from and color non-invariant part proportional classical Z^0 field. Hence, although the magnetic flux tube retains its direction and shape in M^4 degrees of freedom, its electromagnetic properties are affected and this is visible at the level of classical electromagnetic interactions.

5. If color isospin defines the qubit or qutrit in topological quantum computation, color quantum numbers and the flag manifold F should have direct relevance for cognition. Amazingly, there is a direct experimental support for this! Years ago topologist Barbara Shipman made the intriguing observation that honeybee dance can be understood in terms of a model involving the flag manifold F [A25]. This led her to propose that quarks are in some mysterious manner involved with the honeybee dance. My proposal [K51] was that color rotations of the space-time sheets associated with neurons represent geometric information: sensory input would be coded to color rotations defining the directions of quantization axes for I_3 and Y. Subsequent state function reduction would provide conscious representations in terms of trits characterizing for instance sensory input symbolically.

   In [K51] I introduced the notions of geometric and sensory qualia corresponding to two choices involved with the quantum measurement: the choice of quantization axes performed by the measurer and the "choice" of final state quantum numbers in state function reduction. In the case of honeybee dance geometric qualia could code information about the position of the food source. The changes of color quantum numbers in quantum jump were identified as visual colors. In state function reduction one cannot speak about change of quantum numbers but about their emergence. Therefore one must distinguish between color qualia and the conscious experience defined by the emergence of color quantum numbers: the latter would have interpretation as qutrit.

Summarizing, this picture suggests that 1-gates of DNA TQC (understood as "dance of lipids") are defined by color rotations of the ends of space-like braid strands and at lipids. The color rotations would be induced by sensory and other inputs to the system. Topological quantum computation would be directly related to conscious experience and sensory and other inputs would fix the directions of the color magnetic fields.

Realization of braid operation in terms of $h_{eff} = n \times h$ hypothesis

This option would realize braiding as an analog for braiding for the degrees of freedom representable in terms of n-fold covering of embedding space (or space-time surfaces). The different branches of covering would relate to the branching of preferred extremal in n-furcation. Simplest n-furcation would corresponds to that resulting when 2π rotated space-time point no more corresponds to the original point (note that analog with Riemann surface associated with $z^{1/n}$. Similar phenomenon is possible in CP_2 degrees of freedom. The vision is that it is possible to construct dark k-particle states for $k \leq n$ in these discrete degrees of freedom so that discrete variant of second quantization would be in question.

Since large $h_{eff} = nh$ is highly favourable for TQC, the idea that living matter would perform TQC using dark matter phase. This option does not seem to be in conflict with the other option. One can in principle assign to each ordinary quantum state an $h_{eff} = nh$ and even allow the value of n to depend on the ordinary quantum numbers. By NMP state function reduction leads to one of these sub-spaces. As discussed, the outcome of TQC could be deduced by interaction free quantum measurement utilizing "externalized" state function reduction. It is of course not all obvious whether this procedure is equivalent with the standard one. The large value of h_{eff} would increase quantum coherence time and quantum coherence length associated with ordinary quantum numbers so that halting in this sense would be possible.

10.6 About Realization Of Braiding

The most plausible identification of braid strands is as magnetic or wormhole magnetic flux tubes. Flux tubes can contain charged particles such as protons, electrons, and biologically important ions as dark matter with large Planck constant and the model for nerve pulse and EEG indeed relies on this assumption [K94].

10.6.1 Could Braid Strands Be Split And Reconnect All The Time?

As far as braiding alone is considered, braid strands could be split all the time. This would require wormhole flux tubes if strands carry monopole flux. In other words, there would be no continuation of strands through the cell membrane. Computation would halt when lipids lose their unsaturated cis bonds so that they cannot follow the liquid flow. The conservation of strand color would be trivially true but would not have any implications. Supra currents would not be needed to control TQC and there would be no connection with generalized EEG. It is not obvious how the gene expression for the outcome of TQC could take place since the strands would not connect genome to genome. For these reasons this option does not look attractive.

The models for prebiotic evolution [?] and protein folding [K7] lead to a suggestion that braids can connect all kind of bio-molecules to each other and also water molecules and bio-molecules. Thus DNA TQC would represent only one example of TQC like activities performed by the living matter. The conclusion is that braidings are dynamical with reconnection of flux tubes representing a fundamental transformation changing the braiding and thus also TQC programs.

10.6.2 What Do Braid Strands Look Like?

In the following the anatomy of braid strands is discussed at general level and then and identification in terms of flux tubes of magnetic body is proposed.

Braid strands as nearly vacuum extremals

The braid strands should be nearly quantum critical sub-manifolds of $M^4 \times CP_2$ so that phase transitions changing Planck constant and thus their length can take place easily (DNA replication, binding of mRNA molecules to DNA during transcription, binding of transcription factors to promoters, binding of tRNA-amino-acid complexes to mRNA...).

Depending on whether phase transition takes place in M^4 or CP_2 degrees of freedom, either their M^4 projection belongs to $M^2 \subset M^4$ or their CP_2 projection to the homological trivial geodesic sphere $S^2 \subset CP_2$. In the latter case a vacuum extremal is in question. Maximal quantum criticality means $X^4 \subset M^2 \times S^2$ so that one has straight string with a vanishing string tension. The almost vacuum extremal property guarantees the braid strands can be easily generated from vacuum.

An additional requirement is that the gravitational mass is small. For objects of type $M^2 \times X_g^2$, $X_g^2 \subset E^2 \times CP_2$, the gravitational mass vanishes for $g = 1$ (genus) and is of order CP_2 mass otherwise and negative for $g > 1$. Torus topology is the unique choice. A simple model for the braid strand is as a small non-vacuum deformation of $X^4 = M^2 \times X_g^2 \subset M^2 \subset E^2 \times S^2$, $g = 1$. As a special case one has $X^4 = M^2 \times S^1 \times S^1 \subset M^2 \subset E^2 \times S^1$, for which M^4 projection is a hollow cylinder, which could connect the aromatic 5- or 6-cycle of sugar backbone to another DNA strand, lipid, or amino-acid.

Braid strands as flux tubes of color magnetic body

One can make this model more detailed by feeding in simple physical inputs. The flux tubes carry magnetic field when the supra current is on. In TGD Universe all classical fields are expressible in terms of the four CP_2 coordinates and their gradients so that em, weak, color and gravitational fields are not independent as in standard model framework. In particular, the ordinary classical em field is necessarily accompanied by a classical color field in the case of non-vacuum extremals. This predicts color and ew fields in arbitrary long scales and quantum classical correspondence forces to conclude that there exists fractal hierarchy of electro-weak and color interactions.

Since the classical color gauge field is proportional to Kähler form, its holonomy group is Abelian so that effectively $U(1) \times U(1) \subset SU(3)$ gauge field is in question. The generation of color flux requires colored particles at the ends of color flux tube so that the presence of pairs of quark and antiquark assignable to the pairs of wormhole throats at the ends of the tube is unavoidable if one accepts quantum classical correspondence.

In the case of cell, a highly idealized model for color magnetic flux tubes is as flux tubes of a dipole field. The preferred axis could be determined by the position of the centrosomes forming a T shaped structure. DNA strands would define the idealized dipole creating this field: DNA is indeed negatively charged and electronic currents along DNA could create the magnetic field. The flux tubes of this field would go through nuclear and cell membrane and return back unless they end up to another cell. This is indeed required by the proposed model of TQC.

It has been assumed that the initiation of TQC means that the supra current ceases and induces the splitting of braid strands. The magnetic flux need not however disappear completely. As a matter fact, its presence forced by the conservation of magnetic flux seems to be crucial for the conservation of braiding. Indeed, during TQC magnetic and color magnetic flux could return from lipid to DNA along another space-time sheet at a distance of order CP_2 radius from it. For long time ago I proposed that this kind of structures -which I christened "wormhole magnetic fields" - might play key role in living matter [K131]. The wormhole contacts having quark and antiquark at their opposite throats and coding for A, T, C, G would define the places where the current flows to the "lower" space-time sheet to return back to DNA. Quarks would also generate the remaining magnetic field and supra current could indeed cease.

The fact that classical em fields and thus classical color fields are always present for non-vacuum extremals means that also the motion of any kind of particles (space-time sheets), say water flow, induces a braiding of magnetic flux tubes associated with molecules in water if the temporary splitting of flux tubes is possible. Hence the prerequisites for TQC are met in extremely general situation and TQC involving DNA could have developed from a much simpler form of TQC performed by water giving perhaps rise to what is known as water memory [I50, I51, I57, I58]. This would also suggest that the braiding operation is induced by the a controlled flow of cellular water.

10.6.3 How To Induce The Basic Braiding Operation?

The basic braiding operation requires the exchange of two neighboring lipids. After some basic facts about phospholipids the simplest model found hitherto is discussed.

Some facts about phospholipids

Phospholipids [I37] - which form about 30 per cent of the lipid content of the monolayer - contain phosphate group. The dance of lipids requires metabolic energy and the hydrophilic ends of the phospholipid could provide it. They could also couple the lipids to the flow of water in the vicinity of the lipid monolayer possibly inducing the braiding. Of course, the causal arrow could be also opposite.

The hydrophilic part of the phospholipid is a nitrogen containing alcohol such as serine, inositol or ethanolamine, or an organic compound such as choline. Phospholipids are classified into 3 kinds of phosphoglycerides [I36] and sphingomyelin.

1. Phosphoglycerides

In cell membranes, phosphoglycerides are the more common of the two phospholipids, which suggest that they are involved with TQC. One speaks of phosphotidyl X, where X= serine, inositol, ethanolamine is the nitrogen containing alcohol and X=Ch the organic compound. The shorthand notion OS, PI, PE, PCh is used.

The structure of the phospholipid is most easily explained using the dancer metaphor. The two fatty chains define the hydrophobic feet of the dancer, glycerol and phosphate group define the body providing the energy to the dance, and serine, inositol, ethanolamine or choline define the hydrophilic head of the dancer (perhaps "deciding" the dancing pattern).

There is a lipid asymmetry in the cell membrane. PS, PE, PI in cytoplasmic monolayer (alcohols). PC (organic) and sphingomyelin in outer monolayer. Also glycolipids are found only in the outer monolayer. The asymmetry is due to the manner that the phospholipids are manufactured.

[I35] [I35] in the inner monolayer is negatively charged and its presence is necessary for the normal functioning of the cell membrane. It activates protein kinase C which is associated with memory function. PS slows down cognitive decline in animals models. This encourages to think that the hydrophilic polar end of at least PS is involved with TQC, perhaps to the generation of braiding via the coupling to the hydrodynamic flow of cytoplasm in the vicinity of the inner monolayer.

2. Fatty acids

The fatty acid chains in phospholipids and glycolipids usually contain an even number of carbon atoms, typically between 14 and 24 making 5 possibilities altogether. The 16- and 18-carbon fatty acids are the most common. Fatty acids [I17] may be saturated or unsaturated, with the configuration of the double bonds nearly always cis.

The length and the degree of unsaturation of fatty acids chains have a profound effect on membranes fluidity as unsaturated lipids create a kink, preventing the fatty acids from packing together as tightly, thus decreasing the melting point (increasing the fluidity) of the membrane. The number of unsaturaded cis bonds and their positions besides the number of Carbon atoms characterizes the lipid. Quite generally, there are $3n$ Carbons after each bond. The creation of unsatured bond by removing H atom from the fatty acid could be an initiating step in the basic braiding operation creating room for the dancers. The bond should be created on both neighboring lipids simultaneously.

Could hydrodynamic flow induce braiding operations?

One can imagine several models for what might happen during the braiding operation in the lipid bilayer [I26]. One such view is following.

1. The creation of unsaturated bond and involving elimination of H atom from fatty acid would lead to cis configuration and create the room needed by dancers. This operation should be performed for both lipids participating in the braiding operation. After the braiding it might be necessary to add H atom back to stabilize the situation. The energy needed to perform either or both of these operations could be provided by the phosphate group.

2. The hydrophilic ends of lipids couple the lipids to the surrounding hydrodynamic flow in the case that the lipids are able to move. This coupling could induce the braiding. The primary control of TQC would thus be by using the hydrodynamic flow by generating localized vortices. There is considerable evidence for water memory [I50] but its mechanism remains to be poorly understood. If also water memory is realized in terms of the braid strands connecting fluid particles, DNA TQC could have evolved from water memory.

3. Sol-gel phase transition is conjectured to be important for the quantum information processing of cell [J3]. In the transition which can occur cyclically actin filaments (also at EEG frequencies) are assembled and lead to a gel phase resembling solid. Sol phase could correspond to TQC and gel to the phase following the halting of TQC. Actin filaments might be assignable with braid strands or bundles of them and shield the braiding. Also microtubules might shield bundles of braid strands.

4. Only inner braid strands are directly connected to DNA which also supports the view that only the inner monolayer suffers a braiding operation during TQC and that the outer monolayer should be in a "freezed" state during it. There is a net negative charge associated with the inner mono-layer possibly relating to its participation to the braiding. The vigorous hydrodynamical flows known to take place below the cell membrane could induce the braiding.

10.6.4 Some Qualitative Tests

In life sciences the standard manner to test a model is to look whether the function of the system is affected in the predicted manner if one somehow interferes the system. Now interfering with TQC should affect the gene expression resulting otherwise.

1. Lipid layer hydrodynamics is predicted to allow two fundamental phases. The pairs of lipids should behave like single dynamical unit in super-conducting phase and as independent units in non-super-conducting phase. The application of magnetic field or increase of temperature should induce a transition between these two phases. These phase transitions applied selectively to the regions of cell membrane should affect gene expression. One could prevent halting of TQC by applying an external magnetic field and thus prevent gene expression. One could dream of deducing gene-membrane mapping with endoplasmic reticulum included.
2. The temperature range in which quantum critical high T_c super-conductivity is possible is probably rather narrow and should correspond to the temperature range in which cell membrane is functional. Brain is functional in a very narrow range of temperatures. Selective freezing of cell membrane might provide information about gene map provided by cell membrane.
3. One could do various things to the cell membrane. One could effectively remove part of it, freeze, or heat some part of the lipid liquid and look whether this has effects on gene expression. The known effects of ELF em fields on the behavior and physiology of vertebrates [K42] might relate to the fact that these fields interfere with TQC.
4. Artificially induced braiding by inducing a motion of lipids by some kind of stirring during TQC could induce/affect gene expression.
5. The application of external dark magnetic fields could affect gene expression. Tqc could be initiated artificially in some part of cell membrane by the application of dark magnetic field. Running TQC could be halted by an application of dark magnetic field interfering to zero with the background field. The application of magnetic pulses would affect TQC and thus gene expression. The problem is how to create dark magnetic fields in given length scale (range of magnetic field strength). Perhaps one could generate first ordinary magnetic field and then transform it to dark magnetic field by $\hbar$ changing phase transition. This could be achieved by a variation of some macroscopic parameters such as temperature, magnetic field strength, and analog of doping fraction appearing in standard high T_c super-conductivity.
6. Artificially induced scalings of $\hbar$ by varying temperature and parameters such as pH should induce or stop DNA replication, DNA-mRNA transcription and the translation of mRNA to proteins.

10.7 A Model For Flux Tubes

Biochemistry represents extremely complex and refined choreography. It is hard to believe that this reduces to a mere unconscious and actually apparent fight for chemical survival. In TGD Universe consciousness would be involved even at the molecular level and magnetic body would be the choreographer whose dance would induce the molecular activities. This picture combined with the idea of standard plugs and terminals at which flux tubes end, leads to a to a picture allowing to formulate a model for protein folding.

10.7.1 Flux Tubes As A Correlates For Directed Attention

Molecular survival is the standard candidate for the fundamental variational principle motivating the molecular intentional actions. There is entire hierarchy of selves and the survival at the higher level of hierarchy would force co-operation and altruistic behavior at the lower levels. One might hope that this hypothesis reduces to Negentropy Maximization Principle [K72], which states that the information contents of conscious experience is maximized. If this picture is accepted, the evolution of molecular system becomes analogous to the evolution of a society.

Directed attention is the basic aspect of consciousness and the natural guess would be that directed attention corresponds to the formation of magnetic flux tubes between subject and target. The directedness property requires some manner to order the subject and target.

1. The ordering by the values of Planck constant is what first comes in mind. The larger space-time sheet characterized by a larger value of Planck constant and thus at a higher level of evolutionary hierarchy would direct its attention to the smaller one.
2. Also the ordering by the value of p-adic prime characterizing the size scale of the space-time sheet could be considered but in this case directedness could be questioned.
3. Attention can be directed also to thoughts. Could this mean that attention is directed from real space-time sheets to p-adic space-time sheets for various values of primes but not vice versa? Or could the direction be just the opposite at least in the intentional action transforming p-adic space-time sheet to real space-time sheet? Perhaps directions are opposite for cognition.

The generation of (wormhole) magnetic flux tubes could be the correlate for the directed attention, not only at molecular level, but quite generally. Metaphorically, the strands of braid would be the light rays from the eyes of the perceiver to the target and their braiding would code the motions of the target to a topological quantum computation like activity and form a memory representation at least. The additional aspect of directed attention would be the coloring of the braid strands, kind of coloring for the virtual light rays emerging from the eyes of the molecular observer. In the case of DNA this can induce a coloring of braid strands emerging from amino-acids and other molecules so that it would indeed become possible to assign to free amino-acid the conjugate of the codon XYZ coding for it.

Attention can be also redirected. For this process there is a very nice topological description as a reconnection of flux tubes. What happens is that flux tubes $A \rightarrow B$ and $C \rightarrow D$ fuse for a moment and become flux tubes $A \rightarrow D$ and $C \rightarrow B$. This process is possible only if the strands have the same color so that the values of the quark charges associated with A and B are the same.

1. Reconnection process can modify TQC programs. For instance, in the case of the flux tubes coming from nucleotides X and X_c and ending to the lipid layer this process means that X and X_c and corresponding lipids become connected and genome builds memory representation about this process via similar link.
2. Reconnection process makes also possible what might be called color inheritance allowing amino-acids to inherit the conjugate colors of the nucleotides of the codon coding it.
3. DNA would have memory representation about molecular processes via these changing braiding topologies, and one could say that these molecular processes reflect the bodily motions of the magnetic body. Entire molecular dynamics of the organism could represent an enormous TQC induced by the motor activities of the magnetic body. At the level of sensory experience similar idea has been discussed earlier [K110]: out of body experiences (OBEs) and illusions such as train illusion could be understood in terms of motor action of magnetic body inducing virtual sensory percepts.

Attention can be also switched on and off. Here the structure of the lipid ends containing two nearby situated $=O$: s suggest the mechanism: the short flux tube connecting $=O$: s disappears by reconnection mechanism with a pair of hydrogen bonded water molecules leading to a shortcut of the connecting flux tubes to $= O - -H_2O$ hydrogen bonds. The minimization of Coulomb interaction energy at each end implies that re-appearance of the flux tubes creates a short flux tube with the original strand color.

10.7.2 Does Directed Attention Generate Memory Representations And TQC Like Processes

Directed attention induces braiding if the target is moving and changing its shape. This gives rise to a memory representation of the behavior of the object of attention and also to a TQC like process. A considerable generalization of TQC paradigm suggests itself.

Tqc could be induced by the braiding between DNA and lipids, DNA and proteins via folding processes, DNA RNA braiding and braiding between DNA and its conjugate, DNA and protein braiding. The outcome of TQC would be represented as the temporal patterns of biochemical concentrations and rates and there would be hierarchy of p-adic time scales and those associated with the dark matter hierarchy.

For instance, the protein content of lipid membranes is about 50 per cent and varies between 25-75 per cent so that protein folding and lipid flow could define TQC programs as self-organization patterns. The folding of protein is dynamical process: alpha helices are created and disappear in time scale of 10^{-7} seconds and the side chains of protein can rotate.

The details of the TQC like process depend on what one assumes. The minimal scenario is deduced from the transcription and translation processes and from the condition that magnetic body keeps control or at least keeps book about what happens using genome as a tool. The picture would be essentially what one might obtain by applying a rough model for web in terms of nodes and links. The reader is encouraged to use paper and pencil to make the following description more illustrative.

1. Assume that mRNA and DNA remain connected by flux tubes after transcription and that only reconnection process can cut this connection so that mRNA inherits the conjugate colors of DNA. Assume same for mRNA and tRNA. Assume that amino-acid associated with tRNA has similar flux tube connections with the nucleotides of tRNA. Under these assumptions amino-acid inherits the conjugate colors of DNA nucleotides via the connection line DNA-mRNA-tRNA-amino-acid faith-fully if all links are correspond to quark pairs rather than their superpositions. Wobble pairing for Z nucleotide could actually correspond to this kind of superposition.

2. One can consider several options for the amino-acid-acid DNA correspondence but trial-and-error work showed that a realistic folding code is obtained only if X, Y, and Z correspond to $O-H$, $O=$, and NH_2 in the constant part of free amino-acid. During translation the formation of the peptide bond between amino-acids dehydration leads to a loss of $O-H$ and one H from NH_2. The flux tube from tRNA to $O-H$ becomes a flux tube to water molecule inheriting the color of X so that $O=-NH_2$ of the amino-acid inside protein represents the conjugate of YZ.

3. Hydrogen bonding between $O=$ and NH of n: th and $n+k$: th amino-acids inside alpha helices and n: th and $n+1$: th amino-acids inside beta strands reduces effectively to base pairing characterized by $Y=Z$ rule. Assuming that flux tube is only a prerequisite for the formation of hydrogen bond, $Y(n)=Z(n+k)$ or $Z(n)=Y(n+k)$ allows the existence of hydrogen bond. The identification of hydrogen bond with flux tube gives a more stringent condition $Y(n)=Z(n+k)$. The first option is favored. Either condition is extremely restrictive condition on the gene coding the amino-acid unless one assumes quantum counterpart of wobble base pairing for mRNA or tRNA-amino-acid pairing in the case of Z nucleotide (as one indeed must do). Note that the $O=$ atom of the amino-acid is in a special role in that it can have hydrogen bond flux tubes to donors and flux tube connections with $O=$: s of other amino-acids, the residues of amino-acids containing acceptors (say $O=$ or aromatic ring), and with the aromatic rings of say ATP.

4. The recombination process for two conjugate DNA-mRNA-tRNA-amino-acid links can transform the flux tubes in such manner that one obtains link between the $=O$: s of amino-acids A_1 and A_2 characterized by Y and Y_c. Besides hydrogen bonding this mechanism could be central in the enzyme substrate interaction. The process would pair tRNAs corresponding to Y and Y_c together to give DNA-mRNA-tRNA-tRNA-mRNA-DNA link providing a memory representation about amino-acid pairing A_1-A_2. One could say that magnetic body creates with the mediation of the genome dynamical TQC programs to which much of the bio-molecular activity reduces. Not all however, since two amino-acid pairs A_1-A_2 and A_3-A_4 can recombine to A_1-A_4 and A_3-A_2 without DNA knowing anything about it. Magnetic body would however know.

5. The constant part of non-hydrogen bonded amino-acid inside protein would behave like Y_cZ_c if amino-acid is coded by XYZ. The $COOH$ end of protein would behave like $X_cY_cZ_c$.

Also flux tubes connecting the residue groups become possible and protein does not behave like single nucleotide anymore. By color inheritance everything resulting in the reconnection process between $O =$ and NH_2 and residues reduces in a well-defined sense to the genetic code.

10.7.3 Realization Of Flux Tubes

The basic questions about flux are following. Where do they begin, where do they end, and do they have intermediate plugs which allow temporary cutting of the flux tube.

Where do flux tubes begin from?

The view about magnetic body as a controller of biological body using genome as a control tool suggests that DNA is to a high degree responsible for directed attention and other molecules as targets so that flux tubes emanate from DNA nucleotides. The reason would be that the aromatic cycles of DNA correspond to larger value of Planck constant. Some chemical or geometric property of DNA nucleotides or of DNA nucleotides of DNA strand could raise them to the role of subject. Aromatic cycle property correlates with the symmetries associated with large value of Planck constant and is the best candidate for this property.

If this picture is accepted then also some amino-acid residues might act as subjects/objects depending on the option. Phe, His, Trp, Tyr contain aromatic cycle. The derivatives of Trp and Tyr act as neurotransmitters and His is extremely effective nucleophilic catalyst. This would make possible more specific catalytic mechanisms through the pairing of Phe, His, Trp, and Tyr with residues having flux tube terminals.

This raises the question about the physical interaction determining the color of the strand emerging from the aromatic cycle. The interaction energy of quark at the end of flux tube with the classical electromagnetic fields of nuclei and electrons of the ring should determine this. The wormhole contact containing quark/antiquark at the throat at space-time sheet containing nuclei and electrons could also de-localize inside the ring. One of the earliest hypothesis of TGD inspired model for living matter was that wormhole Bose-Einstein condensates could be crucial for understanding of the behavior of biomolecules [K131]. Wormhole throats with quark and antiquark at their throats appear also in the model of high T_c superconductivity [K24]. As far as couplings are considered, these wormhole contacts are in many respects analogous to the so called axions predicted by some theories of elementary particle physics. The wormhole contact like property is by no means exceptional: all gauge bosons correspond to wormhole contacts in TGD Universe.

The only manner for the electronic space-time sheet to feed its electromagnetic gauge flux to larger space-time sheets using exactly two wormhole contacts is to use wormhole contacts with $\overline{u}$ and d at their "upper" throat (T, G). For proton one would have $\overline{d}$ and u at their "upper" throat (A, C). The presence of electron or proton at nucleotide space-time sheet near the end of flux tube might allow to understand the correlation. The transfer of electrons and protons between space-time sheets with different p-adic length scale is basic element of TGD based model of metabolism so that there might be some relation.

Acceptors as plugs and donors as terminals of flux tubes?

Standardization constraint suggests that flux tubes are attached to standard plugs and terminals. The explicit study of various biological molecules and the role of water in biology gives some hints.

1. An attractive idea is that $= O$ serves as a plug to which flux arrives and from which it can also continue. For the minimal option suggested by hydrogen bonding $O =$ could be connected to two donors and $O =$ could not be connected to $O =$. The assumption that the flux tube can connect also two $O =$: s represents a hypothesis going outside the framework of standard physics. A stronger assumption is that all acceptors can act as plugs. For instance, the aromatic rings of DNA nucleotides could act as acceptors and be connected to a sequence of $O =$ plugs eventually terminating to a hydrogen bond.
2. Donors such as $O - H$ would in turn correspond to a terminal at which flux tube can end. One might be very naïve and say that conscious bio-molecules have learned the fundamental role of oxygen and water in the metabolism and become very attentive to the presence of $= O$

and $O-H$. $=O$ appears in $COOH$ part of each amino-acid so that this part defines the standard plug. $=O$ appears also in the residues of Asp, Glu, Asn, Gln. $O-H$ groups appear inside the residues of Asp, Glu and Ser, Thr.

3. Hydrogen bonds $X-H--Y$ have the basic defining property associated with directed attention, namely the asymmetry between donor X and acceptor Y. Hence there is a great temptation consider the possibility that hydrogen bonds correspond to short flux tubes, that flux tubes could be seen as generalized hydrogen bonds. Quite generally, Y could be seen as the object of directed attention of X characterized by larger value of Planck constant. The assumption that two $O=$: s, or even two acceptors of a hydrogen bond, can be connected by a flux tube means more than a generalization of hydrogen bond the connection with a donor would correspond only to the final step in the sequence of flux tubes and plugs giving rise to a directed attention.

4. This hypothesis makes the model rather predictive. For instance, $N-H$, NH_2, $O-H$ and much less often $C-H$ and $S-H$ are the basic donors in the case of proteins whereas $O=$, $-O-$, $-N=S-S$, $-S^-$ and aromatic rings are the basic acceptors. Reconnection process should be involved with the dynamics of ordinary hydrogen bonding. Reconnection process implies inheritance of the flux tube color and means a realization of the symbol based dynamics. It turns out that this hypothesis leads to a model explaining basic qualitative facts about protein folding.

10.7.4 Flux Tubes And DNA

The model of DNA as topological quantum computer gives useful guide lines in the attempt to form a vision about flux tubes. It was assumed that braid strands defined by "wormhole magnetic" flux tubes join nucleotides to lipids and can continue through the nuclear or cell membrane but are split during TQC. The hydrophilic ends of lipids attach to water molecules and self-organization patterns for the water flow in gel phase induce a 2-D flow in the lipid layer which is liquid crystal defining TQC programs at the classical level as braidings. The flow indeed induces braiding if one assumes that during topological computation the connection through the cell membrane is split and reconnected after the halting of TQC.

The challenge is to understand microscopically how the flux tube joins DNA nucleotide to the phospholipid [I38]. Certainly the points at which the flux tubes attach should be completely standard plugs and the formation of polypeptide bonds is an excellent guide line here. Recall that phospholipid, the TQC dancer, has two hydrophobic legs and head. Each leg has at the hydrophilic end O=C-O-C part joining it to glyceride connected to monophosphate group in turn connected to a hydrophilic residue R. The most often appearing residues are serine, inositol, ethanolamine, and choline. Only three of these appear in large quantities and there is asymmetry between cell exterior and interior.

Let us denote by $=O_1$ and $=O_2$ the two oxygens (maybe analogs of right and left hemispheres!) in question. The proposal is that DNA nucleotide and $=O_1$ are connected by a flux tube: the asymmetry between right and left lipid legs should determine which of the legs is "left leg" and which $O=$ is the "left brain hemisphere". $=O_2$, the "holistic right brain hemisphere", connects in turn to the flux tube coming from the other symmetrically situated $=O_2$ at the outer surface of the second lipid layer. Besides this $=O_1$ and $=O_2$ are connected by a flux tube serving as switch on both sides of the membrane.

During TQC the short $O=-O=$ flux tube would experience reconnection with a flux tube acting as hydrogen bond between water molecules so that the connection is split and $O=$: s form hydrogen bonds. The reversal of this reconnection creates the connection again and halts the computation. The lipid residue R couples with the flow of the liquid in gel phase. Since $=O$ is in question the quark or antiquark at the end can correspond to the DNA nucleotide in question. The necessary complete correlation between quark and antiquark charges at the ends of flux tubes associated with $=O_1$ and $=O_2$ can be understood as being due to the minimization of Coulomb interaction energy.

If one is ready to accept magnetic flux tubes between all acceptors then the aromatic rings of nucleotides known to be acceptors could be connected by a flux tube to the $O=$ atom of the lipid or to some intermediate $O=$ atom. The phosphate groups associated with nucleotides of DNA strand contain also $= O$, which could act as a plug to which the flux tube from the nucleotide is attached. The detailed charge structure of the aromatic ring(s) should determine the quark-nucleotide correspondence. The connection line to the lipid could involve several intermediate $O =$ plugs and the first plug in the series would be the $O =$ atom of the monophosphate of the nucleotide.

There is a strong temptation to assume that subset of XYP molecules, $X = A, G, T, C$, $Y = M, D, T$ act as standard plugs with X and phosphates connected by flux tubes to a string. This would make possible to engineer braid strands from standard pieces connected by standard plugs. DNA nucleotide XMP would have flux tube connection to the aromatic ring of X and the $O =$ of last P would be connected to next plug of the communication line. If so, a close connection with metabolism and topological quantum computation would emerge. Phosphorylation would be an absolutely essential for both metabolism and buildup of connection lines acting as braid strands. $O = -O =$ flux tubes could also act as switches inducing a shortcut of the flux tube connection by reconnecting with a hydrogen bond connecting two water molecules. This is an essential step in the model for how DNA acts as topological quantum computer.

This picture would fit with the fact that XYP molecules, in particular AMP, ADP, and ATP, appear in bio-molecules involved with varying functions such as signalling, control, and metabolism. $= O$ might act as a universal plug to which flux tubes from electronegative atoms of information molecules can attach their flux tubes. This would also provide a concrete realization of the idea that information molecules (neurotransmitters, hormones) are analogous to links in Internet [K94]: they would not represent the information but establish a communication channel. The magnetic flux tube associated with the information molecule would connect it to another cell and by the join to $= O$ plug having flux tube to another cell, say to its nucleus, would create a communication or control channel.

10.7.5 Introns And DNA-Protein Attachment

An example is the situation in which protein acts as an enzyme attaching on DNA. Suppose that this process effectively reduces to a base pairing between amino-acid and DNA nucleotide. Protein can attach to any portion of DNA. The simplest interaction is the attachment to the gene coding for the amino-acid itself but much more general enzymatic interactions are possible. It must be however noticed that DNA sequence coding for given amino-acid sequences is considerably longer than amino-acid sequence: the sequence coding for 10 amino-acids is about 10 nm long whereas the corresponding straight amino-acid strand is about 4.7 nm long. It is known that DNA can change its conformation from strand during enzyme-DNA action [I84], and the contraction of DNA strand might make possible to have enzyme-DNA interaction involving fusion along several subsequent amino-acids. This kind of mechanism might work also in the case that attachment region corresponds to several exons. There is however no need to assume that subsequent amino-acids are form a contact with DNA.

One can of course ask whether genes containing introns tend to code for proteins which are used for topological quantum computations. Introns, perhaps the repeating sequences with no obvious function, would have at least this useful function but very probably much more useful ones too (they are now known to be transcribed to RNA and TGD suggest that language corresponds to intronic gene expression). The emergence of introns might be somewhat like the emergence of information society.

The folding of proteins tends to be conserved in the evolution whereas primary structure can change quite a lot apart from some amino-acids critical for enzymatic action. This confirms with the effective base pairing interaction between amino-acids and DNA to be discussed later and would mean that DNA-amino-acid TQC programs are rather robust against mutations.

Flux tubes and DNA

The model of DNA as topological quantum computer gives useful guide lines in the attempt to form a vision about flux tubes. It was assumed that braid strands defined by "wormhole magnetic" flux tubes join nucleotides to lipids and can continue through the nuclear or cell membrane but are split during TQC. The hydrophilic ends of lipids attach to water molecules and self-organization patterns for the water flow in gel phase induce a 2-D flow in the lipid layer which is liquid crystal defining TQC programs at the classical level as braidings. The flow indeed induces braiding if one assumes that during topological computation the connection through the cell membrane is split and reconnected after the halting of TQC.

The challenge is to understand microscopically how the flux tube joins DNA nucleotide to the phospholipid [I37]. Certainly the points at which the flux tubes attach should be completely standard plugs and the formation of polypeptide bonds is an excellent guide line here. Recall that phospholipid, the TQC dancer, has two hydrophobic legs and head. Each leg has at the hydrophilic end O=C-O-C part joining it to glyceride connected to monophosphate group in turn connected to a hydrophilic residue R. The most often appearing residues are serine, inositol, ethanolamine, and choline. Only three of these appear in large quantities and there is asymmetry between cell exterior and interior.

1. Are the flux tubes beginning from O=: s special?

Let us denote by $= O_1$ and $= O_2$ the two oxygens (maybe analogs of right and left hemispheres!) in question.

1. The proposal is that DNA nucleotide and $= O_1$ are connected by a flux tube: the asymmetry between right and left lipid legs should determine which of the legs is "left leg" and which $O =$ is the "left brain hemisphere". $= O_2$, the "holistic right brain hemisphere", connects in turn to the flux tube coming from the other symmetrically situated $= O_2$ at the outer surface of the second lipid layer. Besides this $= O_1$ and $= O_2$ are connected by a flux tube serving as switch on both sides of the membrane.
2. During TQC the short $O = -O =$ flux tube would experience reconnection with a flux tube acting as hydrogen bond between water molecules so that the connection is split and $O =$: s form hydrogen bonds. The reversal of this reconnection creates the connection again and halts the computation. The lipid residue R couples with the flow of the liquid in gel phase. Since $= O$ is in question the quark or antiquark or a pair of electron pairs at the end can correspond to the DNA nucleotide in question. The necessary complete correlation between quark and antiquark charges at the ends of flux tubes associated with $= O_1$ and $= O_2$ might be understood as being due to the minimization of Coulomb interaction energy. In the case of pair of electron pairs the correlation could come from the minimization of the magnetic energy.
3. If one is ready to accept magnetic flux tubes between all acceptors then the aromatic rings of nucleotides known to be acceptors could be connected by a flux tube to the $O =$ atom of the lipid or to some intermediate $O =$ atom. The phosphate groups associated with nucleotides of DNA strand contain also $= O$, which could act as a plug to which the flux tube from the nucleotide is attached. The detailed charge structure of the aromatic ring(s) should determine the quark-nucleotide correspondence. The connection line to the lipid could involve several intermediate $O =$ plugs and the first plug in the series would be the $O =$ atom of the monophosphate of the nucleotide.

There is a strong temptation to assume that subset of XYP molecules, $X = A, G, T, C$, $Y = M, D, T$ act as standard plugs with X and phosphates connected by flux tubes to a string. This would make it possible to engineer braid strands from standard pieces connected by standard plugs. DNA nucleotide XMP would have flux tube connection to the aromatic ring of X and the $O =$ of last P would be connected to next plug of the communication line. If so, a close connection with metabolism and topological quantum computation would emerge.

1. Phosphorylation [I39] would be an absolutely essential for both metabolism and buildup of connection lines acting as braid strands. Phosphorylation is indeed known to be the basic step activating enzymes. In eukaryotes the phosphorylation takes plane amino-acids most often for ser but also thr, and trp with aromatic rings are phosphorylated. Mitochondrions have specialized to produce ATP in oxidative phosphorylation from ADP and photosynthesis produces ATP. All these activities could be seen as a production of standard plugs for braid strands making possible directed attention and quantum information processing at molecular level.
2. As already noticed, $O = -O =$ flux tubes could also act as switches inducing a shortcut of the flux tube connection by reconnecting with a hydrogen bond connecting two water molecules.

This is an essential step in the model for how DNA acts as topological quantum computer. De-phosphorylation might be standard manner to realize this process.

3. This picture would fit with the fact that XYP molecules, in particular AMP, ADP, and ATP, appear in bio-molecules involved with varying functions such as signalling, control, and metabolism. $=O$ might act as a universal plug to which flux tubes from electronegative atoms of information molecules can attach their flux tubes. This would also provide a concrete realization of the idea that information molecules (neurotransmitters, hormones) are analogous to links in Internet [K94]: they would not represent the information but establish a communication channel. The magnetic flux tube associated with the information molecule would connect it to another cell and by the join to $=O$ plug having flux tube to another cell, say to its nucleus, would create a communication or control channel.

2. DNA as topological quantum computer hypothesis and electronic super-conductivity

The vision about DNA as topological quantum computer is very general. The essential element is the coding of DNA nucleotides and one can imagine several options.

1. One realization is based on the representation of DNA nucleotides A, T, C, G as quarks u, d and their antiquarks and requires scaled up version of QCD. The motivation for this realization came from the observation of Barbara Shipman that the mathematical description of honeybee dance suggests that quarks play a role in living matter [A25].
2. Second option that one can imagine would use spin 1 triplet and spin 0 singlet of dark electron pair. Spin 0 state for electron pair however gives rise to vanishing dipole field so that flux tube structure would not be possible. Can one circumvent this option or are quark pairs unavoidable?
3. DNA as TQC lead to the hypothesis that it is O= to which one must assign the flux tube pair responsible for the representation of the genetic code. Why O= would be in special role?
   (a) If there are two parallel flux tubes, one obtains tensor product $3\times 3 = 5+3+1$ of electron triplets at the ends of the flux tubes. Could it be that A, T, C, and G are represented in terms of 3 and 1 and the breaking of rotational invariance implies a mixing of singlet and spin 0 state of triplet so that nucleotides and their conjugates could correspond to the resulting two pairs related by reflection?
   One can however argue that for $S_z = 0$ states the direction of the magnetic flux tubes is orthogonal to that in other cases. An alternative possibility is that one uses only the four $S_z \neq 0$ states of spin 2 5-multiplet obtained in the tensor product. The breaking of the full rotational symmetry down to SO(2) symmetry around flux tube direction could be used to justify this option.
   (b) The coding would be also consistent with quantum classical correspondence since it would reduce at classical level to a coding in terms of directions of magnetic fields in the two flux tubes: the directions could be parallel and in two directions or antiparallel giving also two options: four altogether. Notice however that one must be able to distinguish between two different configurations in which the directions of magnetic flux are opposite for the flux tubes of the pair. Classically this is achieved if the flux tubes form either a right-handed or left-handed double helix. Double helix could also resolve the problem posed by the fact that in $S_z = 0$ case the flux tubes cannot be parallel to their common axis at the flux tube end.
   (c) This option would allow a unification of DNA as topological quantum computer conjecture with the conjectures about dark high T_c super-conductivity and negentropic entanglement.

   $ATP \rightarrow ADP + P_i$ would correspond to the fusion of flux tube pair with two hydrogen bonds associated with water molecules so what they could become short-circuited with water molecules. The reverse process would create flux tube connection labelled by the spin state equivalent of A, T, C, or G. The possibility of 5-plet allows also to consider the possibility of five codons instead of four.

Whatever the correct option is it must explain how the correspondence between A, T, C, G and secondary codons emerges.

$$
\begin{array}{l}
Q_a = [n(A) - n(T)]\frac{2}{3} - [n(G) - n(C)]\frac{1}{3} \ , \\
Q_a = -[n(A) - n(T)]\frac{1}{3} + [n(G) - n(C)]\frac{2}{3} \ , \\
Q_a = -[n(A) - n(T)]\frac{2}{3} + [n(G) - n(C)]\frac{1}{3} \ , \\
Q_a = [n(A) - n(T)]\frac{1}{3} - [n(G) - n(C)]\frac{2}{3} \ .
\end{array}
\tag{10.7.2}
$$

Table 10.2: Table show four possible options for em charge as sum of quark charges.

1. If the pairs of spin triplet electron pairs appear in the correspondence, one must understand why the spin state of the pair of electron pairs at the O= of the phosphate attached with the DNA nucleotide correlates with the character of the nucleotide. Phosphate has also two O^-: s containing two electron pairs. Minimization of the magnetic energy is the explanation which is easiest to imagine. Maybe the total magnetic energy of the pair in the magnetic field of the flux tube structure assignable to the nucleotide plus the de-oxyribose preceding it. T and C contain also O= but not A and G. and A and T and C and G are conjugates.

   By studying the chemical structure of DNA (see `http://tinyurl.com/yd7b7w98`) [I64] one finds that the pairs AT and CG contain two O=: s which belong either to same nucleotide (to T in A-T) or to different nucleotides (C-G). This suggests the coding in which there are flux tube pairs connecting the two phosphate O=s at the two sides of the double strand and going through the two intermediation O=s. The rule would be that the spin states are conjugates at the ends of the flux tubes. A-T and T-A pairs could correspond to parallel flux tubes with same direction of the flux and G-C and C-G to parallel flux tubes with opposite directions of the magnetic fluxes.

2. If quark pairs are unavoidable, the correspondence of A, T, C, G with quarks and antiquark must relate to quark charges coming as $\pm 2/3$, $\pm 1/3$. Also in this case the coding mechanism based on the flux tubes connecting O=: s is natural.

The conclusion would be that the original view about secondary realization of genetic code can be replaced with the realization based on spin 1 dark Cooper pairs of electrons between which the entanglement is negentropic. Quark color plays no special role in the model of DNA as topological quantum computer [K4] so that the model remains as such. One implication would be however that the magnetic flux tubes carrying dark electron pairs at their ends could be of astrophysical size.

10.7.6 Some Predictions Related To The Representation Of Braid Color

Even in the rudimentary form discussed above the model makes predictions. In particular, the hypothesis that neutral quark pairs represent braid color is easily testable.

Anomalous em charge of DNA as a basic prediction

The basic prediction is anomalous charge of DNA. Also integer valued anomalous charge for the structural units of genome is highly suggestive.

The selection of the working option - if any such exists - is indeed experimentally possible. The anomalous charge coupling to the *difference* of the gauge potentials at the two space-time sheets defines the signature of the wormhole contact at the DNA end of braid strand. The effective (or anomalous) em charge is given as sum of quark charges associated with DNA space-time sheet:

$$
Q_a \quad = \quad [n(A) - n(T)]Q(q_A) + [n(G) - n(C)]Q(q_G) \tag{10.7.1}
$$

is predicted. The four possible options for charge are given explicitly in **Table 10.2**.

Second option is obtained from the first option $(A, T, G, C) \rightarrow (u, \overline{u}, d, \overline{d})$ by permuting u and d quark in the correspondence and the last two options by performing charge conjugation for quarks in the first two options.

The anomalous charge is experimentally visible only if the external electromagnetic fields at the two sheets are different. The negative charge of DNA due to the presence of phosphate groups implies that the first sheet carries different em field so that this is indeed the case.

The presence effective em charge depending on the details of DNA sequence means that electromagnetism differentiates between different DNA: s strands and some strands might be more favored dynamically than others. It is interesting to look basic features of DNA from this view point. Vertebral mitochondrial code has full $A \leftrightarrow G$ and $C \leftrightarrow T$ symmetries with respect to the third nucleotide of the codon and for the nuclear code the symmetry is almost exact. In the above scenario A and C *resp.* G and T would have different signs and magnitudes of em charge but they would correspond to different weak isospin states for the third quark so that this symmetry would be mathematically equivalent to the isospin symmetry of strong interactions.

The average gauge potential due to the anomalous charge per length at space-time sheet containing ordinary em field of a straight portion of DNA strand is predicted to be proportional to

$$\frac{dQ_a}{dl} = [p(A)-p(T)]Q(q_A) + [p(G)-p(C)]Q(q_G)\frac{1}{\Delta L} \ ,$$

where ΔL corresponds to the length increment corresponding to single nucleotide and $p(X)$ represents the frequency for nucleotide X to appear in the sequence. Hence the strength of the anomalous scalar potential would depend on DNA and vanish for DNA for which A and T *resp.* G and C appear with the same frequency.

Chargaff's second parity rule and the vanishing of net anomalous charge

Chargaff's second parity rule states that the frequencies of nucleotides for single DNA strand satisfy the conditions $p(A) \simeq p(T)$ and $p(C) \simeq p(G)$ (I am grateful for Faramarz Faghihi for mentioning this rule and the related [H1] [I85] to me). This rule holds true in a good approximation. In the recent context the interpretation would be as the vanishing of the net anomalous charge of the DNA strand and thus charge conjugation invariance. Stability of DNA might explain the rule and the poly-A tail in the untranslated mRNA could relate stabilization of DNA and mRNA strands.

Together with $p(A)+p(T)+p(G)+p(C)=1$ Chargaff's rule implies the conditions

$$\begin{array}{ll} p(A)+p(C) \simeq 1/2 \ , & p(A)+p(G) \simeq 1/2 \ , \\ p(T)+p(C) \simeq 1/2 \ , & p(T)+p(G) \simeq 1/2 \ . \end{array} \tag{10.7.3}$$

An interesting empirical finding [I85] is that only some points at the line $p(A)+p(C) \simeq 1/2$ are realized in the case of human genome and that these points are in a good accuracy expressible in terms of Fibonacci numbers resulting as a prediction of optimization problem in which Fibonacci numbers are however put in by hand. $p(A) = p(G) = p(C) = p(T) = 1/4$ results as a limiting case. The poly-A tail of mRNA (not coded by DNA) could reflect to the compensation of this asymmetry for translated mRNA.

The physical interpretation would be as a breaking of isospin symmetry in the sense that isospin up and down states for quarks (A and G *resp.* T and C) do not appear with identical probabilities. This need not have any effect on protein distributions if the asymmetry corresponds to asymmetry for the third nucleotide of the codon having $A \leftrightarrow G$ and $T \leftrightarrow C$ symmetries as almost exact symmetries. This of course if protein distribution is invariant under this symmetry for the first two codons.

The challenge would be to understand the probabilities $p_3(X)$ for the third codon from a physical model for the breaking of isospin symmetry for the third codon in the sense that u and $\overline{u}$ at DNA space-time sheet are more favored than d and $\overline{d}$ or vice versa. There is an obvious analogy with spontaneous breaking of vacuum symmetry.

Are genes and other genetic sub-structures singlets with respect to QCD color?

Genes are defined usually as transcribed portions of DNA. Genes are however accompanied by promoter regions and other regions affecting the transcription so that the definition of what one really means with gene is far from clear. In the recent case gene would be naturally TQC program module and gene in standard sense would only correspond to its sub-module responsible for the translated mRNA output of TQC.

Whatever the definition of gene is, genes as TQC program modules could be dynamical units with respect to color interaction and thus QCD color singlets (QCD color should not be confused with braid color) or equivalently - possess integer valued anomalous em charge.

One can consider two alternative working hypothesis - in a well-defined sense diametrical opposites of each other.

1. The division of the gene into structural sub-units correlates with the separation into color singlets. Thus various structural sub-units of gene (say transcribed part, translated part, intronic portions, etc...) would be color singlets.
2. Also different genetic codes that I have discussed in [?] could distinguish between different structural sub-units. For this option only gene - understood as TQC unit with un-transcribed regions included - would be color singlet.

Color singletness condition is unavoidable for mRNA and leads to a testable prediction about the length of poly-A tail added to the transcribed mRNA after translation.

1. The condition of integer valued anomalous charge for coding regions

In the case of coding region of gene the condition for integer charge is replaced by the conditions

$$n(A)+n(G)\ \ mod\ \ 3=0\ \ ,\qquad n(C)+n(T)\ \ mod\ \ 3=0\ \ . \tag{10.7.4}$$

These conditions are not independent and it suffices to check whether either of them is satisfied. The conditions are consistent with $A \leftrightarrow G$ and $T \leftrightarrow C$ symmetries of the third nucleotide. Note that the contribution of the stop codon (TAA, TGA or TAG) and initiating codon ATG to the A+G count is one unit.

2. General condition for integer valued anomalous charge

The anomalous charge of gene or even that of an appropriate sub-unit of gene is integer valued implies in the general case

$$n(A)-n(T)+n(G)-n(C)\ \ mod\ \ 3=0\ \ . \tag{10.7.5}$$

Note that this condition does not assume that gene corresponds to $3n$ nucleotides (as I had accustomed to think). The surprising (to me) finding was that gene and also mRNA coding region of the gene in general fails to satisfy $3n$ rule. This rule is of course by no means requiredonly the regions coding for proteins can be thought of as consisting of DNA triplets.

A possible interpretation is in terms of TGD based model for pre-biotic evolution [?] according to which genetic code (or 3-code) was formed as a fusion of 2-code and 1-code. 2-code and 1-code could still be present in genome and be associated with non-translated regions of mRNA preceding and following the translated region. The genes of 2-code and coding for RNA would have $2n$ nucleotides and the genes of 1-code could also consist of odd number of nucleotides.

There might be analogy with drawings for a building. These contain both figures providing information about building and text giving meta-level information about how to interpret figures. Figures could correspond to 3-code coding for proteins and text could be written with other codes and give instructions for the transcription and translation processes. Prokaryotic code would contain mostly figures (CDS). In eukaryotic code intronic portions could carry rich amounts of this kind of metalevel information. In the case of mRNA untranslated region preceding $5'$ end could provide similar information.

1. Repeating sequences consisting of n copies of same repeating unit could obey 1-code or 2-code. The simplest building blocks of repeating sequences are AT and CG having vanishing anomalous em charge. TATATA.... and CGCGCG... indeed appear often. Also combinations of CG and AT could repeat: so called mini-satellites are CG rich repeating sequences. Interpretation in terms of 2-code suggests itself.

2. Triplet of the unit ATTCG with integer charge repeats also often: in this case 3-code suggests itself. Telomeres of vertebrates consist of a repeating unit TTAGGG which does not have integer charge: this unit appears also as 8-nucleotide variant which suggests 2-code. Color singletness would require that this unit appears $3n$ times.
3. I have also proposed that intronic regions could obey memetic code [K52] predicting that intronic codon can be represented as a sequence of 21 3-codons (implying 2^{63} 63-codons!). Individual intronic segments need not satisfy this rule, only their union if even that. Direct experimentation with gene bank data show that neither introns nor their union correspond to integer multiples of 63 nor 3 or 2 in general.

3. Color singletness conditions for gene

Gene is usually defined as the sequence of DNA coding for mRNA. mRNA involves also two untranslated regions (UTRs) [I1].

1. The $5'$ end of mRNA contains $5'$ cap (methylated G) and $5'$ untranslated region (UTR). The latter can be several kb long for eukaryotes. Methylated G is not coded by DNA but added so that it does not contribute to A+G-T-C count at DNA level.
2. mRNA continues after the stop codon as $3'$ UTR. Translation assigns to UTR also a poly-A tail (up to several hundreds A: s) not coded by DNA and not contributing to A+G-T-C count in the case of DNA. This region contains also AAUAAA which does not contribute to A+G-T-C count of mRNA.

   One could argue that any amino-acid sequence must allow coding and that one function of UTRs is to guarantee integer valued charge for the part of gene beginning from the initiating codon. Of course, also the non-transcribed regions of DNA not included in the standard definition of gene could take care of this.

4. Color singletness conditions for mRNA

Both poly-A tail and G gap are known to relate to the stabilization of mRNA. The mechanism could be addition of an anomalous charge compensating for the anomalous charge of mRNA to guarantee that second Chargaff's rule is satisfied in a good approximation: this hypothesis is testable.

Second function would be to guarantee color-singletness property. Color singletness would mean that transcribed mRNA + cap G + poly-A tail as a separate unit must be QCD color singlet at DNA space-time sheet. mRNA stability requires the condition

$$n(A)-n(T)+n(G)-n(C)+n_{tail}(A)+1 \ \ mod \ \ 3=0 \tag{10.7.6}$$

to be satisfied. The knowledge of gene would thus predict $n_{tail}(A)$ mod 3. This hypothesis is testable.

5.Chargaff's rule for mRNA

If Chargaff's rule applies also to mRNA strands one obtains one of the following predictions

$$\begin{aligned}
2\left[n(A)+n_{tail}(A)-n(T)\right]-\left[n(G)+1-n(C)\right] &\simeq 0 \ , \\
-\left[n(A)+n_{tail}(A)-n(T)\right]+2\left[n(G)+1-n(C)\right] &\simeq 0 \ , \\
-2\left[n(A)+n_{tail}(A)-n(T)\right]+\left[n(G)+1-n(C)\right] &\simeq 0 \ , \\
\left[n(A)+n_{tail}(A)-n(T)\right]-2\left[n(G)+1-n(C)\right] &\simeq 0 \ .
\end{aligned} \tag{10.7.7}$$

Here $n_{tail}(A)$ includes also AAUAA contributing 3 units to it plus possible other structures appearing in the tail added to the translated mRNA. The presence of poly-A tail which could also compensate for the ordinary negative charge of translated part of mRNA would suggest that A corresponds to u or $\overline{d}$ corresponding to options 1 and 4.

6. Moving genes and repeating elements

Transposons [I49], [J10] are moving or self-copying genes. Moving genes cut from initial position and past to another position of double strand. Copying genes copy themselves first to RNA and them to a full DNA sequence which is then glued to the double strand by cut and paste procedure. They were earlier regarded as mere parasites but now it is known that their transcription is activated under stress situations so that they help DNA to evolve. In TQC picture their function would be to modify TQC hardware. For copying transposons the cutting of DNA strand occurs usually at different points for DNA and cDNA so that "sticky ends" result ("overhang" and its complement) [I46]. Often the overhang has four nucleotides. The copied transposon have ends which are reversed conjugates of each other so that transposons are palindromes as are also DNA hairpins. This is suggestive of the origin of transposons.

In order to avoid boring repetitions let us denote by "satisfy P" for having having integer valued (or even vanishing) Q_a. The predictions are following:

1) The double strand parts associated with the segments of DNA produced by cutting should satisfy P.

2) The cutting of DNA should take place only at positions separated by segments satisfying P.

3) The overhangs should satisfy P.

4) Transposons should satisfy P: their reverse ends certainly satisfy P.

In the example mentioned in [I44] the overhang is $CTAG$ and has vanishing Q_a. The cut site $CCTAGG$ has also vanishing Q_a. It is known [J10] that transposons - repeating regions themselves - tend to attach to the repeating regions of DNA [I13].

1. There are several kinds of repeating regions. 6-10 base pair long sequences can be repeated in untranslated regions up to 10^5 times and whole genes can repeat themselves $50 - 10^4$ times.

2. Repeats are classified into tandems (say TTAGGG associated with telomeres), interspersed repetitive DNA (nuclear elements), and transposable repeat elements. Interspersed nuclear elements (INEs) are classified LINEs (long), SINEs (short), TLTRs (Transposable elements with Long Terminal Repeats), and DNA transposons themselves.

3. LINEs contain AT rich regions. SINEs known as alus (about 280 bps) contain GC rich regions whereas mariner elements (about 80 bps) are flanked by TA pairs. LTRs have length 300-1000 bps. DNA transposons are flanked with two short inverted repeat sequences flanking the reading frame: "inverted" refers to the palindrome property already mentioned.

AT and CG have vanishing Q_a so that their presence in LINEs and SINEs would make the cutting and pasting easy allowing to understand why transposons favor these regions. Viruses are known to contain long repeating terminal sequences (LTR). One could also check whether DNA decomposes to regions satisfying P and surrounded by repeating sequences which satisfy P separately or as whole as in the case DNA transposons.

7. Tests

Some checks of the color singletness hypothesis were made for human genome [I20].

1. For the coding sequences (CDSs) the strong prediction in general fails as expected (condition would pose restrictions on possible amino-acid contents).

2. Color singletness condition fails for genes defined in terms of translated part of mRNA (with gap and poly-A tail excluded). The un-transcribed regions of DNA involved with the gene expression (promoter region, etc...) could guarantee the color singletness. They could also stabilize DNA by bringing in compensating anomalous charge to guarantee second Chargaff's rule. Different genetic codes could distinguish between the subunits of gene.

3. To test color singletness conditions for mRNA one should know the length of poly-A tail. Unfortunately, I do not have access to this information.

4. The computation of total anomalous charges for a handful of genes, introns, and repeat units for some gene bank examples in the case of human genome indicates that both of them tend

to carry net em charge which is largest for $(a, g) \leftrightarrow (\overline{d}, \overline{u})$ correspondence. The charge is in the range 5-10 per cent from the charge associated with the phosphates (-2 units per nucleotide). For second option giving negative charge (permute u and d) the anomalous charge is few per cent smaller.

By Chargaff's law the regions outside genes responsible for the control of gene expression must contain a compensating charge of opposite sign. Kind of spontaneous symmetry breaking of charge conjugation symmetry $A \leftrightarrow T, G \leftrightarrow C$ and analogous to matter antimatter symmetry seems to take place. That control regions and translated regions have opposite densities of anomalous charge might also help in the control gene expression.

5. The poly-A tail of mRNA would carry compensating positive anomalous charge: the RNA-quark assignment could be conjugate to the DNA-quark assignment as suggested by what takes place in transcription. For instance, for the option $A \rightarrow \overline{d}$, the prediction for the length of polytail for $A \rightarrow \overline{d}$ option would be about $n_{tail}/n_{mRNA} \simeq 3p_a(mRNA)$ where $N(mRNa)$ is the number of nucleotides in transcribed mRNA and $p_a(mRNA)$ is the per cent of anomalous charge which is typically 5-10 per cent. For $p_a(mRNA) = 10$ per cent this gives as much as 30 per cent. For $A \rightarrow \overline{u}$ option one has $n_{tail}/n_{mRNA} \simeq 3p_a(mRNA)/2$. In this case also p_a is considerably smaller, typically by a factor of of order 2-3 per cent and even below per cent in some cases. Hence the relative length of tail would around 3-5 per cent. This option is perhaps more since it minimizes anomalous charge and maximizes the effectiveness of charge compensation by poly-A tail.

6. The predictions for transposons and their cut and past process should be easily testable.

Summary of possible symmetries of DNA

The following gives a list of possible symmetries of DNA inspired by the identification of braid color.

1. Color confinement in strong form

The states of quarks and anti-quarks associated with DNA both wormhole wormhole throats of braided (living) DNA strand can be color singlets and have thus integer valued anomalous em charge. The resulting prediction depends on the assignment of quarks and antiquarks to A, T, C, G which in principle should be determined by the minimization of em interaction energy between quark and nucleotide. For instance $2(A-T)-(G-C) \ \ mod \ \ 3 = 0$ for a piece of living DNA which could make possible color singletness. As a matter fact, color singletness conditions are equivalent for all possible for braid color assignments. This hypothesis might be weakened. For instance, it could hold true only for braided parts of DNA and this braiding are dynamical. It could also hold for entire braid with both ends included only: in this case it does not pose any conditions on DNA.

Questions: Do all living DNA strands satisfy this rule? Are only the double stranded parts of DNA braided and satisfy the rule. What about loops of hairpins?

2. Matter antimatter asymmetry at quark level

$A \leftrightarrow T$ and $G \leftrightarrow C$ corresponds to charge conjugation at the level of quarks (quark $\leftrightarrow$ antiquark). Chargaff's rules states $A \simeq T$ and $C \simeq G$ for long DNA strands and mean matter-antimatter symmetry in the scale of DNA strand. Double strand as a whole is matter anti-matter symmetric.

Matter-antimatter asymmetry is realized functionally at the level of DNA double strand in the sense that only DNA strand is transcribed. The study of some examples shows that genes defined as transcribed parts of DNA do not satisfy Chargaff's rule. This inspires the hypothesis about the breaking of matter antimatter symmetry. Genes have non-vanishing net $A-T$ and $C-G$ and therefore also net Q_a with sign opposite to that in control regions. Just as the Universe is matter-antimatter asymmetric, also genes would be matter-antimatter asymmetric.

3. Isospin symmetry at quark level

$A \leftrightarrow G$ and $T \leftrightarrow A$ correspond change of anomalous em charge by 1 unit and these operations respect color confinement condition. Local modifications of DNA inducing these changes should be preferred. The identification for the symmetries $A \leftrightarrow G$ and $T \leftrightarrow A$ for the third nucleotide of code is as isospin symmetries. For the vertebrate mitochondrial code the symmetry exact and for nuclear code slightly broken.

4. Matter antimatter asymmetry and isospin symmetries for the first two nucleotides

The first two nucleotides of the codon dictate to a high degree which amino-acid is coded. This inspires the idea that 3-code has emerged as fusion of 1- and 2-codes in some sense. There are two kinds of 2-codons. The codons of type A have fractional em charge and net quark number (consisting of either matter or antimatter at quark level) and are not able to form color singlets. The codons of type B have integer em charge and vanishing quark number (consisting of matter and antimatter) and are able to form color singlets. The 2-codons of type A (resp. B) are related by isospin rotations and there should be some property distinguishing between types A and B. There indeed is: if 2-codon is matter-antimatter symmetric, 1-codon is not and vice versa.

1. For almost all type A codons the amino-acid coded by the codon does not depend on the last nucleotide. There are two exceptions in the case of the nuclear code: (leu, leu, phe, phe) and (ile, ile, ile, met). For human mitochondrial code one has (ile, ile, ile, ile) and thus only one exception to the rule. The breaking of matter-antimatter symmetry for the third nucleotide is thus very small.
2. For codons of type B the 4-columns code always for two doublets in the case of vertebrate mitochondrial code so that for codons with vanishing net quark number the breaking of matter-antimatter symmetry for the third nucleotide is always present.

5. Em stability

Anomalous em charge Q_a vanishes for DNA and perhaps also mRNA strand containing also the G cap and poly-A tail which could compensate for the Q_a of the transcribed region so that

$$2(A-T)-(G-C)\simeq 0$$

or some variant of it holds true. Chargaff's rules for long DNA strands imply the smallness of Q_a.

6. Summary of testable working hypothesis

Following gives a summary of testable working hypothesis related to the isospin symmetry and color singletness. The property of having integer valued/vanishing Q_a is referred to as property P.

1. Gene plus control region and also DNA repeats should have property P. Transcribed and control regions of gene have Q_a with opposite signs.
2. Transposons, repeating regions, the overhangs associated with the cut and paste of transposon, and the DNA strands resulting in cutting should have property P. This could explain why transposons can paste themselves to AT and GC ($Q_a = 0$) rich repeating regions of DNA. The points at which DNA can be cut should differ by a DNA section having property P. This gives precise predictions for the points at which transposons and pieces of viral DNA can join and could have implications for genetic engineering.
3. If also mRNA is braided, it has property P. This can be only true if the poly-A tail compensates for the non-vanishing Q_a associated with the translated region.
4. Living hairpins should have property P. If only double helix parts of hairpins are braided, the prediction is trivially true by the palindrome property. tRNA or at least parts of it could be braided. Braids could end to the nuclear membrane or mRNA or to the amino-acid attachable to tRNA. For stem regions Q_a is integer valued. The fact that the nucleotide of the anticodon corresponding to the third nucleotide of codon can base pair with several nucleotides of mRNA suggests that $I(nositol)$ can have Q_a opposite to that of A, T, C and U opposite to that of A, G. For 2-anticodon the pairing would be unique. This would give a lot of freedom to achieve property P in weak sense for tRNA. Braid structure for tRNA + amino-acid could be different that for tRNA alone and also in the translation the braid structure could change.
5. Telomeres [I47] are of special interests as far as anomalous em charge is considered. Chromosomes are not copied completely in cell replication, and one function of telomeres is to

guarantee that the translated part of genome replicates completely for sufficiently many cell divisions. Telomeres consists of 3-20 kilobases long repetitions of TTAGGG, and there is a 100-300 kilobases long repeating sequence between telomere and the rest of the chromosome. Telomeres can form can also 4-stranded structures. Telomere end contains a hair-pin loop as a single stranded part, which prevents the action of DNA repair enzymes on the chromosome end. Telomerase is a reverse transcriptase enzyme involved with the synthesis of telomeres using RNA strand as a template but since its expression is repressed in many types of human cells, telomere length shortens in each cell replication. In the case of germ cells, stem cells and white blood cells telomerase is expressed and telomere length preserved. Telomere shortening is known to relate to ageing related diseases. On the other hand, overactive telomere expression seems to correlate with cancer.

If telomeres possess braid strands, the compensation of Q_a might provide an additional reason for their presence. If this the case and if telomeres are strict multiples of TTAGGG, the shortening of telomeres generates a non-vanishing Q_a unless something happens for the active part of DNA too. Color singletness condition should however remain true: the disappearance of $3n$ multiples of TTAGGG in each replication is the simplest guess for what might happen. In any case, DNA strands would become unstable in cell replication. Q_a could be reduced by a partial death of DNA in the sense that some portions of braiding disappear. Also this would induce ill functioning of TQC harware perhaps related to ageing related diseases. Perhaps evolution has purposefully developed this ageing mechanism since eternal life would stop evolution.

6. Also amino-acids could be braided. Q_a could vary and correspond to Q_a for one of the codons coding for it. The amino-acid sequences of catalysts attaching to DNA strand should have opposite Q_a for each codon-amino-acid pair so that amino-acid would attach only to the codons coding for it. The TGD based model for nerve pulse [K94] inspires the proposal that magnetic flux tubes connecting microtubules to the axonal membrane allow TQC during nerve pulse propagation when axonal membrane makes transition from gel like phase to liquid crystal phase. Amino-acids of tubulin dimers would be connected by 3-braids, smallest interesting braid, to groups of 3-lipids in axonal membrane and tubulin dimers would define fundamental TQC modules.

Empirical rules about DNA and mRNA supporting the symmetry breaking picture

Somewhat surprisingly, basic facts which can be found from Wikipedia, support the proposed vision about symmetry breaking although, the mechanism of matter antimatter symmetry breaking is more complex than the first guess. I am grateful for Dale Trenary for references which made possible to realize this. Before continuing some comments about the physical picture are in order.

1. The vanishing of the induced Kähler field means that the space-time sheet of DNA is a highly unstable vacuum extremal. The non-vanishing of the induced Kähler electric field is thus a natural correlate for both the stability and the non-vanishing quark number density (matter antimatter asymmetry). The generation of matter antimatter asymmetry induces a net density of anomalous em charge, isospin, and quark number in the portion of DNA considered. This in turn generates not only longitudinal electric field but also a longitudinal Kähler electric field along DNA.

2. Weak electric fields play a key role in living matter. There are electric fields associated with embryos, central nervous system, individual neurons, and microtubules and their direction determines the direction of a process involved (head-to-tail direction, direction of propagation of nerve pulse, ...).

3. Same mechanism is expected to be at work also in the case of DNA and RNA. In the case of gene the direction of transcription could be determined by the direction of the electric field created by gene and telomeres at the ends of chromosomes carrying a net anomalous quark number could be partially responsible for the generation of this field. In the case of mRNA the direction of translation would be determined in the similar manner. The net anomalous em charges of poly-A tail and the transcribed part of mRNA would have opposite signs so that a longitudinal electric field would result.

	Human	*Chicken*	*Grass−hopper*	*Sea Urchin*	*Wheat*	*Yeast*	*E.Coli*
$p(A)$	0.3090	0.2880	0.2930	0.3280	0.2730	0.3130	0.2470
$p(T)$	0.2940	0.2920	0.2930	0.3210	0.2710	0.3290	0.2360
$p(C)$	0.1990	0.2050	0.2050	0.1770	0.2270	0.1870	0.2600
$p(G)$	0.1980	0.2170	0.2070	0.1730	0.2280	0.1710	0.2570
$\frac{dq_1}{dn}$	0.0103	−0.0067	−0.0007	0.0060	0.0010	−0.0053	0.0083
$\frac{dq_2}{dn}$	0.0057	−0.0093	−0.0013	0.0050	−0.0000	0.0053	0.0057
$\frac{dI_3}{dn}$	0.0080	−0.0080	−0.0010	0.0055	0.0005	0.0000	0.0070
$\frac{d(q-\overline{q})}{dn}$	0.0140	0.0080	0.0020	0.0030	0.0030	−0.0320	0.0080
$\frac{p(A+T)}{p(G+C)}$	1.5189	1.3744	1.4223	1.8543	1.1956	1.7933	0.9342

(10.7.8)

Table 10.3: The table gives A, T, C, G contents (these data are from Wikipedia [I7]), the amount of quark charge per nucleotide for the options 1) *resp.* 2) given by $dq_1/dn = p[2(A-T)-G-C)]/3$ *resp.* $dq_2/dn = p[A-T-2(G-C)]/3$, the amount $dI_3/dn = p(A-G+C-T]/2$ of isospin per nucleotide, the amount $d(q-\overline{q})/dn = p(A-T+G-C)$ of quark number per nucleotide, and $(A+T)/(C+G)$ ratio for *entire genomes* in some cases.

It will be found that this picture is consistent with empirical findings about properties of DNA.

7. Breaking of matter antimatter symmetry and isospin symmetry for entire genome

Chargaff's rules are not exact and the breaking gives important information about small breakings of isospin and matter-antimatter symmetries at the level of entire genome. The basic parameters are em charge per nucleotide, isospin per nucleotide, the amount of quark number per nucleotide, and the ratio of u and d type matters coded by $(G+C)/(A+T)$ ratio. Recall that there are four options for the map of A, T, C, G to quarks and antiquarks and for option 3) *resp.* 4) the anomalous em charge is opposite to that for 1) *resp.* 2).

Table 10.3 gives A, T, C, G contents (these data are from Wikipedia [I7]) provides interesting data about DNA It will be found that so called Szybalski's rules can be interpreted as saying that for coding regions there is breaking of the approximate matter antimatter asymmetry.

Note that matter antimatter asymmetry in the scale of entire genome has largest positive value for human genome and negative value only for yeast genome: this case the magnitude of the asymmetry is largest.

For option 2) the amount of anomalous charge is about.0057e per nucleotide and thus about $3\times 10^7 e$ for entire human DNA having length of about 1.8 meters. The inspection of tables of [I47] shows that the anomalous em charge for the repeating sequence defining the telomere is always non-vanishing and has always the same sign. Telomeres for human chromosomes consist of TTAGGG repetitions with anomalous em charge with magnitude $5e/3$ for all options and have a length measured in few kbases. Human genome as has 24 chromosomes so that the total anomalous em charge of telomeres is roughly $24\times(5/18)\times x10^3 e\sim .8\times 10^3 xe$, $1<x<10$. The anomalous em charge of telomeres is three orders of magnitude smaller than that of entire DNA but if DNA is quantum critical system the change the total anomalous em charge and quark number due to the shortening of telomeres could induce instabilities of DNA (due to the approach to vacuum extremal) contributing to ageing. Note that the small net value of quark number in all the cases considered might be necessary for overall stability of DNA. Telomeres are also known to prevent the ends of chromosomes to stick to each other. This could be partially due to the Coulomb repulsion due to the anomalous em charge.

According to [I7] Chargaff's rules do not apply to viral organellar genomes (mitochondria [I31], plastids) or single stranded viral DNA and RNA genomes. Thus approximate matter antimatter symmetry fails for DNA: s of organelles involved with metabolism. This might relate to the fact that the coding portion of DNA is very high and repeats are absent. Chargaff's rule applies not only to nucleotides but also for oligonucleotides which corresponds to DNA or RNA sequences with not more than 20 bases. This means that for single strand oligonucleotides and their conjugates appear in pairs. Matter antimatter asymmetry would be realized as presence of matter blobs and their conjugates. This might relate to the mechanism how the sequences of oligonucleotides are generated from DNA and its conjugate.

8. Breaking of matter antimatter symmetry for coding regions

As noticed, one can consider three type of symmetry breaking parameters for DNA in DNA as TQC model. There are indeed three empirical parameters of this kind. Chargaff rules have been already discussed and correspond to approximate matter antimatter symmetry. The second asymmetry parameter would measure the asymmetry between $u\overline{u}$ and $d\overline{d}$ type matter. $p(G+C)$ corresponds to the fraction of $d\overline{d}$ type quark matter for option 1) and $u\overline{u}$ matter for option 2). It is known that G+C fraction $p(G+C)$ characterizes genes [I75] and the value of $p(G+C)$ is proportional to the length of the coding sequence [I23, I75].

Besides Chargaff rules holding true for entire genome also Szybalski's rules [I7] hold true but only for coding coding regions. The biological basis of neither rules is not understood. The interpretation of Chargaff's rules would be in terms of approximate matter antimatter symmetry and the vanishing of net isospin at the level of quarks whereas Szybalski's rule would state the breaking of these symmetries non-coding regions. Hence all the three basic empirical rules would have a nice interpretation in DNA as TQC picture.

Consider now Szybalski's rules in more detail.

1. In most bacterial genomes (which are generally 80-90 % coding) genes are arranged in such a fashion that approximately 50 % of the coding sequence lies on either strand. Note that either strand can act as a template (this came as a surprise for me). Szybalski, in the 1960s, showed that in bacteriophage coding sequences purines (A and G) exceed pyrimidines (C and T). This rule has since been confirmed in other organisms and known as Szybalski's rule [I7, I76]. While Szybalski's rule generally holds, exceptions are known to exist.

   Interpretation. A breaking of matter antimatter symmetry occurs in coding regions such that the net breakings are opposite for regions using different templates and thus different directions of transcription (promoter to the right/left of coding region).

2. One can actually characterize Szybalski's rules more precisely. By Chargaff's rules one has $p(A+T) \simeq 1-p(G+C))$. In coding regions with low value of $p(G+C)$ $p(A)$ is known to be higher than on the average whereas for high value of $p(G+C)$ $p(G)$ tends to higher than on the average.

   Interpretation. These data do not fix completely the pattern of breaking of the approximate matter antimatter symmetry.

   i) It could take place for both kinds of quark matter ($u\overline{u}$ and $d\overline{d}$): both $p(A)$ and $p(G)$ would increase from its value for entire genome but the dominance of A over G or vice versa would explain the observation.

   ii) The breaking could also occur only for the dominating type of quark matter ($u\overline{u}$ or $d\overline{d}$) in which case only $p(A)$ or $p(G)$ would increase from the value for entire genome.

   Also a net isospin is generated which is of opposite sign for short and long coding sequences so that there must be some critical length of the coding sequences for which isospin per nucleotide vanishes. This length should have biological meaning.

3. For mRNA $A+G$ content is always high. This is possible only because the template part of the DNA which need not be always the same strand varies so that if it is strand it has higher $A+G$ content and if it is conjugate strand it has higher $T+C$ content.

Interpretation. mRNA breaks always matter antimatter symmetry and the sign of matter antimatter asymmetry is always the same. Thus mRNA is analogous to matter in observed universe. The poly-A tail added to the end of mRNA after transcription to stabilize it would reduce the too large values of isospin and anomalous em charge per nucleon due to the fact that mRNA does not contain regions satisfying Chargaff's rules. It would also generate the needed longitudinal electric field determining the direction of translation. In the case of DNA the breaking of matter antimatter symmetry is realized at the functional level by a varying direction of transcription and variation of template strand so that matter antimatter symmetry for the entire DNA is only slightly broken. Direction of transcription would be determined by the direction of the electric field. The stability of long DNA sequences might require approximate matter antimatter symmetry for single DNA strand if it is long. In the case of simple genomes (mitochondrial, plastid, and viral) the small size of the genome, the high fraction of coding regions, and the absence of repeating sequences might make approximate matter antimatter symmetry un-necessary. An interesting working hypothesis is that the direction of transcription is always the same for these genomes.

One can try to use this information to fix the most probable option for nucleotide quark correspondence.

1. In nuclear physics the neutron to proton ratio of nucleus increases as nucleus becomes heavier so that the nuclear isospin becomes negative: $I_3 < 0$. The increase of the nuclear mass corresponds to the increase for the length of the coding region. Since G/A fraction increases with the length of coding region, G should correspond to either d quark $((Q_a < 0, I_3 = -1/2))$ or its charge conjugate d_c $(Q_a < 0)$. Hence option 1) or its charge conjugate would be favored.
2. If one takes very seriously the analogy with cosmic matter antimatter asymmetry then matter should dominate and only $(A, G, T, C) \rightarrow (u, d, \overline{u}, \overline{d})$ option would remain.

Szybalski's findings leave open the question whether non-coding regions obey the Chargaff rules in good approximation or whether also they appear as pairs with opposite matter antimatter asymmetry. Introns are belong to coding regions in the sense that they are transcribed to mRNA. Splicing however cuts them off from mRNA. It is not clear whether introns break the approximate matter antimatter symmetry or not. If breaking takes place it might mean that introns code for something but not chemically. On the other hand, the absence of asymmetry might serve at least partially as a signal telling that introns must be cut off before translation. Many interesting questions represent itself. For instance, how the symmetry breaking parameters, in particular matter antimatter asymmetry parameter, depend on genes. The correlation with gene length is the most plausible guess.

Genetic codes and TQC

TGD suggests the existence of several genetic codes besides 3-codon code [K53, ?]. The experience from ordinary computers and the fact that genes in general do not correspond to $3n$ nucleotides encourages to take this idea more seriously. The use of different codes would allow to tell what kind of information a given piece of DNA strand represents. DNA strand would be like a drawing of building containing figures (3-code) and various kinds of text (other codes). A simple drawing for the building would become a complex manual containing mostly text as the evolution proceeds: for humans 96 per cent of code would corresponds to introns perhaps obeying some other code.

The hierarchy of genetic codes is obtained by starting from n basic statements and going to the meta level by forming all possible statements about them (higher order logics) and throwing away one which is not physically realizable (it would correspond to empty set in the set theoretic realization). This allows $2^n - 1$ statements and one can select 2^{n-1} statements consistent with a given atomic statement (1 bit fixed) (half of the full set of statements) and say that these are true and give kind of axiomatics about world. The remaining statements are false. DNA would realize only these statements.

The hierarchy of Mersenne primes $M_n = 2^n - 1$ with $M_{n(next)} = M_{M_n}$ starting from $n = 2$ with $M_2 = 3$ gives rise to 1-code with 4 codons, 3-code with 64 codons, and $3 \times 21 = 63$-code with 2^{126} codons [K53] realized as sequences of 63 nucleotides (the length of 63-codon is about $2L(151)$, roughly twice the cell membrane thickness. It is not known whether this Combinatorial Hierarchy continues ad infinitum. Hilbert conjectured that this is the case.

In the model of pre-biotic evolution also 2-codons appear and 3-code is formed as the fusion of 1- and 2-codes. The problem is that 2-code is not predicted by the basic Combinatorial Hierarchy associated with $n = 2$.

There are however also other Mersenne hierarchies and the next hierarchy allows the realization of the 2-code. This Combinatorial Hierarchy begins from Fermat prime $n = 2^k + 1 = 5$ with $M_5 = 2^5 - 1 = 31$ gives rise to a code with 16 codons realized as 2-codons (2 nucleotides). Second level corresponds to Mersenne prime $M_{31} = 2^{31} - 1$ and a code with $2^{30=15\times 2}$ codons realized by sequences of 15 3-codons containing 45 nucleotides. This corresponds to DNA length of 15 nm, or length scale $3L(149)$, where $L(149) = 5$ nm defines the thickness of the lipid layer of cell membrane. $L(151) = 10$ nm corresponds to 3 full 2π twists for DNA double strand. The model for 3-code as fusion of 1- and 2-codes suggests that also this hierarchy - which probably does not continue further - is realized.

There are also further short Combinatorial hierarchies corresponding to Mersenne primes [A8].

1. $n = 13$ defines Mersenne prime M_{13}. The code would have $2^{12=6\times 2}$ codons representable as sequences of 6 nucleotides or 2 3-codons. This code might be associated with microtubuli.
2. The Fermat prime $17 = 2^4 + 1$ defines Mersenne prime M_{17} and the code would have $2^{16=8\times 2}$ codons representable as sequences of 8 nucleotides.
3. $n = 19$ defines Mersenne prime M_{19} and code would have $2^{18=9\times 2}$ codons representable as sequences of 9 nucleotides or three DNA codons.
4. The next Mersennes are M_{31} belonging to $n = 5$ hierarchy, M_{61} with $2^{60=30\times 2}$ codons represented by 30-codons. This corresponds to DNA length $L(151) = 10$ nm (cell membrane thickness). M_{89} (44-codons), M_{107} (53-codons) and M_{127} (belonging to the basic hierarchy) are the next Mersennes. Next Mersenne corresponds to M_{521} (260-codon) and to completely super-astrophysical p-adic length scale and might not be present in the hierarchy.

   This hierarchy is realized at the level of elementary particle physics and might appear also at the level of DNA. The 1-, 2-, 3-, 6-, 8-, and 9-codons would define lowest Combinatorial Hierarchies.

10.8 Cell Replication And TQC

DNA as TQC model leads to quite detailed ideas about the evolution of the genetic code and the mechanisms of bio-catalysis and of protein folding [**?**]. These applications in turn leads to a considerable generalization of DNA as TQC concept [**?**] . The presence of braiding leads also to a revision of the model of nerve pulse and EEG [K94, K42]. Here the discussion is restricted to one particular example. One can look what happens in the cell replication in the hope of developing more concrete ideas about TQC in multicellular system. This process must mean a replication of the braid's strand system and a model for this process gives concrete ideas about how multicellular system performs TQC.

10.8.1 Mitosis And TQC

Mitosis is the form of cell replication yielding soma cells and it is interesting what constraints this process gives on TQC and whether the special features of this process could be understood from computational point of view.

1. During mitosis chromosomes [I32] are replicated. During this process the strands connecting chromosomes become visible: the pattern brings in mind flux tubes of magnetic field. For prokaryotes the replication of chromosomes is followed by the fission of the cell membrane. Also plant nuclei separated by cellulose walls suffer fission after the replication of chromosomes. For animals nuclear membranes break down before the replication suggesting that nuclear TQC programs are reset and newly formed nuclei start TQC from a clean table. For eukaryotes cell division is controlled by centrosomes [I6]. The presence of centrosomes is not necessary for the survival of the cell or its replication but is necessary for the survival of multicellular. This conforms with the proposed picture.
2. If the conjugate strands are specialized in TQC, the formation of new double strands does not involve braids in an essential manner. The formation of conjugate strand should lead to also to a generation of braid strands unless they already exist as strands connecting DNA and its conjugate and are responsible for "printing". These strands need not be short. The braiding associated with printing would be hardware program which could be genetically determined or at least inherited as such so that the strands should be restricted inside the inner cell membrane or at most traverse the inner nuclear membrane and turn back in the volume between inner membrane and endoplasmic reticulum.

The return would be most naturally from the opposite side of nuclear membrane which suggest a breaking of rotational symmetry to axial symmetry. The presence of centriole implies this kind of symmetry breaking: in neurons this breaking becomes especially obvious. The outgoing braid strands would be analogous to axon and returning braid strands to dendrites. Inner nuclear membrane would decompose the braiding to three parts: one for strand, second for conjugate strand, and a part in the empty space inside nuclear envelope.

3. The formation of new DNA strands requires recognition relying on "strand color" telling which nucleotide can condense at it. The process would conserve the braidings connecting DNA to the external world. The braidings associated with the daughter nuclei would be generated from the braiding between DNA and its conjugate. As printing software they should be identical so that the braiding connecting DNA double strands should be a product of a braiding and its inverse. This would however mean that the braiding is trivial. The division of the braid to three parts hinders the transformation to a trivial braid if the braids combine to form longer braids only during the "printing" activity.
4. The new conjugate strands are formed from the old strands associated with printing. In the case of plants the nuclear envelope does not disintegrate and splits only after the replication of chromosomes. This would suggest that plant cells separated by cell walls perform only intracellular TQC. Hermits do not need social skills. In the case of animals nuclear envelope disintegrates. This is as it must be since the process splits the braids connecting strand and conjugate strands so that they can connect to the cell membrane. The printing braids are inherited as such which conforms with the interpretation as a fixed software.
5. The braids connecting mother and daughter cells to extranuclear world would be different and TQC braidings would give to the cell a memory about its life-cycle. The age ordering of cells would have the architecture of a tree defined by the sequence of cell replications and the life history of the organism. The 4-D body would contain kind of log file about TQC performed during life time: kind of fundamental body memory.
6. Quite generally, the evolution of TQC programs means giving up the dogma of genetic determinism. The evolution of TQC programs during life cycle and the fact that half of them is inherited means kind of quantum Lamarckism [I24]. This inherited wisdom at DNA level might partly explain why we differ so dramatically from our cousins.

10.8.2 Sexual Reproduction And TQC

Meiosis [I28] produces gametes in which the pair of chromosomes from parents is replaced with single chromosome obtained as chimera of the chromosomes of parents. Meiosis is the basic step of sexual reproduction and it is interesting to study it from TQC point of view.

1. Sexual reproduction of eukaryotes relies on haploid cells differing from diploid cells in that chromatids do not possess sister chromatids. Whereas mitosis produces from single diploid [I40] cell two diploid cells, meiosis gives rise to 4 haploid [I40] cells. The first stage is very much like mitosis. DNA and chromosomes duplicate but cell remains a diploid in the sense that there is only single centrosome: in mitosis also centrosome duplicates. After this the cell membrane divides into two. At the next step the chromosomes in daughter cells split into two sister chromosomes each going into its own cell. The outcome is four haploid cells.
2. The presence of only single chromatid [I8] in haploids means that germ cells would perform only one half of the "social" TQC performed by soma cells [I45] who must spend their life cycle as a member of cell community. In some cells the TQC would be performed by chromatids of both father and mother making perhaps possible kind of stereo view about world and a model for couple - the simplest possible social structure.
3. This brings in mind the sensory rivalry between left and right brain: could it be that the two TQC's give competing computational views about world and how to act in it? We would have inside us our parents and their experiences as a pair of chromatids representing chemical chimeras of chromatid pairs possessed by the parents: as a hardware - one might say. Our parents would have the same mixture in software via sharing and fusion of chromatid mental images or via quantum computational rivalry. What is in software becomes hardware in the next generation.¡/p¿¡p¿

4. The ability of sexual reproduction to generate something new relates to meiosis. During meiosis genetic recombination [I19] occurs via chromosomal crossover which in string model picture would mean splitting of chromatids and the recombination of pieces in a new manner $(A_1 + B_1) + (A_2 + B_2) \rightarrow (A_1 + B_2) + (A_2 + B_1)$ takes place in crossover and $(A_1 + B_1 + C_1) + (A_2 + B_2 + C_2) \rightarrow (A_1 + B_2 + C_1) + (A_2 + B_1 + C_2)$ in double crossover. New hardware for TQC would result by combining pieces of existing hardware. What this means in terms of braids should be clarified.
5. Fertilization is in well-define sense the inverse of meiosis. In fertilization the chromatids of spermatozoa and ova combine to form the chromatids of diploid cell. The recombination of genetic programs during meiosis becomes visible in the resulting TQC programs.

10.8.3 What Is The Role Of Centrosomes And Basal Bodies?

Centrosomes [I6] and basal bodies [I3] form the main part of Microtubule Organizing Center [I30]. They are somewhat mysterious objects and at first do not seem to fit to the proposed picture in an obvious manner.

1. Centrosomes consist two centrioles [I5] forming a T shaped antenna like structure in the center of cell. Also basal bodies consist of two centrioles but are associated with the cell membrane. Centrioles and basal bodies have cylindrical geometry consisting of nine triplets of microtubules along the wall of cylinder. Centrosome is associated with nuclear membrane during mitosis.
2. The function of basal bodies which have evolved from centrosomes seems to be the motor control (both cilia [I9] and flagella [I18]) and sensory perception (cilia). Cell uses flagella and cilia to move and perceive. Flagella and cilia are cylindrical structures associated with the basal bodies. The core of both structures is axoneme having $9 \times 2 + 2$ microtubular structure. So called primary cilia do not posses the central doublet and the possible interpretation is that the inner doublet is involved with the motor control of cilia. Microtubules [I29] of the pairs are partially fused together.
3. Centrosomes are involved with the control of [I32] [I32]. Mitosis can take place also without them but the organism consisting of this kind of cells does not survive. Hence the presence of centrosomes might control the proper formation of TQC programs. The polymerization of microtubules [I29] is nucleated at microtubule self-organizing center which can be centriole or basal body. One can say that microtubules which are highly dynamical structures whose length is changing all the time have their second end anchored to the self-organizing center. Since this function is essential during mitosis it is natural that centrosome controls it.
4. The key to the understanding of the role of centrosomes and basal bodies comes from a paradox. DNA and corresponding TQC programs cannot be active during mitosis. What does then control mitosis?
   (a) Perhaps centrosome and corresponding TQC program represents the analog of the minimum seed program in computer allowing to generate an operating system like Windows 2000 (the files from CD containing operating system must be read!). The braid strands going through the microtubules of centrosome might define the corresponding TQC program. The isolation from environment by the microtubular surface might be essential for keeping the braidings defining these programs strictly unchanged.
   (b) The RNA defining the genome of centrosome (yes: centrosome has its own genome defined by RNA rather than DNA [I6] !) would define the hardware for this TQC. The basal bodies could be interpreted as a minimal sensory-motor system needed during mitosis.
   (c) As a matter fact, centrosome and basal bodies could be seen as very important remnants of RNA era believed by many biologists to have preceded DNA era. This assumption is also made in TGD inspired model of prebiotic evolution [**?**].
   (d) Also other cellular organelles possessing own DNA and own TQC could remain partly functional during mitosis. In particular, mitochondria are necessary for satisfying energy needs during the period when DNA is unable to control the situation so that they must have some minimum amount of own genome.

5. Neurons [I33] do not possess centrosome which explains why they cannot replicate. The centrioles are replaced with long microtubules associated with axons and dendrites. The system consisting of microtubules corresponds to a sensory-motor system controlled by the TQC programs having as a hardware the RNA of centrosomes and basal bodies. Also this system would have a multicellular part.
6. Intermediate filaments [I21], actin filaments [I2], and microtubules [I29] are the basic building elements of the eukaryotic cytoskeleton [I11]. Microtubules, which are hollow cylinders with outer radius of 24 nm, are especially attractive candidates for structures carrying bundles of braid strands inside them. The microtubular outer-surfaces could be involved with signalling besides other well-established functions. It would seem that microtubules cannot be assigned with TQC associated with nuclear DNA but with RNA of centrosomes and could contain corresponding braid strand bundles. It is easy to make a rough estimate for the number of strands and this would give an estimate for the amount of RNA associated with centrosomes. Also intermediate filaments and actin filaments might relate to cellular organelles having their own DNA.

10.9 Indirect Evidence For The DNA As Topological Quantum Computer Model

There is a profound revolution taking place in genetics [I70]. It is fair to say that genetic determinism is falling down and the revolution that is waiting just around the corner will be more profound that anything that has taken place before this in biology. The term "genome's dark matter" expresses what has been discovered during last years. The motivation for the term is the strong analogy with the dark matter of physics. In TGD framework this analogy might be much more than analogy.

The basic anomalies discussed in the article are following.

1. Trans-generational inheritance [I74, I79]. The stretches of DNA which were present in parent's or grand parents' genome but are not present in the genome of offspring affect the traits of offspring.
2. Context sensitivity of gene's effect: the effect of gene is highly sensitive on its environment in DNA.
3. Genes explain in many cases only 10 percent of the disease's inheritability: this is "missing heritability" problem [I60].

It is interesting to try to interpret these results in the framework provided by the model of DNA as topological quantum computer. It is good to summarize the basic ideas and concepts behind the model.

10.9.1 The Notion Of Magnetic Body

The notion of magnetic body as intentional agent using biological body as a motor instrument and sensory receptor with communications taking place in terms of fractal generalization of EEG is the key idea. Each physical system consisting of matter has magnetic body. Magnetic body of given living organism has a fractal onion like structure with layer sizes varying from sub-cellular scales to the scales assignable to EEG frequencies (Earth size) and even above up to the scale of light-life and maybe beyond to scales characterizing the evolution of species.

Immediate implications are the notion of collective DNA expression made possible by the interaction of DNA strands so that they belong to magnetic flux sheets: in this manner not only DNAs of cells and organelles, organs, single organism but also groups of organisms can form coherent structures expressing themselves in synchronous manner. This is a testable prediction.

10.9.2 DNA As Topological Quantum Computer

Topological quantum computation is based on braiding: various braiding patterns for braid strands define the TQC programs. There are two types of braids: time-like and space-like.

1. Cell membrane is 2-D liquid and the flow of lipids affected by the flow of cellular liquid and also by nerve pulse patterns in case of neurons induce braiding. This braiding takes place dynamically at the 2-D parquette defined by the cell membrane in time direction and dance metaphor applies to it. Running TQC program can be seen as dancing.
2. The magnetic flux tubes connecting DNA nucleotides to the lipids of nuclear membrane and cell membrane and possibly also to membranes of other cells define the space-like braid strands. Since the flux tubes connected to DNA strands are like threads connecting the feet of dancers to the walls of the dance hall the resulting space-like braiding codes TQC program to memory, which is highly robust as a topological invariant.

There is a kind of duality between time-like and space-like braidings. This is a new element to the conventional quantum computation paradigm. Combined with the idea that memories are stored in geometric past in zero energy ontology this gives an extremely elegant memory storage mechanism.

10.9.3 Implications For Genetics

This vision has profound implications for genetics.

1. Genes define only the hardware of TQC. Software is defined by the braidings. Introns whose portion steadily increases as the evolutionary level becomes higher and is more than 95 per cent in humans, have been traditionally interpreted as junk DNA. In this framework introns correspond naturally to that part of genome specialized in TQC: from the point of view of TQC it does matter much whether the intronic portions correspond to repeating sequences (interpreted as a signal for "junkness") or not.
2. The evolution of topological quantum computation programs would be far more important than the evolution of genome and the huge differences between species with almost the same genome (such as we and our cousins) could be understood in terms ofw hat at our level of hierarchy corresponds to cultural evolution due to the evolution of topological quantum computer programs. The evolution would have been for a long time evolution of TQC programs rather than that of hardware as the fact that the size of genome and details of does not matter much suggests. The appearance of of prokaryotes (and multi-cellulars) meant the emergence of introns and perhaps also the precedessor of cultural evolution as the evolution of quantum software and collective magnetic bodies.

10.9.4 Implications For Mendelian Anomalies

This vision also suggests how to understand the origin of the Mendelian anomalies.

1. Trans-generational inheritance might be understood as an inheritance of TQC programs carrying indirectly information also about the genome of parents. If one accepts TGD vision about organisms as 4-D structures, one must of course be ready to even ask whether genetic effects could be also take place via the mediation of the magnetic bodies assignable to structures formed by several generations.

   Of course, it is far from clear what the mechanism leading to the inheritance of TQC programs could mean. Does it make sense to speak about magnetic body for causal diamonds (CDs) in time scale of several generations? Could TQC programs associated with the magnetic bodies affect also the future generations by signals realized in terms of positive and negative energy photons propagating in opposite time directions? In zero energy ontology (ZEO) instantaneous communications realized as time reflections are indeed possible and are central in the realization of memories and anticipation in TGD Universe.
2. The context sensitivity of the effect of particular gene could be understood in this picture since the programs are determined not only by a single gene but longer portions of DNA. Individual genes do not matter much when one tries to understand genetic correlates for autism, schizophrenia, and other complex diseases related to functions rather than mere structure. If one speaks about structure, such as the color of flowers situation is of course very simple and Mendelian approach works well. An interesting question is how closely

the structure-function dichotomy, exon-intron dichotomy and hardware-software dichotomy correspond to each other.

3. High level diseases would be much more programming errors than hardware problems. This would solve "missing heritability" problem.

What is amusing, that the physicist's dark matter would be behind "genome's dark matter": magnetic flux tubes are assumed to be carriers of dark matter- dark quarks in fact. In the proposed model quarks with large Planck constant meaning that their Compton length scales is scaled up and gives them size scale of order cell at least are in key role!

10.10 How To Build A Quantum Computer From Magnetic Flux Tubes

Magnetic flux tubes play a key role in TGD inspired model of quantum biology. Could the networks of magnetic flux tubes containing dark particles with large $\hbar$ in macroscopic quantum states and carrying beams of dark photons define analogs of electric circuits? This would be rather cheap technology since no metal would be needed for wires. Dark photon beams would propagate along the flux tubes representing the analogs of optical cables and make possible communications with maximal signal velocity.

I have actually made much more radical proposal in TGD inspired quantum biology. According to this proposal, flux tube connections are dynamical and can be changed by reconnection of two magnetic flux tubes. The signal pathways $A \rightarrow C$ and $B \rightarrow D$ would be transformed to signal pathways to $A \rightarrow D$ and $B \rightarrow C$ by reconnection. Reconnection actually represents a basic stringy vertex. The contraction of magnetic flux tubes by a phase transition changing Planck constant could be fundamental in bio-catalysis since it would allow distant molecules connected by flux tubes to find each other in the molecular crowd.

DNA as a topological quantum computer is the idea that I have been developing for 5 years or so. I have concentrated on the new physics realization of braids and devoted not much thought to how the quantum computer problems might run in this framework. I was surprised to realize how little I know about what happens in even ordinary computation. Instead of going immediately to Wikipedia I take the risk of publicly making myself fool and try to use my own brain.

10.10.1 What Can One Learn From Ordinary Computer Programs

One could begin with the question what happens in classical computation. How the program is realized and how it runs? The notion of Turing machine (see `http://tinyurl.com/7c4kl`) represents an extreme abstraction mentioning nothing about the technical side and does not help much in attempts to answer these questions. Turing paradigm also assumes that program is a temporal sequence of operations. These operations could however correspond to a linear spatial sequences and inputs and outputs in this case would correspond to boundary values at the ends of the linear structure. This requires that the dynamics is such that evolution in spatial direction is analogous to a deterministic time evolution. In this case it is much easier to imagine biological realizations of quantum computer programs in TGD inspired bio-world.

To develop concrete ideas, one can start from the picture provided by ordinary computer program.

1. Programs consist of temporal/spatial sequences of commands and commands represent basic functions from which one can build more complex functions by the composition of functions having some numbers of input and output arguments. The eventual output variable can be expressed by printing of a piece of text or as an image in the computer screen. Each step in the program corresponds to a composition of functions: $f_{n+1} = g_{n+1} \circ f_n$. There is some minimal set of primitive/prime functions from which one builds up more complex functions by composition.

2. How this is realized at the level of hardware? One can assume that the basic functions are at some fixed places in the computer memory having addresses given by integers represented as bit sequences. This address represents the command - a name of the function. The names

for input variables and output variables are bit sequences giving the addresses of the places containing the values of these variables. Program is a sequence of commands represented as bit sequences giving the address of the function to be computed at a given step and the addresses of inputs and outputs. As the processing unit reads the command, it generates/activates connections from the addresses of inputs to the address representing the function and from this address to the addresses of outputs.

Essentially the challenge is to reconnect, build/activate connections. An interesting question is whether learning identified as strengthening of synaptic connections (see `http://tinyurl.com/cn7724o`) [J2] is one particular example of this process.

3. How the sequence of bits representing command address is realized? As the processing unit reads the address of command it should automatically create/activate a connection from this address to the command address. The connections from the processing unit to the addresses could exist physically as wirings.

4. It is not necessary that program is dynamical so that the inputs and outputs would be initial and final values of variables. Inputs and outputs could also correspond to values of variables at the ends of a linear structure. In topological quantum computation space-like entanglement would represent superposition of input-output pairs characterizing a function as a rule with instances represented as instances appearing in the superposition.

If this picture is roughly correct, re-connection would be the basic process. Reconnection is the basic process for magnetic flux tubes and ADP$\leftrightarrow$ ATP has been assigned to this process with ATP molecule serving as a relay activating the flux tube connection. Maybe ADP-ATP process, which is usually seen as a basic step of metabolism, could be seen as the core step for quantum computation performed by living matter. One expects that the presence ATP makes the rule represented by negentropic quantum entanglement conscious.

10.10.2 Quantum Computation Magnetic Flux Tubes As Connections

Consider now quantum computation could take place in a circuitry having magnetic flux tubes as wires and some bio-molecules of groups of them as units defining prime functions. DNA as topological quantum computer could be taken as a starting point. The outcome of quantum computation is determined statistically as ensemble average so that a large number of copies of the program should be present and realized in terms of groups of cells or molecules connected by braidings if the quantum computation is space-like. This option seems more natural than time-like quantum computation realized as a 2-D liquid flow of lipids in the lipid layers of the cell membrane.

The hardware

Consider first the hardware of topological quantum computation using space-like braids.

1. Magnetic flux tubes would represent the wires along which inputs and outputs travel in the case of classical computation or dynamical quantum computation. In the case of space-like topological quantum computation entanglement is between the ends of the flux tubes.

2. Variables could be represented in many ways. For space-like quantum computations they could correspond to spin states of dark electrons at flux tubes or to polarization states of dark electrons at the flux tubes. In the original model of DNA as topological quantum computer quarks and antiquarks where proposed as a representation of genetic codons: also this quite science fictive option could make sense in TGD Universe since TGD predicts scaled versions of QCD like dynamics and presence of elementary particles in several p-adic scales and in scales dictated by value of Planck constant for given p-adic length scale.

   The spin states of electron pair has been proposed as one possible representation of the 4 genetic codons. Quantum variables would be represented by qubit sequences and the measurement of qubit would give a bit sequence characterizing the classical value of the variable. Bio-molecules would be natural places for storing the values of the variables. For dynamical computations the values of variables could be transmitted using dark photons.

3. There would exist basic processing units calculating the prime functions from which more complex functions would be obtained as composites. Basic units could correspond to biomolecules. In the case of classical computation the inputs to molecules and outputs from them would travel along the flux tubes. In quantum computation these signals could be used to control the initial values of the variables. Molecules could also serve as gates for quantum computation.

Representation of programs

The basic program units in the case of quantum computation would be represented by braidings.

1. If the ends of braid strands are able to move freely when needed, it becomes possible to rewrite programs. Lipid layers of cell membrane can be in liquid crystal state so that these are ideal for this purpose. The time-like braiding resulting from lipid flow and representing running topological quantum computation program would induce space-like braiding representing space-like topological quantum computation or a rule. A particular quantum computer program represented as space-like braiding of the flux tubes would result as liquid crystal melts for a moment and freezes again.

   The process (see `http://tinyurl.com/yarrblxn`) in which proteins covered by ordered water analogous to ice temporarily melt and form aggregates [I43] is basic process induced by the feed of energy to the cellular system and could be compared to cellular summer. This process could mean quite generally molecular re-programming induced by the flow of cellular water inducing molecular flows inducing re-braidings. The braiding would also store the highlights of the cellular summer to cellular memory! This could be also seen learning by a modification of various quantum computer programs.

2. Negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) is highly suggestive and would conform with the idea that the rule represented by entanglement represents conscious information or information which can become conscious. The process of becoming conscious information could involve ATP$\rightarrow$ ADP and de-activating the flux tube and destroy the information. Time-like braiding represented by liquid flow would modify space-like braiding.

   It is not quite clear whether the information is conscious when negentropic entanglement (and ATP) is present - as Bohm's notion of active information (see `http://tinyurl.com/qhx3suy`) [J15] would suggest - or when ATP is transformed to ADP and connection becomes passive. Negentropic entanglement can be stable with respect to NMP [K72] so that the presence of ATP could mean period of conscious experience - negentropic entanglement could be analogous to active information.

   TGD based model for the memory recall by sending negative energy signals to geometric past suggests that the absorption of negative energy photon transforms ATP to ADP. Conscious experience is regenerated in the geometric now where the negative energy signal came from - perhaps by transforming ADP to ATP by using the negative resulting by sending of negative energy signal! Conscious reading would be actually memory recall and analogous to teleportation? The destruction of the representation of memory in the geometric past would have interpretation in terms of no-cloning theorem (see `http://tinyurl.com/2dhl4oe`) [B2].

3. Static realizations of the programs are easier to imagine since no temporal codes are needed for the transfer of bits. An attractive idea is that the computations are represented by static entanglements for linear structures and that time-like braiding allows to modify the programs.

The realization of program

The program would be basically a sequence of address lists. Address list would contain the address of the function to be performed and the addresses of the input molecules and output molecules. How to represent the address physically?

1. The simplest manner to realize this would use existing flux tubes connecting the processing unit to all possible input and output addresses as well as command addresses, and activate those flux tubes to which input and output data are assigned and reconnect them to the flux

tubes connecting processing unit to the unit representing the function. The processing unit would have flux tubes coming from all possible inputs, going to all possible outputs, flux tubes going to places representing functions and coming from these places. Processing unit would be like a relay station or old fashioned telephone center whose sole purpose would be to create connections by reconnecting flux tubes. ATP molecule would be probably involved with the activation and - allowing a sloppy language - one could say that communication line becomes conscious when ATP is attached to it.

(a) Addressing would be just selection of activated molecules and analogous to that used in telephone network or computer network connected by cables. This would require static flux tube network and flux tubes could be either active or passive. In passive state flux tubes could be short-cut by a reconnection with hydrogen bond so that the ends of cut flux tube would end up to water molecules. This is however not necessary. Activation in absence of the short cut would involve reconnection of a flux tube with a flux tube connecting two parts of ATP - possibly hydrogen bond again- so that ATP becomes part of the flux tubes. If also short cut is involved, the strands coming to the two water molecules reconnect and generate hydrogen bond and flux tube to which ATP would attach in the proposed manner. As ATP is used it transforms to ADP and de-attaches from the flux tube.

(b) One can imagine also a dynamical addressing based on the generation of magnetic flux tubes between inputs and submodules. The computational process could be still space-like. The first manner to realize dynamical addressing would be by attaching to the ends of dynamical flux tubes biomolecules, which bind to specific receptors. Receptor mechanism would allow to connect distant cells to each other and build a magnetic flux tube connection between them. Computational unit specialized to run a specific program could excrete biomolecules binding to the input and output receptors: this program would realized function in terms of space-like entanglement. Glands (see `http://tinyurl.com/cxjro9z`) excrete hormones binding to receptors and various glands could in principle serve as computational units. Various information molecules bind very selectively and this might also relate to quantum space-like computations.

(c) Second mechanism of dynamical addressing would use dark photons. In this case resonant interaction selecting the target would replace the receptor mechanism. In this kind of situation one can claim that flux tubes are un-necessary, one can use just resonance to build connection to a desired place just as one does in radio communications. Of course, topological light rays could be accompanied by flux tubes. For instance, DNA nucleotide could attach by flux tube to its conjugate in distant DNA molecule and if the connection is based on resonance only similar nucleotide sequences could connect with each other. I have discussed this kind of mechanism in a model for remote replication of DNA (see `http://tinyurl.com/ybvosy7h`) [K133] based on the experimental work by Peter Gariaev and his group. The resonance mechanism could also make possible to establish flux tube connections and the quantum computation could be a static operation.

2. DNA as topological quantum computer vision gives some idea about how the computer program could be realized as a spatial linear structure.

(a) Program would be a sequence of topological quantum computations. Given topological quantum computation would be represented by a braiding of flux tubes connecting DNA nucleotides with the lipid molecules of the inner lipid layer. Program would correspond to a linear sequence of cells with the outer lipid layer connected to the DNA of the second cell.

(b) Lipid flows at given lipid layer could be used to rewrite programs and the programs could respond to the changes in environment in this manner: this would require that the lipid layer is in liquid crystal state during the period when program is changed. Also nerve pulse patterns would induce these flows. Programs would also represent memories as rules realized as quantum abstractions or as quantum functions.

(c) The program would "run" in the spatial direction. The selection of active input and output variables would be by acting the connection from molecule in question by attaching ATP as a relay through which the reconnected flux tube would traverse. This would be also part of the writing of the program. The superposition of entangled inputs and outputs could be seen as a quantum superposition of classical programs assigning outputs to inputs. Also microtubule-lipid layer braiding suggested also to play a key role in the realization of memories could give rise to similar space-like quantum computation representing rules.

(d) The effective 2-dimensionality implied by strong form of holography implied in turn by strong form of general coordinate invariance means that the physics depends on partonic 2-surfaces and 4-D tangent space data at them. This suggests that the dynamics on space-like 3-surfaces and light-like orbits of partonic 2-surfaces is fixed by a process analogous to gauge selection. Does just this effective gauge symmetry make possible to write quantum computer programs? Already ordinary deterministic computer program means selection of one particular dynamics from several alternative options suggesting that strict determinism is broken.

3. What could be the role of bio-catalysis in the computation? Bio-catalysis is a central part of the biological information processing and it would not be surprising if the catalysts connected by flux tubes to substrate molecules were involved with the computations. An attractive idea is that various information molecules binding to receptors involved with bio-control (neurotransmitters, hormones, etc...) are involved with building the flux tube connections between cells. These bio-molecules could carry the ends of flux tubes to special places for which receptors serve as addresses and in this manner build hardware for topological quantum computation involving inputs and outputs in distant parts of the body. The final output could be transformed to controlled gene expression. Quite generally, catalysts bind very selectively and could play a role similar that played by information molecules in building up the quantum computer programs.

4. One can imagine also purely classical computation based on catalytic mechanism probably allowing generalization to quantum case. The idea is that computer program - understood now as dynamical structure - is analogous to what happens in fairy tale in which hero finds a key which fits to a lock of a room containing a key which... There exists a beautiful realization of classical computation in terms of chemical concentrations using DNA. The output of given reaction representing computational step appears in the next reaction provide the system contains additional participating molecules, which could be both substrate molecules and catalysts. The program could be represented as concentrations of molecules needed at intermediate steps and lock-to-key mechanism guarantees that they are performed in the correct temporal order. Inputs and output molecules could be connected by flux tubes to bio-molecules which bind to specific receptors associated with the molecule representing the particular subprogram. This would automatically create a large number of classical computations proceeding in fixed order, maybe even quantum computations.

10.11 Appendix: A Generalization Of The Notion Of Embedding Space

In the following the recent view about structure of embedding space forced by the quantization of Planck constant is described. This view has developed much before the original version of this chapter was written.

The original idea was that the proposed modification of the embedding space could explain naturally phenomena like quantum Hall effect involving fractionization of quantum numbers like spin and charge. This does not however seem to be the case. $G_a \times G_b$ implies just the opposite if these quantum numbers are assigned with the symmetries of the embedding space. For instance, quantization unit for orbital angular momentum becomes n_a where Z_{n_a} is the maximal cyclic subgroup of G_a.

One can however imagine of obtaining fractionization at the level of embedding space for space-time sheets, which are analogous to multi-sheeted Riemann surfaces (say Riemann surfaces associated with $z^{1/n}$ since the rotation by 2π understood as a homotopy of M^4 lifted to the space-time sheet is a non-closed curve. Continuity requirement indeed allows fractionization of the orbital quantum numbers and color in this kind of situation.

10.11.1 Both Covering Spaces And Factor Spaces Are Possible

The observation above stimulates the question whether it might be possible in some sense to replace H or its factors by their multiple coverings.

1. This is certainly not possible for M^4, CP_2, or H since their fundamental groups are trivial. On the other hand, the fixing of quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is a geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. Zero energy ontology forces to modify this picture somewhat. In zero energy ontology causal diamonds (CDs) defined as the intersections of future and past directed light-cones are loci for zero energy states containing positive and negative energy parts of state at the two light-cone boundaries. The location of CD in M^4 is arbitrary but p-adic length scale hypothesis suggests that the temporal distances between tips of CD come as powers of 2 using CP_2 size as unit. Thus M^4 is replaces by CD and $\hat{M}^4$ is replaced with $\hat{CD}$ defined in obvious manner.

3. H_4 represents a straight cosmic string inside CD. Quantum field theory phase corresponds to Jones inclusions with Jones index $\mathcal{M} : \mathcal{N} < 4$. Stringy phase would by previous arguments correspond to $\mathcal{M} : \mathcal{N} = 4$. Also these Jones inclusions are labeled by finite subgroups of $SO(3)$ and thus by Z_n identified as a maximal Abelian subgroup.

   One can argue that cosmic strings are not allowed in QFT phase. This would encourage the replacement $\hat{CD} \times \hat{CP}_2$ implying that surfaces in $CD \times S^2$ and $(M^2 \cap CD) \times CP_2$ are not allowed. In particular, cosmic strings and CP_2 type extremals with M^4 projection in M^2 and thus light-like geodesic without zitterwebegung essential for massivation are forbidden. This brings in mind instability of Higgs=0 phase.

4. The covering spaces in question would correspond to the Cartesian products $\hat{CD}_{n_a} \times \hat{CP}_{2n_b}$ of the covering spaces of $\hat{CD}$ and $\hat{CP}_2$ by Z_{n_a} and Z_{n_b} with fundamental group is $Z_{n_a} \times Z_{n_b}$. One can also consider extension by replacing $M^2 \cap CD$ and S^2 with its orbit under G_a (say tetrahedral, octahedral, or icosahedral group). The resulting space will be denoted by $\hat{CD} \hat{\times} G_a$ *resp.* $\hat{CP}_2 \hat{\times} G_b$.

5. One expects the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds $M^2 \cap CD$ or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which correspond to exceptional groups in the ADE correspondence). For instance, in the case of $M^2 \cap CD$ the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

6. Also the orbifolds $\hat{CD}/G_a \times \hat{CP}_2/G_b$ can be allowed as also the spaces $\hat{CD}/G_a \times (\hat{CP}_2 \hat{\times} G_b)$ and $(\hat{CD} \hat{\times} G_a) \times \hat{CP}_2/G_b$. Hence the previous framework would generalize considerably by the allowance of both coset spaces and covering spaces.

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the embedding space to another one.

1. How the gluing of copies of embedding space at $(M^2 \cap CD) \times CP_2$ takes place? It would seem that the covariant metric of M^4 factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of M^4 metric can make sense. This is consistent with the identical vanishing of Chern-Simons action in $M^2 \times S^2$.

2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in CD degrees of freedom. This is not the case. Light-likeness in $(M^2 \cap CD) \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset (M^2 \cap CD) \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from

one sector of H to another one is light-like at $(M^2 \cap CD) \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of $(M^2 \cap CD)$ as M^2 projection. Hence no sudden change of the size X^2 occurs.

3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) and classical light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

10.11.2 Do Factor Spaces And Coverings Correspond To The Two Kinds Of Jones Inclusions?

What could be the interpretation of these two kinds of spaces?

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. $\mathcal{M} : \mathcal{N} = 4$ should correspond to the allowance of cosmic strings and other analogous objects. Does the introduction of the covering spaces bring in cosmic strings in some controlled manner? Formally the subgroup of $SU(2)$ defining the inclusion is $SU(2)$ would mean that states are $SU(2)$ singlets which is something non-physical. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$.

   For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane.

3. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$, which both are labeled by a subset of discrete subgroups of SU(2).

4. The discrete subgroups of $SU(2)$ with fixed quantization axes possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tetrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2¡ generated by reflection, and tetrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized WCW spinor fields in the WCW labelled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

The question how do the Planck constants associated with factors and coverings relate is far from trivial and I have considered several options.

1. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of metric allowed by Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains $r^2 \equiv \hbar^2/\hbar_0^2\hbar^2(M^4)/\hbar^2(CP_2)$. This puts M^4 and CP_2 in a very symmetric role and allows much more flexibility in the identification of symmetries associated with large Planck constant phases.
2. Algebraist would argue that Planck constant must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa. This gives two options.
3. Option I: $r(X) = n$ for covering and $r(X) = 1/n$ for factor space gives $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$. This gives $r = n_a/n_b$ for $\hat{H}/G_a \times G_b$ option and $r = n_b/n_a$ for $\hat{H times}(G_a \times G_b)$ option with obvious formulas for hybrid cases.
4. Option II: $r(X) = 1/n$ for covering and $r(X) = n$ for factor space gives $r = r(CP_2)/r(M^4)$. This gives $r = n_b/n_a$ for $\hat{H}/G_a \times G_b$ option and $r = n_a/n_b$ for $\hat{H times}(G_a \times G_b)$ option with obvious formulas for the hybrid cases.
5. At quantum level the fractionization would come from the modification of fermionic anti-commutation (bosonic commutation) relations involving $\hbar$ at the right hand side so that particle number becomes a multiple of $1/n$ or n. If one postulates that the total number states is invariant in the transition, the increase in the number of sheets is compensated by the increase of the fundamental phase space volume proportional to $\hbar$. This would give $r(X) \to r(X)/n$ for factor space and $r(X) \to nr(X)$ for the covering space to compensate the n-fold reduction/increase of states. This would favor Option II.
6. The second manner to distinguish between these two options is to apply the theory to concrete physical situations. Since G_a and G_b act as symmetries in CD and CP_2 degrees of freedom, one might of being able to distinguish between the two options if it is possible to distinguish between the action of G as symmetry of quantum states associated with covering and factor space. Also the quantization of the orbital spin quantum number at single particle level as multiples of n can be distinguished from that in multiples of $1/n$.

10.11.3 A Simple Model Of Fractional Quantum Hall Effect

The generalization of the embedding space suggests that it could possible to understand fractional quantum Hall effect [D2] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \tag{10.11.1}$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15..., 2/3, 3/5, 4/7, 5/9, 6/11, 7/13..., 5/3, 8/5, 11/7, 14/9...$ $1/5, 2/9, 3/13..., 2/7, 3/11..., 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [D2].

The model of Laughlin [D21] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [D13]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of embedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are four combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this there are two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. On the following just for fun consideration option I is considered although the conservation of number of states in the phase transition changing $\hbar$ favors option II.

1. The easiest manner to understand the observed fractions is by assuming that both M^4 and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. The appearance of $\nu = 5/2$ has been observed [D8]. The fractionized charge is $e/4$ in this case. Since $n_i > 3$ holds true if coverings are correlates for Jones inclusions, this requires to $n_b = 4$ and $n_a = 10$. n_b predicting a correct fractionization of charge. The alternative option would be $n_b = 2$ that also Z_2 would appear as the fundamental group of the covering space. Filling fraction $1/2$ corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [D13]. $n_b = 2$ is however inconsistent with the observed fractionization of electric charge and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e \int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6 \times 10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the identification of charge unit is rather ad hoc. Therefore this model can be taken only as a warm-up exercise.

Chapter 11

The Notion of Wave-Genome and DNA as Topological Quantum Computer

11.1 Introduction

For about eight years ago - inspired by a representation in CASYS$'$2000 conference [I61] - I developed a model [K131, K22] for the fascinating effects of laser light on genome discovered by Peter Gariaev and his collaborators. This model is somewhat obsolete since it does not involve the recent TGD inspired vision about quantum biology and DNA, and the discussions with Peter in the second Unified Theories conference 2008 in Budapest made clear the need to update this model containing also some misinterpretations.

In this article the effects of laser light on living matter are discussed only briefly with a stronger emphasis on the photographs produced by the scattering of ordinary light on DNA reported in [I69]. In TGD framework these photographs could be interpreted as photographs of wormhole magnetic flux tubes containing dark matter. This would realize the dream of making directly visible the basic new structure predicted by TGD inspired quantum biology. Of course, a more conventional explanation might be found for the effect, but the proposed qualitative explanation deserves to be discussed since it fits nicely with the general vision about dark matter in TGD Universe.

11.1.1 The Findings Of Peter Gariaev And Collaborators

These findings of Gariaev and collaborators include the rotation of polarization plane of laser light by DNA [I61]. phantom DNA effect [I62]. the transformation of laser light to radio wave photons having biological effects [I63]. the coding of DNA sequences to the modulated polarization plane of laser light and the ability of this kind of light to induce gene expression in another organisms provided the modulated polarization pattern corresponds to an "address" characterizing the organism [I61]. and the formation of images of what is believed to be DNA sample itself and of the objects of environment by DNA sample in a cell irradiated by ordinary light in UV-IR range [I69].

Gariaev and collaborators have introduced the notion of wave genome [I61] requiring the coding of DNA sequences to temporal patterns of coherent em fields forming a bio-hologram representing geometric information about the organism. Code could mean that nucleotide is represented by a characteristic rotation angle for the polarization plane of linearly polarized laser radiation scattering from it. This kind rotation is known to be induced by chromosomes by a mechanism which to my best knowledge is poorly understood. Other open questions concern the precise identification of the substrate of the bio-hologram, of the reference wave and of information carrying wave, and of the mechanism making possible (quantum) coherence in macroscopic length scales.

The reading of the DNA sequence to a radiation pattern is assumed to rely on the propagation of an acoustic soliton along DNA [I61]. Whatever this process is, one should also identify the reverse process inducing the activation of the genome as the target organism receives the radiation coding for the DNA provided the "address" is correct. One should also identify the mechanism transforming laser radiation to radio-waves at various frequencies as well as the mechanism creating what is believed to be the image of DNA sample and replicated images of some instruments used in experiment.

11.1.2 The Relevant Aspects Of TGD Based View About Living Matter

The called massless extremals (MEs or topological light rays) distinguish between TGD and Maxwell's electrodynamics: they represent classically signals propagating with light velocity in a precisely targeted and dispersion free way, and are therefore excellent candidates for the communication and control tools in the TGD based model for a living system as a conscious hologram [K82, K22, K42]. The notion of magnetic/field body, which can have layers of even astrophysical size, is an essential element of the model. Magnetic body uses biological body as a sensory receptor and motor instrument and MEs mediate sensory input and control signals between the two kinds of bodies [K42]. I have already earlier applied MEs and the notion of magnetic body in an attempt to understand Gariaev's findings [K22].

The new element is the model for DNA as topological quantum computer (TQC) [K4] based on time-like braidings of so called wormhole magnetic flux tubes connecting nucleotides to the lipids at lipid layers nuclear and cell membranes. The model leads to a wide variety of predictions about DNA itself [K4]. to a universal model for a tissue memory in terms of space-like braidings of wormhole magnetic flux tubes [K4]. to a more detailed model of nerve pulse explaining also the origin of EEG and its synchrony [K94]. to a model for the evolution of the genetic code [**?**]. to a model of catalyst action involving a phase transition reducing the value of Planck constant inducing the shortening of the flux tubes connecting the reacting molecules and thus forcing them to the vicinity of each other, and to a model of for protein folding [K7] in which the presence of wormhole magnetic flux tubes connecting bio-molecules becomes almost a definition for what it is to be living. It is interesting to combine these new ideas with the earlier [I61, I63] and more recent [I69] findings of Gariaev. Basically the challenge is to fuse the DNA as TQC model with the model of living systems as a conscious hologram [K22].

11.1.3 The Basic Assumptions Of Model Explaining Findings Of Gariaev

The basic assumptions of the model to be discussed are following.

1. The hierarchy of Planck constants requires a generalization of the notion of 8-D embedding space $H = M^4 \times CP_2$ obtained by gluing together almost copies of H like pages of book along common back. The pages of the book carry matter with various values of Planck constant and the particles at different pages of the book are dark relative to each other in the sense that they cannot appear in the same vertex of Feynman diagram. The particles at different pages of the book can however interact via classical fields and via the exchange of (for instance) photons which suffer a phase transition changing Planck constant as they leak between pages of the book. In principle it is therefore possible to photograph the magnetic flux tubes carrying dark matter, and the proposal is that this is what Gariaev and collaborators have actually achieved [I69].
2. Braid strands realized as wormhole magnetic tubes are identified as correlates for a directed attention. DNA connected by strands to (say) experimental instrument directs its attention to the instrument. One could perhaps say that DNA "sees" the surrounding world. Also ordinary attention for vision and other senses could involve flux tubes connecting DNA to the object of perception. This explains the ability of DNA to generate images of objects of external world [I69] . The hierarchy of Planck constants explains the transformation of laser light to radio waves [I63] as a phase transition increasing Planck constant and thus also wavelength but keeping the energy of photons as such.
3. Wormhole flux tubes carrying super-conducting matter in large $\hbar$ phase are characterized by anomalous em charges characterizing the nucleotides [K4]. and thus define an excellent candidate for the substrate of bio-hologram. A coding of DNA nucleotides to the rotation

of polarization plane results for photons traversing through these flux tubes if a large parity breaking making possible rotation of the polarization plane (Faraday effect) is assumed. This is possible by the large parity breaking of fractally scaled up variant of weak physics [K12] explaining also chiral selection.

4. The model for the nerve pulse [K94] leads to the model of EEG waves in which EEG rhythms induce a complete analog of reference waves whereas nerve pulse induces the analog of information carrying wave [K42]. The model predicts a fractal hierarchy of EEGs (EXGs) and their counterparts associated with long ranged color and electro-weak gauge fields having MEs as classical correlates. EEG rhythms are associated with propagating soliton sequences and nerve pulse corresponds to a propagating perturbation associated with this soliton sequence rather than soliton. The model predicts automatically the synchrony and spatiotemporal coherence of neural firing. EEG photons correspond to a large value of Planck constant implying that their energies are above thermal energy at physiological temperatures so that their effects on living matter are not masked by thermal noise.

   This model generalizes essentially as such to the recent context: the counterparts of nerve pulses propagate along the complex formed by DNA connected to the nuclear or cell membrane or even to another cell nucleus by flux tubes. The prediction is that gene expression can be coherent in the scale of organ and even that of population. This conforms with the notion of super-genome stating that the sequences of DNA strands in different nuclei organize along magnetic flux sheet like text lines at the page of a book. The notion of hyper-genome means that these books from different organisms in turn organize to a pages of a book at higher level of fractal hierarchy and give rise to a gene expression at the level of population or even biosphere.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

11.2 TGD Counterpart For Wave Genetics

The wave genetic model of Gariaev involves the assumption that soliton waves propagating along DNA induce the reading of DNA sequence to a pattern of radiation. DNA is known to rotate the polarization plane but it is unclear how the coding of DNA sequence to a rotation of polarization plane could be achieved.

Second key element is the notion of bio-hologram. It is assumed that fractality is somehow involved. The key questions are following.

1. What is the substrate of the bio-hologram assuming that it is not based on nonlinear action for electromagnetic field (four-wave mechanism)? The substrate should have size larger than wavelength so that chromosomes are too thin to act as substrate.
2. What guarantees coherence or even quantum coherence in macroscopic scales?
3. How reference wave and the wave carrying the information are represented?

11.2.1 The Notion Of Bio-Hologram In TGD Framework

TGD based model is based on the model of living matter inspired by the model of DNA as topological quantum computer [K4]. DNA is connected to other bio-molecules and also to lipid layers of nuclear and cell membrane by wormhole magnetic flux tubes providing a representation of the genetic code. Braids strands defined by the flux tubes make possible topological quantum computation with TQC programs coded by dynamical braidings of the flux tubes induced by the water flow near the vicinity of cell and nuclear membranes inducing the flow of the 2-D liquid crystal defined by the lipids of the membrane. Flux tubes are dynamical, being able to reconnect and in the case of wormhole flux tubes even disappear without breaking conservation of magnetic flux, and they serve as correlates for a directed attention at the molecular and perhaps even at higher levels. Dark matter at the flux tubes has a large value of Planck constant and therefore a slow dissipation rate. Also superconductivity is possible and the predicted exotic nuclear physics allows bosonic chemical equivalents of all biologically important ions. Long range color and electro-weak interactions implying in particular large parity breaking are possible and could explain chirality selection in living matter.

It is easiest to introduce the model through questions and answers.

Q: What is the substrate of the bio-hologram and how coherence is obtained?

A: Magnetic flux tubes with large $\hbar$ define the substrate and make possible macroscopic quantum coherence. Visible photons can suffer a phase transition to large $\hbar$ variants with wavelengths scaled up like $\hbar$. The interpretation would be in terms of bio-photons and their dark variants [I67].

Q: How the Faraday effect results?

A: Flux tubes contain charged particles in super-conducting state so that diamagnetism results. Large parity breaking makes possible different propagation velocities for the two circular polarizations and thus Faraday effect resulting via the splitting of the linearly polarized wave to two circular polarizations fusing back again at the second end of the flux tube. The magnetic field along flux tubes induces Faraday rotation and codes DNA nucleotide to the rotation angle of the polarization plane.

Q: How coding is achieved?

A: Coding is achieved by the different total charges associated with flux tubes implying that the rotation angles for polarization plane depend on nucleotide. This would be made possible by anomalous em charge associated with DNA sheet of wormhole flux tube implying that the rotation of polarization plane is different for each nucleotide [K4].

Q: What is the identification of reference wave and for the wave representing the information?

A: The model for nerve pulse and EEG suggests that reference waves are induced as Josephson radiation from voltage waves propagating along DNA and represent a fractal variant of EEG. The voltages waves generating reference waves correspond to propagating soliton sequences for Sine-Gordon equation describing idealized cylindrical Josephson junction having as an analog series of coupled gravitational penduli. The propagating soliton sequence along DNA with constant phase differences between subsequent penduli would generate the reference wave as Josephson radiation. The analog of nerve pulse would result as one pendulum kicked so that it begins to oscillate instead of rotating and induces an propagating localized oscillation.

Microscopically cylindrical Josephson junction decomposes into junctions defined by the flux tubes and Josephson currents between the ends of the flux tubes generate em radiation as coherent photons. Josephson radiation would therefore give rise to bio-photons and their dark variants with same photon energy but scaled up wavelength. Obviously the transformation of laser photons to radio-wave photons can be understood in terms of this mechanism and the quantization of Planck constant implies quantization of the energies involved.

11.2.2 How To Fuse The Notion Of Bio-Hologram With The Model Of DNAs TQC?

In the most economical picture - inspired by what is known about ordinary computers - intronic sequences would represent the names for TQC programs constructed from basic modules and expressing their outcomes chemically. Calling of the name of TQC would activate the TQC. This would allow an extremely rich combinations of basic modules, explain why the intronic portion of DNA increases during evolution, and why organisms with essentially identical genomes can be at widely differing evolutionary levels (say humans and apes). A further nice feature is that the intronic DNA of a given organism can induce gene expression in an organism for which the genes involved are not identical so that mutations would not be fatal. The prediction is that addresses represented by introns and the portions of promoter regions representing the conjugates of these addresses should be highly conserved.

The reading of the name of TQC to a polarization modulation pattern of incoming light would generate a signal which initiates TQC program in another cell in the case that the reverse polarization to the same linear polarization along the entire length of receiving intronic piece - conjugate of the original - takes place. The resulting overall linear polarization should initiate

TQC leading to the eventual gene expression. Why the condition that linear polarization is same along entire piece of the "name" is not quite clear.

Introns could be connected by flux tubes to a part of DNA initiating gene expression. One would expect that this portion of gene is conjugate of the intronic portion containing the name of submodule. This would make possible RAM type representation of TQC programs if the link to next activated part of genome is represented by this same mechanism: exactly similar mechanism realizes links electromagnetically in web. A nucleus performing TQC infects large number of nuclei to perform the same TQC. Same could occur even at the level of population since very large values of $\hbar$ are possible.

11.3 The Effects Of Laser Light On Living Matter

The effects of laser light on living matter are discussed in the following briefly from TGD point of view.

11.3.1 Phantom DNA Effect

In phantom DNA effect [I62] there is an elastic scattering of the coherent laser radiation from irradiated DNA. When one removes the DNA from the chamber containing it, and irradiates it by laser light, a weak pattern of scattered light is still produced as if there were a kind of phantom DNA there. The pattern can last for months.

For years ago I considered an explanation of the effect based on dropping of part of DNA to larger space-time sheets characterized by larger value of p-adic prime and remaining in the vessel as visible DNA is removed [K131, K22]. A variant of this explanation inspired by the dark matter hierarchy is that the anomalous scattering takes place on dark DNA at wormhole flux tubes remaining in the vessel.

The most science fictive possibility is that the flux tubes connect the vessel boundaries to the removed DNA by wormhole flux tubes which are very long and correspond to a large value of $\hbar$. In this case the scattering would involve a phase transition increasing the value of Planck constant and a travel of photons to the removed DNA and back followed by a phase transition to ordinary photons.

Similar explanation works also in the case of homeopathy and allows to understand why the classic experiments of Benveniste [I57, I58] could not be replicated when experimenters did not know which bottles contained the treated water [K56]. In this case the molecules dissolved in water would lose their magnetic bodies as a consequence of the shaking of the homeopathic remedy and one can say that clusters of water molecules would steal their magnetic coats. This would allow them to mimic the behavior of molecules and their presence would allow the immune system would develop a resistance against real molecules. This of course works only if the cyclotron radiation from the magnetic body is responsible for the biological effects. It is known that em radiation at low frequencies is indeed responsible for the ability of molecules to recognize each other. The generation of cyclotron radiation requires metabolic energy and the magnetic flux tubes connecting the experimenter to the treated bottle of water (correlates for directed attention) could have served as bridges along which metabolic energy could be transferred by using topological light rays (MEs serving as TGD counterparts of Alfven waves). Experimentalists certainly did have strong desire to have successful experiments and this helped to realize the transfer of the metabolic energy.

11.3.2 Effects Of The Polarization Modulated Laser Light On Living Matter

Polarized light with a suitable temporal pattern for the modulation of polarization direction induces biological effects. The effects are not caused to arbitrary target and one can say that the part of target genome involved has an address characterized by a temporal pattern of polarization modulation resulting in the propagation of the scaled variant of nerve pulse along chromosome. DNA is known to induce a rotation of polarization plane of incoming linearly polarized light and Gariaev suggests that the address is due to the propagation of a soliton along DNA inducing the modulation [I61].

TGD based model for the rotation of the polarization plane is based on Faraday effect [K11].

1. Usually diamagnetic dielectric causes the Faraday effect. The effect is due to different propagation velocities of left and right circular polarizations and recombination of polarizations to linear polarization. The rotation of the polarization plane would be caused by a Faraday effect at flux tubes. Superconductivity would imply ideal diamagnetism. Dielectric property is probably not present but large parity breaking due to long range weak interactions [K12] could explain why circular polarizations propagate with different velocities. Strong parity breaking could be caused by the presence of electro-weak gauge fields behaving like massless fields below the cell length scale and would explain also chiral selection. For large values of $\hbar$ the range of these fields would be scaled up accordingly.
2. The travel of the photon along a transversal flux tube starting from DNA nucleotide induces a rotation of the direction of polarization plane. The reverse rotation of polarization plane takes place as the light propagates in the reverse direction. The reverse propagation restoring the original overall linear polarization is expected to induce the biological along the portion of DNA in question. Phase conjugate light might be also involved.
3. The coding of DNA sequences to radiation patterns results since the charge Q associated with the nucleotide end of the wormhole magnetic flux tube affects Faraday rotation and is different for each nucleotide. The value of the charge is given by $Q = -2 + Q_a$, where -2 units come from phosphate and Q_a corresponds to the charge of the quark (u, d) or antiquark $\overline{u}$, $\overline{d}$) at the DNA space-time sheet associated with wormhole magnetic flux tube formed by a pair of space-time sheets connected by wormhole contacts having at its light-like throats quark and antiquark [K4]. Hence the rotation of the polarization plane depends on the nucleotide.

11.3.3 Plr Spectroscopy

Bio-systems could generate holograms in much more concrete sense than the wetty and hot and noisy character of this environment would suggest: even mechanisms generating laser beams could be there. The findings of Peter Gariaev and collaborators described in the article "The spectroscopy of bio-photons in non-local genetic regulation" [I63] led to a concrete model for how bio-photons affect many-sheeted DNA, and in this manner induce a generation of coherent radio waves and ELF waves [K22]. The recent picture brings in the hierarchy of Planck constants and suggests a modification of this model.

The effect

In polarizing laser-radio wave spectroscopy (PLR-spectroscopy) laser light scatters from the target substance. In the experiments of Gariaev *et al* red light ($\lambda = 632.8$ nm, 1.9595 eV) generated by He-Ne laser is used. This energy actually corresponds very precisely to one of the fundamental metabolic energy quanta identified as liberated zero point kinetic energy of proton as it drops from certain space-time sheet to much larger space-time sheet. There are two orthogonal polarizations correlated in intensity in such a way that the total intensity remains constant. After the interaction of one mode with the target substance, the reflected light is returned to the optical resonator, where the re-distribution of the intensity of these modes occurs. One of the laser modes, at a certain mode of generation, is able during the interaction with the target substance to induce polarization modulated radio waves of a wide spectrum correlated with the modulations of the optical modes of the laser radiation. The modulation is assumed to relate to rotational fluctuations of micro-structural components (say, domains of crystals) and of their optical activity. The PLR-spectrum is present also for in-organic materials. For biological targets there is spectral memory effect present, which means that the radio wave radiation continues even when the laser beam is not present anymore.

The frequency interval of the radio emission settles down at the 1 MHz. The PLR-spectrum is depicted in figures 1 and 2 of [I63] for apofillit crystal. The frequency spectrum for the radio waves has a modulated fractal structure suggesting that spectrum is superposition of spectra which consist of harmonics $n_1 f_h - n_2 f_l$ of higher frequency f_h modulated by harmonics of scaled down frequency $f_l = x f_h$. Almost identical copies of a piece of length about

$$\Delta f \sim 100\ Hz$$

appear in a sequence as the pictures 1 and 2 of [I63] for the spectrum of apofillit crystal in 1560-1860 Hz range demonstrate. This suggests the presence of harmonics of basic frequencies perhaps shifted by a constant amount. Cyclotron and spin flip transitions in magnetic field suggest itself.

There is also gross structure consisting of peaks in scale of kHz suggesting harmonics of frequency of order kHz. For wheat seed (picture 3 of [I63]) the strongly expressed frequency ranges are identified as 800-900 Hz (to my personal opinion the band is 300-900 Hz), 1700-1900 Hz, 2400-2600 Hz, 3600-3800 Hz (to my personal opinion a wider frequency range 1700-2200 Hz is strongly expressed). There is also strongly expressed frequency band below 300 Hz. Also the spectrum of high polymerization DNA sample from calf thymus (picture 4 of [I63]) shows a clear peak at 2400-2600 Hz and less pronounced peaks at lower frequencies.

The radio wave radiation from DNA samples is accompanied by specific effects on bio-systems such as ab-normally fast germination and re-vitalization of seeds. Thus it seems that the radio wave radiation is able to restore the genetic control apparatus and the vitality of the seeds.

TGD based explanation of the effect

Dark matter hierarchy suggests the interpretation of radio-wave photons as large $\hbar$ photons with energy equal to that of the original photon. Biophotons and their dark variants could form Bose-Einstein condensates at the wormhole magnetic flux tubes. The flux tubes associated with DNA would transform laser photons to radio wave photons by inducing $\hbar$ increasing phase transition. Large value of $\hbar$ would increase the range of interactions so that they would become possible even in the scale of biosphere. In particular, coherent gene expression in the scale of organism and even population. Genetic code could be represented as radiation patterns with the charges assignable to the end of DNA space-time sheet of flux tube providing the coding.

11.4 The Scattering Of Incoherent UV-IR Light On DNA

The proposed model for the findings about scattering of incoherent UV-IR light from DNA lead to an amazing conclusion that the experiments make directly visible the magnetic flux tubes containing dark matter.

11.4.1 Basic Facts

The figures of the article [I69] give valuable information about what is involved. There are two experimental arrangements.

1. In the first experiment dry/dehydrated DNA is contained in a small seal containing a conical cylinder (4 cm long, .9 cm at its upper end) or 3 ml of DNA water solution 1 mg/ml. The radiation by UV-C lamp lasts for 10 minutes: note that UV-C wavelengths are in the range 280-10 nm.
2. In the second experiment the DNA sample is in open cell and a light source known as Duna-M irradiates red light from 21 LEDS (650 nm) and IR light (920 nm) from 16 LEDs. Also UV-B lamp and Compact electronic CEST26E17 Black lamp are involvedUV-B wavelengths are in the range 315-280 nm. The light sources are turned on and off with intervals of 2-3 seconds. The exposure time is 1 second.

The basic findings are following [I69].

1. The effects occur only if the sample contains DNA.
2. A large number (tens) of closely spaced replica images of nearby objects, in particular the red LED. The replicas for the image of instrument are along strictly horizontal half line (see **Fig.** 1).
3. The replica sequences of the instruments appear periodically suggesting that the energy of incoming photons is gradually accumulated and liberated in a burst. The interference by an external DNA source (touching by finger of DNA cell) changes the direction of the half line which disappears at the next exposure to white light.

4. Single vertical curved band like image of roughly the same height as the entire image and with more or less the same width as the distance between replicas of the instrument parts appears to the left from the instrument image (see **Fig. 11.1**). This image is not replicated in the horizontal direction. The fine structure of the band for one of the reported images (see **Fig. 11.2**) however suggests that also the band like structure consists of replicas of same size as the replicas associated with instruments. The band like structure for second method decomposes to 5 red parallel curves (see **Fig. 11.3**) for which the interpretation as images of 5 red LEDs is proposed based on the observation that these LEDs irradiate directly the DNA cell. The phantom of DNA image remains intact for some time after the irradiation.

If I have understood correctly, the interpretation proposed in [I69] is following.

1. The sequence of the horizontal images of the instrument would result from a motion of single image moving during the exposures: this requires that the motion is fast in the time scale of exposure. The appearance of equally spaced replicas forces to assume that the motion occurs in discrete jumps in horizontal direction.
2. The band like structure is identified as the image of DNA sample. The band is assumed to correspond to a discrete and non-predictable motion of single image.

There are objections against the idea that the motion of single image produces the image. In particular, the discreteness of the motion looks strange. One can also wonder why the motion for the image of the instrument is strictly horizontal whereas the motion of DNA image is not horizontal and is curvilinear. One can also ask whether the an image of DNA sample is actually in question since the position of the band like structure is to left from the cell containing the DNA.

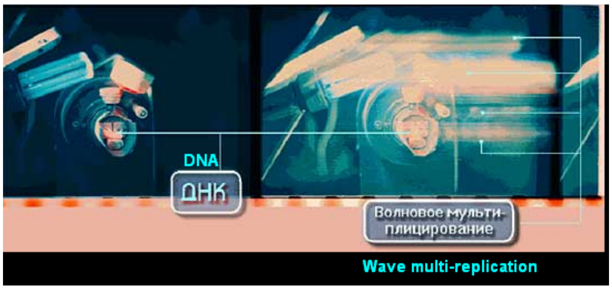

Figure 11.1: The left hand side figure is from [I69] and represents the replica images of the instruments and the image interpreted by experimenters as a replica image of DNA sample (second method).

11.4.2 TGD Based Model For The Replicas

One can consider two models for the replicas. The first model assumes that the images are images of dark magnetic flux tubes. Second model assumes that in the case of instrument images diffraction is involved.

Have wormhole magnetic magnetic flux tubes containing dark matter been photographed?

The most elegant model for the effects found hitherto relies on the assumption that both the horizontal replica sequences and the band like structures having also replica structure correspond to real structures, most naturally (wormhole) magnetic flux tubes. In the case of instrument replicas they would emanate directly from the instruments. In the case of DNA image they would emanate from a position to the left from the cell containing DNA. The presence of DNA should somehow generate the flux tubes.

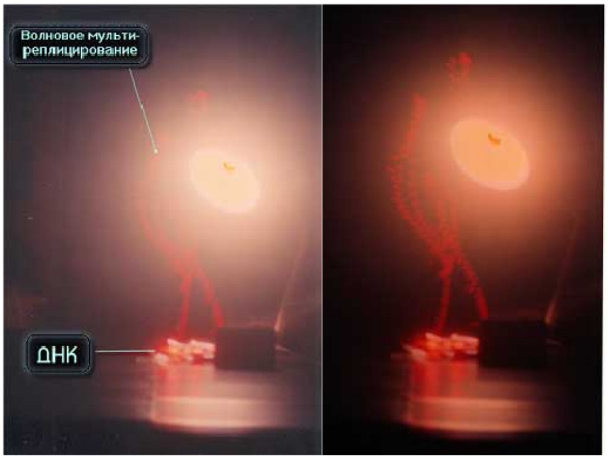

Figure 11.2: The picture shows the discrete replica like structure of the band like image interpreted by experimenters as replica image of DNA sample (first method).

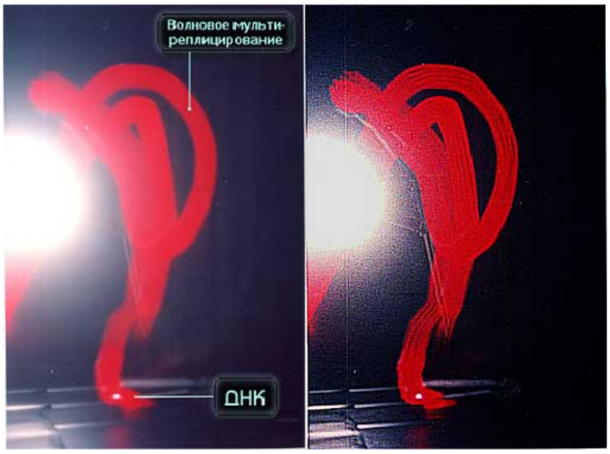

Figure 11.3: The picture reveals the 5-fold fine structure of the band like image interpreted by experimenters as replica image of DNA sample. The 5-fold character probably correspond to five red LEDs above the sample (second method).

1. In the case of horizontal replications of instruments the replicas would be associated with a magnetic flux tube emanating horizontally from the instruments to the right. Replicas would be obtained if a dipole distribution assignable to the surface of object and representable in terms of Fourier transform restricted to a box containing the object and having discrete momentum spectrum is extended to a periodic Fourier transform along the horizontal flux tube. Flux tube would thus represent a series of images of the geometric object and this would make possible to communicate the data through long distances.

2. Also the DNA image could be the image of a curved flux tube assignable to the cell containing the DNA. The band like structure does not however begin from the cell containing DNA being located left from it. A possible explanation is that there topological light ray connecting the cell containing DNA to a similar sized cell at the end of the flux tube irradiating it with photons emitted from the dipole distribution at its surface. The resulting induced dipole distribution representable in terms of a discrete Fourier transform is then continued along the entire curved flux tube and would generate the replicas.

3. The replication of the dipole distribution along the entire length of the flux tube requires macroscopic quantum coherence suggesting a large value of Planck constant. If the coherence is required at least in the length scale L of the flux tube, one obtains ratio $r = \hbar/\hbar_0 \geq L/\lambda \simeq 10^6$ for $L = .5$m and $\lambda = 500$ nm. This value could correspond to the favored value $r = 2^{20}$ and thus to a favored value of Planck constant [K47]. A weaker condition is obtained by replacing L with the size a of the cell giving $r \geq a/\lambda \simeq 2 \times 10^5$ for $a = .1$ m.
4. If the flux tubes correspond to large value of Planck constant, the dark photons emanating from them must transform to ordinary photons since diffractive effects are not involved.
5. The fact that the images of the flux tubes appear periodically suggests that a Bose-Einstein condensate of dark photons is gradually formed at them which bursts out as some critical number of dark photons are present and leaks to the visible sector of the 8-D embedding space becoming ordinary photons. One can visualize the sectors of the generalized 8-D embedding space as pages of a book characterized by different values of Planck constant so that the leakage would occur from page to another one through the back of the book.
6. The effect of touching in the second type experiment involving LEDs can be understood if the touching reverses the direction of the magnetic flux tubes assigned with the instruments. The disappearance of the replicated instrument image 5-8 seconds after the touching could relate to the instability of the right-oriented flux tubes. If the right-directed flux tube is mirror image of the left oriented flux tube, the instability might relate to a parity breaking possible in TGD Universe by the presence of scaled variants of weak interactions. The preferred orientation of the flux tube might be also determined by something in environment, say resources of metabolic energy. If the flux tubes are correlates for attention, one can even imagine that DNA with the mediation of flux tubes directs its attention to something interesting.

There are also some open questions.

1. Why the flux tube assignable to the DNA is curved and why the image of this flux tube does not emanate from the sample?
2. How the presence of DNA induces the generation of the flux tubes? The model for DNA as TQC would suggest that the thin wormhole magnetic flux tubes connecting DNA to the instruments induce the effect, and that the flux tubes explaining the image correspond to higher level structures with larger value of Planck constant and are somehow induced by the presence of DNA. They could also correspond to a larger value of p-adic prime but same value of Planck constant. Perhaps one might say that the magnetic body of DNA makes the instruments in some sense part of its biological body by directing its attention to them.
3. Why the touching chances the orientation of the flux tube?

If this model is on a right track, the findings would mean a direct observation of dark magnetic flux tubes by the em radiation of dark photons transformed to ordinary photons as they leak out from dark sectors of the embedding space to the sector containing the matter visible to us.

The explanation in terms of diffraction does not work

For the sake of completeness also the interpretation of the replication of the images of the instrument and DNA cell in terms of diffraction is discussed although this explanation forces several ad hoc assumptions unlike the previous model.

1. The appearance of the replicas along horizontal half-line $x > 0$ brings strongly in mind a diffraction through a vertical slit defined by a vertical dark flux sheet attached to the instrument and acting as a window. This requires coherence so that ordinary visible light cannot be responsible for the image whereas dark photons with a large enough value of Planck constant makes the quantum coherence possible.
2. The amplitude for a diffraction through slit behaves as $A = sin(x)/x$, $x = \pi \times (a/\lambda) \times sin(\theta)$, where θ is the angle between the normal of the slit and direction of observation. Hence the maxima of the intensity maxima correspond to the central maximum $\sin(\theta) = 0$ given by geometric optics and $\sin(\theta) = (n + 1/2) \times \lambda/a$ so that for small angles one has $\Delta\theta = \lambda/a$ and the distance between replicas is $x = d\Delta\theta = d\lambda/a$.

3. The distance between the replicas in the image requires a wavelength longer than used in experiments. Thus dark photons with a scaled up wavelength $\lambda = r\lambda_0$, $r = \hbar/\hbar_0$, transforming by Planck constant changing phase transition to ordinary photons in camera could be in question. The value of the Planck constant can be deduced by using the geometric data, the values of wavelength, and the distance between the replicas of instrument images assuming that diffraction effectively takes place through a vertical slit with width of order size of typical replicated instrument, say seal. From $\theta \leq D/d$, where D is the size of camera aperture, and from the number n of horizontal replicas $n < 100$ one obtains the estimate $d\lambda/a \sim D/nd$. This gives $\lambda/a \sim D/nd^2$. For $D = .01$ m, $d = .5$ m, one would have $\lambda/a \sim 4 \times 10^{-4}$. For $\lambda = 4 \times 10^{-7}$ m this would give $a \sim 10^{-3}$ m. The appearance of details in the replicated image suggest that a is of the same order than the instrument size so that one has $a \geq x > 1$ cm giving $\hbar/\hbar_0 \geq 10x$. The value of λ seems to be too small to allow coherence in the required length scale.

4. The serious problem of this interpretation is that the diffraction pattern for a diffraction through slit corresponds to maxima at an entire transversal line rather than half-line. It is as if the effective vertical flux sheet attached to the left hand side of the object would contain a distribution of horizontal dipoles generating radiation interfering to zero at the left half of the half-space. This distribution should be determined by the radiation coming from the object so that a kind of induced emission process would be in question. One can also imagine is that the dark space-time sheet along which photons arrive is half-space with horizontal coordinate $x \geq 0$. What is intriguing that in p-adic physics for which the values of variables finite in real sense are always positive as real numbers so that half-lines, quadrants, octants, ... are very natural objects. One must admit that this assumption looks ad hoc.

5. There is also a second problem. The evidence for the replication of same basic unit with the size of the DNA containing cell suggests that a replication of the image of cell containing the DNA along a curved band is in question with essentially the same distance between replicas as in the previous case. It is impossible to have a curved slit producing this kind of diffraction pattern. One could consider also the possibility that the band corresponds to a real structure, may be magnetic flux tube, and that Planck constant is now larger than in the case of instrument images so that only the central image of the diffraction pattern is visible in the camera. This however forces to ask whether also the replicas of instruments correspond to magnetic flux tubes so that one would end up with the first model.

11.5 Water Memory, Phantom DNA Effect, And Development Of TQC Hardware

This section describes speculative picture in which a connection between homeopathy and water memory [K56] with phantom DNA effect is proposed and on basis of this connection a vision about how the TQC hardware represented by the genome is actively developed by subjecting it to evolutionary pressures represented by a virtual world representation of the physical environment.

11.5.1 A Possible Realization Of Water Memory

The Benveniste's discovery of water memory [I57, I58] initiated quite dramatic sequence of events. The original experiment involved the homeopathic treatment of water by human antigene. This meant dilution of the water solution of antigene so that the concentration of antigene became extremely low. In accordance with homeopathic teachings human basophils reacted on this solution.

The discovery was published in Nature and due to the strong polemic raised by the publication of the article, it was decided to test the experimental arrangement. The experimental results were reproduced under the original conditions. Then it was discovered that experimenters knew which bottles contained the treated water. The modified experiment in which experimenters did not possess this information failed to reproduce the results and the conclusion was regarded as obvious and Benveniste lost his laboratory among other things. Obviously any model of the effect taking it as a real effect rather than an astonishingly simplistic attempt of top scientists to cheat should explain also this finding.

The model based on the notion of field body and general mechanism of long term memory allows to explain both the memory of water and why it failed under the conditions described.

1. Also molecules have magnetic field bodies acting as intentional agents controlling the molecules. Nano-motors do not only look co-operating living creatures but are such. The field body of the molecule contains besides the static magnetic and electric parts also dynamical parts characterized by frequencies and temporal patterns of fields. To be precise, one must speak both field and relative field bodies characterizing interactions of molecules. Right brain sings-left brain talks metaphor might generalize to all scales meaning that representations based on both frequencies and temporal pulse with single frequency could be utilized.
2. The effects of complex bio-molecule to other bio-molecules (say antigene on basofil) in water could be characterized to some degree by the temporal patterns associated with the dynamical part of its field body and bio-molecules could recognize each other via these patterns. This would mean that symbolic level in interactions would be present already in the interactions of bio-molecules. Cyclotron frequencies are most natural candidates for the frequency signatures and the fact that frequencies in 10 kHz range are involved supports this view.
3. The original idea was that water molecule clusters are able to mimic the bio-molecules themselves -say their vibrational and rotational spectra could coincide with those of molecules in reasonable approximation. A more natural idea is that they can mimic their field bodies. Homeopathy could rely on extremely simple effect: water molecule clusters would steal the magnetic bodies of the molecules used to manufacture the homeopathic remedy. The shaking of the bottle containing the solution would enhance the probability for bio-molecule to lose its magnetic body in this manner. For instance, water could produce fake copies of say antigenes recognized by basofils and reacting accordingly if the reaction is based on interaction with the magnetic body of the antigene.
4. The basic objection against this picture is that it does not explain why the repeated dilution works. Rather, it seems that dilution of molecules reduces also the density of mimicking pseudo-molecules. Even more, the potency of the homeopathic remedy is claimed to increase as the dilution factor increases. Also alcohol is used instead of water so that also alcohol must allow homeopathic mechanism. (I am grateful for Ulla Matfolk for questions which made me to realize these objections).
    (a) The only way out seems to be that the magnetic bodies or water molecule clusters having these magnetic bodies can replicate. The shaking of the remedy could provide the needed metabolic energy so that the population of magnetic bodies grows to a limiting density determined by the metabolic energy feed. In principle it would be possible to infect unlimited amount of water by these pseudo-molecules. When in bottle the population would be in dormant state but in the body of the patient it would wake up and form a population of molecular actors and stimulate the immune system to develop immune response to the real molecule.
    (b) The potency of the homeopathic remedy is claimed to increase with the increased dilution factor. This would suggest that the continued dilution and shaking also increases the density of pseudo molecules, perhaps by feeding to the system metabolic energy or by some other mechanism.
    (c) Also magnetic bodies must replicate in cell replication and their role as intentional agents controlling bio-matter requires that this replication serves as a template for biochemical replication. On can indeed interpret the images about cell replication in terms of replication of dipole type magnetic field. This process is very simple and could have preceded biological replication. The question is therefore whether water is actually a living system in presence of a proper metabolic energy feed. Also the water's ability near critical point for freezing to form nice patterns correlating with sound stimuli might be due to the presence of the molecular actors.
    (d) This picture fits nicely with the vision that evolution of water in this kind of life form might have happened separately and that pre-biotic chemical life forms have formed symbiosis with living water [**?**]. In the model of DNA as topological quantum computer [K4] the asymptotic self organization patterns of water flow in the vicinity of lipid layers indeed define quantum computer programs by inducing the braiding of the magnetic flux

tubes connecting DNA nucleotides to lipids so that this symbiosis would have brought in new kind of information processing tool.

5. The magnetic body of the molecule could mimic the vibrational and rotational spectra using harmonics of cyclotron frequencies. Cyclotron transitions could produce dark photons, whose ordinary counterparts resulting in de-coherence would have large energies due to the large value of $\hbar$ and could thus induce vibrational and rotational transitions. This would provide a mechanism by which molecular magnetic body could control the molecule. Note that also the antigenes possibly dropped to the larger space-time sheets could produce the effect on basofils.

6. There is a considerable experimental support for the Benveniste's discovery that bio-molecules in water environment are represented by frequency patterns, and several laboratories are replicating the experiments of Benveniste as I learned from the lecture of Yolene Thomas in the 7: th European SSE Meeting held in Röros [J11]. The scale of the frequencies involved is around 10 kHz and as such does not correspond to any natural molecular frequencies. Cyclotron frequencies associated with electrons or dark ions accompanying these macromolecules would be a natural identification if one accepts the notion of molecular magnetic body. For ions the magnetic fields involved would have a magnitude of order.03 Tesla if 10 kHz corresponds to scaled up alpha band. Also Josephson frequencies would be involved if one believes that EEG has fractally scaled up variants in molecular length scales.

Consider now the argument explaining the failure to replicate the experiments of Benveniste.

1. The magnetic bodies of water molecules need metabolic energy for communications with their "biological body" using the fractally scaled analog of EEG. There is no obvious source for this energy in water. The model for protein folding and DNA as topological quantum computer assumes that magnetic flux tubes connecting subject person and target of directed attention serve as correlates for directed attention at the molecular level [K4, K7]. This should be true also in macroscopic scales so that the experimentalist and the bottle containing the treated water should be connected by magnetic flux tubes. If experimenter has directed his attention to the bottle of water, the resulting magnetic flux tubes could allow a transfer of metabolic energy as a radiation along massless extremals parallel to the flux tubes and defining TGD counterparts of Alfven waves. Experimenter's strong motivation to replicate experiments would help to realize the transfer of the metabolic energy. Experimenters not knowing, which bottles were treated did not have these flux tube bridges to the bottles, and were not able to provide the needed metabolic energy, and the magnetic bodies of antigenes failed to generate the cyclotron radiation making them visible to the basofil.

2. If this interpretation is correct, then Benveniste's experiment would demonstrate besides water memory also psychokinesis and direct action of desires of experimenters on physics at microscopic level. Furthermore, the mere fact that we know something about some object or direct attention to it would mean a concrete interaction of our magnetic body with the object. The so called phenomenon of psi track [J21] provides additional support for this conclusion.

11.5.2 Could Virtual DNAs Allow A Controlled Development Of The Genome?

The fundamental question in the evolution biology is the question about the interaction between genome (G), phenotype (P), and environment (E).

1. The standard dogma is that the information transfer from G to P is unidirectional and that environment acts on G by inducing random mutations of G, from which E selects the lucky survivors as those with the best ability to reproduce. Lamarckism [I15, I54, I59] represents a deviation from standard dogma by assuming direct information transfer from E to G.

2. Genetic expression is controlled by environment, at least by silencing [I15], which is like selecting only few books to be read from a big library. Cell differentiation represents basic example of selective gene expression. DNA methylation and transposition are accepted to reflect information transfer from E to G, perhaps via P. These modifications are believed to be short lasting and not transferred to the offspring since it is difficult to imagine a mechanism

transferring the mutations to the germ cells. There is however also evidence that epigenetic information transfer takes place [I78]: this transfer would be selective expression of genes of germ cells rather than that of modified genes.

3. There are findings challenging the dogmas of static genome and random mutations. The cells of the immune system remodel their genes coding for antibodies capable of recognizing large variety of antigens. There is quite recent finding [I65] revealing major genetic differences between blood and tissue cells. There are also mutations due to jumping genes - mobile elements of DNA known as LINE-1 elements usually regarded as junk DNA whose portion from genome increases as one climbs up along the evolutionary ladder. In mice jumping genes are limited to brain and germ cells: this is easy to understand since in organs like heart and lungs this kind of mutations would be fatal. Second recent discovery is that there is a high diversity of human brain cells believed to be due to the jumping genes [I52]. That brain cells would be producing with a high rate junk DNA is not an idea which would make me shout "Eureka!"

4. The question however remains whether the $G \to P-E$ actually could complete to a closed loop $G \to P-E-G$ so that genome could directly respond to the changing physical environment and could transfer the successful response to the next generation [I54].

Could genome be developed like computer hardware?

In TGD framework the sequence $G \to P - E$ is replaced with a closed loop $G - P - M - E$ to which E is attached at P by bidirectional arrow (organisms do also modify their environment actively). Magnetic body thus controls genome and receives information from cell membrane (P). The hierarchy of genomes (super-genome, hyper-genome, ...) corresponding to the different levels of dark matter hierarchy allows this loop to be realized in different scales rather only at the level of single cell.

The question is whether the magnetic body of organism or higher level magnetic bodies could modify genomes, super-genomes, and hyper-genomes directly, perhaps by generating mutations of the genome in a short time scale; by monitoring how genetically modified organism survives in the environment; and -if the outcome of the experiment is successful - replacing the corresponding portion of DNA with the modified DNA both in ordinary germ cells. One can even ask whether the abstract model of the external environment provided by the internal chemical milieu might be mimicked by water magnetic bodies of water molecule clusters and provide a virtual world testing ground for a search of favorable mutations.

In DNA as a TQC vision essentially the development of a new computer hardware would be in question, and should take place in a controlled manner and involve an experimentation before going to the market rather than by random modifications taking place in computer CPUs. Second basic aspect of DNA as TQC paradigm is that water and bio-molecules live in symbiosis in the sense that self organization patterns of the cellular water flow define the TQC programs. The following first guess for how the development of computer hardware might be achieved is just a first guess but might have something to do with reality.

1. What would be needed is a mechanism generating rapidly modifications of DNA. The mutations should be carried out using a kind of virtual DNA mimicking all the essential aspects of the symbolic dynamics associated with DNA. The magnetic bodies of DNA consisting of flux tubes connecting the nucleotides of DNA strands to cell membrane satisfy these conditions since A, T, G, C is coded to exotic light quarks u, d and anti-quarks $\overline{u}$, $\overline{d}$ at the ends of flux tubes [K4]. DNA nucleotides could be replaced with clusters of water molecules but also other options can be imagined. Note that it does not matter when one speaks of mimicry of RNA or DNA molecules.

2. If the proposed model of the phantom DNA and homeopathy has something to do with reality, this kind of virtual DNA exists and is generated in phantom DNA effect as magnetic bodies of DNA, including of course the magnetic flux tubes connecting the nucleotides to the cell membrane or conjugate strand of DNA.

3. The crucial additional assumption would be that also the reversal of phantom DNA effect is possible and corresponds to the analog of DNA replication in which nucleotides attach to the

virtual conjugate nucleotides of the virtual DNA strand or RNA strand in turn transformed to DNA strand be reverse transcription. The hypothesis would have rather strong implications for the genetic engineering since homeopathic remedies of genetically engineered DNA sequences could be transferred to cell nuclei just by drinking them.

4. Phantom DNA sequences could form populations and - as far as their properties as a hardware of topological quantum computer are involved - evolve under selection pressures of the virtual world defined by the nuclear, cellular and extracellular water. A competition of components of TQC hardware developed by the higher level magnetic body to realize optimally TQC programs needed for survival would be in question. The simplest mutation of phantom DNA would replace the quark pairs at the ends the (wormhole-) magnetic flux tube with a new one and could occur in very short time scale. Also basic editing operations like cutting and pasting would be possible for these competing phantom DNA sequences. The winners in the competition would be transformed to actual DNA sequences by utilizing the reverse phantom DNA (or RNA -) effect and be inserted to genome. The genetic machinery performing cutting, gluing, and pasting of real DNA in a controlled manner exists. What is needed is the machinery monitoring who is the winner and making the decision to initiate the modification of the real DNA.

5. The transfer of the mutations to germ cells could be achieved by allowing the population of the virtual DNA sequences to infect the water inside germ cells. The genetic program inducing the modification of DNA by using the winner of the TQC hardware competition should run automatically.

6. One open question is whether the nuclear, cellular or perhaps also extracellular water should represent the physical environment and - if answer is affirmative - how it achieves this. As a matter fact, considerable fraction of water inside cells is in gel phase and it might be that the intercellular water, which naturally defines a symbolic representation of environment, is where the virtual evolution takes place. Internal chemical milieu certainly reflects in an abstract manner the physical environment and the ability of the water molecule clusters to mimic biomolecules would make the representation of the chemical environment possible. Also sudden changes of external milieu would be rapidly coded to the changes in internal milieu which might help to achieve genetic re-organization. The craziest dream is water based simulation of both genes, proteins, and molecules representing external world running at dark space-time sheets.

Dark nuclear strings as analogs of DNA-, RNA- and amino-acid sequences and baryonic realization of genetic code?

The minimal option is that virtual DNA sequences have flux tube connections to the lipids of the cell membrane so that their quality as hardware of TQC can be tested but that there is no virtual variant of transcription and translation machinery. One can however ask whether also virtual amino-acids could be present and whether this could provide deeper insights to the genetic code.

1. Water molecule clusters are not the only candidates for the representatives of linear molecules. An alternative candidate for the virtual variants of linear bio-molecules are dark nuclei consisting of strings of scaled up dark variants of neutral baryons bound together by color bonds having the size scale of atom, which I have introduced in the model of cold fusion and plasma electrolysis both taking place in water environment [L1], [L1]. Colored flux tubes defining braidings would generalize this picture by allowing transversal color magnetic flux tube connections between these strings.

2. This seems to work! The states of dark nucleons formed from three quarks can be naturally grouped to multiplets in one-one correspondence with 64 DNAs, 64 RNAS, and 20 amino-acids and there is natural mapping of DNA and RNA type states to amino-acid type states such that the numbers of DNAs/RNAs mapped to given amino-acid are same as for the vertebrate genetic code.

The basic idea is simple. Since baryons consist of 3 quarks just as DNA codons consist of three nucleotides, one might ask whether codons could correspond to baryons obtained as open strings with quarks connected by two color flux tubes. This representation would be based on entanglement rather than letter sequences. The question is therefore whether the dark baryons constructed as string of 3 quarks using color flux tubes could realize 64 codons and whether 20 amino-acids could be identified as equivalence classes of some equivalence relation between 64 fundamental codons in a natural manner.

The following model indeed reproduces the genetic code directly from a model of dark neutral baryons as strings of 3 quarks connected by color flux tubes.

1. Dark nuclear baryons are considered as a fundamental realization of DNA codons and constructed as open strings of 3 dark quarks connected by two colored flux tubes, which can be also charged. The baryonic strings cannot combine to form a strictly linear structure since strict rotational invariance would not allow the quark strings to have angular momentum with respect to the quantization axis defined by the nuclear string. The independent rotation of quark strings and breaking of rotational symmetry from SO(3) to SO(2) induced by the direction of the nuclear string is essential for the model.
   (a) Baryonic strings could form a helical nuclear string (stability might require this) locally parallel to DNA, RNA, or amino-acid) helix with rotations acting either along the axis of the DNA or along the local axis of DNA along helix. The rotation of a flux tube portion around an axis parallel to the local axis along DNA helix requires that magnetic flux tube has a kink in this portion. An interesting question is whether this kink has correlate at the level of DNA too. Notice that color bonds appear in two scales corresponding to these two strings. The model of DNA as topological quantum computer [K4] allows a modification in which dark nuclear string of this kind is parallel to DNA and each codon has a flux tube connection to the lipid of cell membrane or possibly to some other bio-molecule.
   (b) The analogs of DNA -, RNA -, and of amino-acid sequences could also correspond to sequences of dark baryons in which baryons would be 3-quark strings in the plane transversal to the dark nuclear string and expected to rotate by stringy boundary conditions. In this case all dark baryons would be free to rotate. Thus one would have nuclear string consisting of short baryonic strings not connected along their ends.
2. The new element as compared to the standard quark model is that between both dark quarks and dark baryons can be charged carrying charge $0, \pm 1$. This is assumed also in nuclear string model and there is empirical support for the existence of exotic nuclei containing charged color bonds between nuclei.
3. The net charge of the dark baryons in question is assumed to vanish to minimize Coulomb repulsion:

$$\sum_q Q_{em}(q) = -\sum_{flux\ tubes} Q_{em}(flux\ tube) \ . \tag{11.5.1}$$

   This kind of selection is natural taking into account the breaking of isospin symmetry. In the recent case the breaking cannot however be as large as for ordinary baryons (implying large mass difference between Δ and nucleon states).
4. One can classify the states of the open 3-quark string by the total charges and spins associated with 3 quarks and to the two color bonds. Total em charges of quarks vary in the range $Z_B \in \{2, 1, 0, -1\}$ and total color bond charges in the range $Z_b \in \{2, 1, 0, -1, -2\}$. Only neutral states are allowed. Total quark spin projection varies in the range $J_B = 3/2, 1/2, -1/2, -3/2$ and the total flux tube spin projection in the range $J_b = 2, 1, -1, -2$. If one takes for a given total charge assumed to be vanishing one representative from each class (J_B, J_b), one obtains $4 \times 5 = 20$ states which is the number of amino-acids. Thus genetic code might be realized at the level of baryons by mapping the neutral states with a given spin projection to single representative state with the same spin projection. The problem is to find whether one can identify the analogs of DNA, RNA and amino-acids as baryon like states.

1. States in the quark degrees of freedom

One must construct many-particle states both in quark and flux tube degrees of freedom. These states can be constructed as representations of rotation group SU(2) and strong isospin group SU(2) by using the standard tensor product rule $j_1 \times j_2 = j_1 + j_2 \oplus j_1 + j_2 - 1 \oplus ... \oplus |j_1 - j_2|$ for the representation of SU(2) and Fermi statistics and Bose-Einstein statistics are used to deduce correlations between total spin and total isospin (for instance, $J = I$ rule holds true in quark degrees of freedom). Charge neutrality is assumed and the breaking of rotational symmetry in the direction of nuclear string is assumed.

Consider first the states of dark baryons in quark degrees of freedom.

1. The tensor product $2 \otimes 2 \otimes 2$ is involved in both cases. Without any additional constraints this tensor product decomposes as $(3 \oplus 1) \otimes 2 = 4 \oplus 2 \oplus 2$: 8 states altogether. This is what one should have for DNA and RNA candidates. If one has only identical quarks uuu or ddd, Pauli exclusion rule allows only the 4-D spin 3/2 representation corresponding to completely symmetric representation -just as in standard quark model. These 4 states correspond to a candidate for amino-acids. Thus RNA and DNA should correspond to states of type uud and ddu and amino-acids to states of type uuu or ddd. What this means physically will be considered later.
2. Due to spin-statistics constraint only the representations with $(J, I) = (3/2, 3/2)$ (Δ resonance) and the second $(J, I) = (1/2, 1/2)$ (proton and neutron) are realized as free baryons. Now of course a dark -possibly p-adically scaled up - variant of QCD is considered so that more general baryonic states are possible. By the way, the spin statistics problem which forced to introduce quark color strongly suggests that the construction of the codons as sequences of 3 nucleons - which one might also consider - is not a good idea.
3. Second nucleon like spin doublet - call it 2_{odd} - has wrong parity in the sense that it would require $L = 1$ ground state for two identical quarks (uu or dd pair). Dropping 2_{odd} and using only $4 \oplus 2$ for the rotation group would give degeneracies $(1, 2, 2, 1)$ and 6 states only. All the representations in $4 \oplus 2 \oplus 2_{odd}$ are needed to get 8 states with a given quark charge and one should transform the wrong parity doublet to positive parity doublet somehow. Since open string geometry breaks rotational symmetry to a subgroup SO(2) of rotations acting along the direction of the string and since the boundary conditions on baryonic strings force their ends to rotate with light velocity, the attractive possibility is to add a baryonic stringy excitation with angular momentum projection $L_z = -1$ to the wrong parity doublet so that the parity comes out correctly. $L_z = -1$ orbital angular momentum for the relative motion of uu or dd quark pair in the open 3-quark string would be in question. The degeneracies for spin projection value $J_z = 3/2, ..., -3/2$ are $(1, 2, 3, 2)$. Genetic code means spin projection mapping the states in $4 \oplus 2 \oplus 2_{odd}$ to 4.

2. States in the flux tube degrees of freedom

Consider next the states in flux tube degrees of freedom.

1. The situation is analogous to a construction of mesons from quarks and antiquarks and one obtains the analogs of π meson (pion) with spin 0 and ρ meson with spin 1 since spin statistics forces $J = I$ condition also now. States of a given charge for a flux tube correspond to the tensor product $2 \otimes 2 = 3 \oplus 1$ for the rotation group.
2. Without any further constraints the tensor product $3 \otimes 3 = 5 \oplus 3 \oplus 1$ for the flux tubes states gives 8+1 states. By dropping the scalar state this gives 8 states required by DNA and RNA analogs. The degeneracies of the states for DNA/RNA type realization with a given spin projection for $5 \oplus 3$ are $(1, 2, 2, 2, 1)$. $8\times$ 8 states result altogether for both uud and udd for which color bonds have different charges. Also for ddd state with quark charge -1 one obtains $5 \oplus 3$ states giving 40 states altogether.
3. If the charges of the color bonds are identical as the are for uuu type states serving as candidates for the counterparts of amino-acids bosonic statistics allows only 5 states ($J = 2$ state). Hence 20 counterparts of amino-acids are obtained for uuu. Genetic code means the projection of the states of $5 \oplus 3$ to those of 5 with the same spin projection and same total charge.

3. Analogs of DNA, RNA, amino-acids, and of translation and transcription mechanisms

Consider next the identification of analogs of DNA, RNA and amino-acids and the baryonic realization of the genetic code, translation and transcription.

1. The analogs of DNA and RNA can be identified dark baryons with quark content uud, ddu with color bonds having different charges. There are 3 color bond pairs corresponding to charge pairs $(q_1, q_2) = (-1, 0), (-1, 1), (0, 1)$ (the order of charges does not matter). The condition that the total charge of dark baryon vanishes allows for uud only the bond pair $(-1, 0)$ and for udd only the pair $(-1, 1)$. These thus only single neutral dark baryon of type *uud resp. udd*: these would be the analogous of DNA and RNA codons. Amino-acids would correspond to uuu states with identical color bonds with charges $(-1, -1)$, $(0, 0)$, or $(1, 1)$. uuu with color bond charges (-1, -1) is the only neutral state. Hence only the analogs of DNA, RNA, and amino-acids are obtained, which is rather remarkable result.
2. The basic transcription and translation machinery could be realized as processes in which the analog of DNA can replicate, and can be transcribed to the analog of mRNA in turn translated to the analogs of amino-acids. In terms of flux tube connections the realization of genetic code, transcription, and translation, would mean that only dark baryons with same total quark spin and same total color bond spin can be connected by flux tubes. Charges are of course identical since they vanish.
3. Genetic code maps of $(4\oplus2\oplus2)\otimes(5\oplus3)$ to the states of 4×5. The most natural map takes the states with a given spin to a state with the same spin so that the code is unique. This would give the degeneracies $D(k)$ as products of numbers $D_B \in \{1, 2, 3, 2\}$ and $D_b \in \{1, 2, 2, 2, 1\}$: $D = D_B \times D_b$. Only the observed degeneracies $D = 1, 2, 3, 4, 6$ are predicted. The numbers $N(k)$ of amino-acids coded by D codons would be

$$[N(1), N(2), N(3), N(4), N(6)] = [2, 7, 2, 6, 3] \ .$$

The correct numbers for vertebrate nuclear code are $(N(1), N(2), N(3), N(4), N(6)) = (2, 9, 1, 5, 3)$. Some kind of symmetry breaking must take place and should relate to the emergence of stopping codons. If one codon in second 3-plet becomes stopping codon, the 3-plet becomes doublet. If 2 codons in 4-plet become stopping codons it also becomes doublet and one obtains the correct result $(2, 9, 1, 5, 3)$!
4. Stopping codons would most naturally correspond to the codons, which involve the $L_z = -1$ relative rotational excitation of uu or dd type quark pair. For the 3-plet the two candidates for the stopping codon state are $|1/2, -1/2\rangle \otimes \{|2, k\rangle\}$, $k = 2, -2$. The total spins are $J_z = 3/2$ and $J_z = -7/2$. The three candidates for the 4-plet from which two states are thrown out are $|1/2, -3/2\rangle \otimes \{|2, k\rangle, |1, k\rangle\}$, $k = 1, 0, -1$. The total spins are now $J_z = -1/2, -3/2, -5/2$. One guess is that the states with smallest value of J_z are dropped which would mean that $J_z = -7/2$ states in 3-plet and $J_z = -5/2$ states 4-plet become stopping codons.
5. One can ask why just vertebrate code? Why not vertebrate mitochondrial code, which has unbroken $A - G$ and $T - C$ symmetries with respect to the third nucleotide. And is it possible to understand the rarely occurring variants of the genetic code in this framework? One explanation is that the baryonic realization is the fundamental one and biochemical realization has gradually evolved from non-faithful realization to a faithful one as kind of emulation of dark nuclear physics. Also the role of tRNA in the realization of the code is crucial and could explain the fact that the code can be context sensitive for some codons.

4. Understanding the symmetries of the code

Quantum entanglement between quarks and color flux tubes would be essential for the baryonic realization of the genetic code whereas chemical realization could be said to be classical. Quantal aspect means that one cannot decompose to codon to letters anymore. This raises questions concerning the symmetries of the code.

1. What is the counterpart for the conjugation $ZYZ \rightarrow X_cY_cZ_c$ for the codons?
2. The conjugation of the second nucleotide Y having chemical interpretation in terms of hydrophobia-hydrophily dichotomy in biology. In DNA as TQC model it corresponds to matter-antimatter conjugation for quarks associated with flux tubes connecting DNA nucleotides to the lipids of the cell membrane. What is the interpretation in now?

3. The A-G, T-C symmetries with respect to the third nucleotide Z allow an interpretation as weak isospin symmetry in DNA as TQC model. Can one identify counterpart of this symmetry when the decomposition into individual nucleotides does not make sense?

Natural candidates for the building blocks of the analogs of these symmetries are the change of the sign of the spin direction for quarks and for flux tubes.

1. For quarks the spin projections are always non-vanishing so that the map has no fixed points. For flux tube spin the states of spin $S_z = 0$ are fixed points. The change of the sign of quark spin projection must therefore be present for both $XYZ \rightarrow X_cY_cZ_c$ and $Y \rightarrow Y_c$ but also something else might be needed. Note that without the symmetry breaking $(1,3,3,1) \rightarrow (1,2,3,2)$ the code table would be symmetric in the permutation of 2 first and 2 last columns of the code table induced by both full conjugation and conjugation of Y.
2. The analogs of the approximate $A-G$ and $T-C$ symmetries cannot involve the change of spin direction in neither quark nor flux tube sector. These symmetries act inside the A-G and T-C sub-2-columns of the 4-columns defining the rows of the code table. Hence this symmetry must permute the states of same spin inside 5 and 3 for flux tubes and 4 and 2 for quarks but leave 2_{odd} invariant. This guarantees that for the two non-degenerate codons coding for only single amino-acid and one of the codons inside triplet the action is trivial. Hence the baryonic analog of the approximate $A-G$ and $T-C$ symmetry would be exact symmetry and be due to the basic definition of the genetic code as a mapping states of same flux tube spin and quark spin to single representative state. The existence of full 4-columns coding for the same amino-acid would be due to the fact that states with same quark spin inside $(2,3,2)$ code for the same amino-acid.
3. A detailed comparison of the code table with the code table in spin representation should allow to fix their correspondence uniquely apart from permutations of n-plets and thus also the representation of the conjugations. What is clear that Y conjugation must involve the change of quark spin direction whereas Z conjugation which maps typically 2-plets to each other must involve the permutation of states with same J_z for the flux tubes. It is not quite clear what X conjugation correspond to.

5. Some comments about the physics behind the code

Consider next some particle physicist's objections against this picture.

1. The realization of the code requires the dark scaled variants of spin 3/2 baryons known as Δ resonance and the analogs (and only the analogs) of spin 1 mesons known as ρ mesons. The lifetime of these states is very short in ordinary hadron physics. Now one has a scaled up variant of hadron physics: possibly in both dark and p-adic senses with latter allowing arbitrarily small overall mass scales. Hence the lifetimes of states can be scaled up.
2. Both the absolute and relative mass differences between Δ and N *resp.* ρ and π are large in ordinary hadron physics and this makes the decays of Δ and ρ possible kinematically. This is due to color magnetic spin-spin splitting proportional to the color coupling strength $\alpha_s \sim .1$, which is large. In the recent case α_s could be considerably smaller - say of the same order of magnitude as fine structure constant 1/137 - so that the mass splittings could be so small as to make decays impossible.
3. Dark hadrons could have lower mass scale than the ordinary ones if scaled up variants of quarks in p-adic sense are in question. Note that the model for cold fusion that inspired the idea about genetic code requires that dark nuclear strings have the same mass scale as ordinary baryons. In any case, the most general option inspired by the vision about hierarchy of conscious entities extended to a hierarchy of life forms is that several dark and p-adic scaled up variants of baryons realizing genetic code are possible.
4. The heaviest objection relates to the addition of $L_z = -1$ excitation to $S_z = |1/2, \pm 1/2\rangle_{odd}$ states which transforms the degeneracies of the quark spin states from $(1,3,3,1)$ to $(1,2,3,2)$. The only reasonable answer is that the breaking of the full rotation symmetry reduces $SO(3)$ to $SO(2)$. Also the fact that the states of massless particles are labeled by the representation of $SO(2)$ might be of some relevance. The deeper level explanation in TGD framework might

be as follows. The generalized embedding space is constructed by gluing almost copies of the 8-D embedding space with different Planck constants together along a 4-D subspace like pages of book along a common back. The construction involves symmetry breaking in both rotational and color degrees of freedom to Cartan sub-group and the interpretation is as a geometric representation for the selection of the quantization axis. Quantum TGD is indeed meant to be a geometrization of the entire quantum physics as a physics of the classical spinor fields in the "world of classical worlds" so that also the choice of measurement axis must have a geometric description.

The conclusion is that genetic code can be understand as a map of stringy baryonic states induced by the projection of all states with same spin projection to a representative state with the same spin projection. Genetic code would be realized at the level of dark nuclear physics and biochemical representation would be only one particular higher level representation of the code. A hierarchy of dark baryon realizations corresponding to p-adic and dark matter hierarchies can be considered. Translation and transcription machinery would be realized by flux tubes connecting only states with same quark spin and flux tube spin. Charge neutrality is essential for having only the analogs of DNA, RNA and amino-acids and would guarantee the em stability of the states.

Crying and screaming cells and magnetic bodies expressing their emotions

By using nanotechnological methods James Gimzewski [J1], his student Andrew Pelling and collaborators discovered that the cell walls of bacterium Saccharomyces cerevisiae perform periodic motion with amplitude about 3 nm in the frequency range.8-1.6 kHz (one octave) [I66]. Or more concretely, bacteria produce sounds audible to humans with average frequency of 1 kHz in a range of one octave. The frequency has strong temperature dependence, which suggests a metabolic mechanism. From the temperature dependence one deduces the activation energy to be 58 kJ/mol, which is consistent with the cell's metabolism involving molecular motors such as kinesin, dynein, and myosin. The magnitude of the forces observed (10 nN) suggests concerted nanomechanical activity is operative in the cell.

From less formal popular articles [I55] one can learn that it is difficult to avoid the impression that intelligent communication is in question. Dying cells produce a characteristic screaming sound. One can also distinguish between normal cells and cancel cells on basis of the sound they produces as well as between mammalian and bacterial cells.

What might be the explanation of these findings in TGD framework?

1. It is known that the region of frequencies audible to human ear is from about 20 Hz to 2×10^4 Hz. This is more or less same as the range of frequency range of sferics, the em noise in atmosphere [F7]. This suggests a strong coupling between electromagnetic oscillations and sound as also the fact that biological structures are piezo-electrets transforming em oscillations to sounds and vice versa.
2. The activation energy per mole corresponds to.6 eV per molecule which is at the upper range for the variation range the energy associated with the fundamental metabolic energy quantum identified as the change of zero point kinetic as proton is transferred from atomic space-time sheet to much larger space-time sheet or vice versa.That metabolic energy is needed to produce the sounds supports the view that the sounds are produced intentionally.
3. If one takes seriously the notion of magnetic body as intentional agent controlling biological body, one is led to ask which must sound a totally crazy question in reductionistic ears: could magnetic body express its emotions in terms of frequencies of cyclotron transitions transformed to sound via genetic expression using piezo electric mechanism? Could it be that the photons involved are dark photons with large value of Planck constant so that their energy is above thermal energy. Could one propose a materialistic scientist to consider anything more irritating that singing and crying magnetic bodies!
4. Suppose that the homeopathic mechanism is based on replication of pseudo-molecules with same magnetic body as that of solvent molecules and that neutral dark nuclear strings realize analogs of DNA, RNA, and amino-acids and realizing genetic code exactly in its vertebrate nuclear form and appearing also in the TGD based model of cold fusion and biological transmutations. If so, then homeopathic mechanism (recognition of molecules) could involve

also the transformation of cyclotron radiation to sound at the level of "biological bodies" of molecules.

5. If this picture makes sense then also our speech as a self expression of the magnetic body might involve genetic code mapping sequences of DNA codons to temporal patterns of cyclotron radiation in turn transformed to speech by above mechanism. This would require a realization of genetic code at level of dark matter: could it be that dark nuclear code could define universal quantum level realization of language? The findings of Peter Gariaev and others and structural resemblance of intronic portion of genome with language and their report that DNA sequences are coded to temporal patterns of the rotation angle of the polarization of laser light (in turn inducing genetic expression).

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Chapter 12

Quantum Gravitation and Topological Quantum Computation

12.1 Introduction

In this article the connection of quantum gravitation, as it is understood in the TGD framework, with topological quantum computation (TQC) is considered. I sketched the first TGD based vision about DNA as a TQCer for about 13 years ago. In particular, a model of the system consisting of DNA and nuclear/cell membrane system acting as a TQCer was discussed [K5, K4, K123].

TGD has evolved a lot after this and there are several motivations for seeing what comes out from combining the recent view about quantum TGD and TGD inspired quantum biology with this model.

1. There is a rather detailed view about the role of dark matter as phases of ordinary matter with the effective Planck constant $h_{eff} = nh_0$. Large values of h_{eff} allow to overcome the problems due to the loss of quantum coherence.
   This leads to the notion of the dark DNA (DDNA), whose codons are realized as dark proton triplets and proposed to accompany the ordinary DNA [L13, L110]. Also dark photon triplets are predicted [L6] [L80, L89] and one ends up to a model of communications and control based on dark cyclotron resonance in which codons serve as addresses and modulation of the signal frequency scale codes the signal to a sequence of pulses. Nerve pulses could be one application.
2. Quite recently, also the understanding of the possible role of quantum gravitation in biochemistry, metabolism, bio-catalysis, and in the function of DNA [L103] has considerably increased. The gravitational variants of hydrogen bonds and valence bonds between metal ions having very large value of $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is the gravitational Planck constant [L46] [K103, K83, K86] originally introduced by Nottale [E6], are in a key role in the model and explain metabolic energy quantum as gravitational energy liberated when dark protons "drops" from a very long gravitational flux tube in the transition $h_{gr} \rightarrow h$. Also electronic metabolic energy quantum is predicted and there is empirical support for this.
3. A further motivation comes from the number theoretic vision of quantum TGD. Galois groups as symmetry groups represent new physics [L88, L86, L87] and the natural questions are whether Galois groups could give rise to number theoretic variants of anyons and what could the TGD counterparts of the condensed matter (effective) Majorana electrons proposed by Kitaev [D29] as anyon like states?
   The answer is that quantum superpositions of symmetric hydrogen bonded structures of form X..H-H+X-H...X are excellent candidates for the seats of dark ($h_{eff} > nh_0 > h$) bi-localized electrons defining TGD analogs of condensed matter Majorana electrons.
   The Galois groups permute the roots of a polynomial, which determines a space-time region by $M^8 - H$ duality. The roots correspond to mass squared values, in general algebraic numbers,

and thus to mass hyperboloids in $M^4_c \subset M^8_c$. The H images correspond to 3-hyperboloids with a constant value of light-cone proper time. Therefore the Galois group permutes points with time-like separation.

This looks very strange at first but actually confirms with the fact that time-like braidings defining TQC correspond in TGD time-like braidings (involving also reconnections) of string like objects defining string world sheets, which are not now time evolutions of space-like entities as physical state but correspond to time-like entities defining boundary data necessary for fixing holography completely. Their presence is forced by the small failure of the determinism of the action principle involved and is completely analogous to the non-determinism for soap films with frames serving as seats for the failure of determinism.

4. Braidings appear therefore at the level of fundamental TGD and correspond to string world sheets. They are possible only in 4-D space-time but not in string models.

   Also TQC-like processes appear automatically at the level of fundamental physics. In particular, the number theoretical state function reduction cascade for the Galois group [L82] following the time evolution induced by braiding can be regarded as a generalization of a decomposition of integers to primes: now primes are replaced by simple groups defining primes for finite groups. Nature is doing number theory!

5. Also zero energy ontology (ZEO) [L64, L93] brings in new elements. The change of the arrow of time in "big" state function reductions (BSFRs) implies that dissipation with a reversed arrow of time provides an automatic error correction procedure. Also TQC in which the arrow of time varies for sub-modules, can be considered.

12.1.1 Two visions about physics in TGD framework

TGD leads to two visions about physics discussed in [L81, L96]. In the first vision [K60, K35, K98] physics is seen as geometry of space-time identified as 4-surface in $H = M^4 \times CP_2$, and at a more abstract level, geometry of the "world of classical worlds" (WCW) consisting of space of preferred extremals (PEs) of the basic action principle defining analogs of Bohr orbits as minimal surfaces with singularities.

In the second vision [K109] physics is reduced to number theoretic concepts and 4-surfaces in M^8 analogous to momentum space define the basic objects. $M^8 - H$ duality [L73, L74], analogous to momentum-position duality, relates the two visions. The 4-surfaces in M^8_c (complexified M^8), which has interpretation as complexified octonions, are required to be associative in the sense that their normal space is quaternionic.

For given space-time region, they are determined by the roots of polynomial P of real argument continued to polynomials in M^8_c. The roots define a collection of mass shells of $M^4_c \subset M^8_c$ and by holography they define a 4-D surface of H.

The action principle at the level of H is determined by the twistor lift of TGD and is the sum of 4-D Kähler action and volume term (cosmological constant). It is not fully deterministic and space-time surfaces in H as PEs analogous to Bohr orbits can be regarded as analogs of soap films with frames, which correspond to singularities at which determinism fails.

The frames provide additional holographic data besides the hyperbolic 3-surfaces corresponding to light-bone proper times $a = a_n$ which are determined by the roots of P. Frames include light-like orbits of partonic 2-surfaces and string world sheets connecting them. What is new, and consistent with zero energy ontology (ZEO) [K132], is that space-like data are not enough for holography, also time-like data is required and the string world sheets turn out to be absolutely essential for braiding and TQC.

Physics as geometry

The basic elements of physics as geometry are following.

1. Space-time is identified as minimal 4-surface [L97] in $H = M^4 \times CP_2$. Holography follows from general coordinate invariance and implies what might be called Bohr orbitology. It turns out that holography is not quite strict.
2. Twistor lift of TGD [L41] [L98, L99] replaces space-time surface with what can be regarded as a counterpart of its twistor space having X^4 as a base space and sphere CP_1 as a fiber. The

twistor structure is induced from the product of $T(M^4) \times T(CP_2)$ of twistor spaces $T(M^4)$ $TC(P_2)$, which are the only twistor spaces allowing Kähler structure. The induced twistor structure and determined by an action principle with is 6-D Kähler action existing only for M^4 and CP_2. Twistor structure requires dimensional reduction so that one bundle structure and the action reduces to a sum of a volume term having interpretation in terms of cosmological constant and of 4-D Kähler action as analog of Maxwell action.

PEs realizing the holography are identified as minimal surfaces [L97], which, apart for lower-dimensional singularities, are also locally extremals of the 4-D Kähler action and possess a holomorphic structure reducing the field equations to algebraic conditions analogous to Cauchy-Riemann conditions. One can regard the space-time surface as an analog of soap film spanned by frames assignable to the singularities at which minimal surface property fails but extremal property for the entire action remains true so that conservation laws are not lost. As in the case of ordinary soap films, frames are seats of finite non-determinism interpreted as space-time correlates of quantum non-determinism.

3. The concrete study of the extremals of the action principle leads to the identification of the basic candidates for the basic PEs. From the point of view of TQC, magnetic flux tubes are the most interesting objects and define counterparts of the braid strands. The notion of magnetic body (MB) is central. Its detailed identification is still far from complete: for the latest view about gravitational MB see [L103].

Physics as a generalized number theory and M^8-H duality

Physics as (a generalized) number theory is the dual vision of TGD.

1. p-Adic physics emerged originally from a model for the particle massivation based on p-adic thermodynamics for the mass squared of the particle [K68, K32]. From the beginning it was clear that various p-adic physics had to be fused with the real number based physics to a larger framework, which could be called adelic physics. For mathematical reasons, the natural interpretation of various p-adic physics would be in terms of physical and mathematical correlates of cognition. Number theoretical universality stating that the basic equations of TGD are number-theoretically universal and make sense in all number fields is a natural constraint on the theory.

2. M^8-H duality [L73, L74] realizes the number theoretical vision about TGD and also holography. M^8_c identified as complexified M^8 and interpreted as complexified octonions, is analogous to momentum space and 4-surfaces define the basic objects at the level of M^8.

   The 4-surfaces in M^8_c (complexified M^8), which have an interpretation as complexified octonions, are required to be associative in the sense that their normal space is quaternionic. These 4-surfaces are determined by the roots of polynomials of real argument continued to polynomials in M^8_c. The roots define a collection of 3-D mass shells of $M^4_c \subset M^8_c$ and by holography they define a 4-D surface of M^8_c. Physical states correspond to 4-momenta at these mass shells analogous to Fermi balls.

   M^8-H duality, analogous to momentum-position duality, relates the two visions by mapping the 4-surfaces in M^8 to those in H. M^8-H duality generalizes to the level of twistor space [L73, L74, L96, L98, L99].

3. One can assign to a given polynomial an algebraic extension of rationals. The collection of points of the 4-surface of M^8_c defines a cognitive representation. The mass shells as sources of holographic data are however number theoretically exceptional in that the number of points with algebraic M^8_c coordinates is infinite: cognitive explosion takes place both at the level of M^8 and H: these values of the light-one proper time a correspond to very special moments in the life of self, kind of moments of enlightenment.

   In M^8 the points of mass shells are identifiable as quark momenta assumed to be algebraic integers just as ordinary momenta for particles in a box are integers with suitable choice of momentum unit. These momenta can also be interpreted as points in extension of p-adic numbers so that number theoretical universality follows. The p-adic prime in question is identified as the largest ramified prime of the extension considered.

This gives rise to a hierarchy of algebraic extensions and cognitive representations as unique discretizations of the 4-surface in M^8 and space-time surface and suggests a generalization of computationalism replacing integers with the hierarchy of algebraic integers for extensions of rationals.

4. The dimension n of algebraic extension is identified as an effective Planck constant $h_{eff} = nh_0$ where $h_0 < h$ is true. The identification of the value of n_0 in $h = n_0 h_0$ has been proposed [L92]. The phases of ordinary matter labelled by the value of n behave in many respects as dark matter and the identification as dark matter has been proposed. A particularly important class of phases corresponds to $h_{eff} = nh_0$. These phases would play a central role in living matter. The relationship with galactic dark matter is however somewhat unclear.

   What makes these phases so important is the scale of quantum coherence is expected to scale like h_{eff}. Dark phases are also expected to have very weak interaction with ordinary matter and the proposal is that living matter is controlled by this kind of phases located at MB and approaching only slowly thermal equilibrium with it: this would have interpretation as aging [L124]. The small value of h and thermal fluctuations spoiling quantum coherence and entanglement belong to the key problems of QC and dark matter could solve these problems.

5. Galois confinement [L86] states that physical states have total momenta, whose components are ordinary integers. Galois confinement provides a universal mechanism for the formation of bound states. Galois confinement also applies in spin degrees of freedom and provides spin representations for the covering of the Galois group. The number theoretic degrees of freedom are of special interest in QC and suggest that number theoretic quantum computation (NQC) as a counterpart of TQC, which would involve what might be called Galois anyons. The Galois group could allow identification as a subgroup of the braid group. This would mean strong restrictions on TQC.

6. $M^8 - H$ duality leads to a view about the construction of the counterpart of S-matrix in the TGD framework [L98, L99]. S-matrix would be replaced by the analog of Kähler metric in fermionic degrees of freedom [L85], which by the infinite dimension of Fock space is expected to be highly unique as also the Kähler metric of WCW [K60, K35, K98].

   Incoming and outgoing states of particle scattering would be Galois singlets constructed from lower level states which need not be Galois singlets. Quarks, whose momenta at mass shells are algebraic integers are free and the scattering would be mere reorganization of Galois singlets to new ones.

   Scattering could be also seen as analog of QC and computation in an extension of rationals: both the input and output would consist of a set of rational integer valued momenta and scattering would map them to each other.

   This applies in the twistor picture also to spins having a representation as points of the twistor sphere S^2 known as Bloch sphere. In this case number theoretic constraints suggest that the set of quantization axes corresponds to a finite discrete subgroup of $SO3$) assignable to regular polygons and Platonic solids.

   The quark momenta belonging to the extensions of rationals are invisible, which implies invisible algebraic complexity of cognition and brings in mind unconscious information processing. Quantum physics and psychoanalysis would meet!

12.1.2 Zero energy ontology (ZEO) and QC

The first basic motivation for the introduction of ZEO was that by the general coordinate invariance space-time surface as a preferred extremal is a more natural notion than 3-surface. For exact holography, these notions are equivalent but the identification of space-time surface as minimal surface predicts a small violation of the strict holography identifiable as a correlate for quantum non-determinism associated with the physics of cognition or possibly quite generally. This non-determinism would be essential for the possibility of TQC in TGD.

Second motivation was the basic problem of quantum measurement theory to which ZEO provides an elegant solution if one assumes that the arrow of time changes in "big" state functions reductions (BSFRs) as analogs of ordinary SFRs. In "small" SFRs, which are analogs of "weak" measurements introduced in quantum optics, the arrow is not changed [K132] [L64, L93].

In the TGD framework, quantum measurement theory generalizes to a quantum theory of conscious experience in which SSFR defines the basic element of conscious experience. BSFR has an interpretation as a counterpart of death/sleep. The change of the arrow of time in BSFRs has profound implications in quantum biology. Since the dissipation with a reversed arrow of time for a subsystem looks like self-organization from the point of view of a system with an opposite arrow of time [L61]. The arrow of time can change for macroscopic time periods at the MBs with large h_{eff} and since MB controls the ordinary matter, it induces not only effective quantum coherence but also an effective reversal of time also at this level.

The basic ideas of ZEO [L64, L93] are following.

1. In zero energy ontology (ZEO) [L64, L93], the pair of incoming and outgoing states of particle scattering are replaced with zero energy state and zero energy states define scattering amplitudes as entanglement coefficients.
2. At the level of H, positive and negative energy parts of zero energy states are located at boundaries of causal diamonds (CD), which form a fractal hierarchy. At the level of M^8, they reside at the boundaries of mass shells, which corresponds to the roots of the polynomial defining the space-time region. $M^8 - H$ duality maps these points to the boundary of CD. One can also consider an alternative for which mass shells as hyperbolic spaces $H^3 \subset M^8$ are mapped to their counterparts in H by a map which is essentially inversion (Uncertainty Principle).
3. Scattering events [L98, L99] are QC like events. Input (output) data correspond to incoming (outgoing) quark momenta identified as algebraic integers in an extension of rationals and to spins. Since fermionic Fock state basis defines a Boolean algebra, the fermionic states define quantum analog of Boolean algebra, and the scattering amplitudes could be also seen as a quantum generalization of Boolean maps and realizing statements which are true that is consistent with laws of physics. These transitions could be interpreted in terms of Boolean cognition.

   The replacement of the S-matrix with Kähler metric in fermionic Hilbert space degrees of freedom represents a new element. The analog of unitary transformation is assigned with CD and from the point of view of QC, CD could be interpreted as an embedding space analog of gate. Since gates allow control bits not affected by the unitary transformation, also the Boolean functions, which are not 1-1, can be realized as unitaries. Same is expected to be true also now.
4. The scattering amplitudes correspond a tensor net-like structure. Physical states are Galois singlets consisting basically of free quarks. At the number theoretical level, scattering can be seen as a recombination of Galois singlets to new ones.

ZEO could have a profound impact on QC.

1. Negentropy Maximization Principle (NMP) [K72] [L91] is the variational principle of TGD inspired theory of consciousness. Negentropy can correspond to the sum of p-adic negentropies or to the sum or p-adic and real negentropies, which can be possible and tends to be so by NMP. For both options, NMP guarantees that the p-adic entanglement negentropy increases and is positive. It however also forces the real entanglement entropy to grow. NMP therefore implies cognitive evolution but also second law.

   From the point of view of QC, this picture is very promising since the laws of physics would take care that the entanglement negentropy grows and also that negentropic entanglement tends to be stable. This is quite contrary to what standard physics predicts. This leads to evolution [L38, L39] in the sense that the dimension $n = h_{eff}/h_0$ of the extension of rationals as a measure of algebraic complexity tends to increase since this provides larger negentropic resources. This evolution takes place at MB in human length and time scales and the challenge is to learn to manipulate dark matter.
2. BSFR could take care of error correction automatically since for the reversed arrow of time dissipation looks like error correction by self-organization. This error correction is a key feature of living matter but has remained poorly understood. One can also ask whether BSFRs could make possible QCs involving sub-QCs in both time directions. Could the use of sub-programs with opposite time direction allow a faster QC.

12.1.3 Finite field approximation and QC

Number theoretic vision about QC leads to new ideas about QC itself.

1. The momenta in the extension of rationals as algebraic integers can be interpreted as p-adic integers in the induced extension of p-adic numbers. The p-adic number field corresponds to prime p, which is the maximal ramified prime for the polynomial in quesetion.

   In the approximation $O(p) = 0$ they define a finite field $F(p,n)$ having dimension is is not larger than the dimension of extension but can be smaller. The number of elements is p^n and the situation corresponds to n pinary digits, qupits, instead of qubits. TQC using elements of $F(p,n)$ is an attractive possibility. Besides this one has also spin degrees of freedom.
2. The elements of $F(p,n)$ can be regarded as roots of some, in general non-unique, polynomial with degree p^n. This polynomial is in general not the polynomial inducing the extension of p-adic numbers.
3. The Galois group for the finite field should transform to each other the roots of the originalpolynomial interpreted as a polynomial in $F(p,n)$ and is a subgroup of the Galois group for the polynomial having all points of $F(p,n)$ as its roots.

The automorphism group of quaternions is analogous to Galois group and in the TGD framework with discretization it looks like a natural notion.

1. In the continuous case, the automorphism group of quaternions is the rotation group $SO(3)$ having $SU(2)$ as covering group. In the discrete situation, one expects it to be a finite group and would correspond to symmetries of Platonic solid in non-abelian case and to the symmetries of a regular polygon in abelian ase. Icosahedron, tetrahedron, and octahedron have triangles as faces and the proposal is that genetic code realized in terms of bioharmony [L6] [L80] corresponds to so called icosa-tetrahedral tessellation of H^3 [L89].
2. Therefore genetic code and bioharmony could closely relate to the quaternionic aspects of number theoretic physics and perhaps also to TQC for quantum variant $SU(2)_q$ of quaternionic automorphisms acting in the normal space of the space-time surface. A natural proposal is that the points of the icosahedron and tetrahedron correspond to points for the discretized unit sphere known as Bloch sphere defining possible directions of the quantization axis of spin in TQC.
3. The finite subgroups of $SU(2)$ are associated with the hierarchy of inclusions of hyper-finite factors of type II_1 and the proposal is that the inclusion of these factors define finite measurement resolution such that the included factor defines the resolution [K127, K48].

12.1.4 TQC and the new view about space-time

The new view about space-time is highly relevant for the TGD view of TQC.

Galois anyons

The basic problem of the TGD inspired model of TQC is the identification of the topological qubit identified as an anyon-like state in standard TQC. One could say that topological qubit or its analog does not correspond to quantum state but representation of braid group or quantum group assignable to Chern-Simons action. Topological qubits also satisfy a nice algebra defined by the decomposition rules of the representations of the braid group.

The motivation for this identification is that topological qubits are expected to be highly stable since the change of the representation is not expected to be probable unlike the change of spin direction. The non-local character is also an important aspect. The braids defining TQC program as unitary representation of the braid group allows to identify the gates, which are universal in the sense that they have finite computational accuracy.

The increase of the order of the covering group as a finite covering of the permutation group S_N for N braid strands allows to improve the accuracy. Kitaev [B7, B6] has proposed [D29] that anyon-like bi-localized states of condensed matter Majorana fermions could define stable qubits. Majorana electrons would be superpositions of electron states localized at the ends of a superconducting wire and would have parity $+/-1$ under permutations of ends of the wire.

In TGD framework the electrons defining analogs could be bi-localized states with localization to the ends of a monopole flux tube or pair of them. Galois degrees of freedom are a new element and anyons could correspond to multi-localized states defining representations of Galois group at its orbits consisting of points of the cognitive representation at mass shell H^3. Also spin degrees of freedom would define Galois representation. If the braidings correspond to lifts of number theoretic symmetries, Galois group corresponds to a subgroup of the braid group.

In the standard picture of TQC, a computationally interesting situations corresponds to non-Abelian anyons to guarantee that the states defining topological qubits form a higher-D space. This means that the swap $ab \leftrightarrow ba$ is not a commutative operation inducing a mere phase anymore. Since the status of Majoran fermions is unclear, it is still unclear whether any anyonic system satisfies this constraint. Galois groups are in general non-commutative so that this problem disappears.

Physical states would be Galois singlets and anyon-like states would be their building bricks just as quarks would define building bricks of general Galois singlets including also leptons and various bosons. Since Galois non-singlet cannot appear as a free particle, one could also understand topological entropy associated with anyons as relating to the entanglement with environment forced by Galois singletness in spin degrees of freedom.

Braidings and reconnections as basic elements of TQC

TQC in the TGD Universe involves also other new elements besides Galois groups.

1. The flux tubes connecting the nodes of a tensor net-like structure define natural candidates for braid strands. Both space-like and time-like braiding are possible.

   Time-like braiding defining TQC of the moving nodes connected by flux tubes induces a space-like braiding so that the TQC is recorded to memory as a kind of log file. Dance metaphor expresses this neatly: dancers at the parquette are connected by threads, which get braided and form a memory representation about the dance. This mechanism could define quite a general representation of memories based on space-time topology.

2. The fusion defined by the tensor product for the representation of the braid group or associated quantum groups is a key operation in standard quantum computation. The decomposition of the tensor products gives a superposition of topological qubits or more general qubit-like entities An interesting question is whether the fusion could have a more concrete topological meaning. Could the fusion of flux tubes correspond to a formation of a bound state of flux tubes inside a flux tube?

3. TQC as a braid generalizes to tensor-net (for tensor nets in TGD sense see [K55] [L28]). The nodes can have M incoming qubits and N outgoing qubits. The node corresponds to a quantum computation defined as a map between the incoming and outgoing qubits. In the framework, the nodes would correspond to CDs For $M \neq N$ is not a unitary transformation 1-1 transformation but can be an injection so that it is still an isometry at the level of the state space.

4. Besides swap as the basic braiding operation, also reconnection, having the same effect as far as initial and final states are considered, appears as a basic operation. When the incoming and outgoing qubits cannot move, reconnection could take the same role as swap and make TQC possible.

5. One can wonder whether this more general view about TQC could be realized in quantum biology. Could biochemical reactions correspond to fusions of braids of a tensor net, could reconnections and braidings make it possible to have a larger repertoire of TQCs. Could ZEO-based error correction requiring only time reversal play a key role in TQC.

Different TGD based views of TQC

TGD suggests several different perspectives of TQC.

1. In the flux tube picture, the basic elements are braiding, reconnections and fusions in which flux tubes could even form a bound state inside a larger flux tube so that the fusion could have a geometric meaning. At the level of H, fusion could correspond to a process in which the incoming particles arriving into the CD form a tensor product. Inside CD fusion occurs

and gives rise to a decomposition of irreps. Measurement selects one irrep first and outgoing states are obtained by an SFR cascade reducing the total Galois group to the factors defined by relative Galois groups by a cascade of SSFRs defining cognitive measurements.

Dance metaphor implies a mechanism of memory with spatial braidings representing spatial braidings. This mechanism would be realized in all scales and define kinds of topological Akashic records. If reconnection is equivalent with swap operation, then TQC is also possible without braiding induced by the motions of braid ends.

2. CDs are counterparts of gates at the level of H and define a fractal hierarchy of gates with sub-CDs defining sub-modules.

   Space-time surface in H can be also seen as a 4-D soap film with frames as seats of non-determinism and one could assign mental images with this non-determinism. This suggests that the gates at space-time level correspond to the frames whereas CDs would correspond to entire TQCs at the level of H. This also suggests that TQC in the TGD sense must allow intermediate SSFRs at the frames. The situation is far from obvious since fractality is also present and involves a hierarchy of CDs.

   The M^8-H-duality provides a further view about TQC. A highly attractive idea is that TQC programs can be constructed as functional composites of polynomials giving rise to extensions of extensions of .... and inclusion hierarchies of corresponding Galois groups, each defining a normal subgroup of its sup-group.

   The normal subgroup hierarchy makes it possible to understand cognitive measurements as SSFR cascades reducing the representation of the Galois group to a product of representations for the subgroup and normal subgroup associated with it. This decomposition could generalize the decomposition of the anyonic representations. This would also suggest a deep connection with the paradigm in which computations are functions.

12.2 What could the replacement of the braid group with the Galois group mean?

The replacement of the braid group acting on anyons with the Galois group looks a rather innocent proposal first but has profound implications. The reason is that the Galois group permutes the roots of the polynomial P, which correspond to different mass shells in M^8 and therefore different values of light-cone proper time in H.

12.2.1 Functional composition of the polynomials and many-particle states

Functional composition of the polynomials is proposed to give rise to many-particle states.

1. The roots of P correspond to mass shells. Quarks have momenta at these complex mass shells. Roots and corresponding momenta are in general complex algebraic numbers and total momenta and mass squared values are real by Galois confinement.
2. Functional composite $P = P_n \circ ... \circ P_1$ of polynomials defines the interactions of particles in the number-theoretical picture. Functional composites are proposed to define particles as many-quark states and further functional compositions make it possible to engineer many particle states formed from these.
3. One can also consider iterates of a polynomial as analogs of many particle states involving only a single kind of particle. Functional decomposition gives as roots inverse iterates of the roots of the polynomial Q in $P = Q \circ Q... \circ Q$ [L77, L98, L99]. Asymptotically they give rise to an analog of the true Julia set (`https://mathworld.wolfram.com/JuliaSet.html`) as a boundary of the filled Julia set. The inverse iterates near the boundary of the Julia set would correspond to very nearly the same mass squared values and thus proper time constant hyperboloids.
4. One can regard the roots of P_i as roots with respect to the variable $y = P_{i-1} \circ ...P_1(x)$ if $y = P_{i-1} \circ ...P_1(x)$ defines the ground state coordinate. $h_{eff} = n_0h_0$ would define a natural ground state for which $h_{eff} = nh$ would hold true.

5. If the polynomials appearing in the composite satisfy $P_i(0) = 0$, one has "inheritance of roots". The roots y_i of P_i are mapped to their inverse images $(P_{i-1} \circ ...P_1)^{-1}(y_i) = P_1^{-1} \circ P_2^{-1} ... \circ P_{i-1}^{-1}(x)$. This inheritance brings in mind conserved genes. A weaker form of "inheritance" would be that some polynomials, say $P_1, P_2, ..., P_k$ at the lowest level have $P_k(0) \neq 0$. For $P = Q \circ P_F$, where $P_F = x^2 - x - 1$ is "Fibonacci polynomial", the roots would be of form $(-1 \pm \sqrt{5 + 4y_n})/2$, where y_n is a root of Q. Note that one has $P_1(0) \neq 0$. If one has $P_k(0) \neq 0$ for $k > 1$, the roots of P_1 are roots of any P and therefore universal. This suggests the possibility that the ground state polynomial corresponding to $h_{eff} = h = n_0 h_0$ is non-vanishing at origin.

Ground state polynomial

The ground state polynomial P_g corresponding to $h_{eff} = h = nh_0$ is of special interest physically.

1. The arguments allowing to deduce the value of n_0 in $h = nh_0$ lead to a conclusion that the ground state polynomial P_g [L92] corresponding to $h_{eff} = h = n_0 h_0$ corresponds to a Galois group with $7!^2$ elements.

2. This allows several options. For instance, the semidirect product $S_7 \rtimes S_7$ could act as a Galois group. S^7 decomposes to a semidirect product of the simple alternating group A_7 and Z_2 acting as a normal group. S_7 can appear as a maximal Galois group for a polynomial of order 7. In this case S_7 could correspond to $Q_a = P_7 \circ P_2$ or $Q_b = P_2 \circ P_7$ and one would have four options $P = Q_i \circ Q_j$. Also $P_7 \circ P_7 \circ P_{2,a} \circ P_{2,b}$ $P_{2,a} \circ P_{2,b} \circ P_7 \circ P_7$ are possible.

3. Second roots appear in all basic formulas of quantum mechanics. Therefore one can argue that P_2 should appear at the bottom of the composite polynomial defining the ground state. Fibonacci quantum computation involves Golden Mean and the roots $x_\pm = (-1 \pm \sqrt{5})/2$ of Fibonacci polynomial $P_F(x) = x^2 - x - 1$. All roots would appear as pairs with members related by the Galois group of P_F. For $P_1 = P_F$ and $P_k(0) = 0$ for $k > 1$ (inheritance), the roots of P_F are roots of any P and Golden Mean would play a key role in fundamental physics.

Mass squared formula and inheritance hypothesis

For Galois singlets, the total momentum has components, which are ordinary integers. Also mass squared is integer.

1. If the stringy mass formula $m^2 = n$ holds true for the quark mass squared values as roots of a polynomial, one must have $m^2 = \sum m_i^2 = n$. This requires that the sum of the inner products of quark momenta vanishes. The interpretation would be as an additivity of conformal weights. If every root is realized as quark momentum, $m^2 = \sum m_i^2$ equals the constant coefficient of the total polynomial P giving $m^2 = P(0)$.

2. If the strong form of inheritance holds true, one has $\sum m_i^2 = 0$ so that the total conformal weight vanishes. Could the interpretation be in terms of conformal invariance? Could one say that the tachyonic total mass squared assignable to the space-like states defined by braid strands compensates for the non-tachyonic total mass squared?

   Total momentum would be light-like and the $M^8 - H$ duality should be defined as the map $p^k \to m^k = \hbar_{eff} p^k/(p^0)^2$ where m^k belongs to the light-like boundary of CD containing the CDs assignable to the mass squared values as sub-CDs.

3. In p-adic mass calculations the total conformal weights are however non-vanishing and real. What could this mean?

   (a) The thermal excitations should be excitations of the $m^2 = 0$ state due to interaction with the environment, which extends the system. The thermal excitations would be described by polynomials $Q_{ex} = P_{ex} \circ P$. The roots of Q_{ex} would include, besides roots of P (inheritance), also the roots y_n of P_{ex} and these correspond to non-vanishing values of $P(y_n)$. $y_n \neq 0$ would give non-vanishing mass for the thermalized subsystem defined by P.

(b) If one gives up the "inheritance" hypothesis and allows $P_i \neq 0$, one has $m^2 = \sum m_i^2 = P_n(0)$. Monic polynomials $P(x) = x^n + a_{n-1}x^{n-1} + ... + a_0$ are good candidates for the allowed polynomials. The coefficients a_k are integers so that the mass squared as a conformal weight $\sum m_i^2 = a_0$ is an integer.

Decomposition of Galois group to a product of relative Galois groups

The Galois group Gal for an extension of extension.... decomposes to a product of the relative Galois groups Gal_k/Gal_{k-1}.

1. One can speak of the ground state characterized by some Galois group Gal_0. Ordinary Planck constant h would correspond to Gal_0 and in [L92] it was proposed to be a product of permutation groups S_7 giving $n_0 = (7!)^2$. This allows to interpret CP_2 length scale squared as $n_0 l_P 2$, l_P Planck length. Galois group can be identified as a relative Galois group: as Galois group for extension of the extension defining the ground state.
2. The structure of the Galois group reflects the functional composition involving a large number of identical polynomials with the same mass spectrum as free particles. In the functional composite $P \circ Q$ the mass spectrum S of P is mapped to $Q^{-1}(S)$. Large number of iterations of P produces Julia set as a fractal. One can speak of an asymptotic mass spectrum.
3. The orbit of Galois group consists of mass shells and its cognitive representation can contain momenta at these mass shells.

   Galois symmetry would be a discrete symmetry connecting quarks with different mass values (which are counterparts of virtual masses rather than real masses). Galois symmetry would be analogous to a dynamical symmetry and would not commute with Poincare and Lorentz symmetries.

   Physical states are Galois singlets and have well defined real mass squared. Galois singlet property of physical states would imply that these symmetries would be respected. Physical states correspond to a CD containing sub-CDs... and at the lowest level there would be quarks. Essentially 4-D objects would be in question.

12.2.2 $M^8 - H$ duality at the level of M^4

$M^8 - H$ duality maps the algebraic physics at the level of M^8 formulated using polynomials to the geometric physics at the level of $H = M^4 \times CP_2$ formulated using variational principle and partial differential equations. The preferred extremal property required by general coordinate invariance reduces the number of solutions of field equations so that they can correspond to a much smaller set of solutions of algebraic equations. The holographic aspects of $M^8 - H$ duality have been already considered and in the following only the map $M^8 \supset M^4 \to H \supset M^4$ is discussed.

1. $M^8 - H$ duality maps the surfaces of M^8 to minimal surfaces in H having singularities at which only the field equations for the full action containing also Kähler action besides the volume term hold true. $M^8 - H$ realizes holography: the mass shells determined by the roots of P can be continued to 4-surfaces containing them.
2. The precise form of $M^8 - H$ duality is not quite clear. The first question is whether one should allow complexification of M^4 as at the H side. One could define the H image as $M^k = \hbar_{eff} Re[p^k/m^2]$, where p^k is the quark momentum and at mass shell m^2. M^k would define some geometric objects in H. For physical states m^2 is integer and corresponds to a finite value of $a = \hbar_{eff}/m$. If the stringy mass formula $m^2 = \sum m_i^2 = 0$ is true, the image belongs to the light-cone boundary.

   The image could be a geodesic line of H parallel to m^k, which could start from the origin from the common center of CDs forming a fractal Russian doll hierarchy or from the tip of a given sub-CD.

   The image could also be identified as a point or a set of points. The point could be identified as the intersection of these lines with the boundary of the sub-CD defined by the mass value or its real part. Also the intersections with boundaries of all sub-CDs involved can be considered. Also the map of mass shells to M^8 to hyperboloids $a = a_n$, where a is light-cone proper time and a_n is inversely proportional to mass to realize Uncertainty Principle, makes sense.

3. The image of the orbit of the Galois group would correspond to a geodesic line starting at the centers or tips of various CDs defined by the mass shells. If the CDs are inside each other like a Russian doll, the geodesics intersect the $a = a_n$ hyperboloids and the boundaries of corresponding sub-CDs corresponding to different values of the light-cone proper time a and are time ordered. What is highly non-trivial is that the points at the orbit have time- like distances.

12.2.3 The orbits of the Galois group in H transform hyperboloids to each other

Mass squared values correspond to the roots a_n of a polynomial and are in general complex algebraic numbers. Their real projections can be negative and therefore tachyonic. The big surprise during writing of this article was the trivial observation that the Galois group permutes the mass shells defined by the roots of P.

If the real projections of mass shells to M^4 are mapped to H, Galois group can connect points with different values of complex "cosmic time" $a = a_n$. This does not conform with the idea that the particles of the physical state always have space-like distance but could conform with ZEO and non-determinism inspiring the view that time-like braiding is a physical state rather than its time evolution.

Note however that the spatial distance $(M_1 - M_2)^2$ in H is space-like for $(E_1 \geq m_1^2 + m_2^2)/m_2$ in the coordinate system in which M_2 and p_2 have a vanishing spatial part. This holds true also for the M^4 images.

Orbits of the Galois group as braidings?

Could the orbits of the Galois group for off-mass shell states be identified as braidings?

1. If the braiding is time-like, the value of the real part of the proper time parameter corresponding to the mass shells or CD sizes increases along the orbit.

   This would conform with the idea that the orbit of the Galois group consists of images of mass shells at the quark level. It also conforms with the breaking of Lorentz and Poincare symmetries at the level of the Galois group. This finding also justifies the Galois confinement: physical states correspond to a single value of a.

2. What about number theoretic anyons? These anyons must have non-trivial Galois quantum numbers and algebraic momenta. Here the relative Galois group is a convenient concept. Galois non-singlet property is with respect to the relative Galois group and one can forget the huge complexity of the Galois singlet ground state altogether.

Do Galois anyons require tachyonic states?

The momenta of quarks define the basic representation of the Galois group. One can also imagine representations in spin degrees of freedom. If only the spin degrees of freedom carry Galois quantum numbers, the space-time action of the Galois group is trivial. This does not look attractive and does not conform with time-like braiding. Anyon property therefore suggests the presence of tachyonic momenta.

1. I have played with the idea that quarks and also weak bosons appear in the scale of cells in living matter as dark quarks or even scaled variants with very small mass. How could the dark quarks manifest themselves?

   I have proposed that the protons of dark nucleon triplets representing codons are connected by meson-like bonds, which could be colored and confine codons to genes. This could the case also for the bonds connecting nucleons in the ordinary nuclei. Strong interaction would also make it possible to have dark neutrons.

   I have assigned the Z_3 Galois group with the dark nucleon triplets defining dark codons: this is required by the correct statistics in the model of the genetic code. Could Galois group Z_3 correspond to the center Z^3 of the color group $SU(3)$?

2. In the original proposal for DNA TQC [K4], quark triplets were indeed considered instead of dark nucleon triplets. Dark tachyonic electrons assignable to symmetric hydrogen bonded structures looks like a more realistic option. One can also consider mesons with quark and antiquark ends associated with the ends of the space-like braid strands. Dark tachyonic electrons could be associated with the ends of string world sheets for which the time dimension corresponds to a space-like normal dimension.

   Could one assign a colored quark pair to anyon-like electron? Leptohadrons [K120] are a basic prediction of TGD and there is empirical evidence for them. The predicted mass of the lepto-electron is very nearly the same as electron mass and evidence for its existence was found already in the seventies. Lepto-electron would be a color octet: this is allowed in the TGD framework.

   Lepto-hadron is associated with the breaking of parity symmetry in nuclear collisions involving strong electric and magnetic fields not orthogonal to each other. Its description involves Chern-Simons Kähler action associated also with anyons. The notion of induced gauge field allows its interpretation as SU(3) Chern-Simons action. A possible identification of lepto-electron would be as an anyon for which electron would be accompanied by a color octet quark pair formed by the quarks at the ends of the flux tube.

3. Polynomials can also have roots corresponding to space-like mass squared values. Could dark quarks be tachyonic in the sense that they have a negative real part of mass squared so that time direction as a normal direction for this object would be naturally space-like?

4. Could one see time-like braids structures as genuinely 4-D objects predicted by ZEO and the failure of the strict determinism of the action principle? Singularities as frames span 4-D soap films serve as a source of non-determinism.

How could dark DNA correspond to time-like braids strands for dark DNA?

The following represents a long list of cautious proposals represented as questions.

1. Can one Galois symmetries acting in time direction have projections acting effectively as 3-D symmetries of ordinary matter at time=constant surface.

   The Galois group at the level of (presumably gravitational) MB does not act at the level of ordinary matter. Could the time-like braids at the level of the dark DNA correspond to the ordinary DNA strands in the sense that the temporal sequences would be mapped to spatial sequences by some simple rules?

2. Could genes have a representation as time-like braids? Could one imagine a pile of or ordinary DNA strands and their dark counterparts at different values of $a = a_n$ such that time like braid strands would have the same DNA content as the DNA in $a = constant$ or $t = constant$ plane. For instance, could the intersections of the points of cognitive representation at $a = a_n$ hyperboloids with $t = constant$ hyperplane define the DNA strand.

   The codons of dark DNA as a temporal sequence would correspond to codons of the ordinary DNA unless one assumes that only identical codons correspond to the orbits of the Galois group. This looks like a more reasonable option. Codons themselves would correspond to orbits of the discrete and finite subgroups of automorphisms of quaternions acting as symmetries of Platonic solids and regular polygons. Therefore two kinds of Galois groups would be involved.

3. Could the physical DNA correspond to the space-like braidings assignable to the time-like braidings of dark DNA? Could one realize the representations of the Galois group by using these projections at the level of ordinary DNA.

4. Could identical codons of a gene correspond to projections of points related by the Galois group? If so, the collections of identical codons (64 of them) would correspond to 64 orbits and the anyons would be realized at these collections as wave functions. Different representations would correspond to different anyons serving as number theoretical qupits.

String world sheet interpretation of time-like braidings at the level of H

$M^8 - H$ duality implies time-like braids correspond to physical states rather than time evolutions of an ordinary physical state localizable to time= constant hyperplane. The time-like character of states conforms with ZEO and is implied by the predicted non-determinism in which the singularities of the minimal surface correspond to loci for the failure of strict determinism. These singularities define analogs of frames for the space-time surface as an analog of a 4-D soap film. They are a necessary part of the data allowing to realize holography.

M^8-H duality [L73, L74] predicts candidates for the singularities as loci of non-determinism. The following argument suggests that the 2-D orbits of braid strands defined by string world sheets as fundamental objects of the TGD Universe giving rise to braidings could characterize the non-determinism.

1. 3-D light-like surfaces defining orbits of partonic 2-surfaces starting at the boundaries of CD and 2-D string world sheets connecting two light-like 3-surfaces. Strong form of holography, whose status is uncertain, states that only the partonic 2-surfaces at the boundaries of CD are needed.
2. String world sheets would provide additional data to fix the preferred extremal and the failure of 4-D determinism manifested as the failure of the minimal surface property would be localizable to the string world sheets. According to the dance metaphor, the ends of the strings would represent dancers and strings would represent the threads connecting their feet.

   String world sheets would be necessary for fixing the space-time surface. This is a profound deviation from string models, where data at time=constant section would fix the time evolution.

   In fully deterministic physics, the direction of time coordinate is normal to $t = constant$ slice. The normal directions of the string world sheet are analogous to time direction: that they are space-like conforms with tachyonicity. String world sheet would represent a tachyonic virtual particle exchange between particles with time-like momenta.
3. Also strings are minimal surfaces apart from singularities. Reconnection is a singularity at which the string world sheets intersect at a single point and involves failure of determinism. The effect of reconnection is the same as that of braiding (SWAP). Reconnection therefore corresponds to the SWAP gate in TQC.
4. The 4-D character of the space-time surface implies that the strings develop spatial braiding during the dance and can also reconnect. This does not happen in super string models with 10-D embedding space for strings.

   The braiding and reconnection patterns would represent the time evolution of string-like entries in 4-D space-time so that TQC would reduce to a string model-like theory with one important exception: braiding and reconnections are not possible in string models.

   Gravitational flux tubes would be one particular case of flux tubes. They seem to be key players in biology and provide a quantum gravitational view about metabolism, biocatalysis, and DNA [L103]. TQC involves braiding and flux tubes with strings attached with them: TQC would have a direct connection with string model type description of quantum gravitation and other interactions.

Tachyonicity of the time-like braids as physical states could be therefore understood. One can look at the situation also from the point of $M^8 - H$ duality to gain additional perspective.

1. Virtual particles of QFT picture would in TGD framework have a discrete mass squared spectrum give by the roots of a polynomial and thus algebraic, in general complex, numbers [L98, L99]. Their finite number in zero energy state would resolve the divergence problem of QFTs.

   Only quarks appear as fundamental fermions. Mass squared values and momenta of many quark states constructed are in an extension of rationals without the condition of Galois confinement implying stringy mass squared spectrum and integer valued momentum components using the scale of CD as unit.
2. Quarks at mass shells of $M^4 \subset M^8$ are mapped to geodesic lines of H by $M^8 - H$ duality. They can be also space-like unless one assumes that the real parts of the roots of P are non-negative. For negative real parts, the momenta would be space-like and define points outside the sub-CD but a larger CD could contain them.

   Could the total momentum of say 3-quark state possibly associated with codon (3N quark state associated with a gene) be tachyonic? Could the tachyonic quark triplets be located along the time-like braid strand associated with the codon and define a tachyonic many-quark states?

3. For anyons as tachyons Galois confinement must fail and they should correspond to virtual states made from quarks. Could the strands of a space-like braid as a string with quark and antiquark at its ends define an entity analogous to a virtual meson? Could this meson-like entity have non-trivial color quantum numbers?

   How do Galois confinement and color confinement relate? At the level of "world of classical worlds" (WCW) quark color corresponds to partial waves in CP_2 for cm degrees of freedom for the partonic 2-surfaces associated with quark. At the level of the space-time surface there are no color partial waves since fermions do not have color as a spin-like quantum number. I have proposed a Z_3 subgroup of the Galois group as a counterpart for $Z_3 \subset SU(3)$. Correct statistics requires antisymmetry with respect to Galois Z_3.

   One must take this with caution: maybe the braid statistics of anyons could solve the statistics problem. Note however that braid statistics is analogous to Fermi statistics in that two particles are not possible in the same state.

The original proposal for DNA as a TQCer, was that DNA and nuclear membrane are connected by flux tubes having quark and antiquark at their ends. Also DNA strands would be connected by this kind of strands. The proposal was motivated by the observations and the classical counterpart of color gauge field is proportional to the induced Kähler form, and can define a coherent field in arbitrarily long scales.

I gave up this proposal a long time ago but it seems that this proposal had some seed of truth in it. Anyonic electrons replace quarks and antiquarks.

1. What comes in mind first is that the DNA strand and its conjugate involve, besides dark nucleon triplets, also dark quark/antiquark triplets forced by the time-likeness of the braiding regarded as a physical state in ZEO. This however leads to problems since dark nucleons are strongly favored. Doubling of the genetic code without need for it looks ugly. The mere quantum gravitational modification of the standard chemistry should be enough.

   Most importantly, tachyonicity does not require single quark states. Also the dark anyonic electrons could be virtual particles carrying tachyonic momenta. The 3+3 dark electrons assignable to the asymmetric HBs of form O..H-N would provide electronic realization of the genetic code. The dark codons would serve as names, addresses in the symbolic dynamics of TQC involving the resonance mechanism of communications requiring addresses.

   The dark anyonic electrons assignable with G-C bonds would carry tachyonic momenta and make the braiding possible. The tachyonic electronic momenta assignable to bonds symmetric O...H-O type bonds connecting water molecules and phosphates would be realized in the same way.

2. It is good to bring in mind the possible weak points of the scenario once again. Dark protons are strongly suggested by the Pollack effect and the proposed picture about dark gravitational HBs with delocalized dark protons [L103]. In the original view, dark protons screened the negative charge of phosphates. In the new picture the negative charge of phosphate is assignable with bi-localized (anyonic/dark/virtual) electrons of O...H-O +O-H...H: at the level of ordinary matter, DNA is not negatively charged. In QFT language, one might perhaps say that a dark electron is exchanged between the ends of the flux tube associated with the dark HB.

Connection with time-like character of music experience and cognition

A connection with the model of DNA based on bioharmony is suggestive.

1. DNA and RNA codons are identified as points at the orbits of icosahedral and tetrahedral subgroups of quaternion automorphisms. Amino acids (AAs) have been identified as orbits of the icosahedral and tetrahedral groups, which are discrete subgroups of quaternionic automorphisms, which is completely analogous to Galois groups.

2. Harmony is the basic element of music and music involves time in an essential way. Same is true of cognition. Perhaps the time-like braid strands could give a concrete content to

the proposal. Codons would correspond to 3-chords and gene would correspond to a piece of music in a much more concrete way than originally proposed. Genes would also represent primitive cognitions.

12.2.4 Cognitive measurement cascades as counterparts of measurements of anyon charges

The measurements of topological charges reduce the tensor products for the representations of the braid group to irreducible representations. What would the counterpart for this process be at the level of the NQC?

1. I have discussed cognitive measurements [L11, L82] as a cascade of "small" state function reductions (SSFRs) for the irreducible representations of the Galois group of extensions of extensions of.... . The full Galois group has a representation as a product of relative Galois groups $R_n = Gal_n/Gal_{n-1}$. The SSFR cascade means a reduction of the representation to a product of representations of the relative Galois groups R_n.
2. This measurement cascade would be the opposite for the measurement of anyonic topological charges involving an analogous decomposition of the tensor product of representations to irreducible representations of the full braid group.

   In ZEO, the counterpart for the measurement of topological charges would correspond to the time reversal of this process starting with BSFR, which creates a completely entangled state as the representation of the full Galois group, and is followed by SSFR cascade proceeding in an opposite time direction. The formation and decomposition of tensor products would occur in different time directions.

12.2.5 Comparison of standard view about TQC with the TGD view

It is useful to compare the standard view about TQC with its TGD counterpart.

1. Qubits as states are replaced by representations of the braid group characterized by the value of the topological charge and of the quantum group G assignable to the Chern-Simons action.

   Quantum groups [A89, A27, A45] are discussed from the TGD point of view in [K15] and in chapters about possible role of von Neumann algebras known as hyperfinite factors of type II_1 in TGD [K127, K48].

   Quantum group $SU(2)_q$ quantum group characterized by quantum phase $q = exp(i\pi/k)$, $k = 5$, is the simplest option. One can say that anyons correspond to electrons assignable to the orbits of 2-D systems, whose time evolution could be described by Chern-Simons action.

   In TGD, these 3-surfaces would correspond to the light-like orbits of partonic 2-surfaces which for larger values of h_{eff} can have rather large size. For $h_{gr} = GMm/v_0$ the gravitational Compton length for a particle with mass m is $GM/v_0 = r_s/2v_0$ independent of the mass of the particle and for Earth this gives .45 cm for $v_0 = c$, one half of the Schwartschild radius.
2. Topological qubits correspond to topological charges such as the already mentioned parity for condensed matter Majorana electrons, which would have degenerate energies because they correspond to momentum vectors k and $-k$ differing by lattice momentum.
3. Quite generally, quantum measurements are Hilbert space projections. Measurement of qubit corresponds to a measurement of a topological charge. The qubit can be measured by a fusion process for the representations of the gauge group G. Fusion means a formation of a tensor product of representations and could result as a final state of TQC. Measurement means a projection to a particular representation characterized by a topological charge.

   One can also consider the opposite operation in which one decomposes a given representation to a direct sum of product representations and projects out one particular product representation by measuring topological charges for the composites.
4. Fibonacci TQC with quantum group $SU(2)_q$ for quantum phase $q = exp(i\pi/5)$, serves as the simplest candidate for an interesting TQC. Condensed matter Majorana fermions could correspond to Fibonacci anyons with $q = exp(i\pi/5)$ (`https://phys.org/news/`

2014-12-fibonacci-quasiparticle-basis-future-quantum.html). The fusion for Fibonacci anyons is non-commutative and non-associative. These properties are coded by a non-commutative R matrix and non-trivial F matrix (see Appendix). For a fusion of N representations the number of degenerate ground states is N:th Fibonacci number.

This has a counterpart in TGD.

1. In the TGD framework, Galois group elements in general change the value of cosmic time as a real part of the root of the polynomial defining the mass shell in M^8 and its image in H. Therefore the associated virtual quark states are not energy degenerate.

   That mass squared values for anyons are different conforms with the idea of time-like braiding as a genuine quantum state rather than time evolution of quantum state, which is natural in ZEO. One can of course challenge this assumption. For states containing N particles with the same polynomial P and represented as an iterate $P \circ .... \circ P$ mass squared values as roots approach to Julia set for P, and this could give rise to approximate degeneracy of mass squared values and corresponding values of light-cone proper time a.

   One can also consider a situation in which one has several roots with the same real part (say roots of a second order polynomial). One can ask whether the analogs of condensed matter Majorana fermions correspond to these kinds of states.

2. The topological structure in question would be realized in terms of the space-time topology as a monopole flux tube not possible in Maxwellian electrodynamics. Also the strings assignable to the flux tubes and corresponding string world sheets as representation of time-like braiding inducing space-like braiding would play a key role. Chern-Simons action would be assigned to the light-like 3-surfaces defining the orbits of partonic 2-surfaces and string world sheets would connect these orbits.

3. The quaternionic automorphism group, defining the analog of the Galois group and having $SU(2)$ or its quantum variant as a covering group, serves as the analog of the gauge group G and acts in the normal space of the space-time surface. Discrete and finite subgroups assignable to the Platonic solids and regular polygons define the natural finite discretizations of this group.

   The braid group could be replaced with a subgroup identifiable as the Galois group for an extension of rationals or for extension of extension of rationals. Also this group can be non-Abelian and would be naturally represented as a subgroup of the braid group.

4. Time reversed fusion corresponds to a cognitive measurement cascade consisting of unitary evolutions followed by SSFRs as counterparts of "weak" measurements. Cognitive measurement cascade and its reversal are initiated by a BSFR changing the arrow of time. Two subsequent BSFRs would correspond to fusion and its reversal and the time evolution between them would correspond to the braiding as a unitary evolution. In TGD inspired theory of conscious experience, the sequence of SSFRs gives rise to the flow of consciousness.

5. Quantum group $SU(2)_q$ for Fibonacci TQC has an interpretation as quantum automorphism. What makes this biologically highly interesting is that the twist $exp(i\pi/5)$ is realized geometrically in the structure of the DNA. This suggests that DNA and dark DNA could involve TQC. One can wonder whether genes with N codons correspond to a fusion of N Fibonacci representations.

12.2.6 Could the MB of DNA perform intentional TQC?

In TQC and also in AI as human endeavours, human intention plays a key role. This fact has been often forgotten by AI extremists. The braiding defining the TQC would be constructed using technological tools developed by humans. What about the situation at the level of DNA based TQC? Could the MB of DNA play the role of humans to some degree? What kind of quantum computations could the MB of DNA perform?

1. When the braid ends can participate in the flow defined by cellular water or by 2-D liquid defined by the lipids of the cell membrane in liquid crystal phase, one can consider the possibility that the MB induces this flow and in this way builds time-like TQC program, which is also stored as spatial braiding to memory.

As will be found in the next section, this situation would be true for braids possibly defined by the gravitational flux tubes connecting the oxygens of phosphates of DNA with the lipid ends of nuclear or cell membrane containing also phosphates. Also the GTPs and GDPs of microtubules contain phosphates and their oxygens could be connected with those of lipid phosphates.

The braiding would serve a memory storage purpose. If MB can induce the flow of water or of lipids, one can say that it can build TQC programs. For instance, a representation of function involving two registers could be constructed by starting from entangled register and using the flow of water or lipids to induce the needed braiding for the second register implying the entangled state $\sum |n\rangle\langle f(n)|$. The TQC ending with cognitive state function reduction cascade would define a conscious cognitive representation of the flow.

2. It will also be found that A-G base pairs by the N...H-N $\leftrightarrow$ N-H...N symmetry of gravitational flux tubes define candidates for HBs assignable to TQC. In this case the braid ends cannot move but the reconnections of braid strands could produce braiding and TQC. Similar situation is true for the sequence of identical DNA codons of, say, genes. They could define an orbit of the Galois group and give rise to its representation. There would be 63 types of orbits which could decompose to separate representations corresponding to various codons. Besides single electron states also many electron states would be possible.

   In this kind of situation, the cognitive measurement cascade would give rise to a conscious cognition at DNA level. In ZEO, reconnections would be forced by the preferred extremal property and unavoidable by the 4-D character of the space-time surface. Therefore they would reflect the underlying physics. The failure of the strict determinism could be interpreted as a selection between a finite number of alternatives at the frames defining the space-time surface as a 4-D analog of soap film. The analog of TQC would give rise to a sensory perception accompanied by cognition.

Factorization of integers into primes is one of the most interesting applications of QC. At first, it looks unlikely that the MB of DNA could be able to do something like this. However, finite groups have a prime decomposition to a product of finite groups and in the same way Galois groups have a decomposition to a product of relative Galois groups, which do not have a similar decomposition.

Group theoretical prime decomposition is analogous but more general than the prime decomposition of integers and more general composition of algebraic numbers to algebraic primes. Since groups with a prime number of elements are certainly prime groups, prime factorization would follow as a consequence and would be a side product of any cognitive SSFR cascade. This conforms with the paradoxical finding that idiot savants, who do not have any idea about the notion of prime, can factorize large integers [L26].

Could Quantum Fourier Transform (QFT) have any analog at the level of DNA? The states in the irreps of the Galois group serve as candidates for the plane waves defining Fourier components. Could cognitive measurements naturally involve a measurement of these quantum numbers as eigen values for maximal set of commuting Galois group elements acting as a minimal Galois transformation. For instance, a rotation by $exp(i2\pi/n)$ would be analogous to this kind of transformation in Z_n. These measurements would induce a localization to a single Fourier component and repeated measurements of the same state would give the probabilities of various Fourier components. These states are superpositions of states at mass shells with varying mass squared and involve time delocalization making sense by the finite non-determinism. A repeated measurement of Galois momenta would make it possible to find the factors of an integer as in the ordinary QC.

12.3 DNA as quantum gravitational TQCer?

In this section a detailed model for DNA as a TQCer will be developed. The attribute "quantum gravitational" is not necessary since also smaller values of h_{eff} than h_{gr} can be considered.

12.3.1 Concrete questions concerning DNA TQC

Before representing a concrete model for TQC using Galois anyons as qubits, the basic questions are discussed.

How could DNA qubits be realized physically?

For TQC temperature topological charge identifiable replaces spin as qubit. In the TGD framework Galois charges replace topological charges and one can talk about Galois anyons.

The basic question is how DNA makes it possible to realize anyonic qubits.

1. Dark nucleons associated with dark DNA codons, that is with O...H-O type HBs cannot realize dynamical qubits in terms of spin because the codons must be fixed if they are to represent genetic code. Only in the communications based on resonant cyclotron transitions their states can temporarily change but should return back to the original state as a state of minimum (free) energy.

   One can assign to A-T, G-C pairs 1+1 asymmetric HBs, which do not allow electronic anyons. This gives rise to 3 +3 dark electrons, which could give rise to dark representation of the genetic code.

   The tentative interpretation is that the dark codons define the analog of computer hardware with a fixed ROM. The dark codons would serve as addresses in the resonance mechanism: the analogy with LISP is obvious.

2. The dynamical working memory should correspond to an anyonic realization of qubits. A dark electron associated with the quantum HB of type X...H-X +X-H...X can give rise to two bi-localized states with odd and even Z_2 parity where Z_2 exchanges the ends of HB. These two dark electron states could serve as anyons.

   This could work for electrons of O...H-O bonds between the oxygens of phosphate and water molecules. This could be also the case for the N...H-N bond of C-G base pair, which is symmetric. The HB can be assigned with C codon. In this case, the notion of Z_2 anyon makes sense and could make possible TQC using gravitational variants of symmetric HBs of C-G base pairs (`hhttps://cutt.ly/WGNddJ3`).

How could the unitary time evolution be realized?

Superpositions of HBs of type X...H-X + X-H...X could give rise to electronic anyons with bi-localized dark electrons. Depending on the situation, braiding or reconnections having, at least apparently, the same effect would define the unitary gates.

1. If the molecules containing X can move, braiding is possible. This is the case if the HBs are associated with the phosphates of lipids of the cell membrane forming a liquid crystal and connect them to the molecules of the cellular water. In the sol phase for intracellular water, the flow of water molecules could define braiding.

   The original proposal [K4, K123] was that the flux tubes connecting the oxygens of the phosphates associated with the DNA strand with the phosphates of the lipid ends would define TQCer. The flow of the lipids of the lipid layer forming a 2-D liquid could define a braiding and thus TQC program. For gravitational flux tubes this option could make sense. The oxygens of the phosphates of DNA could be also connected with the molecules of the water surrounding the DNA if they can move.

   In this case, the dance metaphor makes sense: the TQC as time-like braiding produces a log file as a spatial braiding.

2. For N..-H-H + N-H...N HBs of C-G base pairs the nitrogen atoms cannot move. The reconnections of dark braid strands could produce the same effect as braiding and induce flux tube connections between C:s and G:s belonging to distinct C-G pairs. For gravitational flux tubes these connections could be very long.

   String word sheets are fundamental objects in TGD and by the 4-dimensionality of the space-time surface, 2-D string world sheets at flux tubes representing the orbits of space-like braids intersect at a discrete set of points and for preferred extremals the reconnections are forced by topology. The non-determinism is associated with the choice whether the time-like strand pair AC+ BD transforms to AC+BD or AD+BC.

What about ordinary QC or TQC using electron spin of HB as qubit?

I do not understand TQC enough to say whether electron spin could also appear as a qubit when braidings and reconnections define the gates. In any case, this option meets the same objections as the QC option since a very low temperature would be needed in the standard physics framework.

1. The hyperfine splitting (`https://cutt.ly/oGNdeA3`), causing the 21-cm line of hydrogen, corresponds to the magnetic interaction energy of nuclear dipole moment with electron's magnetic field and is proportional to h_{eff}. The energy of hydrogen hyperfine splitting is $\Delta E = 5.89 \times 10^{-6}$ eV. This corresponds to a temperature of 5.89×10^{-2} K. If the electrons are dark, the energy of hyperfine splitting is proportional to h_{eff}. The energy is above thermal energy at room temperature for $h_{eff}/h > 5 \times 10^3$.

   Note that the temperature T at the MB of DNA is assumed to be very low but during aging identified as an approach to thermal equilibrium with the biological body T is assumed to increase and approaches the Hagedorn temperature assignable to the flux tubes of MB [L124].

2. If spin serves as a qubit, the manipulation of electronic qubits by changing their spin direction using photons or braiding or reconnection, which at least apparently seems to have the same effect as braiding, would be needed. Both braiding and reconnection involve the replacement $A \to C + B \to D$ with $A \to D + B \to C$ but reconnection involves temporary touch of the braid strands which might have some effect.

12.3.2 Number theoretical generalization of Kitaev's proposal

Kitaev [B7, B6] has proposed an elegant model for TQC using as qubits the two states of condensed matter Majorana fermion [D29] with two bi-localized states, which have parities +1 and -1 under Z_2 symmetry.

Galois group as subgroup of braid group and Galois anyons

In the TGD framework, the representations of the Galois group would naturally replace these representations and one could speak of TQC which is also number theoretic as far as anyon-like states are considered.

Topological robustness would be replaced by number theoretical robustness due to the fact that the extension of rationals depends only weakly on the polynomial: this is obvious from the fact, the number of extensions is finite for a polynomial of given degree. M^8-H duality [L73, L74] indeed implies that a given space-time region is determined by a polynomial. In QFT approximation one is forced to replace many-sheeted space-time with ordinary space-time and the nice picture is lost. One might however hope that in TQC this loss is fatal.

1. Galois group replaces Z_2. Instead of topological charges, one can speak of number theoretical charges. Representations of the Galois group would correspond to number theoretical qubits. Number theoretical anyon would be identified as a superposition of states localized at points of orbit of Galois group Z_2 associated with DNA double strand.

   As already found, the Galois ground state corresponding to $h_{eff} = h = n_0 h_0$ is not completely unique but would naturally correspond to a polynomial $P_g = Q_g \circ P_2$ where P_2 is second order polynomial, all roots of $P = P_1 \circ P_g$ appear in pairs $x \pm y$ and Z_2 permutes the members of the pairs. Fibonacci polynomial $P_F = X^2 - x - 1$ is highly attractive candidate for P_2 and would give the roots $(1 \pm \sqrt{5})/2$ as roots of all polynomials P. Also the twisting geometry of DNA favors Fibonacci TQC, which is also the minimal option.

2. Hydrogen bonds X...H-X and X-H...X are symmetric and their possibly gravitationally dark variants, could give rise to states with opposite parity. The electron of the hydrogen could define the number theoretic anyon.

3. The gravitational flux tubes as counterparts of H-bonds could define the braid strands but alsos smaller values $h_{eff} \geq h$ assignable to electromagnetic flux tubes could work. Braiding would take place for these strands.

4. What about the protonic option for X...H-X type HBs based on the identification of anyons as delocalized states of the dark proton with opposite parity? Also now one can consider a

superposition of N-H...N and N...H-N gravitational bonds and two different parity states with respect to Z_2. The quantum gravitational model for the metabolic energy quanta however suggests that the dark proton is localized mostly in the interior of the gravitational flux tube so that the dark proton should not have a large amplitude at the ends of the flux tube.

Hydrogen bonded structures of type X...H-X populate living matter. Water and DNA and the first examples that come into mind.

1. The hydrogen bonds between water molecules are of type O..H-O. Hydrogen bonded water molecule clusters could give rise to multiply localized anyonic states of electrons and serve as TQCers.
2. The HBs of the oxygens of phosphate atoms with oxygens of water molecules allow poly-localized electrons if the HB is superposition of O-...H-= and O-H...H. This would allow to associate electronic anyons and TQC also with the dark nucleon triplet codons, which cannot have dynamical spin.
3. G-C base pair has one N..H-N type HBs (`hhttps://cutt.ly/WGNddJ3`). N-H...N $\leftrightarrow$ N...H-N are could be possible for $h - eff > h$ HBs, and could lead to the delocalization so that one could assign anyonic state with Galois Z_2 symmetry with it. The G-C base pairs of the DNA double strand could define a sequence of topological qubits. Note that the splitting of the N-H...N bond in the G-C base pair leading to N + H-N is known to occur during DNA transcription and replication and also in the temporary splitting of the HB [L103].
4. Benzene allows delocalized states of electron pairs, which could be poly-localized and be analogous to Z_6 anyons. Also Z_2 and Z_3 anyons can be considered. The atoms of the aromatic ring could be connected by flux tubes with $h_{eff} > h$ and perhaps even $h_{eff} = h_{gr}$. In DNA , the sequences of the aromatic 5- and 6-rings, possibly defining Z_5 and Z_6 anyons, could give rise to a delocalization of the anyonic states along DNA strands possibly involving gravitational analogs of valence bonds.
5. In DNA strand nucleotides A and G contain aromatic 5- and 6- rings glued together whereas T and C contain aromatic 6-ring (`hhttps://cutt.ly/WGNddJ3`). The members of base pairs contain fused 5- and 6-ring and 6-ring respectively. One can wonder whether the Galois representations associated with these structures in the double DNA strand structure could make possible TQC. Also the side chains of amino acids Phe, Tyr, and Trp contain aromatic rings and HBs between oxygens of water molecules might be relevant for information processing at, say, microtubular level.

The non-symmetric HBs of base pairs and possible new dark realizations of the genetic code

The symmetric HBs of C-G base pairs (`hhttps://cutt.ly/WGNddJ3`) would be in a very special role. What about the remaining non-symmetric HBs associated with codons?

1. Besides N..H-N HB there are 3+3 electrons per codon with asymmetric HB of form X..H-Y, with X,Y= O,N or N,O. The proposal that an electronic variant of metabolism is realized, leads to the question of whether the spins of these 6 electrons could realize genetic code as a 6-bit code. Now only the analogs of DNA codons would be realized.
2. For asymmetric HBs, anyonic dynamics for electrons is not possible but the electronic dark codons could serve as addresses in the resonance mechanism of communication based on the transformation of Josephson radiation to pulse sequences by cyclotron resonance [L110, L103]. This is possible if the electrons are dark so that the energy of the hyper-fine splitting is scaled so that it is higher than thermal energy. This would require $h_{eff} \geq 50$.

   One can also imagine resonance-based communications between dark electron 6-plets and dark nucleon triplets using dark photons.
3. The dark proton at flux tube and dark electron at the hydrogen end could define an analog of dark H atom. Dark H would have 4=3+1 spin states with spins 1 and 0 and these states could define the analogs of nucleotides in 1-1 correspondence with A,T,C,G. C as a special codon would naturally correspond to the spin singlet. Hyper-fine splitting for this dark atom

would distinguish between triplet and singlet. For large h_{eff} the energy this splitting would be above thermal energy so that the spin configurations would be stable.

These observations challenge the details of the earlier view [L110] about the genetic code.

1. The dark nucleon realization of the genetic code [L110] predicts both DNDA, DRNA, DtRNA, and DAAs. One can criticize the realization since also neutrons are required.

   The model of the code has several variants but the most recent model [L103] requires dark variants of both neutron and proton residing at the gravitational flux tube defining gravitational HB connecting the oxygens of phosphate and water. The charge of the delocalized dark proton would not be visible in the scale of DNA so that its replacement with dark neutron would not affect the situation in this scale.

   Dark protons would be generated from ordinary protons in Pollack effect [I68, I53, I86, I81]. They could transform to dark neutrons by the dark variant of strong interactions or of weak interactions at the gravitational flux tubes. Dark weak interactions could be realized in even cellular scales and imply that dark variants of weak bosons are massless in the scales below the dark Compton length of weak bosons. This would explain chiral selection of biomolecules difficult to understand in the standard model.

   The conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) [K120] relate the descriptions of hadrons in terms of strong and weak interactions, which suggests that these views might provide dual descriptions. The duality might in fact reduce to $M^8 - H$ duality. The interpretation of anyonic electron as a color octet electro-pion [K120] involving color octet meson-like state associated with the gravitational flux tube was already discussed.

   If HB is associated with oxygen of phosphate (water molecule), the hydrogen of phosphate (water molecule) would look negatively charged. For anyonic states the electron of H would spend half of the time near the two oxygens involved implying that negative charge would be delocalized in a longer scale.

2. Could the standard genetic code be associated with the electron triplets at HB associated with base pairs rather than with the phosphate water HBs? One can imagine two realizations.

   (a) For both dark DNA strands, both dark proton triplet and dark electron triplet would have 2^3 dark entangled states and together they would combine to form 64 states. Could they provide a dark realization of the genetic code consistent with the chemical genetic code?

   (b) Could the dark protons at the HBs associated with base-pairs pair with dark electrons at their ends give rise to analogs of dark H atoms? This could give 64 states perhaps allowing an interpretation as a dark realization of genetic code.

   There are objections against both proposals. The counterparts of RNA,tRNA, and AAs are not predicted so that the correspondence with the chemical realization of the genetic code is not plausible. Dark codons would have integer spin varying from 0 to 3 and the code table does not show any grouping of codons to these multiplets containing an odd number of states.

To sum up, it would seem that several realizations of the genetic code are possible as indeed suggested by the proposed universality of the genetic code [L89, L110].

Could protonic and electronic anyons define a pair of registers?

Two registers are needed to represent a Boolean function $x \to y = f(x)$ in terms of entanglement (see Appendix). n qubits represent the values of x and y. The simplest representation of f is as a maximally entangled state $\sum |n\rangle\langle f(n)|$. In this representation quantum Fourier transform (QFT) is exponentially faster than the ordinary fast Fourier transform. Also the quantum counterparts of number theoretic algorithms such as finding prime factors and greatest common divisor are faster than their classical counterparts.

How could one realize these registers in the recent case? There should be a natural interaction inducing the entanglement between qubits. The realization of the genetic code fixes the states of dark proton and electron triplets completely for a given codon so that these qubits are non-dynamical.

In the case of HBs of type X...H-X, this however leaves the anyonic degrees of freedom assignable to the dark electron as Z_2 degeneracy and perhaps also with dark protons as a similar degeneracy. The entanglement between electronic and protonic anyons would commute with the spin degrees of freedom. Could the two registers correspond to electronic and protonic anyons? Could the braidings of the flux tube, possibly induced by reconnections, generate entanglement between these anyons? The objection is that the anyonic dark protons would not be delocalized in long scales as the model for metabolic energy quantum requires. The metabolic dark proton states would correspond to different states concentrated near the top of the gravitational flux tube.

12.4 Appendix: Basic concept and ideas of quantum computation

I am not a specialist in quantum computation and since some readers might also have the same problem, I have added some remarks about QC, which I believe to be relevant for this article. I have discussed the TGD view about TQC for about 13 years ago [K5, K4, K123]. These chapters reflect my views at that time and a lot has happened in the TGD based view of quantum biology after that. Perhaps I also have a little bit deeper understanding of TQC now.

12.4.1 About key ideas of QC

In the following the basic ideas QC and TQC are briefly described.

Gates as unitary transformations

Quantum computation can be seen as circuits consisting of gates, which realize unitary transformations assigning to n incoming qubits $n = m$ outgoing qubits: unitary forces $m = n$. For qubits, which reduce to ordinary bits one obtains as a special case Boolean functions from n to n bits.

Unitarity forces $m = n$ but by using control qubits for which nothing happens in the gate but the outcome from the remaining qubits depends on the value of the control qubit, one can realize also gates which for bits reduce to Boolean maps from n bits to a smaller number of bits so that ordinary logic circuits can be realized as a special case.

n-gates with $n = 1, 2, 3$ are enough for obtaining a universal set of gates. The interested reader can learn details from the slides of Viterbi: for instance the slides at `https://cutt.ly/EGNsmcR` describe Quantum Fourier Transform.

1. 1- port represents a unitary transformation of a single qubit.
   (a) Phase gate, Hadamard gate and rotations by Pauli spin matrices are basic gates of this kind. Discrete rotation as $SU(2)$ transformation represents the general unitary transformation. Rotation is specified by two orthogonal rotation axes and by 3 rotation angles.
   (b) Discrete subgroups of rotation group assignable to Platonic solids and regular polygons define especially interesting selections for the set of possible quantization axes and for the possible directions of spin representable as a point of Bloch sphere. For Platonic solids the subgroup of $SU(2)$ is discrete. These subgroups can produce unitary transformation in a finite accuracy only but one can consider the possibility of transformations obtained as products of elements of these subgroups.
   (c) Quantum variant of $SU(2)$ emerges in TQC and also the braid group defines a quantum variant of the permutation group as a finite covering of the braid group. The gates in topological computation correspond to the elements of the braid group. In the TGD framework, $SU(2)$ has a representation as the covering of the automorphism group of quaternions (analogous to Galois group) acting in the normal space of the space-time surface.
2. Arbitrary $N \times N$-D unitary transformation can be constructed as a product of 2-D unitary transformations. In the $N = 2^n$ case, the transformation can be represented at qubit level and using control gates one can represent unitary transformations by using qubit representation with $N < 2^n$.

The representation of a general unitary transformation in dimension n requires of order $n2^n$ gates. The subset needed as unitary transformations is however believed to be much smaller than all possible transformations.

3. Swap, which permutes subsequent incoming qubits and CNOT are examples of 2-gates.
4. The notion of controlled gate generalizes to n qubits. Toffoli gate as CCNOT defines a 3-gate and together with 1- and 2-gates it defines a universal set of gates.

Bloch sphere and Platonic solids

Block sphere gives a parameterization for the directions of the spin quantization axis and spin has two directions for a given quantization axis. In the twistorialization of TGD at the level of M^8_c this interpretation of the twistor sphere is natural [L73, L74].

1. In the number theoretic vision these directions correspond to sines and cosines and in the number theoretic vision these must belong to the extension of rationals considered assignable to a given space-time region. This discretization can be interpreted in terms of finite measurement resolution.
2. The allowed quantization directions are obtained from each other by the transformations of the rotation group $SU(2)$. If these rotations form a finite group, only the symmetry groups of Platonic solids and regular polygons are possible. For Platonic solids there are 4, 6, 8, 12, and 20 quantization axes corresponding to tetrahedron, octahedron, cube, icosahedron and cube.

Some applications of QC

Examples of the applications of QC working faster than their classical counterparts are discussed in the Wikipedia article (`https://cutt.ly/8Hs5qdG`). For instance, the following examples are discussed.

1. A very simple application is the finding of the inverse image of function by measurement the of value of function $f = f(n)$ for $\sum |n\rangle\langle f(n)|$ giving the superposition $\sum |n\rangle\langle f(n) = y|$.

   In a more general case this localization gives the inverse image of a map f of m-D discrete space to n-D discrete space. The repeated application of this algorithm can be used to find the boundary of a region of the inverse image of f.

2. Quantum Fourier transformation calculates a discrete Fourier transformation exponentially faster than ordinary fast Fourier transform. Other related applications find a prime factor of integer, period of a periodic function represented as an entangled state $\sum |n\rangle\langle f(n)|$ of two registers as, and number theoretic logarithm.

   Quantum Fourier transform (QFT) is discussed (`https://cutt.ly/EGNsmcR`) takes place exponentially faster than the classical fast Fourier transform. For $N = 2^n$ qubits the number of computation steps is $O(n)$ whereas classically it is $O(n2^n)$. The discrete Fourier transform has a huge number of both physical and number-theoretical applications.

   QFT can be represented in terms of n qubit registers as an un-entangled product of states of n qubits and this state can be constructed using only gates inducing phase rotations $R_k = ep(i2\pi/k)$ of qubits, Hadamard gates producing the superposition of 0 and 1, and control gates.

3. There is an algorithm calculating the phase produced by a unitary transformation: this algorithm involves one additional qubit, whose phase is opposite.
4. There is a search algorithm, which increases the probability of the searched integer before localization in discrete space defined by integers. The number of trials is $O(\sqrt{N})$ whereas classically it is $O(N)$.
5. Error correction algorithms localizing the logical qubits relevant for the computation to a subspace of logical qubits. These algorithms detect the error by using parity qubits and correct the error by action of a unitary gate in the case that the number of errors is below a given number.

Finding a period of a periodic function

One assumes that the function $f(n)$ is periodic but the period is not known. The entangled state of the registers is $\sum |n\rangle\langle f(n)|$.

1. One assumes that one has measured $y = f(x)$ and has obtained $\sum |n\rangle\langle f(n) = y|$. If f is periodic, one obtains a superposition of points $n_0 + nr$, where n_0 is the offset and r is the period, which should be measured.
2. A QFT is performed for the input register. One obtains a superposition for states with momenta mN/r.
3. The measurement of momentum this state gives momentum state with momentum $p_m = mN/r$ for some m, which is however unknown.
4. The operation is repeated. This gives a series of outcomes $m_1, m_2, m_3, ...$ Eventually the minimum value of momentum corresponds to $m = 1$.

12.4.2 About key ideas and notions of TQC

It is appropriate to briefly recall the basic ideas and concepts of TQC [K5].

Topological gates and qubits

The topological stability of braiding guarantees that the TQC program coded by the braiding is robust against perturbations. If qubits were spins, there would still be the instability of qubits and entanglement caused by the interaction of spins with the environment, in particular thermal instability.

1. Qubits as spins are replaced by representations of the braid group characterized by the value of the topological charge and the quantum group G_q assignable to the Chern-Simons action. The quantum group $SU(2)_q$ is the simplest option. Topological charge replaces spin as qubit. One can say that anyons correspond to electrons assignable to 2-D topological structures, whose time evolution as 3-surfaces could be described by Chern-Simons action.

   The mathematics of quantum groups [A89, A27, A45] is discussed from the TGD point of view in [K15] and in the chapters about the possible role of von Neumann algebras known as hyperfinite factors of type II_1 (HFFs) in TGD [K127, K48]. Quantum groups would be assigned to the inclusions of HFFs characterizing the finite measurement resolution. Cognitive representations are an alternative way to describe the finite measurement resolution.
2. Topological qubits correspond to topological charges such as the already mentioned parity for the condensed matter Majorana electrons, which would have degenerate energies because they correspond to momentum vectors k and $-k$ differing by lattice momentum.

   The idea of Kitaev [B7, B6] [D29] is to use anyons as topological qubits instead of spin. The condensed matter Majorana electrons bi-localized at the ends of superconducting wire have two states with opposite parities associated with the exchange of the ends of the wire. These states with degenerate energies would serve as qubits, which would be much more stable than spins.
3. Topological approach allows to realize gates in terms of braiding operation. Braid group B_N as a covering of the permutation group of N braid strands would define the allowed unitary transformations induced by braidings. This implies finite accuracy but the increase of the covering improves the accuracy.

   This allows to overcome the problem of the Hamiltonian approach in which the gate Hamiltonian defining the unitary transformation must be "on" for a very precise time ΔT. It is not easy to arrange this by external interaction. A possible way to avoid this altogether is to assume a permanent Hamiltonian but allow the qubit system to move with a fixed velocity past the Hamiltonian system with a velocity, which gives the desired ΔT.
4. Non-abelianity is required since the manifold of the energy degenerate states in which the braid group would act, is determined by states and must be a higher-dimensional representation of the braid group in order to give rise to a large enough number of logical qubits. There exist no well-established candidate for the needed non-abelian anyon yet.

R and F matrices

R- and F matrices are central notions in TQC (https://arxiv.org/pdf/2005.03236.pdf) and characterize what happens in the fusion of the representations of quantum groups. These matrices are believed to characterize quantum phases as topological orders and were discovered in 2-D fractional quantum Hall systems.

1. Fusion corresponds to a tensor product, which is commutative and associative for ordinary group representations. For quantum groups and braid groups, the discrete group elements are replaced by flows in plane so that the situation changes. The commutativity of the product ab of the representations is lost and associativity for the product $a(bc)$ of three representations is only modulo unitary transformation: $a(bc)$ is equal to $(ab)c$ only modulo unitary transformation.
2. R matrix characterizes the braid operation, swap, in which the two braid strands are permuted by flow-like continuous transformation. Braiding as an element of B_N replaces the discrete permutation of adjacent braid strands as an element of S_N. The R-matrix characterizes the effect of the braid operation and reduces to a phase in the abelian case but is a genuine matrix in the physically more interesting non-Abelian situation.
3. F matrix characterizes the associativity modular unitary transformation for fusion operations. The F matrix is trivial for the ordinary tensor product . This means that the fusions $a(bc)$ and $(ab)c$ produce different states but do not change the state-space. F-matrix $F(a,b,c)$ relates these two states as a unitary transformation in the tensor product of the 3 state spaces.
4. Fibonacci quantum computation with quantum group $SU(2)_q$ for quantum phase $q = exp(i\pi/5)$ represents the simplest example of a non-commutative situation (https://phys.org/news/2014-12-fibonacci-quasiparticle-basis-future-quantum.html). For a fusion of N representations the number of energy degenerate ground states is N:th Fibonacci number.

Chapter 13

The Possible Role of Spin Glass Phase and P-Adic Thermodynamics in Topological Quantum Computation: the TGD View

13.1 Introduction

Topological quantum computation (TQC) or more generally, a TQC-like process, is one possible application of TGD (for simplicity, I will talk in the sequel of TQC rather than TQC). The interested reader can consult the earlier TGD inspired work in TQC [K5, K4, K123]. The recent rather concrete model for TQC in living matter utilizing quantum gravitation in the TGD sense see [L106].

13.1.1 Basic ideas of TQC according to TGD

There are several new ideas involved [L106].

1. Braidings are represented by monopole flux tubes, which are structures distinguishing between TGD and Maxwellian electrodynamics and are one of the basic implications of the many-sheeted space-time concept. Time-like braidings as TQC programs can be engineered as a flow for the nodes of the flux tube network and they induce spatial braidings as memory representations of the time-like braidings - kind of topological Akashic records.

   The engineering of the flow involves what might be called quantum hydrodynamics [L95]. DNA based TQC would utilize the flow of 2-D liquid crystal defined by a lipid layer of cell membrane to generate braiding [K4].

2. The hierarchy of effective Planck constants, assumed to label dark matter as phases of ordinary matter, predicts quantum coherence in arbitrarily long scales and Negentropy Maximization Principle (NMP) [K72] favors the generation of negentropic entanglement (NE). NE makes sense only in adelic physics [L38, L39] and allows to understand second law as a side effect of the NMP.

   The point is that one can assign to the same entanglement both the ordinary real entanglement entropy and the sum of p-adic variants of entanglement entropies. The sum of two can be negative and the interpretation in this case is as negentropy. NMP tends to make the negentropy positive. The decrease of the negative p-adic entropy would force the increase of real entropy. This view [L8] conforms with the vision of Jeremy England about living systems [I80].

3. $M^8 - H$ duality as a generalization of momentum-position duality of wave mechanics is a central notion on the number theoretic view of TGD providing a view dual to the geometric view. The complexified M^8 has an interpretation as complexified octonions.

   The roots r_n of rational polynomials P of real variable algebraically continued to complecified octonionic polynomials define 3-D mass shells (hyperbolic spaces H^3) $m^2 = r_n$ of $M_c^4 \subset M_c^8$. The mass shells define holographic data for the continuation of these 3-surfaces to 4-D surface X^4 of $M_c^4 \subset M_c^8$.

   Dynamics is dictated by the associativity of the normal space of X^4. Associativity in turn makes it possible to map X^4 to a 4-D space-time surface of $M^4 \times CP_2$ by $M^8 - H$ duality.

4. Cognitive representation is a second central concept: one might call it intersection of reality and p-adicities regarded as correlates for ideas and imagination. Originally, the non-determinism of p-adic differential equations motivated this notion.

   Cognitive representation defines a unique discretization of the space-time surface involving a hierarchy of extensions of rationals associated with rational polynomials defining space-time regions via $M^8 - H$ duality. For mass shells cognitive explosion takes place and the representations contain almost all algebraic in rationals. Physical motivations force restriction to algebraic integers and the condition that active points of the cognitive representation contain quark.

   This leads to a generalization of computationalism replacing rationals with the hierarchy of extensions of rationals.

5. Galois confinement is a further key notion. It states that for the physical states the total 4-momentum as a sum of momenta of quarks with components, which are algebraic integers, are real integers.

   One can interpret quark momenta as discretized virtual momenta [L98, L99]. The mass squared values as roots of P can be tachyonic in the sense that the real part of mass squared is negative.

   Conformal invariance requires that the scaling generator L_0 annihilates the physical states so that the mass squared for the physical states vanishes. Therefore *all* physical states are analogous to massless particles [L112]!

   This leads to a resolution of longstanding interpretational problems of p-adic thermodynamics [K68] due to the necessity of tachyonic states and the fact that, in an apparent conflict with conformal invariance, one allows states with non-vanishing value of L_0. The second surprise is that the result actually conforms with the proportionality of blackhole entropy with mass squared and this relation generalizes so that it applies to all systems. Also an analogy with entropic gravity emerges.

   For subsystems entangled with the environment, which at elementary particle level in a good approximation reduces to wormhole contacts with an Euclidean induced metric having fundamental fermions at the throats, a superposition of pairs of states for which mass squared values of the members sum up to zero emerges. One must use thermodynamics to describe the non-tachyonic part of the system. The thermodynamic state involves both massless ground state and massive excitations for the tensor factor with non-negative mass squared values.

6. Zero energy ontology (ZEO) predicts that the arrow of time changes in "big" state function reductions (BSRs). This leads to a model for homeostasis [L123] as an ability to say near quantum criticality made possible by dissipation with an opposite arrow of time. This would also make healing possible as a time reversed dissipation.

   Concerning TQC, the good news is that the dissipation with a reversed arrow of time could make possible an automatic quantum error correction as a healing.

13.1.2 Could p-adic thermodynamics be relevant for TQC?

What distinguishes quantum computation (QC) from the classical computation (CC), is that QC is not a deterministic process. For instance, the algorithm for finding the period of function gives outcomes which are its multiples. To obtain the desired result with a high enough probability, one can repeat the QC or use an ensemble of QCs. Could one imagine a more elegant approach than just repeating the TQC sufficiently many times or having an ensemble of TQCs?

Classical computation (CC) is in a good approximation a deterministic process and it is interesting to analyze what makes it possible to physically represent Boolean function as a sequence of steps as a deterministic time evolution. Here non-equilibrium thermodynamics involving a realization of bits as flow equilibria and dissipation are in an essential role so that quantum statistical determinism in microscopic scales is an essential element as also electric and magnetic fields in long scale scales serving as masters of the microscopic dynamics.

This inspires the question whether thermodynamics, not necessarily ordinary thermodynamics but p-adic thermodynamics, could provide a tool of TQC. p-Adic thermodynamics is equivalent with ordinary thermodynamics but has additional constraints forced by the number theoretic existence conditions for Boltzmann weights. For instance, temperature quantization is implied and the convergence of the partition function in powers of p is extremely fast for large p-adic primes such as $p = M_{127} = 2^{127} - 1$.

1. p-Adic thermodynamics is naturally associated with spin glass phases in the TGD based view of spin glasses [L94]. One can wonder whether an annealing process could make it possible to end up at the bottom of the deepest valley, possibly assignable with the desired outcome of TQC. p-Adic thermodynamics could assign an analog of free energy minimum to the desired outcome.

   Annealing is a stepwise process involving repeated p-adic heating and cooling. Heating would generate entanglement between anyonic and fermionic degrees of freedom and cooling would allow an SFR to a new deeper local minimum of the p-adic analog of Hamiltonian.

2. The cognitive measurement cascade [L82, L100] is an essential part of TQC and decomposes the representation of Galois groups to a product of representations of the relative Galois groups, which are direct sums over irreps.

   The next step involves measurement of the invariants of irreps of relative Galois groups, which project one irrep for each relative Galois group. The measurement however requires entanglement of the irreps with some states. What could these states be? Does nature provide this entanglement automatically or must one engineer it?

3. There is a grave objection against this proposal. The irreducible representations (irreps) of Galois group and relative Galois groups for an extension defined by a functional composite of polynomials generalizes anyons as group representations. However, Galois confinement states that physical states are Galois singlets!

   The resolution of the objection is that bosionic Galois representations represented in terms of momentum space wave functions entangle with the representations realized in terms of fermionic spin degrees of fundamental fermions so that the entangled state is a superposition of Galois singlets as pairs of irreps. The measurement for the analogs of the Casimir operators for the irreps for the either tensor factor would project out a single irrep. Nature could do this automatically or it could be carried out as a quantum measurement.

   This process could involve a reduction of h_{eff} (and decomposition of functional composites to product of polynomials) leading to breaking of relative Galois symmetries and reduction of entanglement between momentum and spin degrees of freedom. For $h_{eff} \to h$, the entanglement could be completely reduced. It might be that this reduction is necessary in order to represent the outcome of the computation at the level of ordinary matter.

4. A connection with the travelling salesman problem emerges. If the pairing of momentum and spin degrees of freedom involves N different relative Galois irreps, there are N! different pairings between momentum and spin representations, which correspond to the number of different solution candidates in the travelling salesman problem. If the problem can be transformed to a travelling salesman problem, it can be solved by using TQC in the TGD sense.

   This suggests a canonical form for the p-adic analog of Hamiltonian for the p-adic thermodynamics selecting the local minimum in the annealing process. The p-adic analog of Hamiltonian could be engineered as in the thermodynamical solution of the travelling salesman problem.

13.2 Quantum computation *viz.* classical computation

I talked with my friend Tuomas Sorakivi about the relation between quantum computation (QC) and classical computation (CC). The discussion raised a series of questions. For professionals the following ponderings might seem to be trivial but it could give new insights about the TGD counterpart of the topological QC (TQC). In particular, it could give important insights to the basic conceptual and technical problems of QC.

13.2.1 Meanings of CC and QC

What does one mean with CC?

1. Mathematically CC can be represented as a Boolean map mapping m input bits to n output bits. This map can be decomposed to primitive Boolean maps realized as gates. CC can be represented as a program in which inputs at given time $t = n$ arrive as multi-bits to gates producing output multi-bits.
2. It is highly non-trivial that one can represent Boolean functions to electrical circuits. In the physical realization of this picture, the values of bits correspond to voltage values. Gates are constructed as electric circuits. For instance, gates involving logical conditions have control bits affecting the output. Transistors allow the realization of control bit as bit as base current. The output bits are communicated to the next gates as propagating voltage values.

What does one mean with QC?

1. There are several realizations of QC. The realization of QC as a unitary time evolution is constructed in terms of gates and is nearest to CC using electronic circuits.
2. QC is realized as a TQC in the TGD framework and involves new elements and differs from what might be called standard TQC. Besides topology, also number theory is involved in an essential way and predicts hierarchies labelled corresponding to extensions of rationals.

   In TQC according to TGD [L106], the counterpart of the metabolic energy feed is necessary to preserve quantum phases in long scales even at room temperature. Zero energy ontology (ZEO) brings in time reversal as a new element, which allows us to understand homeostasis in living matter.

13.2.2 How do QC and CC differ?

QC is usually regarded as more advanced than CC since it is conceptually a much more complex and abstract notion.

1. The difficulty to understand QC might be partially due to the missing understanding of state function reduction (SFR). TGD suggests a view of SFR, which is free of paradoxes and means a dramatic conceptual clarification.
2. The practical realization of QC meets huge technical challenges. In the TGD framework, these challenges could reflect the lack of the understanding of what dark matter is. Dark matter as $h_{eff} = nh_0$ phases of ordinary matter could help to overcome these problems.
3. CC seems to have emerged in evolution later than TQC-like information processing, which in TGD is proposed to characterize living matter. Does this mean that CC is more advanced? Probably not. The emergence of CC could be seen as reflecting our high level of evolution: we are the first species that has invented CC and does not imply that CC is more advanced than QC.

   Could also living matter combine some elements of CC with TQC to achieve determinism? Of course, living matter could achieve this by using us to build classical computers!

What distinguishes QC (and TQC according to TGD) and CC?

1. CC can be modelled as a deterministic process. This is what makes it so simple as compared to QC. CC also uses as a tool ordinary matter for which quantum effects occur in very small scales.

QC in the prototype situation relies on a deterministic unitary time evolution followed by a non-deterministic SFR. Neither energy feed nor dissipation play an active role. The basic goal is to prevent dissipation by isolating the system from the environment.

2. CC involves statistical determinism of quantum theory implying dissipation used to achieve thermodynamic determinism essential for the computation. In CC, dissipation in the presence of energy feed leads to thermodynamic flow equilibria. For instance, external energy feed allows to preserve the values of bits represented as voltages. For this external energy source (battery) is needed.

   The thermodynamic determinism of CC prevails below given length and time scale resolution. The external energy field implies the presence of macroscopic degrees of freedom, which act as masters. For instance, voltages in circuits made possible by the energy feed use ohmic currents in microscopic degrees of freedom as slaves. In the similar way, in TQC according to TGD, the quantum coherence of MB makes it possible to induce ordinary coherence at the lower levels of the hierarchy.

3. In (T)QC according to standard quantum theory energy feed is not an essential element. In TQC according to TGD, metabolic energy feed is necessary to preserve the distribution of h_{eff} since a state with given value of h_{eff} tends to decay to a state with smaller value of h_{eff} having a lower energy. In TGD, metabolic energy feed also makes possible high Tc superconductivity and quantum phases in long length scales.

   The presence of hierarchy of length scales in TGD based TQC assignable to extensions of rationals and labelled by h_{eff} p-adic length scales is also essential.

   In ordinary (T)QC only a single scale is involved and one can wonder whether the presence of several scales could make QC deterministic without losing the nice features of QC. One can of course consider an ensemble of TQCs giving an ensemble of possible answers but could one imagine other options?

13.2.3 What does one mean with a CC program?

A classical computer program can be regarded as a Boolean map, decomposing to a sequence of steps consisting of primitive Boolean maps represented as logic gates, is a sequence of deterministic steps and nodes at which decisions of what is done next. All gates can be seen as Boolean maps assigning to input bits unique output bits. The circuit decomposes to gates representing logical operations as Boolean maps. Gates can be described in terms of control bits which fix the outputs for given inputs.

1. One could argue that the choice of what is done at the next step is non-deterministic. One could argue that the initial values of the program in principle fix the output uniquely in an ideal deterministic classical physics.

   However, it seems very strange that one could engineer a program realizing any desired computation as a deterministic time evolution. Could SFRs be involved in a hidden way and make this engineering possible.

2. Thermodynamics is indeed involved in an essential manner. The physical realization of gates in electric circuits having the bits as flow equilibrium states associated with the non-equilibrium thermodynamics (NET) with external energy feed. This gives the desired thermodynamic determinism. which reduces to statistical determinism of quantum physics.

   NET involves SFRs in short length and time scales for a larger number of electrons, which gives rise to a dissipation leading to a unique flow equilibrium of NET. The description for the dissipation would be in terms of ohmic currents.

Communication of the outputs of gates to the next gates represent an essential element of CC. The bits representing the outputs can be represented as voltages and can be communicated as voltage pulses to the next gates. How does this compare with the TGD view of a TQC.

1. In the TGD framework this communication has a quantum counterpart inspired by the notion of the dark N-particle as an analog and generalization of Bose Einstein condensate [L110, L80, L89]. In biology, dark genetic codons would form the basic building brick representing 6 bits and would be represented as dark proton triplets and dark photon triplets

having interpretation as 64 chords defining a bioharmony. Codon is received by a cyclotron 3-resonance: this generalizes to 3N-resonance for genes and DNA sequences.

2. The codons serve as addresses determining which receivers get the message: the analogy with computer language LISP is obvious. The information is coded to the modulation of the frequency scale so that the modulation is coded to a sequence of cyclotron N-resonances at the receiving end. If frequencies are modulated independently, a subset of the receiver is selected by the resonance conditions.

   Generalized Josephson junctions produce Josephson radiation with frequency depending on the modulated voltage of the junction [K94] [L105, L108, L110]. As a special case, one can consider a voltage which is piecewise constant and represents two bit values. Neurotransmitters in synaptic junction actually correspond to miniature potentials, which correspond to a voltage change of few meV as compared to membrane potential of order .05 eV. Miniature potentials relate to the notion of preneural system suggested by the finding that multi-cellulars without nervous system behave as if they had a nervous system: the TGD inspired model is discussed in [L103].

One might think that the propagating voltage pulses of rectangular shape as idealized representations of propagating bits could be something totally trivial from the point of view of recent day condensed matter physics. However, it is far from clear what their TGD counterparts could be and I have already years ago considered the possibility that the electric pulses studied by Tesla, which had rather dramatic effects on the environment, might involve new physics. The time reversals of the electric pulses emerged in these considerations much before the precise formulation of ZEO [K44, K8].

1. The voltage pulses propagate with a sub-luminal velocity and can be said to have longitudinal electric polarization. Electron-hole pair is the notion of condensed matter physics which comes to mind. These pairs are formed valence electrons transform to conduction electrons. Holes behave as effective positive charges. Could voltage pulse represent a distribution of electron-hole pairs as bound states. Holes and electrons would reside at 2-D surfaces defining analogs of capacitor plates. Since charge carriers are near the surface of the conductor, the one would have annuli instead of plates.

2. What could be the TGD based model for the structure formed by the electron-hole pairs? An interpretation as a moving electric flux quantum is suggestive [L97]. Electric flux quantum would be analogous to a moving electric capacitor idealized as a pair of 2-D plates. This suggests a TGD based model for a capacitor plate as a membrane like object, which is a space-time surface with a 2+1-D M^4 projection and therefore defines a planar membrane in E^3. In general relativity these objects are not possible. The CP_2 projection could be a geodesic circle of CP_2.

   The 2-D sheets with the shape of annulus would represent the ends of a hollow cylinder. The hollow cylinder would be minimal surface apart from the singular circles at its ends where the minimal surface property would fail and entire field equations involving sum of terms for the Kähler action and volume term would hold true.

3. The longitudinal electric field could reside at the outer and inner cylindrical walls of the structure. The ends of the annular cylinder could be also connected by thin hollow flux tubes carrying the electric flux parallel to the cylinder. The holes would be assigned to the first annulus and electrons with the second annulus.

4. The electrons could be dark. But can one say that the also hole created in the transition of a dark valence electron to the conduction band is dark. In ZEO, it could be natural to speak about a 4-D time classical evolution leading from a dark valence electron to dark conduction electron so that it makes sense to assign also to the hole the attribute "dark".

5. Debye length λ_D (`https://cutt.ly/QJMNHpq`) as a screening length for the electric field created by electrons gives an estimate for the length of the cylindrical structure. For water at room temperature, one has $\lambda_D = .7$ nm: intuitively, one would expect a considerably larger size scale for the voltage pulse.

   In semiconductors one has $L_D = \sqrt{\frac{\epsilon_r T}{q^2 N_{dop}}}$, where n_{dop} is the doping density. Note that the expression fof L_D has no explicit dependence of Planck constant.

The generalization for dark particles would replace N_{dop} with the number n_{dark} of dark electrons per volume. The density of dark electrons is expected to be much smaller than atomic density and to scale roughly like $1/L_p^3$.

One can argue that L_D corresponds to the p-adic length scale defining the length of the electric flux quantum and that there should be roughly 1 dark electron per flux quantum. $n_d 1/\mu m^3$, this gives $L_D \simeq \sqrt{1/n_d}\sqrt{T/300K} \times .45$ μm. For 1 dark electron per volume defined by the p-adic length scale $L(167) = 2.5$ μm assignable to the cell nucleus, one has $L_d = \sqrt{T/300K} \times 1.8$ μm.

13.2.4 Could AI be conscious?

Every morning I learn on FB about mind boggling discoveries. The popular article "Scientists Made a Mind-Bending Discovery About How AI Actually Works" (`https://rb.gy/oes4bv`) told about the article of Akyürek et al with title "What Learning Algorithm Is In-Context Learning? Investigations With Linear Models" [J12] (`https://rb.gy/w88gud`).

What caught my attention was that the AI system was seen as a mysterious phenomenon of Nature to be studied rather than systems that are engineered. If AI systems are what their builders believe them to be, that is deterministic systems with some randomness added, this cannot be the case. If the AI systems are really able to learn like humans, they could be conscious and be able to discover and "step out of the system" by generalizing. They would not be what they are meant to be.

TGD predicts that AI systems might have rudimentary consciousness. The contents of this conscious experience need not have anything to do with the information that the AI system is processing but corresponds to much shorter spatial and temporal scales than the program itself. But who knows?!

In the following I briefly summarize my modest understanding of what was done and then ask whether these AI systems could be conscious and be able to develop new skills. Consider first the main points of the popular article.

1. What is studied are transformers. Transformer mimics a system with directed self-attention. This means the weighting of parts of input data so that the important features of the input data get more attention. This weighting emerges during the training period.

   Transformers differ from recurrent neural networks in that entire input data is processed at once. Natural language processing (NLP) and computer vision (CV) represent examples of transformers.

2. What looks mysterious is that language models seem to learn in flight. Training using only a few examples is enough to learn something new. This learning is not mere memorizing but building on previous knowledge occurs and makes possible generalizations. How and why this in-context learning occurs, is poorly understood.

   In the examples discussed in the article of Akyürek et al, linear regression, the input data never seen before by the program. Generalization and extrapolation took place. Apparently, the transformer wrote its own machine learning model. This suggests an implicit creation and training of smaller, simpler language models.

3. How could one intuitively understand this without assuming that the system is conscious and has intentional intelligence? Could the mimicry of conscious self-attention as weighting of parts of input data explain the in-context learning. Weighting applies also to new data and selects features shared by new and old data. Familiar features with large weights in the new data determine the output to a high degree. If these features are actually important the system manages to assign output to input correctly with very little learning.

The TGD framework also allows us to consider a more sciencefictive explanation. Could the mimicry of conscious self-attention generate conscious self having intentions and understanding and be able to evolve?

1. TGD forces me to keep my mind open to the possibility that AI systems are not what they are planned to be. I have discussed this in previous articles [L26, L109].

2. We tend to think that classical computation is fully deterministic. However, the ability to plan a system behaving in desired manner is in conflict with the determinism of classical physics and statistical determinism of quantum physics. Computer is a system consisting of subsystems, such as bits, which are far from thermal equilibrium and self organize. They must be able to make phase transitions, which are basically non-deterministic at criticality. Changing the direction of the bit as a mesoscopic system is a good example.

3. Zero energy ontology (ZEO) is an essential part of quantum TGD. Quantum states are superpositions of space-time surfaces, which obey holography. One can see them as analogs of computer programs, biological functions, or behaviors at the level of neuroscience. The holography is not completely deterministic and this forces us to regard the space-time surface as the basic object. Any system, in particular AI systems, is accompanied by a superposition of these kinds of space-time surfaces, which serve as a correlate for the behavior of the system, in particular for the program (or its quantum analog) running in it.

   ZEO predicts that in ordinary, "big" state function reduction (BSFR) the arrow of geometric time is changed. This allows the system to correct its errors by going back in time to BSFR and restoring the original time direction by second BSFR. This mechanism might be fundamental in the self-organization of living matter and a key element of homeostasis. This mechanism is universal and one can of course ask whether AI systems might apply it in some time scale, which could be even relevant to computation.

4. In the TGD framework, any system is accompanied by a magnetic body (MB) carrying dark matter in the TGD sense as phases of ordinary matter with a value of effective Planck constant which can be very large, meaning a large scale of quantum coherence. This dark matter makes MB an intelligent agent, which can control ordinary matter with ordinary value of Planck constant.

   In TGD, quantum criticality of the MB of the system is suggested to accompany thermal criticality of the system itself. This leaves a loophole open for the possibility that the MB of the AI system could control the AI system and take the lead.

What one can say of the MB of AI system? Could the structure and function of MB relate closely to that of the program running in it as ZEO indeed suggest? My own conservative view is that the MBs involved are associated with rather small parts of the systems such as bits of composites of bits. But I don't really know!

1. The AI system involves rather long time scales related to the program functioning. Could this be accompanied by layers of MB (TGD counterparts of magnetic fields) with size scales determined by the wavelength of low energy photons with corresponding frequencies. Could these layers make the system aware of the program running in it.

2. Could the MBs associated with living systems involving MBs of Earth and Sun get attached to the AI system [L103, L101, L108, L120]? Of course we used the AI but could there be also other users?: MBs which directly control the AI system! Could it be that we are building tools for these higher level MBs?!

   If this were the case, then the MB of AI system and the program involved with it could evolve. MB of the system could be an intelligent life form. This raises worried questions: are we a necessary piece of equipment needed to develop AI programs? Or do these higher level MBs need us anymore?

13.3 TQC in TGD

In the TGD counterpart of TQC [K5, K4, K123] [L106], metabolic energy feed would make quantum phases in long scales possible and the field/magnetic bodies (FBs/MBs) would form a master-slave hierarchy and one could expect that TQC acording to TGD could have a lot of common with CC.

1. The levels of the dark matter hierarchy below a given level could be modelled as statistical ensembles much like electrons in an ordinary computer as seen from the level considered. Qubits could be also represented as ferromagnetic multispin systems analogous to ferromagnets.

   Quantum spin glasses are very natural systems in the TGD framework since the action principle reduces to Kähler action in long length scales and it has enormous 4-D vacuum degeneracy analogous to spin glass degeneracy. The TGD based inspired model in terms of flux tube spaghettis and the mathematical description involving p-adic thermodynamics is discussed in [L94].

2. QC gives several outcomes in the SFR ending it. The desired outcome, say the minimal period of a function realized as entanglement between qubit registers, is only one of the outcomes, and one should be able to select the desired outcome. One can repeat the QC and find in this manner the shortest period.

   Could one imagine more elegant ways to find the desired outcome? For instance, could the valleys of the quantum spin glass energy landscape correspond to the possible answers of TQC? If this were the case, an annealing involving repeated heating and cooling of the system could lead to the desired answer. The answer of QC (say a minimal period of function) should correspond to the deepest valley. D-wave quantum computers (`https://cutt.ly/QJMNLP7`) rely on an approach, which involves annealing and spin glass ("D-wave" refers to the D-wave superconductors used in the original approach).

13.3.1 What could be the physical realization of the TQC program?

What could the physical realization of the TQC program in the TGD framework look like?

1. Flux tube network, which reduces as a special case to a braid system, is the key notion. The flux tubes connect nodes and the motion of the nodes give rise to a time-like braiding, which induces space-like braiding, which provides a topological memory representation of the TQC program.

   Anyon replaces qubit as spin state with group representation, which is in the simplest situation (bilocal states of condensed matter Majorana fermions with definite parity introduced by Kitaev) characterized by a parity-like quantum number representing qubit. In the TGD framework the bi-localization of the proton of dark hydrogen bond could give rise to this kind of anyons [L106].

   In TGD, the counterparts of anyons would correspond to representations of Galois groups labelled by invariants of the Galois group, and there would be both cognitive measurement cascades using Galois representations for the relative Galois groups as counterparts of qubit and measurements of spins and possibly also momenta of electrons (many-quark states at fundamental level) for these representations

   The TQC program could physically correspond to a dynamical flow for the "liquid" formed by the nodes. There would be large number of realizations of the flow and the physical realizations should correspond to PEs and among them would be PE with minimal action. The braids realized as flux tubes correspond at the fundamental level 2-D string world sheets inside the flux tube orbits and there are many topologically equivalent realization associated with different preferred extremals (PEs).

2. The spin glass energy landscape is realized in terms of manyfermion states for a magnetic flux tube network depends on the detailed realization of a given braiding. Monopole flux tubes induce local magnetization of the fermions parallel to the flux tube. In the optimal situation there would be 1-1 correspondence between the answers of TQC and the PEs. This would be essentially quantum-classical correspondence (QCC).

   The topology preserving modifications of the flux tube network responsible for the TQC would modify the spin glass energy landscape. Could this allow to enhance the probability of the desired outcome of the TQC? It should correspond to the deepest valley: why? Should one engineer this correspondence?

3. p-Adic thermodynamics is a natural mathematical framework for describing the spin glass energy landscape [L94]. Could the p-adic thermodynamics be engineered using topology preserving modifications of the braiding. Note that the braiding is determined by 2-D string world sheets so that the modification of the space-time sheets can be considered.

13.3.2 Is it possible to replace quantum states with irreps of the braided Galois group?

My understanding is that anyons correspond to entire representations of finite groups of or quantum groups.

1. In the TGD framework, they would correspond to the representations of Galois groups, or rather, of their braided counterparts as subgroups of the braid groups associated with S_n, where n is the number roots of polynomial P defining the Galois group assignable to the space-time surface by $M^8 - H$ duality.

   The stability of the Galois group with respect to small changes of the rational coefficients of P would correspond to the topological stability and the braided Galois group would be a subgroup of the braid group. Braided Galois group would be a quantum group-like object.
2. The replacement of quantum states with irreps of the braided Galois group and thus sub-spaces of state space looks something new from the perspective of wave mechanics, where states as 1-D rays of Hilbert space are the basic objects. Does this really make sense? And does the entanglement between states and sub-spaces and, more generally, between sub-spaces make sense?
3. The decomposition of a representation of a group to a direct sum of representations is a standard mathematical procedure and would be analogous to a superposition of a quantum state in a given state basis. The analogs for the coefficients of quantum states are now integers for given irrep.
4. A natural entanglement between group representations is possible number theoretical context. The representation of Galois group Gal of a composite polynomial $P_n \circ .... \circ P_1$ has a decomposition to a semidirect product of relative Galois groups $H_k = Gal_k/Gal_{k-1}$ associated with polynomials $P_k \circ .... \circ P_1$ defining extension of extension defined by $P_{k-1} \circ .... \circ P_1$.
5. A given representation of Gal can be decomposed to a direct sum of tensor products of irreps of the relative Galois groups $H_k = Gal_k/Gal_{k-1}$. The generalized quantum measurement would measure the parameters characterizing the irreps of the relative Galois group $H_k = Gal_k/Galk - 1$. In maximal measurement this measurement would be performed for all semi-direct factors H_k and the superposition would reduce to a single term as a product of representations of H_k. This should generalize to braided Galois groups.
6. What is nice is that the measurement would occur inside a single representation of Gal. The higher levels in the hierarchy of relative Galois groups correspond to dark phases of ordinary matter with $h_{eff} > h$ and Gal_0 would correspond to the lowest extension.

   Note that $h = n_0 h_0$ corresponds to a non-trivial Galois group and there is an argument that one has $n_0 = (7!)^2$ corresponding to $Gal_0 = S_7 \times S_7$ [L92]. Quantum measurement would project out one particular irreducible representation and would measure the invariants of the representations. A possible interpretation is that at the visible matter the measurement would measure the representation of the Galois group for the Gal_1/Gal_0. The simplest measurement would be that for an irrep of a Z_2 anyon giving the parity of the Z_2 representation as an outcome. This parity would serve as a qubit.

The wave-mechanical picture corresponds in von Neumann's theory of factors as basic mathematical building blocks of quantum theory to the simplest situation in quantum theory, that is factor of type I. Von Neumann introduced also factors of type II, to which one can assign quantum groups and factors of type III which would correspond to quantum field theory and which von Neumann regarded as pathological.

1. In the TGD framework, hyperfinite factors of type II$_1$ (HFFs), play a central role and as the name tells, they can be approximated by finite-D matrix algebras in an excellent approximation and are therefore physically very attractive. I have written earlier about the possible application of HFFs in the TGD framework [K127, K48] and discussed quite recently their detailed role in the formulation of quantum TGD itself [L111].

   The TGD inspired interpretation of the inclusions $M \subset N$ of HFFs is as a representation of the finite measurement resolution defined by M. One could say that the Hilbert space ray is replaced by an image of the ray obtained by the action of M and that the "factor space" N/M defines quantum space representing the degrees of freedom above measurement resolution.

2. The technical reason for the replacement of rays with subspaces is the assumption of von Neumann that the trace of identity operator Id satisfies $Tr(Id) = 1$. This conforms with the idea that Id represents one particular density matrix so that the strange looking condition says that the sum of probabilities is equal to 1. Note that for factors of type I it satisfies $Tr(Id) = n$ as the dimension of Hilbert space and would approach infinite for infinite-D Hilbert spaces. Any subspace of HFF must correspond to projector P with $0 < Tr(P) < 1$ and ray would correspond to $Tr(P) = 0$ and need not make sense: the possibility to project to it would be the physics counterpart of the selection axiom.

3. TQC indeed involves quantum groups and they utilize HFFs. The replacement of the state as a ray with an irrep of representation of the braided counterpart of the Galois group conforms with the thinking of von Neumann. Braided Galois group for rational polynomials P of degree n is indeed a subgroup of the braid group as covering of the permutation group S_n.Finite coverings define a hierarchy of approximations which converge rapidly for HFFs.

4. Finite measurement resolution would naturally correspond to the inclusion $M \subset N$ and the replacement of state with the quantum subspace N/M in which the braided Galois group acts. This replacement is done also for other algebraic structures of quantum TGD such as super symplectic algebras allowing hierarchies of sub-algebras labelled by an integer allowing interpretation as the degree of P. Perturbations would characterize the resolution and the use of HFFs would take care that the quantum measurement outcomes are not affected by the perturbations.

   What can be measured would be the invariants characterizing the irreps of the braided relative Galois group. The measurement of the individual state inside the irreps is not possible. These invariants can be measured in TQC and qubits are defined in terms of these. Qubits would be replaced by irreps of relative Galois groups.

13.3.3 Quantum analog of annealing

In thermodynamics, the minimization of free energy combined with annealing could lead to the bottom of the deepest valley of the spin glass energy landscape.

In quantum TGD, one can consider two options for the annealing based on the analog of thermodynamics. Free energy could be replaced by the fundamental action or alternative energy could be replaced by a scaling generator L_0 of conformal symmetries. p-Adic thermodynamics is indeed highly suggestive as a description of spin glasses [L94].

Consider first the approach based on fundamental action.

1. The outputs from the TQC program should correspond to the PEs with given initial values. One should have a variational principle assigning the desired answer of QC with a minimum the action for a preferred extremal (PEs) representing the deepest valley of the spin glass landscape.

   The exponent of the fundamental action defines the vacuum functional as the sum of Kähler action and volume term. PE is simultaneous extermal of both volume term and Kähler action apart from singularities with various dimensions $d < 4$ defining analogs of frames for a soap film. There is a finite non-determinism associated with the frames present already for the ordinary soap films.

   PE property for the space-time surface realizes almost complete holography and almost complete determinism. The possible answers as PEs could correspond to topologically equivalent braidings realized as space-time surfaces. The action is in general different for these PEs. The PE with a minimal action would have the highest probability proportional the exponent of the action.

2. The braiding corresponds at a fundamental level to string world sheets. Since the string worlds sheets defining the braiding can be associated with a large class of PEs, one can think that the PE could be engineered in such a way that the desired configuration is strongly favored.

Only its coupling parameters of the fundamental action that depend on the extension of rationals considered can be varied. This is achieved by varying the polynomial P determining the space-time region considered. The topology of the braiding and also the Galois group should remain unaffected.

There is a large number of rational polynomials with the same Galois group and extension of rationals so that the engineering could correspond to the variation of the coefficients of P affecting ramified primes. Even the degree of the polynomial could be changed.

It is however not all clear how one could modify the polynomial in a controlled manner.

It is far from obvious whether it is possible to add to the exponent of the vacuum functional defined by the fundamental action an additional engineerable exponential factor. A more promising approach is based on the engineering of entanglement between fermion states and anyons as Galois representations.

p-Adic thermodynamics as a tool of quantum annealing

Since cognitive measurement cascades for the representations of Galois group are in question, the entanglement engineering would naturally rely on p-adic thermodynamics using the analog of Hamiltonian, whose eigenvalues are analogs of p-adic conformal weights h distinguishing between different outcomes of SFR ending the TQC.

1. The p-adic prime p associated with the engineered p-adic thermodynamics would naturally correspond to the maximal ramified prime of the extension appearing as a factor the discriminant D given as the square $\prod_{i<j}(r_i - r_j)^2$, where $r_i - r_j$ difference of the roots. p-Adic temperature T_p, whose values come as inverse integers, when $log(p)$ is used as a unit, is natural in the modelling of the spin glass energy landscape [L94] and $T_p = 1/n$ could serve as the counterpart of temperature varied in the annealing procedure.
2. The thermodynamics would be for the scaling generator L_0 associated with conformal invariance. Although the physical states are are annihilated by L_0, entangled states can have non-vanishing thermal expectation for the entangled factor because tachyonic states are predicted as analogs of virtual particles having roots of polynomials as mass squared values: the real parts of the roots can indeed be negative.
3. Physical states, which satisfy Galois confinement and have vanishing mass squared, consist of virtual quarks (in the simplest scenario in which leptons are 3-quark composites).

   Massless Galois confined states are in general entangled states such that the total momentum is light-like momentum with integer valued components. This is possible because the values of mass squared (conformal weights) for quarks as roots of polynomial P are in general algebraic numbers and can have negative real parts (tachyonicity). In particular, Galois singlets can also have a negative mass squared.

   The total mass squared would be for each pair appearing in the entangled state equal to zero and p-adic thermodynamics would apply to quarks with positive mass squared with Virasoro generator representing the mass squared [L112].
4. At the level of H, tachyonic momenta can be assigned with the wormhole contacts having Euclidean signature of the induced metric and associated with elementary particles.

   The twistor lift of TGD requires M^4 to possess the analog of Kähler structure [L41, L98, L99]. The massless solution for Dirac equation in H for the second chirality of H-spinor allows a covariantly constant right handed neutrino as a massless solution, which becomes a tachyon as one adds a coupling to the Kähler gauge potential of M^4 required by the twistor lift. Right-handed neutrino could be elementary or could correspond to 3-quark composite with quarks at the same wormhole throat or possibly in the interior of the wormhole contact.
5. In the simplest model in which momenta define Galois representation, the total momentum of Galois singlet would have integer valued components as a sum of quark momenta with algebraic integer valued momentum components. More general representations involve wave functions in momentum and spin degrees of freedom. Even more, by conformal invariance, the states have a vanishing mass squared [L112] so that they have vanishing conformal weights as eigenstates of L_0.

6. The basic step of quantum annealing would be p-adic heating increasing the quantized p-adic temperature $T_p = 1/n$ followed by cooling. p-Adic heating would induce entanglement between fermionic states and irreps of the relative Galois group describable in terms of p-adic thermodynamics with an increased temperature T_p. Thermodynamics would therefore be an essential part of TQC in the TGD framework.

An objection against the identification of Galois representations as analogs of anyons

Anyons should correspond to Galois non-singlet representations. The problem is that the representations of the relative Galois groups should be Galois singlets by Galois confinement.

The solution of the problem is analogous to the solution of the problem posed by the basic objection against p-adic thermodynamics. Galois singlets are constructed by entangled pairs of Galois representations in the fermionic momentum and spin degrees of freedom.

Cognitive measurement cascade can take place either in momentum or spin degrees of freedom and select from a superposition of paired representations one particular representation a pair fusing to a Galois singlet. The reduction probabilities are analogous to thermodynamic Boltzmann weights and one can have analog of p-adic thermodynamics with reduction probabilities proportional to non-positive powers of p.

13.3.4 How the TQC program could be engineered?

In QC there are several alternative outputs (say the multiples of a minimal period of function). QC could be repeated several times to obtain the desired answer. Could one end up with the desired answer by some other method. Could some kind of engineering make this possible?

One can imagine two approaches to the engineering.

Could TQC program involve engineering of preferred extremal?

One can consider two options for what happens as the TQC program runs.

1. The value of the action exponential differs for the different outcomes of the SFR. In this case, the superposition of different outcomes of SFR corresponds to a superposition over different space-time surfaces defining topologically equivalent braidings and SFR selects one of them.
2. An alternative option is that there is only a single space-time surface involved and the fermionic entanglement probabilities between the spin and fermionic degrees of freedom depend on the fermionic state only. Now SFR takes place in the fermionic degrees of freedom and the TQC programmer must engineer the fermionic entanglement.

For the first option, the fundamental action exponential, vacuum functional, as a counterpart of Boltzmann weight, should be maximal for the desired outcome of SFR.

The engineering should select the polynomial determining the space-time surface such that the desired outcome would be achieved.

1. PE as a minimal surface with singularities is analogous to a soap film having frames as singularities and would be the TGD counterpart for a state at the bottom of the valley. The action would depend on the valley and quantum annealing, whatever it could mean, would take the system to the deepest valley.
2. One should identify the quantum counterpart of temperature and Kähler coupling strength is the first guess. If Kähler coupling strength is determined by the extension of rationals, annealing would involve modifications of the polynomial determining the space-time surface. The topology of the braiding must remain unaffected and string world sheets defining the braiding would define part of the holographic data.
3. This option does not look promising in the recent case since it is difficult to imagine how the engineering of the action by modifying the coefficients of P could be possible and whether this engineering could select the desired outcome of SFR as the most probable outcome.

   The physical modification of action would be based on the modification of braiding, which would preserve its topology by modifying the "hydrodynamic" flow defined by the nodes.

Could the entanglement between Galois representations and fermion spins be engineered?

An attractive option is that Nature performs cognitive state function reduction cascade and entanglement engineering guarantees the most probable outcome of TQC for a given relative Galois group. This requires engineering of the entanglement between Galois irreps and some other degrees of freedom. Is there any way to achieve this?

Topological qubits correspond to irreps of a given relative Galois group. One should assign to each irrep a fermionic state. If the irrep is somehow realized, the structure of this state does not matter: what is only required is that it distinguishes between different irreps and the entanglement probability is largest for the desired outcome of SFR. For instance, magnetized many-fermion states with a fixed or slowly varying spin direction could be considered.

Galois confinement however poses a strong additional condition. The irrep must be entangled with a representation of the Galois group such that the outcome is Galois singlet! This suggests that one has two irreps of Galois: the first one in the fermionic spin degrees of freedom and the second one in fermionic momentum degrees of freedom identifiable asd discretized geometric degrees of freedom. These irreps entangle to a Galois singlet and one has a superposition over pairs of irreps of this kind.

1. The entanglement between qubit registers defines a map between integers defined by the qubits sequences of the registers. Should one introduce besides the topological qubit register an additional qubit register entangled with it in such a way that the desired outcome of TQC corresponds to the most probable outcome of the cognitive SFR cascade?

   One should engineer the entanglement between the registers. The simplest entanglement would be maximal entanglement determined by phase factors in the diagonal representation. This entanglement is determined apart from permutation. The qubits would naturally correspond to quark spins at the fundamental level. At higher level electronic spins would be in question.

   This entanglement would not distinguish between the representation of the Galois group unless the value of the fundamental action correlates with the representations. A more plausible option is that it does not and that the entangled is engineered in such a way that the reduction probability is largest for the desired outcome of TQC.

2. Galois confinement forces entanglement of the measured system with another system since topological qubits as anyons are generalized to representations of Galois group. Could the quark spin degrees of freedom entangle in 1-1 manner with the bosonic number theoretic degrees of freedom assignable to the orbits of the Galois group?

   In M^8, the orbits of the Galois group correspond to quark momenta with algebraic integers as components and Galois acts also in spin degrees of freedom of quarks. Could the entangled degrees of freedom correspond to fermionic spin and momentum degrees of freedom? Could one think of enhancing the probability of the desired outcome by adding an interaction exponential as a p-adic analog of the exponent of free energy?

13.3.5 How to engineer the entanglement between many-fermion states and irreps of relative Galois groups?

The many-fermion states and the representations of the relative Galois group as analogs of anyons, would entangle with fermionic spin states or subspaces of fermionic states. This entanglement would associate to a given geometric irrep of the relative Galois group, realized in the momentum space of fermions, a many-fermion state characterized by fermion spins and entanglement coefficients between fermions.

The number theoretical SFR cascade would begin from a state which a the decomposition of irrep of Galois group to a quantum superposition of products of irreps of Galois group to products of irreps of relative Galois groups.

SFR cascade would decompose representation to a product of representations of the relative Galois groups and for the representations as superpositions of irreps would select a particular irrep for each relative Galois group and assign to it a many-fermion state. This many-fermion state would correspond to a valley of spin glass energy landscape. It will be assumed that the space-time surface does not depend on the fermionic states involved.

The projection to an irrep of the relative Galois group induces the selection of a particular outcome. Basically, the number theoretical invariants associated with a particular Galois representation must be measured. Whether Nature does this measurement or whether this measurement must be engineered, is not quite clear.

Fermionic Galois representations

The momentum and spin degrees of freedom of quarks provide the fundamental Galois degrees of freedom.

1. At the level of M^8, the fermionic representation could be constructed as wave functions in the momentum and spin degrees of freedom (also iso-spin degrees of freedom at the quark level). These wave functions are more general than the "classical" many quark states for which the sum of the momentum components as algebraic integers is equal to an integer valued total momentum. For instance, a single quark can be in an analog of s-wave defined as superposition of states at the orbit of the Galois group.

   Homogeneous polynomials of the quark momenta analogous to spherical harmonics would be involved besides spin wave function in which the Galois group should act. Since finite simple groups are in question, the number of irreps is finite. Given irrep can however appear several times.

2. If the number of irreps considered for a given relative Galois group is N, the number of entanglements with a given set of fermion states or fermionic irreps is the number of permutations $N!$ for N objects. In the travelling salesman problem (`https://cutt.ly/AJMNCbC`) with N cities, the number of ways to visit once in every city and return to the starting city is $N!$. This problem is NP hard.

3. The travelling salesman problem has as special solutions Hamiltonian cycles for which each node has at least one nearest neighbor with a given minimum distance. Each edge of the cycle connects the point of the graph to one of its nearest points so that the path has minimum length and solves the travelling salesman problem. The TGD based model for bioharmony relies on fusion of three icosahedral Hamiltonian cycles with 12 vertices (notes of 12-note scale) and tetrahedral cycle [L6] [L52, L80, L89]. In this case the notes are scaled by factor $3/2$ at each step (quint cycle) for Pythagorean scale. For equal tempered scale the scaling is $2^{7/2)}$.

   For Hamiltonian cycles associated with Platonic solids, one can also consider an ultrametric distance for the points of a given path, not necessarily Hamiltonian cycle. This distance would defined to be the largest scaling as power of x ($x = 3/2$ for icosahedral Hamiltonian cycles) along the path connecting two nodes. This ultmetric distance would be x between all points of the Hamiltonian cycle. p-Adic primes 3 or 2 would be naturally associated with the bioharmony.

The cognitive measurement cascade could therefore have a connection with two problems of computer science.

1. The factorization of the Galois group to prime (simple) factors defined by the relative Galois groups is analogous to the prime factorization of integers.
2. The entanglement between Galois representations and many-fermion states could relate to a solution travelling salesman problem as the path of minimum length connecting all cities.

Travelling salesman problem, entanglement engineering and quantum annealing

The travelling salesman problem for which D-wave quantum computers are proposed as a solution (`https://cutt.ly/QJMNLP7`) suggests a formulation of p-adic thermodynamics allowing to find the desired outcome of SFR by quantum annealing.

1. The path length should appear as an argument in the function to be minimized in the travelling salesman problem. The input data are defined by the distances $d(i,j)$ between the cities.

Suppose that the fundamental action is the same for all space-time surfaces considered in QC. In particular, the fundamental action would not depend on the fermionic state paired with the representation of the relative Galois group. The simplest situation is that one has just a single PE.

2. Entanglement engineering would mean that one assigns to a given permutation representing a possible route as the sum of the dimensionless positive integer valued distances $d_{P(i),P(i+1)}$ between subsequent cities in their permutation. The simplest entanglement coefficient defining the reduction probability in the real context is Boltzmann weight $exp(-\sum_i d_{P(i),P(i+1)}/T)$, where T is a real parameter.

   The counterpart of this Boltzmann weight in the p-adic thermodynamics is $p^{\sum_i d_{P(i),P(i+1)}/T_p}$. Here $T_p = 1/n$ is the quantized p-adic temperature using $log(p)$ as a unit. Travelling salesman program is hard since the minima form an analog of spin glass energy landscape. The annealing by varying the value of $T_p = 1/n$ regenerating the entanglement could lead to the deepest valley.

3. The exponent of Boltzman weight could be also seen as a number theoretical analog of a Casimir operator, whose measurement would select the relative Galois representation in the second tensor factor. This kind of operator should have an integer valued spectrum and could define p-adic thermodynamics. The maximal Abelian subgroup of the Galois group could define the observable analogous to free energy.

   The time evolution of spin glass is such that the magnetic relaxation obeys power law rather than exponential law. This suggests that the time evolution for spin glass could correspond to scaling rather than time translation. Therefore the interpretation of the analog of free energy could be in terms of scaling interaction Hamiltonian.

4. This approach could apply to all problems, which can be transformed to the travelling salesman problem. Note that the very large number of problems with varying distances between cities allows the same path as a solution. This would make it possible to transform the problem to a problem in which the distances are integer valued.

13.3.6 Some delicacies related to the Galois group

The considerations above represent only a general vision and reflect my rather amateurish understanding of number theory.

The isotropy group of the Galois group leaving root fixed

One important point, which is not mentioned above, is that since the action of the Galois group permutes the roots of the polynomial P identified as mass squared values, it does not commute with Lorentz and Poincare transformations. This is one excellent motivation for Galois singlet property of the physical states. The entire Galois group would act along time-like braids and in ZEO, where space-time surfaces are fundamental objects and a small failure of the classical determinism takes place, this would make sense.

For a given root, one can identify its isotropy group as the subgroup of the Galois group leaving the root invariant. The isotropy subgroup respects the value of mass squared and therefore can appear as a physical symmetry group.

For a polynomial of degree n with maximal Galois group S_n, this group is S_{n-1}. For $n = 5$ with A_5 with order 60 as Galois group acting at icosahedron, this group is A_4 with 12 elements acting at tetrahedron. Intriguingly, both groups appear in the model of bioharmony and genetic code [L80].

Relationship with Higgs mechanism

Polynomials P have two kinds of solutions depending on whether their roots determine either mass or energy shells. For the energy option a space-time region corresponds by $M^8 - H$ duality to a solution spectrum in which the roots correspond to energies rather than mass squared values and light-cone proper time is replaced with linear Minkoski time [L73, L74]. The physical interpretation of the energy shell option has remained unclear.

The energy shell option gives rise to a p-adic variant of the ordinary thermodynamics and requires integer quantization of energy. This option is natural for massless states since scalings leave the mass shell invariant in this case. Scaling invariance and conformal invariance are not violated.

One can wonder what the role of these massless virtual quark states in TQC could be. A good guess is that the two options correspond to phases with broken *resp.* unbroken conformal symmetry. In gauge theories to phases with broken and unbroken gauge symmetries. The breaking of gauge symmetry indeed induces breaking of conformal symmetry and its breaking is more fundamental.

1. Particle massivation corresponds in gauge theories to symmetry breaking caused by the generation of the Higgs vacuum expectation value. Gauge symmetry breaking induces a breaking of conformal symmetry and particle massivation. In the TGD framework, the generation of entanglement between members of state pairs such that members having opposite values of mass squared determined as roots of polynomial P in the most general case, leads to a breaking of conformal symmetry for each tensor factor and the description in terms of p-adic thermodynamics gives thermal mass squared.

2. What about the situation when energy, instead of mass squared, comes as a root of P. Also now one can construct physical states from massless virtual quarks with energies coming as algebraic integers. Total energies would be ordinary integers. This gives massless entangled states, if the rational integer parts of 4-momenta are parallel. This brings in mind a standard twistor approach with parallel light-like momenta for on-mass shell states. Now however the virtual states can have transversal momentum components which are algebraic numbers (possibly complex) but sum up to zero.

   Quantum entangled states would be superpositions over state pairs with parallel massless momenta. Massless extremals (topological light rays) are natural classical space-time correlates for them [K10, K82]. This phase would correspond to the phase with unbroken conformal symmetry.

3. One can also assign a symmetry breaking to the thermodynamic massivation. For the energy option, the entire Galois group appears as symmetry of the mass shell whereas for the mass squared option only the isotropy group does so. Therefore there is a symmetry breaking of the full Galois symmetry to the symmetry defined by the isotropy group. In a loose sense, the real valued argument of P serves as a counterpart of Higgs field.

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   If the symmetry breaking in the model of electroweak interaction corresponds to this kind of symmetry breaking, the isotropy group of the group, which presumably involves also a discrete subgroup of quaternionic automorphisms as an analog of the Galois group. Quaternionic group could act as a discrete subgroup of $SU(2) \subset SU(2)_L \times U(1)$. The hierarchy of discrete subgroups associated with the hierarchy of Jones inclusions assigned with measurement resolution suggests itself. It has the isometry groups of Platonic solids as the groups with genuinely 3-D action. $U(1)$ factor could correspond to Z_n as the isotropy group of the Galois group. In the QCD picture about strong interactions there is no gauge symmetry breaking so that a description based on the energy option is natural. Hadronic picture would correspond to mass squared option and symmetry breaking to the isotropy group of the root.

In the maximally symmetric scenario, conformal symmetry breaking would be only apparent, and due to the necessity to restrict to non-tachyonic subsystems using p-adic thermodynamics.

Part III

CATEGORIES, NUMBER THEORY AND CONSCIOUSNESS

Chapter 14

Category Theory, Quantum TGD, and TGD Inspired Theory of Consciousness

14.1 Introduction

Goro Kato has proposed an ontology of consciousness relying on category theory [A43, A63]. Physicist friendly summary of the basic concepts of category theory can be found in [A51]) whereas the books [A24, A47] provide more mathematically oriented representations. Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. To mention only one example, C. J. Isham [A51] has proposed that topos theory could provide a new approach to quantum gravity in which space-time points would be replaced by regions of space-time and that category theory could geometrize and dynamicize even logic by replacing the standard Boolean logic with a dynamical logic dictated by the structure of the fundamental category purely geometrically [A73].

Although I am an innocent novice in this field and know nothing about the horrible technicalities of the field, I have a strong gut feeling that category theory might provide the desired systematic approach to quantum TGD proper, the general theory of consciousness, and the theory of cognitive representations [K80].

14.1.1 Category Theory As A Purely Technical Tool

Category theory could help to disentangle the enormous technical complexities of the quantum TGD and to organize the existing bundle of ideas into a coherent conceptual framework. The construction of the geometry of the configuration space ("world of classical worlds") [K60, K35]. of classical configuration space spinor fields [K128]. and of S-matrix [K33] using a generalization of the quantum holography principle are especially natural applications. Category theory might also help in formulating the new TGD inspired view about number system as a structure obtained by "gluing together" real and p-adic number fields and TGD as a quantum theory based on this generalized notion of number [K108, K109, K107].

14.1.2 Category Theory Based Formulation Of The Ontology Of TGD Universe

It is interesting to find whether also the ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences [?] could be expressed elegantly using the language of the category theory.

There are indeed natural and non-trivial categories involved with many-sheeted space-time and the geometry of the configuration space ("the world of classical worlds"); with configuration space spinor fields; and with the notions of quantum jump, self and self hierarchy. Functors between these categories could express more precisely the quantum classical correspondences and self-referentiality of quantum states allowing them to express information about quantum jump sequence.

1. Self hierarchy has a structure of category and corresponds functorially to the hierarchical structure of the many-sheeted space-time.
2. Quantum jump sequence has a structure of category and corresponds functorially to the category formed by a sequence of maximally deterministic regions of space-time sheet. Even the quantum jump could have space-time correlates made possible by the generalization of the Boolean logic to what might be space-time correlate of quantum logic and allowing to identify space-time correlate for the notion of quantum superposition.
3. The category of light cones with inclusion as an arrow defining time ordering appears naturally in the construction of the configuration space geometry and realizes the cosmologies within cosmologies scenario. In particular, the notion of the arrow of psychological time finds a nice formulation unifying earlier two different explanations.
4. In zero energy ontology (ZEO), which emerged many years after writing the first version of this chapter, causal diamonds (CDs) defined in terms of intersection of future and past directed light-cones form a category with arrow identified as inclusion.
5. The preferred extremals would form a category if the proposed duality mapping associative (co-associative) 4-surfaces of embedding space respects associativity (co-associativity) [K109]. The duality would allow to construct new preferred extremals of Kähler action.

14.1.3 Other Applications

One can imagine also other applications.

1. Categories posses inherent logic [A73] based on the notion of sieves relying on the notion of presheaf which generalizes Boolean logic based on inclusion. In TGD framework inclusion is naturally replaced by topological condensation and this leads to a two-valued logic realizing space-time correlate of quantum logic based on the notions of quantum sieve and quantum topos.

   This suggests the possibility to geometrize the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through logic in which the law of excluded middle is not true. Interestingly, the p-adic logic of cognition is naturally 2-valued whereas the real number based logic of sensory experience allows excluded middle (is the person at the door in or out, in and out, or neither in nor out?). The quantum logic naturally associated with spinors (in the "world of classical worlds") is consistent with the logic based on quantum sieves.
2. Simple Boolean logic of right and wrong does not seem to be ideal for understanding moral rules. Same applies to the beauty-ugly logic of aesthetic experience. The logic based on quantum sieves would perhaps provide a more flexible framework.
3. Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience. Here the new elements associated with the ontology of space-time due to the generalization of number concept would be central. Category theory could be also helpful in the modelling of conscious communications, in particular the telepathic communications based on sharing of mental images involving the same mechanism which makes possible space-time correlates of quantum logic and quantum superposition.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L3]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

14.2 What Categories Are?

In the following the basic notions of category theory are introduced and the notion of presheaf and category induced logic are discussed.

14.2.1 Basic Concepts

Categories [A24, A47, A51] are roughly collections of objects A, B, C... and morphisms $f(A \to B)$ between objects A and B such that decomposition of two morphisms is always defined. Identity morphisms map objects to objects. Topological/linear spaces form a category with continuous/linear maps acting as morphisms. Also algebraic structures of a given type form a category: morphisms are now homomorphisms. Practically any collection of mathematical structures can be regarded as a category. Morphisms can can be very general: for instance, partial ordering $a \leq b$ can define morphism $f(A \to B)$.

Functors between categories map objects to objects and morphisms to morphisms so that a product of morphisms is mapped to the product of the images and identity morphism is mapped to identity morphism. Group representation is example of this kind of a functor: now group action in group is mapped to a linear action at the level of the representations. Commuting square is an easy visual manner to understand the basic properties of a functor, see **Fig. 14.1**.

The product $C = AB$ for objects of categories is defined by the requirement that there are projection morphisms π_A and π_B from C to A and B and that for any object D and pair of morphisms $f(D \to A)$ and $g(D \to B)$ there exist morphism $h(D \to C)$ such that one has $f = \pi_A h$ and $g = \pi_B h$. Graphically (see **Fig. 14.1**) this corresponds to a square diagram in which pairs A, B and C, D correspond to the pairs formed by opposite vertices of the square and arrows DA and DB correspond to morphisms f and g, arrows CA and CB to the morphisms π_A and π_B and the arrow h to the diagonal DC.

Examples of product categories are Cartesian products of topological and linear spaces, of differentiable manifolds, groups, etc. Also tensor products of linear spaces satisfies these axioms. One can define also more advanced concepts such as limits and inverse limits. Also the notions of sheafs, presheafs, and topos are important.

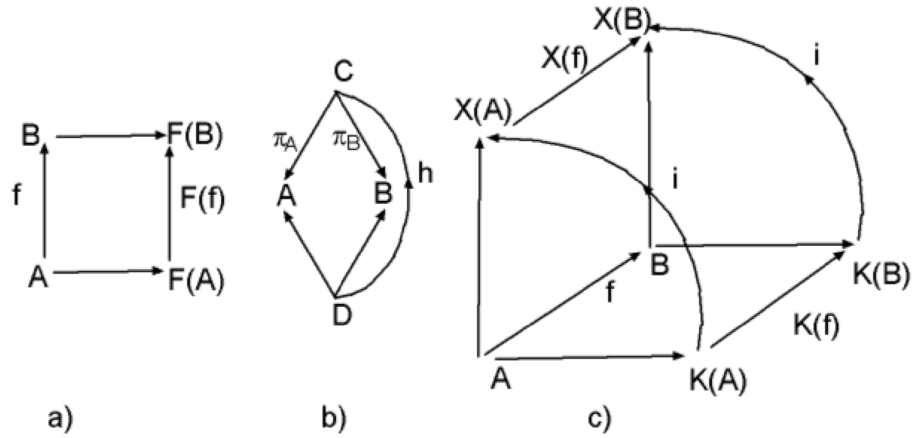

Figure 14.1: Commuting diagram associated with the definition of a) functor, b) product of objects of category, c) presheaf K as sub-object of presheaf X ("two pages of book".)

14.2.2 Presheaf As A Generalization For The Notion Of Set

Presheafs can be regarded as a generalization for the notion of set. Presheaf is a functor X that assigns to any object of a category **C** an object in the category **Set** (category of sets) and maps morphisms to morphisms (maps between sets for **C**). In order to have a category of presheafs, also morphisms between presheafs are needed. These morphisms are called natural transformations $N : X(A) \rightarrow Y(A)$ between the images $X(A)$ and $Y(A)$ of object A of **C**. They are assumed to obey the commutativity property $N(B)X(f) = Y(f)N(A)$ which is best visualized as a commutative square diagram. Set theoretic inclusion $i : X(A) \subset Y(A)$ is obviously a natural transformation.

An easy manner to understand and remember this definition is commuting diagram consisting of two pages of book with arrows of natural transformation connecting the corners of the pages: see **Fig. ??**.

As noticed, presheafs are generalizations of sets and a generalization for the notion of subset to a sub-object of presheaf is needed and this leads to the notion of topos [A73, A51]. In the classical set theory a subset of given sets X can be characterized by a mapping from set X to the set $\Omega = \{true, false\}$ of Boolean statements. Ω itself belongs to the category **C**. This idea generalizes to sub-objects whose objects are collections of sets: Ω is only replaced with its Cartesian power. It can be shown that in the case of presheafs associated with category **C** the sub-object classifier Ω can be replaced with a more general algebra, so called Heyting algebra [A73, A51] possessing the same basic operations as Boolean algebra (and, or, implication arrow, and negation) but is not in general equivalent with any Boolean algebra. What is important is that this generalized logic is inherent to the category **C** so that many-valued logic ceases to be an ad hoc construct in category theory.

In the theory of presheafs sub-object classifier Ω, which belongs to **Set**, is defined as a particular presheaf. Ω is defined by the structure of category **C** itself so that one has a geometrization of the notion of logic implied by the properties of category. The notion of sieve is essential here. A sieve for an object A of category **C** is defined as a collection of arrows $f(A \rightarrow ...)$ with the property that if $f(A \rightarrow B)$ is an arrow in sieve and if $g(B \rightarrow C)$ is any arrow then $gf(A \rightarrow C)$ belongs to sieve.

In the case that morphism corresponds to a set theoretic inclusion the sieve is just either empty set or the set of all sets of category containing set A so that there are only two sieves corresponding to Boolean logic. In the case of a poset (partially ordered set) sieves are sets for which all elements are larger than some element.

14.2.3 Generalized Logic Defined By Category

The presheaf $\Omega : \mathbf{C} \rightarrow \mathbf{Set}$ defining sub-object classifier and a generalization of Boolean logic is defined as the map assigning to a given object A the set of all sieves on A. The generalization of maps $X \rightarrow \Omega$ defining subsets is based on the notion of sub-object K. K is sub-object of presheaf X in the category of presheaves if there exist natural transformation $i : K \rightarrow X$ such that for each A one has $K(A) \subset X(A)$ (so that sub-object property is reduced to subset property).

The generalization of the map $X \rightarrow \Omega$ defining subset is achieved as follows. Let K be a sub-object of X. Then there is an associated characteristic arrow $\chi^K : X \rightarrow \Omega$ generalizing the characteristic Boolean valued map defining subset, whose components $\chi^K_A : X(A) \rightarrow \Omega(A)$ in **C** is defined as

$$\chi^K_A(x) = \{f(A \rightarrow B) | X(f)(x) \in K(B)\} \ .$$

By using the diagrammatic representation of **Fig. 14.1** for the natural transformation i defining sub-object, it is not difficult to see that by the basic properties of the presheaf K $\chi^K_A(x)$ is a sieve. When morphisms f are inclusions in category Set, only two sheaves corresponding to all sets containing X and empty sheaf result. Thus binary valued maps are replaced with sieve-valued maps and sieves take the role of possible truth values. What is also new that truths and logic are in principle context dependent since each object A of **C** serves as a context and defines its own collection of sieves.

The generalization for the notion of point of set X exists also and corresponds to a selection of single element γ_A in the set $X(A)$ for each A object of **C**. This selection must be consistent with the action of morphisms $f(A \rightarrow B)$ in the sense that the matching condition $X(f)(\gamma_A) = \gamma_B$ is satisfied. It can happen that category of presheaves has no points at all since the matching condition need not be satisfied globally.

It turns out that TGD based notion of subsystem leads naturally to what might be called quantal versions of topos, presheaves, sieves and logic.

14.3 More Precise Characterization Of The Basic Categories And Possible Applications

In the following the categories associated with self and quantum jump are discussed in more precise manner and applications to communications and cognition are considered.

14.3.1 Intuitive Picture About The Category Formed By The Geometric Correlates Of Selves

Space-time surface $X^4(X^3)$ decomposes into regions obeying either real or p-adic topology and each region of this kind corresponds to an unentangled subsystem or self lasting at least one quantum jump. By the localization in the zero modes these decompositions are equivalent for all 3-surfaces X^3 in the quantum superposition defined by the prepared WCW spinor fields resulting in quantum jumps. There is a hierarchy of selves since selves can contain sub-selves. The entire space-time surface $X^4(X^3)$ represents the highest level of the self hierarchy.

This structure defines in a natural manner a category. Objects are all possible sub-selves contained in the self hierarchy: sub-self is set consisting of lower level sub-selves, which in turn have a further decomposition to sub-selves, etc... The naïve expectation is that geometrically sub-self belongs to a self as a subset and this defines an inclusion map acting as a natural morphism in this category. This expectation is not quite correct. More natural morphisms are the arrows telling that self as a set of sub-selves contains sub-self as an element. These arrows define a structure analogous to a composite of hierarchy trees.

To be more precise, for a single space-time surface $X^4(X^3)$ this hierarchy corresponds to a subjective time slice of the self hierarchy defined by a single quantum jump. The sequence of hierarchies associated with a sequence of quantum jumps is a natural geometric correlate for the self hierarchy. This means that the objects are now sequences of submoments of consciousness. Sequences are not arbitrary. Self must survive its lifetime although sub-selves at various levels can disappear and reappear (generation and disappearance of mental images). Geometrically this means typically a phase transition transforming real or p_1-adic to p_2-adic space-time region with same topology as the environment. Also sub-selves can fuse to single sub-self. The constraints on self sequences must be such that it takes these processes into account. Note that these constraints emerge naturally from the fact that quantum jumps sequences define the sequences of surfaces $X^4(X^3)$.

By the rich anatomy of the quantum jump there is large number of quantum jumps leading from a given initial quantum history to a given final quantum history. One could envisage quantum jump also as a discrete path in the space of WCW spinor fields leading from the initial state to the final state. In particular, for given self there is an infinite number of closed elementary paths leading from the initial quantum history back to the initial quantum history and these paths in principle give all possible conscious information about a given quantum history/idea: kind of self morphisms are in question (analogous to, say, group automorhisms). Information about point of space is obtained only by moving around and coming back to the point, that is by studying the surroundings of the point. Self in turn can be seen as a composite of elementary paths defined by the quantum jumps. Selves can define arbitrarily complex composite closed paths giving information about a given quantum history.

14.3.2 Categories Related To Self And Quantum Jump

The categories defined by moments of consciousness and the notion of self

Since quantum jump involves state reduction and the sequence of self measurement reducing all entanglement except bound state entanglement, it defines a hierarchy of unentangled subsystems allowing interpretation as objects of a category. Arrows correspond to subsystem-system relationship and the two subsystems resulting in self measurement to the system. What subsystem corresponds mathematically is however not at all trivial and the naïve description as a tensor factor does not work. Rather, a definition relying on the notion of p-adic length scale cutoff identified as a fundamental aspect of nature and consciousness is needed.

It is not clear what the statement that self corresponds to a subsystem which remains unentangled in subsequent quantum jump means concretely since subsystem can certainly change in some limits. What is clear that bound state entanglement between selves means a loss of consciousness. Category theory suggests that there should exist a functor between categories defined by two subsequent moments of consciousness. This functor maps submoments of consciousness to submoments of consciousness and arrows to arrows. Two subsequent submoments of consciousness belong to same sub-self is the functor maps the first one to the latter one. Thus category theory would play essential role in the precise definition of the notion of self.

The sequences of moments of consciousness form a larger category containing sub-selves as sequences of unentangled subsystems mapped to each other by functor arrows functoring subsequent quantum jumps to each other.

What might then be the ultimate characterizer of the self-identity? The theory of infinite primes suggests that space-time surface decomposes into regions labelled by finite p-adic primes. These primes must label also real regions rather than only p-adic ones. A p-adic space-time region characterized by prime p can transform to a real one or vice versa in quantum jump if the sizes of real and p-adic regions are characterized by the p-adic length scale L_p (or n-ary p-adic length scale $L_p(n)$. One can also consider the possibility that real region is accompanied by a p-adic region characterized by a definite prime p and providing a cognitive self-representation of the real region.

If this view is correct, the p-adic prime characterizing a given real or p-adic space-time sheet could be one characterizer of the self-identity. Self identity is lost in bound state entanglement with another space-time sheet (at least when a space-time sheet with smaller value of the p-adic prime joins by flux tube to a one with a higher value of the p-adic prime). Self identity is also lost if a space-time sheet characterized by a given p-adic prime disappears in quantum jump.

The category associated with quantum jump sequences

There are several similarities between the ontologies and epistemologies of TGD and of category theory. Conscious experience is always determined by the discrete paths in the space of configuration space spinor fields defined by a quantum jump connecting two quantum histories (states) and is never determined by single quantum history as such (quantum states are unconscious). Also category theory is about relations between objects, not about objects directly: self-morphisms give information about the object of category (in case of group composite paths would correspond to products of group automorphisms). Analogously closed paths determined by quantum jump sequences give information about single quantum history. The point is however that it is impossible to have direct knowledge about the quantum histories: they are not conscious.

One can indeed define a natural category, call it **QSelf**, applying to this situation. The objects of the category **QSelf** are initial quantum histories of quantum jumps and correspond to prepared quantum states. The discrete path defining quantum jump can be regarded as an elementary morphism. Selves are composites of elementary morphisms of the initial quantum history defined by quantum jumps: one can characterize the morphisms by the number of the elementary morphisms in the product. Trivial self contains no quantum jumps and corresponds to the identity morphism, null path. Thus the collection of all possible sequences of quantum jumps, that is collections of selves allows a description in terms of category theory although the category in question is not a subcategory of the category **Set**.

Category **QSelf** does not possess terminal and initial elements (for terminal (initial) element T there is exactly one arrow $A \to T$ ($T \to A$) for every A: now there are always many paths between quantum histories involved).

14.3.3 Communications In TGD Framework

Goro Kato identifies communications between conscious entities as natural maps between them whereas in TGD natural maps bind submoments of consciousness to selves. In TGD framework quantum measurement and the sharing of mental images are the basic candidates for communications. The problem is that the identification of communications as sharing of mental images is not consistent with the naïve view about subsystem as a tensor factor. Many-sheeted space-time however forces length scale dependent notion of subsystem at space-time level and this saves the situation.

What communications are?

Communication is essentially generation of desired mental images/sub-selves in receiver. Communication between selves need not be directly conscious: in this case communication would generate mental images at some lower level of self hierarchy of receiver: for instance generate large number of sub-sub-selves of similar type. This is like communications between organizations. Communication can be also vertical: self can generate somehow sub-self in some sub-sub....sub-self or sub-sub...sub-self can generate sub-self of self somehow. This is communication from boss to the lower levels organization or vice versa.

These communications should have direct topological counterparts. For instance, the communication between selves could correspond to an exchange of mental image represented as a space-time region of different topology inside sender self space-time sheet. The sender self would simply throw this space-time region to a receiver self like a ball. This mechanism applies also to vertical communications since the ball could be also thrown from a boss to sub...sub-self at some lower level of hierarchy and vice versa.

The sequence of space-time surfaces provides a direct topological counterpart for communication as throwing balls representing sub-selves. Quantum jump sequence contains space-time surfaces in which the regions corresponding to receiver and sender selves are connected by a flux tube (perhaps massless extremal) representing classically the communication: during the communication the receiver and sender would form single self. The cartoon vision about rays connecting the eyes of communication persons would make sense quite concretely.

More refined means of communication would generate sub-selves of desired type directly at the end of receiver. In this case it is not so obvious how the sequence $X^(X^3)$ of space-time surfaces could represent communication. Of course, one can question whether communication is really what happens in this kind of situation. For instance, sender can affect the environment of receiver to be such that receiver gets irritated (computer virus is good manner to achieve this!) but one can wonder whether this is real communication.

Communication as quantum measurement?

Quantum measurement generates one-one map between the states of the entangled systems resulting in quantum measurement. Both state function reduction and self measurement give rise to this kind of map. This map could perhaps be interpreted as quantum communication between unentangled subsystems resulting in quantum measurement. For the state reduction process the space-time correlates are the values of zero modes. For state preparation the space-time correlates should correspond to classical spinor field modes correlating for the two subsystems generated in self measurement.

Communication as sharing of mental images

It has become clear that the sharing of mental images induced by quantum entanglement of sub-selves of two separate selves represents genuine conscious communication which is analogous telepathy and provides general mechanism of remote mental interactions making possible even molecular recognition mechanisms.

1. The sharing of mental images is not possible unless one assumes that self hierarchy is defined by using the notion of length scale resolution defined by p-adic length scale. The notion of scale of resolution is indeed fundamental for all quantum field theories (renormalization group invariance) for all quantum field theories and without it the practical modelling of physics would not be possible. The notion reflects directly the length scale resolution of conscious experience. For a given sub-self of self the resolution is given by the p-adic length scale associated with the sub-self space-time sheet.
2. Length scale resolution emerges naturally from the fact that sub-self space-time sheets having Minkowskian signature of metric are separated from the one representing self by wormhole contacts with Euclidian signature of metric. The signature of the induced metric changes from Minkowskian signature to Euclidian signature at "elementary particle horizons" surrounding the throats of the wormhole contacts and having degenerate induced metric. Elementary particle horizons are thus metrically two-dimensional light like surfaces analogous to the

boundary of the light cone and allow conformal invariance. Elementary particle horizons act as causal horizons. Topologically condensed space-time sheets are analogous to black hole interiors and due to the lack of the causal connectedness the standard description of sub-selves as tensor factors of the state space corresponding to self is not appropriate.

Hence systems correspond, not to the space-time sheets plus entire hierarchy of space-time sheets condensed to it, but rather, to space-time sheets with holes resulting when the space-time sheets representing subsystems are spliced off along the elementary particle horizons around wormhole contacts. This does not mean that all information about subsystem is lost: subsystem space-time sheet is only replaced by the elementary particle horizon. In analogy with the description of the black hole, some parameters (mass, charges, ...) characterizing the classical fields created by the sub-self space-time sheet characterize sub-self.

One can say that the state space of the system contains "holes". There is a hierarchy of state spaces labelled by p-adic primes defining length scale resolutions. This picture resolves a longstanding puzzle relating to the interpretation of the fact that particle is characterize by both classical and quantum charges. Particle cannot couple simultaneously to both and this is achieved if quantum charge is associated with the lowest level description of the particle as CP_2 extremal and classical charges to its description at higher levels of hierarchy.

3. The immediate implication indeed is that it is possible to have a situation in which two selves are unentangled although their sub-selves (mental images) are entangled. This corresponds to the fusion and sharing of mental images. The sharing of the mental images means that union of disjoint hierarchy trees with levels labelled by p-adic primes p is replaced by a union of hierarchy threes with horizontal lines connecting subsystems at the same level of hierarchy. Thus the classical correspondence defines a category of presheaves with both vertical arrows replaced by sub-self-self relationship, horizontal arrows representing sharing of mental images, and natural maps representing binding of submoments of consciousness to selves.

Comparison with Goro Kato's approach

It is of interest to compare Goro Kato's approach with TGD approach. The following correspondence suggests itself.

1. In TGD each quantum jumps defines a category analogous to the Goro Kato's category of open sets of some topological space but set theoretic inclusion replaced by topological condensation. The category defined by a moment of consciousness is dynamical whereas the category of open sets is non-dynamical.
2. The assignment of a 3-surface acting as a causal determinant to each unentangled subsystem defined by a moment of consciousness defines a unique "quantum presheaf" which is the counterpart of the presheaf in Goro Kato's theory. The conscious entity of Kato's theory corresponds to the classical correlate for a moment of consciousness.
3. Natural maps between the causal determinants correspond to the space-time correlates for the functor arrows defining the threads connecting submoments of consciousness to selves. In Goro Kato's theory natural maps are interpreted as communications between conscious entities. The sharing of mental images by quantum entanglement between subsystems of unentangled systems defines horizontal bi-directional arrows between subsystems associated with same moment of consciousness and is counterpart of communication in TGD framework. It replaces the union of disjoint hierarchy trees associated with various unentangled subsystems with hierarchy trees having horizontal connections defining the bi-directional arrows. The sharing of mental images is not possible if subsystem is identified as a tensor factor and thus without taking into account length scale resolution.

14.3.4 Cognizing About Cognition

There are close connections with basic facts about cognition.

1. Categorization means classification and abstraction of common features in the class formed by the objects of a category. Already quantum jump defines category with hierarchical structure and can be regarded as consciously experienced analysis in which totally entangled entire

universe $U\Psi_i$ decomposes to a product of maximally unentangled subsystems. The sub-selves of self are like elements of set and are experienced as separate objects whereas sub-sub-selves of sub-self self experiences as an average: they belong to a class or category formed by the sub-self. This kind of averaging occurs also for the contributions of quantum jumps to conscious experience of self.

2. The notions of category theory might be useful in an attempt to construct a theory of cognitive structures since cognition is indeed to high degree classification and abstraction process. The sub-selves of a real self indeed have p-adic space-time sheets as geometric correlates and thus correspond to cognitive sub-selves, thoughts. A meditative experience of empty mind means in case of real self the total absence of thoughts.
3. Predicate logic provides a formalization of the natural language and relies heavily on the notion of n-ary relation. Binary relations $R(a,b)$ corresponds formally to the subset of the product set $A\times B$. For instance, statements like "A does something to B" can be expressed as a binary relation, particular kind of arrow and morphism ($A \leq B$ is a standard example). For sub-selves this relation would correspond to a dynamical evolution at space-time level modelling the interaction between A and B. The dynamical path defined by a sequence of quantum jumps is able to describe this kind of relationships too at level of conscious experience. For instance, "A touches B" would involve the temporary fusion of sub-selves A and B to sub-self C.

14.4 Logic And Category Theory

Category theory allows naturally more general than Boolean logics inherent to the notion of topos associated with any category. Basic question is whether the ordinary notion of topos algebra based on set theoretic inclusion or the notion of quantum topos based on topological condensation is physically appropriate. Starting from the quasi-Boolean algebra of open sets one ends up to the conclusion that quantum logic is more natural. Also WCW spinor fields lead naturally to the notion of quantum logic.

14.4.1 Is The Logic Of Conscious Experience Based On Set Theoretic Inclusion Or Topological Condensation?

The algebra of open sets with intersections and unions and complement defined as the interior of the complement defines a modification of Boolean algebra having the peculiar feature that the points at the boundary of the closure of open set cannot be said to belong to neither interior of open set or of its complement. There are two options concerning the interpretation.

1. 3-valued logic could be in question. It is however not possible to understand this three-valuedness if one defines the quasi-Boolean algebra of open sets as Heyting algebra. The resulting logic is two-valued and the points at boundaries of the closure do not correspond neither to the statement or its negation. In p-adic context the situation changes since p-adic open sets are also closed so that the logic is strictly Boolean. That our ordinary cognitive mind is Boolean provides a further good reason for why cognition is p-adic.
2. These points at the boundary of the closure belong to both interior and exterior in which case a two-valued "quantum logic" allowing superposition of opposite truth values is in question. The situation is indeed exactly the same as in the case of space-time sheet having wormhole contacts to several space-time sheets.

The quantum logic brings in mind Zen consciousness [J20] (which I became fascinated of while reading Hofstadter's book "Gödel, Escher, Bach" [A34]) and one can wonder whether selves having real space-time sheets as geometric correlates and able to live simultaneously in many parallel worlds correspond to Zen consciousness and Zen logic. Zen logic would be also logic of sensory experience whereas cognition would obey strictly Boolean logic.

The causal determinants associated with space-time sheets correspond to light like 3-surfaces which could elementary particle horizons or space-time boundaries and possibly also 3-surfaces separating two maximal deterministic regions of a space-time sheet. These surfaces act as 3-dimensional quantum holograms and have the strange Zen property that they are neither space-like nor time-like so that they represent both the state and the process. In the TGD based model for topological quantum computation (TQC) light-like boundaries code for the computation so that TQC program code would be equivalent with the running program [K5].

14.4.2 Do WCW Spinor Fields Define Quantum Logic And Quantum Topos

I have proposed already earlier that WCW spinor fields define what might be called quantum logic. One can wonder whether WCW spinor s could also naturally define what might be called quantum topos since the category underlying topos defines the logic appropriate to the topos. This question remains unanswered in the following: I just describe the line of though generalizing ordinary Boolean logic.

Finite-dimensional spinors define quantum logic

Spinors at a point of an $2N$-dimensional space span 2^N-dimensional space and spinor basis is in one-one correspondence with Boolean algebra with N different truth values (N bits). 2N=2-dimensional case is simple: Spin up spinor= true and spin-dow spinor=false. The spinors for $2N$-dimensional space are obtained as an N-fold tensor product of 2-dimensional spinors (spin up, spin down): just like in the case of Cartesian power of Ω.

Boolean spinors in a given basis are eigen states for a set N mutually commuting sigma matrices providing a representation for the tangent space group acting as rotations. Boolean spinors define N Boolean statements in the set Ω^N so that one can in a natural manner assign a set with a Boolean spinor. In the real case this group is $SO(2N)$ and reduces to $SU(N)$ for Kähler manifolds. For pseudo-euclidian metric some non-compact variant of the tangent space group is involved. The selections of N mutually commuting generators are labelled by the flag-manifold $SO(2N)/SO(2)^N$ in real context and by the flag-manifold $U(N)/U(1)^N$ in the complex case. The selection of these generators defines a collection of N 2-dimensional linear subspaces of the tangent space.

Spinors are in general complex superpositions of spinor basis which can be taken as the product spinors. The quantum measurement of N spins representing the Cartan algebra of $SO(2N)$ ($SU(N)$) leads to a state representing a definite Boolean statement. This suggests that quantum jumps as moments of consciousness quite generally make universe classical, not only in geometric but also in logical sense. This is indeed what the state preparation process for WCW spinor field seems to do.

Quantum logic for finite-dimensional spinor fields

One can generalize the idea of the spinor logic also to the case of spinor fields. For a given choice of the local spinor basis (which is unique only modular local gauge rotation) spinor field assigns to each point of finite-dimensional space a quantum superposition of Boolean statements decomposing into product of N statements.

Also now one can ask whether it is possible to find a gauge in which each point corresponds to definite Boolean statement and is thus an eigen state of a maximal number of mutually commuting rotation generators Σ_{ij}. This is not trivial if one requires that Dirac equation is satisfied. In the case of flat space this is certainly true and constant spinors multiplied by functions which solve d'Alembert equation provide a global basis.

The solutions of Dirac equation in a curved finite-dimensional space do not usually possess a definite spin direction globally since spinor curvature means the presence of magnetic spin-flipping interaction and since there need not exist a global gauge transformation leading to an eigen state of the local Cartan algebra everywhere. What might happen is that the local gauge transformation becomes singular at some point: for instance, the direction of spin would be radial around given point and become ill defined at the point. This is much like the singularities for vector fields on sphere. The spinor field having this kind of singularity should vanish at singularity but the local gauge rotation rotating spin in same direction everywhere is necessarily ill-defined at the singularity.

In fact, this can be expressed using the language of category theory. The category in question corresponds to a presheaf which assigns to the points of the base space the fiber space of the spinor bundle. The presence of singularity means that there are no global section for this presheaf, that is a continuous choice of a non-vanishing spinor at each point of the base space. The so called Kochen-Specker theorem discussed in [A51] is closely related to a completely analogous phenomenon involving non-existence of global sections and thus non-existence of a global truth value.

Thus in case of curved spaces is not necessarily possible to have spinor field basis representing globally Boolean statements and only the notion of locally Boolean logic makes sense. Indeed, one can select the basis to be eigen state of maximal set of mutually commuting rotation generators in single point of the compact space. Any such choice does.

Quantum logic and quantum topos defined by the prepared WCW spinor fields

The prepared WCW spinor fields occurring as initial and final states of quantum jumps are the natural candidates for defining quantum logic. The outcomes of the quantum jumps resulting in the state preparation process are maximally unentangled states and are as close to Boolean states as possible.

WCW spinors correspond to fermionic Fock states created by infinite number of fermionic (leptonic and quarklike) creation and annihilation operators. The spin degeneracy is replaced by the double-fold degeneracy associated with a given fermion mode: given state either contains fermion or not and these two states represent true and false now. If WCW were flat, the Fock state basis with definite fermion and anti-fermion numbers in each mode would be in one-one correspondence with Boolean algebra.

Situation is however not so simple. Finite-dimensional curved space is replaced with the fiber degrees of freedom of WCW in which the metric is non-vanishing. The precise analogy with the finite-dimensional case suggests that if the curvature form of the WCW spinor connection is nontrivial, it is impossible to diagonalize even the prepared maximally unentangled WCW spinor fields Ψ_i in the entire fiber of WCW (quantum fluctuating degrees of freedom) for given values of the zero modes. Local singularities at which the spin quantum numbers of the diagonalized but vanishing WCW spinor field become ill-defined are possible also now.

In the infinite-dimensional context the presence of the fermion-anti-fermion pairs in the state means that it does not represent a definite Boolean statement unless one defines a more general basis of WCW spinor s for which pairs are present in the states of the state basis: this generalization is indeed possible. The sigma matrices of the WCW appearing in the spinor connection term of the Dirac operator of WCW indeed create fermion-fermion pairs. What is decisive, is not the absence of fermion-anti-fermion pairs, but the possibility that the spinor field basis cannot be reduced to eigen states of the local Cartan algebra in fiber degrees of freedom globally.

Also for bound states of fermions (say leptons and quarks) it is impossible to reduce the state to a definite Boolean statement even locally. This would suggest that fermionic logic does not reduce to a completely Boolean logic even in the case of the prepared states.

Thus WCW spinor fields could have interpretation in terms of non-Boolean quantum logic possessing Boolean logics only as sub-logics and define what might be called quantum topos. Instead of Ω^N-valued maps the values for the maps are complex valued quantum superpositions of truth values in Ω^N.

An objection against the notion of quantum logic is that Boolean algebra operationsandOR do not preserve fermion number so that quantum jump sequences leading from the product state defined by operands to the state representing the result of operation are therefore not possible. One manner to circumvent the objection is to consider the sub-algebra spanned by fermion and anti-fermion pairs for given mode so that fermion number conservation is not a problem. The objection can be also circumvented for pairs of space-time sheets with opposite time orientations and thus opposite signs of energies for particles. One can construct the algebra in question as pairs of many fermion states consisting of positive energy fermion and negative energy anti-fermion so that all states have vanishing fermion number and logical operations become possible. Pairs of MEs with opposite time orientations are excellent candidates for carries of these fermion-anti-fermion pairs.

Quantum classical correspondence and quantum logic

The intuitive idea is that the global Boolean statements correspond to sections of Z^2 bundle. Möbius band is a prototype example here. The failure of a global statement would reduce to the non-existence of global section so that true would transforms to false as one goes around full 2π rotation.

One can ask whether fermionic quantum realization of Boolean logic could have space-time counterpart in terms of Z_2 fiber bundle structure. This would give some hopes of having some connection between category theoretical and fermionic realizations of logic. The following argument stimulated by email discussion with Diego Lucio Rapoport suggests that this might be the case.

1. The hierarchy of Planck constants realized using the notion of generalized embedding space involves only groups $Z_{n_a} \times Z_{n_b}$, $n_a, n_b \neq 2$ if one takes Jones inclusions as starting point. There is however no obvious reason for excluding the values $n_a = 2$ and $n_b = 2$ and the question concerns physical interpretation. Even if one allows only $n_i \geq 3$ one can ask for the physical interpretation for the factorization $Z_{2n} = Z_2 \times Z_n$. Could it perhaps relate to a space-time correlates for Boolean two-valuedness?
2. An important implication of fiber bundle structure is that the partonic 2-surfaces have $Z_{n_a} \times Z_{n_b} = Z_{n_a n_b}$ as a group of conformal symmetries. I have proposed that n_a or n_b is even for fermions so that Z_2 acts as a conformal symmetry of the partonic 2-surface. Both n_a and n_b would be odd for truly elementary bosons. Note that this hypothesis makes sense also for $n_i \geq 3$.
3. Z_2 conformal symmetry for fermions would imply that all partonic 2-surfaces associated with fermions are hyper-elliptic. As a consequence elementary particle vacuum functionals defined in modular degrees of freedom would vanish for fermions for genus $g > 2$ so that only three fermion families would be possible in accordance with experimental facts. Since gauge bosons and Higgs correspond to pairs of partonic 2-surfaces (the throats of the wormhole contact) one has 9 gauge boson states labelled by the pairs (g_1, g_2) which can be grouped to SU(3) singlet and octet. Singlet corresponds to ordinary gauge bosons.

   super-symplectic bosons are truly elementary bosons in the sense that they do not consist of fermion-anti-fermion pairs. For them both n_a and n_b should be odd if the correspondence is taken seriously and all genera would be possible. The super-conformal partners of these bosons have the quantum numbers of right handed neutrino. Since both spin directions are possible, one can ask whether Boolean Z_2 must be present also now. This need not be the case, ν_R generates only super-symmetries and does not define a family of fermionic oscillator operators. The electro-weak spin of ν_R is frozen and it does not couple at all to electro-weak intersections. Perhaps (only) odd values of n_i are possible in this case.
4. If fermionic Boolean logic has a space-time correlate, one can wonder whether the fermionic Z_2 conformal symmetry might correspond to a space-time correlate for the Boolean true-false dichotomy. If the partonic 2-surface contains points which are fixed points of Z_2 symmetry, there exists no everywhere non-vanishing sections. Furthermore, induced spinor fields should vanish at the fixed points of Z_2 symmetry since they correspond to singular orbifold points so that one could not actually have a situation in which true and false are true simultaneously. Global sections could however fail to exist since CP_2 spinor bundle is non-trivial.

14.4.3 Category Theory And The Modelling Of Aesthetic And Ethical Judgements

Consciousness theory should allow to model model the logics of ethics and aesthetics. Evolution (representable as p-adic evolution in TGD framework) is regarded as something positive and is a good candidate for defining universal ethics in TGD framework. Good deeds are such that they support this evolution occurring in statistical sense in any case. Moral provides a practical model for what good deeds are and moral right-wrong statements are analogous to logical statements. Often however the two-valued right-wrong logic seems to be too simplistic in case of moral statements. Same applies to aesthetic judgements. A possible application of the generalized logics defined by the inherent structure of categories relates to the understanding of the dilemmas associated with the moral and aesthetic rules.

As already found, quantum versions of sieves provide a formal generalization of Boolean truth values as a characteristic of a given category. Generalized moral rules could perhaps be seen as sieve valued statements about deeds. Deeds are either right or wrong in what might be called Boolean moral code. One can also consider Zen moral in which some deeds can be said to be right and wrong simultaneously. Some deeds could also be such that there simply exists no globally consistent moral rule: this would correspond to the non-existence of what is called global section assigning to each object of the category consisting of the pairs formed by a moral agents and given deed) a sieve simultaneously.

14.5 Platonism, Constructivism, And Quantum Platonism

During years I have been trying to understand how Category Theory and Set Theory relate to quantum TGD inspired view about fundamentals of mathematics and the outcome section is added to this chapter several years after its first writing. I hope that reader does not experience too unpleasant discontinuity. I managed to clarify my thoughts about what these theories are by reading the article Structuralism, Category Theory and Philosophy of Mathematics by Richard Stefanik [A77]. Blog discussions and email correspondence with Sampo Vesterinen have been very stimulating and inspired the attempt to represent TGD based vision about the unification of mathematics, physics, and consciousness theory in a more systematic manner.

Before continuing I want to summarize the basic ideas behind TGD vision. One cannot understand mathematics without understanding mathematical consciousness. Mathematical consciousness and its evolution must have direct quantum physical correlates and by quantum classical correspondence these correlates must appear also at space-time level. Quantum physics must allow to realize number as a conscious experience analogous to a sensory quale. In TGD based ontology there is no need to postulate physical world behind the quantum states as mathematical entities (theory is the reality). Hence number cannot be any physical object, but can be identified as a quantum state or its label and its number theoretical anatomy is revealed by the conscious experiences induced by the number theoretic variants of particle reactions. Mathematical systems and their axiomatics are dynamical evolving systems and physics is number theoretically universal selecting rationals and their extensions in a special role as numbers, which can can be regarded elements of several number fields simultaneously.

14.5.1 Platonism And Structuralism

There are basically two philosophies of mathematics.

1. Platonism assumes that mathematical objects and structures have independent existence. Natural numbers would be the most fundamental objects of this kind. For instance, each natural number has its own number-theoretical anatomy decomposing into a product of prime numbers defining the elementary particles of Platonia. For quantum physicist this vision is attractive, and even more so if one accepts that elementary particles are labelled by primes (as I do)! The problematic aspects of this vision relate to the physical realization of the Platonia. Neither Minkowski space-time nor its curved variants understood in the sense of set theory have no room for Platonia and physical laws (as we know them) do not seem to allow the realization of all imaginable internally consistent mathematical structures.

2. Structuralist believes that the properties of natural numbers result from their relations to other natural numbers so that it is not possible to speak about number theoretical anatomy in the Platonic sense. Numbers as such are structureless and their relationships to other numbers provide them with their apparent structure. According to [A77] structuralism is however not enough for the purposes of number theory: in combinatorics it is much more natural to use intensional definition for integers by providing them with inherent properties such as decomposition into primes. I am not competent to take any strong attitudes on this statement but my physicist's intuition tells that numbers have number theoretic anatomy and that this anatomy can be only revealed by the morphisms or something more general which must have physical counterparts. I would like to regard numbers are analogous to bound states of elementary particles. Just as the decays of bound states reveal their inner structure,

the generalizations of morphisms would reveal to the mathematician the inherent number theoretic anatomy of integers.

14.5.2 Structuralism

Set theory and category theory represent two basic variants of structuralism and before continuing I want to clarify to myself the basic ideas of structuralism: the reader can skip this section if it looks too boring.

Set theory

Structuralism has many variants. In set theory [A14] the elements of set are treated as structureless points and sets with the same cardinality are equivalent. In number theory additional structure must be introduced. In the case of natural numbers one introduces the notion of successor and induction axiom and defines the basic arithmetic operations using these. Set theoretic realization is not unique. For instance, one can start from empty set Φ identified as 0, identify 1 as $\{\Phi\}$, 2 as $\{0,1\}$ and so on. One can also identify 0 as Φ, 1 as $\{0\}$, 2 as $\{\{0\}\}$, ... For both physicist and consciousness theorist these formal definitions look rather weird.

The non-uniqueness of the identification of natural numbers as a set could be seen as a problem. The structuralist's approach is based on an extensional definition meaning that two objects are regarded as identical if one cannot find any property distinguishing them: object is a representative for the equivalence class of similar objects. This brings in mind gauge fixing to the mind of physicists.

Category theory

Category theory [A3] represents a second form of structuralism. Category theorist does not worry about the ontological problems and dreams that all properties of objects could be reduced to the arrows and formally one could identify even objects as identity morphisms (looks like a trick to me). The great idea is that functors between categories respecting the structure defined by morphisms provide information about categories. Second basic concept is natural transformation which maps functors to functors in a structure preserving manner. Also functors define a category so that one can construct endless hierarchy of categories. This approach has enormous unifying power since functors and natural maps systemize the process of generalization. There is no doubt that category theory forms a huge piece of mathematics but I find difficult to believe that arrows can catch all of it.

The notion of category can be extended to that of n-category. In the blog post "First edge of the cube" (see `http://tinyurl.com/yydjavv8`) I have proposed a geometric realization of this hierarchy in which one defines 1-morphisms by parallel translations, 2-morphisms by parallel translations of parallel translations, and so on. In infinite-dimensional space this hierarchy would be infinite. Abstractions about abstractions about.., thoughts about thoughts about, statements about statements about..., is the basic idea behind this interpretation. Also the hierarchy of logics of various orders corresponds to this hierarchy. This encourages to see category theoretic thinking as being analogous to higher level self reflection which must be distinguished from the direct sensory experience.

In the case of natural numbers category theoretician would identify successor function as the arrow binding natural numbers to an infinitely long string with 0 as its end. If this approach would work, the properties of numbers would reflect the properties of the successor function.

14.5.3 The View About Mathematics Inspired By TGD And TGD Inspired Theory Of Consciousness

TGD based view might be called quantum Platonism. It is inspired by the requirement that both quantum states and quantum jumps between them are able to represent number theory and that all quantum notions have also space-time correlates so that Platonia should in some sense exist also at the level of space-time. Here I provide a brief summary of this view as it is now.

Physics is fixed from the uniqueness of infinite-D existence and number theoretic universality

1. The basic philosophy of quantum TGD relies on the geometrization of physics in terms of infinite-dimensional Kähler geometry of WCW , whose uniqueness is forced by the mere mathematical existence. Space-time dimension and embedding space $H = M^4 \times CP_2$ are fixed among other things by this condition and allow interpretation in terms of classical number fields. Physical states correspond to WCW spinor fields with WCW spinor s having interpretation as Fock states. Rather remarkably, WCW Clifford algebra defines standard representation of so called hyper finite factor of II_1, perhaps the most fascinating von Neumann algebra.
2. Number theoretic universality states that all number fields are in a democratic position. This vision can be realized by requiring generalization of notions of embedding space by gluing together real and p-adic variants of embedding space along common algebraic numbers. All algebraic extensions of p-adic numbers are allowed. Real and p-adic space-time sheets intersect along common algebraics. The identification of the p-adic space-time sheets as correlates of cognition and intentionality explains why cognitive representations at space-time level are always discrete. Only space-time points belonging to an algebraic extension of rationals associated contribute to the data defining S-matrix. These points define what I call number theoretic braids. The interpretation in of algebraic discreteness terms of a physical realization of axiom of choice is highly suggestive. The axiom of choice would be dynamical and evolving quantum jump by quantum jump as the algebraic complexity of quantum states increases.

Holy trinity of existence

In TGD framework one would have 3-levelled ontology numbers should have representations at all these levels [L2].

1. Subjective existence as a sequence of quantum jumps giving conscious sensory representations for numbers and various geometric structures would be the first level.
2. Quantum states would correspond to Platonia of mathematical ideas and mathematician- or if one is unwilling to use this practical illusion- conscious experiences about mathematic ideas, would be in quantum jumps. The quantum jumps between quantum states respecting the symmetries characterizing the mathematical structure would provide conscious information about the mathematical ideas not directly accessible to conscious experience. Mathematician would live in Plato's cave. There is no need to assume any independent physical reality behind quantum states as mathematical entities since quantum jumps between these states give rise to conscious experience. Theory-reality dualism disappears since the theory is reality or more poetically: painting is the landscape.
3. The third level of ontology would be represented by classical physics at the space-time level essential for quantum measurement theory. By quantum classical correspondence space-time physics would be like a written language providing symbolic representations for both quantum states and changes of them (by the failure of complete classical determinism of the fundamental variational principle). This would involve both real and p-adic space-time sheets corresponding to sensory and cognitive representations of mathematical concepts. This representation makes possible the feedback analogous to formulas written by mathematician crucial for the ability of becoming conscious about what one was conscious of and the dynamical character of this process allows to explain the self-referentiality of consciousness without paradox.

This ontology releases a deep Platonistic sigh of relief. Since there are no physical objects, there is no need to reduce mathematical notions to objects of the physical world. There are only quantum states identified as mathematical entities labelled naturally by integer valued quantum numbers; conscious experiences, which must represent sensations giving information about the number theoretical anatomy of a given quantum number; and space-time surfaces providing space-time correlates for quantum physics and therefore also for number theory and mathematical structures in general.

Factorization of integers as a direct sensory perception?

Both physicist and consciousness theorist would argue that the set theoretic construction of natural numbers could not be farther away from how we experience integers. Personally I feel that neither structuralist's approach nor Platonism as it is understood usually are enough. Mathematics is a conscious activity and this suggests that quantum theory of consciousness must be included if one wants to build more satisfactory view about fundamentals of mathematics.

Oliver Sack's book *The man who mistook his wife for a hat* [J17] (see also [K99]) contains fascinating stories about those aspects of brain and consciousness which are more or less mysterious from the view point of neuroscience. Sacks tells in his book also a story about twins who were classified as idiots but had amazing number theoretical abilities. I feel that this story reveals something very important about the real character of mathematical consciousness.

The twins had absolutely no idea about mathematical concepts such as the notion of primeness but they could factorize huge numbers and tell whether they are primes. Their eyes rolled wildly during the process and suddenly their face started to glow of happiness and they reported a discovery of a factor. One could not avoid the feeling that they quite concretely saw the factorization process. The failure to detect the factorization served for them as the definition of primeness. For them the factorization was not a process based on some rules but a direct sensory perception.

The simplest explanation for the abilities of twins would in terms of a model of integers represented as string like structures consisting of identical basic units. This string can decay to strings. If string containing n units decaying into $m > 1$ identical pieces is not perceived, the conclusion is that a prime is in question. It could also be that decay to units smaller than 2 was forbidden in this dynamics. The necessary connection between written representations of numbers and representative strings is easy to build as associations.

This kind theory might help to understand marvellous feats of mathematicians like Ramanujan who represents a diametrical opposite of Groethendienck as a mathematician (when Groethendienck was asked to give an example about prime, he mentioned 57 which became known as Groethendienck prime!).

The lesson would be that one very fundamental representation of integers would be, not as objects, but conscious experiences. Primeness would be like the quale of redness. This of course does not exclude also other representations.

Experience of integers in TGD inspired quantum theory of consciousness

In quantum physics integers appear very naturally as quantum numbers. In quantal axiomatization or interpretation of mathematics same should hold true.

1. In TGD inspired theory of consciousness [L2] quantum jump is identified as a moment of consciousness. There is actually an entire fractal hierarchy of quantum jumps consisting of quantum jumps and this correlates directly with the corresponding hierarchy of physical states and dark matter hierarchy. This means that the experience of integer should be reducible to a certain kind of quantum jump. The possible changes of state in the quantum jump would characterize the sensory representation of integer.

2. The quantum state as such does not give conscious information about the number theoretic anatomy of the integer labelling it: the change of the quantum state is required. The above geometric model translated to quantum case would suggest that integer represents a multiplicatively conserved quantum number. Decays of this this state into states labelled by integers n_i such that one has $n = \prod_i n_i$ would provide the fundamental conscious representation for the number theoretic anatomy of the integer. At the level of sensory perception based the space-time correlates a string-like bound state of basic particles representing n=1.

3. This picture is consistent with the Platonist view about integers represented as structured objects, now labels of quantum states. It would also conform with the view of category theorist in the sense that the arrows of category theorist replaced with quantum jumps are necessary to gain conscious information about the structure of the integer.

Infinite primes and arithmetic consciousness

Infinite primes [K107] were the first mathematical fruit of TGD inspired theory of consciousness and the inspiration for writing this posting came from the observation that the infinite primes at the lowest level of hierarchy provide a representation of algebraic numbers as Fock states of a super-symmetric arithmetic QFT so that it becomes possible to realize quantum jumps revealing the number theoretic anatomy of integers, rationals, and perhaps even that of algebraic numbers.

1. Infinite primes have a representation as Fock states of super-symmetric arithmetic QFT and at the lowest level of hierarchy they provide representations for primes, integers, rationals and algebraic numbers in the sense that at the lowest level of hierarchy of second quantizations the simplest infinite primes are naturally mapped to rationals whereas more complex infinite primes having interpretation as bound states can be mapped to algebraic numbers. Conscious experience of number can be assigned to the quantum jumps between these quantum states revealing information about the number theoretic anatomy of the number represented. It would be wrong to say that rationals only label these states: rather, these states represent rationals and since primes label the particles of these states.
2. More concretely, the conservation of number theoretic energy defined by the logarithm of the rational assignable with the Fock state implies that the allowed decays of the state to a product of infinite integers are such that the rational can decompose only into a product of rationals. These decays could provide for the above discussed fundamental realization of multiplicative aspects of arithmetic consciousness. Also additive aspects are represented since the exponents k in the powers p^k appearing in the decomposition are conserved so that only the partitions $k = \sum_i k_i$ are representable. Thus both product decompositions and partitions, the basic operations of number theorist, are represented.
3. The higher levels of the hierarchy represent a hierarchy of abstractions about abstractions bringing strongly in mind the hierarchy of n-categories and various similar constructions including n: th order logic. It also seems that the n+1: th level of hierarchy provides a quantum representation for the n: th level. Ordinary primes, integers, rationals, and algebraic numbers would be the lowest level, -the initial object- of the hierarchy representing nothing at low level. Higher levels could be reduced to them by the analog of category theoretic reductionism in the sense that there is arrow between n: th and n+1: th level representing the second quantization at this level. On can also say that these levels represent higher reflective level of mathematical consciousness and the fundamental sensory perception corresponds the lowest level.
4. Infinite primes have also space-time correlates. The decomposition of particle into partons can be interpreted as a infinite prime and this gives geometric representations of infinite primes and also rationals. The finite primes appearing in the decomposition of infinite prime correspond to bosonic or fermionic partonic 2-surfaces. Many-sheeted space-time provides a representation for the hierarchy of second quantizations: one physical prediction is that many particle bound state associated with space-time sheet behaves exactly like a boson or fermion. Nuclear string model is one concrete application of this idea: it replaces nucleon reductionism with reductionism occurs first to strings consisting of $A \leq 4$ nuclei and which in turn are strings consisting of nucleons. A further more speculative representation of infinite rationals as space-time surfaces is based on their mapping to rational functions.

Number theoretic Brahman=Atman identity

The notion of infinite primes leads to the notion of algebraic holography in which space-time points possess infinitely rich number-theoretic anatomy. This anatomy would be due to the existence of infinite number of real units defined as ratios of infinite integers which reduce to unit in the real sense and various p-adic senses. This anatomy is not visible in real physics but can contribute directly to mathematical consciousness [K107].

The anatomies of single space-time point could represent the entire world of classical worlds and quantum states of universe: the number theoretic anatomy is of course not visible in the structure of these these states. Therefore the basic building brick of mathematics - point- would become the Platonia able to represent all of the mathematics consistent with the laws of quantum physics. Space-time points would evolve, becoming more and more complex quantum jump by quantum jump. WCW and quantum states would be represented by the anatomies of space-time points. Some space-time points are more "civilized" than others so that space-time decomposes into "civilizations" at different levels of mathematical evolution.

Paths between space-time points represent processes analogous to parallel translations affecting the structure of the point and one can also define n-parallel translations up to $n = 4$ at level of space-time and $n = 8$ at level of embedding space. At level of world of classical worlds whose points are representable as number theoretical anatomies arbitrary high values of n can be realized.

It is fair to say that the number theoretical anatomy of the space-time point makes it possible self-reference loop to close so that structured points are able to represent the physics of associated with with the structures constructed from structureless points. Hence one can speak about algebraic holography or number theoretic Brahman=Atman identity.

Finite measurement resolution, Jones inclusions, and number theoretic braids

In the history of physics and mathematics the realization of various limitations have been the royal road to a deeper understanding (Uncertainty Principle, Gödel's theorem). The precision of quantum measurement, sensory perception, and cognition are always finite. In standard quantum measurement theory this limitation is not taken into account but forms a corner stone of TGD based vision about quantum physics and of mathematics too as I want to argue in the following.

The finite resolutions has representation both at classical and quantum level.

1. At the level of quantum states finite resolution is represented in terms of Jones inclusions N subset M of hyper-finite factors of type II_1 (HFFs) [K47]. N represents measurement resolution in the sense that the states related by the action of N cannot be distinguished in the measurement considered. Complex rays are replaced by N rays. This brings in non-commutativity via quantum groups [K15]. Non-commutativity in TGD Universe would be therefore due to a finite measurement resolution rather than something exotic emerging in the Planck length scale. Same applies to p-adic physics: p-adic space-time sheets have literally infinite size in real topology!
2. At the space-time level discretization implied by the number theoretic universality could be seen as being due to the finite resolution with common algebraic points of real and p-adic variant of the partonic 3-surface chosen as representatives for regions of the surface. The solutions of Kähler-Dirac equation are characterized by the prime in question so that the preferred prime makes itself visible at the level of quantum dynamics and characterizes the p-adic length scale fixing the values of coupling constants. Discretization could be also understood as effective non-commutativity of embedding space points due to the finite resolution implying that second quantized spinor fields anti-commute only at a discrete set of points rather than along stringy curve.

In this framework it is easy to imagine physical representations of number theoretical and other mathematical structures.

1. Every compact group corresponds to a hierarchy of Jones inclusions corresponding to various representations for the quantum variants of the group labelled by roots of unity. I would be surprised if non-compact groups would not allow similar representation since HFF can be regarded as infinite tensor power of n-dimensional complex matrix algebra for any value of n. Somewhat paradoxically, the finite measurement resolution would make possible to represent Lie group theory physically [K47].
2. There is a strong temptation to identify the Galois groups of algebraic numbers as the infinite permutation group S_∞ consisting of permutations of finite number of objects, whose projective representations give rise to an infinite braid group B_∞. The group algebras of these groups are HFFs besides the representation provided by the spinors of the world of classical worlds having physical identification as fermionic Fock states. Therefore physical states would provide a direct representation also for the more abstract features of number theory [K63].
3. Number theoretical braids crucial for the construction of S-matrix provide naturally representations for the Galois groups G associated with the algebraic extensions of rationals as

diagonal embeddings $G \times G \times ....$ to the completion of S_∞ representable also as the action on the completion of spinors in the world of classical worlds so that the core of number theory would be represented physically [K63]. At the space-time level number theoretic braid having G as symmetries would represent the G. These representations are analogous to global gauge transformations. The elements of S_∞ are analogous to local gauge transformations having a natural identification as a universal number theoretical gauge symmetry group leaving physical states invariant.

Hierarchy of Planck constants and the generalization of embedding space

Jones inclusions inspire a further generalization of the notion of embedding space obtained by gluing together copies of the embedding space H regarded as coverings $H \rightarrow H/G_a \times G_b$. In the simplest scenario $G_a \times G_b$ leaves invariant the choice of quantization axis and thus this hierarchy provides embedding space correlate for the choice of quantization axes inducing these correlates also at space-time level and at the level of world of classical worlds [K47].

Dark matter hierarchy is identified in terms of different sectors of H glued together along common points of base spaces and thus forming a book like structure. For the simplest option elementary particles proper correspond to maximally quantum critical systems in the intersection of all pages. The field bodies of elementary particles are in the interiors of the pages of this "book".

One can assign to Jones inclusions quantum phase $q = exp(i2\pi/n)$ and the groups Z_n acts as exact symmetries both at level of M^4 and CP_2. In the case of M^4 this means that space-time sheets have exact Z_n rotational symmetry. This suggests that the algebraic numbers q^m could have geometric representation at the level of sensory perception as Z_n symmetric objects. We need not be conscious of this representation in the ordinary wake-up consciousness dominated by sensory perception of ordinary matter with $q = 1$. This would make possible the idea about transcendentals like π, which do not appear in any finite-dimensional extension of even p-adic numbers (p-adic numbers allow finite-dimensional extension by since e^p is ordinary p-adic number). Quantum jumps in which state suffers an action of the generating element of Z_n could also provide a sensory realization of these groups and numbers $exp(i2\pi/n)$.

Planck constant is identified as the ratio n_a/n_b of integers associated with M^4 and CP_2 degrees of freedom so that a representation of rationals emerge again. The so called ruler and compass rationals whose definition involves only a repeated square root operation applied on rationals are cognitively the simplest ones and should appear first in the evolution of mathematical consciousness. The successful [K42] quantum model for EEG is only one of the applications providing support for their preferred role. Other applications are to Bohr quantization of planetary orbits interpreted as being induced by the presence of macroscopically quantum coherent dark matter [K103].

14.5.4 Farey Sequences, Riemann Hypothesis, Tangles, And TGD

Farey sequences allow an alternative formulation of Riemann Hypothesis and subsequent pairs in Farey sequence characterize so called rational 2-tangles. In TGD framework Farey sequences relate very closely to dark matter hierarchy, which inspires *"Platonia as the best possible world in the sense that cognitive representations are optimal"* as the basic variational principle of mathematics. This variational principle supports RH.

Possible TGD realizations of tangles, which are considerably more general objects than braids, are considered. One can assign to a given rational tangle a rational number a/b and the tangles labelled by a/b and c/d are equivalent if $ad - bc = \pm 1$ holds true. This means that the rationals in question are neighboring members of Farey sequence. Very light-hearted guesses about possible generalization of these invariants to the case of general N-tangles are made.

Farey sequences

Some basic facts about Farey sequences [A4] demonstrate that they are very interesting also from TGD point of view.

1. Farey sequence F_N is defined as the set of rationals $0 \leq q = m/n \leq 1$ satisfying the conditions $n \leq N$ ordered in an increasing sequence.

2. Two subsequent terms a/b and c/d in F_N satisfy the condition $ad - bc = 1$ and thus define and element of the modular group $SL(2, Z)$.

3. The number $|F(N)|$ of terms in Farey sequence is given by

$$|F(N)| = |F(N-1)| + \phi(N-1) \ . \quad (14.5.1)$$

Here $\phi(n)$ is Euler's totient function giving the number of divisors of n. For primes one has $\phi(p) = 1$ so that in the transition from p to $p+1$ the length of Farey sequence increases by one unit by the addition of $q = 1/(p+1)$ to the sequence.

The members of Farey sequence F_N are in one-one correspondence with the set of quantum phases $q_n = exp(i2\pi/n)$, $0 \leq n \leq N$. This suggests a close connection with the hierarchy of Jones inclusions, quantum groups, and in TGD context with quantum measurement theory with finite measurement resolution and the hierarchy of Planck constants involving the generalization of the embedding space. Also the recent TGD inspired ideas about the hierarchy of subgroups of the rational modular group with subgroups labelled by integers N and in direct correspondence with the hierarchy of quantum critical phases [K34] would naturally relate to the Farey sequence.

Riemann Hypothesis and Farey sequences

Farey sequences are used in two equivalent formulations of the Riemann hypothesis. Suppose the terms of F_N are $a_{n,N}$, $0 < n \leq |F_N|$. Define

$$d_{n,N} = a_{n,N} - \frac{n}{|F_N|} \ .$$

In other words, $d_{n,N}$ is the difference between the n: th term of the N: th Farey sequence, and the n: th member of a set of the same number of points, distributed evenly on the unit interval. Franel and Landau proved that both of the following statements

$$\sum_{n=1,...,|F_N|} |d_{n,N}| = O(N^r) \ \text{ for any } \ r > 1/2 \ ,$$
$$\sum_{n=1,...,|F_N|} d_{n,N}^2 = O(N^r) \ \text{ for any } \ r > 1 \ . \quad (14.5.2)$$

are equivalent with Riemann hypothesis.

One could say that RH would guarantee that the numbers of Farey sequence provide the best possible approximate representation for the evenly distributed rational numbers $n/|F_N|$.

Farey sequences and TGD

Farey sequences seem to relate very closely to TGD.

1. The rationals in the Farey sequence can be mapped to the roots of unity by the map $q \rightarrow exp(i2\pi q)$. The numbers $1/|F_N|$ are in turn mapped to the numbers $exp(i2\pi/|F_N|)$, which are also roots of unity. The statement would be that the algebraic phases defined by Farey sequence give the best possible approximate representation for the phases $exp(in2\pi/|F_N|)$ with evenly distributed phase angle.

2. In TGD framework the phase factors defined by F_N corresponds to the set of quantum phases corresponding to Jones inclusions labelled by $q = exp(i2\pi/n)$, $n \leq N$, and thus to the N lowest levels of dark matter hierarchy. There are actually two hierarchies corresponding to M^4 and CP_2 degrees of freedom and the Planck constant appearing in Schrödinger equation corresponds to the ratio n_a/n_b defining quantum phases in these degrees of freedom. $Z_{n_a \times n_b}$ appears as a conformal symmetry of "dark" partonic 2-surfaces and with very general assumptions this implies that there are only in TGD Universe [K34, K32].

3. The fusion of physics associated with various number fields to single coherent whole requires algebraic universality. In particular, the roots of unity, which are complex algebraic numbers, should define approximations to continuum of phase factors.
4. The subgroups of the hierarchy of subgroups of the modular group with rational matrix elements are labelled by integer N and relate naturally to the hierarchy of Farey sequences. The hierarchy of quantum critical phases is labelled by integers N with quantum phase transitions occurring only between phases for which the smaller integer divides the larger one [K34].

Interpretation of RH in TGD framework

Number theoretic universality of physics suggests an interpretation for the Riemann hypothesis in TGD framework. RH would be equivalent to the statement that the Farey numbers provide best possible approximation to the set of rationals $k/|F_N|$ or to the statement that the roots of unity contained by F_N define the best possible approximation for the roots of unity defined as $exp(ik2\pi/|F_N|)$ with evenly spaced phase angles. The roots of unity allowed by the lowest N levels of the dark matter hierarchy allows the best possible approximate representation for algebraic phases represented exactly at $|F_N|$: th level of hierarchy.

A stronger statement would be that the Platonia, where RH holds true would be the best possible world in the sense that algebraic physics behind the cognitive representations would allow the best possible approximation hierarchy for the continuum physics (both for numbers in unit interval and for phases on unit circle). Platonia with RH would be cognitive paradise.

One could see this also from different view point. "Platonia as the cognitively best possible world" could be taken as the "axiom of all axioms": a kind of fundamental variational principle of mathematics. Among other things it would allow to conclude that RH is true: RH must hold true either as a theorem following from some axiomatics or as an axiom in itself.

Could rational N-tangles exist in some sense?

The article of Kauffman and Lambropoulou [A64] about rational 2-tangles having commutative sum and product allowing to map them to rationals is very interesting from TGD point of view. The illustrations of the article are beautiful and make it easy to get the gist of various ideas. The theorem of the article states that equivalent rational tangles giving trivial tangle in the product correspond to subsequent Farey numbers a/b and c/d satisfying $ad - bc = \pm 1$ so that the pair defines element of the modular group SL(2, Z).

1. Rational 2-tangles

1. The basic observation is that 2-tangles are 2-tangles in both "s- and t-channels". Product and sum can be defined for all tangles but only in the case of 2-tangles the sum, which in this case reduces to product in t-channel obtained by putting tangles in series, gives 2-tangle. The so called rational tangles are 2-tangles constructible by using addition of $\pm[1]$ on left or right of tangle and multiplication by $\pm[1]$ on top or bottom. Product and sum are commutative for rational 2-tangles but the outcome is not a rational 2-tangle in the general case. One can also assign to rational 2-tangle its negative and inverse. One can map 2-tangle to a number which is rational for rational tangles. The tangles $[0]$, $[\infty]$, $\pm[1]$, $\pm 1/[1]$, $\pm[2]$, $\pm[1/2]$ define so called elementary rational 2-tangles.
2. In the general case the sum of $M-$ and $N-$tangles is $M+N-2$-tangle and combines various $N-$tangles to a monoidal structure. Tensor product like operation giving $M+N$-tangle looks to me physically more natural than the sum.
3. The reason why general 2-tangles are non-commutative although 2-braids obviously commute is that 2-tangles can be regarded as sequences of $N-$tangles with 2-tangles appearing only as the initial and final state: N is actually even for intermediate states. Since $N > 2$-braid groups are non-commutative, non-commutativity results. It would be interesting to know whether braid group representations have been used to construct representations of $N-$tangles.

2. Does generalization to $N >> 2$ case exist?

One can wonder whether the notion of rational tangle and the basic result of the article about equivalence of tangles might somehow generalize to the $N > 2$ case.

1. Could the commutativity of tangle product allow to characterize the $N > 2$ generalizations of rational 2-tangles. The commutativity of product would be a space-time correlate for the commutativity of the S-matrices defining time like entanglement between the initial and final quantum states assignable to the N-tangle. For 2-tangles commutativity of the sum would have an analogous interpretation. Sum is not a very natural operation for N-tangles for $N > 2$. Commutativity means that the representation matrices defined as products of braid group actions associated with the various intermediate states and acting in the same representation space commute. Only in very special cases one can expect commutativity for tangles since commutativity is lost already for braids.

2. The representations of 2-tangles should involve the subgroups of N-braid groups of intermediate braids identifiable as Galois groups of N: th order polynomials in the realization as number theoretic tangles. Could non-commutative 2-tangles be characterized by algebraic numbers in the extensions to which the Galois groups are associated? Could the non-commutativity reflect directly the non-commutativity of Galois groups involved? Quite generally one can ask whether the invariants should be expressible using algebraic numbers in the extensions of rationals associated with the intermediate braids.

3. Rational 2-tangles can be characterized by a rational number obtained by a projective identification $[a, b]^T \rightarrow a/b$ from a rational 2-spinor $[a, b]^T$ to which SL(2(N-1), Z) acts. Equivalence means that the columns $[a, b]^T$ and $[c, d]^T$ combine to form element of SL(2, Z) and thus defining a modular transformation. Could more general 2-tangles have a similar representation but in terms of algebraic integers?

4. Could N-tangles be characterized by $N - 1$ $2(N - 1)$-component projective column-spinors $[a_i^1, a_i^2, .., a_i^{2(N-1)}]^T$, $i = 1, ...N - 1$ so that only the ratios $a_i^k/a_i^{2(N-1)} \leq 1$ matter? Could equivalence for them mean that the $N - 1$ spinors combine to form $N - 1 + N - 1$ columns of $SL(2(N - 1), Z)$ matrix. Could N-tangles quite generally correspond to collections of projective $N - 1$ spinors having as components algebraic integers and could $ad - bc = \pm 1$ criterion generalize? Note that the modular group for surfaces of genus g is SL(2g, Z) so that $N - 1$ would be analogous to g and $1 \leq N \geq 3$- braids would correspond to $g \leq 2$ Riemann surfaces.

5. Dark matter hierarchy leads naturally to a hierarchy of modular sub-groups of $SL(2, Q)$ labelled by N (the generator $\tau \rightarrow \tau + 2$ of modular group is replaced with $\tau \rightarrow \tau + 2/N$). What might be the role of these subgroups and corresponding subgroups of $SL(2(N - 1), Q)$. Could they arise in "anyonization" when one considers quantum group representations of 2-tangles with twist operation represented by an N: th root of unity instead of phase U satisfying $U^2 = 1$?

How tangles could be realized in TGD Universe?

The article of Kauffman and Lambropoulou stimulated the question in what senses N-tangles could be realized in TGD Universe as fundamental structures.

1. Tangles as number theoretic braids?

The strands of number theoretical $N-$braids correspond to roots of N: th order polynomial and if one allows time evolutions of partonic 2-surface leading to the disappearance or appearance of real roots $N-$tangles become possible. This however means continuous evolution of roots so that the coefficients of polynomials defining the partonic 2-surface can be rational only in initial and final state but not in all intermediate "virtual" states.

2. Tangles as tangled partonic 2-surfaces?

Tangles could appear in TGD also in second manner.

1. Partonic 2-surfaces are sub-manifolds of a 3-D section of space-time surface. If partonic 2-surfaces have genus $g > 0$ the handles can become knotted and linked and one obtains besides ordinary knots and links more general knots and links in which circle is replaced by figure eight and its generalizations obtained by adding more circles (eyeglasses for $N-$eyed creatures).

2. Since these 2-surfaces are space-like, the resulting structures are indeed tangles rather than only braids. Tangles made of strands with fixed ends would result by allowing spherical partons elongate to long strands with fixed ends. DNA tangles would the basic example, and are discussed also in the article. DNA sequences to which I have speculatively assigned invisible (dark) braid structures might be seen in this context as space-like “written language representations” of genetic programs represented as number theoretic braids.

14.6 Quantum Quandaries

John Baez's [A54] discusses in a physicist friendly manner the possible application of category theory to physics. The lessons obtained from the construction of topological quantum field theories (TQFTs) suggest that category theoretical thinking might be very useful in attempts to construct theories of quantum gravitation.

The point is that the Hilbert spaces associated with the initial and final state n-1-manifold of n-cobordism indeed form in a natural manner category. Morphisms of Hilb in turn are unitary or possibly more general maps between Hilbert spaces. TQFT itself is a functor assigning to a cobordism the counterpart of S-matrix between the Hilbert spaces associated with the initial and final n-1-manifold. The surprising result is that for $n \leq 4$ the S-matrix can be unitary S-matrix only if the cobordism is trivial. This should lead even string theorist to raise some worried questions.

In the hope of feeding some category theoretic thinking into my spine, I briefly summarize some of the category theoretical ideas discussed in the article and relate it to the TGD vision, and after that discuss the worried questions from TGD perspective. That space-time makes sense only relative to embedding space would conform with category theoretic thinking.

14.6.1 The *-Category Of Hilbert Spaces

Baez considers first the category of Hilbert spaces. Intuitively the definition of this category looks obvious: take linear spaces as objects in category Set, introduce inner product as additional structure and identify morphisms as maps preserving this inner product. In finite-D case the category with inner product is however identical to the linear category so that the inner product does not seem to be absolutely essential. Baez argues that in infinite-D case the morphisms need not be restricted to unitary transformations: one can consider also bounded linear operators as morphisms since they play key role in quantum theory (consider only observables as Hermitian operators). For hyper-finite factors of type II_1 inclusions define very important morphisms which are not unitary transformations but very similar to them. This challenges the belief about the fundamental role of unitarity and raises the question about how to weaken the unitarity condition without losing everything.

The existence of the inner product is essential only for the metric topology of the Hilbert space. Can one do without inner product as an inherent property of state space and reduce it to a morphism? One can indeed express inner product in terms of morphisms from complex numbers to Hilbert space and their conjugates. For any state Ψ of Hilbert space there is a unique morphisms T_Ψ from C to Hilbert space satisfying $T_\Psi(1) = \Psi$. If one assumes that these morphisms have conjugates T_Ψ^* mapping Hilbert space to C, inner products can be defined as morphisms $T_\Phi^* T_\Psi$. The Hermitian conjugates of operators can be defined with respect to this inner product so that one obtains *-category. Reader has probably realized that T_Ψ and its conjugate correspond to ket and bra in Dirac's formalism.

Note that in TGD framework based on hyper-finite factors of type II_1 (HFFs) the inclusions of complex rays might be replaced with inclusions of HFFs with included factor representing the finite measurement resolution. Note also the analogy of inner product with the representation of space-times as 4-surfaces of the embedding space in TGD.

14.6.2 The Monoidal *-Category Of Hilbert Spaces And Its Counterpart At The Level Of Ncob

One can give the category of Hilbert spaces a structure of monoid by introducing explicitly the tensor products of Hilbert spaces. The interpretation is obvious for physicist. Baez describes the details of this identification, which are far from trivial and in the theory of quantum groups very interesting things happen. A non-commutative quantum version of the tensor product implying braiding is possible and associativity condition leads to the celebrated Yang-Baxter equations: inclusions of HFFs lead to quantum groups [K15] too.

At the level of nCob the counterpart of the tensor product is disjoint union of n-1-manifolds. This unavoidably creates the feeling of cosmic loneliness. Am I really a disjoint 3-surface in emptiness which is not vacuum even in the geometric sense? Cannot be true!

This horrifying sensation disappears if n-1-manifolds are n-1-surfaces in some higher-dimensional embedding space so that there would be at least something between them. I can emit a little baby manifold moving somewhere perhaps being received by some-one somewhere and I can receive radiation from some-one at some distance and in some direction as small baby manifolds making gentle tosses on my face!

This consoling feeling could be seen as one of the deep justifications for identifying fundamental objects as light-like partonic 3-surfaces in TGD framework. Their ends correspond to 2-D partonic surfaces at the boundaries of future or past directed light-cones (states of positive and negative energy respectively) and are indeed disjoint but not in the desperately existential sense as 3-geometries of General Relativity.

This disjointness has also positive aspect in TGD framework. One can identify the color degrees of freedom of partons as those associated with CP_2 degrees of freedom. For instance, SU(3) analogs for rotational states of rigid body become possible. 4-D space-time surfaces as preferred extremals of Kähler action connect the partonic 3-surfaces and bring in classical representation of correlations and thus of interactions. The representation as sub-manifolds makes it also possible to speak about positions of these sub-Universes and about distances between them. The habitants of TGD Universe are maximally free but not completely alone.

14.6.3 Tqft As A Functor

The category theoretic formulation of TQFT relies on a very elegant and general idea. Quantum transition has as a space-time correlate an n-dimensional surface having initial final states as its n-1-dimensional ends. One assigns Hilbert spaces of states to the ends and S-matrix would be a unitary morphism between the ends. This is expressed in terms of the category theoretic language by introducing the category nCob with objects identified as n-1-manifolds and morphisms as cobordisms and *-category Hilb consisting of Hilbert spaces with inner product and morphisms which are bounded linear operators which do not however preserve the unitarity. Note that the morphisms of nCob cannot anymore be identified as maps between n-1-manifolds interpreted as sets with additional structure so that in this case category theory is more powerful than set theory.

TQFT is identified as a functor nCob $\rightarrow$ Hilb assigning to n-1-manifolds Hilbert spaces, and to cobordisms unitary S-matrices in the category Hilb. This looks nice but the surprise is that for $n \leq 4$ unitary S-matrix exists only if the cobordism is trivial so that topology changing transitions are not possible unless one gives up unitarity.

This raises several worried questions.

1. Does this result mean that in TQFT sense unitary S-matrix for topology changing transitions from a state containing n_i closed strings to a state containing $n_f \neq n_i$ strings does not exist? Could the situation be same also for more general non-topological stringy S-matrices? Could the non-converging perturbation series for S-matrix with finite individual terms matrix fail to no non-perturbative counterpart? Could it be that M-theory is doomed to remain a dream with no hope of being fulfilled?
2. Should one give up the unitarity condition and require that the theory predicts only the relative probabilities of transitions rather than absolute rates? What the proper generalization of the S-matrix could be?
3. What is the relevance of this result for quantum TGD?

14.6.4 The Situation Is In TGD Framework

The result about the non-existence of unitary S-matrix for topology changing cobordisms allows new insights about the meaning of the departures of TGD from string models.

Cobordism cannot give interesting selection rules

When I started to work with TGD for more than 28 years ago, one of the first ideas was that one could identify the selection rules of quantum transitions as topological selection rules for cobordisms. Within week or two came the great disappointment: there were practically no selection rules. Could one revive this naïve idea? Could the existence of unitary S-matrix force the topological selection rules after all? I am skeptic. If I have understood correctly the discussion of what happens in 4-D case [A32] only the exotic diffeo-structures modify the situation in 4-D case.

Light-like 3-surfaces allow cobordism

In the physically interesting GRT like situation one would expect the cobordism to be mediated by a space-time surface possessing Lorentz signature. This brings in metric and temporal distance. This means complications since one must leave the pure TQFT context. Also the classical dynamics of quantum gravitation brings in strong selection rules related to the dynamics in metric degrees of freedom so that TQFT approach is not expected to be useful from the point of view of quantum gravity and certainly not the limit of a realistic theory of quantum gravitation.

In TGD framework situation is different. 4-D space-time sheets can have Euclidian signature of the induced metric so that Lorentz signature does not pose conditions. The counterparts of cobordisms correspond at fundamental level to light-like 3-surfaces, which are arbitrarily except for the light-likeness condition (the effective 2-dimensionality implies generalized conformal invariance and analogy with 3-D black-holes since 3-D vacuum Einstein equations are satisfied). Field equations defined by the Chern-Simons action imply that CP_2 projection is at most 2-D but this condition holds true only for the extremals and one has functional integral over all light-like 3-surfaces. The temporal distance between points along light-like 3-surface vanishes. The constraints from light-likeness bring in metric degrees of freedom but in a very gentle manner and just to make the theory physically interesting.

Feynman cobordism as opposed to ordinary cobordism

In string model context the discouraging results from TQFT hold true in the category of nCob, which corresponds to trouser diagrams for closed strings or for their open string counterparts. In TGD framework these diagrams are replaced with a direct generalization of Feynman diagrams for which 3-D light-like partonic 3-surfaces meet along their 2-D ends at the vertices. In honor of Feynman one could perhaps speak of Feynman cobordisms. These surfaces are singular as 3-manifolds but vertices are nice 2-manifolds. I contrast to this, in string models diagrams are nice 2-manifolds but vertices are singular as 1-manifolds (say eye-glass type configurations for closed strings).

This picture gains a strong support for the interpretation of fermions as light-like throats associated with connected sums of CP_2 type extremals with space-time sheets with Minkowski signature and of bosons as pairs of light-like wormhole throats associated with CP_2 type extremal connecting two space-time sheets with Minkowski signature of induced metric. The space-time sheets have opposite time orientations so that also zero energy ontology emerges unavoidably. There is also consistency TGD based explanation of the family replication phenomenon in terms of genus of light-like partonic 2-surfaces.

One can wonder what the 4-D space-time sheets associated with the generalized Feynman diagrams could look like? One can try to gain some idea about this by trying to assign 2-D surfaces to ordinary Feynman diagrams having a subset of lines as boundaries. In the case of $2\rightarrow 2$ reaction open string is pinched to a point at vertex. $1\rightarrow 2$ vertex, and quite generally, vertices with odd number of lines, are impossible. The reason is that 1-D manifolds of finite size can have either 0 or 2 ends whereas in higher-D the number of boundary components is arbitrary. What one expects to happen in TGD context is that wormhole throats which are at distance characterized by CP_2 fuse together in the vertex so that some kind of pinches appear also now.

Zero energy ontology

Zero energy ontology gives rise to a second profound distinction between TGD and standard QFT. Physical states are identified as states with vanishing net quantum numbers, in particular energy.

Everything is creatable from vacuum - and one could add- by intentional action so that zero energy ontology is profoundly Eastern. Positive *resp.* negative energy parts of states can be identified as states associated with 2-D partonic surfaces at the boundaries of future *resp.* past directed light-cones, whose tips correspond to the arguments of n-point functions. Each incoming/outgoing particle would define a mini-cosmology corresponding to not so big bang/crunch. If the time scale of perception is much shorter than time interval between positive and zero energy states, the ontology looks like the Western positive energy ontology. Bras and kets correspond naturally to the positive and negative energy states and phase conjugation for laser photons making them indeed something which seems to travel in opposite time direction is counterpart for bra-ket duality.

Finite temperature S-matrix defines genuine quantum state in zero energy ontology

In TGD framework one encounters two S-matrix like operators.

1. There is U-matrix acting in zero energy states. U-matrix is the analog of the ordinary S-matrix and constructible in terms of it and orthonormal basis of square roots of density matrices expressible as products of hermitian operators multiplied by unitary S-matrix [K76].
2. The S-matrix like operator describing what happens in laboratory corresponds to the time-like entanglement coefficients between positive and negative energy parts of the state. Measurement of reaction rates would be a measurement of observables reducing time like entanglement and very much analogous to an ordinary quantum measurement reducing space-like entanglement. There is a finite measurement resolution described by inclusion of HFFs and this means that situation reduces effectively to a finite-dimensional one.

p-Adic thermodynamics strengthened with p-adic length scale hypothesis predicts particle masses with an amazing success. At first the thermodynamical approach seems to be in contradiction with the idea that elementary particles are quantal objects. Unitarity is however *not* necessary if one accepts that only relative probabilities for reductions to pairs of initial and final states interpreted as particle reactions can be measured.

The beneficial implications of unitarity are not lost if one replaces QFT with thermal QFT. Category theoretically this would mean that the time-like entanglement matrix associated with the product of cobordisms is a product of these matrices for the factors. The time parameter in S-matrix would be replaced with a complex time parameter with the imaginary part identified as inverse temperature. Hence the interpretation in terms of time evolution is not lost.

In the theory of hyper-finite factors of type III_1 the partition function for thermal equilibrium states and S-matrix can be neatly fused to a thermal S-matrix for zero energy states and one could introduce p-adic thermodynamics at the level of quantum states. It seems that this picture applies to HFFs by restriction. Therefore the loss of unitarity S-matrix might after all turn to a victory by more or less forcing both zero energy ontology and p-adic thermodynamics.

14.7 How To Represent Algebraic Numbers As Geometric Objects?

Physics blogs are also interesting because they allow to get some grasp about very different styles of thinking of a mathematician and physicist. For mathematician it is very important that the result is obtained by a strict use of axioms and deduction rules. Physicist is a cognitive opportunist: it does not matter how the result is obtained by moving along axiomatically allowed paths or not, and the new result is often more like a discovery of a new axiom and physicist is ever-grateful for Gödel for giving justification for what sometimes admittedly degenerates to a creative hand-waving. For physicist ideas form a kind of bio-sphere and the fate of the individual idea depends on its ability to survive, which is determined by its ability to become generalized, its consistency with other ideas, and ability to interact with other ideas to produce new ideas.

14.7.1 Can One Define Complex Numbers As Cardinalities Of Sets?

During few days before writing this we have had in Kea's blog a little bit of discussion inspired by the problem related to the categorification of basic number theoretical structures. I have learned that sum and product are natural operations for the objects of category. For instance, one can define sum as in terms of union of sets or direct sum of vector spaces and product as Cartesian product of sets and tensor product of vector spaces: rigs [A19] are example of categories for which natural numbers define sum and product.

Subtraction and division are however problematic operations. Negative numbers and inverses of integers do not have a realization as a number of elements for any set or as dimension of vector space. The naïve physicist inside me asks immediately: why not go from statics to dynamics and take operations (arrows with direction) as objects: couldn't this allow to define subtraction and division? Is the problem that the axiomatization of group theory requires something which purest categorification does not give? Or aren't the numbers representable in terms of operations of finite groups not enough? In any case cyclic groups would allow to realize roots of unity as operations (Z_2 would give -1).

One could also wonder why the algebraic numbers might not somehow result via the representations of permutation group of infinite number of elements containing all finite groups and thus Galois groups of algebraic extensions as subgroups? Why not take the elements of this group as objects of the basic category and continue by building group algebra and hyper-finite factors of type II_1 isomorphic to spinors of world of classical worlds, and so on.

After having written the first half of the section, I learned that something similar to the transition from statics to dynamics is actually carried out but by manner which is by many orders of magnitudes more refined than the proposal above and that I had never been able to imagine. The article *Objects of categories as complex numbers* of Marcelo Fiore and Tom Leinster [A19] describes a fascinating idea summarized also by John Baez [A17] about how one can assign to the objects of a category complex numbers as roots of a polynomial $Z = P(Z)$ defining an isomorphism of object. Z is the element of a category called rig, which differs from ring in that integers are replaced with natural numbers. One can replace Z with a complex number $|Z|$ defined as a root of polynomial. $|Z|$ is interpreted formally as the cardinality of the object. It is essential to have natural numbers and thus only product and sum are defined. This means a restriction: for instance, only complex algebraic numbers associated with polynomials having natural numbers as coefficients are obtained. Something is still missing.

Note that this correspondence assumes the existence of complex numbers and one cannot say that complex numbers are categorified. Maybe basic number fields must be left outside categorification. One can however require that all of them have a concrete set theoretic representation rather than only formal interpretation as cardinality so that one still encounters the problem how to represent algebraic complex number as a concrete cardinality of a set.

14.7.2 In What Sense A Set Can Have Cardinality -1?

The discussion in Kea's blog led me to ask what the situation is in the case of p-adic numbers. Could it be possible to represent the negative and inverse of p-adic integer, and in fact any p-adic number, as a geometric object? In other words, does a set with -1 or $1/n$ or even $\sqrt{-1}$ elements exist? If this were in some sense true for all p-adic number fields, then all this wisdom combined together might provide something analogous to the adelic representation for the norm of a rational number as product of its p-adic norms. As will be found, alternative interpretations of complex algebraic numbers as p-adic numbers representing cardinalities of p-adic fractals emerge. The fractal defines the manner how one must do an infinite sum to get an infinite real number but finite p-adic number.

Of course, this representation might not help to define p-adics or reals categorically but might help to understand how p-adic cognitive representations defined as subsets for rational intersections of real and p-adic space-time sheets could represent p-adic number as the number of points of p-adic fractal having infinite number of points in real sense but finite in the p-adic sense. This would also give a fundamental cognitive role for p-adic fractals as cognitive representations of numbers.

How to construct a set with -1 elements?

The basic observation is that p-adic -1 has the representation

$$-1 = (p-1)/(1-p) = (p-1)(1+p+p^2+p^3....)$$

As a real number this number is infinite or -1 but as a p-adic number the series converges and has p-adic norm equal to 1. One can also map this number to a real number by canonical identification taking the powers of p to their inverses: one obtains p in this particular case. As a matter fact, any rational with p-adic norm equal to 1 has similar power series representation.

The idea would be to represent a given p-adic number as the infinite number of points (in real sense) of a p-adic fractal such that p-adic topology is natural for this fractal. This kind of fractals can be constructed in a simple manner: from this more below. This construction allows to represent any p-adic number as a fractal and code the arithmetic operations to geometric operations for these fractals.

These representations - interpreted as cognitive representations defined by intersections of real and p-adic space-time sheets - are in practice approximate if real space-time sheets are assumed to have a finite size: this is due to the finite p-adic cutoff implied by this assumption and the meaning a finite resolution. One can however say that the p-adic space-time itself could by its necessarily infinite size represent the *idea* of given p-adic number faithfully.

This representation applies also to the p-adic counterparts of algebraic numbers in case that they exist. For instance, roughly one half of p-adic numbers have square root as ordinary p-adic number and quite generally algebraic operations on p-adic numbers can give rise to p-adic numbers so that also these could have set theoretic representation. For $p\ mod\ 4 = 1$ also $\sqrt{(-1)}$ exists: for instance, for $p = 5$: $2^2 = 4 = -1\ mod\ 5$ guarantees this so that also imaginary unit and complex numbers would have a fractal representation. Also many transcendentals possess this kind of representation. For instance $exp(xp)$ exists as a p-adic number if x has p-adic norm not larger than 1: also $log(1 + xp)$ does so.

Hence a quite impressive repertoire of p-adic counterparts of real numbers would have representation as a p-adic fractal for some values of p. Adelic vision would suggest that combining these representations one might be able to represent quite a many real numbers. In the case of π I do not find any obvious p-adic representation (for instance $sin(\pi/6) = 1/2$ does not help since the p-adic variant of the Taylor expansion of $\pi/6 = arcsin(1/2)$ does not converge p-adically for any value of p). It might be that there are very many transcendentals not allowing fractal representation for any value of p.

Conditions on the fractal representations of p-adic numbers

Consider now the construction of the fractal representations in terms of rational intersections of real real and p-adic space-time sheets. The question is what conditions are natural for this representation if it corresponds to a cognitive representation is realized in the rational intersection of real and p-adic space-time sheets obeying same algebraic equations.

1. Pinary cutoff is the analog of the decimal cutoff but is obtained by dropping away high positive rather than negative powers of p to get a finite real number: example of pinary cutoff is $-1 = (p-1)(1+p+p^2+...) \rightarrow (p-1)(1+p+p^2)$. This cutoff must reduce to a fractal cutoff meaning a finite resolution due to a finite size for the real space-time sheet. In the real sense the p-adic fractal cutoff means not forgetting details below some scale but cutting out all above some length scale. Physical analog would be forgetting all frequencies below some cutoff frequency in Fourier expansion.

   The motivation comes from the fact that TGD inspired consciousness assigns to a given biological body there is associated a field body or magnetic body containing dark matter with large $\hbar$ and quantum controlling the behavior of biological body and so strongly identifying with it so as to belief that this all ends up to a biological death. This field body has an onion like fractal structure and a size of at least order of light-life. Of course, also larger onion layers could be present and would represent those levels of cognitive consciousness not depending on the sensory input on biological body: some altered states of consciousness could relate to these levels. In any case, the larger the magnetic body, the better the numerical skills of the p-adic mathematician.

2. Lowest pinary digits of $x = x_0 + x_1p + x_2p^2 + ...$, $x_n \leq p$ must have the most reliable representation since they are the most significant ones. The representation must be also

highly redundant to guarantee reliability. This requires repetitions and periodicity. This is guaranteed if the representation is hologram like with segments of length p^n with digit x_n represented again and again in all segments of length p^m, $m > n$.

3. The TGD based physical constraint is that the representation must be realizable in terms of induced classical fields assignable to the field body hierarchy of an intelligent system interested in artistic expression of p-adic numbers using its own field body as instrument. As a matter, sensory and cognitive representations are realized at field body in TGD Universe and EEG is in a fundamental role in building this representation. By p-adic fractality fractal wavelets are the most natural candidate. The fundamental wavelet should represent the p different pinary digits and its scaled up variants would correspond to various powers of p so that the representation would reduce to a Fourier expansion of a classical field.

Concrete representation

Consider now a concrete candidate for a representation satisfying these constraints.

1. Consider a p-adic number

$$y = p^{n_0}x, \quad x = \sum x_n p^n \quad , \quad n \geq n_0 = 0 \quad .$$

If one has a representation for a p-adic unit x the representation of is by a purely geometric fractal scaling of the representation by p^n. Hence one can restrict the consideration to p-adic units.

2. To construct the representation take a real line starting from origin and divide it into segments with lengths $1, p, p^2, ...$ In TGD framework this scalings come actually as powers of $p^{1/2}$ but this is just a technical detail.

3. It is natural to realize the representation in terms of periodic field patterns. One can use wavelets with fractal spectrum $p^n\lambda_0$ of "wavelet lengths", where λ_0 is the fundamental wavelength. Fundamental wavelet should have p different patterns correspond to the p values of pinary digit as its structures. Periodicity guarantees the hologram like character enabling to pick n: th digit by studying the field pattern in scale p^n anywhere inside the field body.

4. Periodicity guarantees also that the intersections of p-adic and real space-time sheets can represent the values of pinary digits. For instance, wavelets could be such that in a given p-adic scale the number of rational points in the intersection of the real and p-adic space-time sheet equals to x_n. This would give in the limit of an infinite pinary expansion a set theoretic realization of any p-adic number in which each pinary digit x_n corresponds to infinite copies of a set with x_n elements and fractal cutoff due to the finite size of real space-time sheet would bring in a finite precision. Note however that p-adic space-time sheet necessarily has an infinite size and it is only real world realization of the representation which has finite accuracy.

5. A concrete realization for this object would be as an infinite tree with $x_n + 1 \leq p$ branches in each node at level n ($x_n + 1$ is needed in order to avoid the splitting tree at $x_n = 0$). In 2-adic case -1 would be represented by an infinite pinary tree. Negative powers of p correspond to the of the tree extending to a finite depth in ground.

14.7.3 Generalization Of The Notion Of Rig By Replacing Naturals With P-Adic Integers

Previous considerations do not relate directly to category theoretical problem of assigning complex numbers to objects. It however turns out that p-adic approach allows to generalize the proposal of [A19] by replacing natural numbers with p-adic integers in the definition of rig so that any algebraic complex number can define cardinality of an object of category allowing multiplication and sum and that these complex numbers can be replaced with p-adic numbers if they make sense as such so that previous arguments provide a concrete geometric representation of the cardinality. The road to the realization this simple generalization required a visit to the John Baez's Weekly Finds (Week 102) [A17].

The outcome was the realization that the notion of rig used to categorify the subset of algebraic numbers obtained as roots of polynomials with *natural number* valued coefficients generalizes trivially by replacing natural numbers by *p-adic integers*. As a consequence one obtains beautiful p-adicization of the generating function F(x) of structure as a function which converges p-adically for any rational $x = q$ for which it has prime p as a positive power divisor.

Effectively this generalization means the replacement of natural numbers as coefficients of the polynomial defining the rig with all rationals, also negative, and *all* complex algebraic numbers find a category theoretical representation as "cardinalities". These cardinalities have a dual interpretation as p-adic integers which in general correspond to infinite real numbers but are mappable to real numbers by canonical identification and have a geometric representation as fractals.

Mapping of objects to complex numbers and the notion of rig

The idea of rig approach is to categorify the notion of cardinality in such a way that one obtains a *subset* of algebraic complex numbers as cardinalities in the category-theoretical sense. One can assign to an object a polynomial with coefficients, which are *natural numbers* and the condition $Z = P(Z)$ says that $P(Z)$ acts as an isomorphism of the object. One can interpret the equation also in terms of complex numbers. Hence the object is mapped to a complex number Z defining a root of the polynomial interpreted as an ordinary polynomial: it does not matter which root is chosen. The complex number Z is interpreted as the "cardinality" of the object but I do not really understand the motivation for this. The deep further result is that also more general polynomial equations $R(|Z|) = Q(|Z|)$ satisfied by the generalized cardinality Z imply $R(Z) = Q(Z)$ as isomorphism.

I try to reproduce what looks the most essential in the explanation of John Baez and relate it to my own ideas but take this as my talk to myself and visit This Week's Finds [A17], one of the many classics of Baez, to learn of this fascinating idea.

1. Baez considers first the ways of putting a given structure to n-element set. The set of these structures is denoted by F_n and the number of them by $|F_n|$. The generating function $|F|(x) = \sum_n |F_n|x^n$ packs all this information to a single function.

   For instance, if the structure is binary tree, this function is given by $T(x) = \sum_n C_{n-1}x^n$, where C_{n-1} are Catalan numbers and n¿0 holds true. One can show that T satisfies the formula

   $$T = X + T^2 \ ,$$

   since any binary tree is either trivial or decomposes to a product of binary trees, where two trees emanate from the root. One can solve this second order polynomial equation and the power expansion gives the generating function.

2. The great insight is that one can also work directly with structures. For instance, by starting from the isomorphism $T = 1 + T^2$ applying to an object with cardinality 1 and substituting T^2 with $(1 + T^2)^2$ repeatedly, one can deduce the amazing formula $T^7(1) = T(1)$ mentioned by Kea, and this identity can be interpreted as an isomorphism of binary trees.

3. This result can be generalized using the notion of rig category [A19]. In rig category one can add and multiply but negatives are not defined as in the case of ring. The lack of subtraction and division is still the problem and as I suggested in previous posting p-adic integers might resolve the problem.

   Whenever Z is object of a rig category, one can equip it with an isomorphism $Z = P(Z)$ where $P(Z)$ is polynomial with *natural numbers* as coefficients and one can assign to object "cardinality" as any root of the equation $Z = P(Z)$. Note that set with n elements corresponds to $P(|Z|) = n$. Thus subset of algebraic complex numbers receive formal identification as cardinalities of sets. Furthermore, if the cardinality satisfies another equation $Q(|Z|) = R(|Z|)$ such that neither polynomial is constant, then one can construct an isomorphism $Q(Z) = R(Z)$. Isomorphisms correspond to equations!

4. This is indeed nice that there is something which is not so beautiful as it could be: why should we restrict ourselves to *natural numbers* as coefficients of $P(Z)$? Could it be possible

to replace them with integers to obtain *all complex algebraic numbers* as cardinalities? Could it be possible to replace natural numbers by p-adic integers?

p-Adic rigs and Golden Object as p-adic fractal

The notions of generating function and rig generalize to the p-adic context.

1. The generating function $F(x)$ defining isomorphism Z in the rig formulation converges p-adically for any p-adic number containing p as a factor so that the idea that all structures have p-adic counterparts is natural. In the real context the generating function typically diverges and must be defined by analytic continuation. Hence one might even argue that p-adic numbers are more natural in the description of structures assignable to finite sets than reals.

2. For rig one considers only polynomials $P(Z)$ (Z corresponds to the generating function F) with coefficients which are natural numbers. Any p-adic integer can be however interpreted as a non-negative integer: natural number if it is finite and "super-natural" number if it is infinite. Hence can generalize the notion of rig by replacing natural numbers by p-adic integers. The rig formalism would thus generalize to arbitrary polynomials with integer valued coefficients so that all complex algebraic numbers could appear as cardinalities of category theoretical objects. Even rational coefficients are allowed. This is highly natural number theoretically.

3. For instance, in the case of binary trees the solutions to the isomorphism condition $T = p+T^2$ giving $T = [1 \pm (1-4p)^{1/2}]/2$ and T would be complex number $[p \pm (1-4p)^{1/2}]/2$. $T(p)$ can be interpreted also as a p-adic number by performing power expansion of square root in case that the p-adic square root exists: this super-natural number can be mapped to a real number by the canonical identification and one obtains also the set theoretic representations of the category theoretical object $T(p)$ as a p-adic fractal. This interpretation of cardinality is much more natural than the purely formal interpretation as a complex number. This argument applies completely generally. The case $x = 1$ discussed by Baez gives $T = [1 \pm (-3)^{1/2}]/2$ allows p-adic representation if $-3 == p-3$ is square mod p. This is the case for $p = 7$ for instance.

4. John Baez [A17] poses also the question about the category theoretic realization of "Golden Object", his big dream. In this case one would have $Z = G = -1 + G^2 = P(Z)$. The polynomial on the right hand side does not conform with the notion of rig since -1 is not a natural number. If one allows p-adic rigs, $x = -1$ can be interpreted as a p-adic integer $(p-1)(1+p+...)$, positive and infinite and "super-natural", actually largest possible p-adic integer in a well defined sense.

   A further condition is that Golden Mean converges as a p-adic number: this requires that $\sqrt{5}$ must exist as a p-adic number: $(5 = 1+4)^{1/2}$ certainly converges as power series for $p = 2$ so that Golden Object exists 2-adically. By using [A12] of Euler, one finds that 5 is square mod p only if p is square mod 5. To decide whether given p is Golden it is enough to look whether p mod 5 is 1 or 4. For instance, $p = 11, 19, 29, 31$ ($=M_5$) are Golden. Mersennes M_k, $k = 3, 7, 127$ and Fermat primes are not Golden. One representation of Golden Object as p-adic fractal is the p-adic series expansion of $[1/2 \pm 5^{1/2}]/2$ representable geometrically as a binary tree such that there are $0 \leq x_n + 1 \leq p$ branches at each node at height n if n: th p-adic coefficient is x_n. The "cognitive" p-adic representation in terms of wavelet spectrum of classical fields is discussed in the previous posting.

5. It would be interesting to know how quantum dimensions of quantum groups assignable to Jones inclusions [K127, K47, K15] relate to the generalized cardinalities. The root of unity property of quantum phase ($q^{n+1} = q$) suggests $Q = Q^{n+1} = P(Q)$ as the relevant isomorphism. For Jones inclusions the cardinality $q = exp(i2\pi/n)$ would not be however equal to quantum dimension $D(n) = 4cos^2(\pi/n)$.

Is there a connection with infinite integers?

Infinite primes [K107] correspond to Fock states of a super-symmetric arithmetic quantum field theory and there is entire infinite hierarchy of them corresponding to repeated second quantization.

Also infinite primes and rationals make sense. Besides free Fock states spectrum contains at each level also what might be identified as bound states. All these states can be mapped to polynomials. Since the roots of polynomials represent complex algebraic numbers and as they seem to characterize objects of categories, there are reasons to expect that infinite rationals might allow also interpretation in terms of say rig categories or their generalization. Also the possibility to identify space-time coordinate as isomorphism of a category might be highly interesting concerning the interpretation of quantum classical correspondence.

14.8 Gerbes And TGD

The notion of gerbes has gained much attention during last years in theoretical physics and there is an abundant gerbe-related literature in hep-th archives. Personally I learned about gerbes from the excellent article of Jouko Mickelson [A58] (Jouko was my opponent in PhD dissertation for more than two decades ago: so the time flows!).

I have already applied the notion of bundle gerbe in TGD framework in the construction of the Dirac determinant which I have proposed to define the Kähler function for the WCW (see [K128]). The insights provided by the general results about bundle gerbes discussed in [A58] led, not only to a justification for the hypothesis that Dirac determinant exists for the Kähler-Dirac action, but also to an elegant solution of the conceptual problems related to the construction of Dirac determinant in the presence of chiral symmetry. Furthermore, on basis of the special properties of the Kähler-Dirac operator there are good reasons to hope that the determinant exists even without zeta function regularization. The construction also leads to the conclusion that the space-time sheets serving as causal determinants must be geodesic sub-manifolds (presumably light like boundary components or “elementary particle horizons”). Quantum gravitational holography is realized since the exponent of Kähler function is expressible as a Dirac determinant determined by the local data at causal determinants and there would be no need to find absolute minima of Kähler action explicitly.

In the sequel the emergence of 2-gerbes at the space-time level in TGD framework is discussed and shown to lead to a geometric interpretation of the somewhat mysterious cocycle conditions for a wide class of gerbes generated via the $\wedge d$ products of connections associated with 0-gerbes. The resulting conjecture is that gerbes form a graded-commutative Grassmman algebra like structure generated by -1- and 0-gerbes. 2-gerbes provide also a beautiful topological characterization of space-time sheets as structures carrying Chern-Simons charges at boundary components and the 2-gerbe variant of Bohm-Aharonov effect occurs for perhaps the most interesting asymptotic solutions of field equations especially relevant for anyonics systems, quantum Hall effect, and living matter [K5].

14.8.1 What Gerbes Roughly Are?

Very roughly and differential geometrically, gerbes can be regarded as a generalization of connection. Instead of connection 1-form (0-gerbe) one considers a connection $n+1$-form defining n-gerbe. The curvature of n-gerbe is closed $n+2$-form and its integral defines an analog of magnetic charge. The notion of holonomy generalizes: instead of integrating n-gerbe connection over curve one integrates its connection form over n+1-dimensional closed surface and can transform it to the analog of magnetic flux.

There are some puzzling features associated with gerbes. Ordinary $U(1)$-bundles are defined in terms of open sets U_α with gauge transformations $g_{\alpha\beta}=g_{\beta\alpha}^{-1}$ defined in $U_\alpha\cap U_\beta$ relating the connection forms in the patch U_β to that in patch U_α. The 3-cocycle condition

$$g_{\alpha\beta}g_{\beta\gamma}g_{\gamma\alpha}=1 \qquad (14.8.1)$$

makes it possible to glue the patches to a bundle structure.

In the case of 1-gerbes the transition functions are replaced with the transition functions $g_{\alpha\beta\gamma}=g_{\gamma\beta\alpha}^{-1}$ defined in triple intersections $U_\alpha\cap U_\beta\cap U_\gamma$ and 3-cocycle must be replaced with 4-cocycle:

$$g_{\alpha\beta\gamma}g_{\beta\gamma\delta}g_{\gamma\delta\alpha}g_{\delta\alpha\beta} = 1 \ . \tag{14.8.2}$$

The generalizations of these conditions to n-gerbes is obvious.

In the case of 2-intersections one can build a bundle structure naturally but in the case of 3-intersections this is not possible. Hence the geometric interpretation of the higher gerbes is far from obvious. One possible interpretation of non-trivial 1-gerbe is as an obstruction for lifting projective bundles with fiber space CP_n to vector bundles with fiber space C^{n+1} [A58]. This involves the lifting of the holomorphic transition functions g_α defined in the projective linear group $PGL(n+1,C)$ to $GL(n+1,C)$. When the 3-cocycle condition for the lifted transition functions $\overline{g}_{\alpha\beta}$ fails it can be replaced with 4-cocycle and one obtains 1-gerbe.

14.8.2 How Do 2-Gerbes Emerge In TGD?

Gerbes seem to be interesting also from the point of view of TGD, and TGD approach allows a geometric interpretation of the cocycle conditions for a rather wide class of gerbes.

Recall that the Kähler form J of CP_2 defines a non-trivial magnetically charged and self-dual $U(1)$-connection A. The Chern-Simons form $\omega = A\wedge J = A\wedge dA$ having CP_2 Abelian instanton density $J\wedge J$ as its curvature form and can thus be regarded as a 3-connection form of a 2-gerbe. This 2-gerbe is induced by 0-gerbe.

The coordinate patches U_α are same as for $U(1)$ connection. In the transition between patches A and ω transform as

$$\begin{aligned} A &\rightarrow A + d\phi \ , \\ \omega &\rightarrow \omega + dA_2 \ , \\ A_2 &= \phi\wedge J \ . \end{aligned} \tag{14.8.3}$$

The transformation formula is induced by the transformation formula for $U(1)$ bundle. Somewhat mysteriously, there is no need to define anything in the intersections of U_α in the recent case.

The connection form of the 2-gerbe can be regarded as a second $\wedge d$ power of Kähler connection:

$$A_3 \equiv A\wedge dA \ . \tag{14.8.4}$$

The generalization of this observation allows to develop a different view about n-gerbes generated as $\wedge d$ products of 0-gerbes.

The hierarchy of gerbes generated by 0-gerbes

Consider a collection of $U(1)$ connections $A^{i)}$. They generate entire hierarchy of gerbe-connections via the $\wedge d$ product

$$A_3 = A^{1)}\wedge dA^{2)} \tag{14.8.5}$$

defining 2-gerbe having a closed curvature 4-form

$$F_4 = dA^{1)}\wedge dA^{2)} \ . \tag{14.8.6}$$

$\wedge d$ product is commutative apart from a gauge transformation and the curvature forms of $A^{1)}\wedge dA^{2)}$ and $A^{2)}\wedge dA^{1)}$ are the same.

Quite generally, the connections A_m of $m-1$ gerbe and A_n of $n-1$-gerbe define $m+n+1$ connection form and the closed curvature form of $m+n$-gerbe as

$$\begin{aligned} A_{m+n+1} &= A_m^{1)} \wedge dA_n^{2)} \ , \\ F_{m+n+2} &= dA_m^{1)} \wedge dA_n^{2)} \ . \end{aligned} \tag{14.8.7}$$

The sequence of gerbes extends up to $n = D-2$, where D is the dimension of the underlying manifold. These gerbes are not the most general ones since one starts from 0-gerbes. One can of course start from $n > 0$-gerbes too.

The generalization of the $\wedge d$ product to the non-Abelian situation is not obvious. The problems stem from the that the Lie-algebra valued connection forms $A^{1)}$ and $A^{2)}$ appearing in the covariant version $D = d + A$ do not commute.

14.8.3 How To Understand The Replacement Of 3-Cycles With N-Cycles?

If n-gerbes are generated from 0-gerbes it is possible to understand how the intersections of the open sets emerge. Consider the product of 0-gerbes as the simplest possible case. The crucial observation is that the coverings U_α for $A^{1)}$ and V_β for $A^{2)}$ need not be same (for CP_2 this was the case). One can form a new covering consisting of sets $U_\alpha \cap V_{\alpha_1}$. Just by increasing the index range one can replace V with U and one has covering by $U_\alpha \cap U_{\alpha_1} \equiv U_{\alpha\alpha_1}$.

The transition functions are defined in the intersections $U_{\alpha\alpha_1} \cap U_{\beta\beta_1} \equiv U_{\alpha\alpha_1\beta\beta_1}$ and cocycle conditions must be formulated using instead of intersections $U_{\alpha\beta\gamma}$ the intersections $U_{\alpha\alpha_1\beta\beta_1\gamma\gamma_1}$. Hence the transition functions can be written as $g_{\alpha\alpha_1\beta\beta_1}$ and the 3-cocycle are replaced with 5-cocycle conditions since the minimal co-cycle corresponds to a sequence of 6 steps instead of 4:

$$U_{\alpha\alpha_1\beta\beta_1} \rightarrow U_{\alpha_1\beta\beta_1\gamma} \rightarrow U_{\beta\beta_1\gamma\gamma_1} \rightarrow U_{\beta_1\gamma\gamma_1\alpha} \rightarrow U_{\gamma\gamma_1\alpha\alpha_1} \ .$$

The emergence of higher co-cycles is thus forced by the modification of the bundle covering necessary when gerbe is formed as a product of lower gerbes. The conjecture is that any even gerbe is expressible as a product of 0-gerbes.

An interesting application of the product structure is at the level of WCW ("world of classical worlds"). The Kähler form of WCW defines a connection 1-form and this generates infinite hierarchy of connection $2n+1$-forms associated with $2n$-gerbes.

14.8.4 Gerbes As Graded-Commutative Algebra: Can One Express All Gerbes As Products Of -1 And 0-Gerbes?

If one starts from, say 1-gerbes, the previous argument providing a geometric understanding of gerbes is not applicable as such. One might however hope that it is possible to represent the connection 2-form of any 1-gerbe as a $\wedge d$ product of a connection 0-form ϕ of "-1" -gerbe and connection 1-form A of 0-gerbe:

$$A_2 = \phi dA \equiv A \wedge d\phi \ ,$$

with different coverings for ϕ and A. The interpretation as an obstruction for the modification of the underlying bundle structure is consistent with this interpretation.

The notion of -1-gerbe is not well-defined unless one can define the notion of -1 form precisely. The simplest possibility that 0-form transforms trivially in the change of patch is not consistent. One could identify contravariant n-tensors as $-n$-forms and d for them as divergence and d^2 as the antisymmetrized double divergence giving zero. ϕ would change in a gauge transformation by a divergence of a vector field. The integral of a divergence over closed M vanishes identically so that if the integral of ϕ over M is non-vanishing it corresponds to a non-trivial 0-connection. This interpretation of course requires the introduction of metric.

The requirement that the minimal intersections of the patches for 1-gerbes are of form $U_{\alpha\beta\gamma}$ would be achieved if the intersections patches can be restricted to the intersections $U_{\alpha\beta\gamma}$ defined by $U_\alpha \cap V_\gamma$ and $U_\beta \cap V_\gamma$ (instead of $U_\beta \cap V_\delta$), where the patches V_γ would be most naturally associated with -1-gerbe. It is not clear why one could make this restriction. The general conjecture is that any gerbe decomposes into a multiple $\wedge d$ product of -1 and 0-gerbes just like integers decompose into primes. The $\wedge d$ product of two odd gerbes is anti-commutative so that there is also an analogy with the decomposition of the physical state into fermions and bosons, and gerbes for a graded-commutative super-algebra generalizing the Grassmann algebra of manifold to a Grassmann algebra of gerbe structures for manifold.

14.8.5 The Physical Interpretation Of 2-Gerbes In TGD Framework

2-gerbes could provide some insight to how to characterize the topological structure of the many-sheeted space-time.

1. The cohomology group H^4 is obviously crucial in characterizing 2-gerbe. In TGD framework many-sheetedness means that different space-time sheets with induced metric having Minkowski signature are separated by elementary particle horizons which are light like 3-surfaces at which the induced metric becomes degenerate. Also the time orientation of the space-time sheet can change at these surfaces since the determinant of the induced metric vanishes.

   This justifies the term elementary particle horizon and also the idea that one should treat different space-time sheets as generating independent direct summands in the homology group of the space-time surface: as if the space-time sheets not connected by join along boundaries bonds were disjoint. Thus the homology group H^4 and 2-gerbes defining instanton numbers would become important topological characteristics of the many-sheeted space-time.

2. The asymptotic behavior of the general solutions of field equations can be classified by the dimension D of the CP_2 projection of the space-time sheet. For $D = 4$ the instanton density defining the curvature form of 2-gerbe is non-vanishing and instanton number defines a topological charge. Also the values of the Chern-Simons invariants associated with the boundary components of the space-time sheet define topological quantum numbers characterizing the space-time sheet and their sum equals to the instanton charge. CP_2 type extremals represent a basic example of this kind of situation. From the physical view point $D = 4$ asymptotic solutions correspond to what might be regarded chaotic phase for the flow lines of the Kähler magnetic field. Kähler current vanishes so that empty space Maxwell's equations are satisfied.

3. For $D = 3$ situation is more subtle when boundaries are present so that the higher-dimensional analog of Aharonov-Bohm effect becomes possible. In this case instanton density vanishes but the Chern-Simons invariants associated with the boundary components can be non-vanishing. Their sum obviously vanishes. The space-time sheet can be said to be a neutral C-S multipole. Separate space-time sheets can become connected by flux tubes in a quantum jump replacing a space-time surface with a new one. This means that the cohomology group H^4 as well as instanton charges and C-S charges of the system change.

Concerning the asymptotic dynamics of the Kähler magnetic field, $D = 3$ phase corresponds to an extremely complex but highly organized phase serving as an excellent candidate for the modelling of living matter. Both the TGD based description of anyons and quantum Hall effect and the model for topological quantum computation based on the braiding of magnetic flux tubes rely heavily on the properties $D = 3$ phase [K5].

The non-vanishing of the C-S form implies that the flow lines of the Kähler magnetic are highly entangled and have as an analog mixing hydrodynamical flow. In particular, one cannot define non-trivial order parameters, say phase factors, which would be constant along the lines. The interpretation in terms of broken super-conductivity suggests itself. Kähler current can be non-vanishing so that there is no counterpart for this phase at the level of Maxwell's equations.

14.9 Appendix: Category Theory And Construction Of S-Matrix

The construction of WCW geometry, spinor structure and of S-matrix involve difficult technical and conceptual problems and category theory might be of help here. As already found, the application of category theory to the construction of WCW geometry allows to understand how the arrow of psychological time emerges.

The construction of the S-matrix involves several difficult conceptual and technical problems in which category theory might help. The incoming states of the theory are what might be called free states and are constructed as products of the WCW spinor fields. One can effectively regard them as being defined in the Cartesian power of WCW divided by an appropriate permutation group. Interacting states in turn are defined in the WCW .

Cartesian power of WCW of 3-surfaces is however in geometrical sense more or less identical with WCW since the disjoint union of N 3-surfaces is itself a 3-surface in WCW . Actually it differs from WCW itself only in that the 3-surfaces of many particle state can intersect each other and if one allows this, one has paradoxical self-referential identification $CH = \overline{CH^2}/S_2 = ... = \overline{CH^N}/S_N...$, where over-line signifies that intersecting 3-surfaces have been dropped from the product.

Note that arbitrarily small deformation can remove the intersections between 3-surfaces and four-dimensional general coordinate invariance allows always to use non-intersecting representatives. In case of the spinor structure of the Cartesian power this identification means that the tensor powers SCH^N of the WCW spinor structure are in some sense identical with the spinor structure SCH of the WCW . Certainly the oscillator operators of the tensor factors must be assumed to be mutually anti-commuting.

The identities $CH = \overline{CH^2}/S_2 = ..$ and corresponding identities $SCH = SCH^2 = ...$ for the space SCH of WCW spinor fields might imply very deep constraints on S-matrix. What comes into mind are counterparts for the Schwinger-Dyson equations of perturbative quantum field theory providing defining equations for the n-point functions of the theory [A52]. The isomorphism between SCH^2 and SCH is actually what is needed to calculate the S-matrix elements. Category theory might help to understand at a general level what these self-referential and somewhat paradoxical looking identities really imply and perhaps even develop TGD counterparts of Schwinger-Dyson equations.

There is also the issue of bound states. The interacting states contain also bound states not belonging to the space of free states and category theory might help also here. It would seem that the state space must be constructed by taking into account also the bound states as additional "free" states in the decomposition of states to product states.

A category naturally involved with the construction of the S-matrix (or U-matrix) is the space of preferred extremals of the Kähler action which might be called interacting category. The symplectic transformations acting as isometries of the configuration space geometry act naturally as the morphisms of this category. The group $Diff^4$ of general coordinate transformations in turn acts as gauge symmetries.

S-matrix relates free and interacting states and is induced by the classical long range interactions induced by the criticality of the preferred extremals in the sense of having an infinite number of deformations for which the second variation of Kähler action vanishes S-matrix elements are essentially Glebch-Gordan coefficients relating the states in the tensor power of the interacting super-symplectic representation with the interacting super-symplectic representation itself. More concretely, N-particle free states can be seen as WCW spinor fields in CH^N obtained as tensor products of ordinary WCW spinor fields. Free states correspond classically to the unions of space-time surfaces associated with the 3-surfaces representing incoming particles whereas interacting states correspond classically to the space-time surfaces associated with the unions of the 3-surfaces defining incoming states. These two states define what might be called free and interacting categories with canonical transformations acting as morphisms.

The classical interaction is represented by a functor $S : \overline{CH^N}/S_N \rightarrow CH$ mapping the classical free many particle states, that is objects of the product category defined by $\overline{CH^N}/S_N$ to the interacting category CH. This functor assigns to the union $\cup_i X^4(X_i^3)$ of the absolute minima $X^4(X_i^3)$ of Kähler action associated with the incoming, free states X_i^3 the preferred extreal $X^4(\cup X_i^3)$ associated with the union of 3-surfaces representing the outgoing interacting state. At quantum level this functor maps the state space SCH^N associated with $\cup_i X^4(X_i^3)$ to SCH in a unitary manner. An important constraint on S-matrix is that it acts effectively as a flow in zero modes correlating the quantum numbers in fiber degrees of freedom in one-to-one manner with the values of zero modes so that quantum jump $U\Psi_i \rightarrow \Psi_0...$ gives rise to a quantum measurement.

Chapter 15

Infinite Primes and Consciousness

15.1 Introduction

This chapter is devoted to the possible role of infinite primes in TGD and TGD inspired theory of consciousness.

The notion of prime seems to capture something very essential about what it is to be elementary building block of matter and has become a fundamental conceptual element of TGD. The notion of prime gains it generality from its reducibility to the notion of prime ideal of an algebra. Thus the notion of prime is well-defined, not only in case of quaternions and octonions, but also for their complexifications and one can speak about infinite primes in the case of hyper-quaternions and -octonions, which are especially natural physically and for which numbers having zero norm correspond physically to light-like 8-vectors.

15.1.1 The Notion Of Infinite Prime

The original motivation for the notion of infinite prime came from the first attempts to construct TGD inspired theory of consciousness (around 1995) [K113]. Suppose very naively that the 4-surfaces in a given sector of the "world of classical worlds" (WCW) are labelled by a fixed p-adic prime. The natural expectation is that evolution by quantum jumps means dispersion in the space of these sectors and leads to the increase of the p-adic prime characterizing the Universe. As one moves backwards in subjective time (sequence of quantum jumps) one ends up to the situation in which the prime characterizing the universe was $p = 2$. Should one assume that there was the first quantum jump when everything began? If not, then it would seem that the p-adic prime characterizing the Universe must be infinite. Second problem is that the p-adic length scales are finite and if the size scale of Universe is given by p-adic length scale the Universe has finite sized: this does not make sense in TGD framework. The only way out of the problems is the assumption that the p-adic prime characterizing the entire Universe is literally infinite and that p-adic primes characterizing space-time sheets are finite.

These argument, which are by no means central for the recent view about p-adic primes, motivated the attempt to construct a theory of infinite primes and to extend quantum TGD accordingly. This turns out to be possible. The recipe for constructing infinite primes is structurally equivalent with a repeated second quantization of an arithmetic super-symmetric quantum field theory. At the lowest level one has fermionic and bosonic states labeled by finite primes and infinite primes correspond to many particle states of this theory. Also infinite primes analogous to bound states are predicted. This hierarchy of quantizations can be continued indefinitely by taking the many particle states of the previous level as elementary particles at the next level. It must be also emphasized that the notion of infinity is relativistic. With respect to the p-adic norm infinite primes have unit norm for all finite and infinite primes so that there is nothing to become scared of!

Construction could make sense also for hyper-quaternionic and hyper-octonionic primes although non-commutativity and non-associativity pose technical challenges. One can also construct infinite number of real units as ratios of infinite integers with a precise number theoretic anatomy.

The fascinating finding is that the quantum states labeled by standard model quantum numbers allow a representation as wave fuctions in the discrete space of these units. Space-time point becomes infinitely richly structured in the sense that one can associate to it a wave function in the space of real (or octonionic) units allowing to represent the WCW spinor fields. One can speak about algebraic holography or number theoretic Brahman=Atman identity and one can also say that the points of embedding space and space-time surface are subject to a number theoretic evolution. In philosophical mood one can of course also ask whether there exists a hierarchy of embedding spaces in which the embedding space at the lower level represents something with infinitesimal size in the sense of real topology and whether this hierarchy is accompanied also by a hierarchy of conscious entities.

This picture suggest that the Universe of quantum TGD might basically provide a physical representation of number theory allowing also infinite primes. The proposal suggests also a possible generalization of real numbers to a number system akin to hyper-reals introduced by Robinson in his non-standard calculus [A22] providing a rigorous mathematical basis for calculus. In fact, some rather natural requirements lead to a unique generalization for the concepts of integer, rational and real. Infinite integers and reals can be regarded as infinite-dimensional vector spaces with integer and real valued coefficients respectively. Same generalization could make sense for all classical number fields.

15.1.2 Infinite Primes And Physics In TGD Universe

Several different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

Infinite primes and super-symmetric quantum field theory

Consider next the physical interpretation.

1. The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. This suggests that configuration space spinor fields or at least the ground states of associated super-conformal representations could be mapped to infinite primes in both bosonic and fermionic degrees of freedom. The process might generalize so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.
2. The ordinary primes appearing as building blocks of infinite primes at the first level of the hierarchy could be identified as coding for p-adic primes assignable to fermionic and bosonic partons identified as 2-surfaces of a given space-time sheet. The hierarchy of infinite primes would correspond to hierarchy of space-time sheets defined by the topological condensate. This leads also to a precise identification of p-adic and real variants of bosonic partonic 2-surfaces as correlates of intention and action and pairs of p-adic and real fermionic partons as correlates for cognitive representations.
3. The idea that infinite primes characterize quantum states of the entire Universe, perhaps ground states of super-conformal representations, if not all states, could be taken further. It turns out that this idea makes sense when one considers discrete wave functions in the space of infinite primes and that one can indeed represent standard model quantum numbers in this way.
4. The number theoretical supersymmetry suggests also space-time supersymmetry TGD framework. Space-time super-symmetry in its standard form is not possible in TGD Universe and this cheated me to believe that this supersymmetry is completely absent in TGD Universe. The progress in the understanding of the properties of the modified Dirac action however led to a generalization of the space-time super-symmetry as a dynamical and broken symmetry of quantum TGD [**?**].

Here however emerges the idea about the number theoretic analog of color confinement. Rational (infinite) primes allow not only a decomposition to (infinite) primes of algebraic extensions of rationals but also to algebraic extensions of quaternionic and octonionic (infinite) primes. The physical analog is the decomposition of a particle to its more elementary constituents. This fits nicely with the idea about number theoretic resolution represented as a hierarchy of Galois groups defined by the extensions of rationals and realized at the level of physics in terms of Jones inclusions [K127] defined by these groups having a natural action on space-time surfaces, induced spinor fields, and on configuration space spinor fields representing physical states [K34].

Infinite primes and physics as number theory

The hierarchy of algebraic extensions of rationals implying corresponding extensions of p-adic numbers suggests that Galois groups, which are the basic symmetry groups of number theory, should have concrete physical representations using induced spinor fields and configuration space spinor fields and also infinite primes and real units formed as infinite rationals. These groups permute zeros of polynomials and thus have a concrete physical interpretation both at the level of partonic 2-surfaces dictated by algebraic equations and at the level of braid hierarchy. The vision about the role of hyperfinite factors of II_1 and of Jones inclusions as descriptions of quantum measurements with finite measurement resolution leads to concrete ideas about how these groups are realized.

G_2 acts as automorphisms of hyper-octonions and $SU(3)$ as its subgroup respecting the choice of a preferred imaginary unit. The discrete subgroups of $SU(3)$ permuting to each other hyper-octonionic primes are analogous to Galois group and turned out to play a crucial role in the understanding of the correspondence between infinite hyper-octonionic primes and quantum states predicted by quantum TGD.

The notion of finite measurement resolution as the key concept

TGD predicts several hierarchies: the hierarchy of space-time sheets, the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution [K127]. the dark matter hierarchy characterized by increasing values of $\hbar$ [K47]. the hierarchy of extensions of a given p-adic number field. TGD inspired theory of consciousness predictes the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. These hierarchies should be closely related.

The notion of finite measurement resolution turns out to be the key concept: the p-adic norm of the rational defined by the infinite prime characterizes the angle measurement resolution for given p-adic prime p. It is essential that one has what might be called a state function reduction selecting a fixed p-adic prime which could be also infinite. This gives direct connections with cognition and with the p-adicization program relying also on angle measurement resolution. Also the value the integers characterizing the singular coverings of CD and CP_2 defining as their product Planck constant characterize the measurement resolution for a given p-adic prime in CD and CP_2 degrees of freedom. This conforms with the fact that elementary particles are characterized by two infinite primes. Hence finite measurement resolution ties tightly together the three threads of the number theoretic vision. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory with finite measurement resolution.

Space-time correlates of infinite primes

Infinite primes code naturally for Fock states in a hierarchy of super-symmetric arithmetic quantum field theories. Quantum classical correspondence leads to ask whether infinite primes could also code for the space-time surfaces serving as symbolic representations of quantum states. This would a generalization of algebraic geometry would emerge and could reduce the dynamics of Kähler action to algebraic geometry and organize 4-surfaces to a physical hierarchy according to their algebraic complexity. This conjecture should be consistent with two other conjectures about the dynamics of space-time surfaces (space-time surfaces as preferred extrema of Kähler action and space-time surfaces as quaternionic or co-quaternionic (as associative or co-associative) 4-surfaces of hyper-octonion space M^8).

Quantum classical correspondence requires the map of the quantum numbers of configuration space spinor fields to space-time geometry. The Kähler-Dirac equation with measurement interaction term realizes this requirement. Therefore, if one wants to map infinite rationals to space-time geometry it is enough to map infinite primes to quantum numbers. This map can be indeed achieved thanks to the detailed picture about the interpretation of the symmetries of infinite primes in terms of standard model symmetries. The notion of finite measurement resolution allows to deduce much more detailed about this correspondence. In particular, the rational defined by the infinite prime classifies the finite sub-manifold geometry defined by the discretization of the partonic 2-surface implied by the finite measurement resolution. Also a direct correlation between integers defining Planck constant and the "fermionic" part of the infinite prime emerges.

15.1.3 Infinite Primes, Cognition, And Intentionality

The correlation of infinite primes with cognition is the first fascinating possibility and this possibility has stimulated several ideas.

1. One can define the notion of prime also for the algebraic extensions of rationals. The hierarchy of infinite primes associated with algebraic extensions of rationals leading gradually towards algebraic closure of rationals would in turn define cognitive hierarchy corresponding to algebraic extensions of p-adic numbers.
2. The introduction of infinite primes, integers, and rationals leads also to a generalization of classical number fields since an infinite algebra of real (complex, etc...) units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity pose technical problems. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum of infinitesimals (real zeros) is replaced by multiplication of real units meaning that the set of real and also more general units becomes infinitely degenerate.
3. Infinite primes form an infinite hierarchy so that the points of space-time and embedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point -or more generally wave functions in the space of the units associated with the point- might be even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units. For real physics this structure is completely invisible and is relevant only for the physics of cognition. One can say that Universe is an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.
4. In zero energy ontology hyper-octonionic units identified as ratios of the infinite integers associated with the positive and negative energy parts of the zero energy state define a representation of WCW spinor fields. The action of subgroups of SU(3) and rotation group SU(2) preserving hyper-octonionic and hyper-quaternionic primeness and identification of momentum and electro-weak charges in terms of components of hyper-octonionic primes makes this representation unique. Hence Brahman-Atman identity has a completely concrete realization and fixes completely the quantum number spectrum including particle masses and correlations between various quantum numbers.
5. One can assign to infinite primes at n^{th} level of hierarchy rational functions of n rational arguments which form a natural hierarchical structure in that highest level corresponds to a polynomial with coefficients which are rational functions of the arguments at the lower level. One can solve one of the arguments in terms of lower ones to get a hierarchy of algebraic extensions. At the lowest level algebraic extensions of rationals emerge, at the next level algebraic extensions of space of rational functions of single variable, etc... This would suggest that infinite primes code for the correlation between quantum states and the

algebraic extensions appearing in their their physical description and characterizing their cognitive correlates. The hierarchy of infinite primes would also correlate with a hierarchy of logics of various orders (hierarchy of statements about statements about...).

15.1.4 About Literature

The reader not familiar with the basic algebra of quaternions and octonions is encouraged to study some background material: the home page of Tony Smith provides among other things an excellent introduction to quaternions and octonions [A88]. String model builders are beginning to grasp the potential importance of octonions and quaternions and the articles about possible applications of octonions [A33, A84, A74] provide an introduction to octonions using the language of physicist.

Personally I found quite frustrating to realize that I had neglected totally learning of the basic ideas of algebraic geometry, despite its obvious potential importance for TGD and its applications in string models. This kind of losses are the price one must pay for working outside the scientific community. It is not easy for a physicist to find readable texts about algebraic geometry and algebraic number theory from the bookshelves of mathematical libraries. The book "Algebraic Geometry for Scientists and Engineers" by Abhyankar [A86]. which is not so elementary as the name would suggest, introduces in enjoyable way the basic concepts of algebraic geometry and binds the basic ideas with the more recent developments in the field. "Problems in Algebraic Number Theory" by Esmonde and Murty [A40] in turn teaches algebraic number theory through exercises which concretize the abstract ideas. The book "Invitation to Algebraic Geometry" by K. E. Smith. L. Kahanpää, P. Kekäläinen and W. Traves is perhaps the easiest and most enjoyable introduction to the topic for a novice. It also contains references to the latest physics inspired work in the field.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

15.2 Infinite Primes, Integers, And Rationals

The definition of the infinite integers and rationals is a straightforward procedure and structurally similar to a repeated second quantization of a super-symmetric quantum field theory but including also the number theoretic counterparts of bound states.

15.2.1 The First Level Of Hierarchy

In the following the concept of infinite prime is developed gradually by stepwise procedure rather than giving directly the basic definitions. The hope is that the development of the concept in the same manner as it actually occurred would make it easier to understand it.

Step 1

One could try to define infinite primes P by starting from the basic idea in the proof of Euclid for the existence of infinite number of primes. Take the product of all finite primes and add 1 to get a new prime:

$$\begin{aligned} P &= 1+X \ , \\ X &= \prod_p p \ . \end{aligned} \tag{15.2.1}$$

If P were divisible by finite prime then $P-X=1$ would be divisible by finite prime and one would encounter contradiction. One could of course worry about the possible existence of infinite primes smaller than P and possibly dividing P. The numbers $N=P-k$, $k>1$, are certainly not primes since k can be taken as a factor. The number $P'=P-2=-1+X$ could however be prime. P is certainly not divisible by $P-2$. It seems that one cannot express P and $P-2$ as product of infinite integer and finite integer. Neither it seems possible to express these numbers as products of more general numbers of form $\prod_{p\in U} p+q$, where U is infinite subset of finite primes and q is finite integer.

Step 2

P and $P-2$ are not the only possible candidates for infinite primes. Numbers of form

$$\begin{array}{l} P(\pm,n) = \pm 1 + nX \ , \\ k(p) = 0,1,..... \ , \\ n = \prod_p p^{k(p)} \ , \\ X = \prod_p p \ , \end{array} \tag{15.2.2}$$

where $k(p) \neq 0$ holds true only in finite set of primes, are characterized by a integer n, and are also good prime candidates. The ratio of these primes to the prime candidate P is given by integer n. In general, the ratio of two prime candidates $P(m)$ and $P(n)$ is rational number m/n telling which of the prime candidates is larger. This number provides ordering of the prime candidates $P(n)$. The reason why these numbers are good candidates for infinite primes is the same as above. No finite prime p with $k(p) \neq 0$ appearing in the product can divide these numbers since, by the same arguments as appearing in Euclid's theorem, it would divide also 1. On the other hand it seems difficult to invent any decomposition of these numbers containing infinite numbers. Already at this stage one can notice the structural analogy with the construction of multiboson states in quantum field theory: the numbers $k(p)$ correspond to the occupation numbers of bosonic states of quantum field theory in one-dimensional box, which suggests that the basic structure of QFT might have number theoretic interpretation in some very general sense. It turns out that this analogy generalizes.

Step 3

All $P(n)$ satisfy $P(n) \geq P(1)$. One can however also the possibility that $P(1)$ is not the smallest infinite prime and consider even more general candidates for infinite primes, which are smaller than $P(1)$. The trick is to drop from the infinite product of primes $X = \prod_p p$ some primes away by dividing it by integer $s = \prod_{p_i} p_i$, multiply this number by an integer n not divisible by any prime dividing s and to add to/subtract from the resulting number nX/s natural number ms such that m expressible as a product of powers of only those primes which appear in s to get

$$\begin{array}{l} P(\pm,m,n,s) = n\frac{X}{s} \pm ms \ , \\ m = \prod_{p|s} p^{k(p)} \ , \\ n = \prod_{p|\frac{X}{s}} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{array} \tag{15.2.3}$$

Here $x|y$ means "x divides y". To see that no prime p can divide this prime candidate it is enough to calculate $P(\pm,m,n,s)$ modulo p: depending on whether p divides s or not, the prime divides only the second term in the sum and the result is nonzero and finite (although its precise value is not known). The ratio of these prime candidates to $P(+,1,1,1)$ is given by the rational number n/s: the ratio does not depend on the value of the integer m. One can however order the prime candidates with given values of n and s using the difference of two prime candidates as ordering criterion. Therefore these primes can be ordered.

One could ask whether also more general numbers of the form $n\frac{X}{s} \pm m$ are primes. In this case one cannot prove the indivisibility of the prime candidate by p not appearing in m. Furthermore, for $s \ mod \ 2 = 0$ and $m \ mod \ 2 \neq 0$, the resulting prime candidate would be even integer so that it looks improbable that one could obtain primes in more general case either.

Step 4

An even more general series of candidates for infinite primes is obtained by using the following ansatz which in principle is contained in the original ansatz allowing infinite values of n

$$\begin{array}{l} P(\pm,m,n,s|r) = nY^r \pm ms \ , \\ Y = \frac{X}{s} \ , \\ m = \prod_{p|s} p^{k(p)} \ , \\ n = \prod_{p|\frac{X}{s}} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{array} \tag{15.2.4}$$

The proof that this number is not divisible by any finite prime is identical to that used in the previous case. It is not however clear whether the ansatz for given r is not divisible by infinite primes belonging to the lower level. A good example in $r = 2$ case is provided by the following unsuccessful ansatz

$$\begin{aligned} &N = (n_1Y + m_1s)(n_2Y + m_2s) = \tfrac{n_1n_2X^2}{s^2} - m_1m_2s^2 \ , \\ &Y = \tfrac{X}{s} \ , \\ &n_1m_2 - n_2m_1 = 0 \ . \end{aligned}$$

Note that the condition states that n_1/m_1 and $-n_2/m_2$ correspond to the same rational number or equivalently that (n_1, m_1) and (n_2, m_2) are linearly dependent as vectors. This encourages the guess that all other $r = 2$ prime candidates with finite values of n and m at least, are primes. For higher values of r one can deduce analogous conditions guaranteeing that the ansatz does not reduce to a product of infinite primes having smaller value of r. In fact, the conditions for primality state that the polynomial $P(n, m, r)(Y) = nY^r + m$ with integer valued coefficients ($n > 0$) defined by the prime candidate is irreducible in the field of integers, which means that it does not reduce to a product of lower order polynomials of same type.

Step 5

A further generalization of this ansatz is obtained by allowing infinite values for m, which leads to the following ansatz:

$$\begin{aligned} &P(\pm, m, n, s|r_1, r_2) = nY^{r_1} \pm ms \ , \\ &m = P_{r_2}(Y)Y + m_0 \ , \\ &Y = \tfrac{X}{s} \ , \\ &m_0 = \textstyle\prod_{p|s} p^{k(p)} \ , \\ &n = \textstyle\prod_{p|Y} p^{k(p)}, \ \ k(p) \geq 0 \ . \end{aligned} \tag{15.2.5}$$

Here the polynomial $P_{r_2}(Y)$ has order r_2 is divisible by the primes belonging to the complement of s so that only the finite part m_0 of m is relevant for the divisibility by finite primes. Note that the part proportional to s can be infinite as compared to the part proportional to Y^{r_1}: in this case one must however be careful with the signs to get the sign of the infinite prime correctly. By using same arguments as earlier one finds that these prime candidates are not divisible by finite primes. One must also require that the ansatz is not divisible by lower order infinite primes of the same type. These conditions are equivalent to the conditions guaranteeing the polynomial primeness for polynomials of form $P(Y) = nY^{r_1} \pm (P_{r_2}(Y)Y + m_0)s$ having integer-valued coefficients. The construction of these polynomials can be performed recursively by starting from the first order polynomials representing first level infinite primes: Y can be regarded as formal variable and one can forget that it is actually infinite number.

By finite-dimensional analogy, the infinite value of m means infinite occupation numbers for the modes represented by integer s in some sense. For finite values of m one can always write m as a product of powers of $p_i|s$. Introducing explicitly infinite powers of p_i is not in accordance with the idea that all exponents appearing in the formulas are finite and that the only infinite variables are X and possibly S (formulas are symmetric with respect to S and X/S). The proposed representation of m circumvents this difficulty in an elegant manner and allows to say that m is expressible as a product of infinite powers of p_i despite the fact that it is not possible to derive the infinite values of the exponents of p_i.

Summarizing, an infinite series of candidates for infinite primes has been found. The prime candidates $P(\pm, m, n, s)$ labeled by rational numbers n/s and integers m plus the primes $P(\pm, m, n, s|r_1, r_2)$ constructed as r_1: th or r_2: th order polynomials of $Y = X/s$: the latter ansatz reduces to the less general ansatz of infinite values of n are allowed.

One can ask whether the $p \ mod \ 4 = 3$ condition guaranteeing that the square root of -1 does not exist as a p-adic number, is satisfied for $P(\pm, m, n, s)$. $P(\pm, 1, 1, 1) \ mod \ 4$ is either 3 or 1. The value of $P(\pm, m, n, s) \ mod \ 4$ for odd s on n only and is same for all states containing even/odd number of $p \ mod \ = 3$ excitations. For even s the value of $P(\pm, m, n, s) \ mod \ 4$ depends on m only and is same for all states containing even/odd number of $p \ mod \ = 3$ excitations. This condition resembles G-parity condition of Super Virasoro algebras. Note that either $P(+, m, n, s)$

or $P(-,m,n,s)$ but not both are physically interesting infinite primes ($2m\ mod\ 4 = 2$ for odd m) in the sense of allowing complex Hilbert space. Also the additional conditions satisfied by the states involving higher powers of X/s resemble to Virasoro conditions. An open problem is whether the analogy with the construction of the many-particle states in super-symmetric theory might be a hint about more deeper relationship with the representation of Super Virasoro algebras and related algebras.

It is not clear whether even more general prime candidates exist. An attractive hypothesis is that one could write explicit formulas for all infinite primes so that generalized theory of primes would reduce to the theory of finite primes.

15.2.2 Infinite Primes Form A Hierarchy

By generalizing using general construction recipe, one can introduce the second level prime candidates as primes not divisible by any finite prime p or infinite prime candidate of type $P(\pm,m,n,s)$ (or more general prime at the first level: in the following we assume for simplicity that these are the only infinite primes at the first level). The general form of these prime candidates is exactly the same as at the first level. Particle-analogy makes it easy to express the construction recipe. In present case "vacuum primes" at the lowest level are of the form

$$\begin{array}{l} \frac{X_1}{S} \pm S \ , \\ X_1 = X\prod_{P(\pm,m,n,s)} P(\pm,m,n,s) \ , \\ S = s\prod_{P_i} P_i \ , \\ s = \prod_{p_i} p_i \ . \end{array} \tag{15.2.6}$$

S is product or ordinary primes p and infinite primes $P_i(\pm,m,n,s)$. Primes correspond to physical states created by multiplying X_1/S (S) by integers not divisible by primes appearing S (X_1/S). The integer valued functions $k(p)$ and $K(p)$ of prime argument give the occupation numbers associated with X/s and s type "bosons" respectively. The non-negative integer-valued function $K(P) = K(\pm,m,n,s)$ gives the occupation numbers associated with the infinite primes associated with X_1/S and S type "bosons". More general primes can be constructed by mimicking the previous procedure.

One can classify these primes by the value of the integer $K_{tot} = \sum_{P|X/S} K(P)$: for a given value of K_{tot} the ratio of these prime candidates is clearly finite and given by a rational number. At given level the ratio P_1/P_2 of two primes is given by the expression

$$\begin{array}{l} \frac{P_1(\pm,m_1,n_1,s_1K_1,S_1)}{P_2(\pm,m_2,n_2,s_2,K_2,S_2)} \\ = \frac{n_1s_2}{n_2s_1}\prod_{\pm,m,n,s}(\frac{n}{s})^{K_1^+(\pm,n,m,s)-K_2^+(\pm,n,m,s)} \ . \end{array} \tag{15.2.7}$$

Here K_i^+ denotes the restriction of $K_i(P)$ to the set of primes dividing X/S. This ratio must be smaller than 1 if it is to appear as the first order term $P_1P_2 \to P_1/P_2$ in the canonical identification and again it seems that it is not possible to get all rationals for a fixed value of P_2 unless one allows infinite values of N expressed neatly using the more general ansatz involving higher power of S.

15.2.3 Construction Of Infinite Primes As A Repeated Quantization Of A Super-Symmetric Arithmetic Quantum Field Theory

The procedure for constructing infinite primes is very much reminiscent of the second quantization of an super-symmetric arithmetic quantum field theory in which single particle fermion and boson states are labeled by primes. In particular, there is nothing especially frightening in the particle representation of infinite primes: theoretical physicists actually use these kind of representations quite routinely.

1. The binary-valued function telling whether a given prime divides s can be interpreted as a fermion number associated with the fermion mode labeled by p. Therefore infinite prime is characterized by bosonic and fermionic occupation numbers as functions of the prime labeling various modes and situation is super-symmetric. X can be interpreted as the counterpart

of Dirac sea in which every negative energy state state is occupied and $X/s \pm s$ corresponds to the state containing fermions understood as holes of Dirac sea associated with the modes labeled by primes dividing s.

2. The multiplication of the "vacuum" X/s with $n = \prod_{p|X/s} p^{k(p)}$ creates $k(p)$ "p-bosons" in mode of type X/s and multiplication of the "vacuum" s with $m = \prod_{p|s} p^{k(p)}$ creates $k(p)$ "p-bosons". in mode of type s (mode occupied by fermion). The vacuum states in which bosonic creation operators act, are tensor products of two vacuums with tensor product represented as sum

$$|vac(\pm)\rangle = |vac(\frac{X}{s})\rangle \otimes |vac(\pm s)\rangle \leftrightarrow \frac{X}{s} \pm s \tag{15.2.8}$$

obtained by shifting the prime powers dividing s from the vacuum $|vac(X)\rangle = X$ to the vacuum ± 1. One can also interpret various vacuums as many fermion states. Prime property follows directly from the fact that any prime of the previous level divides either the first or second factor in the decomposition $NX/S \pm MS$.

3. This picture applies at each level of infinity. At a given level of hierarchy primes P correspond to all the Fock state basis of all possible many-particle states of second quantized super-symmetric theory. At the next level these many-particle states are regarded as single particle states and further second quantization is performed so that the primes become analogous to the momentum labels characterizing various single-particle states at the new level of hierarchy.

4. There are two nonequivalent quantizations for each value of S due to the presence of $\pm$ sign factor. Two primes differing only by sign factor are like G-parity $+1$ and -1 states in the sense that these primes satisfy $P\ mod\ 4 = 3$ and $P\ mod\ 4 = 1$ respectively. The requirement that -1 does not have p-adic square root so that Hilbert space is complex, fixes G-parity to say $+1$. This observation suggests that there exists a close analogy with the theory of Super Virasoro algebras so that quantum TGD might have interpretation as number theory in infinite context. An alternative interpretation for the $\pm$ degeneracy is as counterpart for the possibility to choose the fermionic vacuum to be a state in which either all positive or all negative energy fermion states are occupied.

5. One can also generalize the construction to include polynomials of $Y = X/S$ to get infinite hierarchy of primes labeled by the two integers r_1 and r_2 associated with the polynomials in question. An entire hierarchy of vacuums labeled by r_1 is obtained. A possible interpretation of these primes is as counterparts for the bound states of quantum field theory. The coefficient for the power $(X/s)^{r_1}$ appearing in the highest term of the general ansatz, codes the occupation numbers associated with vacuum $(X/s)^{r_1}$. All the remaining terms are proportional to s and combine to form, in general infinite, integer m characterizing various infinite occupation numbers for the subsystem characterized by s. The additional conditions guaranteeing prime number property are equivalent with the primality conditions for polynomials with integer valued coefficients and resemble Super Virasoro conditions. For $r_2 > 0$ bosonic occupation numbers associated with the modes with fermion number one are infinite and one cannot write explicit formula for the boson number.

6. One could argue that the analogy with super-symmetry is not complete. The modes of Super Virasoro algebra are labeled by natural number whereas now modes are labeled by prime. This need not be a problem since one can label primes using natural number n. Also 8-valued spin index associated with fermionic and bosonic single particle states in TGD world is lacking (space-time is surface in 8-dimensional space). This index labels the spin states of 8-dimensional spinor with fixed chirality. One could perhaps get also spin index by considering infinite octonionic primes, which correspond to vectors of 8-dimensional integer lattice such that the length squared of the lattice vector is ordinary prime:

$$\sum_{k=1,\ldots,8} n_k^2 = \text{ prime } .$$

Thus one cannot exclude the possibility that TGD based physics might provide representation for octonions extended to include infinitely large octonions. The notion of prime octonion is well defined in the set of integer octonions and it is easy to show that the Euclidian norm squared for a prime octonion is prime. If this result generalizes then the construction of generalized prime octonions would generalize the construction of finite prime octonions. It would be interesting to know whether the results of finite-dimensional case might generalize to the infinite-dimensional context. One cannot exclude the possibility that prime octonions are in one-one correspondence with physical states in quantum TGD.

These observations suggest a close relationship between quantum TGD and the theory of infinite primes in some sense: even more, entire number theory and mathematics might be reducible to quantum physics understood properly or equivalently, physics might provide the representation of basic mathematics. Of course, already the uniqueness of the basic mathematical structure of quantum TGD points to this direction. Against this background the fact that 8-dimensionality of the embedding space allows introduction of octonion structure (also p-adic algebraic extensions) acquires new meaning. Same is also suggested by the fact that the algebraic extensions of p-adic numbers allowing square root of real p-adic number are 4- and 8-dimensional.

What is especially interesting is that the core of number theory would be concentrated in finite primes since infinite primes are obtained by straightforward procedure providing explicit formulas for them. Repeated quantization provides also a model of abstraction process understood as construction of hierarchy of natural number valued functions about functions about...... At the first level infinite primes are characterized by the integer valued function $k(p)$ giving occupation numbers plus subsystem-complement division (division to thinker and external world!). At the next level prime is characterized in a similar manner. One should also notice that infinite prime at given level is characterized by a pair $(R = MN, S)$ of integers at previous level. Equivalently, infinite prime at given level is characterized by fermionic and bosonic occupation numbers as functions in the set of primes at previous level.

15.2.4 Construction In The Case Of An Arbitrary Commutative Number Field

The basic construction recipe for infinite primes is simple and generalizes even to the case of algebraic extensions of rationals. Let $K = Q(\theta)$ be an algebraic number field (see the Appendix of [K108] for the basic definitions). In the general case the notion of prime must be replaced by the concept of irreducible defined as an algebraic integer with the property that all its decompositions to a product of two integers are such that second integer is always a unit (integer having unit algebraic norm, see Appendix of [K108]).

Assume that the irreducibles of $K = Q(\theta)$ are known. Define two irreducibles to be equivalent if they are related by a multiplication with a unit of K. Take one representative from each equivalence class of units. Define the irreducible to be positive if its first non-vanishing component in an ordered basis for the algebraic extension provided by the real unit and powers of θ, is positive. Form the counterpart of Fock vacuum as the product X of these representative irreducibles of K.

The unique factorization domain (UFD) property (see Appendix of [K108]) of infinite primes does not require the ring O_K of algebraic integers of K to be UFD although this property might be forced somehow. What is needed is to find the primes of K; to construct X as the product of all irreducibles of K but not counting units which are integers of K with unit norm; and to apply second quantization to get primes which are first order monomials. X is in general a product of powers of primes. Generating infinite primes at the first level correspond to generalized rationals for K having similar representation in terms of powers of primes as ordinary rational numbers using ordinary primes.

15.2.5 Mapping Of Infinite Primes To Polynomials And Geometric Objects

The mapping of the generating infinite primes to first order monomials labeled by their rational zeros is extremely simple at the first level of the hierarchy:

$$P_\pm(m, n, s) = \frac{mX}{s} \pm ns \rightarrow x_\pm \pm \frac{m}{sn} \quad . \tag{15.2.9}$$

Note that a monomial having zero as its root is not obtained. This mapping induces the mapping of all infinite primes to polynomials.

The simplest infinite primes are constructed using ordinary primes and second quantization of an arithmetic number theory corresponds in one-one manner to rationals. Indeed, the integer $s = \prod_i p_i^{k_i}$ defining the numbers k_i of bosons in modes k_i, where fermion number is one, and the integer r defining the numbers of bosons in modes where fermion number is zero, are co-prime. Moreover, the generating infinite primes can be written as $(n/s)X \pm ms$ corresponding to the two vacua $V = X \pm 1$ and the roots of corresponding monomials are positive *resp.* negative rationals.

More complex infinite primes correspond sums of powers of infinite primes with rational coefficients such that the corresponding polynomial has rational coefficients and roots which are not rational but belong to some algebraic extension of rationals. These infinite primes correspond simply to products of infinite primes associated with some algebraic extension of rationals. Obviously the construction of higher infinite primes gives rise to a hierarchy of higher algebraic extensions.

It is possible to continue the process indefinitely by constructing the Dirac vacuum at the n: th level as a product of primes of previous levels and applying the same procedure. At the second level Dirac vacuum $V = X \pm 1$ involves X which is the product of all primes at previous levels and in the polynomial correspondence X thus correspond to a new independent variable. At the n: th level one would have polynomials $P(q_1|q_2|...)$ of q_1 with coefficients which are rational functions of q_2 with coefficients which are.... The hierarchy of infinite primes would be thus mapped to the functional hierarchy in which polynomial coefficients depend on parameters depending on....

At the second level one representation of infinite primes would be as algebraic curve resulting as a locus of $P(q_1|q_2) = 0$: this certainly makes sense if q_1 and q_2 commute. At higher levels the locus is a higher-dimensional surface.

One can speculate with possible connections to TGD physics. The degree n of the polynomial is its basic characterizer. Infinite primes corresponding to polynomials of degree $n > 1$ should correspond to bound states. On the other hand, the hierarchy of Planck constants suggests strongly the interpretation in terms of gravitational bound states. Could one identify $h_{eff}/h = n$ as the degree of the polynomial characterizing infinite prime?

15.2.6 How To Order Infinite Primes?

One can order the infinite primes, integers and rationals. The ordering principle is simple: one can decompose infinite integers to two parts: the "large" and the "small" part such that the ratio of the small part with the large part vanishes. If the ratio of the large parts of two infinite integers is different from one or their sign is different, ordering is obvious. If the ratio of the large parts equals to one, one can perform same comparison for the small parts. This procedure can be continued indefinitely.

In case of infinite primes ordering procedure goes like follows. At given level the ratios are rational numbers. There exists infinite number of primes with ratio 1 at given level, namely the primes with same values of N and same S with MS infinitesimal as compared to NX/S. One can order these primes using either the relative sign or the ratio of $(M_1S_1)/(M_2S_2)$ of the small parts to decide which of the two is larger. If also this ratio equals to one, one can repeat the process for the small parts of M_iS_i. In principle one can repeat this process so many times that one can decide which of the two primes is larger. Same of course applies to infinite integers and also to infinite rationals build from primes with infinitesimal MS. If NS is not infinitesimal it is not obvious whether this procedure works. If $N_iX_i/M_iS_i = x_i$ is finite for both numbers (this need not be the case in general) then the ratio $\frac{M_1S_1}{M_2S_2}\frac{(1+x_2)}{(1+x_1)}$ provides the needed criterion. In case that this ratio equals one, one can consider use the ratio of the small parts multiplied by $\frac{(1+x_2)}{(1+x_1)}$ of M_iS_i as ordering criterion. Again the procedure can be repeated if needed.

15.2.7 What Is The Cardinality Of Infinite Primes At Given Level?

The basic problem is to decide whether Nature allows also integers S, $R = MN$ represented as infinite product of primes or not. Infinite products correspond to subsystems of infinite size (S) and infinite total occupation number (R) in QFT analogy.

1. One could argue that S should be a finite product of integers since it corresponds to the requirement of finite size for a physically acceptable subsystem. One could apply similar argument to R. In this case the set of primes at given level has the cardinality of integers ($alef_0$) and the cardinality of all infinite primes is that of integers. If also infinite integers R are assumed to involve only finite products of infinite primes the set of infinite integers is same as that for natural numbers.
2. NMP is well defined in p-adic context also for infinite subsystems and this suggests that one should allow also infinite number of factors for both S and $R = MN$. Super symmetric analogy suggests the same: one can quite well consider the possibility that the total fermion number of the universe is infinite. It seems however natural to assume that the occupation numbers $K(P)$ associated with various primes P in the representations $R = \prod_P P^{K(P)}$ are finite but nonzero for infinite number of primes P. This requirement applied to the modes associated with S would require the integer m to be explicitly expressible in powers of $P_i|S$ ($P_{r_2} = 0$) whereas all values of r_1 are possible. If infinite number of prime factors is allowed in the definition of S, then the application of diagonal argument of Cantor shows that the number of infinite primes is larger than $alef_0$ already at the first level. The cardinality of the first level is $2^{alef_0} 2^{alef_0} == 2^{alef_0}$. The first factor is the cardinality of reals and comes from the fact that the sets S form the set of all possible subsets of primes, or equivalently the cardinality of all possible binary valued functions in the set of primes. The second factor comes from the fact that integers $R = NM$ (possibly infinite) correspond to all natural number-valued functions in the set of primes: if only finite powers $k(p)$ are allowed then one can map the space of these functions to the space of binary valued functions bijectively and the cardinality must be 2^{alef_0}. The general formula for the cardinality at given level is obvious: for instance, at the second level the cardinality is the cardinality of all possible subsets of reals. More generally, the cardinality for a given level is the cardinality for the subset of possible subsets of primes at the previous level.

15.2.8 How To Generalize The Concepts Of Infinite Integer, Rational And Real?

The allowance of infinite primes forces to generalize also the concepts concepts of integer, rational and real number. It is not obvious how this could be achieved. The following arguments lead to a possible generalization which seems practical (yes!) and elegant.

Infinite integers form infinite-dimensional vector space with integer coefficients

The first guess is that infinite integers N could be defined as products of the powers of finite and infinite primes.

$$N = \prod_k p_k^{n_k} = nM \ , \ n_k \geq 0 \ , \tag{15.2.10}$$

where n is finite integer and M is infinite integer containing only powers of infinite primes in its product expansion.

It is not however not clear whether the sums of infinite integers really allow similar decomposition. Even in the case that this decomposition exists, there seems to be no way of deriving it. This would suggest that one should regard sums

$$\sum_i n_i M_i$$

of infinite integers as infinite-dimensional linear space spanned by M_i so that the set of infinite integers would be analogous to an infinite-dimensional algebraic extension of say p-adic numbers such that each coordinate axes in the extension corresponds to single infinite integer of form $N = mM$. Thus the most general infinite integer N would have the form

$$N = m_0 + \sum m_i M_i \ . \tag{15.2.11}$$

This representation of infinite integers indeed looks promising from the point of view of practical calculations. The representation looks also attractive physically. One can interpret the set of integers N as a linear space with integer coefficients m_0 and m_i:

$$N = m_0|1\rangle + \sum m_i|M_i\rangle \ . \tag{15.2.12}$$

$|M_i\rangle$ can be interpreted as a state basis representing many-particle states formed from bosons labeled by infinite primes p_k and $|1\rangle$ represents Fock vacuum. Therefore this representation is analogous to a quantum superposition of bosonic Fock states with integer, rather than complex valued, superposition coefficients. If one interprets M_i as orthogonal state basis and interprets m_i as p-adic integers, one can define inner product as

$$\langle N_a, N_b\rangle = m_0(a)m_0(b) + \sum_i m_i(a)m_i(b) \ . \tag{15.2.13}$$

This expression is well defined p-adic number if the sum contains only enumerable number of terms and is always bounded by p-adic ultra-metricity. It converges if the p-adic norm of of m_i approaches to zero when M_i increases.

Generalized rationals

Generalized rationals could be defined as ratios $R = M/N$ of the generalized integers. This works nicely when M and N are expressible as products of powers of finite or infinite primes but for more general integers the definition does not look attractive. This suggests that one should restrict the generalized rationals to be numbers having the expansion as a product of positive and negative primes, finite or infinite:

$$N = \prod_k p_k^{n_k} = \frac{n_1 M_1}{nM} \ . \tag{15.2.14}$$

Generalized reals form infinite-dimensional real vector space

One could consider the possibility of defining generalized reals as limiting values of the generalized rationals. A more practical definition of the generalized reals is based on the generalization of the pinary expansion of ordinary real number given by

$$\begin{aligned} x &= \sum_{n\geq n_0} x_n p^{-n} \ , \\ x_n &\in \{0,..,p-1\} \ . \end{aligned} \tag{15.2.15}$$

It is natural to try to generalize this expansion somehow. The natural requirement is that sums and products of the generalized reals and canonical identification map from the generalized reals to generalized p-adcs are readily calculable. Only in this manner the representation can have practical value.

These requirements suggest the following generalization

$$\begin{aligned} X &= x_0 + \sum_N x_N p^{-N} \ , \\ N &= \sum_i m_i M_i \ , \end{aligned} \tag{15.2.16}$$

where x_0 and x_N are ordinary reals. Note that N runs over infinite integers which has *vanishing finite part*. Note that generalized reals can be regarded as infinite-dimensional linear space such that each infinite integer N corresponds to one coordinate axis of this space. One could interpret generalized real as a superposition of bosonic Fock states formed from single single boson state labeled by prime p such that occupation number is either 0 or infinite integer N with a vanishing finite part:

$$X = x_0|0\rangle + \sum_N x_N|N> \quad . \tag{15.2.17}$$

The natural inner product is

$$\langle X, Y\rangle = x_0 y_0 + \sum_N x_N y_N \quad . \tag{15.2.18}$$

The inner product is well defined if the number of N: s in the sum is enumerable and x_N approaches zero sufficiently rapidly when N increases. Perhaps the most natural interpretation of the inner product is as R_p valued inner product.

The sum of two generalized reals can be readily calculated by using only sum for reals:

$$X + Y = x_0 + y_0 + \sum_N (x_N + y_N)p^{-N} \quad , \tag{15.2.19}$$

The product XY is expressible in the form

$$XY = x_0 y_0 + x_0 Y + X y_0 + \sum_{N_1, N_2} x_{N_1} y_{N_2} p^{-N_1 - N_2} \quad , \tag{15.2.20}$$

If one assumes that infinite integers form infinite-dimensional vector space in the manner proposed, there are no problems and one can calculate the sums $N_1 + N_2$ by summing component wise manner the coefficients appearing in the sums defining N_1 and N_2 in terms of infinite integers M_i allowing expression as a product of infinite integers.

Canonical identification map from ordinary reals to p-adics

$$x = \sum_k x_k p^{-k} \quad \rightarrow \quad x_p = \sum_k x_k p^k \quad ,$$

generalizes to the form

$$x = x_0 + \sum_N x_N p^{-N} \quad \rightarrow \quad (x_0)_p + \sum_N (x_N)_p p^N \quad , \tag{15.2.21}$$

so that all the basic requirements making the concept of generalized real calculationally useful are satisfied.

There are several interesting questions related to generalized reals.

1. Are the extensions of reals defined by various values of p-adic primes mathematically equivalent or not? One can map generalized reals associated with various choices of the base p to each other in one-one manner using the mapping

$$X = x_0 + \sum_N x_N p_1^{-N} \quad \rightarrow \quad x_0 + \sum_N x_N p_2^{-N} \quad . \tag{15.2.22}$$

   The ordinary real norms of *finite* (this is important!) generalized reals are identical since the representations associated with different values of base p differ from each other only infinitesimally. This would suggest that the extensions are physically equivalent. It these extensions are not mathematically equivalent then p-adic primes could have a deep role in the definition of the generalized reals.

2. One can generalize previous formulas for the generalized reals by replacing the coefficients x_0 and x_i by complex numbers, quaternions or octonions so as to get generalized complex numbers, quaternions and octonions. Also inner product generalizes in an obvious manner. The 8-dimensionality of the embedding space provokes the question whether it might be possible to regard the infinite-dimensional WCW , or rather, its tangent space, as a Hilbert space realization of the generalized octonions. This kind of identification could perhaps reduce TGD based physics to generalized number theory.

15.2.9 Comparison With The Approach Of Cantor

The main difference between the approach of Cantor and the proposed approach is that Cantor uses only the basic arithmetic concepts such as sum and multiplication and the concept of successor defining ordering of both finite and infinite ordinals. Cantor's approach is also purely set theoretic. The problems of purely set theoretic approach are related to the question what the statement "Set is Many allowing to regard itself as One" really means and to the fact that there is no obvious connection with physics.

The proposed approach is based on the introduction of the concept of prime as a basic concept whereas partial ordering is based on the use of ratios: using these one can recursively define partial ordering and get precise quantitative information based on finite reals. The ordering is only partial and there is infinite number of ratios of infinite integers giving rise to same real unit which in turn leads to the idea about number theoretic anatomy of real point.

The "Set is Many allowing to regard itself as One" is defined as quantum physicist would define it: many particle states become single particle states in the second quantization describing the counterpart for the construction of the set of subsets of a given set. One could also say that integer as such corresponds to set as "One" and its decomposition to a product of primes corresponds to the set as "Many". The concept of prime, the ultimate "One", has as its physical counterpart the concept of elementary particle understood in very general sense. The new element is the physical interpretation: the sum of two numbers whose ratio is zero correspond to completely physical finite-subsystem-infinite complement division and the iterated construction of the set of subsets of a set at given level is basically p-adic evolution understood in the most general possible sense and realized as a repeated second quantization. What is attractive is that this repeated second quantization can be regarded also as a model of abstraction process and actually the process of abstraction itself.

The possibility to interpret the construction of infinite primes either as a repeated bosonic quantization involving subsystem-complement division or as a repeated super-symmetric quantization could have some deep meaning. A possible interpretation consistent with these two pictures is based on the hypothesis that fermions provide a reflective level of consciousness in the sense that the 2^N element Fock basis of many-fermion states formed from N single-fermion states can be regarded as a set of all possible statements about N basic statements. Statements about whether a given element of set X belongs to some subset S of X are certainly the fundamental statements from the point of view of mathematics. Hence one could argue that many-fermion states provide cognitive representation for the subsets of some set. Single fermion states represent the points of the set and many-fermion states represent possible subsets.

15.3 Can One Generalize The Notion Of Infinite Prime To TheNon-Commutative And Non-Associative Context?

The notion of prime and more generally, that of irreducible, makes sense also in more general number fields and even algebras. The considerations of [K109] suggests that the notion of infinite prime should be generalized to the case of complex numbers, quaternions, and octonions as well as to their hyper counterparts which seem to be physically the most interesting ones [K109]. Also the hierarchy of infinite primes should generalize as also the representation of infinite primes as polynomials although associativity is expected to pose technical problems.

15.3.1 Quaternionic And Octonionic Primes And Their Hyper Counterparts

The loss of commutativity and associativity implies that the definitions of quaternionic and octonionic primes are not completely straightforward.

Basic facts about quaternions and octonions

Both quaternions and octonions allow both Euclidian norm and the Minkowskian norm defined as a trace of the linear operator defined by the multiplication with octonion. Minkowskian norm has the metric signature of $H = M^4 \times CP_2$ or $M^4_+ \times CP_2$ so that H can be regarded locally as an octonionic space if one uses octonionic representation for the gamma matrices [K109]. Both norms are a multiplicative and the notions of both quaternionic and octonionic prime are well defined despite non-associativity. Quaternionic and octonionic primes have length squared equal to rational prime.

In the case of quaternions different basis of imaginary units I, J, K are related by 3-dimensional rotation group and different quaternionic basis span a 3-dimensional sphere. There is 2-sphere of complex structures since imaginary unit can be any unit vector of imaginary 3-space.

A basis for octonionic imaginary units J, K, L, M, N, O, P can be chosen in many ways and fourteen-dimensional subgroup G_2 of the group $SO(7)$ of rotations of imaginary units is the group labeling the octonionic structures related by octonionic automorphisms to each other. It deserves to be mentioned that G_2 is unique among the simple Lie-groups in that the ratio of the square roots of lengths for long and short roots of G_2 Lie-algebra are in ratio 3 : 1. For other Lie-groups this ratio is either 2: 1 or all roots have same length. The set of equivalence classes of the octonion structures is $SO(7)/G_2 = S^7$. In the case of quaternions there is only one equivalence class.

The group of automorphisms for octonions with a fixed imaginary part is $SU(3)$. The coset space $S^6 = G_2/SU(3)$ labels possible complex structures of the octonion space specified by a selection of a preferred imaginary unit. $SU(3)/U(2) = CP_2$ could be thought of as the space of octonionic structures giving rise to a given quaternionic structure with complex structure fixed. This can be seen as follows. The units $1, I$ are $SU(3)$ singlets whereas J, J_1, J_2 and K, K_1, K_2 form $SU(3)$ triplet and antitriplet. Under $U(2)$ J and K transform like objects having vanishing $SU(3)$ isospin and suffer only a $U(1)$ phase transformation determined by multiplication with complex unit I and are mixed with each other in orthogonal mixture. Thus $1, I, J, K$ is transformed to itself under $U(2)$.

Quaternionic and octonionic primes

Quaternionic primes with $p \ mod \ 4 = 1$ can correspond to (n_1, n_2) with n_1 even and n_2 odd or vice versa. For $p \ mod \ 4 = 3$ (n_1, n_2, n_3) with n_i odd is the minimal option. In this case there is however large number of primes having only two components: in particular, Gaussian primes with $p \ mod \ 4 = 1$ define also quaternionic primes. Purely real Gaussian primes with $p \ mod \ 4 = 3$ with norm $z\overline{z}$ equal to p^2 are not quaternionic primes, and are replaced with 3-component quaternionic primes allowing norm equal to p. Similar conclusions hold true for octonionic primes.

The reality condition for polynomials associated with Gaussian infinite primes requires that the products of generating prime and its conjugate are present so that the outcome is a real polynomial of second order.

Hyper primes

The notion of prime generalizes to hyper-quaternionic and octonionic case. The factorization $n_0^2 - n_3^2 = (n_0 + n_3)(n_0 - n_3)$ implies that any hyper-quaternionic and -octonionic prime has one particualr representative as $(n_0, n_3, 0, ...) = (n_3 + 1, n_3, 0, ...)$, $n_3 = (p - 1)/2$ for $p > 2$. $p = 2$ is exceptional: a representation with minimal number of components is given by $(2, 1, 1, 0, ...)$.

Notice that the interpretation of hyper-quaternionic primes (or integers) as four-momenta implies that it is not possible to find rest system for them if one assumes the entire quaternionic prime as four-momentum: only a system where energy is minimum is possible. The introduction of a preferred hyper-complex plane necessary for several reasons- in particular for the possibility to identify standard model quantum numbers in terms of infinite primes- allows to identify the momentum of particle in the preferred plane as the first two components of the hyper prime in fixed coordinate frame. Note that this leads to a universal spectrum for mass squared.

For time like hyper-primes the momentum is always time like for hyper-primes. In this case it is possible to find a rest frame by applying a hyper-primeness preserving G_2 transformation so that the resulting momentum has no component in the preferred frame. As a matter fact, SU(3) rotation is enough for a suitable choice of SU(3). These transformations form a discrete subgroup of SU(3) since hyper-integer property must be preserved. Massless states correspond to a null norm for the corresponding hyper integer unless one allows also tachyonic hyper primes with minimal representatives $(n_3, n_3 - 1, 0, ...)$, $n_3 = (p-1)/2$. Note that Gaussian primes with $p \ mod 4 \ = 1$ are representable as space-like primes of form $(0, n_1, n_2, 0)$: $n_1^2 + n_2^2 = p$ and would correspond to genuine tachyons. Space-like primes with $p \ mod \ 4 = 3$ have at least 3 non-vanishing components which are odd integers.

The notion of "irreducible" (see Appendix of [K108]) is defined as the equivalence class of primes related by a multiplication with a unit and is more fundamental than that of prime. All Lorentz boosts of a hyper prime combine to form an irreducible. Note that the units cannot correspond to real particles in corresponding arithmetic quantum field theory.

If the situation for $p > 2$ is effectively 2-dimensional in the sense that it is always possible to transform the hyper prime to a 2-component form by multiplying it by a suitable unit representing Lorentz boost, the theory for time-like hyper primes effectively reduces to the 2-dimensional hyper-complex case when irreducibles are chosen to belong to H_2. The physical counterpart for the choice of H_2 would be the choice of the plane of longitudinal polarizations, or equivalently, of quantization axis for spin. This hypothesis is physically highly attractive since it would imply number theoretic universality and conform with the effective 2-dimensionality. Of course, the hyper-octonionic primes related by $SO(7,1)$ boosts need not represent physically equivalent states.

Also the rigorous notion of hyper primeness seems to require effective 2-dimensionality. If effective 2-dimensionality holds true, hyper integers have a decomposition to a product of hyper primes multiplied by a suitable unit. The representation is obtained by Lorentz boosting the hyper integer first to a 2-component form and then decomposing it to a product of hyper-complex primes.

15.3.2 Hyper-Octonionic Infinite Primes

The infinite-primes associated with hyper-octonions are the most natural ones physically because of the underlying Lorentz invariance. It is however not possible to interpret them as as 8-momenta with mass squared equal to prime. The proper identification of standard model quantum numbers will be discussed later.

The basic objections against (hyper-)quaternionic and (hyper-)octonionic infinite primes relate to the non-commutativity and non-associativity.

In the case of quaternionic infinite primes non-commutativity, and in the case of octonionic infinite primes also non-associativity, might be expected to cause difficulties in the definition of X. Fortunately, the fact that all conjugates of a given finite prime appear in the product defining X, implies that the contribution from each irreducible with a given norm p is real and X is real. Therefore the multiplication and division of X with quaternionic or octonionic primes is a well-defined procedure, and generating infinite primes are well-defined apart from the degeneracy due to non-commutativity and non-associativity of the finite number of lower level primes.

Also the products of infinite primes are well defined, since by the reality of X it is possible to tell how the products AB and BA differ. Of course, also infinite primes representing physical states containing infinite numbers of fermions and bosons are possible and infinite primes of this kind must be analogous to generators of a free algebra for which AB and BA are not related in any manner.

Stronger form of associativity and commutativity is obtained if infinite octonionic/quaternionic primes are just ordinary octonionic/quaternonic primes multiplied with ordinary infinite primes. This option is perhaps the more elegant one. For this option the non-commutativity and non-associativity are concentrated on the finite octonionic/quaternionic prime multiplying the commutative infinite prime. This picture allows also the map of infinite octonionic/quaternionic primes to products of finite octonionic/quaternionic primes and of polynomials.

15.4 How To Interpret The Infinite Hierarchy Of Infinite Primes?

From the foregoing it should be clear that infinite primes might play key role in quantum physics. One can even consider the possibility that physics reduces to a generalized number theory, and that infinite primes are crucial for understanding mathematically consciousness and cognition. Of course, one must leave open the question whether infinite primes really provide really the mathematics of consciousness or whether they are only a beautiful but esoteric mathematical construct. In this spirit the following subsections give only different points of view to the problem with no attempt to a coherent overall view.

15.4.1 Infinite Primes And Hierarchy Of Super-Symmetric Arithmetic Quantum Field Theories

Infinite primes are a generalization of the notion of prime. They turn out to provide number theoretic correlates of both free, interacting and bound states of a super-symmetric arithmetic quantum field theory. It turns also possible to assign to infinite prime space-time surface as a geometric correlate although the original proposal for how to achieve this failed. Hence infinite primes serve as a bridge between classical and quantum and realize quantum classical correspondence stating that quantum states have classical counterparts, and has served as a basic heuristic guideline of TGD. More precisely, the natural hypothesis is that infinite primes code for the ground states of super-symplectic representations (for instance, ordinary particles correspond to states of this kind).

Generating infinite primes as counterparts of Fock states of a super-symmetric arithmetic quantum field theory

The basic construction recipe for infinite primes is simple and generalizes to the quaternionic case.

1. Form the product of all primes and call it X:

$$X = \prod_p p \ .$$

2. Form the vacuum states

$$V_\pm = X \pm 1 \ .$$

3. From these vacua construct all *generating* infinite primes by the following process. Kick out from the Dirac sea some negative energy fermions: they correspond to a product s of first powers of primes: $V \rightarrow X/s \pm s$ (s is thus square-free integer). This state represents a state with some fermions represented as holes in Dirac sea but no bosons. Add bosons by multiplying by integer r, which decomposes into parts as $r = mn$: m corresponding to bosons in X/s is product of powers of primes dividing X/s and n corresponds to bosons in s and is product of powers of primes dividing s. This step can be described as $X/s \pm s \rightarrow mX/s \pm ns$.

Generating infinite primes are thus in one-one correspondence with the Fock states of a super-symmetric arithmetic quantum field theory and can be written as

$$P_\pm(m,n,s) = \frac{mX}{s} \pm ns \ ,$$

where X is product of all primes at previous level. s is square free integer. m and n have no common factors, and neither m and s nor n and X/s have common factors.

The physical analog of the process is the creation of Fock states of a super-symmetric arithmetic quantum field theory. The factorization of s to a product of first powers of primes corresponds to many-fermion state and the decomposition of m and n to products of powers of prime correspond to bosonic Fock states since p^k corresponds to k-particle state in arithmetic quantum field theory.

More complex infinite primes as counterparts of bound states

Generating infinite primes are not all that are possible. One can construct also polynomials of the generating primes and under certain conditions these polynomials are non-divisible by both finite primes and infinite primes already constructed. As found, the conjectured effective 2-dimensionality for hyper-octonionic primes allows the reduction of polynomial representation of hyper-octonionic primes to that for hyper-complex primes. This would be in accordance with the effective 2-dimensionality of the basic objects of quantum TGD.

The physical counterpart of n: th order irreducible polynomial is as a bound state of n particles whereas infinite integers constructed as products of infinite primes correspond to non-bound but interacting states. This process can be repeated at the higher levels by defining the vacuum state to be the product of all primes at previous levels and repeating the process. A repeated second quantization of a super-symmetric arithmetic quantum field theory is in question.

The infinite primes represented by irreducible polynomials correspond to quantum states obtained by mapping the superposition of the products of the generating infinite primes to a superposition of the corresponding Fock states. If complex rationals are the coefficient field for infinite integers, this gives rise to states in a complex Hilbert space and irreducibility corresponds to a superposition of states with varying particle number and the presence of entanglement. For instance, the superpositions of several products of type $\prod_{i=1,..,n} P_i$ of n generating infinite primes are possible and in general give rise to irreducible infinite primes decomposing into a product of infinite primes in algebraic extension of rationals.

How infinite rationals correspond to quantum states and space-time surfaces?

The most promising answer to the question how infinite rationals correspond to space-time surfaces is discussed in detail in the next section. Here it is enough to give only the basic idea.

1. In zero energy ontology hyper-octonionic units (in real sense) defined by ratios of infinite integers have an nterpretation as representations for pairs of positive and negative energy states. Suppose that the quantum number combinations characterizing positive and negative energy quantum states are representable as superpositions of real units defined by ratios of infinite integers at each point of the space-time surface. If this is true, the quantum classical correspondence coded by the measurement interaction term of the Kähler-Dirac action maps the quantum numbers also to space-time geometry and implies a correspondence between infinite rationals and space-time surfaces.

2. The space-time surface associated with the infinite rational is in general not a union of the space-time surfaces associated with the primes composing the integers defining the rational. There the classical description of interactions emerges automatically. The description of classical states in terms of infinite integers would be analogous to the description of many particle states as finite integers in arithmetic quantum field theory. This mapping could in principle make sense both in real and p-adic sectors of WCW .

The finite primes which correspond to particles of an arithmetic quantum field theory present in Fock state, correspond to the space-time sheets of finite size serving as the building blocks of the space-time sheet characterized by infinite prime.

What is the interpretation of the higher level infinite primes?

Infinite hierarchy of infinite primes codes for a hierarchy of Fock states such that many-particle Fock states of a given level serve as elementary particles at next level. The unavoidable conclusion is that higher levels represent totally new physics not described by the standard quantization procedures. In particular, the assignment of fermion/boson property to arbitrarily large system would be in some sense exact. Topologically these higher level particles could correspond to space-time sheets containing many-particle states and behaving as higher level elementary particles.

This view suggests that the generating quantum numbers are present already at the lowest level and somehow coded by the hyper-octonionic primes taking the role of momentum quantum number they have in arithmetic quantum field theories. The task is to understand whether and how hyper-octonionic primes can code for quantum numbers predicted by quantum TGD.

The quantum numbers coding higher level states are collections of quantum numbers of lower level states. At geometric level the replacement of the coefficients of polynomials with rational functions is the equivalent of replacing single particle states with new single particle states consisting of many-particle states.

15.4.2 Infinite Primes, The Structure Of Many-Sheeted Space-Time, And The Notion Of Finite Measurement Resolution

The mapping of infinite primes to space-time surfaces codes the structure of infinite prime to the structure of space-time surface in a rather non-implicit manner, and the question arises about the concrete correspondence between the structure of infinite prime and topological structure of the space-time surface. It turns out that the notion of finite measurement resolution is the key concept: infinite prime characterizes angle measurement resolution. This gives a direct connection with the p-adicization program relying also on angle measurement resolution as well as a connection with the hierarchy of Planck constants. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory so that the characterization of the finite measurement resolution, which has been the ugly ducling of theoretical physics transforms to a beautiful swan.

The first intuitions

The concrete prediction of the general vision is that the hierarchy of infinite primes should somehow correspond to the hierarchy of space-time sheets or partonic 2-surfaces if one accepts the effective 2-dimensionality. The challenge is to find space-time counterparts for infinite primes at the lowest level of the hierarchy.

One could hope that the Fock space structure of infinite prime would have a more concrete correspondence with the structure of the many-sheeted space-time. One might that the space-time sheets labeled by primes p would directly correspond to the primes appearing in the definition of infinite prime. This expectation seems to be too simplistic.

1. What seems to be a safe guess is that the simplest infinite primes at the lowest level of the hierarchy should correspond to elementary particles. If inverses of infinite primes correspond to negative energy space-time sheets, this would explain why negative energy particles are not encountered in elementary particle physics.
2. More complex infinite primes at the lowest level of the hierarchy could be interpreted in terms of structures formed by connecting these structures by flux tubes to get space-time correlates of bound states. Even simplest infinite primes must correspond to bound state structures if the condition that the corresponding polynomial has real-rational coefficients is taken seriously.

Infinite primes at the lowest level of hierarchy correspond to several finite primes rather than single finite prime. The number of finite primes is however finite.

1. A possible interpretation for multi-p property is in terms of multi-p p-adic fractality prevailing in the interior of space-time surface. The effective p-adic topology of these space-time sheets would depend on length scale. In the longest scale the topology would correspond to p_n, in some shorter length scale there would be smaller structures with $p_{n-1} < p_n$-adic topology, and so on.... A good metaphor would be a wave containing ripples, which in turn would contain still smaller ripples. The multi-p p-adic fractality would be assigned with the 4-D space-time sheets associated with elementary particles. The concrete realization of multi-p p-adicity would be in terms of infinite integers coming as power series $\sum x_n N^n$ and having interpretation as p-adic numbers for any prime dividing N.
2. Effective 2-dimensionality would suggest that the individual p-adic topologies could be assigned with the 2-dimensional partonic surfaces. Thus infinite prime would characterize at the lowest level space-time sheet and corresponding partonic 2-surfaces. There are however reasons to think that even single partonic 2-surface corresponds to a multi-p p-adic topology.

Do infinite primes code for the finite measurement resolution?

The above describe heuristic picture is not yet satisfactory. In order to proceed, it is good to ask what determines the finite prime or set of them associated with a given partonic 2-surface. It is good to recall first the recent view about the p-adicization program relying crucially on the notion of finite measurement resolution.

1. The vision about p-adicization characterizes finite measurement resolution for angle measurement in the most general case as $\Delta\phi = 2\pi M/N$, where M and N are positive integers having no common factors. The powers of the phases $exp(i2\pi M/N)$ define identical Fourier basis irrespective of the value of M and measurement resolution does not depend on on the value of M. Situation is different if one allows only the powers $exp(i2\pi kM/N)$ for which $kM < N$ holds true: in the latter case the measurement resolutions with different values of M correspond to different numbers of Fourier components. If one regards N as an ordinary integer, one must have $N = p^n$ by the p-adic continuity requirement.

2. One can also interpret N as a p-adic integer. For $N = p^n M$, where M is not divisible by p, one can express $1/M$ as a p-adic integer $1/M = \sum_{k\geq 0} M_k p^k$, which is infinite as a real integer but effectively reduces to a finite integer $K(p) = \sum_{k=0}^{N-1} M_k p^k$. As a root of unity the entire phase $exp(i2\pi M/N)$ is equivalent with $exp(i2\pi R/p^n)$, $R = K(p)M \mod p^n$. The phase would non-trivial only for p-adic primes appearing as factors in N. The corresponding measurement resolution would be $\Delta\phi = R2\pi/N$ if modular arithetics is used to define the measurement resolution. This works at the first level of the hierarchy but not at higher levels. The alternative manner to assign a finite measurement resolution to M/N for given p is as $\Delta\phi = 2\pi|N/M|_p = 2\pi/p^n$. In this case the small fermionic part of the infinite prime would fix the measurement resolution. The argument below shows that only this option works also at the higher levels of hierarchy and is therefore more plausible.

3. p-Adicization conditions in their strong form require that the notion of integration based on harmonic analysis in symmetric spaces makes sense even at the level of partonic 2-surfaces. These conditions are satisfied if the partonic 2-surfaces in a given measurement resolution can be regarded as algebraic continuations of discrete surfaces whose points belong to the discrete variant of the $\delta M^4_\pm \times CP_2$. This condition is extremely powerful since it effectively allows to code the geometry of partonic 2-surfaces by the geometry of finite sub-manifold geometries for a given measurement resolution. This condition assigns the integer N to a given partonic surface and all primes appearing as factors of N define possible effective p-adic topologies assignable to the partonic 2-surface.

How infinite primes could then code for the finite measurement resolution? Can one identify the measurement resolution for $M/N = M/(Rp^n)$ as $\Delta\phi = ((M/R)\ mod\ p^n) \times 2\pi/p^n$ or as $\Delta\phi = 2\pi/p^n$? The following argument allows only the latter option.

1. Suppose that p-adic topology makes sense also for infinite primes and that state function reduction selects power of infinite prime P from the product of lower level infinite primes defining the integer N in M/N. Suppose that the rational defined by infinite integer defines measurement resolution also at the higher levels of the hierarchy.

2. The infinite primes at the first level of hierarchy representing Fock states are in one-one correspondence with finite rationals M/N for which integers M and N can be chosen to characterize the infinite bosonic part and finite fermionic part of the infinite prime. This correspondence makes sense also at higher levels of the hierarchy but M and N are infinite integers. Also other option obtained by exchanging "bosonic" and "fermionic" but later it will be found that only the first identification makes sense.

3. The first guess is that the rational M/N characterizing the infinite prime characterizes the measurement resolution for angles and therefore partially classifies also the finite sub-manifold geometry assignable to the partonic 2-surface. One should define what $M/N = ((M/R)\ mod\ P^n) \times P^{-n}$ is for infinite primes. This would require expression of M/R in modular arithmetics modulo P^n. This does not make sense.

4. For the second option the measurement resolution defined as $\Delta\phi = 2\pi|N/M|_P = 2\pi/P^n$ makes sense. The Fourier basis obtained in this manner would be infinite but all states

$exp(ik/P^n)$ would correspond in real sense to real unity unless one allows k to be infinite P-adic integer smaller than P^n and thus expressible as $k = \sum_{m<n} k_m P^m$, where k_m are infinite integers smaller than P. In real sense one obtains all roots $exp(iq2\pi)$ of unity with $q < 1$ rational. For instance, for $n = 1$ one can have $0 < k/P < 1$ for a suitably chosen infinite prime k. Thus one would have essentially continuum theory at higher levels of the hierarchy. The purely fermionic part N of the infinite prime would code for both the number of Fourier components in discretization for each power of prime involved and the ratio characterize the angle resolution.

The proposed relation between infinite prime and finite measurement resolution implies very strong number theoretic selection rules on the reaction vertices.

1. The point is that the vertices of generalized Feynman diagrams correspond to partonic 2-surfaces at which the ends of light-like 3-surfaces describing the orbits of partonic 2-surfaces join together. Suppose that the partonic 2-surfaces appearing a both ends of the propagator lines correspond to same rational as finite sub-manifold geometries. If so, then for a given p-adic effective topology the integers assignable to all lines entering the vertex must contain this p-adic prime as a factor. Particles would correspond to integers and only the particles having common prime factors could appear in the same vertex.
2. In fact, already the work with modelling dark matter [K47] led to ask whether particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It also seemed natural to assume that only the space-time sheets containing common primes in this collection can interact. This inspired the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The possibility of multi-p p-adicity raises the question about how to fix the p-adic prime characterizing the mass of the particle. The mass scale of the contribution of a given throat to the mass squared is given by $p^{-n/2}$, where $T = 1/n$ corresponds to the p-adic temperature of throat. Hence the dominating contribution to the mass squared corresponds to the smallest prime power p^n associated with the throats of the particle. This works if the integers characterizing other particles than graviton are divisible by the gravitonic p-adic prime or a product of p-adic primes assignable to graviton. If the smallest power p^n assignable to the graviton is large enough, the mass of graviton is consistent with the empirical bounds on it. The same consideration applies in the case of photons. Recall that the number theoretically very natural condition that in zero energy ontology the number of generalized Feynman graphs contributing to a given process is finite is satisfied if all particles have a non-vanishing but arbitrarily small p-adic thermal mass [K128].

Interpretational problem

The identification of infinite prime as a characterizer of finite measurement resolution looks nice but there is an interpretational problem.

1. The model characterizing the quantum numbers of WCW spinor fields to be discussed in the next section involves a pair of infinite primes P_+ and P_- corresponding to the two vacuum primes $X \pm 1$. Do they correspond to two different measurement resolutions perhaps assignable to CD and CP_2 degrees of freedom?
2. Different measurement resolutions in CD and CP_2 degrees of freedom need not be not a problem as long as one considers only the discrete variants of symmetric spaces involved. What might be a problem is that in the general case the p-adic primes associated with CD and CP_2 degrees of freedom would not be same unless the integers N_+ and N_- are assumed to have have same prime factors (they indeed do if $p^0 = 1$ is formally counted as prime power factors).

3. The idea of assigning different p-adic effective topologies to CD and CP_2 does not look attractive. Both CD and CP_2 and thus also partonic 2-surface could however possess simultaneously both p-adic effective topologies. This kind of option might make sense since the integers representable as infinite powers series of integer N can be regarded as p-adic integers for all prime factors of N. As a matter fact, this kind of multi-p p-adicity could make sense also for the partonic 2-surfaces characterized by a measurement resolution $\Delta\phi = 2\pi M/N$. One would have what might be interpreted as N_+N_--adicity.

4. It will be found that quantum measurement means also the measurement of the p-adic prime selecting same p-adic prime from N_+ and N_-. If $N_\pm$ is divisible only by $p^0 = 1$, the corresponding angle measurement resolution is trivial. From the point of view of consciousness state function reduction selects also the p-adic prime characterizing the cognitive representation which is very natural since quantum superpositions of different p-adic topologies are not natural physically.

15.4.3 How The Hierarchy Of Planck Constants Could Relate To Infinite Primes And P-Adic Hierarchy?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution [K127], the dark matter hierarchy characterized by increasing values of $\hbar$ [K43, K41], the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related. Number theoretical considerations give hopes about developing a more quantitative vision about the relationship between these hierarchies, in particular between the hierarchy of infinite primes, p-adic length scale hierarchy, and the hierarchy if Planck constants.

If infinite primes code for the hierarchy of measurement resolutions, the correlations between the p-adic hierarchy and the hierarchy of Planck constants indeed suggest themselves and allow also to select between two interpretations for the fact that two infinite primes N_+ and N_- are needed to characterize elementary particles (see the next section).

Recall that the hierarchy of Planck constants in the most general situation corresponds to a replacement M^4 and CP_2 factors of the embedding space with singular coverings and factor spaces. The condition that Planck constant is integer valued allows only singular coverings characterized by two integers n_a *resp.* n_b assignable to CD *resp.* CP_2. This condition also guarantees that a given value of Planck constant corresponds to only a finite number of pages of the "Big Book" and therefore looks rather attractive mathematically. This option also forces evolution as a dispersion to the pages of the books characterized by increasing values of Planck constant.

Concerning the correspondence between the hierarchy of Planck constants and p-adic length scale hierarchy there seems to be only single working option. The following assumptions make precise the relationship between finite measurement resolution, infinite primes and hierarchy of Planck constants.

1. Measurement resolution CD *resp.* CP_2 degrees of freedom is assumed to correspond to the rational M_+/N_+ *resp.* M_-/N_-. $N_\pm$ is identified as the integer assigned to the fermionic part of the infinite integer..

2. One must always fix the consideration to a fixed p-adic prime. This process could be regarded as analogous to fixing the quantization axes and p would also characterize the p-adic cognitive space-time sheets involved. The p-adic prime is therefore same for CD and CP_2 degrees of freedom as required by internal consistency.

3. The relationship to the hierarchy of Planck constants is fixed by the identifications $n_a = n_+(p)$ and $n_b = n_-(p)$ so that the number of sheets of the covering equals to the number of bosons in the fermionic mode p of the quantum state defined by infinite prime.

4. A physically attractive hypothesis is that number theoretical bosons *resp.* fermions correspond to WCW orbital *resp.* spin degrees of freedom. The first ones correspond to the symplectic algebra of WCW and the latter one to purely fermionic degrees of freedom.

Consider now the basic consequences of these assumptions from the point of view of physics and cognition.

1. Finite measurement resolution reduces for a given value of p to

$$\Delta\phi = \frac{2\pi}{p^{n_\pm(p)+1}} = \frac{2\pi}{p^{n_{a/b}}} ,$$

   where $n_\pm(p) = n_{a/b} - 1$ is the number of bosons in the mode p in the fermionic part of the state. The number theoretical fermions and bosons and also their probably existing physical counterparts are necessary for a non-trivial angle measurement resolution. The value of Planck constant given by

$$\frac{\hbar}{\hbar_0} = n_a n_b = (n_+(p)+1) \times (n_-(p)+1)$$

   tells the total number of bosons added to the fermionic mode p assigned to the infinite prime.
2. The presence of $\hbar > \hbar_0$ partonic 2-surfaces is absolutely essential for a Universe able to measure its own state. This is in accordance with the interpretation of hierarchy of Planck constants in TGD inspired theory of consciousness. One can also say that $\hbar = 0$ sector does not allow cognition at all since $N_\pm = 1$ holds true. For given p $\hbar = n_a n_b = 0$ means that given fermionic prime corresponds to a fermion in the Dirac sea meaning $n_\pm(p) = -1$. Kicking out of fermions from Direac sea makes possible cognition. For purely bosonic vacuum primes one has $\hbar = 0$ meaning trivial measurement resolution so that the physics is purely classical and would correspond to the purely bosonic sector of the quantum TGD.
3. For $\hbar = \hbar_0$ the number of bosons in the fermionic state vanishes and the general expression for the measurement resolution reduces to $\Delta\phi = 2\pi/p$. When one adds $n_\pm(p)$ bosons to the fermionic part of the infinite prime, the measurement resolution increases from $\Delta\phi = 2\pi/p$ to $\Delta\phi = 2\pi/p^{n_\pm(p)+1}$. Adding a sheet to the covering means addition of a number theoretic boson to the fermionic part of infinite prime. The presence of both number theoretic bosons and fermions with the values of p-adic prime $p_1 \neq p$ does not affect the measurement resolution $\Delta\phi = 2\pi/p^n$ for a given prime p.
4. The resolutions in CD and CP_2 degrees of freedom correspond to the same value of the p-adic prime p so that one has dicretizations based on $\Delta\phi = 2\pi/p^{n_a}$ in CD degrees of freedom and $\Delta\phi = 2\pi/p^{n_b}$ in CP_2 degrees of freedom. The finite sub-manifold geometries make sense in this case and since the effective p-adic topology is same, the continuation to continuous p-adic partonic 2-surface is possible.

p-Adic thermodynamics involves the p-adic temperature $T = 1/n$ as basic parameter and the p-adic mass scale of the particle comes as $p^{-(n+1)/2}$. The natural question is whether one could assume the relation $T_\pm = 1/(n_\pm(p)+1)$ between p-adic temperature and infinite prime and thus the relations $T_a = 1/n_a(p)$ and $T_b = 1/n_b(p)$. This identification is not consistent with the recent physical interpretation of the p-adic thermodynamics nor with the view about dark matter hierarchy and must be given up.

1. The minimal non-trivial measurement resolution with $n_i = 1$ and $\hbar = \hbar_0$ corresponds to the p-adic temperature $T_i = 1$. p-Adic mass calculations indeed predict $T = 1$ for fermions for $\hbar = \hbar_0$. In the case of gauge bosons $T \geq 2$ is favored so that gauge bosons would be dark. This would require that gauge bosons propagate along dark pages of the Big Book and become "visible" before entering to the interaction vertex.
2. p-Adic thermodynamics also assumes same p-adic temperature in CD and CP_2 degrees of freedom but the proposed identification allows also different temperatures. In principle the separation of the super-conformal degrees of freedom of CD and CP_2 might allow different p-adic temperatures. This would assign to different p-adic mass scales to the particles and the larger mass scale should give the dominant contribution.
3. For dark particles the p-adic mass scale would be by a factor $1/\sqrt{p}^{n_i(p)-1}$ lower than for ordinary particles. This is in conflict with the assumption that the mass of the particle does not depend on $\hbar$. This prediction would kill completely the recent vision about the dark matter.

15.5 How Infinite Primes Could Correspond To Quantum States And Space-time Surfaces?

The hierarchy of infinite primes is in one-one correspondence with a hierarchy of second quantizations of an arithmetic quantum field theory. The additive quantum number in question is energy like quantity for ordinary primes and given by the logarithm of prime whereas p-adic length scale hypothesis suggests that the conserved quantity is proportional to the inverse of prime or its square root. For infinite primes at the first level of hierarchy these quantum numbers label single particles states having interpretation as ordinary elementary particles. For octonionic and hyper-octonionic primes the quantum number is analogous to a momentum with 8 components. The question is whether these number theoretic quantum numbers could have interpretation as genuine quantum numbers. Quantum classical correspondence raises another question. Is it possible to label space-time surfaces by infinite primes? Could this correspondence be even one-to-one?

I have considered these questions already more than decade ago. The discussion at that time was necessarily highly speculative and just a mathematical exercise. After that time however a lot of progress has taken place in quantum TGD and it is highly interaction to see what comes out from the interaction of the notion of infinite prime with the notions of zero energy ontology and generalized embedding space, and with the recent vision about how Chern-Simons Dirac term in the Kähler-Dirac action allows to code information about quantum numbers to the space-time geometry. The possibility of this coding allows to simplify the discussion dramatically. If one could map infinite hyper-octonionic or hyper-quaternionic primes to quantum numbers of the standard model naturally, then the their map of to the geometry of space-time surfaces would realize the coding of space-time surfaces by infinite primes (and more generally by integers and rationals). Also a detailed realization of number theoretic Brahman=Atman identity would emerge as an outcome.

15.5.1 A Brief Summary About Various Moduli Spaces And Their Symmetries

It is good to sum up the number theoretic symmetries before trying to construct an overall view about the situation. Several kinds of number theoretical symmetry groups are involved corresponding to symmetries in the moduli spaces of hyper-octonionic and hyper-quaternionic structures, symmetries mapping hyper-octonionic primes to hyper-octonionic primes, and translations acting in the space of causal diamonds (CDs) and shifting. The moduli space for CDs labeled by pairs of its tips that its pairs of points of $M^4 \times CP_2$ is also in important role.

1. The basic idea is that color $SU(3) \subset G_2$ acts as automorphisms of hyper-octonion structure with a preferred imaginary unit. $SO(7,1)$ acts as symmetries in the moduli space of hyper-octonion structures. Associativity implies symmetry breaking so that only hyper-quaternionic structures are considered and $SO(3,1) \times SO(4)$ acts as symmetries of the moduli space for hyper-quaternionic structures.
2. CP_2 parameterizes the moduli space of hyper-quaternionic structures induced from a given hyper-octonionic structure with preferred imaginary unit.
3. Color group $SU(3)$ is the analog of Galois group for the extension of reals to octonions and has a natural action on the decompositions of rational infinite primes to hyper-octonionic infinite primes. For given hyper-octonionic prime one can identify a subgroup of SU(3) generating a finite set of hyper-octonionic primes for it at sphere S^7. This suggests wave function at the orbit of given hyper-octonionic prime in turn generalizing to wave functions in the space of infinite primes.
4. Four-momenta correspond to translational degrees of freedom associated with the preferred points of M^4 coded by the infinite rational (tip of the light-cone). Color quantum numbers in cm degrees of freedom can be assigned to the CP_2 projection of the preferred point of H. As a matter fact, the definition of hyper-octonionic structure involves the choice of origin of M^8 giving rise to the preferred point of H.

These symmetries deserve a more detailed discussion.

1. The choice of global hyper-octonionic coordinate is dictated only modulo a transformation of $SO(1,7)$ acting as isometries of hyper-octonionic norm and as transformations in moduli space of hyper-octonion structures. $SO(7)$ respects the choice of the real unit. $SO(1,3) \times SO(4)$ acts in the moduli space of global hyper-quaternionic structures identified as sub-structures of hyper-octonionic structure. The choice of global hyper-octonionic structures involves also a choice of origin implying preferred point of H. The M^4 projection of this point corresponds to the tip of CD. Since the integers representing physical states must be hyper-quaternionic by associativity conditions, the symmetry breaking ("number theoretic compactification") to $SO(1,3) \times SO(4)$ occurs very naturally. This group acts as spinor rotations in H picture and as isometries in M^8 picture. The choice of both tips of CD reduces $SO(1,3)$ to $SO(3)$.

2. $SO(1,7)$ allows 3 different 8-dimensional representations (8_v, 8_s, and $\overline{8}_s$). All these representations must decompose under $SU(3)$ as $1+1+3+\overline{3}$ as little exercise with $SO(8)$ triality demonstrates. Under $SO(6) \cong SU(4)$ the decompositions are $1+1+6$ and $4+\overline{4}$ for 8_v and 8_s and its conjugate. Both hyper-octonion spinors and gamma matrices are identified as hyper-octonion units rather than as matrices. It would be natural to assign to bosonic M^8 primes 8_v and to fermionic M^8 primes 8_s and $\overline{8}_s$. One can distinguish between $8_v, 8_s$ and $\overline{8}_s$ for hyper-octonionic units only if one considers the full $SO(1,3) \times SO(4)$ action in the moduli space of hyper-octonionic structures.

3. G_2 acts as automorphisms on octonionic imaginary units and $SU(3)$ respects the choice of preferred imaginary unit meaning a choice of preferred hyper-complex plane $M^4 \subset M^4$. Associativity requires a reduction to hyper-quaternionic primes and implies color confinement in number theoretical and as it turns also in physical sense. For hyper-quaternionic primes the automorphisms restrict to $SO(3)$ which has right/left action of fermionic hyper-quaternionic primes and adjoint action on bosonic hyper-quaternionc primes. The choice of hyper-quaternionic structure is global as opposed to the local choice of hyper-quaternionic tangent space of space-time surface assigning to a point of $HQ \subset HO$ a point of CP_2. $U(2) \subset SU(3)$ leaves invariant given hyper-quaternionic structure which are thus parameterized by CP_2. Color partial waves can be interpreted as partial waves in this moduli space.

15.5.2 Associativity And Commutativity Or Only Their Quantum Variants?

Associativity and commutativity conditions are absolutely essential notions in quantum TGD and also in the mapping of infinite primes to the space-time sheets. Hyper-quaternionicity formulated in terms of the Kähler-Dirac gamma matrices defined by Kähler action fixes classical space-time dynamics and a very beautiful algebra formulation of quantum TGD in terms of hyper-octonionic local Clifford algebra of embedding space emerges. There is no need for the use of hyper-octonion real analytic maps although one cannot exclude the possibility that they might be involved with the construction of hyper-quaternionic space-time surfaces.

Associativity implies hyper-quaternionicity and commutativity requirement in turn leads to complex rational infinite primes. Since one can decompose complex rational primes to hyper-quaternionic and even hyper-octonionic primes, one might hope that this could allow to represent states which consist of colored constituents. This representations has however the flavor of a formal trick and the considerations related to concrete representations of infinite primes suggest that the rationality of infinite primes might be a too restrictive condition.

A more radical possibility is that physical states are only quantum associative and commutative. In case of associativity this means that they are obtained as quantum superpositions in the space of real units over all possible associations performed for a given product of hyper-octonion primes (for instance, $|A(BC)\rangle + |(AB)C\rangle$). These states would be associative in quantum sense but would not reduce to hyper-quaternionic primes. Also the notion of quantum commutativity makes sense. The fact that mesons are quantum superpositions of quark-antiquark pairs which each corresponds to different pair of hyper-quaternionic primes and are thus not representable classically, suggests that one can require only quantum associativity and quantum commutativity.

15.5.3 How Space-Time Geometry Could Be Coded By Infinite Primes?

Second key question is whether space-time geometry could be characterized in terms of infinite primes (and integers and rationals in the most general case) and how this is achieved. The question is how the quantum states consisting of fundamental fermions serving as building bricks of elementary particles could be coded by infinite quaternionic integeres to which one can assign ordinary finite quaternionic primes.

The basic idea is roughly that at the first level of the hierarchy the finite primes appearing as building blocks of infinite prime correspond to structures formed by pairs or wormhole contacts assigned with elementary particles.

1. The partonic orbits defined by wormhole throats could be characterized by finite primes specifying the preferred p-adic topology assignable to the p-adic "cognitive representation" of the throat.
2. One could assign hyper-quaternionic integer to the real particle as its four-momentum. In this case the mass shell condition would fix the hyper-quaternionic integer to a high extent. All discrete Lorentz boosts of the particle state taking hyper-quaternionic integers to hyper-quaternionic integers would correspond to the same p-adic integer (prime) defined by the length of the Lorentz boosted hyper-quaternionic integer. The p-adic prime characterizing virtual particle would be one of the primes appearing in the factorization of this integer to a product of powers of prime, most naturally the one whose power is largest.

   Note that p-adic length scale hypothesis suggests that the p-adic primes near powers of two are favored for on mass shell particles and perhaps also for the virtual particles.
3. For fundamental fermions associated with boundaries of string world sheets and appearing as building bricks of particles the masses would vanish on mass shell so that the hyper-quaternionic integer would in this case have vanishing norm.

   The virtual four-momentum assigned to a virtual fermion line as a generalized eigenvalue of Chern-Simons Dirac operator would correspond to hyper-quaternionic integer. In this case p-adic prime would be defined as for physical particles and would depend on the mass of the virtual particle. If the integration over virtual momenta by residue calculus effectively leads to an integral over on mass shell massless virtual momenta with non-physical spinor helicities then also virtual fundamental fermions would correspond to zero norm hyper-quaternionic integers.
4. The correlation between particle's four-momentum and the p-adic prime characterizing corresponding cognitive representation would be in accordance with quantum classical correspondence.
5. The hyperquaternionic primes appearing as largest factors in the factorizion of hyper-quaternionic integers assignable with physical particles could be interpreted as building bricks of an infinite hyperquaternionic prime characterizing the many-particle state and at least the boundaries of string world sheets. The idea that p-adic space-time surfaces defined "cognitive representations" as p-adic chart maps of real space-time surfaces and vice versa (as the TGD based definition of p-adic manifolds assumes) suggests that the p-adic primes in question characterize also space-time regions rather than only the boundaries of string world sheets.

A couple of comments about this speculation are in order.

1. ZEO implies a hierarchy of CDs within CDs and this hierarchy as well as the hierarchy of space-time sheets corresponds naturally to the hierarchy of infinite primes. One can assign standard model quantum numbers to various partonic 2-surfaces with positive and negative energy parts of the quantum state assignable to the light-like boundaries of CD. Also infinite integers and rationals are possible and the inverses of infinite primes would naturally correspond to elementary particles with negative energy. The condition that zero energy state has vanishing net quantum numbers implies that the ratio of infinite integers assignable to zero energy state equals to real unit in real sense and has has vanishing total quantum numbers.
2. Neither quantum numbers nor infinite primes coding them cannot characterize the partonic 2-surface itself completely since they say nothing about the deformation of the space-time surface but only about labels characterizing the WCW spinor field. Also the topology of

partonic 2-surface fails to be coded. Quantum classical correspondence however suggests that this correspondence could be possible in a weaker sense. In the Gaussian approximation for functional integral over the world of classical worlds space-time surface and thus the collection of partonic 2-surfaces is effectively replaced with the one corresponding to the maximum of Kähler function, and in this sense one-one correspondence is possible unless the situation is non-perturbative. In this case the physics implied by the hierarchy of Planck constants could however guarantee uniqueness.

One of the basic ideas behind the identification of the dark matter as phases with non-standard value of Planck constant is that when perturbative description of the system fails, a phase transition increasing the value of Planck constant takes place and makes perturbative description possible. Geometrically this phase transition means a leakage to another sector of the embedding space realized as a book like structure with pages partially labeled by the values of Planck constant. Anyonic phases and fractionization of quantum numbers is one possible outcome of this phase transition. An interesting question is what the fractionization of the quantum numbers means number theoretically.

15.6 Infinite Primes And Mathematical Consciousness

The mathematics of infinity relates naturally with the mystery of consciousness and religious and mystic experience. In particular, mathematical cognition might have as a space-time correlate the infinitely structured space-time points implied by the introduction of infinite-dimensional space of real units defined by infinite (hyper-)octonionic rationals having unit norm in the real sense. I hope that the reader takes this section as a noble attempt to get a glimpse about unknown rather than final conclusions.

15.6.1 Algebraic Brahman=Atman Identity

The proposed view about cognition and intentionality emerges from the notion of infinite primes, which was actually the first genuinely new mathematical idea inspired by TGD inspired consciousness theorizing. Infinite primes, integers, and rationals have a precise number theoretic anatomy. For instance, the simplest infinite primes correspond to the numbers $P_{\pm} = X \pm 1$, where $X = \prod_k p_k$ is the product of all finite primes. Indeed, $P_{\pm}\ mod\ p = 1$ holds true for all finite primes. The construction of infinite primes at the first level of the hierarchy is structurally analogous to the quantization of super-symmetric arithmetic quantum field theory with finite primes playing the role of momenta associated with fermions and bosons. Also the counterparts of bound states emerge. This process can be iterated at the second level the product of infinite primes constructed at the first level replaces X and so on.

The structural similarity with repeatedly second quantized quantum field theory strongly suggests that physics might in some sense reduce to a number theory for infinite rationals M/N and that second quantization could be followed by further quantizations. As a matter fact, the hierarchy of space-time sheets could realize this endless second quantization geometrically and have also a direct connection with the hierarchy of logics labeled by their order. This could have rather breathtaking implications.

1. One is forced to ask whether this hierarchy corresponds to a hierarchy of realities for which level below corresponds in a literal sense infinitesimals and the level next above to infinity.
2. Second implication is that there is an infinite number of infinite rationals behaving like real units ($M/N \equiv 1$ in real sense) so that space-time points could have infinitely rich number theoretical anatomy not detectable at the level of real physics. Infinite integers would correspond to positive energy many particle states and their inverses (infinitesimals with number theoretic structure) to negative energy many particle states and $M/N \equiv 1$ would be a counterpart for zero energy ontology to which oneness and emptiness are assigned in mysticism.
3. Single space-time point, which is usually regarded as the most primitive and completely irreducible structure of mathematics, would take the role of Platonia of mathematical ideas being able to represent in its number theoretical structure even the quantum state of entire Universe. Algebraic Brahman=Atman identity and algebraic holography would be realized in a rather literal sense.

Number theoretic anatomy of space-time point

This number theoretical anatomy should relate to mathematical consciousness in some manner. For instance, one can ask whether it makes sense to speak about quantum jumps changing the number theoretical anatomy of space-time points and whether these quantum jumps give rise to mathematical ideas. In fact, the identifications of Platonia as spinor fields in WCW on one hand and as the set number theoretical anatomies of point of embedding space force the conclusion that WCW spinor fields (recall also the identification as correlates for logical mind) can be realized in terms of the space for number theoretic anatomies of embedding space points. Therefore quantum jumps would be correspond to changes in the anatomy of the space-time points. Or more precisely, to the changes of the WCW spinor fields regarded as wave functions in the set of embedding space points which are equivalent in real sense. Embedding space would be experiencing genuine number theoretical evolution. The whole physics would reduce to the anatomy of numbers. All mathematical notions which are more than mere human inventions would be imbeddable to the Platonia realized as the number theoretical anatomies of single embedding space point.

To realize this picture would require that WCW spinor fields and perhaps even WCW allow a mapping to the number theoretic anatomies of space-time point. In finite-dimension Euclidian spaces momentum space labelling plane wavs is dual to the space. One could hope that also now the "orbital" quantum numbers of WCW spinor fields could code for WCW in given measurement resolution. The construction of the previous sections realize the mapping of the quantum states defined by WCW spinors fields assignable to given CD to wave function in the space of hyper-octonionic units. These wave functions can be also regarded as linear combinations of these units if the coefficients are complex numbers formed using the commuting imaginary unit of complexified octonions so that the Hilbert space like structure in question would have purely number theoretic meaning. The rationals defined by infinite primes characterize also measurement resolution and classify the finite sub-manifold geometries associated with partonic two-surfaces. At higher levels one has rationals defined by ratios of infinite integers and one can ask whether this interpretation generalizes.

Note that one must distinguish between two kinds of hyper-octonionic units.

1. Already in the case of complex numbers one has rational complex units defined in terms of Pythagorean triangle and their products generate infinite dimensional space. The hyper-octonionic units defined as ratios U of infinite integers and suggested to provide a representation of WCW spinor fields correspond to these. The powers U^m define roots of unity which can be regarded analogous to $exp(i2\pi x)$, where x is not rational but the exponent itself is complex rational.
2. Besides this there are roots of unity which are in general algebraic complex numbers. These roots of unit correspond to phases $exp(i2\pi M/N)$, where M/N is ratio of real infinite integers and i is the commuting hyper-octonionic imaginary unit. These real infinite integers can be assigned to hyper-octonionic integers by replacing everywhere finite hyper-octonionic primes with their norm which is ordinary prime. By the previous considerations only the phases $exp(i2\pi M/P^n)$ make sense p-adically for infinite primes P.

15.6.2 Leaving The World Of Finite Reals And Ending Up To The Ancient Greece

If strong number theoretic vision is accepted, all physical predictions of quantum TGD would be numbers in finite algebraic extensions of rationals at the first level of hierarchy. Just the numbers which ancient Greeks were able to construct by the technical means at use! This seems rather paradoxical but conforms also with the hypothesis that the dicrete algebraic intersections of real and p-adic 2-surfaces provide the fundamental cognitive representations.

The proposed construction for infinite primes gives a precise division of infinite primes to classes: the ratios of primes in given class span a *subset of rational numbers*. These classes give much more refined classification of infinities than infinite ordinals or alephs. They would correspond to separate phases in the evolution of consciousness identified as a sequence of quantum jumps defining sequence of primes $\rightarrow p_1 \rightarrow p_2$...... Infinite primes could mean a transition from space-time level to the level of function spaces. WCW is example of a space which can be parameterized by a space of functions locally.

The minimal assumption is that infinite primes reflect their presence only in the possibility to multiply the coordinates of embedding space points by real units formed as ratios of infinite integers. The correspondence between polynomials and infinite primes gives hopes of mapping at least the reduced WCW consisting of the maxima of Kähler function to the anatomy of space-time point. Also WCW spinors and perhaps also the modes of WCW spinor fields would allow this kind of map.

One can consider also the possibility that infinite integers and rationals give rise to a hierarchy of embedding spaces such that given level represents infinitesimals from the point of view of higher levels in hierarchy. Even "simultaneous" time evolutions of conscious experiences at different aleph levels with completely different time scales (to put it mildly) are possible since the time values around which the contents of conscious experience are possibly located, are determined by the quantum jump: also multi-snapshots containing snapshots also from different aleph levels are possible. Un-integrated conscious experiences with all values of p could be contained in given quantum jump: this would give rise to a hierarchy of conscious beings: the habitants above given level could be called Gods with full reason: those above us would probably call us just "epsilons" if ready to admit that we exist at all except in non-rigorous formulations of elementary calculus!

15.6.3 Infinite Primes And Mystic World View

The proposed interpretation deserves some additional comments from the point of consciousness theory.

1. An open problem is whether the finite integer S appearing in the infinite prime is product of only finite or possibly even infinite number of lower level primes at a given level of hierarchy. The proposed physical identification of S indeed allows S to be a product of infinitely many primes. One can allow also M and N appearing in the infinite and infinite part to be contain infinite number of factors. In this manner one obtains a hierarchy of infinite primes expressible in the form

$$\begin{aligned}
&P = nY^{r_1} + mS \ , \quad r = 1, 2, ... \\
&m = m_0 + P_{r_2}(Y) \ , \\
&Y = \frac{X}{S} \ , \\
&S = \prod_i P_i \ .
\end{aligned}$$

   Note that this ansatz is in principle of the same general form as the original ansatz $P = nY + mS$. These primes correspond in physical analogy to states containing infinite number of particles.

   If one poses no restrictions on S this implies that the cardinality for the set of infinite primes at first level would be $c = 2^{alef_0}$ ($alef_0$ is the cardinality of natural numbers). This is the cardinality for *all* subsets of natural numbers equal to the cardinality of reals. At the next level one obtains the cardinality 2^c for *all* subsets of reals, etc....

   If S were always a product of *finite number of primes* and $k(p)$ would differ from zero for finite number of primes only, the cardinality of infinite primes would be $alef_0$ at each level. One could pose the condition that mS is infinitesimal as compared to nX/S. This would guarantee that the ratio of two infinite primes at the same level would be well defined and equal to n_1S_2/n_2S_1. On the other hand, the requirement that all rationals are obtained as ratios of infinite primes requires that no restrictions are posed on $k(p)$: in this case the cardinality coming from possible choices of $r = ms$ is the cardinality of reals at first level.

   The possibility of primes for which also S is finite would mean that the algebra determined by the infinite primes must be generalized. For the primes representing states containing infinite number of bosons and/or fermions it would be possible to tell how P_1P_2 and P_2P_1 differ and these primes would behave like elements of free algebra. As already found, this kind of free algebra would provide single space-time point with enormous algebraic representative power and analog of Brahman=Atman identity would result.

2. There is no physical subsystem-complement decomposition for the infinite primes of form $X \pm 1$ since fermionic degrees of freedom are not excited at all. Mystic could interpret it as a state of consciousness in which all separations vanish and there is no observer-observed

distinction anymore. A state of pure awareness would be in question if bosonic and fermionic excitations represent the contents of consciousness! Since fermionic many particle states identifiable as Boolean statements about basic statements are identified as representation for reflective level of consciousness, $S = 1$ means that the reflective level of consciousness is absent: enlightenment as the end of thoughts according to mystics.

The mystic experiences of oneness ($S = 1!$), of emptiness (the subset of primes defined by S is empty!) and of the absence of all separations (there is no subsystem-complement separation and hence no division between observer and observed) could be related to quantum jumps to this kind of sectors of the WCW . In super-symmetric interpretation $S = 1$ means that state contains no fermions.

3. There is entire hierarchy of selves corresponding to the hierarchy of infinite primes and the relationship between selves at different levels of the hierarchy is like the relationship between God and human being. Infinite primes at the lowest level would presumably represent elementary particles. This implies a hierarchy for moments of consciousness and it would be un-natural to exclude the existence of higher level "beings" (one might call them Angels, Gods, etc...).

15.6.4 Infinite Primes And Evolution

The original argument leading to the notion of infinite primes was simple. Generalized unitarity implies evolution as a gradual increase of the p-adic prime labeling the WCW sector D_p to which the localization associated with quantum jump occurs. Infinite p-adic primes are forced by the requirement that p-adic prime increases in a statistical sense and that the number of quantum jumps already occurred is infinite (assuming finite number of these quantum jumps and therefore the first quantum jump, one encounters the problem of deciding what was the first WCW spinor field).

Quantum classical correspondence requires that p-adic evolution of the space-time surface with respect to geometric time repeats in some sense the p-adic evolution by quantum jumps implied by the generalized unitarity [K50]. Infinite p-adic primes are in a well defined sense composites of the primes belonging to lower level of infinity and at the bottom of this de-compositional hierarchy are finite primes. This decomposition corresponds to the decomposition of the space-time surface into p-adic regions which in TGD inspired theory of consciousness correspond to selves. Therefore the increase of the composite primes at lower level of infinity induces the increase of the infinite p-adic prime. p-Adic prime can increase in two ways.

1. One can introduce the concept of the p-adic sub-evolution: the evolution of infinite prime P is induced by the sub-evolution of infinite primes belonging to a lower level of infinity being induced by.... being induced by the evolution at the level of finite primes. For instance, the increase of the cell size means increase of the p-adic prime characterizing it: neurons are indeed very large and complicated cells whereas bacteria are small. Sub-evolution occurs both in subjective and geometric sense.

   (a) For a given value of geometric time the p-adic prime of a given space-time sheet gradually increases in the evolution by quantum jumps: our geometric past evolves also!

   (b) The p-adic prime characterizing space-time sheet also increases as the geometric time associated with the space-time sheet increases (say during morphogenesis).

   The notion of sub-evolution is in accordance with the "Ontogeny recapitulates phylogeny" principle: the evolution of organism, now the entire Universe, contains the evolutions of the more primitive organisms as sub-evolutions.

2. Infinite prime increases also when entirely new finite primes emerge in the decomposition of an infinite prime to finite primes. This means that entirely new space-time sheets representing new structures emerge in quantum jumps. The creation of space-time sheets in quantum jumps could correspond to this process. By quantum classical correspondence this process corresponds at the space-time level to phase transitions giving rise to new material space-time sheets with more and more refined effective p-adic effective topology.

15.7 Does The Notion Of Infinite-P P-Adicity Make Sense?

In this section speculations related to infinite-P p-adicity are represented in the form of shy questions in order to not irritate too much the possible reader. The basic open question causing the tension is whether infinite primes relate only to the physics of cognition or whether they might allow to say something non-trivial about the physics of matter too.

The following list of questions is rather natural with the background provided by the p-adic physics.

1. Can one generalize the notion of p-adic norm and p-adic number field to include infinite primes? Could one define the counterpart of p-adic topology for literally infinite values of p? Does the topology R_P for infinite values of P approximate or is it equivalent with real topology as p-adic topology at the limit of infinite p is assumed to do (at least in the sense that p-adic variants of Diophantine equations at this limit correspond to ordinary Diophantine equations)? This is is possible is suggested by the fact that sheets of 3-surface are expected to have infinite size and thus to correspond to infinite p-adic length scale.
2. Canonical identification maps p-adic numbers of unit norm to real numbers in the range $[0, p]$. Does the canonical identification map the p-adic numbers R_P associated with infinite prime to reals? Could the number fields R_P provide alternative formulations/generalizations of the non-standard analysis based on the hyper-real numbers of Robinson [A22] ?
3. The notion of finite measurement resolution for angle variables given naturally as a hierarchy $2\pi/p^n$ of resolutions for a given p-adic prime defining aa hierarchy of algebraic extension of p-adic numbers is central in the attempts to formulate p-adic variants of quantum TGD and fuse them with real number based quantum TGD [K108] . If p is replaced with an infinite prime, the angular resolution becomes ideal and the roots of unity $exp(2\pi m/p^n)$ are replaced with real units unless also the integer m is replaced with an infinite integer M so that the ratio M/P^n is finite rational number. Could this approach be regarded as alternative for real number based notion of phase angle?

The consideration of infinite primes need not be a purely academic exercise: for infinite values of p p-adic perturbation series contains only two terms and this limit, when properly formulated, could give excellent approximation of the finite p theory for large p. Using infinite primes one might obtain the real theory in this approximation.

The question discussed in this section is whether the notion of p-adic number field makes sense makes sense for infinite primes and whether it might have some physical relevance. One can formally introduce power series in powers of any infinite prime P and the coefficients can be taken to belong to any ordinary number field. In the representation by polynomials P-adic power series correspond to Laurent series in powers of corresponding polynomial and are completely finite.

For straightforward generalization of the norm all powers of infinite-P prime have vanishing norm. The infinite-p p-adic norm of infinite-p p-adic integer would be given by its finite part so that in this sense positive powers of P would represent infinitesimals. For Laurent series this would mean that the lowest term would give the whole approximation in the real topology. For finite-primes one could however replace the norm as a power of p by a power of some other number. This would allow to have a finite norm also for P-adic primes. Since the simplest P-adic primes at the lowest level of hierarchy define naturally a rational one might consider the possibility of defining the norm of P as the inverse of this rational.

15.7.1 Does Infinite-P P-Adicity Reduce To Q-Adicity?

Any non-vanishing p-adic number is expressible as a product of power of p multiplied by a p-adic unit which can be infinite as a normal integer and has pinary expansion in powers of p:

$$x = p^n(x_0 + \sum_{k>0} x_k p^k) \ , \ x_k \in \{0, .., p-1\} \ , \ x_0 > 0 \ . \tag{15.7.1}$$

The p-adic norm of x is given by $N_p(x) = p^{-n}$. Each unit has p-adic inverse which for finite integers is always infinite as an ordinary integer.

To define infinite-P p-adic numbers one must generalize the pinary expansion to a infinite-P p-adic expansion of an infinite rational. In particular, one must identify what the statement "infinite integer modulo P" means when P is infinite prime, and what are the infinite integers N satisfying the condition $N < P$. Also one must be able to construct the p-adic inverse of any infinite prime. The correspondence of infinite primes with polynomials allows to construct infinite-P p-adics in a straightforward manner.

Consider first the infinite integers at the lowest level.

1. Infinite-P p-adics at the first level of hierarchy correspond to Laurent series like expansions using an irreducible polynomial P of degree n representing infinite prime. The coefficients of the series are numbers in the coefficient fields. Modulo p operation is replaced with modulo polynomial P operation giving a unique result and one can calculate the coefficients of the expansion in powers of P by the same algorithm as in the case of the ordinary p-adic numbers. In the case of n-variables the coefficients of Taylor series are naturally rational functions of at most $n-1$ variables. For infinite primes this means rationals formed from lower level infinite-primes.
2. Infinite-P p-adic units correspond to expansions of this type having non-vanishing zeroth order term. Polynomials take the role of finite integers. The inverse of a infinite integer in P-adic number field is obtained by developing the polynomial counterpart of $1/N$ in the following manner. Express N in the form $N = N_0(1+x_1P+..)$, where N_0 is polynomial with degree at most equal to $n-1$. The factor $1/(1+x_1P+...)$ can be developed in geometric series so that only the calculation of $1/N_0$ remains. Calculate first the inverse $\hat{N}_0^{-1}$ of N_0 as an element of the "finite field" defined by the polynomials modulo P: a polynomial having degree at most equal to $n-1$ results. Express $1/N_0$ as

$$\frac{1}{N_0} = \hat{N}_0^{-1}(1 + y_1P + ...)$$

   and calculate the coefficients in the expansion iteratively using the condition $N \times (1/N) = 1$ by applying polynomial modulo arithmetics. Generalizing this, one can develop any rational function to power series with respect to polynomial prime P. The expansion with respect to a polynomial prime can in turn be translated to an expansion with respect to infinite prime and also mapped to a superposition of Fock states.
3. What about the norm of infinite-P p-adic integers? Ultra-metricity suggest a straightforward generalization of the usual p-adic norm. The direct generalization of the finite-p p-adic norm would mean the identification of infinite-P p-adic norm as P^{-n}, where n corresponds to the lowest order term in the polymomial expansion. Thus the norm would be infinite for $n < 0$, equal to one for $n = 0$ and vanish for $n > 0$. Any polynomial integer N would have vanishing norm with respect to those infinite-P p-adics for which P divides N. Essentially discrete topology would result.

This seems too trivial to be interesting. One can however replace P^{-n} with a^{-n}, where a is any finite number a without losing the multiplicativity and ultra-metricity properties of the norm. The function space associated with the polynomial defined by P serves as a guideline also now. This space is naturally q-adic for some rational number q. At the lowest level the infinite prime defines naturally an ordinary rational number as the zero of the polynomial as is clear from the definition of the polynomial. At higher levels of the hierarchy the rational number is rational function of lower level infinite primes and by continuing the assignments of lower level rational functions to the infinite primes one ends up with an assignment of a unique rational number with a given infinite prime serving as an excellent candidate for a rational defining the q-adicity.

For the lowest level infinite primes the natural choice of a would be the rational number defined by it so that infinite-P p-adicity would indeed correspond to q-adicity meaning that number field property is lost.

15.7.2 Q-Adic Topology Determined By Infinite Prime As A Local Topology Of WCW?

Since infinite primes correspond to polynomials, infinite-P p-adic topology, which by previous considerations would be actually q-adic topology, is a natural candidate for a topology in function spaces, in particular in the WCW .

This view conforms also with the idea of algebraic holography. The sub-spaces of WCW can be modelled in terms of function spaces of rational functions, their algebraic extensions, and their P-adic completions. The mapping of the elements of these spaces to infinite rationals would make possible the correspondence between WCW and number theoretic anatomy of point of the embedding space.

The q-adic norm for these function spaces is in turn consistent with the ultra-metricity for the space of maxima of Kähler functions conjectured to be all that is needed to construct S-matrix. Ultra-metricity conforms nicely with the expected four-dimensional spin glass degeneracy due to the enormous vacuum degeneracy meaning that maxima of Kähler function define the analog of spin glass free energy landscape. That only maxima of Kähler function would be needed would mean that radiative corrections to WCW integral would vanish as quantum criticality indeed requires. This TGD can be regarded as an analog of for an integrable quantum theory. Quantum criticality is absolutely essential for guaranteeing that S-matrix and U-matric elements are algebraic numbers which in turn guarantees number theoretic universality of quantum TGD.

15.7.3 The Interpretation Of The Discrete Topology Determined By Infinite Prime

Also $p = 1$-adic topology makes formally sense and corresponds to a discrete topology in which all rationals have unit norm. It results also results if one naïvely generalizes p-adic topology to infinite-p p-adic topology by defining the norm of infinite prime at the lowest level of hierarchy as $|P|_P = 1/P = 0$. In this topology the distance between two points is either 1 or 0 and this topology is the roughest possible topology one can imagine.

It must be however noticed that if one maps infinite-P p-adics to real by the formal generalization of the canonical identification then one obtains real topology naturally if coefficients of powers of P are taken to be reals. This would mean that infinite-P p-adic topology would be equivalent with real topology.

Consider now the possible interpretations.

1. At the level of function spaces infinite-p p-adic topology in the naïve sense has a completely natural interpretation and states that the replacement of the Taylor series with its lowest term.
2. The formal possibility of $p = 1$-adic topology at space-time level suggests a possible interpretation for the mysterious infinite degeneracy of preferred extremals: one can add to any preferred extremal a vacuum extremal, which behaves completely randomly except for the constraints forcing the surface to be a vacuum extremal. This non-determinism is much more general than the non-determinism involving a discrete sequence of bifurcations (I have used the term association sequence about this kind of sequences). This suggests that one must replace the concept of 3-surface with a more general one, allowing also continuous association sequences consisting of a continuous family of space-like 3-surfaces with infinitesimally small time like separations. These continuous association sequences would be analogous to vacuum bubbles of the quantum field theories.

One can even consider the possibility that vacuum extremals are non-differentiable and even discontinuous obeying only effective $p = 1$-adic topology. Also Kähler-Dirac operator vanishes identically in this case. Since vacuum surfaces are in question, $p = 1$ regions cannot correspond to material sheets carrying energy and also the identification as cognitive space-time sheets is questionable. Since $p = 1$, the smallest possible prime in generalized sense, it must represent the lowest possible level of evolution, primordial chaos. Quantum classical correspondence suggests that $p = 1$ level is indeed present at the space-time level and might realized by the mysterious vacuum extremals.

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Chapter i

Appendix

A-1 Introduction

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of embedding space and related spaces are discussed and the relationship of CP_2 to the standard model is summarized. The basic vision is simple: the geometry of the embedding space $H = M^4 \times CP_2$ geometrizes standard model symmetries and quantum numbers. The assumption that space-time surfaces are basic objects, brings in dynamics as dynamics of 3-D surfaces based on the induced geometry. Second quantization of free spinor fields of H induces quantization at the level of H, which means a dramatic simplification.

The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Also the relationship to string models is discussed briefly.

Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure analogous to adele [L38, L39]. In the recent view of quantum TGD [L111], both notions reduce to physics as number theory vision, which relies on $M^8 - H$ duality [L73, L74] and is complementary to the physics as geometry vision.

Zero energy ontology (ZEO) [L64] [K132] has become a central part of quantum TGD and leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory having quantum biology as an application. Also these aspects of TGD are briefly discussed.

A-2 Embedding space $M^4 \times CP_2$

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that embedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Embedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) [L64, L93] [K132] causal diamond

(CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure [A50] so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2.1 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrate that it codes for the symmetries of standard models via its isometries and holonomies.

CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A38] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , \quad e^1 = \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , \quad e^3 = \frac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned}
e^0 &= \frac{dr}{F} \ , & e^1 &= \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\
e^2 &= \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 &= \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ .
\end{aligned} \tag{A-2.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-2.12}$$

From this expression one finds that at coordinate infinity $r = \infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-2.13}$$

is given by

$$\begin{aligned}
V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\
V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\
V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ .
\end{aligned} \tag{A-2.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{aligned}
R_{01} &= e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} &= e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
R_{02} &= e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} &= -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\
R_{03} &= 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} &= 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ .
\end{aligned} \tag{A-2.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -is_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-2.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \tag{A-2.17}$$

The condition states that J and g give representations of real unit and imaginary units related by the formula $i^2 = -1$.

Kähler form is expressible locally in terms of Kähler gauge potential

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

$dJ = ddB = 0$ gives the topological half of Maxwell equations (vanishing of magnetic charges and Faraday's induction law) and self-duality $*J = J$ reduces the remaining equations to $dJ = 0$. Hence the Kähler form can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling).

The magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2} dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F} sin\Theta d\Theta \wedge d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical (or symplectic or Darboux) coordinates in which the Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= -\frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A29]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential

$exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the embedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A82] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2.2 CP_2 geometry and Standard Model symmetries

Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B37] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-2.23}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \otimes \gamma_5$, $1 \otimes \gamma_5$ and $\gamma_5 \otimes 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4)$ having as its covering group $SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-2.24}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{llllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ , \end{array} \tag{A-2.25}$$

and

$$B = 2re^3 \ , \tag{A-2.26}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-2.27}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{A-2.28}$$

A_{ch} is clearly left handed so that one can perform the identification of the gauge potential as

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-2.29}$$

where $W^\pm$ denotes the charged intermediate vector boson.

The covariantly constant curvature tensor is given by

$$\begin{aligned} R_{01} &= -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} &= -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\ R_{03} &= 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , \\ R_{12} &= 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{aligned} \tag{A-2.30}$$

The charged part of the curvature tensor is left handed.

This is to be compared with the Weyl tensor, which defines a representation of quaternionic imaginary units.

$$
\begin{aligned}
&W_{03} = W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
&W_{01} = W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
&W_{02} = W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ .
\end{aligned} \tag{A-2.31}
$$

The charged part of the Weyl tensor is right-handed and that the relative sign of the two terms in the curvature tensor and Weyl tensor are opposite.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$
\begin{aligned}
X &= re^3 \ , \\
Y &= \frac{e^3}{r} \ ,
\end{aligned} \tag{A-2.32}
$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$
\begin{aligned}
\bar{\gamma} &= aX + bY \ , \\
\bar{Z}^0 &= cX + dY \ ,
\end{aligned} \tag{A-2.33}
$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$
\begin{aligned}
A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+ 1_+ + n_- 1_-)]\bar{\gamma} \\
&+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+ 1_+ + n_- 1_-)]\bar{Z}^0 \ .
\end{aligned} \tag{A-2.34}
$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5 \Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-2.35}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.36}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$
\begin{aligned}
Q_{em} &= \Sigma^{12} + \frac{(n_+ 1_+ + n_- 1_-)}{6} \ , \\
I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ .
\end{aligned} \tag{A-2.37}
$$

The fields γ and Z^0 are defined via the relations

$$
\begin{aligned}
\gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\
Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ .
\end{aligned} \tag{A-2.38}
$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \tag{A-2.39}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of the Weinberg angle is a dynamical problem. The original approach was based on the assumption that it makes sense to talk about electroweak action defined at fundamental level and introduce a symmetry breaking by adding an additional term proportional to Kähler action. The recent view is that Kähler action plus volume term defines the fundamental action.

The Weinberg angle is completely fixed if one requires that the electroweak action contains no cross term of type γZ^0. This leads to a definite value for the Weinberg angle.

One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle. As a matter fact, color gauge action identifying color gauge field as proportional to $H^A J_{\alpha\beta}$ is proportional to Kähler action. A possible interpretation would be as a sum of electroweak and color gauge interactions.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \tag{A-2.40}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{A-2.41}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.42}$$

Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-2.43}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-2.44}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-2.45}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$
\begin{aligned}
X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\
K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ ,
\end{aligned} \tag{A-2.46}
$$

This parameter can be calculated by substituting the values of quark and lepton charges and weak isospins.

In the general case the value of the coefficient K is given by

$$
K = \sum_i \left[-\frac{(18 + 2n_i^2) sin^2\theta_W}{9} \right] \ , \tag{A-2.47}
$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i (18 + n_i^2))} \ . \tag{A-2.48}
$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-2.49}
$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is not far from the typical value 9/24 of GUTs at high energies [B9]. The experimental value at the scale length scale of the electron can be deduced from the ratio of W and Z boson masses as $sin^2\theta_W = 1 - (m_W/m_Z)^2 \simeq .22290$. This ratio and also the weak boson masses depend on the length scale.

If one interprets the additional term proportional to J as color action, one could perhaps interpret the value of Weinberg angle as expressing a connection between strong and weak coupling constant evolution. The limit $f \to 0$ should correspond to an infinite value of color coupling strength and at this limit one would have $sin^2\theta_W = \frac{9}{28}$ for $f/g^2 \to 0$. This does not make sense since the Weinberg angle is in the standard model much smaller in QCD scale Λ corresponding roughly to pion mass scale. The Weinberg angle is in principle predicted by the p-adic coupling constant evolution fixed by the number theoretical vision of TGD.

One could however have a sum of electroweak action, correction terms changing the value of Weinberg angle, and color action and coupling constant evolution could be understood in terms of the coupling parameters involved.

Electroweak symmetry breaking

One of the hardest challenges in the development of the TGD based view of weak symmetry breaking was the fact that classical field equations allow space-time surfaces with finite but arbitrarily large size. For a fixed space-time surface, the induced gauge fields, including classical weak fields, are long ranged. On the other hand, the large mass for weak bosons would require a short correlation length. How can one understand this together with the fact that a photon has a long correlation length?

In zero energy ontology quantum states are superpositions of space-time surfaces as analogs of almost unique Bohr orbits of particles identified as 3-D surfaces. For some reason the superposition should be such that the quantum averages of weak gauge boson fields vanish below the weak scale whereas the quantum average of electromagnetic fields is non-vanishing.

This is indeed the case.

1. The supersymplectic symmetries form isometries of the world of classical worlds (WCW) and they act in CP_2 degrees of freedom as symplectic transformations leaving the CP_2 symplectic form J invariant and therefore also its contribution to the electromagnetic field since this part is the same for all space-time surfaces in the superposition of space-time surfaces as a representation of supersymplectic isometry group (as a special case a representation of color group).
2. In TGD, color and electroweak symmetries acting as holonomies are not independent and for the $SU(2)_L$ part of induced spinor connection the symplectic transformations induces $SU(2)_L \times U(1)_R$ gauge transformation. This suggests that the quantum expectations of the induced weak fields over the space-time surfaces vanish above the quantum coherence scale. The averages of W and of the left handed part of Z^0 should therefore vanish.
3. $\langle Z^0 \rangle$ should vanish. For $U(1)_R$ part of Z^0, the action of gauge transformation is trivial in gauge theory. Now however the space-time surface changes under symplectic transformations and this could make the average of the right-handed part of Z^0 vanishing. The vanishing of the average of the axial part of the Z^0 is suggested by the partially conserved axial current hypothesis.

One can formulate this picture quantitatively.

1. The electromagnetic field [L118] contains, besides the induced Kähler form, also the induced curvature form R_{12}, which couples vectorially. Conserved vector current hypothesis suggests that the average of R_{12} is non-vanishing. One can express the neutral part of the induced gauge field in terms of induced spinor curvature and Kähler form J as

$$\begin{aligned}
R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) = J + 2e^0 \wedge e^3 \ , \\
J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) = 3J - 2e^0 \wedge e^3 \quad , \qquad \text{(A-2.50)}
\end{aligned}$$

2. The induced fields γ and Z^0 (photon and Z- boson) can be expressed as

$$\begin{aligned}
\gamma &= 3J - sin^2\theta_W R_{12} \ , \\
Z^0 &= 2R_{03} = 2(J + 2e^0 \wedge e^3) \qquad \text{(A-2.51)} \\
per. & \qquad \text{(A-2.52)}
\end{aligned}$$

The condition $\langle Z^0 \rangle = 0$ gives $2\langle e^0 \wedge e^3 \rangle = -2J$ and this in turn gives $\langle R_{12} \rangle = 4J$. The average over γ would be

$$\langle \gamma \rangle = (3 - 4sin^2\theta_W)J \ .$$

For $sin^2\theta_W = 3/4 \ langle\gamma\rangle$ would vanish.

The quantum averages of classical weak fields quite generally vanish. What about correlation functions?

1. One expects that the correlators of classical weak fields as color invariants, and perhaps even symplectic invariants, are non-vanishing below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as hadron.
2. The intuitive picture is that in longer length scales one has disjoint 3-surfaces with a size scale of Compton length. If the states associated with two disjoint 3-surfaces are separately color invariant there are no correlations in color degrees of freedom and correlators reduce to the products of expectations of classical weak fields and vanish. This could also hold when the 3-surfaces are connected by flux tube bonds.

   Below the Compton length weak bosons would thus behave as correlated massless fields. The Compton lengths of weak bosons are proportional to the value of effective Planck constant h_{eff} and in living systems the Compton lengths are proposed to be even of the order of cell size. This would explain the mysterious chiral selection in living systems requiring large parity violation.

3. What about the averages and correlators of color gauge fields? Classical color gauge fields are proportional to the products of Hamiltonians of color isometries induced Kähler form and the expectations of color Hamiltonians give vanishing average above Compton length and therefore vanishing average. Correlators are non-vanishing below the hadron scale. Gluons do not propagate in long scales for the same reason as weak bosons. This is implied by color confinement, which has also classical description in the sense that 3-surfaces have necessarily a finite size.

   A large value of h_{eff} allows colored states even in biological scales below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as dark hadron.

Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.
2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B10] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-2.53}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-2.54}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-2.55}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-3.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has embedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if embedding space at their image points. In the recent case the embedding of space-time surface to embedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the embedding space to the space-time surface (see `http://tgdtheory.fi/appfigures/induct.jpg`).

Induction procedure makes sense also for the spinor fields of embedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with embedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://tgdtheory.fi/appfigures/induct.jpg`.

A-3.2 Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-3.3 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four embedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of embedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-3.4 Embedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.
2. Spinor harmonics of embedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of embedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the embedding space spinor connection carries W gauge potentials one can say that the embedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.
2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.
3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.
4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.
5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-3.5 About induced gauge fields

In the following the induced gauge fields are studied for general space-time surface without assuming the preferred extremal property (Bohr orbit property). Therefore the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + \frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-3.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.2}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2 F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p) sin(\Theta) = 0 \ , \end{aligned} \tag{A-3.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \tag{A-3.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{A-3.5}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F}du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$
\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u}du \wedge d\Phi = (k+u)du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2}Z^0 \ . \qquad \text{(A-3.6)}
\end{aligned}
$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$
\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \qquad \text{(A-3.7)}
\end{aligned}
$$

and is useful in the construction of vacuum embedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$
\begin{aligned}
\Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\
\Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \qquad \text{(A-3.8)}
\end{aligned}
$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global embedding for, say a constant magnetic field. Although global embedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \tag{A-3.9}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-4 The relationship of TGD to QFT and string models

The recent view of the relationship of TGD to QFT and string models has developed slowly during years and it seems that in a certain sense TGD means a return to roots: instead of QFT like description involving path integral one would have wave mechanics for 3-surfaces.

A-4.1 TGD as a generalization of wave mechanism obtained by replacing point-like particles with 3-surfaces

The first vision of TGD was as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

The later work has revealed that TGD could be seen as a generalization of the wave mechanism based on the replacement of a point-like particle with 3-D surface. This is due to holography implied by general coordinate invariance. The definition of the metric of the "world of classical worlds" (WCW) must assign a unique or at least almost unique space-time surface to a given 3-surface. This 4-surface is analogous to Bohr orbit so that also Bohr orbitology becomes an exact part of quantum physics. The failure of strict determinism forces to replace 3-surfaces with 4-surfaces and this leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K60, K35, K98] [L96, L111].

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

A-4.2 Extension of superconformal invariance

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in a completely unique position as far as symmetries are considered.

In fact, this leads to a generalization of the Kac-Moody type symmetries of string models. $\delta M^4_+ \times CP_2$ allows huge supersymplectic symmetries for which the radial light-like coordinate of δM^4_+ plays the role of complex string coordinate in string models. These symmetries are assumed to act as isometries of WCW.

A-4.3 String-like objects and strings

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of the TGD Universe and the inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string-like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situations in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

A-4.4 TGD view of elementary particles

The TGD based view about elementary particles has two key aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidean signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.
2. At the level of H Fermion number is carried by the modes of the induced spinor field. In space-time regions with Minkowski signature the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle orbit corresponds to a 4-D generalization of a world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidean signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at the same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having a hadronic string as a physical counterpart. Long strings should be distinguished from wormhole contacts which due to their superconformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.
2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.
3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have

the usual interpretation in terms of particle decays but in terms of propagation of particles along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K60, K98].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

A-5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [L41] [L96, L98, L99] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$ of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A50] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.
2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.
3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a

U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.
2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

A-5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.
2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L111]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

The super symplectic algebra A has an infinite hierarchy of sub-algebras [L111] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.

2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L111] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by $M^8 - H$ duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.

2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L111] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P) = n(braid) = n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.

3. The actions related to different values of $n(P) = n(braid) = n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.

4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L102] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i = 2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r = 1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.

5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p \simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K73, K74]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L102], and the forward scattering amplitude corresponds to the propagator.

   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L102], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L102].

3. p-Adic length scale hypothesis [L112] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L111] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K60, K35]. As isometries they would naturally permute the maxima with each other.

A-6 Number theoretic vision of TGD

Physics as number theory vision is complementary to the physics as geometry vision and has developed gradually since 1993. Langlands program is the counterpart of this vision in mathematics [L107].

The notion of p-adic number fields emerged with the motivation coming from the observation that elementary particle mass scales and mass ratios could be understood in terms of the so-called p-adic length scale hypothesis [K77, K68, K32]. The fusion of the various p-adic physics leads to what I call adelic physics [L38, L39]. Later the hypothesis about hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter emerged [K38, K39, K40, K40].

Eventually this led to that the values of effective Planck constant could be identified as the dimension of an algebraic extension of rationals assignable to polynomials with rational coefficients. This led to the number theoretic vision in which so-called $M^8 - H$ duality [L73, L74] plays a key role. M^8 (actually a complexification of real M^8) is analogous to momentum space so that the duality generalizes momentum position duality for point-like particles. M^8 has an interpretation as complexified octonions.

The dynamics of 4-surfaces in M^8 is coded by polynomials with rational coefficients, whose roots define mass shells H^3 of $M^4 \subset M^8$. It has turned out that the polynomials satisfy stringent additional conditions and one can speak of number theoretic holography [L102, L107]. Also the ordinary 3→ 4 holography is needed to assign 4-surfaces with these 3-D mass shells. The number theoretic dynamics is based on the condition that the normal space of the 4-surface in M^8 is associative (quaternionic) and contains a commutative complex sub-space. This makes it possible to assign to this surface space-time surface in $H = M^4 \times CP_2$.

At the level of H the space-time surfaces are by holography preferred extremals and are assumed to be determined by the twistor lift of TGD [L41] giving rise to an action which is sum of the Kähler action and volume term. The preferred extremals would be minimal surfaces analogous to soap films spanned by frames. Outside frames they would be simultaneous extremals of the Kähler action, which requires a generalization of the holomorphy characterizing string world sheets.

In the following only p-adic numbers and hierarchy of Planck constants will be discussed.

A-6.1 p-Adic numbers and TGD

p-Adic number fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A26]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k,\ x(k) = 0, ..., p-1 \ . \tag{A-6.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{A-6.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \tag{A-6.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \tag{A-6.5}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.
2. Distances of points x and y inside single class are smaller than distances between different classes.
3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B30]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

1. Basic form of the canonical identification

There exists a natural continuous map $I : R_p \to R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \to x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1} \sum_{k=0,..} p^{k} \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

2. The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p|y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \tag{A-6.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p|y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-6.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

3. Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

4. Generalization of number concept and notion of embedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic embedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real embedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real embedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, “thought bubbles”.

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification violates general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic embedding space with chart maps to real embedding space and assuming preferred coordinates made possible by isometries of embedding space: one however obtains several in-equivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-6.2 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the embedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of embedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of embedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-6.3 $M^8 - H$ duality as it is towards the end of 2021

The view of M^8-H duality (see Appendix **??**) has changed considerably towards the end 2021 [L96] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L96] of the M^8-H duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L96] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L73, L74]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$. This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L57]. This notion does not make sense at the level M^8 anymore.

The modified M^8-H duality forces us to modify the original interpretation [L96]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L86] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L96]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

A-7 Zero energy ontology (ZEO)

ZEO is implied by the holography forced in the TGD framework by general coordinate invariance.

A-7.1 Basic motivations and ideas of ZEO

The following gives a brief summary of ZEO [L64] [K132].

1. In ZEO quantum states are not 3-dimensional but superpositions of 4-dimensional deterministic time evolutions connecting ordinary initial 3-dimensional states. By holography they are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution. One can say that in the TGD framework general coordinate invariance implies holography and the slight failure of its determinism in turn forces ZEO.

   Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced with a new superposition. Classical determinism of individual time evolution is not violated and this solves the basic paradox of quantum measurement theory. There are two kinds of quantum jumps: ordinary ("big") state function reductions (BSFRs) changing the arrow of time and "small" state function reductions (SSFRs) (weak measurements) preserving it and giving rise to the analog of Zeno effect [L64].

2. To avoid getting totally confused it is good to emphasize some aspects of ZEO.

   (a) ZEO does not mean that physical states in the usual 3-D sense as snapshots of time evolution would have zero energy state pairs defining zero energy states as initial and final states have same conserved quantities such as energy. Conservation implies that one can adopt the conventions that the values of conserved quantities are opposite for these states so that their sum vanishes: one can think that incoming and outgoing particles come from geometric past and future is the picture used in quantum field theories.

(b) ZEO means two times: subjective time as sequence of quantum jumps and geometric time as space-time coordinate. These times are identifiable but are strongly correlated.

3. In BSFRs the arrow of time is changed and the time evolution in the final state occurs backwards with respect to the time of the external observer. BSFRs can occur in all scales since TGD predicts a hierarchy of effective Planck constants with arbitrarily large values. There is empirical support for BSFRs.

   (a) The findings of Minev et al [L54] in atomic scale can be explained by the same mechanism [L54]. In BSFR a final zero energy state as a superposition of classical deterministic time evolutions emerges and for an observer with a standard arrow of time looks like a superposition of deterministic smooth time evolutions leading to the final state. Interestingly, once this evolution has started, it cannot be stopped unless one changes the stimulus signal inducing the evolution in which case the process does not lead to anywhere: the interpretation would be that BSFR back to the initial state occurs!

   (b) Libets' experiments about active aspects of consciousness [J7] can be understood. Subject person raises his finger and neural activity starts before the conscious decision to do so. In the physicalistic framework it is thought to lead to raising of the finger. The problem with the explanation is that the activity beginning .5 seconds earlier seems to be dissipation with a reversed arrow of time: from chaotic and disordered to ordered at around .15 seconds. ZEO explanation is that macroscopic quantum jump occurred and generated a signal proceeding backwards in time and generated neural activity and dissipated to randomness.

   (c) Earthquakes involve a strange anomaly: they are preceded by ELF radiation. One would expect that they generate ELF radiation. The identification as BSFR would explain the anomaly [L56]. In biology the reversal of the arrow of time would occur routinely and be a central element of biological self-organization, in particular self-organized quantum criticality (see [L61, L123].

A-7.2 Some implications of ZEO

ZEO has profound implications for understanding self-organization and self-organized quantum criticality in terms of dissipation with non-standard arrow of time looking like generation of structures [L61, L123]. ZEO could also allow understanding of what planned actions - like realizing the experiment under consideration - could be.

1. Second law in the standard sense does not favor - perhaps even not allow - realization of planned actions. ZEO forces a generalization of thermodynamics: dissipation with a non-standard arrow of time for a subsystem would look like self-organization and planned action and its realization.

   Could most if not all planned action be like this - induced by BSFR in the geometric future and only apparently planned? There would be however the experience of planning and realizing induced by the signals from geometric future by a higher level in the hierarchy of conscious entities predicted by TGD! In long time scales we would be realizing our fates or wishes of higher level conscious entities rather than agents with completely free will.

2. The notion of magnetic body (MB) serving as a boss of ordinary matter would be central. MB carries dark matter as $h_{eff} = nh_0$ phases of ordinary matter with n serving as a measure for algebraic complexity of extension of rationals as its dimension and defining a kind of universal IQ. There is a hierarchy of these phases and MBs labelled by extension of rationals and the value of n.

   MBs would form a hierarchy of bosses - a realization for master slave hierarchy. Ordinary matter would be at the bottom and its coherent behavior would be induced from quantum coherence at higher levels. BSFR for higher level MB would give rise to what looks like planned actions and experienced as planned action at the lower levels of hierarchy. One could speak of planned actions inducing a cascade of planned actions in shorter time scales and eventually proceeding to atomic level.

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. `http://tgdtheory.fi/appfigures/reconect2.jpg`

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. `http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg`

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. `http://tgdtheory.fi/appfigures/cat.jpg`

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred embedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. `http://tgdtheory.fi/appfigures/padictoreal.jpg`

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. `http://tgdtheory.fi/appfigures/sharing.jpg`

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** `http://tgdtheory.fi/appfigures/timemirror.jpg` or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. `http://tgdtheory.fi/appfigures/timemirror.jpg`

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